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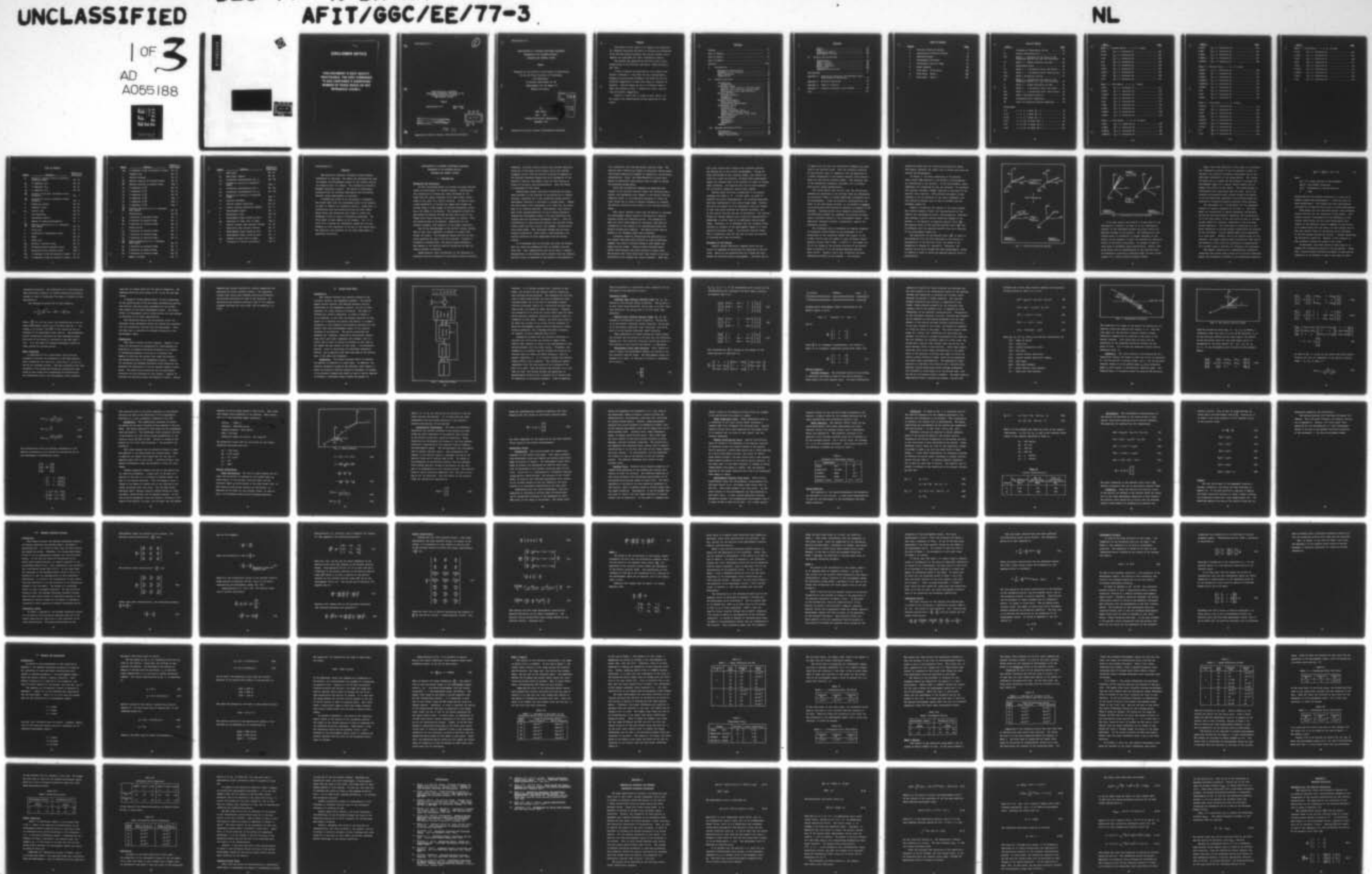

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APPLICATION OF A MAXIMUM LIKELIHOOD PARAMETER ESTIMATOR TO AN ADVANCED MISSILE GUIDANCE AND CONTROL SYSTEM

THESIS

AFIT/GGC/EE/77-3

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APPLICATION OF A MAXIMUM LIKELIHOOD PARAMETER ESTIMATOR TO AN ADVANCED MISSILE GUIDANCE AND CONTROL SYSTEM

THESIS

Presented to the Faculty of the School of Engineering
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in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by
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Preface

The intent of this study is to examine the feasibility of a Maximum Likelihood Estimator to evaluate the parameters of an advanced missile guidance and control system, and to compare two algorithms for their information content.

The project was sponsored by the Fire Control Technology Group of the Avionics Laboratory, Wright-Patterson AFB, Ohio.

I wish to express my appreciation to my sponsor and advisor, Professor J. Gary Reid for his responsiveness, guidance and assistance throughout the study and for his excellent suggestions to make this report as complete as possible. Special thanks go also to Professor James E. Negro and Professor Peter S. Maybeck for their contribution and helpful suggestions.

Finally, I would like to thank my wife, Malca, for her support and understanding during completion of this thesis.

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Model 2 -- With Noise - ϵ = 5, 10, 15 mrad.

List of Symbols

Symbol	Meaning	Defined or First Used
$\underline{A}$	Missile's specific force in aircraft reference frame	Eq. 32
A_X	X component of $\underline{A}$	Eq. 36
A_Y	Y component of $\underline{A}$	Eq. 37
A_Z	Z component of $\underline{A}$	Eq. 38
C_M^A	Missile to aircraft coordinate transformation	Eq. 7
C_A^M	Aircraft to missile coordinate transformation	Eq. 6
D	Dispersion matrix	Eq. 56
H	Gradient vector	Eq. 54
K_D	Control type	Eq. 26
L	Lift magnitude	Eq. 12
M	Information matrix	Eq. 2
N	Number of sampled measurements	Eq. 2
$\frac{\partial P}{\partial \underline{\varepsilon}}$	Position sensitivity w.r.t. misalignment angles	Eq. 39
q	Pitch rate	Eq. 8
R	Covariance of measurement noise	Eq. 2
R	Range	Eq. 20
$\dot{R}$	Range rate	Eq. 21
$\underline{SF}$	Missile's specific force	Eq. 25
$\hat{SF}$	Estimated missile specific force	Fig. 2
$\tilde{SF}$	Measured missile specific force	Fig. 2
SF_1	X component of $\underline{SF}$ (in missile's frame)	Eq. 44
SF_2	Y component of $\underline{SF}$ (in missile's frame)	Eq. 44

Symbol	Meaning	Defined or First Used
SF_3	Z component of $\underline{SF}$ (in missile's frame)	Eq. 44
t	Elapsed time	Eq. 26
V	Missile airspeed	Eq. 12
$\underline{V}^A$	Missile velocity in aircraft frame	Eq. 45
$\underline{V}^M$	Missile velocity in missile frame	Fig. 5
V_X	X component of $\underline{V}^A$	Eq. 33
V_Y	Y component of $\underline{V}^A$	Eq. 34
V_Z	Z component of $\underline{V}^A$	Eq. 35
V_1	X component of $\underline{V}^M$	Fig. 5
V_2	Y component of $\underline{V}^M$	Fig. 5
V_3	Z component of $\underline{V}^M$	Fig. 5
$\frac{\partial \underline{V}}{\partial \underline{\varepsilon}}$	Velocity sensitivity w.r.t. misalignment angles	Eq. 40
w	Pseudo-noise	Eq. 58
X_A	X position in aircraft frame	Eq. 4
X_M	X position in missile frame	Eq. 4
X_V	X position in velocity frame	Eq. 13
$\underline{Y}$	Output vector	Eq. 2
Y_A	Y position in aircraft frame	Eq. 4
Y_M	Y position in missile frame	Eq. 4
Y_V	Y position in velocity frame	Eq. 13
$\frac{\partial \underline{Y}}{\partial \underline{\varepsilon}}$	Output sensitivity w.r.t. misalignment angles	Eq. 47
Z_A	Z position in aircraft frame	Eq. 4
Z_M	Z position in missile frame	Eq. 4
Z_V	Z position in velocity frame	Eq. 13
α	Angle of attack	Eq. 9

Symbol	Meaning	Defined or First Used
β	Bank angle	Eq. 11
β_c	Bank angle command	Eq. 26
Γ_A	Coordinate transformation for γ_A	Eq. 13a
γ_A	Azimuth of velocity in missile's frame	Fig. 5
Γ_β	Coordinate transformation for β	Eq. 13c
Γ_E	Coordinate transformation for γ_E	Eq. 13b
γ_E	Elevation of velocity in missile's frame	Fig. 5
ΔP	Position increment	Fig. 1
δ	Control surface deflection	Eq. 10
δ_c	Control surface deflection command	Eq. 27
$\underline{\varepsilon}$	Misalignment angles	Eq. 3
η	Measurement noise	Eq. A-2
θ	Misalignment angle around Y_A axis	Eq. 3
θ_R	Elevation of radar line of sight	Fig. 6
λ	Control surface time constant inverse	Eq. 10
ν	Bank servo time constant inverse	Eq. 11
ϕ	Misalignment angle around X_A axis	Eq. 3
ψ	Misalignment angle around Z_A axis	Eq. 3
ψ_R	Azimuth of radar line of sight	Fig. 6
ω	Frequency of control oscillation	Eq. 27

Abstract

The problem of parameter estimation using tracking information is examined. Two models are developed and used to estimate the misalignment angles of the inertial system of a missile after its launch. The estimation is based on maximum likelihood concepts. The amount of information extracted from the tracking measurements and the missile specific forces measurements is analysed.

A feasibility study of the two models is conducted. The second model uses the aerodynamic model of the missile in order to enhance its estimation ability. Doing this, it incorporates more non-linearities than the first model. These severe non-linearities were found to offset the advantage it had in terms of information gathering. The first model is much simpler in its concept. Yet, it is still able to gather the information needed and its performance is very comparable to the one of the second model. The simplicity and linearity of the first model make it especially attractive.

APPLICATION OF A MAXIMUM LIKELIHOOD PARAMETER ESTIMATOR TO AN ADVANCED MISSILE GUIDANCE AND CONTROL SYSTEM

I. Introduction

Background and Motivation

An ever increasing amount of research is being directed toward the development of standoff weapons. Unfortunately, the probability of hitting the target decreases as the range from the target increases or in other words, safety comes at the price of a lack of accuracy. In order to keep a high probability of hitting the target even though the release range is safe enough, an accurate navigation system has to be added to the missile. The navigation system will then guide the missile through the mid-course part of the flight until close vicinity to the target is reached. Then the homing system of the missile, responsible for the terminal guidance, will guide it to the target accurately.

One of the disadvantages of the various homing systems used is that their operation is limited to a small area around the target. The navigation system used for the mid-course guidance has then to be very accurate. Because of economical considerations, the missile being expendable, only medium to low quality inertial navigation systems are usually mounted on missiles.

Among numerous other contributors to the existence of erroneous inertial navigation of the missile (such as gravity anomalies, aircraft initial position and attitude uncertainties), two of the major error sources can be the transfer alignment errors (initial aircraft to missile alignment of the inertial reference frames), and error parameters of the missile's inertial instruments (such as scale factors or biases of the gyros and accelerometers). Only the former is considered in this study.

Surprisingly enough, prelaunch calibration and alignment procedure of the platform of the missile has a limited accuracy, expressly due to the relatively benign aircraft environment (basically a perturbed 1 g flight path). Indeed, one would expect this environment to be ideal for the alignment of an inertial platform. Nevertheless, this environment causes unidentifiability of error sources, therefore causing the misalignment angles to be less observable and slowing down the estimation convergence (Ref 1). With the incorporation of sophisticated avionics for aircraft navigational update and fire control (for example, the Electronically Agile Radar - EAR, developed through the Air Force Avionics Laboratory), there exists a new possibility for further reduction of error magnitudes from the various sources.

It is presumed that the aircraft can track the missile with its radar and communicate with it through a two-way data-link. This communication link can be used to command accelerations to the missile and to receive back the achieved specific forces as measured by the missile's accelerometers in a stabilized, non-rotating missile inertial frame. The missile shall undergo much higher accelerations during launch and through maneuvering commands as compared to its prelaunch accelerations. Hence, the identifiability of error parameters is potentially much greater than it might be during the prelaunch benign phase.

Since the acceleration commands are generated with respect to the aircraft inertial frame, but received, interpreted, and implemented with respect to the missile inertial reference frame, it is potentially possible to "close the loop" around the aircraft to missile alignment by tracking the missile trajectory with respect to the aircraft inertial frame.

This report concerns itself with the ability to estimate the misalignment angles caused to the missile's inertial platform during the launching period. The assumption is made that after these angles are estimated, they would be transmitted to the missile to be incorporated within the missile navigational computer. The missile would then be guided to its intended target more accurately.

A possible scenario illustrating the application of the concept described above involves an area surveillance, command, and control aircraft flying at safe height and range from the target. Various strike aircrafts would launch their missiles and return quickly to their bases. The missiles will first boost until they arrive to the area controlled by the command and control aircraft. Then they will coast toward their targets with erroneous attitude and heading due to the initial misalignment. During the first few seconds of this coasting flight, the control aircraft will track the missiles, estimate their misalignment angles and transmit them to the missiles in order to realign their platforms. The missiles will then be well equipped for an accurate navigation toward their targets.

Several works have been accomplished in the general tracking area (Refs 2, 3). In most cases, in order to simplify the filter implementation, the tracking algorithms have been implemented in the line-of-sight frame. However, this does not appear to be an appropriate choice of frames in this case which involved two inertial platforms, one on the control aircraft and one on the missile. Two inertial frames are thereby defined and it would be computationally cumbersome to relate them by an additional line-of-sight frame. Therefore, it is desirable that the tracking filter provides an estimate of the misalignment angles in a local inertial/stationary frame -- the aircraft inertial frame. This is especially true for the case in which the aircraft is tracking more than one missile.

Statement of the Problem

Control surface deflection commands which are the source of commanded accelerations, are generated in the aircraft. There are two possibilities for closing the loop around the aircraft-missile misalignment. The first way is to ignore the fact that the acceleration commands are generated in the aircraft frame. Then the trajectory observed by the aircraft radar is compared to the one generated by a double integration of the missile specific force measurements (in the missile inertial frame). This discards some of the information inherently available, but it allows a particularly simple mechanization.

The second method uses the fact that the acceleration commands are generated in the aircraft frame but implemented in the missile frame to provide additional information about the misalignment. Since the missile's characteristics are known to the controlling aircraft, it is possible to predict the accelerations which should be achieved in response to the commands. Ignoring missile plant dynamics uncertainties, the difference between the commanded and achieved accelerations is directly related to the misalignment angles. This additional information should, conceptually, improve the estimation accuracy.

All techniques used to determine the angular alignment of coordinate frames depend on the measurement of the orientation of the coordinate frames with respect to two or more vectors that are common to each coordinate frames -- physical vectors (Ref 4:685). In method 1, the common vector is the change in the relative position of the missile while in method 2 it is the missile measured specific force vector. Figures 1 and 2 will illustrate the basic differences between the two methods -- the position prediction method and the acceleration prediction method. The figures represent the simple case in which noise does not perturb the measurements.

In Figure 1, the three components of the specific force vector ($\underline{SF}$) measured by the missile's accelerometers are transmitted to the aircraft. In the first method, those components are integrated as they are received. To calculate the position increment ($\Delta\underline{P}(\underline{SF})$) expected on the basis of the specific force received. This position increment is then compared to the one measured by the radar ($\Delta\underline{P}$(Radar)) and the difference between them is used to estimate the misalignment angles ($\underline{\varepsilon}$).

Figure 2 illustrates the concept of the second method. The aircraft computer "knows" the characteristics of the missile (its aerodynamical model, and parameters); therefore, upon sending a control surface deflection command to the missile, it can generate an expected specific force ($\underline{\hat{SF}}$). The measured value of the achieved specific force ($\underline{\tilde{SF}}$) will be different than the expected specific force as a function of the misalignment angles ($\underline{\varepsilon}$).

The expectation of the specific force ($\underline{\hat{SF}}$) is based on the assumption that the lift operating on the missile is perpendicular to the velocity vector (as stated in the assumption to Chapter II, section 3). Therefore, the velocity vector built through the radar measurements, serves to indicate a plane to which the expected specific force is perpendicular.

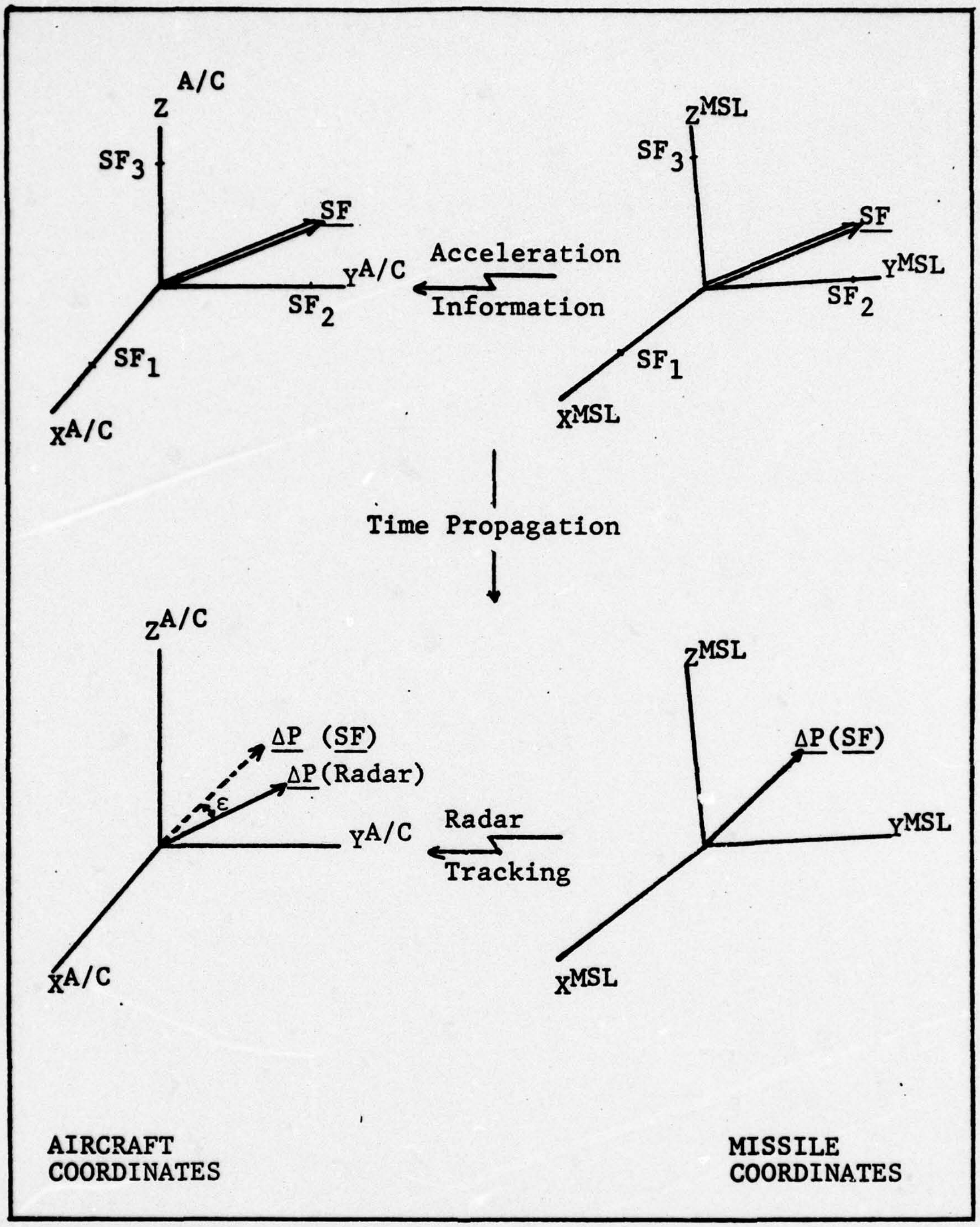

Fig. 1 Position Prediction Method

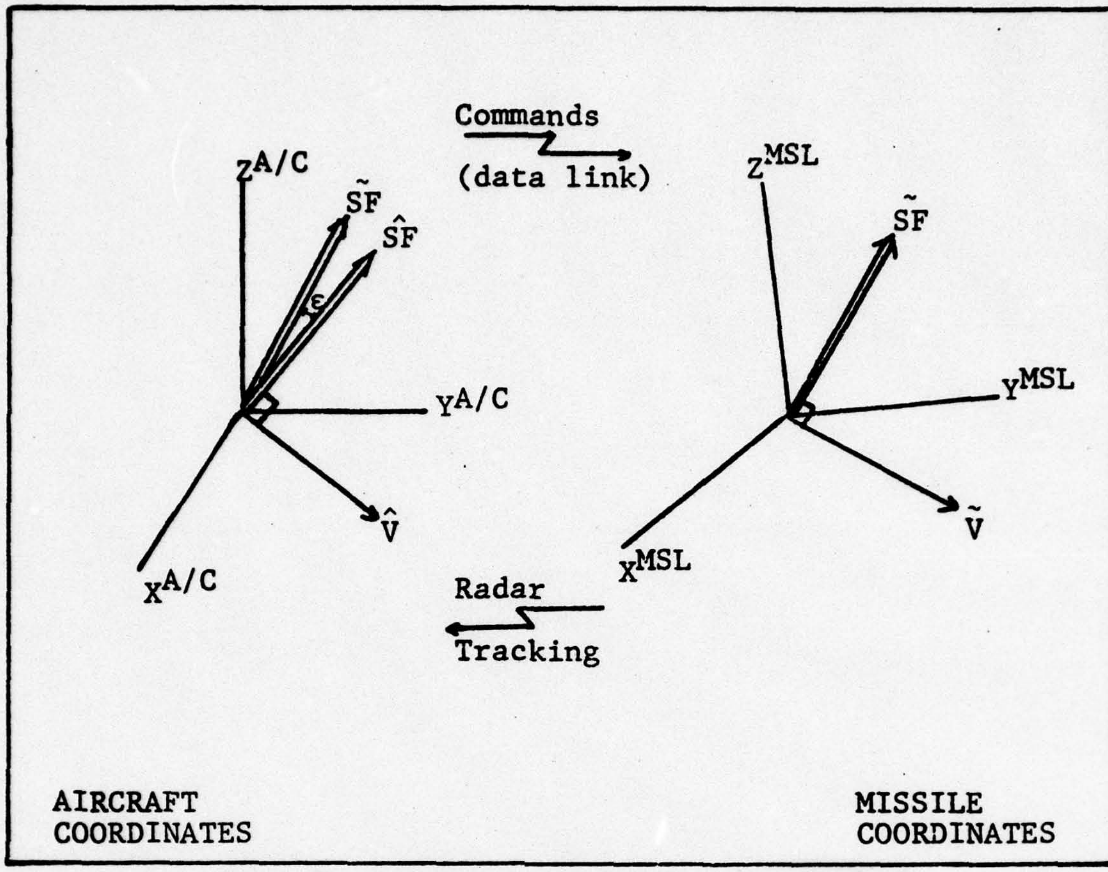

Fig. 2 Acceleration Prediction Method

It is seen clearly that Method 2 in fact uses all the information of Method 1 and adds to it the "information content" of the difference between the actual versus the expected response of the missile. The usefulness of this second source of information is highly dependent on the ability of the designer to model the acceleration characteristics of the missile accurately. No attempt is made in this study to determine these sensitivities. Rather, this study deals with the determination of the relative level of information potentially available under the best circumstances (i.e. perfect missile dynamics model).

Hence, the prime objective of this study is to evaluate the "information content" of the two methods in order to determine the potential estimation enhancement due to the additional information of the second method. The secondary objective is to apply the two methods to estimate the misalignment angles in a typical launch scenario and to compare the quality of the results. The second method is considerably more complicated than the first, and so the research investigation shall provide insight into whether or not the additional complexity of the second method has substantial estimation accuracy potential worth pursuing.

The development of a "truth model" for the engagement scenario is another component of the problem. The truth model is a description of missile parameters and dynamics and system (missile/aircraft) kinematics and uncertainties. It is the "best model" of the real world in that it includes as many effects as possible, regardless of the resulting system complexity. The truth model is exercised in a computer simulation to provide nominal trajectories for analysis of the two models. A more complex model can be considered as a truth model. However, due to the limited scope of this research, the simpler generic model developed by TASC (Ref 5) and modified in this study was preferred as representative of a broader range of missiles to illustrate the techniques.

The nominal trajectory is a deterministic (reference) trajectory that starts from a known set of initial conditions, $\underline{X}_n(t_0)$, and propagates according to the differential equation:

$$\dot{\underline{X}}_n(t) = \underline{f}[\underline{X}_n(t), \underline{u}(t), t] \quad (1)$$

where

$\underline{f}[\ \ ]$ = a known function of the arguments

$\underline{X}_n(t)$ = the nominal trajectory

$\underline{u}(t)$ = deterministic forcing function

t = time

Associated with the nominal trajectory is a sequence of nominal sampled data measurements -- radar and specific force measurements. Both models use those measurements corrupted by an additive white noise as output measurements.

Both models are also exercised against three different control patterns for the trajectory of the missile. This allows a partial investigation of the difference in the various sensitivities of the output measurements with respect to the different types of commands to the missile. It is indeed known from the theory of input design (Ref 6) that the control inputs to the missile can be optimized to provide maximum information content -- thereby maximizing the estimation capability. However, a full analysis of this problem is beyond the scope of this study.

As mentioned, the prime intent of this study is to compare the information content of the two methods. Therefore, a basic maximum likelihood estimator is used and the comparison of the methods is made on the basis of their information matrices. The feasibility of a full-scale maximum likelihood estimator is thereby demonstrated though no attempt is made to investigate the means to achieve on line applicability.

The information matrix (M) is then formed as:

$$M = \sum_{n=1}^{N} \left(\frac{\partial \underline{y}}{\partial \varepsilon} \right)^T (n) \cdot R^{-1}(n) \cdot \left(\frac{\partial \underline{y}}{\partial \varepsilon} \right)(n) \qquad (2)$$

where $\left(\frac{\partial \underline{y}}{\partial \underline{\varepsilon}} \right)(n)$ are the output ($\underline{y}$) sensitivities to the unknown misalignment angles ($\underline{\varepsilon}$) at discrete time (n) -- assuming $\underline{\varepsilon}$ is constant, and $R^{-1}(n)$ is the inverse of the covariance of the measurement noise (Ref 6). The information matrix conceptually describes how much information about the state of the system is contained in the data (Ref 7: 231). N is the number of sampled measurements which are taken during the tracking period.

Major Techniques

A simulation of the "truth model" and of the two estimation processes is performed on a CDC 6600 computer. Euler integration was used with a step size of .01 sec on the 50 sec tracking interval. This was dictated by the time constants of the system and though the integration steps could be much longer when integrating the sensitivities, the information matrix, and the gradient vector elements, they were all chosen equal for the sake of simplicity. The sampling period was also chosen to be .01 sec for the same reason.

A system of "state-sensitivities" is built consisting of the sensitivities of the six states representing position and velocity (the only states considered in this study) with respect to the three misalignment angles. The output vector (or measurement vector) sensitivities are then defined as functions of the state sensitivities.

The information matrix and the gradient vector are developed using information theory and taking into consideration the uncertainty involved with the measurement. These are then used to calculate the optimal correction to the misalignment angles.

Organization

This thesis consists of four chapters. Chapter I outlines the motivation and background for investigating the feasibility of maximum likelihood estimation to a problem of estimating parameters involved in a nonlinear use. Chapter II develops the system truth model and presents the assumptions made in the engagement scenario. Chapter III introduces the Maximum Likelihood Filter Equations and presents the formulation of the two specific models investigated. The Output Error Functional and the performance criterion are also discussed in this chapter. Chapter IV includes the discussion about the numerical results. Having demonstrated concept feasibility, several suggestions are presented for future optional studies. The appendices contain some theory about Maximum Likelihood Estimation, the detailed derivation of some of the equations, the description and computer listing of each of the computer programs developed for this study, and the numerical results.

II. System Truth Model

Introduction

This chapter develops the realistic models of the aircraft, missile, and engagement geometry. The system models should represent the dominant dynamics and non-linearities of the engagement scenario, and yet be representative of a wide spectrum of missiles. The model is divided into several components, as shown in Figure 3. It is important to note that the primary objective of this report is to describe and analyze the ability to estimate parameters in the navigation and guidance equations for the missile (the three misalignment angles of the inertial platform of the missile). The missile subsystems and dynamics are, therefore, modelled in much greater details than other truth model components (for example, the aircraft) and yet kept as simple as possible so that they do not shadow the main thrust of this study -- the estimation problem. Prior to discussing the individual components, however, the assumptions that have been made in the development of the model are presented.

Assumptions. The control aircraft which is tracking the missile is modelled as a point mass. In addition, the inertial navigation system of the aircraft, with respect to which the missile's inertial system is evaluated, is assumed to be perfectly aligned and stable so that it can be regarded as forming a stationary frame in which the missile is

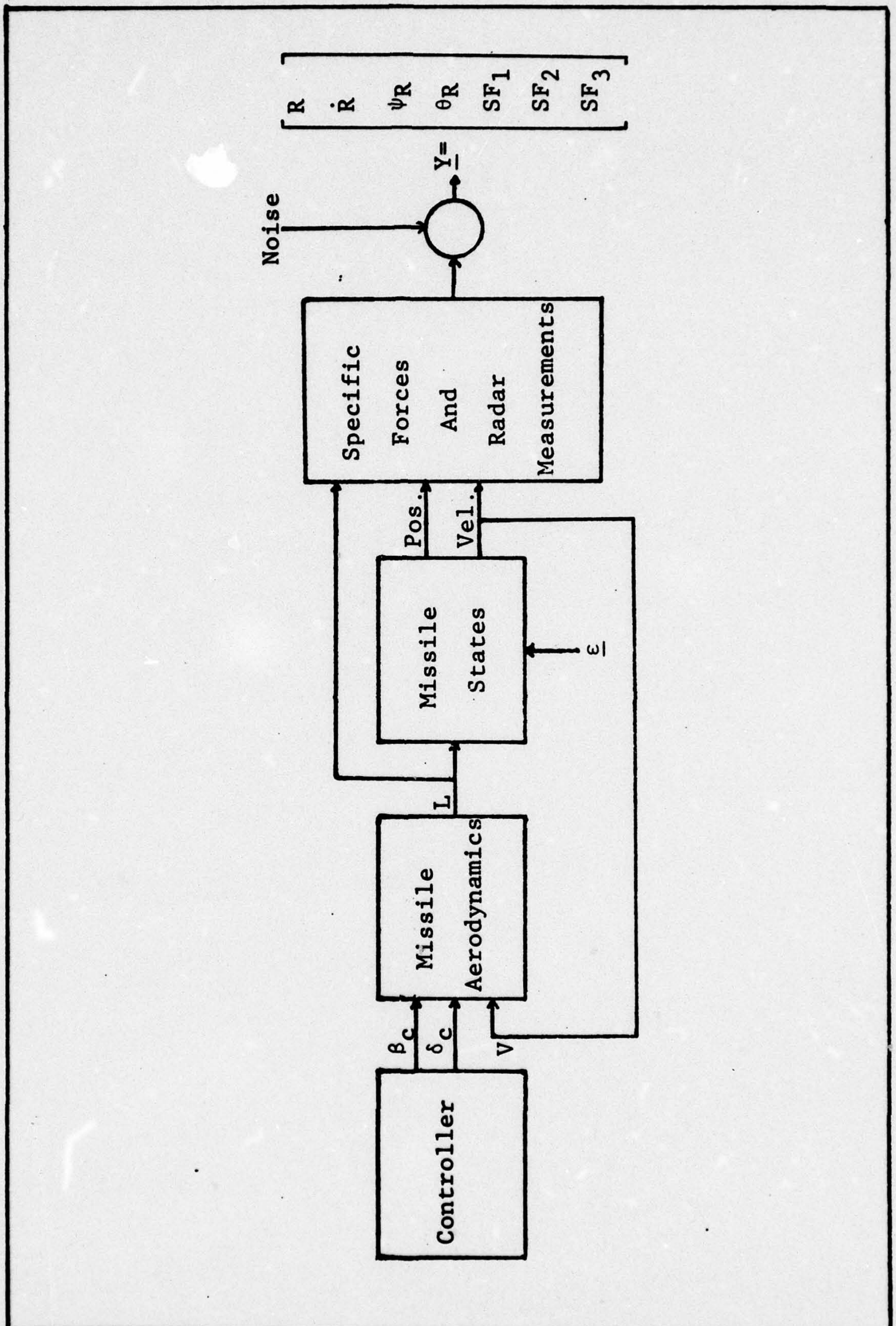

Fig. 3 Simplified Model

observed. It is further assumed that, relative to each other, the aircraft and the missile inertial frames are non-rotating and non-drifting. The assumption of non-rotation is really made without any loss of generality since rotating frames (as in the case of the wander azimuth inertial platforms) consist of a controlled rotation that, since known, can always be taken into account. Likewise, the assumption of no drift has little effect upon the information analysis (parameter sensitivity analysis) which is conducted in this study. However, the drift would have to be taken into account in an actual on-line estimator in which the misalignment angles would be modelled as slowly varying parameters, and a maximum likelihood estimator, for example, would be employed (Ref 8:10-9).

Navigation systems that are currently available in aircraft have errors that are second or higher order effects when compared to errors in the measurements of the missile dynamics with the radar tracker. Therefore, the assumption of a perfect inertial navigation system on the aircraft does not affect the model integrity, while it greatly simplifies it. It is further assumed that meteorological effects can be ignored. This assumption appears reasonable since the time period of the tracking of the missile is short, both the missile and aircraft are in the same air mass, and finally because the magnitudes of meteorological effects will in general be of second order in comparison to the missile dynamics. Other assumptions that are peculiar to a particular model component will be discussed in the applicable section.

Coordinate Frames

Aircraft Local Vertical Inertial Frame (X_A, Y_A, Z_A). Located at the aircraft center of gravity. The X_A axis is in the "true" north direction, the Y_A axis is in the "true" east direction, and the Z_A axis is in the "true" down direction.

Missile Local Vertical Inertial Frame (X_M, Y_M, Z_M). Located at the missile's center of gravity. The X_M axis is in the missile indicated "north" direction, the Y_M axis is in the missile indicated "east" direction, and the Z_M axis is in the missile indicated "down" direction.

These definitions of frames assume the range between the aircraft and the missile to be such that the difference between the "true inertial" direction (north-east-down) at the location of the aircraft and missile, is negligible.

Misalignment Euler Angle Rotation. Euler angles are used to define the misalignment between the missile's and the aircraft's inertial frame. The misalignment angles are assumed to be small, so they may be treated as components of a vector $\underline{\varepsilon}$.

$$\underline{\varepsilon} = \begin{bmatrix} \psi \\ \theta \\ \phi \end{bmatrix} \tag{3}$$

X_i, Y_i, Z_i (i = 1, 2) are intermediate unit vectors for the transformation that represents the Euler angle rotations in sequence (Ref 9:37):

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} \text{Cos}\ \psi & \text{Sin}\ \psi & 0 \\ -\text{Sin}\ \psi & \text{Cos}\ \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} \qquad (4a)$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \text{Cos}\ \theta & 0 & -\text{Sin}\ \theta \\ 0 & 1 & 0 \\ \text{Sin}\ \theta & 0 & \text{Cos}\ \theta \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \qquad (4b)$$

$$\begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}\ \phi & \text{Sin}\ \phi \\ 0 & -\text{Sin}\ \phi & \text{Cos}\ \phi \end{bmatrix} \cdot \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \qquad (4c)$$

The transformation C_A^M is defined as the product of the three matrices in Equations (4):

$$C_A^M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}\phi & \text{Sin}\phi \\ 0 & -\text{Sin}\phi & \text{Cos}\phi \end{bmatrix} \cdot \begin{bmatrix} \text{Cos}\theta & 0 & -\text{Sin}\theta \\ 0 & 1 & 0 \\ \text{Sin}\theta & 0 & \text{Cos}\theta \end{bmatrix} \cdot \begin{bmatrix} \text{Cos}\psi & \text{Sin}\psi & 0 \\ -\text{Sin}\psi & \text{Cos}\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi+\sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi+\cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi+\cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{bmatrix} \quad (5)$$

Assuming small misalignment angles (represented by the general angles α and β):

$$\sin\alpha \simeq \alpha; \quad \sin\alpha\sin\beta \simeq 0; \quad \cos\alpha \simeq 1$$

and so:

$$C_A^M \simeq \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \quad (6)$$

Since C_A^M is an orthogonal transformation, the inverse is equal to its transpose (neglecting second order terms) and:

$$C_M^A \simeq \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \quad (7)$$

<u>Missile Guidance</u>

<u>Airframe Dynamics</u>. The rotational motion of an airframe is generally described in terms of six state variables -- three angles and three angular rates. The exact differential

equations of motion for these variables are nonlinear and are also coupled to the translational motion of the airframe through such quantities as airspeed and altitude, which describe the missile's flight condition. The coupling between translation and rotation is simplified if the former, being affected primarily by the relatively long response time of the guidance loop, is regarded as being independent of the autopilot characteristics. Consequently, in the rotational equations of motion, altitude and airspeed can be regarded as time-varying parameters that are independent of the missile airframe response characteristics. In the case covered in this study, the missile is supposed to cruise for quite a long range. This would require wider wings for a better lift coefficient but would deny the designer the ability to assume instantaneous roll stability. The roll dynamics are therefore taken as a first order lag system with a shorter time constant than the one for the pitch dynamics (bank-to-turn missile). After these simplifications there still remain nonlinearities in the dependence of the equations of motion upon angle of attack and control surface deflection angle; these can be eliminated with small angle approximations. The mathematical model used here to describe pitch motion is for a missile which develops lateral maneuvering forces through aerodynamic lift provided by fixed wings or by the missile body, with the aid of tail-mounted control surfaces. The model neglects longitudinal-lateral coupling and assumes a second order airframe and a first order actuator dynamics with equations of motion given by (Ref 5:2-2 to 2-5):

$$\dot{q}(t) = M_q \cdot q(t) + M_\alpha \cdot \alpha(t) + M_\delta \cdot \delta(t) \tag{8}$$

$$\dot{\alpha}(t) = q(t) - L_\alpha \cdot \alpha(t) - L_\delta \cdot \delta(t) \tag{9}$$

$$\dot{\delta}(t) = -\lambda \cdot \delta(t) + \lambda \cdot \delta_c(t) \tag{10}$$

$$\dot{\beta}(t) = -\nu \cdot \beta(t) + \nu \cdot \beta_c(t) \tag{11}$$

$$L(t) = -V(t)(\dot{\alpha}(t) - q(t)) \tag{12}$$

where M_q, M_α, M_δ, L_α, and L_δ are stability derivatives and:

$\alpha(t)$ - angle of attack,

$q(t)$ - pitch rate,

$L(t)$ - normal acceleration (lift),

$V(t)$ - air speed,

$\delta(t)$ - control surface deflection,

$\delta_c(t)$ - control surface deflection command,

$\beta(t)$ - bank angle,

$\beta_c(t)$ - bank angle command,

$1/\lambda$ - pitch actuator time constant,

$1/\nu$ - bank servo time constant,

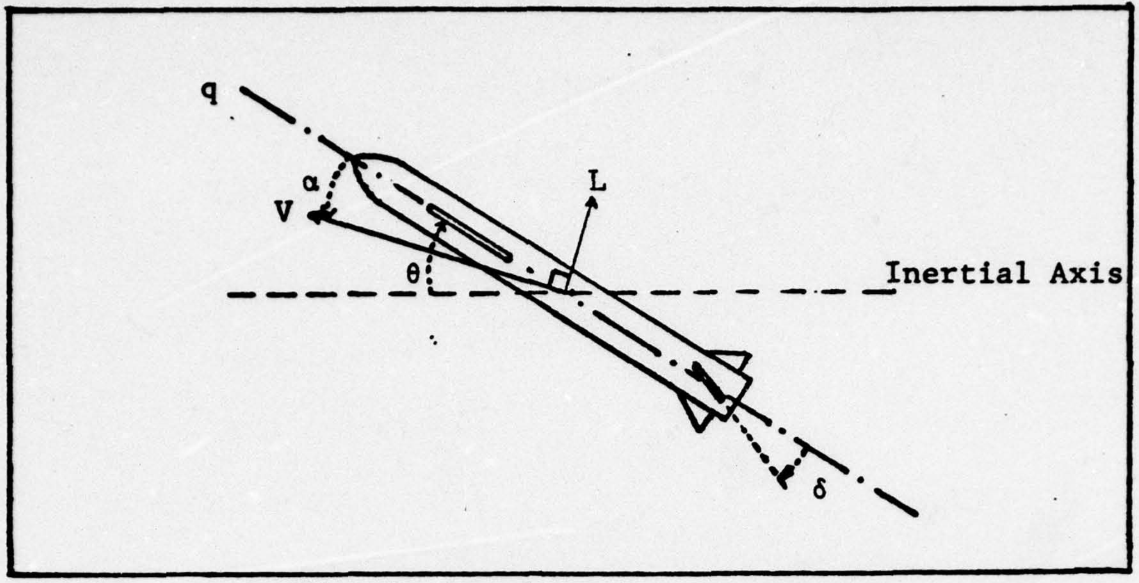

Fig. 4 Aerodynamics Variables

The stabilized roll angle of the missile is measured by its inertial system and defined with respect to it. Once the roll angle (in the missile's inertial frame) is known, deflection commands can be resolved to the pitch or yaw control surfaces. This study does not deal with the resolution of the commanded deflection between the two pairs of fins. $\delta(t)$ is then the effective control surface deflection (Fig. 4).

