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A PRELIMINARY ANALYSIS OF SURFACE DISPLACEMENT BY TIDES OF THE --ETC(U).
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE — READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: TR-3801
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): (6) A PRELIMINARY ANALYSIS OF SURFACE DISPLACEMENT BY TIDES OF THE SOLID EARTH.
5. TYPE OF REPORT & PERIOD COVERED: (9) Final rept.,
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): (10) Walter J. Groeger
8. CONTRACT OR GRANT NUMBER(s): (12) 48p.
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Surface Weapons Center (K11), Dahlgren Laboratory, Dahlgren, Virginia 22448
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: OMDA
11. CONTROLLING OFFICE NAME AND ADDRESS: Headquarters, Defense Mapping Agency, Washington, D. C. 20360
12. REPORT DATE: (11) May 1978
13. NUMBER OF PAGES: 52
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):

18. SUPPLEMENTARY NOTES:
19. KEY WORDS (Continue on reverse side if necessary and identify by block number):
Satellite geodesy
Tide of the solid earth
Tidal surface displacement

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A computer routine is specified for the dislocation of points on land by the tides of the solid earth. The action of the moon as well as that of the sun are accounted for. Included are a discussion of the theoretical background and some trial data. The data suggest that typical displacements of a surface station are on the order of several ten centimeters, in the radial direction.

FOREWORD

Recent progress in instrumentation and computing methods suggested that satellite geodesy ought to account for even such minute effects as the surface displacement caused by the tides of the solid earth. To reflect the earth's deformation due to the gravitational action of moon and sun in our satellite work at the Naval Surface Weapons Center (NSWC), a suitable physical model was established for the tidal surface. Based on this model, a computer algorithm was written which specifies the tidal perturbation, in position, for any geodetic station on the earth's surface and, in particular, for the satellite observing stations. The numerical trial runs resulted in surface displacements which were indeed significant, and the just mentioned algorithm was subsequently included into the CELEST system of computer programs.

The work documented here was done in the Astronautics and Geodesy Division and was funded as part of the CELEST development effort. Mrs. Louise Gordon of the Computer Program Division coded the algorithm and made the required computer runs.

This report was reviewed by Richard J. Anderle, Head, Astronautics and Geodesy Division.

Released by:

R. A. NIEMANN, Head
Strategic Systems Department

CONTENTS

INTRODUCTION

CELEST[1,2] is a new system of computer programs to calculate precise ephemerides and associated diagnostic data for artificial earth satellites. Main data source are the observations conducted at a worldwide network of geodetic stations of the Doppler frequency shift on radio signals transmitted by the orbiting satellites.

CELEST goes to a great length to accomplish its accuracy goals. For example, dynamical accuracy is assured by a force model for the satellite motion, which accounts not only for the customary causes of orbit perturbations such as a very detailed earth gravitation, air drag, and light pressure but, in addition, specifies several rather minute perturbing accelerations like those caused by the gravitational effects of the tidal deformations experienced by the ocean,[3] the atmosphere,[4,5] and the solid earth.[6] For the sake of geometrical accuracy, there is a flexible refraction subroutine in that part of the physical model which formulates the radio-wave propagation from the orbit down to the individual observing station. State-of-the-art values are used for the observing station coordinates, and just like the dynamical part of CELEST was augmented by certain tidal terms, there was recently a tidal correction attached to the range and range-rate data. This correction introduces the displacement due to the lunar and solar tides of the solid earth of the surface under the observing stations. The nature of this correction shall be discussed.

DETAILS OF THE PHYSICAL MODEL

LIMITING ASSUMPTIONS

The computer algorithm below was derived under the following assumptions.

The Love numbers are constants. Only those Love numbers that regulate the surface deformation appear in the algorithm. They shall be named "geometrical" Love numbers and shall be symbolized by h_i to distinguish them from the "dynamical" Love numbers k_i that will appear closer to the end of this report.

Our algorithm is based on the customary linear relationships amongst the harmonic constituents of the tide potential and the corresponding constituents of the surface deformation. Each of the constants appearing in these relationships is one of the geometrical Love numbers. The just mentioned relationships themselves are being accepted without any detailed scrutiny of the relevant geophysical elasticity theory (Reference 7, Chapter VII; Reference 8, Chapter 3, Paragraph III; Reference 9).

The earth is approximated by a sphere of radius R. R is the semimajor axis of some suitable reference ellipsoid, the particular choice of which is uncritical. All terms are neglected which are of the order of the square of the reference ellipsoid eccentricity or of higher order. This assumption is valid because the surface distortion produced by our theory will not need to be known to better than a centimeter, which implies that no more than two significant digits will be needed.

Only the radial component of the tidal deformation of the land surface is significant. This can be rationalized from the fact that only a few terms will be considered in the surface harmonic expansion of the surface displacement. Of these, only one term will actually appear in the algorithm because it alone is sufficient to produce centimeter accuracy. These terms (and especially the one actually used) have a very long wavelength (thousands of kilometers) and a rather small amplitude (a few centimeters at most), causing the surface tilt to be negligible.

The study, of which the present report is a summary, was strictly limited by the requirement to produce a computer algorithm for the displacement of stations on the surface by the tidal mechanism. This objective was rather single-mindedly pursued. In particular, no attempt was made to formulate the tidal surface tilt for possible use with certain classes of observational data sensitive to the tilt angle (for example, astronomical latitude or zenith angle). Neither was it felt advisable to introduce ocean tide loading at the present time.

The sun and moon positions are specified by the contents of certain ephemeris computer tapes and by an interpolation routine for the tapes. These ephemerides are readily available because they are frequently used elsewhere in CELEST.

To facilitate those derivations which concern the tide potential and the tidal surface, it was at first assumed that the tide is an equilibrium tide.* As explained in detail below, the missing tide lag was introduced later by interpreting the equilibrium tide potential (and also the resulting surface distortion) as being produced by a fictitious moon and sun. It will be seen how a fairly simple transformation from the lunar and solar ephemeris coordinates to those of the fictitious moon and sun will change the equilibrium potential field and the equilibrium tidal surface into a potential and a surface which contain the desired effects of tidal lag.

*An equilibrium tide is a stationary solution of the mathematical tide problem. It is different from the real tide mainly because it ignores the effects of the inertia and viscosity of the earth's masses which in the real world would cause a time lag to exist between the motion of sun and moon in the sky and the mass redistribution occurring as a consequence within the earth.

EXPANSION OF THE LUNAR GRAVITY POTENTIAL

With the sole exception of the thermal component* of the air tide, all earth tides are in essence mass relocations within the solid earth, ocean, and atmosphere caused by lunar and/or solar gravitation. To be precise, the earth as a whole rotates within the fields of lunar and solar mass attraction. These fields impart on each terrestrial particle an acceleration which is a function of the position of the particle within the field. Generally, the lunar and solar force fields are variable across the earth. They also depend on time in accordance with the motions of the two celestial bodies relative to each particle. The acceleration of the particle relative to the earth as a whole in turn gives rise to certain geophysical forces which balance it, thus enabling the particle to remain attached to the earth. The latter forces constitute the tidal stress field which is responsible for the tidal deformation of the earth and, in particular, for the distortion of the earth's surface.

First, we shall discuss the lunar field in the earth's vicinity. The next paragraph will introduce the tide potential associated with the lunar gravitation. The TIDE POTENTIAL AND SURFACE DEFORMATION section will establish the surface deformation (tidal bulge). Then, the tidal lag will be formulated in close connection with the procedure for lunar position and, finally, the solar contribution to the tide will be explained.

Let $y^{(i)}$, i = 1, 2, 3, be an earth-fixed Cartesian coordinate frame. i = 1 and i = 2 indicate the equator; $y^{(1)}$ and $y^{(3)}$ define the Greenwich meridian. COE is the center of the earth in the sense of CELEST and TERRA.** The general point is P. Later on, P will be restricted to the surface.

Lunar position is specified in terms of inertial Cartesian coordinates, $x_L^{(i)}$, i = 1, 2, 3, by the moon ephemeris tapes available for use with CELEST and TERRA. Using one of the coordinate transformation computer routines available for that purpose in CELEST and/or TERRA, the inertial lunar position may be transformed for any desired time instant to the corresponding earth-fixed moon coordinates $y_L^{(i)}$.

*Not a tide in the strict sense. (See Reference 4, last paragraph of INTRODUCTION.)
**A new system of computer programs for satellite geodesy being developed at NSWC.

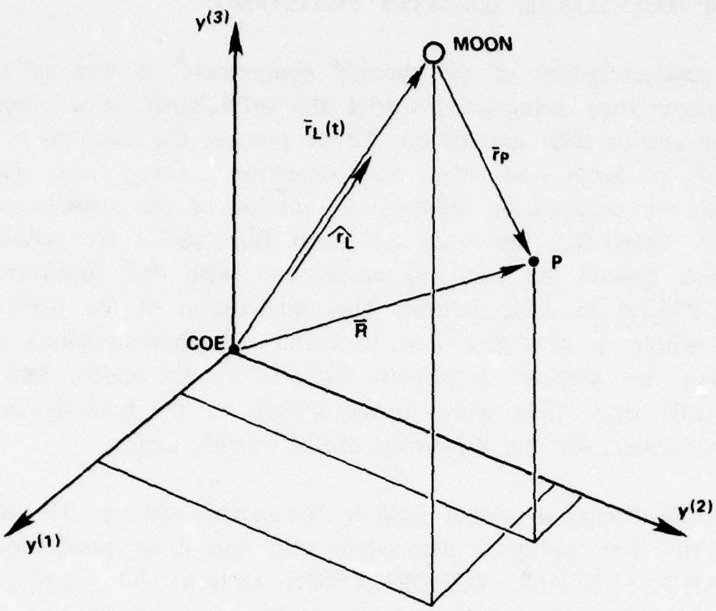

Figure 1. Position of Moon Relative to the Earth

Let now M_L be the moon's mass. G is Newton's constant. To find the moon's gravitational potential about the COE, ϕ_L, consider

$$\left.\begin{aligned} \bar{r}_p &= \bar{R} - \bar{r}_L \\ \bar{R} &= R\,\hat{r} \\ \phi_L &= \frac{\mu_L}{R} = \frac{GM_L}{r_p} = \frac{\mu_L}{|\bar{R} - \bar{r}_L|} \end{aligned}\right\} \quad (1)$$

As already stated, P may safely be assumed to be within the close vicinity of the earth

$$\left.\begin{aligned} r_L &\approx 50\ R \\ r_L &\gg R \\ R &\approx 6000\ \text{km} \end{aligned}\right\} \quad (2)$$

Let γ be the angular distance between the moon and point P. Let φ be latitude* and λ be longitude. Considering the familiar relationships

$$|\bar{R} - \bar{r}_L| = |\bar{r}_L - \bar{R}| = +\sqrt{r_L^2 + R^2 - 2\, r_L R \cos\gamma} \tag{3a}$$

and

$$\frac{1}{|\bar{r}_L - \bar{R}|} = \sum_{n=0}^{\infty} \frac{R^n}{r_L^{n+1}} P_n(\cos\gamma) \tag{3b}$$

the potential of lunar mass attraction takes on the form

$$\phi_L = \mu_L \sum_{n=0}^{\infty} \frac{R^n}{r_L^{n+1}} P_n(\cos\gamma) \tag{4}$$

From Figure 2 follows

$$\begin{aligned} \cos\gamma &= \cos\vartheta_1 \cos\vartheta_2 + \sin\vartheta_1 \sin\vartheta_2 \cos(\lambda_2 - \lambda_1) \\ &= \sin\varphi_1 \sin\varphi_2 + \cos\varphi_1 \cos\varphi_2 \cos(\lambda_2 - \lambda_1) \end{aligned} \tag{5}$$

Also, it can be shown that

$$\begin{aligned} P_n(\cos\gamma) &= P_n(\sin\varphi_1)P_n(\sin\varphi_2) \\ &\quad + 2 \sum_{m=1}^{n} \frac{(n-m)!}{(n+m)!} P_n^m(\sin\varphi_1)P_n^m(\sin\varphi_2) \cos m(\lambda_2 - \lambda_1) \end{aligned} \tag{6}$$

Finally, letting the lunar position correspond to index "1" and the surface point to index "2," the expansion of the moon's potential is

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*We intentionally avoid distinguishing between the various types of latitude because we expect that any inaccuracy arising from this simplification will be masked by the much larger uncertainties which otherwise prevail in current tidal models.

