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SYSTEMS OPTIMIZATION LABORATORY: RESEARCH ON LARGE-SCALE SYSTEMS

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FINAL REPORT

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The Systems Optimization Laboratory (SOL) is a major component of the Department of Operations Research of Stanford University. The SOL carries on an integrated program of theoretical research, algorithmic development, and sophisticated software production for the optimization of large-scale systems. This program was partially supported by this contract.

The contract provided direct support primarily to one of the SOL Research Associates (Dr. John A. Tomlin) for work on computational methods and software development in mathematical programming. Some support also went to one of the SOL Co-Investigators, Professor Frederick S. Hillier, who served as Principal Investigator for this contract. He directs the SOL research program in integer programming, including algorithmic development, decomposition methods, heuristics, and software development. Three of his students working in this area -- Kevin J. Reardon, Paul F. McCoy, and Gary A. Kochman -- received their Ph.D. during the term of this contract.

Abstracts of the technical reports issued under this contract are provided. are attached.

Kevin J. Reardon, "A Multi-Stage Model for Capital Budgeting with Uncertain Future Investment Opportunities," Technical Report SOL 74-9, August 1974, 48 pages.

One application of dual-angular integer programming which has received considerable attention is in the area of multi-stage capital budgeting. Research in this area is concerned with one of the most important decisions for any economic unit, public or private -- that of allocating its limited financial resources in a manner which best supports the attainment of its goals. Nearly always, such decisions must be made in an environment characterized by incomplete information, uncertainty, complex interactions among activities, imperfect capital markets, and many other complicating factors. Explicit consideration of these factors and the influence on current decisions of partial results from preceding ones often leads to complex problems with large numbers of integer decision variables.

Many mathematical programming approaches to capital budgeting have relied on assumptions that all investment opportunities which will become available within the planning horizon can be identified at the outset. Through the use of additional constraints and decision variables, this assumption is relaxed in our problem formulation to one which only requires that those future opportunities which may arise be identified, and that probabilities be assigned to the events that they become available. This approach leads to rather large numbers of diagonal submatrices in the model's dual-angular constraint matrix. The author has developed a decomposition method for solving such dual-angular integer programs; a complete discussion of this method is presented in Technical Report SOL 74-10.

Kevin J. Reardon, "A Decomposition Method for the Solution of Dual-Angular Integer Programs," Technical Report SOL 74-10, August 1974, 80 pages.

Integer linear programming problems whose constraint matrices have dual-angular structures arise in several types of applications, particularly those which seek to specify a strategy for future actions based on observed results of previous decisions. An implicit enumeration method for integer linear programming is developed for application to problems with such dual-angular constraint matrices. The method employs a pre-solution relaxation which allows decomposition of the given problem into groups of smaller, less difficult subproblems. Reimposition of the relaxed requirements is accomplished automatically by the enumerative solution procedure. The method has been implemented within the framework of a penalty-based zero-one integer programming algorithm, and has enjoyed considerable success in reducing the solution effort for a variety of test problems. A detailed discussion of this specific implementation and its associated computational results is presented.

Ikuyo Kaneko, "On the Unboundedness of the Set of Integral Points in a Polyhedral Region," Technical Report SOL 74-12. September 1974, 21 pages.

Let $X = \{x : Ax \geq b, x \geq 0, x : \text{integral}\}$, where A and b are a given m by n matrix and m vector, respectively. This paper studies some properties concerning the unboundedness of X. The properties are in terms of (i) the existence of a "discrete ray in X" and (ii) the nonemptiness of the sets X and $\{x : Ax \geq 0, x \geq 0\} - \{0\}$. It is also shown that if A is rational, then (i) or (ii) characterizes the unboundedness of X.

P.F. McCoy and J.A. Tomlin, "Some Experiments on the Accuracy of Three Methods of Updating the Inverse in the Simplex Method," Technical Report SOL 74-21, December 1974, 27 pages.