Kinematics. The total velocity of the missile $\underline{V}^M$ (the superscript denotes the frame in which the vector is expressed, M - for the missile's inertial frame, and A - for the aircraft inertial frame) is at an azimuth angle γ_A and an elevation angle γ_E with respect to the missile's inertial frame. The banking angle β is measured around the positive $\underline{V}^M$ direction,

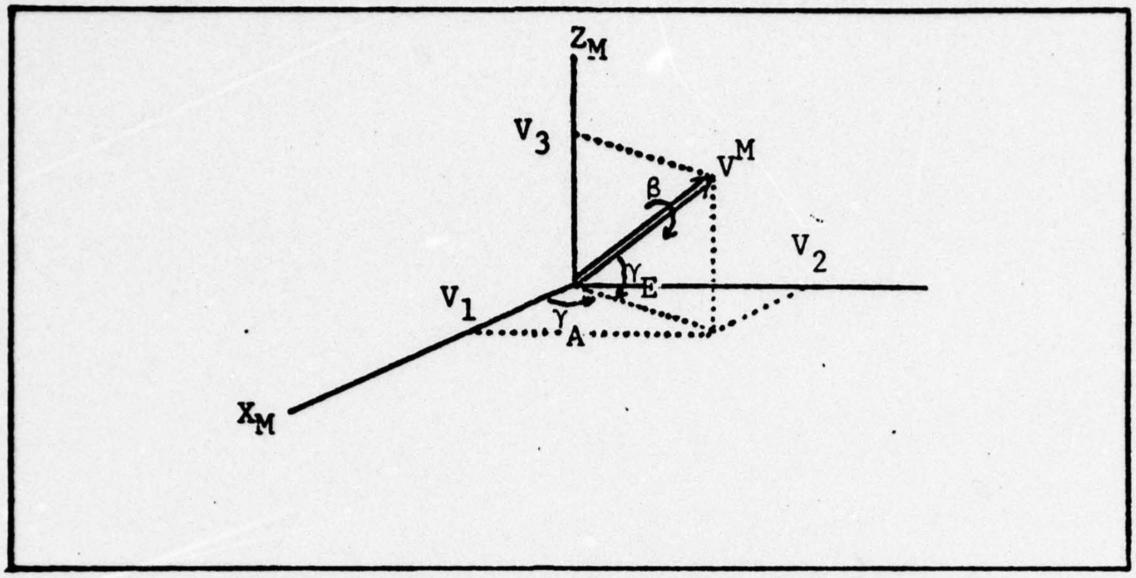

Fig. 5 The Missile Inertial Frame

from the horizontal plane (Fig. 5). X_v, Y_v, Z_v define a coordinate frame such that X_v is in the $\underline{V}^M$ direction, Y_v is in the direction defined by the right wing of the missile on the horizontal plane (at zero bank angle) and Z_v is determined by the cross product ($X_v \times Y_v$). X_i, Y_i, Z_i ($i = 1, 2$) are intermediate unit vectors for the transformation:

$$\begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \Gamma_A \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} \cos\gamma_A & \sin\gamma_A & 0 \\ -\sin\gamma_A & \cos\gamma_A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \qquad (13a)$$

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \Gamma_E \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \mathrm{Cos}\gamma_E & 0 & \mathrm{Sin}\gamma_E \\ 0 & 1 & 0 \\ -\mathrm{Sin}\gamma_E & 0 & \mathrm{Cos}\gamma_E \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \qquad (13b)$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \Gamma_\beta \begin{bmatrix} X_v \\ Y_v \\ Z_v \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}\beta & \mathrm{Sin}\beta \\ 0 & -\mathrm{Sin}\beta & \mathrm{Cos}\beta \end{bmatrix} \begin{bmatrix} X_v \\ Y_v \\ Z_v \end{bmatrix} \qquad (13c)$$

$$\begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \begin{bmatrix} \mathrm{Cos}\gamma_A & \mathrm{Sin}\gamma_A & 0 \\ -\mathrm{Sin}\gamma_A & \mathrm{Cos}\gamma_A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{Cos}\gamma_E & 0 & \mathrm{Sin}\gamma_E \\ 0 & 1 & 0 \\ -\mathrm{Sin}\gamma_E & 0 & \mathrm{Cos}\gamma_E \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}\beta & \mathrm{Sin}\beta \\ 0 & -\mathrm{Sin}\beta & \mathrm{Cos}\beta \end{bmatrix} \begin{bmatrix} X_v \\ Y_v \\ Z_v \end{bmatrix}$$

$$= \Gamma_A \cdot \Gamma_E \cdot \Gamma_\beta \begin{bmatrix} X_v \\ Y_v \\ Z_v \end{bmatrix} \qquad (14)$$

As seen in Fig. 3, Γ_A and Γ_E can resolve the total missile velocity (V) into its components in the missile inertial frame -- V_1, V_2, V_3 (Fig. 5):

$$\mathrm{Sin}\ \gamma_A = \frac{V_2}{(V_1^2 + V_2^2)^{\frac{1}{2}}} \qquad (15)$$

$$\text{Cos } \gamma_A = \frac{V_1}{(V_1{}^2 + V_2{}^2)^{\frac{1}{2}}} \tag{16}$$

$$\text{Sin } \gamma_E = \frac{V_3}{V} \tag{17}$$

$$\text{Cos } \gamma_E = \frac{(V_1{}^2 + V_2{}^2)^{\frac{1}{2}}}{V} \tag{18}$$

The transformation from the aircraft coordinates to the missile coordinates can be carried out through the use of the misalignment transformation matrix:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = C_A^M \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \cdot \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix}$$

$$= \begin{bmatrix} V_X + \psi V_Y - \theta V_Z \\ -\psi V_X + V_Y + \phi V_Z \\ \theta V_X - \phi V_Y + V_Z \end{bmatrix} \tag{19}$$

This representation of the three components of the missile velocity are used in the definition of the trigonometric functions of γ_A and γ_E defined in Equations (15)-(18).

Assumptions. The translational equations of motion are driven by the sum of external forces applied to the airframe. The forces modelled in this study are the aerodynamic force and gravity. The aerodynamic force acting on the body of the missile is a lift force acting perpendicular to the velocity vector $\underline{V}^M$ (Ref 10:328). Gravity is acting on the missile in the "true" down direction -- the aircraft's Z_A direction.

This study assumes that the missile's thrust is equal and opposite to its drag during the tracking phase. Those assumptions are indeed valid especially for the type of missiles dealt with in this study -- missiles having a very smooth aerodynamical shape and designed to coast for long ranges.

Another assumption regards the mass of the missile and its stability parameters. A great part of the mass of a missile is fuel and as it is burned to develop thrust, the mass of the missile decreases. With the change of mass, a change of the moment of inertia and of the location of the center of gravity will also occur. For the same reason mentioned above, having a missile tracked during its coasting phase, those factors will be assumed constant. As for the stability parameters, they are usually a function of the speed of the missile, its mass, its altitude, and the dynamic pressure in the air mass around it (Ref 5:D-2). This study will assume those parameters to be constant. Their values will match the following flight conditions:

Altitude - 35000 ft

Velocity - 1500-3000 ft/sec

Dynamic pressure - 3146 lb/ft^2

Mass - 28 slugs

Centerline moment of inertia - 497 slug-ft^2

The parameters values used for this study for the flight conditions mentioned above are:

M_q - -.462 1/sec

M_α - -5.81 1/sec^2

M_δ - -72. 1/sec^2

L_α - .379

L_δ - .0699

Missile Measurements

Radar Measurements. The line of sight between the aircraft and the missile is defined by the azimuth angle ψ_R (with respect to the aircraft inertial frame) and the elevation angle θ_R (with respect to the same frame) (Fig. 6). The relative position and velocity of the missile, as measured by the radar (in the aircraft frame), as seen in Fig. 6 are therefore given by the following equations:

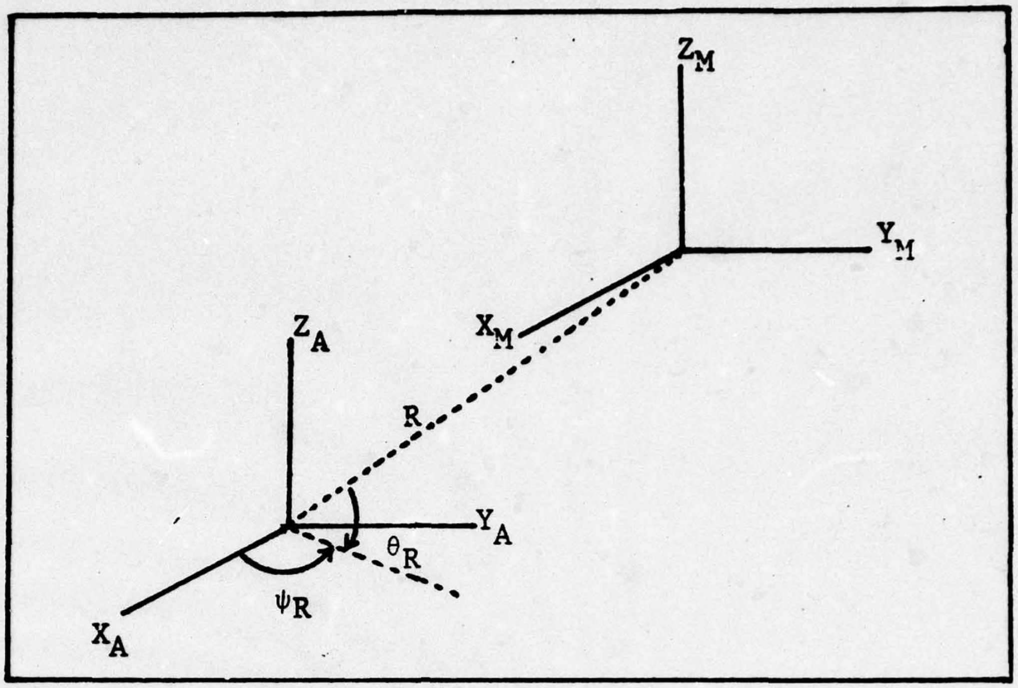

Fig. 6 Radar Geometry

$$R = (X^2 + Y^2 + Z^2)^{\frac{1}{2}} \tag{20}$$

$$R = \frac{2X\dot{X} + 2Y\dot{Y} + 2Z\dot{Z}}{2R}$$

$$= \frac{X}{R}\dot{X} + \frac{Y}{R}\dot{Y} + \frac{Z}{R}\dot{Z}$$

$$= V_X\frac{X}{R} + V_Y\frac{Y}{R} + V_Z\frac{Z}{R} \tag{21}$$

$$\text{Sin } \psi_R = \frac{Y}{(X^2 + Y^2)^{\frac{1}{2}}} \tag{22}$$

$$\text{Sin } \theta_R = \frac{Z}{R} \tag{23}$$

where X, Y, Z, V_X, V_Y, and V_Z are all relative to the aircraft position and velocity. It is noted that the radar measurements are non-linear functions of the relative position and velocity of the aircraft.

Acceleration Measurements. The three accelerometers mounted on the inertial platform of the missile are measuring the three components of the specific force acting on the missile along their sensitive directions. These directions are misaligned with respect to the true cardinal directions (represented by the aircraft inertial frame) by the misalignment angles. The accelerometers are instruments used to measure specific forces. The acceleration with respect to the inertial space is therefore the sum of the specific force and of gravity (Ref 11:62). As stated in the assumption of the previous section, the only external force (except gravity) acting on the missile is the lift, and it is perpendicular to the velocity vector. The velocity being in the X_v direction (Equation (14)), the lift is acting in the $-Z_v$ direction. The lift vector in the velocity frame can therefore be described as:

$$\text{Lift}^v = \begin{bmatrix} 0 \\ 0 \\ -L \end{bmatrix} \qquad (24)$$

Using the transformations defined in Equation (13) coordinatizes the lift vector in the missile inertial frame:

$$\underline{SF} = \Gamma_A \, \Gamma_E \, \Gamma_\beta \begin{bmatrix} 0 \\ 0 \\ -L \end{bmatrix} \tag{25}$$

The three components of the vector SF are the three specific forces sensed by the missile accelerometers.

Measurement Noises

Introduction. This section models the random noise component of the system truth model. This study considers only measurement noises and neglect any system noises which might be present in reality. All measurement noises which might be present are represented by additive white noise over the bandwidth of the systems considered (Ref 12:A-10, A-14). Since this is a "deterministic" information analysis (using apriori sensitivities), the accuracy of the "truth" model, in terms of the individual measurement error sources is not of much concern so that the lumping of the error sources is a justifiable simplification for this study.

Considering first the radar measurements, they will generally be corrupted by various types of noises which can be categorized according to the dependency of their RMS levels on the range to the missile. The actual noise levels and bandwidths are dependent on the exact form of the measurement signal processor, missile attitude and characteristics, environmental conditions and a multitude of related system effects. However, using measurements obtained from actual hardware or mathematical models, most of the observed measurement noise can be lumped into one of three assumed forms: receiver noise, range independent noise, or angular seintillation noise. This study lumps all error sources in one error process modelled as white, Gaussian, and with constant covariance. The justification for ignoring time correlated error sources such as angular scintillation is in the fact that most of the target dependent error sources -- as scintillation, will be eliminated if a beacon is used on the missile to aid the tracking process. The dominant remaining error source is then receiver noise.

Receiver Noise. Receiver noise consists primarily of thermal noise generated by the antenna and receiver electronics on board the aircraft. The effective amplitude of this noise increases with increasing range because of the corresponding decreasing signal-to-noise ratio. The noise bandwidth is dictated by the post-detection bandwidth of the receiver, which in general tends to be much larger than the signal bandwidth. Consequently, it can be assumed that the noise is "white" over the signal spectrum of interest without loss of generality. In the case of a sampled data system, errors in the sampled and held values are assumed to be uncorrelated from sample to sample.

Range Independent Noise. Range independent noise is a collection of all noise sources which contribute a constant RMS error throughout the tracking phase. Typical sources include servo noise generated by the seeker servo. It is assumed that the noise is also "white" over the receiver bandwidth.

Angular Scintillation Noise. Angular scintillation noise is caused by the wandering of the apparent centroid of radiation across the visible surface of the missile. This is generally a narrow band source and is often modelled as a first order Markov process, the result of "white" noise passed through a low pass filter with a time constant which depends primarily on the missile motion spectrum. On the other hand, if the radar frequency is changed in pseudo random manner from sample to sample, then the apparent centroid of the target radiation will tend to be independent from sample to sample.

Accelerometer Specific Force Noise. These include accelerometer input axis misalignment, accelerometer bias and scale factor error, and higher order errors (Ref 13: 105-117). This study assumes a zero bias error. All the error sources in the accelerometers are then modelled as pure white noise. In most current generation inertial navigation systems, the uncompensated accelerometer bias is at least 50-100 μg (Ref 13:139,163). For a lower quality inertial system, as the one which might be mounted on the missile, a typical value for the standard deviation of the noise representing this uncertainty would be 1 mg.

Noise Generator. The additive "white" noise for the seven measurements (four radar measurements and three specific forces) are produced using a call to Subroutine Noise (see Appendix D). The essential procedure is to generate a unit variance Gaussian noise and to multiply it by the strength desired. For a "white" noise, the strength is σ (σ is the standard deviation). The various values for the different strengths are listed in Table I:

Table I

Strengths of Measurement Noises

Measurement	σ
Range - (ft)	10
Range Rate - (ft/sec)	10
Azimuth - (Rad)	10^{-3}
Elevation - (Rad)	10^{-3}
Specific Force - (ft/sec^2)	$32.2 \cdot 10^{-3}$

System Equations

The equations of the system aerodynamics and kinematics are described in this section. A state space representation is used for the development of the aerodynamics and kinematics equations.

<u>Controller</u>. As shown in Fig. 1, a controller will be the function generator for the commands generated in the aircraft and transmitted to the missile. This transmission is assumed to be without loss or perturbation. Two angles controlling the aerodynamics of the missile are transmitted from the aircraft -- δ_c (Eq. (10)), and β_c (Eq. (11)).

Different controls are expected to excite different modes of the system according to the various control frequencies and natural frequencies of the system, and also to the state which is most perturbed by the control. Three different control types are used in this simulation, though no attempt is made to go into the broad field of input design -- a field which investigates the techniques involved in choosing the proper inputs according to optimal identification criteria (Ref 6). A separate computer simulation run is made for each type of control. The specific type of control is defined to the controller by the integer variable K_D such that:

$K_D = 1$:

$$\beta_c = K_1 \cdot t \tag{26}$$

$$\delta_c = K_3 + K_4 \cdot \text{Sin}\ (\omega_2 \cdot t) \tag{27}$$

$K_D = 2$:

$$\beta_c = K_1 \cdot t + K_2 \cdot \text{Sin}\ (\omega_1 \cdot t) \tag{28}$$

$$\delta_c = K_3 \tag{29}$$

$K_D = 3$: $$\beta_c = K_1 \cdot t + K_2 \cdot \mathrm{Sin}\,(\omega_1 \cdot t) \quad (30)$$

$$\delta_c = K_3 + K_4 \cdot \mathrm{Sin}\,(\omega_2 \cdot t) \quad (31)$$

where t is the elapsed time since the start of the controlling process. K_1, K_2, K_3, K_4, ω_1 and ω_2 are constants which result in the controls described in Table II.

K_1 - .063 rad/sec

K_2 - .03 rad

K_3 - .007 rad

K_4 - .002 rad

ω_1 - .1 rad/sec

ω_2 - .2 rad/sec

Table II

Control Characteristics

Control	Max. Deflection in 100 sec. (Rad)	Amp. of Oscillations (when applicable) (Rad)	Freq. of Oscillations (when applicable) (Hz)
β_c	6.28	.03	.016
δ_c	.009	.002	.032

Aerodynamics. The aerodynamical characteristics of the missile are described in the third section of this chapter (the section dealing with the Missile Guidance). The equations are repeated here for completeness:

$$\dot{q}(t) = M_q \cdot q(t) + M_\alpha \cdot \alpha(t) + M_\delta \cdot \delta(t) \tag{8}$$

$$\dot{\alpha}(t) = q(t) - L_\alpha \cdot \alpha(t) - L_\delta \cdot (\delta t) \tag{9}$$

$$\dot{\delta}(t) = -\lambda \cdot \delta(t) + \lambda \cdot \delta_c(t) \tag{10}$$

$$\dot{\beta}(t) = -\nu \cdot \beta(t) + \nu \cdot \beta_c(t) \tag{11}$$

$$L(t) = -V(t) \ (\dot{\alpha}(t) - q(t)) \tag{12}$$

$$\underline{SF} = \Gamma_A \cdot \Gamma_E \cdot \Gamma_\beta \cdot \begin{bmatrix} 0 \\ 0 \\ -L \end{bmatrix} \tag{25}$$

The three components of the specific force vector ($\underline{SF}$) defined in Equation (25) are in the missile inertial frame.

Kinematics. Since the position and velocity states of the missile are defined in the aircraft frame (as stated also in the radar measurement subsection of this chapter), the specific force vector has to be put into the aircraft inertial frame before its integration to generate the missile velocity. This is done by premultiplying the vector $\underline{SF}$ by the misalignment matrix C_M^A. Gravity has to be added to the third component of the resultant vector in the aircraft frame:

$$\underline{A} = C_M^A \cdot \underline{SF} \tag{32}$$

$$\dot{X}(t) = V_X(t) \tag{33}$$

$$\dot{Y}(t) = V_Y(t) \tag{34}$$

$$\dot{Z}(t) = V_Z(t) \tag{35}$$

$$\dot{V}_X(t) = A_X(t) \tag{36}$$

$$\dot{V}_Y(t) = A_Y(t) \tag{37}$$

$$\dot{V}_Z(t) = A_Z(t) + g \tag{38}$$

Summary

The real world model of the engagement scenario's systems, parameters, and noises has been developed in Chapter II. It has been possible to describe a complex, non-linear system and simulate it using a simple rectangular integration scheme and a fast enough sample rate. The modelling emphasis has been on the missile system and its associated parameters and coefficients.

The system equations were developed throughout the chapter. The truth model flowchart and computer listing are in Appendix D. Finally, the "true states" were generated for the determination of "true" measurement values to be used in the maximum likelihood estimation of the parameters -- the three misalignment angles.

III. Maximum Likelihood Filters

Introduction

This chapter presents the Maximum Likelihood Estimator (or Filter) equations and develops Model 1 and Model 2 representations. As it will be seen, both of these filters are highly non-linear. Therefore, the conventional Kalman Filter is not an appropriate estimator for this particular system. In the case for which the covariances of the states would remain Gaussian, it could be possible to use an Extended Kalman Filter. This assumption is not realistic for a non-linear system (Ref 8:10-7). On the other hand, the advantage of the Extended Kalman Filter is that it is recursive so that its implementation is more realizable. Nevertheless, as stated in the introduction to this thesis, the prime objective is in the evaluation of the information content of the two models. The design of a realizable filter is only a secondary objective. For the non-linear system at hand, the maximum likelihood estimator becomes then the best choice because a statistical measure of the information content between the two models is inherently contained in their respective terminal information matrix.

Sensitivity States

As shown in Appendix A, the maximum likelihood estimator is based on the sensitivity operators made up of the output sensitivities, which are in turn functions of the state sensitivities. The state sensitivities to the misalignment angles are defined in this section. The position states sensitivities ($\frac{\partial \underline{P}}{\partial \underline{\varepsilon}}$) are:

$$\frac{\partial \underline{P}}{\partial \underline{\varepsilon}} = \begin{bmatrix} \frac{\partial X}{\partial \psi} & \frac{\partial X}{\partial \theta} & \frac{\partial X}{\partial \phi} \\ \frac{\partial Y}{\partial \psi} & \frac{\partial Y}{\partial \theta} & \frac{\partial Y}{\partial \phi} \\ \frac{\partial Z}{\partial \psi} & \frac{\partial Z}{\partial \theta} & \frac{\partial Z}{\partial \phi} \end{bmatrix} \tag{39}$$

The velocity states sensitivities ($\frac{\partial \underline{V}}{\partial \underline{\varepsilon}}$) are:

$$\frac{\partial \underline{V}}{\partial \underline{\varepsilon}} = \begin{bmatrix} \frac{\partial V_X}{\partial \psi} & \frac{\partial V_X}{\partial \theta} & \frac{\partial V_X}{\partial \phi} \\ \frac{\partial V_Y}{\partial \psi} & \frac{\partial V_Y}{\partial \theta} & \frac{\partial V_Y}{\partial \phi} \\ \frac{\partial V_Z}{\partial \psi} & \frac{\partial V_Z}{\partial \theta} & \frac{\partial V_Z}{\partial \phi} \end{bmatrix} \tag{40}$$

Using state space representation, the relationship between $\frac{\partial \underline{P}}{\partial \underline{\varepsilon}}$ and $\frac{\partial \underline{V}}{\partial \underline{\varepsilon}}$ is:

$$\left(\frac{\partial \dot{\underline{P}}}{\partial \underline{\varepsilon}} \right) = \frac{\partial \underline{V}}{\partial \underline{\varepsilon}} \tag{41}$$

such as (for example):

$$\left(\frac{\partial \dot{X}}{\partial \psi} \right) = \frac{\partial V_X}{\partial \psi}$$

while the derivative in time of $\frac{\partial \underline{\underline{V}}}{\partial \underline{\varepsilon}}$ is:

$$\left(\frac{\partial \dot{\underline{\underline{V}}}}{\partial \underline{\varepsilon}} \right) = \frac{\partial \dot{\underline{\underline{V}}}}{\partial \underline{\varepsilon}} \Bigg|_{\text{Evaluated along the nominal path}} \tag{42}$$

where $\dot{\underline{V}}$ is the acceleration vector in the aircraft inertial frame defined by Equations (36-38), and $\underline{\varepsilon}$ is the vector representing the misalignment angles (Eq. (3)).

Using Equations (7), (32), (38), (42) and the chain rule of partial derivatives:

$$\frac{\partial \dot{\underline{V}}}{\partial \underline{\varepsilon}} = \frac{\partial}{\partial \underline{\varepsilon}} \left(C_M^A \cdot \underline{SF} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \right)$$

$$= \frac{\partial C_M^A}{\partial \underline{\varepsilon}} \cdot \underline{SF} + C_M^A \frac{\partial \underline{\underline{SF}}}{\partial \underline{\varepsilon}} \tag{43}$$

Using Equation (7), the first term of Equation (43) reduces to: (See Appendix B for detailed derivation)

$$\frac{\partial C_M^A}{\partial \underline{\varepsilon}} \cdot \underline{SF} = \begin{bmatrix} -SF_2 & SF_3 & 0 \\ SF_1 & 0 & -SF_3 \\ 0 & -SF_1 & SF_2 \end{bmatrix} \qquad (44)$$

where SF_1, SF_2, and SF_3 are the three components of the specific force vector $\underline{SF}$ (defined in the missile inertial frame). From Equations (15-18, 25) it is seen that $\underline{SF}$ is a function of the missile velocity in the missile inertial frame ($\underline{V}^M$) which is in turn a function of the missile velocity in the aircraft inertial frame ($\underline{V}^A$) and of the misalignment vector ($\underline{\varepsilon}$). The second term of Equation (43) expands then to:

$$\frac{\partial \underline{SF}}{\partial \underline{\varepsilon}} = \frac{\partial \underline{SF}}{\partial \underline{V}^M} \frac{\partial \underline{V}^M}{\partial \underline{V}^A} \frac{\partial \underline{V}^A}{\partial \underline{\varepsilon}} + \frac{\partial \underline{SF}}{\partial \underline{V}^M} \frac{\partial \underline{V}^M}{\partial \underline{\varepsilon}} \qquad (45)$$

Equation (43) expands then to the following expression (for detailed derivation see Appendix B):

$$\frac{\partial \dot{\underline{V}}}{\partial \underline{\varepsilon}} = \frac{\partial C_M^A}{\partial \underline{\varepsilon}} \underline{SF} + C_M^A \left(\frac{\partial \underline{SF}}{\partial \underline{V}^M} \frac{\partial \underline{V}^M}{\partial \underline{V}^A} \frac{\partial \underline{V}}{\partial \underline{\varepsilon}} + \frac{\partial \underline{SF}}{\partial \underline{V}^M} \frac{\partial \underline{V}^M}{\partial \underline{\varepsilon}} \right) . \qquad (46)$$

Output Sensitivities

Looking upon the seven measured values -- four radar measurements and three specific forces, as outputs of the system, it is possible in a way similar to the one used in the previous section to define the output sensitivities ($\frac{\partial \underline{y}}{\partial \underline{\varepsilon}}$) as:

$$\frac{\partial \underline{y}}{\partial \underline{\varepsilon}} = \begin{bmatrix} \frac{\partial R}{\partial \psi} & \frac{\partial R}{\partial \theta} & \frac{\partial R}{\partial \phi} \\ \frac{\partial \dot{R}}{\partial \psi} & \frac{\partial \dot{R}}{\partial \theta} & \frac{\partial \dot{R}}{\partial \phi} \\ \frac{\partial \mathrm{Sin}\psi_R}{\partial \psi} & \frac{\partial \mathrm{Sin}\Psi_R}{\partial \theta} & \frac{\partial \mathrm{Sin}\psi_R}{\partial \phi} \\ \frac{\partial \mathrm{Sin}\theta_R}{\partial \psi} & \frac{\partial \mathrm{Sin}\theta_R}{\partial \theta} & \frac{\partial \mathrm{Sin}\theta_R}{\partial \phi} \\ \frac{\partial SF_1}{\partial \psi} & \frac{\partial SF_1}{\partial \theta} & \frac{\partial SF_1}{\partial \phi} \\ \frac{\partial SF_2}{\partial \psi} & \frac{\partial SF_2}{\partial \theta} & \frac{\partial SF_2}{\partial \phi} \\ \frac{\partial SF_3}{\partial \psi} & \frac{\partial SF_3}{\partial \theta} & \frac{\partial SF_3}{\partial \phi} \end{bmatrix} \qquad (47)$$

Using the chain rule of partial derivatives, the elements of $\frac{\partial \underline{y}}{\partial \underline{\varepsilon}}$ are derived as follows -- using Equations (19-23), (25):

$$\frac{\partial R}{\partial \underline{\varepsilon}} = \left[\frac{X}{R} \quad \frac{Y}{R} \quad \frac{Z}{R} \right] \cdot \frac{\partial \underline{P}}{\partial \underline{\varepsilon}} \tag{48}$$

$$\frac{\partial(\dot{R})}{\partial \underline{\varepsilon}} = \begin{bmatrix} \frac{V_X}{R} - \frac{X}{R^3}(X \cdot V_X + Y \cdot V_Y + Z \cdot V_Z) \\ \frac{V_Y}{R} - \frac{Y}{R^3}(X \cdot V_X + Y \cdot V_Y + Z \cdot V_Z) \\ \frac{V_Z}{R} - \frac{Z}{R^3}(X \cdot V_X + Y \cdot V_Y + Z \cdot V_Z) \end{bmatrix} \cdot \frac{\partial \underline{P}}{\partial \underline{\varepsilon}} \tag{49}$$

$$+ \left[\frac{X}{R} \quad \frac{Y}{R} \quad \frac{Z}{R} \right] \cdot \frac{\partial \underline{V}}{\partial \underline{\varepsilon}}$$

$$\frac{\partial(\mathrm{Sin}\psi_R)}{\partial \underline{\varepsilon}} = \left[\frac{-XY}{(X^2 + Y^2)^{3/2}} \quad \frac{X^2}{(X^2 + Y^2)^{3/2}} \quad 0 \right] \cdot \frac{\partial \underline{P}}{\partial \underline{\varepsilon}} \tag{50}$$

$$\frac{\partial(\mathrm{Sin}\theta_R)}{\partial \underline{\varepsilon}} = \left[\frac{-XZ}{R^3} \quad \frac{-YZ}{R^3} \quad \frac{X^2 + Y^2}{R^3} \right] \cdot \frac{\partial \underline{P}}{\partial \underline{\varepsilon}} \tag{51}$$

This defines the four radar measurements sensitivities, detailed derivation can be found in Appendix B). The specific forces sensitivities were already defined in the previous section -- Equation (45):

$$\frac{\partial \underline{\underline{SF}}}{\partial \underline{\varepsilon}} = \frac{\partial \underline{\underline{SF}}}{\partial \underline{V}^M} \cdot \frac{\partial \underline{V}^M}{\partial \underline{V}^A} \cdot \frac{\partial \underline{V}}{\partial \underline{\varepsilon}} + \frac{\partial \underline{\underline{SF}}}{\partial \underline{V}^M} \cdot \frac{\partial \underline{V}^M}{\partial \underline{\varepsilon}} \tag{45}$$

Model 1

As stated in the introduction to this thesis, Model 1 disregards the fact that the acceleration commands, which are the source of the specific force vector ($\underline{SF}$), are generated in the aircraft inertial frame and implemented in the missile inertial frame. The significance of this statement is that $\underline{SF}$ is not recognized to be a function of the misalignment angle and so Equation (45) is set identically to zero.

Equation (43) reduces then for Model 1 to (using Equation (44)):

$$\frac{\partial \dot{\underline{V}}}{\partial \underline{\varepsilon}} = \frac{\partial C_M^A}{\partial \underline{\varepsilon}} \cdot \underline{SF}$$

$$= \begin{bmatrix} -SF_2 & SF_3 & 0 \\ SF_1 & 0 & -SF_3 \\ 0 & -SF_1 & SF_2 \end{bmatrix} \tag{52}$$

Since there is a nominal state trajectory that remains unperturbed, small scale sensitivities are used here. SF_1, SF_2, and SF_3 are the specific force measurements received directly from the missile.

Model 1 uses the three measured specific forces as inputs for the generation of its trajectory. Hence, they are not treated as measurements of the states as it will be done in Model 2. As shown in Appendix A, the difference between the "true" measurement vector and the one generated from the model trajectory, plays an important role in the determination of the gradient vector. In the case of Model 1, such a difference would not exist for the three components of the measurement vector constituting the three specific forces. Therefore, for the case of Model 1, the dimension of the measurement vectors, "true" and modelled, is reduced to four and includes only the radar measurement.

The calculations of the information matrix and of the gradient vector as described in Appendix A include only the measurement noise covariance R. This is correct when it is assumed that there is no plant noise in the system, or that it is at least negligible. Model 1 uses the specific forces received through the aircraft's communication system as inputs to the generation of the estimated trajectory. As stated in Chapter II, Gaussian white noise is added to the measurements before they are transmitted to the aircraft. This essentially means that the assumption about the zero plant noise is, in fact, not valid for Model 1. This study, nevertheless uses the assumption of zero plant noise. Since the comparison of the information matrices between Model 1 and Model 2 is seeked, they should be compared on a direct basis (both without plant noise). However, if one were to build the estimator using the Model 1 approach, he would have to include plant noise in the maximum likelihood estimator.

Model 2

As stated in the introduction to this thesis, Model 2 is an upgraded model as compared to Model 1, in that it does account for the specific forces measured by the missile accelerometers, being a function of the misalignment angles. The difference between Model 1 and Model 2 lies then in the second term of Equation (43) which appears only in the case of Model 2.

Model 2 does not use the specific forces of the missile transmitted to the aircraft as inputs to the generation of the expected trajectory (as Model 1 does). As mentioned in the introduction, since the characteristics of the missile are known to the aircraft's computer, expected specific forces can be generated (using the nominal expected misalignment angles) and used as inputs to the generation of the expected trajectory. Those specific forces are then compared to the one transmitted from the missile in the process of building the gradient vector needed for the estimation of the misalignment angles. The output (measurement) vectors, "true" and estimated, for Model 2, have then a dimension of seven. On top of the four radar measurements, the three specific forces are now added to the measurement vector. In contrast to what was said in the case of Model 1, the assumption of zero plant noise for Model 2 is correct.

It can be seen that Model 2 is indeed using an inherent source of information (at the price of additional complexity). As stated in the introduction, the main thrust of this study is to determine if the additional complexity is worth the degree of information added to the model. It should also be noted that the predicted states in Model 2 are a far more non-linear function of the error angles $\underline{\varepsilon}$ than they are in Model 1. Thus, in going from Model 1 to Model 2, plant noise has been traded for significant non-linearities. Both, as it will be seen, can cause considerable difficulties in the prediction and estimation capabilities.

Information Matrix

The first sections of this chapter have derived what is called in the literature a "sensitivity system" (Refs 6, 14, 15). The states of this system are the position and velocity sensitivities ($\frac{\partial \underline{\underline{P}}}{\partial \underline{\varepsilon}}$, $\frac{\partial \underline{\underline{V}}}{\partial \underline{\varepsilon}}$) and the outputs are the output sensitivities:

$$\left(\frac{\partial R}{\partial \underline{\varepsilon}}, \frac{\partial \dot{R}}{\partial \underline{\varepsilon}}, \frac{\partial (\mathrm{Sin}\psi_R)}{\partial \underline{\varepsilon}}, \frac{\partial (\mathrm{Sin}\theta_R)}{\partial \underline{\varepsilon}}, \frac{\partial SF_1}{\partial \underline{\varepsilon}}, \frac{\partial SF_2}{\partial \underline{\varepsilon}}, \text{ and } \frac{\partial SF_3}{\partial \underline{\varepsilon}} \right)$$

Once the output sensitivities have been evaluated, the information matrix can be derived. The information matrix is in fact (Appendix A):

$$M = \sum_{n=1}^{N} \left(\frac{\partial \underline{y}}{\partial \underline{\varepsilon}} \right)^T \cdot R^{-1} \cdot \left(\frac{\partial \underline{y}}{\partial \underline{\varepsilon}} \right) \tag{53}$$

Using the output sensitivities and the difference between the "true" output vector versus the estimated one, the gradient vector is evaluated:

$$H = \sum_{n=1}^{N} \left(\frac{\partial \underline{y}}{\partial \underline{\varepsilon}} \right)^T \cdot R^{-1} \cdot (\underline{y}_{true} - \underline{y}_{est.}) \tag{54}$$

The role played by the output sensitivities in the building of the information matrix, and the gradient vector, and in the estimation process is clearly important. Sensitivities can be analogous to signal strength and the information matrix --to signal to noise ratio. Then by making sensitivities large, the signal to noise ratio can be increased thereby enhancing the estimation capability. The next step is to evaluate the correction in the estimation of the misalignment angles. As stated in Appendix A, this correction is:

$$\delta\underline{\varepsilon} = M^{-1}H \tag{55}$$

Performance Criteria

In view of the prime objective of this study -- the comparison of the information matrices for Model 1 and Model 2, this comparison is by itself a performance criterion. The comparison is based on the trace of the "dispersion matrix" defined as the inverse of the information matrix:

$$tr(d) = tr\ (M^{-1}) \tag{56}$$

In view of the secondary objective -- the estimation of the misalignment angles, the quality of this estimation (how close do the estimated angles get to the true angles), is another performance criterion.

As shown in Appendix A (Eq. (A-10)), the estimation process includes in fact a linearization about a nominal trajectory (defined by a nominal estimated misalignment angle vector). Therefore, generally speaking, the estimation process should be iterative and hopefully convergent. Each iteration uses the measurements of the full tracking period. The estimation is then reprocessed about the corrected nominal path so that the estimated misalignment angles vector gets as close as possible to the "true" vector. Three iterations were made. In the case of Model 1, the specific forces transmitted from the missile (and which are the source for the generation of the estimated trajectory) are assumed not to be functions of the misalignment angles. Disregarding gravity, Model 1 trajectory is described by:

$$\begin{bmatrix} \dot{\underline{P}} \\ \dot{\underline{V}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{P} \\ \underline{V} \end{bmatrix} + \begin{bmatrix} 0 \\ \underline{\varepsilon} \end{bmatrix} \underline{SF} \tag{57}$$

where $\underline{SF}$ is assumed not to be a function of $\underline{\varepsilon}$, I is the identity matrix, $\underline{P}$ is the position vector and $\underline{V}$ is the velocity vector.

It is then clear that the system can in fact be reformulated such that the misalignment angles are states (modelled as random biases) in the augmented system ($\underline{W}$ is the pseudo-noise which would be added if an Extended Kalman Filter was to be used):

$$\begin{bmatrix} \dot{\underline{P}} \\ \dot{\underline{V}} \\ \dot{\underline{\varepsilon}} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & \underline{SF} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{P} \\ \underline{V} \\ \underline{\varepsilon} \end{bmatrix} + \underline{W} \tag{58}$$

Presuming that the $\underline{P}$ vector is directly measurable (i.e. there exists a one to one correspondence between the position vector and the radar measurement vector) then it can be shown that the position covariance can be calculated from the Kalman Filter covariance propagation equation so that the estimation process would need just one iteration.

This, of course, is not true for Model 2 for which the dependence of the specific force vector on the misalignment is expressly exploited as a source of further information.

IV. Results and Conclusions

Introduction

As stated in the introduction to this study and in Appendix A, the maximum likelihood estimation is based on the evaluation of state and output sensitivities with respect to unknown parameters -- the misalignment angles -- while the missile follows a nominal trajectory. This nominal trajectory is a function of the parameters, the type of control applied to the missile, and time (Eq. (A-1)).

The complete set of numerical results is gathered in Appendix C. Tables C-I to C-VI describe the trajectories for the Truth Model. Tables C-I to C-III show the nominal path for the arbitrary set of misalignment angles:

$$\psi = 1 \text{ mrad}$$
$$\theta = 2 \text{ mrad}$$
$$\phi = 3 \text{ mrad}$$

but with three different types of control. Likewise, Tables C-IV to C-VI show the nominal path for a different set of arbitrary misalignment angles:

$$\psi = 5 \text{ mrad}$$
$$\theta = 10 \text{ mrad}$$
$$\phi = 15 \text{ mrad}$$

and again, with three types of control.

The two angles β_c and δ_c are transmitted from the aircraft to the missile. Using them, the aircraft in fact controls the missile. As described in the section of Chapter II dealing with the controller, β_c is the bank angle command while δ_c is the control surface deflection command. The control characterized by $K_D = 1$ is described by:

$$\beta_c = K_1 \cdot t \qquad (26)$$

$$\delta_c = K_3 + K_4 \cdot \mathrm{Sin}(\omega_2 \cdot t) \qquad (27)$$

Specific values for the various constants are given in Chapter II. For the second type of control ($K_D = 2$) the commanded angles are:

$$\beta_c = K_1 \cdot t + K_2 \cdot \mathrm{Sin}(\omega_1 \cdot t) \qquad (28)$$

$$\delta_c = K_3 \qquad (29)$$

Finally, the third type of control is described by:

$$\beta_c = K_1 \cdot t + K_2 \cdot \mathrm{Sin}(\omega_1 \cdot t) \tag{30}$$

$$\delta_c = K_3 + K_4 \cdot \mathrm{Sin}(\omega_2 \cdot t) \tag{31}$$

In all cases, the simulation starts when the relative position of the missile with respect to the aircraft is:

$$X(0) = 1000 \text{ ft}$$
$$Y(0) = 1000 \text{ ft}$$
$$Z(0) = 100 \text{ ft}$$

The range (R) measured by the radar is then (without noise):

$$R(0) = 1417.7 \text{ ft}$$

The relative velocity of the missile with respect to the aircraft at the beginning of all simulations is:

$$V_X(0) = 2000 \text{ ft/sec}$$
$$V_Y(0) = 1500 \text{ ft/sec}$$
$$V_Z(0) = 10 \text{ ft/sec}$$

The range-rate ($\dot{R}$) measured by the radar is then (without noise):

$$\dot{R}(0) = 2469.4 \text{ ft/sec}$$

At the beginning, before any commands are transmitted to the missile, its accelerometers are assumed to be measuring no specific force. Tables C-I to C-VI show the missile relative position and velocity, its range and range-rate and its specific forces every 10 seconds for the 50 seconds during which the missile is tracked. It is seen that the range between the aircraft and the missile at the end of the 50 seconds is about 22 nautical miles. This represents a conservative range at which most modern airborne fire control radars would still be able to beacon track a target.

As stated in Appendix A, the trace of the dispersion matrix (which is the inverse of the information matrix) was chosen as the primary measure of merit in comparing the two different modelling techniques. Additionally, using the dispersion matrix and the gradient vector, a new estimate of the misalignment angles vector is computed and printed together with the trace of the dispersion matrix every 10 seconds.

Using Equation (A-10), it is possible to express each of the output differences (true measured output minus estimated output) as the sum of three terms:

$$\Delta y = \frac{\partial y}{\partial \underline{\varepsilon}} \Delta \underline{\varepsilon} + \eta + \text{HOT}(y) \qquad (59)$$

where Δy denotes the output difference, $\frac{\partial y}{\partial \underline{\varepsilon}}$ -- the sensitivity of this particular output to the misalignment angles vector $\underline{\varepsilon}$, $\Delta\underline{\varepsilon}$ -- the actual misalignment (for the current iteration), η -- the measurement noise, and HOT(y) -- the higher order terms in the Taylor's series expansion used for the linearization process. It is important to note that in the "noisy" cases the HOT terms and the noise are lumped together. Therefore, in order to appraise the quality of the linearization (how much is neglected in the higher order terms) one has to look at the results generated for the cases without noise. In those cases, an examination of the HOT terms gives a direct evaluation of the local accuracy of the linearization process. Indeed, if the HOT term becomes a significant percentage of the first order sensitivity term, it would be expected to have a poor estimation capability (in one iteration) in spite of the fact that the dispersion matrix might be very small at that point. Therefore, the dispersion matrix alone will not supply the entire basis of comparison of the two methods as these other indicators must also be considered.

Model 1 Results

The results of the simulation using Model 1 are shown in Tables C-VII to C-XXXIII. In the case of Model 1, the output vector consists of the range between the aircraft and the missile, the range rate, the sine of the azimuth angle, and the sine of the elevation angle. The difference between the estimated and actual output values are calculated and printed every 10 seconds. Next to those differences, the appropriate HOT terms are also printed.

Comparing the trace of the dispersion matrix convergence between the various iterations and with any type of control it is seen that the trace indeed gets smaller as the amount of information increases with time. Table III shows as an example the case without noise and with $K_D = 3$, for the first and third iteration.

Table III

Model 1 -- Decrease of the Trace of the Dispersion Matrix (First and Third Iteration)

Time (sec)	Trace of Dispersion Matrix First Iteration	Third Iteration
10.	$.186 \cdot 10^{-1}$	$.186 \cdot 10^{-1}$
20.	$.188 \cdot 10^{-3}$	$.189 \cdot 10^{-3}$
30.	$.942 \cdot 10^{-5}$	$.942 \cdot 10^{-5}$
40.	$.210 \cdot 10^{-5}$	$.208 \cdot 10^{-5}$
50.	$.750 \cdot 10^{-6}$	$.736 \cdot 10^{-6}$

In the case of Model 1, the system is in fact linear -- augmenting the system by looking at the misalignments as states (Eqs. (56) and (57)). Therefore, there is no more advantage in redoing the estimation in more than one iteration. Indeed, comparing Tables C-VII to C-XXXIII between iterations, it is seen that the convergence of the trace of the dispersion matrix almost does not change from one iteration to the next (Table III). This is basically due to the fact that the state sensitivities in Model 1 are independent of the nominal parameter values selected (i.e., the state sensitivities do not change when the parameter values change).

The radar measurements are non-linear functions of the relative position and velocity of the missile (as stated in the section of Chapter II dealing with the radar measurements). Therefore, the output differences are expected to decrease from one iteration to the next, and indeed they do (Tables C-VII to C-XV). The higher order terms are seen to be increasing due to the output (and output differences) being non-linear. Table IV shows for example, the values for the range difference and HOT term for the first and second iteration of the case without noise and with $K_D = 3$. In the noisy cases (Tables C-XVI to C-XXIV) the output differences are not seen to be decreasing anymore (from one iteration to the next). The reason is, of course, the noise which is now added to the output and which can be seen exclusively at the initial time for each output difference (Table V).

