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PARAMETER ESTIMATION FOR GENERATOR SIMULATION STUDIES

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This report presents a detailed study of alternator parameter estimation procedures, including comparison of theoretical results to test data and to simulation results.

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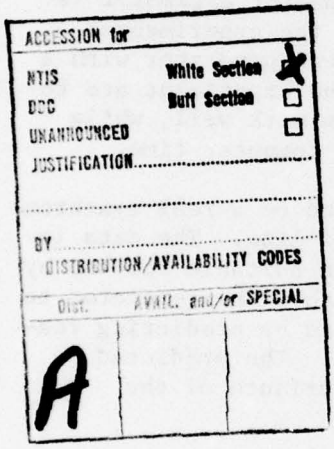
FOREWORD

This final report was submitted by Georgia Institute of Technology, Atlanta GA 30332, and Louisiana State University, Baton Rouge LA 70803, under contract F33615-76-C-2050. The effort was sponsored by the U. S. Air Force Aero Propulsion Laboratory, Air Force Systems Command, Wright-Patterson AFB OH under Project 3145, Task 32 and Work Unit 11 with Capt Hugh L. Southall, AFAPL/POD-1, as the Project Engineer. Dr. R. P. Webb and Mr. C. W. Brice performed part of the work at Georgia Tech and Dr. O. T. Tan performed part of the work at LSU. This work was performed under the auspices of the U. S. Air Force Aero Propulsion Laboratory's Senior Investigator Program.



SUMMARY

The purpose of this research is to model a synchronous generator, using statistical estimation theory to determine the parameters of the model from experimental data. An ideal generator model is represented in state-space formulation, and the method of quasilinearization is used to develop an optimal parameter estimation algorithm, implemented on a small digital computer.

The experiment to produce data for the parameter estimator is designed with the aid of a computer simulation of the experimental setup. The simulation includes a model of a typical generator with a switched load. The objectives of the design of the experiment are to produce sufficient data to permit the estimator to work well, while recognizing economic constraints on equipment and computer time.

The experiment thus designed, is implemented on a real synchronous generator with data recorded and digitized off-line. The data is then stored on a magnetic disk cartridge in a form suitable for use by the digital computer. The parameter estimates enable the generator to be modeled. The adequacy of the model is validated by predicting response of the generator to a larger change in load. The predicted response matches the actual response within the variance of the measurement noise.

The standard methods of machine parameter estimation have been formulated primarily to provide data for utility system stability studies. The models employed for this purpose are extracted from rather simplified measurements, requiring simplifying assumptions, and do not account for measurement noise and inaccuracies. This report presents two methods of estimating the generator model parameters from switched-load test data corrupted by considerable measurement noise.

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SECTION I

INTRODUCTION

1.1 MOTIVATION

In developing electric generation equipment to meet changing requirements such as those posed by military applications where the machine is used to power a variety non-standard loads, it is essential that the machine and load be compatible. It is essential, therefore, that techniques to analyze various generator configurations in specific loading environments be available. Such techniques can be implemented using digital computer simulation if the parameters of mathematical models of the machines are known. Therefore, a problem of great practical importance is the determination of the parameters of models of synchronous generators.

The parameters of an electrical machine can be calculated, at least in principle, if the internal geometry and material properties are known. However, such detailed information is often not available in a form amenable to system analysis. In this case, one must subject the machine to tests, recording data from measurements at the terminals. This data is invariably corrupted with noise during recording and processing. Therefore, the basic problem of determining machine parameters from tests is to develop an estimation algorithm to extract the parameters from noisy data and to design an experiment that produces sufficient data for this algorithm to work accurately.

1.2 REVIEW OF PAST APPROACHES

A coupled circuit model of a three-phase synchronous generator

derived by Park [1] is in general used by power system analysts. This model is obtained from a linear lumped-parameter model by transforming the armature quantities onto a two-axis coordinate frame that rotates with the rotor. Rotor circuits consisting of a field winding and two damper windings are invariant under the transformation. The result of Park's transformation is a set of stationary differential equations in the two-axis coordinate frame, related to the terminal voltages and currents by a set of time-varying measurement equations. The parameters of this model are the inductances and resistances of the two-axis circuits.

Since these parameters include mutual inductance between rotor and stator circuits and inductances of rotor circuits which are not accessible, a simplified parameter set is often used [2,3]. The simplified parameters can be determined from results of relatively simple tests [4]. An approximate analysis is carried out by assuming that a transient in the armature current initially affects the damper windings and then later affects the field winding. Also, the time constants of the dampers are assumed to be much shorter than the time constant of the field. These assumptions are generally accepted but tend to force the characteristics of a standard machine on all machines [2].

Several attempts to overcome the drawbacks of this simplified analysis have been published recently. Canay [5] added a parameter to account for a different coupling between the various rotor and armature circuits. The results predicted rotor quantities with greater accuracy. Yu and Moussa [6] then described several approaches for determining Canay's reactance from tests. Transfer functions for synchronous generators have been determined from sinusoidal perturbations about an

operating point which are introduced by a fast-response static exciter [7] and from low-voltage measurements with the rotor at standstill [8]. The first method requires a source of field current (an exciter) which has a very fast response time while the second requires a variable frequency source of rather large current. In these frequency response tests, Bode plot construction techniques were used. This method requires judgment of the analyst in locating the breakpoint, since it is essentially a graphical method. No treatment of errors was given. Stanton [9] presented a statistical treatment of estimating a transform function that was an empirical relation between rotor speed and electric power output. The result is not directly applicable to determining the parameters of a physical model. Lee and Tan [10] recently implemented a least-squares algorithm to estimate the parameters of the simplified analysis plus Canay's reactance. This approach assumed that the generator was subjected to a sudden three-phase short circuit test. The resulting estimator was tested on simulated data without the effects of measurement noise.

1.3 DEFINITION OF THE PROBLEM

The problem considered in this research is the estimation of the inductance and resistance parameters of Park's model of a synchronous generator. No intermediate parameter set requiring additional assumptions about the machine characteristics is imposed. Data is taken from the terminals of a synchronous generator under a resistive load. A transient is induced by a sudden switching of the load resistance to a smaller value. A statistical formulation of a parameter estimation algorithm is studied to enable an essessment of the effect of measurement noise upon the experiment. This approach overcomes many of the objections to previous

approaches by applying estimation theory in a simple experiment to determine the parameters of a synchronous generator model directly.

A state-space approach is taken by casting the model in the form

$$\frac{dx}{dt} = f(x,y,u,t) \tag{1}$$

$$z = h(x,y,t,t) + w \quad , \tag{2}$$

where x is the state vector, u is the input vector, y is the parameter vector, z is the measurement vector and w is an additive noise term. The problem is to estimate y based on measurements z.

The first step is to derive a parameter estimator. This is an algorithm implemented on a digital computer for recursively computing estimates of the model parameters. A weighted least-squares approach is taken to minimize an error criterion that is the weighted sum of the square of the error. The error is defined as the difference between the observed output and the output computed from the model using the current parameter estimates. An optimal estimator is derived from this formulation by choosing the weighting matrix as the inverse of the noise covariance matrix.

The second step is to design an experiment which produces sufficient data to permit the estimator to work effectively. The success of the parameter estimator is directly dependent upon the experimental conditions. In particular, the load and excitations as well as the data sampling rate must be adequately chosen. To approach this problem, an experiment was designed with the aid of computer simulations. That is, a simulation of a

generator with nominal parameters is used to produce data for the parameter estimator. This enabled the experimental conditions to be designed with maximum flexibility.

Finally, an experiment was implemented on an actual generator. Data recorded on an FM instrumentation tape recorder, was digitized and put into a form suitable for use by the estimation algorithm. The parameter estimator was then used to estimate the generator parameters. These estimates were used to predict the response of the generator to a larger change in load resistance. This final step validated the results and enabled the overall procedure to be evaluated.

SECTION II

DEVELOPMENT OF THE ESTIMATOR

2.1 INTRODUCTION

The object of this chapter is to develop the mathematical models needed to estimate the parameters of a synchronous generator. The approach taken is to model the generator, use this model to predict outputs based on the current parameter estimates, and then to adjust the parameter estimates systematically to minimize the weighted square of the difference between the measured outputs of the generator and the model outputs. This is the essence of a weighted least-squares estimation algorithm. Furthermore, incorporating the statistics of the noise which inevitably corrupts the measurements into the estimation algorithm leads to the maximum a posteriori probability estimator.

This chapter is divided into two main sections. The first presents the model of the generator in a form suitable for digital computer simulation. These equations are cast into a state-space notation for convenience. The second section develops weighted least-squares estimation algorithm by the method of quasilinearization. The statistics of the noise are used to derive an optimal estimator, which is the maximum a posteriori probability estimator. This formulation is a special case of weighted least-squares estimation with the weighting matrices determined from noise statistics and a priori information about parameter error statistics.

2.2 GENERATOR MODEL

A typical three-phase, alternating-current generator consists of

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three armature windings placed symmetrically on the stator surrounding a rotor that is driven externally. The rotor consists of an iron core with a field winding excited by direct current. Additional rotor circuits called damper windings, consisting of short-circuited bars, are often imbedded in the rotor surface. Such a machine, considered here, is drawn schematically in Figure 1a.

Due to the effect of the iron rotor, the armature inductances have components that vary with the rotor angle. By assuming symmetry of the rotor about the pole axis, or direct axis, and about the interpole axis (or quadrature axis) and by assuming sinusoidally distributed armature windings along the air gap, the fundamental component of the air gap flux linking the armature is proportional to $A + B\cos 2\theta$ [1]. As a result, the armature self and mutual inductances are

$$L_a = L_{a0} + L_{a1}\cos 2\theta \tag{3}$$

$$L_{ab} = -[L_{ab0} + L_{a1}\cos 2(\theta + \tfrac{\pi}{6})] \quad . \tag{4}$$

By projecting the armature circuit quantities onto the d-q coordinate frame, which is fixed in the rotor, Park [1] obtained a set of differential equations with constant coefficients. Fictitious windings along the d and q axes have constant inductances since the paths of flux have constant permeance. The self inductances of the d-q model are

$$L_D = L_{a0} + L_{ab0} + \frac{3}{2} L_{a1} \tag{5}$$

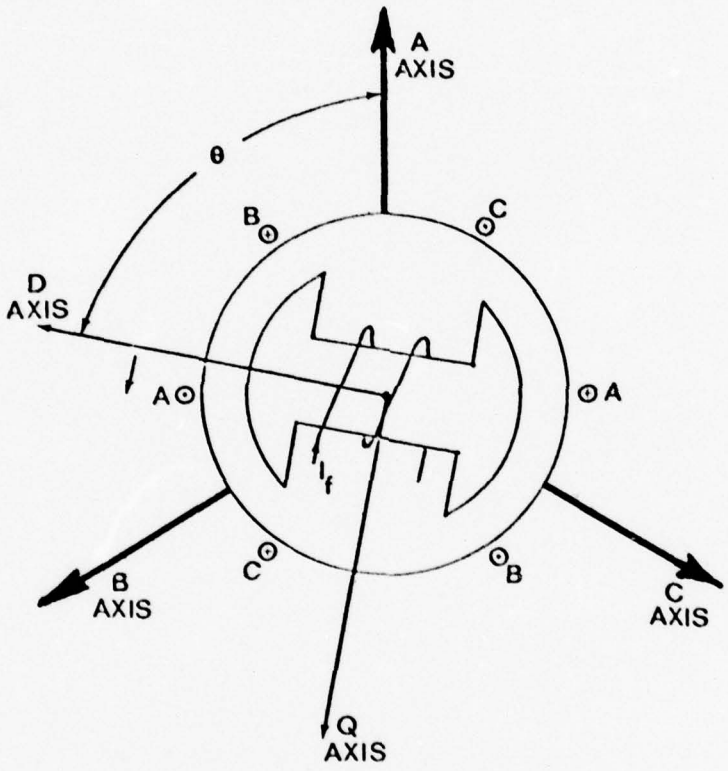

Figure 1a. Schematic Cross-Section of a Synchronous Generator.

$$L_Q = L_{a0} + L_{ab0} - \frac{3}{2} L_{a1} \quad . \tag{6}$$

Defining

$$L_1 \triangleq L_{a0} + L_{ab0} \tag{7}$$

and

$$L_2 \triangleq \frac{3}{2} L_{a1} \quad , \tag{8}$$

then L_D and L_Q are computed from

$$L_D = L_1 + L_2 \tag{9}$$

$$L_Q = L_1 - L_2 \tag{10}$$

The net result of the preceding is the circuit model illustrated in Figure 1b. It is described by a set of time-invariant, linear differential equations and a set of time-varying measurement equations. These are given by:

$$\frac{d\psi}{dt} = -(RL^{-1} + \Omega)\psi(t) + v(t) \tag{11}$$

$$z(t) = T(t)L^{-1}\psi(t) \tag{12}$$

where

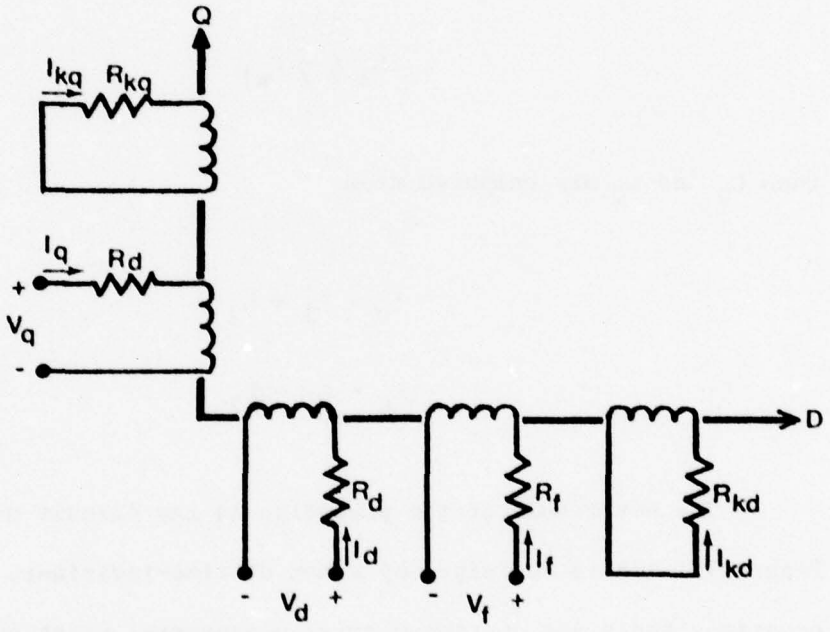

Figure 1b. D-Q Circuit Model of Synchronous Generator Due to Park.

$\psi = (\psi_D \ \psi_F \ \psi_{KD} \ \psi_Q \ \psi_{KQ})^T$, the flux linkage vector

$v = (v_D \ v_F \ 0 \ v_Q \ 0)^T$, the voltage vector

$z = (i_A \ i_B \ i_C \ i_F)^T$, the current vector

$$L = \begin{bmatrix} L_D & L_{DF} & L_{DKD} & 0 & 0 \\ L_{DF} & L_F & L_{FKD} & 0 & 0 \\ L_{DKD} & L_{FKD} & L_{KD} & 0 & 0 \\ 0 & 0 & 0 & L_Q & L_{QKQ} \\ 0 & 0 & 0 & L_{QKQ} & L_{KQ} \end{bmatrix}$$, the inductance matrix

$$R = \begin{bmatrix} R_D & & & & \\ & R_F & & 0 & \\ & & R_{KD} & & \\ & & & R_D & \\ & 0 & & & R_{KQ} \end{bmatrix}$$, the resistance matrix

$$\Omega = \begin{bmatrix} 0 & 0 & 0 & -\omega & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$, the speed matrix

$$T = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & 0 & 0 & -\sin\theta & 0 \\ \cos(\theta - \frac{2\pi}{3}) & 0 & 0 & -\sin(\theta - \frac{2\pi}{3}) & 0 \\ \cos(\theta + \frac{2\pi}{3}) & 0 & 0 & -\sin(\theta + \frac{2\pi}{3}) & 0 \\ 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 \end{bmatrix}$$

and $\theta = \omega t + \theta_0$. The measurement equations correspond to a transformation of coordinates from the D-Q reference frame to the three-phase armature reference frame, denoted by the subscripts A, B, and C. The rotor circuit quantities, denoted by subscript F for the field and by the subscripts KD and KQ for the damper windings, are invariant under this transformation.

These equations are not the same as those derived by Park [1] but have been modified as suggested by Lewis [11].

The virtue of this particular transformation is that the mutual reactances are always reciprocal. This is not true of Park's original equations.

These equations are valid for balanced three-phase operation of the machine. If an unbalanced load is connected, a zero-sequence circuit must be added to the D and Q circuits to represent the armature circuits. The zero-sequence equations are given below.

$$v_0 = \frac{1}{3}(V_a + V_b + V_c) \tag{13}$$

$$i_0 = \frac{1}{3}(i_a + i_b + i_c) \tag{14}$$

$$\psi_0 = \frac{1}{3}(\psi_a + \psi_b + \psi_c) \tag{15}$$

The zero-sequence variables are uncoupled from the rest of the equations, and

$$\frac{d\psi_o}{dt} = -\frac{R_D}{L_o}\psi_o + v_o \quad . \tag{16}$$

Since the analysis and experiments will be carried out under balanced conditions, the zero-sequence equation is not considered further.

Generator parameters are usually expressed in per unit, that is in dimensionless ratios to base values. The base values are ordinarily chosen to be the rated values to facilitate comparison of machines of different sizes and ratings. Although Equations (11) and (12) are valid in any consistent set of units, such as the MKS system, a per unit system is used subsequently to simplify the computations and to be consistent with accepted practices.

Since $i = L^{-1}\psi$ and $p = \frac{d}{dt}$, the direct axis part of Equation (11) can be rewritten in operational form

$$\begin{bmatrix} v_D \\ v_F \\ 0 \end{bmatrix} = \begin{bmatrix} R_D + L_D p & L_{DF} p & L_{DKD} p \\ L_{DF} p & R_F + L_F p & L_{FKD} p \\ L_{DKD} p & L_{FKD} p & R_{KD} + L_{KD} p \end{bmatrix} \begin{bmatrix} i_D \\ i_F \\ i_{KD} \end{bmatrix} + \begin{bmatrix} -\omega\psi_Q \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

Dividing each row by the corresponding base voltage and dividing and multiplying each column by the corresponding base current yields the equations in per unit.

$$\begin{bmatrix} v_d \\ v_f \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{i_B}{v_B}(R_D + L_D p) & \frac{i_{FB}}{v_B}(L_{DF} p) & \frac{i_{KDB}}{v_B}(L_{DKD} p) \\ \frac{i_B}{v_{FB}}(L_{DF} p) & \frac{i_{FB}}{v_{FB}}(R_F + L_F p) & \frac{i_{KDB}}{v_{FB}}(L_{FKD} p) \\ \frac{i_B}{v_{KDB}}(L_{DKD} p) & \frac{i_{FB}}{v_{KDB}}(L_{FKD} p) & \frac{i_{KDB}}{v_{KDB}}(R_{KD} + L_{KD} p) \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{kd} \end{bmatrix} + \begin{bmatrix} -\omega\psi_q \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

where

$$v_d = \frac{v_D}{v_B} \quad , \quad i_d = \frac{i_D}{i_B} \quad , \quad \psi_q = \frac{\psi_Q}{v_B}$$

$$v_f = \frac{v_F}{v_{FB}} \quad , \quad i_f = \frac{i_F}{i_{FB}} \quad , \quad \text{and} \quad i_{kd} = \frac{i_{KD}}{i_{KDB}} \quad .$$

Base current and voltage for the armature circuits are the rated values. Base field current is chosen to be the value of current which causes rated open-circuit armature voltage. Two constraints are placed on the three remaining arbitrary rotor base quantities,

$$v_{KDB}i_{KDB} = v_{FB}i_{FB} = v_B i_B \quad . \tag{19}$$

These relations imply that the base power is the same on all circuits and that the per unit inductance matrix is symmetric. One more constraint will be placed on the base quantities of the damper winding circuit. Choosing

$$i_{KDB} = \frac{L_{DKD}}{L_{KD}} i_B \tag{20}$$

results in

$$\frac{L_{DKD}}{v_{KDB}} i_B = \frac{L_{KD}}{v_{KDB}} i_{KDB}$$

or

$$L_{dkd} = L_{kd} \quad , \tag{21}$$

in per unit. By similar reasoning, choice of $v_{KQB}i_{KQB} = v_B i_B$ results in a reciprocal quadrature axis inductance matrix and choice $i_{KQB} = \frac{L_{QKQ}}{L_{KQ}} i_B$ results in $L_{qkq} = L_{kq}$, in per unit. As a result, Equations (11) and (12) are valid with the following per-unit quantities:

$$L_d = \frac{L_D i_B}{v_B} \quad , \quad L_f = \frac{L_F i_{FB}}{v_{FB}} \quad , \quad L_{df} = \frac{L_{DF} i_{FB}}{v_B} \quad , \quad L_{kd} = \frac{L_{KD} i_{KDB}}{v_{KDB}} = \frac{L_{DKD} i_B}{v_{KDB}} \quad ,$$

$$L_{fkd} = \frac{I_{FKD} i_{KDB}}{v_{FB}} \quad , \quad L_q = \frac{L_Q i_B}{v_B} \quad , \quad L_{kq} = \frac{L_{KQ} i_{KQB}}{v_{KQB}} = L_{QKQ} \frac{i_B}{v_{KQB}} \quad ,$$

$$R_d = \frac{R_D i_B}{v_B} \quad , \quad R_f = \frac{R_F i_{FB}}{v_{FB}} \quad , \quad R_{kd} = \frac{R_{KD} i_{KDB}}{v_{KDB}} \quad , \text{ and } \quad R_{kq} = \frac{R_{KQ} i_{KQB}}{v_{KQB}} \quad ;$$

$$L = \begin{bmatrix} L_d & L_{df} & L_{kd} & 0 & 0 \\ L_{df} & L_f & L_{fkd} & 0 & 0 \\ L_{kd} & L_{fkd} & L_{kd} & 0 & 0 \\ 0 & 0 & 0 & L_q & L_{kq} \\ 0 & 0 & 0 & L_{kq} & L_{kq} \end{bmatrix} \quad , \quad R = \begin{bmatrix} R_d & & & & \\ & R_f & & 0 & \\ & & R_{kd} & & \\ & & & R_d & \\ 0 & & & & R_{kq} \end{bmatrix}$$

$$\psi_q = \frac{\psi_Q}{v_B} \quad , \quad \psi_d = \frac{\psi_D}{v_B} \quad , \quad \psi_f = \frac{\psi_F}{v_{FB}} \quad , \quad \psi_{kd} = \frac{\psi_{KD}}{v_{KDB}} \quad , \quad \psi_{kq} = \frac{\psi_{KQ}}{v_{KQB}}$$

Equation (17) has two more parameters than the per unit equations since the damper winding currents are scaled by a factor containing a ratio of damper winding inductances. This choice of base currents results in a reduction in the number of inductance parameters and the addition of an equal number of parameters in the base rotor quantities. The object of this model is to represent the generator by a circuit equivalent at the terminals. Since the damper windings are inaccessible short-circuited turns embedded in the rotor, the current in them does

not need to be determined absolutely to model the generator at the terminals. The net result of this particular per unit representation is that two parameters are arbitrary (any two parameters may be arbitrarily specified) but the resulting circuit is still equivalent at the armature and field terminals.

Since the experiment was conducted with the generator under a balanced resistive load, as described in Section 3, this special case is now considered. The field is excited by a regulated voltage source providing a constant v_f. The components of the armature voltage are proportional to the corresponding currents.

$$v_d = -R_L i_d$$

$$v_q = -R_L i_q \tag{22}$$

Therefore, Equation (11) still holds if R_d is replaced by $R_d + R_L$, and if $v = (0\ v_f\ 0\ 0\ 0)^T$.

For convenience in the following derivation, the model Equations (11) and (12) are written in more general notation.

$$\frac{dx}{dt} = f(x,y,v) = F(y)x + v; \qquad x(0) = x_o \tag{23}$$

$$z(t) = h(x,y,\theta_o,t) \tag{24}$$

where

$x = x(x_o,y,v,t)$ = state vector (flux linkages) =
$(\psi_d \psi_q \psi_f \psi_{dq} \psi_{df})^T$

y = parameter vector

v = input vector (field voltage)

t = time

$\theta_o = \theta_o(x_o)$ = initial rotor angle

$x_o = x_o(y)$ = initial state vector

z = measurement vector (terminal currents)

The matrix $F = -(RL^{-1} + \Omega)$ and the parameter vector is

$$y = (L_1\ L_{df}\ L_{kd}\ L_f\ L_{fkd}\ L_2\ L_{kq}\ R_d\ R_f\ R_{kd}\ R_{kq})^T \tag{25}$$

where

$$L_d = L_1 + L_2$$

$$L_q = L_1 - L_2 \tag{26}$$

These equations assume the generator is in steady state at time t=0. Therefore, the initial state vector x_o and the initial rotor angle θ_o are computed from the parameters by the relations

$$x(0) = x_o(y) = -F(y)^{-1}v \tag{27}$$

and

$$\theta_o = \theta_o(x_o) = \arctan\left(\frac{i_d}{i_q}\bigg|_{t=0}\right) = \arctan\left(\frac{\omega L_q}{R_d+R_L}\right) \quad . \tag{28}$$

For t>0, a transient is induced by a step change in load resistance and the state vector is found by numerical solution of Equation (23).

Equations (23) and (24) describe a continuous-time, deterministic state-space model of the generator. Since a digital computer estimation algorithm is developed in the sequel, consider the measurement Equation (24) to be a function of t_k for $k=1,2,\ldots,k_f$, a set of discrete time points. Finally, since inaccuracies and noise inevitably corrupt the measurement process, a term representing an additive measurement noise sequence is included. The result is expressed as

$$z(t_k) = h(x,y,\theta_o,t_k) + w_k \quad , \quad k = 1,2, \ldots ,k_f \tag{29}$$

where t_k is the k^{th} discrete time point, and w_k is the k_{th} discrete noise sample.

2.3 Weighted Least-Squares Estimation

Consider a parameter estimator, an algorithm not yet specified, which recursively generates an estimate of the parameter vector. If the current parameter estimate is denoted by $\hat{y}$, then the output from a model of the generator is $h(\hat{x},\hat{y},\theta_o,t_k)$, where $\hat{x}$ is the state vector estimate computed from numerical solution of the model Equation (23) using $\hat{y}$. If the measured output of the actual generator is $z(t_k)$, then the weighted least-squares error criterion is given by

$$J = \frac{1}{2} \sum_{k=1}^{k_f} \{[z(t_k)-h(\hat{x},\hat{y},\theta_o,t_k)]^T Q[z(t_k)-h(\hat{x},\hat{y},\theta_o,t_k)]\} \quad . \tag{30}$$

Here Q is a non-negative definite weighting matrix. The interpretation of this error criterion is that minimizing J also minimizes the weighted

squares of the error between the observed output and the modeled output.

In the estimation algorithm, the method of quasilinearization [12,13] will be applied. Let the current parameter estimate $\hat{y}$ be updated to produce the new estimate

$$\hat{y}_{new} = \hat{y} + \Delta y \quad . \tag{31}$$

Expanding Equation (29) in a Taylor series about $\hat{y}$ and retaining only the first two terms gives the linear approximation

$$z(t_k) \simeq \hat{h}_k + \frac{d\hat{h}_k}{dy} \Delta y + w_k \tag{32}$$

where

$$\hat{h}_k = h(\hat{x},\hat{y},\theta_o,t_k)$$

and

$$\frac{d\hat{h}_k}{dy} = \left.\frac{dh(x,y,\theta_o,t_k)}{dy}\right|_{\hat{x},\hat{y}} \quad .$$

Repeated application of the chain rule for partial derivatives give

$$\frac{dh}{dy} = \frac{\partial h}{\partial y} + \frac{\partial h}{\partial x}\left(\frac{\partial x}{\partial y} + \frac{\partial x}{\partial x_o}\frac{dx_o}{dy}\right) + \frac{\partial h}{\partial \theta_o}\frac{d\theta_o}{dx_o}\frac{dx_o}{dy} \tag{33}$$

Likewise, the right hand side of Equation (23), f(x,y,v) may be expanded Differentiating Equation (23) with respect to y and x_o yields the

sensitivity matrix differential equations below,

$$\frac{d}{dt}\left(\frac{\partial x}{\partial y}\right) = \frac{\partial f}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial f}{\partial y} \quad , \quad \left.\frac{\partial x}{\partial y}\right|_{t=0} = \frac{\partial x_o}{\partial y} \tag{34}$$

$$\frac{d}{dt}\left(\frac{\partial x}{\partial x_o}\right) = \frac{\partial f}{\partial x}\frac{\partial x}{\partial x_o} \quad , \quad \left.\frac{\partial x}{\partial x_o}\right|_{t=0} = I \quad . \tag{35}$$

The solutions to Equations (34) and (35) are the sensitivity matrices

$$\frac{\partial x}{\partial y} \quad \text{and} \quad \frac{\partial x}{\partial x_o} \quad . \tag{36}$$

A necessary condition for minimizing J is that its gradient with respect to y vanish. Evaluating the gradient at the new parameter estimate $\hat{y} + \Delta y$ results in

$$\left.\frac{dJ}{dy}\right|_{\hat{y}+\Delta y} = 0 = -\sum_{k=1}^{k_f} \left(\frac{d\hat{h}_k}{dy}^T Q\{z(t_k) - [\hat{h}_k + \frac{d\hat{h}_k}{dy}\Delta y]\}\right) \tag{37}$$

Solving for Δy gives

$$\Delta y = \left[\sum_{k=1}^{k_f} \frac{d\hat{h}_k}{dy}^T Q \frac{d\hat{h}_k}{dy}\right]^{-1} \sum_{k=1}^{k_f} \left\{\frac{d\hat{h}_k}{dy} Q[z(t_k) - \hat{h}_k]\right\} \quad . \tag{38}$$

Setting the new estimate equal to $y + \Delta y$ completes the recursive algorithm for computing the parameter estimates.

In summary, the algorithm consists of solving the differential Equations (23), (34), and (35) numerically, computing sensitivity matrices and solving the system of linear algebraic Equations (38).

This recursive algorithm is ideally suited for implementation on a digital computer.

2.4 Statistical Estimation Theory

The least-squares approach of the previous section ignores the statistical nature of the estimation problem. If information about the statistics of the noise is available, the estimation process can be improved. This section presents a derivation and discussion of relevant aspects of stochastic parameter estimation theory.

2.4.1 Bayesian Approach

Consider an error defined to be

$$y - \hat{y}(z) \tag{39}$$

where $\hat{y}(z)$ is some estimate based on measuring z. Let $C(y-\hat{y}(z))$ be the cost function describing the penalty for making that error. The Bayesian risk [14,15] is defined as the conditional mean of the error

$$\mathrm{BR} = E\{C(y - \hat{y})|z\} = \int_{-\infty}^{\infty} C(y - \hat{y})p(y|z)dy \quad . \tag{40}$$

For the case of the uniform cost function,

$$C(y-\hat{y}) = \begin{cases} \frac{1}{\epsilon} & \text{if } ||y-\hat{y}|| \geq \epsilon \\ 0 & \text{otherwise} \end{cases} \quad . \tag{41}$$

Consider the limit of the cost function as $\epsilon \to 0$,

$$C(y - \hat{y}) = \prod_{j=1}^{N} \delta(y_j - \hat{y}_j) \quad . \tag{42}$$

As a result

$$BR = -p(\hat{y}(z)|z) \quad . \tag{43}$$

Minimizing the Bayesian risk is equivalent to maximizing the posterior probability density.

