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INVESTIGATION INTO THE EFFECTS OF WEAPON SETBACK
ON VARIOUS MATERIALS AND GEOMETRIES

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by

Richard S. Simak

Munitions Division

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
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19. KEY WORDS (Contd)

Dynamic testing
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Mechanical-pneumatic forming
Air-activated-ram forming
Strain hardening
Dynamic mechanics

Plastic deformation
High-velocity stress
Strength factors
Impact loading
Displacement mechanisms
Grain distortion
Slip
Twinning
Metal flow
Dislocation dipoles
Geometric influence

20. ABSTRACT (Contd)

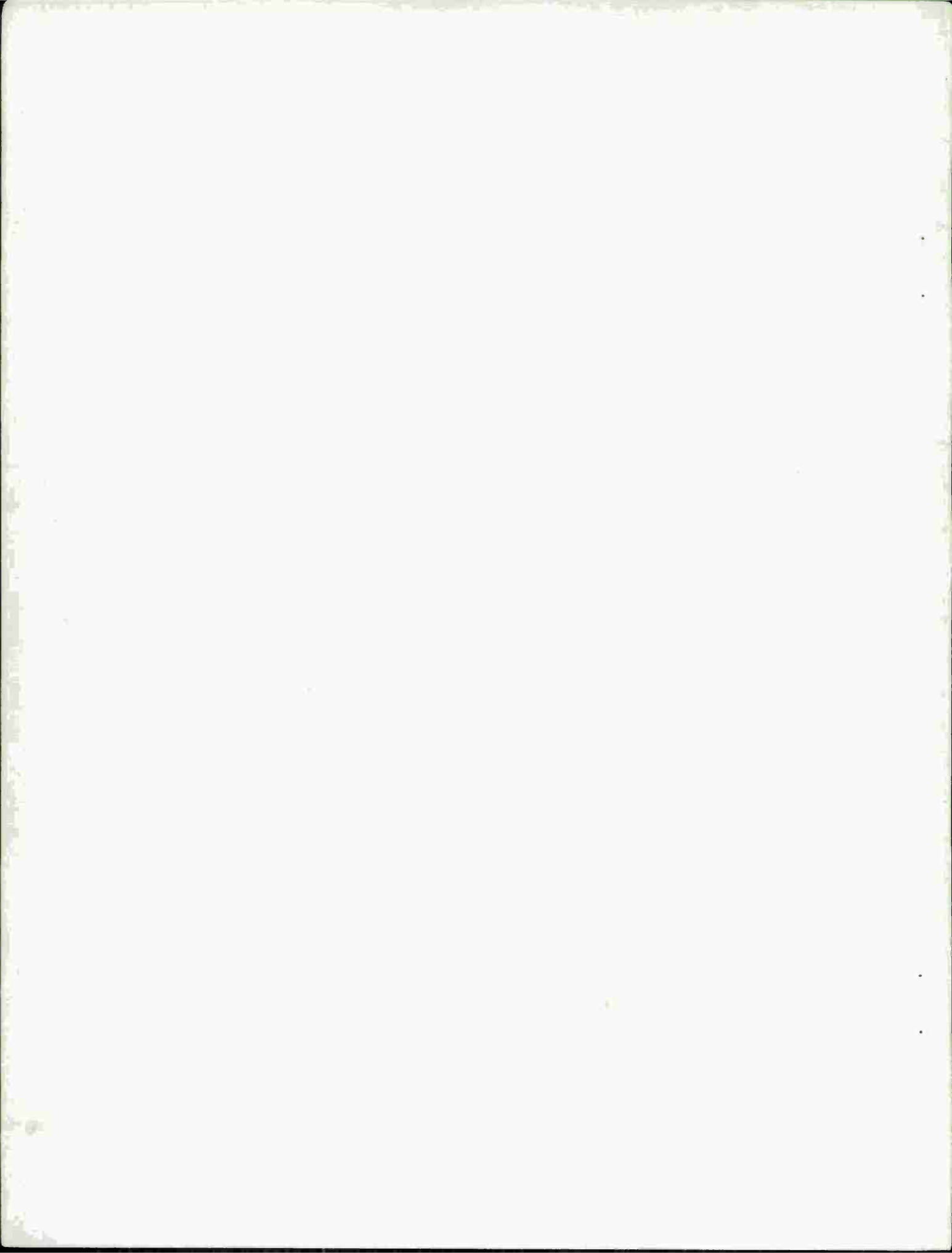
predicted by the mathematical model. The mathematical model selected was a viscoelastic model involving viscous, elastic, and in one instance, plastic parameters. This model accurately predicted the measured values in five material-geometry combinations actually tested. The methodology described in this report can be used to predict accurately the structural integrity of munition components when exposed to weapon setback environments.

PREFACE

The literature search and experimental work described in this report were authorized under project/task 1T161101A91A/7HA91X47, an in-house independent research task, Investigation into the Effects of Weapon Setback on Various Materials and Geometries. This work was carried out from March 1977 to January 1978. Data are recorded in Notebook 9612.

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INVESTIGATION INTO THE EFFECTS OF WEAPON SETBACK ON VARIOUS MATERIALS AND GEOMETRIES

I. INTRODUCTION.

The design of chemical munitions involves several factors and among them is the structural behavior of the components when subjected to various loads. In some cases the requirement is that the component withstand the maximum stress or weapon setback force while others require failure at minimum setbacks yet survive the logistical cycle. In either event, the structural integrity of the component must be evaluated as part of the development of an item.

At present, this evaluation involves the calculation of the component's structural integrity using statically obtained strength values, followed by bench testing, if possible, and the testing of munitions. The emphasis is on system testing which yields specific solutions for specific configurations, and if the configuration is changed the system must be retested. This method has resulted in increased project costs and, at times, in schedule slippages.

What is required is a reliable method for analyzing the effects of the launch environment on munition components. Therefore, an investigation was undertaken to develop such a method and, thereby, reduce the amount of munition flight testing and/or dynamic bench testing required with its associated costs.

This work was divided into two phases, the first of which was to develop a mathematical model which predicts the mechanical behavior of materials subjected to setback. This involved a literature search followed by a mathematical modeling effort. The second phase involved comparing the model prediction with experimental data which were generated as part of this effort.

II. BACKGROUND.

Plastic deformation of solids was first considered in 1904 by Hopkinson who observed that iron and copper wires subjected to rapidly applied tensile stress could be stressed beyond the static elastic limit and breaking loads and still remain in the elastic range, providing that the duration of such stress was of the order of 0.001 second or less.¹ But it was not until 1941 when Von Karman and Taylor each independently established the theory of plastic strain propagation in metals that this phenomenon was seriously studied.² This theory assumes that stress is a unique function of strain and that this functional relationship can be obtained by solving the equation of motion for whatever case is under consideration. The original paper used the simple case of a slender rod subjected to longitudinal impact. Two important features of this theory are that the velocity of propagation is less than that of an elastic strain and that an upper or critical impact velocity exists above which the material fails.

Since the publication of this theory, a body of research data has grown; in general it can be placed into four categories:

(1) The determination of the dynamic properties of various materials. The materials tested have been mostly high-strength ferrous alloys³ and lightweight alloys for use in the aerospace industry⁴ with a small amount of data on annealed nonferrous alloys. In general, a material's strength increases when subjected to impact loads (relative to statically applied loads), but by varying relative amounts for various materials. For example, cold-rolled low-carbon steel has an ultimate strength of 87,750 psi under static conditions and 125,250 psi under dynamic conditions or a 43% increase while 24-ST aluminum has an ultimate strength of 65,150 psi under static conditions and 68,600 psi under dynamic conditions or only a 5% increase. It was also noted that the strain rate affects both the strength and ultimate elongation of a material.

(2) Various investigations into the behavior of materials subjected to impact loads. This work usually involves solving the equation of motion for a geometry of interest, having one or more "dynamic" parameters, usually viscous or flow parameters. This is followed by dynamic bench tests to determine the specific value of parameters for a specific material.

(3) High-energy-forming technology for manufacturing applications. This work is concentrated in two areas; the development of techniques for applying high-velocity stress waves and the development of process parameters. The techniques developed include both high- and low-explosive-forming, exploding-wire (capacitor rapidly discharging through a fine wire which vaporizes) forming, magnetic-pulse (quickly changing electromagnetic field) forming and mechanical-pneumatic (air-activated-ram) forming techniques.⁵ The parametric process studies involved relating the depth of draw, material thickness, die diameter and standoff to the material's mechanical properties and the amount of forming energy involved.⁴

(4) Physical descriptions of the effect of impact loading of metals. The displacement mechanisms for metal subjected to impact loads are similar to that for statically applied loads, that is, grain distortion, slip* and twinning.** But in the case of impact loads, the deformation is greatest near the point of application and reduces with distance from that point.⁶ There is also a time delay between the application of an impact load and the subsequent yielding. This phenomenon has been explained in part by the presence of foreign atoms in a metal lattice and grain boundaries, both of which constitute obstacles to the flow of metals.¹ Another obstacle to flow is strain hardening which is due to the buildup of dislocation dipoles† into a network which anchors dislocations.⁷

In summary: (1) The displacement mechanisms for metals subjected to impact loads are similar to those for statically applied loads, but with different distributions, (2) there exists a time delay between the application of impact loads and subsequent plastic yielding, (3) materials' strength increases when subjected to impact loads, but by varying relative amounts for various materials, and (4) previous investigations showed that flow or viscous parameters are important.