Table IV

Model 1 -- Range Difference and HOT

Iteration #	Time (sec)	DELTA(R) (ft)	HOT(R) (ft)
1	10	-.18	$.47 \cdot 10^{-2}$
	20	.39	$.41 \cdot 10^{-1}$
	30	-.67	.12
	40	-.64	.23
	50	-.21	.37
2	10	$-.31 \cdot 10^{-2}$	$.79 \cdot 10^{-7}$
	20	$-.42 \cdot 10^{-1}$	$.78 \cdot 10^{-6}$
	30	-.12	$.24 \cdot 10^{-5}$
	40	-.23	$.52 \cdot 10^{-5}$
	50	-.37	$.90 \cdot 10^{-5}$

Table V

Measurement Noise

Output	Variance	Actual Added Noise
Range (ft)	10	11.25
Range Rate (ft/sec)	10	9.21
Azimuth (mrad)	1	1.56
Elevation (mrad)	1	.90

For the same reason, the higher order terms do not appear to be lower than the output differences anymore.

The errors made in estimating the misalignment angles are seen to decrease remarkably (by 4 to 6 orders of magnitude) from one iteration to the other in the no-noise cases. Table VI shows the evolution of this error for the estimation of the misalignment angle ψ after 50 seconds for a no-noise case (with $K_D = 3$).

Table VI

Model 1 -- ψ Estimation Error (No Noise)

Iteration #1	Iteration #2	Iteration #3
$-.12 \cdot 10^{-1}$	$-.24 \cdot 10^{-6}$	$-.55 \cdot 10^{-9}$

On the other hand, in the noisy cases, the estimation errors seem to stay more or less constant from one iteration to the next. Table VII shows the evolution of this error for the estimation of the misalignment angle ψ for a noisy case (with $K_D = 3$) after 50 seconds.

Table VII

Model 1 -- ψ Estimation Error (With Noise)

Iteration #1	Iteration #2	Iteration #3
$-.16 \cdot 10^{-1}$	$-.11 \cdot 10^{-1}$	$-.11 \cdot 10^{-1}$

The reason that these errors stay essentially constant is that the presence of the noise in the measurements tends to create a bias in the estimation error. This result was, in fact, expected out of a single run of a "Monte Carlo simulation" as conducted in this study (even though the additive measurement noise was specified as zero mean).

The ability of the estimator to estimate the misalignment angles was checked for the angles 1, 2, and 3 milliradians. Tables C-XXV to C-XXXIII include the results for the case in which the misalignment angles are 5, 10, and 15 milliradians. Table VIII ($K_D = 3$, with noise) shows that the performance of the estimator was not degraded in this case. No attempt has been made to check for the maximum misalignment angles which can still be estimated adequately using this small angle approximation model.

Table VIII

Model 1 Estimation Ability

Misalignment Angles			Estimation Error		
ψ	θ (mrad)	ϕ	ψ	θ (μrad)	ϕ
1	2	3	-11.	-21.	-47.
5	10	15	-10.	-21.	-48.

<u>Model 2 Results</u>

The results of the simulation using Model 2 are included in Tables C-XXXIV to C-LX. In the case of Model 2, the output vector consists of the four radar tracking components included in Model 1 as well as the three specific forces which are now regarded as measurements to be compared to the predicted values of the specific forces.

Comparing the trace of the dispersion matrix between the various iterations and with any type of control, it is seen that the trace indeed gets smaller as the amount of information increases with time. On the other hand, the trace stays essentially the same from one iteration to the next (Table IX).

Table IX

Model 2 -- Decrease of the Trace of the Dispersion Matrix (First and Third Iteration)

Time (sec)	Trace of Dispersion Matrix First Iteration	Third Iteration
10.	$.46 \cdot 10^{-3}$	$.47 \cdot 10^{-3}$
20.	$.72 \cdot 10^{-4}$	$.68 \cdot 10^{-4}$
30.	$.44 \cdot 10^{-4}$	$.42 \cdot 10^{-4}$
40.	$.96 \cdot 10^{-5}$	$.90 \cdot 10^{-5}$
50.	$.46 \cdot 10^{-5}$	$.43 \cdot 10^{-5}$

Comparison of the estimation errors show that they seem to increase with time within each iteration. The reason for this is in the non-linearities which are inherent to Model 2. The error made in neglecting the higher order terms in the process of linearization, builds up in time and deteriorate the validity of the linearized model. The closer the estimated misalignment angles are from the true ones, the longer the linearization stays valid and the error in the estimate decreases. Table C-XL-b (first iteration), for example, shows a minimum estimation error occurring after 10 seconds while Table C-XLII-b (third iteration) shows a minimum estimation error occurring after 20 seconds.

As in Model 1, the output differences are non-linear functions of the relative position and velocity of the missile. The higher order terms therefore increase accordingly; they are still smaller than the appropriate output difference after 10 seconds on the first iteration but later on they increase. As the non-linearity decreases in significance on the second iteration (the nominal misalignment being closer to the "true" one), they are now seen to stay below the output differences during all the tracking period. Table X shows, for example, the values for the range difference and HOT term for the first and second iteration of the case without noise and with $K_D = 3$. It is seen that on the first iteration after 10 seconds the HOT term is still smaller than the output difference by two orders of magnitude but later it becomes bigger and closer to the range difference. On the second iteration the HOT term remains smaller than the output difference than it was in the first iteration.

As in Model 1, here too, the additive measurement noise masks any decrease in the output differences seen before

Table X

Model 2 -- Range Difference and HOT

Iteration #	Time (sec)	DELTA(R) (ft)	HOT(R) (ft)
1	10	$-.11$	$-.95\cdot10^{-3}$
	20	$-.34$	$.42\cdot10^{-1}$
	30	$-.17\cdot10^{1}$	$.30$
	40	$-.36\cdot10^{1}$	$.15\cdot10^{1}$
	50	$-.63\cdot10^{1}$	$.51\cdot10^{1}$
2	10	$.89\cdot10^{-2}$	$.15\cdot10^{-2}$
	20	$-.37\cdot10^{-1}$	$-.41\cdot10^{-2}$
	30	$-.28$	$-.39\cdot10^{-1}$
	40	$-.17\cdot10^{1}$	$-.22$
	50	$-.56\cdot10^{1}$	$-.88$

from one iteration to the next. Tables C-XLIII to C-LX include the results for the noisy cases. Table V which shows the additive measurement noise as it appears at the initial time of each iteration, applies to Model 2 too. This is because the same pseudo-random sequence is used for the generation of the additive noise in both models.

The ability of the algorithm to estimate misalignment angles was checked for the angles 1, 2, and 3 milliradians. The results are included in Tables C-XXXIV to C-LI. The errors made in estimating the misalignment angles are seen to decrease from one iteration to the next in the no-noise cases. Table XI shows the evolution of this error for the estimation of the misalignment angle ψ after 50 seconds for a no-noise case (with $K_D = 3$).

Table XI

Model 2 -- ψ Estimation Error (No Noise)

Iteration #1	Iteration #2	Iteration #3
-.63	.14	$-.31 \cdot 10^{-1}$

On the other hand, in the noisy cases, the estimating errors seem to stay essentially constant from one iteration to the next. Table XII shows the evolution of this error for the estimation of the misalignment angle ψ for a noisy case (with $K_D = 3$) after 50 seconds.

Table XII

Model 2 -- ψ Estimation Error (With Noise)

Iteration #1	Iteration #2	Iteration #3
$.85 \cdot 10^{-1}$	$-.19 \cdot 10^{-1}$	$-.14 \cdot 10^{-1}$

It is seen that the errors stay more or less constant and the reason for it is as stated in the case of Model 1 -- the additive noise.

Tables C-LII to LX include the results for the case in which the misalignment angles are 5, 10, and 15 milliradians. Table XIII ($K_D = 3$, with noise) shows that the performance of the estimator was not degraded in this case. No attempt has been made to check for the maximum misalignment angles which can still be estimated adequately using this small angle approximation model.

Table XIII

Model 2 Estimation Ability

Misalignment Angles			Estimation Error		
ψ	θ (mrad)	ϕ	ψ	θ (μrad)	ϕ
1	2	3	-14	-24	-28
5	10	15	-14	-25	-30

<u>Models Comparison</u>

Model 1 is definitely simpler in its concept than Model 2. Model 2 was expected to perform better in the misalignment angles estimation since it inherently tried to incorporate more apriori knowledge in the estimation -- i.e., it had more information content. Comparing the performance of Model 1 and Model 2 for similar cases (for example, K_D = 3 with noise) it is seen that the two different models estimate the misalignment angles with errors as given in Table XIV.

Comparing the "information content" of Model 1 and 2, it is seen that Model 2 (as expected) draws more information from the measurements. This is observed from the comparison

Table XIV

Estimation Error Comparison

	Model 1 Error (μrad)			Model 2 Error (μrad)		
	ψ	θ	ϕ	ψ	θ	ϕ
Iteration #1	-16.	-20.	-46.	85.	-170.	-140.
Iteration #3	-11.	-21.	-46.	-14.	- 24.	- 28.

of the traces of the dispersion matrices as shown in Table XV (K_D = 3 with noise).

Table XV

Trace of Dispersion Matrix Comparison

Time (sec)	Model 1-Trace of Dispersion Matrix	Model 2-Trace of Dispersion Matrix
10	$.19 \cdot 10^{-2}$	$.17 \cdot 10^{-4}$
20	$.29 \cdot 10^{-4}$	$.14 \cdot 10^{-5}$
30	$.33 \cdot 10^{-5}$	$.38 \cdot 10^{-6}$
40	$.64 \cdot 10^{-6}$	$.17 \cdot 10^{-6}$
50	$.21 \cdot 10^{-6}$	$.11 \cdot 10^{-6}$

Conclusions

In regard to the prime objective of this study -- the comparison of the information content of the two models, it is clear that Model 2 starts indeed with a higher level of information than Model 1 (see the traces of the dispersion matrix at 10 sec. in Table XV), but they both tend to approximately equal information after 50 seconds of tracking.

In regard to the estimation objective, Model 1 appears to be much more advantageous than Model 2. It is not only simpler (uses only 65 seconds of the CDC 6600 central processor time to be executed in the noisy case for $K_D = 3$, versus 145 seconds for the case of Model 2), but it also does not require many iterations to take care of inaccuracies caused by the linearization process.

The HOT terms are in fact an indicator of the quality of the linearization process which turns out to be much better in the case of Model 1 than in Model 2 (this is seen from general trends and orders of magnitude in Tables IV and X). The final results of the estimation are also very comparable between Model 1 and Model 2 (Table XIV). Therefore, it can be said that in the process of parameters estimation, it is not enough to look at the trace of the dispersion matrix and one has also to be concerned about the quality of the linearization.

Finally, it has been seen that to the limited extent at which it was evaluated, neither the size of the initial misalignment angles nor the various input control had any major influence on the results.

Suggested Future Study

This study can really be characterized as a feasibility study meant to investigate the amount of information included in each one of the two methods checked. Expanding the feasibility issue, one could investigate, in much greater depth than was done in this study, the broad field of input design applied to this problem. In this way, one could try to optimize the inputs in order to get maximum sensitivities -- or in other words, maximum information -- to optimize the estimation accuracy.

Another prospective subject of investigation is the influence of different initial error on the estimation accuracy as a function of time.

Using a Monte Carlo analysis, one could investigate the difference in the two models between the trace of the dispersion matrix and an error ellipsoid for the misalignment angles variances.

Finally, expanding this study in the direction of implementation, one could use Model 1 for example, and try to design a realistic estimator without assuming zero plant noise (as it was done in this study) and without assuming perfect knowledge of the initial conditions.

Bibliography

1. Ryles, J.C. and R.M. Reeves. "Transfer Alignment for Motion Stabilization of Coherent Radars", NAECON '75, IEEE National Aerospace and Electronics, June 1975.

2. Fitts, John Murray. "Aided Tracking as Applied to High Accuracy Pointing Systems", IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-9, No. 3, May 1973.

3. Pearson, John B., III and E.B. Stead. "Kalman Filter Applications in Airborne Radar Tracking", IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-10, No. 3, May 1974

4. Harris, R.A. and C.D. Wakefield. "Coordinate Alignment for Elastic Bodies", NAECON '77, IEEE National Aerospace and Electronics, June 1977.

5. Price, C.F. and W.D. Koeningsburg. Adaptive Control with Explicit Parameter Identification for Tactical Missiles. TASC TR-170-3. Reading, Massachusetts: The Applied Sciences Corporation, December 1971.

6. Mehra, R.K. "Optimal Inputs for Linear System Identification", IEEE Transactions on Automatic Control, AC-19:192-200, June 1974.

7. Jazwinski, A.H. Stochastic Processes and Filtering Theory. New York:Academic Press, 1970.

8. Maybeck, P.S. Stochastic Models, Estimation and Control. Unpublished Textbook Manuscript, 1975.

9. Wrigley, W., et al. Gyroscopic Theory, Design and Instrumentation. Cambridge, Massachusetts:The M.I.T. Press, 1969.

10. Blakelock, John H. Automatic Control of Aircraft and Missiles. New York:John Wiley and Sons, Inc., April 1965.

11. Britting, Kenneth R. Inertial Navigation Systems Analysis. New York:John Wiley and Sons, Inc., 1971.

12. Price, C.F. and R.S. Warren. Performance Evaluation of Homing Guidance Laws for Tactical Missiles. TASC TR-170-4. Reading, Massachusetts:The Analytic Sciences Corporation, January 1973.

13. Widnall, W.S. and P.A. Grundy. Inertial Navigation System Error Models. TR-03-73. Cambridge, Massachusetts:Intermetrics Inc., May 1973.

14. Gupta, N.K., and W.E. Hall. Input Design for Identification of Aircraft Stability and Control Derivatives", NASA-CR-2493, February 1975.

15. Reid, G.J. Sensitivity Operators and Associated System Concepts for Linear Dynamic Systems. Ph.D. Dissertation. AFAL-TR-76-118. Wright-Patterson Air Force Base, Ohio:Air Force Avionics Laboratory/RWT, July 1976.

16. Sage, A.P. and J.L. Melsa. System Identification. New York:Academic Press, 1971.

17. Luenberger, D.G. Optimization by Vector Space Methods. New York:John Wiley, 1969.

Appendix A

Sensitivity Operators and Maximum Likelihood Parameter Estimation

For many applications of interest, including the case dealt with in this study, various techniques can be used to produce an adequate system description in the form of a linear system model driven by known inputs and white Gaussian noises. Using this linear system model, it is possible to develop an optimal state estimator and/or controller. However, the optimality of these devices is dependent upon complete knowledge of the parameters which defined the best model for system dynamics, output relations, and statistical description of uncertainties. Thus, in order to improve the quality of the state estimates, it would be desirable to estimate the unknown parameters in an online fashion. For the problem considered in this thesis, this would be the three misalignment angles of the platform of the missile. This is often termed combined state estimation and system identification (Ref 8:10-1). The concept of maximum likelihood estimation is used when estimating the parameters without any apriori statistics but assuming that even though they are unknown, the parameters are practically constant (Ref 16:40-42, 7:156-157).

The system can be described by the following vector linear differential equation:

$$\dot{\underline{X}}(t,u) = A(X,b,u)\ \underline{X}(t,u) + B(X,b,u)\ \underline{u}(t) \qquad \text{(A-1)}$$

$$\underline{X}(0) = \underline{d}(b)$$

The measurement vector is described by:

$$\underline{y}(t,b,u) = H(x,b,u)\ \underline{X}(t,u) + \underline{n}(t) \qquad \text{(A-2)}$$

where X(t) is an n- dimensional state vector, $\underline{u}(t)$ is an r-dimensional control input, $\underline{y}(t)$ is an m-dimensional output vector, and b is a p-dimensional real parameter vector which parametrizes the matrices A and B and the initial condition vector $\underline{d}$. It can be seen that the system described here is less than the most general one since it assumes no plant noise so that the only uncertainty present is in the measurements -- $\underline{n}(t)$. The measurement noise is modelled as additive noise.

It is assumed that the matrices A, B, and H are continuously differentiable with respect to the individual components b_i at a nominal value of the parameter vector b_0. Then the state sensitivities may be computed from the so-called "sensitivity system":

$$\dot{\underline{X}}(t,u) = \bar{A}\, \underline{X}(t,u) + \bar{B}\, \underline{u}(t)$$

$$\underline{X}(0) = \underline{d} \qquad \text{(A-3)}$$

The measurement sensitivity vector is:

$$\underline{Y}(t,u) = \bar{H}\, \underline{X}(t, u) \qquad \text{(A-4)}$$

where $\underline{X}(t,u)$ is a $((1 + P) \times n)$-dimensional state sensitivity vector, and $\underline{Y}(t,u)$ is a $((1 + P) \times m)$-dimensional output sensitivity vector. The control input, u, is assumed to be fixed and completely known. Therefore, to emphasize that the control is fixed, the explicit dependency of the system output (measurement vector) upon the control, u, will be deleted. $\hat{b}_0$ denotes an apriori estimate of b and $y(\hat{b}_0)$ denotes the corresponding mathematical model output function. The system output sensitivities $v^{(i)}$, $i = 1, \ldots p$ are defined as the p m-dimensional output sensitivity vectors, and they are assumed to be evaluated along the mathematical model output and to exist and be continuous for all $\hat{b}$.

The estimation criterion adopted is the commonly used output error functional:

$$J_e(\hat{b}) = \int_{ti}^{tf} \{(\underline{y}(t) - \underline{y}(\hat{b},t))^T R^{-1} (\underline{y}(t) - \underline{y}(\hat{b},t))\} dt \quad \text{(A-5)}$$

where $\underline{y}$ is the vector of actual true measurements and R^{-1} is the inverse of the covariance of the zero-mean additive white Gaussian measurement noise:

$$E(\eta(t)\ \eta^T(\tau)) = R^{-1}(t) \quad \delta(t-\tau) \quad \text{(A-6)}$$

where $E(\cdot)$ is the expectation operator, and $\delta(\cdot)$ is the unit sampling function defined by $\delta(t) = 0$ for $t = 0$, and:

$$\int_{-\varepsilon}^{\varepsilon} \delta(t)\ g(t)\ dt = g(0) \quad \text{(A-7)}$$

for any arbitrary function g. The superscript T denotes the transpose of a vector. The best estimate, $\hat{b}_{opt}$, is then the $\hat{b}$ which minimizes $J_e(\hat{b})$.

Since the existence and continuity of the sensitivity operators at $\hat{b}_0$ are assumed, the true system output, $\underline{y}$, may be linearized about the nominal output $\underline{y}(\hat{b}_0)$, through the generalized Taylor's formula as follows:

$$\underline{y} = \underline{y}(\hat{b}_0) + V\Delta b + \eta + \text{H.O.T.}$$

$$= \underline{y}(\hat{b}_0) + \sum_{i=1}^{P} v^{(i)} \Delta b_i + \eta + \text{H.O.T.} \tag{A-8}$$

where Δb is $(b - \hat{b}_0)$, H.O.T. denotes "higher order terms" (assumed negligible), and η is the additive measurement noise. Defining the output error as:

$$\underline{Z} = \underline{y} - \underline{y}(\hat{b}_0) \tag{A-9}$$

the linearized measurement equation is obtained:

$$\underline{Z} = V\Delta b + \eta \tag{A-10}$$

This equation, although quite simple, is of fundamental importance as it clearly demonstrates the importance of the sensitivity operator in the parameter identification problem. To first order, the sensitivity operators tell one how much the system output will be perturbed by small changes in the system parameters. If the sensitivity is high, then, in some sense, one can more accurately estimate the true parameter values (Ref 15:20-32).

The output error functional now becomes:

$$J_R(\Delta b) = \int_{t_i}^{t_f} \left[Z - V \cdot \Delta b\right]^T R^{-1} \left[Z - V \cdot \Delta b\right] dt \qquad \text{(A-11)}$$

It can be shown using the theory of least squares (Ref 17:160) that the unique minimizing solution for the functional defined above is:

$$\Delta b_{opt.} = (V^* R^{-1} V)^{-1} (V^* R^{-1} Z) \qquad \text{(A-12)}$$

where V^* is the V adjoint vector, $(V^* R^{-1} V)$ is the $(p \times p)$ Gram matrix also called the information matrix (M), and $V^* R^{-1} Z$ is the p-dimensional gradient vector (H).

$$M = V^* R^{-1} V = \int_{t_i}^{t_f} \left[v^{(i)}(t)\right]^T R^{-1} \left[v^{(i)}(t)\right] dt \qquad \text{(A-13)}$$

$$H = V^* R^{-1} Z = \int_{t_i}^{t_f} \left[v^{(i)}(t)\right]^T R^{-1} Z(t)\, dt \qquad \text{(A-14)}$$

(The reader who lacks the background in functional analysis should see Ref 15). The information matrix has general importance in estimation theory because the covariance of the estimation error for any unbiased estimator is found to be limited by the Cramer-Rao lower bound which is equal to the inverse of M. This is one of the advantages of maximum likelihood estimation. Another one is the fact that the optimal solution $\Delta b_{opt.}$ is unique (Ref 8:10-1 to 10-7). Since this study deals with a non-linear estimation problem, the maximum likelihood estimation is especially advantageous in that it uses all the information gathered between t_i to t_f to generate the estimates about the nominal path versus linearizing along the path while estimating the parameters (as it is done in the Extended Kalman Filter).

Thus, it is relatively easy to compute the minimizing solution $\Delta b_{opt.}$. The updated parameter estimate is then determined from the equation:

$$\hat{b}_1 = \hat{b}_0 + \Delta b_{opt.} \tag{A-15}$$

The system output may now be linearized about $\hat{b}_1$ and $y(\hat{b}_1)$, and the process of obtaining a new $\Delta b_{opt.}$ repeated.

Because the information matrix is of $p \times p$ dimension, some suitable scalar measure must be chosen as an optimization criterion. From the information theory viewpoint any linear functional of the dispersion matrix (the inverse of the information matrix) is the most appropriate criterion (Ref 15:36-38). As stated previously, the dispersion matrix is the lower bound for the covariance matrix of the

Appendix B

Detailed Derivation

Derivatives for the Velocity Sensitivity

The sensitivity states were defined in Chapter III. As stated in Equation (41), the derivatives needed for the integration of the position sensitivities are the velocity sensitivities. The algorithm for the evaluation of the derivatives for the integration of the velocity sensitivities is given in Equation (43).

The transformation matrix (C_M^A) from the missile inertial frame to the aircraft inertial frame is a three-by-three matrix defined in Equation (7). Its derivative with respect to the misalignment vector $\underline{\varepsilon}$ defined in Equation (3) is therefore a tensor. Deriving C_M^A with respect to each of the components of $\underline{\varepsilon}$ and multiplying the results by the specific force vector $\underline{SF}$:

$$C_M^A = \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \tag{B-1}$$

$$\frac{\partial C_M^A}{\partial \psi} \cdot \underline{SF} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} SF_1 \\ SF_2 \\ SF_3 \end{bmatrix} = \begin{bmatrix} -SF_2 \\ SF_1 \\ 0 \end{bmatrix} \tag{B-2}$$

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$$\frac{\partial C_M^A}{\partial \theta} \cdot \underline{SF} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} SF_1 \\ SF_2 \\ SF_3 \end{bmatrix} = \begin{bmatrix} SF_3 \\ 0 \\ -SF_1 \end{bmatrix} \qquad \text{(B-3)}$$

$$\frac{\partial C_M^A}{\partial \phi} \cdot \underline{SF} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} SF_1 \\ SF_2 \\ SF_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -SF_3 \\ SF_2 \end{bmatrix} \qquad \text{(B-4)}$$

Combining the three columns of Equations (B-2), (B-3), and (B-4) to a matrix:

$$\frac{\partial C_M^A}{\partial \underline{\varepsilon}} \cdot \underline{SF} = \begin{bmatrix} -SF_2 & SF_3 & 0 \\ SF_1 & 0 & -SF_3 \\ 0 & -SF_1 & SF_2 \end{bmatrix} \qquad \text{(B-5)}$$

This is in fact the expression for the derivatives of the velocity sensitivities as used in Equation (52) for Model 1.

The second term of Equation (43), added in the case of Model 2, involves the derivative of the specific force vector $\underline{SF}$ with respect to the misalignment vector $\underline{\varepsilon}$. The specific force vector is a function of the missile velocity direction with respect to its inertial frame -- Γ_A, Γ_E, and Γ_β (as stated in Equation (25)):

$$\underline{SF} = \Gamma_A \cdot \Gamma_E \cdot \Gamma_\beta \begin{bmatrix} 0 \\ 0 \\ -L \end{bmatrix} \quad \text{(B-6)}$$

The first two of these direction cosine matrices are in turn expressed in terms of the missile velocity components in its inertial frame (Equations (15) - (18)):

$$\Gamma_A = \begin{bmatrix} \dfrac{V_1}{(V_1{}^2 + V_2{}^2)^{\frac{1}{2}}} & \dfrac{-V_2}{(V_1{}^2 + V_2{}^2)^{\frac{1}{2}}} & 0 \\ \dfrac{V_2}{(V_1{}^2 + V_2{}^2)^{\frac{1}{2}}} & \dfrac{V_1}{(V_1{}^2 + V_2{}^2)^{\frac{1}{2}}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(B-7)}$$

$$\Gamma_E = \begin{bmatrix} \dfrac{(V_1{}^2 + V_2{}^2)^{\frac{1}{2}}}{V} & 0 & \dfrac{-V_3}{V} \\ 0 & 1 & 0 \\ \dfrac{V_3}{V} & 0 & \dfrac{(V_1{}^2 + V_2{}^2)^{\frac{1}{2}}}{V} \end{bmatrix} \quad \text{(B-8)}$$

Each one of these velocity terms is in turn a function of the misalignment vector $\underline{\varepsilon}$ and of the velocity components in the aircraft inertial frame (Equation (19)):

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_X + \psi V_Y - \theta V_Z \\ -\psi V_X + V_Y + \phi V_Z \\ \theta V_X - \phi V_Y + V_Z \end{bmatrix} \quad \text{(B-9)}$$

The derivative of the specific force vector with respect to the misalignment vector $\underline{\varepsilon}$ is then (as in Equation (45)):

$$\frac{\partial \underline{\underline{SF}}}{\partial \underline{\varepsilon}} = \frac{\partial \underline{\underline{SF}}}{\partial \underline{V}^M} \cdot \frac{\partial \underline{V}^M}{\partial \underline{V}^A} \cdot \frac{\partial \underline{V}^A}{\partial \underline{\varepsilon}} + \frac{\partial \underline{\underline{SF}}}{\partial \underline{V}^M} \cdot \frac{\partial \underline{V}^M}{\partial \underline{\varepsilon}}$$

$$= \frac{\partial \underline{\underline{SF}}}{\partial \underline{V}^M} \left[\frac{\partial \underline{V}^M}{\partial \underline{V}^A} \cdot \frac{\partial \underline{V}^A}{\partial \underline{\varepsilon}} + \frac{\partial \underline{V}^M}{\partial \underline{\varepsilon}} \right] \quad \text{(B-10)}$$

The first term in the brackets is a Jacobian matrix which can be expressed in terms of the transformation matrix C_M^A. Using Equation (B-9):

$$\frac{\partial \underline{V}^M}{\partial \underline{V}^A} = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} = (C_M^A)^T = C_A^M \quad \text{(B-11)}$$

The second term in the brackets is the velocity sensitivity ($\frac{\partial P}{\partial \underline{\varepsilon}}$) defined in Equation (40). The third term in the brackets can be derived using Equation (B-9):

$$\frac{\partial \underline{V}^M}{\partial \underline{\varepsilon}} = \begin{bmatrix} V_Y & -V_Z & 0 \\ -V_X & 0 & V_Z \\ 0 & V_X & -V_Y \end{bmatrix} \qquad \text{(B-12)}$$

The derivative of the specific force vector ($\underline{SF}$) with respect to $\underline{V}^M$ involves differentiation of the direction cosine matrices Γ_A and Γ_E with respect to $\underline{V}^M$ (tensors). Again, taking the derivatives with respect to each component of $\underline{V}^M$ at a time:

$$\frac{\partial \underline{SF}}{\partial \underline{V}^M} = \left(\frac{\partial \Gamma_A}{\partial \underline{V}^M} \Gamma_E + \Gamma_A \frac{\partial \Gamma_E}{\partial \underline{V}^M} \right) \Gamma_\beta \begin{bmatrix} 0 \\ 0 \\ -L \end{bmatrix} \qquad \text{(B-13)}$$

Calculating the derivatives of the various entries of Γ_A and Γ_E (using Equations (B-7) and (B-8):

$$\frac{\partial \frac{V_1}{(V_1^2+V_2^2)^{1/2}}}{\partial V_1} = \frac{1}{(V_1^2+V_2^2)^{1/2}} - \frac{V_1^2}{(V_1^2+V_2^2)^{3/2}} = \frac{V_2^2}{(V_1^2+V_2^2)^{3/2}}$$

(B-14a)

$$\frac{\partial \dfrac{V_2}{(V_1{}^2+V_2{}^2)^{\frac{1}{2}}}}{\partial V_1} = \frac{-V_1V_2}{(V_1{}^2+V_2{}^2)^{3/2}} \qquad \text{(B-14b)}$$

$$\frac{\partial \dfrac{V_1}{(V_1{}^2+V_2{}^2)^{\frac{1}{2}}}}{\partial V_2} = \frac{-V_1V_2}{(V_1{}^2+V_2{}^2)^{3/2}} \qquad \text{(B-14c)}$$

$$\frac{\partial \dfrac{V_2}{(V_1{}^2+V_2{}^2)^{\frac{1}{2}}}}{\partial V_2} = \frac{1}{(V_1{}^2+V_2{}^2)^{\frac{1}{2}}} - \frac{V_2{}^2}{(V_1{}^2+V_2{}^2)^{3/2}}$$

$$= \frac{V_1{}^2}{(V_1{}^2+V_2{}^2)^{3/2}} \qquad \text{(B-14d)}$$

$$\frac{\partial \dfrac{(V_1{}^2+V_2{}^2)^{\frac{1}{2}}}{(V_1{}^2+V_2{}^2+V_3{}^2)^{\frac{1}{2}}}}{\partial V_1} = \frac{V_1}{V(V_1{}^2+V_2{}^2)^{\frac{1}{2}}} - \frac{V_1(V_1{}^2+V_2{}^2)^{\frac{1}{2}}}{V^3}$$

$$= \frac{V_1V^2-V_1(V_1{}^2+V_2{}^2)}{V^3(V_1{}^2+V_2{}^2)^{\frac{1}{2}}}$$

$$= \frac{V_1V_3{}^2}{V^3(V_1{}^2+V_2{}^2)^{\frac{1}{2}}} \qquad \text{(B-14e)}$$

$$\frac{\partial \dfrac{(V_1^2+V_2^2)^{\frac{1}{2}}}{(V_1^2+V_2^2+V_3^2)^{\frac{1}{2}}}}{\partial V_2} = \frac{V_2 V_3^2}{V^3 (V_1^2+V_2^2)^{\frac{1}{2}}} \tag{B-14f}$$

$$\frac{\partial \dfrac{(V_1^2+V_2^2)^{\frac{1}{2}}}{(V_1^2+V_2^2+V_3^2)^{\frac{1}{2}}}}{\partial V_3} = -\frac{V_3 (V_1^2+V_2^2)^{\frac{1}{2}}}{V^3} \tag{B-14g}$$

$$\frac{\partial \dfrac{V_3}{(V_1^2+V_2^2+V_3^2)^{\frac{1}{2}}}}{\partial V_1} = -\frac{V_1 V_3}{V^3} \tag{B-14h}$$

$$\frac{\partial \dfrac{V_3}{(V_1^2+V_2^2+V_3^2)^{\frac{1}{2}}}}{\partial V_2} = -\frac{V_2 V_3}{V^3} \tag{B-14i}$$

$$\frac{\partial \dfrac{V_3}{(V_1^2+V_2^2+V_3^2)^{\frac{1}{2}}}}{\partial V_3} = \frac{1}{V} - \frac{V_3^2}{V^3} = \frac{V_1^2+V_2^2}{V^3} \tag{B-14j}$$

where

$$V = (V_1^2 + V_2^2 + V_3^2)^{\frac{1}{2}}$$

Using Equations (B-14) in Equation (B-13), the matrix $\frac{\partial \underline{SF}}{\partial \underline{V}^M}$ is built as a Jacobian matrix for the derivatives of the specific force vector with respect to the misalignment vector. Using Equations (B-11), (B-12), and (B-13) in Equation (B-10) and then Equation (B-15) and (B-10) build together the total term for the derivative of velocity sensitivity (as in Equation (46)):

$$\frac{\partial \dot{\underline{V}}}{\partial \underline{\varepsilon}} = \frac{\partial C_M^A}{\partial \underline{\varepsilon}} \cdot \underline{SF} + C_M^A \left(\frac{\partial \underline{SF}}{\partial \underline{V}^M} \left(\frac{\partial \underline{V}^M}{\partial \underline{V}^A} \cdot \frac{\partial \underline{V}}{\partial \underline{\varepsilon}} + \frac{\partial \underline{V}^M}{\partial \underline{V}^A} \right) \right) \qquad \text{(B-15)}$$

Derivation of the Output Sensitivities

The output (measurement) vector consists of seven components, the first four of which are radar measurements, and the other three are specific force measurements. The sensitivity of the specific force measurements to the misalignment angles vector $\underline{\varepsilon}$ is derived in the previous section of this appendix and given in Equation (B-10). The radar measurements sensitivities are stated in Equations (48) to (51) and will be derived in this section.

The four radar measurements are the range, the range rate, and the line of sight azimuth and elevation angles. Their expression as a function of the position and velocity states of the missile in the aircraft inertial frame is given in Equations (20) to (23) (repeated here for completeness):

$$R = (X^2 + Y^2 + Z^2)^{\frac{1}{2}} \qquad \text{(B-16)}$$

$$\dot{R} = V_X\frac{X}{R} + V_Y\frac{Y}{R} + V_Z\frac{Z}{R} \qquad \text{(B-17)}$$

$$\text{Sin}\psi_R = \frac{Y}{(X^2 + Y^2)^{\frac{1}{2}}} \qquad \text{(B-18)}$$

$$\text{Sin}\theta_R = \frac{Z}{R} \qquad \text{(B-19)}$$

The position and velocity states are functions of the misalignment vector thereby defining the sensitivity states. The radar measurements, being functions of the position and velocity states are therefore functions of the position and velocity sensitivity states. The range sensitivity is:

$$\frac{\partial R}{\partial \psi} = \frac{\partial R}{\partial X}\cdot\frac{\partial X}{\partial \psi} + \frac{\partial R}{\partial Y}\cdot\frac{\partial Y}{\partial \psi} + \frac{\partial R}{\partial Z}\cdot\frac{\partial Z}{\partial \psi} \qquad \text{(B-20a)}$$

$$\frac{\partial R}{\partial \theta} = \frac{\partial R}{\partial X}\cdot\frac{\partial X}{\partial \theta} + \frac{\partial R}{\partial Y}\cdot\frac{\partial Y}{\partial \theta} + \frac{\partial R}{\partial Z}\cdot\frac{\partial Z}{\partial \theta} \qquad \text{(B-20b)}$$

$$\frac{\partial R}{\partial \phi} = \frac{\partial R}{\partial X}\cdot\frac{\partial X}{\partial \phi} + \frac{\partial R}{\partial Y}\cdot\frac{\partial Y}{\partial \phi} + \frac{\partial R}{\partial Z}\cdot\frac{\partial Z}{\partial \phi} \qquad \text{(B-20c)}$$

Using Equation (B-16):

$$\frac{\partial R}{\partial X} = \frac{X}{R} \qquad \text{(B-21a)}$$

$$\frac{\partial R}{\partial Y} = \frac{Y}{R} \qquad \text{(B-21b)}$$

$$\frac{\partial R}{\partial Z} = \frac{Z}{R} \qquad \text{(B-21c)}$$

The derivatives of the position states in Equations (B-20) are recognized to be the position sensitivity states -- $\frac{\partial \underline{P}}{\partial \underline{\varepsilon}}$ (Equation (39)). Substituting Equations (B-21) in Equations (B-20), results in a matrix form of the range sensitivity with respect to the misalignment vector:

$$\frac{\partial R}{\partial \underline{\varepsilon}} = \begin{matrix} \frac{X}{R} & \frac{Y}{R} & \frac{Z}{R} \end{matrix} \cdot \frac{\partial \underline{P}}{\partial \underline{\varepsilon}} \qquad \text{(B-22)}$$

The range rate sensitivity is derived in a similar manner (using Equation (B-17)):

$$\frac{\partial \dot{R}}{\partial \psi} = \frac{\partial \dot{R}}{\partial X}\frac{\partial X}{\partial \psi} + \frac{\partial \dot{R}}{\partial Y}\frac{\partial Y}{\partial \psi} + \frac{\partial \dot{R}}{\partial Z}\frac{\partial Z}{\partial \psi}$$

$$+ \frac{\partial \dot{R}}{\partial V_X}\frac{\partial V_X}{\partial \psi} + \frac{\partial \dot{R}}{\partial V_Y}\frac{\partial V_Y}{\partial \psi} + \frac{\partial \dot{R}}{\partial V_Z}\frac{\partial V_Z}{\partial \psi} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial \psi}$$

$$= \left(\frac{\partial \dot{R}}{\partial X} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial X}\right)\frac{\partial X}{\partial \psi} + \left(\frac{\partial \dot{R}}{\partial Y} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial Y}\right)\frac{\partial Y}{\partial \psi}$$

$$+ \left(\frac{\partial \dot{R}}{\partial Z} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial Z}\right)\frac{\partial Z}{\partial \psi} + \frac{\partial \dot{R}}{\partial V_X}\frac{\partial V_X}{\partial \psi} + \frac{\partial \dot{R}}{\partial V_Y}\frac{\partial V_Y}{\partial \psi} + \frac{\partial \dot{R}}{\partial V_Z}\frac{\partial V_Z}{\partial \psi} \quad \text{(B-23a)}$$

$$\frac{\partial \dot{R}}{\partial \theta} = \left(\frac{\partial \dot{R}}{\partial X} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial X}\right)\frac{\partial X}{\partial \theta} + \left(\frac{\partial \dot{R}}{\partial Y} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial Y}\right)\frac{\partial Y}{\partial \theta}$$

$$+ \left(\frac{\partial \dot{R}}{\partial Z} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial Z}\right)\frac{\partial Z}{\partial \theta} + \frac{\partial \dot{R}}{\partial V_X}\frac{\partial V_X}{\partial \theta} + \frac{\partial \dot{R}}{\partial V_Y}\frac{\partial V_Y}{\partial \theta} + \frac{\partial \dot{R}}{\partial V_Z}\frac{\partial V_Z}{\partial \theta} \quad \text{(B-23b)}$$

$$\frac{\partial \dot{R}}{\partial \phi} = \left(\frac{\partial \dot{R}}{\partial X} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial X}\right)\frac{\partial X}{\partial \phi} + \left(\frac{\partial \dot{R}}{\partial Y} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial Y}\right)\frac{\partial Y}{\partial \phi}$$

$$+ \left(\frac{\partial \dot{R}}{\partial Z} + \frac{\partial \dot{R}}{\partial R}\frac{\partial R}{\partial Z}\right)\frac{\partial Z}{\partial \phi} + \frac{\partial \dot{R}}{\partial V_X}\frac{\partial V_X}{\partial \phi} + \frac{\partial \dot{R}}{\partial V_Y}\frac{\partial V_Y}{\partial \phi} + \frac{\partial \dot{R}}{\partial V_Z}\frac{\partial V_Z}{\partial \phi} \quad \text{(B-23c)}$$

Using Equation (B-17):

$$\frac{\partial \dot{R}}{\partial X} = \frac{V_X}{R} \tag{B-24a}$$

$$\frac{\partial \dot{R}}{\partial Y} = \frac{V_Y}{R} \tag{B-24b}$$

$$\frac{\partial \dot{R}}{\partial Z} = \frac{V_Z}{R} \tag{B-24c}$$

$$\frac{\partial \dot{R}}{\partial V_X} = \frac{X}{R} \tag{B-24d}$$

$$\frac{\partial \dot{R}}{\partial V_Y} = \frac{Y}{R} \tag{B-24e}$$

$$\frac{\partial \dot{R}}{\partial V_Z} = \frac{Z}{R} \tag{B-24f}$$

$$\frac{\partial \dot{R}}{\partial R} = \frac{-(XV_X + YV_Y + ZV_Z)}{R^2} \tag{B-24g}$$

Substituting Equations (B-24) in Equations (B-23):

$$\frac{\partial \dot{R}}{\partial \psi} = \left(\frac{V_X}{R} - \frac{(XV_X + YV_Y + ZV_Z)}{R^2} \cdot \frac{X}{R} \right) \frac{\partial X}{\partial \psi}$$

$$+ \left(\frac{V_Y}{R} - \frac{(XV_X + YV_Y + ZV_Z)}{R^2} \cdot \frac{Y}{R} \right) \frac{\partial Y}{\partial \psi}$$

$$+ \left(\frac{V_Z}{R} - \frac{(XV_X + YV_Y + ZV_Z)}{R^2} \cdot \frac{Z}{R} \right) \frac{\partial Z}{\partial \psi}$$

$$+ \frac{X}{R} \cdot \frac{\partial V_X}{\partial \psi} + \frac{Y}{R} \cdot \frac{\partial V_Y}{\partial \psi} + \frac{Z}{R} \cdot \frac{\partial V_Z}{\partial \psi} \quad \text{(B-25)}$$

The expressions for the derivatives with respect to θ and ϕ can be derived in a similar way. In matrix form, the range rate sensitivity is:

$$\frac{\partial \dot{R}}{\partial \underline{\varepsilon}} = \underline{VR} \cdot \frac{\partial \underline{P}}{\partial \underline{\varepsilon}} + \underline{PR} \cdot \frac{\partial \underline{V}}{\partial \underline{\varepsilon}} \quad \text{(B-26)}$$

where the $\underline{PR}$ row vector is defined as:

$$\underline{PR} = \begin{bmatrix} \frac{X}{R} & \frac{Y}{R} & \frac{Z}{R} \end{bmatrix} \quad \text{(B-27)}$$

and the $\underline{VR}$ row vector is defined as:

$$\underline{VR}_i = \frac{V_i}{R} - \frac{(X \cdot V_X + Y \cdot V_Y + Z \cdot V_Z)}{R^2} \cdot PR_i \qquad \text{(B-28)}$$

where i = X, Y, or Z.

The sensitivity of the sine of the azimuth angle is calculated using Equation (B-18):

$$\frac{\partial(\text{Sin}\psi_R)}{\partial \underline{\varepsilon}} = \left(\frac{1}{(X^2+Y^2)^{1/2}} - \frac{Y^2}{(X^2+Y^2)^{3/2}} \right) \frac{\partial Y}{\partial \underline{\varepsilon}} - \frac{XY}{(X^2+Y^2)^{3/2}} \frac{\partial X}{\partial \underline{\varepsilon}}$$

$$= \frac{1}{(X^2+Y^2)^{3/2}} \left(-XY \frac{\partial X}{\partial \underline{\varepsilon}} + X^2 \frac{\partial Y}{\partial \underline{\varepsilon}} \right) \qquad \text{(B-29)}$$

The sensitivity of the sine of the elevation angle is calculated using Equation (B-19):

$$\frac{\partial(\text{Sin}\theta_R)}{\partial \underline{\varepsilon}} = \frac{1}{R} \frac{\partial Z}{\partial \underline{\varepsilon}} - \frac{Z}{R^2} \left(\frac{X}{R} \frac{\partial X}{\partial \underline{\varepsilon}} + \frac{Y}{R} \frac{\partial Y}{\partial \underline{\varepsilon}} + \frac{Z}{R} \frac{\partial Z}{\partial \underline{\varepsilon}} \right)$$

$$= \frac{1}{R^3} \left(-XZ \frac{\partial X}{\partial \underline{\varepsilon}} - YZ \frac{\partial Y}{\partial \underline{\varepsilon}} + (X^2 + Y^2) \frac{\partial Z}{\partial \underline{\varepsilon}} \right) \qquad \text{(B-30)}$$

parameter estimation error (by the Cramer-Rao theorem), therefore the trace of the dispersion matrix was chosen among many other criteria as the optimization criterion (Ref 14:20). It can be shown that it is directly related to the covariance matrix of the estimation errors.