2.4.2 Maximum A Posteriori Probability Estimator

Denote the set of all measurement vectors in the sequence by $Z = \{z(t_1), z(t_2), \ldots, z(t_f)\}$. If the measurement noise is represented as a vector stochastic process and the parameter vector as a random vector, the posterior probability density is given by Bayes' rule as

$$p(y|Z) = \frac{p(Z|y)p(y)}{p(Z)} \quad . \tag{44}$$

Since $p(Z)$ is not dependent on y, maximizing (44) by choice of y is equivalent to maximizing $p(Z|y)p(y)$.

If the measurement noise is normally distributed with zero mean and covariance matrix V_w, the conditional probability of the k^{th} measurement vector is given by [14]

$$p[z(t_k)|y] = [(2\pi)^m \det(V_w)]^{-\frac{1}{2}} \cdot \exp\{-\tfrac{1}{2}[z(t_k) - h(x,y,t_k)]^T V_w^{-1} [z(t_k) - h(x,y,t_k)]\} \tag{45}$$

If the noise samples are uncorrelated with each other, that is from a white noise sequence, the probability of observing the entire sequence Z is simply the product of k_f terms like the right side of Equation (45).

$$p[Z|y] = [(2\pi)^m \det(V_w)]^{-k_f/2} \cdot \exp\{-\tfrac{1}{2} \sum_{k=1}^{k_f} [z(t_k)-h(x,y,t_k)]^T V_w^{-1} [z(t_k)-h(x,y,t_k)]\} \tag{46}$$

If the random parameter vector y is now assumed to be normally distributed with mean M_y and variance V_y, then

$$p(y) = [(2\pi)^m |V_y|]^{-\frac{1}{2}} \exp[-\tfrac{1}{2}(y-M_y)^T V_y^{-1}(y-M_y)] \quad . \tag{47}$$

Since the exponential function is monotonic in its argument and since the functions multiplying the exponentials in Equation (46) and (47) do not contain y, maximizing $p(Z|y)p(y)$ is equivalent to minimizing the error criterion

$$J = \frac{1}{2} \sum_{k=1}^{k_f} [z(t_k)-h(x,y,t_k)]^T V_w^{-1} [z(t_k)-h(x,y,t_k)] + \frac{1}{2}(y-M_y)^T V_y^{-1}(y-M_y) \quad , \tag{48}$$

resulting in

$$\Delta y = \left[\sum_{k=1}^{k_f} \left(\frac{d\hat{h}_k}{dy} V_w^{-1} \frac{d\hat{h}_k}{dy}\right) + V_y^{-1}\right]^{-1} \left\{\sum_{k=1}^{k_f} \left[\frac{d\hat{h}_k}{dy} V_w^{-1} (z(t_k) - \hat{h}_k)\right] - V_y^{-1} (\hat{y} - M_y)\right\} \quad . \tag{49}$$

Thus the maximum a posteriori probability estimator can be considered a special case of a least-squares estimator, including prior information about the parameter vector, if the weighting matrices are chosen equal to the inverse of the error covariance matrices.

2.4.3 Lower Bound on the Error Covariance

A lower bound on the error covariance of the estimator derived is easily obtained from a generalization of Fisher's information matrix [15,16].

$$E[(y-\hat{y})(y-\hat{y})^T] \leq \left\{\sum_{k=1}^{k_f} \left[\frac{d\hat{h}_k}{dy}^T V_w^{-1} \frac{d\hat{h}_k}{dy}\right] + V_y^{-1}\right\}^{-1} \tag{50}$$

The derivation of this bound is presented in Appendix B. This lower bound is computationally inexpensive since the right side of Equation (50) is already computed in the estimation in the algorithm, Equation (49).

2.5 Summary

The derivation of an algorithm suitable for estimating synchronous generator parameters is approached from the point of view of weighted least-squares estimation theory. First, the equations describing the generator are cast into a state-variable form. Next, the method of quasilinearization is used to derive a least-squares estimation algorithm. By considering a stochastic formulation, the maximum a posteriori probability estimator is shown to be a special

case of the weighted least-squares algorithm with correct choice of the weighting matrices.

While the direct implementation of the weighted least-squares algorithm results from weights chosen arbitrarily or by physical intuition, the optimal estimator improves the quality of the estimates by using the prior error statistics to choose the proper weights. Unfortunately, the exact statistics of the noise are not known. The approach taken here is to study the effect of incorrect prior statistics using a computer simulation. Then, when processing data from the actual generator, estimates of the relative magnitudes of error variances can be made to enable intelligent choice of weights. If these estimates of the prior statistics are correct, the result should approach the performance of the optimal estimator. If the statistics are in error, at least the resulting suboptimal estimate satisfies the least-squares error criterion. In any case, the algorithm minimizes the weighted square of the output error.

SECTION III

DESIGN OF THE EXPERIMENT

3.1 Introduction

The success of an iterative parameter estimation algorithm, such as the one previously described, depends on the quality of the input data. In other words, the criterion of goodness for the experiment that produces this data must be the performance of the estimator. In this research the experiment was designed with the aid of computer simulations. This enabled various experimental conditions to be tested with maximum flexibility.

First, certain constraints on available equipment and on computing and processing time were recognized. These were primarily economic limitations. The experiment was designed within these constraints by selecting such factors as machine load and excitation, data sampling rate and overall data record length. To choose these conditions intelligently, a computer simulation of a typical generator was used to model the experimental setup. The resulting test data were used to assess the performance of the estimator. Therefore, the effect of the undetermined experimental condition was easily established and the experiment thereby designed.

3.2 Equipment Constraints

The test generator was a three-phase, four-pole, alternating current, synchronous machine rated at 3 KVA and 230 volts at 40 hertz. The drive motor was a 15 horsepower direct current machine rated at 1200 revolutions per minute. The dc supply, from a large motor-generator set, had only open-loop control. As a result, the speed of the drive motor was manually adjustable with no provision for automatic regulation.

The load bank consisted of three power resistors connected in wye with an additional resistor suddenly switched in parallel with each leg to induce a transient. This arrangement is illustrated in Figure 2. The closing of the three-pole switch, labeled S, causes the load resistance to suddenly decrease from R_1+R_3 to $\frac{R_1R_2}{R_1+R_2} + R_3$. Resistors R_3 are current shunts used to obtain voltage signals proportional to the load current in each leg. These signals, along with a similar one proportional to field current, were recorded and processed as described in the next section.

Excitation of the field was supplied by a regulated electronic dc power supply rated at 0.5 amp and 500 volts. This was adequate to supply up to the rated field current of .525 amps at the rated field voltage of 125 volts.

The main constraint posed by the available equipment was to limit armature current to values within the generator ratings to prevent damage to the windings. The second constraint was a limit on the maximum change in armature current during the test. To insure that these constraints were met, the field voltage was reduced below its rated value, reducing the magnitude of the load currents.

3.3 Simulation Using Nominal Parameters

Within the constraints already posed, several experimental conditions remain to be selected. First, the magnitude of the step load change must be selected large enough to enable all the parameters to be estimated. Some parameters, notably the damper circuit rectances and resistances, affect the terminal currents only during transients.

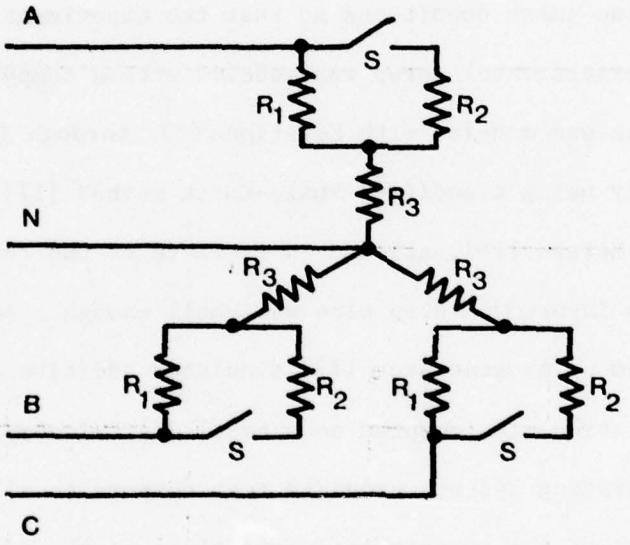

Figure 2. Schematic Diagram of Load Bank.

Thus a significant transient must be introduced by the sudden load changes. Second, the data sampling rate must be chosen fast enough to accurately represent the terminal current waveforms. The well-known sampling theorem states that the minimum rate is twice the highest frequency component of the waveform. Experience shows that the practical minimum is somewhat faster than the theoretical limit. A balancing constraint is the limitation of the total amount of data storage space available. Finally, the length of the data record and the number of load switchings must be determined.

To determine these conditions so that the experiment would be successful, the experimental setup was modeled with a computer simulation. The machine was modeled with Equations (23) through (29) solved numerically using a modified Runge-Kutta method [17]. The modification, due to Merson [18], allowed an estimate of the roundoff error to be computed to insure the step size was small enough. A multiplicative type pseudo-random noise generator [19] simulated additive measurement noise. The simulation, implemented on a small digital computer with a magnetic disk operating system, provided test outputs to allow assessment of the effect of the experimental conditions on the parameter estimator. This information allowed the experiment to be designed.

To implement the numerical solution of the machine model, a set of typical parameters was determined from nameplate data, a few simple tests, and a list of typical machine constants [3]. The parameters X_d, X_{df}, R_d and R_f were measured by simple ac and dc steady-state tests. The remaining parameters were roughly estimated from the typical parameter list. The results of this nominal parameter

computation are given in Table 1. The results of the steady-state tests and the details of the calculation are presented in Appendix C. It should be emphasized that the nominal-parameter model of the machine is not an accurate representation of this generator but is similar to a typical generator.

The first run of the simulation modeled the generator during a sudden three-phase short circuit on the armature terminals. The main purpose of this run was to test the operation of the estimator. No measurement noise was added and the measurements were weighted equally. The step size was 2 msec. and the field voltage was the rated value. The parameter estimates converged rapidly as shown in Figure 3a and 3b, which are plots of the computed relative error versus iteration. The relative error is defined as

$$\epsilon = \frac{\hat{y}-y}{y} , \tag{51}$$

where $\hat{y}$ is the parameter estimate, and y is the actual parameter value, i.e. the parameter value of the model simulation. Figure 3c shows the instantaneous phase A current and the field current using parameter values which are estimated. The outputs based on the parameter estimates match the data within graphical error.

The second test is similar to the first; however, a transient was induced by switching load resistance rather than a sudden short circuit. The switched load test is a more realistic simulation of the actual experimental setup. With the machine initially in steady state, the load resistance was suddenly switched from 1.0 to 0.25 per unit. The load was switched back to one per unit, 400 milliseconds later.

TABLE 1

NOMINAL PARAMETERS IN PER UNIT

Parameter Number	Parameter	Nominal Value (per unit)
1	L_l	2.69×10^{-2}
2	L_{df}	4.87×10^{-3}
3	L_{kd}	2.2×10^{-3}
4	L_f	10.8×10^{-3}
5	L_{fkd}	4.01×10^{-3}
6	L_2	$.565 \times 10^{-3}$
7	L_{kq}	1.17×10^{-3}
8	R_d	1.42×10^{-2}
9	R_f	2.31×10^{-2}
10	R_{kd}	7.59×10^{-2}
11	R_{kq}	7.59×10^{-2}

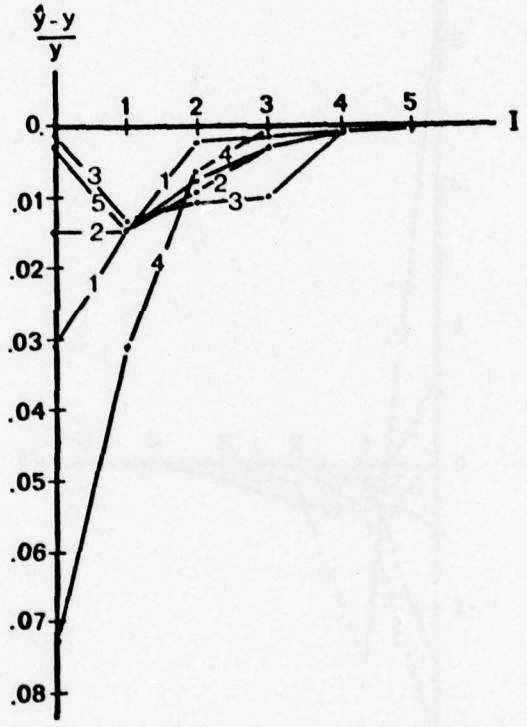

Figure 3a. Parameter Error for Sudden Short-Circuit, Test One, Parameters 1-5

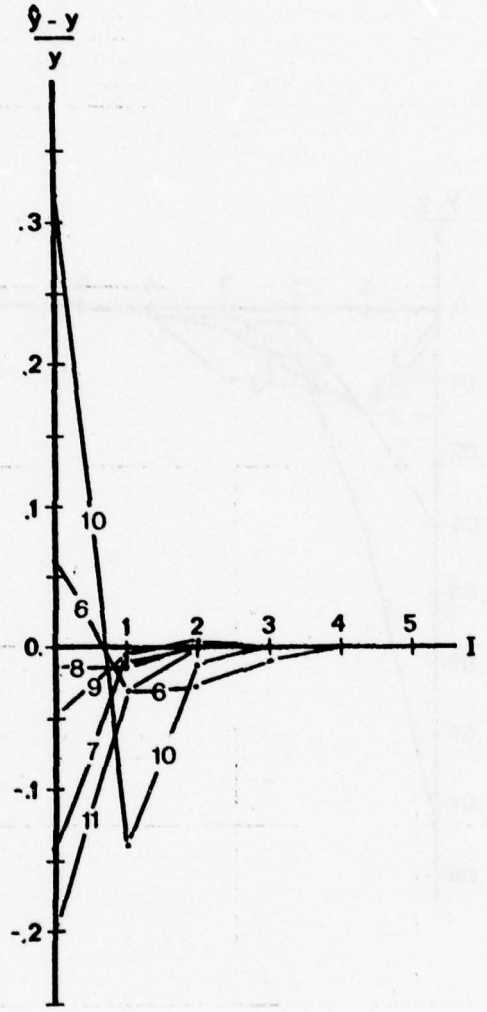

Figure 3b. Parameter Error for Sudden Short-Circuit, Test One, Parameters 6-11

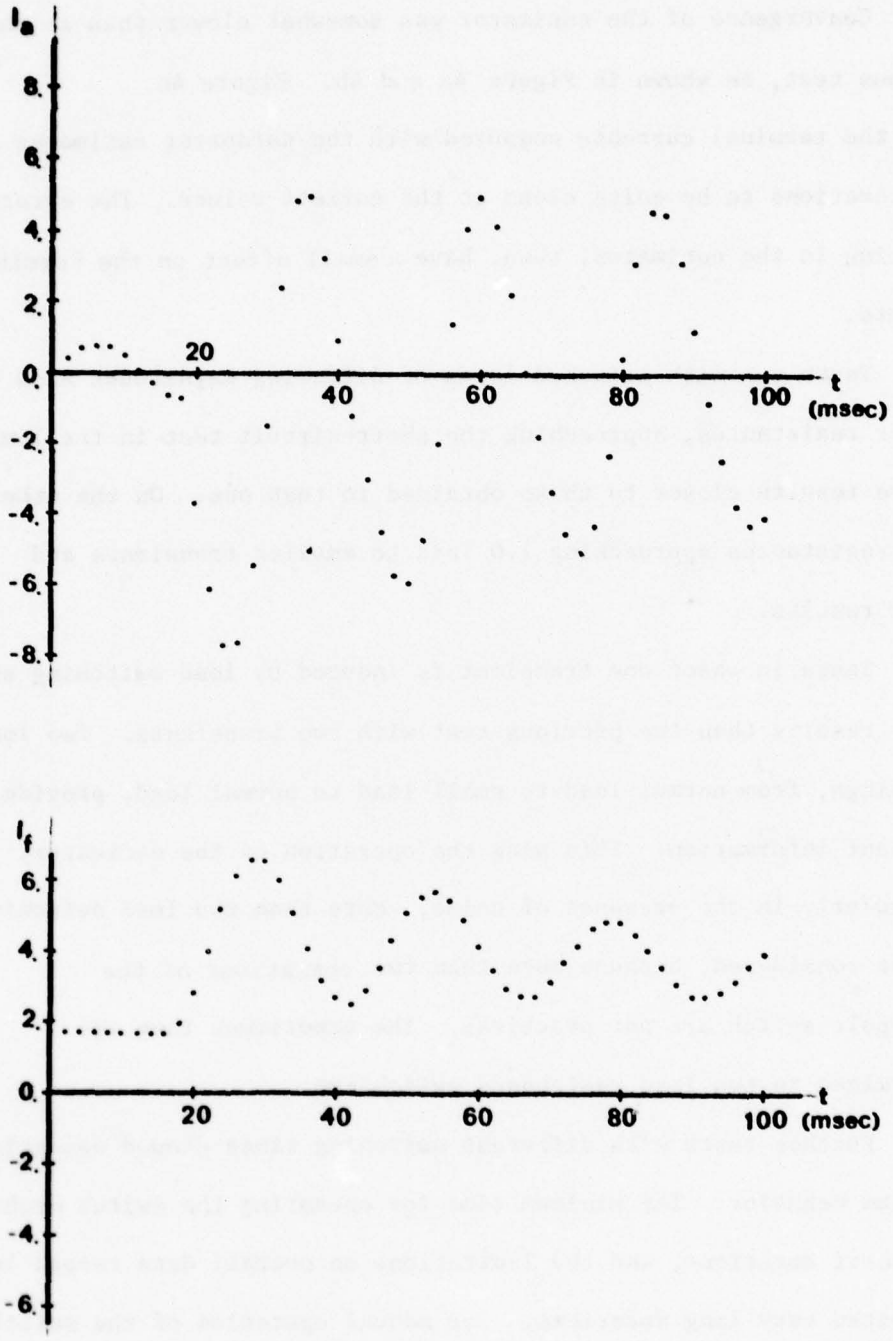

Figure 3c. Armature and Field Current for Suddent Short-Circuit

Convergence of the estimator was somewhat slower than in the previous test, as shown in Figure 4a and 4b. Figure 4c shows the terminal currents computed with the parameter estimates after ten iterations to be quite close to the correct values. The errors remaining in the estimates, then, have a small effect on the terminal currents.

Tests run with switched loads of differing magnitudes show that smaller resistances, approaching the short-circuit test in the limit, produce results closer to those obtained in test one. On the other hand, resistances approaching 1.0 lead to smaller transients and poorer results.

Tests in which one transient is induced by load switching show poorer results than the previous test with two transients. Two load switchings, from normal load to small load to normal load, provide redundant information. This aids the operation of the estimator, particularly in the presence of noise. More than two load switchings are not considered, because more than two operations of the three-pole switch are not practical. The experiment then was constrained to two load resistance switchings.

Further tests with different switching times showed essentially the same behavior. The minimum time for operating the switch prohibited very short durations, and the limitations on overall data record length prohibited very long durations. For manual operation of the switch, a duration of several hundred milliseconds is reasonable.

The next test, illustrated in Figure 5a shows the effect of ten percent errors in the initial parameter guesses and measurement

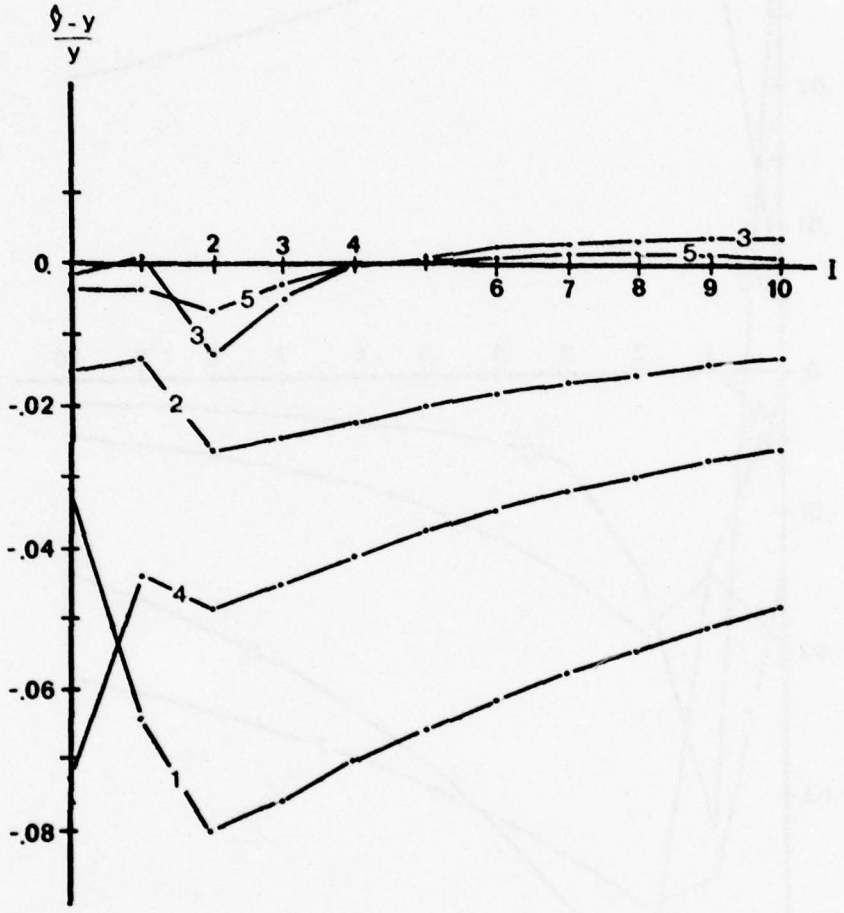

Figure 4a. Parameter Error for Switched Resistive Load, Test Two, Parameters 1-5

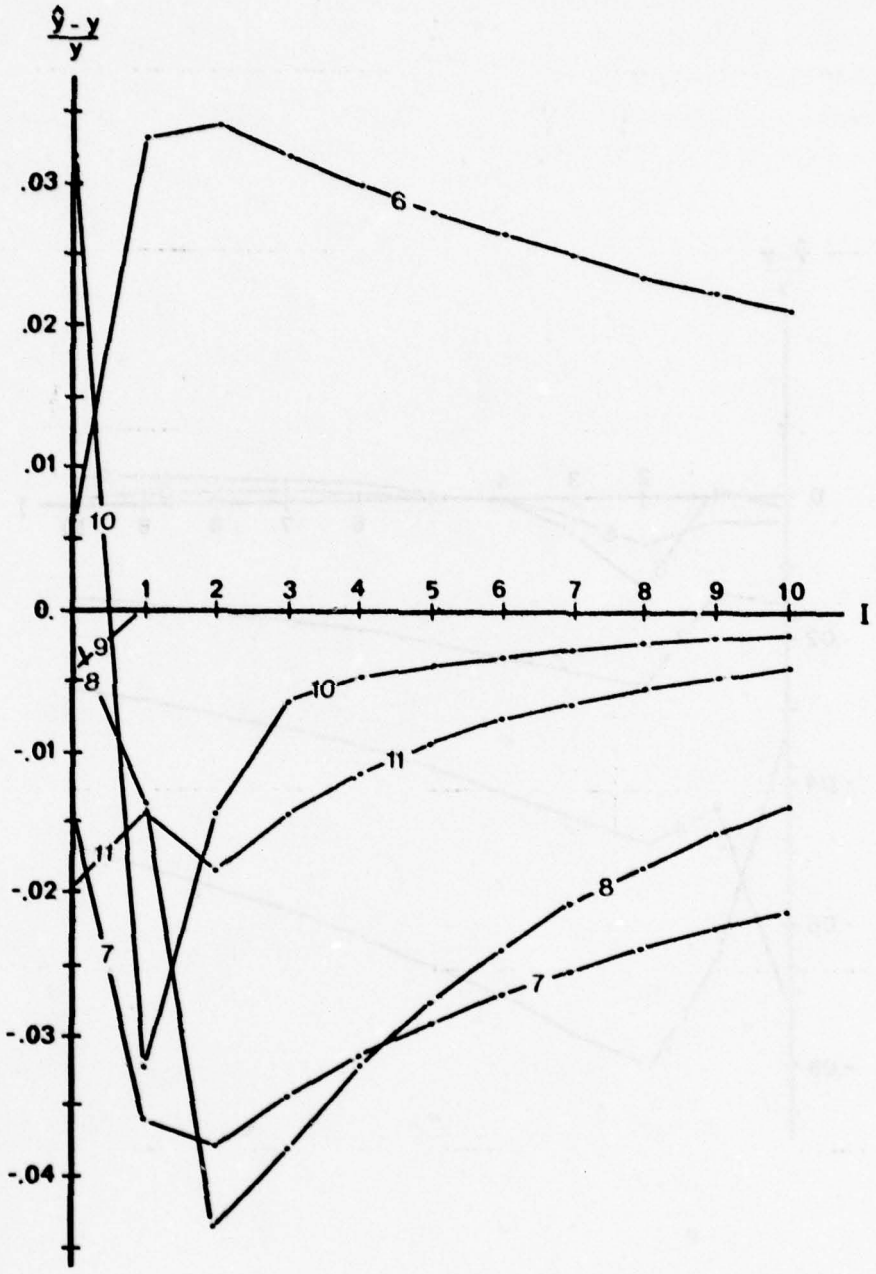

Figure 4b. Parameter Error for Switched Resistive Load, Test Two, Parameters 6-11

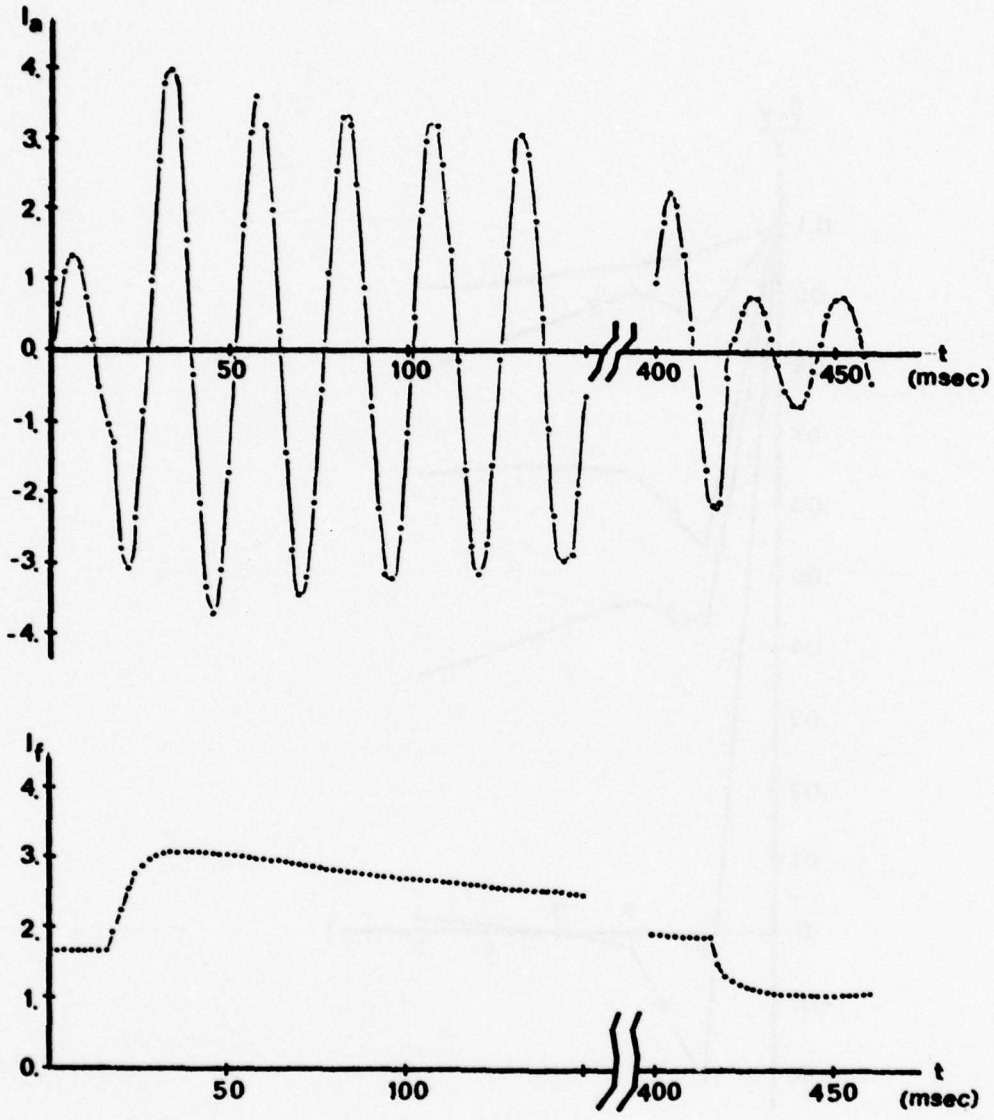

Figure 4c. Armature and Field Current for Switched Resistive Load, Test Two

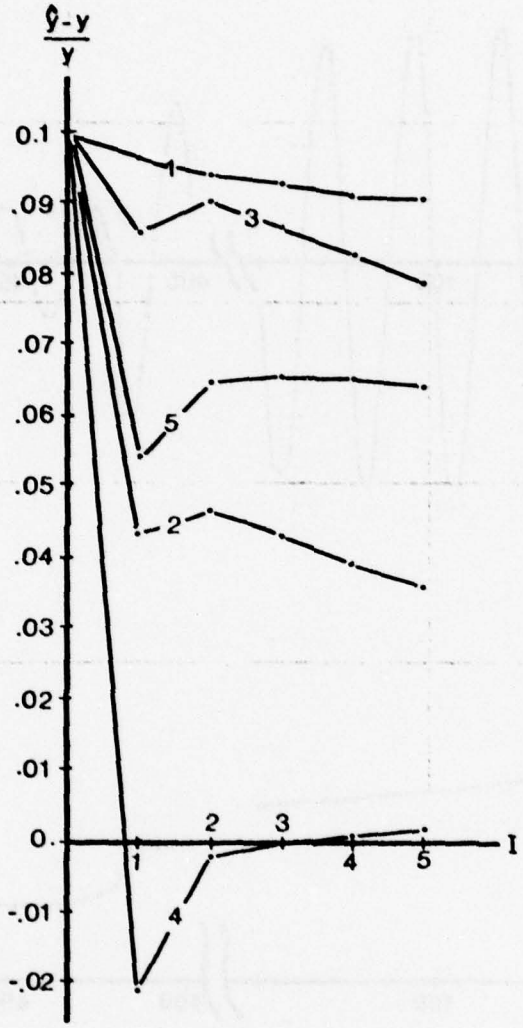

Figure 5a. Parameter Error Versus Iteration for Test Three, Parameters 1-5

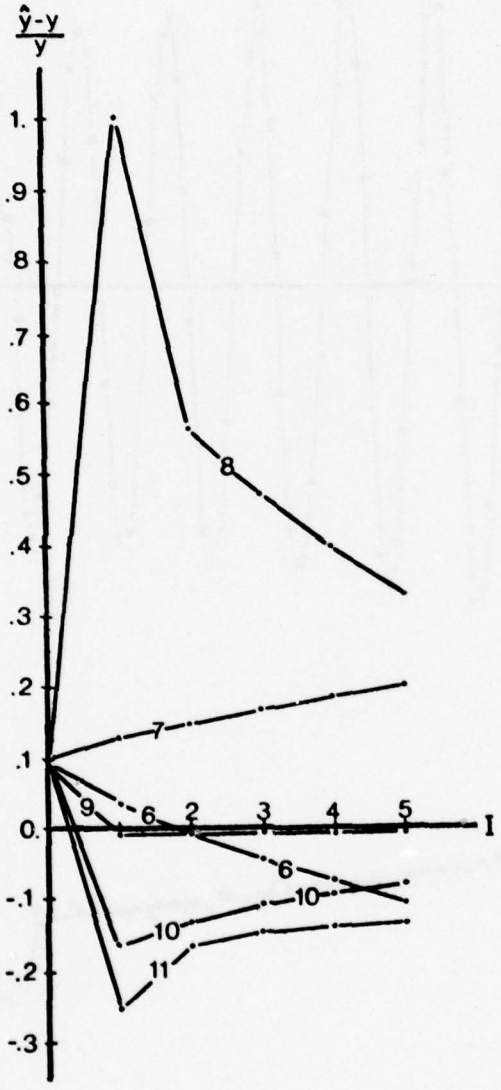

Figure 5b. Parameter Error Versus Iteration for Test Three, Parameters 6-11

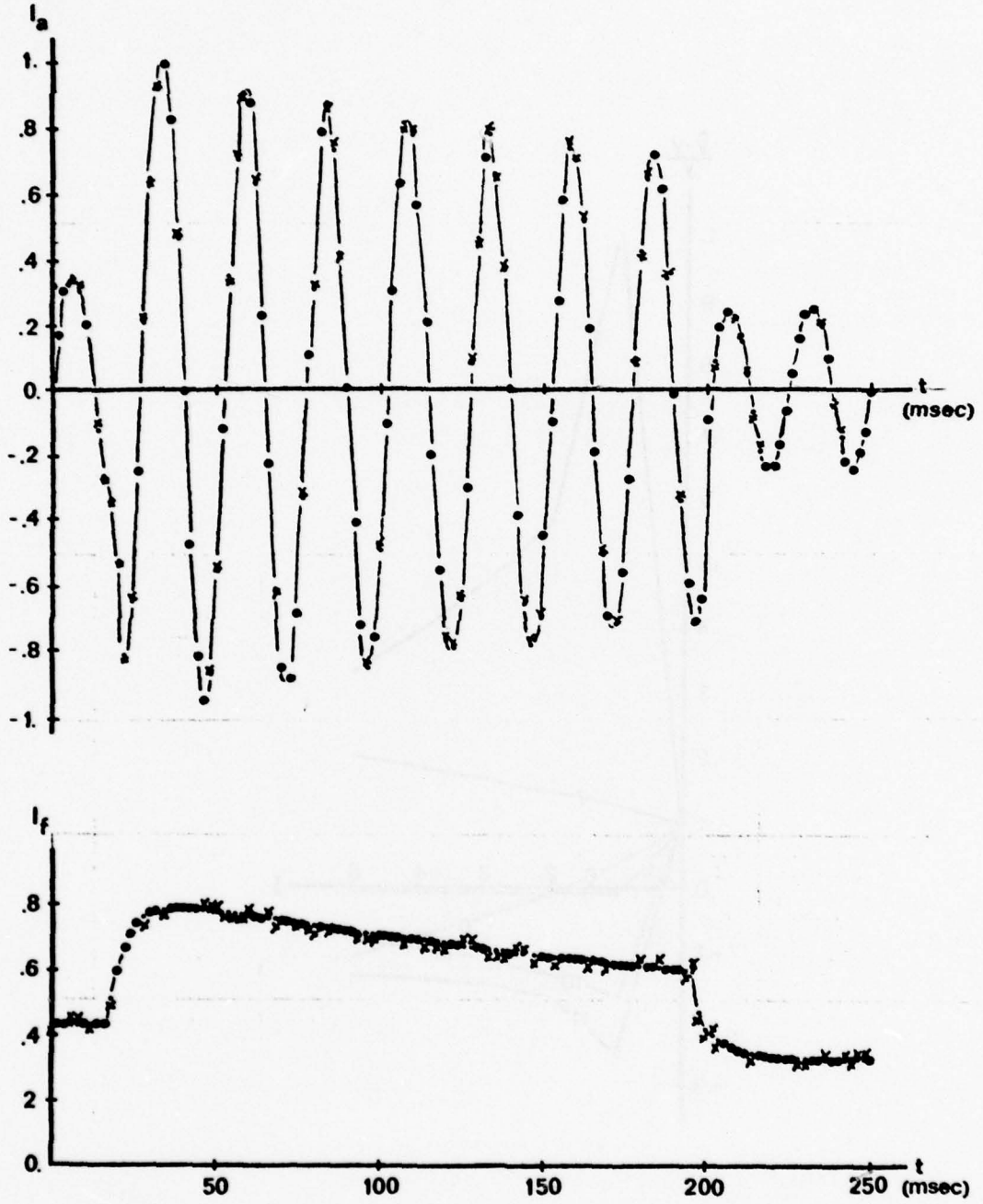

Figure 5c. Armature and Field Current for Test Three

noise with zero mean and standard deviation 1×10^{-2}. Figures 5c and 5d show instantaneous test values of a phase A current and field current, respectively. All measurements and parameters were weighted equally.

