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THE QUASI-NEAR FIELDS ABOVE A CONDUCTING SLAB.(U)
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The Quasi-Near Fields Above a Conducting Slab

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Submarine Electromagnetic
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24 April 1978

NUSC
NAVAL UNDERWATER SYSTEMS CENTER
Newport, Rhode Island • New London, Connecticut

PREFACE

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Approximate expressions for the quasi-near fields in air produced by electric and magnetic antennas located within a conducting slab have been obtained by applying the quasi-static and quasi-near approximations to the basic Sommerfeld integrals. The resulting expressions are particularly applicable to subsurface to air propagation for the shallow sea case.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

THE QUASI-NEAR FIELDS ABOVE A CONDUCTING SLAB

INTRODUCTION

During the last few years, considerable interest has developed in determining the quasi-static and quasi-near field components of antennas located above, or buried within, the earth's surface. The quasi-static range is defined as the range where the measurement distance is much less than a free-space wavelength; the quasi-near range is defined as the asymptotic part of the quasi-static range (i.e., where the measurement distance is much greater than a skin depth in the conducting medium and much greater than the burial depth of the transmitting and receiving antennas). Quasi-static and quasi-near range results are useful for submarine radio communication and detection purposes, as well as in dealing with the problem of buried miners. They are also helpful to geophysicists in determining the electrical properties of the earth.

The quasi-near fields of dipole antennas located above or buried within a semi-infinite conducting medium (i.e., air and single layered earth) were derived a number of years ago.[1,2] (Most of the results are summarized by Kraichman.[3]) Recently, we derived approximate expressions for the quasi-static and quasi-near field components within a conducting slab produced by various subsurface antennas.[4]

This report applies the quasi-static and quasi-near approximations to the basic Sommerfeld integrals to derive approximate expressions for the quasi-near fields in air produced by various subsurface antennas such as the vertical electric dipole (VED), horizontal electric dipole (HED), vertical magnetic dipole (VMD), horizontal magnetic dipole (HMD), and long horizontal line antenna, with each of these antennas located in the upper layer of a two-layer conducting earth. For mathematical convenience, we shall establish the conductivity of the bottom layer as being equal to zero. Thus the problem is reduced to calculating the fields outside of a conducting slab. The fact that the bottom layer conductivity is equal to zero is not restrictive because, for many practical cases, the conductivity of the upper layer (σ_1) is much greater than the conductivity of the bottom layer (σ_2). This is particularly true in the sea-sea bed case. However, the assumption does limit the results to measurement distances ($R = \sqrt{\rho^2 + z^2}$) of approximately $\delta_2/2$, where δ_2 is the skin depth in the bottom layer. For example, at a frequency of 1 Hz, if $\sigma_1 = 4$ S/m (sea) and $\sigma_2 = 10^{-2}$ S/m (sea bed), then $\delta_1 \sim 250$ m and $\delta_2 \sim 5$ km. Thus, in this particular example, the results should be valid up to a measurement distance of approximately 2.5 km. Furthermore, if the conductivity of the sea bed is 10^{-4} S/m ($\delta_2 \sim 50$ km), the results should be valid up to a measurement distance of approximately 25 km.

For the purpose of this report, all four dipole sources are considered to be located at depth h with respect to a cylindrical coordinate system

(ρ, ϕ, z) and are assumed to carry a constant current I. The VED and HED antennas (of infinitesimal length ℓ) are oriented in the z and x directions, respectively, and the axes of the VMD and HMD antennas (of infinitesimal area, A) are oriented in the z and y directions, respectively. Free space occupies the regions $z \geq 0$ and $z \geq \ell_1$, whereas the conducting slab occupies the region $0 < z < \ell_1$ (see fig. 1). Displacement currents are ignored in the slab and, for most cases, in the air. The magnetic permeability of the conducting slab is assumed to equal μ_o, the permeability of free space. Meter-kilogram-second (mks) units are employed and a suppressed time factor of exp $(i\omega t)$ is assumed.

