

AD-A056 920

SCRIPPS INSTITUTION OF OCEANOGRAPHY LA JOLLA CALIF
CLIMATE FORECAST VERIFICATION VIA MULTINOMIAL STOCHASTERS, (U)
DEC 77 R W PREISENDORFER

F/G 4/2

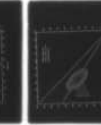
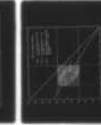
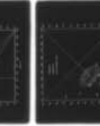
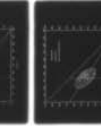
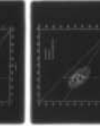
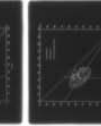
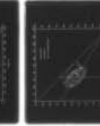
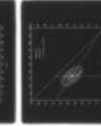
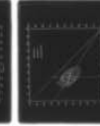
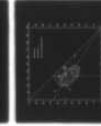
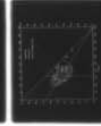
UNCLASSIFIED

SIO-REF-77-33

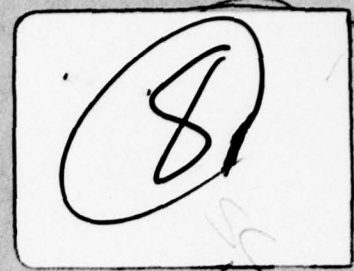
N00014-75-C-0152

NL

1 of 2
AD
A056 920

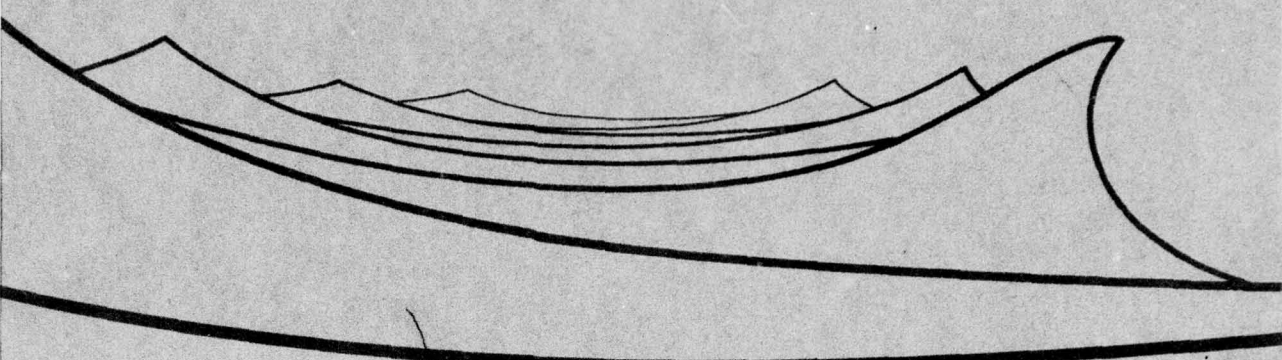


28 07 18 032



LEVEL

AD A 056920

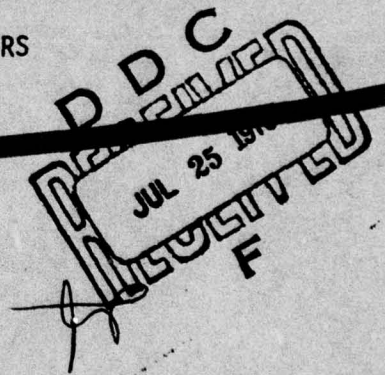


SIO REFERENCE SERIES

77-33

CLIMATE FORECAST VERIFICATION VIA MULTINOMIAL STOCHASTERS

AD No. _____
DDC FILE COPY



RUDOLPH W. PREISENDORFER

This document has been approved
for public release and sale; its
distribution is unlimited.

University of California Scripps Institution of Oceanography

December 1977

78 07 18 032

UNIVERSITY OF CALIFORNIA, SAN DIEGO
Scripps Institution of Oceanography
La Jolla, California

6 CLIMATE FORECAST VERIFICATION
VIA
MULTINOMIAL STOCHASTERS

15 Contract No. 014-75-G-0152, ✓ NSF-ATM75-18699

10 Rudolph W. Preisendorfer

This work was done while on leave to the Climate Research Group from the Pacific Marine Environmental Laboratory of The Environmental Research Laboratories, National Oceanic and Atmospheric Administration.

Sponsored by:

National Science Foundation contract ATM75-18699

14 SIO-Ref-77-33

11 December 1977

12 136p.

319 100

LB

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CLIMATE FORECAST VERIFICATION VIA MULTINOMIAL STOCHASTERS		5. TYPE OF REPORT & PERIOD COVERED
7. AUTHOR(s) Rudolph W. Preisendorfer		6. PERFORMING ORG. REPORT NUMBER 77-33
9. PERFORMING ORGANIZATION NAME AND ADDRESS Scripps Institution of Oceanography La Jolla, CA 92093		8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0152
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, VA 22217		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE July 1978
		13. NUMBER OF PAGES 140 pages
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) <div style="border: 1px solid black; padding: 5px; display: inline-block;">This document has been approved for public release and sale; its distribution is unlimited.</div>		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) Approved for public release: distribution unlimited		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The problem of attaching some quantitative measure of skill to forecasts of temperature, precipitation and other physical fields over extensive regions of the atmosphere and hydrosphere is examined. It is suggested that to each forecaster we may assign a competitive stochaster, a device or person that performs the same forecast over the same regions of space and time as the forecaster, but using a specially designed random procedure. This notion is illustrated for the case of a multinomial stochaster, by means of numerical...		

ABSTRACT

↙
The problem of attaching some quantitative measure of skill to forecasts of temperature, precipitation and other physical fields over extensive regions of the atmosphere and hydrosphere is examined. It is suggested that to each forecaster we may assign a competitive stochaster, a device or person that performs the same forecast over the same regions of space and time as the forecaster, but using a specially designed random procedure. This notion is illustrated for the case of a multinomial stochaster, by means of numerical studies of actual temperature and precipitation forecasts over the U.S. mainland for various seasons over the past three years. Specially designed tables and charts show how quantitative judgments of forecaster skills can be made in a variety of ways.

↖

ACCESSION NO.	
NTIS	W. H. H. ✓
DDC	B. H. Search 13
UNANNOUNCED	<input type="checkbox"/>
JUST. KATON	
BY	
DISTRIBUTION/AVAILABILITY NOTES	
A	

- i -

78 07 18 03 2

TABLE OF CONTENTS

0. Introduction. 1

1. Forecaster vs the Mean Stochaster 4

2. Forecaster vs the Binomial Stochaster 6

3. Forecaster vs the Trinomial Stochaster (unsigned errors). 8

4. Forecaster vs the Trinomial Stochaster (signed errors). 12

5. Forecaster vs the Multinomial Stochaster (the concept χ^2) 15

6. The Problem of Ranking Forecasting Skill in the Context of Trinomial Stochasters 18

 A. χ^2 -Ellipses and Their Associated Probabilities. 18

 B. Various Performance Regions on the Trinomial Domain 19

 C. Examples of Performances by Forecasters 21

 D. Examples of Forecaster vs Stochaster. 23

 E. Ranking Performances by Moments and χ^2 27

7. Construction of Tables A-E and EXP. 31

 A. Table A 32

 B. Table B 34

 C. Table C 34

 D. Table D 35

 E. Table E 36

 F. Table EXP 37

8. Construction of the Skill Charts. 39

9. Acknowledgments 42

10. References. 43

 Tables and Figures. 44

CLIMATE FORECAST VERIFICATION VIA MULTINOMIAL STOCHASTERS

Rudolph W. Preisendorfer

0. Introduction

In this work we develop a general approach to the problem of forecast verification in physical climatology. This problem has already been the subject of numerous studies (cf. e.g., Brier and Allen, 1951; Namias, 1953; Panofsky and Brier, 1958; and their references). We are encouraged to make another essay in this direction because these studies have only presented partial solutions of the problem by omitting essential stochastic elements; or if the latter were included, then the appropriate common geometric setting of the forecast and its realization (the predictand) was not developed. Moreover, some early studies of the problem have confused its formulation by introducing elements of subjectivity and qualitative reasoning into what is a matter requiring objectivity and quantitative reasoning.

In what follows we will take the point of view that both the forecast and its realization must be treated within the same quantitative framework: the *forecast* will be viewed as *the numerical specification of values that a geophysical field* (e.g., temperature, pressure, precipitation or some combination thereof) *will take, at some specified times in the future over a specified set of spatially distributed points.* The predictand field will be couched in precisely the same framework and so will be wholly commensurate with the forecast field. For example, if the predictand field is atmospheric pressure at n points of the U.S. mainland and the values of pressure are classified into r categories at each point, then so too will the forecast field be presented in r -tile form at each of these n points.

Moreover, in what follows we will solve the problem of finding a suitably general reference forecaster, i.e., a verifier against which the skill of all forecasters

can be gauged. We will do this by choosing the *stochaster* as a worthy competitor of the forecaster. That is, we choose a stochastic forecaster (a person or device) which is assigned precisely the forecast problem faced by its competitor, and proceeds in a purely random way to solve it: both forecaster and stochaster, each in his own characteristic way, must predict the future state of the same geophysical field over the same set of spatial points and same set of future times. *There is accordingly, in principle, a stochaster assignable to each forecaster whose efforts are to be verified.*

For us, then, a *verification* of a forecast consists of two parts, namely the application of: (i) *a quantitative measure of the degree of match between a given predictand and the forecaster's prediction; and (ii) a probability measure of attaining the same degree of match between the given predictand and the associated stochaster's prediction.* In every practical instance these two parts of the verification are required to be readily converted into tabular or graphical form. In particular the forecaster's skill may be depicted as a point (in a suitably dimensioned euclidean point- or subset-space) to which has been assigned a level of statistical significance via the performance of the competing stochaster at that same point. *Thus when two different forecaster's skills are to be compared, this must be done on the same geometric-probabilistic background, namely that of their common stochastic competitor.* In this way we can also solve the problem of comparing the relative merits of a wide range of possible different forecasters* all attempting to predict the same geophysical field's configuration over the same space points and same set of future times. This, obviously, requires the appropriate cooperative preliminary arrangements by two or more forecasters to insure that their recorded efforts will fall into the common geometric-probabilistic verification framework.

* These can range from the simplest, such as the persisters and advectors, to the most advanced of current prediction strategies.

We will use our general approach to develop several of these frameworks so as to attain a hierarchy of ever-increasing stringency, appropriate parts of which may be adopted by each forecaster, who, as his mastery increases, can then apply ever more rigorous tests of his forecast skills. Moreover, it will be possible for him to compare his skills with those of other forecasters who attempt the same forecasts in a common framework, such as any of those given in the hierarchy below.

The general principles utilized in the present approach are of sufficient breadth so as to allow their extension to virtually every problem setting the climate forecaster may encounter. However, in the interests of brevity we will in this study explore only a specific class of stochasters, namely the class of multinomial stochasters. This class is already so broad that it will cover many, if not all, of the cases encountered in usual practice. Yet we should mention that there exist settings which require classes of stochasters that are not multinomial per se. For example, global skill scores for analog forecasts require stochasters that are not multinomial, but rather an immediate generalization of these, i.e., the multivectorial stochasters. It will be noted, however, that the verification principles (i), (ii) enunciated above are still applicable as guides to attain the appropriate forms of the match and significance quantities, now in the analog setting.

Having attained a general objective and quantitative overview of forecast verification, one can now go on to apply it in various ways to the practical aspects of economics and administrative problems contingent on sound forecasts and their verifications. These problems, of course, are beyond the immediate scope of this study and will be reserved for a future time. Yet we wish to make one important observation in this regard: if we possess an objective, quantitative verification system of adjustable stringency, such as that developed below, then it will always be possible to extract from it auxiliary quantitative, or even qualitative measures of forecast verification applicable to the specific needs of the less quantitative

fields of economics and administration. In other words, we can more easily, in such matters as these, descend the ladder from objectivity to subjectivity and from quantity to quality rather than ascend it, and we now possess the basis for such descents.

1. Forecaster vs the Mean Stochaster

The simplest form of competition between forecaster and stochaster uses the expected value of the stochaster's performance as a point of reference. This is exemplified in the popular form of the skill S_n given by the Heidke formula (Brier and Allen, 1951):

$$S_n = \frac{u - \bar{u}}{n - \bar{u}} \quad (1.1)$$

where u is the number of 0-class errors (number of correct predictions) made by the forecaster in a set of n forecasts, and \bar{u} is the expected number of 0-class errors made by the stochaster.

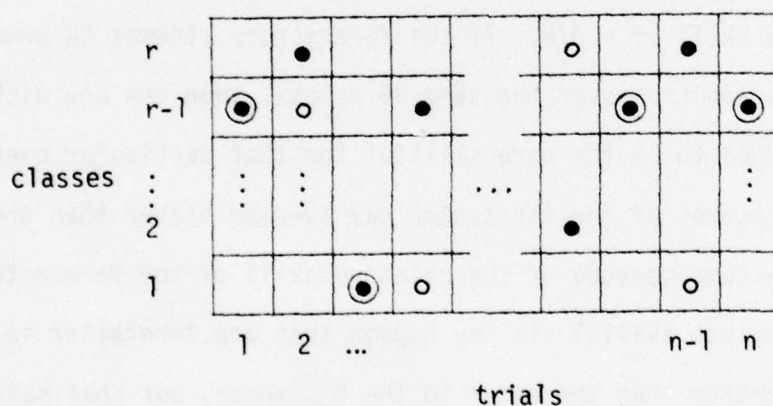
In viewing (1.1) within the framework of our verification principles (i) and (ii) defined in §0, the quantitative degree of match between predictand and prediction is u , but the probability measure associated with the stochaster is missing. The stochaster's mean \bar{u} is, of course, a statistical point of reference that serves in (1.1) to tell whether the forecaster has positive or negative skill according as $u > \bar{u}$ or $u < \bar{u}$. But what is missing is some number (a confidence level, e.g.) that tells how much better or worse, respectively, the forecaster's efforts are than blind chance. Thus (1.1) serves only to tell whether one is doing better or worse than chance, but not by how much, in a probabilistic sense.

Consider, e.g., the lists of skill scores in Table 9. These may be associated

with the following hypothetical situation: A temperature field (say) is to be predicted over the U.S. mainland at 99 selected points (cf Fig. 1). Thus $n=99$ in (1.1). The predictions are to be made by stating that, at each point, the temperature will be either above normal (A), normal (N), or below normal (B), where 'normal' is some previously established climatological mean. For this purpose the range of temperatures occurring in the record at each station is divided or partitioned into three equal classes (or intervals): one that contains the normal temperatures, and an interval each that contains the above normal and below normal temperatures. In this way the data have been 'terciled' at each point. Subsequently, the predictions are compared with the actual temperatures realized at each of the 99 points. Let u be the number of correct predictions (e.g., if A is predicted and A occurs, the prediction is correct). On the average, by chance, one would expect to guess $1/3$ of the temperatures at the points, so that $99 \times (1/3) = 33 = \bar{u}$ in (1.1). If, e.g., $u=41$, then the skill S_{99} would be $+0.121$. If $u=33$, then S_{99} would be 0, while 28 correct would have a skill of -0.076 . If two forecasters attempt to predict the same temperature distribution over the same 99 points, then the one with the higher S_{99} value may be judged to be the more skillful for that particular event. In the long run, the skill scores of one forecaster may average higher than the other, and hence S_{99} would give some measure of the relative skill of the forecasters. But what about their absolute skills? It may happen that one forecaster is in the long run uniformly better than the other in the S_{99} sense, but that neither is better at forecasting events than a thrown die attempting to do the same job! In what follows we shall explore the ideas inherent in this last observation, with the goal in mind of attaining one form of an absolute measure of skill against which forecasters' efforts may be pitted.

2. Forecaster vs the Binomial Stochaster

One way to improve on the skill score formula in (1.1) is to attach to S_n the missing statistical significance of the score. This is done by assigning to the forecaster's problem a competitive stochaster. For example, if the physical field has n points at which it is to be predicted, and the predictions consist in specifying one of r possible values at each point, then the associated stochaster takes the following form (in the preceding example, $n=99$, $r=3$): at each point the stochaster chooses randomly one of the r possible values. Hence the probability of choosing any one of the r values is $1/r$. At the next point he starts again and independently of his previous decision, the stochaster chooses randomly from the r possibilities at that point. He continues this way through all n points. Now imagine that the predictand is depicted as n appropriately distributed dots in the following abstract diagram of the prediction problem:



The open circles are forecasts by the stochaster. Sometimes he has a hit (circled dot) and sometimes not. Since his trials of choice are independent of each other, the probability of u correct predictions is $(1/r)^u$. The remaining $n-u$ predictions are incorrect and have probability $(1-\frac{1}{r})^{n-u}$ of occurring. The probability of this particular set of u correct and $n-u$ incorrect predictions is $(\frac{1}{r})^u (1-\frac{1}{r})^{n-u}$. The total probability $P_n(u)$ of u correct and $n-u$ incorrect predictions, regardless of

which u dots are circled and which $n-u$ are not, is given by

$$P_n(u) = \frac{n!}{u!(n-u)!} \left(\frac{1}{r}\right)^u \left(1-\frac{1}{r}\right)^{n-u} \quad (2.1)$$

where $n!/u!(n-u)!$ accounts for the number of distinct ways the stochaster can achieve u correct predictions in the set of n dots. Eq (2.1) defines the performance of the *binomial stochaster*: He can have only two outcomes: correct prediction, or wrong prediction.

This probability function supplies the missing information needed in the use of (1.1) to gauge how much better are the forecaster's efforts than the stochaster's. For example, Table 10 lists* values of $P_{99}(u)$ and its cumulative probability function

$$Q_{99}(u) = \sum_{j=0}^u P_{99}(j) \quad (2.2)$$

for the case $n=99$ and $r=3$. By (1.1) we can find the values of skill S , now associated with u and \bar{u} for the case $n=99$. Thus comparing Tables 9, 10, we see that skill scores of (say) $+1.06$ or greater are statistically significant at the 95% level. The column '1' in Table 10 corresponds to the u column in Table 9. Another index of skill in Table 9 is the critical ratio (where σ is the standard deviation):

$$C_{99}(u) = \frac{u-\bar{u}}{\sigma}$$

which is closely related to the approximating gaussian distribution to (2.1) for large n . The skill number S_n or the critical ratio are evidently but two of an infinite number of equivalent apparent-skill indicators. Moreover this skill S as reckoned by (1.1) changes with \bar{u} and n , so that $+1.06$ need no longer be associated with statistical significance at the 95% level.

The main observation to make here is that *skill numbers like the critical ratio $C_{99}(u)$ or like $S_{99}=S(u,\bar{u},99)$ are by themselves not the true indicators of forecasting skill*. The true indicators (relative to the competing stochaster) are given

* See Preface to Tables 10-15, just before them.

via the cumulative probabilities $Q_{99}(u)$. Thus, associated with $u=40$ is $Q_{99}(u)=.9433$, (of Table 10) which says that 94.33% of the stochaster's predictions are below 41 correct. Or putting it another way, for every 100 tries by the stochaster to attain 41 or more correct predictions at 99 points, only $100 - 94.33 = 5.67$ times (on the average) will he be able to do so. Hence if a forecaster *consistently* obtains $u=41$ or more as a score in the present experimental setting, he is doing well relative to the stochaster, i.e., blind chance.

There is an important point illustrated here which is perhaps too implicitly buried in part (ii) of the verification principle of §0 and which we now draw out in detail: in practice the stochaster works very hard at establishing his level of performance; experiment after experiment (under fixed conditions) goes by as he gradually establishes empirically the $P_n(u)$ distribution which we so glibly assembled, by logical argument, in (2.1). In an identical practical sense, *a forecaster's true skill emerges only after a sufficient number of experiments have determined (under fixed conditions) his own $P_n(u)$ distribution relative to that of the stochaster.* If the forecaster is consistently skillful, his 'scatter diagram' of predictions, when superimposed on that of the stochaster, will show some distinctive and favorable form of departure from the latter. This will be illustrated in some discussions below.

3. Forecaster vs the Trinomial Stochaster (unsigned errors)

The next step up the ladder of ever more potentially stringent verification tests brings us to the trinomial stochaster. Returning to the diagram in §2 we now look not only at the correct number of predictions by the forecaster and stochaster, but also the number of 1-class, 2-class, ..., (r-1)-class errors they may commit. A *j-class error*, $0 \leq j \leq r-1$, is committed if the prediction circle and predictand dot are in classes whose indexes differ by j . For example a 0-class error ($j=0$) is a

correct prediction, a 1-class error ($j=1$) is a miss by one class. Clearly, for an r -tile classification of the predictand values, there can be up to $(r-1)$ -class errors.

A *trinomial stochaster* is a stochaster whose scores are registered in three categories, namely as 0-class errors, 1-class errors, and $\bar{2}$ -class errors. The latter are all errors of class 2,3,..., up to $r-1$, lumped together. We thus see that the trinomial stochaster is the next step higher than the binomial stochaster; the latter's scores are registered as 0-class errors and $\bar{1}$ -class errors, where the latter are all errors of class 1, 2, ..., $r-1$, lumped together.

We can determine the probability a_j that a stochaster may commit a j -class error, as follows. Clearly $a_0=1/r$. Another way to see this is to reckon a_0 as:

$$\begin{aligned} a_0 &= \sum_{i=1}^r (\text{prob. that stochaster chooses cell } i) \times (\text{prob. that predictand is in cell } i) \\ &= \sum_{i=1}^r \frac{1}{r} \times \frac{1}{r} = \underbrace{\frac{1}{r^2} + \frac{1}{r^2} + \dots + \frac{1}{r^2}}_{r \text{ terms}} = \frac{1}{r} \end{aligned}$$

Continuing in this way:

$$\begin{aligned} a_1 &= \sum_{i=1}^r (\text{prob. that stochaster chooses cell } i) \times (\text{prob. that predictand is in cell } (i-1) \text{ or cell } (i+1)) \\ &= \frac{1}{r} \left(\frac{1}{r}\right) + \underbrace{\frac{1}{r} \left(\frac{1}{r} + \frac{1}{r}\right) + \dots + \frac{1}{r} \left(\frac{1}{r} + \frac{1}{r}\right)}_{(r-2) \text{ terms}} + \frac{1}{r} \left(\frac{1}{r}\right) \\ &= \frac{2(r-1)}{r^2} \end{aligned}$$

Again,

$$\begin{aligned}
 a_2 &= \sum_{i=1}^r (\text{prob. that stochaster chooses cell } i) \times (\text{prob. that predictand is} \\
 &\quad \text{in cell } (i-2) \text{ or cell } (i+2)) \\
 &= \frac{1}{r} \left(\frac{1}{r}\right) + \frac{1}{r} \left(\frac{1}{r}\right) + \underbrace{\frac{1}{r} \left(\frac{1}{r} + \frac{1}{r}\right) + \cdots + \frac{1}{r} \left(\frac{1}{r} + \frac{1}{r}\right)}_{(r-4) \text{ terms}} + \frac{1}{r} \left(\frac{1}{r}\right) + \frac{1}{r} \left(\frac{1}{r}\right) \\
 &= \frac{2(r-2)}{r^2}
 \end{aligned}$$

From these we can guess the general pattern for the probability a_j , namely:

$$a_j = \frac{2(r-j)}{r^2}, \quad 1 \leq j \leq r-1. \quad (3.1)$$

This may be checked, and a formal proof devised, by considering in detail, e.g., the cases for $r = 6, 7$. Another check consists in seeing that the sum of the a_j is unity

$$a_0 + \sum_{j=1}^{r-1} a_j = \frac{1}{r} + \sum_{j=1}^{r-1} \frac{2(r-j)}{r^2} = 1$$

As an example, if $r=3$, so that we tercale the field values at each point, then $a_0=1/3$, $a_1=4/9$, $a_2=2/9$.

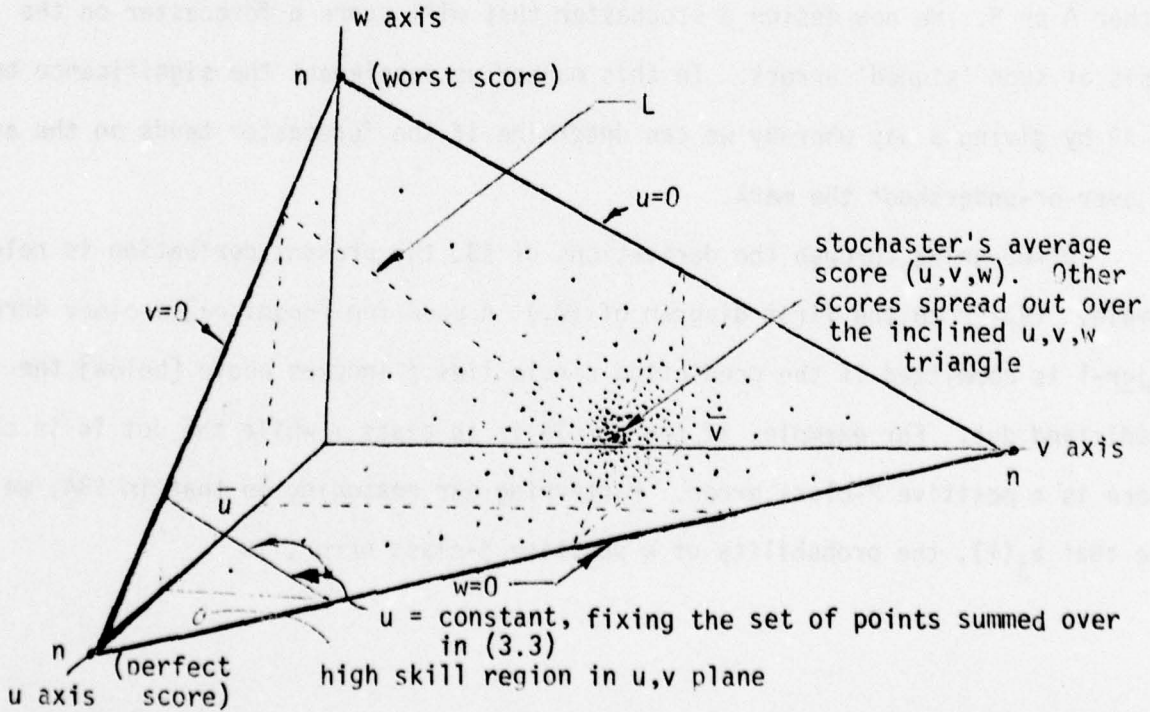
Now suppose that, in the context of the diagram of §2, the stochaster makes n predictions. Let u, v, w be the resulting number, respectively, of 0-class, 1-class, and 2-class errors. The probability of committing each type of error singly at a time is, respectively a_0, a_1 , and $a_2 (=1-(a_0+a_1))$. Hence the joint probability of u, v, w is

$$\begin{aligned}
 p(u,v,w) &= \frac{n!}{u!v!w!} a_0^u a_1^v a_2^w \\
 a_0 + a_1 + a_2 &= 1 \quad \text{r-tile} \\
 &\quad \text{(classification)} \\
 u + v + w &= n
 \end{aligned}
 \tag{3.2}$$

It may be verified that we recover the binomial $P_n(u)$ of (2.1) if we fix u and sum $p(u,v,w)$ over all possible values of v,w . That is, we fix u ; then

$$P_n(u) = \sum_{v=0}^{n-u} p(u,v, n-(u+v))
 \tag{3.3}$$

This process of summation may be viewed in the diagram below which gives an overview of the trinomial stochaster's domain. By fixing u , we fix a plane through u and parallel to the vw plane. The summation in (3.3) is over the lattice points of line L .



The scores of the trinomial stochaster are represented as triples (u,v,w) of integers u,v,w which sum to n . Hence the set of all possible scores lies on the finite triangular portion of the inclined plane through the three n -points on each axis. The probability of each score is given by (3.2). A perfect score is one for which $u=n$ and $v=w=0$, i.e., the point on the u -axis, a distance n from the origin. The worst score is the n -point on the w -axis, and a score of intermediate skill is the n -point of the v -axis. The stochaster, after many experiments of length n , each experiment resulting in a triple (u,v,w) , begins to accumulate a cloud of points on the inclined triangle and centered on the average point $(\bar{u}, \bar{v}, \bar{w}) = (na_0, na_1, na_2)$. For example, if $n=99$, and we choose terciles (so that $r=3$), then $(\bar{u}, \bar{v}, \bar{w}) = (33, 44, 22)$.

4. Forecaster vs the Trinomial Stochaster (signed errors)

Suppose we are not only interested in the number of j -class errors committed by a forecaster, but also whether his errors were above or below the predictand mark. That is, e.g., if the predictand in a tercile classification were 'N', and the forecast error were of class 1, we would like to know specifically if it were either A or B. We now design a stochaster that will score a forecaster on the basis of such 'signed' errors. In this manner we supplement the significance tests of §3 by giving a way whereby we can determine if the forecaster tends on the average to over-or-undershoot the mark.

After going through the derivations of §3, the present derivation is relatively simple. (Refer to the first diagram of §2.) A *positive* [*negative*] j -class error $1 \leq j \leq r-1$ is committed if the prediction circle lies j indexes above [below] the predictand dot. For example, if the circle is in class 3 while the dot is in class 1, there is a positive 2-class error. Patterning our reasoning on that in §3A, we can see that $a_j(+)$, the probability of a positive j -class error, is

$$a_j(+)=\frac{(r-j)}{r^2} \quad 0 \leq j \leq r-1 \quad (4.1)$$

and similarly

$$a_j(-)=\frac{(r-j)}{r^2} \quad 0 \leq j \leq r-1 \quad (4.2)$$

is the probability of a negative j -class error. For example, if $r=3$, then $a_0=1/3$, $a_1(+)=a_1(-)=2/9$, $a_2(+)=a_2(-)=1/9$.

Our test for predictive symmetry in forecasting is supplied by the trinomial stochaster whose elementary probabilities are

$$a(0) = 1/r \quad (4.3)$$

$$a(+)=\sum_{j=1}^{r-1} a_j(+)=\frac{1}{2}\left(1-\frac{1}{r}\right) \quad (4.4)$$

$$a(-)=\sum_{j=1}^{r-1} a_j(-)=\frac{1}{2}\left(1-\frac{1}{r}\right) \quad (4.5)$$

Here $a(0)$ is the probability of a 0-class error. $a(+)$ gives the probability of a positive-class error, while $a(-)$ is the probability of a negative-class error. The joint probability $p(u(0), v(+), v(-))$ of $u(0)$ 0-class errors, $v(+)$ positive-class errors and $v(-)$ negative-class errors incurred in a set of n independent trials by the stochaster is

$$p(u(0), v(+), v(-)) = \frac{n!}{u(0)!v(+)!v(-)!} [a(0)]^{u(0)} [a(+)]^{v(+)} [a(-)]^{v(-)}$$

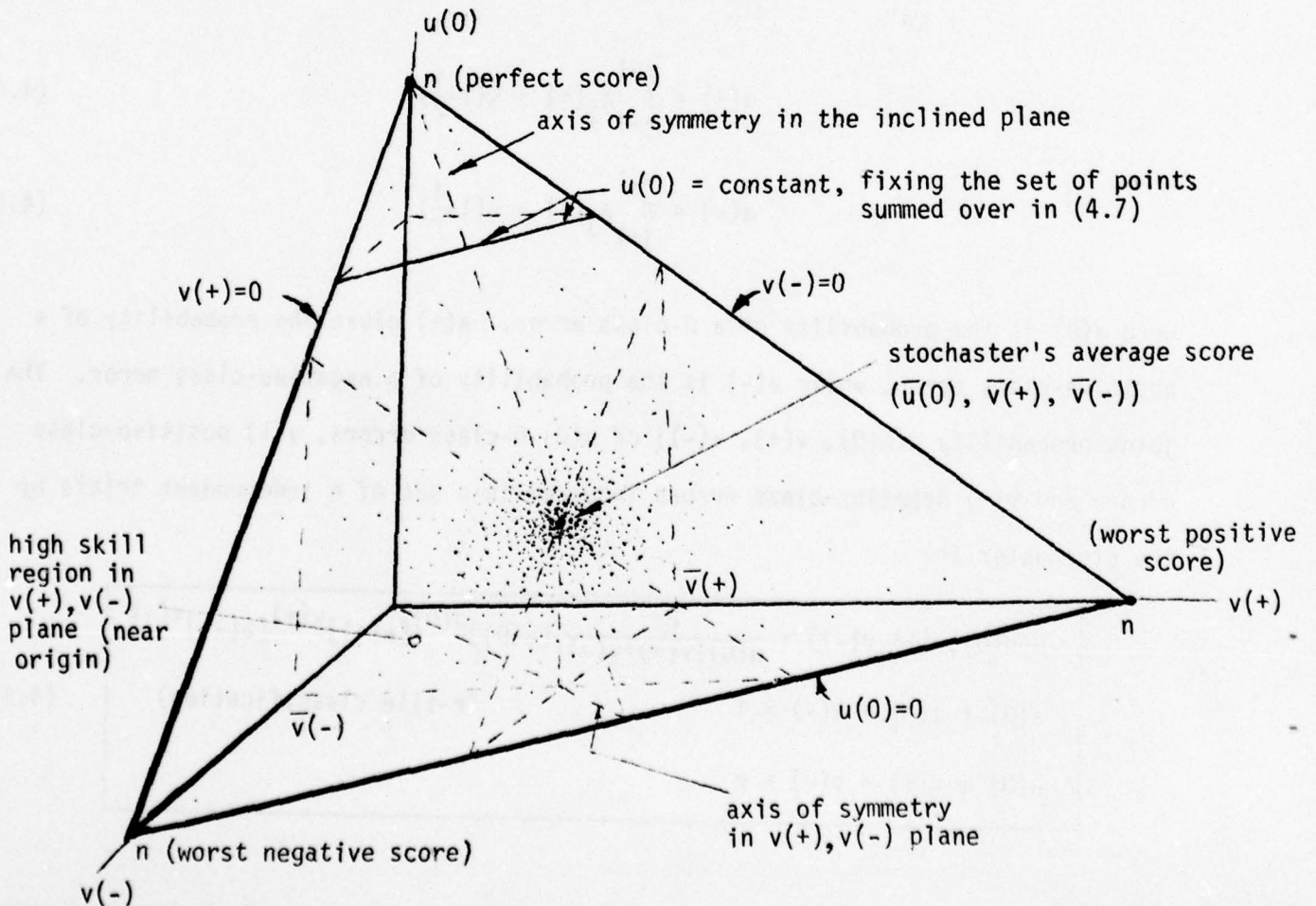
$$a(0) + a(+) + a(-) = 1 \quad (r\text{-tile classification}) \quad (4.6)$$

$$u(0) + v(+) + v(-) = n$$

It may be verified that we recover the form of $p_n(u)$ of (2.1) if we fix $u(0)$ and sum $p(u(0), v(+), v(-))$ over all possible values of $v(+), v(-)$. That is, suppose we fix $u(0)$; then

$$P_n(u(0)) = \sum_{v(+)=0}^{n-u(0)} p(u(0), v(+), n-(u(0)+v(+))) \quad (4.7)$$

The process of summation may be visualized in the diagram below which gives an overview of the trinomial stochaster's domain.



We now have an axis of symmetry in the stochaster's domain, either on the tipped triangular area or in the $v(+)$, $v(-)$ plane, about which the stochaster's scores lie. For example, the expected (average) scores of the stochaster for the case $n=99$, $r=3$ are $(na(0), na(+), na(-)) = (33, 33, 33)$.

5. Forecaster vs the Multinomial Stochaster; (the concept χ^2)

We will now explicitly consider more than three j -class errors in our search for the significance of forecaster skills. Of course, we can no longer visualize the skills in simple geometric diagrams, but we gain instead a useful parameter, the χ^2 value, belonging to the forecaster's performance. We shall turn this parameter back into our preceding analyses to help solve the problem of ordering the skills when given in trinomial form. Thus the following excursion into the domain of the multinomial stochaster, while of possible interest in later studies, is actually our present means of introducing, in a natural way, the χ^2 quantity into the theory of the trinomial stochaster.

We return to the first diagram of §2 and let the stochaster perform an experiment of n independent prediction trials. Let u_0, u_1, \dots, u_{r-1} be respectively the number of 0, 1, \dots , $r-1$ class errors he commits in that experiment. Let a_0, a_1, \dots, a_{r-1} be the elementary probabilities that he commits such errors, respectively. Values for these were derived in §3. Therefore we can in principle compute the joint probability for the r values u_j :

$$p(u_0, u_1, \dots, u_{r-1}) = \frac{n!}{u_0! u_1! \dots u_{r-1}!} a_0^{u_0} a_1^{u_1} \dots a_{r-1}^{u_{r-1}}$$

$$u_0 + u_1 + \dots + u_{r-1} = n$$

(r - tile
classification)

(5.1)

$$a_0 + a_1 + \dots + a_{r-1} = 1$$

By approximating the factorials in this expression, using Sterling's formula, by writing

$$'x_j' \text{ for } \frac{(u_j - na_j)}{(na_j)^{1/2}}, \quad (5.2)$$

and by making some further algebraic reductions, we find that, to good approximation,

$$p(u_0, u_1, \dots, u_{r-1}) = (2\pi n)^{(1-r)/2} (a_0 a_1 \dots a_{r-1})^{-1/2} \exp\{-\frac{1}{2} \sum_{i=0}^{r-1} x_i^2\} \quad (5.3)$$

In this way we condense all the j -class scores u_j into a single number of the form

$$\chi^2 \equiv \sum_{i=0}^{r-1} x_i^2 = \sum_{r=0}^{r-1} \frac{(u_j - na_j)^2}{na_j} \quad (5.4)$$

This quantity, as is well known,* is governed by the χ^2 -distribution (using our r -tile notation):

$$T_{r-1}(\chi^2) d(\chi^2) = \frac{(\chi^2)^{(r-3)/2} e^{-\frac{1}{2}\chi^2}}{2^{(r-1)/2} \Gamma(\frac{r-1}{2})} \cdot d(\chi^2) \quad (5.5)$$

Since the u_j are constrained to add up to n , there are only $r-1$ degrees of freedom associated with (5.5).

