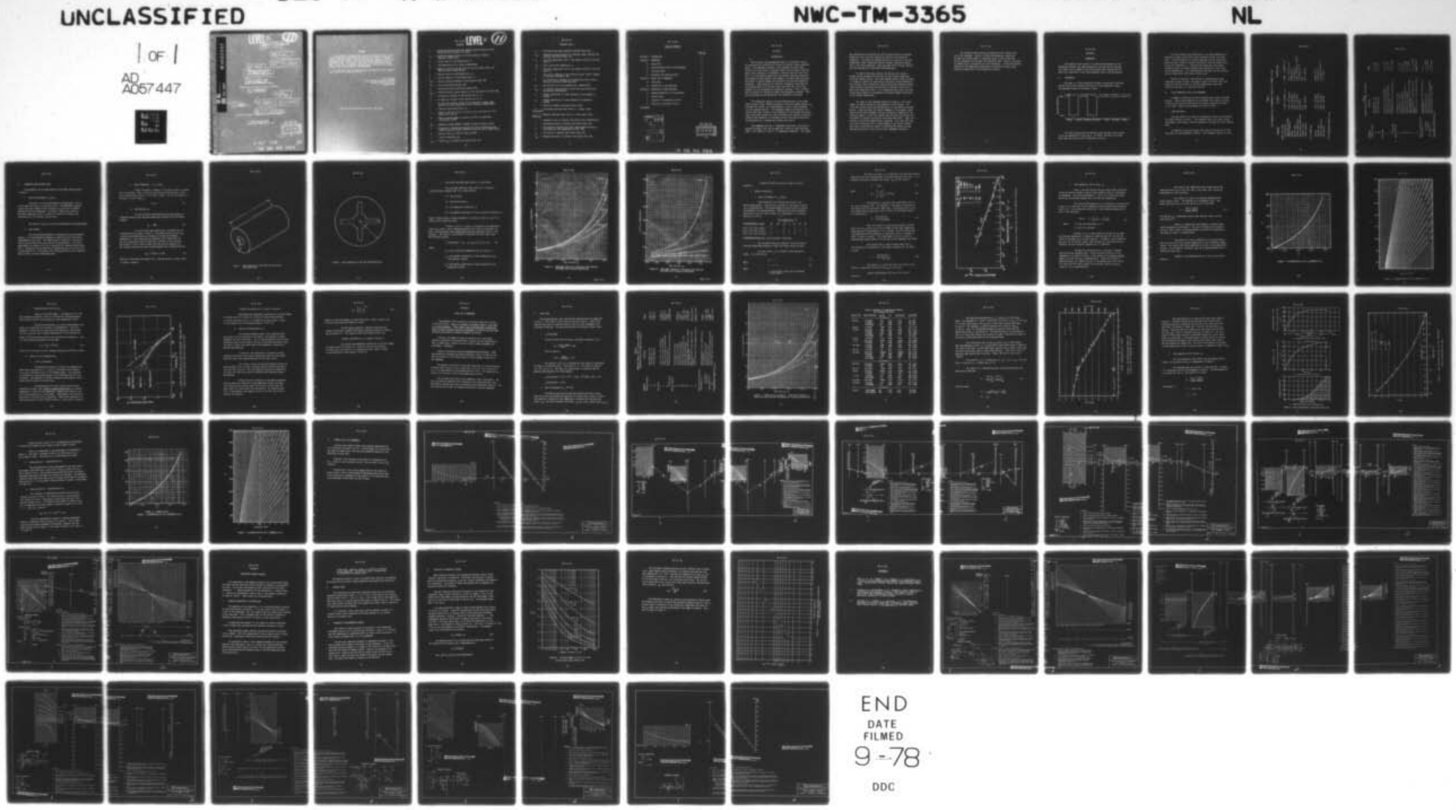
AD-A057 447 AEROJET SOLID PROPULSION CO SACRAMENTO CALIF F/G 21/9.2
HANDBOOK, STRUCTURAL DESIGN NOMOGRAPH FOR THERMAL CYCLING OF TA--ETC(U)
DEC 77 K W BILLS N00123-76-C-1283
UNCLASSIFIED NWC-TM-3365 NL

1 OF 1
AD
A057447



END
DATE
FILMED
9-78
DDC

NWC Technical Memorandum 3365

NWC

TM-3365

HANDBOOK,

STRUCTURAL DESIGN MONOGRAPH FOR THERMAL CYCLING OF TACTICAL ROCKET PROPELLANTS.

by

K. W. Bills, Jr
Aerojet Solid Propulsion Company

Prepared for the Ordnance Systems Department

December 1977

76p.

N00123-76-C-1283

[redacted]

F31300

WF31300300

Approved for public release; distribution unlimited.

NAVAL WEAPONS CENTER
China Lake, California 93555

FOREWORD

This handbook was prepared for use at the working level as an analytical device for predicting the conditions for failure of tactical rocket propellant grains. The work on which this handbook is based was performed by the Aerojet Solid Propulsion Company under Navy contract N00123-76-C-1283. The work was sponsored by the Naval Weapons Center (NWC), China Lake, California, and supported by the Naval Air Systems Command under AirTask A03W3300/008B/6F31300300.

Mr. Ron Vetter was the Navy Technical Coordinator and has reviewed this document for technical accuracy.

R. A. MILLER
Head, Propulsion Systems Division
Ordnance Systems Department
3 January 1978

NWC TM 3365, published by Code 327, 100 copies

LEVEL II

11

GLOSSARY

A = Normalized damage fraction per thermal cycle (of motor or SEC) multiplied by a constant, min./cycle

A_s = Normalized damage fraction in failure testing of tensile specimen, dimensionless

a = Inside radius of case-bonded grain, cm

a_T = Time-temperature shift factor, dimensionless

B = Negative reciprocal of the slope of log true stress versus log time-to-failure, dimensionless

b = Outside radius of case-bonded grain, cm

D = Outside diameter of case-bonded grain, cm

$E(1)$ = Tensile relaxation modulus at one minute at 25°C, MPa

$E(t)$ = Tensile relaxation modulus, MPa

E_e = Equilibrium tensile relaxation modulus, MPa

E_{eff} = Effective biaxial tensile modulus at the inner bore of a grain, MP_a

f_1 = An empirical constant in the relation for a_T, °C

f_2 = An empirical constant in the relation for a_T, °C

K = An empirical constant related to the reduction in grain inner-bore strain due to propellant strain dilatation, dimensionless

L = Length of case-bonded grain, cm

N = Number of thermal cycles to failure at the inner-bore of a case bonded grain, cycles

$\bar{N}$ = Mean or average number of cycles to failure in logarithmic distribution, cycles

$\bar{N}_g$ = Geometric average number of thermal cycles to failure, cycles

q = An empirical constant that accounts for strain softening of the propellant and for moduli gradients within the grain, dimensionless

R = Ratio of log a_T at -40°C to log a_T at 60°C

t = Time in relaxation, min.

t_{tm} = Time to σ_{tm} in constant rate tensile test, min.

GLOSSARY (Cont.)

t_f = Time-to-failure under constantly applied stress, min.

t_{fe} = Equivalent time-to-failure at a constant stress, derived from constant rate tensile data, min.

T_L = The lower temperature limit in the thermal cycling of the test motor, °C

T_{SF} = Grain strain-free temperature, °C

T_u = The upper temperature limit in the thermal cycling of the test motor, °C

V = The elastic component of the effective biaxial tensile modulus at the grain inner-bore, MPa

W = The viscoelastic component of the effective biaxial tensile modulus at the grain inner-bore, MPa

w_f = Web fraction of the case-bonded grain, dimensionless

w_{fe} = An effective web fraction for grain with non-circular bore perforation, dimensionless

α_c = Thermal coefficient of linear expansion of case material, cm/cm/K

α_p = Thermal coefficient of linear expansion of propellant, cm/cm/K

ε = Strain in constant rate tensile test, cm/cm

ε_θ (Calculated) = Calculated inner-bore hoop strain for a motor, cm/cm

ε_θ (Measured) = Measured inner-bore hoop strain in a test motor, cm/cm

λ = Elongation ratio in constant rate tensile test, dimensionless

σ = Engineering stress in constant rate tensile test, MPa

σ_o = The constantly applied true stress that produces failure at one minute at a test temperature of 25°C, MPa

σ_t = True stress in constant rate tensile test, MPa

σ_{tm} = Maximum true stress in constant rate tensile test, MPa

TABLE OF CONTENTS

SECTION 1

INTRODUCTION

The Structural Design Nomograph (SDN) is an analytical device for predicting the conditions for failure of tactical rocket propellant grains. The nomograph discussed in this handbook considers these failures to be the result of repeated thermal cycling (under conditions where stress-ratcheting is prevented). The most practical use of the SDN is as a preliminary design tool that permits rapid and inexpensive calculations by both chemists and engineers, from which they can assess the effects of design changes and variations in propellant mechanical properties. These analyses are relatively inexpensive since they can be performed about 20 to 50 times faster than the input time to the computer for the corresponding viscoelastic analyses. Also, no mathematical talents are required to conduct the nomographic analyses. They can be performed by non-engineering, non-mathematical personnel.

The nomographic analysis involves approximations to the highly sophisticated, linear, thermo-viscoelastic stress and damage analyses. Although this analysis is an approximation, it has a major advantage over the computer analyses. The nomograph contains two empirical correction terms that account for real behaviors of solid propellant grains. These corrections account for strain dilatation (an increase in material volume as the propellant is stretched) and the associated softening of the propellant (as it becomes more spongy with strain).

The nomograph utilizes 11 independent design, test, and material property variables, plus the two empirical correction terms described above. This comes to a total of 13 independent parameters. This number was obtained after specifying the case and grain density and thermal properties which were held constant, as were the case mechanical properties. The predictive method specifically accounts for cycling temperature limits, propellant tensile strength, relaxation modulus and a_T, the grain dimensions, inner-bore hoop strain and web fraction, plus the two empirical corrections mentioned above.

To simplify the motor testing, the SDN was built around a thermal cycling schedule with 24 hours at each storage temperature (a 48 hour thermal cycle). After two complete thermal cycles the motor is allowed to recover for three days at the upper storage temperature. This recovery step is required to reverse the stress ratcheting effect (grain stresses increasing from one cycle to the next) that occurs in most solid propellant grains. It is recognized that by this plan the larger motors (above 18 cm diameter) do not reach thermal equilibrium at the storage temperatures.

The layout of the nomograph requires six charts. This large number is required because of the complexity of the problem. The first four charts provide the determinations for the effective biaxial modulus. Chart one gives the elastic component of the modulus, while charts two and three together yield the viscoelastic component of the effective modulus. Chart four combines the two terms and accounts for the effects of thermal lag upon the inner-bore hoop strain. Chart five provides a simplified damage analysis procedure. The sixth chart provides the calculations and is a good summary of the overall problem. This chart is particularly useful in evaluating the effects of hypothesized variations in one or more of the independent variables.

The following three sections are designed to be a guide to the use of the nomograph. Section 2 defines the parameters required by the SDN, the minimum number of laboratory tests to obtain them, and how the parameters are obtained from laboratory data. Section 3 provides an example set of calculations on a real propellant. In the final section (Section 4) the nomographic predictions are assessed in terms of the nature and statistics of solid propellant failures.

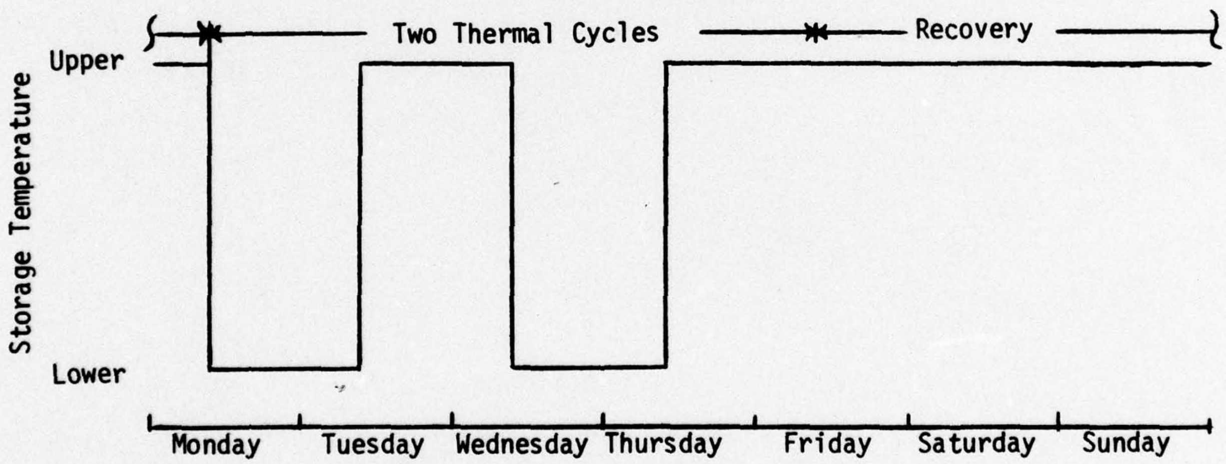
SECTION 2

PARAMETERS

The purpose of this section is to define the parameters of the nomographic analysis and, where appropriate, how they may be obtained. This begins with a summary of the testing constraints imposed upon the motor and a tabulation of the fixed parameters used in the SDN analysis.

A. CONSTRAINTS

All motor, or strain evaluation cylinder, testing must follow the thermal cycling test schedule given below for the nomograph to apply. The nomograph assumes circulating air ovens.



This plan produces only two thermal cycles per week, with 24 hours at the low temperature in each cycle. Of course, the days of the week when these two cycles occur are completely arbitrary.

The long recovery time (three days) at the high temperature is required to anneal any possible stress-ratcheting effect that might occur. Stress-ratcheting is an effect observed in solid propellants where the grain stresses increase significantly from one thermal cycle to the next (Reference 1). Experience has shown that this stress-ratcheting effect is readily annealed upon storage for a short time at high temperatures. This observation was the basis for the recovery period allowed after every second thermal cycle.

Because of the very strong effects of condensed water upon propellant surface failures, care must be taken to prevent frost from forming on the bore surface at low test temperatures, even while inspecting the motor for inner-bore cracking.

B. FIXED PARAMETERS USED IN THE NOMOGRAPH

A number of propellant and case parameters were fixed in the SDN analyses. These fixed parameters are considered to be typical of those of most of the tactical rocket motors and strain evaluation cylinders in use today. These fixed parameters are given in Table 1.

C. USER DATA NEEDS

The data required to conduct the nomographic analysis are tabulated and simply defined in Table 2. Four parameter types are listed, thermal environment, grain design, material properties, and empirical grain response data.

In addition to the definitions, this table also serves as an input data sheet for the nomographic analysis. The nomographs require SI units.