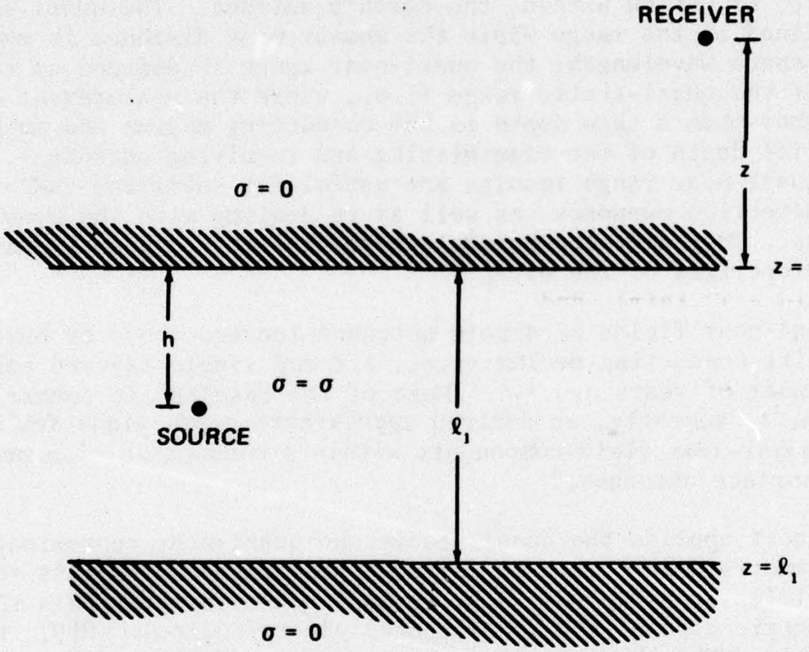

Figure 1. Conducting Slab Geometry

LONG HORIZONTAL LINE ANTENNA

For a long horizontal line antenna oriented in the x direction, the electric field outside the conducting slab (i.e., in air) may be written exactly as

$$E_x = -\frac{i\omega\mu_o I}{2\pi}\int_o^\infty C(\lambda)\, e^{-U_o z} \cos \lambda y \, d\lambda, \tag{1}$$

where

$$C(\lambda) = \frac{2}{U + U_o} \frac{\left[e^{U(\ell_1 - h)} + R_h e^{-U(\ell_1 - h)}\right]}{\left[e^{U\ell_1} - R_h^2 e^{-U\ell_1}\right]} , \qquad (2)$$

$$R_h = \frac{U - U_o}{U + U_o} . \qquad (3)$$

In equation (3),

$$U_o = (\lambda^2 + \gamma_o{}^2)^{\frac{1}{2}} ,$$

$$U = (\lambda^2 + \gamma^2)^{\frac{1}{2}} ,$$

$$\gamma_o = i\omega(\mu_o\varepsilon_o)^{\frac{1}{2}} \text{ (air), and}$$

$$\gamma \sim (i\omega\mu_o\sigma)^{\frac{1}{2}} \text{ (earth).}$$

The magnetic fields within the conducting slab may be determined from

$$H_y = -\frac{1}{i\omega\mu_o}\frac{\partial E_x}{\partial z} \text{ and } H_z = \frac{1}{i\omega\mu_o}\frac{\partial E_x}{\partial y} . \qquad (4)$$

The quasi-static range approximation is establishing the function U_o in the exact integral expression equal to λ, the variable of integration (i.e., $\gamma_o \sim 0$); the quasi-near range approximation is establishing the function U equal to γ, which is the propagation constant in the conducting medium. The quasi-static range approximation is valid when the measurement distance is less than one-twentieth of a free space wavelength; the quasi-near range approximation is valid when the measurement distance, R, is much greater than a skin depth (δ) in the conducting medium and much greater than the depth of burial of the transmitting antenna (h). Generally, R must be greater than 3δ and 3h. For the conducting slab case, R should also be greater than $3\ell_1$.