Appendix C

Numerical Results

This appendix contains the results of the computer simulations for the Truth Model, Model 1 and Model 2. The titles of the tables are appearing in the list of tables that is in the prefatory section of this study.

EPSITRUE denotes the true misalignment angles while EPSIO denotes the nominal (estimated) ones in the following order: ψ, θ, ϕ. KD is the control type used (defined in Eq. (26) to (31)). VX, VY, and VZ are the missile velocities relative to the aircraft and RD is the range rate. SP.FRC.( ) denotes the specific forces measured by the missile accelerometers (not including gravity). DELTA denotes the difference between the true output and the estimated one. HOT is the appropriate higher order term. The various outputs are the range (R), the range rate (RD), the sine of the azimuth angle (AZ.), the sine of the elevation angle (EL.), and the three specific forces (SF_1, SF_2, and SF_3).

Table C-I

TRUTH MODEL EPSITRUE= .10E+01, .20E+01, .30E+01
KD = 1

TIME	X	Y	Z	R
0.	.10000E+04	.10000E+04	.10000E+03	.14177E+04
10.	.21009E+05	.14795E+05	.58759E+04	.26353E+05
20.	.40799E+05	.22326E+05	.21740E+05	.51338E+05
30.	.60488E+05	.22871E+05	.42660E+05	.77471E+05
40.	.80929E+05	.15021E+05	.64627E+05	.10465E+06
50.	.10321E+06	-.18709E+04	.82292E+05	.13202E+06

TIME	VX	VY	VZ	RD
0.	.20000E+04	.15000E+04	.10000E+02	.24694E+04
10.	.19962E+04	.11329E+04	.11641E+04	.24865E+04
20.	.19637E+04	.38396E+03	.19086E+04	.25358E+04
30.	.19893E+04	-.30290E+03	.22238E+04	.26833E+04
40.	.21153E+04	-.13020E+04	.20592E+04	.27205E+04
50.	.23652E+04	-.19441E+04	.14747E+04	.27960E+04

TIME	SP.FRC.(1)	SP.FRC.(2)	SP.FRC.(3)
0.	0.	0.	0.
10.	-.28812E+01	-.70695E+02	.73793E+02
20.	-.16763E+01	-.67123E+02	.15335E+02
30.	.74941E+01	-.83218E+02	-.17808E+02
40.	.17841E+02	-.10047E+03	-.81286E+02
50.	.31203E+02	-.24964E+02	-.82230E+02

Table C-II

TRUTH MODEL EPSITRUE= .10E+01, .20E+01, .30E+01
KD = 2

TIME	X	Y	Z	R
0.	.10000E+04	.10000E+04	.10000E+03	.14177E+04
10.	.21070E+05	.15000E+05	.53484E+04	.26411E+05
20.	.41452E+05	.23705E+05	.19522E+05	.51599E+05
30.	.62239E+05	.24192E+05	.38782E+05	.77220E+05
40.	.83467E+05	.15800E+05	.59105E+05	.10349E+06
50.	.10605E+06	.48947E+02	.76435E+05	.13073E+06

TIME	VX	VY	VZ	RD
0.	.20000E+04	.15000E+04	.10000E+02	.24694E+04
10.	.20189E+04	.12062E+04	.10288E+04	.25040E+04
20.	.20593E+04	.48624E+03	.17396E+04	.25364E+04
30.	.20357E+04	-.40033E+03	.20465E+04	.25923E+04
40.	.21532E+04	-.12559E+04	.19486E+04	.26659E+04
50.	.23935E+04	-.18169E+04	.14632E+04	.27965E+04

TIME	SP.FRC.(1)	SP.FRC.(2)	SP.FRC.(3)
0.	0.	0.	0.
10.	.30921E+01	-.55040E+02	.58495E+02
20.	.41041E+01	-.83974E+02	.18745E+02
30.	.35886E+01	-.90461E+02	-.21131E+02
40.	.12322E+02	-.76589E+02	-.62604E+02
50.	.35425E+02	-.29925E+02	-.94291E+02

Table C-III

TRUTH MODEL EPSITRUE= .10E+01, .20E+01, .30E+01
KD = 3

TIME	X	Y	Z	R
0.	.10000E+04	.10000E+04	.10000E+03	.14177E+04
10.	.21036E+05	.14763E+05	.58624E+04	.26359E+05
20.	.40970E+05	.22153E+05	.21601E+05	.51341E+05
30.	.60876E+05	.22527E+05	.42279E+05	.77465E+05
40.	.81468E+05	.14509E+05	.63968E+05	.10459E+06
50.	.10350E+06	-.26497E+04	.81422E+05	.13180E+06

TIME	VX	VY	VZ	RD
0.	.20000E+04	.15000E+04	.10000E+02	.24694E+04
10.	.20040E+04	.11240E+04	.11590E+04	.24865E+04
20.	.19935E+04	.36684E+03	.18890E+04	.25359E+04
30.	.20112E+04	-.31955E+03	.21960E+04	.26862E+04
40.	.21182E+04	-.13206E+04	.20339E+04	.27105E+04
50.	.23340E+04	-.19810E+04	.14562E+04	.27742E+04

TIME	SP.FRC.(1)	SP.FRC.(2)	SP.FRC.(3)
0.	0.	0.	0.
10.	-.14840E+01	-.72056E+02	.72497E+02
20.	-.91557E+00	-.67357E+02	.14150E+02
30.	.59971E+01	-.83043E+02	-.18263E+02
40.	.14444E+02	-.10143E+03	-.80341E+02
50.	.28717E+02	-.27130E+02	-.82203E+02

Table C-IV

TRUTH MODEL EPSITRUE= .50E+01, .10E+02, .15E+02
KD = 1

TIME	X	Y	Z	R
0.	.10000E+04	.10000E+04	.10000E+03	.14177E+04
10.	.21045E+05	.14748E+05	.58621E+04	.26359E+05
20.	.40345E+05	.22151E+05	.21641E+05	.51337E+05
30.	.60786E+05	.22550E+05	.42394E+05	.77464E+05
40.	.81414E+05	.14591E+05	.64090E+05	.10464E+06
50.	.10339E+06	-.22960E+04	.81379E+05	.13193E+06

TIME	VX	VY	VZ	RD
0.	.20000E+04	.15000E+04	.10000E+02	.24694E+04
10.	.20039E+04	.11233E+04	.11599E+04	.24864E+04
20.	.19772E+04	.36936E+03	.18959E+04	.25355E+04
30.	.20063E+04	-.31683E+03	.22026E+04	.26875E+04
40.	.21353E+04	-.13082E+04	.20260E+04	.27193E+04
50.	.23832E+04	-.19372E+04	.14343E+04	.27939E+04

TIME	SP.FRC.(1)	SP.FRC.(2)	SP.FRC.(3)
0.	0.	0.	0.
10.	-.29834E+01	-.70660E+02	.73813E+02
20.	-.16544E+01	-.67095E+02	.15334E+02
30.	.75636E+01	-.83124E+02	-.17786E+02
40.	.19195E+02	-.10030E+03	-.81084E+02
50.	.31328E+02	-.24892E+02	-.81959E+02

Table C-V

TRUTH MODEL EPSITRUE= .50E+01, .10E+02, .15E+02
KD = 2

TIME	X	Y	Z	R
0.	.10000E+04	.10000E+04	.10000E+03	.14177E+04
10.	.21102E+05	.14959E+05	.53365E+04	.26411E+05
20.	.41576E+05	.23557E+05	.19433E+05	.51537E+05
30.	.62504E+05	.23921E+05	.38516E+05	.77217E+05
40.	.83908E+05	.15442E+05	.58554E+05	.10348E+06
50.	.10568E+06	-.30376E+03	.75516E+05	.13070E+06

TIME	VX	VY	VZ	RD
0.	.20000E+04	.15000E+04	.10000E+02	.24694E+04
10.	.20253E+04	.11981E+04	.10253E+04	.25033E+04
20.	.20711E+04	.47386E+03	.17272E+04	.25353E+04
30.	.21129E+04	-.41179E+03	.20232E+04	.25919E+04
40.	.21817E+04	-.12609E+04	.19153E+04	.26648E+04
50.	.24100E+04	-.18101E+04	.14237E+04	.27938E+04

TIME	SP.FRC.(1)	SP.FRC.(2)	SP.FRC.(3)
0.	0.	0.	0.
10.	.30130E+01	-.55023E+02	.58510E+02
20.	.41097E+01	-.83940E+02	.18744E+02
30.	.37576E+01	-.90367E+02	-.21107E+02
40.	.12577E+02	-.76454E+02	-.62451E+02
50.	.35567E+02	-.29829E+02	-.93974E+02

Table C-VI

TRUTH MODEL EPSITRUE= .50E+01, .10E+02, .15E+02
KD = 3

TIME	X	Y	Z	R
0.	.10000E+04	.10000E+04	.10000E+03	.14177E+04
10.	.21072E+05	.14716E+05	.58480E+04	.26359E+05
20.	.41114E+05	.21981E+05	.21499E+05	.51339E+05
30.	.61173E+05	.22211E+05	.42005E+05	.77459E+05
40.	.81948E+05	.14089E+05	.63420E+05	.10453E+06
50.	.10428E+06	-.30644E+04	.80496E+05	.13177E+06

TIME	VX	VY	VZ	RD
0.	.20000E+04	.15000E+04	.10000E+02	.24694E+04
10.	.20116E+04	.11144E+04	.11547E+04	.24865E+04
20.	.19359E+04	.35251E+03	.18759E+04	.25357E+04
30.	.20280E+04	-.33308E+03	.21745E+04	.26853E+04
40.	.21380E+04	-.13265E+04	.20004E+04	.27098E+04
50.	.23519E+04	-.19742E+04	.14157E+04	.27720E+04

TIME	SP.FRC.(1)	SP.FRC.(2)	SP.FRC.(3)
0.	0.	0.	0.
10.	-.15898E+01	-.72025E+02	.72516E+02
20.	-.30540E+00	-.67327E+02	.14148E+02
30.	.71625E+01	-.82948E+02	-.18239E+02
40.	.14798E+02	-.10127E+03	-.80139E+02
50.	.28852E+02	-.27065E+02	-.81929E+02

Table C-VII

```
MODEL     1      EPSITRUE=  .10000E+01,  .20000E+01,  .30000E+01
W/O  NOISE       EPSIO=     0.         , 0.         , 0.
KD   =   1       ITERATION # 1
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.21727E+00	.47242E-02	.13745E-01	.15814E-02
20.	.23252E+00	.40869E-01	.22372E-01	.55933E-02
30.	-.89958E+00	.11544E+00	-.29050E+00	.91527E-02
40.	-.65354E+01	.22561E+00	-.90775E+00	.12861E-01
50.	-.20584E+02	.36655E+00	-.19573E+01	.14890E-01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.48865E-03	-.91641E-07	-.13390E-03	-.41079E-07
20.	-.11002E-02	-.91514E-07	-.49673E-03	-.33525E-06
30.	-.15414E-02	.56090E-06	-.87024E-03	-.83182E-06
40.	-.16286E-02	.17165E-05	-.12712E-02	-.14130E-05
50.	-.11654E-02	.18531E-05	-.16982E-02	-.19992E-05

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.19173E-01	-.61E-02	.29E-02	.33E-02
20.	.20145E-03	-.60E-02	.46E-02	.54E-02
30.	.10151E-04	-.74E-02	.10E-02	.39E-03
40.	.22912E-05	-.10E-01	-.45E-02	-.30E-02
50.	.79537E-06	-.13E-01	-.77E-02	-.13E-01

Table C-VIII

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .10126E+01, .20077E+01, .30133E+01
KD = 1 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.31249E-02	.10105E-06	-.16584E-02	.35441E-07
20.	-.42514E-01	.99755E-06	-.60065E-02	.14913E-06
30.	-.11678E+00	.30605E-05	-.88905E-02	.25507E-06
40.	-.22705E+00	.64333E-05	-.13278E-01	.41073E-06
50.	-.37025E+00	.11080E-04	-.14242E-01	.49353E-06

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	.22565E-05	-.19887E-11	.62716E-06	-.73450E-12
20.	.53845E-05	.24696E-11	.25467E-05	-.51098E-11
30.	.76840E-05	.28273E-10	.47184E-05	-.14635E-10
40.	.79392E-05	.67780E-10	.71383E-05	-.22459E-10
50.	.48881E-05	.62121E-10	.96757E-05	-.25167E-10

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.19226E-01	-.14E-06	.12E-06	.14E-06
20.	.20249E-03	-.13E-06	.24E-06	.30E-06
30.	.10149E-04	-.16E-06	.14E-06	.15E-06
40.	.22533E-05	-.24E-06	-.76E-08	-.70E-07
50.	.78001E-06	-.33E-06	-.13E-06	-.27E-06

Table C-IX

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .10000E+01, .20000E+01, .30000E+01
KO = 1 ITERATION # 3

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.43772E-07	.33318E-09	-.36700E-07	-.58007E-10
20.	-.10373E-05	-.34340E-08	-.15812E-06	-.31572E-10
30.	-.30803E-05	-.21420E-09	-.25550E-06	.37757E-12
40.	-.63917E-05	.43431E-08	-.41508E-06	.80501E-10
50.	-.11109E-04	.22151E-08	-.49743E-06	.41835E-10

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	.46253E-10	.11043E-13	.12792E-10	-.15886E-13
20.	.11359E-09	.20200E-14	.54120E-10	.22724E-13
30.	.16412E-09	.77041E-14	.10262E-09	.26360E-13
40.	.16930E-09	-.81212E-14	.15786E-09	.37498E-13
50.	.99042E-10	-.19391E-14	.21627E-09	.10709E-12

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.19226E-01	-.39E-08	-.24E-07	-.32E-07
20.	.20248E-03	.22E-08	.13E-08	.25E-08
30.	.10149E-04	.29E-08	.68E-08	.97E-08
40.	.22534E-05	-.34E-09	.11E-08	.32E-09
50.	.78008E-06	-.39E-08	-.43E-08	-.73E-08

Table C-X

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= 0. , 0. , 0.
KD = 2 ITERATION # 1

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.19876E+00	.35291E-02	.13448E-01	.11186E-02
20.	.36519E+00	.29425E-01	.54566E-01	.43267E-02
30.	-.27841E-01	.93326E-01	-.21285E+00	.35241E-02
40.	-.53251E+01	.19907E+00	-.31504E+00	.12463E-01
50.	-.19469E+02	.33679E+00	-.19537E+01	.14683E-01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.41850E-03	-.69415E-07	-.11565E-03	-.27901E-07
20.	-.89531E-03	-.84265E-07	-.44092E-03	-.21298E-06
30.	-.12728E-02	.40859E-06	-.87415E-03	-.50829E-06
40.	-.13391E-02	.12603E-05	-.13224E-02	-.11688E-05
50.	-.96957E-03	.13461E-05	-.17254E-02	-.17570E-05

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.22840E-01	-.52E-02	.41E-03	.19E-03
20.	.29435E-03	-.51E-02	.17E-02	.13E-02
30.	.13649E-04	-.51E-02	.18E-02	.20E-02
40.	.15695E-05	-.54E-02	.11E-02	.85E-03
50.	.34542E-06	-.63E-02	-.23E-03	-.13E-02

Table C-XI

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .10063E+01, .20002E+01, .30013E+01
KD = 2 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.30344E-02	-.25930E-09	-.10945E-02	.39513E-09
20.	-.29440E-01	.12682E-07	-.44324E-02	.32866E-08
30.	-.94085E-01	.82397E-07	-.85116E-02	.10695E-07
40.	-.19901E+00	.24134E-06	-.12411E-01	.21604E-07
50.	-.33885E+00	.50511E-06	-.15279E-01	.31102E-07

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	.23665E-06	.43249E-14	.72055E-07	.18599E-13
20.	.56039E-06	.35916E-12	.39895E-06	.98530E-13
30.	.10300E-05	.17135E-11	.97712E-06	.54397E-12
40.	.94474E-06	.30438E-11	.16639E-05	.18456E-11
50.	.14610E-06	.64500E-12	.22697E-05	.35864E-11

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.22819E-01	.11E-08	.33E-07	.42E-07
20.	.29460E-03	-.17E-08	.94E-08	.11E-07
30.	.13667E-04	.18E-08	.22E-07	.23E-07
40.	.15698E-05	-.10E-08	.17E-07	.21E-07
50.	.34451E-06	-.39E-08	.12E-07	.13E-07

Table C-XII

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .10000E+01, .20000E+01, .30000E+01
KD = 2 ITERATION # 3

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	.12806E-08	-.79567E-09	.58208E-10	.31798E-10
20.	-.14558E-07	-.13249E-08	-.36525E-08	.58096E-10
30.	-.81956E-07	.21654E-08	-.10987E-07	-.17479E-11
40.	-.24533E-06	-.40366E-08	-.21202E-07	-.15904E-09
50.	-.51259E-06	-.34408E-08	-.32538E-07	-.22751E-09

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.20073E-11	-.43612E-13	-.53380E-12	.15014E-13
20.	-.39559E-11	-.29124E-13	-.18812E-11	.13056E-13
30.	-.53486E-11	-.17301E-13	-.34319E-11	-.82572E-14
40.	-.55524E-11	-.25529E-14	-.47713E-11	.29046E-13
50.	-.47230E-11	-.14897E-14	-.59579E-11	.24030E-13

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.22819E-01	-.44E-08	-.13E-07	-.19E-07
20.	.29460E-03	-.20E-08	-.63E-08	-.92E-08
30.	.13667E-04	-.16E-08	-.92E-09	-.23E-08
40.	.15698E-05	-.29E-08	-.44E-08	-.72E-08
50.	.34451E-06	-.46E-09	-.77E-09	-.13E-08

Table C-XIII

```
MODEL    1      EPSITRUE=  .10000E+01,  .20000E+01,  .30000E+01
W/O  NOISE      EPSTO=     0.          , 0.          , 0.
KD   =   3      ITERATION # 1
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.18286E+00	.47165E-02	.22909E-01	.13774E-02
20.	.39021E+00	.40710E-01	.34581E-01	.56565E-02
30.	-.56708E+00	.11500E+00	-.28875E+00	.31448E-02
40.	-.53526E+01	.22554E+00	-.91964E+00	.12983E-01
50.	-.20530E+02	.36892E+00	-.19834E+01	.15246E-01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.48749E-03	-.89437E-07	-.13985E-03	-.40790E-07
20.	-.10898E-02	-.70888E-07	-.51474E-03	-.33012E-06
30.	-.15124E-02	.56758E-06	-.89545E-03	-.81916E-06
40.	-.15824E-02	.16870E-05	-.12984E-02	-.14000E-05
50.	-.11202E-02	.17772E-05	-.17257E-02	-.20027E-05

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.18554E-01	-.58E-02	.30E-02	.34E-02
20.	.18322E-03	-.57E-02	.48E-02	.57E-02
30.	.94236E-05	-.69E-02	.15E-02	.11E-02
40.	.20997E-05	-.97E-02	-.35E-02	-.55E-02
50.	.74909E-06	-.12E-01	-.66E-02	-.12E-01

Table C-XIV

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .10119E+01, .20066E+01, .30117E+01
KD = 3 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.31406E-02	.78963E-07	-.16467E-02	.27554E-07
20.	-.42351E-01	.78453E-06	-.59682E-02	.11790E-06
30.	-.11526E+00	.24243E-05	-.88810E-02	.21199E-06
40.	-.22571E+00	.51505E-05	-.13351E-01	.33535E-06
50.	-.37253E+00	.90039E-05	-.14772E-01	.41940E-06

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	.19853E-05	-.12061E-11	.57786E-06	-.59541E-12
20.	.47295E-05	.26485E-11	.23480E-05	-.45244E-11
30.	.55834E-05	.23750E-10	.43403E-05	-.10513E-10
40.	.68155E-05	.54083E-10	.65408E-05	-.15327E-10
50.	.40461E-05	.47350E-10	.88444E-05	-.17175E-10

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.18608E-01	-.95E-07	.12E-06	.14E-06
20.	.18928E-03	-.87E-07	.22E-06	.27E-06
30.	.94222E-05	-.12E-06	.13E-06	.14E-06
40.	.20784E-05	-.17E-06	.27E-07	-.93E-08
50.	.73642E-06	-.24E-06	-.63E-07	-.15E-06

Table C-XV

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .10000E+01, .20000E+01, .30000E+01
KD = 3 ITERATION # 3

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.35904E-07	-.10544E-09	-.28551E-07	.44124E-10
20.	-.80909E-06	.13266E-08	-.12392E-06	.30514E-09
30.	-.24368E-05	.12814E-08	-.20488E-06	.23381E-09
40.	-.51158E-05	.60826E-06	-.33835E-06	.41337E-09
50.	-.90273E-05	.12452E-07	-.42238E-06	.50292E-09

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	.26517E-10	-.17717E-13	.77121E-11	-.45795E-15
20.	.65454E-10	-.82518E-13	.33623E-10	-.71113E-14
30.	.95849E-10	-.88069E-13	.64656E-10	-.20824E-13
40.	.96907E-10	-.10880E-12	.10010E-09	-.91736E-14
50.	.51357E-10	-.90513E-13	.13749E-09	-.19463E-13

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.18508E-01	.19E-09	-.11E-07	-.13E-07
20.	.18927E-03	.50E-08	.16E-07	.23E-07
30.	.94221E-05	-.22E-08	-.35E-08	-.55E-08
40.	.20784E-05	.52E-09	.14E-08	.20E-08
50.	.73647E-06	-.55E-09	-.11E-09	-.50E-09

Table C-XVI

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= 0. , 0. , 0.
KO = 1 ITERATION # 1

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.15725E+02	.16947E+02	.10658E+02	.10546E+02
20.	.73351E+01	.71435E+01	-.11823E+02	-.11839E+02
30.	-.13156E+02	-.12151E+02	-.11909E+02	-.11610E+02
40.	-.21416E+02	-.14655E+02	.97332E+01	.10554E+02
50.	-.39639E+02	-.16689E+02	-.13562E+02	-.11590E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.88414E-03	.13727E-02	.59303E-03	.72588E-03
20.	-.88023E-03	.21986E-03	-.17939E-02	-.12975E-02
30.	-.17673E-02	-.22535E-03	-.49941E-03	.35998E-03
40.	-.87758E-03	.75277E-03	-.83387E-03	.43585E-03
50.	-.12057E-02	-.39436E-04	-.13656E-02	.33065E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.20948E-02	-.80E+01	.13E+01	-.78E+00
20.	.30248E-04	.74E+00	-.41E+00	-.10E+00
30.	.33748E-05	-.26E-01	-.55E-02	-.20E-01
40.	.64403E-06	.21E-01	-.29E-01	-.23E-01
50.	.20370E-06	-.16E-01	-.20E-01	-.45E-01

Table C-XVII

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .10162E+01, .20202E+01, .30455E+01
KD = 1 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16977E+02	.16942E+02	.10651E+02	.10644E+02
20.	.72141E+01	.71026E+01	-.11836E+02	-.11845E+02
30.	-.12048E+02	-.12266E+02	-.11606E+02	-.11519E+02
40.	-.14517E+02	-.14881E+02	.10658E+02	.10541E+02
50.	-.18432E+02	-.19055E+02	-.11581E+02	-.11605E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13795E-02	.13728E-02	.72868E-03	.72592E-03
20.	.23534E-03	.21995E-03	-.12905E-02	-.12972E-02
30.	-.20386E-03	-.22591E-03	.38253E-03	.37081E-03
40.	.77498E-03	.75106E-03	.45499E-03	.43726E-03
50.	-.23606E-04	-.41298E-04	.35706E-03	.33265E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.20933E-02	-.80E+01	.13E+01	-.79E+00
20.	.30206E-04	.75E+00	-.41E+00	-.10E+00
30.	.33689E-05	-.19E-01	-.70E-02	-.22E-01
40.	.54304E-06	.27E-01	-.30E-01	-.25E-01
50.	.20349E-06	-.11E-01	-.21E-01	-.47E-01

Table C-XVIII

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .10110E+01, .20208E+01, .30475E+01
KO = 1 ITERATION # 3

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16985E+02	.16942E+02	.10653E+02	.10544E+02
20.	.72610E+01	.71026E+01	-.11830E+02	-.11845E+02
30.	-.11930E+02	-.12266E+02	-.11597E+02	-.11619E+02
40.	-.14288E+02	-.14881E+02	.10671E+02	.10541E+02
50.	-.18107E+02	-.19055E+02	-.11564E+02	-.11605E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13795E-02	.13728E-02	.72870E-03	.72592E-03
20.	.23549E-03	.21995E-03	-.12906E-02	-.12972E-02
30.	-.20358E-03	-.22591E-03	.38237E-03	.37081E-03
40.	.77539E-03	.75106E-03	.45450E-03	.43726E-03
50.	-.22652E-04	-.41286E-04	.35640E-03	.33265E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.20933E-02	-.80E+01	.13E+01	-.78E+00
20.	.30206E-04	.75E+00	-.41E+00	-.10E+00
30.	.33689E-05	-.19E-01	-.70E-02	-.22E-01
40.	.64304E-06	.27E-01	-.30E-01	-.25E-01
50.	.20349E-06	-.11E-01	-.21E-01	-.47E-01

Table C-XIX

```
MODEL     1      EPSITRUE=  .10000E+01,  .20000E+01,  .30000E+01
WITH NOISE       EPSIO=     0.          , 0.          , 0.
KD    =   2      ITERATION # 1
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16754E+02	.16947E+02	.10658E+02	.10546E+02
20.	.74707E+01	.71351E+01	-.11780E+02	-.11341E+02
30.	-.12281E+02	-.12160E+02	-.11830E+02	-.11509E+02
40.	-.20185E+02	-.14660E+02	.97266E+01	.10654E+02
50.	-.38495E+02	-.18689E+02	-.13557E+02	-.11589E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.95423E-03	.13728E-02	.51123E-03	.72584E-03
20.	-.67617E-03	.22006E-03	-.17383E-02	-.12376E-02
30.	-.14985E-02	-.22529E-03	-.50363E-03	.35990E-03
40.	-.58538E-03	.75250E-03	-.98541E-03	.43577E-03
50.	-.10108E-02	-.39829E-04	-.13940E-02	.33057E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.27617E-02	-.96E+01	.15E+01	-.10E+01
20.	.33434E-04	.71E+00	-.40E+00	-.34E-01
30.	.29604E-05	-.37E-01	-.16E-02	-.27E-01
40.	.54323E-06	.14E-01	-.26E-01	-.25E-01
50.	.24356E-06	-.19E-01	-.20E-01	-.51E-01

Table C-XX

```
MODEL      1        EPSITRUE=   .10000E+01,   .20000E+01,   .30000E+01
WITH NOISE          EPSIO=      .10169E+01,   .20204E+01,   .30512E+01
KD    =    2        ITERATION # 2
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16984E+02	.16943E+02	.10652E+02	.10545E+02
20.	.72275E+01	.71057E+01	-.11836E+02	-.11845E+02
30.	-.12029E+02	-.12253E+02	-.11605E+02	-.11517E+02
40.	-.14492E+02	-.14859E+02	.10659E+02	.10542E+02
50.	-.18453E+02	-.19026E+02	-.11580E+02	-.11504E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13791E-02	.13728E-02	.72852E-03	.72686E-03
20.	.23397E-03	.22015E-03	-.12909E-02	-.12974E-02
30.	-.20546E-03	-.22570E-03	.38377E-03	.37050E-03
40.	.77322E-03	.75124E-03	.45768E-03	.43694E-03
50.	-.24728E-04	-.41175E-04	.36023E-03	.33234E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.27603E-02	-.96E+01	.15E+01	-.10E+01
20.	.33401E-04	.72E+00	-.40E+00	-.94E-01
30.	.29566E-05	-.32E-01	-.26E-02	-.28E-01
40.	.54744E-06	.20E-01	-.27E-01	-.25E-01
50.	.24328E-06	-.14E-01	-.22E-01	-.53E-01

Table C-XXI

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .10137E+01, .20215E+01, .30525E+01
KD = 2 ITERATION # 3

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16988E+02	.16943E+02	.10653E+02	.10545E+02
20.	.72598E+01	.71057E+01	-.11831E+02	-.11845E+02
30.	-.11932E+02	-.12253E+02	-.11597E+02	-.11617E+02
40.	-.14287E+02	-.14859E+02	.10672E+02	.10542E+02
50.	-.18102E+02	-.19026E+02	-.11564E+02	-.11604E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15551E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13792E-02	.13728E-02	.72854E-03	.72586E-03
20.	.23405E-03	.22015E-03	-.12910E-02	-.12374E-02
30.	-.20542E-03	-.22570E-03	.38352E-03	.37050E-03
40.	.77341E-03	.75124E-03	.45714E-03	.43594E-03
50.	-.24053E-04	-.41175E-04	.35945E-03	.33234E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.27603E-02	-.96E+01	.15E+01	-.10E+01
20.	.33401E-04	.72E+00	-.40E+00	-.94E-01
30.	.29566E-05	-.32E-01	-.26E-02	-.23E-01
40.	.64744E-06	.20E-01	-.27E-01	-.25E-01
50.	.24328E-06	-.14E-01	-.22E-01	-.53E-01

Table C-XXII

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= 0. , 0. , 0.
KD = 3 ITERATION # 1

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16759E+02	.16947E+02	.10657E+02	.10546E+02
20.	.74940E+01	.71446E+01	-.11810E+02	-.11839E+02
30.	-.12929E+02	-.12147E+02	-.11907E+02	-.11509E+02
40.	-.21228E+02	-.14650E+02	.97215E+01	.10554E+02
50.	-.39637E+02	-.18678E+02	-.13583E+02	-.11589E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.88530E-03	.13727E-02	.58707E-03	.72587E-03
20.	-.86990E-03	.21986E-03	-.18119E-02	-.12975E-02
30.	-.17383E-02	-.22533E-03	-.52467E-03	.35394E-03
40.	-.83132E-03	.75276E-03	-.86117E-03	.43580E-03
50.	-.11615E-02	-.39484E-04	-.13931E-02	.33058E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.19437E-02	-.77E+01	.12E+01	-.82E+00
20.	.28517E-04	.70E+00	-.40E+00	-.90E-01
30.	.32713E-05	-.28E-01	-.40E-02	-.20E-01
40.	.64304E-06	.20E-01	-.29E-01	-.22E-01
50.	.20636E-06	-.16E-01	-.20E-01	-.45E-01

Table C-XXIII

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .10164E+01, .20198E+01, .30456E+01
KD = 3 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16978E+02	.16942E+02	.10651E+02	.10645E+02
20.	.72148E+01	.71039E+01	-.11836E+02	-.11345E+02
30.	-.12047E+02	-.12262E+02	-.11606E+02	-.11518E+02
40.	-.14517E+02	-.14875E+02	.10658E+02	.10541E+02
50.	-.18493E+02	-.19047E+02	-.11581E+02	-.11504E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13794E-02	.13728E-02	.72875E-03	.72591E-03
20.	.23518E-03	.21993E-03	-.12903E-02	-.12972E-02
30.	-.20425E-03	-.22591E-03	.38296E-03	.37076E-03
40.	.77437E-03	.75107E-03	.45537E-03	.43720E-03
50.	-.24213E-04	-.41261E-04	.35749E-03	.33258E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.19424E-02	-.77E+01	.12E+01	-.32E+00
20.	.28579E-04	.71E+00	-.40E+00	-.91E-01
30.	.32658E-05	-.21E-01	-.55E-02	-.22E-01
40.	.54205E-06	.26E-01	-.30E-01	-.24E-01
50.	.20613E-06	-.11E-01	-.21E-01	-.47E-01

Table C-XXIV

MODEL 1 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .10112E+01, .20206E+01, .30474E+01
KD = 3 ITERATION # 3

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16985E+02	.16942E+02	.10653E+02	.10545E+02
20.	.72611E+01	.71039E+01	-.11831E+02	-.11945E+02
30.	-.11929E+02	-.12262E+02	-.11537E+02	-.11518E+02
40.	-.14288E+02	-.14875E+02	.10671E+02	.10641E+02
50.	-.18106E+02	-.19047E+02	-.11563E+02	-.11504E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13796E-02	.13728E-02	.72877E-03	.72591E-03
20.	.23532E-03	.21993E-03	-.12904E-02	-.12372E-02
30.	-.20408E-03	-.22591E-03	.38268E-03	.37076E-03
40.	.77477E-03	.75107E-03	.45486E-03	.43720E-03
50.	-.23272E-04	-.41261E-04	.35679E-03	.33258E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.19424E-02	-.77E+01	.12E+01	-.82E+00
20.	.28579E-04	.71E+00	-.40E+00	-.31E-01
30.	.32658E-05	-.21E-01	-.55E-02	-.22E-01
40.	.64205E-06	.26E-01	-.30E-01	-.24E-01
50.	.20613E-06	-.11E-01	-.21E-01	-.47E-01

Table C-XXV

MODEL 1 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= 0. , 0. , 0.
K0 = 1 ITERATION # 1

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.15942E+02	.17060E+02	.10743E+02	.10584E+02
20.	.90326E+01	.81247E+01	-.11625E+02	-.11703E+02
30.	-.14354E+02	-.93838E+01	-.12893E+02	-.11391E+02
40.	-.43072E+02	-.92667E+01	.53809E+01	.10959E+02
50.	-.11424E+03	-.99699E+01	-.21028E+02	-.11238E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	-.10723E-02	.13705E-02	.58126E-04	.72585E-03
20.	-.52845E-02	.21776E-03	-.37812E-02	-.13056E-02
30.	-.79239E-02	-.21174E-03	-.39816E-02	.35009E-03
40.	-.73596E-02	.79374E-03	-.59154E-02	.40232E-03
50.	-.58505E-02	.46457E-05	-.81490E-02	.28355E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.21016E-02	-.81E+01	.13E+01	-.78E+00
20.	.30415E-04	.58E+00	-.39E+00	-.87E-01
30.	.33980E-05	-.19E+00	.30E-01	.11E-01
40.	.54744E-06	-.12E+00	.17E-02	.23E-01
50.	.20429E-06	-.13E+00	-.60E-02	.95E-03

Table C-XXVI

MODEL 1 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= .51343E+01, .10006E+02, .14939E+02
KD = 1 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16793E+02	.16942E+02	.10601E+02	.10545E+02
20.	.61130E+01	.71031E+01	-.11953E+02	-.11845E+02
30.	-.14815E+02	-.12266E+02	-.11811E+02	-.11519E+02
40.	-.19856E+02	-.14881E+02	.10343E+02	.10641E+02
50.	-.27448E+02	-.19056E+02	-.11975E+02	-.11505E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13751E-02	.13728E-02	.72809E-03	.72567E-03
20.	.23150E-03	.21998E-03	-.12893E-02	-.12372E-02
30.	-.20874E-03	-.22584E-03	.38840E-03	.37073E-03
40.	.75447E-03	.75116E-03	.46584E-03	.43719E-03
50.	-.46133E-04	-.41148E-04	.37124E-03	.33258E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.20945E-02	-.79E+01	.12E+01	-.79E+00
20.	.30209E-04	.75E+00	-.41E+00	-.10E+00
30.	.33695E-05	-.19E-01	-.72E-02	-.22E-01
40.	.64264E-06	.27E-01	-.31E-01	-.25E-01
50.	.20330E-06	-.10E-01	-.21E-01	-.49E-01

Table C-XXVII

MODEL 1	EPSITRUE=	.50000E+01,	.10000E+02,	.15000E+02
WITH NOISE	EPSIO=	.50102E+01,	.10021E+02,	.15048E+02
KD = 1	ITERATION # 3			

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16985E+02	.16942E+02	.10653E+02	.10645E+02
20.	.72611E+01	.71031E+01	-.11831E+02	-.11845E+02
30.	-.11930E+02	-.12266E+02	-.11597E+02	-.11519E+02
40.	-.14288E+02	-.14881E+02	.10671E+02	.10641E+02
50.	-.18105E+02	-.19056E+02	-.11563E+02	-.11505E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13797E-02	.13728E-02	.72866E-03	.72587E-03
20.	.23555E-03	.21998E-03	-.12907E-02	-.12972E-02
30.	-.20365E-03	-.22584E-03	.38231E-03	.37073E-03
40.	.77534E-03	.75116E-03	.45443E-03	.43719E-03
50.	-.22593E-04	-.41147E-04	.35631E-03	.33258E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.20944E-02	-.79E+01	.12E+01	-.78E+00
20.	.30209E-04	.75E+00	-.41E+00	-.10E+00
30.	.33694E-05	-.19E-01	-.72E-02	-.22E-01
40.	.54262E-06	.27E-01	-.31E-01	-.25E-01
50.	.20329E-06	-.10E-01	-.21E-01	-.48E-01

Table C-XXVIII

MODEL 1 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= 0. , 0. , 0.
KO = 2 ITERATION # 1

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16054E+02	.17032E+02	.10733E+02	.10573E+02
20.	.94838E+01	.78417E+01	-.11440E+02	-.11737E+02
30.	-.10511E+02	-.99235E+01	-.12514E+02	-.11405E+02
40.	-.37524E+02	-.99074E+01	.63355E+01	.10350E+02
50.	-.10927E+03	-.10674E+02	-.21021E+02	-.11242E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	-.72155E-03	.13711E-02	.14921E-03	.72512E-03
20.	-.42644E-02	.21810E-03	-.35018E-02	-.13028E-02
30.	-.65827E-02	-.21539E-03	-.39989E-02	.35535E-03
40.	-.59140E-02	.78260E-03	-.61695E-02	.40806E-03
50.	-.48733E-02	-.77538E-05	-.82909E-02	.28893E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.27690E-02	-.98E+01	.15E+01	-.10E+01
20.	.33569E-04	.59E+00	-.39E+00	-.89E-01
30.	.29757E-05	-.17E+00	.21E-01	-.57E-02
40.	.65067E-06	-.11E+00	.44E-02	.79E-02
50.	.24424E-06	-.14E+00	.52E-02	-.21E-01

Table C-XXIX

MODEL 1 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= .51402E+01, .99948E+01, .15021E+02
KD = 2 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16888E+02	.16943E+02	.10623E+02	.10645E+02
20.	.64906E+01	.71062E+01	-.11941E+02	-.11845E+02
30.	-.14288E+02	-.12252E+02	-.11806E+02	-.11617E+02
40.	-.19253E+02	-.14860E+02	.10360E+02	.10642E+02
50.	-.25620E+02	-.19027E+02	-.11956E+02	-.11504E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.30217E-03	.30217E-03
10.	.13757E-02	.13729E-02	.72800E-03	.72582E-03
20.	.23194E-03	.22017E-03	-.12901E-02	-.12375E-02
30.	-.20681E-03	-.22563E-03	.38925E-03	.37043E-03
40.	.76809E-03	.75134E-03	.46963E-03	.43586E-03
50.	-.40658E-04	-.41041E-04	.37738E-03	.33227E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.27620E-02	-.96E+01	.15E+01	-.10E+01
20.	.33407E-04	.72E+00	-.40E+00	-.95E-01
30.	.29575E-05	-.32E-01	-.29E-02	-.23E-01
40.	.64691E-06	.20E-01	-.27E-01	-.25E-01
50.	.24290E-06	-.13E-01	-.22E-01	-.53E-01

Table C-XXX

MODEL 1	EPSITRUE=	.50000E+01,	.10000E+02,	.15000E+02
WITH NOISE	EPSIO=	.50128E+01,	.10022E+02,	.15053E+02
KD = 2	ITERATION # 3			

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.15987E+02	.16943E+02	.10653E+02	.10545E+02
20.	.72585E+01	.71062E+01	-.11831E+02	-.11845E+02
30.	-.11932E+02	-.12252E+02	-.11597E+02	-.11517E+02
40.	-.14288E+02	-.14860E+02	.10672E+02	.10542E+02
50.	-.18101E+02	-.19027E+02	-.11563E+02	-.11604E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13793E-02	.13729E-02	.72850E-03	.72682E-03
20.	.23413E-03	.22017E-03	-.12910E-02	-.12975E-02
30.	-.20538E-03	-.22563E-03	.38346E-03	.37042E-03
40.	.77339E-03	.75134E-03	.45706E-03	.43686E-03
50.	-.23980E-04	-.41041E-04	.35936E-03	.33226E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.27620E-02	-.96E+01	.15E+01	-.10E+01
20.	.33407E-04	.72E+00	-.40E+00	-.95E-01
30.	.29574E-05	-.32E-01	-.29E-02	-.23E-01
40.	.64689E-06	.20E-01	-.27E-01	-.25E-01
50.	.24289E-06	-.13E-01	-.22E-01	-.53E-01

Table C-XXXI

MODEL 1 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= 0. , 0. , 0.
KD = 3 ITERATION # 1

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.15114E+02	.17060E+02	.10788E+02	.10584E+02
20.	.98178E+01	.81220E+01	-.11565E+02	-.11703E+02
30.	-.13398E+02	-.93911E+01	-.12881E+02	-.11391E+02
40.	-.42155E+02	-.92632E+01	.53241E+01	.10962E+02
50.	-.11422E+03	-.99018E+01	-.21152E+02	-.11229E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	-.10665E-02	.13706E-02	.28347E-04	.72585E-03
20.	-.52325E-02	.21825E-03	-.38711E-02	-.13055E-02
30.	-.77789E-02	-.21109E-03	-.41073E-02	.35036E-03
40.	-.71298E-02	.79303E-03	-.60514E-02	.40260E-03
50.	-.56261E-02	.28135E-05	-.82869E-02	.28340E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.19499E-02	-.78E+01	.13E+01	-.82E+00
20.	.28774E-04	.55E+00	-.38E+00	-.78E-01
30.	.32934E-05	-.19E+00	.32E-01	.98E-02
40.	.64627E-06	-.12E+00	.37E-02	.22E-01
50.	.20632E-06	-.14E+00	-.31E-02	-.15E-02

Table C-XXXII

MODEL 1 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= .51367E+01, .10003E+02, .15002E+02
KD = 3 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.32098E+01	.32098E+01
10.	.16801E+02	.16942E+02	.10602E+02	.10545E+02
20.	.51253E+01	.71044E+01	-.11968E+02	-.11845E+02
30.	-.14805E+02	-.12262E+02	-.11811E+02	-.11518E+02
40.	-.19861E+02	-.14875E+02	.10340E+02	.10541E+02
50.	-.27500E+02	-.19048E+02	-.11982E+02	-.11504E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13762E-02	.13728E-02	.72814E-03	.72587E-03
20.	.23156E-03	.21996E-03	-.12890E-02	-.12973E-02
30.	-.20871E-03	-.22584E-03	.38895E-03	.37069E-03
40.	.76433E-03	.75118E-03	.46672E-03	.43713E-03
50.	-.45428E-04	-.41122E-04	.37262E-03	.33251E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.19435E-02	-.77E+01	.12E+01	-.82E+00
20.	.28587E-04	.71E+00	-.40E+00	-.91E-01
30.	.32566E-05	-.21E-01	-.58E-02	-.22E-01
40.	.54149E-06	.27E-01	-.30E-01	-.24E-01
50.	.20594E-06	-.10E-01	-.21E-01	-.43E-01

Table C-XXXIII

MODEL 1 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= .50104E+01, .10021E+02, .15048E+02
KO = 3 ITERATION # 3

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16995E+02	.16942E+02	.10653E+02	.10645E+02
20.	.72511E+01	.71044E+01	-.11831E+02	-.11845E+02
30.	-.11930E+02	-.12262E+02	-.11597E+02	-.11618E+02
40.	-.14289E+02	-.14875E+02	.10671E+02	.10641E+02
50.	-.18105E+02	-.19048E+02	-.11563E+02	-.11604E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13796E-02	.13728E-02	.72873E-03	.72587E-03
20.	.23538E-03	.21996E-03	-.12904E-02	-.12973E-02
30.	-.20405E-03	-.22584E-03	.38262E-03	.37068E-03
40.	.77472E-03	.75118E-03	.45478E-03	.43713E-03
50.	-.23203E-04	-.41121E-04	.35669E-03	.33251E-03