TABLE 1

FIXED PARAMETERS USED IN NOMOGRAPHIC ANALYSIS OF THERMAL CYCLING

Parameter	SI Units	English Units
Motor Case, Steel		
Case thickness	$= 0.1758 + 0.03077b^*$, cm	$= 0.0692 + 0.03077b^*$, in.
Coefficient of linear thermal expansion	10.62×10^{-6} cm/cm/K	5.90×10^{-6} in./in./°F
Density	7.833×10^{3} kg/m3	0.2830 lb/in.^{3}
Specific heat	4.857×10^{-2} J/kg·K	0.116 BTu/lb·°F
Thermal conductivity	3.807×10^{1} W/m·K	2.640×10^{2} BTu·in./h·ft^2·°F
Heat transfer coefficient	10.20 W/m^2·K	1.797 BTu/h·ft^2·°F
Poisson's ratio	0.30	0.30
Young's modulus	2.096×10^{5} MPa	30.4×10^{6} psi
Solid Propellant		
Density	1.705×10^{3} kg/m^3	0.0616 lb/in.^{3}
Specific heat	1.319×10^{3} J/kg·K	0.315 BTu/lb·°F
Thermal conductivity	4.847×10^{1} W/m·K	3.361 BTu·in./h·ft^2·°F
Bulk modulus	3.447×10^{3} MPa	5×10^{5} psi

* b is the grain outside radius

TABLE 2

PARAMETERS REQUIRED FOR THERMAL CYCLING ANALYSIS

Parameters	Definitions	Values
Thermal Environment		
T_u	Upper temperature limit	______ °C
T_L	Lower temperature limit	______ °C
T_{SF}	Strain-Free Temperature	______ °C
Grain Design		
a	Inside radius of grain (strain-free)	______ cm
b	Outside radius of grain (strain-free)	______ cm
L	Length of case-bonded grain (strain-free)	______ cm
w_f	Web fraction of grain	______
ε_θ(Calculated)	Calculated maximum inner-bore hoop strain at T_L	______ cm/cm
Material Properties		
a_T	Time-temperature shift factor	
f_1	First constant in a_T function	______ °C
f_2	Second constant in a_T function	______ °C
B	Exponent in stress-time relation	______
E(1)	Relaxation modulus at one minute at 25°C	______ MPa*
E_e	Estimate of equilibrium modulus	______ MPa
σ_o	Stress causing failure at one minute in constant stress testing at 25°C	______ MPa
α_p	Thermal coefficient of linear expansion of the propellant	______ cm/cm/K
α_c	Thermal coefficient of linear expansion of the case	10.62×10^{-6} cm/cm/K
Empirical Grain Response Data		
Subcalculation I: K	Constant defining bore strain reduction due to strain dilatation	______
ε_θ(Calculated)	Calculated bore strain for SEC	______ cm/cm
ε_θ(Measured)	Measured bore strain in SEC	______ cm/cm
Subcalculation II: q	Constant related to grain dilatation softening	______
$\bar{N}_g$	Geometric mean cycles to failure of SEC's	______ cycles

* Conversion MPa = (6.895×10^{-3})(psi)

D. PARAMETERS AND OBTAINING THEM

The parameters are discussed under the four major headings given in Table 2.

1. Thermal Environment, T_U and T_L

The motor is to be cycled between two temperatures, with T_U as the upper limit and T_L as the lower limit. The nomographic analysis assumes the motor thermal cycling to begin at T_U and to be in thermal equilibrium at that temperature. The motor is then shock cycled to the lower temperature limit, T_L, where it is held for 24 hours, then taken back to T_U for 24 hours, and so forth.

The values of T_U and T_L are to be established by the experimenter.

2. Grain Design

The grain design parameters are obtainable from two different sources: (1) from engineering drawings or reports; and (2) from direct measurements on the grain. The former source is required in the case of actual tactical motors with non-circular bore perforations. The latter measurements are normally all that is available for laboratory tests using SEC's. Recognizing the availability of engineering data in the first case and the need for it in the second, the following definitions are limited to those for the circularly perforated grain.

a. Grain Dimensions: a, b, D and L

Figure 1 provides a schematic of the grain, which is assumed to be case bonded. Here, a and b are the inside and outside radii of the grain, respectively, while L is its overall length. The outside diameter of the grain is given simply as

$$D = 2b \tag{1}$$

b. Web Fraction, w_f

For the circularly perforated grain the web fraction is obtainable directly from the grain dimensions according to the following relation

$$w_f = \frac{b-a}{b} \tag{2}$$

For non-circular bore perforations, the definition of a is modified to that shown in Figure 2. The effective web fraction, w_{fe}, in the star-perforated grain was studied by Fourney and Parmerter (Reference 2). They found that the propellant lying inward from the star tips contributed little to the grain stiffness, so the effective web fraction is only slightly larger than that given by Equation (2). The relation they derived could be approximated by the following equation.

$$w_{fe} = 0.949\ w_f + 0.051 \tag{3}$$

Where w_f is obtained using Equation (2), with the values of a and b taken as shown in Figure 2.

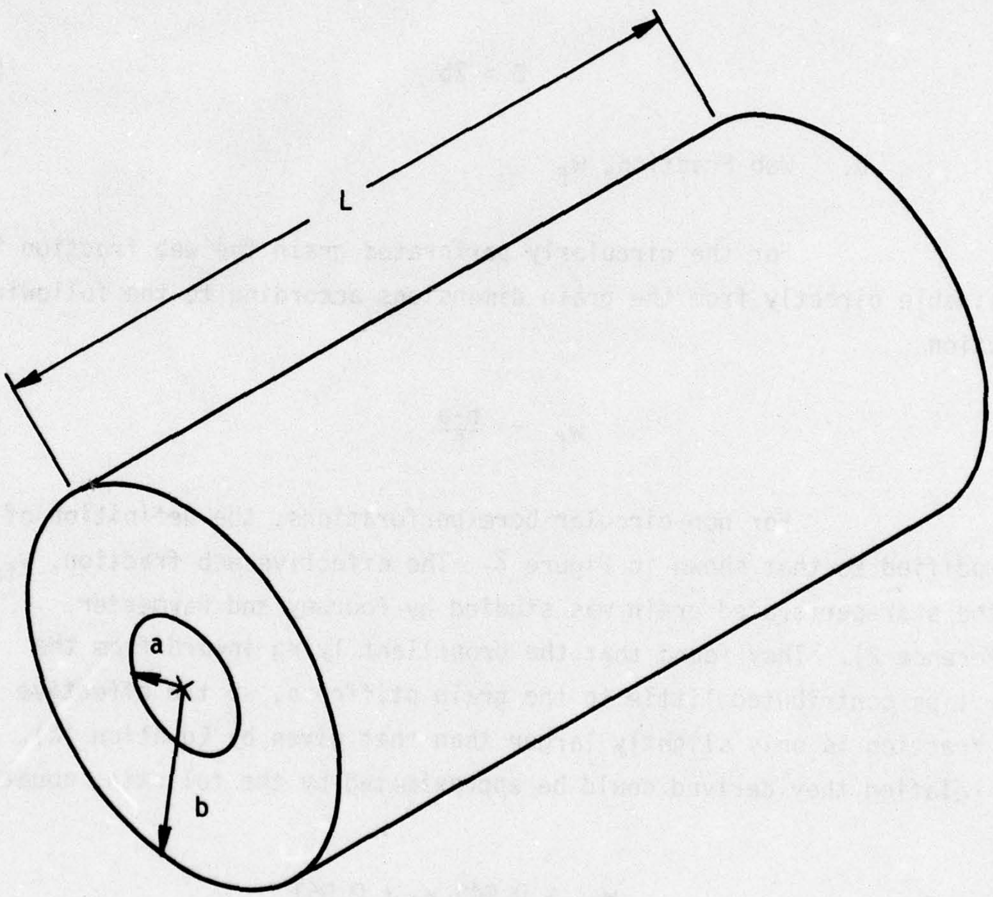


Figure 1. Base Dimensions to be Taken from Circularly Perforated Grain

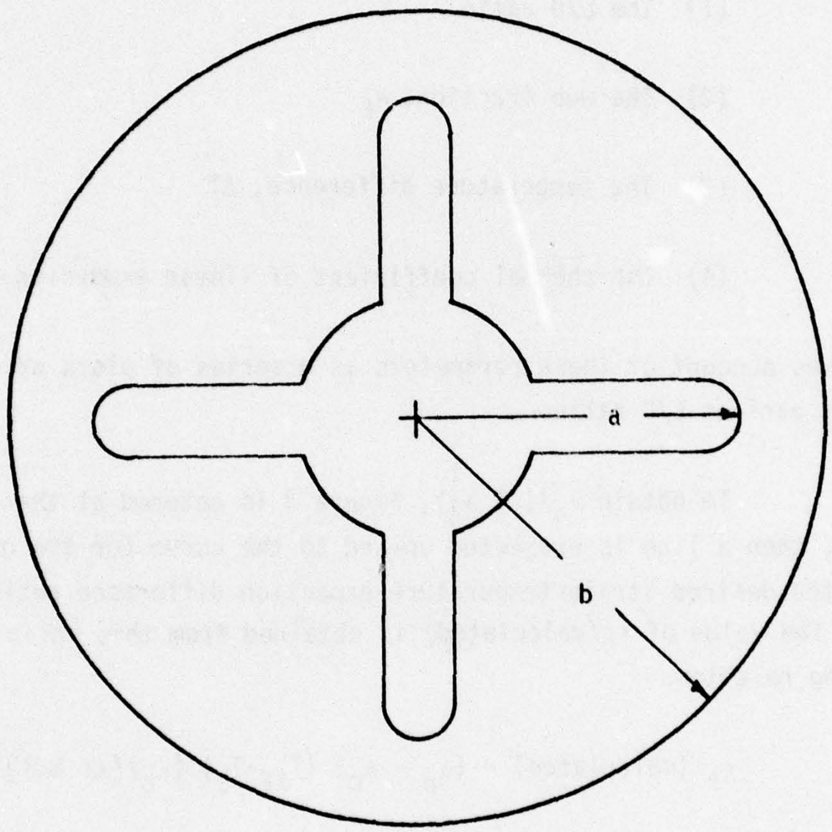


Figure 2. Base Dimensions of the Star-Perforated Grain

c. Calculated Inner-Bore Hoop Strain, ε_θ (calculated)

The calculated inner-bore hoop strain for a circularly perforated grain depends upon four primary factors:

(1) The L/D ratio

(2) The web fraction, w_f

(3) The temperature difference, ΔT

(4) The thermal coefficient of linear expansion difference, $\Delta\alpha$

Figure 3 takes account of these parameters as a series of plots of $\varepsilon_\theta/(\Delta T\ \Delta\alpha)$ versus w_f at various L/D ratios.

To obtain $\varepsilon_\theta/(\Delta T\ \Delta\alpha)$, Figure 3 is entered at the appropriate level of w_f, then a line is projected upward to the curve for the given L/D ratio, and the desired strain-temperature-expansion difference ratio is read directly. The value of ε_θ(calculated) is obtained from this ratio using the following relation.

$$\varepsilon_\theta \text{ (calculated)} = (\alpha_p - \alpha_c)\ (T_{SF} - T_L)\ [\varepsilon_\theta/(\Delta T\ \Delta\alpha)] \quad (4)$$

where

T_{SF} is the strain-free temperature for the grain, °C.