For example, let $n=99$, $r=3$ and consider the signed errors of §3. Then $a_0=1/3$, $a_1=4/9$, $a_2=2/9$, and we now have

$$\chi^2 = \frac{(u-33)^2}{33} + \frac{(v-44)^2}{44} + \frac{(w-22)^2}{22} \quad (5.6)$$

and

$$T_2(\chi^2) d(\chi^2) = \frac{1}{2} e^{-\frac{1}{2}\chi^2} d(\chi^2)$$

* See, e.g., Kenney, J. F., 'Mathematics of Statistics' (part two), D. Van Nostrand Co. N.Y. (1947) (7th printing). p167 has a particularly clear derivation of the χ^2 distribution's form from (5.3).

In this case we have two degrees of freedom.

Equation (5.6) gives the probability of occurrence of all those triples (u,v,w) with χ^2 values in the range $(\chi^2 - \frac{1}{2}d(\chi^2), \chi^2 + \frac{1}{2}d(\chi^2))$. Since (as we shall see below) the set of all (u,v,w) having exactly some fixed χ^2 value generates an ellipse in the uv plane of the diagram of §3, (5.6) gives the element of probability that the triples lie in an elliptical annulus defined by $\chi^2 \pm \frac{1}{2}d(\chi^2)$.

The approximation (5.6) must be examined for accuracy in our present work on the trinomial stochaster. This will be done in detail below (§7). But for the moment, we can view (5.6) as a possible tool for ranking the skill of a forecaster. In general, for a specified n, a_0, a_1, a_2 , we can form the quantity

$$\chi^2 = \frac{(u-\bar{u})^2}{\bar{u}} + \frac{(v-\bar{v})^2}{\bar{v}} + \frac{(w-\bar{w})^2}{\bar{w}} \quad (5.7)$$

where

$$\bar{u} = na_0, \bar{v} = na_1, \bar{w} = na_2$$

and compute the probability of the value χ^2 associated with (u,v,w) . One might expect that of two forecasts, the one with the greater χ^2 value is the better, since its u,v,w values would depart on the average more from the mere chance values $\bar{u}, \bar{v}, \bar{w}$ than the other forecast. Unfortunately, this is not generally correct. Mere departure from the chance point $(\bar{u}, \bar{v}, \bar{w})$ in the triangular score plane of §3 is not enough to insure high skill. As we have seen, triples near the point $(n,0,0)$ are to be preferred by an ambitious forecaster. How to rank the skill value of points in the uv plane is an important and to some extent an elusive problem. It will be taken up next.

6. The Problem of Ranking Forecasting Skill in the Context of Trinomial Stochasters

We shall, in the present context of trinomial stochasters, explore several ways, all more or less objective, in which we can make a judgment that a forecast is good or bad.

A. χ^2 Ellipses and their associated probabilities

As we saw in §5, the χ^2 value associated with a performance triple (u,v,w) resulting from a forecast can in turn have an ellipse and a probability associated with it. Without going through all the mathematics (given in §7, below) we can understand the connection between the ellipse and its probability, as follows.

Let T be the set of all possible triples (u,v,w) , $0 \leq u,v,w \leq n$, enclosed by the scoring pyramid of §3. Since u,v,w can take on only integral values between 0 and n inclusive, there are, in all, exactly $(n+1)(n+2)(n+3)/6$ such triples in T . (For example, in the case of $n=99$, the number of triples is 171,700.) Fortunately, we need not work with all these triples in T , by virtue of the sum constraint $(u+v+w=n)$ on them. We may thus restrict our attention to a subset of them, say the u,v plane. This has only $(n+1)(n+2)/2$ points of interest (for example, in the case of $n=99$, the number of (u,v) pairs is 5050). Each of these points may be envisioned (cf the diagram in §3) as the projection of the triple (u,v,w) , in the triangular plane, down onto its correspondent (u,v) in the uv plane. Some observations follow.

1) To each projected point (u,v) in the uv plane we may uniquely assign the probability of its associated point (u,v,w) , as given by (3.2). For example by Table A (with $a_0 = 1/3$, $a_1 = 4/9$, $a_2 = 2/9$, $n=99$) the point $(33, 33, 33)$ has the probability .00017 and we assign this probability to $(33, 33)$. The point $(33, 44, 22)$ (the 'average' point) has probability .00880, and we assign this to $(33, 44)$. Thus every point (u,v) in the uv plane has a probability, namely that of the unique point $(u,v, n-(u+v))$ above it on the triangular plane.

2) To every point (u,v) in the uv plane there is assignable via (5.7) a

unique χ^2 value, namely that of the unique point $(u,v,n - (u+v))$ above it (For example, for $n=99$, and $r=3$, with $a_0=1/3$, $a_1=4/9$, $a_2=2/9$, the point $(33,33)$ has $\chi^2 = 8.2500$, and the point $(33,44)$ (the 'average' point) has $\chi^2 = 0$). The set of points (u,v) in the uv plane having a χ^2 value not exceeding χ_0^2 form an approximately elliptical region about (\bar{u},\bar{v}) , the average point, as center and with a well defined total probability. (For example, with $n=99$, $r=3$, $a_0=1/3$, $a_1=4/9$, $a_2=2/9$, if we set $\chi_0^2 = 1.4621$, it turns out that there are about 79 points within the ellipse associated with χ_0^2 (see Fig. 24) and moreover the sum of the probabilities of these 79 points, each probability reckoned via (3.2), comes to .50206.) Thus to each value of χ^2 we have assignable a probability, namely the sum of all probabilities of the points caught within the elliptical region defined by χ^2 .

3) Examples of the χ^2 -ellipses may be seen in Figs 26, 27, 28, 29. In particular, in Fig 26 we show the six ellipses associated with probabilities .50, .80, .90, .95, .98, .99 for the case of $n=99$, $r=3$, and $a_0=1/3$, $a_1=4/9$, $a_2=2/9$. Thus, the outermost ellipse contains 99% of all the probability mass generated by the stochaster: that is, if the stochaster makes a large number, say 100 experiments at forecasting tercile values at 99 points with basic probabilities for 0, 1-, and 2-class errors given by $1/3$, $4/9$, $2/9$, respectively, then on the average, 99 of his performance pairs (u,v) will fall within the ellipse. The ellipses in Fig 27 may be described in the same way, but now for the case $n=99$, $r=5$ (i.e., quintiles) for which $a_0=1/5$, $a_1=8/25$, $a_2=12/25$.

B. Various performance regions in the trinomial domain

We now may consider the problem of ranking skill scores, or of grouping them into regions of high or low skill. To fix ideas, consider Fig 2 which depicts the trinomial domain for the case of unsigned tercile errors (§3) in which $n=99$, $a_0=1/3$, $a_1=4/9$, $a_2=2/9$. u is measured along the horizontal axis, v along the vertical axis. The average point is $(\bar{u},\bar{v},\bar{w}) = (33,44,22)$. Point 0 is the projection $(33,44)$ of

this point on the uv plane. The line $d-d$ therefore separates the total triangular region into two parts: those points (u,v) such that $u > 33$ (have positive skill S_{gg} ; cf (1.1)) and those points such that $u < 33$ (have negative skill S_{gg}).

1) Suppose in Fig 2 we consider the region bounded by $d-d$, the heavy portion of the 95% ellipse, the u axis, and the diagonal line $w=0$. This is a roughly triangular region with a portion of an elliptical region removed. Any point (u,v) in this region has an associated χ^2 for which its probability is not less than .95. Hence we may at first believe that points in such a region are statistically significant. Of course, this is a matter of definition. However, we may not wish to consider points on or near $d-d$ in this region as indicative of great skill in forecasting. For while such points may occur very infrequently (about 2½% of the time) a point such as (33,20) with only 33 correct predictions and 20 1-class errors (and hence 46 2-class errors) strikes one as indicative of rather mediocre skill. Nevertheless the region so defined is a candidate for high skill, and we can propose it for further study.

2) Consider next the triangular region in Fig 2 bounded by the line $c-c$, the diagonal line ($w=0$) and the u -axis. Call this region 'A'. Recalling our discussion in §2, we know that a score (u,v) with $u > 41$ occurs only 5% of the time during a stochaster's attempts to predict. That is, the set of all points (u,v) in the domain with $u > 41$ has associated with it a total probability mass of .05. Notice, however, that there are points (u,v) along the dashed portion of $c-c$ that fall rather deep within the 95% χ^2 -ellipse. These particular points are clearly not significant on the 95% level relative to the partitioning of the plane by χ^2 -ellipses.* This shows that using only u values to judge a skill (as in §2) may lead us to misjudge that skill. If we choose that subset of the total triangular

* Observe that there are many subsets of the total trinomial domain whose points have a total probability mass of nearly .95. The complement of A and the 95% ellipse under discussion are but two such subsets. One determines the confidence level of a subset by simply totaling the probability mass within it using Table A.

domain consisting of the smaller triangular region A bounded by c-c, less the segment of the 95% χ^2 -ellipse, we would then have a set of points (u,v) associated with relatively high skill. The statistical significance of the subset would be slightly larger than 95%. (The exact increment of the value, which is near 1%, is not of interest here. It would be found by adding up the probabilities of the points in the elliptical segment removed from the c-c triangular region A. This can be done with the help of Table A.) Thus we have another well-defined candidate for a high-skill region, this one a bit more stringent than in 1) above.

3) The horizontal dashed line b is formed by cutting the pyramidal solid with a plane parallel to the uw plane at a value of v equal to 52, which is the 95% level for a binomial distribution* with probability of success equal to 4/9, i.e., a_1 of §3. The inclined dashed line b is formed by cutting the pyramidal solid with a plane parallel to the uv plane at a value of w equal to 29, which is the 95% level for a binomial distribution* with probability of success equal to 2/9, i.e., a_2 of §3. Together with the heavy portion of the 95% ellipse, these lines (even though they are generous in their restrictions) define a region of high skill somewhat more stringent than the preceding region. Obviously, a still more stringent region is that defined by a-a, since it contains still less probability mass within its region. Similar regions are defined in Fig 3 for the case of $r=5$.

C. Examples of performance by forecasters

1) Sprinkled throughout the domain of Fig 2 are fourteen points representing the scores of a forecaster denoted by 'A' in Table 1. These scores are the results of actual forecasts of temperature over the 99 points of the U.S. mainland depicted in Fig 1. For example, according to Table 1 the predictions of Winter '74 yielded $u=42$ correct predictions, $v=37$ 1-class errors and $w=20$ 2-class errors, and the associated (u,v) pair is denoted by '1' in Fig 2. Observe that point 1 is not

* See binomial probability Tables 11, 12.

significant in any of the three senses 1) - 3) defined above. Neither is point 7, associated with the summer of '75, significant. There are five points 2, 3, 9, 10, 14 that have negative skill and which, moreover, are not significant relative to the 95% contour. The set of six points 4, 5, 6, 8, 11, 12, however, are outside the 95% contour and are situated in high skill regions. In particular the three points 8, 6, 12, especially the latter two, are outstanding forecasts. Point 12, the second most outstanding of them all, was the temperature forecast of the infamous winter of '77. These points are outstanding because they have relatively high u values (number of correct predictions). Moreover their 2-class errors are very small by virtue of being situated near the $w=0$ line. In general we may say that the higher the u value and the smaller the w value the better the skill. But there are exceptions, and we shall explore this situation at the appropriate time later in this study (cf §6E).

2) Another way of looking at Forecaster A's performance is shown in Fig 4. Here the same fourteen points are displayed in the signed-error domain, as defined in §4. $v(-)$, $v(+)$ are along the horizontal and vertical axes, respectively. The regions of various stringency are defined as explained in the diagram. Thus the area of least stringency is defined by the axes and the line $d-d$. Here we are asking the Forecaster to merely perform better than chance in obtaining the number of correct scores u which are measured along the axis normal to the diagram. Recalling the perspective view of the scoring plane in §4, it is clear that the closer in toward the origin a forecaster's score lies, the better is his effort. Notice that Forecaster A's two outstanding performances (points 6, 12) stand closest to the origin. The scattering of the fourteen points is generally well balanced: six are in the overshoot region (above axis x), eight are below, indicating that Forecaster A's performance is generally not to over or under estimate in his forecasts. In this frame, eight points are considered significant and are circled. They all lie in the region bounded by the axes, $c-c$, minus the area in

the 95% ellipse. The region of highest stringency, that bounded by a-a, the 95% ellipse, and the axes, has only three points, 5, 6, 12. This situation should be compared with that of the three points 8, 6, 12 caught in the a-a region of Fig 2. This shows that measures of forecast significance, even in the present relatively objective setting, are near, but not quite, absolute. However, a way of ranking every pair of forecasts will be given below, and which can help remove this ambiguity (cf §6E).

3) The diagrams in Figs 3, 5 are exactly analogous to those in Figs 2, 4, but now for the quintiled-data case. These diagrams have been included here to point up the remarks made earlier that the trinomial scheme of gauging the skill of a forecaster can be made arbitrarily stringent. For example, according to Table 13, for the case of a binomial stochaster with probability of success $1/5$, the 95% level of performance is 26 correct predictions out of 99. But suppose in such a quintiled setting we still demand 41 correct (as in the terceled setting of Table 10) to be the mark of a good forecaster. It is seen that $u=41$ in the context of Table 13 is virtually an impossibly high performance for a *stochaster*. However for an expert competing *forecaster*, $u=41$ in a quintile setting may not at all be impossibly high; it simply would set a relatively higher demand on that forecaster whose method has reached a state of development in which the terceled setting is not sufficiently stringent, not much of a challenge. This stringency manifests itself in Fig 3 by the closer proximity to the origin of the nested set of elliptical contours. Now, to get to the high u , low w places, the forecaster must exert himself considerably more to rise out of the bull's-eye of mediocrity.

D. Examples of forecaster vs stochaster

We shall now compare the relative performances of forecasters and stochasters in actual experiments at prediction of temperatures and precipitations over the U.S. mainland.

1) Table 1 gives performances of Forecaster A in terms of unsigned scores (u,v,w) and also in terms of signed scores $(u(0), v(+), v(-))$, as defined in §§3,4. For example, the prediction scores of Forecaster A for the winter of '74 are $(42, 37, 20) = (u,v,w)$ for the unsigned errors and $(42, 13, 44) = (u(0), v(+), v(-))$ for the signed errors. The pertinent connections between these errors are given below the table. Thus $v(+)$ is the sum of the positive 1- and 2-class errors, while v is the sum of the 1-class errors of positive and negative type. In a similar way we can interpret the remaining Tables 2, 3, 4 for forecasters B, C, D, respectively. All four forecasters were engaged in predicting the temperatures at the 99 points (of Fig 1) over the U.S. mainland for the fourteen seasons listed. These are summarized in Figs 7, 8, 9, 10. The results of their performances in predicting precipitation are summarized graphically in Figs 11, 12, 13, 14, and are tabulated in Tables 5, 6, 7, 8.

2) These latter four figures (11, 12, 13, 14) are worth studying in detail. A first impression is that Forecasters A and B are considerably superior to Forecasters C and D in forecasting precipitation. Of the latter two it appears that D has more points of positive skill than C. Forecaster C has no points in any of the areas of high skill defined in §§ B, C above. Similarly for D, who just barely has a significant point (no. 3) to show for his efforts. Forecasters A and B, however, each have seven significant points: 2, 3, 4, 5, 8, 9, 10 for A and 1, 3, 4, 5, 10, 11, 12 for B. It is remarkable that four of the points they share, namely 3, 4, 5 and 10 lie in just about the same places in each diagram. Also note that each has a common point, namely 7, nearly dead center on the bull's-eye, meaning, of course, a shared poor prediction (the winter of '76). This leads us to conjecture that Forecasters A, B and Forecasters C, D belong to two different classes of ability, and each one in each group is comparable in skill to the other, namely A and B are of comparable skill while C, D are of comparable skill.

3) Turning to Figures 7, 8, 9, 10, we compare the skills of the same four forecasters, now in their attempts to predict temperatures over the 99 U.S. mainland points and over the fourteen seasons listed in Tables 1, 2, 3, 4. Once again Forecasters A, B show definite superiority over C, D. Indeed, Forecasters A, B each have six significant points in high skill areas: Forecaster A has points 4, 5, 6, 8, 11, 12 while B has points 3, 7, 8, 10, 11, 12. Forecaster A showed extraordinary skill at point 6 (spring '75), while B showed such skill at point 3 (summer '74). Forecaster C has points 5, 10, 13 as significant above the 95% level using the χ^2 criterion. However, observe that these are all of negative skill, showing that a high χ^2 value (such as may be encountered in a contingency table of classified observations and predictions) does not necessarily mean high skill. Forecaster D has point 10 above the 95% level, but its u value and v value are undistinguished.

4) In Figures 15, 16, 17, 18, the same temperature skills in Tables, 1, 2, 3, 4 for the four forecasters are plotted, now using signed errors (§4). Thus the information in Fig 7 for A is viewed in a new way in Fig 15. The first impression is that Forecaster A tends to have balanced forecasts on the whole: the number of over estimates above the symmetry axis is six while those below are eight. Forecaster B has the same split but in the opposite sense. To help judge the quality of skill of forecasters B, C, D, the reader may wish to lightly sketch in various regions of high skill, as defined in Fig 4, on the appropriate diagrams. The temperature skills viewed via signed errors in Figs 17, 18 are completely undistinguished. Forecaster C seems to have a scattering of eight significant points, but observe that they are not in the high skill areas. Once again, statistical significance in the χ^2 value doesn't imply quality forecasts.

5) The reader may by now have surmised that forecasters C, D are actually stochasters. We shall describe how they made their predictions.

Stochaster C had five dice before him. The faces that normally had numbers '1' and '2' were marked with 'B', the faces on each die normally marked with '3',

'4', had 'N', and the faces normally marked with '5', '6' had 'A' in the case of temperature predictions. For precipitation 'A,N,B' were replaced by 'L,M,H'. To make a set of five predictions, the stochaster threw all five dice on a smooth flat table.* The symbols on the scattered dice were always read from left to right and recorded. Each such throw therefore produced five random predictions. The throws continued until an accumulation of 99 predictions had been made. Each of these 99 predictions was then compared with its correspondent for the particular season under study whose actual temperatures or precipitations (in tercile form) had been compiled and listed beforehand for each of the 99 stations. From this point-by-point comparison, the class errors were calculated and tabulated. This process of throwing dice and comparing these results with each of the 99 observed field values was repeated until all seasons had been gone through for each set of temperature and precipitation data.

Stochaster D had before him an urn containing nine white balls. Three of the balls had the symbol '0' inscribed on them; two had '+1', two had '-1'; and one had '+2' while another had '-2' inscribed on it. The numbers of balls for each symbol are the relative frequencies with which the j -class signed errors, $j = 0, \pm 1, \pm 2$ occur for terceled data (cf §4). To make predictions the stochaster drew a ball at random from the urn. If it had '0' on it, then it was recorded that he made a correct prediction; if it had '+1' or '+2' on it, he committed ± 1 -class or ± 2 -class errors, respectively for that draw and it was so noted. In all, for a given season he made 99 independent draws from the urn. At the end of the 99 draws the number and type of signed j -class errors were totaled. From these, the unsigned errors could be found. For example for season 1, Table 4 shows he had the score $(u,v,w) = (33, 46, 20)$, obtained from the signed errors as indicated below the table.

6) The differences in appearances between the scatter diagrams of C and D are readily explained: recall that C had a more open pattern than D, signifying

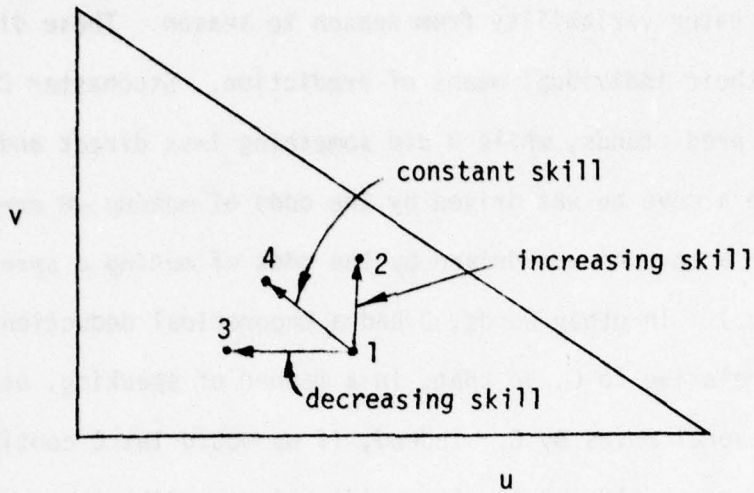
* There is no significance to the number of dice used; they simply were available from a popular game of chance.

more scatter, greater variability from season to season. These differences are clearly due to their individual means of prediction. Stochaster C worked directly with the actual predictands, while D did something less direct and more abstract: each time D made a move he was driven by the odds of *making an error*; by contrast each time C made a move he was driven by the odds of *making a specific prediction* (A, N, or B, e.g.). In other words, D had a theoretical deduction of a higher order built into him relative to C, so that, in a manner of speaking, each move by D was equivalent to several moves by C. Indeed, if we would let C continue indefinitely, his scatter patterns would relatively rapidly tighten like those of D and in the limit be described by the elliptical contours in the diagrams: 50% of his scores would eventually fall within the 50% contours, 80% within the 80% contours, and so on.

E. Ranking performances by moments and χ^2

We may supplement the χ^2 value of a score in judging skill by the following considerations.

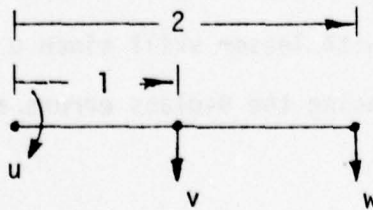
1) In the diagram below, point 1 is given in the uv plane. If on the one hand we rise vertically from 1 so as to leave u fixed, we go to a point 2 which is clearly associated with greater skill since v increases while w decreases; in other words, we are decreasing the 2-class errors and trading them in for 1-class errors, errors which are more palatable to the forecaster. On the other hand, if we move to the left of point 1, horizontally, so as to leave v fixed, we go to a point 3 which is clearly associated with lesser skill since u decreases while w increases; in other words, we are decreasing the 0-class errors and trading them in for 2-class errors.



There must then be an intermediate direction between that of segments 12 and 13, say 14, along which there is no change in the quality of skill. A moment's reflection would show that once we give numerical weights to the importances of the 1- and 2-class errors, this direction of constant skill is fixed. A natural assignment of weights may be made by defining the *moment* m of a trinomial score. We write

$$'m' \text{ for } 0 \cdot u + 1 \cdot v + 2 \cdot w$$

and call it the *moment of (u,v,w) about u*. We may envision the v and w scores as occurring on a lever



thereby producing a turning moment; the values v, w act like masses and their distances 1, 2 respectively act like moment arms. The object of a forecaster is to minimize

this moment, to bring it down to 0, ideally. The smaller m is, the better his performance. Since $u + v + w = n$, we can write

$$m = 2n - 2u - v \quad (6.1)$$

where n is the number of prediction locations, as usual. For a fixed n and m , (6.1) defines a straight line in the uv plane, namely

$$v = -2u + (2n-m) \quad (6.2)$$

along which the moments of the scores are constant, and hence, by agreement, the points (u,v) have equal quality in the moment sense.

2) As a result of this assignment of a *moment* to each (u,v) we can, with the help of the χ^2 ellipses (introduced in § A) lay down a coordinate net over the trinomial uv domain. By means of this network, shown, e.g., in Fig 6, we can locate points and assign to them relative ranks of performance. For example, on Figs 6, 6a we have placed the average* points (\hat{u}, \hat{v}) of the scores given in Tables 1-8. The average temperature scores are given on Fig 6, the average precipitation scores are on Fig 6a. It is seen that our earlier conclusions about the essentially equivalent skills of A and B and their superiority over C and D are graphically borne out using the present coordinate frame. The average points of A and B on Fig 6a lie essentially along the same moment line and on the same χ^2 curve. Each is clearly superior to C and D. However A and B find themselves between the 50% and 80% χ^2 ellipses, as may be seen by comparing with Fig 6b, in which the 50% - 99% ellipses are also drawn in for comparison. Thus, *on the average*, the performances of A and B are mediocre. These average points are also drawn in as the circled crosses in Figs 7-14. In the latter set, the standard deviation of each average score is shown by means of a

* Thus \hat{u} is the average of all u -points and \hat{v} that of all v -points.

dashed circle with radius equal to the standard deviation and centered on the mean point. These statistics are summarized below Table 1. Observe that in these average temperature and precipitation tables, while A has a larger average \hat{u} than B, our agreement to measure performances relative to χ^2 and m in Fig 6 shows that their performances are essentially the same. If an edge has to be given to one over the other, for the present accumulation of scores, A may be judged slightly superior, by looking at the m scores below Table 1 or closely at Fig 6 and seeing that, while A and B lie on the same χ^2 ellipse, A lies on a slightly lower moment line. At this stage of development of the prediction art, these differences are too small either to comfort or discourage A or B, respectively. Observe in particular that the average \hat{u} score of A or B by itself places either forecaster quite close to the 95% level (cf Table 9). If, however, we look not only at the number of correct predictions, but also at the number of 1-class errors (and hence implicitly the 2-class errors) a new perspective on their performances is attained: *In general, a good average score should land in a high skill region and with a relatively small standard deviation circle.* Both forecasters therefore should be concerned with increasing their average \hat{u} and \hat{v} scores; it was these that placed them both in a rather undistinguished area of the skill diagram. Moreover, consistently predicting climate *variations* manifests itself in smaller (tighter) scatter diagrams.

Thus we now have a reasonably objective framework in which to gauge forecasting skill as actual scores begin to accumulate and scatter diagrams begin to fill in.

3) We may summarize the ranking procedure using m and χ^2 as follows

$$\begin{array}{l}
 (u,v) = (u',v') \text{ if } \left[\begin{array}{l} \chi^2(u,v) = \chi^2(u',v') \\ m(u,v) = m(u',v') \end{array} \right. \\
 (u,v) > (u',v') \text{ if } \left[\begin{array}{l} m(u,v) < m(u',v') \\ \text{regardless of } \chi^2 \\ \text{or } m(u,v) = m(u',v') \\ \text{and } \chi^2(u,v) > \chi^2(u',v') \end{array} \right.
 \end{array}$$

In other words, two pairs (u,v) , (u',v') are of equal rank if their moments m and χ^2 values agree. Observe they need not be coincident to be of equal rank. Points C, D in Fig 6 are essentially equivalent. Also points 1, 2 are of equal rank. If the moments of two points agree, then we use χ^2 to break the deadlock, the one with the lesser probability of occurrence (higher χ^2) being of higher rank; e.g., point 2 is of greater rank than 3 in Fig 6. Therefore, in ranking points within a given region or set of points we give precedence to the moment of a score. This is clearly a convention (rather than a logical deduction) but one that is based on the intuitive interpretations of the scores u,v and their probabilities of occurrence. If a reader takes issue with this convention, then this means that he must (i) decide on a new relative weighting of v,w errors (and come up with an alternative to the moment m) and (ii) decide on the relative importance of the new m , and χ^2 . It may be that these relative weights and relative importances would vary with location over the uv plane.

A word of advice can be made here, in conclusion: whatever one convenes as the method of ranking performances, fine differences and subtle nuances in scoring systems will be swept aside and be inessential in the face of truly superior or even just good forecasting. The present method of ranking appears to go far beyond what has already been used. Perhaps then it is time to turn to the really basic problem at hand, the problem of forecasting, to devote more energy to improving *that* art, and perhaps gauging such efforts with the basically adequate ranking scheme we now have at hand.

7. Construction of Tables A-E and EXP

The graphical scoring charts we have used in our studies above are based on some simple analytical geometry and on probability calculations. These latter calculations are summarized in Tables A-E and EXP. They represent a fresh look at the χ^2 quantity by calculating its values and their corresponding exact probabilities

from the trinomial distribution for $p(u,v,w)$ given in §3. In particular we compared the approximate probabilities of χ^2 as given by (5.5) with their exact counterparts given by (3.2) and saw that, except for certain noncritical regions in the uv plane, the classical cumulative probability distribution for χ^2 was adequate to serve as a base for our probability ellipses in the trinomial skill charts. We now discuss the construction of these tables for the benefit of those who may wish to explore analogous skill chart constructions for values of n and a_0, a_1, a_2 not specifically covered in this study.

A. Table A

One of the motivations of this calculation was simple curiosity: to see what the probability was for each of the 5050 possible triples (u,v,w) (ranging from $(0, 0, 99)$ to $(99, 0, 0)$) on the triangular scoring surface depicted in §3. Accordingly a computer was instructed to find $p(u,v,w)$ via (3.2) to five significant figures for the tercile case: $a_0=1/3, a_1=4/9, a_2=2/9, n=99$. It turned out that many of the triples with low u values (≤ 14) and high u values (≥ 54) had probabilities far below 10^{-5} . Removing these from the computed list, we were left with 2644 triples whose probabilities or associated cumulative probabilities were 10^{-5} or greater. The range of these 2644 triples may be seen in graphic form in Fig 25, or directly in Table A, which begins with the triple $(14,52,33)$ and ends with $(54,35,10)$. The triples in Table A are arranged in 'alphabetical' order and may be visualized as progressing through the uv plane as shown in Fig 25. Along with (u,v,w) are given their χ^2 values (in the column marked ' χ^2 '), their probabilities (marked ' $P(A)$ '), and their cumulative probabilities (marked ' $CUM P(A)$ '). In order to understand the connections with later tables, we summarize the present calculations as follows, using the column headings:

TABLE A:

(u,v,w)	χ^2	P(A)	CUM P(A)
(ordered)	(computed)	(computed)	(computed from P(A))

yields

As we progress through Table A, we observe the χ^2 values dipping in value, reaching a minimum, then rising again, over and over again. This may be explained graphically by looking at Fig 6 and imagining the paths taken through its domain as indicated schematically in Fig 25. As we start with (14,52,33) and move along the trajectory suggested in Fig 25, and at the same time keep an eye on the values of P(A), we see that CUM P(A) builds slowly, being fed invisibly by P(A) until, finally, at triple (23,38,38) the triples have probabilities larger than 10^{-5} , and which go on to swell to a maximum at (23,51,25) and then decrease down to 10^{-5} again at (23,63,13). All of this can be followed in imagination on Fig 25 by visualizing a probability haystack centered on (33,44) in the uv plane. Again and again the ordered triples (u,v,w) in Table A slice through the haystack, taking increasingly meatier chunks of probability as the vertical traverses in Fig 25 get closer to the u=33 slice. As this slice is traversed (see p(15) of Table A) we finally attain the maximum value of p(u,v,w) in the entire table at the average point (33,44,22), namely $p(33,44,22) = .00880$. At this point, as the cumulative probability tally shows, we have accumulated half of the total probability mass. After this, the slices cut through the lower slopes of the probability haystack, decreasing steadily in content until eventually, as the traverse of slice u=54 is made, the final readable contributions to the total mass are made.

B. Table B

For this table we ordered the χ^2 values, encountered in Table A, in increasing order. As these χ^2 values were ordered we simply carried along the associated triples (u,v,w) and $P(A)$ values. The net result was a shuffled set of triples and probabilities. From the latter, as we went along, we added them up and formed CUM $P(B)$:

TABLE B:

(u,v,w) (shuffled)	χ^2 (ordered)	$P(A)$ (shuffled)	CUM $P(B)$ (computed from shuffled $P(A)$)

└──────────────────┬──────────────────┘
yields

The net result, CUM $P(B)$, could be visualized as an 'integration' of $P(A)$ using a polar coordinate frame with $(33,44,22)$ as center. As we progressed from smaller to larger χ^2 values we were sweeping up $P(A)$ values in ever larger (essentially elliptical) regions about $(33,44,22)$, and adding them together. Fig 24 shows the 50% ellipse enclosing about 79 points. These 79 points are represented by the first 79 entries of Table B from $(33,44,22)$ to $(31,41,27)$ at which the total probability mass accumulated was .50206. The χ^2 'radius' at this point is 1.4621. In this way we were able to associate to each χ^2 its *exact* associated cumulative probability. This was the primary purpose of Table B. By the time we had moved out to $\chi^2 = 76.0909$, we had essentially accumulated all probability mass (to within 10^{-5}), and could have truncated the table there. The region covered by the associated ellipse may be estimated from Figs 6 and 25. See in particular the points on Fig 25 for χ^2 near 75, 76.

C. Table C

This table is Table B now with ordered triples for easy look up of CUM $P(B)$

at each (u,v,w):

TABLE C

(u,v,w) (ordered)	X2 (as in Table A)	P(A)	CUM P(C) (shuffled CUM P(B))
----------------------	---------------------------	------	--

D. Table D

To see how well the χ^2 -ellipses (to be constructed below) embraced the accumulating probability mass as we swept radially outward from the center (average) point (33,44,22), we returned to Table A and arranged P(A) in *decreasing* order. In this way we nibbled outward from the center of the haystack, accumulating probability in a natural way, going along the 'true' contours of the *discrete* haystack:

TABLE D

(u,v,w) (shuffled)	X2 (shuffled)	P(A) (decreasing order)	CUM P(C) (computed from decreasing order of P(A))
-----------------------	----------------------	-----------------------------------	--

yields

To see what we had, we immediately made from this:

E. Table E

TABLE E

(u,v,w) (ordered)	χ^2 (as in Table A)	P(A)	CUM P(E) (shuffled CUM P(D))
----------------------	---------------------------------	------	--

A spot check was made at several points (u,v,w,) in the uv plane to see how well the cumulative probabilities agreed in Tables C and E. This would give a check on how well the χ^2 contours could describe the enclosed probability mass. The reader is invited to do the same. To start him off, consider the following selection of points

		CUM P(E)	CUM P(C)
a)	(33, 44, 22)	.00880	.00880
b)	(38, 40, 21)	.44133	.45115
c)	(40, 39, 20)	.65669	.66549
d)	(44, 36, 19)	.92697	.93879
e)	(48, 33, 18)	.99199	.99438

These points are shown on Fig 6 radiating outward from the origin. The agreement in cumulative probabilities is within one or two percent. Other checks along different lines show that we may use the χ^2 value as a radial index in terms of which, within a few percent, we may characterize the probability mass within the $\chi^2 = \text{constant}$ elliptical contour. *This then supplied the rigorous basis for the nested elliptical contours in the skill score charts of this study.* Any further constructions wishing to use smooth elliptical contours to summarize constant- χ^2 regions of given probability mass must satisfactorily pass this test. Otherwise the exact constant- χ^2 contours, which will likely be somewhat irregular, must be found by detailed plotting.

F. Table EXP

As a matter of simple curiosity we wanted to see how closely the χ^2 distribution (5.5) approximated the exact trinomial probabilities yielded by (3.2). The form of (5.5) for the terceled trinomial case is obtained by setting $r=3$, resulting in

$$T_2(\chi^2)d(\chi^2) = \frac{1}{2} e^{-\frac{1}{2}\chi^2} d(\chi^2) \quad , \quad (7.1)$$

a simple exponential in the variable χ^2 . How well does (7.1) describe the present state of affairs? In Fig 23 we show a plot of the exact values of $p(u,v,w)$ for various χ^2 values. For example, for $\chi^2=0$ we have from Table B the probability of $P(33,33,22)$ as .00880, and is shown on Fig 23. For $\chi^2 \cong 1-13$, there are several triples associated with each value (cf. e.g., $\chi^2 = 1.0227$). The range of probabilities associated with each χ^2 is indicated by the vertical bar on Fig 23. This points up the important theoretical fact that $T_2(\chi^2)$ does not account for the multiple-valuedness of the exact χ^2 relation defined by Table B. Moreover, a plot of the exponential in (7.1) in Fig 23 does not coincide with the visually-fit exponential going through the mass of points from Table B.

To see how well the *cumulative* probabilities were given by (7.1), the computer was instructed to find

$$(CUM EXP)_n \equiv \frac{1}{2} \sum_{i=1}^n \exp \left[\frac{-A_i}{2} \right] \Delta A_i, \quad n \geq 1 \quad (7.2)$$

$$A_i = \chi_i^2 = (X^2)_i$$

where n denotes the row of Table B. Here χ_i^2 is the ordered χ^2 entry in row i , and $\Delta A_i = A_i - A_{i-1}$, with $A_0=0$. The listing below compares CUM $P(B)$ with (CUM EXP) as found in (7.2), which simulates the discrete indefinite integral of (7.1).

χ^2	CUM P(B)	CUM EXP
0	.00880	.00000
.0530	.02593	.02582
.1667	.06787-.07610	.07861
.2121	.09999-.10797	.09917
.3030	.14672-.15419	.13882
.5303	.23229-.23892	.22986
1.0227	.38958-.40544	.39524
1.5000	.51474-.52730	.52025
2.0076	.62822-.63176	.62341
3.0303	.78191-.78374	.76761
4.0530	.86759-.86883	.85401
5.0303	.92268-.92333	.90402
6.0000	.95200-.95343	.93455
7.0227	.96993-.97076	.95408
8.0076	.98150-.98167	.96551
9.0000	.98916-.98944	.97250
10.0076	.99351-.99360	.97682

This shows that the cumulative probabilities of χ^2 in the third column, as given by (7.1)-(7.2) are reasonably good approximations to the exact values. Strictly speaking, as we saw in Fig 23, there is no one triple associated with a χ^2 value, but actually several. Hence the exact displayed range of values of CUM P(B) for each χ^2 . A similar comparison with CUM P(E) is possible, and shows the same degree of close agreement with CUM EXP. This indicates that for rough practical purposes we can use tables B, C, E, EXP interchangeably when assigning probabilities to χ^2 . However, the exact table for this purpose is B or C. Table A is our basic table from which our numerical knowledge of $p(u,v,w)$ springs.