$$\phi_L = \mu_L \sum_{n=0}^{\infty} \frac{R^n}{r_L^{n+1}} \left\{ P_n(\sin\varphi)P_n(\sin\varphi_L) \right.$$

$$\left. + 2 \sum_{m=1}^{\infty} \frac{(n-m)!}{(n+m)!} P_n^m(\sin\varphi)P_n^m(\sin\varphi_L)\cos m(\lambda - \lambda_L) \right\} \tag{7}$$

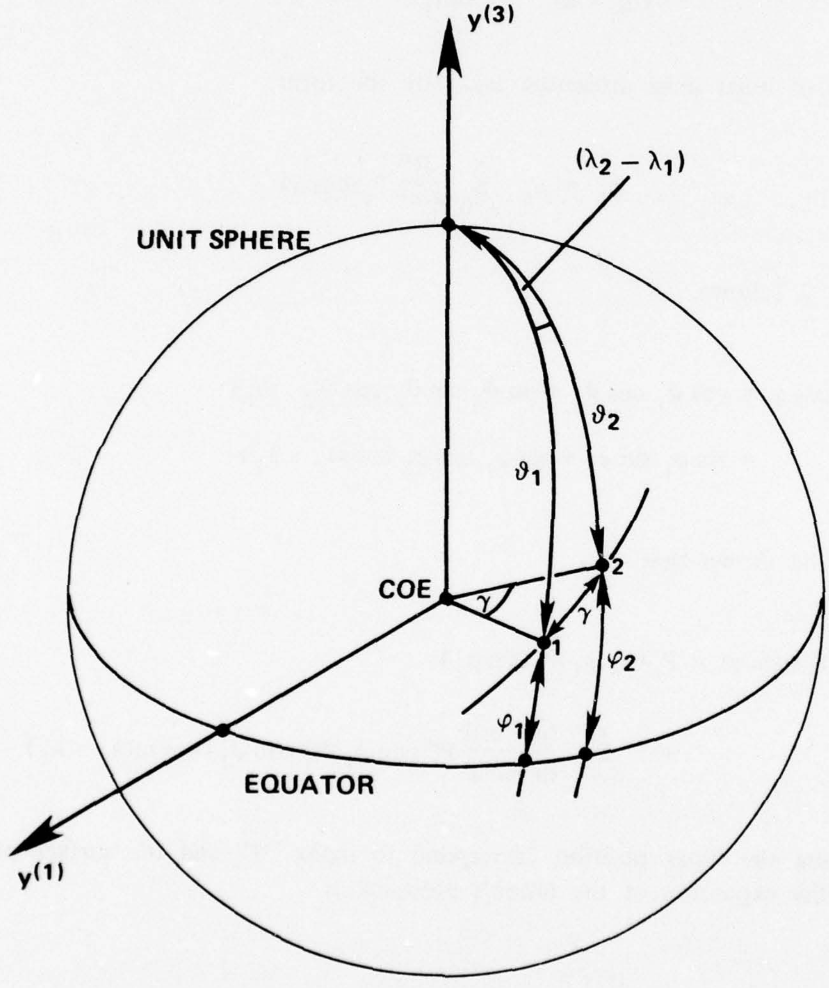

Figure 2. Latitudes, Longitudes, and Distance of Two Points on Unit Sphere About COE

Note that Equation (7) represents the moon's potential as a sum over surface harmonics,

$$\phi_L = \sum_{n=0}^{\infty} \frac{1}{R^{n+1}} \delta_n(\varphi,\lambda) \tag{8}$$

assuming that P has already been restricted to the surface (R = const), the surface harmonics being

$$y_n(\varphi,\lambda) = a_0 P_n(\sin\varphi) + \sum_{m=1}^{n} a_{nm} P_n^m(\sin\varphi)\cos m\lambda$$
$$+ \sum_{m=1}^{\infty} b_{nm} P_n^m(\sin\varphi)\sin m\lambda \tag{9}$$

Optionally, we may interpret Equation (7) as an expansion of the potential in terms of solid harmonics,

$$U_n(R,\varphi,\lambda) = \frac{y_n(\varphi,\lambda)}{R^{n+1}} \tag{10}$$

THE TIDE POTENTIAL

As it stands, ϕ_L is of course not itself the tide potential (tide generating potential), because the earth-fixed-reference system $y^{(i)}$ of Figure 1 is not an inertial system. Rather, the moon's mass attraction imparts to $y^{(i)}$ the "guidance" acceleration

$$\bar{a}_F = + \frac{\mu_L}{r_L^2} r_L . \tag{11}$$

Consequently, any mass when referred to $y^{(i)}$ experiences the acceleration, caused by inertia,

$$\bar{a}_{IN} = - \frac{\mu_L}{r_L^2} r_L . \tag{12}$$

This, in turn, can be represented as the gradient of a potential, ϕ_{IN},

$$\left.\begin{aligned}
\phi_{IN} &= -\frac{\mu_L}{r_L^2}\, r\cos\gamma = -\frac{\mu_L}{r_L^2}\,\hat{r}_L\cdot\bar{r} \\
\bar{a}_{IN} &= \operatorname{grad}\phi_{IN} = -\frac{\mu_L}{r_L^3}\,(\operatorname{grad}\bar{r})\cdot\bar{r}_L \\
&= -\frac{\mu_L}{r_L^3}\, I\cdot\bar{r}_L = -\frac{\mu_L}{r_L^3}\,\bar{r}_L\,.
\end{aligned}\right\} \tag{13}$$

For the present purpose, ϕ_{IN} is of interest only on the surface, $\bar{r} = \bar{R}$, and thus

$$\phi_{IN} = -\frac{\mu_L}{r_L^2}\, R\cos\gamma = -\frac{\mu_L}{r_L^2}\, R\, P_1(\cos\gamma) \tag{14}$$

The total contribution of lunar mass attraction to the mechanism which causes the tidal bulge is then the tide potential,

$$\psi_L = \phi_{IN} + \phi_L \tag{15}$$

Not all of the terms contained in Equation (7) for ϕ_L are needed however. Closely following the reasoning of Reference 8, Page 70, ϕ_L was truncated after the fourth term, resulting in

$$\left.\begin{aligned}
\phi_L &\approx \phi_{L0} + \phi_{L1} + \phi_{L2} + \phi_{L3} \\
\phi_{L0} &= \mu_L\,\frac{P_0(\cos\gamma)}{r_L} = \frac{\mu_L}{r_L} \\
\phi_{L1} &= \mu_L\,\frac{R}{r_L^2}\,P_1(\cos\gamma) \\
\phi_{L2} &= \mu_L\,\frac{R^2}{r_L^3}\,P_2(\cos\gamma) \\
\phi_{L3} &= \mu_L\,\frac{R^3}{r_L^4}\,P_3(\cos\gamma)
\end{aligned}\right\} \tag{16}$$

Thus,

$$\psi_L = \phi_{IN} + \phi_{L0} + \phi_{L1} + \phi_{L2} + \phi_{L3} \qquad (17)$$

Closer scrutiny of Equation (17) shows that the first term cancels the third. Also, the second term is of no consequence because its gradient vanishes. The final version of the tide potential is now quite simple

$$\psi_L \approx \mu_L \frac{R^2}{r_L^3} P_2(\cos\gamma) + \mu_L \frac{R^3}{r_L^4} P_3(\cos\gamma) \qquad (18)$$

The Legendre polynomials in this equation might be specified from Equation (6) as functions of the angular coordinates of both the moon and the surface point. However, because it will greatly simplify the algorithm, it is suggested to use Equation (5) first to calculate

$$\cos\gamma = \sin\varphi \sin\varphi_L + \cos\varphi \cos\varphi_L \cos(\lambda - \lambda_L) \qquad (19)$$

and to subsequently evaluate the two Legendre polynomials.

TIDE POTENTIAL AND SURFACE DEFORMATION

References 7, 8, and 9 relate how elasticity theory may be applied to the geophysics of the solid earth tide. Essentially, the result is as follows. Assume that U_n is one of the harmonic constituents of the tide potential ψ_L. The radial displacement of the earth's surface, caused by the presence of this term, is proportional to the term itself

$$H_n(\varphi,\lambda) = h_n \frac{U_n(r = R,\lambda,\varphi)}{g} \qquad (20)$$

In this equation occurs a proportionality factor "h_n." This is the above mentioned geometrical Love number or, simply, "Love number h_n." We regard it a constant (not dependent on φ or λ). Surface gravity is g. It is also assumed constant ($g \approx g_0 \approx 9.81$ m/sec^2). For further details, see Reference 7, pp 272 and 273; Reference 8, pp 70-72; Reference 9, Chapter 2.

The "tidal bulge" may be visualized as a surface which is displaced by a distance $H(\varphi,\lambda)$ from the unperturbed earth surface, as illustrated in Figure 3. The displacement H is the sum of its components,

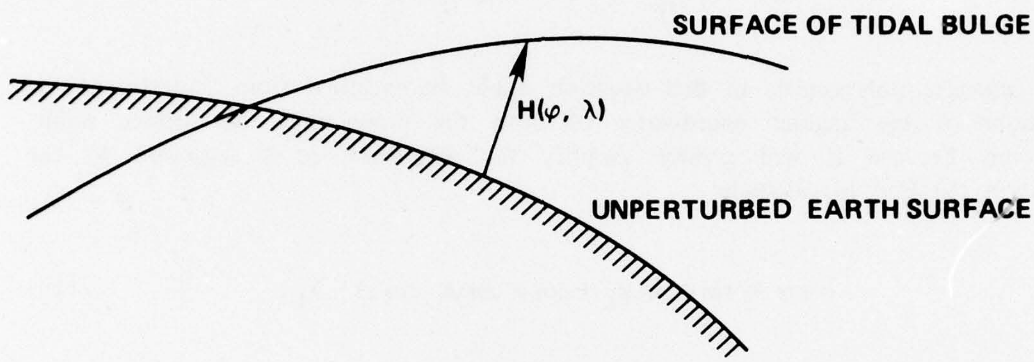

Figure 3. Geometry of Tidal Bulge

$$H(\varphi,\lambda) = \sum_{n=0}^{\infty} H_n(\varphi,\lambda) . \tag{21}$$

The surface displacement now follows from Equations (21), (20), (10), (8), (17), and (18).

$$H(\varphi,\lambda) = h_2 \frac{\mu_L}{g} \frac{R^2}{r_L^3} P_2(\cos \gamma_L) + h_3 \frac{\mu_L}{g} \frac{R^3}{r_L^4} P_3(\cos \gamma_L) \tag{22}$$

where the Legendre polynomials P_2 and P_3 are

$$P_2(x) \equiv \frac{1}{2}(3x^2 - 1) \tag{23a}$$

$$P_3(x) \equiv \frac{1}{2}(5x^3 - 3x) \tag{23b}$$

and

$$\cos \gamma_L = \sin \varphi \sin \varphi_L + \cos \varphi \cos \varphi_L \cos (\lambda - \lambda_L) \tag{19}$$

POSITION OF MOON AND TIDE LAG

The three preceding sections demonstrated how the tidal surface displacement depends, via the tide potential, on the moon ephemerides. The discussion was entirely based on the equilibrium tide concept introduced in the LIMITING ASSUMPTIONS. While thus in our model the tidal elevation H is in phase with the prevailing tide potential ψ_L, there exists in the real world a time lag between H and ψ_L. This time lag has a twofold origin. Firstly, due to the combined effects of the inertia and viscosity inherent in the earth's masses, the tidal bulge will be delayed with respect to the moon's motion. At any given time instant, the shape of the bulge will correspond not to the simultaneous but to a slightly earlier position of the moon overhead. And, secondly, also because of inertia and viscosity, the earth's rotation will "sweep along" the bulge, out from under the moon, in an easterly direction.

It would have been quite impractical to account for the complicated discrepancy between equilibrium tide and actual tide in the derivations leading to H. Instead, it was found convenient (Reference 6, Pages 2 and 9) to resolve it by modifying the lunar position as shown in Reference 6, Page 10, Figure 1. Thus, the tidal retardation may be reflected in our model by introducing a time delay into the lunar ephemeris. The rotational advance of the bulge may be introduced by suitably rotating the orbital plane associated with the motion, relative to the earth, of the moon.

No further mathematical details appear necessary. The equations which reflect the tidal lag in the second paragraph of the next section on the computer algorithm will illustrate the matter sufficiently. But it still ought to be mentioned that there is a mathematical flaw in this part of the algorithm. Namely, the algorithm first retards the moon by setting back the inertial lunar coordinates to a slightly earlier orbital position. Subsequently it computes the earth-fixed coordinates of the moon. The advance of the lunar orbit plane is then accomplished by suitably manipulating the earth-fixed moon coordinates. The author is obliged to Drs. C. Oesterwinter and J. O'Toole of the Astronautics and Geodesy Division for pointing out to him that this procedure is not invariant with respect to the epoch of the lunar ephemeris as any procedure of this type should be.