The effect of increasing measurement noise is to degrade the performance of the estimator. At noise of standard deviation greater than 1.0, the estimator does not converge. With noise of standard deviation of 0.1, the estimator converged very slowly, but large errors in some parameters show that they tend to track the noise.

The final test with simulated data, shown in Figure 6a, was the same as the preceding test; however, the weighting matrices were chosen as the inverse covariance matrices. Figures 6b and 6c show instantaneous test values of phase A current and field current, respectively. This is an implementation of the maximum a posteriori probability estimator. The results show only minor improvements on the preceding weighted least-squares approach, shown in Figures 5a-5c.

The results of the simulated experiment show that the estimator will work satisfactorily with a resistive load switched from 1.0 to 0.25 to 1.0 per unit, with practical switching periods, in the presence of moderate initial parameter errors and measurement noise. This experiment was consistent with the limitations imposed by available equipment. The implementation of this experiment is discussed in the next section.

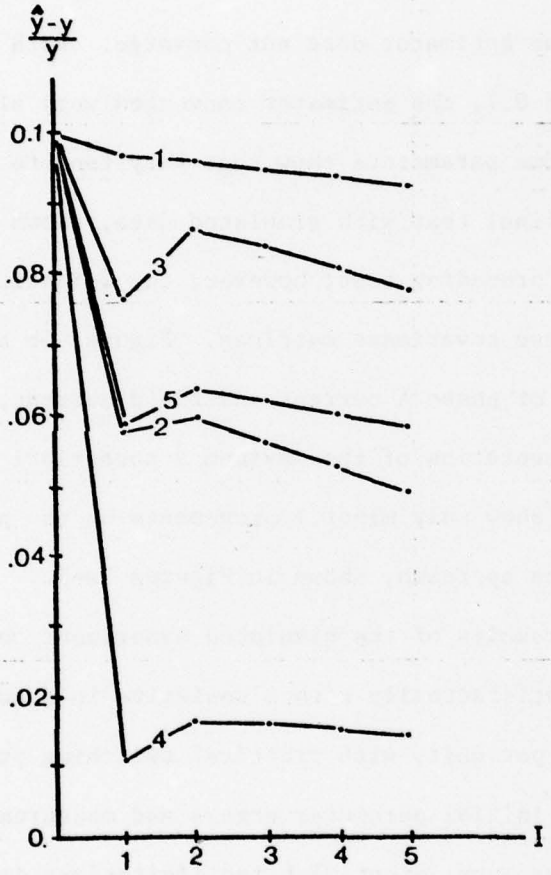

Figure 6a. Parameter Error Versus Iteration for Test Four, Parameters 1-5

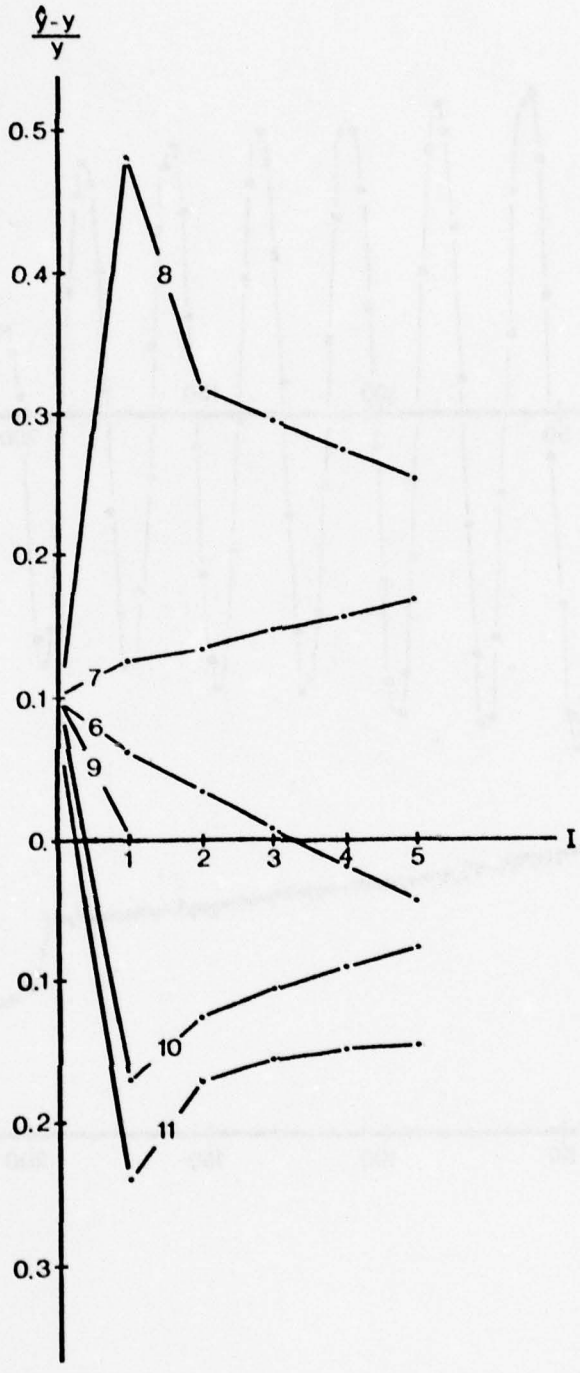

Figure 6b. Parameter Error Versus Iteration for Test Four, Parameters 6-11

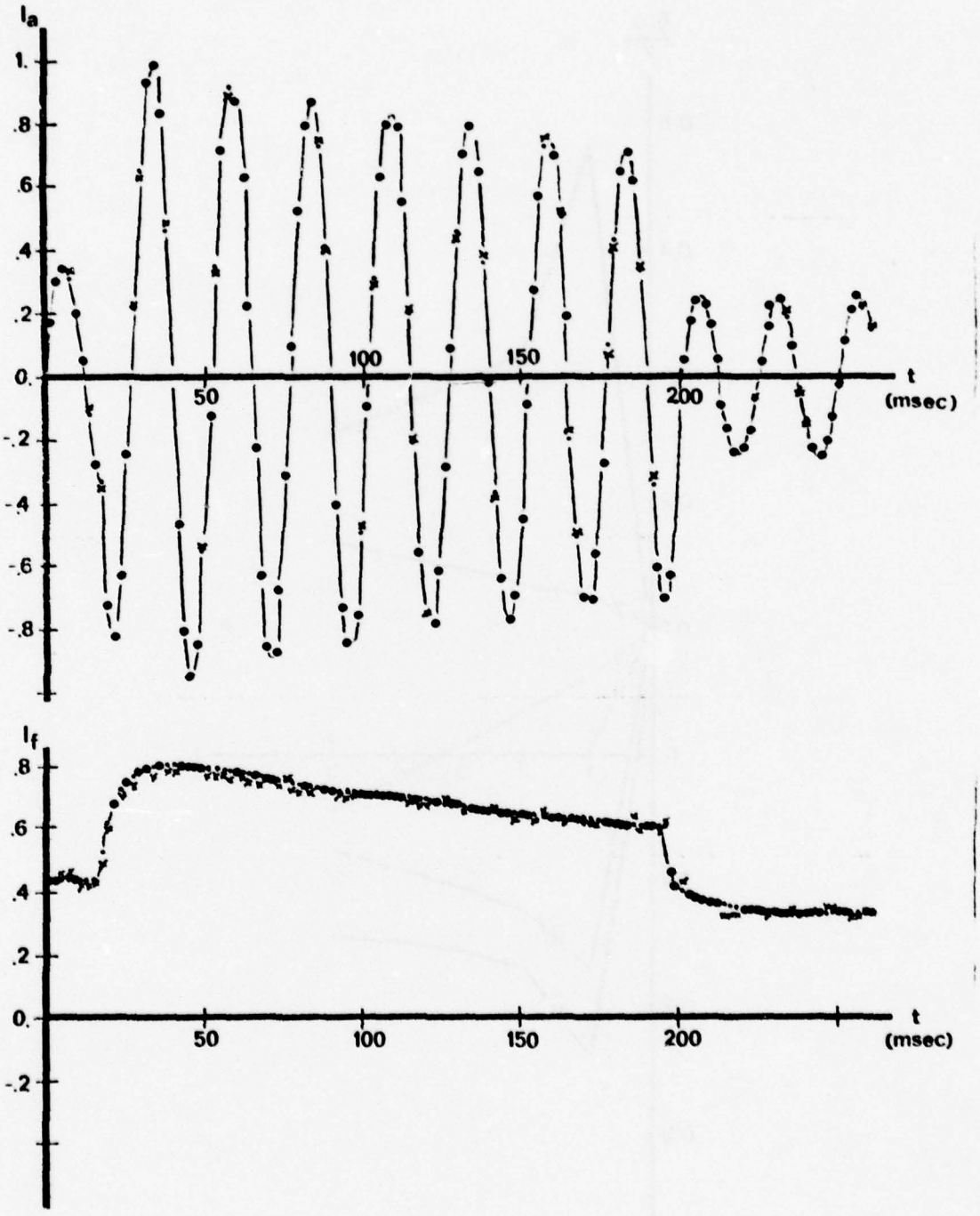

Figure 6c. Armature and Field Current for Test Four

SECTION IV

IMPLEMENTATION OF THE EXPERIMENT

4.1 Introduction

This section describes an experiment to measure the parameters of a synchronous generator in the laboratory. Terminal current measurements are recorded, stored in digitized form, and fed off-line into the parameter estimation algorithm. The resulting parameter estimates are used to predict new results, which are compared to actual measurements to validate the model.

The three main thrusts of this part of the research are reported in the remainder of this section. First the data collection, processing and storage methods are described in detail. Next, the results of the experiment, the parameter estimates, are presented. Finally, the adequacy of the results is assessed by using the parameter estimates in a computer simulation to predict the generator response to a relatively large change in load. This prediction is compared to actual measurements under the same conditions.

4.2 Data Processing Methods

Analog signals proportional to the three armature currents and the field current are produced by current shunts in the four generator terminal circuits. These signals are recorded on separate channels of an FM instrumentation tape recorder. A typical channel is shown schematically in Figure 7.

Since the parameter estimator is implemented on a digital computer, the four channels of analog data are digitized off-line and stored in serial form on a magnetic disk cartridge. Figure 8 is a schematic diagram of the analog-to-digital (A/D) conversion and the multiplexing of the

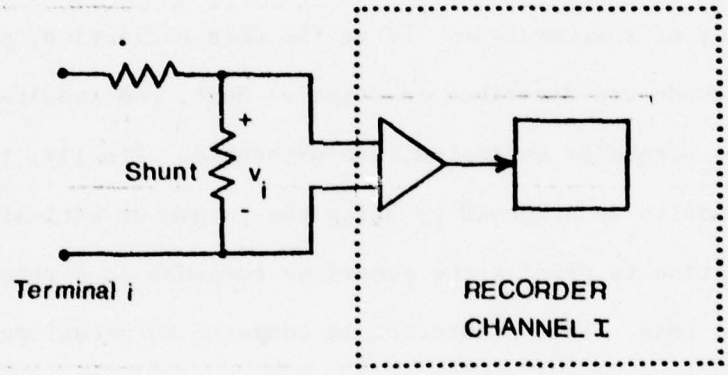

Figure 7. Schematic Diagram of Instrumentation for Channel i

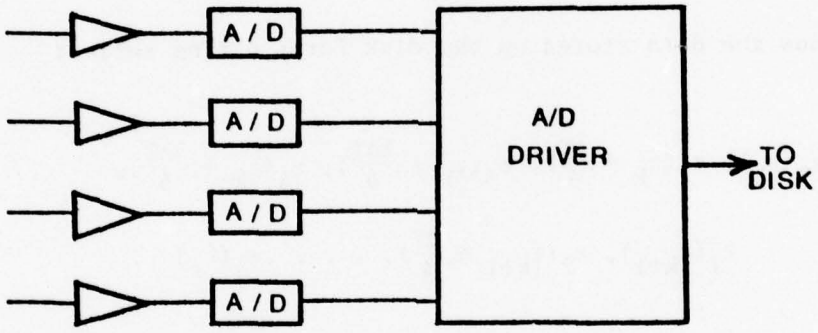

Figure 8. Four Channels of Data Digitized and Written to Disk Serially

four digital data channels into one serial record on the disk. The A/D converters provide sampled-data output in 12 bit two's complement binary numbers. The A/D driver software writes these numbers to the disk serially. The stream of data to disk contains sample one of channel one, then sample one of channel two and so forth. Sample two of channel one follows sample one from the last channel. This process of multiplexing data to the disk is illustrated for two channels of triangular waveform data in Figure 9.

Thus the data stored on the disk forms a time series:

$$z_1(t_k),\ z_2(t_k + \frac{\Delta T}{4}),\ z_3(t_k + \frac{2\Delta T}{4}),\ z_4(t_k + \frac{3\Delta T}{4}),$$

$$z_1(t_{k+1}),\ z_2(t_{k+1} + \frac{\Delta T}{4}),\ \ldots\ ,z_4(t_f)\ .$$

Here ΔT is the overall time step and $t_{k+1} = t_k + \Delta T$. Data in this serial multiplexed format is used directly in a weighted least-squares algorithm by updating the state vector and the sensitivity matrices at all the time points

$$t_k,\ t_k + \frac{\Delta T}{4}\ ,\ t_k + \frac{2\Delta T}{4}\ ,\ t_k + \frac{3\Delta T}{4}\ ,\ t_{k+1}\ ,\ \ldots\ ,t_f\ .$$

The error between observed and computed outputs is defined as

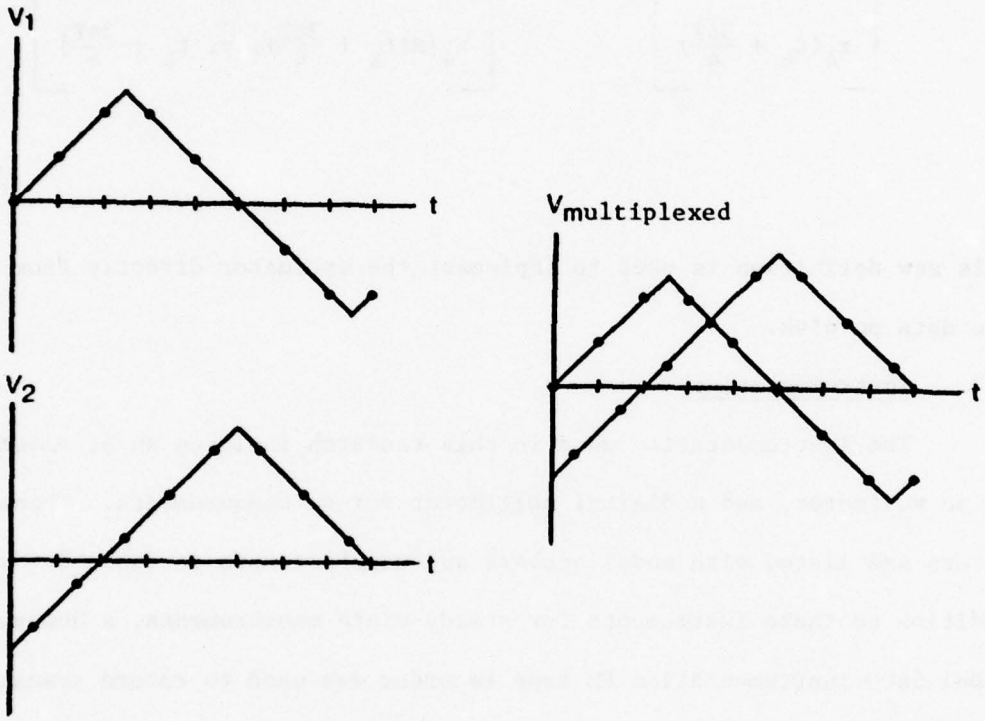

Figure 9. Reduction of Two Channels of Digitized Data to One Serial Record. (a) Two Channels of Data Sampled Alternately, (b) Multiplexed Serial Data.

$$\tilde{z}(k) \triangleq \begin{bmatrix} z_1(t_k) \\ z_2(t_k + \frac{\Delta T}{4}) \\ z_3(t_k + \frac{2\Delta T}{4}) \\ z_4(t_k + \frac{3\Delta T}{4}) \end{bmatrix} - \begin{bmatrix} h_1[x(t_k), \hat{y}, t_k] \\ h_2[x(t_k + \frac{\Delta T}{4}), \hat{y}, t_k + \frac{\Delta T}{4}] \\ h_3[x(t_k + \frac{2\Delta T}{4}), \hat{y}, t_k + \frac{2\Delta T}{4}] \\ h_4[x(t_k + \frac{3\Delta T}{4}), \hat{y}, t_k + \frac{3\Delta T}{4}] \end{bmatrix} \quad (52)$$

This new definition is used to implement the estimator directly from the data on disk.

4.3 Instrumentation

The instrumentation used in this research includes an ac ammeter, an ac voltmeter, and a digital multimeter for dc measurements. These meters are listed with model numbers and manufacturers in Table 2. In addition to these instruments for steady-state measurements, a Honeywell model 5600 instrumentation FM tape recorder was used to record transient data. Analog-to-digital conversion and data processing, described previously, were implemented on a Data General NOVA small computer.

4.4 Result of the Experiment

After operating the generator at normal load for fifteen minutes to minimize variations due to temperature changes, the load resistance was measured as 0.98 per unit. The additional load resistor bank was switched in parallel to this load, and the resistance of the combination measured as 0.23 per unit. Then the load was returned to normal, and

TABLE 2

LIST OF INSTRUMENTS USED

DESCRIPTION	NUMBER	MANUFACTURER
AC ammeter	AA-10	General Electric
AC voltmeter	AV-12	General Electric
Digital multimeter	3476B	Hewlett-Packard

the rotor speed measured as 252 radians per second. The first test was performed under these conditions. The resulting data, recorded and processed as previously described, was sampled at a rate of 500 Hertz per channel or an overall rate of 2KHz for all four channels.

To determine a priori information about the measurement noise, the recorder was run with the inputs shorted to ground. After digitizing, this data indicated approximately equal noise variances in all four channels. As a result the weights for the measurements were selected as unity. Due to lack of good a priori information on the initial parameter error variances, these weights were adjusted empirically to obtain good results from the estimator. By weighting the parameters heavily at first, estimates were prevented from large excursions on the first few iterations, enhancing the stability of the estimator. The weights chosen are summarized in Table 3, while the parameter estimates are plotted versus iteration number in Figure 10a.

Figure 11a shows a comparison of the phase A current and the field current predicted from the parameter estimates to the corresponding data from the experiment. First, in Figure 11a, the results of the initial parameter estimates are compared to the experimental data. Then Figure 11b shows the results of the final parameter estimates compared to the data. The final results follow the data quite well. This indicates that the model fits the data well at these particular operating conditions.

To test these results under another condition and to assess the validity of the model, a second test was run with load resistance

TABLE 3

PARAMETER ERROR WEIGHTS

ITERATION	PARAMETERS	WEIGHT
1-13	Y_1-Y_7	$1x10^7$
1-3	Y_8-Y_{11}	$1x10^6$
4-13	Y_8,Y_9	$1x10^5$
4-13	Y_{10},Y_{11}	$1x10^4$

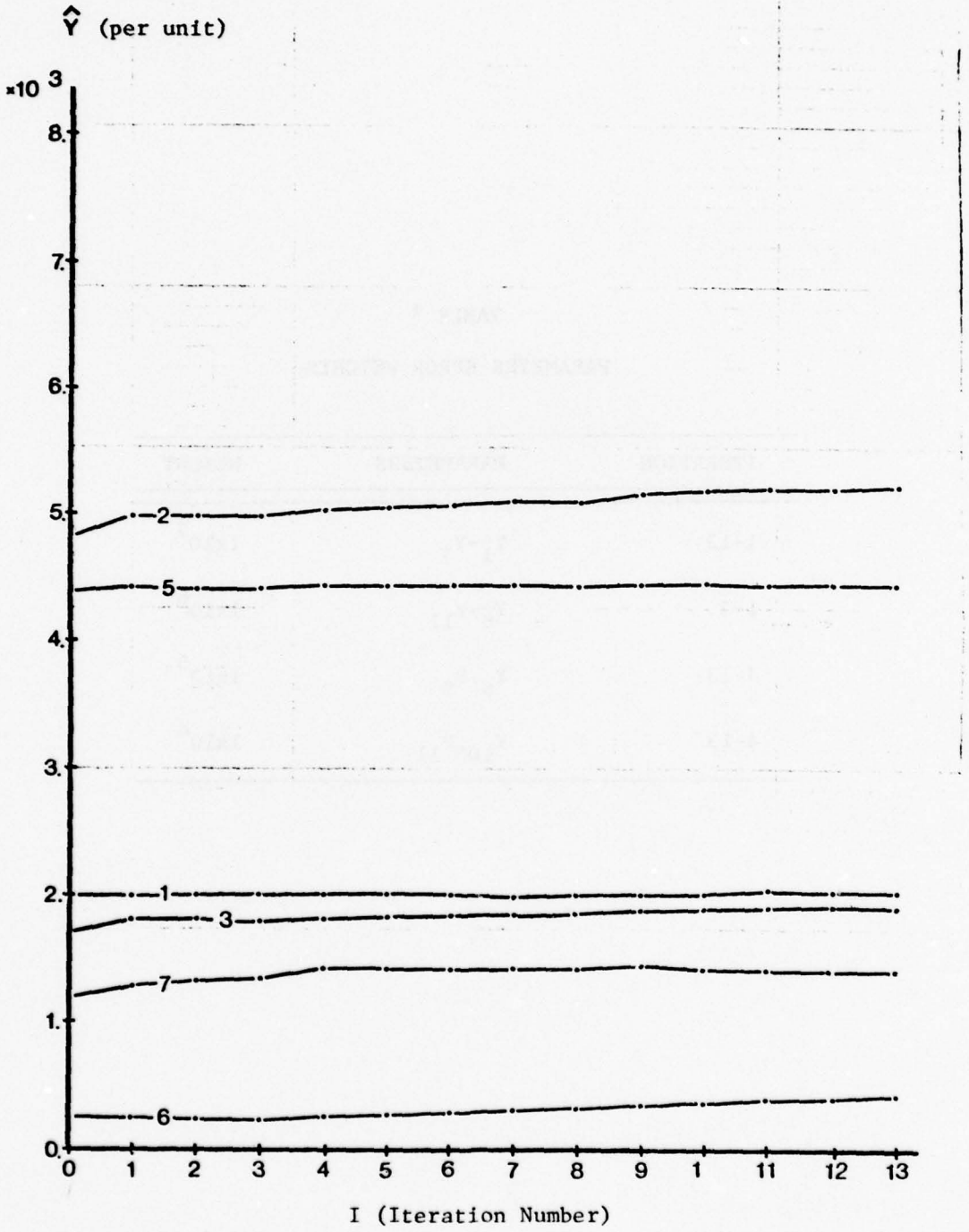

Figure 10a. Parameter Estimates Versus Iteration Number from Experimental Data (Test One), Parameters 1, 2, 3, 5, 6, and 7

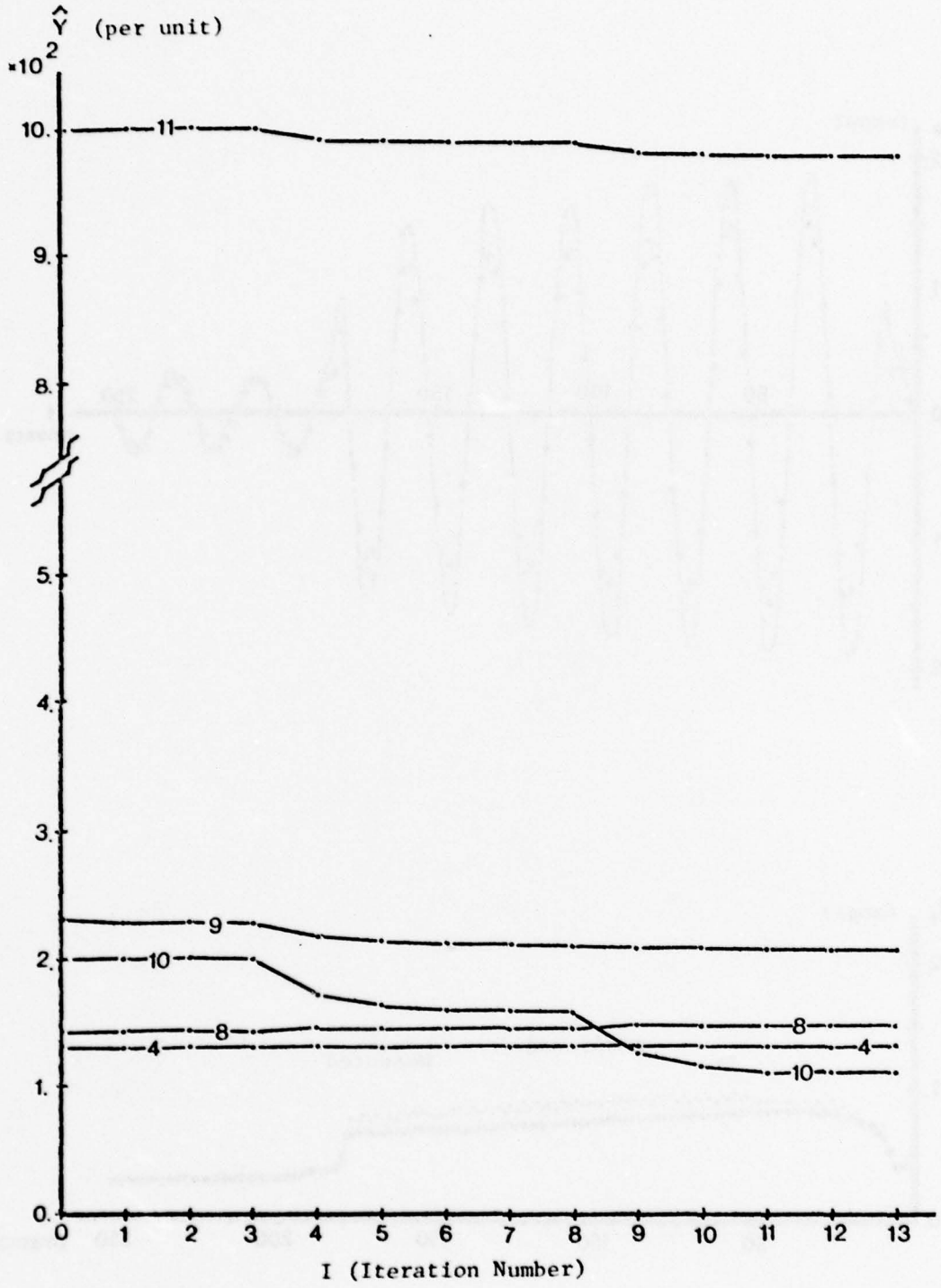

Figure 10b. Parameter Estimates Versus Iteration Number from Experimental Data (Test One), Parameters 4, 8, 9, 10 and 11