8. Construction of the Skill Charts

The elliptical contours in the various figures in this study (as justified by the above results on Table C and Table E) may be found analytically as follows. Imagine the set of all points (u,v,w) in the scoring plane (cf diag. in §3) with a given fixed χ^2 value. Thus we imagine all (u,v,w) in the plane such that

$$\frac{(u-\bar{u})^2}{\bar{u}} + \frac{(v-\bar{v})^2}{\bar{v}} + \frac{(w-\bar{w})^2}{\bar{w}} = \chi^2 \quad (8.1)$$

where

$$\bar{u} = na_0, \quad \bar{v} = na_1, \quad \bar{w} = na_2$$

and a_0, a_1, a_2 are defined in §3. Since

$$u + v + w = n \quad (8.2)$$

there is a corresponding set of points (u,v) in the uv plane having the same constant χ^2 value. Using (8.2) in (8.1) and solving for v as a function of u , we find

$$v = \frac{-\bar{v}}{\bar{v}+\bar{w}} \cdot (u-\bar{u}) + \bar{v} \pm b^{-1} \left\{ (1-ab)(u-\bar{u})^2 + \bar{w}b\chi^2 \right\}^{1/2} \quad (8.3)$$

where

$$a = 1+(\bar{w}/\bar{u}), \quad b = 1+(\bar{w}/\bar{v})$$

The plus sign describes the upper half, the minus sign the lower half of an ellipse centered on the straight line defined by

$$v = \frac{-\bar{v}}{\bar{v}+\bar{w}} \cdot (u-\bar{u}) + \bar{v} \quad (8.4)$$

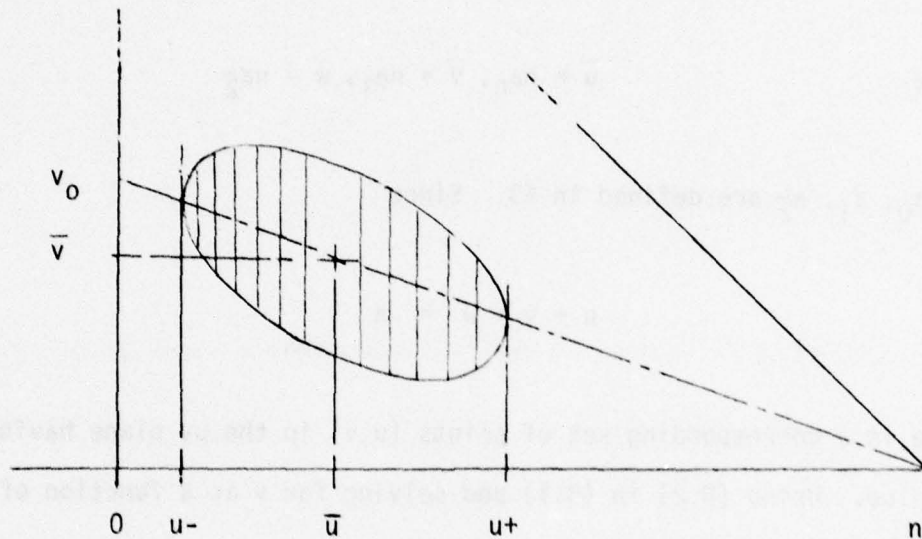
This is the straight line through the average point (\bar{u},\bar{v}) and the point $(n,0)$, the

point of maximum skill in a trinomial diagram. The v intercept v_0 occurs where $u=0$.

In the case of $n=99$, $a_0=1/3$, $a_1=4/9$, $a_2=2/9$, v_0 is given by:

$$v_0 = \bar{v} + \left(\frac{a_0 a_1}{1-a_0} \right) n = 44 + 22 = 66 .$$

A general sketch of the ellipses in the trinomial setting is given below. It is seen that the ellipses are vertically sheared about the line given by (8.4).



The horizontal limits u_{\pm} of the ellipses in these diagrams are obtained by setting the term in curly brackets in (8.3) to zero and solving the resultant quadratic for u :

$$u_{\pm} = \bar{u} \pm \left\{ \frac{\bar{w} b x^2}{ab-1} \right\}^{1/2} \quad (8.5)$$

By construction, each vertical line ($u=u_{\pm}$) is tangent to its ellipse where the line (8.4) pierces the ellipse. A study of Table A shows that the line (8.4) is the locus of maximum probabilities observed by making vertical (const u) slices through the probability haystack based on the uv plane.

The formulations above serve also to define the ellipses in the signed error diagrams, such as in Fig 4. We simply make the following assignments in (8.1) and related equations and carry out the resultant forms of the calculations:

<u>Unsigned errors</u>		<u>Signed errors</u>
u, \bar{u}	pairs with	$v(-), \bar{v}(-)$
v, \bar{v}		$v(+), \bar{v}(+)$
w, \bar{w}		$u(0), \bar{u}(0)$
a_0		$a(-)$
a_1		$a(+)$
a_2		$a(0)$

Finally, to assign a probability to χ^2 values for the purpose of labeling the ellipses with confidence level values, we used Table B as follows: we ran down the table until we encountered cumulative probabilities .50, .80, .90, .95, .98, and .99, and then simply picked off the corresponding χ^2 values, which are summarized below.

Cum prob.	Assoc. χ^2	From std. χ^2 tables (2d. f.)
50%	1.4621	1.386
80	3.2121	3.219
90	4.6667	4.605
95	5.9394	5.991
98	7.8030	7.824
99	9.1667	9.210

It can be seen that our exact χ^2 values agree closely with those obtained from standard

(but approximate) χ^2 tables for two degrees of freedom. But this agreement is not generally known *a priori* for a given n and a_0, a_1, a_2 . For this reason, the cautious chart designer would go through essentially the procedures described in §7, to find not only his own particular χ^2 values for (say) the above probabilities but also to see if the ellipses themselves are adequate to describe the regions in the uv plane with constant χ^2 (recall the concluding remarks of §7E).

Final checks on the accuracy of our computer graphics are made in Figs 24, 25. In particular, note how closely the analytically defined ellipses follow the discretely determined points with constant χ^2 .

9. Acknowledgments

This work was partially supported by the Office for Climate Dynamics of the National Science Foundation under contract ATM75-18699, and as part of the NORPAX program which is jointly sponsored by the National Science Foundation Office of International Decade of Ocean Exploration and the Office of Naval Research. It was also partially supported through the generosity of Director John Apel of the Pacific Marine Environmental Laboratory of the Environmental Research Laboratories of the National Oceanic and Atmospheric Administration.

The computations of Tables A-E and EXP were done by Anthony Tubbs, as also were preliminary computer graphic versions of the skill diagrams. Ron Moe completed the computer graphic versions. The author programmed Tables 10-15. Karen Douglas programmed the Figures 6, 6a, 6b. Madge Sullivan compiled the original meteorologic data from which Tables 1, 2, 5, 6 were made. Eleanor Preisendorfer aided in operating the stochasters C and D leading to Tables 3, 4, 7, 8. Grace Johnston typed the report. Finally, I am grateful to Tim Barnett for helpful discussions during the course of research, and Jerry Namias for supplying the initial inspiration and continued encouragement for the study.

10. References

Brier, G. W., Allen, R. A., 1951, 'Verification of Weather Forecasts,' p. 841 in Compendium of Meteorology. Ed. by T. F. Malone. American Meteorological Society, Boston, Mass.

Namias, J., 1953, Thirty-Day Forecasting, Meteorological Monographs. Vol. 2, No. 6, American Meteorological Society, Boston, Mass.

Panofsky, H. A., Brier, G. W., 1958, Some applications of Statistics to Meteorology, Pennsylvania State University, University Park, Pennsylvania.

TABLE 1
 TERCILED TEMPERATURE
 FORECASTER A

	SEASON	$u(0)$ $= u$	$v_1(+)$	$v_2(+)$	$v(+)$	$v_1(-)$	$v_2(-)$	$v(-)$	v	w
1	Wnt 74	42	13	0	13	24	20	44	37	20
2	Spr 74	27	19	5	24	28	20	48	47	25
3	Sum 74	30	30	11	41	21	7	28	51	18
4	Fal 74	44	15	5	20	28	7	35	43	12
5	Wnt 75	46	29	3	32	19	2	21	48	5
6	Spr 75	70	23	0	23	6	0	6	29	0
7	Sum 75	45	19	15	34	18	3	21	37	18
8	Fal 75	45	20	0	20	30	4	34	50	4
9	Wnt 76	23	8	1	9	40	27	67	48	28
10	Spr 76	30	30	5	35	23	11	34	53	16
11	Sum 76	43	41	6	47	8	1	9	49	7
12	Wnt 77	59	16	2	18	20	2	22	36	4
13	Spr 77	37	1	0	1	36	25	61	37	25
14	Sum 77	27	15	8	23	26	23	49	41	31

$$v(+) = v_1(+) + v_2(+)$$

$$v(-) = v_1(-) + v_2(-)$$

$$v = v_1(+) + v_1(-)$$

$$w = v_2(+) + v_2(-)$$

Forecaster's TEMPERATURE Average Scores

	\hat{u}	\hat{v}	s	m
A	40.5	43.3	14.9	73.7
B	36.1	48.4	16.4	77.4
C	31.3	46.7	8.4	88.7
D	33.6	42.1	6.0	88.7

Forecaster's PRECIPITATION Average Scores

	\hat{u}	\hat{v}	s	m
A	39.3	44.0	11.2	75.4
B	37.3	47.3	7.9	76.1
C	31.3	46.8	6.7	88.1
D	35.9	44.2	5.9	82.0

TABLE 2
 TERCILED TEMPERATURE
 FORECASTER B

	SEASON	$u(0)$ $= u$	$v_1(+)$	$v_2(+)$	$v(+)$	$v_1(-)$	$v_2(-)$	$v(-)$	v	w
1	Wnt 74	29	31	1	32	23	15	38	54	16
2	Spr 74	38	9	0	9	31	21	52	40	21
3	Sum 74	62	20	0	20	16	1	17	36	1
4	Fal 74	19	26	22	48	21	11	32	47	33
5	Wnt 75	43	6	2	8	35	13	48	41	15
6	Spr 75	11	64	24	88	0	0	0	64	24
7	Sum 75	45	25	3	28	23	3	26	48	6
8	Fal 75	43	26	5	31	24	1	25	50	6
9	Wnt 76	33	14	1	15	44	7	51	58	8
10	Spr 76	45	26	3	29	24	1	25	50	4
11	Sum 76	47	25	13	38	13	1	14	38	14
12	Wnt 77	44	42	1	43	9	3	12	51	4
13	Spr 77	32	4	0	4	54	9	63	58	9
14	Sum 77	14	14	8	22	28	35	63	42	43

$$v(+)=v_1(+)+v_2(+)$$

$$v=v_1(+)+v_1(-)$$

$$v(-)=v_1(-)+v_2(-)$$

$$w=v_2(+)+v_2(-)$$

TABLE 3
 TERCILED TEMPERATURE
 FORECASTER C (target: actual predictand)

	SEASON	$u(0)$ = u	$v_1(+)$	$v_2(+)$	$v(+)$	$v_1(-)$	$v_2(-)$	$v(-)$	v	w
1	Wnt 74	29	31	2	33	23	14	37	54	16
2	Spr 74	35	13	3	16	30	18	48	43	21
3	Sum 74	31	17	15	32	24	12	36	41	27
4	Fal 74	26	29	18	47	16	10	26	45	28
5	Wnt 75	31	23	4	27	32	9	41	55	13
6	Spr 75	34	31	21	52	13	0	13	44	21
7	Sum 75	30	25	15	40	20	9	29	45	24
8	Fal 75	41	16	11	27	26	5	31	42	16
9	Wnt 76	27	12	0	12	40	20	60	52	20
10	Spr 76	24	32	2	34	29	12	41	61	14
11	Sum 76	38	29	12	41	17	3	20	46	15
12	Wnt 77	38	31	15	46	13	2	15	44	17
13	Spr 77	29	10	4	14	26	30	56	36	34
14	Sum 77	25	15	8	23	31	20	51	46	28

$$v(+)=v_1(+)+v_2(+)$$

$$v(-)=v_1(-)+v_2(-)$$

$$v=v_1(+)+v_1(-)$$

$$w=v_2(+)+v_2(-)$$

TABLE 4
 TERCILED TEMPERATURE

FORECASTER D (target: idealized predictand)

	SEASON	$u(0)$ = u	$v_1(+)$	$v_2(+)$	$v(+)$	$v_1(-)$	$v_2(-)$	$v(-)$	v	w
1	Wnt 74	33	26	12	38	20	8	28	46	20
2	Spr 74	31	28	13	41	21	6	27	49	19
3	Sum 74	35	22	13	35	18	11	29	40	24
4	Fal 74	38	18	15	33	15	13	28	33	28
5	Wnt 75	35	24	11	35	20	9	29	44	20
6	Spr 75	33	24	15	39	21	6	27	45	21
7	Sum 75	33	19	9	28	25	13	38	44	22
8	Fal 75	36	26	11	37	18	8	26	44	19
9	Wnt 76	30	21	13	34	27	8	35	48	21
10	Spr 76	38	14	12	26	17	18	35	31	30
11	Sum 76	34	21	12	33	22	10	32	43	22
12	Wnt 77	34	24	16	40	13	12	25	37	28
13	Spr 77	32	20	14	34	21	12	33	41	26
14	Sum 77	28	22	8	30	22	19	41	44	27

$$v(+)=v_1(+)+v_2(+)$$

$$v=v_1(+)+v_1(-)$$

$$v(-)=v_1(-)+v_2(-)$$

$$w=v_2(+)+v_2(-)$$

TABLE 5
 TERCILED PRECIPITATION
 FORECASTER A

	SEASON	$u(0)$ = u	$v_1(+)$	$v_2(+)$	$v(+)$	$v_1(-)$	$v_2(-)$	$v(-)$	v	w
1	Sum 74	38	23	11	34	20	7	27	43	18
2	Fal 74	49	15	7	22	24	4	28	39	11
3	Wnt 75	46	11	2	13	25	15	40	36	17
4	Spr 75	40	24	11	35	23	1	24	47	12
5	Sum 75	38	24	8	32	25	4	29	49	12
6	Fal 75	21	33	17	50	17	11	28	50	28
7	Wnt 76	37	33	15	48	9	5	14	42	20
8	Spr 76	43	22	9	31	20	5	25	42	14
9	Sum 76	52	16	5	21	25	1	26	41	6
10	Wnt 77	45	27	16	43	10	1	11	37	17
11	Spr 77	25	36	12	48	24	2	26	60	14
12	Sum 77	37	22	10	32	20	10	30	42	20

$$v(+)=v_1(+)+v_2(+)$$

$$v(-)=v_1(-)+v_2(-)$$

$$v=v_1(+)+v_1(-)$$

$$w=v_2(+)+v_2(-)$$

TABLE 6
 TERCEILED PRECIPITATION
 FORECASTER B

	SEASON	$u(0)$ = u	$v_1(+)$	$v_2(+)$	$v(+)$	$v_1(-)$	$v_2(-)$	$v(-)$	v	w
1	Sum 74	40	32	10	42	15	2	17	47	12
2	Fal 74	34	31	18	49	15	1	16	46	19
3	Wnt 75	46	26	6	32	17	4	21	43	10
4	Spr 75	43	18	3	21	33	2	35	51	5
5	Sum 75	37	24	4	28	29	5	34	53	9
6	Fal 75	32	30	18	48	14	5	19	44	23
7	Wnt 76	35	35	17	52	11	1	12	46	18
8	Spr 76	31	19	27	46	18	4	22	37	31
9	Sum 76	32	30	10	40	23	4	27	53	14
10	Wnt 77	46	29	11	40	10	3	13	39	14
11	Spr 77	37	44	6	50	9	3	12	53	9
12	Sum 77	35	26	2	28	29	7	36	55	9

$$v(+) = v_1(+) + v_2(+)$$

$$v(-) = v_1(-) + v_2(-)$$

$$v = v_1(+) + v_1(-)$$

$$w = w_2(+) + v_2(-)$$

TABLE 7
 TERCILED PRECIPITATION
 FORECASTER C (target: actual predictand)

	SEASON	$u(0)$ = u	$v_1(+)$	$v_2(+)$	$v(+)$	$v_1(-)$	$v_2(-)$	$v(-)$	v	w
1	Sum 74	28	33	13	46	19	6	25	52	19
2	Fal 74	33	19	15	35	19	13	32	38	28
3	Wnt 75	29	18	7	25	28	17	45	46	24
4	Spr 75	35	18	7	25	26	13	39	44	20
5	Sum 75	31	26	8	34	27	7	34	53	15
6	Fal 75	34	23	9	32	24	9	33	47	18
7	Wnt 76	27	19	20	39	24	9	33	43	29
8	Spr 76	27	25	21	46	21	5	26	46	26
9	Sum 76	36	29	11	40	16	7	23	45	18
10	Wnt 77	34	27	20	47	13	5	18	40	25
11	Spr 77	35	31	5	36	20	8	28	51	13
12	Sum 77	26	20	8	28	37	8	35	57	16

$$v(+)=v_1(+)+v_2(+)$$

$$v(-)=v_1(-)+v_2(-)$$

$$v=v_1(+)+v_1(-)$$

$$w=v_2(+)+v_2(-)$$

TABLE 8
 TERCILED PRECIPITATION

FORECASTER D (target: idealized predictand)

	SEASON	$u(0)$ = u	$v_1(+)$	$v_2(+)$	$v(+)$	$v_1(-)$	$v_2(-)$	$v(-)$	v	w
1	Sum 74	34	23	7	30	22	13	35	45	20
2	Fal 74	39	15	13	28	17	15	32	32	28
3	Wnt 75	41	26	3	29	20	9	29	46	12
4	Spr 75	34	25	7	32	24	9	33	49	16
5	Sum 75	35	20	10	30	23	11	34	43	21
6	Fal 75	34	25	15	40	20	5	25	45	20
7	Wnt 76	39	27	11	38	16	6	22	43	17
8	Spr 76	38	20	6	26	26	9	35	46	15
9	Sum 76	30	27	10	37	25	7	32	52	17
10	Wnt 77	38	20	14	34	21	6	27	41	20
11	Spr 77	33	18	11	29	23	14	37	41	25
12	Sum 77	36	23	10	33	24	6	30	47	16

$$v(+)=v_1(+)+v_2(+)$$

$$v(-)=v_1(-)+v_2(-)$$

$$v=v_1(+)+v_1(-)$$

$$w=v_2(+)+v_2(-)$$

TABLE 9
 SKILL SCORES S
 AND CRITICAL RATIOS C vs u
 CASE OF n=99, p=1/3, $\bar{u}=33$, $\sigma=4.69$

u = No. Correct Predictions (0-class errors)	Skill Score $S_{99} = (u - \bar{u})(n - \bar{u})^{-1}$	Critical Ratio $C_{99} = (u - \bar{u})\sigma^{-1}$
15	-.273	
16	-.258	
17	-.242	
18	-.227	
19	-.212	
20	-.197	
21	-.182	
22	-.167	-2.34
		(1%)
23	-.152	-2.13
24	-.136	-1.92
25	-.121	-1.70
		(5%)
26	-.106	-1.49
27	-.091	-1.28
28	-.076	-1.07
29	-.061	-.853
30	-.045	-.640
31	-.030	-.426
32	-.015	-.213
		(50%)
33	.000	.000
34	+.015	+.213
35	+.030	+.426
36	+.045	+.640
37	+.061	+.853
38	+.076	+1.07
39	+.091	+1.28
40	+.106	+1.49
		(95%)
41	+.121	+1.70
42	+.136	+1.92
43	+.152	+2.13
		(99%)
44	+.167	+2.34
45	+.182	
46	+.197	
47	+.212	
48	+.227	
49	+.242	
50	+.258	
51	+.273	

Preface to Tables 10-15

These tables are included for the reader's convenience. In particular, 'K' can stand for u, v, or w, as the case may be, when specialized to the notation of this study. Thus, we have, for terceled data:

In Table 10	K corresponds to u, CUM P(K) to $Q_{99}(u)$.	$P_{99}(K)$ to $p_{99}(u)$, $0.3333333333 = 1/3$
In Table 11	K corresponds to v, CUM P(K) to $Q_{99}(v)$,	$P(K)$ to $p_{99}(v)$, $.0.4444444444 = 4/9$
In Table 12	K corresponds to w, CUM P(K) to $Q_{99}(w)$,	$P(K)$ to $p_{99}(w)$ $0.2222222222 = 2/9$

Similarly, Tables 13, 14, 15 are for quintiled data, with K corresponding respectively to u, v, w, and

$$0.2000000000 = 1/5$$

$$0.3800000000 = 8/25$$

$$0.4800000000 = 12/25$$

Such tables are readily made up for other values of P and N.

THIS PAGE IS BEST QUALITY PRACTICABLE
 FROM COPY FURNISHED TO DDC

TABLE 10

BINOMIAL PROBABILITIES

$$P(K) = [N! / (K!(N-K)!)] [P**K] [(1-P)**(N-K)]$$

N= 99
 P= 0.3333333333

K	P(K)	CUM P(K)	1-CUM P(K)
14	0.00001	0.00001	0.99999
15	0.00002	0.00003	0.99997
16	0.00006	0.00010	0.99990
17	0.00016	0.00025	0.99975
18	0.00035	0.00061	0.99939
19	0.00075	0.00136	0.99864
20	0.00151	0.00287	0.99713
21	0.00284	0.00571	0.99429
22	0.00503	0.01074	0.98926
23	0.00842	0.01916	0.98084
24	0.01333	0.03249	0.96751
25	0.02000	0.05249	0.94751
26	0.02846	0.08095	0.91905
27	0.03848	0.11943	0.88057
28	0.04947	0.16890	0.83110
29	0.06056	0.22945	0.77055
30	0.07065	0.30010	0.69990
31	0.07862	0.37872	0.62128
32	0.08354	0.46226	0.53774
33	0.08480	0.54707	0.45293
34	0.08231	0.62938	0.37062
35	0.07643	0.70581	0.29419
36	0.06794	0.77375	0.22625
37	0.05784	0.83159	0.16841
38	0.04719	0.87877	0.12123
39	0.03690	0.91567	0.08433
40	0.02768	0.94335	0.05665
41	0.01991	0.96326	0.03674
42	0.01375	0.97701	0.02299
43	0.00911	0.98612	0.01388
44	0.00580	0.99192	0.00808
45	0.00354	0.99547	0.00453
46	0.00208	0.99755	0.00245
47	0.00117	0.99872	0.00128
48	0.00064	0.99936	0.00064
49	0.00033	0.99969	0.00031
50	0.00017	0.99985	0.00015
51	0.00008	0.99993	0.00007
52	0.00004	0.99997	0.00003
53	0.00002	0.99999	0.00002
54	0.00001	0.99999	0.00001

TABLE 11

BINOMIAL PROBABILITIES

$$P(K) = \frac{N!}{K!(N-K)!} [P^K] [(1-P)^{N-K}]$$

N = 99
 P = 0.4444444444

K	P(K)	CUM P(K)	1-CUM P(K)
23	0.00001	0.00001	0.99999
24	0.00002	0.00002	0.99998
25	0.00004	0.00006	0.99994
26	0.00008	0.00014	0.99986
27	0.00018	0.00032	0.99968
28	0.00037	0.00069	0.99931
29	0.00073	0.00142	0.99858
30	0.00136	0.00278	0.99722
31	0.00242	0.00521	0.99479
32	0.00412	0.00933	0.99067
33	0.00669	0.01602	0.98398
34	0.01039	0.02641	0.97359
35	0.01544	0.04184	0.95816
36	0.02195	0.06379	0.93621
37	0.02990	0.09370	0.90630
38	0.03903	0.13273	0.86727
39	0.04824	0.18157	0.81843
40	0.05861	0.24018	0.75982
41	0.06747	0.30765	0.69235
42	0.07454	0.38219	0.61781
43	0.07905	0.46123	0.53877
44	0.08048	0.54172	0.45828
45	0.07869	0.62041	0.37959
46	0.07390	0.69432	0.30568
47	0.06667	0.76099	0.23901
48	0.05778	0.81877	0.18123
49	0.04811	0.86688	0.13312
50	0.03849	0.90537	0.09463
51	0.02958	0.93496	0.06504
52	0.02185	0.95680	0.04320
53	0.01550	0.97230	0.02770
54	0.01056	0.98286	0.01714
55	0.00691	0.98978	0.01022
56	0.00435	0.99412	0.00588
57	0.00262	0.99674	0.00326
58	0.00152	0.99826	0.00174
59	0.00084	0.99911	0.00089
60	0.00045	0.99956	0.00044
61	0.00023	0.99979	0.00021
62	0.00011	0.99990	0.00010
63	0.00005	0.99996	0.00004
64	0.00002	0.99998	0.00002
65	0.00001	0.99999	0.00001

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

TABLE 12

BINOMIAL PROBABILITIES

$$P(K) = \frac{N!}{K!(N-K)!} [P^K] [(1-P)^{(N-K)}]$$

N= 99
P= 0.2222222222

K	P(K)	CUM P(K)	1-CUM P(K)
6	0.00001	0.00001	0.99999
7	0.00004	0.00005	0.99995
8	0.00012	0.00016	0.99984
9	0.00034	0.00051	0.99949
10	0.00088	0.00139	0.99861
11	0.00204	0.00344	0.99656
12	0.00428	0.00772	0.99228
13	0.00819	0.01591	0.98409
14	0.01437	0.03028	0.96972
15	0.02327	0.05355	0.94645
16	0.03491	0.08846	0.91154
17	0.04869	0.13715	0.86285
18	0.06338	0.20052	0.79948
19	0.07719	0.27772	0.72228
20	0.08822	0.36594	0.63406
21	0.09482	0.46076	0.53924
22	0.09606	0.55682	0.44318
23	0.09188	0.64870	0.35130
24	0.08313	0.73183	0.26817
25	0.07125	0.80308	0.19692
26	0.05794	0.86102	0.13898
27	0.04476	0.90578	0.09422
28	0.03288	0.93867	0.06133
29	0.02300	0.96167	0.03833
30	0.01534	0.97700	0.02300
31	0.00975	0.98676	0.01324
32	0.00592	0.99268	0.00732
33	0.00343	0.99611	0.00389
34	0.00190	0.99802	0.00198
35	0.00101	0.99903	0.00097
36	0.00051	0.99954	0.00046
37	0.00025	0.99979	0.00021
38	0.00012	0.99991	0.00009
39	0.00005	0.99996	0.00004
40	0.00002	0.99998	0.00002
41	0.00001	0.99999	0.00001

THIS PAGE IS BEST QUALITY PRACTICABLE
 FROM COPY FURNISHED TO DDC

TABLE 13

BINOMIAL PROBABILITIES

$$P(K) = \frac{N!}{K!(N-K)!} [P^K] [(1-P)^{(N-K)}]$$

N= 99
 P= 0.2000000000

K	P(K)	CUM P(K)	1-CUM P(K)
5	0.00002	0.00002	0.99998
6	0.00007	0.00009	0.99991
7	0.00023	0.00032	0.99968
8	0.00067	0.00099	0.99902
9	0.00168	0.00267	0.99733
10	0.00378	0.00645	0.99355
11	0.00765	0.01410	0.98590
12	0.01403	0.02813	0.97187
13	0.02347	0.05160	0.94840
14	0.03605	0.08765	0.91235
15	0.05107	0.13871	0.86129
16	0.06702	0.20574	0.79426
17	0.08181	0.28755	0.71245
18	0.09317	0.38072	0.61928
19	0.09930	0.48002	0.51998
20	0.09930	0.57932	0.42068
21	0.09339	0.67271	0.32729
22	0.08278	0.75548	0.24452
23	0.06928	0.82476	0.17524
24	0.05485	0.87961	0.12039
25	0.04114	0.92075	0.07925
26	0.02927	0.95002	0.04998
27	0.01978	0.96980	0.03020
28	0.01272	0.98252	0.01748
29	0.00778	0.99030	0.00970
30	0.00454	0.99484	0.00516
31	0.00253	0.99737	0.00263
32	0.00134	0.99871	0.00129
33	0.00068	0.99939	0.00061
34	0.00033	0.99973	0.00027
35	0.00015	0.99988	0.00012
36	0.00007	0.99995	0.00005
37	0.00003	0.99999	0.00002
38	0.00001	0.99999	0.00001

BINOMIAL PROBABILITIES

$$P(K) = [N!/K!(N-K)!][P**K][(1-P)**(N-K)]$$

N= 99
P= 0.3200000000

K	P(K)	CUM P(K)	1-CUM P(K)
13	0.00001	0.00001	0.99999
14	0.00003	0.00004	0.99996
15	0.00007	0.00010	0.99990
16	0.00017	0.00028	0.99972
17	0.00039	0.00067	0.99933
18	0.00084	0.00151	0.99849
19	0.00169	0.00321	0.99679
20	0.00319	0.00639	0.99361
21	0.00564	0.01204	0.98796
22	0.00941	0.02145	0.97855
23	0.01483	0.03628	0.96372
24	0.02210	0.05839	0.94161
25	0.03120	0.08959	0.91041
26	0.04179	0.13139	0.86861
27	0.05318	0.18456	0.81544
28	0.06435	0.24891	0.75109
29	0.07414	0.32305	0.67695
30	0.08140	0.40445	0.59555
31	0.08527	0.48972	0.51028
32	0.08527	0.57498	0.42502
33	0.08147	0.65645	0.34355
34	0.07442	0.73087	0.26913
35	0.06504	0.79591	0.20409
36	0.05441	0.85032	0.14968
37	0.04360	0.89392	0.10608
38	0.03348	0.92740	0.07260
39	0.02464	0.95203	0.04797
40	0.01739	0.96943	0.03057
41	0.01178	0.98120	0.01880
42	0.00765	0.98886	0.01114
43	0.00477	0.99363	0.00637
44	0.00286	0.99649	0.00351
45	0.00164	0.99814	0.00186
46	0.00091	0.99905	0.00095
47	0.00048	0.99953	0.00047
48	0.00025	0.99977	0.00023
49	0.00012	0.99990	0.00010
50	0.00006	0.99995	0.00005
51	0.00003	0.99998	0.00002
52	0.00001	0.99999	0.00001

TABLE 15

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDG

BINOMIAL PROBABILITIES

$$P(K) = (N!/K!(N-K)!)[P**K][(1-P)**(N-K)]$$

N= 99
P= 0.4800000000

K	P(K)	CUM P(K)	1-CUM P(K)
27	0.00001	0.00001	0.99999
28	0.00003	0.00004	0.99996
29	0.00007	0.00011	0.99989
30	0.00014	0.00025	0.99975
31	0.00029	0.00054	0.99946
32	0.00058	0.00112	0.99888
33	0.00108	0.00220	0.99780
34	0.00193	0.00413	0.99587
35	0.00331	0.00744	0.99256
36	0.00543	0.01287	0.98713
37	0.00854	0.02142	0.97858
38	0.01286	0.03428	0.96572
39	0.01857	0.05285	0.94715
40	0.02571	0.07856	0.92144
41	0.03416	0.11272	0.88728
42	0.04354	0.15626	0.84374
43	0.05328	0.20954	0.79046
44	0.06259	0.27213	0.72787
45	0.07062	0.34275	0.65725
46	0.07652	0.41927	0.58073
47	0.07965	0.49893	0.50107
48	0.07965	0.57858	0.42142
49	0.07653	0.65510	0.34490
50	0.07064	0.72574	0.27426
51	0.06265	0.78839	0.21161
52	0.05338	0.84177	0.15823
53	0.04370	0.88547	0.11453
54	0.03436	0.91983	0.08017
55	0.02595	0.94578	0.05422
56	0.01882	0.96460	0.03540
57	0.01311	0.97771	0.02229
58	0.00876	0.98647	0.01353
59	0.00562	0.99209	0.00791
60	0.00346	0.99555	0.00445
61	0.00204	0.99759	0.00241
62	0.00115	0.99874	0.00126
63	0.00063	0.99937	0.00063
64	0.00033	0.99969	0.00031
65	0.00016	0.99986	0.00014
66	0.00008	0.99993	0.00007
67	0.00003	0.99997	0.00003
68	0.00002	0.99998	0.00002
69	0.00001	0.99999	0.00001

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDG

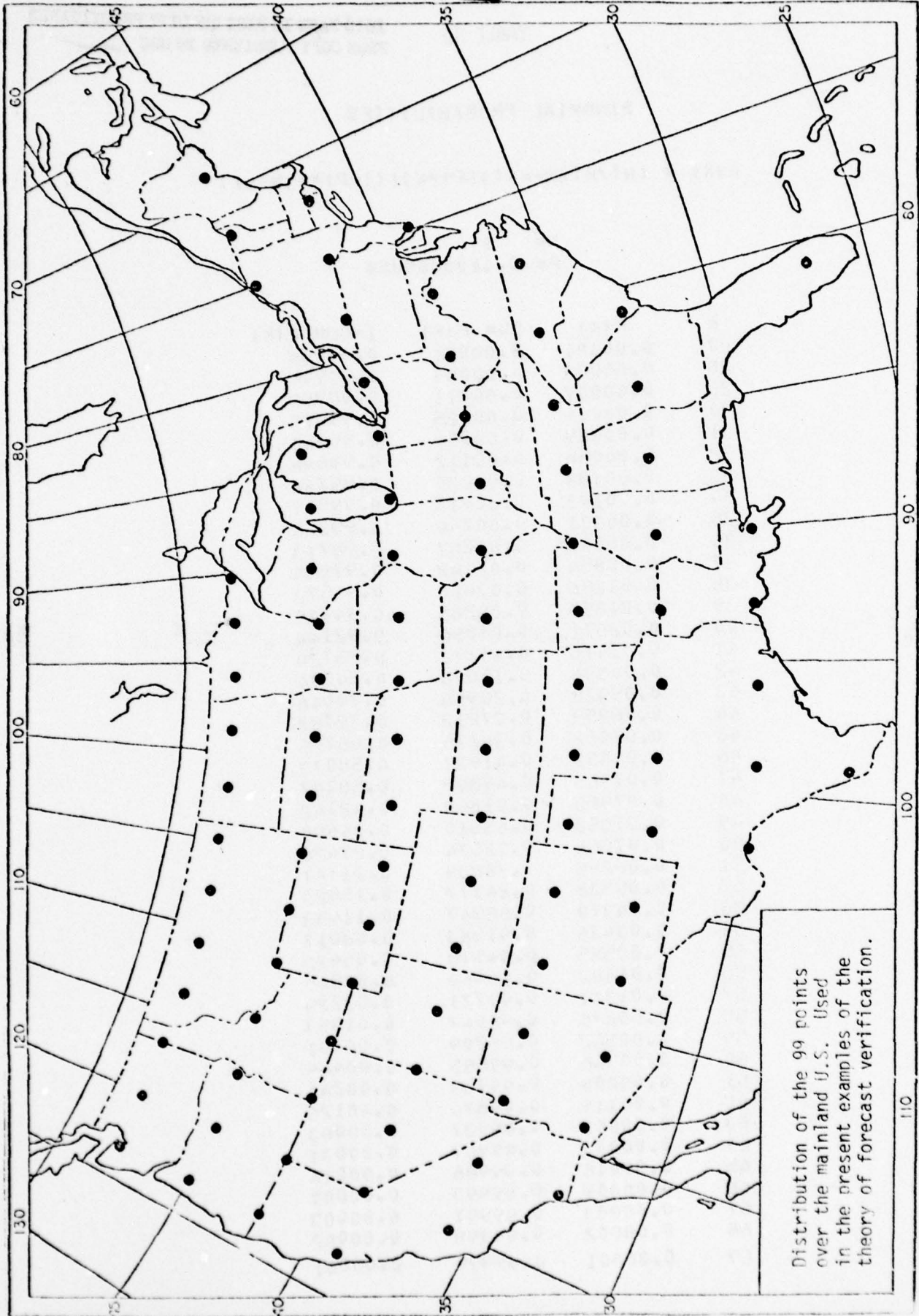
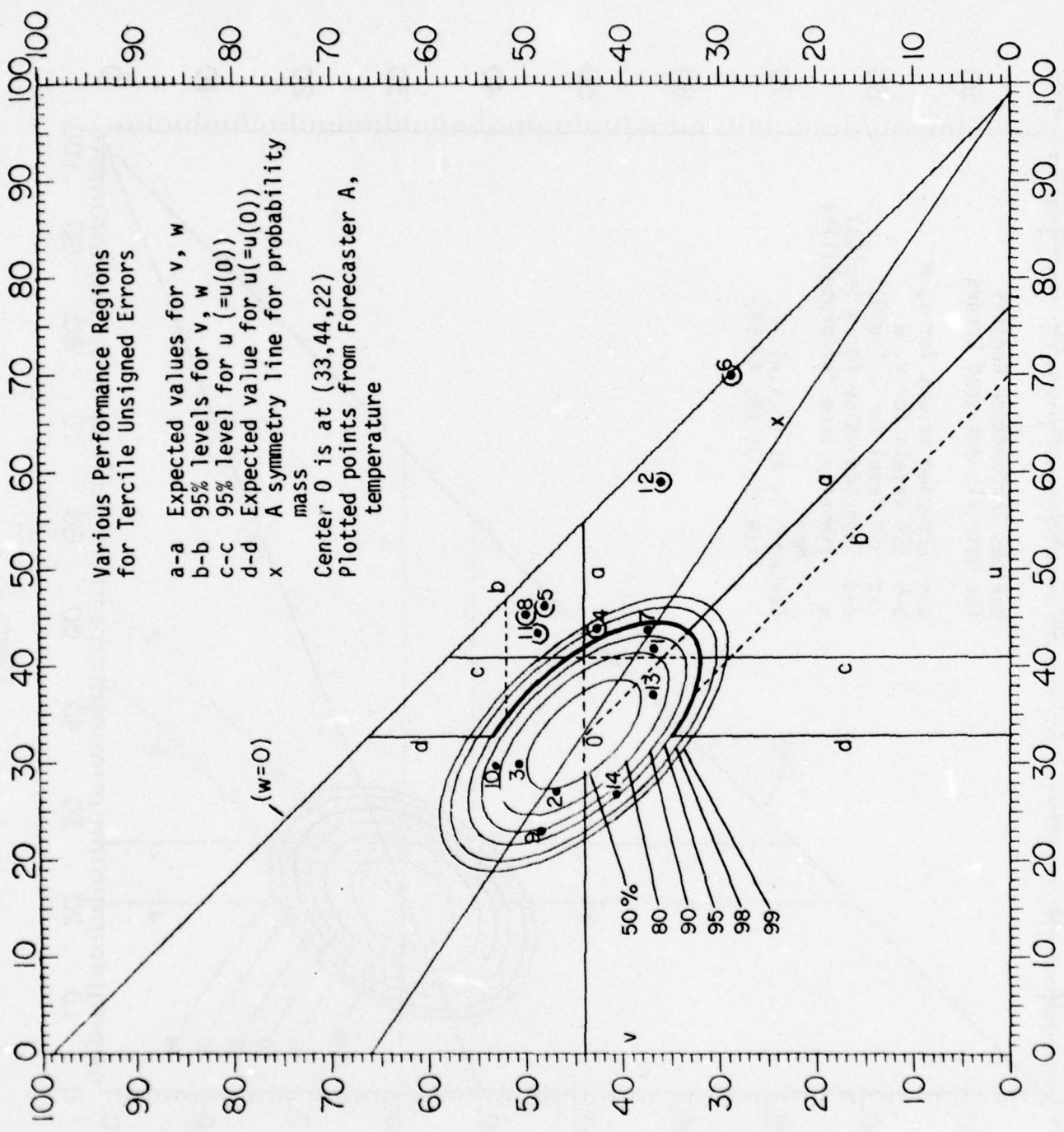