Let X^t be the lunar position vector at time instant t, referred to the inertial frame corresponding to time t. Let Y^t be the corresponding earth-fixed position of the moon. Let the matrix product* ABCD be the transformation matrix leading from the inertial frame to the earth-fixed frame,

$$Y^t = ABCD\,X^t\,. \tag{24}$$

D is the Precession Matrix, leading from the mean equator and equinox of the epoch frame to that of date. The Nutation Matrix is C which continues the just mentioned rotation on to the frame asssociated with the true equator and equinox of date. B continues the transformation to the earth-fixed frame of rotation, and A transforms to the frame of figure (correction for polar motion).

Now, let "40" and "50" indicate two different epochs, say 1940.0 and 1950.0, each defining a particular inertial frame. Then,

$$\left.\begin{aligned} X^t &= D_{50,t}X^{50} \\ X^{50} &= D_{40,50}X^{40} \\ X^t &= D_{50,t}D_{40,50}X^{40} \end{aligned}\right\} \tag{25}$$

and, defining L to be a matrix representing the position difference between the ephemeris moon and the fictitious moon (or part of that position differential, such as the retardation of the moon in its orbit),

$$Y^t = ABC\,D_{50,t}(L\,X^{50}) = ABC\,D_{50,t}L\,D_{40,50}X^{40} \neq ABC\,D_{40,t}L\,X^{40} \tag{26}$$

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*From unpublished working notes by W. Groeger, on the Development of the Equations of Motion for the TERRA Computer Program System, August 1973.

as, generally, L and $D_{40,50}$ will not commute

$$L D_{40,50} \neq D_{40,50} L . \tag{27}$$

However, everyone agrees with the author that the tide lag algorithm appearing below in the COMPUTER ALGORITHM section, because of the small magnitude of the effect, is a legitimate approximation to reality despite its mathematical inconsistency.

CONTRIBUTION BY THE SUN

Everything said in the above sections may be equally applied to the deformation of the earth by the tidal action of the sun. Note especially that the Love numbers h_n are geophysical constants and depend on the structure of the earth alone. They consequently apply equally to the lunar tide and to the solar tide.

The sun is much more distant from the earth than the moon,

$$\frac{R}{r_S} \ll \frac{R}{r_L} \tag{28}$$

suggesting that only the first term be considered in the equation for the surface displacement,

$$H(\varphi,\lambda) = h_2 \frac{\mu_S}{g} \frac{R^2}{r_S^3} P_2(\cos \gamma_S) \tag{29}$$

$$\cos \gamma_S = \sin \varphi \sin \varphi_S + \cos \varphi \cos \varphi_S \cos (\lambda - \lambda_S) \tag{30}$$

COMPUTER ALGORITHM

INPUT DATA

Geophysical Constants

g — Approximation of earth's surface gravity, in terms of m/sec^2. Suggested value: $g = 9.81\ m/sec^2$.

R — Semimajor axis, in meters, of a suitable reference ellipsoid. Use a value compatible with the data for the particular CELEST run. Or else, R = 6 378 150 m ought to be satisfactory.

h_2 — Love number. Suggested value for trial runs: $h_2 = 0.6$.

μ_L — Moon's gravitational constant, in cm^3/sec^2. $\mu_L = G\,M_L$ where M_L is the lunar mass. Use a value compatible with CELEST input. Suggested value for trial runs:, $\mu_L = 4.9177 * 10^{18}\ cm^3/sec^2$.

μ_S — Sun's gravitational constant, in terms of cm^3/sec^2. $\mu_S = G\,M_S$ where M_S is the solar mass. Use a value compatible with the CELEST input. Suggested value for trial runs: $\mu_S = 1.3291 * 10^{26}\ cm^3/sec^2$.

Station Coordinates

φ — Latitude of station (geocentric or geodetic etc.). May be expected to be available in terms of degrees, from a station catalog. Express this either in decimal fraction of degrees or in radians, to be compatible with those trigonometric functions where φ is the argument.

λ — Station longitude. The comments just made for the station latitude apply here also.

Position of Moon and Sun

t — Time instant for which the tidal surface displacement H is to be found. Specify t in terms of year, day UT, and seconds elapsed since beginning of day UT.

Δt Tidal lag parameter in seconds.

$\tilde{\omega}$ Earth's sidereal rate of rotation,

$$\tilde{\omega} = 4.178\,074\,622 * 10^{-3} \text{ deg/sec}$$
$$= 7.292\,115\,855 * 10^{-5} \text{ rad/sec}$$

Have available suitable ephemerides for moon and sun, such as the TERRA/CELEST moon and sun ephemeris tapes, plus the associated interpolation routine.

When to Evaluate the Tidal Displacement During the Data Blocks on the CELEST Observation Tape

t_{CA} Time, in seconds, of closest approach (of satellite to station) during data block (satellite pass) on CELEST observation tape.

t_1 First observing time value, in seconds, for the individual data block on the CELEST observing tape.

τ Time increment, in seconds, necessary to specify the time instants for which the tidal displacement is to be found.

COLLECTION OF EQUATIONS FOR CODING

Position of the Moon

For the time instant $(t - \Delta t)$, obtain the inertial Cartesian coordinates of the moon, x''_L, y''_L, z''_L, by interpolation from the moon ephemeris tape. Note that a suitable interpolation routine exists for use with the tape.

Transform the latter coordinates to the corresponding earth-fixed Cartesian coordinates, x'_L, y'_L, z'_L. The necessary transformation procedure is readily available in CELEST and/or TERRA as a subroutine. Reference is also made to the POSITION OF MOON AND TIDE LAG section. If not already expressed in terms of kilometers, now write x'_L, y'_L and z'_L in terms of this unit.

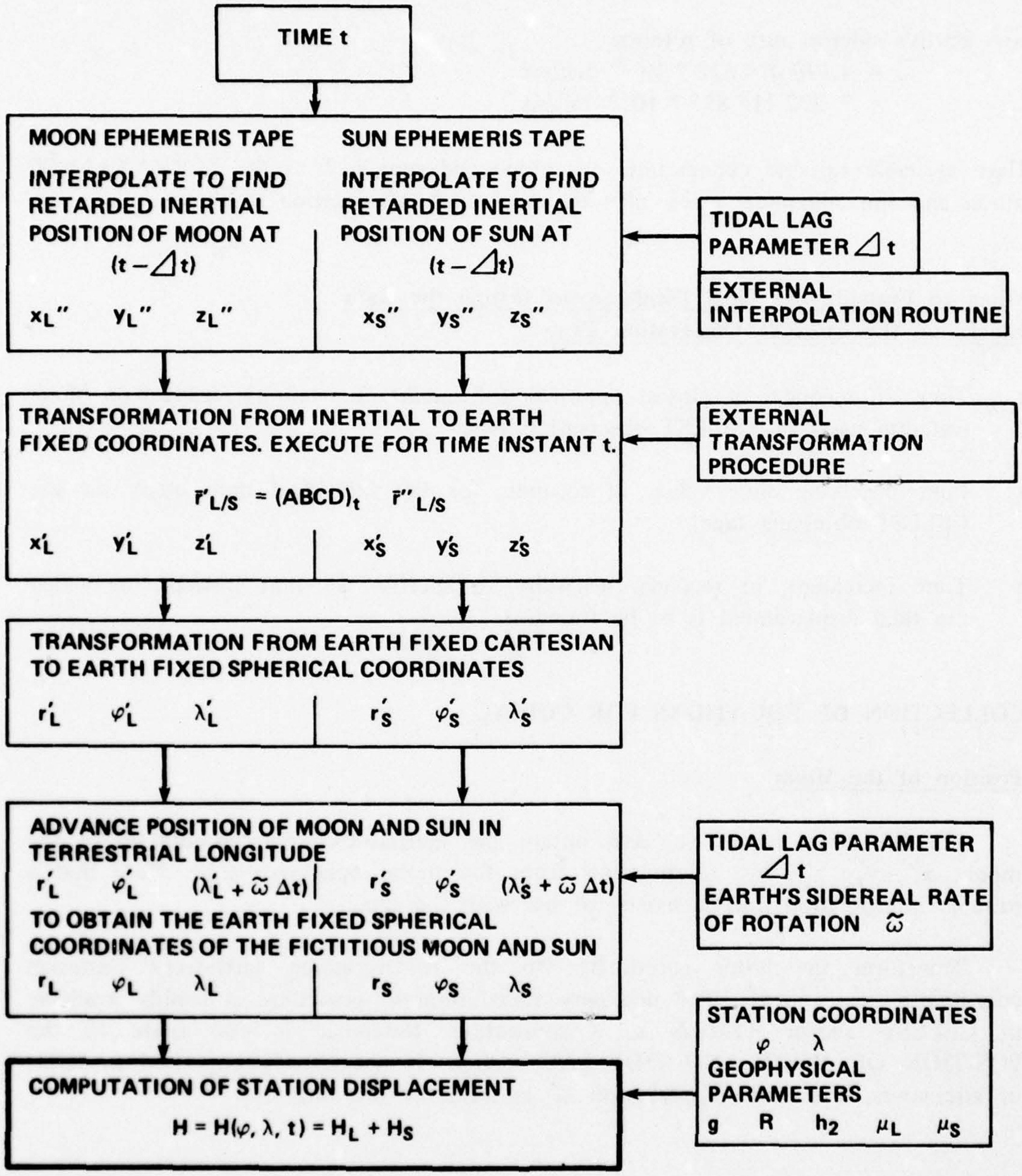

Figure 4. Schematic of the Algorithm

Now transform to earth-fixed spherical coordinates,

$$r'_L = +\sqrt{(x'_L)^2 + (y'_L)^2 + (z'_L)^2} \tag{101A}$$

$$\varphi'_L = \tan^{-1} \frac{Z'_L}{+\sqrt{(x'_L)^2 + (y'_L)^2}} \tag{101B}$$

$$\lambda'_L = \tan^{-1} \frac{y'_L}{x'_L} \tag{101C}$$

taking the customary precautions to assure that the resulting angle values are located in the correct quadrant.

Then advance the position of the moon in terrestrial longitude (longitude east) to obtain the earth-fixed position of the fictitious (tide generating) moon

$$r_L = r'_L \tag{102A}$$

$$\varphi_L = \varphi'_L \tag{102B}$$

$$\lambda_L = \lambda'_L + \tilde{\omega}\Delta t \tag{102C}$$

Sun Position

Now calculate the position of the fictitious sun (tide generating sun). Code for this purpose a procedure similar to that just specified for the moon.

Station Coordinates

The station coordinates can be expected to be read off a catalog for the observing stations directly in terms of longitude and latitude. For the present purpose, geodetic, geographic, and geocentric latitude may be freely interchanged and will in the following be referred to as "latitude." It may reasonably be assumed that any arbitrary surface points, other than observing stations, will also be specified in terms of their latitudes and longitudes.

Computation of Surface Displacement

Now find the surface displacement, H, associated with a particular station or surface point as well as with a specific time instant t. The radial displacement is H. A positive value of H implies that the station has increased its distance from the COE. Tidal movement of the station, parallel to the surface, has been neglected for the purpose of the analysis which led to the present algorithm. From the equations below, H will result in terms of meters.

$$H(\varphi,\lambda,t) = 10^{-15} h_2 \frac{\mu_L}{g} \frac{R^2}{r_L^3} P_2(\cos\gamma_L) + 10^{-15} h_2 \frac{\mu_S}{g} \frac{R^2}{r_S^3} P_2(\cos\gamma_S) \qquad (103)$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1) \qquad (104)$$

$$\cos\gamma_L = \sin\varphi \sin\varphi_L + \cos\varphi \cos\varphi_L \cos(\lambda - \lambda_L) \qquad (105)$$

$$\cos\gamma_S = \sin\varphi \sin\varphi_S + \cos\varphi \cos\varphi_S \cos(\lambda - \lambda_S) \qquad (106)$$

When to Evaluate the Tidal Displacement During the Data Blocks on the CELEST Observation Tape

First Option — For each data block on the observation tape (satellite pass), evaluate H at the time of closest approach (of the satellite to the station) $H = H(t_{CA})$. Regard this as the value of H throughout the satellite pass.