α_c is the thermal coefficient of linear expansion of the case material, cm/cm/K

α_p is the thermal coefficient of linear expansion of the propellant, cm/cm/K

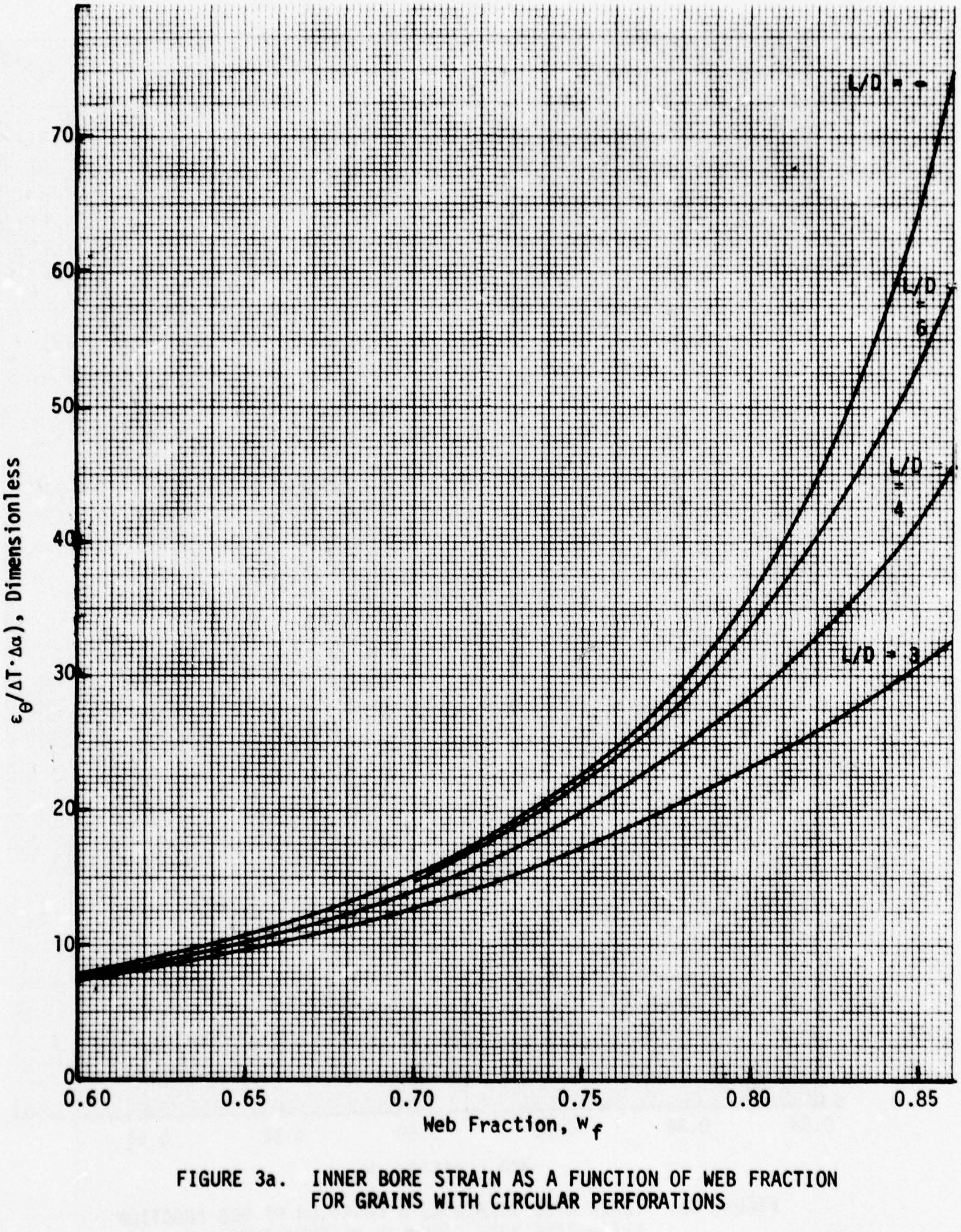


FIGURE 3a. INNER BORE STRAIN AS A FUNCTION OF WEB FRACTION FOR GRAINS WITH CIRCULAR PERFORATIONS

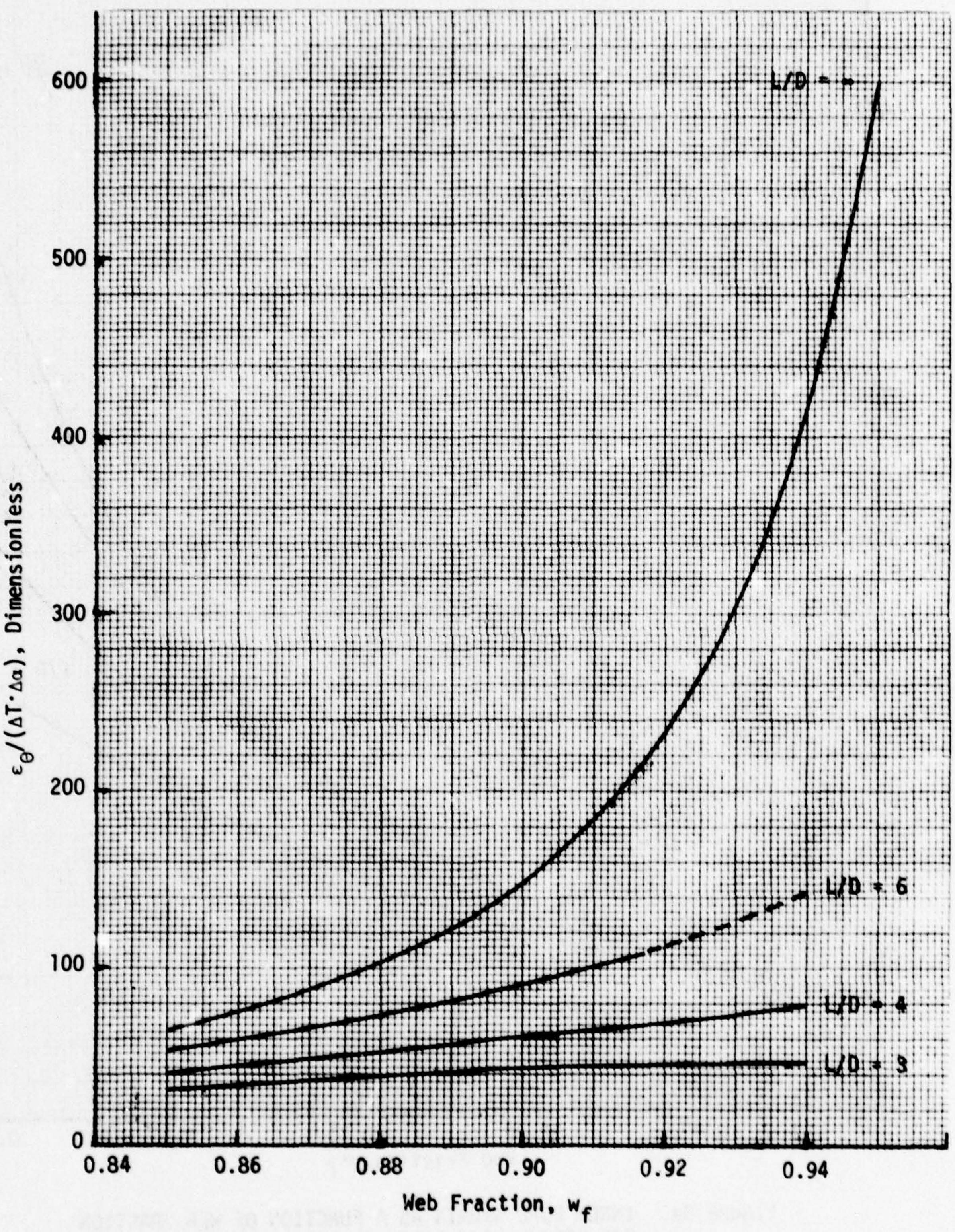


FIGURE 3b. INNER BORE STRAIN AS A FUNCTION OF WEB FRACTION FOR GRAINS WITH CIRCULAR PERFORATIONS

A sample calculation using this graph is given in Section 3.

3. Material Properties

a. Tensile Parameters B, σ_o and A_s

These parameters are obtained from constant rate tensile measurements. They are to be obtained in the course of making the a_T determinations. The minimum number of tests required to define the parameters are tabulated below. Additional tests may be added as required to cover the range of motor test temperatures, or to better define the a_T curve.

	Test Temperatures, °C						
Crosshead Rates, cm/sec	-40	-30	-15	5	25	40	60
8.47×10^{-1} (20 in./min.)	X		X		X		X
8.47×10^{-2} (2 in./min.)	X	X	X	X	X	X	X
8.47×10^{-3} (0.2 in./min.)	X		X		X		X

X indicates tensile test to be performed in duplicate

The raw tensile data are reduced a little differently from the conventional approach in that true stress values are used.

The true stress, σ_t, is related to the engineering stress, σ, by the relation

$$\sigma_t = \lambda \sigma \tag{5}$$

where

$$\lambda = 1 + \varepsilon \tag{6}$$

where

ε is the tensile strain that corresponds to the given σ.

The time to failure, t_f, is defined as the time that would be required to fail the specimen under a constantly applied true stress. It is related to the time to maximum true stress, t_{tm}, by the relation

$$t_f = t_{tm} A_s \tag{7}$$

where

$$A_s = \int_0^1 \frac{(\sigma_t)^B}{(\sigma_{tm})^B} \, d\,(t/t_{tm}) \tag{8}$$

The exponent B is obtained from a preliminary plot of $\log \sigma_{tm}$ vs $\log t_{tm}$ at 25°C. This exponent is the negative reciprocal of the slope of the line defined by these data. Taking two points on the line (identified by subscripts 1 and 2) yields the following relation for calculating B.

$$B = \frac{\log (t_{t2}/t_{t1})}{\log (\sigma_{tm1}/\sigma_{tm2})} \tag{9}$$

After the values of t_f have been estimated, separate plots of $\log \sigma_{tm}$ versus $\log t_f$ will be made for each test temperature. These data will be superposed to fit a straight line and to define the time-temperature shift parameter a_T (referred to 25°C), an example of which is given in Figure 4. This experimental value of a_T must be characterized further before it can be used in the nomograph (see below).

The shifted curve is used to obtain both B and σ_o. The relation for calculating B is the same as Equation (9), but using t_f data.

$$B = \frac{\log (t_{f2}/t_{f1})}{\log (\sigma_{tm1}/\sigma_{tm2})} \tag{10}$$

The quantity σ_o is the true stress at failure at one minute, as taken from the plot of $\log \sigma_{tm}$ vs $\log t_f/a_T$.

Example determinations of B and σ_o are given in Section 3.

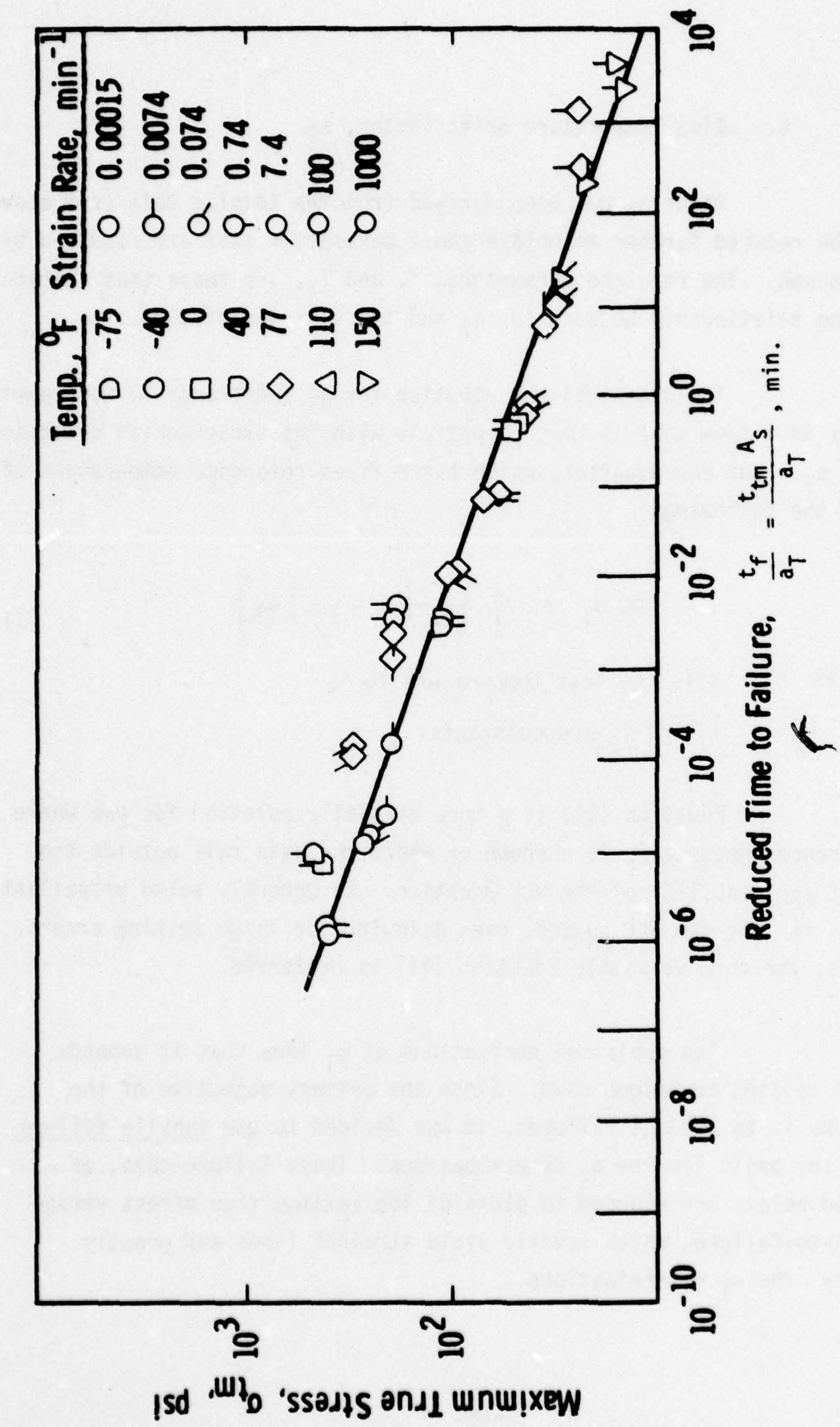


Figure 4. Maximum True Stress vs Reduced Time to Failure for ANB-3241-2 Propellant

b. Time-Temperature Shift Factor, a_T

After a_T has been derived from the tensile data (see above) it must be reduced further to obtain those parameters that are required by the nomograph. The required parameters, f_1 and f_2, are those that characterize the relationship between log a_T and the test temperature.

The classical WLF equation for a_T (Reference 3) was rewritten to put it in a form that is more compatible with the experimental determinations of a_T. The new equation, which has a fixed reference temperature of 25°C, is the following:

$$\log a_T = f_1 \left(\frac{1}{f_2 + T} - \frac{1}{f_2 + 25} \right) \qquad (11)$$

where T is the test temperature in °C

f_1 and f_2 are constants

Equation (11) is a more versatile relation for use where the reference temperature is unknown or where the data fall outside the limits of applicability of the WLF equation. In general, solid propellant data fall far off the WLF curves, even allowing for large testing errors. Therefore, the more versatile Equation (11) is preferred.

The empirical derivations of a_T show that it depends upon the testing technique used. Since the primary objective of the nomographs is to predict failures, it was decided to use <u>tensile failure data</u> as the basis for the a_T determinations. These failure data, as described below, are reduced to plots of log maximum true stress versus log time-to-failure, which usually yield straight lines and greatly simplify the a_T determinations.

The range of test temperatures must exceed that of the required analyses by at least 10°C and, in any event, must include determinations at -40°C and +60°C.

The values of f_1 and f_2 are obtained nomographically using Figures 5 and 6. The parameter f_2 is determined first. This begins with the ratio, R, of the logs of a_T at -40 and +60°C.

$$R = \frac{\log a_T\ (-40°C)}{\log a_T\ (+60°C)} \tag{12}$$

The quantity f_2 is determined directly upon entering Figure 5 at the given value of R.

After f_2 is determined, the parameter f_1 may be conveniently determined using Figure 6. The scale is entered at the value of $\log a_T$ at -40°C, which is projected upward to the curve corresponding to the given f_2, then f_1 is read directly.

The values of f_1 and f_2 obtained in this way may lead to poor curve fits (of $\log a_T$ versus temperature) at the intermediate temperatures. This results from the accumulation of errors in the superposition process. It is essential, therefore, that the a_T determinations be conducted with great care.

Examples of the determinations of f_1 and f_2 are given in Section 3.