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	0.	0.	0.
10.	.19435E-02	-.77E+01	.12E+01	-.92E+00
20.	.28587E-04	.71E+00	-.40E+00	-.91E-01
30.	.32666E-05	-.21E-01	-.58E-02	-.22E-01
40.	.54148E-06	.27E-01	-.30E-01	-.24E-01
50.	.20583E-06	-.16E-01	-.21E-01	-.48E-01

Table C-XXXIV-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= 0. , 0. , 0.
KO = 1 ITERATION # 1

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.11140E+00	-.10122E-02	-.11023E-01	.86163E-04
20.	-.33143E+00	.40300E-01	-.54256E-01	.11042E-01
30.	-.15743E+01	.28397E+00	-.18456E+00	.45730E-01
40.	-.32944E+01	.14026E+01	-.13876E+00	.21426E+00
50.	-.55752E+01	.49308E+01	-.45025E+00	.47140E+00

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.47213E-03	.84891E-08	-.12996E-03	-.22418E-05
20.	-.10504E-02	.26822E-05	-.47858E-03	-.28479E-05
30.	-.14683E-02	.14360E-04	-.84824E-03	-.57740E-05
40.	-.15250E-02	.46656E-04	-.12631E-02	-.78028E-05
50.	-.99237E-03	.10746E-03	-.16995E-02	.81804E-05

Table C-XXXIV-b

```
MODEL     2     EPSITRUE=  .10000E+01,   .20000E+01,   .30000E+01
W/O  NOISE      EPSIO=     0.          , 0.          , 0.
KO   =    1     ITERATION # 1
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	0.	0.	0.	0.	0.	0.
10.	-.23E-01	-.23E-03	.95E-02	.11E-02	.58E-02	-.13E-02
20.	.60E-02	.11E-02	.72E-02	.70E-02	.50E-04	-.15E-02
30.	.46E-01	.42E-02	.24E-01	.20E-01	.53E-02	.41E-02
40.	.74E-01	.17E-01	.44E-01	.42E-01	.52E-01	.38E-01
50.	.33E-01	.74E-03	.20E-01	.12E-01	.71E-01	.51E-01

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.10E+01	.20E+01	.30E+01
10.	.46353E-03	-.57E-02	-.19E-01	-.24E-01
20.	.71510E-04	.26E-01	.14E+00	.15E+00
30.	.43267E-04	.71E-01	.31E+00	.31E+00
40.	.92953E-05	-.31E+00	.49E+00	-.30E+00
50.	.45842E-05	-.58E+00	.51E+00	-.71E+00

Table C-XXXV-a

```
MODEL     2        EPSITRUE=   .10000E+01,   .20000E+01,   .30000E+01
W/O  NOISE         EPSIO=      .15772E+01,   .14913E+01,   .37054E+01
KD    =   1        ITERATION # 2
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	.77229E-02	.12994E-02	.26566E-03	.39597E-04
20.	-.33120E-01	-.34963E-02	-.10350E-01	-.12999E-02
30.	-.26240E+00	-.33119E-01	-.48691E-01	-.57550E-02
40.	-.16311E+01	-.18733E+00	-.25992E+00	-.31325E-01
50.	-.52793E+01	-.74666E+00	-.40800E+00	-.73587E-01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	.36432E-04	.19120E-06	.10258E-04	-.18279E-05
20.	.94892E-04	-.37362E-07	.49718E-04	.47120E-07
30.	.14863E-03	-.11655E-05	.11235E-03	.45285E-06
40.	.14256E-03	-.53313E-05	.21777E-03	.52012E-06
50.	-.27417E-04	-.14050E-04	.36087E-03	-.22579E-05

Table C-XXXV-b

```
MODEL    2     EPSITRUE=  .10000E+01,  .20000E+01,  .30000E+01
W/O  NOISE     EPSIO=     .15772E+01,  .14913E+01,  .37354E+01
KO   =   1     ITERATION # 2
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	0.	0.	0.	0.	0.	0.
10.	-.18E-01	.54E-04	-.32E-01	-.10E-03	-.31E-01	.84E-04
20.	.31E-02	-.15E-03	-.35E-02	-.72E-03	-.12E-01	.16E-03
30.	-.29E-03	-.56E-03	-.60E-02	-.26E-02	.15E-01	-.54E-03
40.	-.30E-02	-.24E-02	-.50E-01	-.68E-02	.46E-01	-.62E-02
50.	.29E-01	.30E-03	-.64E-01	-.22E-02	.18E-01	-.12E-01

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE EPROR IN PSI	ESTIMATE EPROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	-.58E+00	.51E+00	-.71E+00
10.	.47350E-03	.70E-02	.29E-01	.33E-01
20.	.57389E-04	.24E-02	-.46E-02	-.94E-03
30.	.40756E-04	.28E-03	-.35E-01	-.24E-01
40.	.86108E-05	.56E-01	-.65E-01	.61E-01
50.	.42976E-05	.11E+00	-.70E-01	.14E+00

Table C-XXXVI-a

```
MODEL     2      EPSITRUE=   .10000E+01,   .20000E+01,   .30000E+01
W/O  NOISE       EPSIO=      .88767E+00,   .20704E+01,   .28556E+01
KD   =    1      ITERATION # 3
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.21774E-02	-.12970E-03	-.12925E-03	.20782E-05
20.	.30015E-02	.10318E-02	.13190E-02	.31187E-03
30.	.31936E-01	.79389E-02	.67859E-02	.13227E-02
40.	.25059E+00	.42551E-01	.43039E-01	.59327E-02
50.	.85231E+00	.16407E+00	.65771E-01	.15905E-01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.99205E-05	-.33336E-07	-.27910E-05	.22293E-07
20.	-.24413E-04	.19882E-07	-.12259E-04	-.44184E-07
30.	-.36829E-04	.29638E-06	-.25815E-04	-.14806E-05
40.	-.35239E-04	.12769E-05	-.46991E-04	-.16201E-05
50.	-.31695E-05	.32209E-05	-.74743E-04	.44862E-05

Table C-XXXVI-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .88767E+00, .20704E+01, .28566E+01
KD = 1 ITERATION # 3

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	0.	0.	0.	0.	0.	0.
10.	.33E-02	.30E-05	.55E-02	.27E-04	.53E-02	-.34E-04
20.	-.32E-03	.38E-04	.67E-03	.18E-03	.20E-02	-.40E-04
30.	-.11E-03	.12E-03	.12E-02	.61E-03	-.25E-02	.13E-03
40.	.15E-02	.50E-03	.87E-02	.15E-02	-.73E-02	.13E-02
50.	-.48E-02	-.68E-04	.11E-01	.46E-03	-.25E-02	.25E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.11E+00	-.70E-01	.14E+00
10.	.47472E-03	-.79E-03	-.45E-02	-.60E-02
20.	.58029E-04	.24E-03	.26E-02	.25E-02
30.	.41112E-04	.94E-03	.34E-02	.72E-02
40.	.86423E-05	-.12E-01	.15E-01	-.12E-01
50.	.42962E-05	-.23E-01	.16E-01	-.23E-01

Table C-XXXVII-a

```
MODEL    2     EPSITRUE=  .10000E+01,  .20000E+01,  .30000E+01
W/O  NOISE     EPSIO=     0.          , 0.          , 0.
KD   =   2     ITERATION # 1
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.10141E+00	-.81423E-03	-.10342E-01	.92151E-05
20.	-.22180E+00	.26538E-01	-.17767E-01	.35013E-02
30.	-.62386E+00	.33175E+00	-.74828E-01	.52518E-01
40.	-.20768E+01	.15738E+01	-.24014E+00	.20245E+00
50.	-.51687E+01	.46308E+01	-.63065E+00	.41432E+00

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.40597E-03	.12646E-08	-.11262E-03	-.15022E-06
20.	-.86017E-03	.21528E-05	-.42658E-03	-.21173E-05
30.	-.12177E-02	.14649E-04	-.85720E-03	-.50521E-05
40.	-.12553E-02	.48877E-04	-.13183E-02	-.71926E-05
50.	-.82728E-03	.10237E-03	-.17282E-02	.81415E-05

Table C-XXXVII-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= 0. , 0. , 0.
KD = 2 ITERATION # 1

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	0.	0.	0.	0.	0.	0.
10.	-.18E-01	-.24E-03	.50E-02	.72E-03	.40E-02	-.92E-03
20.	.47E-02	.57E-03	.66E-02	.80E-02	.23E-03	-.17E-02
30.	.47E-01	.63E-02	.24E-01	.22E-01	.58E-02	.51E-02
40.	.57E-01	.12E-01	.35E-01	.34E-01	.40E-01	.30E-01
50.	.38E-01	-.16E-03	.26E-01	.16E-01	.83E-01	.72E-01

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.10E+01	.20E+01	.30E+01
10.	.61260E-03	-.53E-02	-.16E-01	-.20E-01
20.	.97801E-04	.17E-01	.10E+00	.11E+00
30.	.54652E-04	-.11E+00	.49E+00	.62E-01
40.	.40968E-04	-.21E+00	.58E+00	-.17E+00
50.	.12427E-04	.54E+00	.83E+00	.11E+01

Table C-XXXVIII-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .46377E+00, .11678E+01, .19300E+01
KD = 2 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.38625E-01	-.24656E-03	-.40549E-02	.59542E-04
20.	-.91931E-01	.12537E-01	-.87906E-02	.39552E-02
30.	-.29922E+00	.13057E+00	-.38733E-01	.23974E-01
40.	-.10328E+01	.60008E+00	-.11848E+00	.75909E-01
50.	-.29503E+01	.17367E+01	-.28305E+00	.15317E+00

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.15256E-03	.70757E-07	-.42282E-04	-.78000E-07
20.	-.32232E-03	.87801E-06	-.15910E-03	-.78949E-06
30.	-.45441E-03	.53961E-05	-.31618E-03	-.22324E-05
40.	-.46951E-03	.17863E-04	-.47997E-03	-.26907E-05
50.	-.32125E-03	.37653E-04	-.62221E-03	.29153E-05

Table C-XXXVIII-b

```
MODEL    2        EPSITRUE=  .10000E+01,   .20000E+01,   .30000E+01
W/O  NOISE        EPSIO=     .46377E+00,   .11676E+01,   .19300E+01
KD   =   2        ITERATION # 2
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	0.	0.	0.	0.	0.	0.
10.	-.21E-01	-.23E-04	-.16E-02	.30E-03	-.10E-02	-.32E-03
20.	-.15E-01	.35E-03	.20E-02	.30E-02	-.14E-02	-.61E-03
30.	-.18E-02	.25E-02	.76E-02	.83E-02	.41E-02	.18E-02
40.	.93E-02	.47E-02	.64E-02	.12E-01	.19E-01	.11E-01
50.	.12E-01	.14E-03	-.63E-02	.59E-02	.34E-01	.26E-01

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.54E+00	.83E+00	.11E+01
10.	.65038E-03	-.16E-02	-.74E-02	-.99E-02
20.	.96597E-04	.10E-01	.49E-01	.55E-01
30.	.52999E-04	-.36E-01	.20E+00	.42E-01
40.	.38108E-04	-.79E-01	.22E+00	-.51E-01
50.	.13006E-04	.15E+00	.30E+00	.36E+00

Table C-XXXIX-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .85421E+00, .16965E+01, .26439E+01
KD = 2 ITERATION # 3

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.13369E-01	-.12766E-03	-.14216E-02	.20131E-04
20.	-.33233E-01	.44822E-02	-.34316E-02	.13691E-02
30.	-.11572E+00	.45629E-01	-.15473E-01	.83047E-02
40.	-.40550E+00	.20711E+00	-.46569E-01	.25994E-01
50.	-.11445E+01	.59451E+00	-.10690E+00	.52030E-01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.52558E-04	.32221E-07	-.14549E-04	-.32124E-07
20.	-.11073E-03	.31079E-06	-.54507E-04	-.27701E-06
30.	-.15572E-03	.18426E-05	-.10758E-03	-.77501E-06
40.	-.16118E-03	.60748E-05	-.16195E-03	-.94829E-05
50.	-.11272E-03	.12824E-04	-.20843E-03	.86594E-05

Table C-XXXIX-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .85421E+00, .16965E+01, .25439E+01
KO = 2 ITERATION # 3

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	0.	0.	0.	0.	0.	0.
10.	-.52E-02	.14E-05	-.11E-02	.11E-03	-.89E-03	-.11E-03
20.	-.31E-02	.14E-03	.73E-03	.10E-02	-.82E-03	-.20E-03
30.	.19E-02	.92E-03	.26E-02	.28E-02	.18E-02	.62E-03
40.	.54E-02	.17E-02	.17E-02	.42E-02	.75E-02	.37E-02
50.	.51E-02	.80E-04	-.29E-02	.20E-02	.12E-01	.86E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.15E+00	.30E+00	.35E+00
10.	.64509E-03	-.69E-03	-.39E-02	-.53E-02
20.	.35010E-04	.37E-02	.17E-01	.19E-01
30.	.51543E-04	-.12E-01	.69E-01	.15E-01
40.	.36362E-04	-.27E-01	.78E-01	-.17E-01
50.	.13233E-04	.44E-01	.10E+00	.11E+00

Table C-XL-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= 0. , 0. , 0.
KD = 3 ITERATION # 1

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.11115E+00	-.95231E-03	-.10971E-01	.12086E-03
20.	-.34389E+00	.42436E-01	-.58279E-01	.11506E-01
30.	-.16820E+01	.29517E+00	-.20072E+00	.47349E-01
40.	-.36458E+01	.14529E+01	-.17108E+00	.22187E+00
50.	-.62842E+01	.51182E+01	-.48677E+00	.49102E+00

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.47103E-03	.19059E-07	-.13567E-03	-.23446E-05
20.	-.10406E-02	.28935E-05	-.49552E-03	-.28975E-05
30.	-.14404E-02	.15095E-04	-.87172E-03	-.57907E-05
40.	-.14805E-02	.50160E-04	-.12875E-02	-.77001E-05
50.	-.95183E-03	.11025E-03	-.17232E-02	.83308E-05

Table C-XL-b

```
MODEL    2        EPSITRUE=  .10000E+01,   .20000E+01,   .30000E+01
W/O  NOISE        EPSIO=     0.          , 0.          , 0.
KD   =    3       ITERATION # 1
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	0.	0.	0.	0.	0.	0.
10.	-.24E-01	-.25E-03	.87E-02	.11E-02	.56E-02	-.14E-02
20.	.55E-02	.11E-02	.75E-02	.73E-02	-.60E-04	-.14E-02
30.	.45E-01	.44E-02	.24E-01	.21E-01	.56E-02	.44E-02
40.	.34E-01	.19E-01	.42E-01	.43E-01	.52E-01	.38E-01
50.	.36E-01	.28E-02	.18E-01	.13E-01	.72E-01	.62E-01

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.10E+01	.20E+01	.30E+01
10.	.46323E-03	-.57E-02	-.19E-01	-.23E-01
20.	.71523E-04	.27E-01	.14E+00	.17E+00
30.	.44079E-04	.79E-01	.30E+00	.31E+00
40.	.96263E-05	-.34E+00	.51E+00	-.34E+00
50.	.45460E-05	-.63E+00	.55E+00	-.73E+00

Table C-XLI-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .16293E+01, .14473E+01, .37897E+01
KO = 3 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	.88575E-02	.15343E-02	.27010E-03	.48708E-04
20.	-.37383E-01	-.41188E-02	-.11377E-01	-.15379E-02
30.	-.27727E+00	-.39183E-01	-.50411E-01	-.68069E-02
40.	-.17105E+01	-.22092E+00	-.27364E+00	-.36850E-01
50.	-.55733E+01	-.87909E+00	-.43357E+00	-.92588E-01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	.42069E-04	.20033E-06	.12405E-04	-.20118E-05
20.	.10829E-03	-.85209E-07	.59238E-04	.52565E-07
30.	.16678E-03	-.14441E-05	.13156E-03	.53193E-05
40.	.15780E-03	-.63109E-05	.25015E-03	.58389E-05
50.	-.28177E-04	-.16438E-04	.41083E-03	-.25523E-05

Table C-XLI-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSI0= .16293E+01, .14473E+01, .37897E+01
K0 = 3 ITERATION # 2

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	0.	0.	0.	0.	0.	0.
10.	-.19E-01	.63E-04	-.33E-01	-.13E-03	-.33E-01	.96E-04
20.	.36E-02	-.17E-03	-.34E-02	-.86E-03	-.12E-01	.17E-03
30.	-.27E-03	-.88E-03	-.68E-02	-.31E-02	.17E-01	-.66E-03
40.	-.10E-01	-.31E-02	-.53E-01	-.79E-02	.48E-01	-.71E-02
50.	.29E-01	-.11E-03	-.70E-01	-.27E-02	.19E-01	-.13E-01

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	-.63E+00	.55E+00	-.79E+00
10.	.47208E-03	.82E-02	.34E-01	.46E-01
20.	.67445E-04	.36E-02	-.56E-02	-.92E-03
30.	.41744E-04	-.72E-03	-.39E-01	-.29E-01
40.	.89355E-05	.70E-01	-.79E-01	.73E-01
50.	.42682E-05	.14E+00	-.90E-01	.13E+00

Table C-XLII-a

```
MODEL     2      EPSITRUE=   .10000E+01,   .20000E+01,   .30000E+01
W/O  NOISE       EPSIO=      .66310E+00,   .20895E+01,   .28219E+01
KD   =    3      ITERATION # 3
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	0.	0.	0.	0.
10.	-.26346E-02	-.25751E-03	-.13999E-03	.33762E-05
20.	.43205E-02	.12762E-02	.16919E-02	.39499E-03
30.	.39057E-01	.10069E-01	.80014E-02	.15879E-02
40.	.29912E+00	.54353E-01	.51427E-01	.33844E-02
50.	.10235E+01	.21073E+00	.79711E-01	.21825E-01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	0.	0.	0.	0.
10.	-.12155E-04	-.40389E-07	-.35870E-05	.27124E-07
20.	-.29809E-04	.30074E-07	-.15727E-04	-.56206E-07
30.	-.44509E-04	.38959E-06	-.32912E-04	-.18346E-05
40.	-.43111E-04	.16346E-05	-.59370E-04	-.19527E-05
50.	-.26606E-05	.41012E-05	-.94142E-04	.56651E-05

Table C-XLII-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
W/O NOISE EPSIO= .86310E+00, .20395E+01, .23219E+01
KD = 3 ITERATION # 3

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	0.	0.	0.	0.	0.	0.
10.	.29E-02	.37E-05	.65E-02	.35E-04	.65E-02	-.43E-04
20.	-.11E-02	.46E-04	.73E-03	.23E-03	.23E-02	-.47E-04
30.	-.11E-03	.15E-03	.15E-02	.77E-03	-.32E-02	.17E-03
40.	.21E-02	.70E-03	.10E-01	.19E-02	-.89E-02	.17E-02
50.	-.55E-02	.11E-04	.13E-01	.62E-03	-.31E-02	.31E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.14E+00	-.90E-01	.18E+00
10.	.47352E-03	-.11E-02	-.60E-02	-.80E-02
20.	.68164E-04	.18E-03	.30E-02	.23E-02
30.	.42132E-04	.13E-02	.10E-01	.39E-02
40.	.89710E-05	-.16E-01	.19E-01	-.17E-01
50.	.42659E-05	-.31E-01	.22E-01	-.40E-01

Table C-XLIII-a

```
MODEL     2      EPSITRUE=  .10000E+01,  .20000E+01,  .30000E+01
WITH NOISE       EPSIO=     0.          , 0.          , 0.
KD    =   1      ITERATION # 1
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16894E+02	.17004E+02	.10649E+02	.10660E+02
20.	.70100E+01	.73817E+01	-.11887E+02	-.11821E+02
30.	-.13500E+02	-.11642E+02	-.11799E+02	-.11568E+02
40.	-.17788E+02	-.13091E+02	.10504E+02	.10857E+02
50.	-.24166E+02	-.13660E+02	-.12048E+02	-.11126E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.90309E-03	.13752E-02	.59938E-03	.72912E-03
20.	-.82665E-03	.22642E-03	-.17735E-02	-.12978E-02
30.	-.16888E-02	-.20617E-03	-.47448E-03	.35598E-03
40.	-.76823E-03	.80539E-03	-.82300E-03	.43234E-03
50.	-.10290E-02	.71117E-04	-.13624E-02	.34527E-03

Table C-XLIII-b

```
MODEL     2      EPSITRUE=   .10000E+01,   .20000E+01,   .30000E+01
WITH NOISE       EPSIO=      0.          , 0.          , 0.
KD   =    1      ITERATION # 1
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.20E-01	.43E-01	-.19E-01	-.27E-01	.67E-02	-.49E-03
20.	.44E-02	-.49E-03	-.40E-01	-.40E-01	.19E-01	.17E-01
30.	.73E-01	.30E-01	.64E-01	.60E-01	.22E-01	.21E-01
40.	.33E-01	.60E-02	.21E-01	.19E-01	.57E-01	.42E-01
50.	.97E-01	.64E-01	-.64E-01	-.72E-01	.78E-01	.68E-01

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.10E+01	.20E+01	.30E+01
10.	.17185E-04	-.11E+00	-.54E+00	-.75E+00
20.	.13880E-05	-.19E-01	-.43E-01	-.75E-01
30.	.37915E-06	-.65E-02	-.18E-01	-.41E-01
40.	.16511E-06	.13E-01	-.11E+00	-.15E+00
50.	.10745E-06	.79E-01	-.17E+00	-.14E+00

Table C-XLIV-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .92144E+00, .21588E+01, .31338E+01
KD = 1 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17012E+02	.17005E+02	.10660E+02	.10560E+02
20.	.73663E+01	.73393E+01	-.11828E+02	-.11833E+02
30.	-.11802E+02	-.11943E+02	-.11598E+02	-.11517E+02
40.	-.14150E+02	-.14574E+02	.10672E+02	.10531E+02
50.	-.17811E+02	-.18861E+02	-.11536E+02	-.11623E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.14030E-02	.13752E-02	.73697E-03	.72937E-03
20.	.28442E-03	.22357E-03	-.12678E-02	-.12948E-02
30.	-.13699E-03	-.22132E-03	.42004E-03	.37412E-03
40.	.84433E-03	.75411E-03	.50517E-03	.44057E-03
50.	.29948E-04	-.42153E-04	.41935E-03	.33674E-03

Table C-XLIV-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .92144E+00, .21688E+01, .31398E+01
KD = 1 ITERATION # 2

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.40E-01	.44E-01	-.26E-01	-.28E-01	.29E-02	.90E-03
20.	-.38E-02	-.17E-02	-.47E-01	-.47E-01	.20E-01	.19E-01
30.	.15E-01	.26E-01	.38E-01	.39E-01	.15E-01	.17E-01
40.	-.25E-01	-.12E-01	-.24E-01	-.26E-01	-.24E-02	.25E-02
50.	.57E-01	.63E-01	-.83E-01	-.85E-01	.12E-02	.42E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.79E-01	-.17E+00	-.14E+00
10.	.17346E-04	-.10E+00	-.54E+00	-.75E+00
20.	.13970E-05	-.19E-01	-.31E-01	-.63E-01
30.	.38091E-06	-.13E-01	.23E-01	.41E-02
40.	.16695E-06	-.12E-01	.71E-02	-.33E-02
50.	.10769E-06	-.19E-01	-.17E-01	-.23E-01

Table C-XLV-a

```
MODEL    2      EPSITRUE=  .10000E+01,  .20000E+01,  .30000E+01
WITH NOISE      EPSIO=     .10192E+01,  .20172E+01,  .30226E+01
KD   =   1      ITERATION # 3
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17006E+02	.17005E+02	.10660E+02	.10660E+02
20.	.73443E+01	.73409E+01	-.11832E+02	-.11833E+02
30.	-.11912E+02	-.11929E+02	-.11612E+02	-.11614E+02
40.	-.14463E+02	-.14505E+02	.10645E+02	.10541E+02
50.	-.18532E+02	-.18630E+02	-.11593E+02	-.11501E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13789E-02	.13752E-02	.73036E-03	.72934E-03
20.	.23199E-03	.22372E-03	-.12912E-02	-.12949E-02
30.	-.20903E-03	-.22063E-03	.38033E-03	.37381E-03
40.	.76859E-03	.75636E-03	.44982E-03	.44020E-03
50.	-.28282E-04	-.37177E-04	.34996E-03	.33703E-03

Table C-XLV-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .10192E+01, .20172E+01, .30225E+01
KD = 1 ITERATION # 3

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.45E-01	.44E-01	-.28E-01	-.28E-01	.87E-03	.85E-03
20.	-.30E-03	-.16E-02	-.47E-01	-.47E-01	.19E-01	.19E-01
30.	.27E-01	.26E-01	.40E-01	.40E-01	.17E-01	.17E-01
40.	-.10E-01	-.11E-01	-.23E-01	-.24E-01	.39E-02	.42E-02
50.	.63E-01	.63E-01	-.84E-01	-.84E-01	.66E-02	.67E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	-.19E-01	-.17E-01	-.23E-01
10.	.17342E-04	-.10E+00	-.54E+00	-.75E+00
20.	.13970E-05	-.19E-01	-.32E-01	-.64E-01
30.	.38094E-06	-.13E-01	.21E-01	.17E-02
40.	.15697E-06	-.10E-01	.16E-02	-.11E-01
50.	.10770E-06	-.15E-01	-.24E-01	-.23E-01

Table C-XLVI-a

```
MODEL     2      EPSITRUE=  .10000E+01,  .20000E+01,  .30000E+01
WITH NOISE       EPSIO=     0.         , 0.         , 0.
KO    =   2      ITERATION # 1
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16904E+02	.17004E+02	.10649E+02	.10560E+02
20.	.71201E+01	.73699E+01	-.11850E+02	-.11823E+02
30.	-.12550E+02	-.11594E+02	-.11689E+02	-.11552E+02
40.	-.16571E+02	-.12920E+02	.10403E+02	.10345E+02
50.	-.24759E+02	-.13960E+02	-.12228E+02	-.11183E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.96924E-03	.13752E-02	.61672E-03	.72919E-03
20.	-.63643E-03	.22589E-03	-.17215E-02	-.12971E-02
30.	-.14382E-02	-.20588E-03	-.48344E-03	.36770E-03
40.	-.49859E-03	.80561E-03	-.37817E-03	.43295E-03
50.	-.86363E-03	.66028E-04	-.13911E-02	.34523E-03

Table C-XLVI-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= 0. , 0. , 0.
KD = 2 ITERATION # 1

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.25E-01	.43E-01	-.23E-01	-.27E-01	.49E-02	-.79E-04
20.	.31E-02	-.10E-02	-.38E-01	-.39E-01	.19E-01	.17E-01
30.	.73E-01	.32E-01	.64E-01	.62E-01	.23E-01	.22E-01
40.	.57E-01	.16E-02	.12E-01	.11E-01	.44E-01	.35E-01
50.	.10E+00	.63E-01	-.58E-01	-.68E-01	.90E-01	.79E-01

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.10E+01	.20E+01	.30E+01
10.	.22566E-04	-.12E+00	-.63E+00	-.87E+00
20.	.19359E-05	-.22E-01	-.41E-01	-.75E-01
30.	.50420E-06	-.12E-01	-.35E-01	-.63E-01
40.	.22737E-06	.39E-02	-.10E+00	-.14E+00
50.	.13519E-06	.63E-01	-.16E+00	-.12E+00

Table C-XLVII-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .93705E+00, .21607E+01, .31221E+01
KJ = 2 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17011E+02	.17006E+02	.10660E+02	.10660E+02
20.	.73576E+01	.73404E+01	-.11830E+02	-.11833E+02
30.	-.11854E+02	-.11943E+02	-.11603E+02	-.11517E+02
40.	-.14226E+02	-.14574E+02	.10674E+02	.10533E+02
50.	-.17862E+02	-.18819E+02	-.11535E+02	-.11517E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.30217E-03	.30217E-03
10.	.13970E-02	.13752E-02	.73536E-03	.72937E-03
20.	.26908E-03	.22361E-03	-.12729E-02	-.12948E-02
30.	-.15754E-03	-.22127E-03	.41571E-03	.37406E-03
40.	.82245E-03	.75433E-03	.50048E-03	.44052E-03
50.	.14629E-04	-.41345E-04	.41153E-03	.33679E-03

Table C-XLVII-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .93705E+00, .21607E+01, .31221E+01
KD = 2 ITERATION # 2

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.42E-01	.44E-01	-.27E-01	-.28E-01	.23E-02	.88E-03
20.	-.39E-02	-.16E-02	-.47E-01	-.47E-01	.20E-01	.19E-01
30.	.15E-01	.26E-01	.39E-01	.39E-01	.15E-01	.17E-01
40.	-.21E-01	-.11E-01	-.24E-01	-.25E-01	-.53E-03	.31E-02
50.	.56E-01	.63E-01	-.83E-01	-.85E-01	.57E-03	.40E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.63E-01	-.16E+00	-.12E+00
10.	.22696E-04	-.11E+00	-.63E+00	-.97E+00
20.	.19476E-05	-.21E-01	-.29E-01	-.63E-01
30.	.50748E-06	-.16E-01	.22E-01	-.15E-02
40.	.22855E-06	-.15E-01	.64E-02	-.11E-01
50.	.13541E-06	-.22E-01	-.21E-01	-.31E-01

Table C-XLVIII-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE. EPSIO= .10222E+01, .20213E+01, .30310E+01
KO = 2 ITERATION # 3

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.32098E+01	.32098E+01
10.	.17006E+02	.17005E+02	.10650E+02	.10560E+02
20.	.73439E+01	.73409E+01	-.11832E+02	-.11833E+02
30.	-.11919E+02	-.11930E+02	-.11613E+02	-.11615E+02
40.	-.14470E+02	-.14511E+02	.10646E+02	.10641E+02
50.	-.18521E+02	-.18640E+02	-.11591E+02	-.11602E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13794E-02	.13752E-02	.73051E-03	.72934E-03
20.	.23271E-03	.22372E-03	-.12905E-02	-.12949E-02
30.	-.20795E-03	-.22067E-03	.38266E-03	.37382E-03
40.	.76980E-03	.75624E-03	.45380E-03	.44022E-03
50.	-.27657E-04	-.37406E-04	.35495E-03	.33701E-03

Table C-XLVIII-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .10222E+01, .20213E+01, .30310E+01
KD = 2 ITERATION # 3

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.44E-01	.44E-01	-.28E-01	-.28E-01	.82E-03	.85E-03
20.	-.51E-03	-.16E-02	-.47E-01	-.47E-01	.19E-01	.19E-01
30.	.27E-01	.26E-01	.40E-01	.40E-01	.17E-01	.17E-01
40.	-.10E-01	-.11E-01	-.24E-01	-.24E-01	.40E-02	.41E-02
50.	.63E-01	.63E-01	-.84E-01	-.84E-01	.63E-02	.64E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	-.22E-01	-.21E-01	-.31E-01
10.	.22634E-04	-.11E+00	-.63E+00	-.37E+00
20.	.19477E-05	-.21E-01	-.29E-01	-.64E-01
30.	.50752E-06	-.16E-01	.20E-01	-.45E-02
40.	.22858E-06	-.14E-01	.22E-02	-.15E-01
50.	.13543E-06	-.19E-01	-.26E-01	-.34E-01

Table C-XLIX-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= 0. , 0. , 0.
KD = 3 ITERATION # 1

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16894E+02	.17004E+02	.10649E+02	.10660E+02
20.	.69975E+01	.73838E+01	-.11891E+02	-.11821E+02
30.	-.13608E+02	-.11631E+02	-.11815E+02	-.11567E+02
40.	-.18140E+02	-.13041E+02	.10472E+02	.10865E+02
50.	-.24875E+02	-.13472E+02	-.12084E+02	-.11107E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.90418E-03	.13752E-02	.59367E-03	.72911E-03
20.	-.81688E-03	.22664E-03	-.17906E-02	-.12978E-02
30.	-.16609E-02	-.20543E-03	-.49797E-03	.36696E-03
40.	-.72384E-03	.80689E-03	-.84742E-03	.43244E-03
50.	-.98818E-03	.73904E-04	-.13861E-02	.34542E-03

Table C-XLIX-b

```
MODEL     2      EPSITRUE=   .10000E+01,   .20000E+01,   .30000E+01
WITH NOISE       EPSIO=      0.          , 0.          , 0.
KD   =    3      ITERATION # 1
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.19E-01	.43E-01	-.19E-01	-.27E-01	.65E-02	-.52E-03
20.	.40E-02	-.50E-03	-.39E-01	-.40E-01	.19E-01	.17E-01
30.	.72E-01	.31E-01	.64E-01	.61E-01	.22E-01	.21E-01
40.	.33E-01	.80E-02	.19E-01	.20E-01	.57E-01	.43E-01
50.	.39E-01	.65E-01	-.66E-01	-.71E-01	.79E-01	.69E-01

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.10E+01	.20E+01	.30E+01
10.	.17087E-04	-.11E+00	-.54E+00	-.75E+00
20.	.13868E-05	-.19E-01	-.42E-01	-.74E-01
30.	.38222E-06	-.57E-02	-.18E-01	-.42E-01
40.	.16810E-06	.14E-01	-.11E+00	-.15E+00
50.	.10811E-06	.85E-01	-.17E+00	-.14E+00

Table C-L-a

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .91491E+00, .21703E+01, .31359E+01
KD = 3 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17012E+02	.17005E+02	.10650E+02	.10560E+02
20.	.73674E+01	.73392E+01	-.11828E+02	-.11833E+02
30.	-.11794E+02	-.11943E+02	-.11597E+02	-.11617E+02
40.	-.14128E+02	-.14576E+02	.10674E+02	.10631E+02
50.	-.17754E+02	-.18868E+02	-.11533E+02	-.11523E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.14027E-02	.13752E-02	.73724E-03	.72937E-03
20.	.28334E-03	.22356E-03	-.12671E-02	-.12348E-02
30.	-.13932E-03	-.22136E-03	.42073E-03	.37412E-03
40.	.84113E-03	.75405E-03	.50545E-03	.44057E-03
50.	.27390E-04	-.42239E-04	.41895E-03	.33674E-03

Table C-L-b

```
MODEL      2      EPSITRUE=   .10000E+01,   .20000E+01,   .30000E+01
WITH NOISE        EPSIO=      .91491E+00,   .21703E+01,   .31359E+01
KD    =    3      ITERATION # 2
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.39E-01	.44E-01	-.26E-01	-.28E-01	.29E-02	.90E-03
20.	-.32E-02	-.17E-02	-.47E-01	-.47E-01	.20E-01	.19E-01
30.	.15E-01	.26E-01	.38E-01	.39E-01	.15E-01	.17E-01
40.	-.26E-01	-.12E-01	-.24E-01	-.26E-01	-.24E-02	.25E-02
50.	.56E-01	.63E-01	-.83E-01	-.85E-01	.12E-02	.42E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.85E-01	-.17E+00	-.14E+00
10.	.17247E-04	-.10E+00	-.54E+00	-.75E+00
20.	.13958E-05	-.18E-01	-.29E-01	-.61E-01
30.	.38399E-06	-.13E-01	.24E-01	.47E-02
40.	.16893E-06	-.11E-01	.75E-02	-.34E-02
50.	.10832E-06	-.19E-01	-.17E-01	-.23E-01

Table C-LI-a

```
MODEL     2     EPSITRUE=  .10000E+01,  .20000E+01,  .30000E+01
WITH NOISE      EPSIO=     .10191E+01,  .20173E+01,  .30227E+01
KO   =    3     ITERATION # 3
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17005E+02	.17005E+02	.10660E+02	.10660E+02
20.	.73444E+01	.73409E+01	-.11832E+02	-.11833E+02
30.	-.11911E+02	-.11929E+02	-.11612E+02	-.11614E+02
40.	-.14460E+02	-.14506E+02	.10645E+02	.10641E+02
50.	-.18527E+02	-.18632E+02	-.11593E+02	-.11502E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13789E-02	.13752E-02	.73041E-03	.72934E-03
20.	.23196E-03	.22372E-03	-.12910E-02	-.12949E-02
30.	-.20919E-03	-.22064E-03	.38055E-03	.37381E-03
40.	.76841E-03	.75635E-03	.45006E-03	.44020E-03
50.	-.28557E-04	-.37203E-04	.35021E-03	.33703E-03

Table C-LI-b

MODEL 2 EPSITRUE= .10000E+01, .20000E+01, .30000E+01
WITH NOISE EPSIO= .10191E+01, .20173E+01, .30227E+01
KD = 3 ITERATION # 3

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.45E-01	.44E-01	-.28E-01	-.28E-01	.87E-03	.85E-03
20.	-.30E-03	-.16E-02	-.47E-01	-.47E-01	.19E-01	.19E-01
30.	.27E-01	.26E-01	.40E-01	.40E-01	.17E-01	.17E-01
40.	-.10E-01	-.11E-01	-.23E-01	-.24E-01	.39E-02	.42E-02
50.	.63E-01	.63E-01	-.64E-01	-.64E-01	.66E-02	.67E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	-.19E-01	-.17E-01	-.23E-01
10.	.17244E-04	-.10E+00	-.54E+00	-.75E+00
20.	.13958E-05	-.18E-01	-.29E-01	-.52E-01
30.	.38402E-06	-.12E-01	.21E-01	.22E-02
40.	.16895E-06	-.10E-01	.20E-02	-.11E-01
50.	.10833E-06	-.14E-01	-.24E-01	-.23E-01

Table C-LII-a

```
MODEL     2      EPSTTRUE=  .50000E+01,   .10000E+02,   .15000E+02
WITH NOISE       EPSIO=      0.          , 0.          , 0.
KD    =   1      ITERATION # 1
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16401E+02	.16953E+02	.10590E+02	.10546E+02
20.	.53583E+01	.72169E+01	-.12143E+02	-.11816E+02
30.	-.20598E+02	-.11307E+02	-.12595E+02	-.11445E+02
40.	-.32690E+02	-.92048E+01	.38092E+01	.11574E+02
50.	-.50216E+02	.23144E+01	-.14107E+02	-.94391E+01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	-.98732E-03	.13734E-02	.80632E-04	.72932E-03
20.	-.50298E-02	.23664E-03	-.36867E-02	-.13080E-02
30.	-.75484E-02	-.13513E-03	-.38654E-02	.34190E-03
40.	-.68394E-02	.10287E-02	-.58710E-02	.40573E-03
50.	-.49991E-02	.50158E-03	-.91557E-02	.39252E-03

Table C-LII-b

```
MODEL     2      EPSITRUE=  .50000E+01,  .10000E+02,  .15000E+02
WITH NOISE       EPSIO=     0.        ,  0.        ,  0.
KD   =    1      ITERATION # 1
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	-.32E-01	.34E-01	.16E-01	-.26E-01	.27E-01	-.89E-02
20.	.16E-01	-.82E-02	-.11E-01	-.12E-01	.18E-01	.99E-02
30.	.24E+00	.31E-01	.16E+00	.14E+00	.44E-01	.38E-01
40.	.44E+00	.52E-01	.19E+00	.18E+00	.26E+00	.19E+00
50.	.22E+00	.59E-01	.80E-02	-.31E-01	.35E+00	.30E+00

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.50E+01	.10E+02	.15E+02
10.	.17185E-04	-.15E+00	-.43E+00	-.54E+00
20.	.13980E-05	-.10E+00	-.18E-01	.31E-01
30.	.37915E-06	-.92E-01	-.12E+00	-.99E-01
40.	.15511E-06	-.27E-01	-.51E+00	-.61E+00
50.	.10745E-06	.28E+00	-.69E+00	-.53E+00

Table C-LIII-a

```
MODEL     2       EPSITRUE=   .50000E+01,   .10000E+02,   .15000E+02
WITH NOISE        EPSIO=      .47204E+01,   .10693E+02,   .15528E+02
KD   =    1       ITERATION # 2
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17036E+02	.17006E+02	.10654E+02	.10560E+02
20.	.74676E+01	.73359E+01	-.11812E+02	-.11335E+02
30.	-.11363E+02	-.11985E+02	-.11545E+02	-.11525E+02
40.	-.12949E+02	-.14803E+02	.10779E+02	.10597E+02
50.	-.15030E+02	-.19621E+02	-.11318E+02	-.11590E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.14844E-02	.13751E-02	.75950E-03	.72953E-03
20.	.46129E-03	.22302E-03	-.11883E-02	-.12945E-02
30.	.10439E-03	-.22368E-03	.55423E-03	.37469E-03
40.	.10966E-02	.74669E-03	.69154E-03	.44141E-03
50.	.22682E-03	-.58294E-04	.65257E-03	.33587E-03

Table C-LIII-b

MODEL 2 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= .47204E+01, .10693E+02, .15523E+02
KD = 1 ITERATION # 2

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.33E-01	.44E-01	-.18E-01	-.28E-01	.10E-01	.89E-03
20.	-.26E-01	-.13E-02	-.47E-01	-.48E-01	.23E-01	.19E-01
30.	-.13E-01	.25E-01	.33E-01	.36E-01	.10E-01	.16E-01
40.	-.65E-01	-.15E-01	-.23E-01	-.32E-01	-.24E-01	-.20E-02
50.	.37E-01	.63E-01	-.77E-01	-.86E-01	-.16E-01	-.32E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.28E+00	-.69E+00	-.53E+00
10.	.17804E-04	-.94E-01	-.53E+00	-.74E+00
20.	.14293E-05	-.17E-01	-.32E-01	-.63E-01
30.	.38760E-06	-.13E-01	.30E-01	.15E-01
40.	.17017E-06	-.15E-01	.25E-01	.24E-01
50.	.10842E-06	-.35E-01	.89E-03	-.49E-02

Table C-LIV-a

```
MODEL     2      EPSITRUE=   .50000E+01,   .10000E+02,   .15000E+02
WITH NOISE       EPSIO=      .50353E+01,   .99991E+01,   .15005E+02
KD    =   1      ITERATION # 3
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17005E+02	.17005E+02	.10660E+02	.10660E+02
20.	.73420E+01	.73409E+01	-.11832E+02	-.11832E+02
30.	-.11925E+02	-.11927E+02	-.11614E+02	-.11614E+02
40.	-.14495E+02	-.14496E+02	.10642E+02	.10643E+02
50.	-.18602E+02	-.18597E+02	-.11599E+02	-.11598E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13757E-02	.13752E-02	.72944E-03	.72933E-03
20.	.22493E-03	.22374E-03	-.12945E-02	-.12949E-02
30.	-.21894E-03	-.22054E-03	.37467E-03	.37377E-03
40.	.75837E-03	.75668E-03	.44171E-03	.44016E-03
50.	-.35640E-04	-.36474E-04	.33946E-03	.33709E-03

Table C-LIV-b

```
MODEL      2        EPSITRUE=   .50000E+01,   .10000E+02,   .15000E+02
WITH NOISE          EPSIO=      .50353E+01,   .99991E+01,   .15005E+02
KD    =    1        ITERATION # 3
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.46E-01	.44E-01	-.28E-01	-.28E-01	.72E-03	.85E-03
20.	.53E-03	-.16E-02	-.47E-01	-.47E-01	.19E-01	.19E-01
30.	.29E-01	.26E-01	.40E-01	.40E-01	.17E-01	.17E-01
40.	-.73E-02	-.11E-01	-.23E-01	-.23E-01	.46E-02	.44E-02
50.	.54E-01	.63E-01	-.83E-01	-.84E-01	.72E-02	.71E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	-.35E-01	.99E-03	-.49E-02
10.	.17799E-04	-.95E-01	-.53E+00	-.74E+00
20.	.14292E-05	-.18E-01	-.32E-01	-.65E-01
30.	.38768E-06	-.12E-01	.21E-01	.15E-02
40.	.17023E-06	-.10E-01	.44E-03	-.13E-01
50.	.10848E-06	-.14E-01	-.25E-01	-.30E-01