This note reports the results of some experiments on measuring the accuracy of a group of methods for updating the inverse in the simplex method. These methods are the standard product form, the Bartels-Golub method and the Forrest-Tomlin update. The experiments, carried out on small to medium size models (depending on your point of view), were somewhat disturbing in that no method showed consistent superiority, and in that the error measurements that we used showed very erratic behavior.

Mohammad Saiidi and J.A. Tomlin, "Some Computational Experiments with Scolnik's Linear Programming Approach," Technical Report SOL 74-22, December 1974, 8 pages.

In this note we describe a reasonably efficient implementation of Scolnik's linear programming approach. We became interested in using the code to test the usefulness of this approach as a starting heuristic or "crashing" technique, as the method is known to fail in general. Our computational experience, however, leads us to believe that the method is too costly even for this modest objective.

J.A. Tomlin, "An Accuracy Test for Updating Triangular Factors," Technical Report SOL 75-9, April 1975, 5 pages.

This note describes a new accuracy test for the Forrest-Tomlin method for updating triangular factors of the basis, based on standard error analysis techniques. This test has proved very useful in practice when the method is applied to digitally unstable models.

J.A. Tomlin, "A Parametric Bounding Method for Finding a Minimum ℓ_∞-Norm Solution to a System of Equations," Technical Report SOL 75-12, May 1975, 16 pages.

This paper presents a method for finding the minimum ℓ_∞-norm solution to a set of consistent linear equations using a form of parametric linear programming. In this application the upper and lower bounds of all the variables are parametrized, and we work with only the original variables and constraints. Computational results indicate that the method is superior to both a primitive linear programming approach to the problem and to other, more specialized methods, which have been suggested.

Paul Franklin McCoy, "An Optimal Algorithm for the Resource Constrained Project Scheduling Problem," Technical Report SOL 75-26, October 1975, 173 pages.

The report presents an algorithm for solving a form of the resource constrained project scheduling problem. This particular form of the problem differs from that usually considered in that the time needed to complete a job depends on the amount of resources applied to that job. Jobs are preemptable and the objective is to minimize the project duration. It is shown that this problem is equivalent to the problem of finding that transportation polytope, defined by the resource constraints, of minimal dimension which has a face specified by the precedence constraints. A theorem is presented which gives conditions under which a face of a specified type exists. Using this theorem, the problem transforms into an integer programming problem with variables representing the completion times for each job. The constraint set is defined by inequalities involving addition and maximum operations on the variables and, without the constraint that the variables be integer, the constraint set forms a nonconvex, polyhedral set.

It is shown that an optimal solution exists in the set of points which form the peaks on the underside of the constraint set. A necessary condition that a point belong to this set is that it be a fixed point of a certain operator. This operator can be used to calculate these points and has the nice property of transforming schedules into schedules with job completion times which are less than or equal to those of the original schedule.

Two solution procedures are presented. Both start with a feasible schedule and iteratively improve upon it. One procedure is based on a branch and bound search over the fixed points of the operator mentioned above and the other procedure is based on a heuristic search over the same set of points.

The solution procedures were programmed and applied to ten problems which were derived from practical construction project scheduling problems and from problems used to test other optimal solution procedures. The number of jobs ranged from 5 to 39 and the number of resource types from 1 to 3. An optimal solution was obtained in less than half a second of computation time for seven of the smaller problems. For the other three problems both procedures improved upon the starting schedule.

Ted Eschenbach and Robert C. Carlson, "The Capacitated Multi-Period Location-Allocation Problem," Technical Report SOL 75-27, October 1975, 19 pages.

The problem is that of locating capacitated plants at some of a number of potential locations, so as to minimize the discounted sum of fixed costs incurred for open plants and variable supply costs. Both the costs and demands may vary arbitrarily over time. The results of optimal LIFO branch-and-bound and of heuristic algorithms using various branching variable selection rules and some new bounds are presented.