* Slip is the displacement of one part of a crystal relative to another.

** Twinning is the sliding of one plane of atoms over the adjacent plane.

† Dislocation dipole is the junction of one dislocation with another dislocation on an intersecting plane.

III. MODELING EFFECT.

Based on the information obtained during the literature search a mathematical model has been developed to predict the time-dependent behavior of materials. This model assumes that materials behave in an elastic manner below the proportional limit (arbitrarily set at 2% elongation) and in a viscoelastic manner above the proportional limit. In addition, the viscoelastic element must have a time delay or, in this case, a viscous resistance feature which is known as a Voigt element. The mechanical analog of the model is given in figure 1, the equations for which are given to the right of the figure.

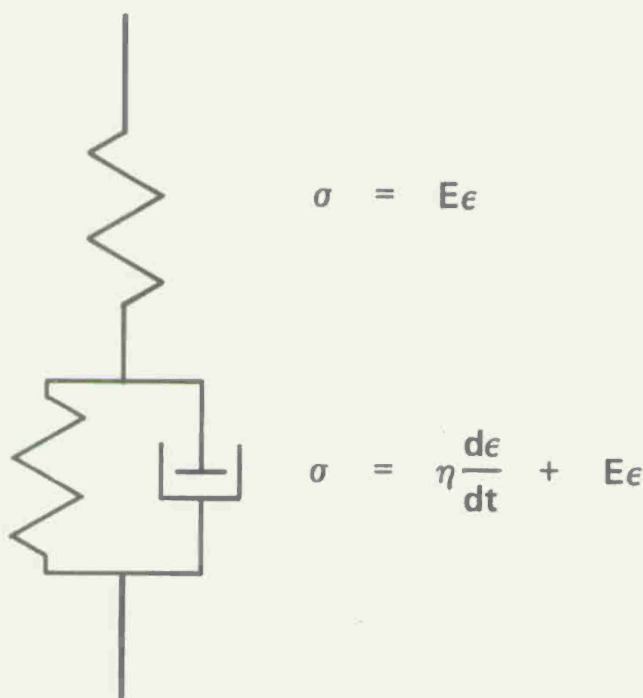


Figure 1. Mechanical Analog of Elastic-Viscoelastic Model

The response equation for this model is $\sigma = (2E)\epsilon + \eta \frac{\partial \epsilon}{\partial t}$ *. The time-dependent strain for this system is found by solving this equation for the model when excited by the forces due to the launch environments (which can be obtained from acceleration/pressure histories of the weapon system). Once the strain relationship is known, the other dynamic values can be determined. This involves taking the Laplace Transform of the dynamic equation, rearrangement and taking the inverse transform to find the time-dependent strain. The "dynamic modulus" is then the stress divided by the strain which is constantly changing with time.

* See the glossary for definition of variables used here and in succeeding sections of this report.

The elastic and viscous constants of a material may be obtained from a statically obtained true stress-strain diagram by assuming that the material behaves elastically below the proportional limit and in a viscous manner above it. The graphical representation of this is shown in figure 2.

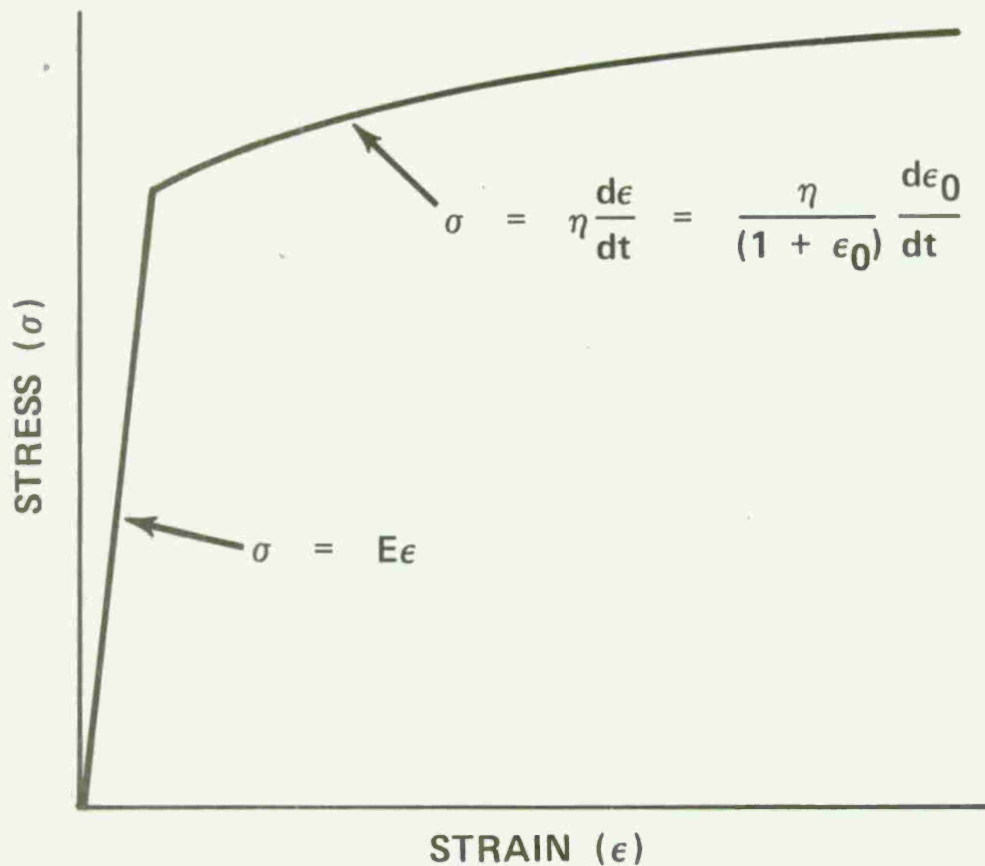


Figure 2. Idealized True Stress-Strain Diagram

IV. EXPERIMENTAL METHODS.

To compare the model predictions with experimental data a series of dynamic bench tests was performed involving two geometries (the thin flat plate as shown in figure 3 and the solid pin as shown in figure 4) and four materials (AISI 1018 and 1020 low carbon steel, type 316 austenitic stainless steel and 6061-T6 aluminum alloy). In addition, the stress-strain diagrams for all four materials were generated to provide elastic and viscous material constants.

The bench test machine used in this work was a Veripulse VP 400 Shock Machine which produces a controlled and reproducible acceleration at a constant pulse rate. The procedure for this testing involves securing a component holding fixture and weight to the bed of the shock machine and varying the bed drop height to achieve the proper acceleration while the neoprene compression pads control the pulse rate. The calibration chart for both pads is given in figure 5, and the estimated standard deviation for each pad is given in the results section of this