Figure 1



Various Performance Regions
for Tercile Unsigned Errors

- a-a Expected values for v, w
- b-b 95% levels for v, w
- c-c 95% level for u (=u(0))
- d-d Expected value for u(=u(0))
- x A symmetry line for probability mass

Center 0 is at (33,44,22)
Plotted points from Forecaster A,
temperature

Figure 2

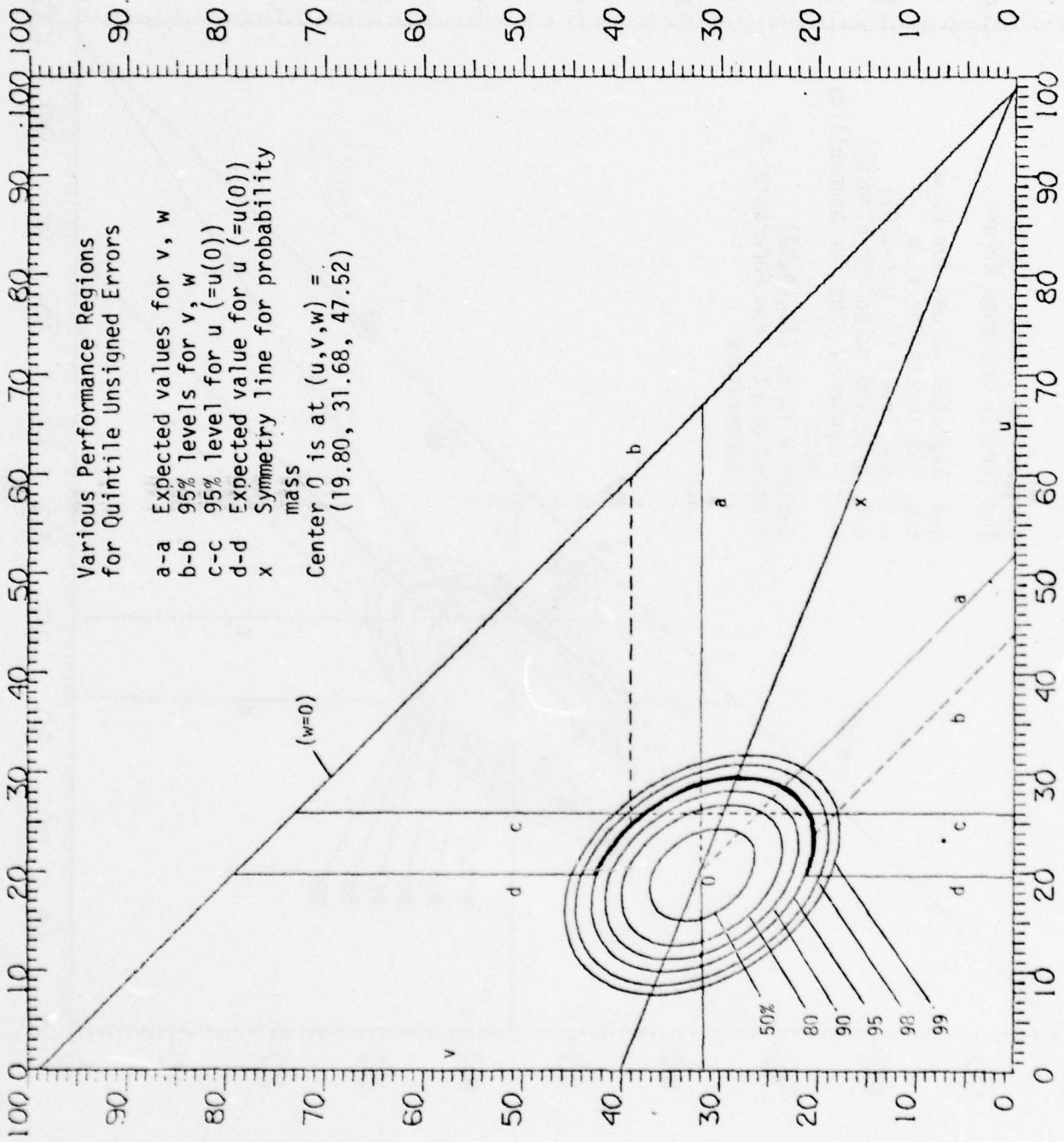


Figure 3

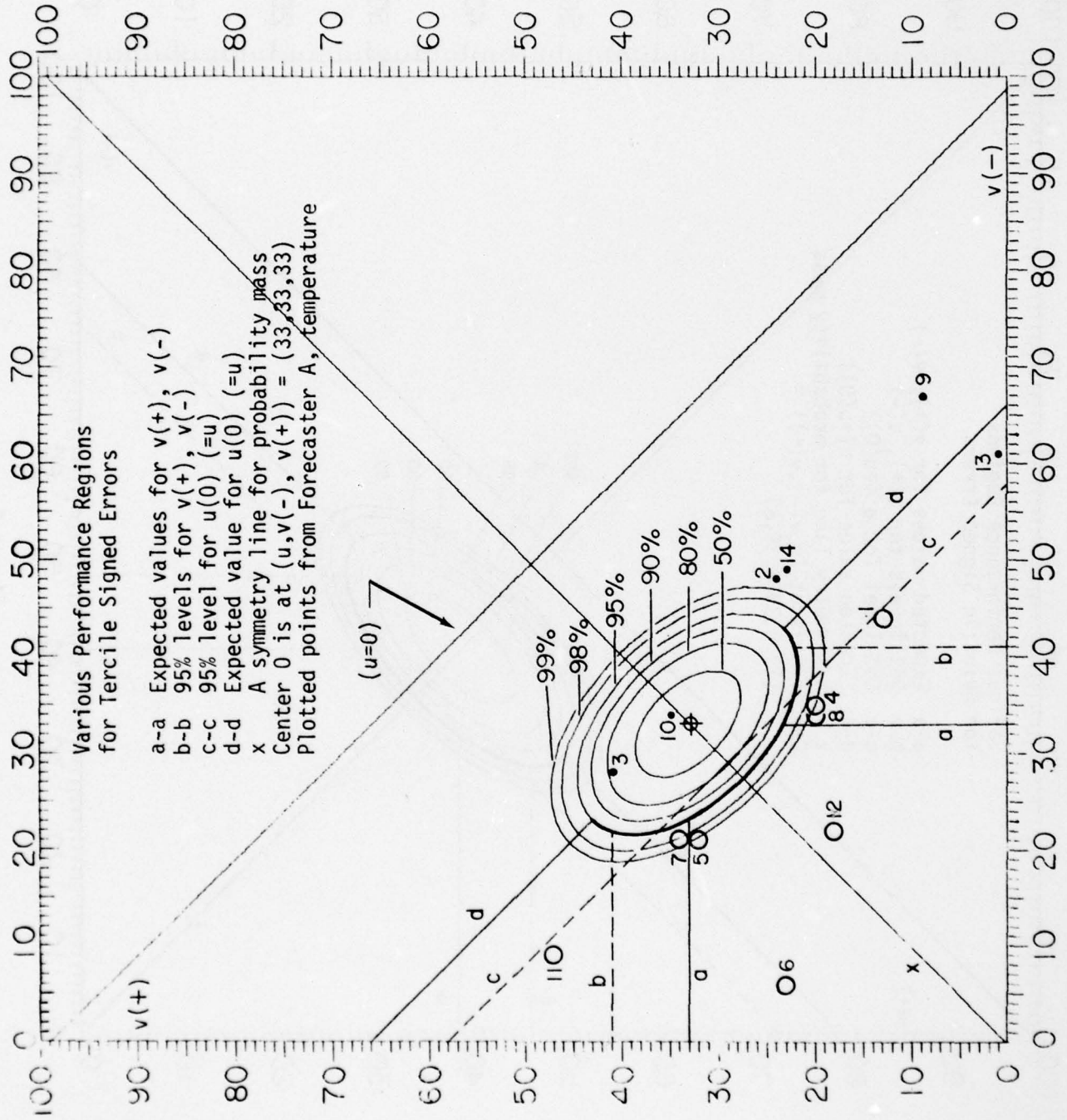
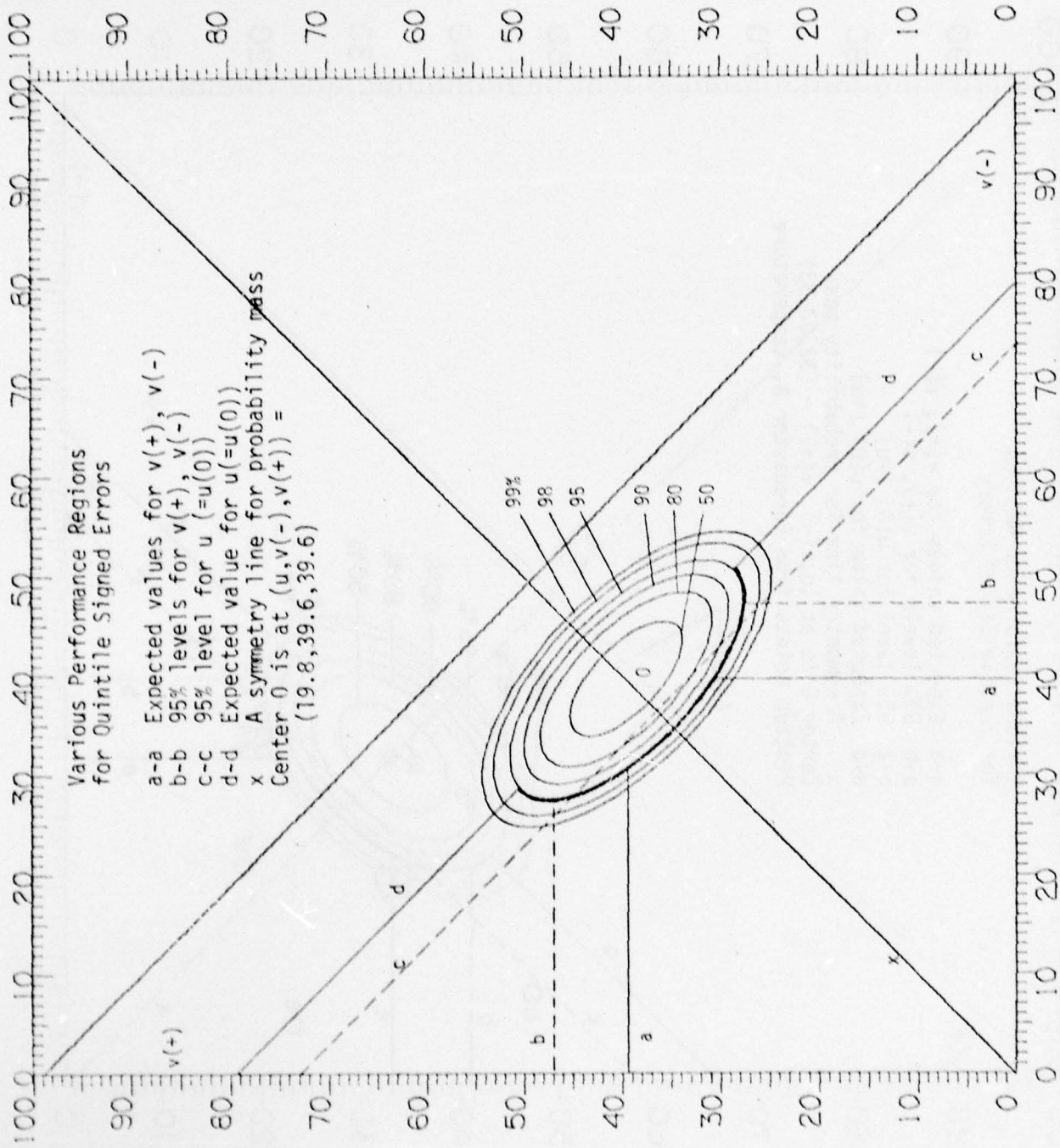


Figure 4



Various Performance Regions for Quintile Signed Errors

- a-a Expected values for $v(+)$, $v(-)$
 - b-b 95% levels for $v(+)$, $v(-)$
 - c-c 95% level for $u(=u(0))$
 - d-d Expected value for $u(=u(0))$
 - x A symmetry line for probability mass
- Center 0 is at $(u, v(-), v(+)) = (19.8, 39.6, 39.6)$