Table C-LV-a

```
MODEL      2      EPSITRUE=  .50000E+01,   .10000E+02,   .15000E+02
WITH NOISE        EPSIO=     0.          , 0.          , 0.
KD    =    2      ITERATION # 1
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16466E+02	.16969E+02	.10599E+02	.10650E+02
20.	.60000E+01	.72492E+01	-.11955E+02	-.11819E+02
30.	-.15797E+02	-.11019E+02	-.12060E+02	-.11373E+02
40.	-.26589E+02	-.83362E+01	.93172E+01	.11530E+02
50.	-.52808E+02	.11897E+01	-.14966E+02	-.97406[illegible]+01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	-.65616E-03	.13737E-02	.16728E-03	.72962E-03
20.	-.40783E-02	.23331E-03	-.34265E-02	-.13042E-02
30.	-.62995E-02	-.13694E-03	-.39089E-02	.34678E-03
40.	-.54979E-02	.10231E-02	-.61453E-02	.41026E-03
50.	-.41726E-02	.47569E-03	-.82990E-02	.38263E-03

Table C-LV-b

```
MODEL     2      EPSITRUE=  .50000E+01,  .10000E+02,  .15000E+02
WITH NOISE       EPSIO=     0.         , 0.         , 0.
KO   =    2      ITERATION # 1
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	-.54E-01	.37E-01	-.58E-02	-.27E-01	.19E-01	-.56E-02
20.	.38E-02	-.12E-01	-.47E-02	-.78E-02	.19E-01	.91E-02
30.	.24E+00	.40E-01	.16E+00	.15E+00	.46E-01	.42E-01
40.	.31E+00	.36E-01	.15E+00	.14E+00	.20E+00	.15E+00
50.	.24E+00	.54E-01	.37E-01	-.94E-02	.41E+00	.35E+00

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.50E+01	.10E+02	.15E+02
10.	.22566E-04	-.17E+00	-.56E+00	-.73E+00
20.	.19359E-05	-.10E+00	-.16E-01	.75E-02
30.	.50420E-06	-.10E+00	-.17E+00	-.13E+00
40.	.22737E-06	-.56E-01	-.46E+00	-.55E+00
50.	.13519E-06	.22E+00	-.64E+00	-.43E+00

Table C-LVI-a

MODEL 2 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= .47802E+01, .10642E+02, .15432E+02
KO = 2 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17028E+02	.17007E+02	.10663E+02	.10660E+02
20.	.74213E+01	.73389E+01	-.11821E+02	-.11834E+02
30.	-.11587E+02	-.11987E+02	-.11566E+02	-.11626E+02
40.	-.13292E+02	-.14791E+02	.10777E+02	.10506E+02
50.	-.15399E+02	-.19421E+02	-.11327E+02	-.11667E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.14570E-02	.13751E-02	.75210E-03	.72952E-03
20.	.39347E-03	.22320E-03	-.12124E-02	-.12946E-02
30.	.13473E-04	-.22336E-03	.52909E-03	.37464E-03
40.	.10006E-02	.74790E-03	.66063E-03	.44136E-03
50.	.15863E-03	-.54449E-04	.60615E-03	.33628E-03

Table C-LVI-b

```
MODEL      2     EPSITRUE=   .50000E+01,   .10000E+02,   .15000E+02
WITH NOISE       EPSIO=      .47802E+01,   .10642E+02,   .15432E+02
KD    =    2     ITERATION # 2
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.40E-01	.44E-01	-.21E-01	-.28E-01	.76E-02	.90E-03
20.	-.25E-01	-.17E-02	-.49E-01	-.48E-01	.23E-01	.19E-01
30.	-.72E-02	.25E-01	.36E-01	.36E-01	.10E-01	.16E-01
40.	-.46E-01	-.13E-01	-.22E-01	-.29E-01	-.16E-01	.14E-03
50.	.34E-01	.63E-01	-.74E-01	-.87E-01	-.18E-01	-.31E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.22E+00	-.64E+00	-.43E+00
10.	.23100E-04	-.10E+00	-.62E+00	-.87E+00
20.	.19914E-05	-.20E-01	-.30E-01	-.64E-01
30.	.52016E-06	-.16E-01	.31E-01	.11E-01
40.	.23300E-06	-.17E-01	.20E-01	.99E-02
50.	.13595E-06	-.34E-01	-.84E-02	-.20E-01

Table C-LVII-a

```
MODEL     2       EPSITRUE=    .50000E+01,   .10000E+02,   .15000E+02
WITH NOISE        EPSIO=       .50344E+01,   .10008E+02,   .15020E+02
KD   =    2       ITERATION # 3
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17006E+02	.17005E+02	.10660E+02	.10660E+02
20.	.73433E+01	.73409E+01	-.11832E+02	-.11832E+02
30.	-.11922E+02	-.11929E+02	-.11614E+02	-.11614E+02
40.	-.14484E+02	-.14505E+02	.10644E+02	.10642E+02
50.	-.18566E+02	-.18621E+02	-.11595E+02	-.11600E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13776E-02	.13752E-02	.72999E-03	.72933E-03
20.	.22894E-03	.22373E-03	-.12924E-02	-.12949E-02
30.	-.21311E-03	-.22061E-03	.37906E-03	.37379E-03
40.	.76432E-03	.75644E-03	.44855E-03	.44019E-03
50.	-.31714E-04	-.36973E-04	.34841E-03	.33704E-03

Table C-LVII-b

```
MODEL       2        EPSITRUE=   .50000E+01,   .10000E+02,   .15000E+02
WITH NOISE           EPSIO=      .50344E+01,   .10008E+02,   .15020E+02
KD    =     2        ITERATION # 3
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.45E-01	.44E-01	-.28E-01	-.28E-01	.72E-03	.85E-03
20.	.56E-03	-.16E-02	-.47E-01	-.47E-01	.19E-01	.19E-01
30.	.28E-01	.26E-01	.40E-01	.40E-01	.17E-01	.17E-01
40.	-.89E-02	-.11E-01	-.23E-01	-.24E-01	.44E-02	.43E-02
50.	.64E-01	.63E-01	-.83E-01	-.84E-01	.69E-02	.67E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	-.34E-01	-.84E-02	-.20E-01
10.	.23097E-04	-.10E+00	-.62E+00	-.85E+00
20.	.19913E-05	-.21E-01	-.31E-01	-.67E-01
30.	.52024E-06	-.16E-01	.20E-01	-.46E-02
40.	.23308E-06	-.14E-01	.13E-02	-.13E-01
50.	.13603E-06	-.18E-01	-.28E-01	-.37E-01

AD-A055 188 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCH--ETC F/G 17/7
APPLICATION OF A MAXIMUM LIKELIHOOD PARAMETER ESTIMATOR TO AN A--ETC(U)
DEC 77 R DAYAN
UNCLASSIFIED AFIT/GGC/EE/77-3 NL

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DDC

Table C-LVIII-a

```
MODEL      2      EPSITRUE=  .50000E+01,  .10000E+02,  .15000E+02
WITH NOISE        EPSIO=     0.          , 0.          , 0.
KD   =     3      ITERATION # 1
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.16403E+02	.16954E+02	.10591E+02	.10546E+02
20.	.52964E+01	.72281E+01	-.12163E+02	-.11814E+02
30.	-.21135E+02	-.11248E+02	-.12677E+02	-.11436E+02
40.	-.34434E+02	-.89399E+01	.96501E+01	.11615E+02
50.	-.53705E+02	.33074E+01	-.14285E+02	-.33958E+01

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	-.98180E-03	.13734E-02	.52075E-04	.72927E-03
20.	-.49796E-02	.23798E-03	-.37717E-02	-.13081E-02
30.	-.74098E-02	-.13139E-03	-.39825E-02	.34216E-03
40.	-.66186E-02	.10350E-02	-.59928E-02	.40651E-03
50.	-.47950E-02	.51443E-03	-.82747E-02	.38312E-03

Table C-LVIII-b

```
MODEL     2       EPSITRUE=  .50000E+01,  .10000E+02,  .15000E+02
WITH NOISE        EPSIO=     0.          , 0.          , 0.
KD   =    3       ITERATION # 1
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	-.96E-01	.34E-01	.12E-01	-.26E-01	.26E-01	-.89E-02
20.	.14E-01	-.82E-02	-.96E-02	-.11E-01	.17E-01	.10E-01
30.	.24E+00	.32E-01	.16E+00	.14E+00	.46E-01	.39E-01
40.	.44E+00	.62E-01	.18E+00	.18E+00	.26E+00	.19E+00
50.	.23E+00	.69E-01	-.68E-03	-.25E-01	.35E+00	.30E+00

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.50E+01	.10E+02	.15E+02
10.	.17087E-04	-.15E+00	-.43E+00	-.54E+00
20.	.13868E-05	-.10E+00	-.17E-01	.27E-01
30.	.38222E-06	-.91E-01	-.13E+00	-.11E+00
40.	.16310E-06	-.23E-01	-.52E+00	-.63E+00
50.	.10811E-06	.31E+00	-.70E+00	-.52E+00

Table C-LIX-a

MODEL 2 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= .46933E+01, .10701E+02, .15523E+02
KO = 3 ITERATION # 2

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.32098E+01	.32098E+01
10.	.17036E+02	.17006E+02	.10664E+02	.10660E+02
20.	.74719E+01	.73355E+01	-.11810E+02	-.11835E+02
30.	-.11333E+02	-.11988E+02	-.11541E+02	-.11625E+02
40.	-.12863E+02	-.14813E+02	.10786E+02	.10595E+02
50.	-.14851E+02	-.19656E+02	-.11308E+02	-.11594E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.14842E-02	.13751E-02	.76083E-03	.72954E-03
20.	.45839E-03	.22297E-03	-.11847E-02	-.12345E-02
30.	.97910E-04	-.22384E-03	.55859E-03	.37470E-03
40.	.10869E-02	.74636E-03	.69521E-03	.44140E-03
50.	.21817E-03	-.58858E-04	.65461E-03	.33587E-03

Table C-LIX-b

```
MODEL      2        EPSITRUE=   .50000E+01,   .10000E+02,   .15000E+02
WITH NOISE          EPSIO=      .46933E+01,   .10701E+02,   .15523E+02
KD    =    3        ITERATION # 2
```

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.31E-01	.44E-01	-.19E-01	-.26E-01	.10E-01	.90E-03
20.	-.28E-01	-.18E-02	-.47E-01	-.48E-01	.23E-01	.19E-01
30.	-.15E-01	.25E-01	.33E-01	.35E-01	.10E-01	.16E-01
40.	-.58E-01	-.15E-01	-.22E-01	-.32E-01	-.24E-01	-.20E-02
50.	.35E-01	.63E-01	-.76E-01	-.86E-01	-.16E-01	-.34E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	.31E+00	-.70E+00	-.52E+00
10.	.17721E-04	-.94E-01	-.53E+00	-.75E+00
20.	.14284E-05	-.16E-01	-.29E-01	-.60E-01
30.	.39072E-06	-.12E-01	.31E-01	.15E-01
40.	.17211E-06	-.15E-01	.26E-01	.25E-01
50.	.10889E-06	-.36E-01	.85E-03	-.54E-02

Table C-LX-a

```
MODEL    2     EPSITRUE=  .50000E+01,  .10000E+02,  .15000E+02
WITH NOISE     EPSIO=     .50362E+01,  .99992E+01,  .15005E+02
KD   =   3     ITERATION # 3
```

TIME	DELTA(R)	HOT(R)	DELTA(RD)	HOT(RD)
0.	.11254E+02	.11254E+02	.92098E+01	.92098E+01
10.	.17005E+02	.17005E+02	.10660E+02	.10660E+02
20.	.73420E+01	.73409E+01	-.11832E+02	-.11832E+02
30.	-.11924E+02	-.11927E+02	-.11614E+02	-.11614E+02
40.	-.14495E+02	-.14497E+02	.10642E+02	.10643E+02
50.	-.18602E+02	-.18598E+02	-.11599E+02	-.11598E+02

TIME	DELTA(AZ.)	HOT(AZ.)	DELTA(EL.)	HOT(EL.)
0.	-.15561E-02	-.15561E-02	.90217E-03	.90217E-03
10.	.13757E-02	.13752E-02	.72947E-03	.72933E-03
20.	.22497E-03	.22374E-03	-.12944E-02	-.12949E-02
30.	-.21876E-03	-.22054E-03	.37484E-03	.37377E-03
40.	.75853E-03	.75667E-03	.44196E-03	.44016E-03
50.	-.35575E-04	-.36493E-04	.33981E-03	.33708E-03

Table C-LX-b

MODEL 2 EPSITRUE= .50000E+01, .10000E+02, .15000E+02
WITH NOISE EPSIO= .50362E+01, .99992E+01, .15005E+02
KO = 3 ITERATION # 3

TIME	DELTA(SF1)	HOT(SF1)	DELTA(SF2)	HOT(SF2)	DELTA(SF3)	HOT(SF3)
0.	-.11E-01	-.11E-01	.35E-01	.35E-01	-.30E-01	-.30E-01
10.	.46E-01	.44E-01	-.28E-01	-.28E-01	.72E-03	.85E-03
20.	.58E-03	-.16E-02	-.47E-01	-.47E-01	.19E-01	.19E-01
30.	.29E-01	.26E-01	.40E-01	.40E-01	.17E-01	.17E-01
40.	-.73E-02	-.11E-01	-.23E-01	-.23E-01	.46E-02	.44E-02
50.	.54E-01	.63E-01	-.83E-01	-.84E-01	.72E-02	.71E-02

TIME	TRACE OF DISPERSION MATRIX	ESTIMATE ERROR IN PSI	ESTIMATE ERROR IN TETA	ESTIMATE ERROR IN PHI
0.	0.	-.36E-01	.85E-03	-.54E-02
10.	.17716E-04	-.95E-01	-.53E+00	-.75E+00
20.	.14284E-05	-.18E-01	-.30E-01	-.63E-01
30.	.39081E-06	-.12E-01	.21E-01	.22E-02
40.	.17219E-06	-.10E-01	.82E-03	-.13E-01
50.	.10896E-06	-.14E-01	-.25E-01	-.30E-01

Appendix D

Computer Flowcharts and Programs

This Appendix contains the computer listing and an abbreviated flowchart of the programs utilized in the study. The main program, SIMUL 1 or 2, initializes the misalignment angles and the appropriate transformation matrix, the random number generator, the position, velocity and specific forces and evaluates the states for the Truth model. Using subroutine MEASURE the output vector is then generated, noise is added to it (using subroutine NOIZE) and it is recorded on Tape 5.

Model 1 takes the specific force values from Tape 5 and uses them as inputs to the model. An estimated output vector is then generated ($\underline{\hat{Rdr}}$) and compared to the true one ($\underline{Rdr}$) read from Tape 5 in subroutine INFOMAT. The state sensitivities and the information matrix components are generated in the main program using the integration routine EULINT. Then, using the output sensitivities generated in subroutine SENSOUT and the output differences (D_{out}) the gradient vector is generated and the optimal misalignment angles correction is calculated.

Model 2 is very similar except for the fact that it generates estimated specific forces ($\underline{\hat{SF}}$) itself and uses all the values from Tape 5 as true output values to be compared to the estimated ones that it evaluates.

The flowcharts that follow are for the Truth Model, Model 1 and Model 2. The listings are of the programs SIMUL 1 (Truth Model and Model 1) and SIMUL 2 (Truth Model and Model 2).

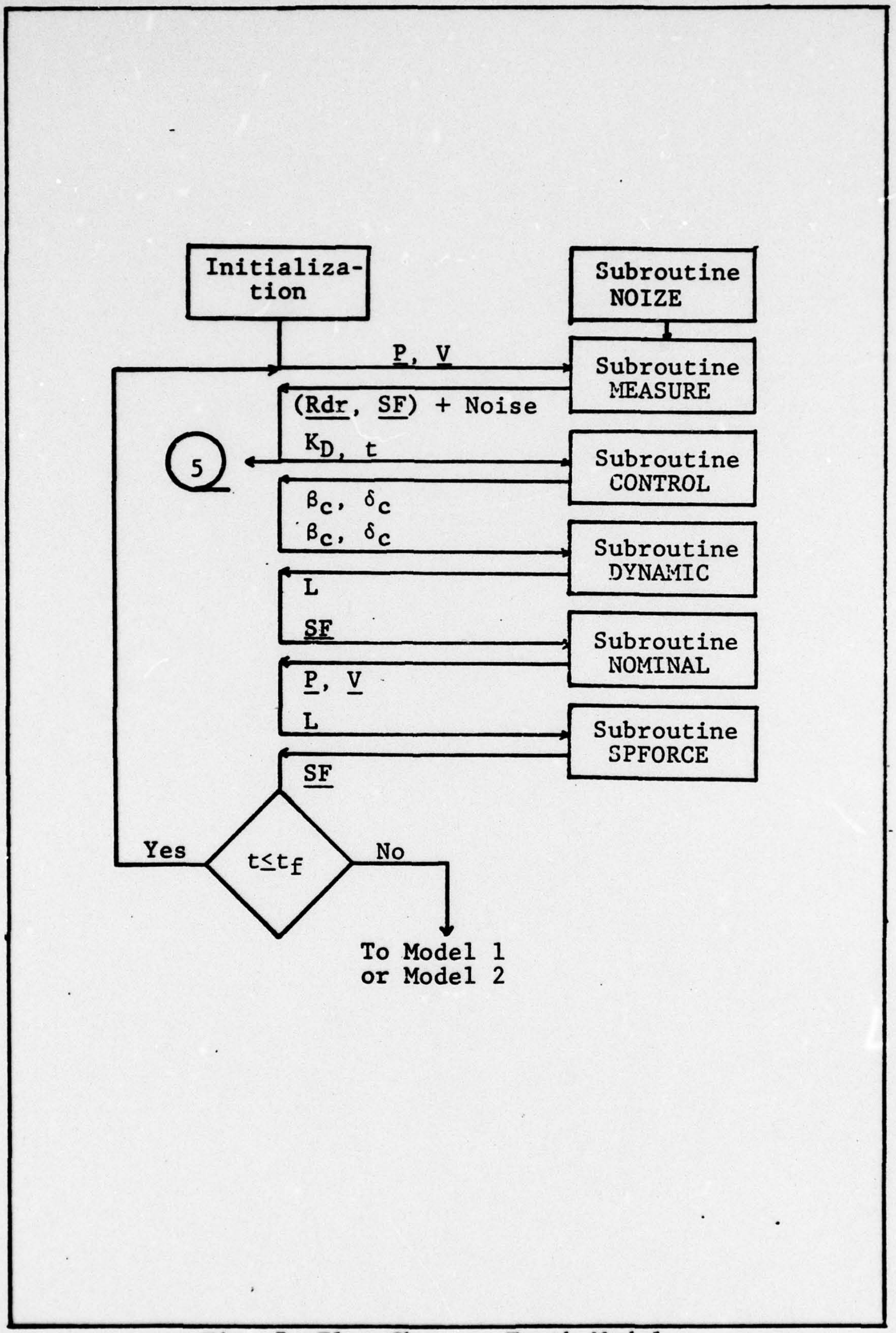

Fig. 7 Flow Chart: Truth Model

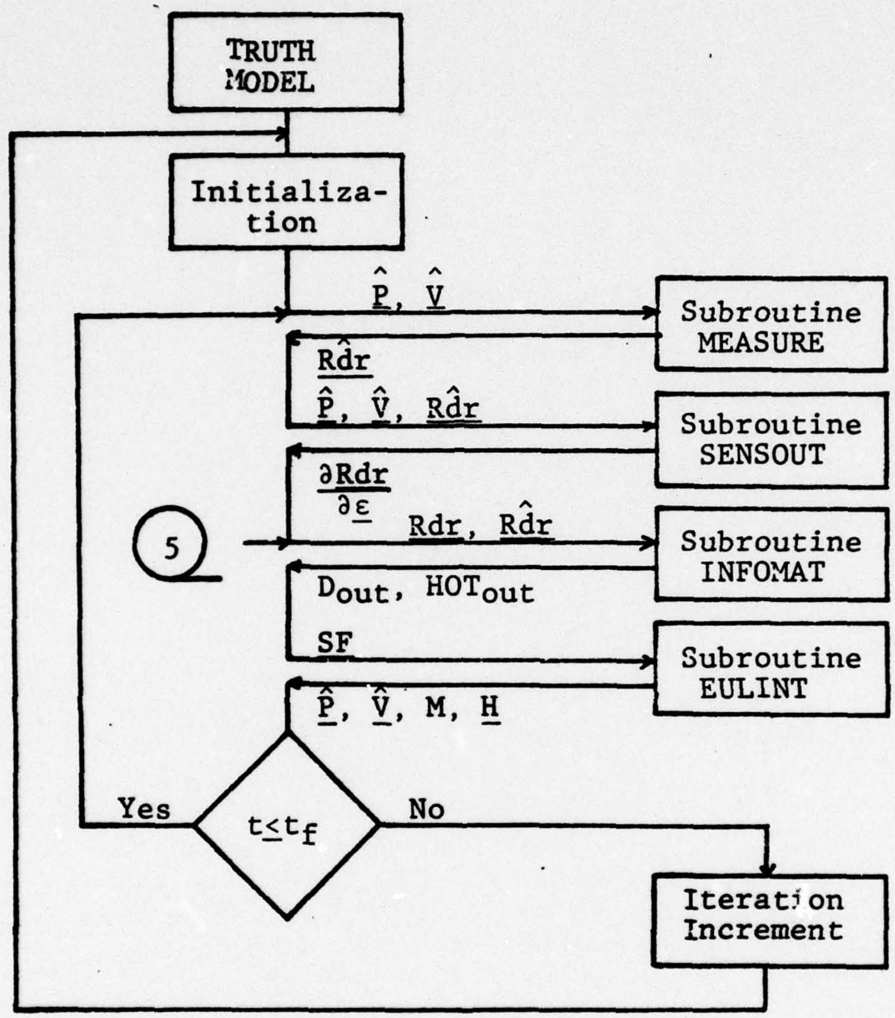

Fig. 8 Flow Chart: Model 1

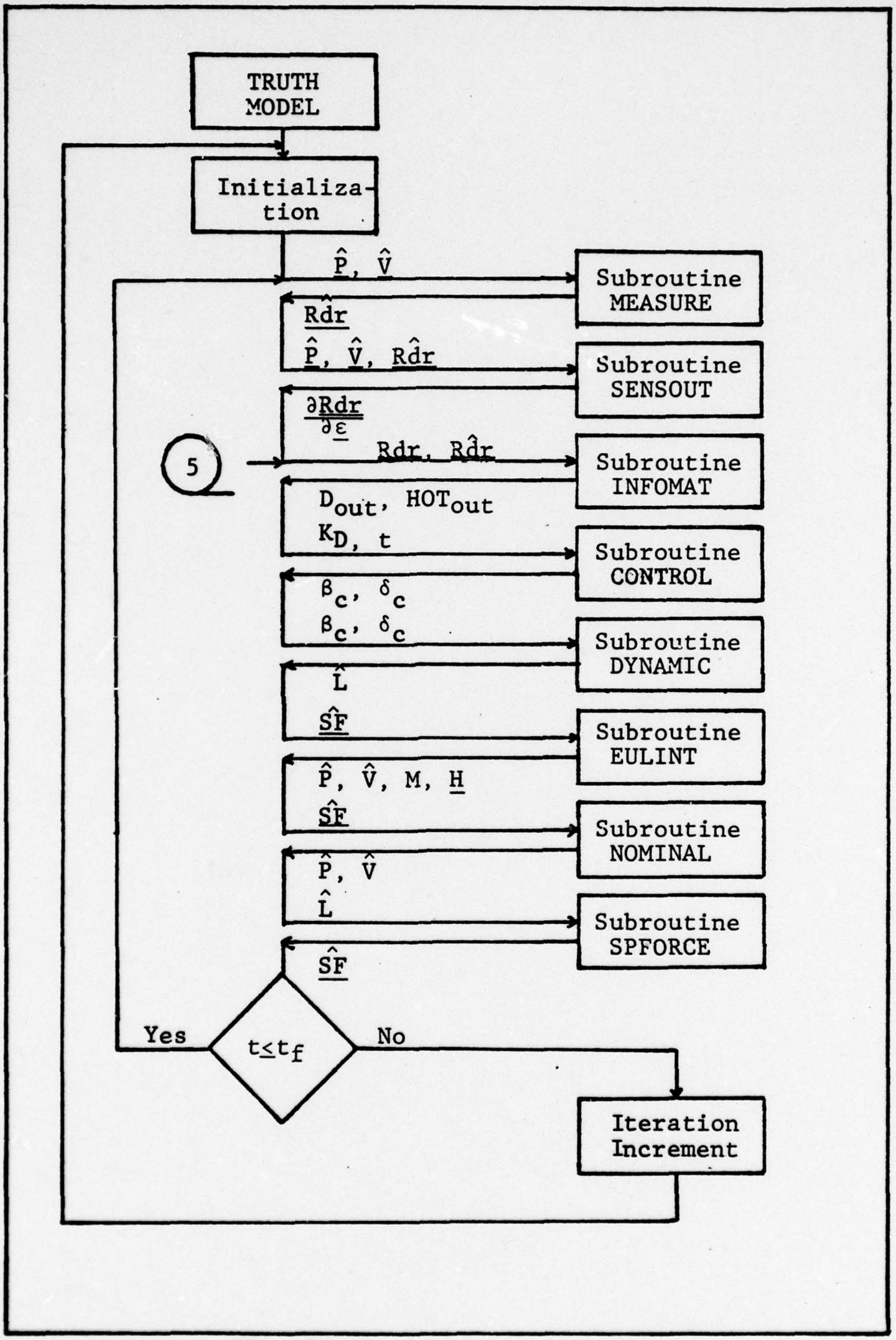

Fig. 9 Flow Chart: Model 2

```
      PROGRAM SIMUL1(INPUT,OUTPUT=10023,TAPE5=10023,TAPE10=OUTPUT)
*               ********************************
*PROGRAM SIMUL IS A SIMULATION OF THE TRACKING AND CONTROL OF
*AN AIR-TO-GROUND MISSILE USING MAXIMUM LIKELIHOOD ESTIMATION
*OF THE THREE ANGLES REPRESENTING ITS INITIAL MISALIGNMENT.
*               ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/EPSTRUE/PSIT,TETAT,PHIT
      COMMON/EPSIO/PSIO,TETAO,PHIO
      COMMON/CONTROL/KD,BETAC,DELTAC
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/INITIAL/XO,YO,ZO,VYO,VYO,VZO
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/SENSPDR/SPEPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/NOISVAR/EN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      COMMON/LOGIC/MODE1
      LOGICAL MODE1
      DIMENSION U(36),WKAREA(18),MI(3,3),TYME(6),DELTOUT(18,4),HOTOUT(
     $18,4),TRD(18),DPSI(18),DTETA(18),DPHI(18),PSIP(3),TETAP(3),PHIP
     $(3),DOUT(4,1),HOT(4),TAYLCR(4,1),EPS(3,1)
      REAL M,LIFT,L,MI
*               ********************************
*INITIALIZATION OF THE RANDOM NUMBER GENERATOR FOR NOISE GENERATION.
*               ********************************
      PRINT(10,98)
      JJ=1
      CALL RANSET(JJ)
      T=0.
      DPRNT=0.
      PRINT*,"***TRUTH MODEL ***"
      MODE1=.FALSE.
*               ********************************
*INITIALIZATION OF THE MISALIGNMENT ANGLES WITH VALUES ASSUMED TO BE
*TRUE.
*               ********************************
      PSI=PSIT
      TETA=TETAT
      PHI=PHIT
      APSI=ATETA=APHI=0.
*               ********************************
*DEFINITION OF THE MISALIGNMENT ANGLES TRANSFORMATION MATRIX.
*
*           1.      -PSI    TETA
*   CMA=    PSI     1.      -PHI
*           -TETA   PHI     1.
*
*               ********************************
      CMA(1,1)=CMA(2,2)=CMA(3,3)=1.
```

```
      CMA(2,1)=PSI
      CMA(3,1)=-TETA
      CMA(3,2)=PHI
      CMA(1,2)=-CMA(2,1)
      CMA(1,3)=-CMA(3,1)
      CMA(2,3)=-CMA(3,2)
*                   ******************************
*INITIALIZATION OF POSITION,VELOCITY,AND SPECIFIC FORCE.
*                   ******************************
      X=X0
      Y=Y0
      Z=Z0
      VX=VX0
      VY=VY0
      VZ=VZ0
      V=SQRT(VX*VX+VY*VY+VZ*VZ)
      SF(1,1)=0.
      SF(2,1)=0.
      SF(3,1)=0.
*                   ******************************
*EVALUATION OF THE VALUE OF THE STATES FOR THE TRUTH MODEL .
*                   ******************************
   10 CALL MEASURE
      WRITE(5)T,(OUT(I,1),I=1,7)
      CALL CONTROL
      FD=-1.
      CALL DYNAMIC
      FD=0.
      CALL NOMINAL
      CALL SPFORCE
   20 IF(ABS(T-DPRNT).LT.1.E-06)GO TO 40
   30 T=T+DT
      IF(T.LE.TF)GO TO 10
      GO TO 50
   40 PRINT(10,99)KD,BETA,DELTA,C,ALPHA,L,X,Y,Z,V,VX,VY,VZ,T,BETAC,
     $DELTAC,SF(1,1),SF(2,1),SF(3,1),OUT(1,1),OUT(2,1),OUT(3,1),OUT(4,1)
     $,ACC(1,1),ACC(2,1),ACC(3,1)
      PRINT(10,79)T,(OUT(I,1),I=1,7)
      DPRNT=DPRNT+DPRT
      GO TO 30
   50 REWIND 5
      PRINT*,"***MODEL 1***"
      MODE1=.TRUE.
      PRINT*,"MODE1=",MODE1
      PSI=PSIO
      TETA=TETAO
      PHI=PHIO
      DO 110 NN=1,3
      N=NN
      T=0.
      DPRNT=0.
*                   ******************************
*INITIALIZATION OF THE MISALIGNMENT ANGLES WITH FIRST ESTIMATED
*NOMINAL VALUES.
*                   ******************************
      PSI=PSI+APSI/1000
      TETA=TETA+ATETA/1000
      PHI=PHI+APHI/1000
      PSIP(N)=1000*PSI
      TETAP(N)=1000*TETA
      PHIP(N)=1000*PHI
      CMA(1,1)=CMA(2,2)=CMA(3,3)=1.
      CMA(2,1)=PSI
      CMA(3,1)=-TETA
      CMA(3,2)=PHI
      CMA(1,2)=-CMA(2,1)
```

```
      CMA(1,3)=-CMA(3,1)
      CMA(2,3)=-CMA(3,2)
*              ******************************
*INITIALIZATION OF POSITION,VELOCITY,SPECIFIC FORCE,AERODYNAMIC
*VALUES,INFORMATION AND DISPERSION MATRIX ELEMENTS AND TRACE,
*AND SENSITIVITIES.
*              ******************************
      U(1)=X0
      U(2)=Y0
      U(3)=Z0
      U(4)=VX0
      U(5)=VY0
      U(5)=VY0
      U(6)=VZ0
      SF(1,1)=0.
      SF(2,1)=0.
      SF(3,1)=0.
      BETA=0.
      DELTA=0.
      Q=0.
      ALPHA=0.
      L=0.
      DO 55 I=1,3
      DO 55 J=1,3
      SFPSP(I,J)=0.
      SFPSV(I,J)=0.
      M(I,J)=0.
      MI(I,J)=0.
   55 CONTINUE
      DO 60 I=7,36
      U(I)=0.
   60 CONTINUE
      OUTN(1,1)=1./(RN*RN)
      OUTN(2,2)=1./(RDN*RDN)
      OUTN(3,3)=1./(PSIRN*PSIRN)
      OUTN(4,4)=1./(TETRN*TETRN)
      OUTN(1,2)=OUTN(1,3)=OUTN(1,4)=0.
      OUTN(2,1)=OUTN(2,3)=OUTN(2,4)=0.
      OUTN(3,1)=OUTN(3,2)=OUTN(3,4)=0.
      OUTN(4,1)=OUTN(4,2)=OUTN(4,3)=0.
      DO 100 KK=1,5001
      READ(5)T,(OUTRU(I),I=1,7)
*              ******************************
*                          STATES  DEFINITION
*                          ------------------
*POSITION:(1-3)
*     U(1)=X,    U(2)=Y,    U(3)=Z,
*VELOCITY:(4-6)
*     U(4)=VX,   U(5)=VY,   U(6)=VZ,
*POSITION SENSITIVITY:(7-15)
*     U(7)=X/PSI,    U(8)=X/TETA,    U(9)=X/PHI,
*     U(10)=Y/PSI,   U(11)=Y/TETA,   U(12)=Y/PHI,
*     U(13)=Z/PSI,   U(14)=Z/TETA,   U(15)=Z/PHI,
*VELOCITY SENSITIVITY:(16-24)
*     U(16)=VX/PSI,   U(17)=VX/TETA,   U(18)=VX/PHI,
*     U(19)=VY/PSI,   U(20)=VY/TETA,   U(21)=VZ/PHI,
*     U(22)=VZ/PSI,   U(23)=VZ/TETA,   U(24)=VZ/PHI,
*INFORMATION MATRIX:(25-33)
*          U(25)   U(28)   U(31)
*     M=   U(26)   U(29)   U(32)
*          U(27)   U(30)   U(33)
*TRANSPOSE OF GRADIENT VECTOR:(34-36)
*     H=   U(34)   U(35)   U(36)
*              ******************************
      X=U(1)
      Y=U(2)
```

```
      Z=U(3)
      VX=U(4)
      VY=U(5)
      VZ=U(6)
*              ********************************
*USING THE RECEIVED SPECIFIC FORCE AS INPUTS TO MODEL 1.
*              ********************************
      DO 70 I=1,3
      SF(I,1)=OUTRU(I+4)
   70 CONTINUE
      CALL MEASURE
      DO 80 I=1,3
      DO 80 J=1,3
      SEPSP(I,J)=U(3*I+J+3)
      SEPSV(I,J)=U(3*I+J+12)
   80 CONTINUE
      IF((KK-1)-1000*((KK-1)/1000).EQ.0)PRINT(10,87)((SEPSP(I,J),J=1,3),
     $I=1,3),((SEPSV(I,J),J=1,3),I=1,3)
      CALL SENSOUT
      CALL INFOMAT
      DO 65 I=1,4
      DOUT(I,1)=OUTRU(I,1)-OUT(I,1)
   65 CONTINUE
      EPS(1,1)=PSIT-PSI
      EPS(2,1)=TETAT-TETA
      EPS(3,1)=PHIT-PHI
      CALL MATM(SOUTEPS,EPS,TAYLOR,4,3,1)
      DO 75 I=1,4
      HOT(I)=DOUT(I,1)-TAYLOR(I,1)
   75 CONTINUE
      FD=0.
      K=36
      CALL EULINT(U,K,.01)
      IF((KK-1)-1000*((KK-1)/1000).EQ.0)PRINT(10,81)DOUT,HOT
      IF((KK-1)-1000*((KK-1)/1000).EQ.0)PRINT(10,79)T,(OUTRU(I),I=1,7)
      IF((KK-1)-1000*((KK-1)/1000).NE.0)GO TO 83
      TYME((KK-1)/1000+1)=(KK-1)/100.
      DO 82 I=1,4
      DELTOUT((6*(N-1)+(KK-1)/1000+1),I)=DOUT(I,1)
      HOTOUT((6*(N-1)+(KK-1)/1000+1),I)=HOT(I)
   82 CONTINUE
   83 DO 90 J=1,3
      DO 90 I=1,3
      M(I,J)=U(21+I+3*J)
   90 CONTINUE
      IF(KK.EQ.1)GO TO 100
      IF((KK-1)-1000*((KK-1)/1000).NE.0)GO TO 100
      CALL LINV2F(M,3,3,MI,10,WKAREA,IER)
      TRD(6*(N-1)+(KK-1)/1000+1)=MI(1,1)+MI(2,2)+MI(3,3)
      PRINT(10,78)TRD(6*(N-1)+(KK-1)/1000+1)
      APSI=(MI(1,1)*U(34)+MI(1,2)*U(35)+MI(1,3)*U(36))*1000
      ATETA=(MI(2,1)*U(34)+MI(2,2)*U(35)+MI(2,3)*U(36))*1000
      APHI=(MI(3,1)*U(34)+MI(3,2)*U(35)+MI(3,3)*U(36))*1000
      DPSI(6*(N-1)+(KK-1)/1000+1)=(PSIT-PSI)*1000-APSI
      DTETA(6*(N-1)+(KK-1)/1000+1)=(TETAT-TETA)*1000-ATETA
      DPHI(6*(N-1)+(KK-1)/1000+1)=(PHIT-PHI)*1000-APHI
      PRINT(10,92)KD,N,DPSI(6*(N-1)+(KK-1)/1000+1),DTETA(6*(N-1)+
     $(KK-1)/1000+1),DPHI(6*(N-1)+(KK-1)/1000+1)
  100 CONTINUE
      REWIND 5
  110 CONTINUE
      PSITP=1000*PSIT
      TETATP=1000*TETAT
      PHITP=1000*PHIT
      DO 15 N=1,3
      PRINT(10,1)PSITP,TETATP,PHITP,PSIP(N),TETAP(N),PHIP(N),N
```

```
 1 FORMAT("1  MODEL     1",6X,"EPSITRUE=",2(E12.5,","),E12.5,/3X,
 $"WITH NOISE",3X,"EPSIO=      ",2(E12.5,","),E12.5,/3X,"<O    =    3",
 $5X,"ITERATION # ",I1,///)
  PRINT(10,2)
 2 FORMAT(4X,"TIME",5X,"DELTA(R)",7X,"HOT(R)",8X,"DELTA(RO)",
 $6X,"HOT(RO)"/)
  DO 4 I=1,6
  PRINT(10,3)TYME(I),DELTOUT((6*(N-1)+I),1),HOTOUT((6*(N-1)+I),1),
 $DELTOUT((6*(N-1)+I),2),HOTOUT((6*(N-1)+I),2)
 4 CONTINUE
 3 FORMAT(4X,F4.0,4X,2(E12.5,2X),2(2X,E12.5)/)
  PRINT(10,5)
 5 FORMAT("0",3X,"TIME",5X,"DELTA(AZ.)",5X,"HOT(AZ.)",7X,
 $"DELTA(EL.)",5X,"HOT(EL.)",/)
  DO 6 I=1,6
  PRINT(10,3)TYME(I),DELTOUT((6*(N-1)+I),3),HOTOUT((6*(N-1)+I),3),
 $DELTOUT((6*(N-1)+I),4),HOTOUT((6*(N-1)+I),4)
 6 CONTINUE
  PRINT(10,11)
11 FORMAT(5X,"TIME",5X,"TRACE   OF",6X,3(8HESTIMATE,4X)/15X,
 $"DISPERSION",6X,3(8HERROR IN,4X)/17X,"MATRIX",10X,"PSI",
 $9X,"TETA",8X,"PHI"/)
  DO 12 I=1,6
  PRINT(10,13)TYME(I),TRD(6*(N-1)+I),DPSI(6*(N-1)+I),DTETA
 $(6*(N-1)+I),DPHI(6*(N-1)+I)
12 CONTINUE
13 FORMAT(5X,F4.0,5X,E12.5,3(3X,E9.2)/)
15 CONTINUE
78 FORMAT(1X,"TRD=",E12.5/)
87 FORMAT(11X,3(E9.2,2X)/3X,"SEPSP=   ",3(E9.2,2X)/11X,3(E9.2,2X)
 $//11X,3(E9.2,2X)/3X,"SEPSV=   ",3(E9.2,2X)/11X,3(E9.2,2X)//)
79 FORMAT(1X,6HOUTRU=,F6.3,4(2X,E9.2)/13X,3(2X,E9.2)//)
92 FORMAT(1X,7HDELEPS:,I1,2X,I1,3E12.5/)
91 FORMAT(1X,"DOUT=",4(E10.3,2X)/1X,"HOT =",4(E10.3,2X)/)
98 FORMAT(5X,"KO",6X,"BETA",6X,"DELTA",8X,"O",8X,"ALPHA",8X,"L",//
 $,5X,"X",10X,"Y",10X,"Z",10X,"V",10X,"VX",9X,"VY",9X,"VZ",//,
 $5X,"T",3X,"BETAC",6X,"DELTAC",17X,"SF1",8X,"SF2",8X,"SF3",//,
 $5X,"R",9X,"RDOT",7X,"PSIR",7X,"TETR",7X,"ACC1",7X,"ACC2",7X,
 $"ACC3",//)
99 FORMAT(5X,I1,6X,5(E10.3,1X)/1X,7(E10.3,1X)/1X,F6.3,5X,2(E10.3,1X),
 $11X,3(E10.3,1X)/1X,7(E10.3,1X)//)
  STOP
  END
**************************************************
  BLOCK DATA
  REAL L,M
  COMMON/TIMER/T,DT,TF,DPRT
  COMMON/EPSTRUE/PSIT,TETAT,PHIT
  COMMON/EPSIO/PSIO,TETAO,PHIO
  COMMON/CONTROL/KO,BETAC,DELTAC
  COMMON/DYNAMIC/BETA,DELTA,O,ALPHA,L
  COMMON/INITIAL/XO,YO,ZO,VXO,VYO,VZO
  COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
  COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
  COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
  DATA DT,TF,DPRT/.01,50.,10./
  DATA PSIT,TETAT,PHIT/1.E-03,2.E-03,3.E-03/
  DATA PSIO,TETAO,PHIO/3*0./
  DATA KO/3/
  DATA BETA,DELTA,O,ALPHA,L/5*0./
  DATA XO,YO,ZO,VXO,VYO,VZO/2*1000.,100.,2000.,1500.,10./
  DATA SEPSP,SEPSV/18*0./
  DATA RN,RDN,PSIRN,TETRN,F1N,F2N,F3N/10.,10.,2*.001,
 $3*32.2E-03/
  DATA M,H/9*0.,3*0./
  END
```

```
      SUBROUTINE CONTROL
*             *******************************
*SUBROUTINE CONTROL GENERATES THREE TYPES OF COMMANDS FOR
*DELTA AND BETA.
*             *******************************
      COMMON/TIMER/T,DT,TF,DPPT
      COMMON/CONTROL/KD,BETAC,DELTAC
      REAL K1,K2,K3,K4
      DATA(K1=.053),(K2=.03),(K3=.007),(K4=.002),(W1=.1),(W2=.2)
      TC=T+DT/2.
      IF(KD.EQ.1)10,20
   10 DELTAC=K3+K4*SIN(W2*TC)
      BETAC=K1*TC
      RETURN
   20 IF(KD.EQ.2)30,40
   30 DELTAC=K3
      BETAC=K1*TC+K2*SIN(W1*TC)
      RETURN
   40 BETAC=K1*TC+K2*SIN(W1*TC)
      DELTAC=K3+K4*SIN(W2*TC)
      RETURN
      END
      SUBROUTINE DYNAMIC
*             *******************************
*SUBROUTINE DYNAMIC CALCULATES THE VALUES OF BETA,DELTA,Q (PITCH
*RATE),ALPHA (ANGLE OF ATTACK),AND L (LIFT) BASED ON AN EULER
*INTEGRATION ROUTINE.
*             *******************************
      COMMON/TIMER/T,DT,TF,DPPT
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/BLANK/OMA(3,3),GAMA(3,3),GAMF(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      DIMENSION U(4)
      REAL L,LIFT
      U(1)=BETA
      U(2)=DELTA
      U(3)=Q
      U(4)=ALPHA
      GAB(1,1)=1.
      GAB(2,2)=GAB(3,3)=COS(U(1))
      GAB(3,2)=SIN(U(1))
      GAB(2,3)=-GAB(3,2)
      GAB(2,1)=GAB(1,2)=GAB(3,1)=GAB(1,3)=0.
      K=4
      CALL EULINT(U,K,.01)
      BETA=U(1)
      DELTA=U(2)
      Q=U(3)
      ALPHA=U(4)
      RETURN
      END
      SUBROUTINE NOMINAL
*             *******************************
*SUBROUTINE NOMINAL CALCULATES THE VALUES OF THE MISSILE POSITION
*AND VELOCITY IN THE A/C FRAME,BASED ON AN EULER ROUTINE.
*             *******************************
      COMMON/TIMER/T,DT,TF,DPPT
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/BLANK/OMA(3,3),GAMA(3,3),GAMF(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      DIMENSION U(6)
      U(1)=X
      U(2)=Y
      U(3)=Z
      U(4)=VX
```