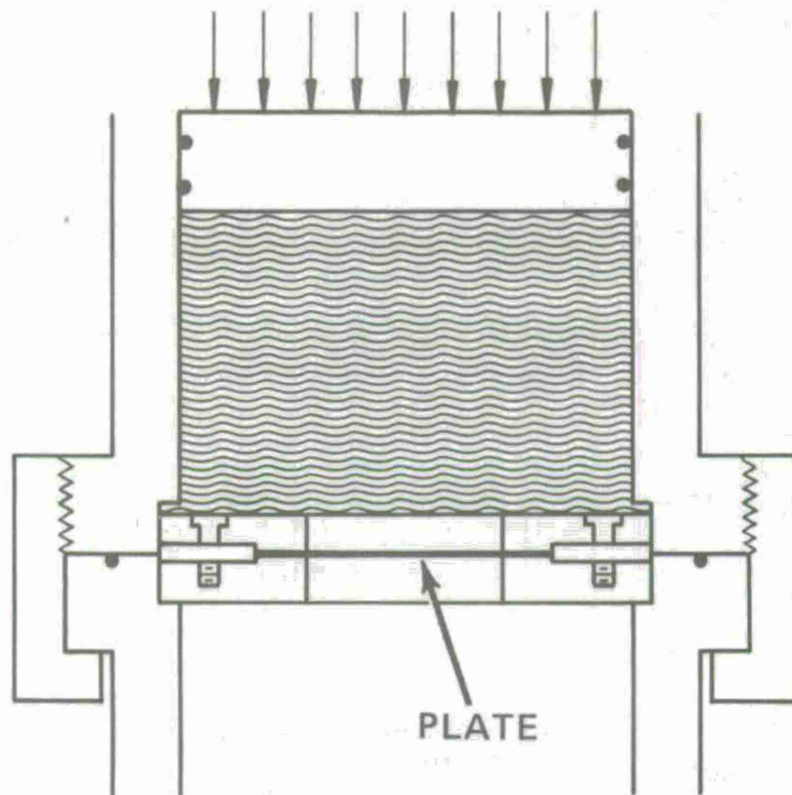


Figure 3. Flat Plate Configuration

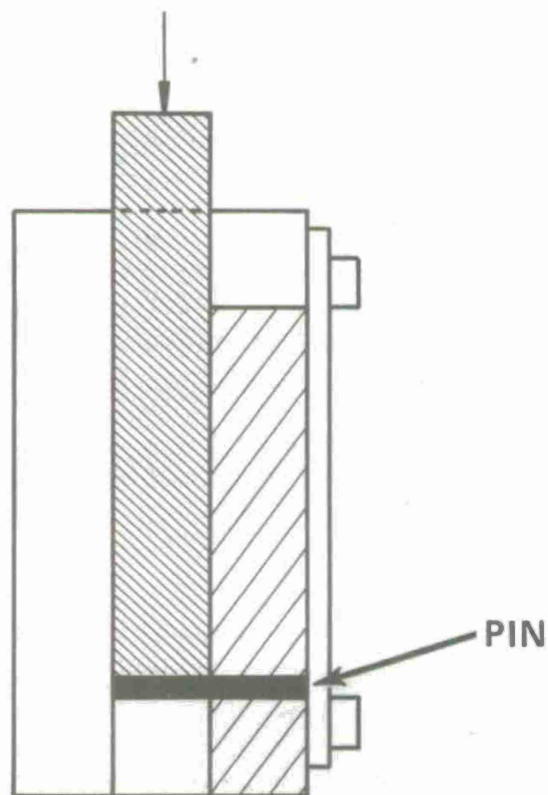


Figure 4. Solid Pin Configuration

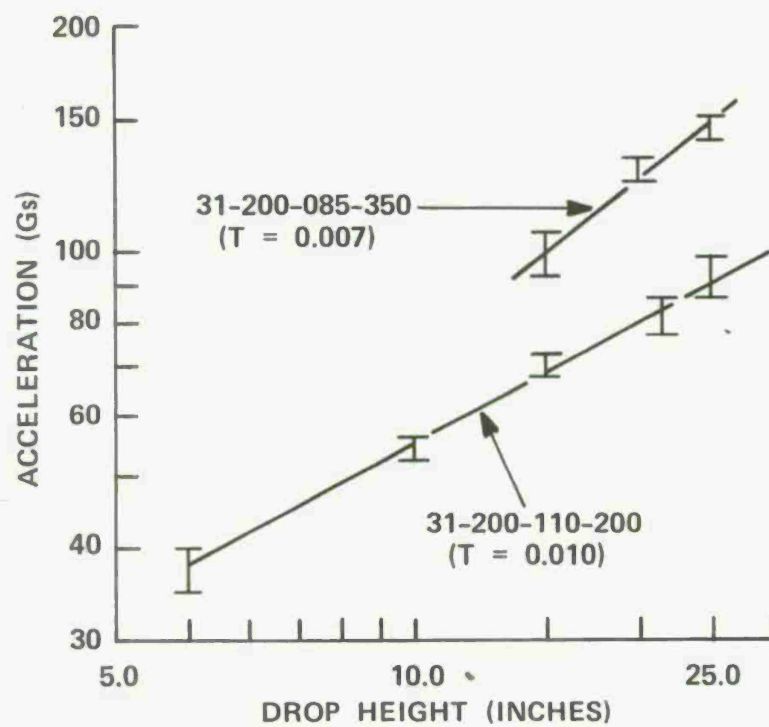


Figure 5. Calibration Chart for Shock Mixture

report. The acceleration history of this machine is a half sine wave pulse ($a(t) = a_p \sin\left(\frac{\pi}{\tau}t\right)$). The "dynamic modulus" for the various materials can be estimated by solving the dynamic equation for the Elastic-Viscoelastic model when excited by the wave pulse, the solution of which is given in appendix A. Note that for loads of short duration, the modulus is dependent on the viscous constant rather than both material constants (elastic and viscous). The stress on a flat plate can then be found by solving the equation of motion for the flat plate given in appendix B. The stress on a solid pin can also be found by solving the equation of motion for that geometry, the solution of which is analogous to that for a flat plate, but with different boundary conditions ($\alpha = \frac{\pi}{2\Lambda}$ and $p = \frac{\pi c}{2\Lambda}$).

V. RESULTS.

The results of the dynamic bench test effort are given in table 1. This work involved five material-geometry combinations and because of the go-no-go nature of the test setup only upper and lower strength values are reported. The Elastic-Viscoelastic-model predictions for all five combinations are also given in table 1. A sample calculation for 1018-flat plate combination is given in appendix C. For comparative purposes, strength predictions based on statically obtained material properties using standard formulas are included in table 1.

Table 1. Comparison Between Measured and Predicted Strengths

Material	Geometry	Bench-tested value	Model prediction	"Static prediction"
1018 Carbon steel	Flat plate (.0153 inch thick)	647–693 psi	697 psi	27 psi
1020 Carbon steel	Solid pin (.125 inch diameter)	1,048–1,082 lb	1,111 lb	388 lb
6061-T6 Aluminum	Flat plate (.025 inch thick)	508–555 psi	486 psi	59 psi
6061-T6 Aluminum	Solid pin (.1875 inch diameter)	1,048–1,119 lb	1,021 lb	714 lb
316 Austenitic steel	Solid pin (.1875 inch diameter)	2,484–2,552 lb	9,232 lb	1,898 lb

The statically obtained true stress-strain diagrams for all four materials are given in figure 6. From these diagrams the dynamic modulus ($\sigma = \frac{\pi}{\tau} \eta$) and velocity of wave propagation $\left[c = \left(\frac{\phi_s}{\rho}\right)^{1/2}\right]$ were calculated and are given in table 2.