Second Option — For each satellite pass (data block on the observation tape), evaluate H for the first observing time value, t_1, and, subsequently for $t_2 = t_1 + \tau$, $t_3 = t_2 + \tau$, ..., $t_{n+1} = t_n + \tau$, ..., up to the end of the satellite pass.

For the observing time t, $t_n \leqslant t < t_{n+1}$, assign to H the value $H(t_n)$.

τ is supposed to be specified by the CELEST user via the program input. Make provision for the user to insert the chosen value of τ together with the input data for the individual computer run. If the present second option is chosen at all, it is quite likely that the value τ = 10 min (600 sec) will be selected because this appears to be a suitable value for a wide range of satellite altitudes (pass lengths between 20 min and 10 hr).

COMPUTER PROGRAM OUTPUT

It can be expected that the present computer program will serve as a subroutine during CELEST runs. Whenever required, CELEST will specify latitude and longitude for a surface point plus a time value. The present routine will then calculate the surface displacement associated with the station position and the time instant.

Optionally, provision may be made to input for any given station position, a table of time values and to automatically find all associated surface displacements. The latter procedure is expected to be useful for the program checkout and also for certain geophysical studies made independently of CELEST.

QUANTITATIVE RESULTS FOR THREE TYPICAL STATION LOCATIONS

The numerical checkout of the just specified computer program was based on the input parameters listed in the first paragraph of the COMPUTER ALGORITHM. Surface displacements (total value H in centimeters, lunar component HMOON in centimeters and solar component HSUN in centimeters) were computed for each of three typical stations at ten-minute intervals for an entire day (Day UT No. 88 of Year 1977).

Table 1. Surface Displacement and Lunar and Solar Components of Surface Displacement for Station No. 1

(Station longitude = 0 deg, station latitude = 0 deg, HMOON, HSUN, and H are in centimeters. Time is listed in terms of seconds from Midnight UT of Day 88 of Year 1977.)

TIME =	0 SEC	HMOON =	-8.05	HSUN =	9.86	H =	1.82
TIME =	600 SEC	HMOON =	-8.59	HSUN =	9.87	H =	1.28
TIME =	1200 SEC	HMOON =	-9.04	HSUN =	9.82	H =	.78
TIME =	1800 SEC	HMOON =	-9.41	HSUN =	9.72	H =	.32
TIME =	2400 SEC	HMOON =	-9.68	HSUN =	9.56	H =	-.11
TIME =	3000 SEC	HMOON =	-9.85	HSUN =	9.35	H =	-.50
TIME =	3600 SEC	HMOON =	-9.93	HSUN =	9.08	H =	-.84
TIME =	4200 SEC	HMOON =	-9.91	HSUN =	8.77	H =	-1.14
TIME =	4800 SEC	HMOON =	-9.79	HSUN =	8.40	H =	-1.39
TIME =	5400 SEC	HMOON =	-9.58	HSUN =	8.00	H =	-1.58
TIME =	6000 SEC	HMOON =	-9.27	HSUN =	7.54	H =	-1.73
TIME =	6600 SEC	HMOON =	-8.87	HSUN =	7.05	H =	-1.81
TIME =	7200 SEC	HMOON =	-8.38	HSUN =	6.53	H =	-1.85
TIME =	7800 SEC	HMOON =	-7.80	HSUN =	5.97	H =	-1.83
TIME =	8400 SEC	HMOON =	-7.14	HSUN =	5.39	H =	-1.75
TIME =	9000 SEC	HMOON =	-6.40	HSUN =	4.78	H =	-1.61
TIME =	9600 SEC	HMOON =	-5.58	HSUN =	4.16	H =	-1.42
TIME =	10200 SEC	HMOON =	-4.71	HSUN =	3.53	H =	-1.18
TIME =	10800 SEC	HMOON =	-3.76	HSUN =	2.88	H =	-.88
TIME =	11400 SEC	HMOON =	-2.77	HSUN =	2.24	H =	-.53
TIME =	12000 SEC	HMOON =	-1.73	HSUN =	1.59	H =	-.14
TIME =	12600 SEC	HMOON =	-.64	HSUN =	.95	H =	.31
TIME =	13200 SEC	HMOON =	.47	HSUN =	.32	H =	.80
TIME =	13800 SEC	HMOON =	1.61	HSUN =	-.29	H =	1.32
TIME =	14400 SEC	HMOON =	2.77	HSUN =	-.88	H =	1.89
TIME =	15000 SEC	HMOON =	3.93	HSUN =	-1.44	H =	2.49
TIME =	15600 SEC	HMOON =	5.10	HSUN =	-1.98	H =	3.12
TIME =	16200 SEC	HMOON =	6.25	HSUN =	-2.48	H =	3.77
TIME =	16800 SEC	HMOON =	7.39	HSUN =	-2.94	H =	4.45
TIME =	17400 SEC	HMOON =	8.51	HSUN =	-3.37	H =	5.14
TIME =	18000 SEC	HMOON =	9.59	HSUN =	-3.74	H =	5.85
TIME =	18600 SEC	HMOON =	10.63	HSUN =	-4.08	H =	6.56
TIME =	19200 SEC	HMOON =	11.63	HSUN =	-4.36	H =	7.27
TIME =	19800 SEC	HMOON =	12.57	HSUN =	-4.59	H =	7.98
TIME =	20400 SEC	HMOON =	13.45	HSUN =	-4.76	H =	8.68
TIME =	21000 SEC	HMOON =	14.26	HSUN =	-4.88	H =	9.37
TIME =	21600 SEC	HMOON =	14.99	HSUN =	-4.95	H =	10.05
TIME =	22200 SEC	HMOON =	15.65	HSUN =	-4.96	H =	10.69
TIME =	22800 SEC	HMOON =	16.23	HSUN =	-4.91	H =	11.32

Table 1. Surface Displacement and Lunar and Solar Components of Surface Displacement for Station No. 1 (Continued)

TIME = 23400 SEC	HMOON = 16.71	HSUN = -4.81	H = 11.91
TIME = 24000 SEC	HMOON = 17.11	HSUN = -4.65	H = 12.46
TIME = 24600 SEC	HMOON = 17.41	HSUN = -4.43	H = 12.98
TIME = 25200 SEC	HMOON = 17.61	HSUN = -4.17	H = 13.45
TIME = 25800 SEC	HMOON = 17.72	HSUN = -3.85	H = 13.87
TIME = 26400 SEC	HMOON = 17.73	HSUN = -3.49	H = 14.24
TIME = 27000 SEC	HMOON = 17.64	HSUN = -3.08	H = 14.56
TIME = 27600 SEC	HMOON = 17.45	HSUN = -2.63	H = 14.82
TIME = 28200 SEC	HMOON = 17.17	HSUN = -2.14	H = 15.03
TIME = 28800 SEC	HMOON = 16.79	HSUN = -1.61	H = 15.17
TIME = 29400 SEC	HMOON = 16.31	HSUN = -1.06	H = 15.26
TIME = 30000 SEC	HMOON = 15.75	HSUN = -.47	H = 15.28
TIME = 30600 SEC	HMOON = 15.10	HSUN = .13	H = 15.23
TIME = 31200 SEC	HMOON = 14.38	HSUN = .75	H = 15.13
TIME = 31800 SEC	HMOON = 13.57	HSUN = 1.39	H = 14.96
TIME = 32400 SEC	HMOON = 12.70	HSUN = 2.03	H = 14.73
TIME = 33000 SEC	HMOON = 11.77	HSUN = 2.68	H = 14.45
TIME = 33600 SEC	HMOON = 10.77	HSUN = 3.32	H = 14.10
TIME = 34200 SEC	HMOON = 9.73	HSUN = 3.96	H = 13.69
TIME = 34800 SEC	HMOON = 8.65	HSUN = 4.59	H = 13.24
TIME = 35400 SEC	HMOON = 7.53	HSUN = 5.20	H = 12.73
TIME = 36000 SEC	HMOON = 6.38	HSUN = 5.79	H = 12.17
TIME = 36600 SEC	HMOON = 5.22	HSUN = 6.35	H = 11.57
TIME = 37200 SEC	HMOON = 4.04	HSUN = 6.89	H = 10.93
TIME = 37800 SEC	HMOON = 2.86	HSUN = 7.39	H = 10.25
TIME = 38400 SEC	HMOON = 1.69	HSUN = 7.85	H = 9.55
TIME = 39000 SEC	HMOON = .54	HSUN = 8.28	H = 8.81
TIME = 39600 SEC	HMOON = -.60	HSUN = 8.65	H = 8.06
TIME = 40200 SEC	HMOON = -1.70	HSUN = 8.98	H = 7.29
TIME = 40800 SEC	HMOON = -2.76	HSUN = 9.26	H = 6.50
TIME = 41400 SEC	HMOON = -3.77	HSUN = 9.49	H = 5.72
TIME = 42000 SEC	HMOON = -4.74	HSUN = 9.67	H = 4.93
TIME = 42600 SEC	HMOON = -5.64	HSUN = 9.79	H = 4.15
TIME = 43200 SEC	HMOON = -6.47	HSUN = 9.85	H = 3.38
TIME = 43800 SEC	HMOON = -7.23	HSUN = 9.86	H = 2.63
TIME = 44400 SEC	HMOON = -7.91	HSUN = 9.81	H = 1.90
TIME = 45000 SEC	HMOON = -8.51	HSUN = 9.71	H = 1.20
TIME = 45600 SEC	HMOON = -9.01	HSUN = 9.55	H = .53
TIME = 46200 SEC	HMOON = -9.43	HSUN = 9.34	H = -.10
TIME = 46800 SEC	HMOON = -9.75	HSUN = 9.07	H = -.68
TIME = 47400 SEC	HMOON = -9.98	HSUN = 8.75	H = -1.22
TIME = 48000 SEC	HMOON = -10.10	HSUN = 8.39	H = -1.71
TIME = 48600 SEC	HMOON = -10.13	HSUN = 7.98	H = -2.15
TIME = 49200 SEC	HMOON = -10.05	HSUN = 7.53	H = -2.52
TIME = 49800 SEC	HMOON = -9.88	HSUN = 7.04	H = -2.84
TIME = 50400 SEC	HMOON = -9.60	HSUN = 6.51	H = -3.09
TIME = 51000 SEC	HMOON = -9.23	HSUN = 5.96	H = -3.27
TIME = 51600 SEC	HMOON = -8.76	HSUN = 5.37	H = -3.39
TIME = 52200 SEC	HMOON = -8.21	HSUN = 4.77	H = -3.44
TIME = 52800 SEC	HMOON = -7.56	HSUN = 4.15	H = -3.42
TIME = 53400 SEC	HMOON = -6.84	HSUN = 3.51	H = -3.32
TIME = 54000 SEC	HMOON = -6.03	HSUN = 2.87	H = -3.16
TIME = 54600 SEC	HMOON = -5.15	HSUN = 2.22	H = -2.93
TIME = 55200 SEC	HMOON = -4.21	HSUN = 1.58	H = -2.63

Table 1. Surface Displacement and Lunar and Solar Components of Surface Displacement for Station No. 1 (Continued)