```
      U(5)=VY
      U(6)=VZ
      K=6
      CALL EULINT(U,K,.01)
      V=SQRT(U(4)*U(4)+U(5)*U(5)+U(6)*U(6))
      X=U(1)
      Y=U(2)
      Z=U(3)
      VX=U(4)
      VY=U(5)
      VZ=U(6)
      RETURN
      END
      SUBROUTINE SFFORCE
*               ********************************
*SUBROUTINE SFFORCE CALCULATES THE SPECIFIC FORCES MEASURED BY THE
*THREE ACCELEROMETERS ON THE MISSILE.
*               ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      DIMENSION GA(3,3),GAF(3,3)
      REAL L,LIFT
      LIFT(3,1)=-L
      LIFT(1,1)=LIFT(2,1)=0.
      A=SQRT(VX*VX+VY*VY)
      V1=VX+PSI*VY-TETA*VZ
      V2=-PSI*VX+VY+PHI*VZ
      V3=TETA*VX-PHI*VY+VZ
      GAMA(1,1)=V1/A
      GAMA(2,2)=GAMA(1,1)
      GAMA(1,2)=-V2/A
      GAMA(2,1)=-GAMA(1,2)
      GAMA(3,3)=1.
      GAMA(1,3)=GAMA(3,1)=GAMA(2,3)=GAMA(3,2)=0.
      GAME(1,1)=A/V
      GAME(3,3)=GAME(1,1)
      GAME(3,1)=V3/V
      GAME(1,3)=-GAME(3,1)
      GAME(2,2)=1.
      GAME(1,2)=GAME(2,1)=GAME(2,3)=GAME(3,2)=0.
      CALL MATM(GAMA,GAME,GA,3,3,3)
      CALL MATM(GA,GAB,GAF,3,3,3)
      CALL MATM(GAF,LIFT,SF,3,3,1)
      RETURN
      END
      SUBROUTINE MEASURE
*               ********************************
*SUBROUTINE MEASURE CALCULATES THE RANGE,RANGE RATE,AZIMUTH,
*AND ELEVATION AS MEASURED BY THE RADAR IN THE A/C.
*               ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      COMMON/LOGIC/MODE1
      LOGICAL MODE1
      WN=0.
      R=SQRT(X*X+Y*Y+Z*Z)
      RD=(X*VX+Y*VY+Z*VZ)/R
      PSIR=Y/SQRT(X*X+Y*Y)
      TETAR=Z/R
```

```
      IF(MODE1) GO TO 10
      CALL NOIZE(RN,0.,WN)
   10 OUT(1,1)=R+WN
      IF(MODE1) GO TO 20
      CALL NOIZE(RDN,0.,WN)
   20 OUT(2,1)=RD+WN
      IF(MODE1) GO TO 30
      CALL NOIZE(PSIRN,0.,WN)
   30 OUT(3,1)=PSIR+WN
      IF(MODE1) GO TO 40
      CALL NOIZE(TETRN,0.,WN)
   40 OUT(4,1)=TETAR+WN
      IF(MODE1) GO TO 50
      CALL NOIZE(F1N,0.,WN)
   50 OUT(5,1)=SF(1,1)+WN
      IF(MODE1) GO TO 60
      CALL NOIZE(F2N,0.,WN)
   60 OUT(6,1)=SF(2,1)+WN
      IF(MODE1) GO TO 70
      CALL NOIZE(F3N,0.,WN)
   70 OUT(7,1)=SF(3,1)+WN
      RETURN
      END
      SUBROUTINE F(U,P)
*               ******************************
*SUBROUTINE F CALCULATES THE DERIVATIVES NEEDED FOR THE EULER
*INTEGRATION FOR SUBROUTINES DYNAMIC NOMINAL,AND SENSTAT.
*               ******************************
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/CONTROL/KD,BETAC,DELTAC
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/SENSTAT/SEPSF(3,3),SEPSV(3,3)
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      COMMON/LOGIC/MODE1
      DIMENSION U(36),P(36),OUTF(3,1)
      LOGICAL MODE1
      REAL LAMDA,NU,L,MQ,MA,MD,LA,LD,LIFT
      DATA(MQ=-.462),(MA=-5.81),(MD=-72.0),(LA=.379),(LD=.0599),
     $(LAMDA=10.),(NU=30.),(G=32.2)
      IF (FD) 10,20,30
*SUBROUTINE F FOR SUBROUTINE DYNAMIC
   10 P(1)=-NU*(U(1)-BETAC)
      P(2)=-LAMDA*(U(2)-DELTAC)
      P(3)=MQ*U(3)+MA*U(4)+MD*U(2)
      P(4)=U(3)-LA*U(4)-LD*U(2)
      L=-V*(P(4)-U(3))
      RETURN
*SUBROUTINE F FOR SUBROUTINE NOMINAL
   20 IF(MODE1)GO TO 25
      P(1)=U(4)
      P(2)=U(5)
      P(3)=U(6)
      CALL MATM(CMA,SF,ACC,3,3,1)
      P(4)=ACC(1,1)
      P(5)=ACC(2,1)
      P(6)=ACC(3,1)+G
      RETURN
   25 P(1)=U(4)
      P(2)=U(5)
      P(3)=U(6)
      OUTF(1,1)=OUT(5,1)
      OUTF(2,1)=OUT(6,1)
```

```
      OUTF(3,1)=OUT(7,1)
      CALL MATM(CMA,OUTF,ACC,3,3,1)
      P(4)=ACC(1,1)
      P(5)=ACC(2,1)
      P(6)=ACC(3,1)+G
   33 DO 35 I=7,15
      P(I)=U(I+9)
   35 CONTINUE
      P(16)=-SF(2,1)
      P(17)=SF(3,1)
      P(19)=SF(1,1)
      P(21)=-SF(3,1)
      P(23)=-SF(1,1)
      P(24)=SF(2,1)
      P(18)=P(20)=P(22)=0.
      DO 36 J=1,3
      DO 36 I=1,3
      P(21+I+3*J)=OM(I,J)
   36 CONTINUE
      DO 37 I=1,3
      P(33+I)=OH(I,1)
   37 CONTINUE
      RETURN
      END
      SUBROUTINE SENSOUT
*                ******************************
*SUBROUTINE SENSOUT CALCULATES THE OUTPUT (RADAR MEASUREMENTS AND
*SPECIFIC FORCES) SENSITIVITIES W.R.T. THE MISALIGNMENT ANGLES.
*                ******************************
      COMMON/TIMER/T,DT,TF,DPFT
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/SENSRDR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      DIMENSION PR(1,3),VR(1,3),XY(1,3),XYZ(1,3),TEMP1(1,3),TEMP2(1,3)
      R=OUT(1,1)
      PR(1,1)=X/R
      PR(1,2)=Y/R
      PR(1,3)=Z/R
      PVR=(X*VX+Y*VY+Z*VZ)/(R*R)
      VR(1,1)=(VX/R-PVR*PR(1,1))
      VR(1,2)=(VY/R-PVR*PR(1,2))
      VR(1,3)=(VZ/R-PVR*PR(1,3))
      CALL MATM(PR,SEPSP,SREPS,1,3,3)
      CALL MATM(VR,SEPSP,TEMP1,1,3,3)
      CALL MATM(PR,SEPSV,TEMP2,1,3,3)
      DO 10 I=1,3
      SRDEPS(1,I)=TEMP1(1,I)+TEMP2(1,I)
   10 CONTINUE
      XY(1,1)=-X*Y/SQRT((X*X+Y*Y)**3)
      XY(1,2)=X*X/SQRT((X*X+Y*Y)**3)
      XY(1,3)=0.
      CALL MATM(XY,SEPSP,SPSREPS,1,3,3)
      XYZ(1,1)=-X*Z/R**3
      XYZ(1,2)=-Y*Z/R**3
      XYZ(1,3)=(X*X+Y*Y)/R**3
      CALL MATM(XYZ,SEPSP,STEREPS,1,3,3)
      RETURN
      END
      SUBROUTINE NOIZE (RMSNOIS,OUTMEAN,WN)
*                ******************************
*SUBROUTINE NOIZE CALCULATES THE VALUES OF THE MEASUREMENT NOISE
*COMPONENTS USING A RANDOM NUMBER GENERATOR MODELLED AS GAUSSIAN.
*                ******************************
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
```

```
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAMF(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      GAUSS=0.
      DO 10 I=1,12
      GAUSS=GAUSS+RANF(DUM)
   10 CONTINUE
      GAUSS=GAUSS-6.+OUTMEAN
      WN=GAUSS*RMSNOIS
      RETURN
      END
      SUBROUTINE INFOMAT
*                 ********************************
*SUBROUTINE INFOMAT CALCULATES THE VALUES OF THE INFORMATION
*MATRIX AND GRADIENT VECTOR INCREMENTS.
*                 ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/EPSTRUE/PSIT,TETAT,PHIT
      COMMON/SENSRDR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAMF(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      COMMON/LOGIC/MODE1
      REAL M
      LOGICAL MODE1
      DIMENSION SOUEPST(3,4),TEMP1(3,4),DOUT(4,1)
      DO 10 I=1,3
      SOUTEPS(1,I)=SREPS(1,I)
      SOUTEPS(2,I)=SRDEPS(1,I)
      SOUTEPS(3,I)=SPSREPS(1,I)
      SOUTEPS(4,I)=STEREPS(1,I)
   10 CONTINUE
      DO 20 I=1,4
      DO 20 J=1,3
      SOUEPST(J,I)=SOUTEPS(I,J)
   20 CONTINUE
      CALL MATM(SOUEPST,OUTN,TEMP1,3,4,4)
      CALL MATM(TEMP1,SOUTEPS,DM,3,4,3)
      DO 30 I=1,4
      DOUT(I,1)=OUTRU(I,1)-OUT(I,1)
   30 CONTINUE
      CALL MATM(TEMP1,DOUT,DH,3,4,1)
      RETURN
      END
      SUBROUTINE EULINT(U,K,DT)
      DIMENSION U(36),P(36)
      CALL F(U,P)
      DO 10 J=1,K
      U(J)=U(J)+P(J)*DT
   10 CONTINUE
      RETURN
      END
      SUBROUTINE MATM(A,B,C,M,K,N)
      DIMENSION A(M,K),B(K,N),C(M,N)
      DO 10 J=1,N
      DO 10 I=1,M
      C(I,J)=0.
      DO 10 L=1,K
   10 C(I,J)=C(I,J)+A(I,L)*B(L,J)
      RETURN
      END
```

```
      PROGRAM SIMUL2(INPUT,OUTPUT=1002B,TAPE2=1002B,TAPE5=1002B,
     $TAPE10=OUTPUT)
*                    ******************************
*PROGRAM SIMUL IS A SIMULATION OF THE TRACKING AND CONTROL OF
*AN AIR-TO-GROUND MISSILE USING MAXIMUM LIKELIHOOD ESTIMATION
*OF THE THREE ANGLES REPRESENTING ITS INITIAL MISALIGNMENT.
*                    ******************************
      COMMON/TIMER/T,DT,TF,DPRT,N,KK
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/EPSTRUE/PSIT,TETAT,PHIT
      COMMON/EPSIO/PSIO,TETAO,PHIO
      COMMON/CONTROL/KD,BETAC,DELTAC
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/RADAR/R,RD
      COMMON/INITIAL/XO,YO,ZO,VXO,VYO,VZO
      COMMON/SENSTAT/SEPSF(3,3),SEPSV(3,3)
      COMMON/SENSRDR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/SENSFRC/SF1EPS(1,3),SF2EPS(1,3),SF3EPS(1,3),SFEPS(3,3)
     $,SFEPSA(3,3),DFDV(3,3),TEMP5(3,3)
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/GMA(3,3),GAMA(3,3),GAMF(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(7,7),OUTRU(7,1),FD,ACC(3,1),V1,V2,V3
     $,SOUTEPS(7,3)
      COMMON/LOGIC/MODE2
      LOGICAL MODE2
      DIMENSION U(35),WKAREA(18),MI(3,3),EPS(3,1),FIRST(3,1),SECOND(3,1)
     $,DS(6),HOTS(3),DVM(3,1),DSF(3,1),HOTSF(3),TEMP1(3,1),HOTVM(3)
     $,TYME(5),DELTOUT(18,7),HOTOUT(18,7),TRD(18),DPSI(18),DTETA(18),
     $DPHI(18),SF1(6),SF2(6),SF3(6),XTT(6),YTT(6),ZTT(6),
     $VXTT(6),VYTT(6),VZTT(6),RTT(6),RDTT(6),PSIP(3),TETAP(3),
     $PHIP(3),DOUT(7,1),HOT(7),TAYLOR(7,1)
      REAL L,LIFT,M,MI
*                    ******************************
*INITIALIZATION OF THE RANDOM NUMBER GENERATOR FOR NOISE GENERATION.
*                    ******************************
      PRINT(10,98)
      JJ=1
      CALL RANSET(JJ)
      DPRNT=0.
      T=0.
      MODE2=.FALSE.
      PRINT*,"***TRUTH MODEL ***"
*                    ******************************
*INITIALIZATION OF THE MISALIGNMENT ANGLES WITH VALUES ASSUMED TO BE
*TRUE.
*                    ******************************
      PSI=PSIT
      TETA=TETAT
      PHI=PHIT
      APSI=ATETA=APHI=0.
*                    ******************************
```

```
*DEFINITION OF THE MISALIGNMENT ANGLES TRANSFORMATION MATRIX.
*
*          1.       -PSI    TETA
*   CMA=  PSI       1.      -PHI
*         -TETA     PHI      1.
*
*                   *******************************
      CMA(1,1)=CMA(2,2)=CMA(3,3)=1.
      CMA(2,1)=PSI
      CMA(3,1)=-TETA
      CMA(3,2)=PHI
      CMA(1,2)=-CMA(2,1)
      CMA(1,3)=-CMA(3,1)
      CMA(2,3)=-CMA(3,2)
*                   *******************************
*INITIALIZATION OF POSITON,VELOCITY,AND SPECIFIC FORCE.
*                   *******************************
      SF(1,1)=0.
      SF(2,1)=0.
      SF(3,1)=0.
      X=XO
      Y=YO
      Z=ZO
      VX=VXO
      VY=VYO
      VZ=VZO
      V=SQRT(VX*VX+VY*VY+VZ*VZ)
*                   *******************************
*EVALUATION OF THE VALUE OF THE STATES FOR THE TRUTH MODEL .
*                   *******************************
   20 CALL MEASURE
      WRITE(2)X,Y,Z,VX,VY,VZ,SF(1,1),SF(2,1),SF(3,1),R,RD
      WRITE(5)KO,T,(OUT(I,1),I=1,7)
      CALL CONTROL
      FD=-1.
      CALL DYNAMIC
      FD=0.
      CALL NOMINAL
      CALL SPFORCE
      IF(ABS(T-DPRNT).LT.1.E-06)GO TO 40
   30 T=T+DT
      IF(T.LE.TF)GO TO 20
      GO TO 50
   40 PRINT(10,99)KO,BETA,DELTA,C,ALPHA,L,X,Y,Z,V,VX,VY,VZ,T,BETAC,
     $DELTAC,SF(1,1),SF(2,1),SF(3,1),OUT(1,1),OUT(2,1),OUT(3,1),OUT(4,1)
     $,ACC(1,1),ACC(2,1),ACC(3,1)
      PRINT(10,79)T,(OUT(I,1),I=1,7)
      DPRNT=DPRNT+DPRT
      GO TO 30
   50 REWIND 5
      PRINT*,"***MODEL 2***"
      MODE2=.TRUE.
      PRINT*,"MODE2=",MODE2
      PSI=PSIO
      TETA=TETAO
      PHI=PHIO
      DO 100 NN=1,3
      N=NN
      REWIND 2
      T=0.
      DPRNT=0.
*                   *******************************
*INITIALIZATION OF THE MISALIGNMENT ANGLES WITH THE ESTIMATED
*NOMINAL VALUES.
*                   *******************************
      PSI=PSI+APSI/1000
```

```
      TETA=TETA+ATETA/1000
      PHI=PHI+APHI/1000
      PSIP(N)=1000*PSI
      TETAP(N)=1000*TETA
      PHIP(N)=1000*PHI
      CMA(1,1)=CMA(2,2)=CMA(3,3)=1.
      CMA(2,1)=PSI
      CMA(3,1)=-TETA
      CMA(3,2)=PHI
      CMA(1,2)=-CMA(2,1)
      CMA(1,3)=-CMA(3,1)
      CMA(2,3)=-CMA(3,2)
*                ******************************
*INITIALIZATION OF POSITION,VELOCITY,SPECIFIC FORCE,AERODYNAMIC
*VALUES,INFORMATION AND DISPERSION MATRIX ELEMENTS AND TRACE,
*AND SENSITIVITIES.
*                ******************************
      X=X0
      Y=Y0
      Z=Z0
      VX=VX0
      VY=VY0
      VZ=VZ0
      SF(1,1)=0.
      SF(2,1)=0.
      SF(3,1)=0.
      BETA=0.
      DELTA=0.
      Q=0.
      ALPHA=0.
      L=0.
      DO 55 I=7,36
      U(I)=0.
   55 CONTINUE
      DO 52 I=1,3
      DO 52 J=1,3
      SFPSP(I,J)=0.
      SFPSV(I,J)=0.
      SFEPS(I,J)=0.
      M(I,J)=0.
      MI(I,J)=0.
   52 CONTINUE
      OUTN(1,1)=1./(RN*RN)
      OUTN(2,2)=1./(RDN*RDN)
      OUTN(3,3)=1./(PSIRN*PSIRN)
      OUTN(4,4)=1./(TETRN*TETRN)
      OUTN(5,5)=1./(F1N*F1N)
      OUTN(6,6)=1./(F2N*F2N)
      OUTN(7,7)=1./(F3N*F3N)
      OUTN(1,2)=OUTN(1,3)=OUTN(1,4)=OUTN(1,5)=OUTN(1,6)=OUTN(1,7)=0.
      OUTN(2,1)=OUTN(3,1)=OUTN(4,1)=OUTN(5,1)=OUTN(6,1)=OUTN(7,1)=0.
      OUTN(2,3)=OUTN(2,4)=OUTN(2,5)=OUTN(2,6)=OUTN(2,7)=0.
      OUTN(3,2)=OUTN(4,2)=OUTN(5,2)=OUTN(6,2)=OUTN(7,2)=0.
      OUTN(3,4)=OUTN(3,5)=OUTN(3,6)=OUTN(3,7)=0.
      OUTN(4,3)=OUTN(5,3)=OUTN(6,3)=OUTN(7,3)=0.
      OUTN(4,5)=OUTN(4,6)=OUTN(4,7)=0.
      OUTN(5,4)=OUTN(6,4)=OUTN(7,4)=0.
      OUTN(5,6)=OUTN(5,7)=0.
      OUTN(6,5)=OUTN(7,5)=0.
      OUTN(6,7)=OUTN(7,6)=0.
      DO 50 KK=1,3001
      READ(5)KD,T,(OUTRU(I),I=1,7)
      DSF(1,1)=OUTRU(5)-SF(1,1)
      DSF(2,1)=OUTRU(6)-SF(2,1)
      DSF(3,1)=OUTRU(7)-SF(3,1)
      CALL MEASURE
```

```
      CALL SENSOUT
      IF((KK-1)-1000*((KK-1)/1000).EQ.0)PRINT(10,86)((SOUTEPS(I,J),J=1,3
     S),I=1,7)
      CALL INFOMAT
      DO 51 I=1,7
      DOUT(I,1)=OUTPU(I,1)-OUT(I,1)
 51   CONTINUE
      EPS(1,1)=PSIT-PSI
      EPS(2,1)=TETAT-TETA
      EPS(3,1)=PHIT-PHI
      CALL MATM(SOUTEPS,EPS,TAYLOR,7,3,1)
      DO 52 I=1,7
      HOT(I)=DOUT(I,1)-TAYLOR(I,1)
 52   CONTINUE
      CALL CONTROL
      FD=-1.
      CALL DYNAMIC
      FD=1.
      K=36
      CALL EULINT(U,K,.01)
      DO 56 I=1,3
      DO 56 J=1,3
      SEPSP(I,J)=U(3*I+J+3)
      SEPSV(I,J)=U(3*I+J+12)
      M(I,J)=U(I+3*J+21)
 56   CONTINUE
      FD=0.
      CALL NOMINAL
      READ(2)XT,YT,ZT,VXT,VYT,VZT,SF1T,SF2T,SF3T,RT,RDT
      IF((KK-1)-1000*((KK-1)/1000).NE.0)GO TO 51
      XTT((KK-1)/1000+1)=XT
      YTT((KK-1)/1000+1)=YT
      ZTT((KK-1)/1000+1)=ZT
      VXTT((KK-1)/1000+1)=VXT
      VYTT((KK-1)/1000+1)=VYT
      VZTT((KK-1)/1000+1)=VZT
      SF1((KK-1)/1000+1)=SF1T
      SF2((KK-1)/1000+1)=SF2T
      SF3((KK-1)/1000+1)=SF3T
      RTT((KK-1)/1000+1)=RT
      RDTT((KK-1)/1000+1)=RDT
 51   CALL SPFORCE
      IF((KK-1)-1000*((KK-1)/1000).EQ.0)PRINT(10,87)((SEPSP(I,J),J=1,3),
     $I=1,3),((SEPSV(I,J),J=1,3),I=1,3),((SFEPS(I,J),J=1,3),I=1,3)
      V1T=VXT+PSIT*VYT-TETAT*VZT
      V2T=-PSIT*VXT+VYT+PHIT*VZT
      V3T=TETAT*VXT-PHIT*VYT+VZT
      DVM(1,1)=V1T-V1
      DVM(2,1)=V2T-V2
      DVM(3,1)=V3T-V3
      CALL MATM(DFDV,DVM,TEMP1,3,3,1)
      DO 53 I=1,3
      HOTSF(I)=DSF(I,1)-TEMP1(I,1)
 53   CONTINUE
      DS(1)=XT-X
      DS(2)=YT-Y
      DS(3)=ZT-Z
      DS(4)=VXT-VX
      DS(5)=VYT-VY
      DS(6)=VZT-VZ
      CALL MATM(SEPSP,EPS,FIRST,3,3,1)
      CALL MATM(SEPSV,EPS,SECOND,3,3,1)
      CALL MATM(TEMP5,EPS,TEMP1,3,3,1)
      DO 54 I=1,3
      HOTS(I)=DS(I)-FIRST(I,1)
      HOTVM(I)=DVM(I,1)-TEMP1(I,1)
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```
 54 CONTINUE
    DO 58 I=4,6
    HOTS(I)=DS(I)-SECOND((I-3),1)
 58 CONTINUE
    IF((KK-1)-1000*((KK-1)/1000).NE.0)GO TO 75
    PRINT(10,77)DS,HOTS
    PRINT(10,73)DVM,HOTVM
    PRINT(10,74)DSF,HOTSF
    PRINT(10,81)DOUT,HOT
    PRINT(10,79)T,(OUTRU(I),I=1,7)
    PRINT(10,39)KD,BETA,DELTA,Q,ALPHA,L,X,Y,Z,V,VX,VY,VZ,T,BETAC,
   $DELTAC,SF(1,1),SF(2,1),SF(3,1),OUT(1,1),OUT(2,1),OUT(3,1),OUT(4,1)
   $,ACC(1,1),ACC(2,1),ACC(3,1)
    TYME((KK-1)/1000+1)=(KK-1)/100.
    DO 57 I=1,7
    DELTOUT((6*(N-1)+(KK-1)/1000+1),I)=DOUT(I,1)
    HOTOUT((6*(N-1)+(KK-1)/1000+1),I)=HOT(I)
 57 CONTINUE
    IF(KK.EQ.1)GO TO 59
    CALL LINV2F(M,3,3,MI,10,WKAREA,IER)
    TRD(6*(N-1)+(KK-1)/1000+1)=MI(1,1)+MI(2,2)+MI(3,3)
 59 CONTINUE
    PRINT(10,78)TRD(6*(N-1)+(KK-1)/1000+1)
    APSI=(MI(1,1)*U(34)+MI(1,2)*U(35)+MI(1,3)*U(36))*1000
    ATETA=(MI(2,1)*U(34)+MI(2,2)*U(35)+MI(2,3)*U(36))*1000
    APHI=(MI(3,1)*U(34)+MI(3,2)*U(35)+MI(3,3)*U(36))*1000
    DPSI(6*(N-1)+(KK-1)/1000+1)=(PSIT-PSI)*1000-APSI
    DTETA(6*(N-1)+(KK-1)/1000+1)=(TETAT-TETA)*1000-ATETA
    DPHI(6*(N-1)+(KK-1)/1000+1)=(PHIT-PHI)*1000-APHI
    PRINT(10,92)KD,N,DPSI(6*(N-1)+(KK-1)/1000+1),DTETA(6*
   $(N-1)+(KK-1)/1000+1),DPHI(6*(N-1)+(KK-1)/1000+1)
 75 CONTINUE
 50 CONTINUE
    REWIND 5
100 CONTINUE
    PSITP=1000*PSIT
    TETATP=1000*TETAT
    PHITP=1000*PHIT
    PRINT(10,14)PSITP,TETATP,PHITP
 14 FORMAT("1  TRUTH  MODEL",5X,"EPSITRUE=",2(E9.2,","),E9.2,/1X,
   $"  KD    =     3",///)
    PRINT(10,16)
 16 FORMAT(2X,"TIME",8X,"X",13X,"Y",13X,"Z",13X,"R",/)
    DO 17 I=1,6
    PRINT(10,18)TYME(I),XTT(I),YTT(I),ZTT(I),RTT(I)
 17 CONTINUE
 18 FORMAT(2X,F4.0,3X,4(E12.5,2X)/)
    PRINT(10,19)
 19 FORMAT(2X,"TIME",8X,"VX",12X,"VY",12X,"VZ",12X,"RD",/)
    DO 21 I=1,6
    PRINT(10,18)TYME(I),VXTT(I),VYTT(I),VZTT(I),RDTT(I)
 21 CONTINUE
    PRINT(10,22)
 22 FORMAT(2X,"TIME",6X,"SP.FRC.(1)",5X,"SP.FRC.(2)",5X,"SP.FRC.(3)"/)
    DO 23 I=1,6
    PRINT(10,24)TYME(I),SF1(I),SF2(I),SF3(I)
 23 CONTINUE
 24 FORMAT(2X,F4.0,5X,3(E12.5,3X)/)
    DO 15 N=1,3
    PRINT(10,1)PSITP,TETATP,PHITP,PSIP(N),TETAP(N),PHIP(N),N
  1 FORMAT("1  MODEL    2",FX,"EPSITRUE=",2(E12.5,","),E12.5,/3X,
   $"WITH NOISE",5X,"EPSIO=    ",2(E12.5,","),E12.5,/3X,"KD    =    3",
   $5X,"ITERATION # ",I1,///)
    PRINT(10,2)
  2 FORMAT(4X,"TIME",6X,"DELTA(R)",7X,"HOT(R)",8X,"DELTA(RD)",
   $6X,"HOT(RD)"/)
```

```
      DO 4 I=1,6
      PRINT(10,3)TYME(I),DELTOUT((6*(N-1)+I),1),HOTOUT((6*(N-1)+I),1),
     $DELTOUT((6*(N-1)+I),2),HOTOUT((6*(N-1)+I),2)
    4 CONTINUE
    3 FORMAT(4X,F4.0,4X,2(E12.5,2X),2(2X,E12.5)/)
      PRINT(10,5)
    5 FORMAT("0",3X,"TIME",5X,"DELTA(AZ.)",5X,"HOT(AZ.)",7X,
     $"DELTA(EL.)",5X,"HOT(EL.)",/)
      DO 6 I=1,6
      PRINT(10,3)TYME(I),DELTOUT((6*(N-1)+I),3),HOTOUT((6*(N-1)+I),3),
     $DELTOUT((6*(N-1)+I),4),HOTOUT((6*(N-1)+I),4)
    6 CONTINUE
      PRINT(10,1)PSITP,TETATP,PHITP,PSIP(N),TETAP(N),PHIP(N),N
      PRINT(10,7)
    7 FORMAT(1X,"TIME",  "DELTA(SF1)",1X,"HOT(SF1)",2X,"DELTA(SF2)",
     $1X,"HOT(SF2)",2X,  LTA(SF3)",1X,"HOT(SF3)"/)
      DO 8 I=1,6
      PRINT(10,9)TYME(I),DELTOUT((6*(N-1)+I),5),HOTOUT((6*(N-1)+I),5),
     $DELTOUT((6*(N-1)+I),6),HOTOUT((6*(N-1)+I),6),DELTOUT((6*(N-1)
     $+I),7),HOTOUT((6*(N-1)+I),7)
    8 CONTINUE
    9 FORMAT(1X,F4.0,1X,2(E9.2,1X),1X,2(E9.2,1X),1X,2(E9.2,2X)/)
      PRINT(10,11)
   11 FORMAT(5X,"TIME",5X,"TRACE    OF"6X,3(9HESTIMATE,4X)/15X,
     $"DISPERSION",6X,3(8HERROR IN,4X)/17X,"MATRIX",10X,"PSI",
     $9X,"TETA",8X,"PHI"//)
      DO 12 I=1,6
      PRINT(10,13)TYME(I),TRD(6*(N-1)+I),DPSI(6*(N-1)+I),DTETA
     $(6*(N-1)+I),DPHI(6*(N-1)+I)
   12 CONTINUE
   13 FORMAT(5X,F4.0,5X,E12.5,3(3X,E9.2)/)
   73 FORMAT(1X,"DVM= ",3(E12.5,2X)/1X,"HOTVM=",3(E12.5,2X)/)
   15 CONTINUE
   36 FORMAT(11X,3(E9.2,2X)/11X,3(E9.2,2X)/11X,3(E9.2,2X)/1X,"SOUTEPS=
     $",3(E9.2,2X)/11X,3(E9.2,2X)/11X,3(E9.2,2X)/11X,3(E9.2,2X)//)
   78 FORMAT(1X,"TRD=",E12.5/)
   74 FORMAT(1X,"DSF= ",3(E12.5,2X)/1X,"HOTSF=",3(E12.5,2X)/)
   77 FORMAT(1X,"DS=  ",3(E10.3,2X)/16X,3(E10.3,2X)/1X,"HOTS=",
     $3(E10.3,2X)/16X,3(E10.3,2X)/)
   91 FORMAT(1X,"DOUT=",4(E10.3,2X)/16X,3(E10.3,2X)/1X,"HOT =",
     $4(E10.3,2X)/16X,3(E10.3,2X)/)
   79 FORMAT(1X,6HOUTPU=,F6.3,4(2X,E9.2)/13X,3(2X,E9.2)//)
   37 FORMAT(11X,3(E9.2,2X)/3X,"SEPSP=  ",3(E9.2,2X)/11X,3(E9.2,2X)//
     $11X,3(E9.2,2X)/3X,"SEPSV=  ",3(E9.2,2X)/11X,3(E9.2,2X)//
     $11X,3(E9.2,2X)/3X,"SFEPS=  ",3(E9.2,2X)/11X,3(E9.2,2X)//)
   92 FORMAT(1X,7HDELEPS ,I1,2X,I1,3E13.5/)
   98 FORMAT(5X,"KD",6X,"BETA",6X,"DELTA",8X,"Q",8X,"ALPHA",8X,"L",//,5X
     $,"X",10X,"Y",10X,"Z",10X,"V",10X,"VX",9X,"VY",9X,"VZ",//,5X,"T",8X
     $,"BETAC",6X,"DELTAC",17X,"SF1",6X,"SF2",8X,"SF3",//,5X"R",9X,"ROOT
     $",7X,"PSIP",7X,"TETR",7X,"ACC1",7X,"ACC2",7X,"ACC3",//)
   99 FORMAT(5X,I1,5X,5(E10.3,1X)/1X,7(E10.3,1X)/1X,F6.3,5X,
     $2(E10.3,1X),11X,3(E10.3,1X)/1X,7(E10.3,1X)//)
      STOP
      END
*********************************************
      BLOCK DATA
      REAL L,M
      COMMON/TIMER/T,DT,TF,DPFT,N,KK
      COMMON/EPSTRUE/PSIT,TETAT,PHIT
      COMMON/EPSIO/PSIO,TETAO,PHIO
      COMMON/CONTROL/KD,BETAC,DELTAC
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/INITIAL/XO,YO,ZO,VXO,VYO,VZO
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/NOISVAR/RN,RCN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
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      DATA DT,TF,OPPT/.01,50.,10./
      DATA PSIT,TETAT,PHIT/1.E-03,2.E-03,3.E-03/
      DATA PSIO,TETAO,PHIO/3*0./
      DATA KD/3/
      DATA BETA,DELTA,Q,ALPHA,L/5*0./
      DATA XO,YO,ZO,VXO,VYO,VZO/2*1000.,100.,2000.,1500.,10./
      DATA SEPSP,SEPSV/18*0./
      DATA RN,RDN,PSIEN,TETEN,F1N,F2N,F3N/2*10.,2*.001,3*32.2E-03/
      DATA M,H/9*0.,3*0./
      END
      SUBROUTINE CONTROL
*                *******************************
*SUBROUTINE CONTROL GENERATES THREE TYPES OF COMMANDS FOR
*DELTA AND BETA.
*                *******************************
      COMMON/TIMER/T,DT,TF,OPFT,N,KK
      COMMON/CONTROL/KD,BETAC,DELTAC
      REAL K1,K2,K3,K4
      DATA(K1=.053),(K2=.03),(K3=.007),(K4=.002),(W1=.1),(W2=.2)
      TC=T+DT/2
      IF(KD.EQ.1)10,20
   10 DELTAC=K3+K4*SIN(W2*TC)
      BETAC=K1*TC
      RETURN
   20 IF(KD.EQ.2)30,40
   30 DELTAC=K3
      BETAC=K1*TC+K2*SIN(W1*TC)
      RETURN
   40 BETAC=K1*TC+K2*SIN(W1*TC)
      DELTAC=K3+K4*SIN(W2*TC)
      RETURN
      END
      SUBROUTINE DYNAMIC
*                *******************************
*SUBROUTINE DYNAMIC CALCULATES THE VALUES OF BETA,DELTA,Q (PITCH
*RATE),ALPHA(ANGLE OF ATTACK),AND L (LIFT) BASED ON AN EULER
*INTEGRATION ROUTINE.
*                *******************************
      COMMON/TIMER/T,DT,TF,OPFT,N,KK
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/BLANK/CMA(3,7),GAMA(3,3),GAMF(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(7,7),OUTRU(7,1),FD,ACC(3,1),V1,V2,V3
      DIMENSION U(4)
      REAL L,LIFT
      U(1)=BETA
      U(2)=DELTA
      U(3)=Q
      U(4)=ALPHA
      GAB(1,1)=1.
      GAB(2,2)=GAB(3,3)=COS(U(1))
      GAB(3,2)=SIN(U(1))
      GAB(2,3)=-GAB(3,2)
      GAB(2,1)=GAB(1,2)=GAB(3,1)=GAB(1,3)=0.
      K=4
      CALL EULINT(U,K,.01)
      BETA=U(1)
      DELTA=U(2)
      Q=U(3)
      ALPHA=U(4)
      RETURN
      END
      SUBROUTINE NOMINAL
*                *******************************
*SUBROUTINE NOMINAL CALCULATES THE VALUES OF THE MISSILE POSITION
*AND VELOCITY IN THE A/C FRAME,BASED ON AN EULER ROUTINE.
*                *******************************
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      COMMON/TIMER/T,DT,TF,DPFT,N,KK
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(7,7),OUTRU(7,1),FD,ACC(3,1),V1,V2,V3
      DIMENSION U(6)
      U(1)=X
      U(2)=Y
      U(3)=Z
      U(4)=VX
      U(5)=VY
      U(6)=VZ
      K=6
      CALL EULINT(U,K,.01)
      V=SQRT(U(4)*U(4)+U(5)*U(5)+U(6)*U(6))
      X=U(1)
      Y=U(2)
      Z=U(3)
      VX=U(4)
      VY=U(5)
      VZ=U(6)
      RETURN
      END
      SUBROUTINE SPFORCE
*                 ********************************
*SUBROUTINE SPFORCE CALCULATES THE SPECIFIC FORCES MEASURED BY THE
*THREE ACCELEROMETERS ON THE MISSILE.
*                 ********************************
      COMMON/TIMER/T,DT,TF,DPFT,N,KK
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/SENSFRC/SF1EPS(1,3),SF2EPS(1,3),SF3EPS(1,3),SFEPS(3,3)
     $,SFEPSA(3,3),DFDV(3,3),TEMP5(3,3)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(7,7),OUTRU(7,1),FD,ACC(3,1),V1,V2,V3
      COMMON/LOGIC/MODE2
      DIMENSION GA(3,1),GAF(3,1),TEMP1(3,1),TEMP2(3,1),TEMP3(3,1),
     $TEMP4(3,3),DGAV1(3,3),DGAV2(3,3),DGEV1(3,3),
     $DGEV2(3,3),DGEV3(3,3),CAM(3,3),DVEPS(3,3)
      REAL L,LIFT
      LOGICAL MODE2
      LIFT(3,1)=-L
      LIFT(1,1)=LIFT(2,1)=0.
      V1=VX+PSI*VY-TETA*VZ
      V2=-PSI*VX+VY+PHI*VZ
      V3=TETA*VX-PHI*VY+VZ
      A=SQRT(V1*V1+V2*V2)
      GAMA(1,1)=V1/A
      GAMA(2,2)=GAMA(1,1)
      GAMA(1,2)=-V2/A
      GAMA(2,1)=-GAMA(1,2)
      GAMA(3,3)=1.
      GAMA(1,3)=GAMA(3,1)=GAMA(2,3)=GAMA(3,2)=0.
      GAME(1,1)=A/V
      GAME(3,3)=GAME(1,1)
      GAME(3,1)=V3/V
      GAME(1,3)=-GAME(3,1)
      GAME(2,2)=1.
      GAME(1,2)=GAME(2,1)=GAME(2,3)=GAME(3,2)=0.
      CALL MATM(GAB,LIFT,GA,3,3,1)
      CALL MATM(GAME,GA,GAF,3,3,1)
      CALL MATM(GAMA,GAF,SF,3,3,1)
      IF(MODE2)GO TO 10
      RETURN
```

```
10 CONTINUE
   A=SQRT(V1*V1+V2*V2)
   DO 20 J=1,3
   DO 20 I=1,3
   DGAV1(I,J)=DGAV2(I,J)=DGEV1(I,J)=DGEV2(I,J)=DGEV3(I,J)=0.
20 CONTINUE
   DGAV1(1,1)=DGAV1(2,2)=V2*V2/A**3
   DGAV1(1,2)=V1*V2/A**3
   DGAV1(2,1)=-DGAV1(1,2)
   DGAV2(1,1)=DGAV2(2,2)=-V1*V2/A**3
   DGAV2(1,2)=-V1*V1/A**3
   DGAV2(2,1)=-DGAV2(1,2)
   DGEV1(1,1)=DGEV1(3,3)=V1*V3*V3/(A*V**3)
   DGEV1(1,3)=V1*V3/V**3
   DGEV1(3,1)=-DGEV1(1,3)
   DGEV2(1,1)=DGEV2(3,3)=V2*V3*V3/(A*V**3)
   DGEV2(1,3)=V2*V3/V**3
   DGEV2(3,1)=-DGEV2(1,3)
   DGEV3(1,1)=DGEV3(3,3)=-A*V3/V**3
   DGEV3(3,1)=A*A/V**3
   DGEV3(1,3)=-DGEV3(3,1)
   CALL MATM(DGAV1,GAF,TEMP1,3,3,1)
   CALL MATM(DGEV1,GA,TEMP2,3,3,1)
   CALL MATM(GAMA,TEMP2,TEMP3,3,3,1)
   DO 30 I=1,3
   DFDV(I,1)=TEMP1(I,1)+TEMP3(I,1)
30 CONTINUE
   CALL MATM(DGAV2,GAF,TEMP1,3,3,1)
   CALL MATM(DGEV2,GA,TEMP2,3,3,1)
   CALL MATM(GAMA,TEMP2,TEMP3,3,3,1)
   DO 40 I=1,3
   DFDV(I,2)=TEMP1(I,1)+TEMP3(I,1)
40 CONTINUE
   CALL MATM(DGEV3,GA,TEMP1,3,3,1)
   CALL MATM(GAMA,TEMP1,TEMP2,3,3,1)
   DO 50 I=1,3
   DFDV(I,3)=TEMP2(I,1)
50 CONTINUE
   DO 60 I=1,3
   DO 60 J=1,3
   GAM(J,I)=GMA(I,J)
60 CONTINUE
   DVEPS(3,2)=VX
   DVEPS(2,1)=-VX
   DVEPS(1,1)=VY
   DVEPS(3,3)=-VY
   DVEPS(2,3)=VZ
   DVEPS(1,2)=-VZ
   DVEPS(1,3)=DVEPS(2,2)=DVEPS(3,1)=0.
   CALL MATM(GAM,SEPSV,TEMP4,3,3,3)
   DO 70 I=1,3
   DO 70 J=1,3
   TEMP5(I,J)=TEMP4(I,J)+DVEPS(I,J)
70 CONTINUE
   CALL MATM(DFDV,TEMP5,SFEPS,3,3,3)
   IF((KK-1)-1000*((KK-1)/1000).NE.0)GO TO 80
   PRINT(10,71)((DVEPS(I,J),J=1,3),I=1,3)
   PRINT(10,72)((TEMP4(I,J),J=1,3),I=1,3)
   PRINT(10,73)((TEMP5(I,J),J=1,3),I=1,3)
   PRINT(10,74)((DFDV(I,J),J=1,3),I=1,3)
71 FORMAT(11X,3(E12.5,2X)/3X,"DVEPS=  ",3(E12.5,2X)/,11X,3(E12.5,2X)/
  $/)
72 FORMAT(11X,3(E12.5,2X)/3X,"TEMP4=  ",3(E12.5,2X)/,11X,3(E12.5,2X)/
  $/)
73 FORMAT(11X,3(E12.5,2X)/3X,"TEMP5=  ",3(E12.5,2X)/,11X,3(E12.5,2X)/
  $/)
```