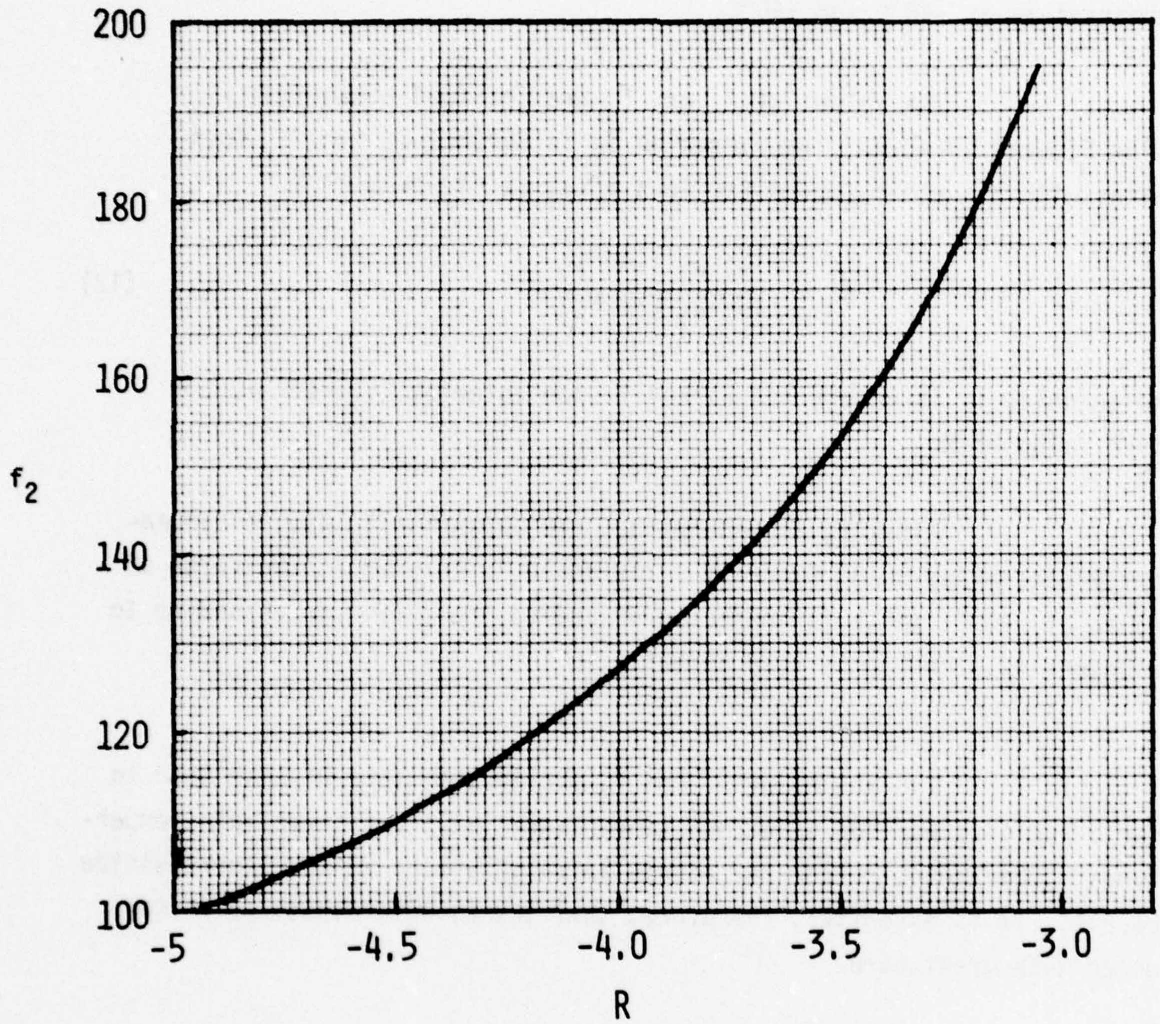


FIGURE 5. DETERMINATION OF THE f_2 PARAMETER OF a_T

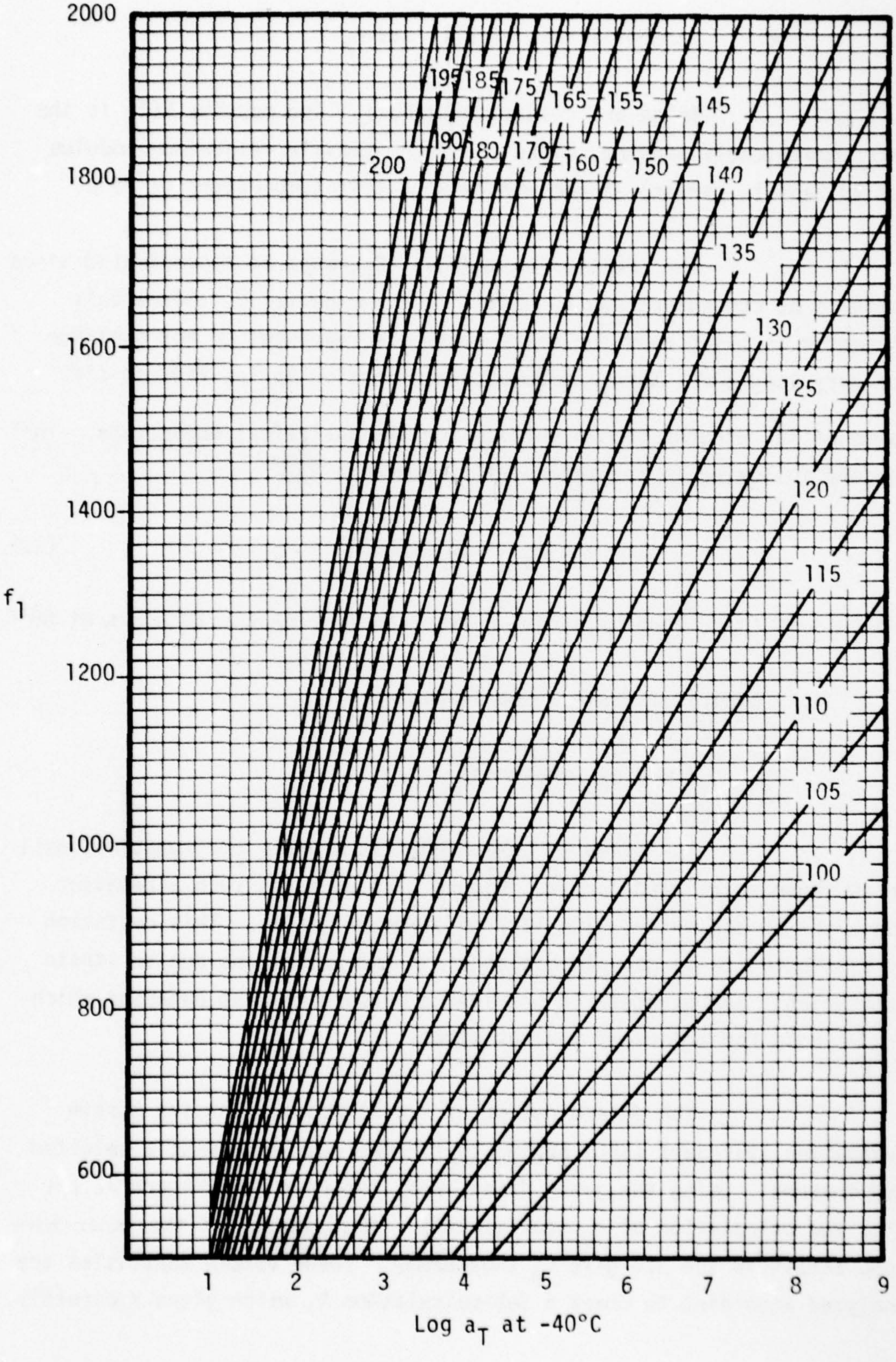


FIGURE 6. DETERMINATION OF THE f_1 PARAMETER OF a_T

c. Determinations of E(1) and E_e

These are relaxation moduli. The modulus E(1) is the most conveniently handled, since it is the tensile relaxation modulus (at 2% tensile strain) at one minute at a test temperature of 25°C.

The equilibrium modulus, E_e, must be approximated since there is no convenient way to obtain it experimentally. Since this parameter is being used for tests where the responses do not continue for very long time, then a modulus determination in the reduced time range of 10^5 minutes should suffice for the analytical objectives. That is, the following approximation will be made

$$E_e \approx E\left(\frac{t}{a_T} = 10^5 \text{ min.}\right) \qquad (13)$$

and this is to be done by stress relaxation testing for 50 hours at 60°C.

4. Empirical Grain Response Data

a. K and ε_θ (measured)

The measured bore strains in a grain are sometimes well below those calculated for it. Figure 7 illustrates such a behavior for a set of ten motors with 12.7 cm diameter grains. This deviation is attributed to large volume changes in the propellant due to strain dilatation. The parameter K provides a measure of that behavior which would hold for motors of various sizes.

The determination of K will usually involve strain evaluation cylinders (SEC) cooled to thermal equilibrium at a selected temperature. Using the grain dimensions, together with Figure 3, permits the calculation of ε_θ (calculated). Measurement of the inner-bore hoop strain in the SEC give ε_θ (measured). These strain quantities are analyzed according to Chart 6 Sub-calculation I, which gives K directly.

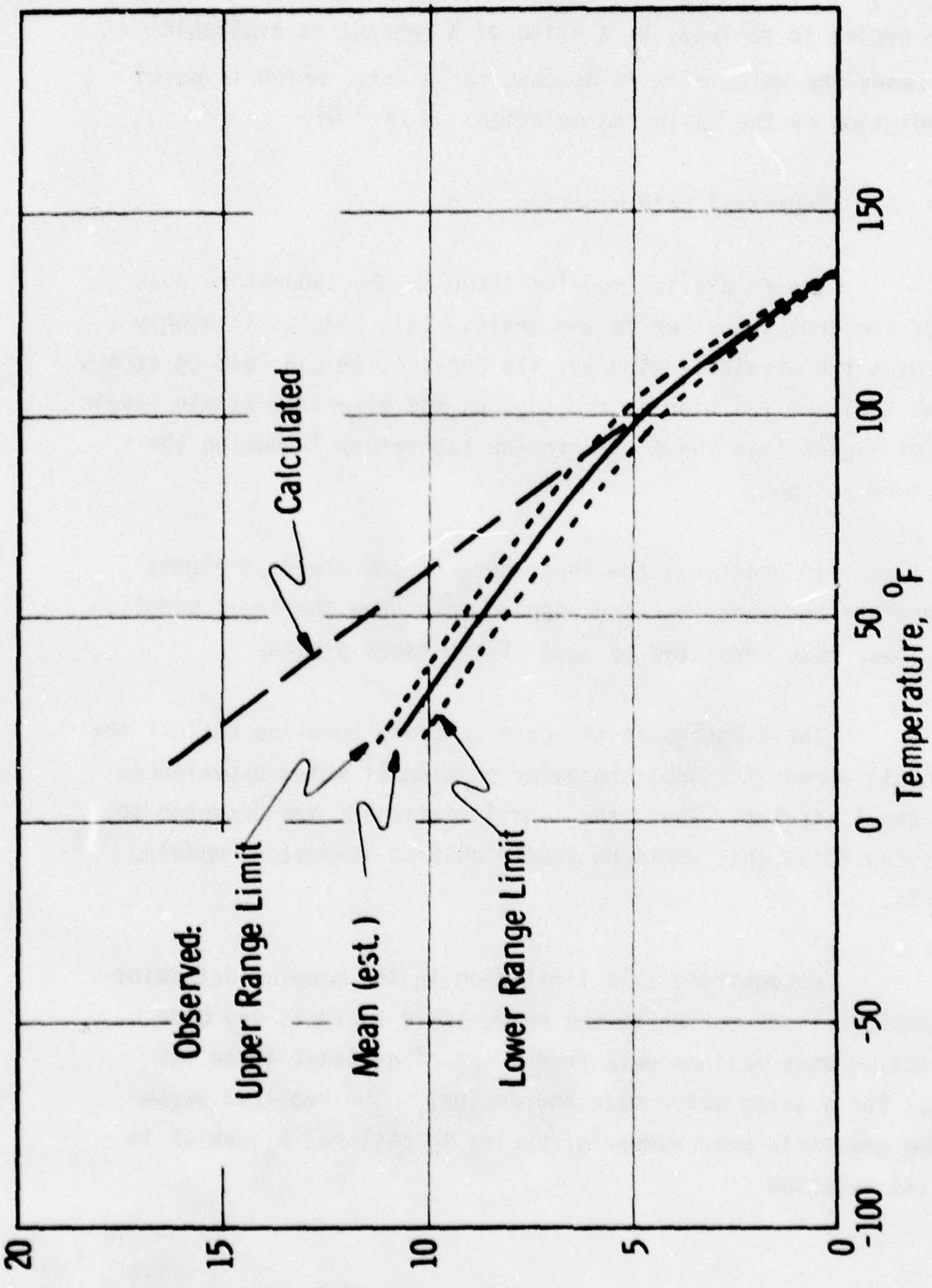


FIGURE 7. COMPARISON BETWEEN CALCULATED AND OBSERVED INNER-BORE HOOP STRAINS FOR TEST MOTORS STEP-WISE COOLED TO 20°F

A sample calculation of K is given in Section 3.

When making the nomographic predictions of the mean number of thermal cycles to failure, $\bar{N}$, a value of K may not be available. For these cases the value of K is assumed to be zero, which is noted in the prediction by the following notation: $\bar{N}$ (K $\cong$ 0).

b. Empirical Determination of q

The relaxation modulus taken in the laboratory does not reflect the true situation in the grain. This modulus is highly dependent upon the strain level. So, its application can lead to stress predictions that are too high or too low, as the effective strain levels are lower or higher than those used in the laboratory in making the modulus determinations.

In practice, the inner-bore of the grain is highly strained and the bore hoop stresses depend only upon the local moduli, which are lower than predicted because of the large strains.

The stresses at the case-to-grain bondline reflect the average moduli across the web, the major portion of which experiences only very small strains. Thus, the bondline stresses are expected to be larger than those that would be predicted from laboratory modulus measurements.

Recognizing this limitation in the modulus determinations, an empirical correction to the nomographic analysis was made. This correction uses failure data from a set of at least three SEC motor tests (of a given motor size and design). The required parameter is the geometric mean number of cycles to failure, $\bar{N}_g$, which is given by the relation

$$\bar{N}_g = \left[\prod_{i-1}^{n} N_i \right]^{1/n} \qquad (14)$$

where n is the total number of tested motors and π means "product from multiplying each observed number of cycles".

The SEC design parameters, material properties and thermal environments, together with $\bar{N}_g$ are analyzed according to Chart 6 Sub-calculation II. This analysis yields the required value of q.

A sample calculation of q is given in Section 3.

When making the nomographic predictions of the mean number of thermal cycles to failure, $\bar{N}$, a value of q may not be available. For those cases the value of q is assumed to be zero, which is noted *in the prediction by the notation:* $\bar{N}$ ($q \cong 0$).

SECTION 3

ACTUAL USE OF NOMOGRAPH

The nomograph itself consists of six charts, which will be used in the order given. Chart 2 is omitted if the motor radius is less than 9 cm (18 cm diameter). Also, chart 4 may be omitted when an additional factor of error of about 1.25 can be tolerated; as would be the case when evaluating the effects of material property variations.

Actually, the number of cycles to failure, N, in real motors follows a statistical distribution that is based upon the logarithm of N. Thus, by experience, a factor of error of two is often acceptable for preliminary assessments.

Example calculations using the nomograph are given below. These calculations are centered upon Sub-calculations I and II of chart 6, but they illustrate all of the steps that are to be followed in the use of the nomograph.

The example calculations involve RV-7 propellant in strain evaluation cylinders (SECs) that are 50.2 cm long with an I.D. of 1.91 cm and an O.D of 12.7 cm and thermally cycled between 60°C and -40°C. The propellant was cured at 57°C and has a strain-free temperature of 65.6°C.

The two subsections which follow summarize these calculations. The first sub-section summarizes the collection of the required design, test, and material property data that are required by the nomographic analyses. The last sub-section provides the example calculations using the nomograph.

A. INPUT DATA

The required design, test, and material property data are summarized in Table 3. This table is the filled-in version of Table 2. Some of the required input data are obtained directly from the given parameters, but a few must be derived. The derived properties are discussed further below.