Figure 5

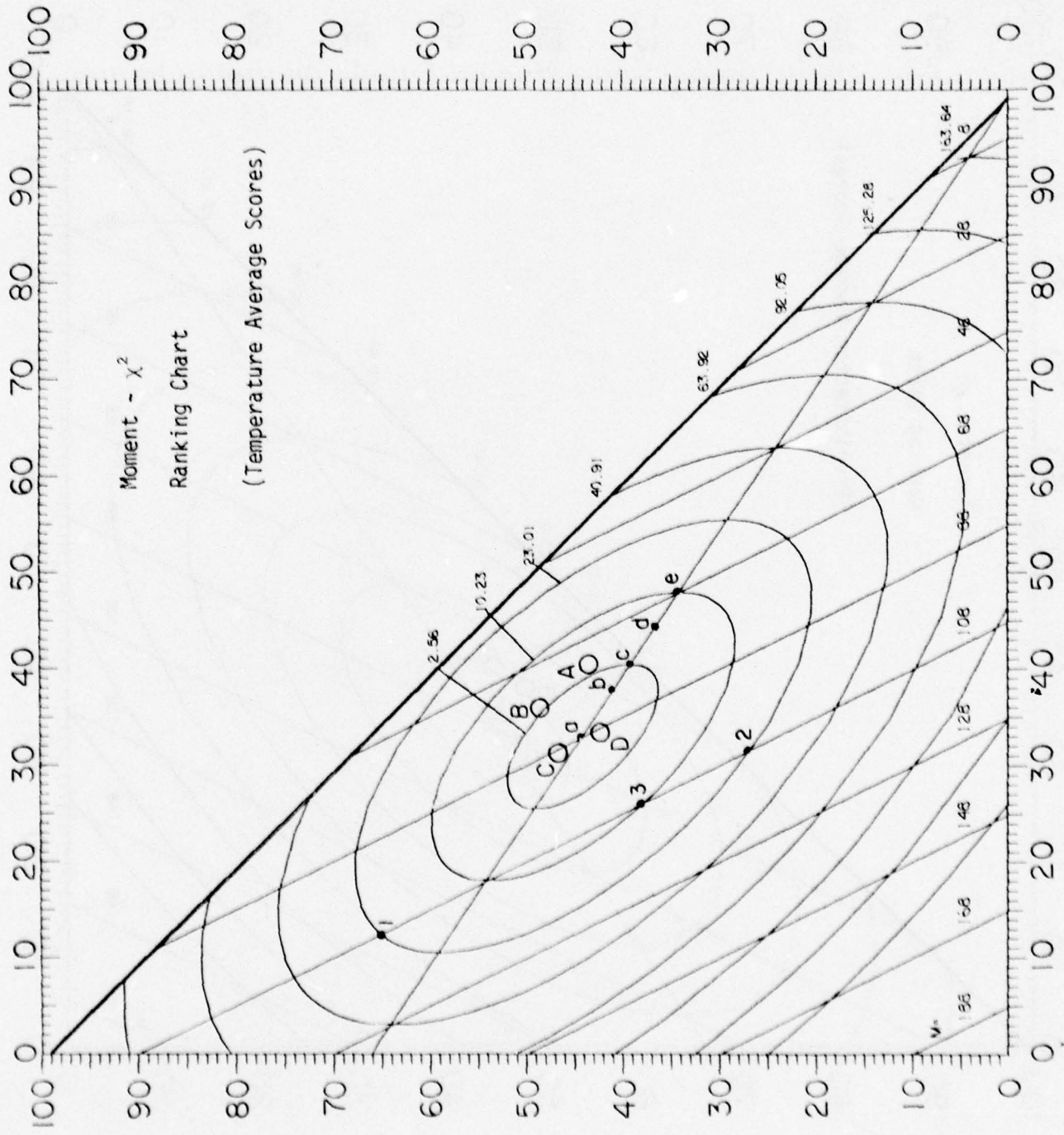


Figure 6

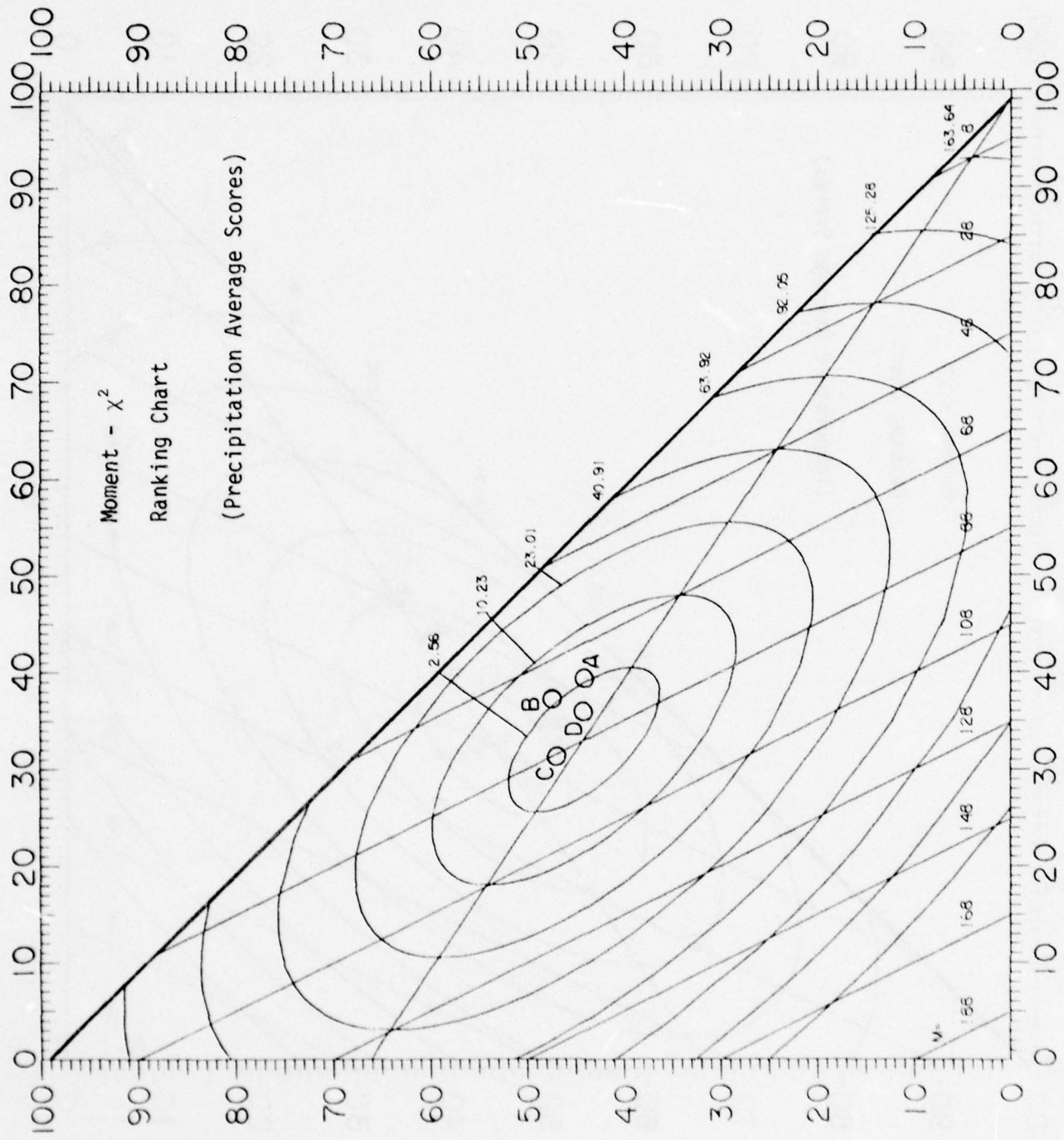


Figure 6a

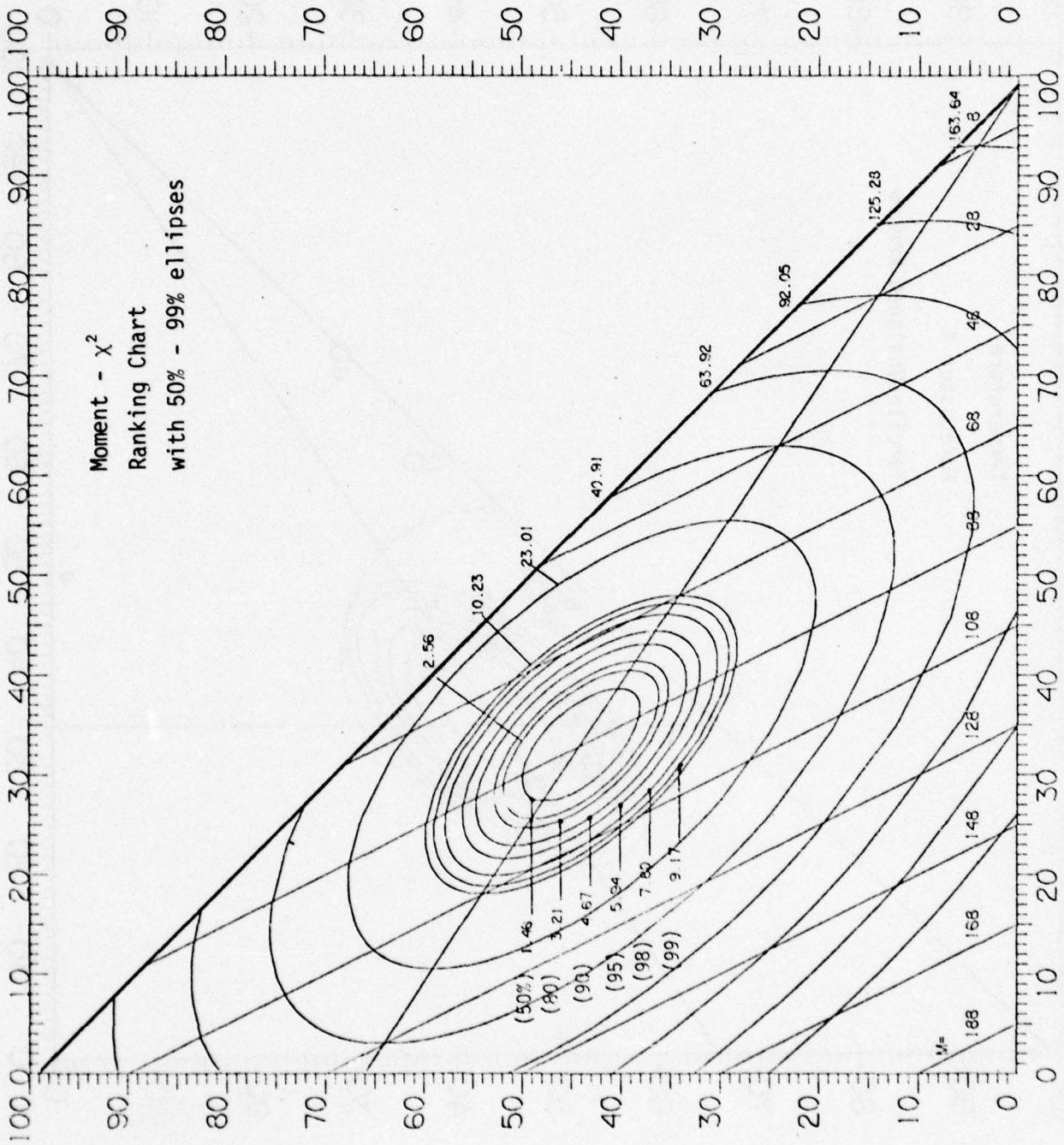


Figure 6b

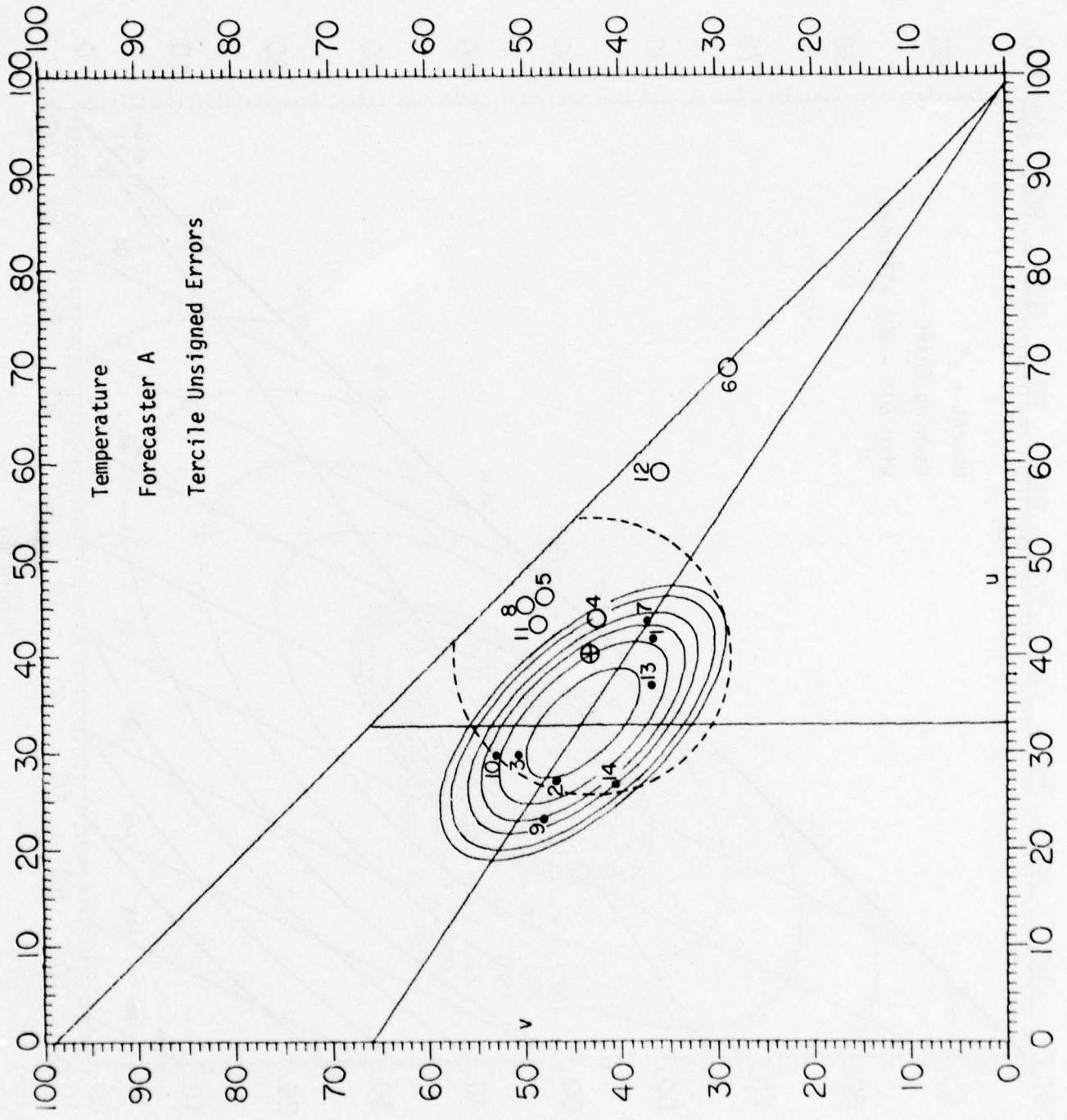


Figure 7

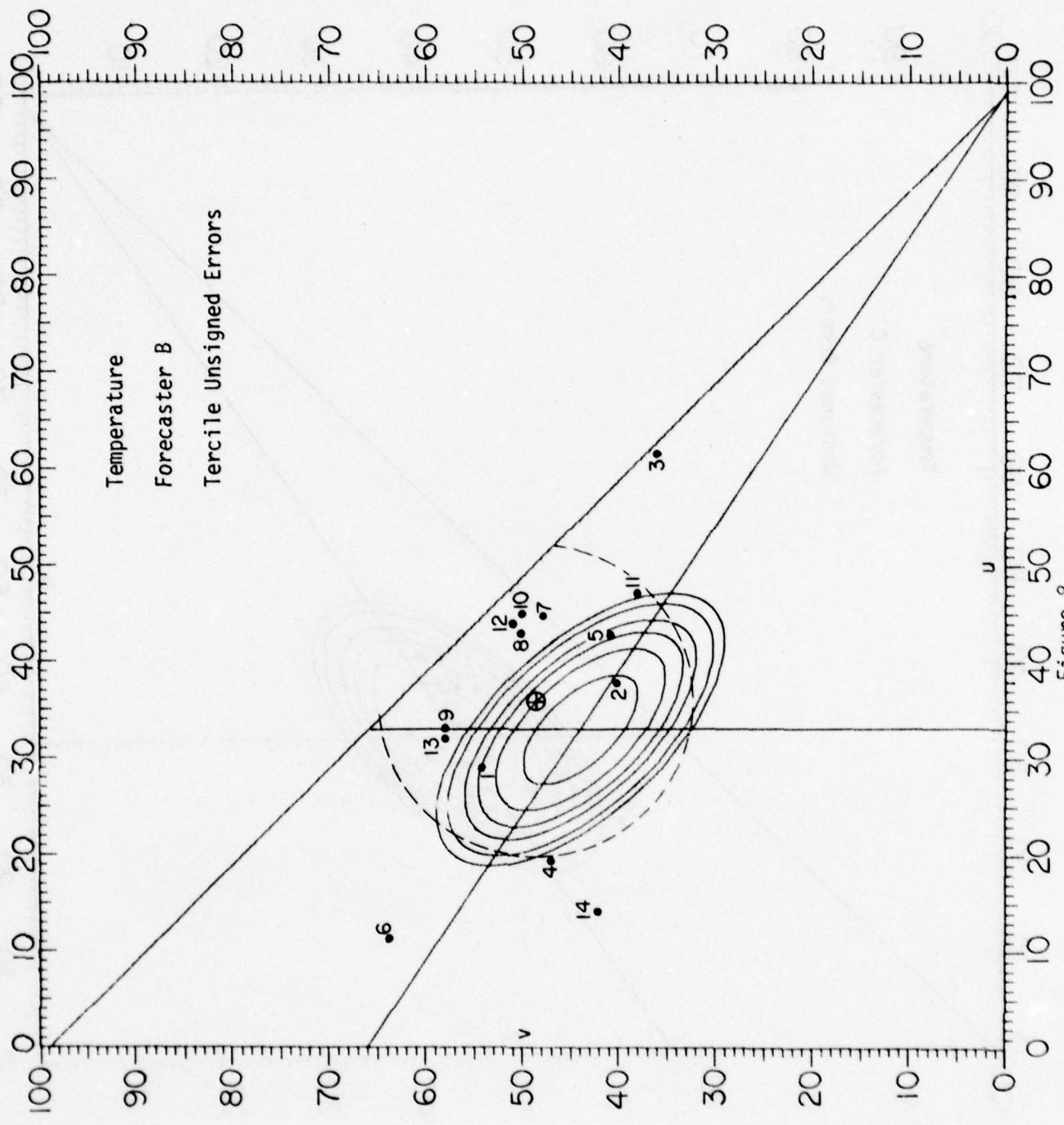


Figure 8

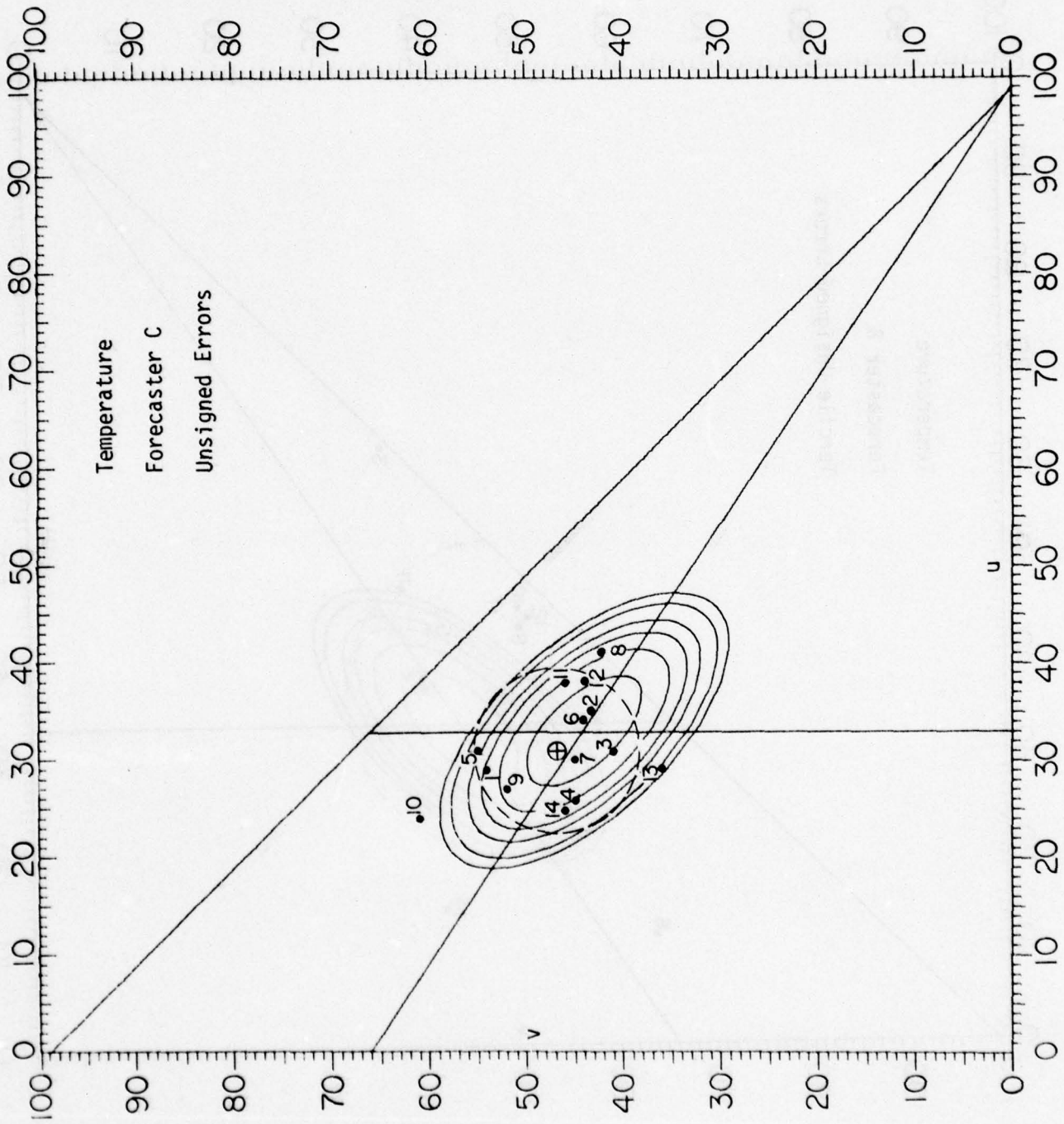


Figure 9

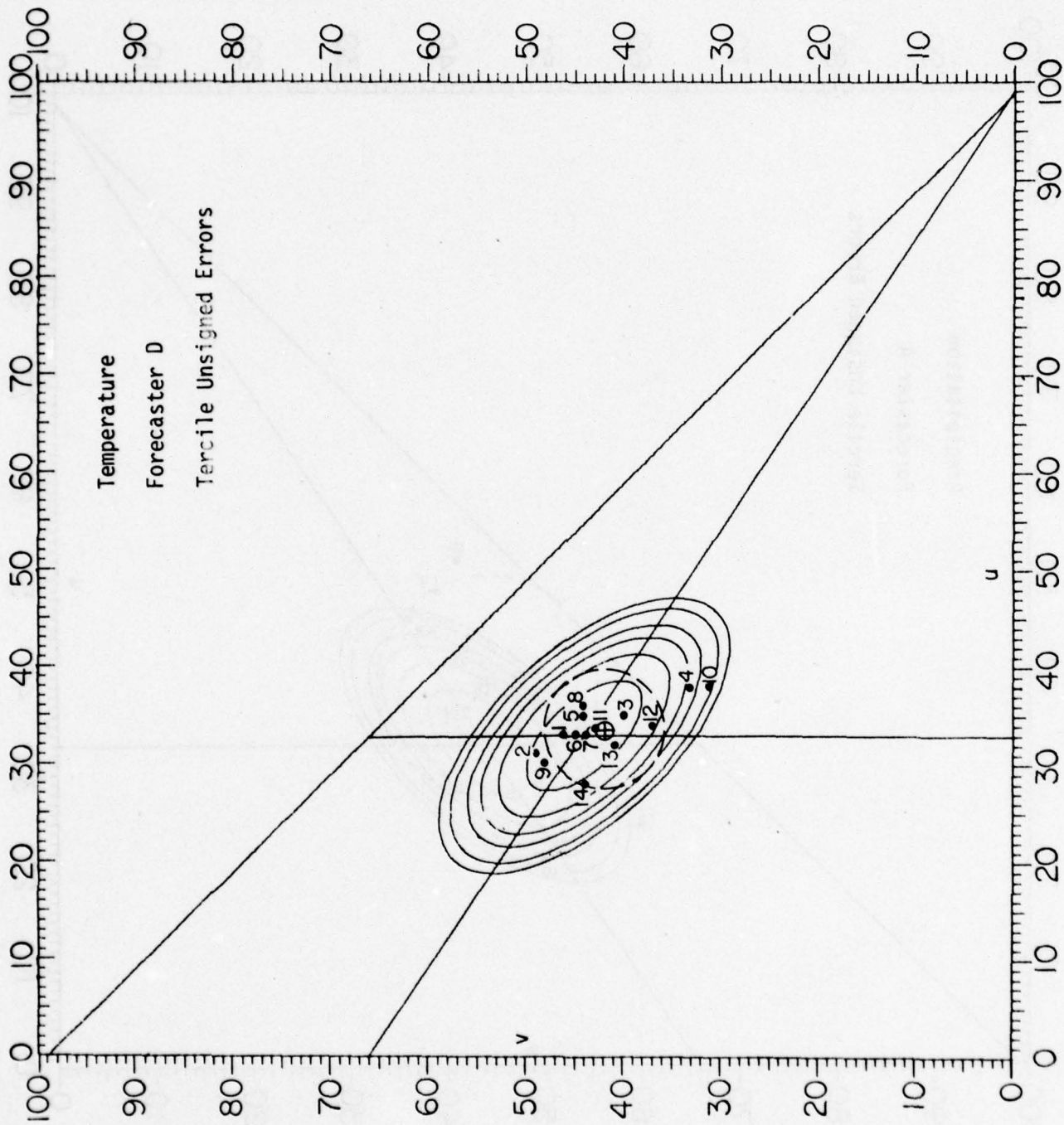


Figure 10

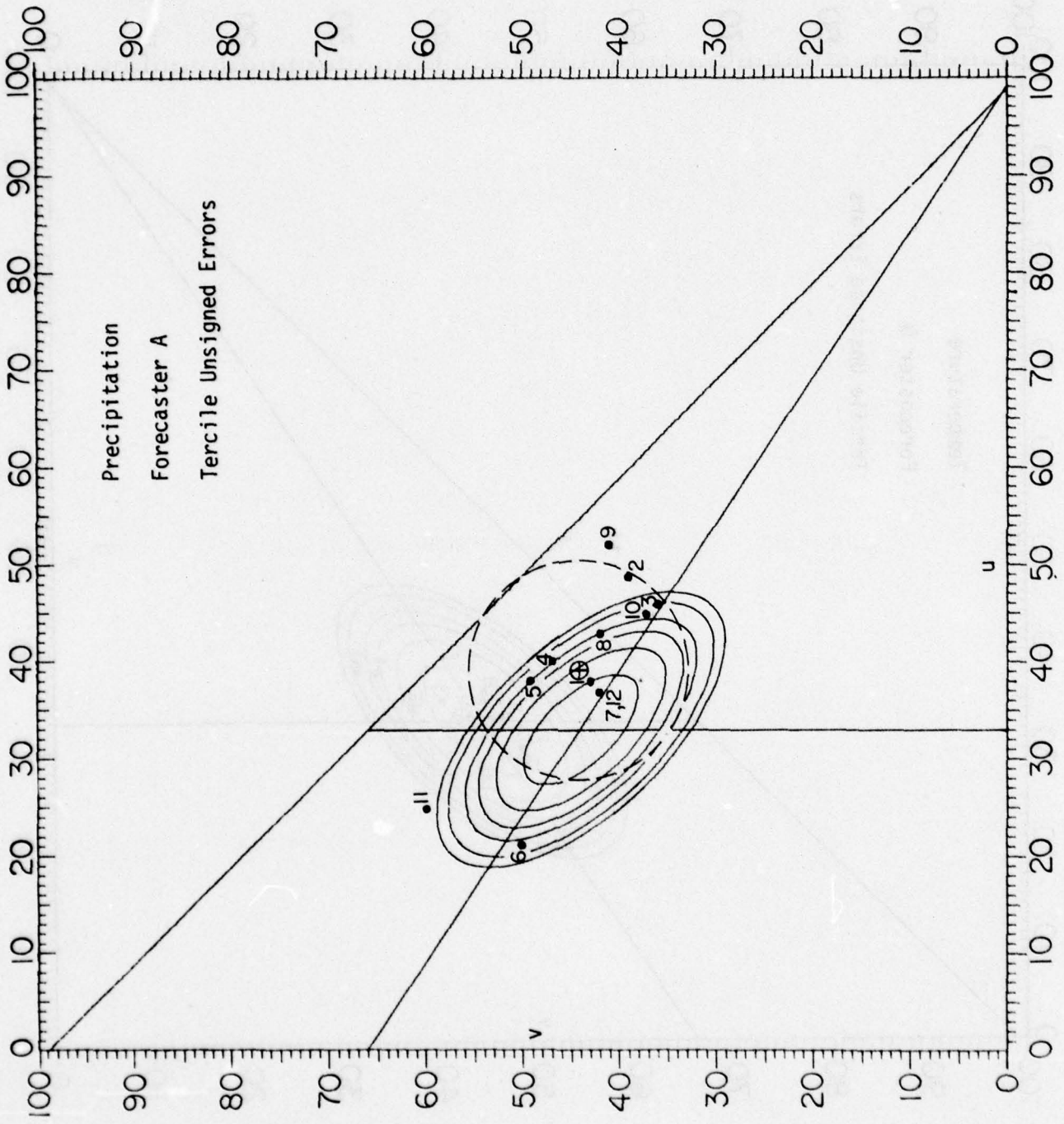


Figure 11

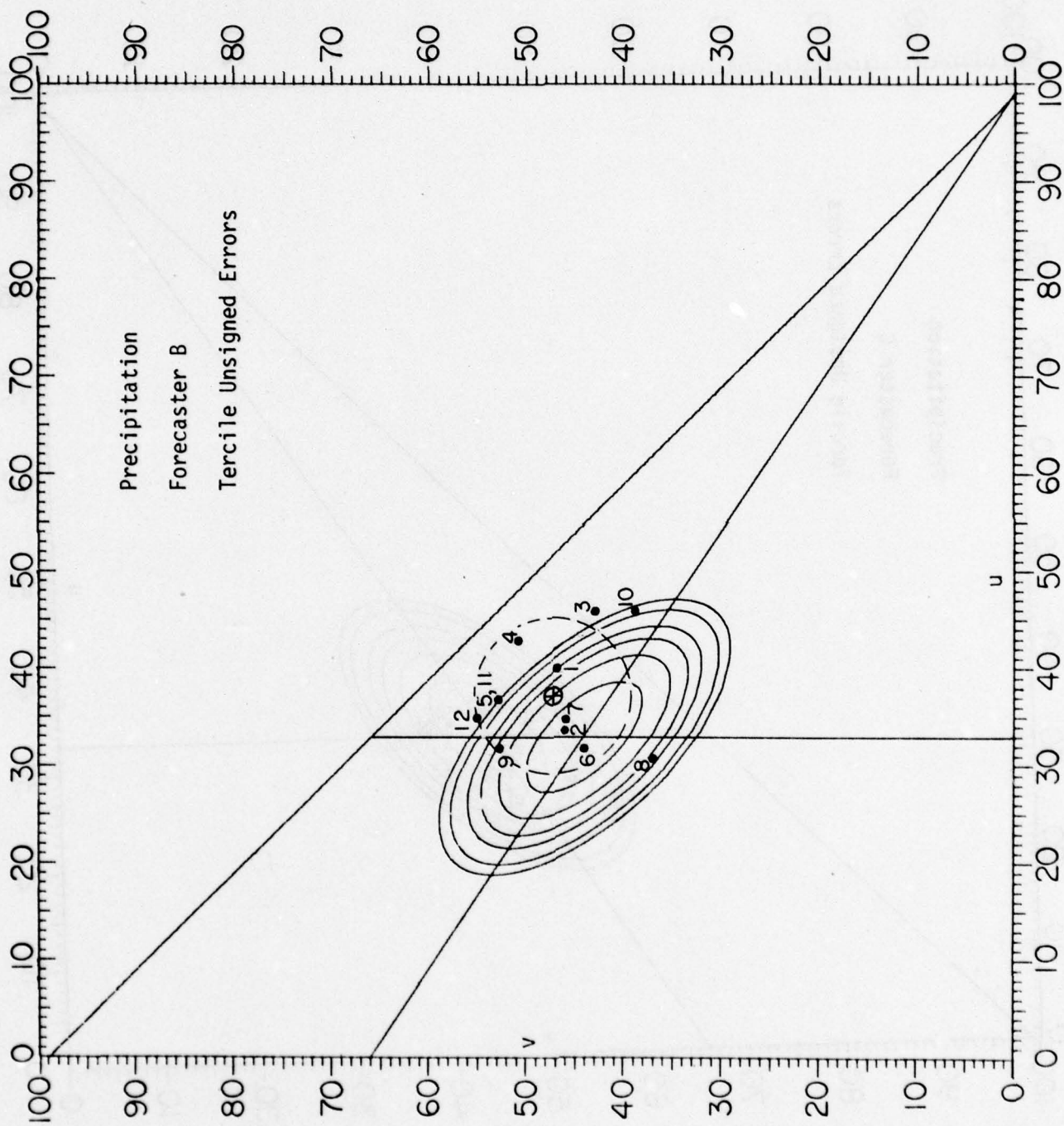


Figure 12

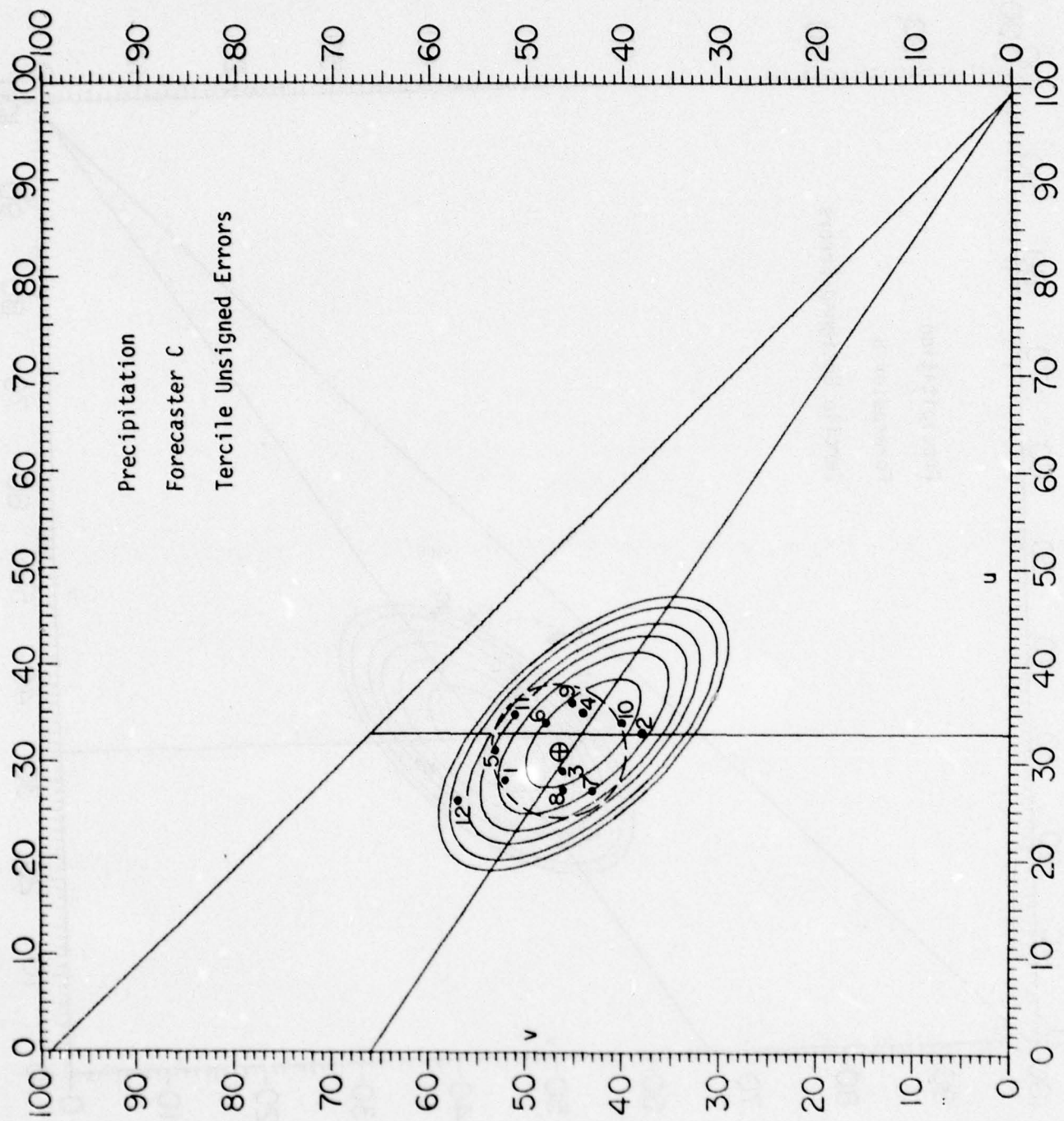


Figure 13

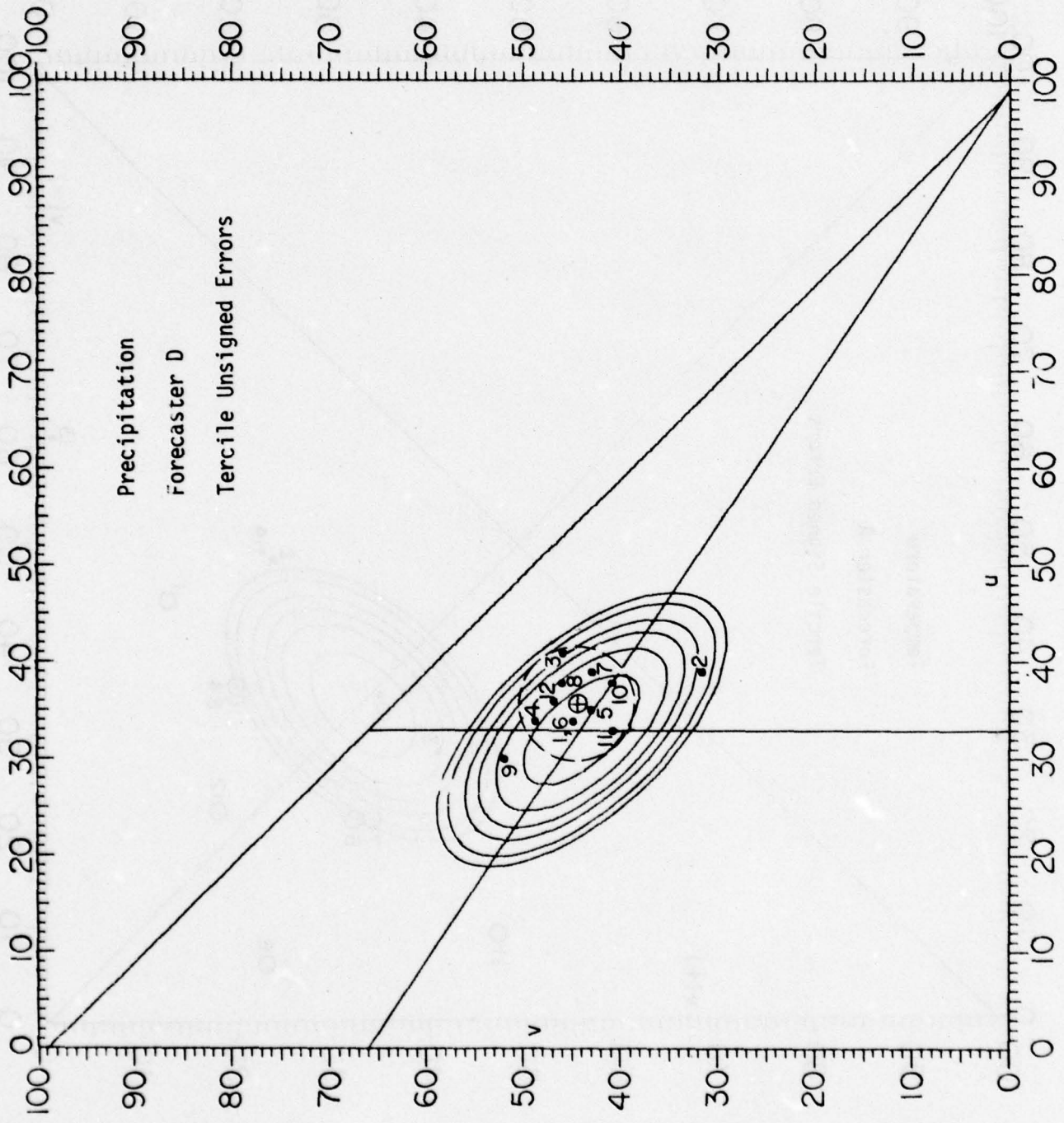


Figure 14

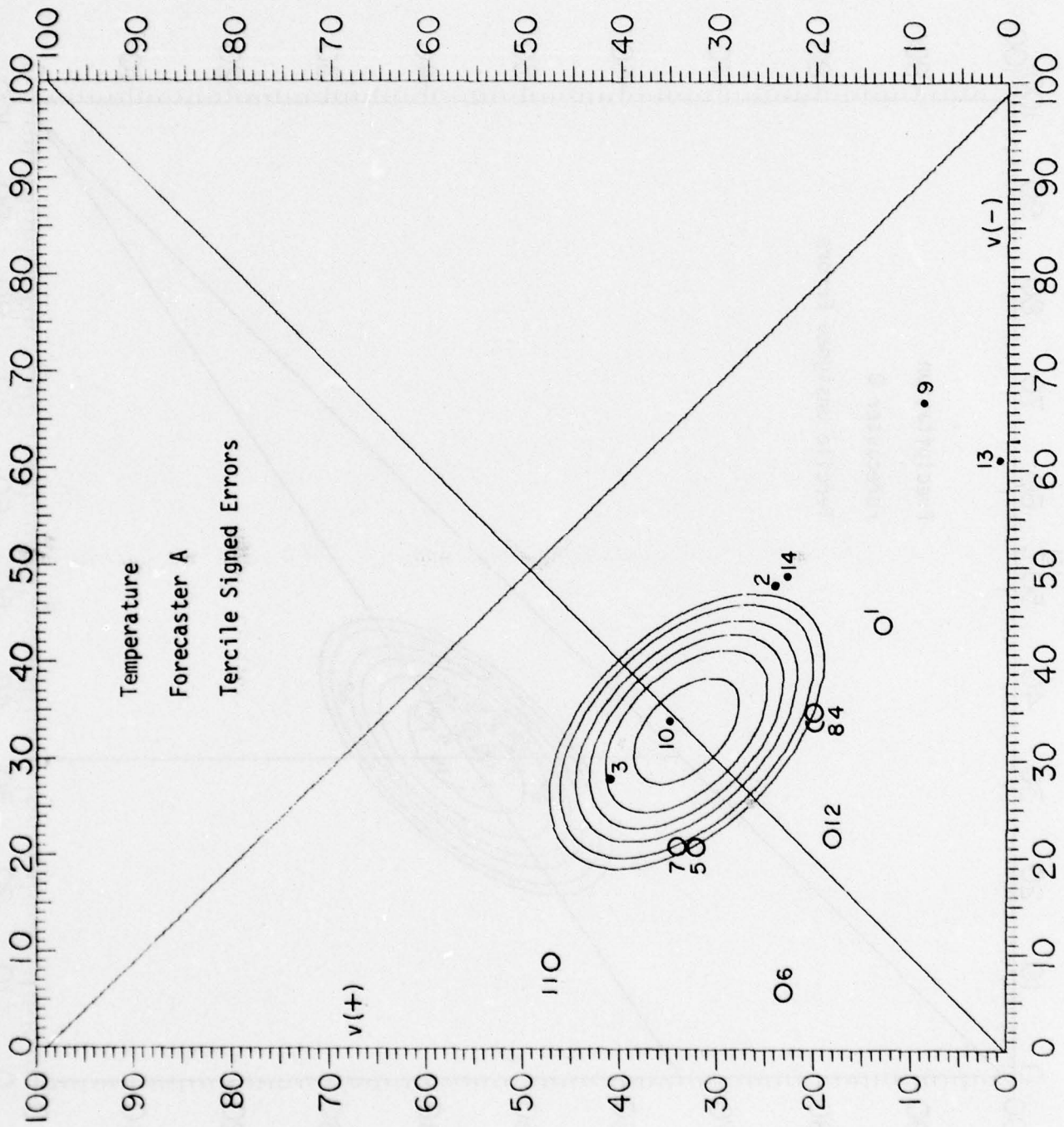


Figure 15

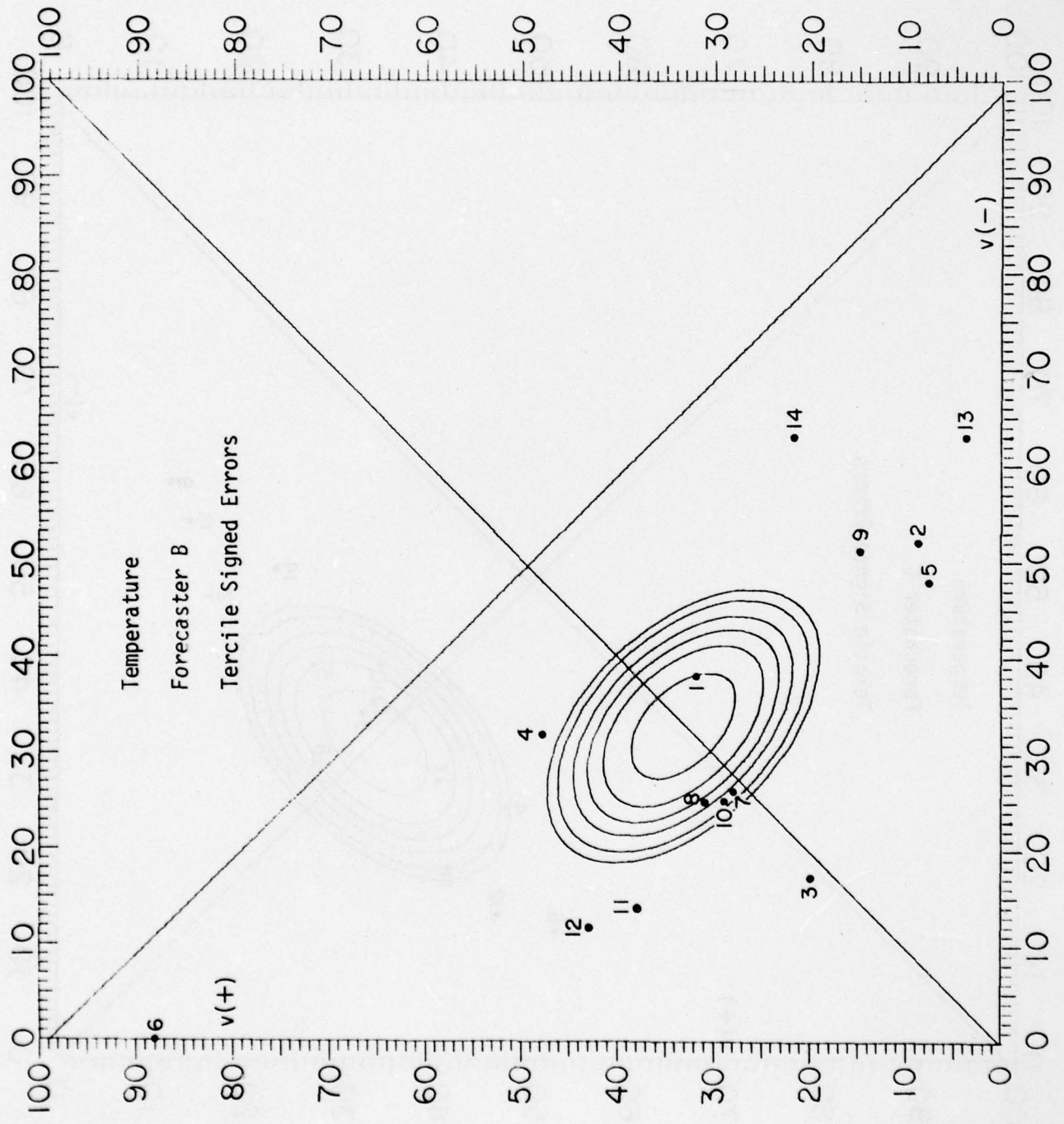


Figure 16

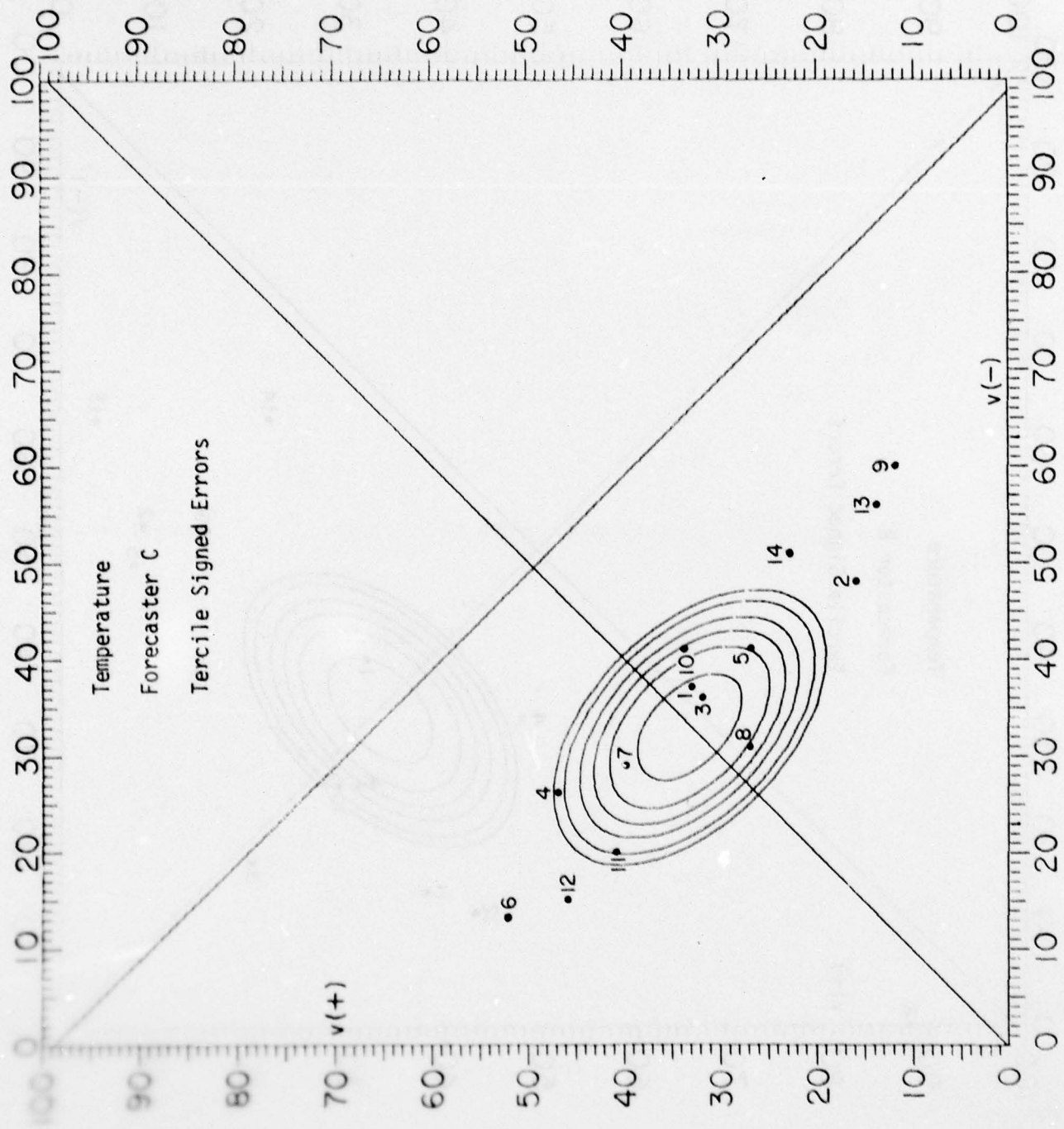


Figure 17

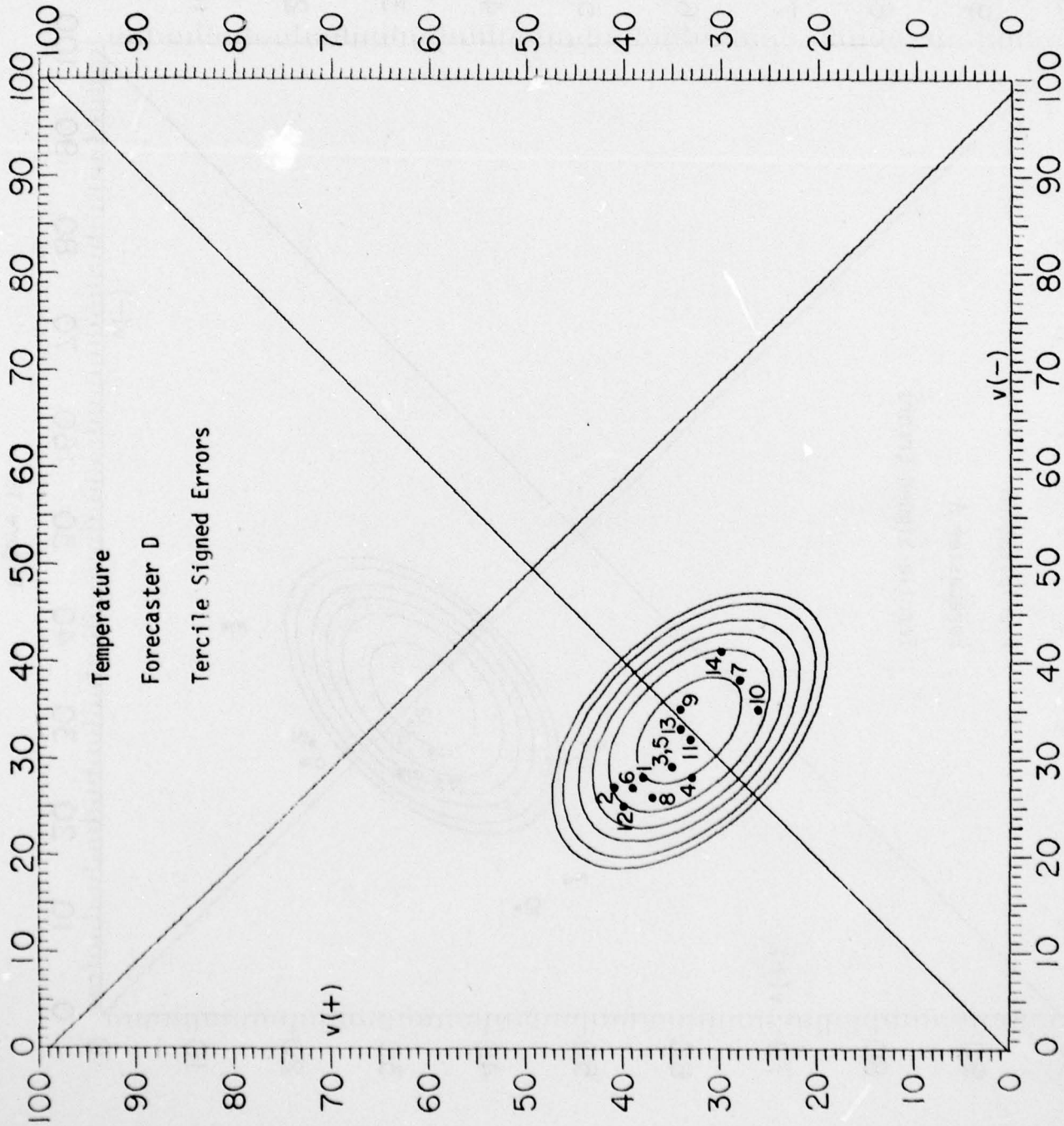


Figure 18

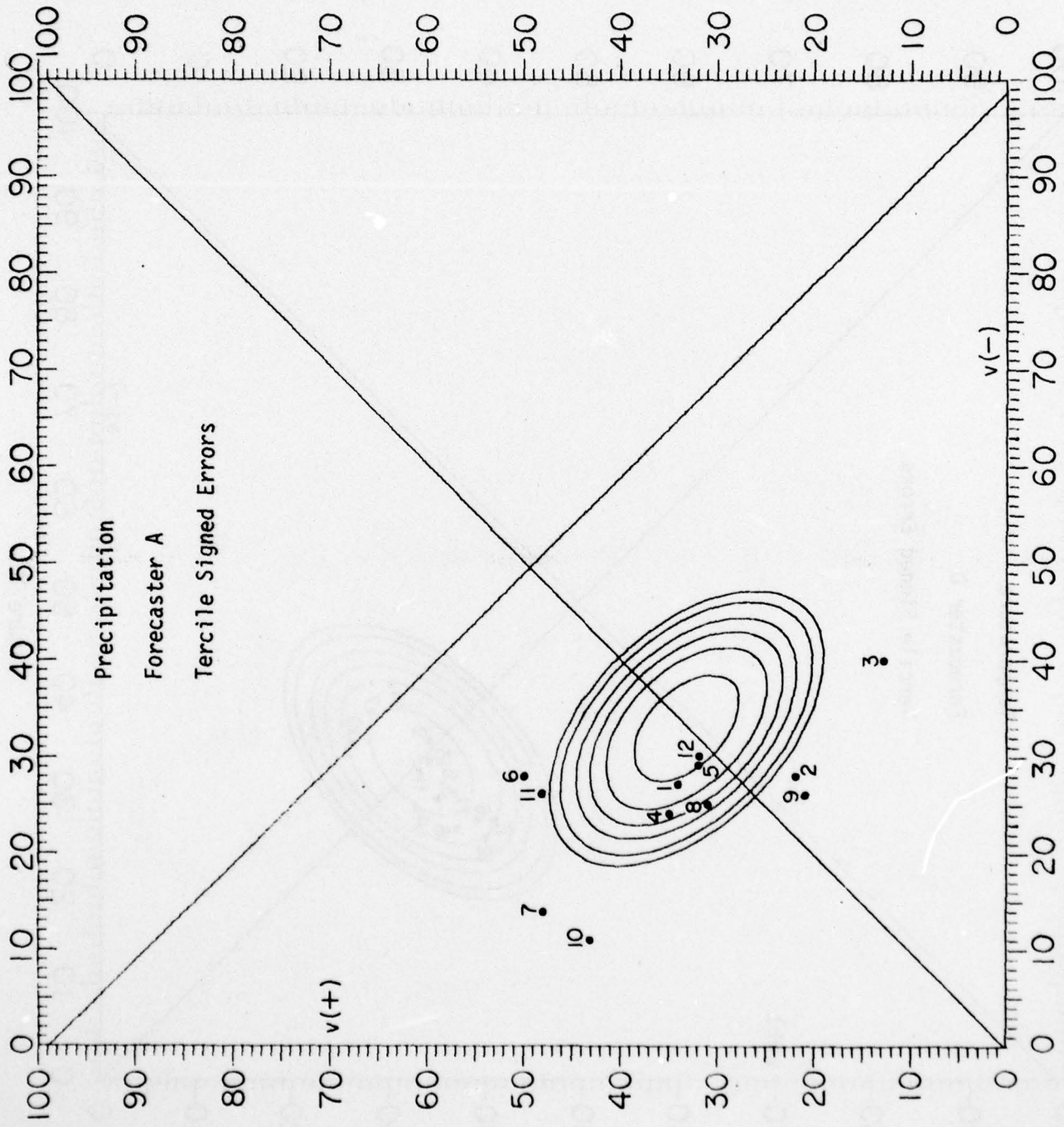


Figure 19

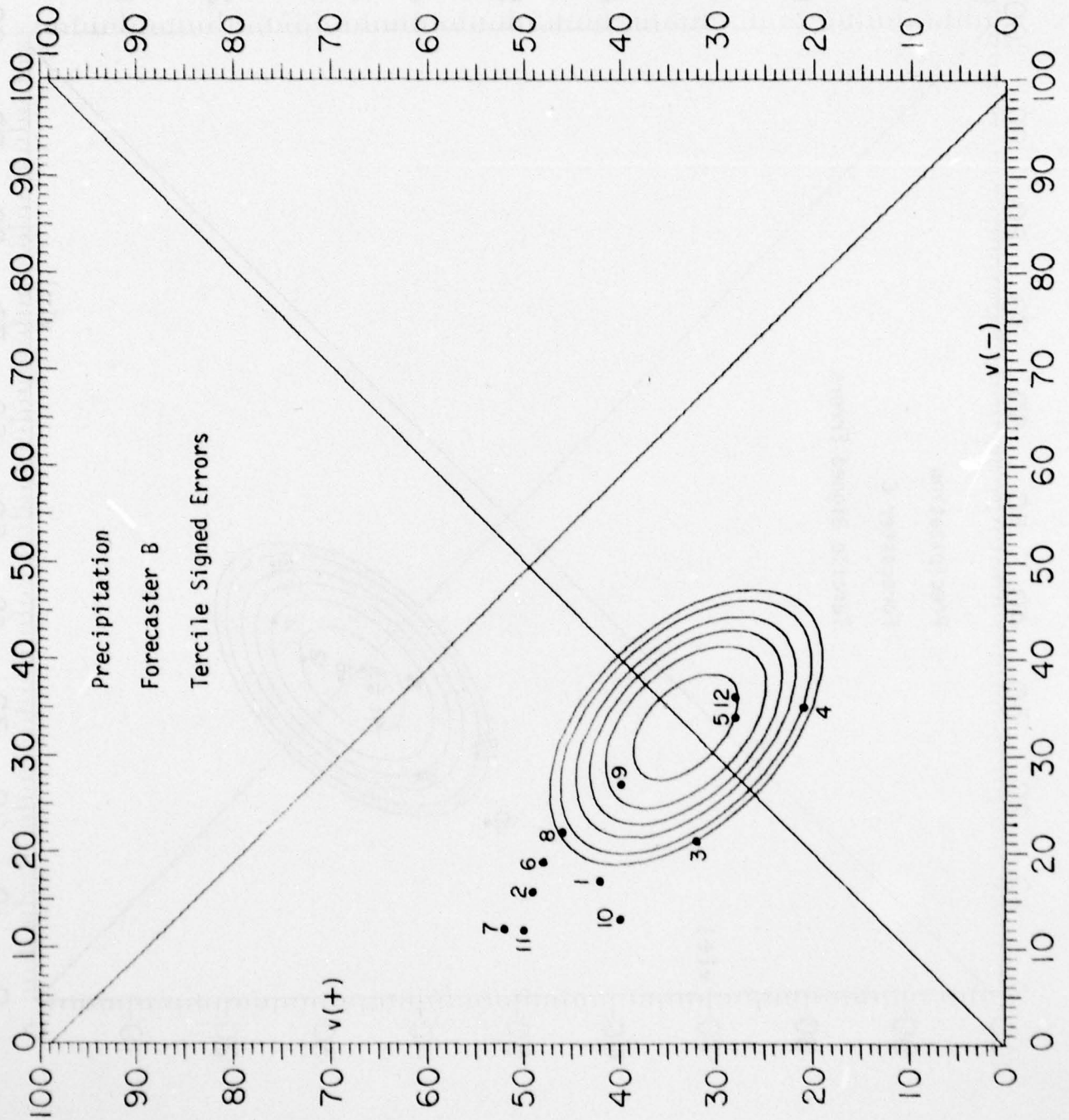


Figure 20

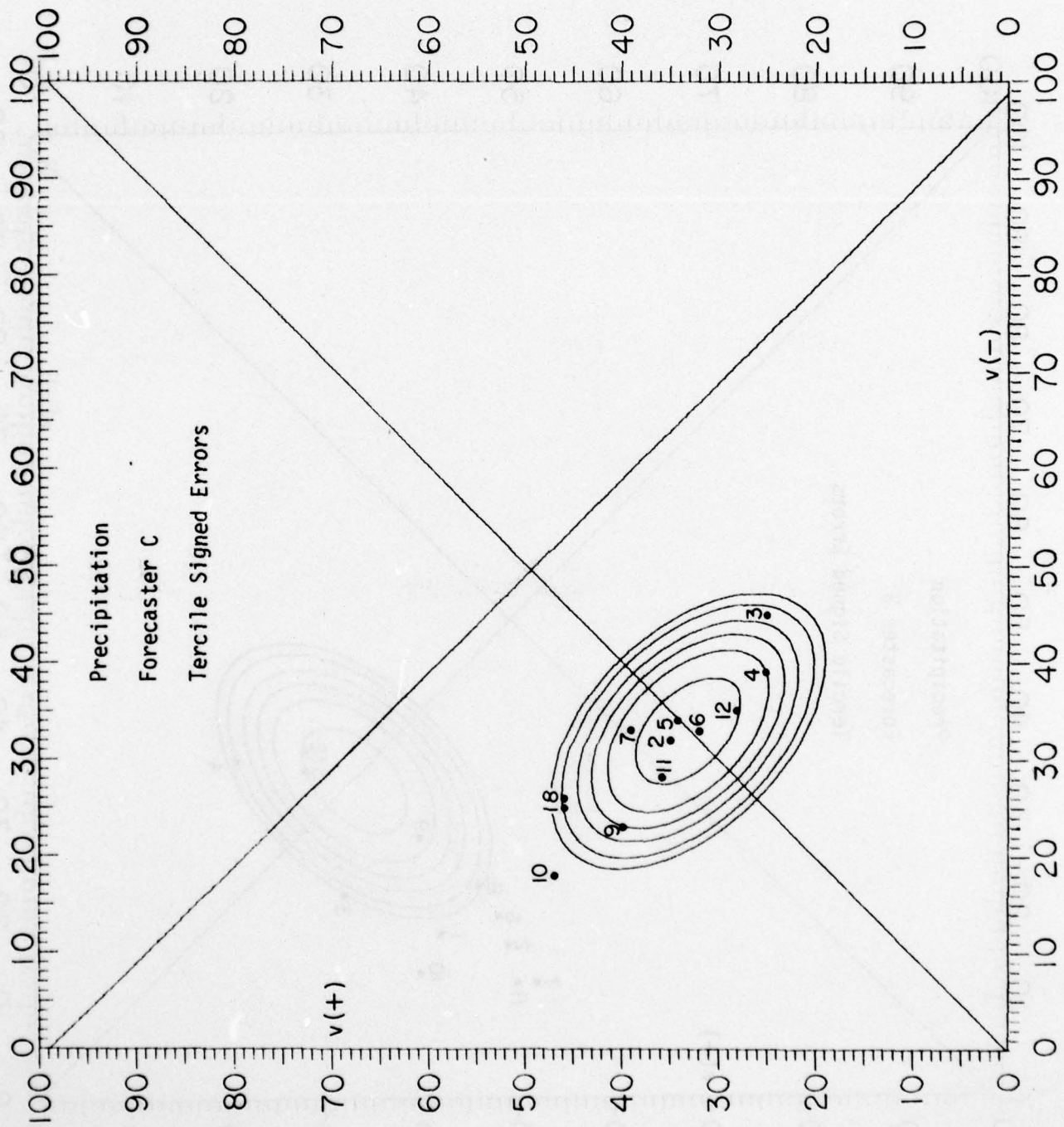


Figure 21

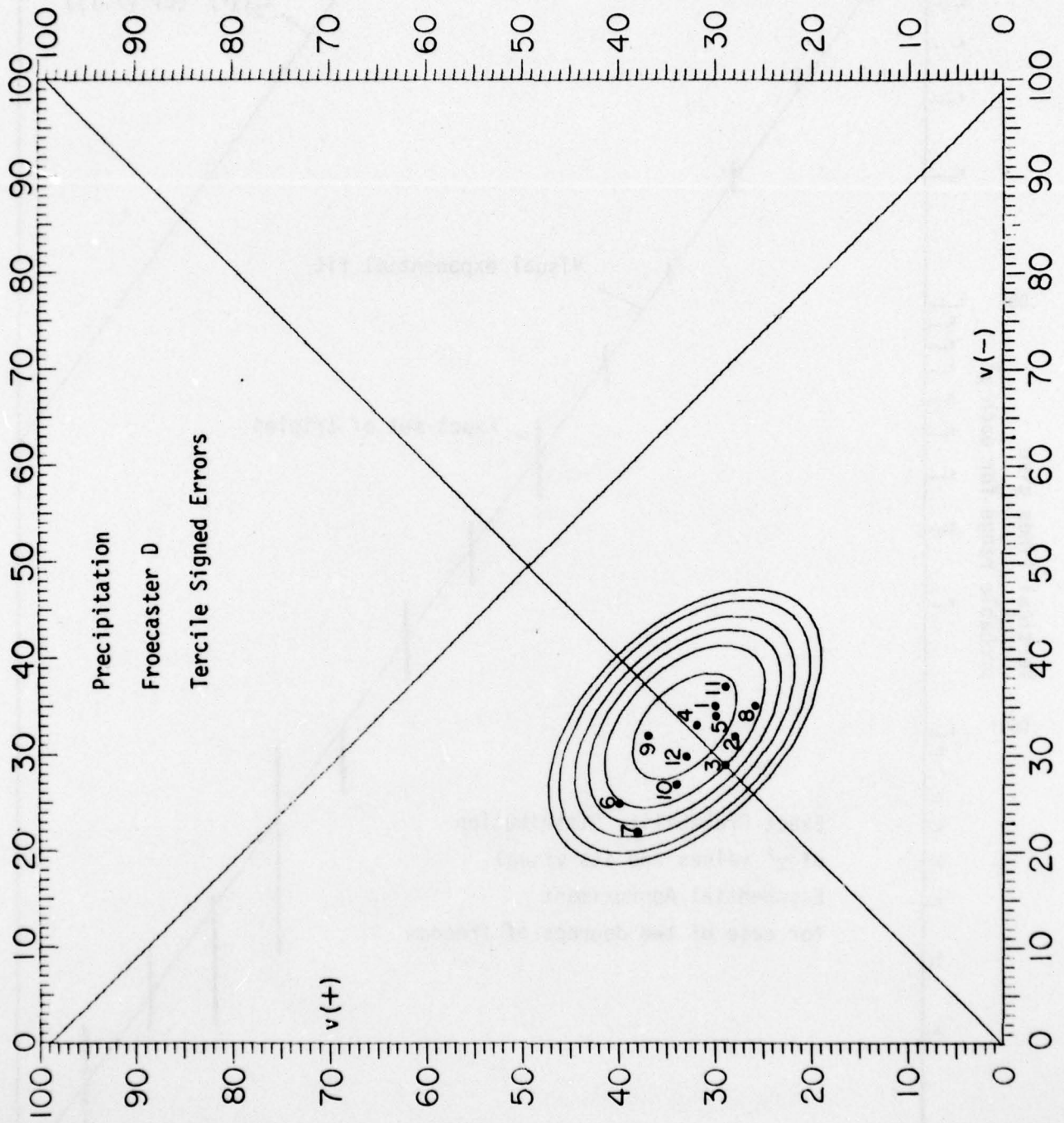


Figure 22

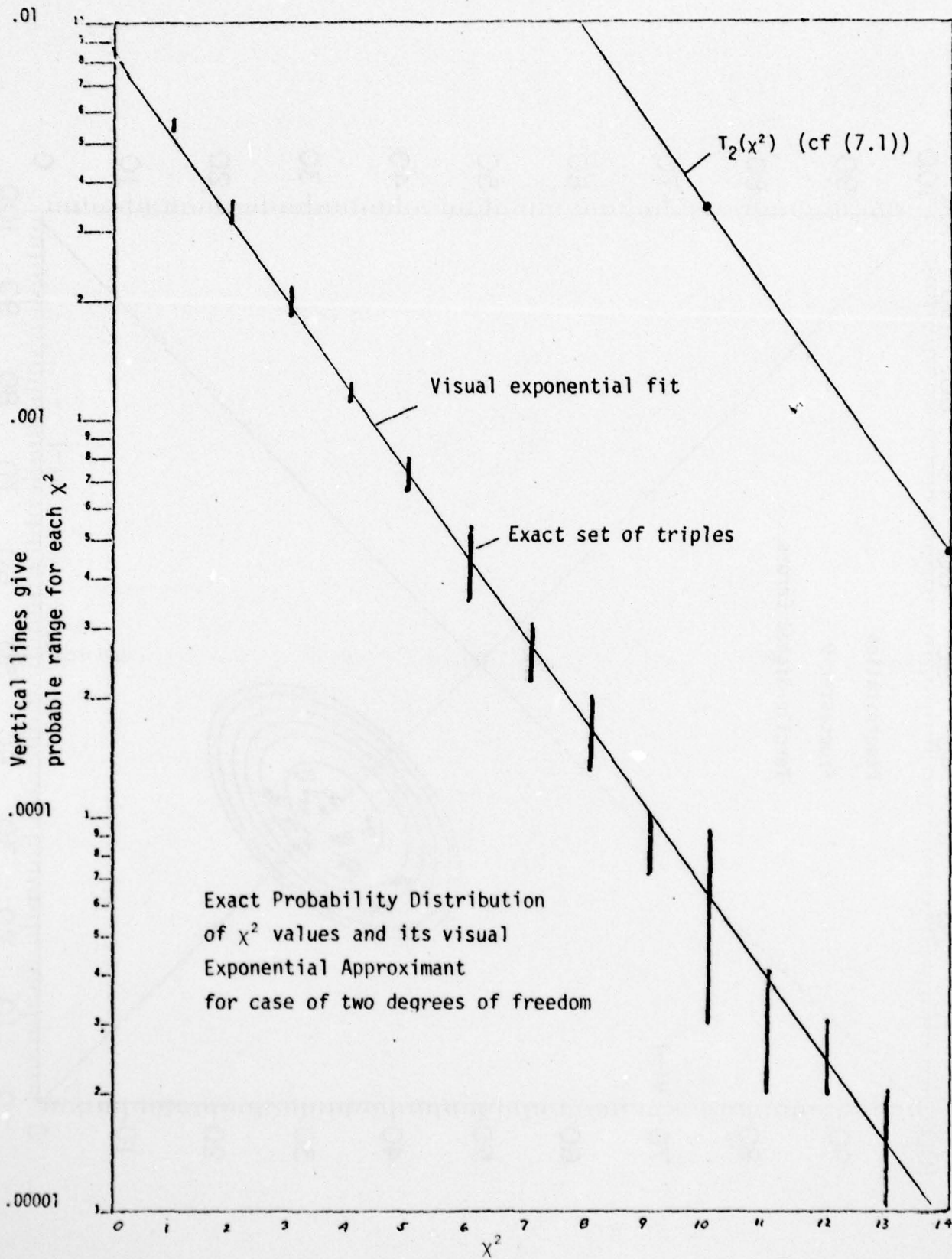
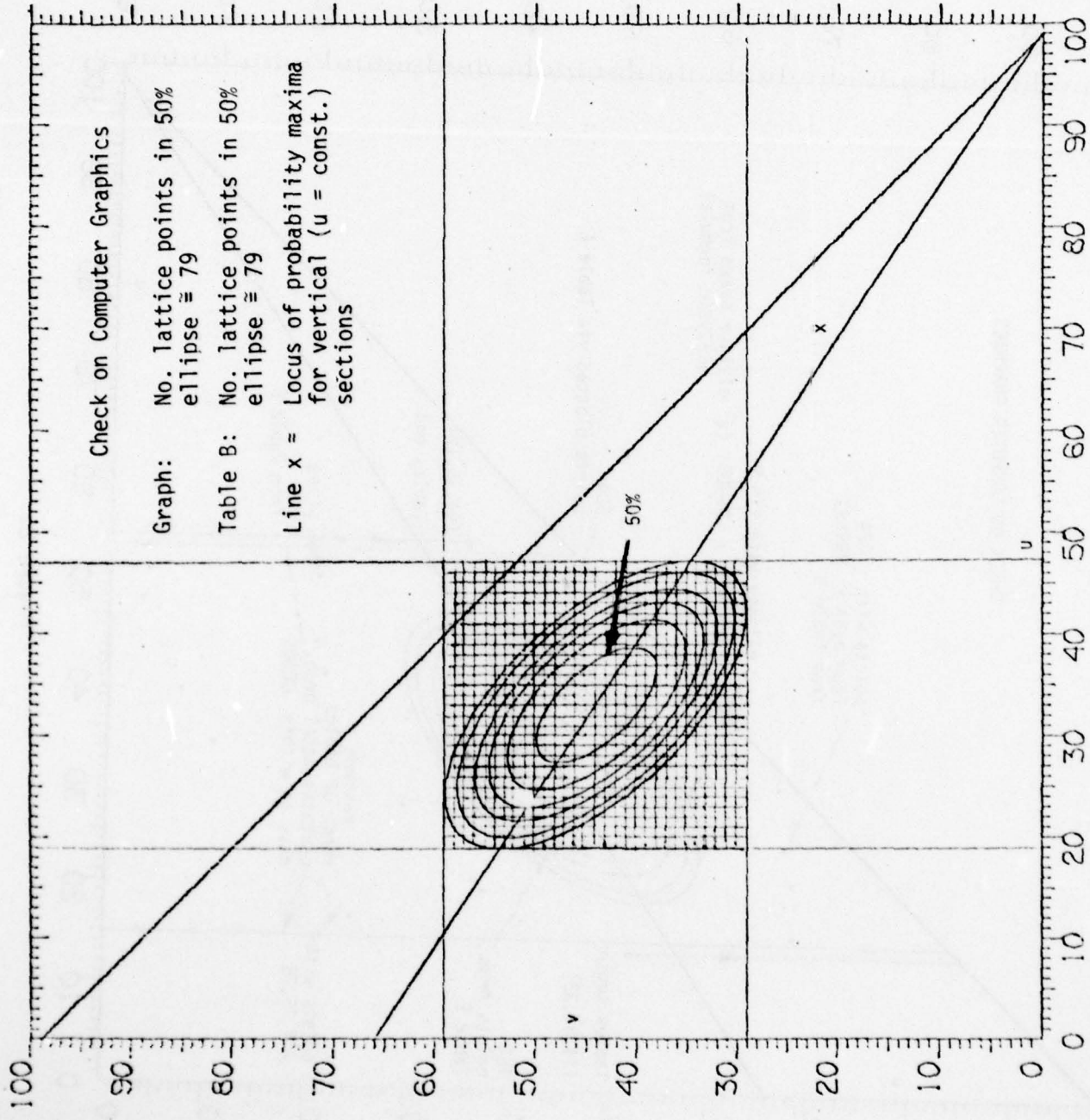


Figure 23



Check on Computer Graphics

- Graph: No. lattice points in 50% ellipse ≈ 79
- Table B: No. lattice points in 50% ellipse ≈ 79
- Line x = Locus of probability maxima for vertical ($u = \text{const.}$) sections

Figure 24

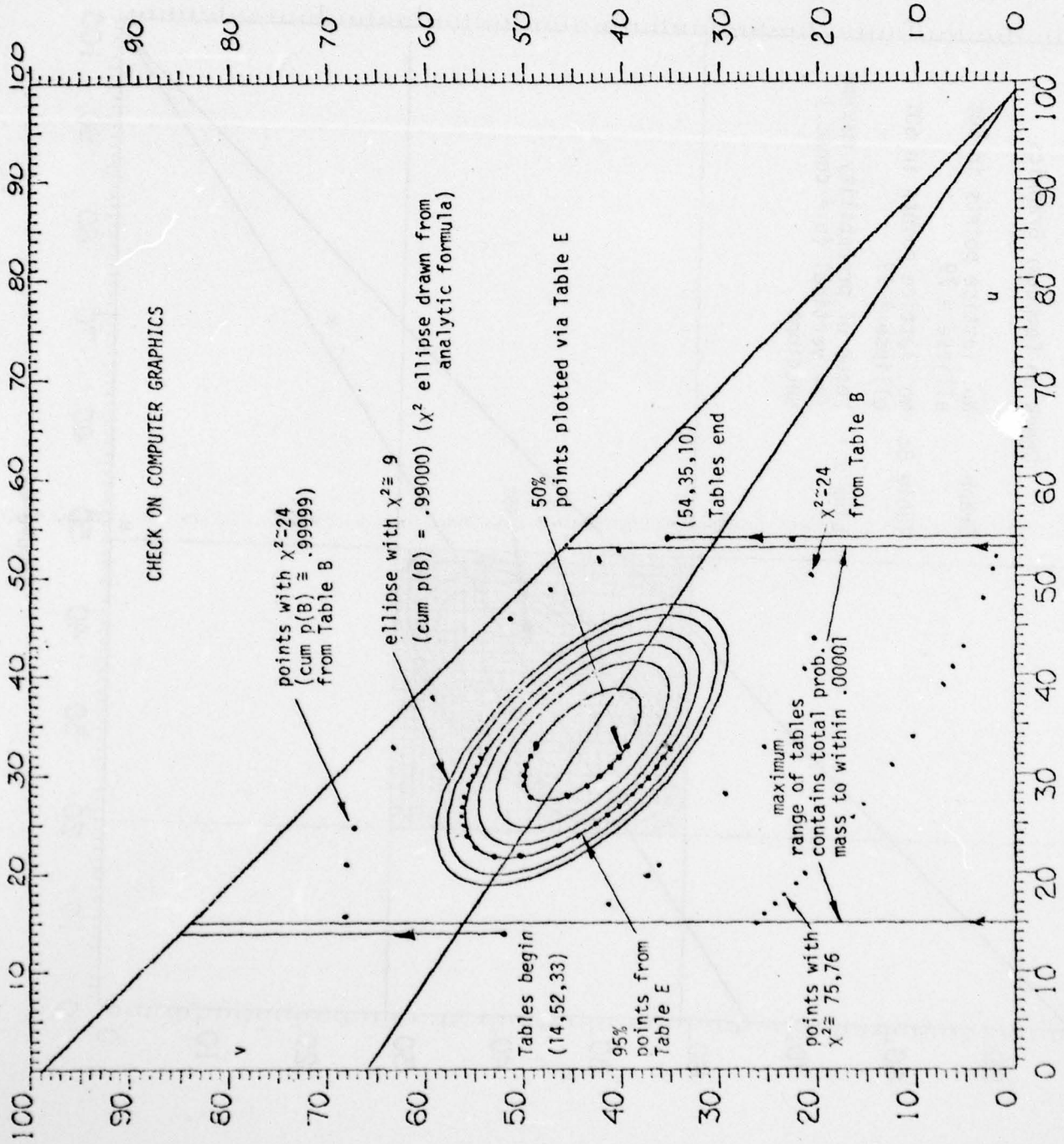


Figure 25

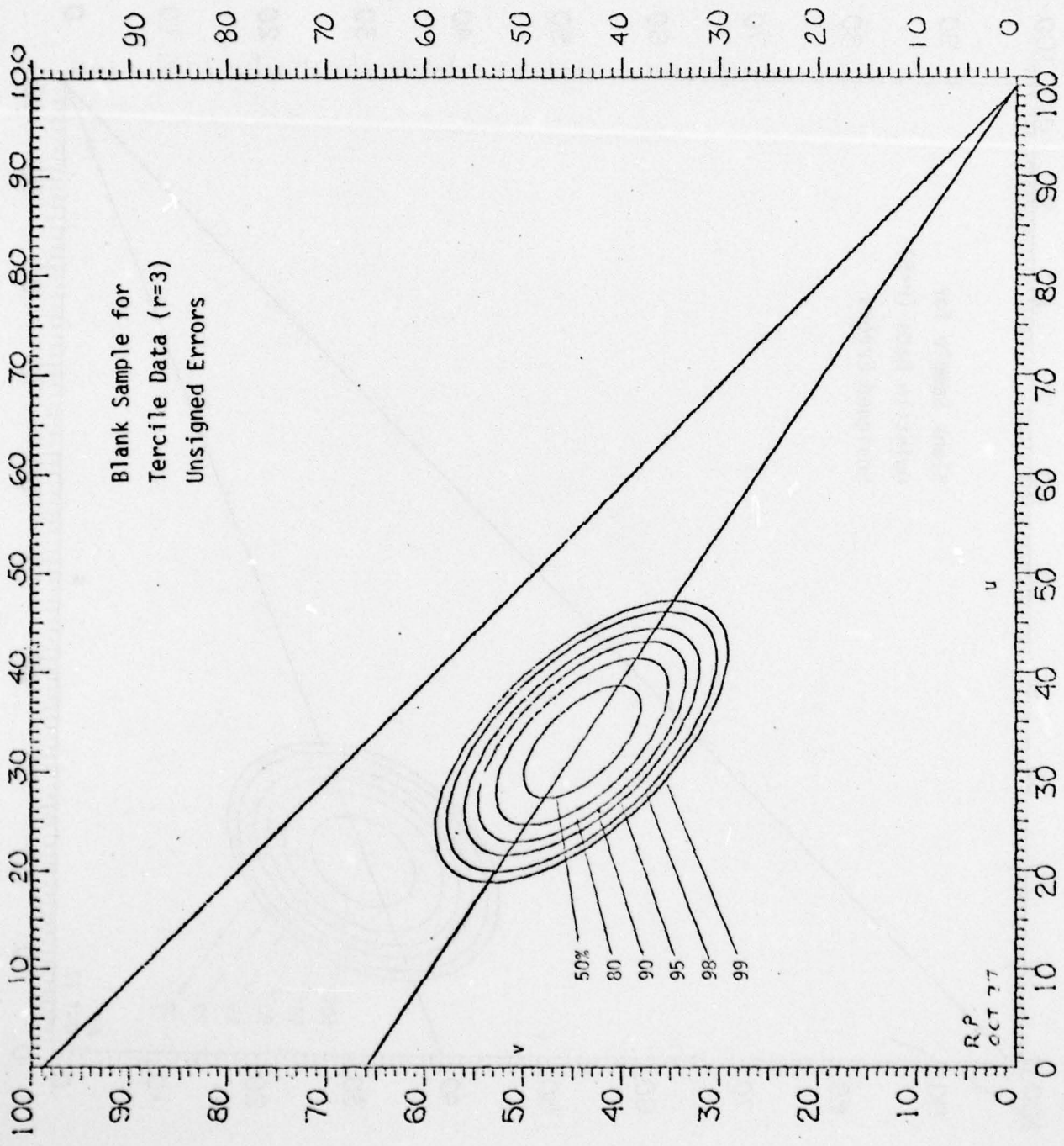


Figure 26

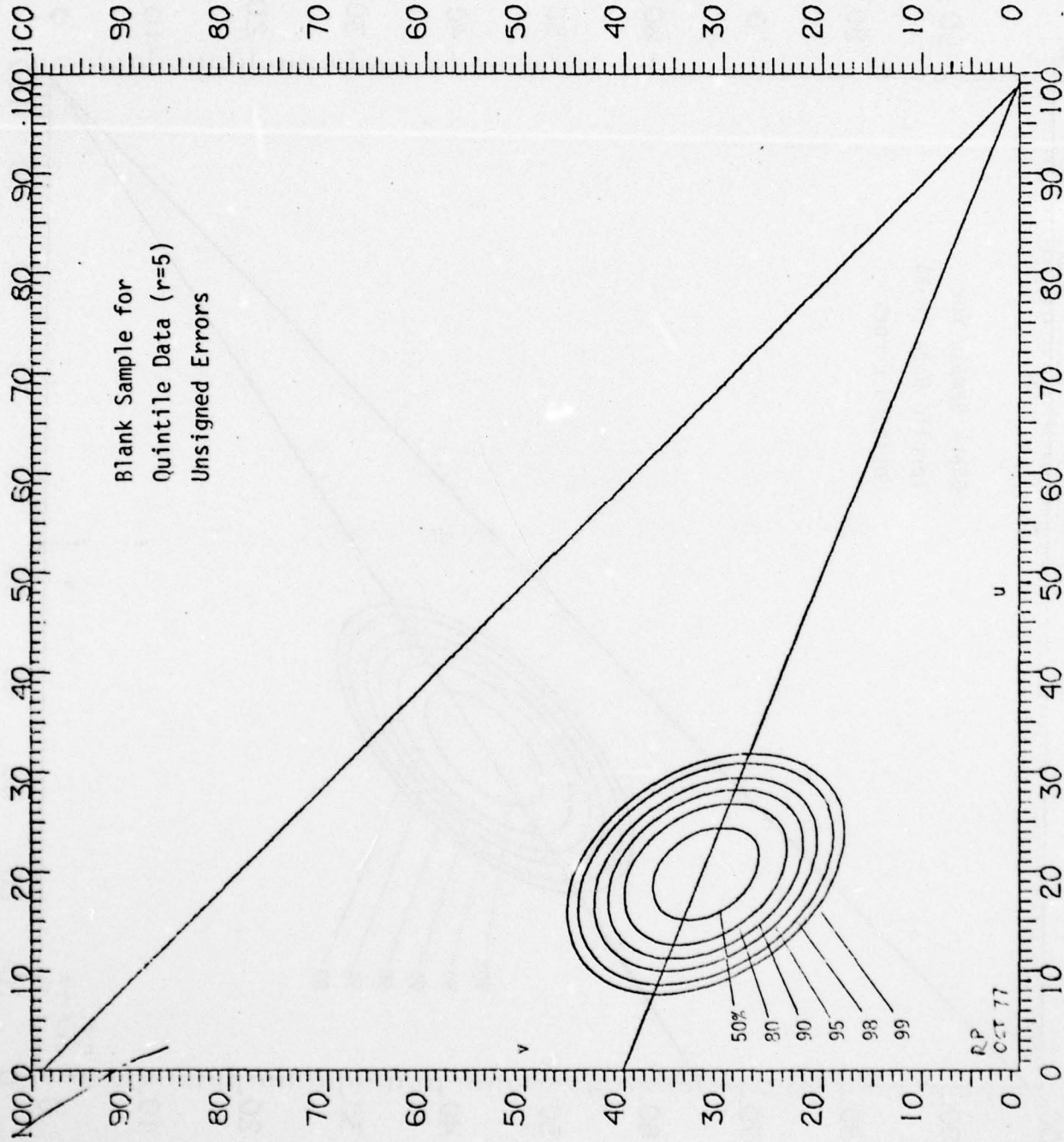


Figure 27

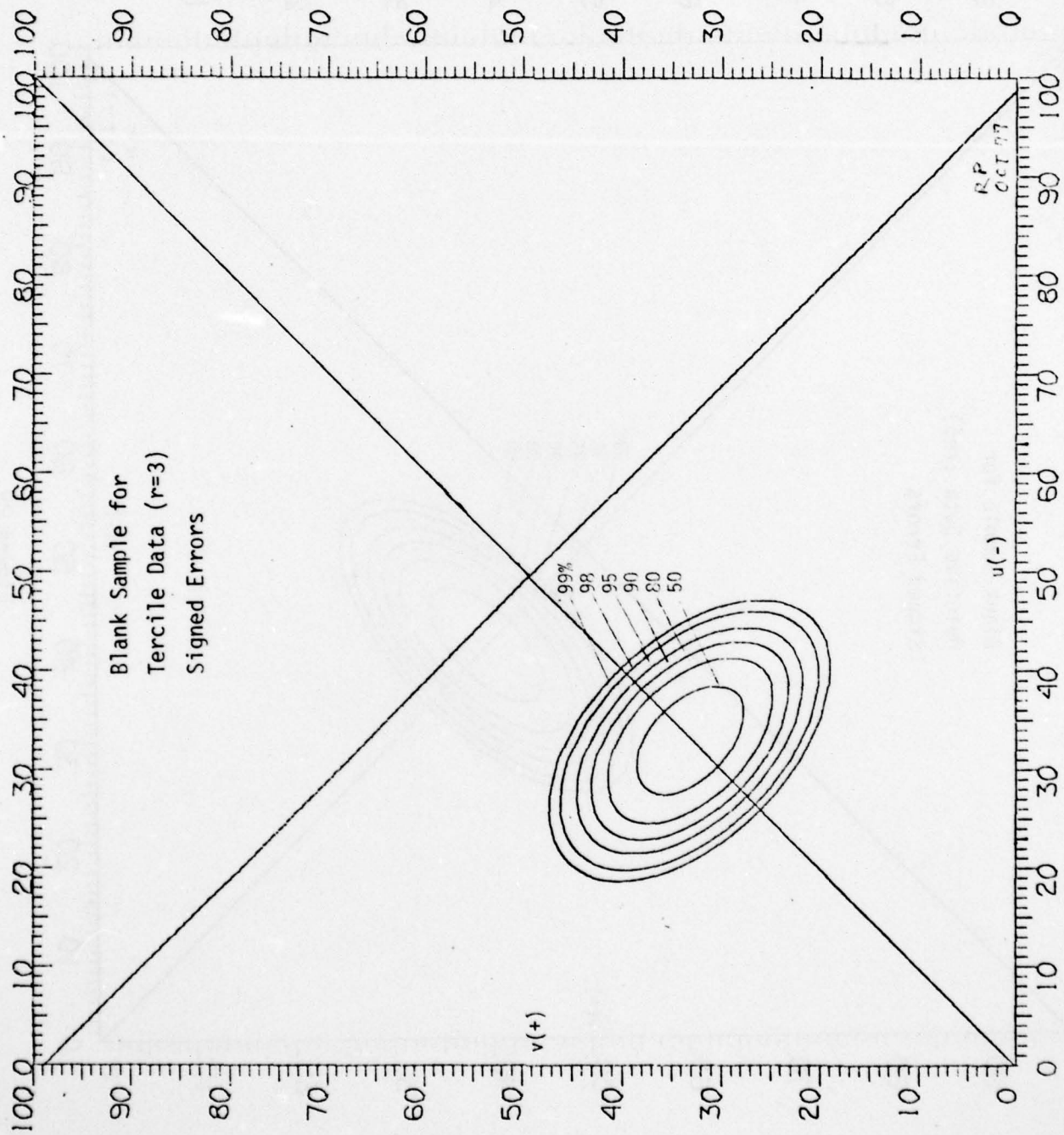


Figure 28

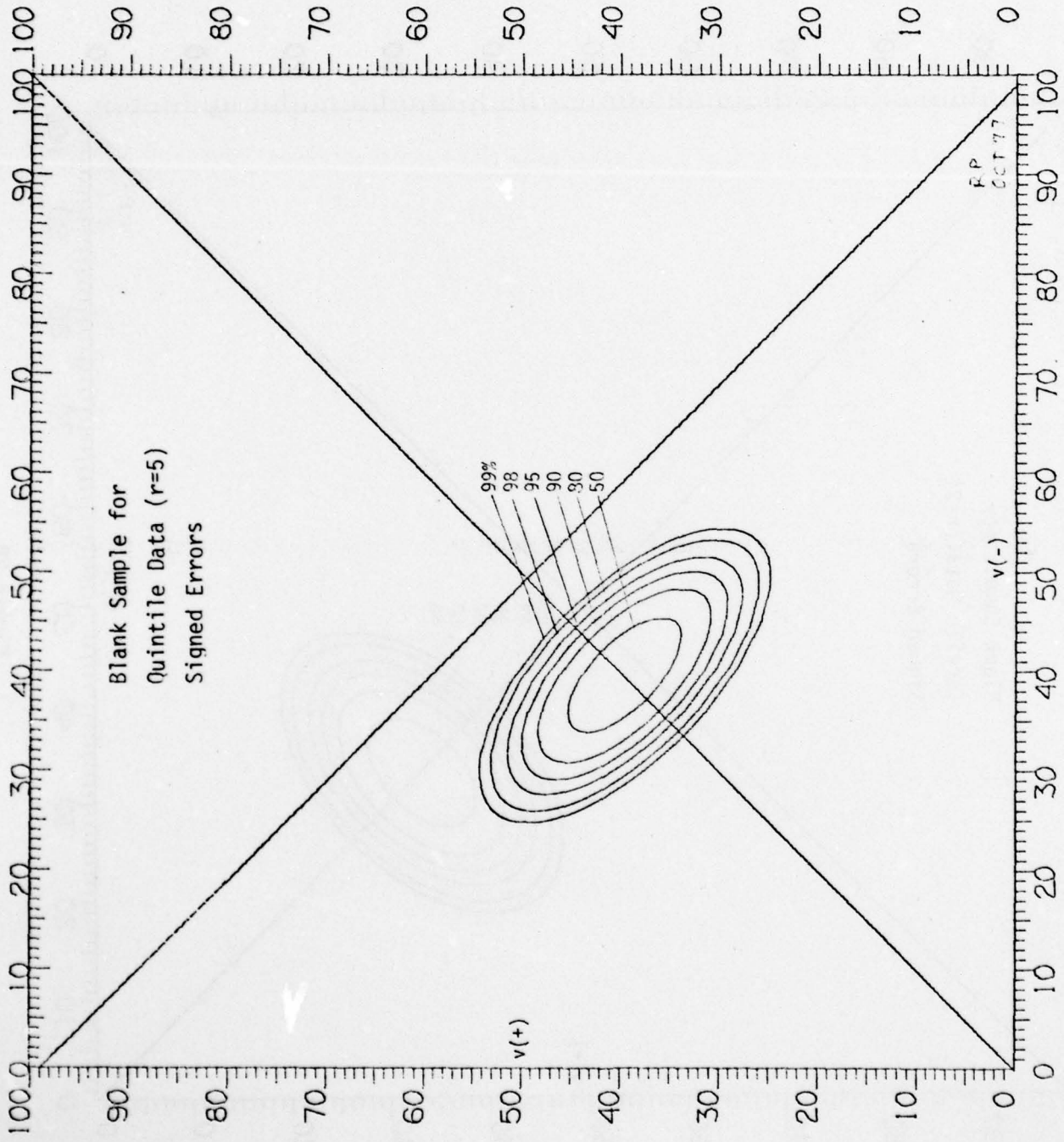


Figure 29

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

TABLE A

CHI SQUARE - P(1/3), P(4/9), P(2/27) N=99

U	V	W	X2	P(A)	CUM P(A)	U	V	W	X2	P(A)	CUM P(A)
15	52	33	17.8939	.00000	.00001	15	16	66	123.6182	.00000	.00001
15	53	34	17.3258	.00000	.00001	15	17	67	118.4318	.00000	.00001
15	54	35	16.8939	.00000	.00001	15	18	68	113.1818	.00000	.00001
15	55	36	16.5895	.00000	.00001	15	19	69	108.0682	.00000	.00001
15	56	37	16.3899	.00000	.00001	15	20	70	103.0909	.00000	.00001
15	57	38	16.2467	.00000	.00001	15	21	71	98.2506	.00000	.00001
15	58	39	16.1533	.00000	.00001	15	22	72	93.5455	.00000	.00001
15	59	40	16.1030	.00000	.00001	15	23	73	88.9773	.00000	.00001
15	60	41	16.0916	.00000	.00001	15	24	74	84.5455	.00000	.00001
15	61	42	16.1167	.00000	.00001	15	25	75	80.2530	.00000	.00001