TIME =	55800 SEC	HMOON =	-3.21	HSUN =	.94	H =	-2.27
TIME =	56400 SEC	HMOON =	-2.15	HSUN =	.31	H =	-1.84
TIME =	57000 SEC	HMOON =	-1.05	HSUN =	-.30	H =	-1.35
TIME =	57600 SEC	HMOON =	.09	HSUN =	-.89	H =	-.80
TIME =	58200 SEC	HMOON =	1.25	HSUN =	-1.45	H =	-.20
TIME =	58800 SEC	HMOON =	2.44	HSUN =	-1.99	H =	.45
TIME =	59400 SEC	HMOON =	3.64	HSUN =	-2.49	H =	1.15
TIME =	60000 SEC	HMOON =	4.84	HSUN =	-2.95	H =	1.89
TIME =	60600 SEC	HMOON =	6.04	HSUN =	-3.37	H =	2.67
TIME =	61200 SEC	HMOON =	7.22	HSUN =	-3.75	H =	3.48
TIME =	61800 SEC	HMOON =	8.39	HSUN =	-4.08	H =	4.31
TIME =	62400 SEC	HMOON =	9.52	HSUN =	-4.36	H =	5.16
TIME =	63000 SEC	HMOON =	10.62	HSUN =	-4.59	H =	6.03
TIME =	63600 SEC	HMOON =	11.67	HSUN =	-4.76	H =	6.91
TIME =	64200 SEC	HMOON =	12.66	HSUN =	-4.88	H =	7.78
TIME =	64800 SEC	HMOON =	13.60	HSUN =	-4.95	H =	8.65
TIME =	65400 SEC	HMOON =	14.47	HSUN =	-4.95	H =	9.52
TIME =	66000 SEC	HMOON =	15.27	HSUN =	-4.91	H =	10.36
TIME =	66600 SEC	HMOON =	15.98	HSUN =	-4.80	H =	11.18
TIME =	67200 SEC	HMOON =	16.62	HSUN =	-4.64	H =	11.98
TIME =	67800 SEC	HMOON =	17.16	HSUN =	-4.43	H =	12.73
TIME =	68400 SEC	HMOON =	17.61	HSUN =	-4.16	H =	13.45
TIME =	69000 SEC	HMOON =	17.97	HSUN =	-3.85	H =	14.12
TIME =	69600 SEC	HMOON =	18.23	HSUN =	-3.48	H =	14.75
TIME =	70200 SEC	HMOON =	18.38	HSUN =	-3.07	H =	15.31
TIME =	70800 SEC	HMOON =	18.44	HSUN =	-2.62	H =	15.82
TIME =	71400 SEC	HMOON =	18.39	HSUN =	-2.13	H =	16.26
TIME =	72000 SEC	HMOON =	18.24	HSUN =	-1.61	H =	16.64
TIME =	72600 SEC	HMOON =	17.99	HSUN =	-1.05	H =	16.95
TIME =	73200 SEC	HMOON =	17.65	HSUN =	-.47	H =	17.18
TIME =	73800 SEC	HMOON =	17.20	HSUN =	.14	H =	17.34
TIME =	74400 SEC	HMOON =	16.66	HSUN =	.76	H =	17.42
TIME =	75000 SEC	HMOON =	16.03	HSUN =	1.40	H =	17.43
TIME =	75600 SEC	HMOON =	15.32	HSUN =	2.04	H =	17.36
TIME =	76200 SEC	HMOON =	14.53	HSUN =	2.68	H =	17.21
TIME =	76800 SEC	HMOON =	13.66	HSUN =	3.33	H =	16.99
TIME =	77400 SEC	HMOON =	12.72	HSUN =	3.97	H =	16.69
TIME =	78000 SEC	HMOON =	11.72	HSUN =	4.59	H =	16.31
TIME =	78600 SEC	HMOON =	10.67	HSUN =	5.20	H =	15.87
TIME =	79200 SEC	HMOON =	9.57	HSUN =	5.79	H =	15.36
TIME =	79800 SEC	HMOON =	8.42	HSUN =	6.36	H =	14.78
TIME =	80400 SEC	HMOON =	7.25	HSUN =	6.89	H =	14.14
TIME =	81000 SEC	HMOON =	6.06	HSUN =	7.39	H =	13.45
TIME =	81600 SEC	HMOON =	4.85	HSUN =	7.85	H =	12.70
TIME =	82200 SEC	HMOON =	3.63	HSUN =	8.27	H =	11.90
TIME =	82800 SEC	HMOON =	2.42	HSUN =	8.65	H =	11.07
TIME =	83400 SEC	HMOON =	1.21	HSUN =	8.98	H =	10.19
TIME =	84000 SEC	HMOON =	.03	HSUN =	9.26	H =	9.29
TIME =	84600 SEC	HMOON =	-1.12	HSUN =	9.49	H =	8.36
TIME =	85200 SEC	HMOON =	-2.24	HSUN =	9.66	H =	7.42
TIME =	85800 SEC	HMOON =	-3.31	HSUN =	9.78	H =	6.47

Table 2. Surface Displacement and Lunar and Solar Components of Surface Displacement for Station No. 2

(Station longitude = 0 deg, station latitude = 30 deg, HMOON, HSUN, and H are in centimeters. Time is listed in terms of seconds from Midnight UT of Day 88 of Year 1977.)

TIME =	0 SEC	HMOON =	-6.04	HSUN =	5.44	H =	-.60
TIME =	600 SEC	HMOON =	-6.74	HSUN =	5.45	H =	-1.29
TIME =	1200 SEC	HMOON =	-7.38	HSUN =	5.41	H =	-1.97
TIME =	1800 SEC	HMOON =	-7.95	HSUN =	5.34	H =	-2.61
TIME =	2400 SEC	HMOON =	-8.45	HSUN =	5.22	H =	-3.23
TIME =	3000 SEC	HMOON =	-8.88	HSUN =	5.06	H =	-3.81
TIME =	3600 SEC	HMOON =	-9.24	HSUN =	4.87	H =	-4.36
TIME =	4200 SEC	HMOON =	-9.52	HSUN =	4.64	H =	-4.88
TIME =	4800 SEC	HMOON =	-9.73	HSUN =	4.38	H =	-5.36
TIME =	5400 SEC	HMOON =	-9.87	HSUN =	4.08	H =	-5.79
TIME =	6000 SEC	HMOON =	-9.94	HSUN =	3.75	H =	-6.18
TIME =	6600 SEC	HMOON =	-9.93	HSUN =	3.40	H =	-6.53
TIME =	7200 SEC	HMOON =	-9.85	HSUN =	3.02	H =	-6.83
TIME =	7800 SEC	HMOON =	-9.71	HSUN =	2.62	H =	-7.09
TIME =	8400 SEC	HMOON =	-9.50	HSUN =	2.20	H =	-7.30
TIME =	9000 SEC	HMOON =	-9.22	HSUN =	1.76	H =	-7.46
TIME =	9600 SEC	HMOON =	-8.89	HSUN =	1.31	H =	-7.57
TIME =	10200 SEC	HMOON =	-8.50	HSUN =	.86	H =	-7.64
TIME =	10800 SEC	HMOON =	-8.06	HSUN =	.40	H =	-7.66
TIME =	11400 SEC	HMOON =	-7.57	HSUN =	-.07	H =	-7.63
TIME =	12000 SEC	HMOON =	-7.04	HSUN =	-.53	H =	-7.56
TIME =	12600 SEC	HMOON =	-6.47	HSUN =	-.98	H =	-7.45
TIME =	13200 SEC	HMOON =	-5.87	HSUN =	-1.43	H =	-7.30
TIME =	13800 SEC	HMOON =	-5.24	HSUN =	-1.86	H =	-7.10
TIME =	14400 SEC	HMOON =	-4.60	HSUN =	-2.28	H =	-6.87
TIME =	15000 SEC	HMOON =	-3.93	HSUN =	-2.67	H =	-6.61
TIME =	15600 SEC	HMOON =	-3.26	HSUN =	-3.05	H =	-6.31
TIME =	16200 SEC	HMOON =	-2.59	HSUN =	-3.39	H =	-5.98
TIME =	16800 SEC	HMOON =	-1.92	HSUN =	-3.71	H =	-5.63
TIME =	17400 SEC	HMOON =	-1.26	HSUN =	-4.00	H =	-5.25
TIME =	18000 SEC	HMOON =	-.61	HSUN =	-4.25	H =	-4.86
TIME =	18600 SEC	HMOON =	.02	HSUN =	-4.47	H =	-4.44
TIME =	19200 SEC	HMOON =	.63	HSUN =	-4.65	H =	-4.02
TIME =	19800 SEC	HMOON =	1.20	HSUN =	-4.79	H =	-3.58
TIME =	20400 SEC	HMOON =	1.74	HSUN =	-4.89	H =	-3.14
TIME =	21000 SEC	HMOON =	2.24	HSUN =	-4.94	H =	-2.70
TIME =	21600 SEC	HMOON =	2.70	HSUN =	-4.96	H =	-2.26
TIME =	22200 SEC	HMOON =	3.11	HSUN =	-4.93	H =	-1.82

Table 2. Surface Displacement and Lunar and Solar Components of Surface Displacement for Station No. 2 (Continued)

TIME = 22800 SEC	HMOON = 3.47	HSUN = -4.87	H = -1.40
TIME = 23400 SEC	HMOON = 3.77	HSUN = -4.75	H = -.98
TIME = 24000 SEC	HMOON = 4.02	HSUN = -4.60	H = -.58
TIME = 24600 SEC	HMOON = 4.21	HSUN = -4.41	H = -.20
TIME = 25200 SEC	HMOON = 4.35	HSUN = -4.18	H = .17
TIME = 25800 SEC	HMOON = 4.42	HSUN = -3.91	H = .51
TIME = 26400 SEC	HMOON = 4.42	HSUN = -3.61	H = .82
TIME = 27000 SEC	HMOON = 4.37	HSUN = -3.27	H = 1.10
TIME = 27600 SEC	HMOON = 4.26	HSUN = -2.90	H = 1.36
TIME = 28200 SEC	HMOON = 4.08	HSUN = -2.50	H = 1.58
TIME = 28800 SEC	HMOON = 3.85	HSUN = -2.08	H = 1.77
TIME = 29400 SEC	HMOON = 3.56	HSUN = -1.63	H = 1.93
TIME = 30000 SEC	HMOON = 3.21	HSUN = -1.16	H = 2.05
TIME = 30600 SEC	HMOON = 2.81	HSUN = -.68	H = 2.13
TIME = 31200 SEC	HMOON = 2.37	HSUN = -.19	H = 2.18
TIME = 31800 SEC	HMOON = 1.88	HSUN = .31	H = 2.19
TIME = 32400 SEC	HMOON = 1.34	HSUN = .82	H = 2.16
TIME = 33000 SEC	HMOON = .77	HSUN = 1.33	H = 2.10
TIME = 33600 SEC	HMOON = .17	HSUN = 1.84	H = 2.01
TIME = 34200 SEC	HMOON = -.46	HSUN = 2.34	H = 1.88
TIME = 34800 SEC	HMOON = -1.11	HSUN = 2.83	H = 1.72
TIME = 35400 SEC	HMOON = -1.77	HSUN = 3.31	H = 1.53
TIME = 36000 SEC	HMOON = -2.45	HSUN = 3.77	H = 1.32
TIME = 36600 SEC	HMOON = -3.13	HSUN = 4.21	H = 1.08
TIME = 37200 SEC	HMOON = -3.81	HSUN = 4.63	H = .82
TIME = 37800 SEC	HMOON = -4.48	HSUN = 5.02	H = .53
TIME = 38400 SEC	HMOON = -5.14	HSUN = 5.38	H = .23
TIME = 39000 SEC	HMOON = -5.78	HSUN = 5.71	H = -.08
TIME = 39600 SEC	HMOON = -6.40	HSUN = 6.00	H = -.40
TIME = 40200 SEC	HMOON = -6.99	HSUN = 6.26	H = -.73
TIME = 40800 SEC	HMOON = -7.54	HSUN = 6.48	H = -1.06
TIME = 41400 SEC	HMOON = -8.05	HSUN = 6.65	H = -1.39
TIME = 42000 SEC	HMOON = -8.51	HSUN = 6.79	H = -1.72
TIME = 42600 SEC	HMOON = -8.92	HSUN = 6.89	H = -2.04
TIME = 43200 SEC	HMOON = -9.28	HSUN = 6.94	H = -2.35
TIME = 43800 SEC	HMOON = -9.58	HSUN = 6.94	H = -2.64
TIME = 44400 SEC	HMOON = -9.82	HSUN = 6.91	H = -2.91
TIME = 45000 SEC	HMOON = -9.99	HSUN = 6.83	H = -3.16
TIME = 45600 SEC	HMOON = -10.09	HSUN = 6.70	H = -3.39
TIME = 46200 SEC	HMOON = -10.12	HSUN = 6.54	H = -3.58
TIME = 46800 SEC	HMOON = -10.08	HSUN = 6.33	H = -3.74
TIME = 47400 SEC	HMOON = -9.96	HSUN = 6.09	H = -3.87
TIME = 48000 SEC	HMOON = -9.77	HSUN = 5.80	H = -3.96
TIME = 48600 SEC	HMOON = -9.50	HSUN = 5.49	H = -4.02
TIME = 49200 SEC	HMOON = -9.16	HSUN = 5.13	H = -4.03
TIME = 49800 SEC	HMOON = -8.75	HSUN = 4.75	H = -3.99
TIME = 50400 SEC	HMOON = -8.26	HSUN = 4.34	H = -3.91
TIME = 51000 SEC	HMOON = -7.70	HSUN = 3.91	H = -3.79
TIME = 51600 SEC	HMOON = -7.07	HSUN = 3.46	H = -3.62
TIME = 52200 SEC	HMOON = -6.38	HSUN = 2.98	H = -3.40
TIME = 52800 SEC	HMOON = -5.63	HSUN = 2.50	H = -3.13
TIME = 53400 SEC	HMOON = -4.81	HSUN = 2.00	H = -2.81
TIME = 54000 SEC	HMOON = -3.95	HSUN = 1.49	H = -2.45
TIME = 54600 SEC	HMOON = -3.03	HSUN = .99	H = -2.05
TIME = 55200 SEC	HMOON = -2.07	HSUN = .48	H = -1.60