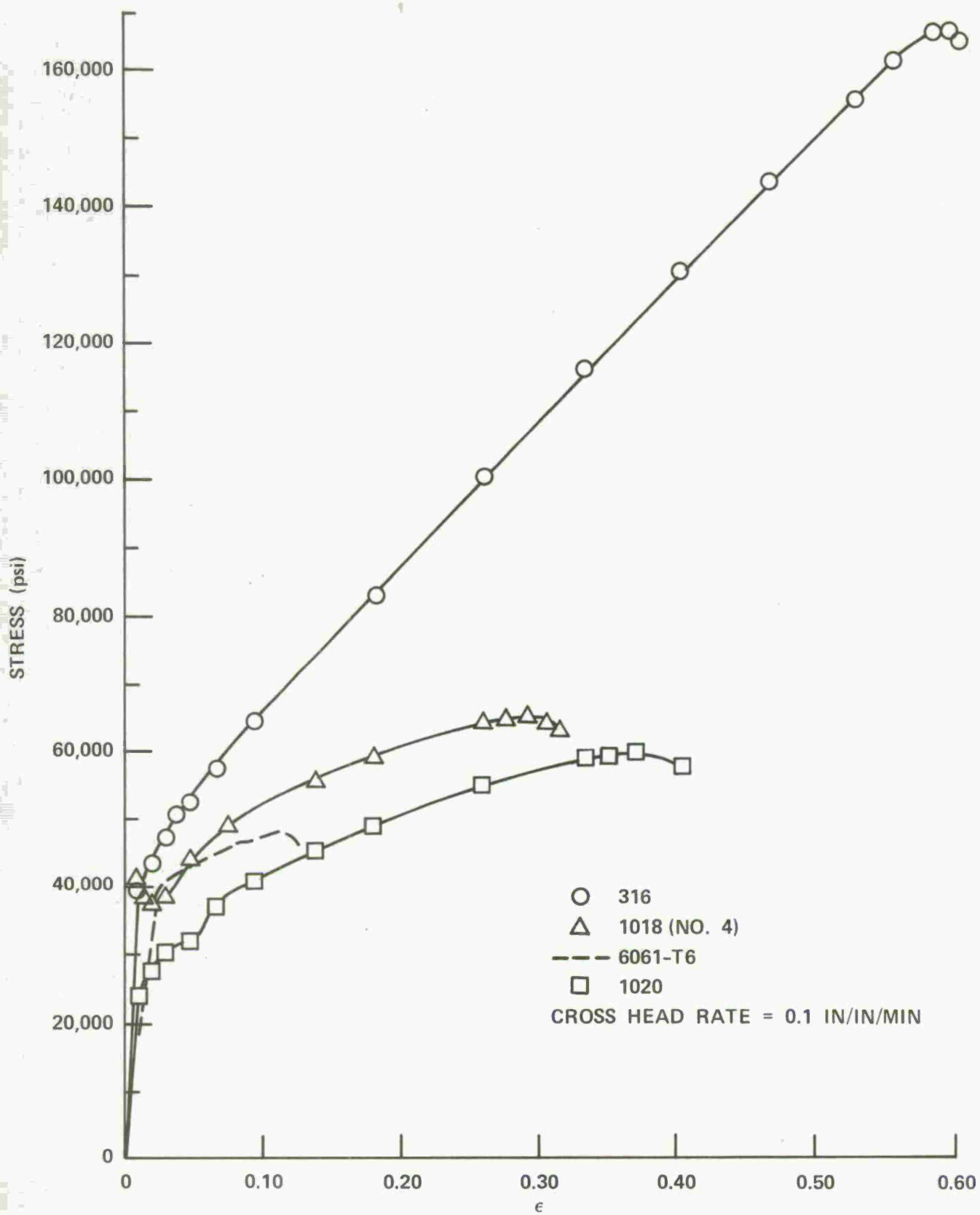


Figure 6. Statically Obtained True Stress-Strain Diagram

Table 2. Calculated Dynamic Material Constants

Material	Geometry	Dynamic modulus	Velocity of wave propagation
		psi	ft
1018 Carbon steel	Flat plate	4,358,000,000	21,367
1020 Carbon steel	Solid pin	4,222,000,000	21,913
6061-T6 Aluminum	Flat plate	1,632,000,000	22,840
6061-T6 Aluminum	Solid pin	1,305,000,000	20,430
316 Austenitic steel	Solid pin	27,390,000,000	54,400

To estimate the precision of the Veripulse VP 400 Shock Machine, a series of instrumented tests was performed on both sets of neoprene compression pads. For pad 31-200-110-200, used in the 1020 steel and 6061-T6 aluminum solid pin tests, the standard deviation was 2.9 g acceleration or 49.0 lb load ($F = ma = 16.9 \text{ lb} \times 2.9g = 49 \text{ lb}$). For pad 31-200-085-350, used in the flat plate and the 316 austenitic steel solid pin tests, the standard deviation was 4.6 g acceleration or 21.2 psi for the plates and 77.7 lb load for the pin.

VI. DISCUSSION OF RESULTS.

For the low carbon steel the Elastic-Viscoelastic model predicted values slightly higher than the upper bench test value for both geometries tested (697 psi versus 693 psi for flat plate and 1,111 lb versus 1,082 lbs for the solid pin). For the 6061-T6 aluminum alloy the Elastic-Viscoelastic model predicted values slightly lower than the lower bench test value for both geometries tested (486 psi versus 508 psi for the flat plate and 1,021 lb versus 1,048 lb for the solid pin). In each case the difference between predicted and measured values are approximately the estimated standard deviation of the shock machine and consequently the differences may not be significant. In comparing the two methods of predicting strength values, it is seen that the predictions based on the Elastic-Viscoelastic model are far superior to those made using standard material strength values and formulas.

For the austenitic stainless steel type 316 the Elastic-Viscoelastic model predicted values considerably higher than the upper bench test value for the solid pin geometry (9,232 lb versus 2,522 lb). The most obvious explanation for this discrepancy is that the model as proposed does not adequately account for the behavior of 316 while it does so for low-carbon steel and the aluminum alloy.

In looking into the structure of the three metals some differences were found. Both the low-carbon steel and 6061-T6 aluminum alloy are basically pure metals doped with small percentages of other atoms to achieve certain desirable mechanical properties. Furthermore, both materials are subject to strain hardening.⁸ Both the presence of impurities in the crystal lattice and the susceptibility to strain hardening tend to produce barriers to metal flow which can be modeled as a viscous drag element.

On the other hand, austenitic stainless steel type 316 is a mixture of iron (Fe), chromium (Cr) and nickel (Ni) where the Cr and Ni atoms substitute for Fe in the crystal lattice. Because of the high nickel content (10 to 14%) this steel does not appreciably strain-harden and is sometimes referred to as free spinning steel.⁹ This relative lack of strain hardening has, in part, been explained by an "easy glide" mechanism where the density of dislocations rises linearly with plastic strain.⁷ In fact, the presence of grain boundaries and impurities would be the only serious barrier to metal flow. Since the austenitic stainless steel does not follow the Elastic-Viscoelastic model previously stated, a new model for this material must be postulated. Based on the above discussion concerning 316, a three-element model shown in figure 7 is proposed. This model involves an elastic element to account for the behavior below the proportional limit followed by a Voigt element to account for grain boundary behavior followed by a friction element to account for the "easy glide" phenomenon. The equation of motion for this system is $\sigma = (2E + \partial)\epsilon + \eta \frac{\partial \epsilon}{\partial t}$ whose equation is analogous to that for the Elastic-Viscoelastic model, i.e., $\phi = \frac{\pi}{\tau} \eta$.