15	62	43	16.1835	.00000	.00001	15	26	76	76.0909	.00000	.00001
15	63	44	16.2879	.00000	.00001	15	27	77	72.0682	.00000	.00001
15	64	45	16.4249	.00000	.00001	15	28	78	68.1818	.00000	.00001
15	65	46	16.5983	.00000	.00001	15	29	79	64.4318	.00000	.00001
15	66	47	16.8085	.00000	.00001	15	30	80	60.8182	.00000	.00001
15	67	48	17.0569	.00000	.00001	15	31	81	57.3409	.00000	.00001
15	68	49	17.3447	.00000	.00001	15	32	82	54.0000	.00000	.00001
15	69	50	17.6730	.00000	.00001	15	33	83	50.7955	.00000	.00001
15	70	51	18.0439	.00000	.00001	15	34	84	47.7273	.00000	.00001
15	71	52	18.4595	.00000	.00001	15	35	85	44.7955	.00000	.00001
15	72	53	18.9230	.00000	.00001	15	36	86	42.0000	.00000	.00001
15	73	54	19.4379	.00000	.00001	15	37	87	39.3409	.00000	.00001
15	74	55	19.9979	.00000	.00001	15	38	88	36.8182	.00000	.00001
15	75	56	20.6081	.00000	.00001	15	39	89	34.4318	.00000	.00001
15	76	57	21.2747	.00000	.00001	15	40	90	32.1818	.00000	.00001
15	77	58	21.9935	.00000	.00001	15	41	91	30.0682	.00000	.00001
15	78	59	22.7616	.00000	.00001	15	42	92	28.0909	.00000	.00001
15	79	60	23.5759	.00000	.00001	15	43	93	26.2530	.00000	.00001
15	80	61	24.4433	.00000	.00001	15	44	94	24.5455	.00000	.00001
15	81	62	25.3616	.00000	.00001	15	45	95	22.9773	.00000	.00001
15	82	63	26.3380	.00000	.00001	15	46	96	21.5455	.00000	.00001
15	83	64	27.3705	.00000	.00001	15	47	97	20.2530	.00000	.00001
15	84	65	28.4583	.00000	.00001	15	48	98	19.0909	.00000	.00001
15	85	66	29.5999	.00000	.00001	15	49	99	18.0682	.00000	.00001
15	86	67	30.7938	.00000	.00001	15	50	100	17.1818	.00000	.00001
15	87	68	32.0394	.00000	.00001	15	51	101	16.4318	.00000	.00001
15	88	69	33.3361	.00000	.00001	15	52	102	15.8182	.00000	.00001
15	89	70	34.6835	.00000	.00001	15	53	103	15.3409	.00000	.00001
15	90	71	36.0811	.00000	.00001	15	54	104	14.9773	.00000	.00001
15	91	72	37.5285	.00000	.00001	15	55	105	14.7000	.00000	.00001
15	92	73	39.0252	.00000	.00001	15	56	106	14.5000	.00000	.00001
15	93	74	40.5708	.00000	.00001	15	57	107	14.3636	.00000	.00001
15	94	75	42.1649	.00000	.00001	15	58	108	14.2818	.00000	.00001
15	95	76	43.8071	.00000	.00001	15	59	109	14.2500	.00000	.00001
15	96	77	45.4971	.00000	.00001	15	60	110	14.1667	.00000	.00001
15	97	78	47.2345	.00000	.00001	15	61	111	14.1250	.00000	.00001
15	98	79	49.0189	.00000	.00001	15	62	112	14.0167	.00000	.00001
15	99	80	50.8500	.00000	.00001	15	63	113	13.9583	.00000	.00001