1. ε_{Θ}(calculated)

The web fraction for this motor, according to Equation (2) is

$$w_f = \frac{6.35 - 0.953}{6.35} = 0.85$$

The L/D ratio is

$$L/D = \frac{50.2}{2 \times 6.35} = 3.95$$

The value of $\varepsilon_{\Theta}/(\Delta T\ \Delta\alpha)$ is obtained for this motor as illustrated in Figure 8, on entering at $w_f = 0.85$ and $L/D = 4$. This yields a value of 41.5 for $\varepsilon_{\Theta}/(\Delta T\ \Delta\alpha)$. The value of ε_{Θ}(calculated) is obtained from this quantity using Equation (4), which becomes

$$\varepsilon_{\Theta}(\text{calculated}) = (9.72 \times 10^{-5} - 10.62 \times 10^{-6})(65.6 + 40) \times 41.5$$

$$\varepsilon_{\Theta}(\text{calculated}) = 0.379$$

2. Tensile Parameters σ_o, B and A_s

The constant rate tensile data were reduced according to Equations (5) to (9) and yielded the tabulation in English units, given in Table 4. The tabulation includes the test temperature, crosshead rate, maximum true stress, σ_{tm}, the time-to-maximum true stress, t_{tm}, Λ_s, and t_f (which equals $t_{tm}\ \Lambda_s$).

TABLE 3

COMPLETED FORM OF TABLE 2 FOR RV-7 PROPELLANT

PARAMETERS REQUIRED FOR THERMAL CYCLING ANALYSIS

Parameters	Definitions	Values
Thermal Environment		
T_u	Upper temperature limit	60 °C
T_L	Lower temperature limit	-40 °C
T_{SF}	Strain-Free Temperature	65.6 °C
Grain Design		
a	Inside radius of grain (strain-free)	0.953 cm
b	Outside radius of grain (strain-free)	6.35 cm
L	Length of case-bonded grain (strain-free)	50.2 cm
w_f	Web fraction of grain	0.85
ε_θ(Calculated)	Calculated maximum inner-bore hoop strain at T_L	0.379 cm/cm
Material Properties		
a_T	Time-temperature shift factor	
f_1	First constant in a_T function	1980 °C
f_2	Second constant in a_T function	183 °C
B	Exponent in stress-time relation	8.2
E(1)	Relaxation modulus at one minute at 25°C	4.14 MPa*
E_e	Estimate of equilibrium modulus	1.73 MPa
σ_o	Stress causing failure at one minute in constant stress testing at 25°C	1.313 MPa
α_p	Thermal coefficient of linear expansion of the propellant	9.72×10^{-5} cm/cm/K
α_c	Thermal coefficient of linear expansion of the case	10.62×10^{-6} cm/cm/K
Empirical Grain Response Data		
Subcalculation I: K	Constant defining bore strain reduction due to strain dilatation	0.25
ε_θ(Calculated)	Calculated bore strain for SEC	.379 cm/cm
ε_θ(Measured)	Measured bore strain in SEC	.33 cm/cm
Subcalculation II: q	Constant related to grain dilatation softening	-0.95
$\overline{N}_g$	Geometric mean cycles to failure of SEC's	15.9 cycles

* Conversion MPa = (6.895×10^{-3})(psi)

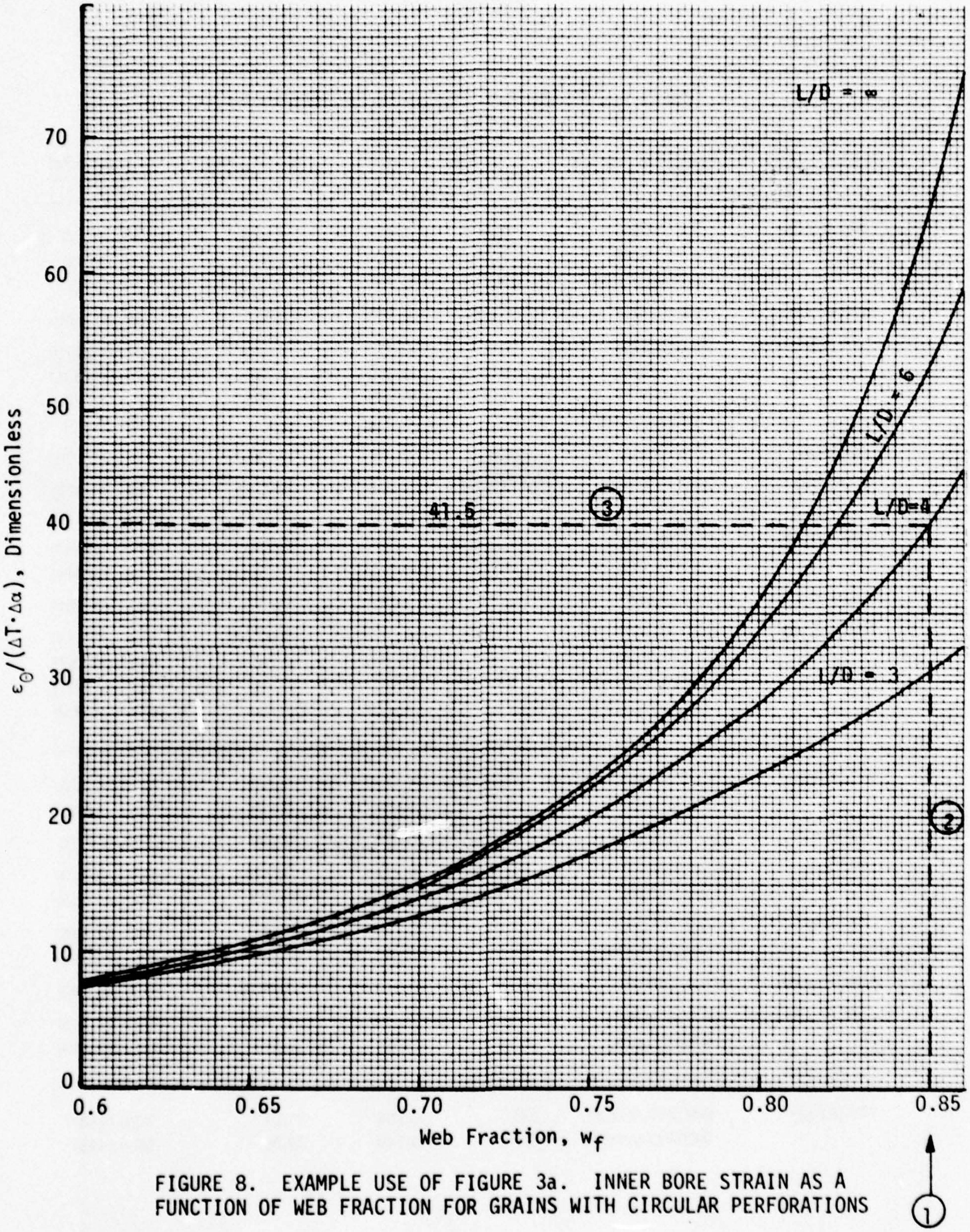


FIGURE 8. EXAMPLE USE OF FIGURE 3a. INNER BORE STRAIN AS A FUNCTION OF WEB FRACTION FOR GRAINS WITH CIRCULAR PERFORATIONS

TABLE 4. Tabulation of True Stress Failure Data for RV-7 Propellant (Mix 7374).

Temp., °F (°C)	Rate, in/min (cm/s)	$t_{\ell m}$, min	A_s	$t_f = t_{\ell m} A_s$, min	$\sigma_{\ell m}$, psi (kPa)
First Data Set					
180 (82.2)	2.0 (0.085)	0.54	0.413	0.223	133 (917)
	0.2 (0.008)	4.72	0.300	1.414	102 (703)
	0.02 (0.0008)	33.8	0.200	6.75	83 (572)
135 (57.2)	2.0 (0.085)	0.608	0.253	0.154	163 (1 124)
78 (25.5)	20.0 (0.846)	0.081	0.310	0.025	307 (2 118)
	2.0 (0.085)	0.702	0.300	0.210	222 (1 531)
	0.2 (0.008)	6.08	0.248	1.506	178 (1 228)
	0.02 (0.0008)	54.0	0.328	17.7	138 (952)
40 (4.4)	2.0 (0.085)	0.716	0.208	0.149	316 (2 180)
0 (−18)	20.0 (0.846)	0.0742	0.278	0.0206	723 (4 988)
	2.0 (0.085)	0.81	0.286	0.232	503 (3 470)
−20 (−28.8)	2.0 (0.085)	0.608	0.305	0.186	629 (4 340)
	0.2 (0.008)	6.75	0.271	1.830	471 (3 249)
−40 (−40)	2.0 (0.085)	0.27	0.433	0.117	693 (4 781)
−65 (−53.8)	20.0 (0.846)	0.00648	0.414	0.00268	938 (6 472)
	2.0 (0.085)	0.115	0.403	0.0464	891 (6 147)
	0.2 (0.008)	1.485	0.391	0.580	751 (5 181)
	0.02 (0.0008)	27.0	0.434	11.72	667 (4 602)
Second Data Set					
165 (73.8)	20.0 (0.846)	0.0675	0.267	0.018	192 (1 324)
	2.0 (0.085)	0.54	0.252	0.136	143 (986)
135 (57.2)	20.0 (0.846)	0.0742	0.256	0.019	228 (1 573)
40 (4.4)	20.0 (0.846)	0.0878	0.214	0.0188	471 (3 249)
	0.2 (0.008)	7.763	0.232	1.803	268 (1 849)
0 (−18)	0.2 (0.008)	7.42	0.235	1.746	369 (2 546)
	0.02 (0.0008)	74.2	0.243	18.03	298 (2 056)
−20 (−28.8)	20.0 (0.846)	0.0405	0.343	0.0139	745 (5 140)
−40 (−53.8)	0.2 (0.008)	4.72	0.349	1.648	602 (4 153)
	0.02 (0.0008)	57.4	0.294	16.87	471 (3 249)
Third Data Set					
179 (81.6)	0.002 (0.00008)	235	0.154	36.21	70.8 (488)
	0.002 (0.00008)	239	0.140	33.39	69.4 (478)

The separate determinations of σ_o, B and A_s are illustrated below. The parameter B is readily determined from plots of log σ_{tm} vs log t_{tm}, or vs log t_f (the plots vs log t_{tm} are a little less accurate than those vs log t_f). The parameter σ_o must be obtained from a plot of log σ_{tm} vs log t_f. Thus, for illustration purposes we used the latter plot to obtain both B and σ_o (see Figure 9). The determination of A_s (see Equation (8)) is a little more complex and involves the steps illustrated in Figure 10. All three of these determinations are discussed below.

The illustrative plot of log σ_{tm} versus log t_f, after making the time-temperature shift, a_T, yielded the bi-linear curve given in Figure 9. This form of data plot, by past experience, usually gives a break in the curve at stress values above about 6.5 MP_a (about 950 psi). This stress value is *above any that would be met in service.* Hence, that part of the curve is ignored.

The value of σ_o is 1.313 MP_a (from log σ_{tm} = 2.28 or σ_{tm} = 191 psi), which is taken at t_f = 1 minute (log t_f = 0).

The value of B is obtained from this curve using Equation (10), which may be rewritten

$$B = \frac{\log t_{f_2} - \log t_{f_1}}{\log \sigma_{tm_1} - \log \sigma_{tm_2}} \qquad (15)$$

For this curve,

$$B = \frac{-7-3}{1.915 - 3.135} = \frac{-10}{-1.22}$$

$$B = 8.2$$

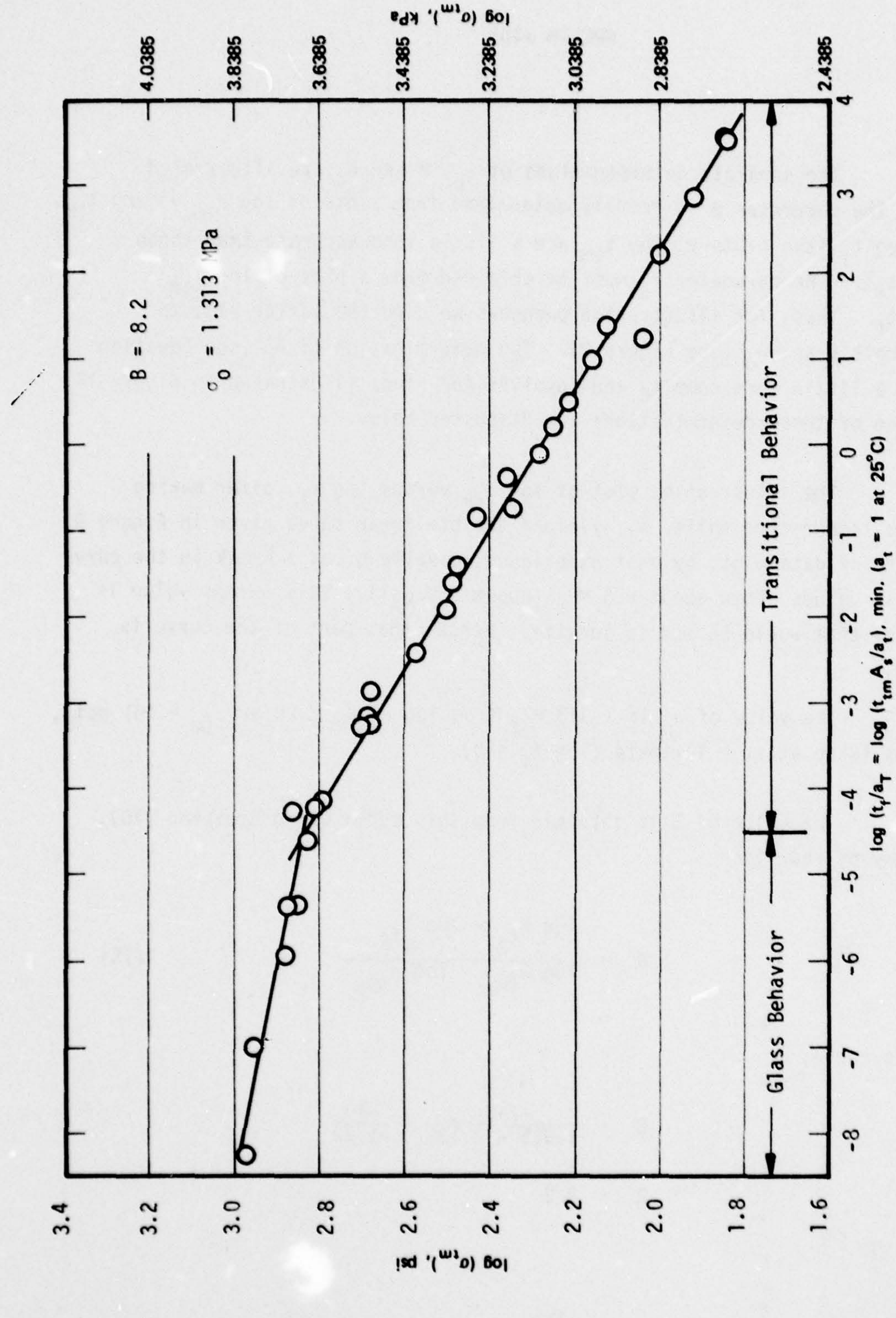


FIGURE 9. TRUE MAXIMUM STRESS VERSUS TIME TO FAILURE FOR RV-7 PROPELLANT (MIX 7374)

The determination of A_s involves the three steps shown in Figure 10. The basic engineering stress-time curve (Figure 10a) is corrected according to Equations (5) and (6) to give the true stress-time curve illustrated in Figure 10b. From this plot we obtain σ_{tm} and t_{tm}. The stress data are reduced (using the given value of B) to give $(\sigma_t/\sigma_{tm})^B$, which is plotted versus t_{tm} in Figure 10c. The integration of the shaded area may be performed graphically, with a planimeter, by calculation, or gravimetrically (a simple process of cutting out and weighing the paper of the outlined unit square, then cutting out and weighing the shaded area. The ratio of the weight of the paper for the shaded area to that of the unit square is numerically equal to A_s).