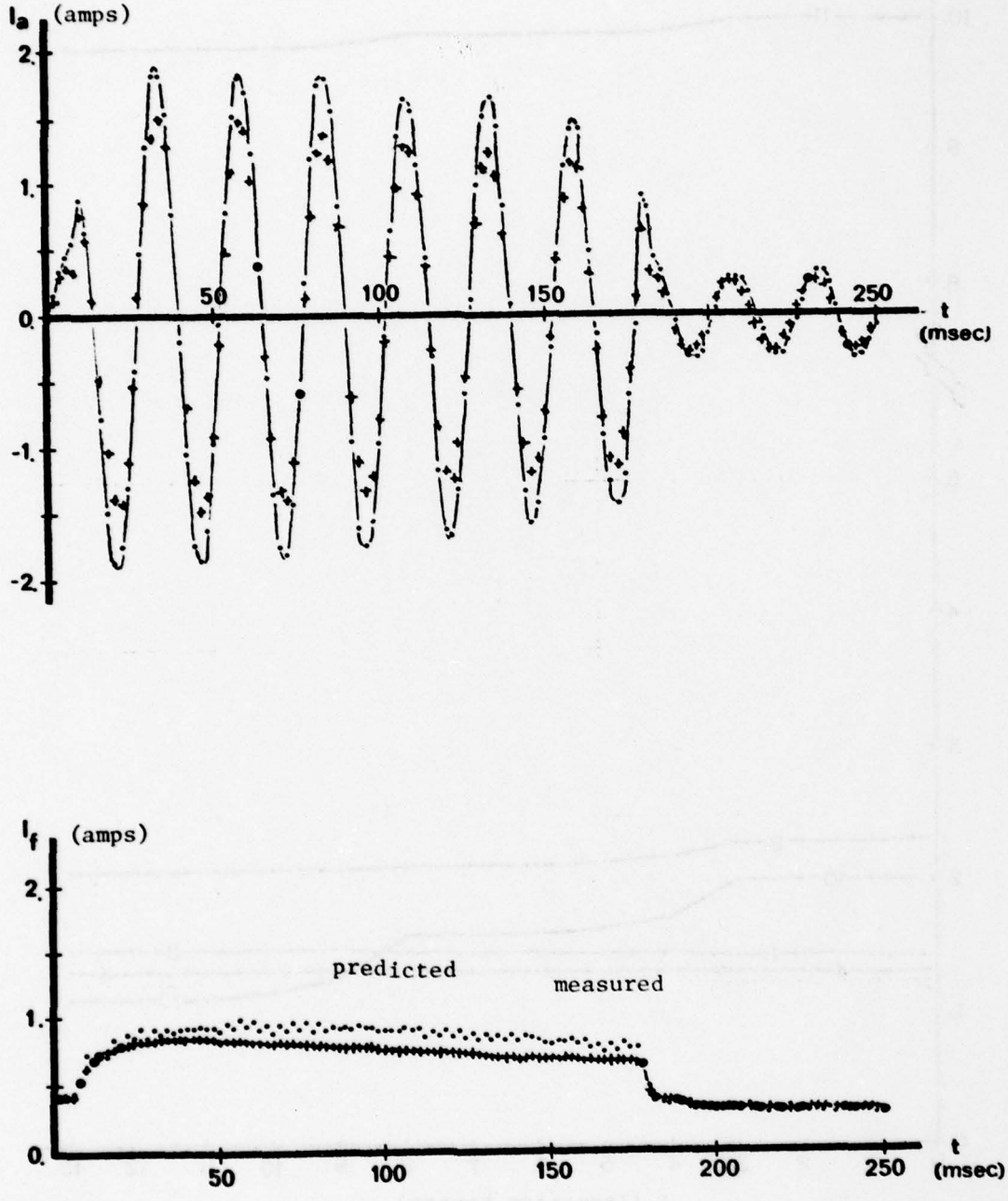

Figure 11a. Phase A and Field Currents for Test One, Iteration Zero

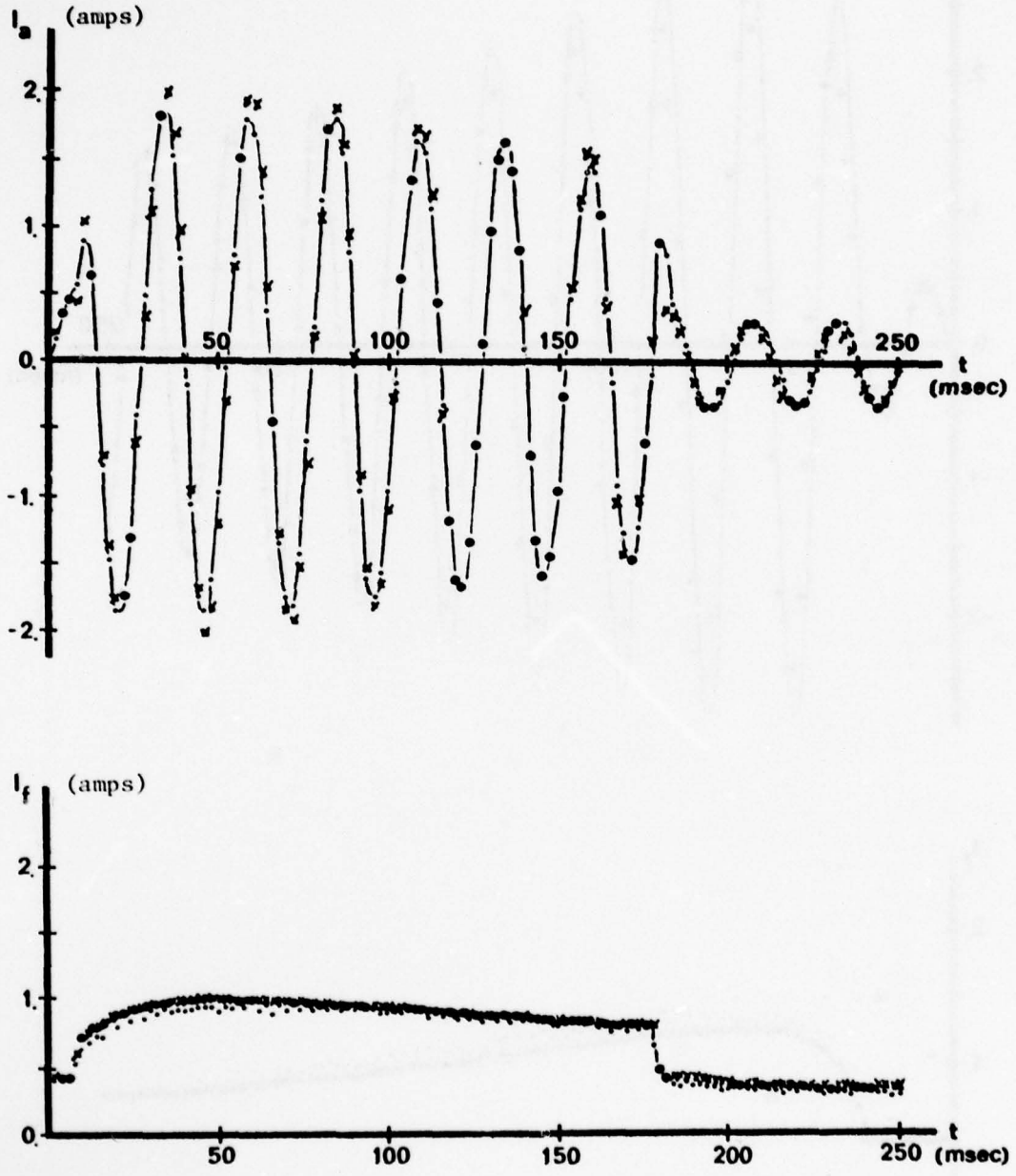

Figure 11b. Phase A and Field Currents for Test One, Iteration 13

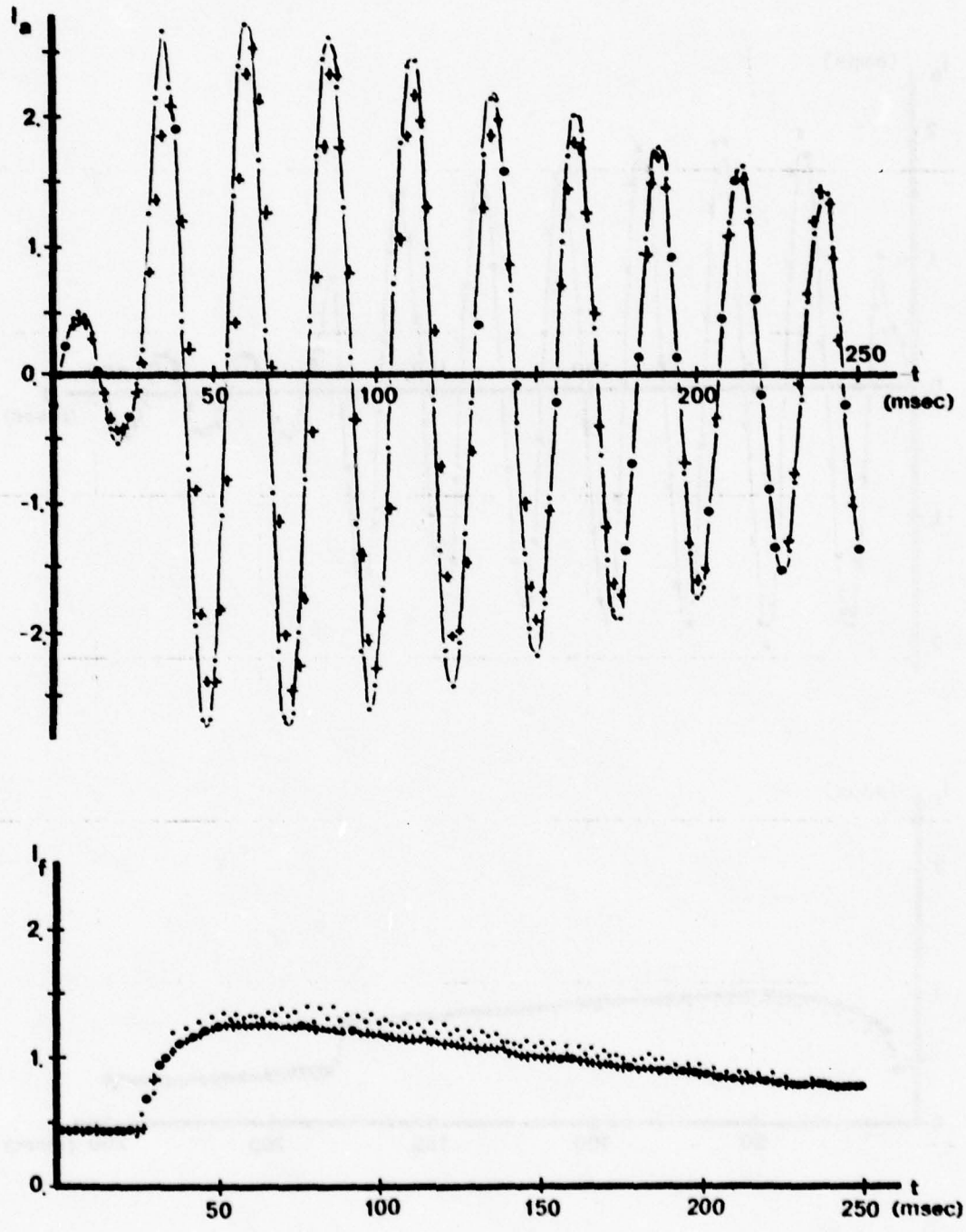

Figure 12. Phase A Armature and Field Currents

switched from 0.99 per unit to 0.175 per unit. This test was first simulated on the digital computer using the parameter estimates from the first test, then implemented in the laboratory. Figure 12 shows the comparison of the simulated results to the experimental results. The reasonable agreement of these results indicates the success of the model of the synchronous generator at similar operating conditions. This does not, however, guarantee validity of this model over much larger changes in operating conditions. In fact, a synchronous generator exhibits nonlinearities; therefore, this linearized representation is probably somewhat inaccurate for very large perturbations. The model is valid, though, about the conditions of the tests.

SECTION V

CONCLUSIONS

5.1 Summary of Results

This work is a new approach to an old problem: system identification theory applied to the experimental determination of synchronous generator parameters. The power of this statistical identification technique allows the parameters of Park's equations, cast in a state-variable formulation, to be determined using current measurements of the three armature phase currents and the field current.

Considering the identification problem as a multiport boundary-value problem and applying the method of quasilinearization leads to a weighted least-squares parameter estimator. Including additive noise corrupting the measurement process, a statistical analysis proves the maximum a posteriori probability estimator is achieved by selecting the weighting matrices as the inverse noise and parameter error covariance matrices. This estimator is the optimal estimator in the sense of minimizing the Bayesian risk. If the error covariances are incorrect, the estimator is no longer optimal but is still a good implementation of a least-squares estimator.

Before the experiment was implemented, several experimental conditions, such as the magnitude and duration of the transient and the data sampling rate and duration, needed to be selected. Also, a set of constraints, mainly economic in nature, were recognized. Within these constraints the experiment was designed with the aid of a digital computer simulation. That is, a numerical solution of the mathematical model of a typical synchronous generator under a switched resistive load, was used to test various experimental conditions. The results

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show that a sufficient amount of data is produced by a load resistance switched from full load value to 0.25 per unit then switched back to full load value. The duration of the transient was consistent with manual operation of a three-pole switch. The resulting experiment provided sufficient data at a sampling rate which could be handled by the data recording and processing facilities available, while being implemented on available laboratory equipment.

Implementing this experiment in the laboratory, recording the terminal current data, and digitizing this data for input to the off-line parameter estimator led to estimates of the parameters of the mathematical model of the generator. These parameter estimates provide a linearized model of the generator which is valid over a range of operating conditions near the experimental conditions. The results of predicting the generator response to a larger step change in load resistance compare favorably to the actual measured response under those conditions. This favorable comparison validates the results, showing that the model can indeed predict the generator behavior under similar load conditions.

5.2 Significance of Results

In fact, the synchronous generator is a nonlinear device, exhibiting saturation and other deviations from the assumed linear model. By assuming this linear model structure and using an iterative estimation algorithm to calculate parameters, the results are effectively linearized about the test conditions. That is, the estimator fits the best linear model to the data. This, of course, presents a limitation to modeling drastically different operating conditions. For relatively

small perturbations in conditions, however, a simple valid solution has been achieved. It should be emphasized that the approach used here is an improvement over methods which assume linearity of the model over large changes in operating condition. For example, estimates of X_d, the synchronous reactance, can be obtained from combining results of open-circuit, unsaturated, armature voltage and of short-circuit armature current. However, such tests assume that results of two tests at drastically different points can be combined to model the generator under still different conditions, such as at unity power factor and full load. This implies that the generator model is linear to large perturbations. Thus, the present work should be viewed as a step toward proper modeling of a nonlinear device by a linear model valid over a small operating region.

The assumption of a linear model was made primarily for simplicity. The estimation of parameters of a dynamical system, such as the generator, can be viewed as an inherently nonlinear problem, even when the model itself is linear. Augment the state vector x with the parameter vector y,

$$\tilde{x} = \begin{bmatrix} x \\ y \end{bmatrix} \quad . \tag{53}$$

Now, in the linear model used, the parameters are multiplied by the states in the right side of Equation (23). As a result, this can be rewritten as a nonlinear dynamical system:

$$\frac{dx}{dt} = f(\tilde{x}) \tag{54}$$

The problem can now be formulated as a nonlinear state estimation problem. As a result of this view of the problem, any nonlinearity that can be formulated can be included by this approach.

Therefore, a general method for determination of synchronous generator parameters from the results of test data has been presented. The particular model used here is Park's linear circuit model of an ideal generator. A solution, using available equipment, shows favorable results.

5.3 Recommendations for Future Research

Inclusion of nonlinearities, especially saturation, in the generator model deserves additional research. The preceding section shows that nonlinear differential equations are easily handled by this algorithm. Consequently, future research should concentrate on finding a suitable parameterization of the nonlinearities. Once this is done, the extension of the present algorithm to include estimating the parameters that describe the nonlinearities is straightforward. In addition, the development of algorithms for parameter estimation for permanent magnet synchronous generators and brushless machines where, in each case, the field current is not available as a measurable quantity are required.

SECTION VI

ESTIMATION OF IEEE STANDARD PARAMETERS

6.1 Introduction

For the purpose of digital simulation and application of modern systems theory, it is most convenient to represent a synchronous machine in state-space form. The electrical behavior of the machine is then determined by coefficient matrices whole elements can be expressed[20] in terms of standard machine parameters as defined in the IEEE code.[21,22]

An output sensitivity analysis study,[23,24] of an alternator-load system showed that its electrical output variables are very sensitive to variations of certain standard machine parameters. Consequently, the set of parameter data with which the model has to be provided, should be sufficiently accurate. Procedures to determine the standard parameters of a synchronous machine have been established[22] where several different tests have to be performed. With increasingly detailed model structures of the machine,[25-27] several methods were introduced to find additional parameter values.[28-30] In general, however, the need for a method to obtain more accurate parameter values for a given model structure is recognized.[24,25] One of the primary tasks of an IEEE Working Group is to suggest standard methods to determine the parameter values for synchronous machine models.[25]

A newly developed test method to accurately determine the coefficients of an alternator transfer function is a frequency-response technique,[31,32] where judgement is required in locating the breakpoints of the plot to get a good fit of the data. Its practical implementation need a variable frequency source capable of supplying relatively large currents.

In this section, a weighted-least-squares and maximum likelihood estimation technique is presented to determine the complete set of standard parameters of a salient-pole synchronous machine with damper winding from the observed output data of a sudden short-circuit. An advantage of identifying standard parameters is that the parameter values obtained can be used for any model formulation of the same order employing standard parameters, which are best known to power engineers. In this connection, it is noted that the values of the coefficient-matrix elements of the corresponding state-space model are uniquely determined from a given set of values of the standard parameters whereas this is generally not true for the reversed case.[24] The chosen test procedure has the feature that regular test equipment can be employed in the practical implementation and, above all, fast convergence of the estimation scheme is achieved while observation time and estimation error are kept to a minimum.

6.2 Parameter Estimation

6.2.1 Weighted-Least-Squares (WLS) Method

Let a real physical system be described by

$$\dot{x} = A(\alpha)x + B(\alpha)u;\ x(t_o) = x_o \quad , \tag{55}$$

$$y = C(\alpha)x$$

where x and y are the state and observable output vectors, respectively, and u represents the input vector assumed to be a given time function. $A(\alpha)$, $B(\alpha)$ and $C(\alpha)$ are coefficient matrices dependent on a parameter vector α, whose value is to be identified. Note that the unknown parameter α does not necessarily correspond directly to the elements of any of the coefficient matrices.

The value of α is chosen to minimize[33] a weighted-least-squares error

$$J = \sum_{i=1}^{N} [y_r(i) - y(i)]^T W(i) [y_r(i) - y(i)] \quad . \tag{56}$$

Herein, $y_r(i)$ and $y(i)$, $i=1,2,\ldots,N$, represent the sampled outputs of the real system and model reference, respectively, both excited with the same input, whereas $W(i)$ is a positive semidefinite symmetric weighting matrix chosen on the basis of engineering judgment. Expanding the output $y(i)$ in a Taylor series about its trajectory at a given value α_o of parameter α gives

$$y(i) = y(i,\alpha_o) + [\partial y(i)/\partial\alpha]_{\alpha_o}(\alpha - \alpha_o)$$

$$+ \text{higher order terms,} \tag{57}$$

where

$$[\partial y(i)/\partial\alpha]_{\alpha_o} = C(\alpha_o)[\partial x(i)/\partial\alpha]_{\alpha_o} + [\partial C(\alpha)/\partial\alpha]_{\alpha_o} x(i,\alpha_o) \tag{58}$$

The time-dependent variables $[\partial x(i)/\partial\alpha]_{\alpha_o}$ are found by solving and sampling the partial derivative of (55) relative to α, evaluated at α_o

$$d(\partial x/\partial\alpha)_{\alpha_o}/dt = A(\alpha_o)(\partial x/\partial\alpha)_{\alpha_o} + [\partial A(\alpha)/\partial\alpha]_{\alpha_o} x(\alpha_o)$$

$$+ [\partial B(\alpha)/\partial\alpha]_{\alpha_o} u; \quad (\partial x/\partial\alpha)_{\alpha_o,t_o} = 0 \ , \tag{59}$$

and equation (55) itself, evaluated at α_o. Substitution of (57) in (56) yields

$$J = \sum_{i=1}^{N} \{y_r(i) - y(i,\alpha_o) - [\partial y(i)/\partial\alpha]_{\alpha_o}(\alpha - \alpha_o)\}^T W(i)$$

$$\cdot \{y_r(i) - y(i,\alpha_o) - [\partial y(i)/\partial\alpha]_{\alpha_o}(\alpha - \alpha_o)\} \ , \tag{60}$$

where the higher-order terms of the expansion are ignored. With $(\alpha - \alpha_o)$ selected such that (60) is minimized,

$$\partial J/\partial(\alpha - \alpha_o) = 0 \tag{61}$$

from which follows a set of linear algebraic equations for $(\alpha - \alpha_o)$:

$$\{\sum_{i=1}^{N} [\partial y(i)/\partial\alpha]^T_{\alpha_o} W(i)/\partial\alpha]\alpha_o\}(\hat{\alpha} - \alpha_o) =$$

$$\sum_{i=1}^{N} [\partial y(i)/\partial\alpha]^T_{\alpha_o} W(i)[y_r(i) - y(i,\alpha_o)] \tag{62}$$

where $\hat{\alpha}$ denotes the α that minimizes (60). From (55), (58), (59) and (62), an iterative solution for $\hat{\alpha}$ can be obtained. Upon writing $(\hat{\alpha} - \alpha_o) = \Delta\hat{\alpha}$, the recursion formula for computing successive estimates of $\hat{\alpha}$ is

$$\hat{\alpha}^k = \hat{a}^{k-1} + \Delta\hat{\alpha}^k \tag{63}$$

where $k(=1,2,\ldots)$ refers to the iteration count and $\hat{\alpha}^o$ is some initial guess made by engineering judgment. Since the derivation assumes a sufficiently small $\Delta\hat{\alpha}$ due to truncating the Taylor series, it is proposed to modify (63) in

$$\hat{\alpha}^k = \hat{\alpha}^{k-1} + G(k)\Delta\hat{\alpha}^k \tag{64}$$

where G(k) is a gain matrix, whose elements are selected on the basis of computational judgment. G(k) controls the extent of correction in each iteration step. An excessively large value of G(k) may cause $\hat{\alpha}^k$ to overshoot and go into oscillation about $\hat{\alpha}$, whereas an overly small value of G(k) causes very slow convergence of $\hat{\alpha}^k$ to $\hat{\alpha}$. One of the many possible variations is to take

$$G(k) = \text{diag}[g_{ii}(k)] \tag{65}$$

where the scalars $g_{ii}(k)$ are chosen in some proper manner.

6.2.2 Maximum Likelihood (ML) Method

It was assumed in the previous section that neither input nor output of the system is corrupted with noise. Considering the physical nature of the case to which the estimation technique is to be applied, ignoring the input noise seems to be quite acceptable. However, the output measurement is generally noise corrupted. Therefore, a more realistic expression for the system output is

$$y = C(\alpha)x + \xi \tag{66}$$

Here, ξ represents a measurement noise which is assumed to be zero-mean gaussian white with

$$E\{\xi(t)\xi^T(\tau)\} = R(t)\delta(t - \tau) \ , \tag{67}$$

where $R(t)$ is positive definite, and $E\{\cdot\}$ and $\delta(t-\tau)$ stand for the expectation operator and Dirac delta function, respectively.

The value of α is chosen from the observed sampled output $y_N = [y^T(1), y^T(2), \ldots, y^T(N)]^T$ such that the likelihood function[34] $l(y_N:\alpha)$, which depends on α, is maximum. A priori, the functional relationship between y_N and α is given by the conditional probability density function $p(y_N|\alpha)$. Thus, using the product rule for probabilities, the likelihood function

$$L(y_N:\alpha) = \prod_{i=1}^{N} p\{y(i)|y_{i-1},\alpha\} \tag{68}$$

where $y_i = [y^T(1), y^T(2), \ldots, y^T(i)]^T$ and $p\{y(i)|y_{i-1},\alpha\}$ is the conditional density function of $y(i)$ for given y_{i-1} and α. For the system described by (55) and (66),

$$p\{y(i)|y_{i-1}\alpha\} = [(2\pi)^m|R(i)|]^{-\frac{1}{2}}$$

$$\cdot \exp\{-\tfrac{1}{2}[y(i)-\bar{y}(i)]^T R^{-1}[y(i)-\bar{y}(i)]\} \tag{69}$$

where the mean output

$$\bar{y}(i) = C(\alpha)x(i) \ , \tag{70}$$

and m is the dimension of output vector y. Therefore, it follows from (68) and (69) that maximization of $L(y_N:\alpha)$ and minimization of

$$J = \sum_{i=1}^{N} [y(i) - \bar{y}(i)]^T R^{-1}[y(i) - \bar{y}(i)] \tag{71}$$

both with respect to α are equivalent. Considering $y = C(\alpha)x$ and (70), expression (71) is identical to (56) if

$$W = R^{-1} \tag{72}$$

Therefore, the same computational algorithm as for the WLS estimation can be applied if the inverse covariance matrix of the measurement noise

determined from the a priori knowledge is used for the weighting matrix.

6.3 Application to Synchronous Machines

6.3.1 Machine Model at Sudden Short-Circuit

A salient-pole synchronous machine with damper winding is considered. In the following, the machine equations are written in state-space form as required by (55) with flux-linkages selected as the state variables[20,23]

$$\dot{\psi}_d = (c_d/T_d)\psi_d + \omega\psi_q + (c_{dkd}/T_d)\psi_{kd} + (c_{df}/T_d)\psi_f + v_d$$

$$\dot{\psi}_q = -\omega\psi_d + (c_q/T_q)\psi_q + (c_{qkq}/T_q)\psi_{kq} + v_q$$

$$\dot{\psi}_{kd} = (c_{kdd}/T''_{do})\psi_d + (c_{kd}/T''_{do})\psi_{kd} + (c_{kdf}/T''_{do})\psi_f$$

$$\dot{\psi}_{kq} = (c_{qkq}/T''_{qo})\psi_q + (c_q/T''_{qo})\psi_{kq}$$

$$\dot{\psi}_f = (c_{fd}/T'_{do})\psi_d + (c_{fkd}/T'_{do})\psi_{kd} + (c_f/T'_{do})\psi_f + v_f \qquad (73)$$

Herein, v_d and v_q are the d,q components of the armature voltage and v_f represents the field voltage. The time constants T_i, i=d,q) and the dimensionless c-constants are explicitly expressed in standard parameters and listed below.

$$T_i = X_i/\omega R_a, \ (i=d,q) \quad c_d = -X_d/X''_d, \quad c_{kd} = -X'_d/X''_d$$

$$c_f = -\frac{X_d}{X''_d} + \frac{(X_d-X_\ell)^2(X'_d-X''_d)}{(X'_d-X_\ell)^2 X''_d} - \frac{(X_d-X'_d)(X'_d-X''_d)X_d}{(X'_d-X_\ell)^2 X''_d}$$

$$c_{dkd} = \frac{(X'_d - X''_d)X_d}{(X'_d - X_\ell)X''_d} \quad , \quad c_{df} = \frac{(X''_d - X_\ell)(X_d - X'_d)X_d}{(X_d - X_\ell)(X'_d - X_\ell)X''_d}$$

$$c_{kdf} = \frac{(X_d - X'_d)X_\ell}{(X_d - X_\ell)X''_d} \quad , \quad c_{fkd} = \frac{(X'_d - X''_d)(X_d - X_\ell)X_\ell}{(X'_d - X_\ell)^2 X''_d}$$

$$c_{kdd} = (X'_d - X_\ell)/X''_d \quad , \quad c_{fd} = \frac{(X''_d - X_\ell)(X_d - X_\ell)}{(X'_d - X_\ell)X''_d}$$

$$c_q = -X_q/X''_q \quad , \quad c_{qkq} = \sqrt{(X_q/X''_q)^2 - X_q/X''_q} \qquad (74)$$

It is noted, that an additional modified armature leakage reactance X_ℓ has been added to the conventional set of standard parameters, since the conventional set alone does not completely describe the machine behavior.[28,30] However, if the "leakage" inductance of damper winding in the d-axis is negligible with respect to its self inductance, it can be readily shown that $X_\ell = X''_d$.

A sudden short-circuit is taken from a steady-state no-load condition, where the field current i_f^o is adjusted to obtain some desired value of the no-load armature voltage E. Thus, before the short-circuit,

$$v_d = 0, \; v_q = E, \; v_f = v_f^o \qquad (75)$$

and if the elements of the 5-state vector ψ are ordered in the same sequence as in (73), the constant state ψ^o is given by

$$\psi^{o} = [L_{md} i_f^{o} = E \quad 0 \quad L_{md} i_f^{o} = E \quad 0 \quad L_f i_f^{o}]^T \quad . \tag{76}$$

In (75), v_f^{o} is the field voltage corresponding to i_f^{o}, whereas L_f and L_{md} in (76) are, respectively, the self inductance of the field winding and the mutual inductance between the three circuits in the d-axis. During short-circuit, the terminal conditions are

$$v_d = 0, \quad v_q = 0, \quad v_f = v_f^{o} \quad . \tag{77}$$

Upon writing

$$\psi^{*} = \psi - \psi^{o} \quad , \tag{78}$$

the equations for the flux-linkage change ψ^{*} during short-circuit can be written as

$$\dot{\psi}_d^{*} = (c_d/T_d)\psi_d^{*} + \omega\psi_q^{*} + (c_{dkd}/T_d)\psi_{kd}^{*} + (c_{df}/T_d)\psi_f^{*}$$

$$\dot{\psi}_q^{*} = -\omega\psi_d^{*} + (c_q/T_q)\psi_q^{*} + (c_{qkq}/T_q)\psi_{kq}^{*} - E$$

$$\dot{\psi}_{kd}^{*} = (c_{kdd}/T''_{do})\psi_d^{*} + (c_{kd}/T''_{do})\psi_{kd}^{*} + (c_{kdf}/T''_{do})\psi_f^{*}$$

$$\dot{\psi}_{kq}^{*} = (c_{qkq}/T''_{qo})\psi_q^{*} + (c_q/T''_{qo})\psi_{kq}^{*}$$

$$\dot{\psi}_f^{*} = (c_{fd}/T'_{do})\psi_d^{*} + (c_{fkd}/T'_{do})\psi_{kd}^{*} + (c_f/T'_{do})\psi_f^{*} \tag{79}$$

with

$$\psi^*(t_o) = 0 \quad . \tag{80}$$

If the three-phase short-circuit current i_{abc}, and thus its d, q components i_d and i_q by applying Park's transformation, and the field current change i_f^* ($= i_f - i_f^o$) are considered as the observable output, then the generally noise corrupted output equations are described by

$$i_d = (\omega/X_d)(c_d\psi_d^* + c_{dkd}\psi_{kd}^* + c_{df}\psi_f^*) + \xi_d$$

$$i_q = (\omega/X_q)(c_q\psi_q^* + c_{qkq}\psi_{kq}^*) + \xi_q$$

$$i_f^* = (-1/R_f T'_{do})(c_{fd}\psi_d^* + c_{fkd}\psi_{kd}^* + c_f\psi_f^*) + \xi_f \tag{81}$$

where R_f is the field winding resistance, and ξ_d, ξ_q and ξ_f represent the corresponding measurement noises.

6.3.2 Machine Parameter Estimator

The standard parameters to be estimated are X_d, X'_d, X''_d, X_ℓ, X_q, X''_q, R_a, T'_{do}, T''_{do} and T''_{qo}. Note that R_f in (81) is also considered not known, therefore, it has to be estimated together with the standard parameters. Equations (79), (80) and (81) are now considered to correspond to the general formulations in (55) and (66). In (79), -E is regarded as an input whose absolute per-unit value is equal to the no-load, per-unit voltage of the generator before short-circuit, thus known from the short-circuit test. The

remaining equations needed to construct the parameter estimator are obtained by applying (58), (59), (62), (64), and (65), where the expressions for $\partial A(\alpha)/\partial\alpha$, $\partial B(\alpha)/\partial\alpha$ and $\partial C(\alpha)/\partial\alpha$ have to be evaluated.