(1)

TABLE A

U	V	W	X2	P(A)	CUM P(A)	U	V	W	X2	P(A)	CUM P(A)
16	31	52	53.5076	.00000	.00004	16	31	52	53.5076	.00000	.00004
16	32	51	50.2576	.00000	.00004	16	32	51	50.2576	.00000	.00004
16	33	50	47.1439	.00000	.00004	16	33	50	47.1439	.00000	.00004
16	34	49	44.1667	.00000	.00004	16	34	49	44.1667	.00000	.00004
16	35	48	41.3258	.00000	.00004	16	35	48	41.3258	.00000	.00004
16	36	47	38.6212	.00000	.00004	16	36	47	38.6212	.00000	.00004
16	37	46	36.0530	.00000	.00004	16	37	46	36.0530	.00000	.00004
16	38	45	33.6212	.00000	.00004	16	38	45	33.6212	.00000	.00004
16	39	44	31.3258	.00000	.00004	16	39	44	31.3258	.00000	.00004
16	40	43	29.1667	.00000	.00004	16	40	43	29.1667	.00000	.00004
16	41	42	27.1439	.00000	.00004	16	41	42	27.1439	.00000	.00004
16	42	41	25.2576	.00000	.00004	16	42	41	25.2576	.00000	.00004
16	43	40	23.5000	.00000	.00004	16	43	40	23.5000	.00000	.00004
16	44	39	21.8759	.00000	.00004	16	44	39	21.8759	.00000	.00004
16	45	38	20.4667	.00000	.00004	16	45	38	20.4667	.00000	.00004
16	46	37	19.1750	.00000	.00004	16	46	37	19.1750	.00000	.00004
16	47	36	17.9712	.00000	.00004	16	47	36	17.9712	.00000	.00004
16	48	35	16.8382	.00000	.00004	16	48	35	16.8382	.00000	.00004
16	49	34	15.7500	.00000	.00004	16	49	34	15.7500	.00000	.00004
16	50	33	14.7000	.00000	.00004	16	50	33	14.7000	.00000	.00004
16	51	32	13.7000	.00000	.00004	16	51	32	13.7000	.00000	.00004
16	52	31	12.7500	.00000	.00004	16	52	31	12.7500	.00000	.00004
16	53	30	11.8438	.00000	.00004	16	53	30	11.8438	.00000	.00004
16	54	29	10.9773	.00000	.00004	16	54	29	10.9773	.00000	.00004
16	55	28	10.1500	.00000	.00004	16	55	28	10.1500	.00000	.00004
16	56	27	9.3636	.00000	.00004	16	56	27	9.3636	.00000	.00004
16	57	26	8.6167	.00000	.00004	16	57	26	8.6167	.00000	.00004
16	58	25	7.9076	.00000	.00004	16	58	25	7.9076	.00000	.00004
16	59	24	7.2345	.00000	.00004	16	59	24	7.2345	.00000	.00004
16	60	23	6.5967	.00000	.00004	16	60	23	6.5967	.00000	.00004
16	61	22	6.0000	.00000	.00004	16	61	22	6.0000	.00000	.00004
16	62	21	5.4438	.00000	.00004	16	62	21	5.4438	.00000	.00004
16	63	20	4.9273	.00000	.00004	16	63	20	4.9273	.00000	.00004
16	64	19	4.4500	.00000	.00004	16	64	19	4.4500	.00000	.00004
16	65	18	4.0117	.00000	.00004	16	65	18	4.0117	.00000	.00004
16	66	17	3.6125	.00000	.00004	16	66	17	3.6125	.00000	.00004
16	67	16	3.2500	.00000	.00004	16	67	16	3.2500	.00000	.00004
16	68	15	2.9231	.00000	.00004	16	68	15	2.9231	.00000	.00004
16	69	14	2.6300	.00000	.00004	16	69	14	2.6300	.00000	.00004
16	70	13	2.3700	.00000	.00004	16	70	13	2.3700	.00000	.00004
16	71	12	2.1400	.00000	.00004	16	71	12	2.1400	.00000	.00004
16	72	11	1.9400	.00000	.00004	16	72	11	1.9400	.00000	.00004
16	73	10	1.7700	.00000	.00004	16	73	10	1.7700	.00000	.00004
16	74	9	1.6200	.00000	.00004	16	74	9	1.6200	.00000	.00004
16	75	8	1.4900	.00000	.00004	16	75	8	1.4900	.00000	.00004
16	76	7	1.3800	.00000	.00004	16	76	7	1.3800	.00000	.00004
16	77	6	1.2900	.00000	.00004	16	77	6	1.2900	.00000	.00004
16	78	5	1.2200	.00000	.00004	16	78	5	1.2200	.00000	.00004
16	79	4	1.1700	.00000	.00004	16	79	4	1.1700	.00000	.00004
16	80	3	1.1300	.00000	.00004	16	80	3	1.1300	.00000	.00004

(2)

TABLE A

CHI SQUARE - P(1/3), P(4/9), P(2/27) N=99

U	V	W	X2	P(A)	CUM P(A)	U	V	W	X2	P(A)	CUM P(A)
17	47	35	15.6439	.00000	.00011	17	47	35	15.6439	.00000	.00011
17	48	34	14.6667	.00000	.00011	17	48	34	14.6667	.00000	.00011
17	49	33	13.8258	.00000	.00011	17	49	33	13.8258	.00000	.00011
17	50	32	13.1212	.00000	.00011	17	50	32	13.1212	.00000	.00011
17	51	31	12.5530	.00000	.00011	17	51	31	12.5530	.00000	.00011
17	52	30	12.1212	.00000	.00011	17	52	30	12.1212	.00000	.00011
17	53	29	11.8258	.00000	.00011	17	53	29	11.8258	.00000	.0001

AD-A056 920

SCRIPPS INSTITUTION OF OCEANOGRAPHY LA JOLLA CALIF
CLIMATE FORECAST VERIFICATION VIA MULTINOMIAL STOCHASTERS, (U)
DEC 77 R W PREISENDORFER

F/G 4/2

N00014-75-C-0152

UNCLASSIFIED

SIO-REF-77-33

NL

2 of 2

AD
A056 920



END

DATE
FILMED

9--78

DDC

THIS PAGE IS BEST QUALITY PRACTICABLE FROM COPY FURNISHED TO DDC

TABLE A

Table with 5 columns: U, V, W, X2, P(A) CUM P(A). Contains data rows for CHI SQUARE - P(41/3), P1(4/9), P2(2/9) N=99.

(5)

TABLE A

Table with 5 columns: U, V, W, X2, P(A) CUM P(A). Contains data rows for CHI SQUARE - P(41/3), P1(4/9), P2(2/9) N=99.

(6)

TABLE A

Table with 5 columns: U, V, W, X2, P(A) CUM P(A). Contains data rows for CHI SQUARE - P(41/3), P1(4/9), P2(2/9) N=99.

(7)

TABLE A

Table with 5 columns: U, V, W, X2, P(A) CUM P(A). Contains data rows for CHI SQUARE - P(41/3), P1(4/9), P2(2/9) N=99.

(8)

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

TABLE A

TABLE A

CHE SQUARE = P.11(73), P14(79), P2(279) N=99

Table with 6 columns: U, V, W, X2, P(AS), CUM P(AS). Contains numerical data for various indices.

Table with 6 columns: U, V, W, X2, P(AS), CUM P(AS). Contains numerical data for various indices.

(9)

(10)

TABLE A

TABLE A

CHE SQUARE = P.11(73), P14(79), P2(279) N=99

Table with 6 columns: U, V, W, X2, P(AS), CUM P(AS). Contains numerical data for various indices.

Table with 6 columns: U, V, W, X2, P(AS), CUM P(AS). Contains numerical data for various indices.

(11)

(12)

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

TABLE A

TABLE A

CHI SQUARE - P(41/31), P(14/9), P(2/2/9) N=99

Table with 5 columns: U, V, W, X2, P(A) CUM P(A). Rows contain numerical data for chi-square tests.

(13)

Table with 5 columns: U, V, W, X2, P(A) CUM P(A). Rows contain numerical data for chi-square tests.

(14)

TABLE A

TABLE A

CHI SQUARE - P(41/31), P(14/9), P(2/2/9) N=99

Table with 5 columns: U, V, W, X2, P(A) CUM P(A). Rows contain numerical data for chi-square tests.

(15)

Table with 5 columns: U, V, W, X2, P(A) CUM P(A). Rows contain numerical data for chi-square tests.

(16)

THIS PAGE IS BEST QUALITY PRACTICABLE FROM COPY FURNISHED TO BDC

TABLE A

TABLE A

CHI SQUARE = P(41/3) P(42/7) P(21/7) N=99

Table with 4 columns: U, V, W, X2. Sub-columns: P(41) CUM P(41), U, V, W, X2, P(42) CUM P(42). Rows contain numerical data for 99 trials.

(17)

TABLE A

TABLE A

CHI SQUARE = P(41/3) P(42/7) P(21/7) N=99

Table with 4 columns: U, V, W, X2. Sub-columns: P(41) CUM P(41), U, V, W, X2, P(42) CUM P(42). Rows contain numerical data for 99 trials.

(19)

Table with 4 columns: U, V, W, X2. Sub-columns: P(41) CUM P(41), U, V, W, X2, P(42) CUM P(42). Rows contain numerical data for 99 trials.

(18)

Table with 4 columns: U, V, W, X2. Sub-columns: P(41) CUM P(41), U, V, W, X2, P(42) CUM P(42). Rows contain numerical data for 99 trials.

(20)

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

TABLE A

TABLE A

CHI SQUARE - P(1/33), P(1/4/9), P(2/2/9) N=99

Table with columns U, V, W, X2, P(A) CUM P(A) and data rows for chi-square values.

(21)

Table with columns U, V, W, X2, P(A) CUM P(A) and data rows for chi-square values.

(22)

TABLE A

TABLE A

CHI SQUARE - P(1/33), P(1/4/9), P(2/2/9) N=99

Table with columns U, V, W, X2, P(A) CUM P(A) and data rows for chi-square values.

(23)

Table with columns U, V, W, X2, P(A) CUM P(A) and data rows for chi-square values.

(24)

THIS PAGE IS BEST QUALITY PRACTICABLE FROM COPY FURNISHED TO DDC

TABLE A

CHE SQUARE - P417(3), P147(3), P212(3) N=99

U	V	W	X2	PIA3	CUM	PIA3	U	V	W	X2	PIA3	CUM	PIA3
49 37 13	12.4553	0.0000	0.0000	0.0000	0.0000	0.0000	50 36 13	13.8939	0.0001	0.0000	0.0000	0.0000	0.0000
49 38 11	13.4212	0.0000	0.0000	0.0000	0.0000	0.0000	50 37 12	14.4167	0.0001	0.0000	0.0000	0.0000	0.0000
49 39 11	13.4208	0.0001	0.0000	0.0000	0.0000	0.0000	50 38 11	15.0758	0.0001	0.0000	0.0000	0.0000	0.0000
49 40 10	14.4467	0.0001	0.0000	0.0000	0.0000	0.0000	50 39 10	15.4712	0.0000	0.0000	0.0000	0.0000	0.0000
49 41 8	15.4439	0.0000	0.0000	0.0000	0.0000	0.0000	50 40 8	16.4070	0.0000	0.0000	0.0000	0.0000	0.0000
49 42 7	16.7576	0.0000	0.0000	0.0000	0.0000	0.0000	50 41 8	17.4712	0.0000	0.0000	0.0000	0.0000	0.0000
49 43 7	18.4478	0.0000	0.0000	0.0000	0.0000	0.0000	50 42 7	19.4758	0.0000	0.0000	0.0000	0.0000	0.0000
49 44 6	19.4939	0.0000	0.0000	0.0000	0.0000	0.0000	50 43 6	21.4967	0.0000	0.0000	0.0000	0.0000	0.0000
49 45 5	20.4967	0.0000	0.0000	0.0000	0.0000	0.0000	50 44 5	21.4939	0.0000	0.0000	0.0000	0.0000	0.0000
49 46 4	22.4978	0.0000	0.0000	0.0000	0.0000	0.0000	50 45 4	23.4978	0.0000	0.0000	0.0000	0.0000	0.0000
49 47 3	24.4978	0.0000	0.0000	0.0000	0.0000	0.0000	50 46 3	25.4978	0.0000	0.0000	0.0000	0.0000	0.0000
49 48 3	26.4978	0.0000	0.0000	0.0000	0.0000	0.0000	50 47 2	27.4978	0.0000	0.0000	0.0000	0.0000	0.0000
49 49 1	28.4978	0.0000	0.0000	0.0000	0.0000	0.0000	50 48 1	29.4978	0.0000	0.0000	0.0000	0.0000	0.0000
49 50 1	30.4978	0.0000	0.0000	0.0000	0.0000	0.0000	50 49 2	31.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 0 49	33.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 0 46	34.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 1 48	35.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 1 47	36.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 2 47	37.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 2 46	38.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 3 46	39.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 3 45	40.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 4 45	41.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 4 44	42.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 5 44	43.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 5 43	44.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 6 43	45.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 6 42	46.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 7 42	47.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 7 41	48.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 8 41	49.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 8 40	50.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 9 40	51.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 9 39	52.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 10 39	53.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 10 38	54.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 11 38	55.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 11 37	56.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 12 37	57.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 12 36	58.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 13 36	59.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 13 35	60.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 14 35	61.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 14 34	62.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 15 34	63.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 15 33	64.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 16 33	65.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 16 32	66.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 17 32	67.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 17 31	68.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 18 31	69.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 18 30	70.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 19 30	71.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 19 29	72.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 20 29	73.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 20 28	74.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 21 28	75.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 21 27	76.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 22 27	77.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 22 26	78.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 23 26	79.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 23 25	80.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 24 25	81.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 24 24	82.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 25 24	83.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 25 23	84.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 26 23	85.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 26 22	86.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 27 22	87.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 27 21	88.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 28 21	89.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 28 20	90.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 29 20	91.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 29 19	92.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 30 19	93.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 30 18	94.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 31 18	95.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 31 17	96.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 32 17	97.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 32 16	98.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 33 16	99.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 33 15	100.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 34 15	101.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 34 14	102.4978	0.0000	0.0000	0.0000	0.0000	0.0000
50 35 14	103.4978	0.0000	0.0000	0.0000	0.0000	0.0000	51 35 13	104.4978	0.0000	0.0000	0.0000	0.0000	0.0000

(25)

TABLE A

U	V	W	X2	PIA3	CUM	PIA3	U	V	W	X2	PIA3	CUM	PIA3
51 36 12	15.8182	0.0000	0.0000	0.0000	0.0000	0.0000	52 37 10	18.5985	0.0000	0.0000	0.0000	0.0000	0.0000
51 37 11	16.4318	0.0000	0.0000	0.0000	0.0000	0.0000	52 38 9	19.4939	0.0000	0.0000	0.0000	0.0000	0.0000
51 38 10	17.1818	0.0000	0.0000	0.0000	0.0000	0.0000	52 39 8	20.4167	0.0000	0.0000	0.0000	0.0000	0.0000
51 39 9	18.4682	0.0000	0.0000	0.0000	0.0000	0.0000	52 40 7	21.5303	0.0000	0.0000	0.0000	0.0000	0.0000
51 40 8	19.4949	0.0000	0.0000	0.0000	0.0000	0.0000	52 41 6	22.7603	0.0000	0.0000	0.0000	0.0000	0.0000
51 41 7	20.4900	0.0000	0.0000	0.0000	0.0000	0.0000	52 42 5	24.1867	0.0000	0.0000	0.0000	0.0000	0.0000
51 42 6	21.8455	0.0000	0.0000	0.0000	0.0000	0.0000	52 43 4	25.4694	0.0000	0.0000	0.0000	0.0000	0.0000
51 43 5	22.9773	0.0000	0.0000	0.0000	0.0000	0.0000	52 44 3	27.3485	0.0000	0.0000	0.0000	0.0000	0.0000
51 44 4	24.5405	0.0000	0.0000	0.0000	0.0000	0.0000	52 45 2	29.1439	0.0000	0.0000	0.0000	0.0000	0.0000
51 45 3	26.2500	0.0000	0.0000	0.0000	0.0000	0.0000	52 46 1	31.0750	0.0000	0.0000	0.0000	0.0000	0.0000
51 46 2	28.0909	0.0000	0.0000	0.0000	0.0000	0.0000	52 47 0	33.1439	0.0000	0.0000	0.0000	0.0000	0.0000
51 47 1	30.4682	0.0000	0.0000	0.0000	0.0000	0.0000	53 0 46	35.3850	0.0000	0.0000	0.0000	0.0000	0.0000
51 48 0	33.1818	0.0000	0.0000	0.0000	0.0000	0.0000	53 1 45	37.4894	0.0000	0.0000	0.0000	0.0000	0.0000
52 0 47	35.3485	0.0000	0.0000	0.0000	0.0000	0.0000	53 2 44	39.4121	0.0000	0.0000	0.0000	0.0000	0.0000
52 1 46	37.4839	0.0000	0.0000	0.0000	0.0000	0.0000	53 3 43	41.1439	0.0000	0.0000	0.0000	0.0000	0.0000
52 2 45	39.4758	0.0000	0.0000	0.0000	0.0000	0.0000	53 4 42	42.6667	0.0000	0.0000	0.0000	0.0000	0.0000
52 3 44	41.4239	0.0000	0.0000	0.0000	0.0000	0.0000	53 5 41	44.5985	0.0000	0.0000	0.0000	0.0000	0.0000
52 4 43	43.3485	0.0000	0.0000	0.0000	0.0000	0.0000	53 6 40	46.6667	0.0000	0.0000	0.0000	0.0000	0.0000
52 5 42	45.4682	0.0000	0.0000	0.0000	0.0000	0.0000	53 7 39	48.5312	0.0000	0.0000	0.0000	0.0000	0.0000
52 6 41	47.4667	0.0000	0.0000	0.0000	0.0000	0.0000	53 8 38	50.5121	0.0000	0.0000	0.0000	0.0000	0.0000
52 7 40	49.4780	0.0000	0.0000	0.0000	0.0000	0.0000	53 9 37	52.1894	0.0000	0.0000	0.0000	0.0000	0.0000
52 8 39	51.5503	0.0000	0.0000	0.0000	0.0000	0.0000	53 10 36	53.5630	0.0000	0.0000	0.0000	0.0000	0.0000
52 9 38	53.4167	0.0000	0.0000	0.0000									

**THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC**

TABLE B
CHI SQUARE - P(1/3), P(1/9), P(2/9) N=99

TABLE B

U	V	W	X ₂	P(A)	CUM P(B)	U	V	W	X ₂	P(A)	CUM P(B)	U	V	W	X ₂	P(A)	CUM P(B)	U	V	W	X ₂	P(A)	CUM P(B)
29	37	33	7.4985	.00033	.97178	25	58	16	8.0303	.00019	.98199	19	53	27	8.9167	.00007	.98887	19	57	23	9.8258	.00005	.99262
45	33	21	7.1591	.00027	.97206	42	46	11	8.0455	.00011	.98211	47	34	18	8.9394	.00013	.98860	46	41	12	9.8712	.00006	.99268
39	49	11	7.1591	.00017	.97222	24	42	33	8.0455	.00020	.98230	19	54	26	8.9394	.00007	.98867	20	47	32	9.8712	.00006	.99274
27	39	13	7.1591	.00032	.97255	42	30	27	8.0455	.00013	.98230	35	54	10	8.9394	.00006	.98873	45	43	11	9.8864	.00005	.99279
21	55	23	7.1591	.00021	.97276	24	58	17	8.0455	.00019	.98262	31	34	34	8.9394	.00014	.98887	45	29	25	9.8864	.00006	.99284
45	39	15	7.1591	.00029	.97305	46	33	20	8.0530	.00015	.98281	46	31	22	8.9621	.00011	.98898	39	51	9	9.8864	.00003	.99287
39	31	29	7.1591	.00021	.97327	20	55	24	8.0530	.00013	.98293	20	57	22	8.9621	.00009	.98916	39	29	31	9.8864	.00005	.99293
27	57	15	7.1591	.00028	.97355	46	38	15	8.1667	.00018	.98312	45	30	24	9.0008	.00009	.98916	27	59	13	9.8864	.00007	.99300
21	49	28	7.1591	.00024	.97376	40	30	29	8.1667	.00012	.98342	21	58	20	9.0000	.00010	.98926	27	37	35	9.8864	.00010	.99310
35	52	11	7.2273	.00016	.97416	26	58	15	8.1667	.00018	.98342	45	42	12	9.0000	.00009	.98955	21	59	19	9.8864	.00007	.99316
35	52	31	7.2273	.00023	.97446	27	50	29	8.1667	.00011	.98355	21	46	32	9.0004	.00010	.98984	21	45	33	9.8864	.00007	.99323
30	56	13	7.2273	.00024	.97439	43	30	26	8.2121	.00012	.98366	47	36	16	9.0303	.00013	.98957	38	52	7	9.8939	.00003	.99326
33	36	33	7.2273	.00031	.97470	23	58	18	8.2121	.00017	.98382	19	52	28	9.0303	.00017	.98964	28	36	35	9.8939	.00010	.99336
45	31	25	7.2803	.00021	.97491	44	43	12	8.2348	.00013	.98395	47	33	19	9.0985	.00012	.98976	40	50	9	9.9848	.00003	.99339
23	57	19	7.2803	.00025	.97516	22	45	32	8.2348	.00015	.98410	19	55	25	9.0985	.00007	.98983	26	38	35	9.9848	.00010	.99348
33	33	34	7.3258	.00025	.97541	45	31	23	8.2500	.00014	.98424	46	40	13	9.1667	.00010	.98993	37	53	9	10.0076	.00003	.99351
32	55	12	7.3258	.00019	.97560	21	57	21	8.2500	.00014	.98438	20	48	31	9.1667	.00008	.99000	29	35	35	10.0076	.00009	.99360
42	48	11	7.3485	.00015	.97576	45	41	13	8.2500	.00015	.98453	44	44	11	9.1667	.00007	.99007	47	39	13	10.1894	.00006	.99367
26	40	33	7.3485	.00029	.97605	33	55	11	8.2500	.00011	.98463	22	44	33	9.1667	.00010	.99017	42	49	9	10.1894	.00003	.99369
48	42	13	7.4394	.00021	.97626	33	33	33	8.2500	.00017	.98486	42	47	10	9.2045	.00005	.99023	25	39	35	10.1894	.00009	.99376
22	46	31	7.4394	.00021	.97647	21	47	31	8.2500	.00013	.98493	42	29	28	9.2045	.00007	.99030	19	49	31	10.1894	.00004	.99382
35	53	11	7.4621	.00015	.97662	30	51	10	8.4167	.00018	.98531	24	59	16	9.2445	.00011	.99040	48	34	17	10.2273	.00007	.99389
31	35	33	7.4621	.00027	.97689	28	37	34	8.4167	.00019	.98520	24	41	34	9.2445	.00012	.99053	26	38	35	10.2273	.00005	.99395
38	51	30	7.5076	.00019	.97707	46	36	21	8.4394	.00015	.98535	47	37	15	9.2445	.00011	.99064	30	58	11	10.2273	.00004	.99399
23	57	14	7.5076	.00023	.97730	20	56	13	8.4394	.00011	.98545	43	29	27	9.2445	.00007	.99071	18	54	27	10.2273	.00003	.99403
45	44	12	7.5758	.00017	.97747	39	50	10	8.4545	.00017	.98533	23	59	17	9.2445	.00010	.99081	30	54	9	10.2273	.00003	.99405
43	44	14	7.5758	.00022	.97769	39	37	30	8.4545	.00011	.98544	19	51	29	9.2445	.00006	.99087	31	34	35	10.2273	.00008	.99413
45	40	14	7.6364	.00022	.97791	27	58	14	8.4545	.00015	.98578	41	29	29	9.2445	.00007	.99094	44	45	10	10.2348	.00003	.99417
45	32	22	7.6364	.00020	.97811	27	36	34	8.4545	.00019	.98597	25	59	15	9.2445	.00010	.99104	22	43	34	10.2348	.00007	.99423
21	56	22	7.6364	.00018	.97829	37	52	16	8.4848	.00017	.98604	34	55	10	9.3258	.00005	.99109	48	35	16	10.2555	.00007	.99431
21	48	30	7.6364	.00017	.97846	29	36	34	8.4848	.00018	.98623	32	33	34	9.3258	.00011	.99120	48	33	18	10.2555	.00007	.99436
41	47	11	7.6439	.00014	.97859	44	30	25	8.5303	.00011	.98634	47	32	20	9.3939	.00010	.99130	18	55	26	10.2555	.00003	.99441
23	42	33	7.6439	.00025	.97884	22	58	14	8.5303	.00013	.98647	19	56	24	9.3939	.00006	.99136	18	53	28	10.2555	.00003	.99444
46	35	18	7.6894	.00024	.97908	43	45	11	8.5530	.00009	.98656	35	31	33	9.4621	.00008	.99144	47	30	22	10.2555	.00003	.99449
48	31	24	7.6894	.00014	.97926	25	43	33	8.5530	.00015	.98671	31	57	11	9.4621	.00006	.99151	19	58	22	10.2555	.00004	.99453
22	57	14	7.6894	.00021	.97945	46	39	14	8.5945	.00014	.98685	37	30	32	9.4621	.00007	.99158	46	29	24	10.4167	.00004	.99456
20	53	28	7.6894	.00024	.97959	40	49	10	8.5945	.00007	.98692	29	58	12	9.4621	.00007	.99165	26	59	24	10.4167	.00004	.99463
46	34	17	7.7121	.00014	.97983	26	34	34	8.5945	.00017	.98709	44	29	26	9.5176	.00016	.99172	38	29	32	10.4167	.00004	.99467
42	52	17	7.7121	.00014	.97997	20	45	30	8.5945	.00010	.98719	40	29	20	9.5176	.00006	.99176	38	31	34	10.4167	.00006	.99473
45	34	19	7.8030	.00022	.98019	36	53	19	8.6541	.00007	.98725	26	59	14	9.5076	.00009	.99187	32	57	10	10.4167	.00003	.99476
45	34	11	7.8030	.00013	.98032	36	31	32	8.6591	.00012	.98737	22	59	18	9.5076	.00009	.99196	28	59	12	10.4167	.00005	.99481
32	34	33	7.8030	.00022	.98054	30	57	12	8.6591	.00011	.98748	46	30	23	9.6214	.00007	.99203	43	28	48	10.4848	.00004	.99484
42	54	25	7.8030	.00014	.98068	30	35	34	8.6591	.00016	.98764	20	58	21	9.6214	.00007	.99209	23	60	16	10.4848	.00004	.99490
45	37	16	7.8712	.00022	.98090	45	32	33	8.8030	.00012	.98777	47	38	14	9.6667	.00009	.99218	48	36	15	10.5000	.00006	.99497
45	31	28	7.8712	.00013	.98103	32	56	11	8.8030	.00008	.98785	43	46	10	9.6667	.00004	.99223	48	32	19	10.5000	.00006	.99503
35	36	14	7.9394	.00018	.98120	41	48	10	8.8485	.00006	.98791	23	54	30	9.6667	.00009	.99232	18	56	25	10.5000	.00003	.99506
31	56	12	7.9394	.00015	.98135	25	40	34	8.8485	.00015	.98806	19	40	34	9.6667	.00005	.99237	18	52	29	10.5000	.00003	.99516
37	31	31	8.0076	.00015	.98150	38	30	31	8.8939	.00009	.98815	33	56	10	9.8182	.00004	.99241	42	48	9	10.5000	.00002	.99511
29	57	13	8.0076	.00017	.98167	28	58	13	8.8939	.00011	.98826	33	32	34	9.8182	.00006	.99249	42	28	29	10.5000	.00004	.99514
41	30	28	8.0383	.00013	.98180	47	35	17	8.9167	.00014	.98840	47	31	21	9.8258	.00007	.99257	24	60	13	10.5000	.00006	.99520

(5)

(6)

TABLE B
CHI SQUARE - P(1/3), P(1/9), P(2/9) N=99

TABLE B

U	V	W	X ₂	P(A)	CUM P(B)	U	V	W	X ₂	P(A)	CUM P(B)	U	V	W	X ₂	P(A)	CUM P(B)	U	V	W	X ₂	P(A)	CUM P(B)
24	46	35	10.5000	.00007	.99527	47	41	11	11.6439	.00002	.99697	23	40	36	12.5030	.00003	.99806	23	62	14	13.3030	.00001	.99874
35	55	9	10.5530	.00002	.99530	19	47	33	11.6439	.00002	.99699	49	30	20	12.3939	.00002	.99809	50	31	18	13.3258	.00002	.99876
31	33	35	10.5530	.00007	.99536	17	55	27	11.6439	.00001	.99700	17	58	24	12.3939	.00001	.99812	16	57	8	13.3258	.00001	.99876
41	28	27	10.6212	.00003	.99540	49	34	16	11.6667	.00004	.99704	46	27	26	12.4167	.00001	.99811	45	45	8	13.3636	.00002	.99877
22	60	17	10.6212	.00005																			

THIS PAGE IS BEST QUALITY PRACTICABLE FROM COPY FURNISHED TO DDC

TABLE B

CHE SQUARE - P. 11/33, P114/93, P212/93 N499

Table with 4 columns: U, V, W, X2. Rows contain numerical data for CHE SQUARE.

(17)

TABLE B

Table with 4 columns: U, V, W, X2. Rows contain numerical data for CHE SQUARE.

(18)

TABLE B

CHE SQUARE - P. 11/33, P114/93, P212/93 N499

Table with 4 columns: U, V, W, X2. Rows contain numerical data for CHE SQUARE.

(19)

TABLE B

Table with 4 columns: U, V, W, X2. Rows contain numerical data for CHE SQUARE.

(20)

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

TABLE B

TABLE B

CHI SQUARE - P(1/3), P(4/9), P(2/9) N=99

U	V	W	X2	P(A)	CUM P(B)	U	V	W	X2	P(A)	CUM P(B)
28	6	63	106.6212	.00000	.99999	28	6	65	117.6212	.00000	.99999
32	5	62	107.3258	.00000	.99999	24	9	66	118.2955	.00000	.99999
28	11	64	107.3864	.00000	.99999	32	3	64	118.4167	.00000	.99999
33	3	61	107.4621	.00000	.99999	15	17	67	118.4318	.00000	.99999
40	1	59	107.7121	.00000	.99999	35	1	63	118.6212	.00000	.99999
29	1	63	108.0076	.00000	.99999	16	16	67	118.6212	.00000	.99999
13	19	65	108.0682	.00000	.99999	17	15	67	118.9167	.00000	.99999
16	18	65	108.1667	.00000	.99999	29	5	65	119.0985	.00000	.99999
17	17	65	108.3712	.00000	.99999	18	14	67	119.3182	.00000	.99999
23	13	64	108.3939	.00000	.99999	25	8	66	119.3939	.00000	.99999
38	1	60	108.4167	.00000	.99999	19	13	67	119.6258	.00000	.99999
18	16	65	108.6818	.00000	.99999	33	2	64	120.2727	.00000	.99999
33	4	62	109.0909	.00000	.99999	20	12	67	120.4394	.00000	.99999
19	15	65	109.0985	.00000	.99999	26	7	66	120.5985	.00000	.99999
36	2	61	109.3600	.00000	.99999	36	0	63	120.6818	.00000	.99999
37	6	63	109.5000	.00000	.99999	30	4	65	120.6818	.00000	.99999
26	9	64	109.5076	.00000	.99999	21	11	67	121.1591	.00000	.99999
20	14	65	109.6212	.00000	.99999	27	6	66	121.9091	.00000	.99999
21	13	65	110.2500	.00000	.99999	22	10	67	121.9848	.00000	.99999
27	8	64	110.7273	.00000	.99999	34	1	64	122.2348	.00000	.99999
39	0	60	110.7273	.00000	.99999	31	3	65	122.3712	.00000	.99999
34	3	62	110.9621	.00000	.99999	23	9	67	122.9167	.00000	.99999
22	12	65	110.9848	.00000	.99999	28	5	66	123.3258	.00000	.99999
31	5	63	111.0985	.00000	.99999	15	16	68	123.8182	.00000	.99999
37	1	61	111.6439	.00000	.99999	24	8	67	123.9545	.00000	.99999
23	11	65	111.8258	.00000	.99999	16	15	68	124.0530	.00000	.99999
28	7	64	112.6530	.00000	.99999	32	2	65	124.1667	.00000	.99999
20	10	65	112.7727	.00000	.99999	35	0	64	124.5030	.00000	.99999
32	4	63	112.8030	.00000	.99999	17	14	68	124.5939	.00000	.99999
35	2	62	112.9394	.00000	.99999	18	13	68	124.6090	.00000	.99999
13	16	66	113.1818	.00000	.99999	29	4	66	124.8485	.00000	.99999
18	17	66	113.3258	.00000	.99999	25	7	67	125.6985	.00000	.99999
28	6	64	113.4848	.00000	.99999	19	12	68	125.3939	.00000	.99999
17	16	66	113.4848	.00000	.99999	20	11	68	126.0530	.00000	.99999
23	9	65	113.8258	.00000	.99999	33	1	65	126.0682	.00000	.99999
38	1	61	113.8939	.00000	.99999	26	6	67	126.3485	.00000	.99999
19	15	66	113.9318	.00000	.99999	30	3	66	126.4773	.00000	.99999
19	14	66	114.3939	.00000	.99999	21	10	68	126.8182	.00000	.99999
33	3	63	114.6136	.00000	.99999	22	9	68	127.6894	.00000	.99999
20	13	66	114.9621	.00000	.99999	27	5	67	127.7045	.00000	.99999
25	8	65	114.9848	.00000	.99999	34	0	65	128.0758	.00000	.99999
36	1	62	115.6227	.00000	.99999	31	2	66	128.2121	.00000	.99999
39	5	64	115.6227	.00000	.99999	23	8	68	128.6667	.00000	.99999
27	7	65	116.2500	.00000	.99999	28	4	67	129.1667	.00000	.99999
22	11	66	116.4167	.00000	.99999	15	15	69	129.3409	.00000	.99999
34	2	63	116.5303	.00000	.99999	16	14	69	129.6212	.00000	.99999
31	4	64	116.6667	.00000	.99999	24	7	68	129.7500	.00000	.99999
37	0	62	117.2121	.00000	.99999	17	13	69	130.0076	.00000	.99999
25	10	66	117.3030	.00000	.99999	32	1	66	130.0530	.00000	.99999
23	10	66	117.3030	.00000	.99999	18	12	69	130.5030	.00000	.99999

U	V	W	X2	P(A)	CUM P(B)	U	V	W	X2	P(A)	CUM P(B)
29	3	67	130.7348	.00000	.99999	24	8	71	147.9545	.00000	.99999
25	6	68	130.9394	.00000	.99999	18	9	72	148.2955	.00000	.99999
19	11	69	131.0985	.00000	.99999	19	8	72	149.0303	.00000	.99999
20	10	69	131.8030	.00000	.99999	29	0	70	149.2121	.00000	.99999
33	0	66	132.0000	.00000	.99999	25	3	71	149.2403	.00000	.99999
26	5	68	132.4348	.00000	.99999	20	7	72	149.8712	.00000	.99999
30	2	67	132.4091	.00000	.99999	26	2	71	150.7121	.00000	.99999
21	9	69	132.6136	.00000	.99999	21	6	72	150.8182	.00000	.99999
22	8	69	133.5303	.00000	.99999	22	5	72	151.8712	.00000	.99999
27	4	68	133.6364	.00000	.99999	27	1	71	152.2500	.00000	.99999
31	1	67	134.1894	.00000	.99999	15	11	73	152.7955	.00000	.99999
23	7	69	134.5530	.00000	.99999	23	4	72	153.0303	.00000	.99999
15	14	70	135.0000	.00000	.99999	16	10	73	153.2576	.00000	.99999
35	3	68	135.1439	.00000	.99999	17	9	73	153.8258	.00000	.99999
16	13	70	135.2258	.00000	.99999	28	0	71	153.8939	.00000	.99999
24	6	69	135.6818	.00000	.99999	24	3	72	154.2955	.00000	.99999
17	12	70	135.7576	.00000	.99999	18	7	73	154.5000	.00000	.99999
32	0	67	136.6756	.00000	.99999	19	7	73	155.2803	.00000	.99999
18	11	70	136.2955	.00000	.99999	25	2	72	155.6667	.00000	.99999
29	2	68	136.7576	.00000	.99999	20	5	73	156.1667	.00000	.99999
25	5	69	136.9167	.00000	.99999	26	1	72	157.1439	.00000	.99999
19	10	70	136.9394	.00000	.99999	21	5	73	157.1591	.00000	.99999
20	9	70	137.6894	.00000	.99999	22	4	73	158.2576	.00000	.99999
26	4	69	138.2576	.00000	.99999	27	0	72	158.7576	.00000	.99999
30	1	68	138.4773	.00000	.99999	15	10	74	159.0000	.00000	.99999
21	8	70	138.5455	.00000	.99999	23	3	73	159.4621	.00000	.99999
22	7	70	139.5076	.00000	.99999	16	9	74	159.5076	.00000	.99999
27	3	69	139.7045	.00000	.99999	17	8	74	160.1212	.00000	.99999
31	0	68	140.3630	.00000	.99999	24	2	73	160.7727	.00000	.99999
23	6	70	140.5758	.00000	.99999	18	7	74	160.8409	.00000	.99999
15	13	71	140.7955	.00000	.99999	19	6	74	161.6667	.00000	.99999
16	12	71	141.1667	.00000	.99999	25	1	73	162.1894	.00000	.99999
28	2	69	141.2576	.00000	.99999	20	5	74	162.5985	.00000	.99999
17	11	71	141.6439	.00000	.99999	21	4	74	163.6364	.00000	.99999
24	5	70	141.7500	.00000	.99999	26	0	73	163.7121	.00000	.99999
18	10	71	142.2273	.00000	.99999	22	3	74	164.7403	.00000	.99999
29	1	69	142.9167	.00000	.99999	15	9	75	165.3409	.00000	.99999
19	9	71	142.9167	.00000	.99999	16	8	75	165.8939	.00000	.99999
25	4	70	143.0303	.00000	.99999	23	2	74	166.0303	.00000	.99999
20	8	71	143.7121	.00000	.99999	17	7	75	166.5530	.00000	.99999
26	3	70	144.4167	.00000	.99999	18	5	75	167.3182	.00000	.99999
21	7	71	144.6136	.00000	.99999	24	1	74	167.3864	.00000	.99999
30	0	69	144.8182	.00000	.99999	19	5	75	168.1894	.00000	.99999
22	6	71	145.6212	.00000	.99999	25	3	74	168.8485	.00000	.99999
27	2	75	145.9091	.00000	.99999	26	4	75	169.1667	.00000	.99999
15	12	72	146.7273	.00000	.99999	21	3	75	170.2500	.00000	.99999
23	5	71	146.7348	.00000	.99999	22	2	75	171.4394	.00000	.99999
16	11	72	147.1439	.00000	.99999	15	8	76	171.8182	.00000	.99999
28	1	70	147.5076	.00000	.99999	16	7	76	172.4167	.00000	.99999
17	10	72	147.6667	.00000	.99999	23	1	75	172.7348	.00000	.99999

(25)

(26)

TABLE B

CHI SQUARE - P(1/3), P(4/9), P(2/9) N=99

U	V	W	X2	P(A)	CUM P(B)
17	6	76	173.1242	.00000	.99999
14	5	76	173.9318	.00000	.99999
29	2	75	174.1364	.00000	.99999
19	4	76	174.8885	.00000	.99999
20	3	76	175.6712	.00000	.99999
21	2	76	177.0000	.00000	.99999
22	1	76	178.2348	.00000	.99999
6	7	77	178.4318	.00000	.99999
15	6	77	179.0758</		

THIS PAGE IS BEST QUALITY PRACTICABLE FROM COPY FURNISHED TO DDC

TABLE C

CHI SQUARE - F(61/31) P(14/91) P(22/74) N=99

Table with 5 columns: U, V, W, X2, P(A) CUM P(C). Contains 20 rows of data.