3. Time-Temperature Shift Factor, a_T

The time-temperature shift factors for the tensile data of Figure 9 are plotted as log a_T vs temperature in Figure 11.

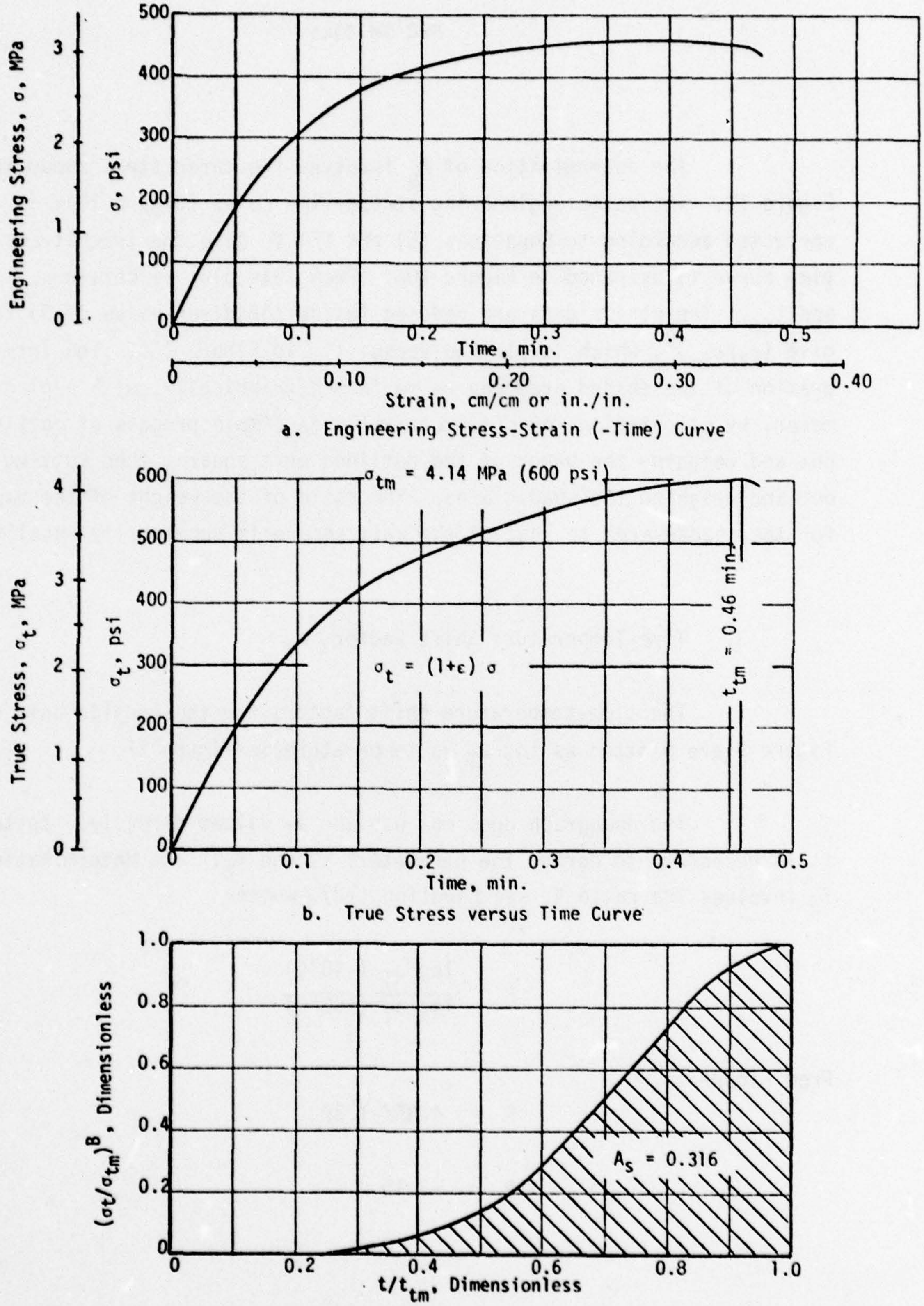
The nomograph does not use the a_T values directly. Instead, it is necessary to derive the parameters f_1 and f_2. The determination of f_2 involves the ratio R, see Equation (12), where

$$R = \frac{\log a_T\ (-40°C)}{\log a_T\ (+60°C)}$$

From Figure 11,

$$R = 4.35/-1.38$$

$$R = -3.15$$



a. Engineering Stress-Strain (-Time) Curve

b. True Stress versus Time Curve

c. Normalized Stress versus Normalized Time

FIGURE 10. STEPS IN DETERMINING A_s FROM SIMPLE TENSILE DATA

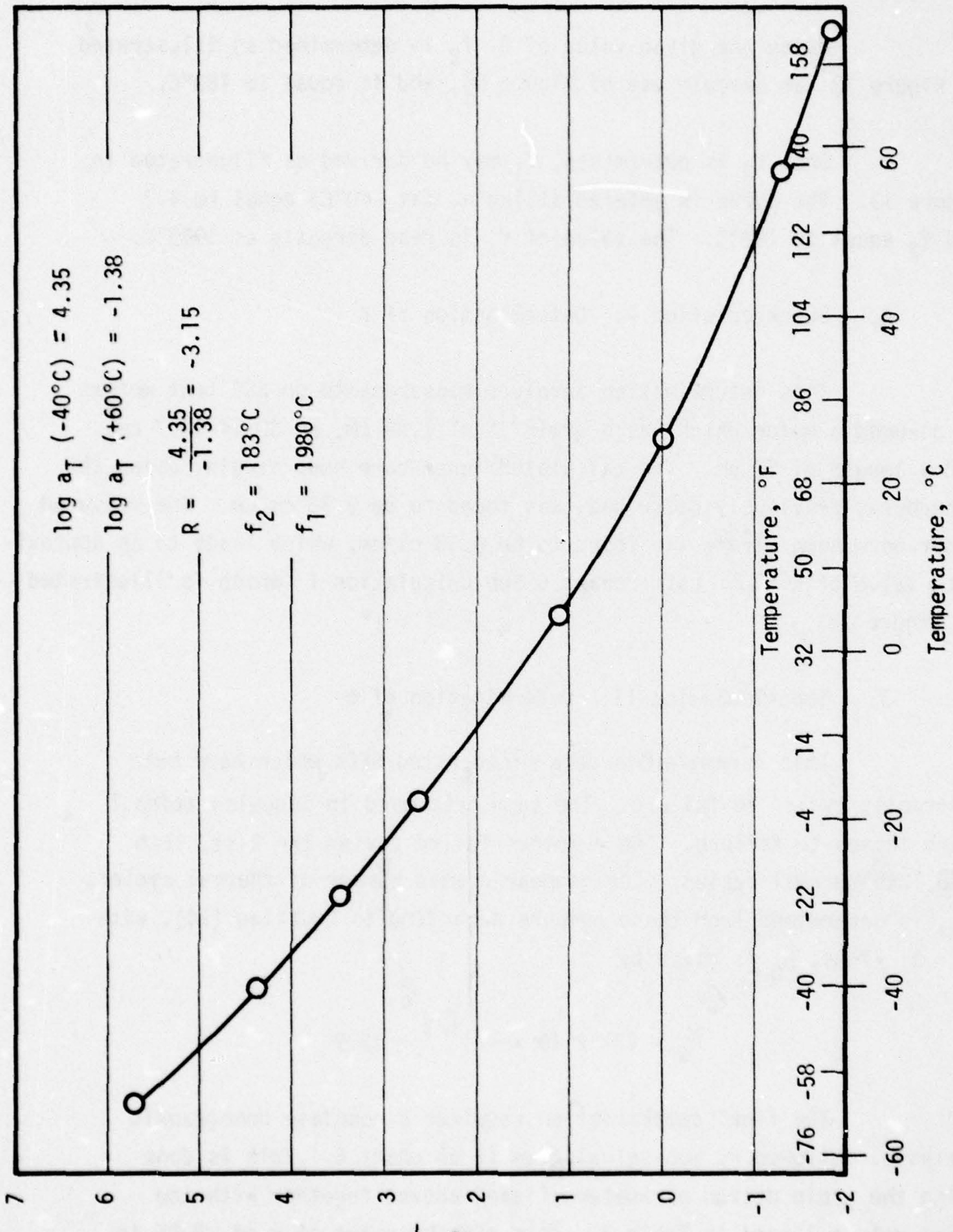


FIGURE 11. EMPIRICAL CURVE FOR LOG a_T VS TEST TEMPERATURE FOR RV-7 PROPELLANT (MIX 7374)

Using the given value of R, f_2 is determined as illustrated in Figure 12 (an example use of Figure 5), and is equal to 183°C.

Once f_2 is determined, f_1 may be derived as illustrated in Figure 13. The curve is entered at log a_T (at -40°C) equal to 4.3 and f_2 equal to 183°C. The value of f_1 is read directly as 1980°C.

4. Subcalculation I: Determination of K

This determination involves measurements on SEC test motors. We assumed a motor which has a grain ID of 1.90 cm, an OD of 12.7 cm, and a length of 50 cm. The calculated inner-bore hoop strain, using the procedures previously described, was found to be 0.38 cm/cm. The measured inner-bore hoop strain was found to be 0.33 cm/cm, which leads to an approximate value of K = .25 using chart 6 Sub-Calculation I (which is illustrated in Figure 18).

5. Subcalculation II: Determination of q

This parameter is determined using SECs which have been thermally cycled to failure. The same SECs used in Subcalculation I were tested to failure. These motors failed during the 21st, 16th and 12th thermal cycles. The geometric mean number of thermal cycles, $\overline{N}_g$, is determined from these numbers according to Equation (14), with n = 3. Thus, $\overline{N}_g$ is given by

$$\overline{N}_g = (21 \times 16 \times 12)^{1/3} = 15.9$$

The final determination requires a complete nomographic analysis, followed by Sub-Calculation II on chart 6. This is done using the grain design parameters listed above, together with the other values listed in Table 3. This gives a value of q of -0.95 in. in Figure 18.