```
74 FORMAT(11X,3(E12.5,2X)/3X,"DFDV=    ",3(E12.5,2X)/,11X,3(E12.5,2X)/
   $/)
80 CALL MATM(CMA,SFEPS,SFEPSA,3,3,3)
   RETURN
   END
   SUBROUTINE MEASURE
*              ********************************
*SUBROUTINE MEASURE CALCULATES THE RANGE,RANGE RATE,AZIMUTH,
*AND ELEVATION AS MEASURED BY THE RADAR IN THE A/C,AND ADDS NOISE
*TO THEM.
*              ********************************
   COMMON/TIMER/T,DT,TF,DPFT,N,KK
   COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
   COMMON/RADAR/R,RD
   COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
   COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
  $LIFT(3,1),OUT(7,1),OUTN(7,7),OUTRU(7,1),FD,ACC(3,1),V1,V2,V3
   COMMON/LOGIC/MODE2
   LOGICAL MODE2
   WN=0.
   R=SQRT(X*X+Y*Y+Z*Z)
   RD=(X*VX+Y*VY+Z*VZ)/R
   PSIR=Y/SQRT(X*X+Y*Y)
   TETAR=Z/R
   IF (MODE2) GO TO 10
   CALL NOIZE(RN,0.,WN)
10 OUT(1,1)=R+WN
   IF (MODE2) GO TO 20
   CALL NOIZE(RDN,0.,WN)
20 OUT(2,1)=RD+WN
   IF (MODE2) GO TO 30
   CALL NOIZE(PSIRN,0.,WN)
30 OUT(3,1)=PSIR+WN
   IF (MODE2) GO TO 40
   CALL NOIZE(TETRN,0.,WN)
40 OUT(4,1)=TETAR+WN
   IF (MODE2) GO TO 50
   CALL NOIZE(F1N,0.,WN)
50 OUT(5,1)=SF(1,1)+WN
   IF (MODE2) GO TO 60
   CALL NOIZE(F2N,0.,WN)
60 OUT(6,1)=SF(2,1)+WN
   IF (MODE2) GO TO 70
   CALL NOIZE(F3N,0.,WN)
70 OUT(7,1)=SF(3,1)+WN
   RETURN
   END
   SUBROUTINE F(U,P)
*              ********************************
*SUBROUTINE F CALCULATES THE DERIVATIVES NEEDED FOR THE EULER
*INTEGRATION FOR SUBROUTINES DYNAMIC NOMINAL,AND SENSTAT.
*              ********************************
   COMMON/EPSILON/PSI,TETA,PHI
   COMMON/CONTROL/KD,BETAC,DELTAC
   COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
   COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
   COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
   COMMON/SENSFRC/SF1EPS(1,3),SF2EPS(1,3),SF3EPS(1,3),SFEPS(3,3)
  $,SFEPSA(3,3),DFDV(3,3),TEMP5(3,3)
   COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
   COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
   COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
  $LIFT(3,1),OUT(7,1),OUTN(7,7),OUTRU(7,1),FD,ACC(3,1),V1,V2,V3
   COMMON/LOGIC/MODE2
   DIMENSION U(36),P(36)
   LOGICAL MODE2
```

```
      REAL LAMDA,MU,L,MQ,MA,MD,LA,LD,LIFT
      DATA(MQ=-.462),(MA=-5.81),(MD=-72.0),(LA=.379),(LD=.0599),
     $(LAMDA=10.),(MU=30.),(G=32.2)
      IF (FD) 10,20,30
*SUBROUTINE F FOR SUBROUTINE DYNAMIC
   10 P(1)=-MU*(U(1)-DETAC)
      P(2)=-LAMDA*(U(2)-DELTAC)
      P(3)=MQ*U(3)+MA*U(4)+MD*U(2)
      P(4)=U(3)-LA*U(4)-LD*U(2)
      L=-V*(P(4)-U(3))
      RETURN
*SUBROUTINE F FOR SUBROUTINE NOMINAL
   20 P(1)=U(4)
      P(2)=U(5)
      P(3)=U(6)
      CALL MATM(CMA,SF,ACC,3,3,1)
      P(4)=ACC(1,1)
      P(5)=ACC(2,1)
      P(6)=ACC(3,1)+G
      RETURN
*SUBROUTINE F FOR THE SENSITIVITY CALCULATIONS
   30 P(7)=U(16)
      P(8)=U(17)
      P(9)=U(18)
      P(10)=U(19)
      P(11)=U(20)
      P(12)=U(21)
      P(13)=U(22)
      P(14)=U(23)
      P(15)=U(24)
      P(16)=-SF(2,1)+SFEPSA(1,1)
      P(17)=SF(3,1)+SFEPSA(1,2)
      P(18)=SFEPSA(1,3)
      P(19)=SF(1,1)+SFEPSA(2,1)
      P(20)=SFEPSA(2,2)
      P(21)=-SF(3,1)+SFEPSA(2,3)
      P(22)=SFEPSA(3,1)
      P(23)=-SF(1,1)+SFEPSA(3,2)
      P(24)=SF(2,1)+SFEPSA(3,3)
      DO 130 I=1,3
      P(33+I)=DH(I,1)
      DO 130 J=1,3
      P(21+I+3*J)=DM(I,J)
  130 CONTINUE
      RETURN
      END
      SUBROUTINE SENSOUT
*                 ********************************
*SUBROUTINE SENSOUT CALCULATES THE OUTPUT (RADAR MEASUREMENTS AND
*SPECIFIC FORCES) SENSITIVITIES W.R.T. THE MISALIGNMENT ANGLES.
*                 ********************************
      COMMON/TIMER/T,DT,TF,DPFT,N,KK
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/SENSRDR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/SENSFRC/SF1EPS(1,3),SF2EPS(1,3),SF3EPS(1,3),SFEPS(3,3)
     $,SFEPSA(3,3),DEDV(3,3),TEMPS(3,3)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAMF(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(7,7),OUTRU(7,1),FD,ACC(3,1),V1,V2,V3
      DIMENSION PR(1,3),VR(1,3),XY(1,3),XYZ(1,3),TEMP1(1,3),TEMP2(1,3)
      R=OUT(1,1)
      PR(1,1)=X/R
      PR(1,2)=Y/R
      PR(1,3)=Z/R
      PVR=(X*VX+Y*VY+Z*VZ)/(R*R)
      VR(1,1)=(VX/R-PVR*PR(1,1))............
```

```
      VR(1,2)=(VY/R-RVR*PR(1,2))
      VR(1,3)=(VZ/R-RVR*PR(1,3))
      CALL MATM(PR,SEPSP,SREPS,1,3,3)
      CALL MATM(VR,SEPSP,TEMP1,1,3,3)
      CALL MATM(PR,SEPSV,TEMP2,1,3,3)
      DO 10 I=1,3
      SRDEPS(1,I)=TEMP1(1,I)+TEMP2(1,I)
   10 CONTINUE
      XY(1,1)=-X*Y/SQRT((X*X+Y*Y)**3)
      XY(1,2)=X*X/SQRT((X*X+Y*Y)**3)
      XY(1,3)=0.
      CALL MATM(XY,SEPSP,SPSREPS,1,3,3)
      XYZ(1,1)=-X*Z/R**3.
      XYZ(1,2)=-Y*Z/R**3.
      XYZ(1,3)=(X*X+Y*Y)/R**3
      CALL MATM(XYZ,SEPSP,STEREPS,1,3,3)
      DO 20 I=1,3
      SF1EPS(1,I)=SFEPS(1,I)
      SF2EPS(1,I)=SFEPS(2,I)
      SF3EPS(1,I)=SFEPS(3,I)
   20 CONTINUE
      RETURN
      END
      SUBROUTINE NOIZE (RMSNOIS,OUTMEAN,WN)
*               ******************************
*SUBROUTINE NOIZE CALCULATES THE VALUES OF THE MEASUREMENT NOISE
*COMPONENTS USING A RANDOM NUMBER GENERATOR MODELLED AS GAUSSIAN.
*               ******************************
      COMMON/NOISVAR/RN,RDN,PSIRN,TETAN,F1N,F2N,F3N
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(7,7),OUTRU(7,1),FD,ACC(3,1),V1,V2,V3
      GAUSS=0.
      DO 10 I=1,12
      GAUSS=GAUSS+RANF(DUM)
   10 CONTINUE
      GAUSS=GAUSS-6.+OUTMEAN
      WN=GAUSS*RMSNOIS
      RETURN
      END
      SUBROUTINE INFOMAT
*               ******************************
*SUBROUTINE INFOMAT CALCULATES THE VALUES OF THE INFORMATION
*MATRIX AND GRADIENT VECTOR INCREMENTS.
*               ******************************
      COMMON/TIMER/T,DT,TF,DPST,N,KK
      COMMON/EPSTRUE/PSIT,TETAT,PHIT
      COMMON/EPSIO/PSIO,TETAO,PHIO
      COMMON/SENSROR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/SENSFRC/SF1EPS(1,3),SF2EPS(1,3),SF3EPS(1,3),SFEPS(3,3)
     $,SFEPSA(3,3),DFDV(3,3),TEMP5(3,3)
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAMF(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(7,7),OUTRU(7,1),FD,ACC(3,1),V1,V2,V3
     $,SOUTEPS(7,3)
      COMMON/LOGIC/MODE2
      REAL M
      LOGICAL MODE2
      DIMENSION SOUEPST(3,7),TEMP1(3,7),DOUT(7,1)
      DO 10 I=1,3
      SOUTEPS(1,I)=SREPS(1,I)
      SOUTEPS(2,I)=SRDEPS(1,I)
      SOUTEPS(3,I)=SPSREPS(1,I)
      SOUTEPS(4,I)=STEREPS(1,I)
      SOUTEPS(5,I)=SF1EPS(1,I)
      SOUTEPS(6,I)=SF2EPS(1,I)
      SOUTEPS(7,I)=SF3EPS(1,I)
```

```
10 CONTINUE
   DO 20 I=1,7
   DO 20 J=1,3
   SOUEPST(J,I)=SOUTEPS(I,J)
20 CONTINUE
   CALL MATM(SOUEPST,OUTN,TEMP1,3,7,7)
   CALL MATM(TEMP1,SOUTEPS,DM,3,7,3)
   DO 30 I=1,7
   DOUT(I,1)=OUTRU(I,1)-OUT(I,1)
30 CONTINUE
   CALL MATM(TEMP1,DOUT,DH,3,7,1)
   RETURN
   END
   SUBROUTINE EULINT(U,K,DT)
   DIMENSION U(36),P(36)
   CALL F(U,P)
   DO 10 J=1,K
   U(J)=U(J)+P(J)*DT
10 CONTINUE
   RETURN
   END
   SUBROUTINE MATM(A,B,C,M,K,N)
   DIMENSION A(M,K),B(K,N),C(M,N)
   DO 10 J=1,N
   DO 10 I=1,M
   C(I,J)=0.
   DO 10 L=1,K
   C(I,J)=C(I,J)+A(I,L)*B(L,J)
10 CONTINUE
   RETURN
   END
```

Vita

Rony Dayan was born on 23 November 1944, in Cairo, Egypt. In 1957, he immigrated with his family to Israel. He was graduated from the Gymnasia Realit High School in Richon Lezion, Israel, in 1963. He then entered the Technion in Haifa, Israel through the ROTC program and was awarded a B.Sc. degree in Electrical Engineering in August, 1967. During his various assignments in the Air Force, he was responsible for maintenance, procurement, development and testing of weapon systems. In June, 1976, he was selected to join the resident graduate Guidance and Control program under the Electrical Engineering department at the U.S. Air Force Institute of Technology. He is married to the former Malca Shapira and they have two sons.

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This thesis was typed by Mrs. Annette Marchand.

```
      PROGRAM SIMUL1(INPUT,OUTPUT=1002B,TAPE5=1002B,TAPE10=OUTPUT)
*                 ********************************
*PROGRAM SIMUL IS A SIMULATION OF THE TRACKING AND CONTROL OF
*AN AIR-TO-GROUND MISSILE USING MAXIMUM LIKELIHOOD ESTIMATION
*OF THE THREE ANGLES REPRESENTING ITS INITIAL MISALIGNMENT.
*                 ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/EPSTRUE/PSIT,TETAT,PHIT
      COMMON/EPSIO/PSIO,TETAO,PHIO
      COMMON/CONTROL/KD,BETAC,DELTAC
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/INITIAL/XO,YO,ZO,VXO,VYO,VZO
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/SENSRDR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      COMMON/LOGIC/MODE1
      LOGICAL MODE1
      DIMENSION U(36),WKAREA(18),MI(3,3),TYME(6),DELTOUT(18,4),HOTOUT(
     $18,4),TRD(18),DPSI(18),DTETA(18),DPHI(18),PSIP(3),TETAP(3),PHIP
     $(3),DOUT(4,1),HOT(4),TAYLOR(4,1),EPS(3,1)
      REAL M,LIFT,L,MI
*                 ********************************
*INITIALIZATION OF THE RANDOM NUMBER GENERATOR FOR NOISE GENERATION.
*                 ********************************
      PRINT(10,98)
      JJ=1
      CALL RANSET(JJ)
      T=0.
      DPRNT=0.
      PRINT*,"***TRUTH MODEL ***"
      MODE1=.FALSE.
*                 ********************************
*INITIALIZATION OF THE MISALIGNMENT ANGLES WITH VALUES ASSUMED TO BE
*TRUE.
*                 ********************************
      PSI=PSIT
      TETA=TETAT
      PHI=PHIT
      APSI=ATETA=APHI=0.
*                 ********************************
*DEFINITION OF THE MISALIGNMENT ANGLES TRANSFORMATION MATRIX.
*
*           1.      -PSI    TETA
*   CMA=    PSI     1.      -PHI
*           -TETA   PHI     1.
*
*                 ********************************
      CMA(1,1)=CMA(2,2)=CMA(3,3)=1.
      CMA(2,1)=PSI
      CMA(3,1)=-TETA
      CMA(3,2)=PHI
      CMA(1,2)=-CMA(2,1)
      CMA(1,3)=-CMA(3,1)
      CMA(2,3)=-CMA(3,2)
*                 ********************************
*INITIALIZATION OF POSITION,VELOCITY,AND SPECIFIC FORCE.
*                 ********************************
      X=XO
      Y=YO
      Z=ZO
      VX=VXO
```

```
      CMA(1,2)=-CMA(2,1)
      CMA(1,3)=-CMA(3,1)
      CMA(2,3)=-CMA(3,2)
*                 ********************************
*INITIALIZATION OF POSITION,VELOCITY,AND SPECIFIC FORCE.
*                 ********************************
      X=XO
      Y=YO
      Z=ZO
      VX=VXO
      VY=VYO
      VZ=VZO
      V=SQRT(VX*VX+VY*VY+VZ*VZ)
      SF(1,1)=0.
      SF(2,1)=0.
      SF(3,1)=0.
*                 ********************************
*EVALUATION OF THE VALUE OF THE STATES FOR THE TRUTH MODEL .
*                 ********************************
   10 CALL MEASURE
      WRITE(5)T,(OUT(I,1),I=1,7)
      CALL CONTROL
      FD=-1.
      CALL DYNAMIC
      FD=0.
      CALL NOMINAL
      CALL SPFORCE
   20 IF(ABS(T-DPRNT).LT.1.E-06)GO TO 40
   30 T=T+DT
      IF(T.LE.TF)GO TO 10
      GO TO 50
   40 PRINT(10,99)KD,BETA,DELTA,Q,ALPHA,L,X,Y,Z,V,VX,VY,VZ,T,BETAC,
     &DELTAC,SF(1,1),SF(2,1),SF(3,1),OUT(1,1),OUT(2,1),OUT(3,1),OUT(4,1)
     &,ACC(1,1),ACC(2,1),ACC(3,1)
      PRINT(10,79)T,(OUT(I,1),I=1,7)
      DPRNT=DPRNT+DPRT
      GO TO 30
   50 REWIND 5
      PRINT*,"***MODEL 1***"
      MODE1=.TRUE.
      PRINT*,"MODE1=",MODE1
      PSI=PSIO
      TETA=TETAO
      PHI=PHIO
      DO 110 NN=1,3
      N=NN
      T=0.
      DPRNT=0.
*                 ********************************
*INITIALIZATION OF THE MISALIGNMENT ANGLES WITH FIRST ESTIMATED
*NOMINAL VALUES.
*                 ********************************
      PSI=PSI+APSI/1000
      TETA=TETA+ATETA/1000
      PHI=PHI+APHI/1000
      PSIP(N)=1000*PSI
      TETAP(N)=1000*TETA
      PHIP(N)=1000*PHI
      CMA(1,1)=CMA(2,2)=CMA(3,3)=1.
      CMA(2,1)=PSI
      CMA(3,1)=-TETA
      CMA(3,2)=PHI
      CMA(1,2)=-CMA(2,1)
      CMA(1,3)=-CMA(3,1)
      CMA(2,3)=-CMA(3,2)
*                 ********************************
*INITIALIZATION OF POSITION,VELOCITY,SPECIFIC FORCE,AERODYNAMIC
*VALUES,INFORMATION AND DISPERSION MATRIX ELEMENTS AND TRACE,
```

2

```
      CMA(2,3)=-CMA(3,2)
*                 *******************************
*INITIALIZATION OF POSITION,VELOCITY,SPECIFIC FORCE,AERODYNAMIC
*VALUES,INFORMATION AND DISPERSION MATRIX ELEMENTS AND TRACE,
*AND SENSITIVITIES.
*                 *******************************
      U(1)=X0
      U(2)=Y0
      U(3)=Z0
      U(4)=VX0
      U(5)=VY0
      U(5)=VY0
      U(6)=VZ0
      SF(1,1)=0.
      SF(2,1)=0.
      SF(3,1)=0.
      BETA=0.
      DELTA=0.
      Q=0.
      ALPHA=0.
      L=0.
      DO 55 I=1,3
      DO 55 J=1,3
      SEPSP(I,J)=0.
      SEPSV(I,J)=0.
      M(I,J)=0.
      MI(I,J)=0.
   55 CONTINUE
      DO 60 I=7,36
      U(I)=0.
   60 CONTINUE
      OUTN(1,1)=1./(RN*RN)
      OUTN(2,2)=1./(RDN*RDN)
      OUTN(3,3)=1./(PSIRN*PSIRN)
      OUTN(4,4)=1./(TETRN*TETRN)
      OUTN(1,2)=OUTN(1,3)=OUTN(1,4)=0.
      OUTN(2,1)=OUTN(2,3)=OUTN(2,4)=0.
      OUTN(3,1)=OUTN(3,2)=OUTN(3,4)=0.
      OUTN(4,1)=OUTN(4,2)=OUTN(4,3)=0.
      DO 100 KK=1,5001
      READ(5)T,(OUTRU(I),I=1,7)
*                 *******************************
*                             STATES  DEFINITION
*                             ------------------
*POSITION:(1-3)
*     U(1)=X,   U(2)=Y,   U(3)=Z,
*VELOCITY:(4-6)
*     U(4)=VX,  U(5)=VY,  U(6)=VZ,
*POSITION SENSITIVITY:(7-15)
*     U(7)=X/PSI,    U(8)=X/TETA,    U(9)=X/PHI,
*     U(10)=Y/PSI,   U(11)=Y/TETA,   U(12)=Y/PHI,
*     U(13)=Z/PSI,   U(14)=Z/TETA,   U(15)=Z/PHI,
*VELOCITY SENSITIVITY:(16-24)
*     U(16)=VX/PSI,   U(17)=VX/TETA,   U(18)=VX/PHI,
*     U(19)=VY/PSI,   U(20)=VY/TETA,   U(21)=VZ/PHI,
*     U(22)=VZ/PSI,   U(23)=VZ/TETA,   U(24)=VZ/PHI,
*INFORMATION MATRIX:(25-33)
*         U(25)  U(28)  U(31)
*     M=  U(26)  U(29)  U(32)
*         U(27)  U(30)  U(33)
*TRANSPOSE OF GRADIENT VECTOR:(34-36)
*     H=  U(34)  U(35)  U(36)
*                 *******************************
      X=U(1)
      Y=U(2)
      Z=U(3)
      VX=U(4)
      VY=U(5)
      VZ=U(6)
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      Z=U(3)
      VX=U(4)
      VY=U(5)
      VZ=U(6)
*                  ********************************
*USING THE RECEIVED SPECIFIC FORCE AS INPUTS TO MODEL 1.
*                  ********************************
      DO 70 I=1,3
      SF(I,1)=OUTRU(I+4)
   70 CONTINUE
      CALL MEASURE
      DO 80 I=1,3
      DO 80 J=1,3
      SEPSP(I,J)=U(3*I+J+3)
      SEPSV(I,J)=U(3*I+J+12)
   80 CONTINUE
      IF((KK-1)-1000*((KK-1)/1000).EQ.0)PRINT(10,87)((SEPSP(I,J),J=1,3),
     &I=1,3),((SEPSV(I,J),J=1,3),I=1,3)
      CALL SENSOUT
      CALL INFOMAT
      DO 65 I=1,4
      DOUT(I,1)=OUTRU(I,1)-OUT(I,1)
   65 CONTINUE
      EPS(1,1)=PSIT-PSI
      EPS(2,1)=TETAT-TETA
      EPS(3,1)=PHIT-PHI
      CALL MATM(SOUTEPS,EPS,TAYLOR,4,3,1)
      DO 75 I=1,4
      HOT(I)=DOUT(I,1)-TAYLOR(I,1)
   75 CONTINUE
      FD=0.
      K=36
      CALL EULINT(U,K,.01)
      IF((KK-1)-1000*((KK-1)/1000).EQ.0)PRINT(10,81)DOUT,HOT
      IF((KK-1)-1000*((KK-1)/1000).EQ.0)PRINT(10,79)T,(OUTRU(I),I=1,7)
      IF((KK-1)-1000*((KK-1)/1000).NE.0)GO TO 83
      TYME((KK-1)/1000+1)=(KK-1)/100.
      DO 82 I=1,4
      DELTOUT((6*(N-1)+(KK-1)/1000+1),I)=DOUT(I,1)
      HOTOUT((6*(N-1) +(KK-1) /1000+1),I)=HOT(I)
   82 CONTINUE
   83 DO 90 J=1,3
      DO 90 I=1,3
      M(I,J)=U(21+I+3*J)
   90 CONTINUE
      IF(KK.EQ.1)GO TO 100
      IF((KK-1)-1000*((KK-1)/1000).NE.0)GO TO 100
      CALL LINV2F(M,3,3,MI,10,WKAREA,IER)
      TRD(6*(N-1)+(KK-1)/1000+1)=MI(1,1)+MI(2,2)+MI(3,3)
      PRINT(10,78)TRD(6*(N-1)+(KK-1)/1000+1)
      APSI=(MI(1,1)*U(34)+MI(1,2)*U(35)+MI(1,3)*U(36))*1000
      ATETA=(MI(2,1)*U(34)+MI(2,2)*U(35)+MI(2,3)*U(36))*1000
      APHI=(MI(3,1)*U(34)+MI(3,2)*U(35)+MI(3,3)*U(36))*1000
      DPSI(6*(N-1)+(KK-1)/1000+1)=(PSIT-PSI)*1000-APSI
      DTETA(6*(N-1)+(KK-1)/1000+1)=(TETAT-TETA)*1000-ATETA
      DPHI(6*(N-1)+(KK-1)/1000+1)=(PHIT-PHI)*1000-APHI
      PRINT(10,92)KD,N,DPSI(6*(N-1)+(KK-1)/1000+1),DTETA(6*(N-1)+
     &(KK-1)/1000+1),DPHI(6*(N-1)+(KK-1)/1000+1)
  100 CONTINUE
      REWIND 5
  110 CONTINUE
      PSITP=1000*PSIT
      TETATP=1000*TETAT
      PHITP=1000*PHIT
      DO 15 N=1,3
      PRINT(10,1)PSITP,TETATP,PHITP,PSIP(N),TETAP(N),PHIP(N),N
    1 FORMAT("1  MODEL    1",5X,"EPSITRUE=",2(E12.5,","),E12.5,/3X,
     &"WITH NOISE",6X,"EPSIO=    ",2(E12.5,","),E12.5,/3X,"KD    =    3",
```

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     &(KK-1)/1000+1),DPHI(6*(N-1)+(KK-1)/1000+1)
 100 CONTINUE
     REWIND 5
 110 CONTINUE
     PSITP=1000*PSIT
     TETATP=1000*TETAT
     PHITP=1000*PHIT
     DO 15 N=1,3
     PRINT(10,1)PSITP,TETATP,PHITP,PSIP(N),TETAP(N),PHIP(N),N
   1 FORMAT("1  MODEL     1",5X,"EPSITRUE=",2(E12.5,","),E12.5,/3X,
    &"WITH NOISE",5X,"EPSIO=    ",2(E12.5,","),E12.5,/3X,"KD    =    3",
    &5X,"ITERATION # ",I1,///)
     PRINT(10,2)
   2 FORMAT(4X,"TIME",6X,"DELTA(R)",7X,"HOT(R)",8X,"DELTA(RD)",
    &6X,"HOT(RD)"/)
     DO 4 I=1,6
     PRINT(10,3)TYME(I),DELTOUT((6*(N-1)+I),1),HOTOUT((6*(N-1)+I),1),
    &DELTOUT((6*(N-1)+I),2),HOTOUT((6*(N-1)+I),2)
   4 CONTINUE
   3 FORMAT(4X,F4.0,4X,2(E12.5,2X),2(2X,E12.5)/)
     PRINT(10,5)
   5 FORMAT("0",3X,"TIME",5X,"DELTA(AZ.)",5X,"HOT(AZ.)",7X,
    &"DELTA(EL.)",5X,"HOT(EL.)",/)
     DO 6 I=1,6
     PRINT(10,3)TYME(I),DELTOUT((6*(N-1)+I),3),HOTOUT((6*(N-1)+I),3),
    &DELTOUT((6*(N-1)+I),4),HOTOUT((6*(N-1)+I),4)
   6 CONTINUE
     PRINT(10,11)
  11 FORMAT(5X,"TIME",5X,"TRACE   OF",6X,3(8HESTIMATE,4X)/15X,
    &"DISPERSION",6X,3(8HERROR IN,4X)/17X,"MATRIX",10X,"PSI",
    &9X,"TETA",8X,"PHI"/)
     DO 12 I=1,6
     PRINT(10,13)TYME(I),TRD(6*(N-1)+I),DPSI(6*(N-1)+I),DTETA
    &(6*(N-1)+I),DPHI(6*(N-1)+I)
  12 CONTINUE
  13 FORMAT(6X,F4.0,5X,E12.5,3(3X,E9.2)/)
  15 CONTINUE
  78 FORMAT(1X,"TRD=",E12.5/)
  87 FORMAT(11X,3(E9.2,2X)/3X,"SEPSP=  ",3(E9.2,2X)/11X,3(E9.2,2X)
    &//11X,3(E9.2,2X)/3X,"SEPSV=  ",3(E9.2,2X)/11X,3(E9.2,2X)//)
  79 FORMAT(1X,6HOUTRU=,F6.3,4(2X,E9.2)/13X,3(2X,E9.2)//)
  92 FORMAT(1X,7HDELEPS:,I1,2X,I1,3E12.5/)
  81 FORMAT(1X,"DOUT=",4(E10.3,2X)/1X,"HOT =",4(E10.3,2X)/)
  98 FORMAT(5X,"KD",8X,"BETA",6X,"DELTA",8X,"Q",8X,"ALPHA",8X,"L",//
    &,5X,"X",10X,"Y",10X,"Z",10X,"V",10X,"VX",9X,"VY",9X,"VZ",//,
    &5X,"T",8X,"BETAC",6X,"DELTAC",17X,"SF1",8X,"SF2",8X,"SF3",//,
    &5X,"R",9X,"RDOT",7X,"PSIR",7X,"TETR",7X,"ACC1",7X,"ACC2",7X,
    &"ACC3",//)
  99 FORMAT(5X,I1,6X,5(E10.3,1X)/1X,7(E10.3,1X)/1X,F6.3,5X,2(E10.3,1X),
    &11X,3(E10.3,1X)/1X,7(E10.3,1X)//)
     STOP
     END
*********************************************
     BLOCK DATA
     REAL L,M
     COMMON/TIMER/T,DT,TF,DPRT
     COMMON/EPSTRUE/PSIT,TETAT,PHIT
     COMMON/EPSIO/PSIO,TETAO,PHIO
     COMMON/CONTROL/KD,BETAC,DELTAC
     COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
     COMMON/INITIAL/XO,YO,ZO,VXO,VYO,VZO
     COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
     COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
     COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
     DATA DT,TF,DPRT/.01,50.,10./
     DATA PSIT,TETAT,PHIT/1.E-03,2.E-03,3.E-03/
     DATA PSIO,TETAO,PHIO/3*0./
     DATA KD/3/
     DATA BETA,DELTA,Q,ALPHA,L/5*0./
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      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/INITIAL/XO,YO,ZO,VXO,VYO,VZO
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      DATA DT,TF,DPRT/.01,50.,10./
      DATA PSIT,TETAT,PHIT/1.E-03,2.E-03,3.E-03/
      DATA PSIO,TETAO,PHIO/3*0./
      DATA KD/3/
      DATA BETA,DELTA,Q,ALPHA,L/5*0./
      DATA XO,YO,ZO,VXO,VYO,VZO/2*1000.,100.,2000.,1500.,10./
      DATA SEPSP,SEPSV/18*0./
      DATA RN,RDN,PSIRN,TETRN,F1N,F2N,F3N/10.,10.,2*.001,
     &3*32.2E-03/
      DATA M,H/9*0.,3*0./
      END
      SUBROUTINE CONTROL
*              ********************************
*SUBROUTINE CONTROL GENERATES THREE TYPES OF COMMANDS FOR
*DELTA AND BETA.
*              ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/CONTROL/KD,BETAC,DELTAC
      REAL K1,K2,K3,K4
      DATA(K1=.063),(K2=.03),(K3=.007),(K4=.002),(W1=.1),(W2=.2)
      TC=T+DT/2.
      IF(KD.EQ.1)10,20
   10 DELTAC=K3+K4*SIN(W2*TC)
      BETAC=K1*TC
      RETURN
   20 IF(KD.EQ.2)30,40
   30 DELTAC=K3
      BETAC=K1*TC+K2*SIN(W1*TC)
      RETURN
   40 BETAC=K1*TC+K2*SIN(W1*TC)
      DELTAC=K3+K4*SIN(W2*TC)
      RETURN
      END
      SUBROUTINE DYNAMIC
*              ********************************
*SUBROUTINE DYNAMIC CALCULATES THE VALUES OF BETA,DELTA,Q (PITCH
*RATE),ALPHA (ANGLE OF ATTACK),AND L (LIFT) BASED ON AN EULER
*INTEGRATION ROUTINE.
*              ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      DIMENSION U(4)
      REAL L,LIFT
      U(1)=BETA
      U(2)=DELTA
      U(3)=Q
      U(4)=ALPHA
      GAB(1,1)=1.
      GAB(2,2)=GAB(3,3)=COS(U(1))
      GAB(3,2)=SIN(U(1))
      GAB(2,3)=-GAB(3,2)
      GAB(2,1)=GAB(1,2)=GAB(3,1)=GAB(1,3)=0.
      K=4
      CALL EULINT(U,K,.01)
      BETA=U(1)
      DELTA=U(2)
      Q=U(3)
      ALPHA=U(4)
      RETURN
      END
      SUBROUTINE NOMINAL
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      K=4
      CALL EULINT(U,K,.01)
      BETA=U(1)
      DELTA=U(2)
      Q=U(3)
      ALPHA=U(4)
      RETURN
      END
      SUBROUTINE NOMINAL
*               ********************************
*SUBROUTINE NOMINAL CALCULATES THE VALUES OF THE MISSILE POSITION
*AND VELOCITY IN THE A/C FRAME,BASED ON AN EULER ROUTINE.
*               ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      DIMENSION U(6)
      U(1)=X
      U(2)=Y
      U(3)=Z
      U(4)=VX
      U(5)=VY
      U(6)=VZ
      K=6
      CALL EULINT(U,K,.01)
      V=SQRT(U(4)*U(4)+U(5)*U(5)+U(6)*U(6))
      X=U(1)
      Y=U(2)
      Z=U(3)
      VX=U(4)
      VY=U(5)
      VZ=U(6)
      RETURN
      END
      SUBROUTINE SPFORCE
*               ********************************
*SUBROUTINE SPFORCE CALCULATES THE SPECIFIC FORCES MEASURED BY THE
*THREE ACCELEROMETERS ON THE MISSILE.
*               ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      DIMENSION GA(3,3),GAF(3,3)
      REAL L,LIFT
      LIFT(3,1)=-L
      LIFT(1,1)=LIFT(2,1)=0.
      A=SQRT(VX*VX+VY*VY)
      V1=VX+PSI*VY-TETA*VZ
      V2=-PSI*VX+VY+PHI*VZ
      V3=TETA*VX-PHI*VY+VZ
      GAMA(1,1)=V1/A
      GAMA(2,2)=GAMA(1,1)
      GAMA(1,2)=-V2/A
      GAMA(2,1)=-GAMA(1,2)
      GAMA(3,3)=1.
      GAMA(1,3)=GAMA(3,1)=GAMA(2,3)=GAMA(3,2)=0.
      GAME(1,1)=A/V
      GAME(3,3)=GAME(1,1)
      GAME(3,1)=V3/V
      GAME(1,3)=-GAME(3,1)
      GAME(2,2)=1.
      GAME(1,2)=GAME(2,1)=GAME(2,3)=GAME(3,2)=0.
      CALL MATM(GAMA,GAME,GA,3,3,3)
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      GAMA(2,1)=-GAMA(1,2)
      GAMA(3,3)=1.
      GAMA(1,3)=GAMA(3,1)=GAMA(2,3)=GAMA(3,2)=0.
      GAME(1,1)=A/V
      GAME(3,3)=GAME(1,1)
      GAME(3,1)=V3/V
      GAME(1,3)=-GAME(3,1)
      GAME(2,2)=1.
      GAME(1,2)=GAME(2,1)=GAME(2,3)=GAME(3,2)=0.
      CALL MATM(GAMA,GAME,GA,3,3,3)
      CALL MATM(GA,GAB,GAF,3,3,3)
      CALL MATM(GAF,LIFT,SF,3,3,1)
      RETURN
      END
      SUBROUTINE MEASURE
*                ********************************
*SUBROUTINE MEASURE CALCULATES THE RANGE,RANGE RATE,AZIMUTH,
*AND ELEVATION AS MEASURED BY THE RADAR IN THE A/C.
*                ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      COMMON/LOGIC/MODE1
      LOGICAL MODE1
      WN=0.
      R=SQRT(X*X+Y*Y+Z*Z)
      RD=(X*VX+Y*VY+Z*VZ)/R
      PSIR=Y/SQRT(X*X+Y*Y)
      TETAR=Z/R
      IF(MODE1) GO TO 10
      CALL NOIZE(RN,0.,WN)
   10 OUT(1,1)=R+WN
      IF(MODE1) GO TO 20
      CALL NOIZE(RDN,0.,WN)
   20 OUT(2,1)=RD+WN
      IF(MODE1) GO TO 30
      CALL NOIZE(PSIRN,0.,WN)
   30 OUT(3,1)=PSIR+WN
      IF(MODE1) GO TO 40
      CALL NOIZE(TETRN,0.,WN)
   40 OUT(4,1)=TETAR+WN
      IF(MODE1) GO TO 50
      CALL NOIZE(F1N,0.,WN)
   50 OUT(5,1)=SF(1,1)+WN
      IF(MODE1) GO TO 60
      CALL NOIZE(F2N,0.,WN)
   60 OUT(6,1)=SF(2,1)+WN
      IF(MODE1) GO TO 70
      CALL NOIZE(F3N,0.,WN)
   70 OUT(7,1)=SF(3,1)+WN
      RETURN
      END
      SUBROUTINE F(U,P)
*                ********************************
*SUBROUTINE F CALCULATES THE DERIVATIVES NEEDED FOR THE EULER
*INTEGRATION FOR SUBROUTINES DYNAMIC NOMINAL,AND SENSTAT.
*                ********************************
      COMMON/EPSILON/PSI,TETA,PHI
      COMMON/CONTROL/KD,BETAC,DELTAC
      COMMON/DYNAMIC/BETA,DELTA,Q,ALPHA,L
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
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      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      COMMON/LOGIC/MODE1
      DIMENSION U(36),P(36),OUTF(3,1)
      LOGICAL MODE1
      REAL LAMDA,NU,L,MQ,MA,MD,LA,LD,LIFT
      DATA(MQ=-.462),(MA=-5.81),(MD=-72.0),(LA=.379),(LD=.0699),
     &(LAMDA=10.),(NU=30.),(G=32.2)
      IF (FD) 10,20,30
*SUBROUTINE F FOR SUBROUTINE DYNAMIC
   10 P(1)=-NU*(U(1)-BETAC)
      P(2)=-LAMDA*(U(2)-DELTAC)
      P(3)=MQ*U(3)+MA*U(4)+MD*U(2)
      P(4)=U(3)-LA*U(4)-LD*U(2)
      L=-V*(P(4)-U(3))
      RETURN
*SUBROUTINE F FOR SUBROUTINE NOMINAL
   20 IF(MODE1)GO TO 25
      P(1)=U(4)
      P(2)=U(5)
      P(3)=U(6)
      CALL MATM(CMA,SF,ACC,3,3,1)
      P(4)=ACC(1,1)
      P(5)=ACC(2,1)
      P(6)=ACC(3,1)+G
      RETURN
   25 P(1)=U(4)
      P(2)=U(5)
      P(3)=U(6)
      OUTF(1,1)=OUT(5,1)
      OUTF(2,1)=OUT(6,1)
      OUTF(3,1)=OUT(7,1)
      CALL MATM(CMA,OUTF,ACC,3,3,1)
      P(4)=ACC(1,1)
      P(5)=ACC(2,1)
      P(6)=ACC(3,1)+G
   30 DO 35 I=7,15
      P(I)=U(I+9)
   35 CONTINUE
      P(16)=-SF(2,1)
      P(17)=SF(3,1)
      P(19)=SF(1,1)
      P(21)=-SF(3,1)
      P(23)=-SF(1,1)
      P(24)=SF(2,1)
      P(18)=P(20)=P(22)=0.
      DO 36 J=1,3
      DO 36 I=1,3
      P(21+I+3*J)=DM(I,J)
   36 CONTINUE
      DO 37 I=1,3
      P(33+I)=DH(I,1)
   37 CONTINUE
      RETURN
      END
      SUBROUTINE SENSOUT
*                   ********************************
*SUBROUTINE SENSOUT CALCULATES THE OUTPUT (RADAR MEASUREMENTS AND
*SPECIFIC FORCES) SENSITIVITIES W.R.T. THE MISALIGNMENT ANGLES.
*                   ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/SENSRDR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
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      SUBROUTINE SENSOUT
*                ********************************
*SUBROUTINE SENSOUT CALCULATES THE OUTPUT (RADAR MEASUREMENTS AND
*SPECIFIC FORCES) SENSITIVITIES W.R.T. THE MISALIGNMENT ANGLES.
*                ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/NOMINAL/X,Y,Z,VX,VY,VZ,V
      COMMON/SENSTAT/SEPSP(3,3),SEPSV(3,3)
      COMMON/SENSRDR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      DIMENSION PR(1,3),VR(1,3),XY(1,3),XYZ(1,3),TEMP1(1,3),TEMP2(1,3)
      R=OUT(1,1)
      PR(1,1)=X/R
      PR(1,2)=Y/R
      PR(1,3)=Z/R
      PVR=(X*VX+Y*VY+Z*VZ)/(R*R)
      VR(1,1)=(VX/R-PVR*PR(1,1))
      VR(1,2)=(VY/R-PVR*PR(1,2))
      VR(1,3)=(VZ/R-PVR*PR(1,3))
      CALL MATM(PR,SEPSP,SREPS,1,3,3)
      CALL MATM(VR,SEPSP,TEMP1,1,3,3)
      CALL MATM(PR,SEPSV,TEMP2,1,3,3)
      DO 10 I=1,3
      SRDEPS(1,I)=TEMP1(1,I)+TEMP2(1,I)
   10 CONTINUE
      XY(1,1)=-X*Y/SQRT((X*X+Y*Y)**3)
      XY(1,2)=X*X/SQRT((X*X+Y*Y)**3)
      XY(1,3)=0.
      CALL MATM(XY,SEPSP,SPSREPS,1,3,3)
      XYZ(1,1)=-X*Z/R**3
      XYZ(1,2)=-Y*Z/R**3
      XYZ(1,3)=(X*X+Y*Y)/R**3
      CALL MATM(XYZ,SEPSP,STEREPS,1,3,3)
      RETURN
      END
      SUBROUTINE NOIZE (RMSNOIS,OUTMEAN,WN)
*                ********************************
*SUBROUTINE NOIZE CALCULATES THE VALUES OF THE MEASUREMENT NOISE
*COMPONENTS USING A RANDOM NUMBER GENERATOR MODELLED AS GAUSSIAN.
*                ********************************
      COMMON/NOISVAR/RN,RDN,PSIRN,TETRN,F1N,F2N,F3N
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      GAUSS=0.
      DO 10 I=1,12
      GAUSS=GAUSS+RANF(DUM)
   10 CONTINUE
      GAUSS=GAUSS-6.+OUTMEAN
      WN=GAUSS*RMSNOIS
      RETURN
      END
      SUBROUTINE INFOMAT
*                ********************************
*SUBROUTINE INFOMAT CALCULATES THE VALUES OF THE INFORMATION
*MATRIX AND GRADIENT VECTOR INCREMENTS.
*                ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/EPSTRUE/PSIT,TETAT,PHIT
      COMMON/SENSRDR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     &LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      COMMON/LOGIC/MODE1
      REAL M
      LOGICAL MODE1
      DIMENSION SOUEPST(3,4),TEMP1(3,4),DOUT(4,1)
      DO 10 I=1,3
```

```
      RETURN
      END
      SUBROUTINE INFOMAT
*                ********************************
*SUBROUTINE INFOMAT CALCULATES THE VALUES OF THE INFORMATION
*MATRIX AND GRADIENT VECTOR INCREMENTS.
*                ********************************
      COMMON/TIMER/T,DT,TF,DPRT
      COMMON/EPSTRUE/PSIT,TETAT,PHIT
      COMMON/SENSRDR/SREPS(1,3),SRDEPS(1,3),SPSREPS(1,3),STEREPS(1,3)
      COMMON/INFOMAT/M(3,3),H(3,1),DM(3,3),DH(3,1)
      COMMON/BLANK/CMA(3,3),GAMA(3,3),GAME(3,3),SF(3,1),GAB(3,3),
     $LIFT(3,1),OUT(7,1),OUTN(4,4),OUTRU(7,1),FD,ACC(3,1),SOUTEPS(4,3)
      COMMON/LOGIC/MODE1
      REAL M
      LOGICAL MODE1
      DIMENSION SOUEPST(3,4),TEMP1(3,4),DOUT(4,1)
      DO 10 I=1,3
      SOUTEPS(1,I)=SREPS(1,I)
      SOUTEPS(2,I)=SRDEPS(1,I)
      SOUTEPS(3,I)=SPSREPS(1,I)
      SOUTEPS(4,I)=STEREPS(1,I)
   10 CONTINUE
      DO 20 I=1,4
      DO 20 J=1,3
      SOUEPST(J,I)=SOUTEPS(I,J)
   20 CONTINUE
      CALL MATM(SOUEPST,OUTN,TEMP1,3,4,4)
      CALL MATM(TEMP1,SOUTEPS,DM,3,4,3)
      DO 30 I=1,4
      DOUT(I,1)=OUTRU(I,1)-OUT(I,1)
   30 CONTINUE
      CALL MATM(TEMP1,DOUT,DH,3,4,1)
      RETURN
      END
      SUBROUTINE EULINT(U,K,DT)
      DIMENSION U(36),P(36)
      CALL F(U,P)
      DO 10 J=1,K
      U(J)=U(J)+P(J)*DT
   10 CONTINUE
      RETURN
      END
      SUBROUTINE MATM(A,B,C,M,K,N)
      DIMENSION A(M,K),B(K,N),C(M,N)
      DO 10 J=1,N
      DO 10 I=1,M
      C(I,J)=0.
      DO 10 L=1,K
   10 C(I,J)=C(I,J)+A(I,L)*B(L,J)
      RETURN
      END
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The problem of parameter estimation using tracking information is examined. Two models are developed and used to estimate the misalignment angles of the inertial system of a missile after its launch. The estimation is based on maximum likelihood concepts. The amount of information extracted from the tracking measurements and the missile specific forces measurements is analysed. A feasibility study of the two models is conducted. The second model uses the aerodynamic model of the missile in order to →

enhance its estimation ability. Doing this, it incorporates more non-linearities than the first model. These severe non-linearities were found to offset the advantage it had in terms of information gathering. The first model is much simpler in its concept. Yet, it is still able to gather the information needed and its performance is very comparable to the one of the second model. The simplicity and linearity of the first model make it especially attractive.