(5)

TABLE C

Table with 5 columns: U, V, W, X2, P(A) CUM P(C). Contains 20 rows of data.

(6)

TABLE C

CHI SQUARE - F(61/31) P(14/91) P(22/74) N=99

Table with 5 columns: U, V, W, X2, P(A) CUM P(C). Contains 20 rows of data.

(7)

TABLE C

Table with 5 columns: U, V, W, X2, P(A) CUM P(C). Contains 20 rows of data.

(8)

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO BDC

TABLE C

CHI SQUARE - P(1/3), P(1/2), P(2/3) N=99

U	V	X2	P(A) CUM P(C)	U	V	X2	P(A) CUM P(C)
29 67	3	26.9267	.00000 .99999				
29 68	2	31.7576	.00000 .99999				
29 69	1	34.7346	.00000 .99999				
29 70	0	37.0465	.00000 .99999				
30 0 69	144.6818	.00000 .99999					
30 1 68	138.4775	.00000 .99999					
30 2 67	132.4091	.00000 .99999					
30 3 66	126.4773	.00000 .99999					
30 4 65	120.6818	.00000 .99999					
30 5 64	115.0227	.00000 .99999					
30 6 63	109.5000	.00000 .99999					
30 7 62	104.1136	.00000 .99999					
30 8 61	98.8636	.00000 .99999					
30 9 60	93.7500	.00000 .99999					
30 10 59	88.7727	.00000 .99999					
30 11 58	83.9318	.00000 .99999					
30 12 57	79.2273	.00000 .99999					
30 13 56	74.6591	.00000 .99999					
30 14 55	70.2273	.00000 .99999					
30 15 54	65.9318	.00000 .99999					
30 16 53	61.7727	.00000 .99999					
30 17 52	57.7500	.00000 .99999					
30 18 51	53.8636	.00000 .99999					
30 19 50	50.1136	.00000 .99999					
30 20 49	46.5000	.00000 .99999					
30 21 48	43.0227	.00000 .99999					
30 22 47	39.6818	.00000 .99999					
30 23 46	36.4773	.00000 .99999					
30 24 45	33.4091	.00000 .99999					
30 25 44	30.4773	.00000 .99999					
30 26 43	27.6818	.00000 .99999					
30 27 42	25.0227	.00000 .99999					
30 28 41	22.5000	.00000 .99999					
30 29 40	20.1136	.00000 .99999					
30 30 39	17.8636	.00000 .99999					
30 31 38	15.7500	.00000 .99999					
30 32 37	13.7727	.00000 .99999					
30 33 36	11.9318	.00000 .99999					
30 34 35	10.2273	.00000 .99999					
30 35 34	8.6591	.00000 .99999					
30 36 33	7.2273	.00000 .99999					
30 37 32	5.9318	.00000 .99999					
30 38 31	4.7727	.00000 .99999					
30 39 30	3.7500	.00000 .99999					
30 40 29	2.8636	.00000 .99999					
30 41 28	2.1136	.00000 .99999					
30 42 27	1.5000	.00000 .99999					
30 43 26	1.0227	.00000 .99999					
30 44 25	.6818	.00000 .99999					
30 45 24	.4773	.00000 .99999					

(13)

TABLE C

U	V	X2	P(A) CUM P(C)	U	V	X2	P(A) CUM P(C)
31 26 42	25.6667	.00000 .99999					
31 27 41	23.9805	.00000 .99999					
31 28 40	22.6667	.00000 .99999					
31 29 39	21.7712	.00000 .99999					
31 30 38	21.2121	.00000 .99999					
31 31 37	20.1898	.00000 .99999					
31 32 36	19.5000	.00000 .99999					
31 33 35	18.5556	.00000 .99999					
31 34 34	18.0399	.00000 .99999					
31 35 33	17.8421	.00000 .99999					
31 36 32	16.1212	.00000 .99999					
31 37 31	14.9167	.00000 .99999					
31 38 30	13.8005	.00000 .99999					
31 39 29	12.9167	.00000 .99999					
31 40 28	12.1212	.00000 .99999					
31 41 27	11.4621	.00000 .99999					
31 42 26	10.9399	.00000 .99999					
31 43 25	10.5556	.00000 .99999					
31 44 24	10.3000	.00000 .99999					
31 45 23	10.1898	.00000 .99999					
31 46 22	10.1212	.00000 .99999					
31 47 21	10.0917	.00000 .99999					
31 48 20	10.0667	.00000 .99999					
31 49 19	10.0399	.00000 .99999					
31 50 18	10.0167	.00000 .99999					
31 51 17	10.0000	.00000 .99999					
31 52 16	10.0000	.00000 .99999					
31 53 15	10.0000	.00000 .99999					
31 54 14	10.0000	.00000 .99999					
31 55 13	10.0000	.00000 .99999					
31 56 12	10.0000	.00000 .99999					
31 57 11	10.0000	.00000 .99999					
31 58 10	10.0000	.00000 .99999					
31 59 9	10.0000	.00000 .99999					
31 60 8	10.0000	.00000 .99999					
31 61 7	10.0000	.00000 .99999					
31 62 6	10.0000	.00000 .99999					
31 63 5	10.0000	.00000 .99999					
31 64 4	10.0000	.00000 .99999					
31 65 3	10.0000	.00000 .99999					
31 66 2	10.0000	.00000 .99999					
31 67 1	10.0000	.00000 .99999					
31 68 0	10.0000	.00000 .99999					
32 0 67	136.0750	.00000 .99999					
32 1 66	130.0530	.00000 .99999					
32 2 65	124.1667	.00000 .99999					
32 3 64	118.4167	.00000 .99999					
32 4 63	112.8000	.00000 .99999					
32 5 62	107.3250	.00000 .99999					
32 6 61	101.9880	.00000 .99999					

(14)

TABLE C

CHI SQUARE - P(1/3), P(1/2), P(2/3) N=99

U	V	X2	P(A) CUM P(C)	U	V	X2	P(A) CUM P(C)
32 57 11	10.4167	.00000 .99999					
32 58 10	10.1667	.00000 .99999					
32 59 9	10.0530	.00000 .99999					
32 60 8	10.0000	.00000 .99999					
32 61 7	10.0000	.00000 .99999					
32 62 6	10.0000	.00000 .99999					
32 63 5	10.0000	.00000 .99999					
32 64 4	10.0000	.00000 .99999					
32 65 3	10.0000	.00000 .99999					
32 66 2	10.0000	.00000 .99999					
32 67 1	10.0000	.00000 .99999					
33 0 66	132.0000	.00000 .99999					
33 1 65	126.6667	.00000 .99999					
33 2 64	121.2727	.00000 .99999					
33 3 63	116.0136	.00000 .99999					
33 4 62	110.8699	.00000 .99999					
33 5 61	105.8455	.00000 .99999					
33 6 60	100.9345	.00000 .99999					
33 7 59	96.1309	.00000 .99999					
33 8 58	91.4363	.00000 .99999					
33 9 57	86.8527	.00000 .99999					
33 10 56	82.3812	.00000 .99999					
33 11 55	78.0240	.00000 .99999					
33 12 54	73.7827	.00000 .99999					
33 13 53	69.6527	.00000 .99999					
33 14 52	65.6363	.00000 .99999					
33 15 51	61.7349	.00000 .99999					
33 16 50	57.9545	.00000 .99999					
33 17 49	54.2945	.00000 .99999					
33 18 48	50.7549	.00000 .99999					
33 19 47	47.3363	.00000 .99999					
33 20 46	44.0363	.00000 .99999					
33 21 45	40.8527	.00000 .99999					
33 22 44	37.7827	.00000 .99999					
33 23 43	34.8363	.00000 .99999					
33 24 42	31.9167	.00000 .99999					
33 25 41	29.0240	.00000 .99999					
33 26 40	26.1549	.00000 .99999					
33 27 39	23.3167	.00000 .99999					
33 28 38	20.5090	.00000 .99999					
33 29 37	17.7349	.00000 .99999					
33 30 36	15.0000	.00000 .99999					
33 31 35	12.3167	.00000 .99999					
33 32 34	9.6812	.00000 .99999					
33 33 33	7.0909	.00000 .99999					
33 34 32	4.5455	.00000 .99999					
33 35 31	2.0455	.00000 .99999					
33 36 30	0.0000	.00000 .99999					
33 37 29	0.0000	.00000 .99999					
33 38 28	0.0000	.00000 .99999					
33 39 27	0.0000	.00000 .99999					
33 40 26	0.0000	.00000 .99999					

(15)

TABLE C

U	V	X2	P(A) CUM P(C)	U	V	X2	P(A) CUM P(C)
34 22 43	31.0758	.00000 .99999					
34 23 42	28.2348	.00000 .99999					
34 24 41	25.5303	.00000 .99999					
34 25 40	22.9621	.00000 .99999					
34 26 39	20.5333	.00000 .99999					
34 27 38	18.2368	.00000 .99999					
34 28 37	16.0758	.00000 .99999					
34 29 36	14.0530	.00000 .99999					
34 30 35	12.1667	.00000 .99999					
34 31 34	10.4167	.00000 .99999					
34 32 33	8.8030	.00000 .99999					
34 33 32	7.3258	.00000 .99999					
34 34 31	5.9909	.00000 .99999					
34 35 30	4.7803	.00000 .99999					
34 36 29	3.7121	.00000 .99999					
34 37 28	2.7803	.00000 .99999					
34 38 27	1.9880	.00000 .99999					
34 39 26	1.3258	.00000 .99999					
34 40 25	0.8030	.00000 .99999					
34 41 24	0.4167	.00000 .99999					
34 42 23	0.1667	.00000 .99999					
34 43 22	0.0530	.00000 .99999					
34 44 21	0.0000	.00000 .99999					
34 45 20	0.0000	.00000 .99999					
34 46 19	0.0000	.00000 .99999					
34 47 18	0.0000	.00000 .99999					
34 48 17	0.0000	.00000 .99999					
34 49 16	0.0000	.00000 .99999					
34 50 15	0.0000	.00000 .99999					
34 51 14	0.0000	.00000 .99999					
34 52 13	0.0000	.00000 .99999					
34 53 12	0.0000	.00000 .99999					
34 54 11	0.0000	.00000 .99999					
34 55 10	0.0000	.00000 .99999					
34 56 9	0.0000	.00000 .99999					
34 57 8	0.0000	.00000 .99999					
34 58 7	0.0000	.00000 .99999					
34 59 6	0.0000	.00000 .99999					
34 60 5	0.0000	.00000 .99999					
34 61 4	0.0000	.00000 .99999					
34 62 3	0.0000	.00000 .99999					
34 63 2	0.0000	.00000 .99999					
34 64 1	0.0000	.00000 .99999					
34 65 0	0.0000	.00000 .99999					
34 66 0	0.0000	.00000 .99999					
34 67 0	0.0000	.00000 .99999					
34 68 0	0.0000	.00000 .99999					
34 69 0	0.0000	.00000 .99999					
34 70 0	0.0000	.00000 .99999					
35 0 67	136.0750	.00000 .99999					
35 1 66	130.0530	.00000 .99999					
35 2 65	124.1667	.00000 .99999					
35 3 64	118.4167	.00000 .99999					
35 4 63	112.8000	.00000 .99999					
35 5 62	107.3250	.00000 .99999					
35 6 61	101.9880	.00000 .99999					

(16)

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

TABLE C

CHI SQUARE - P(41/33), P(14/9), P(24/29) N=39

TABLE C

Table with 16 columns: U, V, W, X2, P(A) CUM P(C), U, V, W, X2, P(A) CUM P(C), U, V, W, X2, P(A) CUM P(C), U, V, W, X2, P(A) CUM P(C). Rows contain numerical data for various statistical calculations.

(21)

(22)

TABLE C

CHI SQUARE - P(41/33), P(14/9), P(24/29) N=39

TABLE C

Table with 16 columns: U, V, W, X2, P(A) CUM P(C), U, V, W, X2, P(A) CUM P(C), U, V, W, X2, P(A) CUM P(C), U, V, W, X2, P(A) CUM P(C). Rows contain numerical data for various statistical calculations.

(23)

(24)

**THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC**

TABLE C

TABLE C

CHI SQUARE - P(1/3), P(4/9), P(2/2/9) N=99

U	V	W	X ²	P(A) CUM P(C)	U	V	W	X ²	P(A) CUM P(C)	U	V	W	X ²	P(A) CUM P(C)
49 37 13	12.5530	.00000	.99815		50 36 13	13.8939	.00001	.99913		51 36 12	15.8182	.00000	.99965	
49 38 14	13.1212	.00002	.99888		50 37 12	14.4167	.00001	.99926		51 37 11	17.4318	.00000	.99972	
49 39 11	13.8258	.00001	.99906		50 38 11	15.9758	.00001	.99950		51 38 10	17.1818	.00000	.99983	
49 40 10	14.6667	.00001	.99930		50 39 10	15.8712	.00000	.99966		51 39 9	16.8667	.00000	.99987	
49 41 9	15.8439	.00000	.99960		50 40 9	16.8030	.00000	.99977		51 40 8	19.8909	.00000	.99992	
49 42 8	16.7576	.00001	.99975		50 41 8	17.8712	.00000	.99986		51 41 7	20.2500	.00000	.99995	
49 43 7	18.0076	.00000	.99987		50 42 7	19.8758	.00000	.99992		51 42 6	21.5455	.00000	.99997	
49 44 6	19.3939	.00000	.99993		50 43 6	20.4167	.00000	.99996		51 43 5	22.9773	.00000	.99998	
49 45 5	21.8167	.00000	.99996		50 44 5	21.8939	.00000	.99996		51 44 4	24.5455	.00000	.99999	
49 46 4	22.5758	.00000	.99998		50 45 4	23.5076	.00000	.99998		51 45 3	26.2500	.00000	.99999	
49 47 3	24.3712	.00000	.99999		50 46 3	25.2576	.00000	.99999		51 46 2	28.0909	.00000	.99999	
49 48 2	26.3030	.00000	.99999		50 47 2	27.1439	.00000	.99999		51 47 1	30.8667	.00000	.99999	
49 49 1	28.3712	.00000	.99999		50 48 1	29.1667	.00000	.99999		51 48 0	32.1818	.00000	.99999	
49 50 0	30.7576	.00000	.99999		50 49 0	31.3258	.00000	.99999		52 0 47	83.3485	.00000	.99999	
50 0 49	83.8939	.00000	.99999		51 0 48	84.5455	.00000	.99999		52 1 46	79.1439	.00000	.99999	
50 1 48	81.5076	.00000	.99999		51 1 47	80.2500	.00000	.99999		52 2 45	75.8758	.00000	.99999	
50 2 47	77.2576	.00000	.99999		51 2 46	76.0909	.00000	.99999		52 3 44	71.1439	.00000	.99999	
50 3 46	73.1439	.00000	.99999		51 3 45	72.0667	.00000	.99999		52 4 43	67.3485	.00000	.99999	
50 4 45	69.1667	.00000	.99999		51 4 44	68.1818	.00000	.99999		52 5 42	63.8899	.00000	.99999	
50 5 44	65.3258	.00000	.99999		51 5 43	64.4318	.00000	.99999		52 6 41	60.1667	.00000	.99999	
50 6 43	61.8212	.00000	.99999		51 6 42	60.8182	.00000	.99999		52 7 40	56.7803	.00000	.99999	
50 7 42	58.4530	.00000	.99999		51 7 41	57.3809	.00000	.99999		52 8 39	53.5383	.00000	.99999	
50 8 41	54.8212	.00000	.99999		51 8 40	54.0000	.00000	.99999		52 9 38	50.4167	.00000	.99999	
50 9 40	51.3258	.00000	.99999		51 9 39	50.7955	.00000	.99999		52 10 37	47.4394	.00000	.99999	
50 10 39	48.1667	.00000	.99999		51 10 38	47.7273	.00000	.99999		52 11 36	44.5985	.00000	.99999	
50 11 38	43.1439	.00000	.99999		51 11 37	44.7955	.00000	.99999		52 12 35	41.8939	.00000	.99999	
50 12 37	42.2576	.00000	.99999		51 12 36	42.8000	.00000	.99999		52 13 34	39.3258	.00000	.99999	
50 13 36	39.5076	.00000	.99999		51 13 35	39.5409	.00000	.99999		52 14 33	36.8939	.00000	.99999	
50 14 35	35.8939	.00000	.99999		51 14 34	36.8182	.00000	.99999		52 15 32	34.5985	.00000	.99999	
50 15 34	34.4167	.00000	.99999		51 15 33	34.4318	.00000	.99999		52 16 31	32.4394	.00000	.99999	
50 16 33	32.1758	.00000	.99999		51 16 32	32.1818	.00000	.99999		52 17 30	30.4167	.00000	.99999	
50 17 32	29.8712	.00000	.99999		51 17 31	30.8667	.00000	.99999		52 18 29	28.5383	.00000	.99999	
50 18 31	27.8030	.00000	.99999		51 18 30	28.0909	.00000	.99999		52 19 28	26.7803	.00000	.99999	
50 19 30	25.8712	.00000	.99999		51 19 29	26.2500	.00000	.99999		52 20 27	25.1667	.00000	.99999	
50 20 29	24.7576	.00000	.99999		51 20 28	24.5455	.00000	.99999		52 21 26	23.8899	.00000	.99998	
50 21 28	22.4167	.00000	.99998		51 21 27	22.9773	.00000	.99998		52 22 25	22.3485	.00000	.99998	
50 22 27	20.8939	.00000	.99996		51 22 26	21.5455	.00000	.99997		52 23 24	21.1439	.00000	.99997	
50 23 26	19.5076	.00000	.99994		51 23 25	20.2500	.00000	.99998		52 24 23	20.8758	.00000	.99995	
50 24 25	18.2576	.00000	.99989		51 24 24	19.0909	.00000	.99992		52 25 22	19.1439	.00000	.99993	
50 25 24	17.1439	.00000	.99981		51 25 23	18.0667	.00000	.99987		52 26 21	18.3485	.00000	.99989	
50 26 23	16.1667	.00000	.99968		51 26 22	17.1818	.00000	.99983		52 27 20	17.6894	.00000	.99986	
50 27 22	15.3258	.00000	.99955		51 27 21	16.4318	.00000	.99972		52 28 19	17.1667	.00000	.99982	
50 28 21	14.6212	.00001	.99929		51 28 20	15.8182	.00000	.99965		52 29 18	16.7803	.00000	.99976	
50 29 20	14.0530	.00001	.99916		51 29 19	15.3409	.00001	.99956		52 30 17	16.5383	.00000	.99974	
50 30 19	13.6212	.00001	.99908		51 30 18	15.0000	.00001	.99945		52 31 16	16.4167	.00000	.99971	
50 31 18	13.3258	.00002	.99906		51 31 17	14.7955	.00001	.99936		52 32 15	16.4394	.00000	.99972	
50 32 17	13.1667	.00002	.99905		51 32 16	14.7273	.00001	.99932		52 33 14	16.5985	.00000	.99974	
50 33 16	13.1439	.00002	.99901		51 33 15	14.7955	.00001	.99935		52 34 13	16.8939	.00000	.99977	
50 34 15	13.2576	.00002	.99888		51 34 14	15.0000	.00001	.99945		52 35 12	17.3258	.00000	.99984	
50 35 14	13.5076	.00002	.99891		51 35 13	15.3409	.00001	.99956		52 36 11	17.8939	.00000	.99987	

U	V	W	X ²	P(A) CUM P(C)	U	V	W	X ²	P(A) CUM P(C)	U	V	W	X ²	P(A) CUM P(C)
52 37 10	18.5985	.00000	.99998		53 37 9	19.4394	.00000	.99994		54 37 8	20.4167	.00000	.99996	
52 38 9	19.4394	.00000	.99994		53 38 8	20.4167	.00000	.99996		54 38 7	21.5500	.00000	.99997	
52 39 8	20.4167	.00000	.99996		53 39 7	21.5500	.00000	.99997		54 39 6	22.7803	.00000	.99998	
52 40 7	21.5500	.00000	.99997		53 40 6	22.7803	.00000	.99998		54 40 5	24.1867	.00000	.99998	
52 41 6	22.7803	.00000	.99998		53 41 5	24.1867	.00000	.99998		54 41 4	25.8894	.00000	.99999	
52 42 5	24.1867	.00000	.99998		53 42 4	25.8894	.00000	.99999		54 42 3	27.3485	.00000	.99999	
52 43 4	25.8894	.00000	.99999		53 43 3	27.3485	.00000	.99999		54 43 2	29.1439	.00000	.99999	
52 44 3	27.3485	.00000	.99999		53 44 2	29.1439	.00000	.99999		54 44 1	31.0758	.00000	.99999	
52 45 2	29.1439	.00000	.99999		53 45 1	31.0758	.00000	.99999		55 0 46	82.3030	.00000	.99999	
52 46 1	31.0758	.00000	.99999		53 46 0	33.1439	.00000	.99999		55 1 45	78.1894	.00000	.99999	
52 47 0	33.1439	.00000	.99999		53 47 0	35.3030	.00000	.99999		55 2 44	74.2121	.00000	.99999	
53 0 46	82.3030	.00000	.99999		53 48 0	37.5076	.00000	.99999		55 3 43	70.3712	.00000	.99999	
53 1 45	78.1894	.00000	.99999		53 49 0	39.8530	.00000	.99999		55 4 42	66.8667	.00000	.99999	
53 2 44	74.2121	.00000	.99999		53 50 0	42.3485	.00000	.99999		55 5 41	63.6885	.00000	.99999	
53 3 43	70.3712	.00000	.99999		53 51 0	44.9909	.00000	.99999		55 6 40	59.6667	.00000	.99999	
53 4 42	66.8667	.00000	.99999		53 52 0	47.7955	.00000	.99999		55 7 39	55.3712	.00000	.99999	
53 5 41	63.6885	.00000	.99999		53 53 0	50.7500	.00000	.99999		55 8 38	51.2121	.00000	.99999	
53 6 40	59.6667	.00000	.99999		53 54 0	54.7576	.00000	.99999		55 9 37	46.1894	.00000	.99999	
53 7 39	55.3712	.00000	.99999		53 55 0	59.5076	.00000	.99999		55 10 36	41.3030	.00000	.99999	
53 8 38	51.2121	.00000	.99999		53 56 0	65.5076	.00000	.99999		55 11 35	36.8530	.00000	.99999	
53 9 37	46.1894	.00000	.99999		53 57 0	72.8530	.00000	.99999		55 12 34	32.8530	.00000	.99999	
53 10 36	41.3030	.00000	.99999		53 58 0	81.5076	.00000	.99999		55 13 33	29.3485	.00000	.99999	
53 11 35	36.8530	.00000	.99999		53 59 0	91.5076	.00000	.99999		55 14 32	26.3485	.00000	.99999	
53 12 34	32.8530	.00000	.99999		53 60 0	103.0000	.00000	.99999		55 15 31	23.8485	.00000	.99999	
53 13 33	29.3485	.00000	.99999		53 61 0	116.0000	.00000	.99999		55 16 30	21.8485	.00000	.99999	
53 14 32	26.3485	.00000	.99999		53 62 0	130.7576	.00000	.99999		55 17 29	20.3485	.00000	.99999	
53 15 31	23.8485	.00000	.99999		53 63 0	147.5076	.00000	.99999		55 18 28	19.3485	.00000	.99999	
53 16 30	21.8485	.00000	.99999		53 64 0	166.5076	.00000	.99999		55 19 27	18.8485	.00000	.99999	
53 17 29	20.3485	.00000	.99999		53 65 0	187.8530	.00000	.999						

THIS PAGE IS BEST QUALITY PRACTICABLE FROM COPY FURNISHED TO DDC

TABLE D
CHI SQUARE - P(11/3), P(14/3), P(2(2/3) N=99

Table with 6 columns: U, V, W, X2, P(A), CUM P(D). Contains 49 rows of numerical data.

(17)

TABLE D

Table with 6 columns: U, V, W, X2, P(A), CUM P(D). Contains 49 rows of numerical data.

(18)

TABLE D
CHI SQUARE - P(11/3), P(14/3), P(2(2/3) N=99

Table with 6 columns: U, V, W, X2, P(A), CUM P(D). Contains 49 rows of numerical data.

(19)

TABLE D

Table with 6 columns: U, V, W, X2, P(A), CUM P(D). Contains 49 rows of numerical data.

(20)

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

TABLE D
CHI SQUARE - P(11/3), P(14/9), P(21/7) N=99

U	V	W	X2	P(A)	CUM P(D)
43	3	56	92.2348	.00000	.99999
17	13	69	150.2076	.00000	.99999
23	8	66	119.2933	.00000	.99999
45	2	52	85.3636	.00000	.99999
25	9	67	122.9167	.00000	.99999
27	7	65	115.2502	.00000	.99999
32	5	62	107.3258	.00000	.99999
39	3	57	98.9775	.00000	.99999
19	12	68	133.5000	.00000	.99999
33	6	60	112.1212	.00000	.99999
21	16	80	126.8182	.00000	.99999
59	1	48	77.3866	.00000	.99999
28	6	64	115.8888	.00000	.99999
44	2	53	87.4398	.00000	.99999
53	1	45	78.1898	.00000	.99999
52	1	46	79.1339	.00000	.99999
19	11	69	131.0985	.00000	.99999
43	2	54	89.6667	.00000	.99999
33	5	58	97.8712	.00000	.99999
15	14	70	135.0000	.00000	.99999
51	1	47	80.2500	.00000	.99999
29	8	67	123.9585	.00000	.99999
35	5	63	111.0985	.00000	.99999
21	7	66	120.8085	.00000	.99999
34	4	61	103.5303	.00000	.99999
22	9	68	127.6898	.00000	.99999
16	13	70	135.5258	.00000	.99999
50	1	48	81.5076	.00000	.99999
28	6	65	117.6212	.00000	.99999
42	2	55	92.6455	.00000	.99999
37	1	49	82.9167	.00000	.99999
39	3	59	100.9167	.00000	.99999
20	10	69	131.8030	.00000	.99999
17	12	70	135.7576	.00000	.99999
33	6	62	109.9699	.00000	.99999
30	5	64	113.0227	.00000	.99999
48	1	50	86.8733	.00000	.99999
48	1	50	86.8733	.00000	.99999
41	2	56	94.5758	.00000	.99999
23	7	67	125.0985	.00000	.99999
23	8	68	128.6667	.00000	.99999
18	11	73	136.2955	.00000	.99999
47	1	51	86.1898	.00000	.99999
36	3	60	109.1136	.00000	.99999
27	6	66	121.9091	.00000	.99999
21	9	69	132.6136	.00000	.99999
40	2	57	97.2576	.00000	.99999
49	1	52	88.1539	.00000	.99999
32	4	63	112.8030	.00000	.99999
15	13	71	140.7955	.00000	.99999
29	5	65	119.0985	.00000	.99999

(25)

TABLE D

U	V	W	X2	P(A)	CUM P(D)
59	1	59	105.3409	.00000	.99999
28	4	67	129.1667	.00000	.99999
10	11	73	152.7955	.00000	.99999
34	2	63	116.5303	.00000	.99999
23	4	70	140.5758	.00000	.99999
45	0	54	94.9091	.00000	.99999
21	7	71	144.6136	.00000	.99999
25	5	69	136.9167	.00000	.99999
38	1	60	108.4167	.00000	.99999
16	10	75	155.2576	.00000	.99999
19	8	72	149.0303	.00000	.99999
30	3	66	126.4773	.00000	.99999
49	0	85	97.1667	.00000	.99999
27	4	68	133.6364	.00000	.99999
33	2	64	120.2727	.00000	.99999
17	1	61	111.6439	.00000	.99999
37	3	73	155.8258	.00000	.99999
43	0	56	99.5758	.00000	.99999
22	4	71	145.6212	.00000	.99999
24	5	70	141.7500	.00000	.99999
20	7	72	149.8712	.00000	.99999
29	3	67	130.7348	.00000	.99999
42	0	57	102.1364	.00000	.99999
32	2	65	124.1667	.00000	.99999
36	1	62	115.0227	.00000	.99999
26	4	69	136.2576	.00000	.99999
18	8	73	154.5000	.00000	.99999
15	10	74	159.0000	.00000	.99999
41	0	58	104.8485	.00000	.99999
28	3	66	135.1439	.00000	.99999
35	1	63	118.5530	.00000	.99999
20	6	72	150.8182	.00000	.99999
23	5	71	146.7348	.00000	.99999
16	9	74	159.5076	.00000	.99999
32	2	66	128.2121	.00000	.99999
19	7	75	155.2885	.00000	.99999
40	0	59	107.7121	.00000	.99999
25	4	70	143.0303	.00000	.99999
17	8	74	160.1212	.00000	.99999
39	1	64	122.2348	.00000	.99999
34	0	60	110.7273	.00000	.99999
37	3	69	139.7045	.00000	.99999
20	2	67	132.4091	.00000	.99999
22	5	72	151.8712	.00000	.99999
20	6	73	156.1667	.00000	.99999
24	4	71	147.9585	.00000	.99999
15	9	75	165.3409	.00000	.99999
33	0	61	113.8939	.00000	.99999
36	1	65	126.0682	.00000	.99999
18	7	74	160.8485	.00000	.99999

(26)

TABLE D
CHI SQUARE - P(11/3), P(14/9), P(21/7) N=99

U	V	W	X2	P(A)	CUM P(D)
29	3	76	175.8712	.00000	.99999
16	5	78	185.8712	.00000	.99999
23	1	73	162.1898	.00000	.99999
22	2	75	171.4398	.00000	.99999
24	0	71	153.8939	.00000	.99999
19	3	77	181.6439	.00000	.99999
29	1	74	167.3866	.00000	.99999
17	4	78	186.6667	.00000	.99999
21	2	76	177.0000	.00000	.99999
15	5	79	192.0682	.00000	.99999
27	0	72	158.7273	.00000	.99999
25	1	75	172.7348	.00000	.99999
18	3	78	187.5682	.00000	.99999
15	4	79	192.8030	.00000	.99999
26	0	73	163.7121	.00000	.99999
23	2	77	182.7121	.00000	.99999
22	1	76	178.2348	.00000	.99999
25	0	74	168.8485	.00000	.99999
17	3	79	193.6439	.00000	.99999
19	2	78	188.5758	.00000	.99999
15	4	80	199.4909	.00000	.99999
21	1	77	183.8866	.00000	.99999
24	0	75	174.1364	.00000	.99999
16	3	80	199.8712	.00000	.99999
18	2	79	194.5909	.00000	.99999
23	1	78	189.6898	.00000	.99999
20	0	76	179.5758	.00000	.99999
17	2	80	208.7576	.00000	.99999
15	3	81	208.2500	.00000	.99999
22	0	77	185.1667	.00000	.99999
19	1	79	195.6439	.00000	.99999
15	2	81	207.0758	.00000	.99999
21	0	78	190.9091	.00000	.99999
18	1	80	201.7500	.00000	.99999
23	0	79	195.8030	.00000	.99999
15	2	82	213.5455	.00000	.99999
17	1	81	208.0076	.00000	.99999
19	0	80	202.8485	.00000	.99999
16	1	82	214.4167	.00000	.99999
15	0	81	209.8485	.00000	.99999
17	1	83	221.8773	.00000	.99999
17	0	82	215.3939	.00000	.99999
15	0	83	221.8939	.00000	.99999
15	0	84	224.5455	.00000	.99999
0	0	99	346.5000	.00000	.00000

(27)

THIS PAGE IS BEST QUALITY PRACTICABLE FROM COPY FURNISHED TO DDC

TABLE E

CHE SQUARE - P(41/3), P(44/9), P(42/9) N=99

Table with 10 columns: U, V, W, X2, P(A), CUM P(E), U, V, W, X2, P(A), CUM P(E). Contains 99 rows of numerical data.

(5)

TABLE E

Table with 10 columns: U, V, W, X2, P(A), CUM P(E), U, V, W, X2, P(A), CUM P(E). Contains 99 rows of numerical data.

(6)

TABLE E

CHE SQUARE - P(41/3), P(44/9), P(42/9) N=99

Table with 10 columns: U, V, W, X2, P(A), CUM P(E), U, V, W, X2, P(A), CUM P(E). Contains 99 rows of numerical data.

(7)

TABLE E

Table with 10 columns: U, V, W, X2, P(A), CUM P(E), U, V, W, X2, P(A), CUM P(E). Contains 99 rows of numerical data.

(8)

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

TABLE EXP

TABLE EXP

CHI SQUARE - P(01/3), P(10/9), P(21/7) N=99

Table with 5 columns: U, V, W, X2, P(IA) CUR(EMP). Rows contain numerical data for chi-square analysis.

(17)

Table with 5 columns: U, V, W, X2, P(IA) CUR(EMP). Rows contain numerical data for chi-square analysis.

(18)

TABLE EXP

TABLE EXP

CHI SQUARE - P(01/3), P(10/9), P(21/7) N=99

Table with 5 columns: U, V, W, X2, P(IA) CUR(EMP). Rows contain numerical data for chi-square analysis.

(19)

Table with 5 columns: U, V, W, X2, P(IA) CUR(EMP). Rows contain numerical data for chi-square analysis.

(20)