Application of the quasi-static and quasi-near approximations to equations (2) and (3) results in

$$C(\lambda) \sim \frac{2\left[\cosh \gamma(\ell_1 - h) + \frac{\lambda}{\gamma} \sinh \gamma(\ell_1 - h)\right]}{\gamma \sinh \gamma\ell_1 \left[1 + \frac{2\lambda}{\gamma} \coth \gamma\ell_1\right]} \; . \tag{5}$$

If the slab is not too thin, the denominator of equation (5) may be approximated as

$$\frac{1}{\gamma \sinh \gamma\ell_1 \left[1 + \frac{2\lambda}{\gamma} \coth \gamma\ell_1\right]} \sim \frac{\left[1 - \frac{2\lambda}{\gamma} \coth \gamma\ell_1\right]}{\gamma \sinh \gamma\ell_1} \; . \tag{6}$$

Substituting equations (5) and (6) into equations (1) and (4), we note that[5]

$$\int_0^\infty e^{-\lambda z} \cos \lambda y \, d\lambda = \frac{z}{r^2} \; , \tag{7}$$

$$\int_0^\infty \lambda e^{-\lambda z} \cos \lambda y \, d\lambda = -\frac{1}{r^2}\left(1 - \frac{2z^2}{r^2}\right) , \tag{8}$$

and

$$\int_0^\infty \lambda^2 e^{-\lambda z} \cos \lambda y \, d\lambda = -\frac{2z}{r^4}\left(2 - \frac{3z^2}{r^2}\right) \tag{9}$$

result in

$$E_x \sim -\frac{IQ^2}{\pi\sigma r^2}\left\{\left(1 - \frac{2z^2}{r^2}\right) A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{10}$$

$$H_y \sim \frac{IQ}{\pi\gamma r^2}\left\{\left(1 - \frac{2z^2}{r^2}\right) B(h,\ell_1)\right\} , \tag{11}$$

and

$$H_z \sim \frac{2IQ^2 y}{\pi\gamma^2 r^4}\left\{\left(1 - \frac{4z^2}{r^2}\right) A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{12}$$

where $r^2 = y^2 + z^2$,

$$A(h,\ell_1) \sim \frac{2 \cosh \gamma(\ell_1 - h)}{\cosh \gamma\ell_1} - \frac{\sinh \gamma(\ell_1 - h)}{Q^2 \sinh \gamma\ell_1} , \tag{13}$$

and

$$B(h,\ell_1) \sim \frac{\cosh \gamma(\ell_1 - h)}{\cosh \gamma\ell_1} . \tag{14}$$

For the conducting slab case,

$$Q \sim \coth \gamma\ell_1 , \tag{15}$$

where Q is the familiar plane wave correction factor employed to account for the presence of stratification in the earth.[6]

When $h = 0$, $B(h,\ell_1) \sim 1$ and

$$A(h,\ell_1) \sim 2 - \tanh^2 \gamma\ell_1 \sim 1 \text{ for } |\gamma\ell_1| > 1. \tag{16}$$

Furthermore, when $h = \ell_1$,

$$A(h,\ell_1) \sim 2\ B(h,\ell_1) \sim \frac{2}{\cosh \gamma\ell_1} . \tag{17}$$

When $|\gamma\ell_1| > 2\sqrt{2}$ (or $\ell_1/\delta > 2$),

$$A(h,\ell_1) \sim 2\, e^{-\gamma\ell_1} \Big[2 \cosh \gamma(\ell_1 - h) - \sinh \gamma(\ell_1 - h)\Big] , \tag{18}$$

and

$$B(h,\ell_1) \sim 2\, e^{-\gamma\ell_1} \cosh \gamma(\ell_1 - h) . \tag{19}$$

Furthermore, if $\ell_1 > 2h$, equations (18) and (19) become

$$A(h,\ell_1) \sim B(h,\ell_1) \sim e^{-\gamma h} , \tag{20}$$

and equations (10) through (12) reduce to the single layered earth quasi-near field equations.[3,7]

If y >> z, equations (10) through (12) become

$$E_x \sim -\frac{IQ^2}{\pi\sigma y^2}\left\{A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \qquad (21)$$

$$H_y \sim \frac{IQ}{\pi\gamma y^2} B(h,\ell_1) , \qquad (22)$$

and

$$H_z \sim \frac{2IQ^2}{\pi\gamma^2 y^3}\left\{A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right\} . \qquad (23)$$

If y → 0, equations (10) through (12) reduce to

$$E_x \sim \frac{IQ^2}{\pi\sigma z^2}\left\{A(h,\ell_1) - \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \qquad (24)$$

$$H_y \sim -\frac{IQ}{\pi\gamma z^2} B(h,\ell_1) , \qquad (25)$$

and

$$H_z \sim -\frac{2IQ^2 y}{\pi\gamma^2 z^4}\left\{3A(h,\ell_1) - \frac{\gamma z}{Q} B(h,\ell_1)\right\} . \qquad (26)$$