A block diagram for the estimation algorithm is given in Figure 13, where the real output currents $i^r_{abc}(A)$ and $i^r_f(A)$ are expressed in amperes. Therefore, these currents are divided by their base values i_B and i_{fB}, respectively, before they are compared with the corresponding per-unit currents of the model reference. The base current i_B is easily computed from the machine rating. Depending on the choice of per-unit system for the rotor quantities,[35] a base current i_{fB} can be obtained, e.g., that value of the field current which produces rated no-load voltage. It is noted, however, that i_{fB} only affects the per-unit value of R_f, but none of the other parameters to be identified.

6.4 Simulation Results

A digital computer program listed in Appendix E has been developed for the WLS and ML estimation and applied to a 120 kVA, 208 V, 400 Hz aircraft generator which was simulated as a fifth order model on a digital computer. Its per-unit parameter values with a time base of 1/2513 s were taken equal to the nominal values listed by the manufacturer as $X_d = 2.10$, $X'_d = 0.216$, $X''_d = 0.186$, $X_\ell = 0.04$, $X_q = 0.786$, $X''_q = 0.105$, $R_a = 0.0189$, $T'_{do} = 522$, $T''_{do} = 18.2$, $T''_{qo} = 115$.

The short-circuit takes place at unity no-load voltage (E=1), which corresponds with a field voltage v^o_f of 0.00210 and current i^o_f of 0.0510 for the simulated generator. The short-circuit current i^r_{abc} and field current i^r_f in per-unit are displayed in Figure 14.

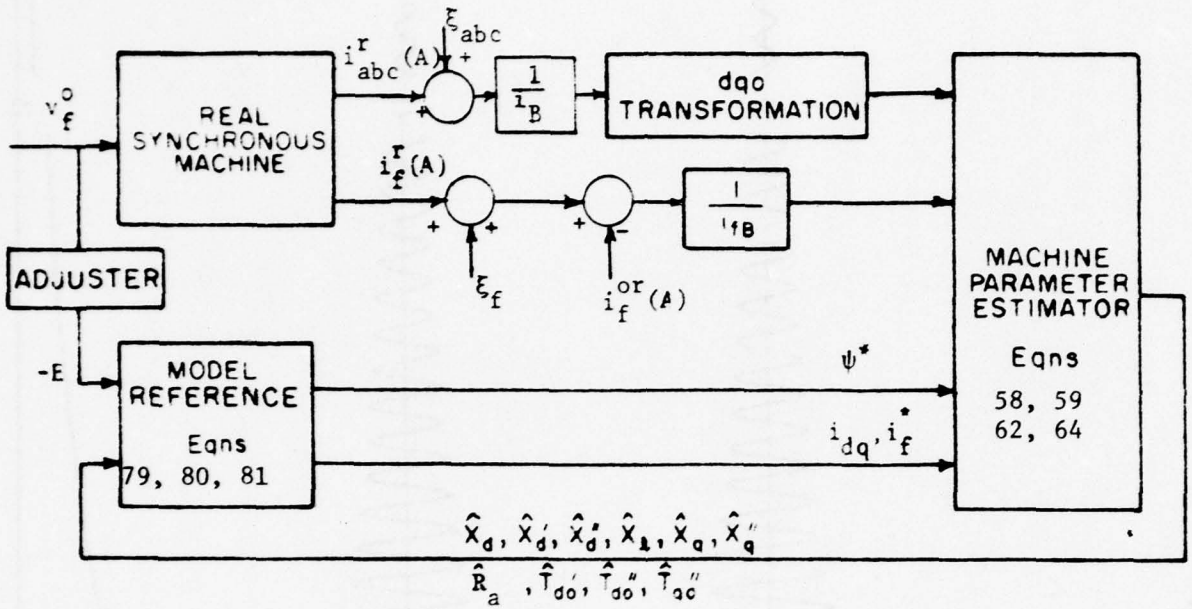

Figure 13. Block Diagram for Machine Parameter Estimation

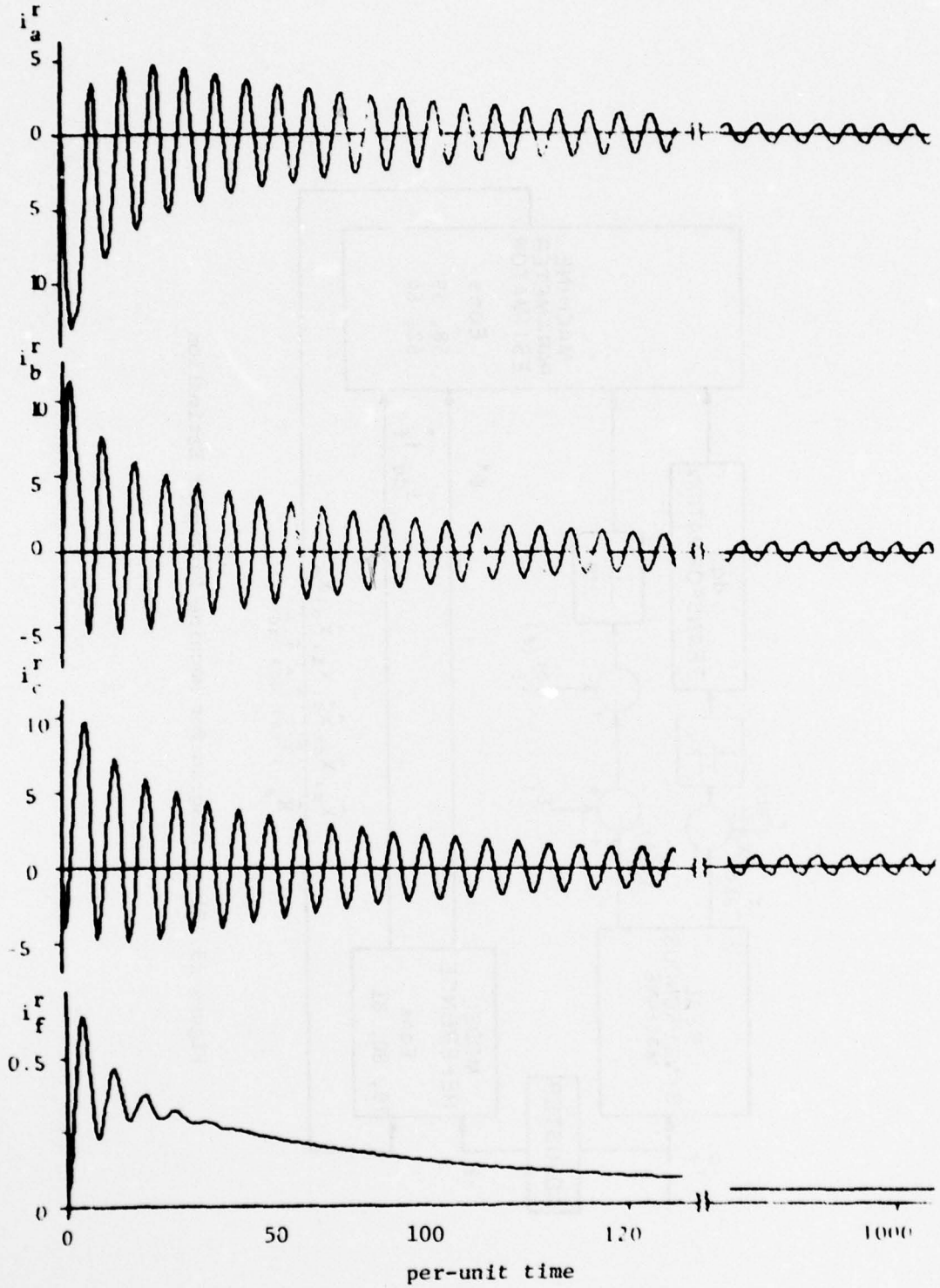

Figure 14. Oscillogram of Short-Circuit Currents.

The parameter estimation algorithm was performed digitally with a sampling time of 0.2 pu, whereas the total number of samples is 900 corresponding to an observation time length of 180 pu. The gain element $g_{ii}(k)$ in (1.2-12) was selected equal to unity if $|\Delta\hat{\alpha}_i^k/\hat{\alpha}_i^{k-1}| < 0.25$, whereas $g_{ii}(k)$ equals $0.25\hat{\alpha}_i^{k-1}/|\Delta\hat{\alpha}_i^k|$ if $|\Delta\hat{\alpha}_i^k/\hat{\alpha}_i^{k-1}| \geq 0.25$ for all parameter α_i.

1.4.1 WLS Estimation

Here, the weighting matrix W was chosen equal to the identity matrix. Several sets of initial parameter estimates were subsequently tried with values equal to 40%, 60%, 80%, 120%, 160% and 200% of the true values, and finally with values chosen at random between 40% and 200% of the true values. The results are summarized in Table 4. The behavior of successive estimates is displayed in Figure 15, which is typical for the entire set of parameters, except X_ℓ, whose behavior is visualized in Figure 16. Figure 17 shows the behavior for all parameters with their initial estimates chosen at random.

The results show that the estimates for all parameters, except X_ℓ, converge to the true values in a few iterations with less than 1% error, while it takes some more iterations for X_ℓ to converge. It is also observed, that the convergence is faster for initial estimates with smaller deviations from the true values.

1.4.2 Output Noise Effect

To investigate the effect of output noise on the WLS estimation, a zero-mean gaussian white noise with constant standard deviation of 5% of the steady-state magnitude of i_{abc} and i_f was properly added to the output of the simulated generator. Keeping all other conditions the

TABLE 4

WLS ESTIMATION

Number of Iteration	Parameter Estimates as Fraction of True Value									
	$\hat{X}_c$	$\hat{X}'_d$	$\hat{X}''_d$	$\hat{X}_\ell$	$\hat{X}_q$	$\hat{X}''_q$	$\hat{R}_a$	$\hat{T}'_{do}$	$\hat{T}''_{do}$	$\hat{T}''_{qo}$
0	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
3	0.947	0.978	0.985	0.845	0.963	0.986	0.984	0.945	0.962	0.952
5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600
3	0.983	0.997	0.998	1.023	0.990	0.998	0.998	0.980	0.992	0.989
5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0	1.200	1.200	1.200	1.200	1.200	1.200	1.200	1.200	1.200	1.200
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800
3	0.999	1.000	1.000	0.938	1.000	1.000	1.000	0.999	0.998	1.000
5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.600
5	1.000	0.999	1.000	0.659	1.000	1.000	1.000	1.000	0.997	1.000
8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
5	0.964	0.991	1.000	0.158	0.770	1.000	0.996	0.970	1.040	0.433
10	0.999	0.999	1.000	0.483	1.000	1.000	1.000	1.000	0.995	1.000
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0	1.200	1.400	1.600	1.800	0.800	0.600	1.000	0.700	0.500	0.400
5	0.988	0.994	1.000	1.372	0.999	1.000	1.000	0.980	0.916	0.998
8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

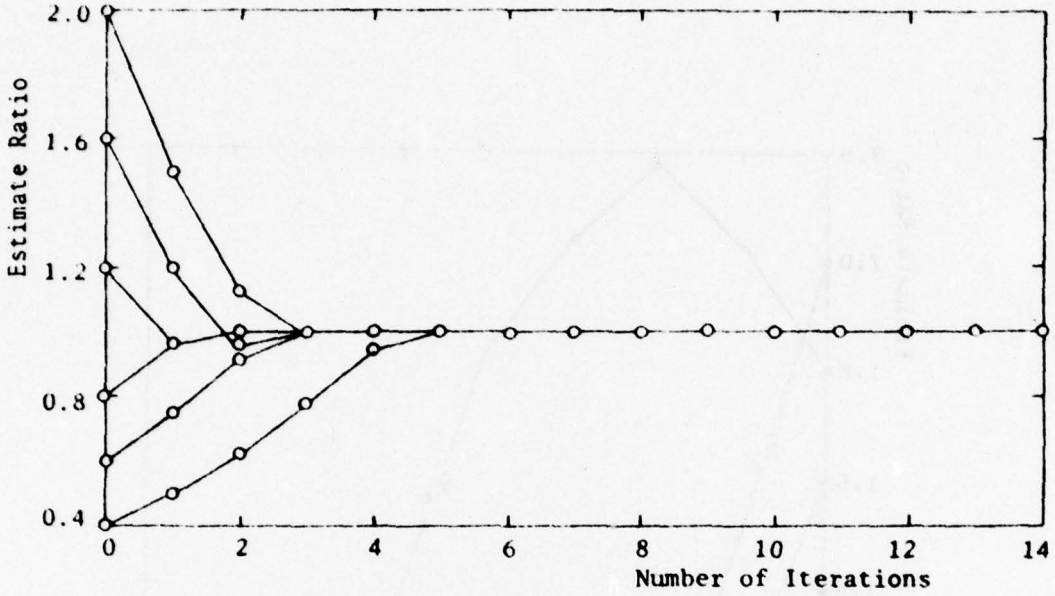

Figure 15. Ratio of WLS Estimate to True Value for X_d''

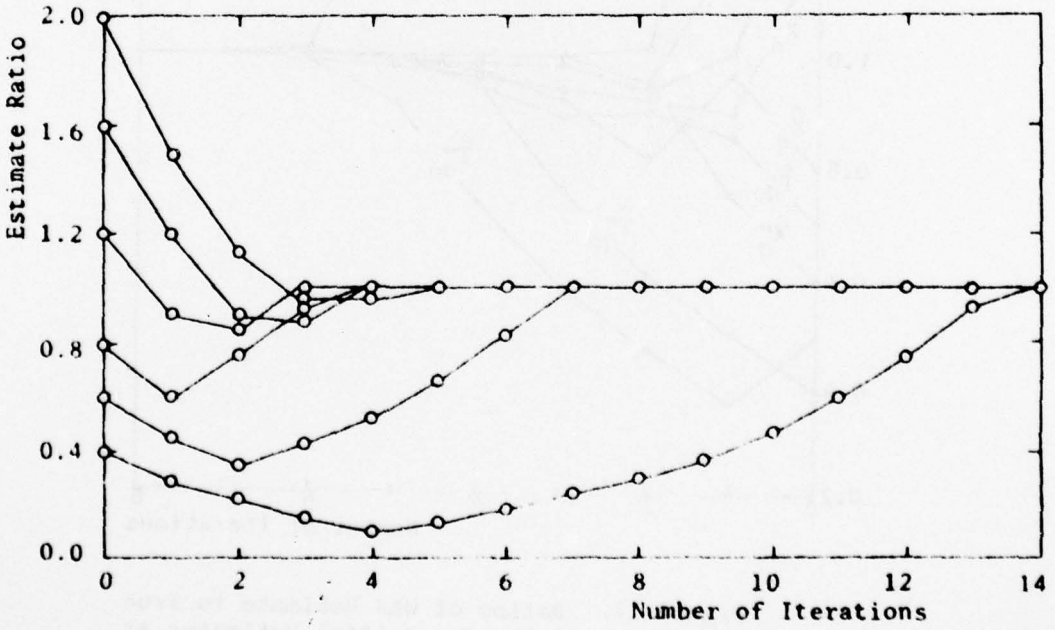

Figure 16. Ratio of WLS Estimate to True Value for X_ℓ

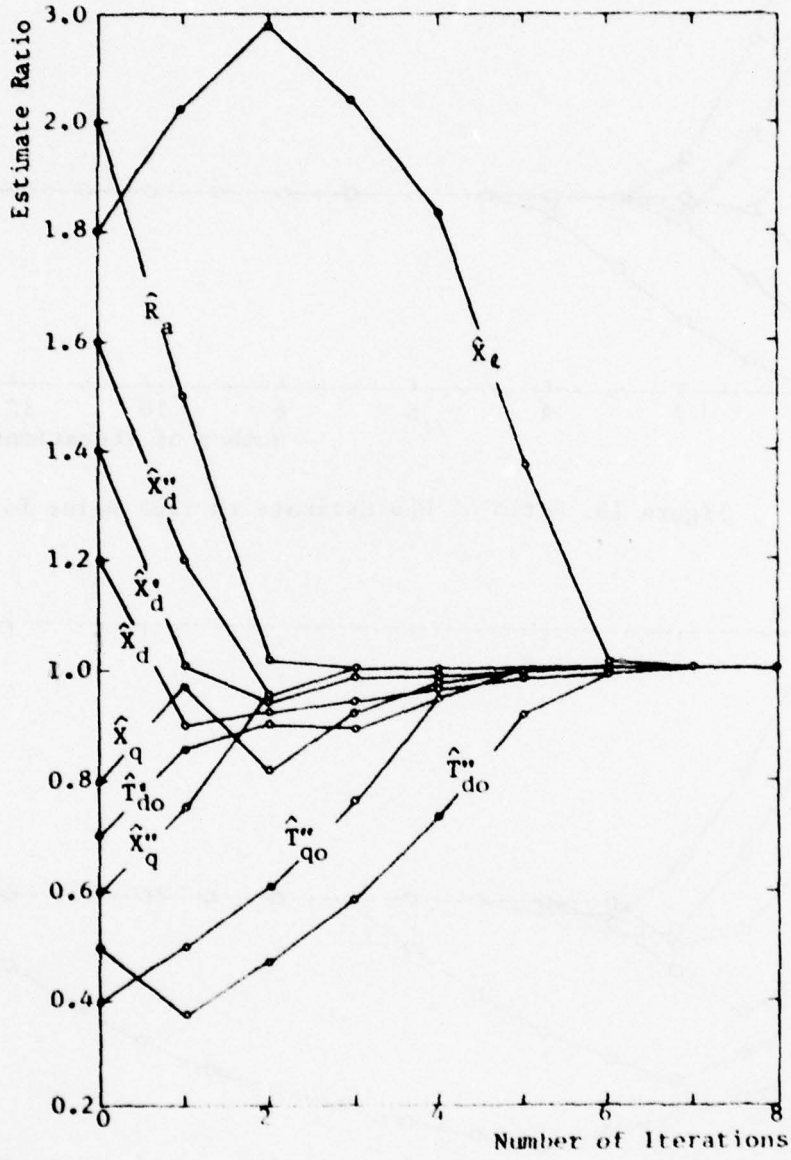

Figure 17. Ratios of WLS Estimate to True Value for Initial Estimates at Random

same as before, the machine parameters were estimated using randomly chosen initial values. The numerical results are summarized in Table 5 and displayed in Figure 18.

From the results, it is observed that the convergence rate is about the same as in the deterministic case. The estimates for parameters X_d, X'_d, X''_d, X''_q, R_a and T'_{do} converge to the true values employed in the generator simulation in a few iterations with less than 1% error, whereas the estimates for X_ℓ, X_q, T''_{do} and T''_{qo} converge to biased values resulting in errors between 2% and 13% with X_ℓ showing the largest error.

6.4.3 ML Estimation

The maximum likelihood estimator was then applied with the same conditions and output noise, whose inverse covariance matrix R^{-1} = diag $(0.02205, 3.845, 0.02205 \times 10^4$. The results for various initial estimates are listed in Table 6. For easy comparison, the convergence behavior of successive estimates for randomly chosen initial values is depicted in Figure 19.

The results show that compared with the WLS estimation, the ML estimator has about the same convergence rate. However, the accuracy is significantly greater, viz., all parameter estimates converge to the true values in a few iterations with less than 1% error, except X_q, T''_{do} and T''_{qo}, which have an error between 1.5% and 2.5%.

6.4.4 Input Noise Effect

The effect of input noise on the estimation results was investigated by adding an input noise to the generator simulation while keeping all other conditions including output noise the same as before. The input noise was generated as a zero-mean gaussian white process with

TABLE 5

OUTPUT NOISE EFFECT ON WLS ESTIMATE

Number of Iteration	Parameter Estimates as Fraction of True Value									
	$\hat{X}_d$	$\hat{X}'_d$	$\hat{X}''_d$	$\hat{X}_\ell$	$\hat{X}_q$	$\hat{X}''_q$	$\hat{R}_a$	$\hat{T}'_{do}$	$\hat{T}''_{do}$	$\hat{T}''_{qo}$
0	1.200	1.400	1.600	1.800	0.800	0.600	2.000	0.700	0.500	0.400
3	0.972	0.986	1.005	2.360	0.912	1.000	0.996	0.936	0.586	0.763
6	0.997	0.995	1.000	1.147	0.980	0.999	1.001	0.996	0.957	0.976
8	0.997	0.996	1.000	1.131	0.980	0.999	1.001	0.997	0.960	0.976

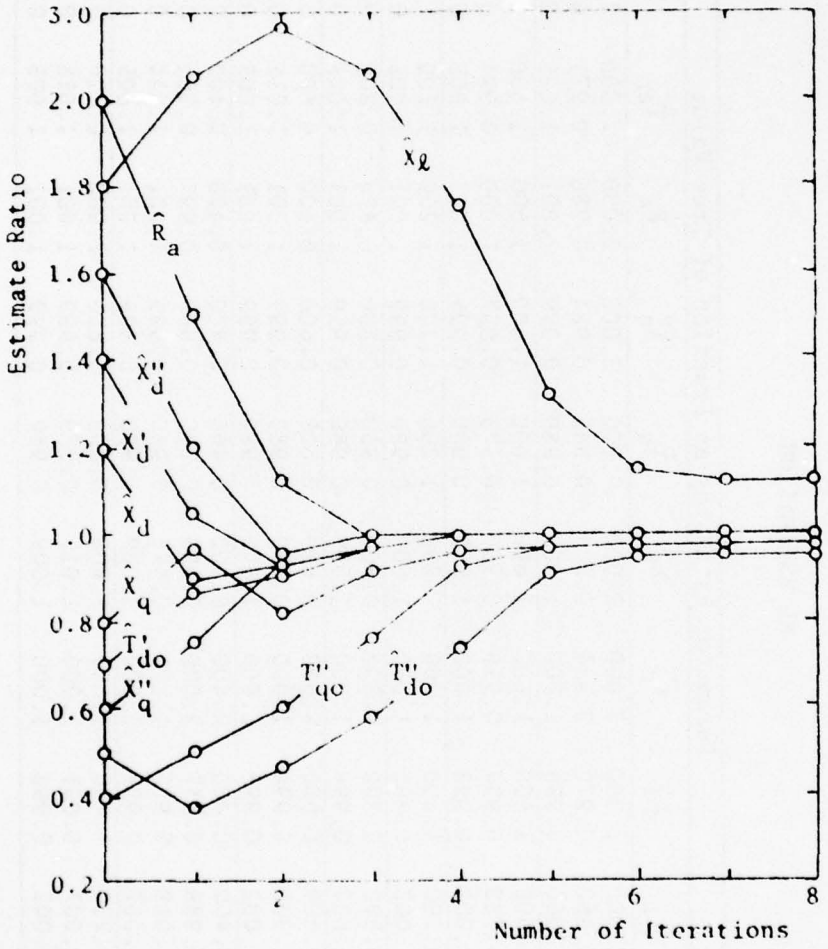

Figure 18. Ratio of WLS Estimate to True Voltage for Random Initial Estimates: Output Noise Effect

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TABLE 6

ML ESTIMATION

Number of Iteration	Parameter Estimates as Fraction of True Value									
	$\hat{X}_d$	$\hat{X}'_d$	$\hat{X}''_d$	$\hat{X}_\ell$	$\hat{X}_q$	$\hat{X}''_q$	$\hat{R}_a$	$\hat{T}'_{do}$	$\hat{T}''_{do}$	$\hat{T}''_{qo}$
0	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
3	0.962	0.970	0.984	0.844	0.942	0.984	0.986	0.971	0.903	0.925
5	1.002	0.998	1.000	1.009	0.980	0.999	1.001	1.005	0.985	0.976
0	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600
3	0.989	0.994	0.998	0.900	0.973	0.997	1.000	0.994	0.998	0.968
5	1.002	0.998	1.000	1.009	0.980	0.999	1.001	1.005	0.985	0.976
0	1.200	1.200	1.200	1.200	1.200	1.200	1.200	1.200	1.200	1.200
4	1.002	0.998	1.000	1.009	0.980	0.999	1.001	1.005	0.985	0.976
0	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800
4	1.002	0.998	1.000	1.009	0.980	0.999	1.001	1.005	0.985	0.976
0	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.600	0.600
5	1.006	0.994	1.000	0.659	0.980	0.999	1.001	1.015	0.964	0.976
8	1.002	0.998	1.000	1.009	0.980	0.999	1.001	1.005	0.985	0.976
0	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400	0.400
5	0.988	0.937	1.009	0.158	0.740	0.999	0.997	0.991	0.253	0.380
10	1.019	0.983	1.001	0.483	0.980	0.999	1.001	1.044	0.771	0.977
14	1.002	0.998	1.000	1.009	0.980	0.999	1.001	1.005	0.985	0.976
0	1.200	1.400	1.600	1.800	0.800	0.600	2.000	0.700	0.500	0.400
5	1.007	0.991	1.000	1.015	0.979	0.999	1.001	1.018	0.916	0.975
8	1.002	0.998	1.000	1.009	0.980	0.999	1.001	1.005	0.985	0.976

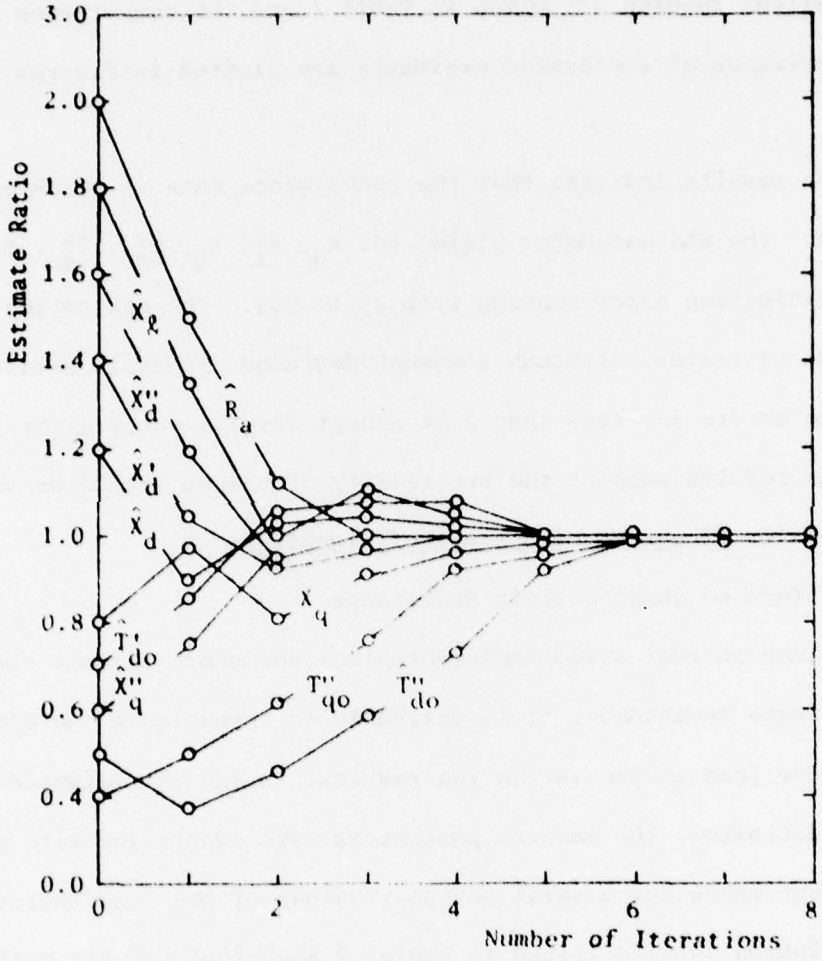

Figure 19. Ratio of ML Estimate to True Value for Random Initial Estimates

standard deviation of 1% of the steady-state value of the corresponding state variables. An identity coefficient matrix for the input noise was used. The parameters were estimated with randomly chosen initial values. The numerical results are given in Table 7 and the convergence characteristics of successive estimates are plotted in Figures 20 and 21.

The results indicate that the convergence rate is practically not affected. The WLS estimator yields for X_d, X_ℓ, X_q, T'_{do}, T''_{do}, T''_{do} and T''_{qo} an estimation error ranging from 2% to 50%. The estimation accuracy of the ML estimator, although somewhat degraded, is still satisfactory, viz., the errors are less than 2.5% except for X_ℓ, whose error is 14%. Thus, the results support the superiority of the ML estimator over the WLS estimator if system noises are of significance.

1.4.5 Effect of Short-Circuit Resistance

Since in real world implementation, the short-circuit connections have a finite resistance, it is desirable to investigate the effect of a resistive load on the estimation results. Using the weighted-least-squares estimator, the machine parameters were identified with random initial estimates for several per-unit values of the load resistance. The simulation results listed in Table 8 show that the estimation accuracy is affected by the load resistance value with the greatest accuracy occuring at $R_L = 0.0$. However, a load resistance of less than $R_L = 0.5$ pu still provides the same accuracy but at the expense of a few more iterations. This is very favorable since in practice the short-circuit connections will have a much smaller value than 0.5 pu.