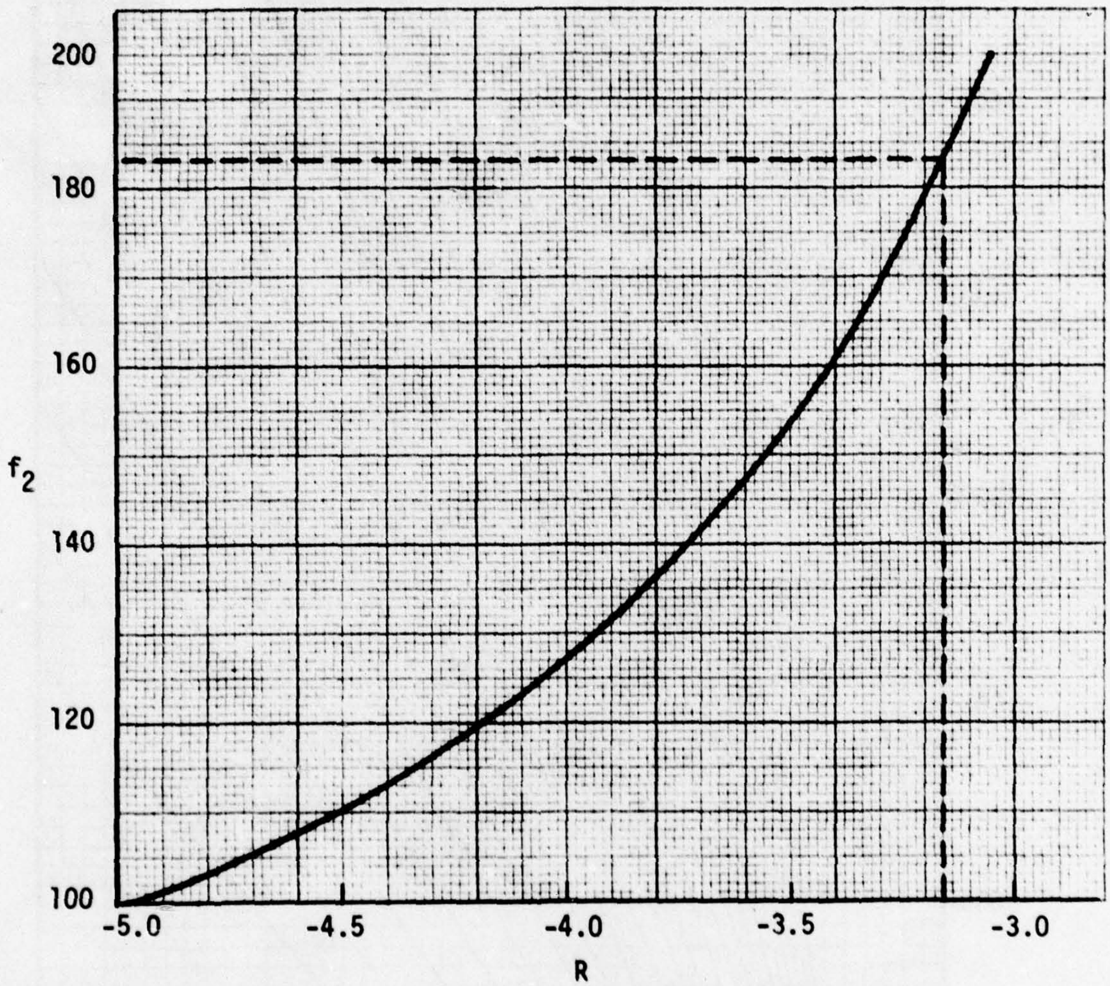


FIGURE 12. EXAMPLE USE OF
FIGURE 5. DETERMINATION OF THE f_2 PARAMETER OF a_T

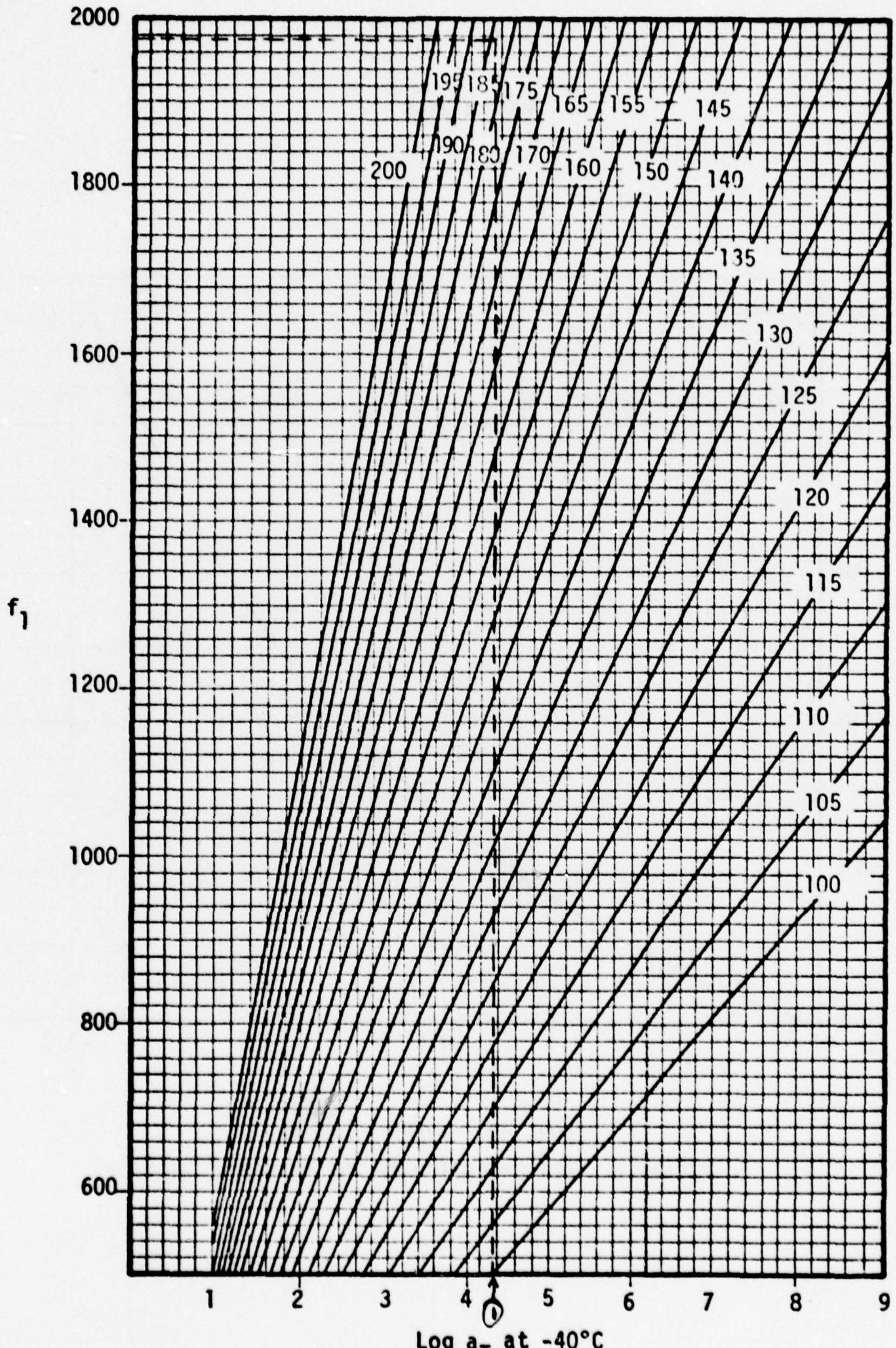


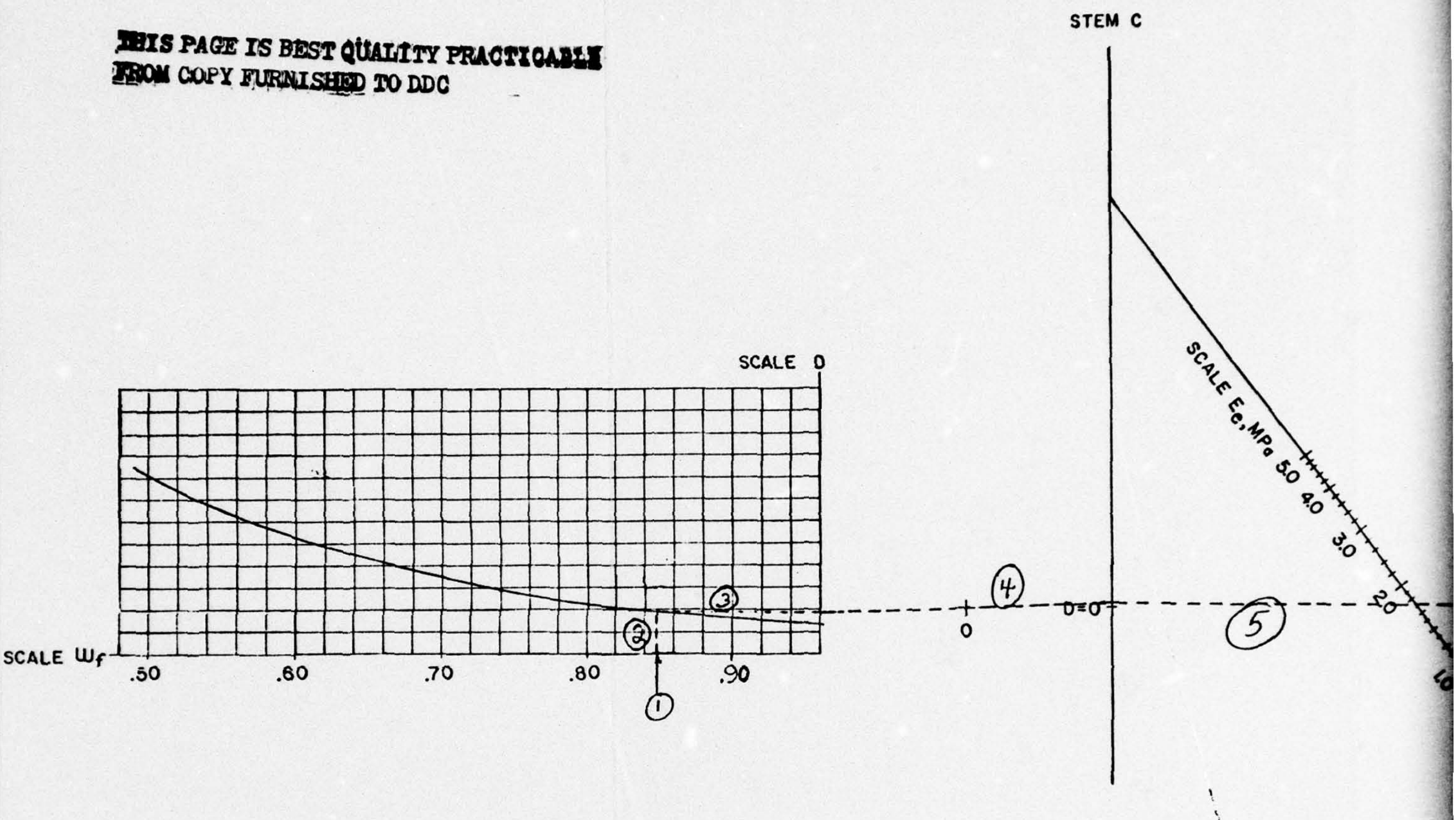
FIGURE 13. DETERMINATION OF THE f_1 PARAMETER OF a_T

B. EXAMPLE USE OF THE NOMOGRAPH

The test case listed in Table 3 was analyzed nomographically as illustrated in Figures 14 to 19. In this example the calculations are shown as dashed lines, with each step numbered according to the directions on each chart.

Chart No. 2 was included in the analysis to demonstrate its use. Actually, the calculated value of δ was too small to be of any interest.

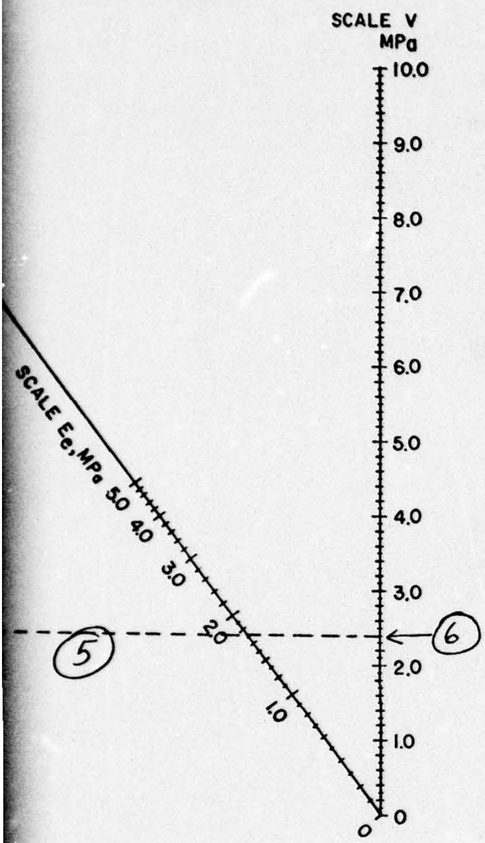
A prediction is not actually demonstrated in the sample but would follow a similar route through the nomograph pages except that K and q would be as determined in the sample if RV-7 propellant were to be predicted in some other size SEC or motor.



DIRECTIONS:

1. ENTER AT GIVEN VALUE ON ω_f SC
2. DRAW A VERTICAL LINE UNTIL I
3. FROM THIS INTERSECTION DRAW UNTIL IT INTERSECTS THE D SCA
4. CONNECT THIS INTERSECTION ON POINT "O" WITH A STRAIGHT LI SECTS THE. STEM C AT POINT c.
   - IF THE VALUE OF D IS ZER THEN STEP 4 MAY BE BY-PA MARKED D=0 ON THE STEM
5. FROM THE INTERSECTION POINT GIVEN VALUE ON THE E_c SCALE
6. THIS INTERSECTION PROVIDES T ANALYSES OF CHART 4.NOTE TH

Figure 14



...NS:

...R AT GIVEN VALUE ON ω_f SCALE.

...W A VERTICAL LINE UNTIL IT INTERSECTS THE CURVE.

...M THIS INTERSECTION DRAW A HORIZONTAL LINE PARALLEL TO THE GRID ...L IT INTERSECTS THE D SCALE.

...NECT THIS INTERSECTION ON THE D SCALE AND THE CENTER OF THE CROSS AT ...NT "O" WITH A STRAIGHT LINE. EXTEND THIS STRAIGHT LINE UNTIL IT INTER-...S THE. STEM C AT POINT c.

...) IF THE VALUE OF D IS ZERO OR NEGLIGIBLE (CLOSE TO OR AT THE BASE LINE) THEN STEP 4 MAY BE BY-PASSED AND STEP 5 WOULD BEGIN AT THE POINT MARKED D=0 ON THE STEM C .

...M THE INTERSECTION POINT c ON THE STEM C EXTENT A LINE THROUGH THE ...EN VALUE ON THE E_e SCALE UNTIL IT INTERSECTS THE V SCALE.

...S INTERSECTION PROVIDES THE VALUE OF V WHICH IS REQUIRED FOR THE ...LYSES OF CHART 4. NOTE THIS QUANTITY ON THAT CHART.

aerojet solid propulsion company
SACRAMENTO, CALIFORNIA

STRUCTURAL DESIGN NOMOGRAPH
FOR THERMAL CYCLING
CHART 1 – ELASTIC COMPONENT

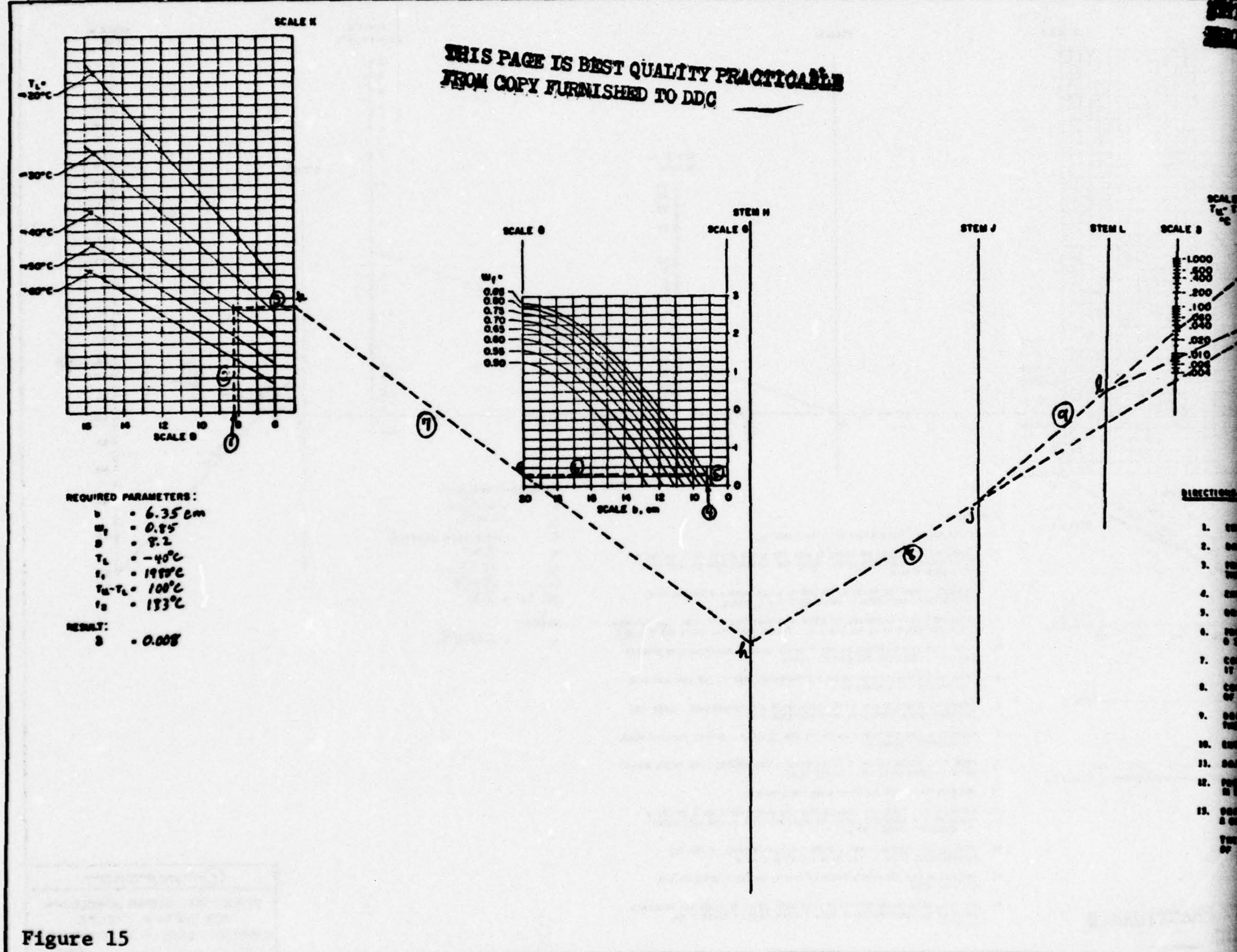
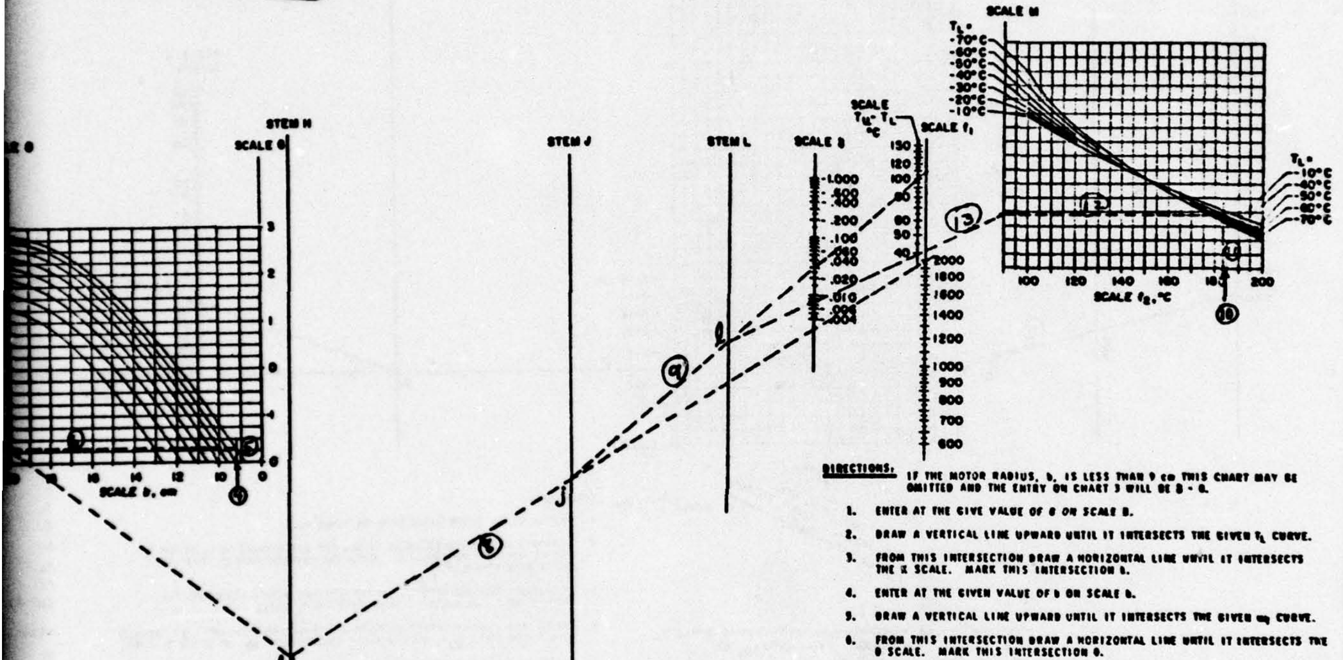