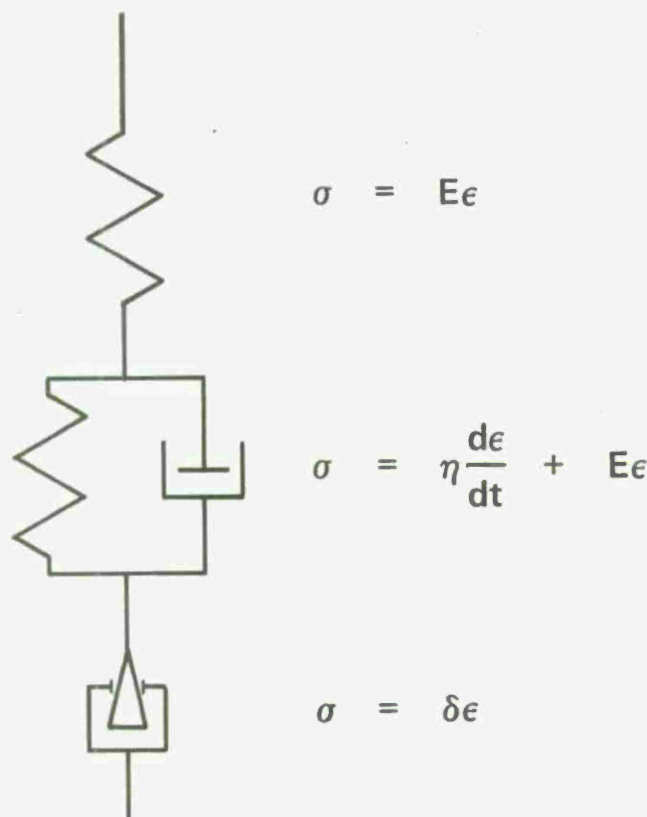


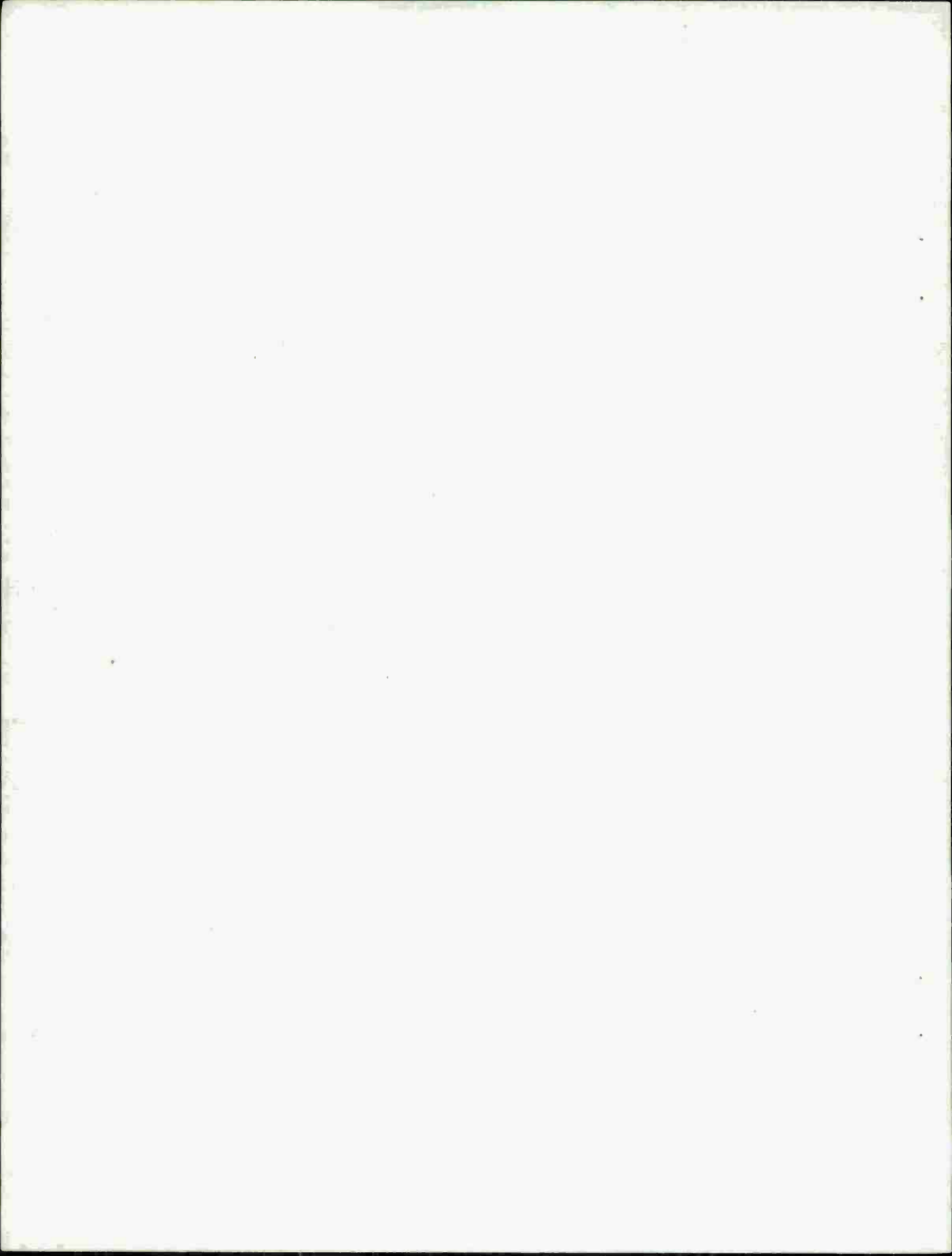
Figure 7. Mechanical Analog for Elastic-Viscoelastic-Plastic Model

In analyzing the true stress-strain diagram it is seen that 316 is elastic from 0 to .01 strain, viscoelastic from .01 to .039 and plastic from .039 to 59. For this case the viscous parameter $\eta = \frac{50,400 - 39,500}{.0033} (1 + .039) = 3.4 \times 10^6$; likewise, $\phi = 5.91 \times 10^8$ and

$c = 13,040$ ft/sec. Substituting this value of c into the equation of motion for the solid pin yields a predicted value of 2,861 lb. In comparing the three methods of predicting strength values it is seen that the prediction based on the three-element Elastic-Viscoelastic-Plastic model is superior to those made using both the standard formulas and two-element Elastic-Viscoelastic Model.

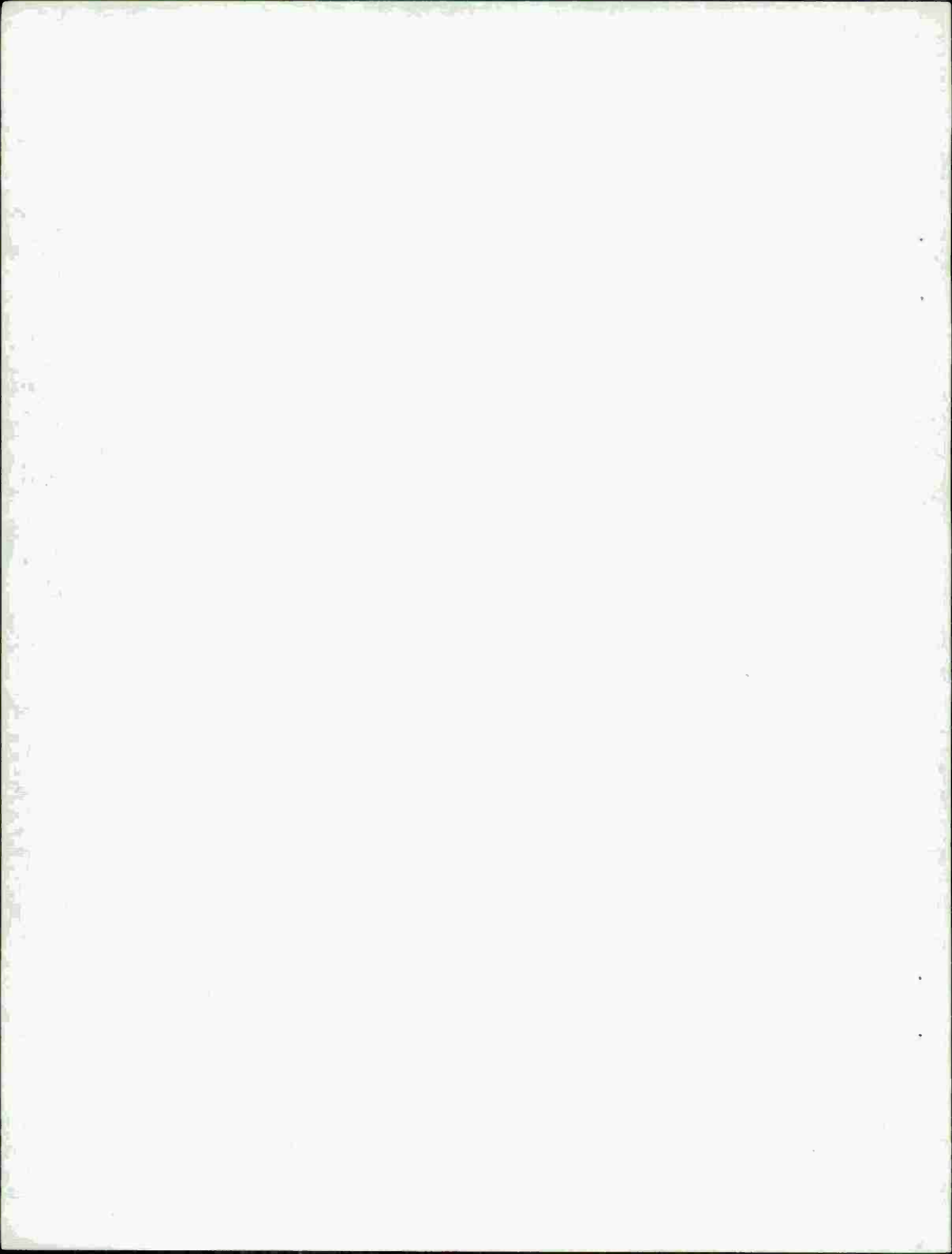
VII. CONCLUSIONS.

The mechanical behavior of materials subjected to impact loads can be predicted by using relatively simple mechanical models to describe their behavior. The model parameters can be obtained from a true stress-strain diagram. In addition, the methodology used in the body of this report can be used to predict accurately the structural integrity of munition components.



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GLOSSARY

σ	stress
ϵ	true strain = $\ln(1 + \epsilon_0)$
ϵ_0	engineering strain
E	modulus of elasticity
η	viscous parameter
δ	plastic modulus
ρ	density
A	shear area
I	moment of inertia
b	thickness
r	radius
ν	Poisson's ratio
ϕ	dynamic shear modulus = $\frac{\phi_E}{2(1+\nu)}$
ϕ_ϵ	dynamic tensile modulus
w	displacement
t	time
Λ	half wave length = $v_0 t$
v_0	impact velocity
a_p	peak acceleration
T	half sine wave period
F	force
M	moment

x	radial component of flat plate
y	circumferential component of flat plate
c_o	initial stress wave propagation velocity = $\left(\frac{\phi}{\rho}\right)^{1/2}$
K_o	radius of gyration = $\left(\frac{I}{A}\right)^{1/2}$
α	wave number
P	pressure
p	angular frequency
C_o	mass $\times$ peak acceleration
m	mass
Ω	factor equal to $(1 + \nu/2\pi)$
D	amplitude of transverse pulse
c_g	group velocity