Table 2. Surface Displacement and Lunar and Solar Components of Surface Displacement for Station No. 2 (Continued)

TIME = 55800 SEC	HMOON = -1.07	HSUN = -.03	H = -1.10
TIME = 56400 SEC	HMOON = -.04	HSUN = -.52	H = -.57
TIME = 57000 SEC	HMOON = 1.01	HSUN = -1.01	H = .01
TIME = 57600 SEC	HMOON = 2.09	HSUN = -1.48	H = .61
TIME = 58200 SEC	HMOON = 3.19	HSUN = -1.93	H = 1.25
TIME = 58800 SEC	HMOON = 4.29	HSUN = -2.36	H = 1.92
TIME = 59400 SEC	HMOON = 5.39	HSUN = -2.77	H = 2.62
TIME = 60000 SEC	HMOON = 6.48	HSUN = -3.15	H = 3.34
TIME = 60600 SEC	HMOON = 7.57	HSUN = -3.50	H = 4.07
TIME = 61200 SEC	HMOON = 8.63	HSUN = -3.81	H = 4.82
TIME = 61800 SEC	HMOON = 9.67	HSUN = -4.09	H = 5.58
TIME = 62400 SEC	HMOON = 10.68	HSUN = -4.34	H = 6.34
TIME = 63000 SEC	HMOON = 11.64	HSUN = -4.54	H = 7.10
TIME = 63600 SEC	HMOON = 12.57	HSUN = -4.71	H = 7.86
TIME = 64200 SEC	HMOON = 13.44	HSUN = -4.83	H = 8.61
TIME = 64800 SEC	HMOON = 14.26	HSUN = -4.91	H = 9.34
TIME = 65400 SEC	HMOON = 15.01	HSUN = -4.95	H = 10.06
TIME = 66000 SEC	HMOON = 15.70	HSUN = -4.95	H = 10.75
TIME = 66600 SEC	HMOON = 16.32	HSUN = -4.91	H = 11.41
TIME = 67200 SEC	HMOON = 16.86	HSUN = -4.82	H = 12.04
TIME = 67800 SEC	HMOON = 17.33	HSUN = -4.70	H = 12.63
TIME = 68400 SEC	HMOON = 17.71	HSUN = -4.53	H = 13.18
TIME = 69000 SEC	HMOON = 18.01	HSUN = -4.33	H = 13.68
TIME = 69600 SEC	HMOON = 18.23	HSUN = -4.09	H = 14.14
TIME = 70200 SEC	HMOON = 18.35	HSUN = -3.81	H = 14.54
TIME = 70800 SEC	HMOON = 18.39	HSUN = -3.51	H = 14.89
TIME = 71400 SEC	HMOON = 18.35	HSUN = -3.17	H = 15.17
TIME = 72000 SEC	HMOON = 18.21	HSUN = -2.81	H = 15.40
TIME = 72600 SEC	HMOON = 17.99	HSUN = -2.42	H = 15.56
TIME = 73200 SEC	HMOON = 17.68	HSUN = -2.02	H = 15.66
TIME = 73800 SEC	HMOON = 17.28	HSUN = -1.59	H = 15.69
TIME = 74400 SEC	HMOON = 16.81	HSUN = -1.15	H = 15.65
TIME = 75000 SEC	HMOON = 16.25	HSUN = -.71	H = 15.55
TIME = 75600 SEC	HMOON = 15.62	HSUN = -.25	H = 15.37
TIME = 76200 SEC	HMOON = 14.92	HSUN = .21	H = 15.13
TIME = 76800 SEC	HMOON = 14.16	HSUN = .67	H = 14.83
TIME = 77400 SEC	HMOON = 13.33	HSUN = 1.12	H = 14.45
TIME = 78000 SEC	HMOON = 12.44	HSUN = 1.57	H = 14.01
TIME = 78600 SEC	HMOON = 11.50	HSUN = 2.01	H = 13.51
TIME = 79200 SEC	HMOON = 10.52	HSUN = 2.43	H = 12.95
TIME = 79800 SEC	HMOON = 9.50	HSUN = 2.83	H = 12.34
TIME = 80400 SEC	HMOON = 8.45	HSUN = 3.22	H = 11.67
TIME = 81000 SEC	HMOON = 7.37	HSUN = 3.58	H = 10.95
TIME = 81600 SEC	HMOON = 6.27	HSUN = 3.91	H = 10.18
TIME = 82200 SEC	HMOON = 5.16	HSUN = 4.21	H = 9.37
TIME = 82800 SEC	HMOON = 4.04	HSUN = 4.48	H = 8.52
TIME = 83400 SEC	HMOON = 2.92	HSUN = 4.72	H = 7.64
TIME = 84000 SEC	HMOON = 1.82	HSUN = 4.92	H = 6.74
TIME = 84600 SEC	HMOON = .72	HSUN = 5.09	H = 5.81
TIME = 85200 SEC	HMOON = -.35	HSUN = 5.21	H = 4.86
TIME = 85800 SEC	HMOON = -1.39	HSUN = 5.30	H = 3.90

Table 3. Surface Displacement and Lunar and Solar Components of Surface Displacement for Station No. 3

(Station longitude = 0 deg, station latitude = 60 deg, HMOON, HSUN, and H are in centimeters. Time is listed in terms of seconds from Midnight UT of Day 88 of Year 1977.)

TIME =	0	SEC	HMOON =	-5.75	HSUN =	-1.95	H =	-7.69
TIME =	600	SEC	HMOON =	-6.18	HSUN =	-1.95	H =	-8.12
TIME =	1200	SEC	HMOON =	-6.59	HSUN =	-1.96	H =	-8.55
TIME =	1800	SEC	HMOON =	-6.98	HSUN =	-1.98	H =	-8.96
TIME =	2400	SEC	HMOON =	-7.35	HSUN =	-2.02	H =	-9.37
TIME =	3000	SEC	HMOON =	-7.69	HSUN =	-2.07	H =	-9.76
TIME =	3600	SEC	HMOON =	-8.02	HSUN =	-2.13	H =	-10.14
TIME =	4200	SEC	HMOON =	-8.31	HSUN =	-2.20	H =	-10.51
TIME =	4800	SEC	HMOON =	-8.59	HSUN =	-2.28	H =	-10.87
TIME =	5400	SEC	HMOON =	-8.84	HSUN =	-2.37	H =	-11.21
TIME =	6000	SEC	HMOON =	-9.06	HSUN =	-2.47	H =	-11.53
TIME =	6600	SEC	HMOON =	-9.25	HSUN =	-2.58	H =	-11.84
TIME =	7200	SEC	HMOON =	-9.43	HSUN =	-2.70	H =	-12.13
TIME =	7800	SEC	HMOON =	-9.57	HSUN =	-2.82	H =	-12.40
TIME =	8400	SEC	HMOON =	-9.69	HSUN =	-2.95	H =	-12.65
TIME =	9000	SEC	HMOON =	-9.79	HSUN =	-3.09	H =	-12.88
TIME =	9600	SEC	HMOON =	-9.87	HSUN =	-3.22	H =	-13.09
TIME =	10200	SEC	HMOON =	-9.92	HSUN =	-3.36	H =	-13.28
TIME =	10800	SEC	HMOON =	-9.95	HSUN =	-3.50	H =	-13.45
TIME =	11400	SEC	HMOON =	-9.96	HSUN =	-3.64	H =	-13.61
TIME =	12000	SEC	HMOON =	-9.96	HSUN =	-3.78	H =	-13.74
TIME =	12600	SEC	HMOON =	-9.93	HSUN =	-3.91	H =	-13.85
TIME =	13200	SEC	HMOON =	-9.90	HSUN =	-4.05	H =	-13.94
TIME =	13800	SEC	HMOON =	-9.84	HSUN =	-4.17	H =	-14.02
TIME =	14400	SEC	HMOON =	-9.78	HSUN =	-4.29	H =	-14.07
TIME =	15000	SEC	HMOON =	-9.70	HSUN =	-4.41	H =	-14.11
TIME =	15600	SEC	HMOON =	-9.61	HSUN =	-4.51	H =	-14.13
TIME =	16200	SEC	HMOON =	-9.52	HSUN =	-4.61	H =	-14.13
TIME =	16800	SEC	HMOON =	-9.42	HSUN =	-4.69	H =	-14.12
TIME =	17400	SEC	HMOON =	-9.32	HSUN =	-4.77	H =	-14.09
TIME =	18000	SEC	HMOON =	-9.22	HSUN =	-4.83	H =	-14.05
TIME =	18600	SEC	HMOON =	-9.11	HSUN =	-4.88	H =	-14.00
TIME =	19200	SEC	HMOON =	-9.01	HSUN =	-4.92	H =	-13.93
TIME =	19800	SEC	HMOON =	-8.91	HSUN =	-4.95	H =	-13.85
TIME =	20400	SEC	HMOON =	-8.81	HSUN =	-4.96	H =	-13.77
TIME =	21000	SEC	HMOON =	-8.72	HSUN =	-4.96	H =	-13.67
TIME =	21600	SEC	HMOON =	-8.63	HSUN =	-4.94	H =	-13.57
TIME =	22200	SEC	HMOON =	-8.55	HSUN =	-4.91	H =	-13.46

Table 3. Surface Displacement and Lunar and Solar Components of Surface Displacement for Station No. 3 (Continued)

TIME = 22800 SEC	HMOON = -8.48	HSUN = -4.87	H = -13.35
TIME = 23400 SEC	HMOON = -8.42	HSUN = -4.81	H = -13.23
TIME = 24000 SEC	HMOON = -8.37	HSUN = -4.73	H = -13.11
TIME = 24600 SEC	HMOON = -8.34	HSUN = -4.65	H = -12.99
TIME = 25200 SEC	HMOON = -8.31	HSUN = -4.55	H = -12.86
TIME = 25800 SEC	HMOON = -8.30	HSUN = -4.44	H = -12.74
TIME = 26400 SEC	HMOON = -8.30	HSUN = -4.32	H = -12.61
TIME = 27000 SEC	HMOON = -8.31	HSUN = -4.18	H = -12.49
TIME = 27600 SEC	HMOON = -8.33	HSUN = -4.04	H = -12.37
TIME = 28200 SEC	HMOON = -8.37	HSUN = -3.89	H = -12.25
TIME = 28800 SEC	HMOON = -8.41	HSUN = -3.72	H = -12.14
TIME = 29400 SEC	HMOON = -8.47	HSUN = -3.56	H = -12.03
TIME = 30000 SEC	HMOON = -8.54	HSUN = -3.38	H = -11.92
TIME = 30600 SEC	HMOON = -8.62	HSUN = -3.20	H = -11.82
TIME = 31200 SEC	HMOON = -8.70	HSUN = -3.02	H = -11.72
TIME = 31800 SEC	HMOON = -8.80	HSUN = -2.84	H = -11.63
TIME = 32400 SEC	HMOON = -8.90	HSUN = -2.65	H = -11.55
TIME = 33000 SEC	HMOON = -9.00	HSUN = -2.46	H = -11.46
TIME = 33600 SEC	HMOON = -9.11	HSUN = -2.28	H = -11.39
TIME = 34200 SEC	HMOON = -9.22	HSUN = -2.10	H = -11.32
TIME = 34800 SEC	HMOON = -9.33	HSUN = -1.92	H = -11.25
TIME = 35400 SEC	HMOON = -9.44	HSUN = -1.75	H = -11.19
TIME = 36000 SEC	HMOON = -9.55	HSUN = -1.58	H = -11.13
TIME = 36600 SEC	HMOON = -9.65	HSUN = -1.42	H = -11.07
TIME = 37200 SEC	HMOON = -9.74	HSUN = -1.27	H = -11.02
TIME = 37800 SEC	HMOON = -9.83	HSUN = -1.13	H = -10.96
TIME = 38400 SEC	HMOON = -9.91	HSUN = -1.00	H = -10.91
TIME = 39000 SEC	HMOON = -9.98	HSUN = -.89	H = -10.86
TIME = 39600 SEC	HMOON = -10.03	HSUN = -.78	H = -10.81
TIME = 40200 SEC	HMOON = -10.07	HSUN = -.69	H = -10.76
TIME = 40800 SEC	HMOON = -10.09	HSUN = -.61	H = -10.70
TIME = 41400 SEC	HMOON = -10.10	HSUN = -.55	H = -10.64
TIME = 42000 SEC	HMOON = -10.08	HSUN = -.50	H = -10.58
TIME = 42600 SEC	HMOON = -10.05	HSUN = -.46	H = -10.51
TIME = 43200 SEC	HMOON = -9.99	HSUN = -.44	H = -10.44
TIME = 43800 SEC	HMOON = -9.92	HSUN = -.44	H = -10.36
TIME = 44400 SEC	HMOON = -9.81	HSUN = -.45	H = -10.27
TIME = 45000 SEC	HMOON = -9.69	HSUN = -.48	H = -10.17
TIME = 45600 SEC	HMOON = -9.54	HSUN = -.52	H = -10.06
TIME = 46200 SEC	HMOON = -9.36	HSUN = -.58	H = -9.95
TIME = 46800 SEC	HMOON = -9.16	HSUN = -.66	H = -9.82
TIME = 47400 SEC	HMOON = -8.94	HSUN = -.74	H = -9.68
TIME = 48000 SEC	HMOON = -8.69	HSUN = -.84	H = -9.53
TIME = 48600 SEC	HMOON = -8.41	HSUN = -.96	H = -9.37
TIME = 49200 SEC	HMOON = -8.11	HSUN = -1.08	H = -9.19
TIME = 49800 SEC	HMOON = -7.79	HSUN = -1.22	H = -9.01
TIME = 50400 SEC	HMOON = -7.44	HSUN = -1.36	H = -8.81
TIME = 51000 SEC	HMOON = -7.07	HSUN = -1.52	H = -8.59
TIME = 51600 SEC	HMOON = -6.68	HSUN = -1.68	H = -8.36
TIME = 52200 SEC	HMOON = -6.27	HSUN = -1.85	H = -8.12
TIME = 52800 SEC	HMOON = -5.84	HSUN = -2.03	H = -7.87
TIME = 53400 SEC	HMOON = -5.40	HSUN = -2.21	H = -7.61
TIME = 54000 SEC	HMOON = -4.94	HSUN = -2.39	H = -7.33
TIME = 54600 SEC	HMOON = -4.46	HSUN = -2.58	H = -7.04
TIME = 55200 SEC	HMOON = -3.98	HSUN = -2.76	H = -6.74