TABLE 7

INPUT NOISE EFFECT ON WLS AND ML ESTIMATES

Number of Iteration	WLS Parameter Estimates as Fraction of True Value									
	$\hat{X}_d$	$\hat{X}'_d$	$\hat{X}''_d$	$\hat{X}_\ell$	$\hat{X}_q$	$\hat{X}''_q$	$\hat{R}_a$	$\hat{T}'_{do}$	$\hat{T}''_{do}$	$\hat{T}''_{qo}$
0	1.200	1.400	1.600	1.800	0.800	0.600	2.000	0.700	0.500	0.400
3	0.956	0.986	1.004	2.384	0.913	1.000	0.996	0.924	0.586	0.763
6	0.971	0.990	0.999	1.49t	0.981	0.999	1.001	0.966	0.869	0.974
8	0.971	0.991	0.999	1.496	0.981	0.999	1.001	0.965	0.870	0.974
Number of Iteration	ML Parameter Estimates as Fraction of True Value									
	$\hat{X}_d$	$\hat{X}'_d$	$\hat{X}''_d$	$\hat{X}_\ell$	$\hat{X}_q$	$\hat{X}''_q$	$\hat{R}_a$	$\hat{T}'_{do}$	$\hat{T}''_{do}$	$\hat{T}''_{qo}$
0	1.200	1.400	1.600	1.800	0.800	0.600	2.000	0.700	0.500	0.400
3	1.023	0.978	1.007	0.939	0.918	1.001	0.995	1.065	0.586	0.774
6	0.997	1.002	1.000	0.862	0.981	0.999	1.001	1.010	1.001	0.974
8	0.996	1.003	1.000	0.861	0.981	0.999	1.001	1.007	1.011	0.974

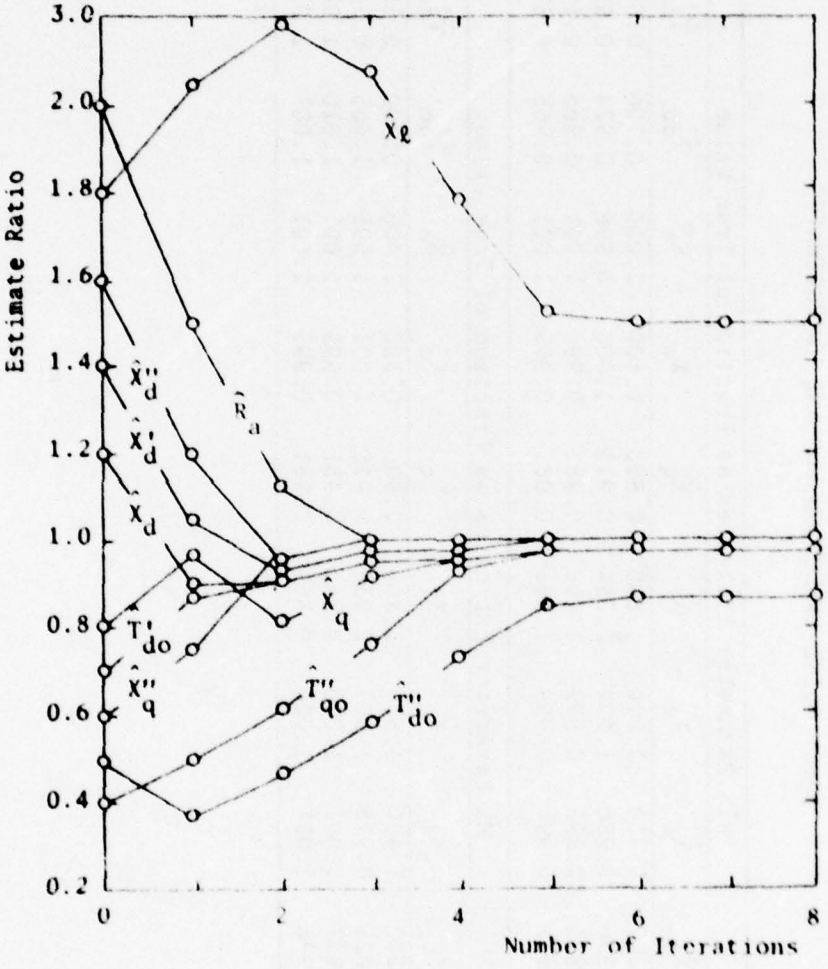

Figure 20. Ratio of WLS Estimate to True Value for Random Initial Estimates: Input and Output Noise Effect

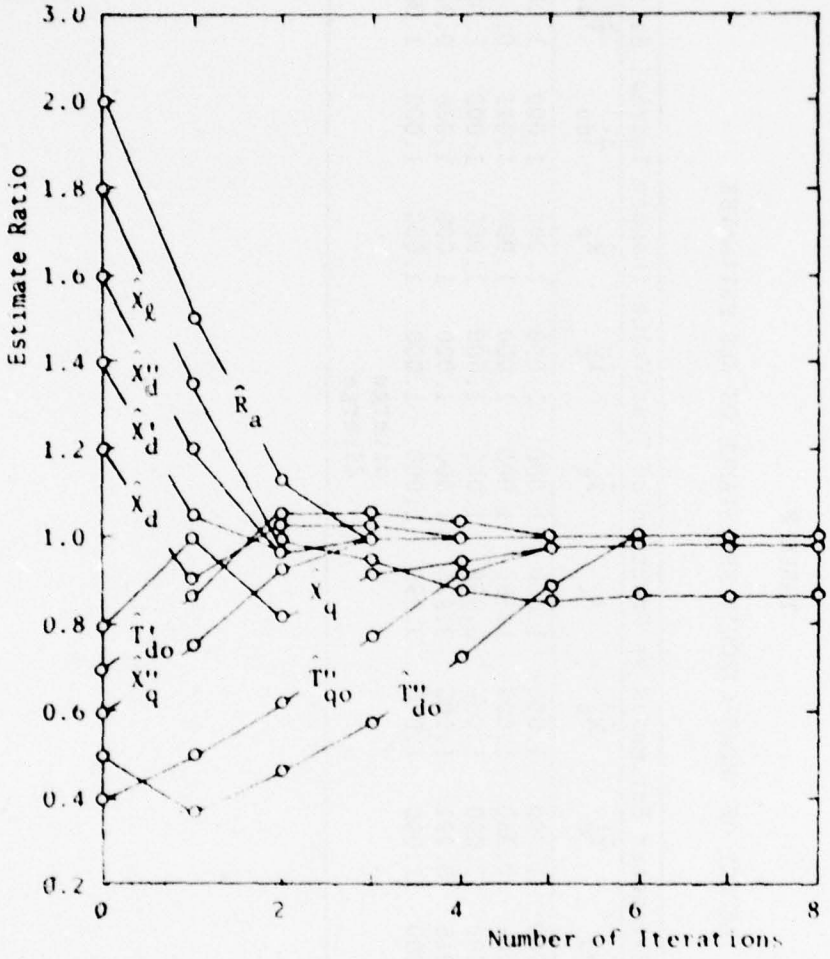

Figure 21. Ratio of ML Estimate to True Value for Random Initial Estimates: Input Noise Effect

TABLE 8

EFFECT OF SHORT-CIRCUIT RESISTANCE ON WLS ESTIMATES

R_L	Number of Iteration	Parameter Estimates as Fraction of True Value (Random Initial Estimates)									
		$\hat{X}_d$	$\hat{X}'_d$	$\hat{X}''_d$	$\hat{X}_\ell$	$\hat{X}_q$	$\hat{X}''_q$	$\hat{R}_a$	$\hat{T}'_{do}$	$\hat{T}''_{do}$	$\hat{T}''_{qo}$
0.00	8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	8	1.009	0.980	1.004	1.390	1.000	1.000	1.000	1.015	0.644	1.000
0.25	13	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
0.50	8	1.015	0.992	1.000	0.926	1.000	1.000	1.000	1.026	0.928	1.000
0.50	11	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
0.75		diverge									
1.00		diverge									

6.5 Conclusions

The numerical examples show that no convergence problems were encountered for initial estimates within the interval from 40% to 200% of the true values employed in the generator model simulation. With the same weighting and gain matrices, similar results have been obtained for different sets of parameter values typical for large 60 Hz machines. For initial guesses outside the above mentioned interval, the algorithm converges slower and may even diverge. This is not surprising since the estimator equations are highly nonlinear in the standard parameters. In the practical implementation, however, initial estimates can be taken equal to the values obtained from a reference table,[36] design data or conventional tests, so that convergence problems should not occur.

The simulation results show that the estimators are very accurate, viz., in the absence of environmental disturbances and with the availability of accurate measuring devices, the estimation error for all parameters is expected to be less than 0.1% of the true value. For noise levels which can be reasonably expected under experimental conditions, reasonably accurate parameter estimation can be anticipated by employing the maximum likelihood estimator which utilizes the a-priori knowledge of measurement noise.

The estimators are very effective considering that all parameters are simultaneously obtained from the data of a single test, where it is not even necessary to reach the steady-state condition. Note also that the q-axis parameters are extremely difficult to determine accurately by conventional test procedures.[25]

The study on the effect of resistive load on parameter estimation indicates that the best results are obtained with a zero load resistance. It is also noted, that the sudden short-circuit test provides superior results if compared with a step-function field-voltage test with short-circuited armature winding. In the latter case, the subtransient behavior of the armature currents is practically not noticeable so that larger estimation error, slower convergence rate or even divergence can be expected as indeed has been found. Consequently, the sudden short-circuit test is an extremely proper choice for parameter estimation.

APPENDIX A

LOWER BOUND ON ERROR COVARIANCE

The inverse of Fisher's information matrix provides a lower bound on the error covariance of the maximum likelihood estimate [35,32]. A similar approach results in a lower bound on the error covariance of the maximum a posteriori estimate [34]. These bounds are generalizations of the Cramer-Rao bound. This appendix outlines derivation of this bound for the estimator used in this research.

If the estimator were to converge exactly to a parameter estimate $\tilde{y}$, then

$$\sum_{k=1}^{k_f} \left\{ \frac{dh(t_k)}{dy}^T \Big|_{\tilde{y}} V_w^{-1} [z(t_k) - h(x,\tilde{y},t_k)] \right\} - V_y^{-1}(\tilde{y}-m_y) = 0 \quad . \tag{A-1}$$

In words, the model output $h(x,\tilde{y},t_k)$ would exactly equal the observed output $z(t_k)$. This condition is the theoretical best estimate of the parameter vector, but is never achieved in the presence of measurement noise. It is reasonable to use this theoretical performance limit to obtain a lower bound on the error covariance. Linearizing $h(x,\tilde{y},t_k)$ about the true parameter vector, y, gives

$$\sum_{k=1}^{k_f} \left\{ \frac{dh(t_k)}{dy}^T \Big|_{\tilde{y}} V_w^{-1} [z(t_k) - h(x,y,t_k) - \frac{dh}{dy}\Big|_{\tilde{y}} (\tilde{y}-y)] \right\} \tag{A-2}$$

$$-V_y^{-1}(\tilde{y}-y) - V_y^{-1}(y-m) = 0 \quad .$$

Solving for $(\tilde{y}-y)$ yields

$$(\tilde{y}-y) = R^{-1}\{\sum_{k=1}^{k_f} \frac{dh(t_k)}{dy}^T\Big|_{\tilde{y}} V_w^{-1}(w_k)] - V_y^{-1}(y-m_y)\} \qquad (A-3)$$

where

$$R = \{\sum_{k=1}^{k_f} \frac{dh(t_k)}{dy}^T\Big|_{\tilde{y}} V_w^{-1} \frac{dh(t_k)}{dy}\Big|_{\tilde{y}}] + V_y^{-1}\} \qquad (A-4)$$

and

$$w_k = z(t_k) - h(x,y,t_k) \quad .$$

Squaring and taking expected value yields the desired error covariance, Equation (7.1-5).

$$\text{cov}(\tilde{y}-y) = R^{-1}\{\sum_{k=1}^{k_f}\sum_{j=1}^{k_f} \frac{dh(t_k)}{dy}^T\Big|_{\tilde{y}} V_w^{-1}E(w_k w_j^T)V_w^{-1} \frac{dh(t_j)}{dy}\Big|_{\tilde{y}}$$

$$-2\sum_{k=1}^{k_f} \frac{dh(t_k)}{dy}^T\Big|_{\tilde{y}} V_w^{-1}E[w_k(y-m_y)^T]V_y^{-1} + V_y^{-1}\}R^{-1} \quad , \qquad (A-5)$$

Now the noise samples are assumed independent

$$E(w_k w_j^T) = \begin{cases} V_w & \text{if } k=j \\ 0 & \text{if } k\neq j \end{cases} \qquad (A-6)$$

and $E[w_k(y-m)^T]$ is assumed to be zero, i.e. the noise is assumed uncorrelated with the random parameter vector. As a result, the desired bound is simply

$$\text{cov}(\tilde{y}-y) = R^{-1}RR^{-1} = R^{-1} \quad . \tag{A-7}$$

APPENDIX B

NOMINAL PARAMETERS FOR SIMULATION

The nominal parameters used in the simulated experiment of Section II were approximations derived from nameplate data, steady-state measurements, and a list of typical machine parameters [3].

From the generator nameplate the rated values of armature voltage and current were

$$v_B = \frac{230}{\sqrt{3}} = 132.79 \text{ volts, line to neutral} \qquad \text{(B-1)}$$

and

$$i_B = \frac{1000}{132.79} = 7.531 \text{ amps} \quad . \qquad \text{(B-2)}$$

The angular speed of the equivalent two-pole machine is

$$\omega_o = 1200 \times \frac{2\pi}{60} \times 2 = 251.33 \text{ radians/second} \quad . \qquad \text{(B-3)}$$

The results of steady-state open-circuit and short-circuit tests are plotted in Figure B-1. The rated value of open-circuit armature voltage corresponds to

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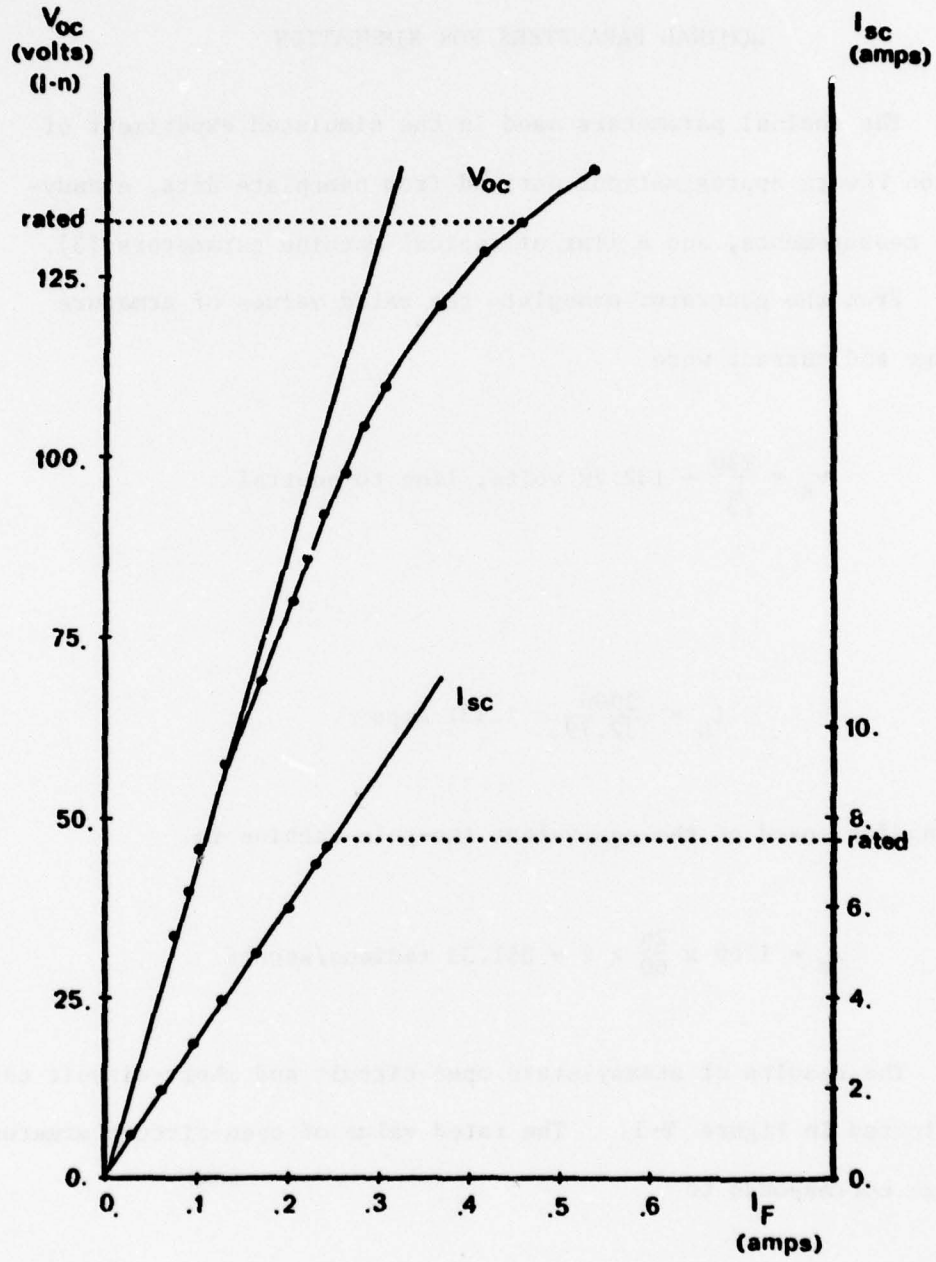

Figure B.1. Open-Circuit Voltage and Short-Circuit Current Versus Field Current

$$i_{FB} = .31 \text{ amps} , \tag{B-4}$$

neglecting saturation. Thus,

$$v_{FB} = \frac{v_B i_B}{i_{FB}} = 3226 \text{ volts} \tag{B-5}$$

and

$$L_{df} \simeq 4.87 \times 10^{-3} \text{ per unit} . \tag{B-6}$$

Rated short-circuit current corresponds to

$$i_{FS} = .253 \text{ amps} ; \tag{B-7}$$

therefore,

$$L_d \simeq \frac{i_{FS}}{i_{FB}} \frac{1}{w_o} = 3.25 \times 10^{-3} \text{ per unit} . \tag{B-8}$$

A typical value of transient reactance is

$$X'_d = .322 \, X_d \quad . \tag{B-9}$$

Therefore, from the definition of transient reactance,

$$X_d - X'_d = \frac{X_{df}^2}{X_f} , \tag{B-10}$$

and

$$L_f \simeq \frac{L_{df}^2}{.678L_d} = 10.78 \times 10^{-3} \text{ per unit} . \qquad \text{(B-11)}$$

The remaining d-axis parameters are roughly approximated by assuming equal coupling from stator to all rotor circuits.

$$L_{kd} = k^2 L_d = .678 \times 3.25 \times 10^{-3} = 2.20 \times 10^{-3} \text{ p.u.} \qquad \text{(B-12)}$$

$$L_{fkd} = \sqrt{k^2 L_f L_{kd}} = \sqrt{k^2 L_{df}^2} = kL_{df} = 4.01 \times 10^{-3} \text{ p.u.} \qquad \text{(B-13)}$$

Typical q-axis parameters give

$$L_q \simeq .652L_d = 2.12 \times 10^{-3} \text{ per unit} \qquad \text{(B-14)}$$

and

$$L_{kq} = L_q - L_q'' = .55 \times 2.12 \times 10^{-3} = 1.17 \times 10^{-3} \text{ per unit} \qquad \text{(B-15)}$$

With the rotor at standstill, direct current measurements give

$$R_D = .25\Omega \qquad \text{and} \qquad R_F = 240\Omega , \qquad \text{(B-16)}$$

or

$$R_d = 1.418 \times 10^{-2} \text{ p.u. and } R_f = 2.30 \times 10^{-2} \text{ p.u.} \qquad \text{(B-17)}$$

The damper resistances are not measurable; however, typical time constants

are

$$T_{do}' = \frac{L_f}{R_f} = .467 \text{ sec.} \tag{B-18}$$

and

$$T_{do}'' \simeq .02\ T_{do}' = .00935 \text{ sec.} \tag{B-19}$$

The result is

$$R_{kd} \simeq \frac{L_{kd} - \frac{L_{fkd}^2}{L_f}}{T_{do}''} = .0759 \text{ p.u.} \tag{B-20}$$

For lack of better information, we assume

$$R_{kq} \simeq R_{kd} \quad . \tag{B-21}$$

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APPENDIX C

PARAMETER IDENTIFICATION PROGRAM

```
COMMENT:   QUAD7.FR A QUASILINEARIZATION PROGRAM
COMMENT:   RANDOM PARAMETERS
COMMENT:   USES MULTIPLEXED DATA
COMMENT:   INITIAL STATE VECTOR COMPUTED FROM PARAMETERS
COMMENT:   SUBROUTINES USED XSS7, GRAD7, SOLV5, DCHY7, DF7, AF7, UDAT7, INV
COMMENT:   OVERLAY OVL1 HAS XSS7,INV AND SOLV5; OVERLAY OVL2 DCHY7,UDAT7,GRAD7,DF7
           COMPILER FREE.DOUBLE PRECISION
           COMMON EPK,RLK,WR
           DIMENSION Y(11),ZP(4),X(5),X1(5),W1(11,4),Q(4,4),W2(11,11)
           DIMENSION W(11,11),WN(11),C0(11),C(4,11),X2(5),WM(11,12)
           DIMENSION DX(5),DX1(5),DX2(5),W3(4),Z(4),W4(11),DM(4,11)
           DIMENSION DXY(5,11),D(4,5),DH(4,11),DEL(11),DY(5,11)
           DIMENSION PXY(5,11),PX0(5,5),R(11),YM(11)
C          EXTERNAL OVL1,OVL2
C  THE FOLLOWING IS EQUIVALENT TO THIS EXTERNAL STATEMENT
           INTEGER OVL1,OVL2
           OVL1=0
           OVL2=1
C
COMMENT:   OPEN I/O FILES
           OPEN 4,"DATA7"
           OPEN 5,"PRM"
           OPEN 6,"PRINT0"
           OPEN 7,"DATGRA"
C
COMMENT:   OPEN OVERLAY FILE QUAD7.OL
           CALL OVOPN (8,"QUAD7.OL",IOR)
           IF (IOR.NE.1) STOP "ERROR IN OVERLAY OPEN"
C
COMMENT:   READ INPUT DATA
           ACCEPT "NO.  OF ITERATIONS=",IT
           READ(4) N,NX,NZ,KF,T,SCA,SCB,WR,T0
           READ(4) EPK,RL1,K1,RL2,K2,K3
           DO 7 L=1,NZ
7          READ(4) (Q(L,M),M=1,NZ)
           READ(4) (R(L),L=1,N)
           READ(4) (Y(J),J=1,N)
           K=1
           DO 8 K=1,KF
           READ(5,1002) (Z(L),L=1,NZ)
           WRITE(7,2003) K,(Z(L),L=1,NZ)
8          WRITE(6,2003) K,(Z(L),L=1,NZ)
           WRITE(6,2004) (Y(L),L=1,N)
           DO 19 J=1,N
19         YM(J)=Y(J)
           IOV=0
           I=0
```

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```
COMMENT: BEGIN ITERATION LOOP
10       I=I+1
         DO 30 L=1,N
         DO 20 J=1,N
20       WM(J,L)=0.
30       WN(L)=0.
         RLK=RL1
COMMENT:  LOAD FIRST OVERLAY OVL1
         CALL OVLOD(8,OVL1,IOV,IOR)
         IF(IOR.NE.1) STOP "ERROR IN LOAD OVL1"
C
         CALL XSS7(X,NX,Y,N,C0,DY,TH)
         TH=TH+WR*T0
         WRITE(6) "X="
         WRITE(6,2004) (X(L),L=1,NX)
         DO 11 L=1,N
         DO 11 J=1,NX

11       PXY(J,L)=DY(J,L)
         DO 12 L=1,NX
         DO 12 J=1,NX
12       PX0(J,L)=0.
         DO 13 J=1,NX
13       PX0(J,J)=1.
         REWIND 5
         TK=0.
         K=1
COMMENT:  LOAD TIME LOOP OVERLAY OVL2
         CALL OVLOD(8,OVL2,IOV,IOR)
         IF (IOR.NE.1) STOP "ERROR IN LOAD OVL2"
C
COMMENT: BEGIN TIME LOOP
40       CONTINUE
COMMENT: SET UP EQUATIONS,  D=DH/DX, C=DH/DY, DXY=DX/DY
         IF(K.EQ.K1) RLK=RL2
         IF(K.EQ.K2) RLK=RL1
         IF(K.EQ.K3) RLK=RL2
         TK=T*(K-1)
         JK=1
         TJ=T/NZ
92       IF(JK.GT.NZ) GO TO 93
         CALL DCHY7(X,NX,Y,N,RLK,NZ,C0,C,D,Z,TK,TH)
         DO 80 L=1,NX
         DO 80 M=1,N
         DXY(L,M)=PXY(L,M)
         DO 80 J=1,NX
80       DXY(L,M)=DXY(L,M)+PX0(L,J)*DY(J,M)
         DO 90 L=1,NZ
         DO 90 M=1,N
         DM(L,M)=C(L,M)
         DO 90 J=1,NX
90       DM(L,M)=DM(L,M)+D(L,J)*DXY(J,M)
         CALL UDAT7(X,NX,Y,N,PXY,PX0,TJ)
         ZP(JK)=Z(JK)
         DO 91 M=1,N
```

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```
91          DH(JK,M)=DM(JK,M)
            TK=TK+TJ
            JK=JK+1
            GO TO 92
93          CONTINUE
            WRITE(7,2003) K,(ZP(J),J=1,NZ)
            DO 100 L=1,N
            DO 100 M=1,NZ
            W1(L,M)=0.
            DO 100 J=1,NZ
100         W1(L,M)=W1(L,M)+DH(J,L)*Q(J,M)
            DO 120 L=1,N
            DO 120 M=1,N
            W2(L,M)=0.
            DO 120 J=1,NZ
120         W2(L,M)=W2(L,M)+W1(L,J)*DH(J,M)
            DO 150 L=1,N
            DO 150 M=1,N
150         WM(L,M)=WM(L,M)+W2(L,M)
            READ(5,1002) (Z(L),L=1,NZ)
            DO 170 L=1,NZ
170         W3(L)=Z(L)-ZP(L)
            DO 190 L=1,N
            W4(L)=0.
            DO 190 J=1,NZ
190         W4(L)=W4(L)+W1(L,J)*W3(J)
            DO 200 L=1,N

200         WN(L)=WN(L)+W4(L)
            WRITE(10) "K=",K,"<13>"
            K=K+1
            IF(K.LE.KF) GO TO 40
COMMENT:    END OF TIME LOOP
COMMENT:     LOAD OVL1 AGAIN
            CALL OVLOD(8,OVL1,IOV,IOR)
            IF (IOR.NE.1) STOP "ERROR IN LOAD OVL1"
C
            DO 699 L=1,N
699         WM(L,L)=WM(L,L)+R(L)
            DO 799 L=1,N
799         WN(L)=WN(L)-R(L)*(Y(L)-YM(L))
            DO 600 J=1,N
            DO 610 L=1,N
610         WM(J,L)=WM(J,L)*SCA
            WN(J)=WN(J)*SCA
600         WM(J,J)=WM(J,J)*SCB
COMMENT:    SOLVE SIMULTANEOUS EQUATIONS
            DO 601 J=1,N
            DO 601 L=1,N
601         W(J,L)=WM(J,L)
            DO 602 J=1,N
602         WM(J,N+1)=WN(J)
            M=N+1
            CALL SOLV5(WM,DEL,N,M)
            DO 719 L=1,N
            W4(L)=0.
            DO 719 J=1,N
```

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```
719         W4(L)=W4(L)+W(L,J)*DEL(J)
            WRITE(6) "WM(L,L)="
            WRITE(6,2004) (W(L,L),L=1,N)
            WRITE(6) "DEL="
            WRITE(6,2004) (DEL(J),J=1,N)
            WRITE(6) "WM*DEL="
            WRITE(6,2004) (W4(J),J=1,N)
            WRITE(6) "WN =    "
            WRITE(6,2004) (WN(J),J=1,N)
            DO 720 L=1,N
720         Y(L)=Y(L)+DEL(L)
            WRITE (6) "PARAMETERS="
            WRITE(6,2004) (Y(L),L=1,N)
            WRITE(10) "PARAMETERS="
            WRITE(10,2004) (Y(L),L=1,N)
            CALL OVERF(IOF)
            IF (IOF.EQ.1) PAUSE "FLOATING POINT OVERFLOW"
            IF (IOF.EQ.3) PAUSE "FLOATING POINT UNDERFLOW"
251         IF (I.LT.IT) GO TO 10
COMMENT:    END OF ITERATION LOOP
            ACCEPT "NO. OF ADDITIONAL ITERATIONS=",IA
            IF(IA.LT.1) GO TO 260
            IT=IT+IA
            GO TO 251
COMMENT:    SOLVE FOR FINAL VALUE OF X AND ZP
260         RLK=RL1
            CALL XSS7(X,NX,Y,N,CO,DY,TH)
            TH=TH+WR*TO
            CALL INV(W,W2,N)
            DO 262 L=1,N
262         W2(L,L)=W2(L,L)*SCA
            WRITE(6) "ERROR VARIANCE ESTIMATES"
            WRITE(6,2004) (W2(L,L),L=1,N)
COMMENT:     LOAD OVL2 AGAIN
            CALL OVLOD(8,OVL2,ICV,IOR)
```

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```
      IF(IOP.NE.1) STOP "ERROR IN LOAD OVL2"
C
      TJ=T/NZ
      WRITE(6,2002)
      DO 250 K=1,KF
      IF(K.EQ.K1) RLK=RL2
      IF(K.EQ.K2) RLK=RL1
      IF(K.EQ.K3) RLK=RL2
      TK=T*(K-1)
      DO 252 JK=1,NZ
      CALL DCHY7(X,NX,Y,N,NZ,C0,C,D,Z,TK,TH)
      TK=TK+TJ
      CALL GRAD7(X,NX,Y,N,DX)
      DO 450 J=1,NX
450   X1(J)=DX(J)*TJ/3.+X(J)
      CALL GRAD7(X1,NX,Y,N,DX1)
      DO 460 J=1,NX
460   X1(J)=(DX1(J)+DX(J))*TJ/6.+X(J)
      CALL GRAD7(X1,NX,Y,N,DX1)
      DO 470 J=1,NX
470   X1(J)=(DX1(J)*3.+DX(J))*TJ/8.+X(J)
      CALL GRAD7(X1,NX,Y,N,DX2)
      DO 480 J=1,NX
480   X1(J)=(DX2(J)*4.-DX1(J)*3.+DX(J))*TJ/2.+X(J)
      CALL GRAD7(X1,NX,Y,N,DX1)
      DO 490 J=1,NX
490   X(J)=(DX2(J)*4.+DX1(J)+DX(J))*TJ/6.+X(J)
252   ZP(JK)=Z(JK)
      WRITE(7,2003) K,(ZP(L),L=1,NZ)
250   WRITE(6,2003) K,(ZP(L),L=1,NZ)
1002  FORMAT (5G16.8)
2002  FORMAT(3X,"K",6X,"Z(K)")
2003  FORMAT(1X,I4,1P5E13.5)
2004  FORMAT(1X,1P5E14.5)
      END
```

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```
      COMPILER DOUBLE PRECISION
      OVERLAY OVL1
      SUBROUTINE XSS7(X,NX,Y,N,C0,DY,TH)
COMMENT: COMPUTE STEADY STATE USING CURRENT PARAMETER ESTIMATE Y
COMMENT:  DY=PART. DER. OF X0 WRT Y
      COMMON EFK,RLK,W
      DIMENSION X(NX),Y(N),DY(NX,N),C0(N)
      I1=1
      I2=2
      I3=3
      I4=4
      I5=5
      I6=6
      I7=7
      I8=8
      I9=9
      R=Y(I3)+RLK
      S=SQRT(2./3.)
      P=3.14159265358979
      P=2.*P/3.
      DL=Y(I1)+Y(I6)
      QL=Y(I1)-Y(I6)
      DX=Y(I9)*(R**2+W**2*DL*QL)
      X(I1)=EFK*Y(I2)*R**2/DX
      X(I2)=EFK*Y(I4)/Y(I9)-Y(I2)**2*QL*EFK*W**2/DX
      X(I3)=EFK*Y(I5)/Y(I9)-Y(I2)*Y(I3)*QL*EFK*W**2/DX
      X(I4)=-EFK*Y(I2)*QL*R*W/DX
      X(I5)=-EFK*Y(I2)*Y(I7)*R*W/DX
      DO 101 I=1,N
      DO 101 J=1,NX
```

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```
101       DY(J,I)=0.
          DY(I1,I1)=-X(I1)*Y(I9)*W**2*(QL+DL)/DX
          DY(I1,I2)=X(I1)/Y(I2)
          DY(I1,I6)=-X(I1)*Y(I9)*W**2*(QL-DL)/DX
          DY(I1,I8)=2*X(I1)*(1/R-R*Y(I9)/DX)
          DY(I1,I9)=-X(I1)/Y(I9)
          DY(I2,I1)=W**4*Y(I2)**2*Y(I9)*EFK*(QL+DL)/DX**2
     *    -Y(I2)**2*EFK*W**2/DX
          DY(I2,I2)=-2*Y(I2)*QL*EFK*W**2/DX
          DY(I2,I4)=EFK/Y(I9)
          DY(I2,I6)=W**4*Y(I2)**2*Y(I9)*EFK*(QL-DL)/DX**2
     *    +Y(I2)**2*EFK*W**2/DX
          DY(I2,I8)=2*Y(I2)**2*QL*R*Y(I9)*EFK*W**2/DX**2
          DY(I2,I9)=-X(I2)/Y(I9)
          DY(I3,I1)=W**4*Y(I2)*Y(I3)*Y(I3)*EFK*QL*(QL-DL)/DX**2
     *    -Y(I2)*Y(I3)*EFK*W**2/DX
          DY(I3,I2)=-Y(I3)*QL*EFK*W**2/DX
          DY(I3,I3)=-Y(I2)*QL*EFK*W**2/DX
          DY(I3,I5)=EFK/Y(I9)
          DY(I3,I6)=+Y(I2)*Y(I3)*EFK*W**2/DX
     *    +W**4*Y(I2)*Y(I3)*Y(I9)*EFK*QL*(QL-DL)/DX**2
          DY(I3,I8)=2*Y(I2)*Y(I3)*QL*R*Y(I9)*EFK*W**2/DX**2
          DY(I3,I9)=-X(I3)/Y(I9)
          DY(I4,I1)=-X(I4)*W**2*(QL+DL)*Y(I9)/DX+X(I4)/QL
          DY(I4,I2)=X(I4)/Y(I2)
          DY(I4,I6)=-X(I4)*W**2*Y(I9)*(QL-DL)/DX-X(I4)/QL
          DY(I4,I8)=-X(I4)*(-1/R+2*R*Y(I9)/DX)
          DY(I4,I9)=-X(I4)/Y(I9)
          DY(I5,I1)=-X(I5)*W**2*(QL+DL)*Y(I9)/DX
          DY(I5,I2)=X(I5)/Y(I2)
          DY(I5,I6)=-X(I5)*W**2*(QL-DL)*Y(I9)/DX
          DY(I5,I7)=X(I5)/Y(I7)
          DY(I5,I8)=-X(I5)*(-1/R+2*R*Y(I9)/DX)
          DY(I5,I9)=-X(I5)/Y(I9)

COMMENT TH = INITIAL ROTOR ANGLE      C0=DERIVATIVE OF TH WRT Y
          TH=ATAN(W*QL/R)
          DO 160 M=1,N
160       C0(M)=0.
          C0(I1)=W*R/(R**2+W**2*QL**2)
          C0(I6)=-C0(I1)
          C0(I9)=-W*QL/(R**2+W**2*QL**2)
          RETURN
          END
```

