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The research has centered on various classes of multi-input multi-output linear systems containing time-varying parameters and/or non-lumped (i.e. infinite-dimensional) elements such as pure or distributed time delays. The primary objective of the research has been to develop new algebraic methods which yield constructive procedures for both analysis and design of complex linear systems. Specific topics studied include representations, realizations, solution of operational differential equations, reachability, controllability, observability, duality, state feedback, assignability, state observers, regulators, and stability. A summary of these results is contained in this report.

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I. Topics Studied under Grants

The research has centered on various classes of multi-input multi-output linear systems containing time-varying parameters and/or non-lumped (i.e. infinite-dimensional) elements such as pure or distributed time delays. The primary objective of the research has been to develop new algebraic methods which yield constructive procedures for both analysis and design of complex linear systems. Specific topics studied include representations, realizations, solution of operational differential equations, reachability, controllability, observability, duality, state feedback, assignability, state observers, regulators, and stability.

II. Summary of Research Findings

The results of the research have been published (or will appear) in journals. A list of the papers is given in Section IV. of this report. Many of the key results are contained in several rather long papers published in well-known journals. A brief summary of these results is presented below. The references indicated below are taken from the list given in Section IV.

The completed work deals with the following classes of systems.

1. Linear infinite-dimensional continuous-time systems

The first part of this work involves the representation of linear infinite-dimensional systems in terms of input/output operators and state equations. Results have been obtained on causality and structural properties of a general class of input/output operators [6]. A good deal of effort has been devoted to the problem of realizing an input/output operator. by an "internal model" given in terms of the notion of state [2,6,9,15]. Since the systems under consideration are infinite dimensional, the state of a system at any fixed time is an element of an infinite-dimensional vector space. However, the work has shown that a large class of input/output operators can be realized by state-space models possessing a type of finiteness. In fact, there are two types of finiteness depending on the internal (state) model considered.

In one of the two possible frameworks, the state space admits a finitely-generated module structure over a convolution ring of generalized functions, even though it is infinite dimensional as a vector space over the field of real numbers [1,6,7]. In has been shown that any causal time-invariant input/output operator (as defined in this work) can be realized by a system whose state space is finitely generated with respect to a module structure. This result is very useful, because the finiteness of the state module can be exploited in the study of system structure and properties. In particular, the approach has produced many new results on the reachability and controllability of states and outputs, including procedures for constructing bounded-time and minimal-time controls [1,6]. These results have been utilized to study the regulator or tracking problem in infinite-dimensional systems [7].

The other type of internal model is given by a first-order vector differential equation with coefficients belonging to a ring of operators, such as a ring of delay operators [2,4,8,9,10,11,12,15]. This framework is a natural model for infinite-dimensional systems containing pure or distributed time delays. Here the finiteness mentioned above is due to the specification of the system by a finite set of first-order differential equations with operator coefficients. Necessary and sufficient conditions for the existence of such a model have been obtained [2,15]. An algorithm has been developed for constructing realizations, starting from the transfer function matrix associated with a given input/output operator [15]. Results have also been obtained on the realization of systems given by input/output operational differential equations with time-varying coefficients [9].

The research has also yielded an algebraic theory for a large class of first-order vector differential equations with operator coefficients. New results have been obtained on the construction of complete solutions and stability [4,12]. The algebraic theory has also been applied to the study of state feedback in systems with time delays. Results here include the development of procedures for spectrum assignment (i.e. pole assignment) or stabilization by employing state feedback [8,10,12].

2. Linear finite-dimensional time-varying systems

A new approach has been developed for the study of linear time-varying discrete-time systems [5,14,16]. The theory is based on the specification of a time-varying finite-dimensional system by a vector difference equation with coefficients belonging to a ring of time functions. This framework leads to a new characterization of system structure in the time-varying case. More precisely, dynamical behavior is given in terms of a special type of nonlinear transformation, called a semilinear transformation. Via the concept of a semilinear transformation, the time variance of a system is directly incorporated into the algebraic structure, resulting in new techniques for both analysis and design. For example, this approach has resulted in the formulation and solution of a "time-varying version" of the important pole-placement theorem [14]. The solution to this problem and other feedback control problems is based on the notion of cyclic and n-cyclic semilinear transformations. The theory has produced constructive procedures for the design of state-feedback controllers to achieve a given performance criterion. It has also been shown that these results can be "dualized," yielding results on the design of state observers for time-varying systems [16].

3. Linear systems over rings

This part of the work deals with a large class of linear systems that can be viewed as a generalization of the class of linear systems defined over fields [15]. In particular, systems given by a vector differential equation over a ring of operators and systems given by a vector difference equation over a ring of time functions (which were discussed above) can be treated as linear systems over rings. Other examples of systems that can be treated as linear systems over rings include discrete-time systems over a ring of scalars such as the integers, large-scale (high-dimensional) discrete-time systems, and linear two-dimensional digital filters.