Table 3. Surface Displacement and Lunar and Solar Components of Surface Displacement for Station No. 3 (Continued)

TIME =	55800 SEC	HMOON =	-3.49	HSUN =	-2.95	H =	-6.43
TIME =	56400 SEC	HMOON =	-2.99	HSUN =	-3.13	H =	-6.12
TIME =	57000 SEC	HMOON =	-2.48	HSUN =	-3.31	H =	-5.79
TIME =	57600 SEC	HMOON =	-1.97	HSUN =	-3.49	H =	-5.46
TIME =	58200 SEC	HMOON =	-1.47	HSUN =	-3.66	H =	-5.13
TIME =	58800 SEC	HMOON =	-.96	HSUN =	-3.82	H =	-4.79
TIME =	59400 SEC	HMOON =	-.47	HSUN =	-3.98	H =	-4.44
TIME =	60000 SEC	HMOON =	.03	HSUN =	-4.12	H =	-4.10
TIME =	60600 SEC	HMOON =	.51	HSUN =	-4.26	H =	-3.76
TIME =	61200 SEC	HMOON =	.97	HSUN =	-4.39	H =	-3.41
TIME =	61800 SEC	HMOON =	1.43	HSUN =	-4.50	H =	-3.08
TIME =	62400 SEC	HMOON =	1.86	HSUN =	-4.61	H =	-2.74
TIME =	63000 SEC	HMOON =	2.28	HSUN =	-4.70	H =	-2.42
TIME =	63600 SEC	HMOON =	2.67	HSUN =	-4.78	H =	-2.10
TIME =	64200 SEC	HMOON =	3.04	HSUN =	-4.84	H =	-1.80
TIME =	64800 SEC	HMOON =	3.39	HSUN =	-4.89	H =	-1.50
TIME =	65400 SEC	HMOON =	3.71	HSUN =	-4.93	H =	-1.22
TIME =	66000 SEC	HMOON =	3.99	HSUN =	-4.95	H =	-.96
TIME =	66600 SEC	HMOON =	4.25	HSUN =	-4.96	H =	-.71
TIME =	67200 SEC	HMOON =	4.47	HSUN =	-4.95	H =	-.48
TIME =	67800 SEC	HMOON =	4.66	HSUN =	-4.93	H =	-.27
TIME =	68400 SEC	HMOON =	4.82	HSUN =	-4.90	H =	-.08
TIME =	69000 SEC	HMOON =	4.94	HSUN =	-4.86	H =	.08
TIME =	69600 SEC	HMOON =	5.02	HSUN =	-4.80	H =	.22
TIME =	70200 SEC	HMOON =	5.06	HSUN =	-4.73	H =	.34
TIME =	70800 SEC	HMOON =	5.07	HSUN =	-4.65	H =	.43
TIME =	71400 SEC	HMOON =	5.04	HSUN =	-4.56	H =	.49
TIME =	72000 SEC	HMOON =	4.98	HSUN =	-4.46	H =	.52
TIME =	72600 SEC	HMOON =	4.88	HSUN =	-4.35	H =	.53
TIME =	73200 SEC	HMOON =	4.74	HSUN =	-4.23	H =	.50
TIME =	73800 SEC	HMOON =	4.56	HSUN =	-4.11	H =	.45
TIME =	74400 SEC	HMOON =	4.35	HSUN =	-3.98	H =	.37
TIME =	75000 SEC	HMOON =	4.11	HSUN =	-3.85	H =	.26
TIME =	75600 SEC	HMOON =	3.85	HSUN =	-3.72	H =	.12
TIME =	76200 SEC	HMOON =	3.52	HSUN =	-3.58	H =	-.06
TIME =	76800 SEC	HMOON =	3.19	HSUN =	-3.45	H =	-.25
TIME =	77400 SEC	HMOON =	2.83	HSUN =	-3.31	H =	-.48
TIME =	78000 SEC	HMOON =	2.44	HSUN =	-3.17	H =	-.74
TIME =	78600 SEC	HMOON =	2.02	HSUN =	-3.04	H =	-1.02
TIME =	79200 SEC	HMOON =	1.59	HSUN =	-2.91	H =	-1.33
TIME =	79800 SEC	HMOON =	1.14	HSUN =	-2.79	H =	-1.66
TIME =	80400 SEC	HMOON =	.66	HSUN =	-2.68	H =	-2.01
TIME =	81000 SEC	HMOON =	.18	HSUN =	-2.57	H =	-2.39
TIME =	81600 SEC	HMOON =	-.32	HSUN =	-2.47	H =	-2.78
TIME =	82200 SEC	HMOON =	-.83	HSUN =	-2.37	H =	-3.20
TIME =	82800 SEC	HMOON =	-1.34	HSUN =	-2.29	H =	-3.63
TIME =	83400 SEC	HMOON =	-1.86	HSUN =	-2.22	H =	-4.08
TIME =	84000 SEC	HMOON =	-2.38	HSUN =	-2.16	H =	-4.54
TIME =	84600 SEC	HMOON =	-2.90	HSUN =	-2.11	H =	-5.01
TIME =	85200 SEC	HMOON =	-3.42	HSUN =	-2.07	H =	-5.49
TIME =	85800 SEC	HMOON =	-3.93	HSUN =	-2.04	H =	-5.97

Table 4. Sample Data from Two-Hand Computations for Computer Program Checkout

STATION #	1	2
STN LONG (deg)	0	0
STN LAT (deg)	0	30
UNIVERSAL TIME (sec) OF DAY 88 IN YEAR 1977	57600	1200
XSUN (km) = x'_S	77220921	-148806042
YSUN = y'_S	-127563246	9783868
ZSUN = z'_S	9143321	8480080
XMOON (km) = x'_L	229338	65180
YMOON = y'_L	300370	-372772
ZMOON = z'_L	103334	113461
RSUN (km)	149395625	149368250
φ SUN (deg)	3.5088129	3.254602
λ SUN (deg)	-58.3934085	176.656074
RMOON (km)	391786	395071
φ MOON (deg)	15.292773	16.689852
λ MOON (deg)	53.055410	-79.6642185
cos γ SUN	0.523101438	-0.834769991
γ SUN (deg)	58.459	146.591896
P_2 (cos γ SUN)	-0.08954733	0.545261408
cos γ MOON	0.579759898	0.292429179
γ MOON (deg)	54.566343	72.9965565
P_2 (cos γ MOON)	0.004182309	-0.371727763
HSUN (cm)	-0.9	5.4
HMOON (cm)	0.085	-7.4

The following are explanations for some of the quantities listed in Table 4.

XSUN, YSUN, ZSUN and XMOON, YMOON, ZMOON are the earth-fixed Cartesian coordinates of the "fictitious" sun and moon at the specified Universal time. These were produced by the computer algorithm while excecuting those computations which led to the results listed in Tables 1 and 2. They were made the starting point of the two hand cases without being recomputed by hand, from the sun and moon ephemerides. Various checks were however made to assure correctness, such as comparison with the positions listed in the ***American Ephemeris*** and ***Nautical Almanac*** as well as tests concerning the direction and rate of rotation of the fictitious sun and moon with respect to inertial space and with respect to the earth.

RSUN, φ SUN, and λ SUN are the geocentric distance, latitude and longitude of the fictitious sun. RMOON, φ MOON, λ MOON are the corresponding quantities for the fictitious moon.

γ SUN is the spherical angle between the station and the fictitious sun. γ MOON is the corresponding quantity for the fictitious moon.

HSUN and HMOON are the surface displacements caused by sun and moon, at the specified value of Universal time.

FUTURE REQUIREMENTS

The computer program specified in this report for the tidal surface elevation is regarded adequate for use with the present version of CELEST. But now already a number of desirable improvements may be anticipated.

The text following Equation (15) relates that the tide potential and thus the surface displacement may be truncated after the term containing h_3. But as Appendix A will demonstrate, even this latter term may safely be deleted unless millimeter accuracy is desired. On the other hand, Reference 6, Page 7 relates that the orbit integration for geodetic satellites already makes use of the dynamical Love numbers up to k_3 and also resorts to surface harmonic expansions for the Love numbers in order to account for the details of the gravitational influence of the tidal bulge on the satellite orbit.

The present version of TERRA considers the dynamical Love numbers program input. But the author expects that in the not too distant future these numbers may be made bias parameters to be "measured" by the least squares procedure involved in the TERRA runs. At present, CELEST also treats the Love numbers h_2 and perhaps h_3 as input quantities. Consequently, it should be interesting to expand h_2 into surface harmonics and to experiment with the thus modified algorithm. And, secondly, in the light of Appendix C, one may attempt to establish a valid relationship amongst the dynamical Love numbers resulting from the geodetic parameter solutions and the yet unmeasured expansion coefficients for the geometrical Love numbers h_2 and h_3. The author feels certain that the necessary relationship $h_{ij} = h_{ij}(k_{lm})$ can be approximated by a rather straightforward extension of the elementary theory involving the geometrical and dynamical Love number concepts. It is believed that this extension will involve essentially a mass density model which deviates from the customary assumption of homogeneity. The author plans to experiment with a three-layer model of the tidal earth. Also, a review is being made of the latest geophysical research relevant to the solid earth tide, with particular attention to work published by P. Melchior (note the summary of the subject in Reference 10, Volume 3).

REFERENCES

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2. J. W. O'Toole, ***CELEST Computer Program for Computing Satellite Orbits***, Naval Surface Weapons Center, Dahlgren Laboratory Technical Report NSWC/DL TR-3565, Dahlgren, VA, October 1976.

3. W. Groeger and R. Manrique, ***Ocean Tides in the Equations of Motion of the TERRA System of Computer Programs for Satellite Geodesy***, Naval Surface Weapons Center, Dahlgren Laboratory Technical Report NSWC/DL TR-3535, Dahlgren, VA, December 1976.