VERTICAL ELECTRIC DIPOLE (VED) ANTENNA

By following the same procedure outlined in the derivation of the long horizontal line antenna equations, we may readily determine the quasi-near range VED antenna field component expressions that are valid outside the conducting slab. They are

$$E_\rho \sim \frac{I\ell\rho}{2\pi\sigma R^5}\left\{3zC(h,\ell_1) - \frac{\gamma_o^2 R^2 Q}{\gamma} D(h,\ell_1)\right\} , \qquad (27)$$

$$E_z \sim -\frac{I\ell}{2\pi\sigma R^3}\left(1 - \frac{3z^2}{R^2}\right)C(h,\ell_1), \tag{28}$$

and

$$H\phi \sim \frac{I\ell\rho}{2\pi R^3}\left(\frac{\gamma_o^2}{\gamma^2}\right)C(h,\ell_1) \quad , \tag{29}$$

where

$$C(h,\ell_1) \sim \frac{\sinh \gamma\ (\ell_1 - h)}{\sinh \gamma\ell_1} \quad , \tag{30}$$

$$D(h,\ell_1) \sim \frac{2 \sinh \gamma(\ell_1 - h)}{\sinh \gamma\ell_1} - \frac{\cosh \gamma(\ell_1 - h)}{\cosh \gamma\ell_1} \quad , \tag{31}$$

and $R^2 = \rho^2 + z^2$.

When $h = 0$, $C(h,\ell_1) \sim D(h,\ell_1) \sim 1$. Furthermore, when $h = \ell_1$, $C(h,\ell_1) \sim 0$ and

$$D(h,\ell_1) \sim \frac{1}{\cosh \gamma\ell_1} \quad . \tag{32}$$

When $|\gamma\ell_1| > 2\sqrt{2}$ (or $\ell_1/\delta > 2$),

$$C(h,\ell_1) \sim 2\, e^{-\gamma\ell_1} \sinh \gamma(\ell_1 - h) \quad , \tag{33}$$

and

$$D(h,\ell_1) \sim 2\, e^{-\gamma\ell_1}\left[2 \sinh \gamma(\ell_1 - h) - \cosh \gamma(\ell_1 - h)\right] \quad . \tag{34}$$

Furthermore, if $\ell_1 > 2h$, equations (33) and (34) become

$$C(h,\ell_1) \sim D(h,\ell_1) \sim e^{-\gamma h} \quad , \tag{35}$$

and equations (27) through (29) are reduced to the single layered earth quasi-near field equations.[1,3]

If $\rho >> z$, equations (27) through (29) become

$$E_\rho \sim \frac{I\ell}{2\pi\sigma\rho^4}\left\{3zC(h,\ell_1) - \frac{\gamma_o^2\rho^2 Q}{\gamma} D(h,\ell_1)\right\} , \tag{36}$$

$$E_z \sim - \frac{I\ell}{2\pi\sigma\rho^3} C(h,\ell_1) \quad , \tag{37}$$

and

$$H_\phi \sim \frac{I\ell}{2\pi\rho^2}\left(\frac{\gamma_o^2}{\gamma^2}\right)C(h,\ell_1) \quad . \tag{38}$$

If $\rho \rightarrow 0$, equations (27) through (29) are reduced to

$$E_\rho \sim \frac{3I\ell\rho}{2\pi\sigma z^4} C(h,\ell_1) \ , \tag{39}$$

$$E_z \sim \frac{I\ell}{\pi\sigma z^3} C(h,\ell_1) \quad , \tag{40}$$

and

$$H_\phi \sim \frac{I\ell\rho}{2\pi z^3}\left(\frac{\gamma_o^2}{\gamma^2}\right)C(h,\ell_1) \quad . \tag{41}$$