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```
          COMPILER DOUBLE PRECISION
          SUBROUTINE GRAD7(X,NX,Y,N,DX)
COMMENT:  THIS SUBROUTINE COMPUTES DX/DT = F(X(T),T)
C         X(1)=LAMBDA D,     X(2)=LAMBDA F,     X(3)=LAMBDA KD
C         X(4)=LAMBDA Q,     X(5)=LAMBDA KQ
          COMMON EFK,RLK,W
          DIMENSION X(NX),Y(N),DX(NX),A(5,5),F(5,5)
          I2=2
          CALL AF7(Y,N,NX,A,F)
          DO 100 I=1,NX
          DX(I)=0.
          DO 100 J=1,NX
100       DX(I)=DX(I)+F(I,J)*X(J)
          DX(I2)=DX(I2)+EFK
          RETURN
          END

          COMPILER DOUBLE PRECISION
          SUBROUTINE SOLV5(A,C,N,M)
COMMENT:  THIS SUBROUTINE COMPUTES SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
COMMENT:  THE EQUATIONS ARE Y = B C
COMMENT:  THE FIRST N COLUMNS OF A CONTAIN THE MATRIX B
COMMENT:  THE N+1 COLUMN OF A CONTAINS THE VECTOR Y
COMMENT:  THE VECTOR C CONTAINS THE SOLUTION
          DIMENSION A(N,M),C(N)
          NM1=N-1
          NP1=N+1
COMMENT:  TRIANGULARIZE A BY SUBTRACTING MULTIPLES OF UPPER ROWS
          DO 10 K=1,NM1
          AKK=A(K,K)
          KP1=K+1
          DO 10 KI=KP1,N
          AKIK=A(KI,K)
          DO 10 L=KP1,NP1
10        A(KI,L)=A(KI,L)-(AKIK/AKK)*A(K,L)
          DO 11 I1=1,1
11        DETA=A(I1,I1)
COMMENT:  STARTING AT BOTTOM, SOLVE FOR C
          C(N)=A(N,N+1)/A(N,N)
          DO 20 I=1,NM1
          C(N-I)=A(N-I,N+1)
          CNI=C(N-I)
          DO 30 J=1,I
30        CNI=CNI-C(J+N-I)*A(N-I,J+N-I)
          C(N-I)=CNI/A(N-I,N-I)
20        DETA=DETA*A(I+1,I+1)
          TYPE "DET=",DETA
          WRITE(6,101) DETA
101       FORMAT(1X,"DETERMINANT=" ,1PE17.5)
          RETURN
          END
```

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```
      COMPILER FREE,DOUBLE PRECISION
      OVERLAY OVL2
      SUBROUTINE DCHY7(X,NX,Y,N,NZ,C0,C,D,ZP,T,TH)
      COMMON EFK,RLK,W
      DIMENSION X(NX),Y(N),C0(N),C(NZ,N),ZP(NZ),D(NZ,NX),A(5,5),F(5,5),DTH(4)
      REAL ID,IQ
      I1=1
      I2=2
      I3=3
      I4=4
      I5=5
      I6=6
      I7=7
      I8=8
      P=3.14159265358979
      P=2.*P/3.
      S=SQRT(2./3.)
      DL=Y(I1)+Y(I6)
      QL=Y(I1)-Y(I6)
      CALL AF7(Y,N,NX,A,F)
      ID=0.
      DO 40 J=1,3
40    ID=ID+A(I1,J)*X(J)
      IQ=0.
      DO 50 J=4,5
50    IQ=IQ+A(I4,J)*X(J)
      DO 90 I=1,N
      DO 90 J=1,NZ
90    C(J,I)=0.
      DO 95 I=1,NX
      DO 95 J=1,NZ
95    D(J,I)=0.
      DO 100 I=1,3
      D(I1,I)=S*A(I1,I)*COS(W*T+TH)
      D(I2,I)=S*A(I1,I)*COS(W*T+TH-P)
      D(I3,I)=S*A(I1,I)*COS(W*T+TH+P)
100   D(I4,I)=A(I2,I)
      DO 105 I=4,5
      D(I1,I)=-S*A(I4,I)*SIN(W*T+TH)
      D(I2,I)=-S*A(I4,I)*SIN(W*T+TH-P)
```

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```
105       C(3,I)=-S*A(I4,I)*SIN(W*T+TH+P)
          DO 110 I=1,NZ
          ZP(I)=0.
          DO 110 J=1,NX
110       ZP(I)=ZP(I)+D(I,J)*X(J)
          DD=DL*(Y(I4)*Y(I3)-Y(I5)**2)-Y(I2)*(Y(I3)*Y(I2)-Y(I5)*Y(I3))
          -Y(I3)*(Y(I4)*Y(I3)-Y(I2)*Y(I5))
          DQ=QL*Y(I7)-Y(I7)**2
          D1=A(I1,I1)*X(I1)+A(I1,I2)*X(I2)+A(I1,I3)*X(I3)
          D4=A(I4,I4)*X(I4)+A(I4,I5)*X(I5)
          C(I1,I1)=-A(I1,I1)*D1
          C(I1,I2)=(-Y(I3)*X(I2)+Y(I5)*X(I3))/DD-2*A(I1,I2)*D1
          C(I1,I3)=(Y(I4)*X(I1)-Y(I2)*X(I2)+Y(I5)*X(I2)-Y(I4)*X(I3))/DD
          -(A(I3,I3)+2*A(I1,I3))*D1
          C(I1,I4)=(Y(I3)*X(I1)-Y(I3)*X(I3))/DD-A(I2,I2)*D1
          C(I1,I5)=(-2*Y(I5)*X(I1)+Y(I3)*X(I2)+Y(I2)*X(I3))/DD-2*A(I2,I3)*D1
          C(I1,I6)=C(I1,I1)
          C(I4,I1)=-A(I4,I4)*D4
          C(I4,I6)=-C(I4,I1)
          C(I4,I7)=(X(I4)-X(I5))/DQ-(A(I5,I5)+2*A(I4,I5))*D4
          DO 120 I=1,N
          C(3,I)=S*(C(I1,I)*COS(W*T+TH+P)-C(I4,I)*SIN(W*T+TH+P))
          C(2,I)=S*(C(I1,I)*COS(W*T+TH-P)-C(I4,I)*SIN(W*T+TH-P))
120       C(1,I)=S*(C(I1,I)*COS(W*T+TH)-C(I4,I)*SIN(W*T+TH))

          DO 130 I=1,N
130       C(I4,I)=0.
          D2=A(I2,I1)*X(I1)+A(I2,I2)*X(I2)+A(I2,I3)*X(I3)
          C(I4,I1)=(Y(I3)*X(I2)-Y(I5)*X(I3))/DD-A(I1,I1)*D2
          C(I4,I2)=(-Y(I3)*X(I1)+Y(I3)*X(I3))/DD-2*A(I1,I2)*D2
          C(I4,I3)=((Y(I5)-Y(I2))*X(I1)+(DL-2*Y(I3))*X(I2)+Y(I2)*X(I3))/DD
     *    -(A(I3,I3)+2*A(I1,I3))*D2
          C(I4,I4)=-A(I2,I2)*D2
          C(I4,I5)=(Y(I3)*X(I1)-DL*X(I3))/DD-2*A(I2,I3)*D2
          DO 30 L=1,NZ
30        DTH(L)=0.
          DTH(I1)=-S*(ID*SIN(W*T+TH)+IQ*COS(W*T+TH))
          DTH(I2)=-S*(ID*SIN(W*T+TH-P)+IQ*COS(W*T+TH-P))
          DTH(I3)=-S*(ID*SIN(W*T+TH+P)+IQ*COS(W*T+TH+P))
          DO 35 L=1,3
          C(L,I1)=C(L,I1)+DTH(L)*C0(I1)
          C(L,I6)=C(L,I6)+DTH(L)*C0(I6)
35        C(L,I8)=C(L,I8)+DTH(L)*C0(I8)
          RETURN
          END
```

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```
      COMPILER DOUBLE PRECISION
      SUBROUTINE DF7(DFX,DFY,X,Y,NX,N)
      COMMON EPK,RLK,W
      DIMENSION X(NX),Y(N),DFX(NX,NX),DFY(NX,N),A(5,5)
      I1=1
      I2=2
      I3=3
      I4=4
      I5=5
      I6=6
      I7=7
      I8=8
      I9=9
      I10=10
      I11=11
      CALL AF7(Y,N,NX,A,DFX)
      DO 200 K=1,N
      DO 200 J=1,NX
  200 DFY(J,K)=0.
      R=Y(I8)+RLK
      DL=Y(I1)+Y(I6)
      QL=Y(I1)-Y(I6)
      D=DL*(Y(I4)*Y(I3)-Y(I5)**2)-Y(I2)*(Y(I3)*Y(I2)-Y(I5)*Y(I3))
     *  -Y(I3)*(Y(I4)*Y(I3)-Y(I2)*Y(I5))
      D1=DFX(I1,I1)*X(I1)+DFX(I1,I2)*X(I2)+DFX(I1,I3)*X(I3)
      DFY(I1,I1)=-A(I1,I1)*D1
      DFY(I1,I2)=R*(Y(I3)*X(I2)-Y(I5)*X(I3))/D-2*A(I1,I2)*D1
      DFY(I1,I3)=R*(-Y(I5)*X(I2)+Y(I4)*X(I3)-Y(I4)*X(I1)+Y(I2)*X(I2))/D
     *  -(A(I3,I3)+2*A(I1,I3))*D1
      DFY(I1,I4)=R*(-Y(I3)*X(I1)+Y(I3)*X(I3))/D-A(I2,I2)*D1
      DFY(I1,I5)=R*(2*Y(I5)*X(I1)-Y(I3)*X(I2)-Y(I2)*X(I3))/D-2*A(I2,I3)*D1
      DFY(I1,I8)=D1/R
      RF=Y(I9)
      D2=DFX(I2,I1)*X(I1)+DFX(I2,I2)*X(I2)+DFX(I2,I3)*X(I3)
      DFY(I2,I1)=RF*(-Y(I3)*X(I2)+Y(I5)*X(I3))/D-A(I1,I1)*D2
      DFY(I2,I2)=RF*(Y(I3)*X(I1)-Y(I3)*X(I3))/D-2*A(I1,I2)*D2
      DFY(I2,I3)=RF*((-Y(I5)+Y(I2))*X(I1)+(2*Y(I3)-DL)*X(I2)-Y(I2)*X(I3))/D
     *  -(A(I3,I3)+2*A(I1,I3))*D2
      DFY(I2,I4)=-A(I2,I2)*D2
      DFY(I2,I5)=RF*(-Y(I3)*X(I1)+DL*X(I3))/D-2*A(I2,I3)*D2
      DFY(I2,I9)=D2/RF
      RKD=Y(I10)
      D3=DFX(I3,I1)*X(I1)+DFX(I3,I2)*X(I2)+DFX(I3,I3)*X(I3)
      DFY(I3,I1)=RKD*(Y(I5)*X(I2)-Y(I4)*X(I3))/D-A(I1,I1)*D3
      DFY(I3,I2)=RKD*(-Y(I5)*X(I1)-Y(I3)*X(I2)+2*Y(I2)*X(I3))/D-2*A(I1,I2)*D3
      DFY(I3,I3)=RKD*(Y(I4)*X(I1)-Y(I2)*X(I2))/D-(A(I3,I3)+2*A(I1,I3))*D3
      DFY(I3,I4)=RKD*(Y(I3)*X(I1)-DL*X(I3))/D-A(I2,I2)*D3
      DFY(I3,I5)=RKD*(-Y(I2)*X(I1)+DL*X(I2))/D-2*A(I2,I3)*D3
      DFY(I3,I10)=D3/RKD
      DQ=QL*Y(I7)-Y(I7)**2
      D4=DFX(I4,I4)*X(I4)+DFX(I4,I5)*X(I5)
      DFY(I4,I6)=A(I4,I4)*D4
      DFY(I4,I7)=R*(X(I5)-X(I4))/DQ-(A(I5,I5)+2*A(I4,I5))*D4
      DFY(I4,I8)=D4/R
      RKQ=Y(I11)
      D5=DFX(I5,I4)*X(I4)+DFX(I5,I5)*X(I5)
      DFY(I5,I6)=RKQ*X(I5)/DQ+A(I4,I4)*D5
      DFY(I5,I7)=RKQ*X(I4)/DQ-(A(I5,I5)+2*A(I4,I5))*D5
      DFY(I5,I11)=D5/RKQ
      RETURN
      END
```

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```
      COMPILER DOUBLE PRECISION
      SUBROUTINE AF7(Y,N,NX,A,F)
COMMENT: THIS SUBROUTINE COMPUTES A=INV(L) AND F=-R*A
      COMMON EFK,RLK,W
      DIMENSION Y(N),A(NX,NX),F(NX,NX),R(5)
      I1=1
      I2=2
      I3=3
      I4=4
      I5=5
      I6=6
      I7=7
      I8=8
      I9=9
      I10=10
      I11=11
      DL=Y(I1)+Y(I6)
      QL=Y(I1)-Y(I6)
      DO 30 I=1,NX
      DO 30 J=1,NX
30    A(J,I)=0.
      DO 40 I=1,NX
      DO 40 J=1,NX
40    F(J,I)=0.
      D1=Y(I4)*Y(I3)-Y(I5)**2
      D2=Y(I3)*Y(I2)-Y(I5)*Y(I3)
      D3=Y(I4)*Y(I3)-Y(I2)*Y(I5)
      D=DL*D1-Y(I2)*D2-Y(I3)*D3
      DQ=QL*Y(I7)-Y(I7)**2
      A(I1,I1)=D1/D
      A(I1,I2)=-D2/D
      A(I1,I3)=-D3/D
      A(I2,I1)=A(I1,I2)
      A(I2,I2)=(DL*Y(I3)-Y(I3)**2)/D
      A(I2,I3)=-(DL*Y(I5)-Y(I2)*Y(I3))/D
      A(I3,I1)=A(I1,I3)
      A(I3,I2)=A(I2,I3)
      A(I3,I3)=(DL*Y(I4)-Y(I2)**2)/D
      A(I4,I4)=Y(I7)/DQ
      A(I4,I5)=-Y(I7)/DQ
      A(I5,I4)=A(I4,I5)
      A(I5,I5)=QL/DQ
      R(I1)=Y(I6)+RLK
      R(I2)=Y(I9)
      R(I3)=Y(I10)
      R(I4)=Y(I8)+RLK
      R(I5)=Y(I11)
      DO 100 I=1,NX
      DO 100 J=1,NX
100   F(I,J)=-R(I)*A(I,J)
      F(I1,I4)=W
      F(I4,I1)=-W
      RETURN
      END
```

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```
      COMPILER FREE,DOUBLE PRECISION
      SUBROUTINE UDAT7(X,NX,Y,N,PXY,PX0,T)
COMMENT: SOLVE FOR PXY(K+1) ,PX0(K+1) AND X(K+1)
      DIMENSION X(NX),Y(N),PXY(NX,N),PX0(NX,NX),DFX(5,5),DFY(5,11)
      DIMENSION X1(5),X2(5),PXY1(5,11)
      DIMENSION DD(5,11),DD1(5,11),DD2(5,11),DX(5),DX1(5),DX2(5)
      CALL DF7(DFX,DFY,X,Y,NX,N)
COMMENT: PXY AND PX0 ARE SENSITIVITY MATRICES
COMMENT:  DD=DUMMY MATRIX=DERIVATIVE WRT TIME OF A SENSITIVITY MATRIX
COMMENT:  USES RUNGE-KUTTA METHOD TO SOLVE DIFF. EQNS.
COMMENT:  FIRST COMPUTE SENSITIVITY TO Y
      DO 700 L=1,NX
      DO 700 M=1,N
      DD(L,M)=DFY(L,M)
      DO 700 J=1,NX
700   DD(L,M)=DD(L,M)+DFX(L,J)*PXY(J,M)
      DO 701 L=1,NX
      DO 701 M=1,N
701   PXY1(L,M)=DD(L,M)*T/3.+PXY(L,M)
      DO 702 L=1,NX
      DO 702 M=1,N
      DD1(L,M)=DFY(L,M)
      DO 702 J=1,NX
702   DD1(L,M)=DD1(L,M)+DFX(L,J)*PXY1(J,M)
      DO 703 L=1,NX
      DO 703 M=1,N
703   PXY1(L,M)=(DD1(L,M)+DD(L,M))*T/6.+PXY(L,M)
      DO 704 L=1,NX
      DO 704 M=1,N
      DD1(L,M)=DFY(L,M)
      DO 704 J=1,NX
704   DD1(L,M)=DD1(L,M)+DFX(L,J)*PXY1(J,M)
      DO 705 L=1,NX
      DO 705 M=1,N
```

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```
705       PX[illegible],M)=(DD1(L,M)*3.+DD(L,M))*T/8.+PXY(L,M)
          DO 706 L=1,NX
          DO 706 M=1,N
          DD2(L,M)=DFY(L,M)
          DO 706 J=1,NX
706       DD2(L,M)=DD2(L,M)+DFX(L,J)*PXY1(J,M)
          DO 707 L=1,NX
          DO 707 M=1,N
707       PXY1(L,M)=(DD2(L,M)*4.-DD1(L,M)*3.+DD(L,M))*T/2.+PXY(L,M)
          DO 709 L=1,NX
          DO 709 M=1,N
          DD1(L,M)=DFY(L,M)
          DO 708   J=1,NX
708       DD1(L,M)=DD1(L,M)+DFX(L,J)*PXY1(J,M)
          DO 709 L=1,NX
          DO 709 M=1,N
709       PXY1(L,M)=(DD2(L,M)*4.+DD1(L,M)+DD(L,M))*T/6.+PXY(L,M)
          DO 710 L=1,NX
          DO 710 M=1,N
710       PXY(L,M)=PXY1(L,M)
COMMENT:   COMPUTE SENSITIVITY TO X0
          DO 500 L=1,NX
          DO 500 M=1,NX
          DD(L,M)=0.
          DO 500 J=1,NX
500       DD(L,M)=DD(L,M)+DFX(L,J)*PX0(J,M)
          DO 501 L=1,NX
          DO 501 M=1,NX
501       PXY1(L,M)=DD(L,M)*T/3.+PX0(L,M)
          DO 502 L=1,NX
```

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```
      DO 502 M=1,NX
      DD1(L,M)=0.
      DO 502 J=1,NX
502   DD1(L,M)=DD1(L,M)+DFX(L,J)*PXY1(J,M)
      DO 503 L=1,NX
      DO 503 M=1,NX
503   PXY1(L,M)=(DD1(L,M)+DD(L,M))*T/6.+PX0(L,M)
      DO 504 L=1,NX
      DO 504 M=1,NX
      DD1(L,M)=0.
      DO 504 J=1,NX
504   DD1(L,M)=DD1(L,M)+DFX(L,J)*PXY1(J,M)
      DO 505 L=1,NX
      DO 505 M=1,NX
505   PXY1(L,M)=(DD1(L,M)*3.+DD(L,M))*T/6.+PX0(L,M)
      DO 506 L=1,NX
      DO 506 M=1,NX
      DD2(L,M)=0.
      DO 506 J=1,NX
506   DD2(L,M)=DD2(L,M)+DFX(L,J)*PXY1(J,M)
      DO 507 L=1,NX
      DO 507 M=1,NX
507   PXY1(L,M)=(DD2(L,M)*4.-DD1(L,M)*3.+DD(L,M))*T/2.+PX0(L,M)
      DO 508 L=1,NX
      DO 508 M=1,NX
      DD1(L,M)=0.
      DO 508 J=1,NX
508   DD1(L,M)=DD1(L,M)+DFX(L,J)*PXY1(J,M)
      DO 509 L=1,NX
      DO 509 M=1,NX
509   PXY1(L,M)=(DD2(L,M)*4+DD1(L,M)+DD(L,M))*T/6.+PX0(L,M)
      DO 510 L=1,NX
      DO 510 M=1,NX
510   PX0(L,M)=PXY1(L,M)
COMMENT:  UPDATE STATE VECTOR
      CALL GRAD7(X,NX,Y,N,DX)
      DO 400 J=1,NX
400   X1(J)=DX(J)*T/3.+X(J)
      CALL GRAD7(X1,NX,Y,N,DX1)
      DO 410 J=1,NX
410   X1(J)=(DX1(J)+DX(J))*T/6.+X(J)
      CALL GRAD7(X1,NX,Y,N,DX1)
      DO 420 J=1,NX
420   X1(J)=(DX1(J)*3.+DX(J))*T/8.+X(J)
      CALL GRAD7(X1,NX,Y,N,DX2)
      DO 430 J=1,NX
430   X1(J)=(DX2(J)*4.-DX1(J)*3.+DX(J))*T/2.+X(J)
      CALL GRAD7(X1,NX,Y,N,DX1)
      DO 440 J=1,NX
440   X(J)=(DX2(J)*4.+DX1(J)+DX(J))*T/6.+X(J)
      RETURN
      END
```

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```
      COMPILER DOUBLE PRECISION
      SUBROUTINE INV(A,B,N)
      DIMENSION A(N,N),B(N,N)
      DO 11 I=1,N
      DO 10 J=1,N
10    B(I,J)=0.
11    B(I,I)=1.
      DETA=1.
      DO 20 K=1,N
      AKK=A(K,K)
      DETA=DETA*AKK
      DO 30 J=1,N
30    B(K,J)=B(K,J)/AKK
      DO 40 J=K,N
40    A(K,J)=A(K,J)/AKK
      DO 50 KI=1,N
      IF (K.EQ. KI) GO TO 50
      DO 60 L=1,N
60    B(KI,L)=B(KI,L)-A(KI,K)*B(K,L)
      IF (K.EQ.N) GO TO 50
      KP1=K+1
      DO 70 L=KP1,N
70    A(KI,L)=A(KI,L)-A(KI,K)*A(K,L)
50    CONTINUE
20    CONTINUE
X     WRITE(6,101) DETA
X101  FORMAT (1X,"DETERMINANT=",1PE15.7)
      END
```

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APPENDIX D

GENERATOR SIMULATION PROGRAM

```
COMMENT:   SIM6.FR A MACHINE SIMULATION PROGRAM
COMMENT:   USES MULTIPLEXED DATA
COMMENT:   INITIAL STATE VECTOR COMPUTED FROM PARAMETERS
COMMENT:   SUBROUTINES USED XSS6, GRAD6, DCHY6, AF6
       COMPILER FREE,DOUBLE PRECISION
       COMMON EFK,PLK,W
       DIMENSION Q(4,4),Y(11),ZP(4),Z(4),X(5),X1(5),X2(5),DX(5),DX1(5),DX2(5)
       DIMENSION DY(5,11),C0(11),C(4,11),D(4,5)
COMMENT:  OPEN I/O FILES
       OPEN 5,"DATA56"
       OPEN 6,"PRT"
COMMENT:  READ INPUT DATA
       READ(5) N,NX,NZ,KF,T,W
       READ(5) EFK,EL1,K1,EL2,K2
       READ(5) (Y(J),J=1,N)
       K=1
       ELK=EL1
       CALL XSS6(X,NX,Y,N,C0,DY,TH)
       TJ=T/NZ
       DO 250 K=1,KF
       IF(K.EQ.K1) ELK=EL2
       IF(K.EQ.K2) ELK=EL1
       TK=T*(K-1)
       DO 252 JK=1,NZ
       CALL DCHY6(X,NX,Y,N,NZ,C0,C,D,Z,TK,TH)
       TK=TK+TJ
       CALL GRAD6(X,NX,Y,N,DX)
       DO 450 J=1,NX
450    X1(J)=DX(J)*TJ/3.+X(J)
       CALL GRAD6(X1,NX,Y,N,DX1)
       DO 460 J=1,NX
460    X1(J)=(DX1(J)+DX(J))*TJ/6.+X(J)
       CALL GRAD6(X1,NX,Y,N,DX1)
       DO 470 J=1,NX
470    X1(J)=(DX1(J)*3.+DX(J))*TJ/8.+X(J)
       CALL GRAD6(X1,NX,Y,N,DX2)
       DO 480 J=1,NX
480    X1(J)=(DX2(J)*4.-DX1(J)*3.+DX(J))*TJ/2.+X(J)
       CALL GRAD6(X1,NX,Y,N,DX1)
       DO 490 J=1,NX
       X(J)=(DX2(J)*4.+DX1(J)+DX(J))*TJ/6.+X(J)
490    X2(J)=.2*ABS(X(J)-X1(J))
       WRITE(10,2003) (X2(J),J=1,NX)
252    ZP(JK)=Z(JK)
250    WRITE(6,2003) (ZP(L),L=1,NZ)
2003   FORMAT(5G16.6)
       END
```

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```
      COMPILER DOUBLE PRECISION
      OVERLAY OVL1
      SUBROUTINE XSS6(X,NX,Y,N,C0,DY,TH)
COMMENT: COMPUTE STEADY STATE USING CURRENT PARAMETER ESTIMATE Y
COMMENT:  DY=PART. DER. OF X0 WRT Y
      COMMON EFK,ELK,W
      DIMENSION X(NX),Y(N),DY(NX,N),C0(N)
      I1=1
      I2=2
      I3=3
      I4=4
      I5=5
      I6=6
      I7=7
      I8=8
      I9=9
      E=Y(I8)*ELK
      S=SQRT(2./3.)
      P=3.14159265358979
      P=2.**3/3.
      DX=Y(I9)*(-E**2+W**2*Y(I1)*Y(I6))
      X(I1)=EFK*Y(I2)*E**2/DX
      X(I2)=EFK*Y(I4)/Y(I9)-Y(I2)**2*Y(I5)*EFK*W**2/DX
      X(I3)=EFK*Y(I5)/Y(I9)-Y(I2)*Y(I3)*Y(I6)*EFK*W**2/DX
      X(I4)=-EFK*Y(I2)*Y(I6)*E*W/DX
      X(I5)=-EFK*Y(I2)*Y(I7)*E*W/DX
      DO 101 I=1,N
      DO 101 J=1,NX
101   DY(J,I)=0.
      DY(I1,I1)=-X(I1)*W**2*Y(I6)*Y(I9)/DX
      DY(I1,I2)=X(I1)/Y(I2)
      DY(I1,I6)=-X(I1)*Y(I1)*W**2*Y(I9)/DX
      DY(I1,I8)=2*X(I1)*(1/E-E*Y(I9)/DX)
      DY(I1,I9)=-X(I1)/Y(I9)
      DY(I2,I1)=W**4*Y(I2)**2*Y(I6)**2*Y(I9)*EFK/DX**2
      DY(I2,I2)=-2*Y(I2)*Y(I6)*EFK*W**2/DX
      DY(I2,I4)=EFK/Y(I9)
      DY(I2,I6)=-Y(I2)**2*EFK*W**2/DX+W**4*Y(I1)*Y(I2)**2*Y(I6)*Y(I9)*EFK/DX**2
      DY(I2,I8)=2*Y(I2)**2*Y(I6)*E*Y(I9)*EFK*W**2/DX**2
      DY(I2,I9)=-X(I2)/Y(I9)
      DY(I3,I1)=W**4*Y(I2)*Y(I3)*Y(I6)**2*Y(I9)*EFK/DX**2
      DY(I3,I2)=-Y(I3)*Y(I6)*EFK*W**2/DX
      DY(I3,I3)=-Y(I2)*Y(I6)*EFK*W**2/DX
      DY(I3,I5)=EFK/Y(I9)
      DY(I3,I6)=-Y(I2)*Y(I3)*EFK*W**2/DX
     *+W**4*Y(I1)*Y(I2)*Y(I3)*Y(I6)*Y(I9)*EFK/DX**2
      DY(I3,I8)=2*Y(I2)*Y(I3)*Y(I6)*E*Y(I9)*EFK*W**2/DX**2
      DY(I3,I9)=-X(I3)/Y(I9)
      DY(I4,I1)=-X(I4)*W**2*Y(I6)*Y(I9)/DX
      DY(I4,I2)=X(I4)/Y(I2)
      DY(I4,I6)=-X(I4)*(-1/Y(I6)+W**2*Y(I1)*Y(I9)/DX)
      DY(I4,I8)=-X(I4)*(-1/E+2*E*Y(I9)/DX)
      DY(I4,I9)=-X(I4)/Y(I9)
      DY(I5,I1)=-X(I5)*W**2*Y(I6)*Y(I9)/DX
      DY(I5,I2)=X(I5)/Y(I2)
      DY(I5,I6)=-X(I5)*W**2*Y(I1)*Y(I9)/DX
      DY(I5,I7)=X(I5)/Y(I7)
      DY(I5,I8)=-X(I5)*(-1/E+2*E*Y(I9)/DX)
      DY(I5,I9)=-X(I5)/Y(I9)
```

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```
      CO(I8)=-W*Y(I6)/(R**2+W**2*Y(I6)**2)
      RETURN
      END
COMMENT TH = INITIAL ROTOR ANGLE      CO=DERIVATIVE OF TH WRT Y
      TH=ATAN(W*Y(I6)/R)
      DO 160 M=1,N
160   CO(M)=0.
      CO(I6)=W*R/(R**2+W**2*Y(I6)**2)

      COMPILER DOUBLE PRECISION
      SUBROUTINE GRAD6(X,NX,Y,N,DX)
COMMENT THIS SUBROUTINE COMPUTES DX/DT = F(X(T),T)
C     X(1)=LAMBDA D,     X(2)=LAMBDA F,     X(3)=LAMBDA KD
C     X(4)=LAMBDA Q,     X(5)=LAMBDA KQ
      COMMON EFK,RLK,W
      DIMENSION X(NX),Y(N),DX(NX),A(5,5),F(5,5)
      I2=2
      CALL AF6(Y,N,NX,A,F)
      DO 100 I=1,NX
      DX(I)=0.
      DO 100 J=1,NX
100   DX(I)=DX(I)+F(I,J)*X(J)
      DX(I2)=DX(I2)+EFK
      RETURN
      END