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APPENDIX A

CONTRIBUTIONS BY TERMS CONTAINING THE LOVE NUMBER h_3

CONTRIBUTIONS BY TERMS CONTAINING THE LOVE NUMBER h_3

To be able to decide which terms to retain and which ones to delete from the formula for the tidal surface elevation, the following (rather simple yet sufficient) trial case was done by hand computation. The results suggest that it is safe at present to feature in the computer algorithm only the two terms containing the Love number h_2 and to omit all the others.

g	9.81	m/sec^2	Approximate surface gravity
G	6.673 E - 08	$cm^3/gr\ sec^2$	Current value
R	6378150	m	Typical value
μ_L	4.9177 E + 18	cm^3/sec^2	Current value
μ_S	1.3291 E + 26	cm^3/sec^2	Current value
h_2	0.6		Typical values, sources
h_3	0.3		indicated in Appendix C
r_L	400 000	km	Typical value
r_S	150 000 000	km	Typical value

$$(H_L)_2 = h_2 \frac{\mu_L}{g} \frac{R^2}{r_L^3} 10^{-15} P_2(\cos \gamma_L) \tag{201}$$

$$(H_L)_3 = h_3 \frac{\mu_L}{g} \frac{R^3}{r_L^4} 10^{-18} P_3(\cos \gamma_L) \tag{202}$$

$$(H_S)_2 = h_2 \frac{\mu_S}{g} \frac{R^2}{r_S^3} 10^{-15} P_2(\cos \gamma_S) \tag{203}$$

$$(H_S)_3 = h_3 \frac{\mu_S}{g} \frac{R^3}{r_S^4} 10^{-18} P_3(\cos\gamma_S) \tag{204}$$

$$(H_L)_2 = 0.191 \text{ m} \quad P_2(\cos\gamma_L) \tag{205}$$

$$(H_L)_3 = 0.001 \text{ m} \quad P_3(\cos\gamma_L) \tag{206}$$

$$(H_S)_2 = 0.098 \text{ m} \quad P_2(\cos\gamma_S) \tag{207}$$

$$(H_S)_3 = 2 \cdot 10^{-6} \text{ m} \quad P_3(\cos\gamma_S) \tag{208}$$

First Case

Station at and sun and moon above intersection of equator and Greenwich meridian.

$$\lambda = \varphi = 0$$

$$\gamma_L = \gamma_S = 0 \rightarrow \cos\gamma_L = \cos\gamma_S = 1$$

$$\rightarrow P_i(\cos\gamma_L) = P_i(\cos\gamma_S) = 1, \quad i = 1,2$$

$$(H_L)_2 = 0.191 \text{ m}$$

$$(H_L)_3 = 0.001 \text{ m}$$

$$(H_S)_2 = 0.098 \text{ m}$$

$$(H_S)_3 = \underline{0.000002 \text{ m}}$$

$$H = \underline{\underline{0.290 \text{ m}}}$$

Second Case

Station at intersection of equator and Greenwich meridian. Moon and sun at intersection of equator and 180° meridian.

$$\lambda = \varphi = 0$$

$$\gamma_L = \gamma_S = 180° \rightarrow \cos\gamma_L = \cos\gamma_S = -1$$

$$P_2(-1) = +1, \quad P_3(-1) = -4$$

$$(H_L)_2 = 0.191 \text{ m}$$

$$(H_L)_3 = -0.006 \text{ m}$$

$$(H_S)_2 = 0.098 \text{ m}$$

$$(H_S)_3 = \underline{0.000 \text{ m}}$$

$$H = \underline{\underline{0.283 \text{ m}}}$$

Third Case

Station at intersection of equator and Greenwich meridian. Moon and sun at intersection of equator and +90° meridian or −90° meridian.

$$\gamma = \varphi = 0$$

$$\gamma_L = \gamma_S = \pm 90° \rightarrow \cos\gamma_L = \cos\gamma_S = 0$$

$$P_2(0) = -1/2, \qquad P_3(0) = 0$$

$$(H_L)_2 = -0.096 \text{ m}$$

$$(H_L)_3 = 0$$

$$(H_S)_2 = -0.049 \text{ m}$$

$$(H_S)_3 = \underline{0}$$

$$H = \underline{\underline{-0.145 \text{ m}}}$$

APPENDIX B

A NOTE CONCERNING THE APPLICATION TO CELEST

A NOTE CONCERNING THE APPLICATION TO CELEST

During the early computer runs made while developing the present algorithm, the tidal surface elevation was recomputed for each data line on the CELEST observation tape. The fact, that these data lines follow each other at rather short intervals, makes the algorithm a costly augmentation of the observation tape. On the other hand, it may be concluded from the data in the section on "QUANTITATIVE RESULTS" that it is unnecessary to update H that frequently as it changes quite slowly.

The CELEST observation tape is divided into data blocks, each data block representing a satellite pass. In actual CELEST applications, two types of data blocks occur, namely those that are about 20 min long and others which are several hours in length (durations in excess of 12 hr being possible). The former are associated with passes over the observation stations of typical geodetic satellites. The latter arise from high altitude satellite orbits.

A simple analysis suggested that for both types of station passes it is equally suitable to calculate H once every 10 min. For the short passes this was shown by entering the trial data listed in Appendix A into the equation for H (Equation 103). The observing station was assumed to be located on the equator with sun and moon in line with each other and also positioned in the equator. The tidal elevation was then

$$H(t) = \alpha(3 \cos^2 \gamma - 1) \qquad (301)$$

$$\alpha \approx 15 \text{ cm} .$$

Assuming

$$\gamma \approx \tilde{\omega} t , \qquad (302)$$

$$\frac{d\gamma}{dt} \approx \tilde{\omega} , \qquad (303)$$

the time rate of H was

$$\frac{dH}{dt} \approx -3\alpha\tilde{\omega} \sin 2\tilde{\omega}t \,, \tag{304}$$

$$|\dot{H}|_{max} \approx 3\alpha\tilde{\omega} \approx 2 \text{ cm/10 min} \,. \tag{305}$$

For a typical long duration pass, the maximum total variation of H was

$$\frac{3\alpha}{6 \text{ hrs}} \approx \frac{45 \text{ cm}}{6 \text{ hrs}} \approx 1.5 \text{ cm/10 min} \,, \tag{306}$$

which is roughly the amount estimated for the 20-min pass. Assuming that a discrepancy of 2 cm in observing station elevation is yet undetectable by the means of satellite geodesy, it was resolved to settle on the 10-min interval length for both types of satellite passes.

APPENDIX C

NOTES ON THE EFFECT OF THE TIDAL MASS RELOCATION ON TERRESTRIAL GRAVITY

NOTES ON THE EFFECT OF THE TIDAL MASS RELOCATION ON TERRESTRIAL GRAVITY

The present analysis of the tidal surface displacement requires that the geometrical Love numbers are available. In particular, h_2 is assumed to be known to about ten percent. The value $h_2 = 0.6$ was specified for use with the computer routine for the surface displacement. It was adopted from Reference 11, Pages 27-29; Reference 12, Page XVII; Reference 13, Page 368; and Reference 14, Page 285.

For the purposes of the computer routine, h_3 is irrelevant. However, to demonstrate that (which is done in Appendix A), a value for this particular Love number had to be known. $h_3 = 0.3$ was finally adopted from data tabulated in Reference 13, Page 370.

By their definition,[7,8,11] the geometrical Love numbers and the dynamical ones are related to each other. For this reason, it was desired to have an algorithm capable of deriving a value for h_2 from the Love numbers k_i. The latter are already being used in various computer programs[6,15] to account for the gravitational perturbation which the tidal bulge of the solid earth exerts on satellites. A suitable algorithm was obtained as follows.

For example, as mentioned in Reference 9, Page 3, the tidal deformation of the earth's surface distorts the gravity field. Specifically, the presence of a particular *solid harmonic* U_n *in the tide generating potential field* causes a surface displacement

$$H_n = h_n \frac{U_n}{g} \tag{20}$$

which in turn produces a distortion

$$T_n(R,\varphi,\lambda) = k_n U_n(R,\varphi,\lambda) \tag{401}$$

of the earth's gravitational potential. The factor k_n is the n^{th} dynamical Love number. T_n is the n^{th} constituent of the expansion (in terms of spherical harmonics) of the potential field by which the masses in the tidal bulge perturb the satellite orbit. To deduce the relationship amongst the h_i and k_j, it is assumed once more that the earth is spherical (surface radius R) and that its density, ρ, is constant. The surface area element is dS. The surface density corresponding to the tidal mass redistribution is σ.

$$\rho\, H\, dS = \sigma\, dS\,, \tag{402}$$

$$\sigma = \rho\, H\,, \tag{403}$$

and

$$\sigma_n = \rho\, h_n \left(\frac{\dot{U}_n}{g}\right)_{r=R} . \tag{404}$$

Considering certain relationships from the main body of this report,

$$\sigma_n = h_n \frac{\rho}{g} \frac{1}{R^{n+1}} \Bigg\{ a_0 P_n(\sin\varphi) + \sum_{m=1}^{n} a_{nm} P_n^m(\sin\varphi) \cos m\lambda + \sum_{m=1}^{n} b_{nm} P_n^m(\sin\varphi) \sin m\lambda \Bigg\} \tag{405}$$

and the perturbing potential (acting on the satellite), T_n, can now be expressed as a Poisson integral,

$$T_n = G \iint_{(S)} \frac{\sigma_n(\varphi',\lambda')\,dS'}{|\bar{r} - \bar{r}'|}\,, \tag{406}$$

$$(T_n)_S = (T_n)_{r=R}\,. \tag{407}$$

Once more using Equations (3b) and (6) and restricting the integration to computing surface values of the potential only,

$$(T_n)_S = (T_n)_1 + (T_n)_2$$

$$(T_n)_1 = \frac{B_n}{R} \iint_{(S)} d\varphi' d\lambda' Y_n(\varphi',\lambda') \cos\varphi' * \sum_{\nu=0}^{\infty} P_\nu(\sin\varphi) P_\nu(\sin\varphi') \quad (408)$$

$$(T_n)_2 = \frac{2B_n}{R} \iint_{(S)} d\varphi' d\lambda' Y_n(\varphi',\lambda') \cos\varphi'$$

$$* \sum_{\nu=0}^{\infty} \sum_{\mu=1}^{\nu} \frac{(\nu-\mu)!}{(\nu+\mu)!} P_\nu^\mu(\sin\varphi) P_\nu^\mu(\sin\varphi') \cos\mu(\lambda-\lambda') \quad (409)$$

$$B_n = h_n \frac{\rho}{g} \frac{G}{R^{n-1}} \quad (410)$$

Thus,

$$(T_n)_1 = \frac{4\pi}{2n+1} a_0 \frac{B_n}{R} P_n(\sin\varphi) \quad (411)$$

$$(T_n)_2 = \frac{2\pi}{R} B_n \sum_{\mu=1}^{n} a_{n\mu} \frac{2}{2n+1} P_n^\mu(\sin\varphi) \cos\mu\lambda$$

$$+ \frac{2\pi}{R} B_n \sum_{\mu=1}^{n} b_{n\mu} \frac{2}{2n+1} P_n^\mu(\sin\varphi) \sin\mu\lambda \quad (412)$$

and, consequently,

$$(T_n)_S = \frac{4\pi}{2n+1} \frac{\rho}{g} RG\, h_n \frac{Y_n(\varphi,\lambda)}{R^{n+1}} . \quad (413)$$

This means that $(T_n)_S$ is proportional to $(U_n)_S$, the proportionality factor being the Love number k_n,

$$(T_n)_S = k_n (U_n)_{r=R} , \quad (414)$$

$$k_n = \frac{4\pi}{2n+1} \frac{\rho}{g} RG\, h_n \,. \tag{415}$$

A familiar theorem of potential theory states that a harmonic function (potential) is uniquely determined by its boundary values on a specified surface. If applied to Equation (401), the relationship

$$T_n(r,\varphi,\lambda) = k_n\, U_n(r,\varphi,\lambda) \tag{416}$$

will appear justified. This latter equation is the one which is frequently invoked to define k_n.

If one enters the values for k_2 and k_3 (actually k_{20} and k_{30} or averages of the variable k_2 and k_3) listed in Reference 15, Page 460 into Equation (415), the following values result for h_2 and h_3:

$$\left.\begin{aligned} k_2 &= 0.3 \;\ldots\ldots\; h_2 = 0.9 \\ k_3 &= 0.1 \;\ldots\ldots\; h_3 = 0.4 \end{aligned}\right\} \tag{417}$$

While the two values for the dynamical Love numbers are generally accepted in magnitude, the two h values are too large. This failure of the just outlined theory is believed to be caused by the assumption of constant density. The author intends to produce a multilayer model which he hopes will yield more realistic h_i values. Although there appears at present no need to refine the model by using surface harmonic expansions for the h_i similar to those used in Reference 11 for the k_j, it is intended to try that while deriving the improved $h_i = h_i(k_j)$ algorithm.