VERTICAL MAGNETIC DIPOLE (VMD) ANTENNA

By following the same procedure outlined in the derivation of the long horizontal line antenna equations, we may readily determine the quasi-near range VMD antenna field component expressions. They are

$$E_\rho \sim - \frac{3IAQ^2\rho}{2\pi\sigma R^5}\left\{\left(1 - \frac{5z^2}{R^2}\right)A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{42}$$

$$H_\rho \sim -\frac{3IAQ\rho}{2\pi\gamma R^5}\left\{\left(1-\frac{5z^2}{R^2}\right)B(h,\ell_1)\right\}, \tag{43}$$

and

$$H_z \sim -\frac{3IAQ^2}{2\pi\gamma^2R^5}\left\{\left(3-30\frac{z^2}{R^2}+35\frac{z^4}{R^4}\right)A(h,\ell_1)+\frac{\gamma z}{Q}\left(3-\frac{5z^2}{R^2}\right)B(h,\ell_1)\right\}. \tag{44}$$

If $\rho >> z$, equations (42) through (44) reduce to

$$E_\rho \sim -\frac{3IAQ^2}{2\pi\sigma\rho^4}\left\{A(h,\ell_1)+\frac{\gamma z}{Q}B(h,\ell_1)\right\}, \tag{45}$$

$$H_\rho \sim -\frac{3IAQ}{2\pi\gamma\rho^4}B(h,\ell_1), \tag{46}$$

and

$$H_z \sim -\frac{9IAQ^2}{2\pi\gamma^2\rho^5}\left\{A(h,\ell_1)+\frac{\gamma z}{Q}B(h,\ell_1)\right\}. \tag{47}$$

If $\rho \rightarrow 0$, equations (42) through (44) become

$$E_\rho \sim \frac{3IAQ^2\rho}{2\pi\sigma z^5}\left\{4A(h,\ell_1)-\frac{\gamma z}{Q}B(h,\ell_1)\right\}, \tag{48}$$

$$H_\rho \sim \frac{6IAQ\rho}{\pi\gamma z^5}B(h,\ell_1), \tag{49}$$

and

$$H_z \sim -\frac{3IAQ^2}{\pi\gamma^2z^5}\left\{4A(h,\ell_1)-\frac{\gamma z}{Q}B(h,\ell_1)\right\}. \tag{50}$$

HORIZONTAL ELECTRIC DIPOLE (HED) ANTENNA

Again, by following the same procedure outlined in the derivation of the long horizontal line antenna equations, we may readily determine the quasi-near range HED antenna field component expressions. They are

$$E_\rho \sim \frac{I\ell Q^2 \cos\phi}{2\pi\sigma R^3}\left\{A(h,\ell_1) - \frac{\gamma z}{Q} B(h,\ell_1)\right\}, \tag{51}$$

$$E_\phi \sim \frac{I\ell Q^2 \sin\phi}{2\pi\sigma R^3}\left[\left(2 - 3\frac{z^2}{R^2}\right)A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right], \tag{52}$$

$$E_z \sim \frac{I\ell \cos\phi}{2\pi\sigma R^3}(\gamma\rho)QB(h,\ell_1) \quad , \tag{53}$$

$$H_\rho \sim \frac{I\ell Q \sin\phi}{2\pi\gamma R^3}\left(2 - 3\frac{z^2}{R^2}\right)B(h,\ell_1), \tag{54}$$

$$H_\phi \sim -\frac{I\ell Q \cos\phi}{2\pi\gamma R^3}B(h,\ell_1), \tag{55}$$

and

$$H_z \sim \frac{3I\ell\rho Q^2 \sin\phi}{2\pi\gamma^2 R^5}\left[\left(1 - 5\frac{z^2}{R^2}\right)A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right] . \tag{56}$$

If $\rho >> z$, equations (51) through (56) become

$$E_\rho \sim \frac{I\ell Q^2 \cos\phi}{2\pi\sigma\rho^3}\left\{A(h,\ell_1) - \frac{\gamma z}{Q} B(h,\ell_1)\right\}, \tag{57}$$

$$E_\phi \sim \frac{I\ell Q^2 \sin\phi}{2\pi\sigma\rho^3}\left\{2A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{58}$$

$$E_z \sim \frac{I\ell \cos\phi}{2\pi\sigma\rho^2} \gamma Q B(h,\ell_1) , \tag{59}$$

$$H_\rho \sim \frac{I\ell Q \sin\phi}{\pi\gamma\rho^3} B(h,\ell_1) , \tag{60}$$

$$H_\phi \sim \frac{I\ell Q \cos\phi}{2\pi\gamma\rho^3} B(h,\ell_1) , \tag{61}$$

and

$$H_z \sim \frac{3I\ell Q^2 \sin\phi}{2\pi\gamma^2\rho^4}\left\{A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right\} . \tag{62}$$