APPENDIX A

SOLUTION OF THE DYNAMIC EQUATION FOR THE ELASTIC- VISCOELASTIC MODEL WHEN EXCITED BY A HALF-CYCLE SINE PULSE

The equation of motion for the Elastic-Viscoelastic Model is:

$$\eta \frac{\partial \epsilon}{\partial t} + (2E)\epsilon = \sigma \quad (1)$$

rearranging

$$\frac{\partial \epsilon}{\partial t} + \frac{(2E)}{\eta} \epsilon = \frac{\sigma}{\eta}$$

or

$$\frac{\partial \epsilon}{\partial t} + \frac{E'}{\eta} \epsilon = \frac{\sigma}{\eta}$$

taking the Laplace transform of this equation

$$sF(s) + \frac{E'}{\eta} F(s) - f(0) = \frac{1}{\eta} F_{\sigma}(s) \quad (2)$$

the stress generated by the half cycle sine pulse is $m a(t) = m a_p \sin \frac{\pi}{\tau} t$ or

$$F(t) = C_0 \sin\left(\frac{\pi}{\tau} t\right) \quad (3)$$

where

$$C_0 = m a_p$$

taking the Laplace transform of the force equation yields

$$F_{\sigma}(s) = C_0 \frac{\pi}{\tau} \left(\frac{1}{s^2 + \left(\frac{\pi}{\tau}\right)^2} \right) \quad (4)$$

substituting equation (4) into equation (2) yields:

$$sF(s) + \frac{E'}{\eta} F(s) - f(0) = \frac{1}{\eta} C_0 \frac{\pi}{\tau} \left(\frac{1}{s^2 + \left(\frac{\pi}{\tau}\right)^2} \right)$$

rearranging and solving for $F(s)$ gives

$$F(s) = \frac{f(0) + \frac{C_0}{\eta} \frac{\pi}{\tau} \frac{1}{s^2 + (\frac{\pi}{\tau})^2}}{s + \frac{E'}{\eta}}$$

$$F(s) = \frac{f(0)}{s + \frac{E'}{\eta}} + \frac{C_0}{\eta} \left(\frac{\pi}{\tau} \right) \left[\frac{1}{(s + \frac{E'}{\eta}) (s^2 + (\frac{\pi}{\tau})^2)} \right] \quad (5)$$

The time dependent strain is found by taking the inverse transform of the above equation:

$$\epsilon(t) = \mathcal{L}^{-1}_{(s)} F(s) = \epsilon_0 \mathcal{L}^{-1} \left[\frac{1}{s + \frac{E'}{\eta}} \right] + \frac{C_0}{\eta} \left(\frac{\pi}{\tau} \right) \mathcal{L}^{-1} \left[\frac{1}{(s + \frac{E'}{\eta}) (s^2 + (\frac{\pi}{\tau})^2)} \right]$$

If we neglect the residual strain on the system prior to the application of the load ($\epsilon_0 = 0$) the above expression simplifies to

$$\epsilon(t) = \frac{C_0}{\eta} \frac{\pi}{\tau} \mathcal{L}^{-1} \left[\frac{1}{(s + \frac{E'}{\eta}) (s^2 + (\frac{\pi}{\tau})^2)} \right]$$

expanding the fraction in bracket by partial fractions yields:

$$\epsilon(t) = \frac{C_0}{\eta} \left(\frac{\pi}{\tau} \right) \mathcal{L}^{-1} \left[\frac{K_1}{s + \frac{E'}{\eta}} + \frac{K_2}{s + i(\frac{\pi}{\tau})} + \frac{K_3}{s - i(\frac{\pi}{\tau})} \right]$$

performing the inverse transform yields:

$$\epsilon(t) = \frac{C_0}{\eta} \left(\frac{\pi}{\tau} \right) \left[K_1 e^{-\left(\frac{E'}{\eta}\right)t} + K_2 e^{-i\left(\frac{\pi}{\eta}\right)t} + K_3 e^{i\left(\frac{\pi}{\eta}\right)t} \right] \quad (6)$$

where

$$i = \sqrt{-1}$$

$$K_1 = \frac{1}{\left(\frac{E'}{\eta}\right)^2 + \left(\frac{\pi}{\tau}\right)^2}$$

$$K_2 = \frac{1}{2\frac{\pi}{\tau} \left(-\frac{\pi}{\tau} + \frac{E'}{\eta}\right)}$$

$$K_3 = \frac{1}{2\frac{\pi}{\tau} \left(-\frac{\pi}{\tau} + i \frac{E'}{\eta}\right)}$$

substituting these constants into equation (6) yields

$$\epsilon(t) = \frac{C_0}{\eta} \left(\frac{\pi}{\tau}\right) \left[\frac{e^{-\left(\frac{E'}{\eta}\right)t}}{\left(\frac{\pi}{\tau}\right)^2 + \left(\frac{E'}{\eta}\right)^2} + \frac{e^{-i\left(\frac{\pi}{\tau}\right)t}}{2\frac{\pi}{\tau} \left(-\frac{\pi}{\tau} - i \frac{E'}{\eta}\right)} + \frac{e^{i\left(\frac{\pi}{\tau}\right)t}}{2\frac{\pi}{\tau} \left(-\frac{\pi}{\tau} + i \frac{E'}{\eta}\right)} \right]$$

rearrangement along with using the relations; $\cos \frac{\pi}{\tau} t = 1/2 \left(e^{i\left(\frac{\pi}{\tau}\right)t} + e^{-i\left(\frac{\pi}{\tau}\right)t} \right)$ and

$\sin\left(\frac{\pi}{\tau}\right)t = -i(1/2) \left(e^{i\left(\frac{\pi}{\tau}\right)t} - e^{-i\left(\frac{\pi}{\tau}\right)t} \right)$ yield the final expression for the strain:

$$\epsilon(t) = \frac{C_0}{\eta} \left[\frac{\frac{E'}{\eta} \sin\left(\frac{\pi}{\tau}\right)t - \frac{\pi}{\tau} \cos\left(\frac{\pi}{\tau}\right)t + \frac{\pi}{\tau} e^{-\left(\frac{E'}{\eta}\right)t}}{\left(\frac{\pi}{\tau}\right)^2 + \left(\frac{E'}{\eta}\right)^2} \right] \quad (7)$$

The "dynamic" modulus is

$$\phi = \frac{\sigma(t)}{\epsilon(t)} = \frac{C_0 \sin\left(\frac{\pi}{\tau}\right)t}{\frac{C_0}{\eta} \left[\frac{\frac{E'}{\eta} \sin\left(\frac{\pi}{\tau}\right)t - \frac{\pi}{\tau} \cos\left(\frac{\pi}{\tau}\right)t + \frac{\pi}{\tau} e^{-\left(\frac{E'}{\eta}\right)t}}{\left(\frac{\pi}{\tau}\right)^2 + \left(\frac{E'}{\eta}\right)^2} \right]}$$

rearrangement yields

$$\phi = \left[\frac{\left(\frac{\pi}{\tau}\right)^2 + \left(\frac{E'}{\eta}\right)^2 \eta \sin\left(\frac{\pi}{\tau}\right)t}{\frac{E'}{\eta} \sin\left(\frac{\pi}{\tau}\right)t - \frac{\pi}{\tau} \cos\left(\frac{\pi}{\tau}\right)t + \frac{\pi}{\tau} e^{-\left(\frac{E'}{\eta}\right)t}} \right]$$

In this case, the burst strength was to be measured and the maximum value of the "dynamic modulus" was of interest. This occurs when $\sin\left(\frac{\pi}{\tau}\right)t = 1$ or $t = \frac{\tau}{2}$, substituting in this value into ϕ gives

$$\phi = \left[\frac{\left(\frac{\pi}{\tau}\right)^2 + \left(\frac{E'}{\eta}\right)^2}{\left(\frac{E'}{\eta} + \frac{\pi}{\tau} e^{-\left(\frac{E'}{\eta}\right)\frac{\tau}{2}}\right)} \right] \eta \quad (8)$$

when

$$\frac{\pi}{\tau} \gg \frac{E'}{\eta} \text{ and } e^{-\left(\frac{E'}{\eta}\right)\frac{\tau}{2}} \cong 1$$

equation (8) can be simplified to

$$\phi = \left(\frac{\pi}{\tau}\right) \eta$$

APPENDIX B

PROPAGATION OF STRESS WAVE IN A FLAT PLATE

Figure B shows the forces acting on a small element of the plate of radial dimension Δx . In this system the bending moment M is balanced by shearing forces f acting perpendicular to the plate thickness.