C.E. Pfefferkorn and J.A. Tomlin, "Design of a Linear Programming System for the Illiac IV," Technical Report SOL 76-8, April 1976, 87 pages.

This paper outlines a design for implementing a linear programming system on the ILLIAC IV computer. The central concern is to take advantage of the special features of the ILLIAC IV (64 parallel processing elements, large fast disk memory and relatively small fast core memory) and at the same time to take advantage of the sparsity of real large-scale linear programs and the (mostly serial) methodology which has been developed to exploit this sparsity. This requires both the adaption of existing techniques to a parallel environment and the development of new parallel techniques for efficient sparse matrix processing. It appears that this can be done successfully and that ILLIAC IV should be able to solve problems considerably larger than those which can be attempted on serial computers.

J.A. Tomlin, "USERS GUIDE FOR LCPL - A Program for Solving Linear Complementarity Problems by Lemke's Method," Technical Report SOL 76-16, August 1976, 33 pages.

This document is a users guide for LCPL, an efficient robust program for solving Linear Complementarity Problems by Lemke's Method.

Gary A. Kochman, "Computer Programs for Decomposition in Integer Programming," Technical Report SOL 76-20, September 1976, 162 pages.

This report gives documentations for two computer codes, DSLC and DMLC for solving block angular integer programming problems. The first code, DSLC, is for the single linking constraint case and the second DMLC is for the multiple linking constraint case.

Gary A. Kochman, "Decomposition in Integer Programming," Technical Report SOL 76-21, September 1976, 156 pages.

Linear programming models in which the constraint matrix has a block angular structure arise frequently in many applications. While much work has been devoted to exploiting this special structure when the problem variables are assumed to be continuous, little consideration has been given to models of this type in which the variables are required to take on only integer values. In this report, an algorithm for the decomposition of block angular integer programs is presented.

The block angular integer program consists of several subproblems which would operate independently except that they are tied together by a set of linking constraints. Conceptually, these linking constraints are viewed as representing common resources which the subproblems must share. The problem thus becomes that of determining an optimal allocation of these resources among the subproblems.

Towards this end, a branch-and-bound search routine is developed. It is shown how the LP-optimal dual multipliers and any slacks which appear in the optimal integer solutions to the subproblems can be used to guide the search, as well as for bounding and fathoming purposes. Special structures which arise when there is only a single linking constraint are discussed in detail.

Since the problem decomposes completely once an allocation of the linking resources is specified, only the subproblems ever need be solved explicitly. Computational results obtained with the decomposition algorithm are reported.

Gary A. Kochman, "On a Class of Concave-Separable Integer Programs," Technical Report SOL 76-22, September 1976, 14 pages.

A class of nonlinear integer programs is introduced. Problems in this class are characterized by a concave and separable objective function subject to a set of linear constraints.

It is shown how by suitably modifying the objective function, the theory of separability in linear programming can be applied to derive efficient solution procedures for problems falling in this class. This work unifies and extends several results previously obtained independently in the literature. Two illustrative applications are discussed in some detail, and specific algorithms are presented for these examples.

J.A. Tomlin, "Robust Implementation of Lemke's Method for the Linear Complementarity Problem," Technical Report SOL 76-24, September 1976, 15 pages.

This note discusses techniques for implementing Lemke's algorithm for the linear complementarity problem in a numerically robust way as well as a method for recovering from loss of feasibility or singularity of the basis. This recovery method is valid for both positive semi-definite M matrices and those with positive principal minors. It also allows a user to start from an advanced basis for such problems.

J.A. Tomlin, "Programmers Guide to LCPL - A Program for Solving Linear Complementarity Problems by Lemke's Method," Technical Report SOL 76-25, 53 pages.

This report gives programmers information and documentation for LCPL--an efficient robust program for solving Linear Complementarity Problems by Lemke's Method.