If $\rho \to 0$, equations (51) through (56) reduce to

$$E_\rho \sim \frac{I\ell Q^2 \cos\phi}{2\pi\sigma z^3}\left\{A(h,\ell_1) - \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{63}$$

$$E_\phi \sim -\frac{I\ell Q^2 \sin\phi}{2\pi\sigma z^3}\left\{A(h,\ell_1) - \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{64}$$

$$E_z \sim \frac{I\ell \cos\phi}{2\pi\sigma z^3} (\gamma\rho) Q B(h,\ell_1) , \tag{65}$$

$$H_\rho \sim -\frac{I\ell Q \sin\phi}{2\pi\gamma z^3} B(h,\ell_1) , \tag{66}$$

$$H_\phi \sim -\frac{I\ell Q\cos\phi}{2\pi\gamma z^3}\,B(h,\ell_1)\ , \tag{67}$$

and

$$H_z \sim -\frac{3I\ell\rho Q^2\sin\phi}{2\pi\gamma^2 z^5}\left\{4A(h,\ell_1) - \frac{\gamma z}{Q}\,B(h,\ell_1)\right\}\ . \tag{68}$$

HORIZONTAL MAGNETIC DIPOLE (HMD) ANTENNA

Again, by following the same procedure outlined in the derivation of the long horizontal line antenna equations, we may readily determine the quasi-near range HMD antenna field component expressions that are valid outside the conducting slab. They are

$$E_\rho \sim \frac{i\omega\mu_o IAQ\cos\phi}{2\pi\gamma R^3}\left\{A(h,\ell_1) - \frac{\gamma z}{Q}\,B(h,\ell_1)\right\}\ , \tag{69}$$

$$E_\phi \sim \frac{i\omega\mu_o IAQ\sin\phi}{2\pi\gamma R^3}\left\{\left(2 - 3\,\frac{z^2}{R^2}\right)A(h,\ell_1) + \frac{\gamma z}{Q}\,B(h,\ell_1)\right\}\ , \tag{70}$$

$$E_z \sim \frac{i\omega\mu_o IA\rho\cos\phi}{2\pi R^3}\,B(h,\ell_1)\ , \tag{71}$$

$$H_\rho \sim \frac{IA\sin\phi}{2\pi R^3}\left(2 - 3\,\frac{z^2}{R^2}\right)B(h,\ell_1)\ , \tag{72}$$

$$H_\phi \sim -\frac{IA\cos\phi}{2\pi R^3}\,B(h,\ell_1)\ , \tag{73}$$

and

$$H_z \sim \frac{3IA\rho Q\sin\phi}{2\pi\gamma R^5}\left\{\left(1 - 5\,\frac{z^2}{R^2}\right)A(h,\ell_1) + \frac{\gamma z}{Q}\,B(h,\ell_1)\right\}\ . \tag{74}$$

If $\rho >> z$, equations (69) through (74) reduce to

$$E_\rho \sim \frac{i\omega\mu_o IAQ \cos\phi}{2\pi\gamma\rho^3}\left\{A(h,\ell_1) - \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{75}$$

$$E_\phi \sim \frac{i\omega\mu_o IAQ \sin\phi}{2\pi\gamma\rho^3}\left\{2A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{76}$$

$$E_z \sim \frac{i\omega\mu_o IA \cos\phi}{2\pi\rho^2} B(h,\ell_1) , \tag{77}$$

$$H_\rho \sim \frac{IA \sin\phi}{\pi\rho^3} B(h,\ell_1) , \tag{78}$$

$$H_\phi \sim - \frac{IA \cos\phi}{2\pi\rho^3} B(h,\ell_1) , \tag{79}$$

and

$$H_z \sim \frac{3IAQ \sin\phi}{2\pi\gamma\rho^4}\left\{A(h,\ell_1) + \frac{\gamma z}{Q} B(h,\ell_1)\right\} . \tag{80}$$