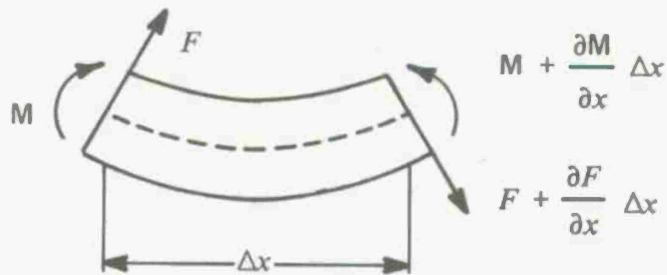


Figure B

From Newton's second law, the equation of motion for the plate is:

$$\Delta x \rho A \frac{\partial^2 w}{\partial t^2} = f + \frac{\partial f}{\partial x} \Delta x - f$$

or

(1)

$$\rho A \frac{\partial^2 w}{\partial t^2} = \frac{\partial f}{\partial x}$$

the relationship between shearing force and bending moment is

$$f = \frac{\partial M}{\partial x}$$

where

$$M = \phi I \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

or

$$f = \phi I \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \equiv \phi I \frac{\partial}{\partial x} \left(\frac{\partial \epsilon_x}{\partial x} + \frac{\partial \epsilon_y}{\partial y} \right) \quad (2)$$

where w is the displacement, x is the radial component, y is the circumferential component and the strain is defined as equal to

$$\epsilon_x = \frac{\partial w}{\partial x}, \quad \epsilon_y = \frac{\partial w}{\partial y}$$

The relationship between the radial and circumferential components are:

$$\frac{\epsilon_y}{\epsilon_x} = \nu$$

and for the circular flat plate

$$y = 2\pi x$$

therefore

$$\frac{\partial \epsilon_y}{\partial y} = \nu \frac{\partial \epsilon_x}{\partial y} = \nu \frac{\partial \epsilon_x}{\partial x} \frac{\partial x}{\partial y} = \frac{\nu}{2\pi} \frac{\partial \epsilon_x}{\partial x}$$

substituting the preceding equation into equation (2) yields

$$f = \phi I \frac{\partial}{\partial x} \left(\frac{\partial \epsilon_x}{\partial x} + \frac{\nu}{2\pi} \frac{\partial \epsilon_x}{\partial x} \right) = \phi I \left(1 + \frac{\nu}{2\pi} \right) \frac{\partial^2 \epsilon_x}{\partial x^2}$$

or

$$f = \phi I \left(1 + \frac{\nu}{2\pi} \right) \frac{\partial^3 w}{\partial x^3} \quad (3)$$

substituting into the equation of motion (1) yields

$$\frac{\partial^2 w}{\partial t^2} = \left(\frac{\phi}{\rho}\right)\left(\frac{1}{A}\right)\left(1 + \frac{\nu}{2\pi}\right)\frac{\partial^4 w}{\partial x^4}$$

let

$$c_0^2 = \left(\frac{\phi}{\rho}\right), K^2 = \left(\frac{1}{A}\right), \Omega^2 = \left(1 + \frac{\nu}{2\pi}\right)$$

substituting these parameters into the equation of motion yield

$$\frac{\partial^2 w}{\partial t^2} = c_0^2 K^2 \Omega^2 \frac{\partial^4 w}{\partial x^4}$$

The solution to the above equation is $w = D\cos(pt - \alpha x)$ or $w = D\sin(pt - \alpha x)$ which are the equations for a transverse pulse where $\alpha = \frac{2\pi}{\Lambda}$, $p = \frac{2\pi c}{\Lambda}$ and Λ is the wave length of the pulse and c is the phase velocity of the stress wave.

Substituting either one of the above equations for w into the equation of motion gives,

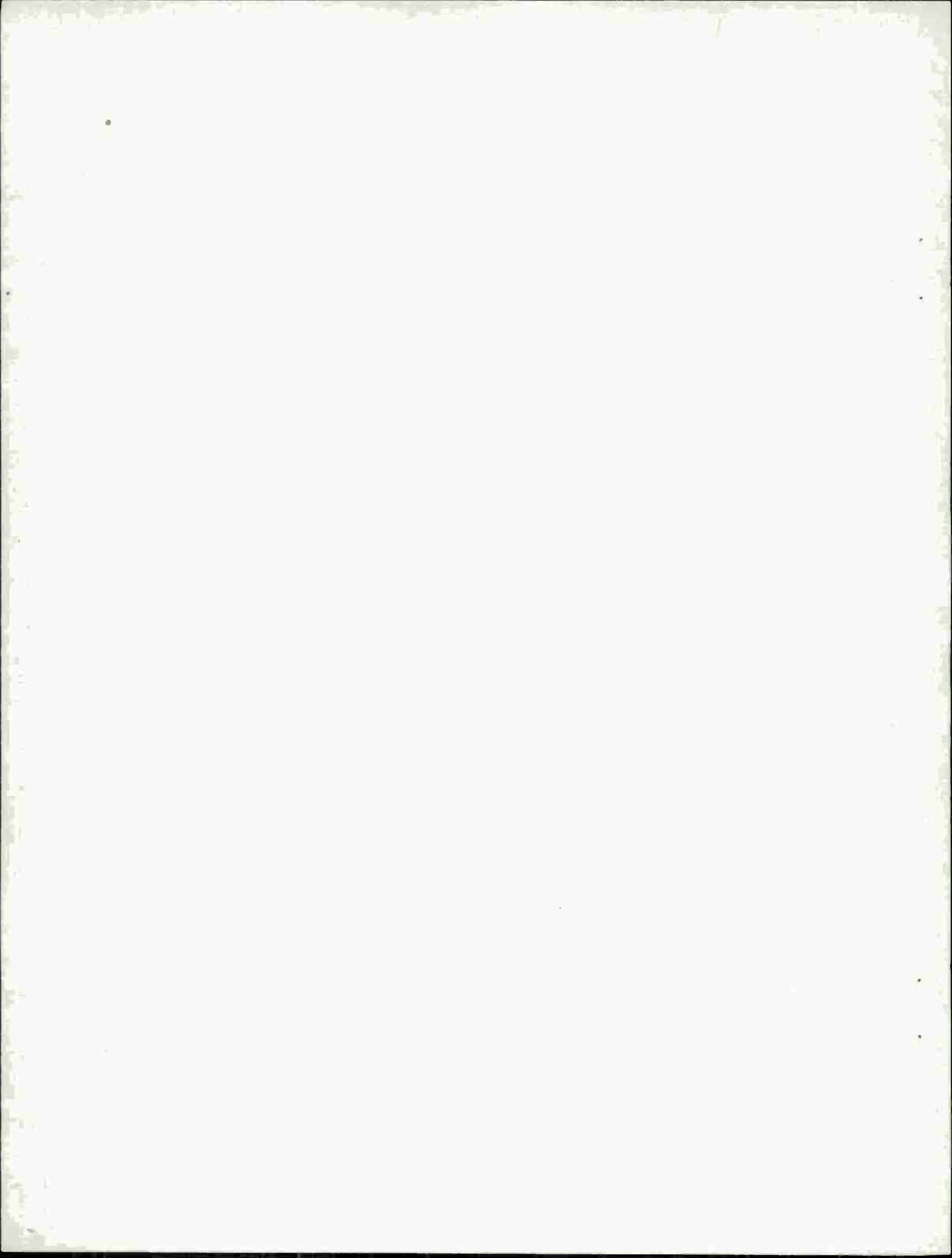
$$p^2 = c_0^2 K^2 \alpha^4 \Omega^2$$

$$c = c_0 \left(\frac{2\pi K \Omega}{\Lambda} \right)$$

From the equation of continuity, that is, the displacement of the disk must equal the total elongation of the disk or $v t = \epsilon x$. Rearranging this equation give $v = \frac{x}{t} \epsilon$ where Von Karman showed that $\frac{x}{t}$ is equal to the velocity of the wave propagation in the plastic region. In this case the velocity of wave propagation (C_g) refers to the velocity with which a packet of waves is propagated, the wavelengths of which are close to Λ . This is related to the phase velocity (c) by $C_g = 2c$.

Finally at the point of rupture

$$\sigma = \phi \epsilon = \phi \frac{v}{c_g} = c_0 v \rho \left(\frac{\Lambda}{4\pi K \Omega} \right)$$



APPENDIX C

CALCULATION OF THE AISI 1018 LOW-CARBON STEEL

CIRCULAR FLAT PLATE

From the stress-strain diagram (figure 6)

$$\Delta\sigma = 65,000 - 41,700 = 23,300 \text{ psi}$$

$$\epsilon_{O,\nu} = (\epsilon_{O,\text{total}} - \epsilon_{O,E})$$

$$\epsilon_{O,\text{total}} = 0.30$$

$$\epsilon_{O,E} = 0.02$$

$$\epsilon_{O,\nu} = 0.30 - 0.02 = 0.28$$

$$\frac{\partial \epsilon_O}{\partial t} = \text{cross head rate} \times \text{distance between jaws}$$

$$= \frac{0.1 \text{ in/in/min}}{1 \text{ in}} \times 2 \text{ in} \times \frac{1 \text{ min}}{60 \text{ sec}} = 0.00333 \text{ sec}^{-1}$$

From Calibration Chart (figure 5, pad 31-200-085-350) half sine wave period, $\tau = 0.007 \text{ sec}$.