Bruce A. Murtagh and Michael A. Saunders, "MINOS - A Large-Scale Nonlinear Programming System (For Problems with Linear Constraints): User's Guide," Technical Report SOL 77-9, 127 pages.

MINOS is a Fortran program designed to minimize a linear or nonlinear function subject to linear constraints, where the constraint matrix is in general assumed to be large and sparse. The User's Guide contains an overview of the MINOS System, including descriptions of the theoretical algorithms as well as the details of implementation. The Guide also provides complete instructions for the use of MINOS, and illustrates the diversity of application by several examples.

Frederick S. Hillier, "A Further Investigation of Efficient Heuristic Procedures for Integer Linear Programming with an Interior," Technical Report SOL 77-13, February 1977, 61 pages.

This paper presents the results of an extensive investigation of algorithmic heuristic procedures for general pure integer linear programming problems having only inequality constraints. Included are a number of promising new variations and extensions of the procedures previously proposed by the author. Extensive computational experimentation has largely succeeded in identifying a flexible package of the most effective approaches, ranging from a very fast streamlined procedure to a very powerful combination of procedures. These procedures are both extremely efficient (comparable to the simplex method) and very effective in identifying good solutions (often obtaining an optimal one). Although they are designed primarily for dealing algorithmically with the frequently encountered problems that are too large to be computationally feasible for exact algorithms, they also can be valuable on smaller problems by quickly providing an advanced starting solution for such algorithms.

Bruce H. Faaland and Frederick S. Hillier, "Interior Path Methods for Heuristic Integer Programming Procedures," Technical Report SOL 77-14, February 1977, 33 pages.

This paper considers heuristic procedures for general mixed integer linear programming with inequality constraints. It focuses on the question of how to most effectively initialize such procedures by constructing an "interior path" from which to search for good feasible solutions. These paths lead from an optimal solution for the corresponding linear programming problem (i.e., deleting integrality restrictions) into the interior of the feasible region for this problem. Previous methods for constructing linear paths of this kind are analyzed from a statistical viewpoint, which motivates a promising new method. These methods are then extended to piecewise linear paths in order to improve the direction of search in certain cases where constraints that are not binding on the optimal linear programming solution become particularly relevant. Computational experience is reported.

James K. Ho and John A. Tomlin, "A Hybrid Approach to Multi-Stage Linear Programs," Technical Report SOL 77-27, September 1977, 12 pages.

This paper presents a hybrid algorithm for multi-stage linear programs arising from time-phased or dynamic models. The hybrid computation is based on a nested decomposition algorithm and the revised simplex method. Initial computational experience is reported.

M.A. Saunders, "MINOS System Manual," Technical Report SOL 77-31, December 1977, 136 pages.

MINOS is a Fortran system for solving large-scale linearly constrained optimization problems. The System Manual gives an overview of the system, the programming conventions used, data structures, tolerances, and error conditions. Details are given of a practical implementation of the method of Bartels and Golub for maintaining a sparse LU factorization. The reduced-gradient approach for handling a nonlinear objective function has been described elsewhere by Murtagh and Saunders; further implementation details are included here. The System Manual should facilitate interfacing of MINOS with other optimization software.

Margaret H. Wright, "A Survey of Available Software for Nonlinearly Constrained Optimization," Technical Report SOL 78-4, January 1978, 25 pages.

The aim of this paper is to present a brief overview of current practice in solving nonlinearly constrained optimization problems. First, we consider selected theoretical and computational aspects of some of the widely used methods for such problems. Second, the status of implementations of such methods will be sketched. The latter discussion will include: factors to be considered in evaluating optimization programs; guidelines concerning the application of a general-purpose routine to any particular problem; and a list of some available software.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This final report summarizes the most important results obtained under this contract. This research was carried on in the Systems Optimization Laboratory as part of an integrated program of theoretical research, algorithmic development, and sophisticated software production for the optimization of large-scale systems. The emphasis here was on computational methods and software development in linear programming, linearly constrained optimization problems, and integer programming.