If $\rho \rightarrow 0$, equations (69) through (74) reduce to

$$E_\rho \sim \frac{i\omega\mu_o IAQ \cos\phi}{2\pi\gamma z^3}\left\{A(h,\ell_1) - \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{81}$$

$$E_\phi \sim - \frac{i\omega\mu_o IAQ \sin\phi}{2\pi\gamma z^3}\left\{A(h,\ell_1) - \frac{\gamma z}{Q} B(h,\ell_1)\right\} , \tag{82}$$

$$E_z \sim \frac{i\omega\mu_o IA\rho \cos\phi}{2\pi z^3} B(h,\ell_1) , \tag{83}$$

$$H_\rho \sim -\frac{IA\sin\phi}{2\pi z^3}B(h,\ell_1) \quad , \tag{84}$$

$$H_\phi \sim -\frac{IA\cos\phi}{2\pi z^3}B(h,\ell_1) \quad , \tag{85}$$

and

$$H_z \sim -\frac{3IA\rho Q\sin\phi}{2\pi\gamma z^5}\left\{4A(h,\ell_1) - \frac{\gamma z}{Q}B(h,\ell_1)\right\} \quad . \tag{86}$$

It should be noted that quasi-near range HMD antenna equations (69) through (86) also could have been derived directly from HED antenna equations (51) through (68) simply by replacing $\ell Q/\gamma$ with A. This is so because a diffused return current exists in the earth that corresponds to the current in the HED antenna. For this reason, a horizontal wire with grounded ends is called a "ground-return antenna." The current in the horizontal wire of length ℓ and the ground return current act as an HMD (vertical loop) antenna of area ℓxW, where $W = Q/\gamma$; that is, the magnitude of the effective area A of this equivalent loop antenna is

$$|A| = \ell|W| = \ell\left|\frac{Q}{\gamma}\right| = \frac{\ell\delta}{\sqrt{2}}\,|\coth\gamma\ell_1| \quad . \tag{87}$$

As a point of interest, a ground-return antenna, either buried or at the surface of the conducting slab, is equivalent to a rectangular, one-turn loop of length ℓ and height $W = Q/\gamma$, as long as the measurement distance R is greater than three skin depths, $R > 3h$ and $R > 3\ell_1$.

CONCLUSIONS

Approximate expressions for the quasi-near fields in air produced by electric and magnetic antennas located within a conducting slab have been derived by applying the quasi-static and quasi-near approximations to the basic Sommerfeld integrals. The results should be valid when the measurement distance is greater than three skin depths and greater than three times the slab depth.

When the slab depth is greater than two skin depths and greater than two times the burial depth of the transmitting antenna, the expressions derived in this report reduce to previously derived quasi-near range single-layered earth expressions.

Although displacement currents in the conducting slab have been ignored in the analysis, they can be included simply by replacing σ with $\sigma + i\omega\varepsilon$ in the field strength equations, providing $|\gamma^2| >> |\gamma_o{}^2|$.

The resulting expressions are particularly applicable to short-range, subsurface-to-air propagation for the shallow sea case. Appropriate equations for the reciprocal problem (i.e., air-to-subsurface propagation) may be determined easily from the expressions derived in this report by utilizing the reciprocity theorem.

REFERENCES

1. P. R. Bannister, "The Quasi-Near Fields of Dipole Antennas," IEEE Transactions on Antennas and Propagation, vol. AP-15, no. 5, 1967, pp. 618-626.

2. A. Baños, Jr., Dipole Radiation in the Presence of a Conducting Half-Space, Pergamon Press, Oxford, 1966.

3. M. B. Kraichman, Handbook of Electromagnetic Propagation in Conducting Media, U. S. Government Printing Office, Washington, DC, 1970 (second edition, 1976).

4. P. R. Bannister and R. L. Dube, Modified Image Theory Quasi-Static Range Subsurface-to-Subsurface Propagation Equations for the Conducting Slab Case, NUSC Technical Report 5807, Naval Underwater Systems Center, New London, CT, (to be published).

5. A. Erdélyi, ed., Tables of Integral Transforms, vol. 1, McGraw-Hill Book Co., Inc., NY, 1954, p. 14.

6. J. R. Wait, Electromagnetic Waves in Stratified Media, Pergamon Press, NY, 1970, chapter 2.

7. P. R. Bannister, The Quasi-Near Fields of a Long Horizontal Line Source, NUSL Report 918, Naval Underwater Systems Center, New London, CT, 17 May 1968.

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