From test data

$$\text{Acceleration at shear} = 145 \text{ g}$$

$$\text{Disk radius} = 1 \text{ in}$$

$$\text{Disk thickness} = 0.0153 \text{ in}$$

Handbook Data

$$E = 30,000,000 \text{ psi}$$

$$\rho = 0.284 \text{ lb/in}^3$$

$$\nu = 0.29$$

(1) Calculation of dynamic modulus:

The viscous parameter η , was calculated by substituting the values given in figure 6 into the equation given in figure 2 of the body of this report, that is:

$$\eta = \frac{23,300}{.00333} (1 + 0.28) = 8,956,200 \frac{\text{lb-sec}}{\text{in}^2}$$

$$\frac{E}{\eta} = \frac{30,000,000}{8,956,200} = 3.35 \text{ sec}^{-1}$$

$$\frac{\pi}{\tau} = \frac{3.1416}{0.007} = 448.8 \text{ sec}^{-1}$$

$$\frac{E}{\eta} \frac{\tau}{2} = 3.35 \times \frac{0.007}{2} = 0.0117$$

since $\frac{\pi}{\tau} \gg \frac{E}{\eta}$ and $e^{-0.0117} \cong 1$, the simplified form was used,

$$\phi_E = \left(\frac{\pi}{\tau}\right) \eta = 448.8 \times 8,956,200 = 4,018,500,000 \text{ psi}$$

However, the plate was in shear while the data were from tensile tests. This was accounted for by using the following relation:

$$\phi_s = \frac{\phi_E}{2(1+\nu)} = \frac{4,018,500,000}{2(1+0.29)} = 1,558,000,000 \text{ psi}$$

or

$$224,352,000,000 \text{ psf (lb/ft}^2\text{)}$$

(2) Calculation of stress wave parameters where:

- the impact velocity* $v = a_p \frac{\tau}{\pi}$, in this case $a_p = 145 \text{ gs}$ acceleration

or

* See Appendix D for response characteristics of the pulse

$$v = 145 \text{ g} \times 32.17 \text{ ft/sec}^2 \times \frac{(0.007 \text{ sec})}{3.1416}$$

$$= 10.39 \text{ ft/sec}$$

- the half wave length $\Lambda = v\tau$

$$\Lambda = 10.39 \text{ ft/sec} \times 0.007 \text{ sec} = .0727 \text{ ft}$$

$$\text{- the radius of gyration } K = \left(\frac{I}{A}\right)^{1/2} = \left(\frac{\rho \pi b \frac{r^4}{4}}{2\pi br}\right)^{1/2} = \left(\frac{\rho r^3}{8}\right)^{1/2}$$

The radius of gyration is

$$K = \left(\frac{0.284 \times 1^3}{8}\right)^{1/2} = 0.188$$

$$-\Omega = \left(1 + \frac{\nu}{2\pi}\right)^{1/2} = \left(1 + \frac{0.29}{2\pi}\right)^{1/2} = 1.023$$

$$\begin{aligned} \text{- the propagation velocity } c_o &= \left(\frac{\phi_s}{\rho}\right)^{1/2} \\ &= \left(\frac{224,352,000,000}{0.284 \times 12^3}\right)^{1/2} \\ &= 21,381 \text{ ft/sec} \end{aligned}$$

(3) Calculation of plate stress and pressure – from Appendix B:

$$\sigma = c_o \rho v \left(\frac{\Lambda}{4\pi K \Omega}\right)$$

$$= 21,381 \times 490.75 \times 10.39 \times \frac{0.0727}{4 \times 3.1416 \times 0.188 \times 1.023}$$

$$= 3,279,396 \text{ psf or } 22,774 \text{ psi stress}$$

The pressure P is equal to the total load (stress $\times$ circumference) divided by the total area of the plate, $P = \frac{\sigma \times 2\pi r b}{\pi r^2} = \sigma \left(\frac{2b}{r} \right)$

for a 0.0153-inch-thick plate having a 1-inch radius:

$$P = 22,774 \left(\frac{2 \times 0.0153}{1} \right) = 697 \text{ psi pressure.}$$

APPENDIX D

THE RESPONSE CHARACTERISTICS OF THE HALF-CYCLE SINE PULSE

ACCELERATION

$$a(t) = a_p \sin \left(\frac{\pi}{\tau} \right) t$$

taking the Laplace transform

$$s F(v) - f(0) = a_p \frac{\pi}{\tau} \left[\frac{1}{\left(s^2 + \frac{\pi}{\tau} \right)^2} \right]$$

$$F(v) = \frac{f(0)}{s} + a_p \frac{\pi}{\tau} \left[\frac{1}{\left(s \left(s^2 + \frac{\pi}{\tau} \right)^2 \right)} \right]$$

taking the inverse transform

$$v(t) = v_0 t + a_p \frac{\pi}{\tau} \frac{1}{\left(\frac{\pi}{\tau} \right)^2} \left(1 - \cos \left(\frac{\pi}{\tau} \right) t \right)$$

$$= v_0 t + a_p \left(\frac{\tau}{\pi} \right) \left(1 - \cos \frac{\pi}{\tau} t \right)$$

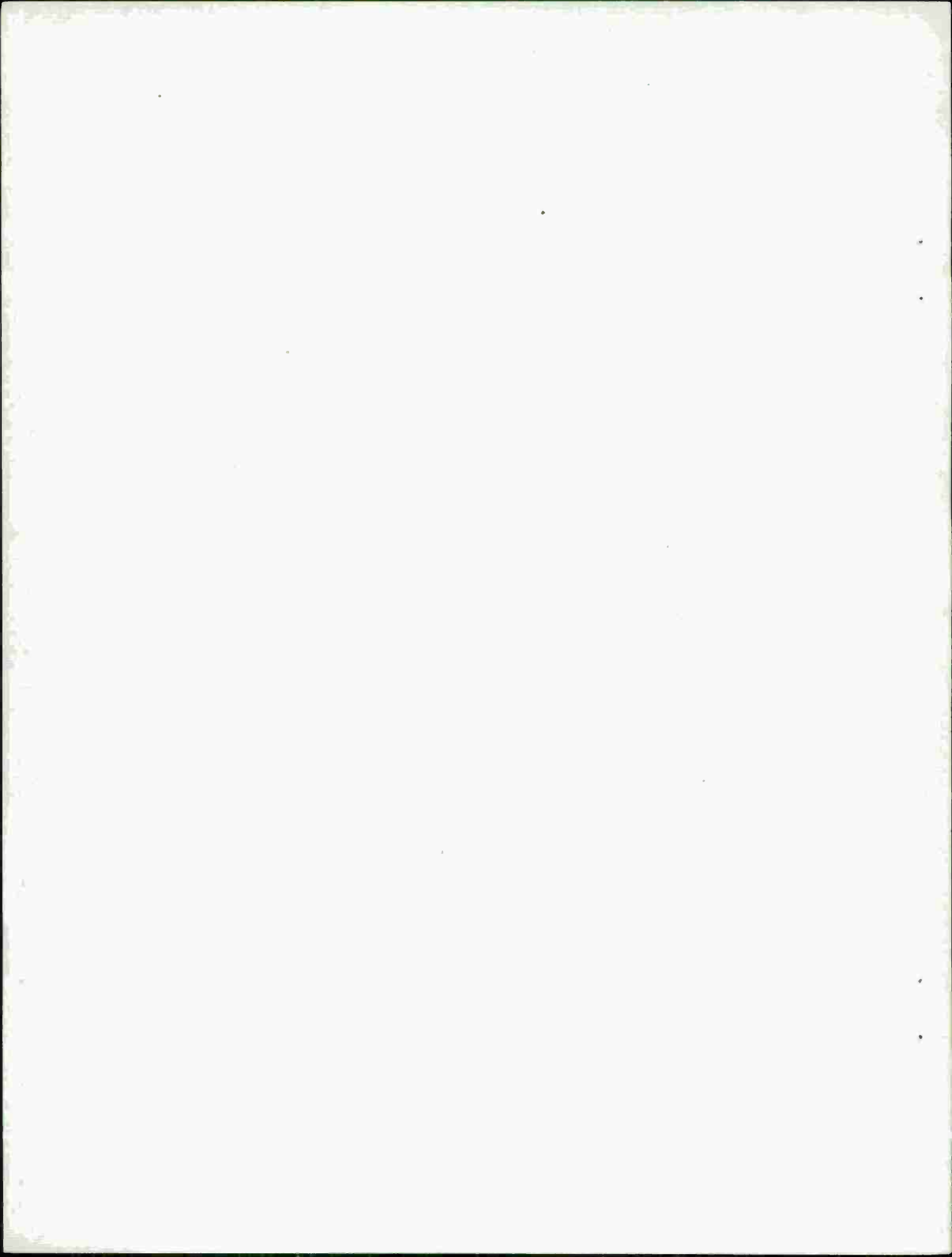
let

$$v_0 = 0$$

$$v(t) = a_p \left(\frac{\tau}{\pi} \right) \left(1 - \cos \frac{\pi}{\tau} t \right)$$

the velocity is maximum at $t = \frac{\tau}{2}$

having $v_m = a_p \left(\frac{\tau}{\pi} \right)$.



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