Viewing the above-mentioned systems as linear systems over rings has resulted in new techniques for both analysis and design. Specific results include methods for computing optimal control functions that drive a given system to desired terminal states and techniques for constructing state-feedback control laws that allow one to specify the asymptotic behavior of the state response resulting from arbitrary initial states [15].

III. Application of Research to U. S. Army Problems

The research on systems with time delays is applicable to U. S. Army problems involving sizable time delays. For instance, delays result from (i) communication links between ground-based substations and drones in the drone control problem at White Sands Missile Range; (ii) reaction or decision times of human operators in radar-tracking systems or systems involving manned vehicles; (iii) on-line computer operations such as data storage and data processing.

During part of the summer of 1977, the principal investigator considered the application of the theory to the drone control problem at White Sands Missile Range. A study of the existing system indicated that the algebraic theory could be utilized to design optimal sampled-data controllers, which could result in improved system performance, especially in those cases in which the drone trajectories include high-g turns.

The research on time-varying systems should be of value in several problem areas, since many systems are best described by equations with time-varying coefficients. Examples include linearized models of nonlinear systems and communication systems with time-varying channels. The theory could also be of use in problems involving the modeling (or identification) of nonstationary discrete-time random processes, such as sampled radar signals.

The work dealing with the treatment of large-scale systems as systems over rings should be applicable to many problems, since high-dimensional systems are a common occurrence. Initial results indicate that this approach to large-scale systems can yield "fast algorithms" for the design of controllers and filters.

IV. List of Research Papers Prepared under Grants.

1. E. W. Kamen, "Control of linear continuous-time systems defined over rings of distributions," in Lecture Notes in Computer Science, Vol. 25, edited by E. Manes, Springer-Verlag, New York, pp. 180-187, 1975.

2. E. W. Kamen, "Linear continuous-time systems over Noetherian operator rings," Proceedings of 1974 IEEE Int. Symposium on Circuits and Systems, pp. 227-229, April 1974.

3. E. W. Kamen, "A new algebraic approach to linear time-varying systems," Technical Report, School of E.E., Georgia Tech, March 1974.

4. E. W. Kamen, "On an operator theory of linear systems with pure and distributed delays," Proceedings of the 1975 Conference on Decision and Control, pp. 77-80, December, 1975.

5. K. M. Hafez, "New results on discrete-time time-varying linear systems," Ph.D. Thesis, School of Information and Computer Science, Georgia Tech, March 1975.

6. E. W. Kamen, "Module structure of infinite-dimensional systems with applications to controllability," SIAM Journal on Control and Optimization, Vol. 14, No. 3, pp. 389-408, 1976.

7. E. W. Kamen, "Finiteness in infinite-dimensional systems applied to regulation," in Lecture Notes in Economics and Mathematical Systems, Vol. 131, edited by G. Marchesini and S. K. Mitter, Springer-Verlag, Berlin, pp. 252-265, 1976.

8. E. W. Kamen, "State and input feedback in systems containing time delays," Proceedings of Fourteenth Allerton Conference on Circuit and System Theory, pp. 310-319, September, 1976.

9. E. W. Kamen, "Representation and realization of operational differential equations with time-varying coefficients," Journal of the Franklin Institute, Vol. 301, No. 6, pp. 559-571, 1976.

10. S. P. Stuk, "An observer and regulator for linear systems with pure and distributed delays," M.S. Thesis, School of Industrial and Systems Engineering, Georgia Tech, Dec. 1976.

11. E. W. Kamen, "Use of algebraic methods in the design of controllers and observers for systems with time delays," Transactions of the Twenty-Third Conference of Army Mathematicians, pp. 625-637, May 1977.

12. E. W. Kamen, "An operator theory of linear functional differential equations," Journal of Differential Equations, Vol. 27, No. 2, pp. 274-297, February, 1978.

13. E. W. Kamen and R. E. Green, "Range-rate estimation with pulse doppler ambiguity correction," Project Report, Instrumentation Directorate, White Sands Missile Range, New Mexico, July, 1977.

14. E. W. Kamen and K. M. Hafez, "Algebraic theory of linear time-varying systems," submitted to Journal of Computer and System Sciences, September 1977.

15. E. W. Kamen, "Introduction to linear systems over rings," submitted to Proceedings of the IEEE, February 1978.

16. K. M. Hafez, "Duality theory of linear time-varying systems," to be submitted for publication.

V. Scientific Personnel Supported by Grants and Degrees Awarded

In addition to the principal investigator, the following students were supported by the grants. The degrees earned by these students while employed on the grants is also given.

1. Khaled M. Hafez, Doctor of Philosophy
2. Stephen P. Stuk, Master of Science