COMPILER FREE,DOUBLE PRECISION
OVERLAY OVL2
SUBROUTINE DCHY6(X,NX,Y,N,NZ,CO,C,D,ZP,T,TH)
COMMON EFK,RLK,W
DIMENSION X(NX),Y(N),CO(N),C(NZ,N),ZP(NZ),D(NZ,NX),A(5,5),F(5,5),DTH(4)
REAL I0,IQ
I1=1
I2=2
I3=3
I4=4
I5=5
I6=6
I7=7
I8=8
P=3.14159265358979
R=2.*P/3.
S=SQRT(2./3.)
CALL AF6(Y,N,NX,A,F)
ID=0.
DO 40 J=1,3
```

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```
40      ID=ID+A(I1,J)*X(J)
        IQ=0.
        DO 50 J=4,5
50      IQ=IQ+A(I4,J)*X(J)
        DO 90 I=1,N
        DO 90 J=1,NZ
90      C(J,I)=0.
        DO 95 I=1,NX
        DO 95 J=1,NZ
95      D(J,I)=0.
        DO 100 I=1,3
        D(I1,I)=S*A(I1,I)*COS(W*T+TH)
        D(I2,I)=S*A(I1,I)*COS(W*T+TH-P)
        D(I3,I)=S*A(I1,I)*COS(W*T+TH+P)
100     D(I4,I)=A(I2,I)
        DO 105 I=4,5
        D(I1,I)=-S*A(I4,I)*SIN(W*T+TH)
        D(I2,I)=-S*A(I4,I)*SIN(W*T+TH-P)
105     D(I3,I)=-S*A(I4,I)*SIN(W*T+TH+P)
        DO 110 I=1,NZ
        ZP(I)=0.
        DO 110 J=1,NX
110     ZP(I)=ZP(I)+D(I,J)*X(J)
        DD=Y(I1)*(Y(I4)*Y(I3)-Y(I5)**2)-Y(I2)*(Y(I3)*Y(I2)-Y(I5)*Y(I3))
     *  -Y(I3)*(Y(I4)*Y(I3)-Y(I2)*Y(I5))
        DQ=Y(I6)*Y(I7)-Y(I7)**2
        D1=A(I1,I1)*X(I1)+A(I1,I2)*X(I2)+A(I1,I3)*X(I3)
        D4=A(I4,I4)*X(I4)+A(I4,I5)*X(I5)
        C(I1,I1)=-A(I1,I1)*D1
        C(I1,I2)=(-Y(I3)*X(I2)+Y(I5)*X(I3))/DD-2*A(I1,I2)*D1
        C(I1,I3)=(Y(I4)*X(I1)-Y(I2)*X(I2)+Y(I5)*X(I2)-Y(I4)*X(I3))/DD
     *  -(A(I3,I3)+2*A(I1,I3))*D1
        C(I1,I4)=(Y(I3)*X(I1)-Y(I3)*X(I3))/DD-A(I2,I2)*D1
        C(I1,I5)=(-2*Y(I5)*X(I1)+Y(I3)*X(I2)+Y(I2)*X(I3))/DD-2*A(I2,I3)*D1
        C(I1,I6)=-A(I4,I4)*D4
        C(I1,I7)=(X(I4)-X(I5))/DQ-(A(I5,I5)+2*A(I4,I5))*D4
        DO 120 I=1,5
        C(I3,I)=S*C(I1,I)*COS(W*T+TH+P)
        C(I2,I)=S*C(I1,I)*COS(W*T+TH-P)
120     C(I1,I)=S*C(I1,I)*COS(W*T+TH)
        DO 130 I=6,7
        C(I3,I)=-S*C(I1,I)*SIN(W*T+TH+P)
        C(I2,I)=-S*C(I1,I)*SIN(W*T+TH-P)
130     C(I1,I)=-S*C(I1,I)*SIN(W*T+TH)
```

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```
      D2=A(I2,I1)*X(I1)+A(I2,I2)*X(I2)+A(I2,I3)*X(I3)
      C(I4,I1)=(Y(I3)*X(I2)-Y(I5)*X(I3))/DD-A(I1,I1)*D2
      C(I4,I2)=(-Y(I3)*X(I1)+Y(I3)*X(I3))/DD-2*A(I1,I2)*D2
      C(I4,I3)=((Y(I5)-Y(I2))*X(I1)+(Y(I1)-2*Y(I3))*X(I2)+Y(I2)*X(I3))/DD
     *-(A(I3,I3)+2*A(I1,I3))*D2
      C(I4,I4)=-A(I2,I2)*D2
      C(I4,I5)=(Y(I3)*X(I1)-Y(I1)*X(I3))/DD-2*A(I2,I3)*D2
      DO 30 L=1,NZ
30    DTH(L)=0.
      DTH(I1)=-S*(ID*SIN(W*T+TH)+IQ*COS(W*T+TH))
      DTH(I2)=-S*(ID*SIN(W*T+TH-P)+IQ*COS(W*T+TH-P))
      DTH(I3)=-S*(ID*SIN(W*T+TH+P)+IQ*COS(W*T+TH+P))
      DO 35 L=1,3
      C(L,I6)=C(L,I6)+DTH(L)*C0(I6)
35    C(L,I8)=C(L,I8)+DTH(L)*C0(I8)
      RETURN
      END
```

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```
      COMPILER DOUBLE PRECISION
      SUBROUTINE AFB(Y,N,NX,A,F)
COMMENT* THIS SUBROUTINE COMPUTES A=INV(L) AND F=-R*A
      COMMON EFK,RLK,W
      DIMENSION Y(N),A(NX,NX),F(NX,NX),R(5)
      I1=1
      I2=2
      I3=3
      I4=4
      I5=5
      I6=6
      I7=7
      I8=8
      I9=9
      I10=10
      I11=11
      DO 30 I=1,NX
      DO 30 J=1,NX
30    A(J,I)=0.
      DO 40 I=1,NX
      DO 40 J=1,NX
40    F(J,I)=0.
      D1=Y(I4)*Y(I3)-Y(I5)**2
      D2=Y(I3)*Y(I2)-Y(I5)*Y(I3)
      D3=Y(I4)*Y(I3)-Y(I2)*Y(I5)
      D=Y(I1)*D1-Y(I2)*D2-Y(I3)*D3
      DQ=Y(I6)*Y(I7)-Y(I7)**2
      A(I1,I1)=D1/D
      A(I1,I2)=-D2/D
      A(I1,I3)=-D3/D
      A(I2,I1)=A(I1,I2)
      A(I2,I2)=(Y(I1)*Y(I3)-Y(I3)**2)/D
      A(I2,I3)=-(Y(I1)*Y(I5)-Y(I2)*Y(I3))/D
      A(I3,I1)=A(I1,I3)
      A(I3,I2)=A(I2,I3)
      A(I3,I3)=(Y(I1)*Y(I4)-Y(I2)**2)/D
      A(I4,I4)=Y(I7)/DQ
      A(I4,I5)=-Y(I7)/DQ
      A(I5,I4)=A(I4,I5)
      A(I5,I5)=Y(I6)/DQ
      R(I1)=Y(I6)+RLK
      R(I2)=Y(I8)
      R(I3)=Y(I10)
      R(I4)=Y(I8)+RLK
      R(I5)=Y(I11)
      DO 100 I=1,NX
      DO 100 J=1,NX
100   F(I,J)=-R(I)*A(I,J)
      F(I1,I4)=W
      F(I4,I1)=-W
      RETURN
      END
```

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```
COMMENT: GRAF1,FR
C        READ IN UP TO 5 COLUMNS OF REAL DATA AND PLOT THEM ON DIGIVIEW
C        HIT ANY KEY TO SEE NEXT PLOTS
C        NP=NO. OF POINTS PER GRAPH,  NG=NO. OF GRAPHS (UP TO 10)
C        S=ARRAY OF SCALE FACTORS FOR EACH GRAPH - S(J)=LARGEST NO. ON PLOT J
C
         DIMENSION X(5,451),IX(5,451),IY(5,451),S(5),JX(451)
         OPEN 5,"DATGRA",ERR=999,ATT="I"
         READ FREE(5) NP,NG
         READ FREE(5) (S(J),J=1,NG)
1        DO 19 K=1,NP
         READ FREE(5,END=900) (X(J,K),J=1,NG)
         DO 19 J=1,NG
19       IX(J,K)=X(J,K)^100/S(J)
         J=1
20       CONTINUE
         CALL PASS(0)
         CALL GMODE
         CALL TPLOT(0,0,0)
         CALL TPLOT(0,250,2)
         CALL TPLOT(500,250,1)
         CALL TAXIS(50,125,450,200,10,10,0,-100)
         DO 30 K=1,NP
30       JX(K)=IX(J,K)
         CALL TPOIN(JX,1.,NP,1,1,0)
         J=J+1
         IF(J-NG) 50,50,60
50       CALL TAXIS(50,375,450,200,10,10,0,-100)
         DO 40 K=1,NP
40       JX(K)=IX(J,K)
         CALL TPOIN(JX,1.,NP,1,1,0)
         J=J+1
         IF(J-NG) 70,70,60
70       CALL AMODE(0)
         PAUSE
         GO TO 20
60       CALL AMODE(0)
         PAUSE
         GO TO 1
900      STOP
999      STOP " ERROR IN OPEN DATGRA"
         END
```

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APPENDIX E

STANDARD PARAMETER ESTIMATION PROGRAM

```
C
C  *****  PFACTR IS PARAMETER ERROR FACTOR OF INITIAL GUESS
C  *****  DELFAC IS FOR THE INCREMENT OF PFACTR
C  *****  NINPAR  THE NUMBER OF REPEAT OF  PFACTR*DELFAC
C  *****  NREPIT IS NUMBER OF ITERATION OF PARAM ESTIMATION
C  *****  NSAMPL IS FOR THE NUMBER  OF SAMPLES
C  *****  NIRUNGE IS THE NUMBER OF RUNGE ROUTINE REPEAT
      PFACTR=2.0
      DELFAC=-0.4
      NINPAR=7
      NREPIT=14
      NIRNGE=1
      PLIMIT=0.25
       RL=0.
       OMEGA=1.
      DELT=0.2
       NV=5
       DIMENSION LLMTX(5),MMMTX(5)
      DIMENSION SSAI(5)
      DIMENSION  XR(5),AR(5,5),U(5),YR(3),CR(3,5)
      DIMENSION  X(5),A(5,5),Y(3),C(3,5)
      DIMENSION SX(5,11),AP(5,11),SY(3,11),CP(3,11)
      DIMENSION  XS(5),US(5),WINV(3,3),SYT(11,3)
      DIMENSION SYTW(11,3),SUM(11,11),TSUM(11,11)
      DIMENSION YD(3),YW(11),TYW(11),TSUMIN(11,11)
      DIMENSION DELP(11),PN(11)
      DIMENSION PTRUE(11),PRATIO(11)
      DIMENSION LMTX(11),MMTX(11)
      DIMENSION ESTMAT(15,11)
      COMMON AR,A,U,US
       U(1)=0.
       U(2)=0.
      U(3)=SQRT(1.)*0.0021
      U(4)=0.
       U(5)=0.
      READ(5,10) P1,P2,P3,P4,P5,P6,P7,P8,P9,P10,P11
   10 FORMAT(4F10.5/3F10.5)
      WRITE(6,104) P1,P2,P3,P4,P5,P6,P7,P8,P9,P10,P11
  104 FORMAT(1H1,10X,'TRUE PARAMETER VALUES'///(10X,1F10.5))
      PTRUE(1)=P1
      PTRUE(2)=P2
      PTRUE(3)=P3
      PTRUE(4)=P4
      PTRUE(5)=P5
      PTRUE(6)=P6
      PTRUE(7)=P7
      PTRUE(8)=P8
      PTRUE(9)=P9
      PTRUE(10)=P10
      PTRUE(11)=P11
      DO 306 I=1,5
      DO 305 J=1,5
      A(I,J)=0.
  306 AR(I,J)=0.
      DO 310 I=1,3
      DO 310 J=1,5
      C(I,J)=0.
  310  CR(I,J)=0.
      CD=-P3/P1
      CDKD=P3*(P2-P1)/(P1*(P2-P4))
      CDF=P3*(P1-P4)*(P3-P2)/(P1*(P3-P4)*(P2-P4))
      CF=CD+CDKD*(1.-P4*(P3-P4)/(P3*(P2-P4)))
      CFD=(P1-P4)*(P3-P4)/(P1*(P2-P4))
      CFKD=(P2-P1)*(P3-P4)*P4/(P1*(P2-P4)**2)
      CKDD=(P2-P4)/P1
      CKD=-P2/P1
       CKDF=(P3-P2)*P4/(P1*(P3-P4))
      CQ=-P8/P7
      CQKQ=SQRT(CQ**2+CQ)
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```
      A(1,1)=((P10+RL)/P3)*CD
      A(1,2)=((P10+RL)/P3)*CDKD
      A(1,3)=((P10+RL)/P3)*CDF
      A(1,4)=OMEGA
      A(2,1)=(1./P5)*CKDD
      A(2,2)=(1./P5)*CKD
      A(2,3)=(1./P5)*CKDF
      A(3,1)=(1./P6)*CFD
      A(3,2)=(1./P6)*CFKD
      A(3,3)=(1./P6)*CF
       A(4,1)=-OMEGA
       A(4,4)=((P10+RL)/P8)*CQ
       A(4,5)=((P10+RL)/P8)*CQKQ
      A(5,4)=(1./P9)*CQKQ
      A(5,5)=(1./P9)*CQ
       C(1,1)=(1./P3)*CD
       C(1,2)=(1./P3)*CDKD
       C(1,3)=(1./P3)*CDF
      C(2,1)=(1./(P11*P6))*(-CFD)
      C(2,2)=(1./(P11*P6))*(-CFKD)
      C(2,3)=(1./(P11*P6))*(-CF)
      C(3,4)=(1./P8)*CQ
      C(3,5)=(1./P8)*CQKQ
      DO 295  I=1,5
       DO 295 J=1,5
  295 AR(I,J)=A(I,J)
      DO 296 I=1,3
       DO 296 J=1,5
  296 CR(I,J)=C(I,J)
      WRITE(6,101)((AR(I,J),J=1,5),I=1,5)
  101 FORMAT(1H0,10X,'MTX AR'///(10X,5F20.7))
      WRITE(6,102) ((CR(I,J),J=1,5),I=1,3)
  102 FORMAT(1H0,10X,'MTX CR'///(10X,5F20.7))
C
CC***************INPUT NOISE IS ADDED*******START
C
      CALL MINV(A,5,DD,LLMTX,MMMTX)
      CALL GMPRD(A,U,SSAI,5,5,1)
       DO 283 I=1,5
  283 SSAI(I)=-SSAI(I)
      WRITE(6,282)(SSAI(I),I=1,5)
  282 FORMAT(1H0,5F20.5)
      DINPUT=0.01
      CGMAU1=SSAI(1)*DINPUT
      CGMAU2=SSAI(2)*DINPUT
      CGMAU3=SSAI(3)*DINPUT
      CGMAU4=SSAI(4)*DINPUT
      CGMAU5=SSAI(5)*DINPUT
      CGMAU1=ABS(CGMAU1)
      CGMAU2=ABS(CGMAU2)
      CGMAU3=ABS(CGMAU3)
      CGMAU4=ABS(CGMAU4)
      CGMAU5=ABS(CGMAU5)
       DO 205 I=1,3
       DO 205 J=1,3
  205 WINV(I,J)=0.
      WINV(1,1)=0.022047
      WINV(2,2)=3.845
      WINV(3,3)=0.022047
      WINV(1,1)=1.0
      WINV(2,2)=1.
      WINV(3,3)=1.0
       DO 888 NPARAM=1,NINPAR
      PN(1)=PTRUE(1)*PFACTR
      PN(2)=PTRUE(2)*PFACTR
      PN(3)=PTRUE(3)*PFACTR
      PN(4)=PTRUE(4)*PFACTR
      PN(5)=PTRUE(5)*PFACTR
      PN(6)=PTRUE(6)*PFACTR
      PN(7)=PTRUE(7)*PFACTR
      PN(8)=PTRUE(8)*PFACTR
      PN(9)=PTRUE(9)*PFACTR
```

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```
      PN(10)=PTRUE(10)*PFACTR
      PN(11)=PTRUE(11)*PFACTR
      IF(NPARAM.LT.7) GO TO 11
      PN(1)=PTRUE(1)*1.6
      PN(2)=PTRUE(2)*1.4
      PN(3)=PTRUE(3)*1.2
      PN(4)=PTRUE(4)*1.8
      PN(5)=PTRUE(5)*0.5
      PN(6)=PTRUE(6)*0.7
      PN(7)=PTRUE(7)*0.6
      PN(8)=PTRUE(8)*0.8
      PN(9)=PTRUE(9)*0.4
      PN(10)=PTRUE(10)*2.0
      PN(11)=PTRUE(11)*0.9
11    DO 3 I=1,11
        PRATIO(I)=PN(I)/PTRUE(I)
3     CONTINUE
      DO 4 I=1,11
4     ESTMAT(1,I)=PRATIO(I)
        DO 999 NIT=1,NREPIT
      IX1=5
      IX2=55
      IX3=555
      IX4=5555
      IX5=55555
      IDN=991
      IFN=333
      IQN=777
      DO 201 I=1,5
      XP(I)=0.
      X(I)=0.
      DO202 J=1,5
202   A(I,J)=0.
      DO 201 J=1,11
        SX(I,J)=0.
201   AP(I,J)=0.
      DO 203 I=1,3
      DO 204 J=1,5
204   C(I,J)=0.
      DO 203 J=1,11
      CP(I,J)=0.
203   SY(I,J)=0.
      DO 206 I=1,11
        DO 206 J=1,11
        TSUM(I,J)=0.
206   SUM(I,J)=0.
      DO 209 I=1,11
209   IYN(I)=0.
      P1=PN(1)
      P2=PN(2)
      P3=PN(3)
      P4=PN(4)
      P5=PN(5)
      P6=PN(6)
      P7=PN(7)
      P8=PN(8)
      P9=PN(9)
      P10=PN(10)
      P11=PN(11)
      CD=-P3/P1
      CDKD=P3*(P2-P1)/(P1*(P2-P4))
      CDF=P3*(P1-P4)*(P3-P2)/(P1*(P3-P4)*(P2-P4))
      CF=CD*(CDKD*(1.-P4*(P3-P4)/(P3*(P2-P4))))
      CFD=(P1-P4)*(P3-P4)/(P1*(P2-P4))
      CFKD=(P2-P1)*(P3-P4)*P4/(P1*(P2-P4)**2)
      CKDD=(P2-P4)/P1
      CKD=-P2/P1
        CKDF=(P3-P2)*P4/(P1*(P3-P4))
      CQ3=-P8/P7
```

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```
      CGKQ=SQRT(CQ**2+CQ)
      A(1,1)=((P10+RL)/P3)*CD
      A(1,2)=((P10+RL)/P3)*CDKD
      A(1,3)=((P10+RL)/P3)*CDF
      A(1,4)=OMEGA
      A(2,1)=(1./P5)*CKDD
      A(2,2)=(1./P5)*CKD
      A(2,3)=(1./P5)*CKDF
      A(3,1)=(1./P6)*CFD
      A(3,2)=(1./P6)*CFKD
      A(3,3)=(1./P6)*CF
       A(4,1)=-OMEGA
       A(4,4)=((P10+RL)/P8)*CQ
       A(4,5)=((P10+RL)/P8)*CQKQ
      A(5,4)=(1./P9)*CQKQ
      A(5,5)=(1./P9)*CQ
       C(1,1)=(1./P3)*CD
       C(1,2)=(1./P3)*CDKD
       C(1,3)=(1./P3)*CDF
      C(2,1)=(1./(P11*P6))*(-CFD)
      C(2,2)=(1./(P11*P6))*(-CFKD)
      C(2,3)=(1./(P11*P6))*(-CF)
      C(3,4)=(1./P8)*CQ
      C(3,5)=(1./P8)*CQKQ
      CDP1=P3/P1**2
      CDP2=0.
      CDP3=-1./P1
      CDP4=0.
      CDKDP1=-P2*P3/(P1**2*(P2-P4))
      CDKDP2=P3*(P1-P4)/(P1*(P2-P4)**2)
      CDKDP3=(P2-P1)/((P2-P4)*P1)
      CDKDP4=(P2-P1)*P3/(P1*(P2-P4)**2)
      CDFP1=P3*P4*(P3-P2)/((P3-P4)*(P2-P4)*P1**2)
      CDFP2=-P3*(P1-P4)/(P1*(P2-P4)**2)
      CDFP3=(P1-P4)*(P3*(P3-P4)+P4*(P2-P3))/(P1*(P2-P4)*(P3-P4)**2)
      CDFP4=P3*(P3-P2)*((P1-P4)*(P2-P4)+(P1-P2)*(P3-P4))/
     *(P1*((P3-P4)*(P2-P4))**2)
       CKDDP1=(P4-P2)/P1**2
      CKDDP2=1./P1
      CKDDP3=0.
      CKDDP4=-1./P1
      CKDP1=P3/P1**2
      CKDP2=-1./P1
      CKDP3=0.
      CKDP4=0.
      CKDFP1=(P2-P3)*P4/((P3-P4)*P1**2)
      CKDFP2=P4/((P4-P3)*P1)
      CKDFP3=P4*(P2-P4)/(P1*(P3-P4)**2)
      CKDFP4=P3*(P3-P2)/(P1*(P3-P4)**2)
      CFDP1=(P3-P4)*P4/((P2-P4)*P1**2)
      CFDP2=(P1-P4)*(P4-P3)/(P1*(P2-P4)**2)
       CFDP3=(P1-P4)/(P1*(P2-P4))
      CFDP4=(P4*(P2-P4)+P2*(P4-P3)+P1*(P3-P2))/(P1*(P2-P4)**2)
      CFKDP1=(P4-P3)*P2*P4/(P1*(P2-P4)**2*P1)
      CFKDP2=P4*(P3-P4)*(2.*P1-P2-P4)/(P1*(P2-P4)**3)
      CFKDP3=(P2-P1)*P4/(P1*(P2-P4)**2)
      CFKDP4=(P2-P1)*(P2*(P3-P4)+P4*(P3-P2))/(P1*(P2-P4)**3)
      CFP1=CDP1+CDKDP1*(1.-P4*(P3-P4)/(P3*P2-P3*P4))
      CFP2=CDP2+CDKDP2*(1.-P4*(P3-P4)/(P3*P2-P3*P4))
     *  +CDKD*P4*(P3-P4)/(P3*(P2-P4)**2)
      CFP3=CDP3+CDKDP3*(1.-P4*(P3-P4)/(P3*P2-P3*P4))
     *  -CDKD*P4**2/((P2-P4)*P3**2)
      CFP4=CDP4+CDKDP4*(1.-P4*(P3-P4)/(P3*P2-P3*P4))
     *  -CDKD*((P3-P4)*P2+(P4-P2)*P4)/(P3*(P2-P4)**2)
      CQP7=P8/P7**2
      CQP8=-1./P7
      CQKQP7=0.5*CQ*(2.*P8/P7**2-1./P7)/CQKQ
      CQKQP8=0.5*(2.*P8/P7**2-1./P7)/CQKQ
      I=0.
      DO 901 NKK=1,900
      DO 60 L=1,NIRNGE
      DO 40 J=1,11
```

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```
      DO 45  I=1,5
      US(I)=AP(I,J)
   45 XS(I)=SX(I,J)
      CALL RUNGE(T,DELT,XS,NV,3)
      DO 50 I=1,5
   50 SX(I,J)=XS(I)
   40  CONTINUE
      CALL GAUSS(IX1,CGMAU1,0.,VV1)
      CALL GAUSS(IX2,CGMAU2,0.,VV2)
      CALL GAUSS(IX3,CGMAU3,0.,VV3)
      CALL GAUSS(IX4,CGMAU4,0.,VV4)
      CALL GAUSS(IX5,CGMAU5,0.,VV5)
       U(1)=VV1
       U(2)=VV2
       U(3)=VV3
       U(4)=VV4   -SQRT(3.)
       U(5)=VV5
      CALL RUNGE(T,DELT,XR,NV,1)
      U(1)=0.
      U(2)=0.
      U(3)=0.
      U(4)=-SQRT(3.)
      U(5)=0.
      CALL RUNGE(T,DELT,X,NV,2)
      AP(1,1)=(CDP1*X(1)+CDKDP1*X(2)+CDFP1*X(3))*(P10+RL)/P3
      AP(1,2)=(CDKDP2*X(2)+CDFP2*X(3))*(P10+RL)/P3
      AP(1,3)=(CDP3*X(1)+CDKDP3*X(2)+CDFP3*X(3))*(P10+RL)/P3-
      (CD*X(1)+CDKD*X(2)+CDF*X(3))*(P10+RL)/P3**2
      AP(1,4)=(CDKDP4*X(2)+CDFP4*X(3))*(P10+RL)/P3
      AP(1,10)=(CD*X(1)+CDKD*X(2)+CDF*X(3))/P3
       AP(2,1)=(CKDDP1*X(1)+CKDP1*X(2)+CKDFP1*X(3))/P5
      AP(2,2)=(CKDDP2*X(1)+CKDP2*X(2)+CKDFP2*X(3))/P5
      AP(2,3)=CKDFP3*X(3)/P5
      AP(2,4)=(CKDDP4*X(1)+CKDFP4*X(3))/P5
      AP(2,5)=-(CKDD*X(1)+CKD*X(2)+CKDF*X(3))/P5**2
      AP(3,1)=(CFDP1*X(1)+CFKDP1*X(2)+CFP1*X(3))/P6
      AP(3,2)=(CFDP2*X(1)+CFKDP2*X(2)+CFP2*X(3))/P6
      AP(3,3)=(CFDP3*X(1)+CFKDP3*X(2)+CFP3*X(3))/P6
      AP(3,4)=(CFDP4*X(1)+CFKDP4*X(2)+CFP4*X(3))/P6
      AP(3,6)=-(CFD*X(1)+CFKD*X(2)+CF*X(3))/P6**2
      AP(4,7)=(CQP7*X(4)+CQKQP7*X(5))*(P10+RL)/P8
      AP(4,8)=(P10+RL)*(CQKQP8/P8-CQKQ/P8**2)*X(5)
      AP(4,10)=(CQ*X(4)+CQKQ*X(5))/P8
      AP(5,7)=(CQKQP7*X(4)+CQP7*X(5))/P9
      AP(5,8)=(CQKQP8*X(4)+CQP8*X(5))/P9
       AP(5,9)=-(CQKQ*X(4)+CQ*X(5))/P9**2
      T=T+DELT
   60 CONTINUE
      CP(1,1)=AP(1,1)/(P10+RL)
      CP(1,2)=AP(1,2)/(P10+RL)
      CP(1,3)=AP(1,3)/(P10+RL)
      CP(1,4)=AP(1,4)/(P10+RL)
      CP(2,1)=-AP(3,1)/P11
      CP(2,2)=-AP(3,2)/P11
      CP(2,3)=-AP(3,3)/P11
      CP(2,4)=-AP(3,4)/P11
      CP(2,6)=-AP(3,6)/P11
      CP(2,11)=CP(2,6)*(P6/P11)
      CP(3,7)=AP(4,7)/(P10+RL)
      CP(3,8)=AP(4,8)/(P10+RL)
      CALL GMPRD(C,SX,SY,3,5,11)
      DO 65 I=1,3
      DO 65 J=1,11
   65 SY(I,J)=SY(I,J)+CP(I,J)
      CALL GMPRD(CR,XR,YR,3,5,1)
      CALL GAUSS(IDN,0.03368,0.,V1)
      CALL GAUSS(IFN,0.0026,0.,V2)
      CALL GAUSS(IQN,0.03368,0.,V3)
      YR(1)=YR(1)+V1
```

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```
      YR(2)=YR(2)+V2
      YR(3)=YR(3)+V3
      CALL GMPRD(C,X,Y,3,5,1)
      DO 80 I=1,3
   80  YD(I)=YR(I)-Y(I)
C
C  FOR 5TH ORDER MODEL WITH 11 MACHINE PARAMETERS
C
      DO 70 I=1,3
      DO 70 J=1,11
   70 SYT(J,I)=SY(I,J)
      CALL GMPRD(SYT,WINV,SYTW,11,3,3)
      CALL GMPRD(SYTW,SY,SUM,11,3,11)
      DO 75 I=1,11
      DO 75  J=1,11
   75 TSUM(I,J)=TSUM(I,J)+SUM(I,J)
      CALL GMPRD(SYTW,YD,YW,11,3,1)
      DO 85 I=1,11
   85 TYW(I)=TYW(I)+YW(I)
  901 CONTINUE
      CALL MINV(TSUM,11,D,LMTX,MMTX)
      CALL GMPRD(TSUM,TYW,DELP,11,11,1)
      DO 401 I=1,11
      PPGAIN=ABS(DELP(I)/PN(I))-PLIMIT
      IF(PPGAIN.GT.0.) DELP(I)=PN(I)*(DELP(I)/
     *ABS(DELP(I)))*PLIMIT
      PN(I)=PN(I)+DELP(I)
  401  CONTINUE
C
C   FR 5TH ORDER MODEL WITH 11 MACHINE PARAMETERS                    END
C
      DO 7 I=1,11
       PRATIO(I)=PN(I)/PTRUE(I)
    7 CONTINUE
      DO 5 I=1,11
    5  ESTMAT(1+NIT,I)=PRATIO(I)
  999 CONTINUE
      NREAP=NREPIT+1
      WRITE(6,9)
    9 FORMAT(1H1,10X,'RATIO'///)
      WRITE(6,6)((ESTMAT(I,J),J=1,11),I=1,NREAP)
    6 FORMAT(20X,11F7.3)
      IF(NPARAM.GT.3) DELFAC=-0.2
      PFACTR=PFACTR+DELFAC
  888 CONTINUE
        STOP
      END




      SUBROUTINE GMPRD(A,B,R,N,M,L)
      DIMENSION A(1),B(1),R(1)
      IR=0
      IK=-M
      DO 10 K=1,L
      IK=IK+M
      DO 10 J=1,N
      IR=IR+1
       JI=J-N
      IB=IK
      R(IR)=0.
      DO 10 I=1,M
      JI=JI+N
      IB=IB+1
   10 R(IR)=R(IR)+A(JI)*B(IB)
      RETURN
      END
```

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```
      SUBROUTINE MINV(A,N,D,L,M)
      DIMENSION A(1),L(1),M(1)
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
   10 IF( ABS(BIGA)-ABS(A(IJ))) 15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
      J=L(K)
      IF(J-K) 35,35,25
   25  KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI)=HOLD
   35 I=M(K)
      IF(I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI)=HOLD
   45 IF(BIGA) 48,46,48
   46 D=0.0
      RETURN
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
      DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
   65 CONTINUE
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
      D=D*BIGA
      A(KK)=1.0/BIGA
   80 CONTINUE
      K=N
  105 K=(K-1)
      IF(K) 150,150,105
```

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```
105   I=L(K)
      IF(I-K) 120,120,108
108   JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
        JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
        A(JK)=-A(JI)
110   A(JI)=HOLD
120     J=M(K)
      IF(J-K)   100,100,125
125     KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
        JI=KI-K+J
      A(KI)=-A(JI)
130   A(JI)=HOLD
      GO TO 100
150   RETURN
      END



      SUBROUTINE GAUSS(IX,S,AM,V)
      A=0.0
      DO 50 I=1,12
      CALL RANDU(IX,IY,Y)
      IX=IY
50    A=A+Y
      V=(A-6.0)*S+AM
      RETURN
      END


      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY)5,6,6
5     IY=IY+2147483647+1
6     YFL=IY
      YFL=YFL*.4656613E-9
       RETURN
      END



       FUNCTION FCN(T,X,K,NDET)
      DIMENSION F(10),X(10)
      DIMENSION AR(5,5),A(5,5),U(5),US(5)
      COMMON AR,A,U,US
      F(K)=0.
      IF(NDET.EQ.1) GO TO 110
      DO 100 J=1,5
100   F(K)=F(K)+A(K,J)*X(J)
      GO TO 150
110   DO 120 I=1,5
120   F(K)=F(K)+AR(K,I)*X(I)
150   IF(NDET.EQ.3) GO TO 300
200   F(K)=F(K)+U(K)
      GO TO 400
300   F(K)=F(K)+US(K)
400   FCN=F(K)
      RETURN
      END
```

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