SCALE K
T_L =
-20°C
-30°C
-40°C
-50°C
-60°C
SCALE B
REQUIRED PARAMETERS:
b = 6.35 cm
w_f = 0.85
B = 8.2
T_L = -40°C
f_c = 199°C
$T_M - T_L$ = 100°C
f_B = 183°C
RESULT:
δ = 0.008
STEM H
SCALE G
SCALE b, cm
STEM J
STEM L
SCALE D
DIRECTIONS

Figure 15

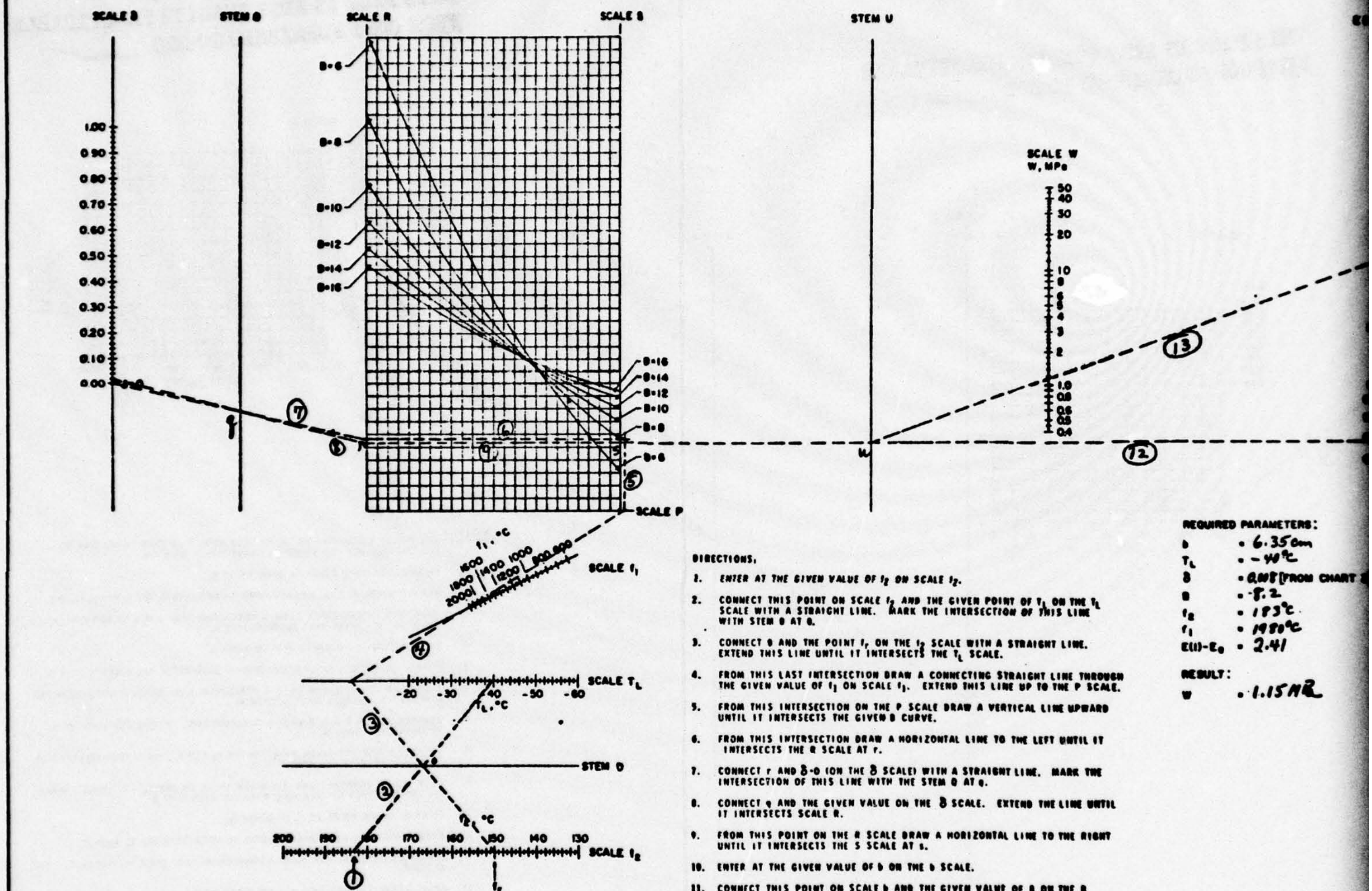


DIRECTIONS: IF THE MOTOR RADIUS, b, IS LESS THAN 9 cm THIS CHART MAY BE OMITTED AND THE ENTRY ON CHART 3 WILL BE δ = 0.

1. ENTER AT THE GIVE VALUE OF θ ON SCALE B.
2. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN T_L CURVE.
3. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE K SCALE. MARK THIS INTERSECTION k.
4. ENTER AT THE GIVEN VALUE OF b ON SCALE b.
5. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN ω_1 CURVE.
6. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE G SCALE. MARK THIS INTERSECTION G.
7. CONNECT POINTS k AND G WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS STEM H AT h.
8. CONNECT h AND THE GIVEN VALUE ON THE f_1 SCALE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM J AT j.
9. DRAW A LINE BETWEEN j AND THE GIVEN VALUE ON THE $T_H - T_L$ SCALE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM L AT ℓ.
10. ENTER AT GIVEN VALUE OF f_2 ON SCALE f_2.
11. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE T_L CURVE.
12. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE M SCALE
13. DRAW A LINE BETWEEN THIS INTERSECTION ON THE M SCALE AND THE POINT ℓ ON STEM L.

THE INTERSECTION OF THIS LINE WITH THE δ SCALE GIVES THE REQUIRED VALUE OF δ THAT IS TO BE USED IN CHART 3, STEP 4.

STRUCTURAL DESIGN NOMOGRAPH
FOR THERMAL CYCLING
CHART 2- TEMPERATURE DIFFERENTIAL



DIRECTIONS,

1. ENTER AT THE GIVEN VALUE OF t_2 ON SCALE t_2.
2. CONNECT THIS POINT ON SCALE t_2 AND THE GIVEN POINT OF T_L ON THE T_L SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH STEM O AT O.
3. CONNECT O AND THE POINT t_r ON THE t_2 SCALE WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE T_L SCALE.
4. FROM THIS LAST INTERSECTION DRAW A CONNECTING STRAIGHT LINE THROUGH THE GIVEN VALUE OF t_1 ON SCALE t_1. EXTEND THIS LINE UP TO THE P SCALE.
5. FROM THIS INTERSECTION ON THE P SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN B CURVE.
6. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE R SCALE AT r.
7. CONNECT r AND δ-O (ON THE δ SCALE) WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM Q AT q.
8. CONNECT q AND THE GIVEN VALUE ON THE δ SCALE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE R.
9. FROM THIS POINT ON THE R SCALE DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE S SCALE AT s.
10. ENTER AT THE GIVEN VALUE OF b ON THE b SCALE.
11. CONNECT THIS POINT ON SCALE b AND THE GIVEN VALUE OF B ON THE B SCALE WITH A STRAIGHT LINE. EXTEND THIS LINE TO THE LEFT UNTIL IT INTERSECTS STEM Y AT y.
12. CONNECT THE POINTS s AND y WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM U AT u.
13. CONNECT u AND THE GIVEN VALUE OF $E(1)-E_0$ ON THE E SCALE WITH A STRAIGHT LINE.
14. THE INTERSECTION OF THIS LAST LINE WITH SCALE W GIVES THE REQUIRED VALUE OF W. THIS QUANTITY PLUS V FROM CHART 3 GIVES S ON CHART 4.

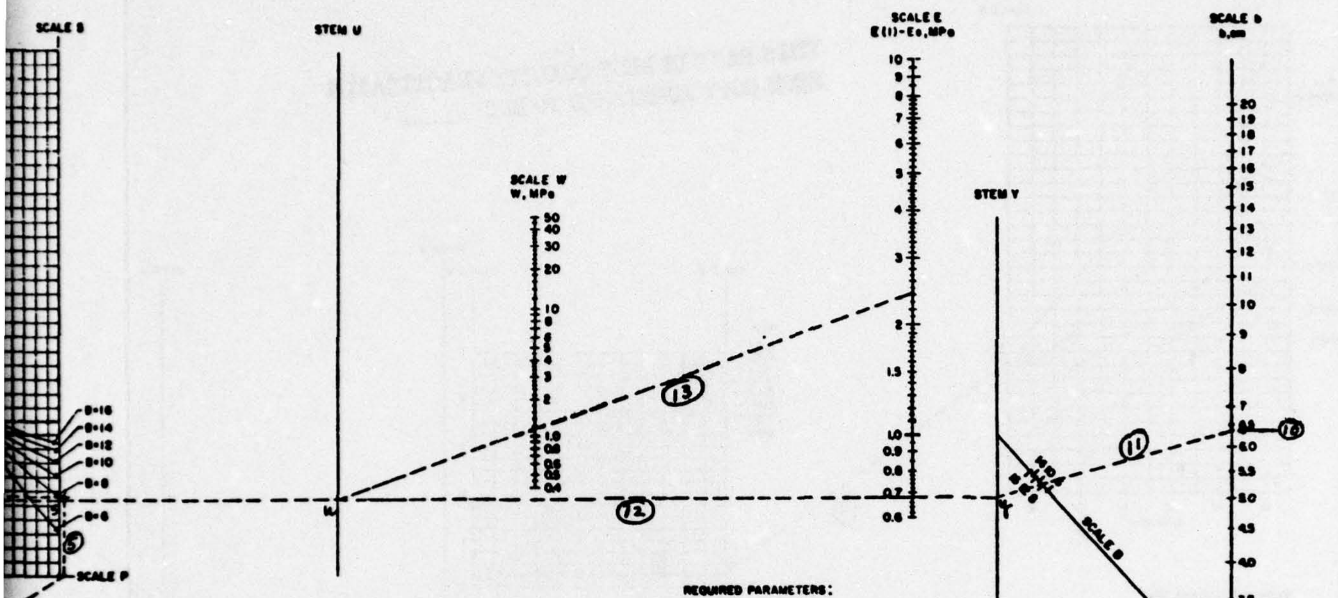
REQUIRED PARAMETERS:

b	= 6.35 cm
T_L	= −40°C
δ	= 0.008 [FROM CHART
B	= 8.2
t_2	= 183°C
t_1	= 1980°C
$E(1)-E_0$	= 2.41

RESULT:

W = 1.15 MR

Figure 16



DIRECTIONS:

1. ENTER AT THE GIVEN VALUE OF t_2 ON SCALE t_2.
2. CONNECT THIS POINT ON SCALE t_2 AND THE GIVEN POINT OF T_L ON THE T_L SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH STEM O AT o.
3. CONNECT o AND THE POINT t_2 ON THE t_2 SCALE WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE T_L SCALE.
4. FROM THIS LAST INTERSECTION DRAW A CONNECTING STRAIGHT LINE THROUGH THE GIVEN VALUE OF t_1 ON SCALE t_1. EXTEND THIS LINE UP TO THE P SCALE.
5. FROM THIS INTERSECTION ON THE P SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN D CURVE.
6. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE R SCALE AT r.
7. CONNECT r AND δ-0 (ON THE δ SCALE) WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM Q AT q.
8. CONNECT q AND THE GIVEN VALUE ON THE δ SCALE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE R.
9. FROM THIS POINT ON THE R SCALE DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE S SCALE AT s.
10. ENTER AT THE GIVEN VALUE OF b ON THE b SCALE.
11. CONNECT THIS POINT ON SCALE b AND THE GIVEN VALUE OF B ON THE B SCALE WITH A STRAIGHT LINE. EXTEND THIS LINE TO THE LEFT UNTIL IT INTERSECTS STEM Y AT y.
12. CONNECT THE POINTS s AND y WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM U AT u.
13. CONNECT u AND THE GIVEN VALUE OF $E(1)-E_0$ ON THE E SCALE WITH A STRAIGHT LINE.
14. THE INTERSECTION OF THIS LAST LINE WITH SCALE W GIVES THE REQUIRED VALUE OF W. THIS QUANTITY PLUS V FROM CHART I GIVES S ON CHART 4.

REQUIRED PARAMETERS:

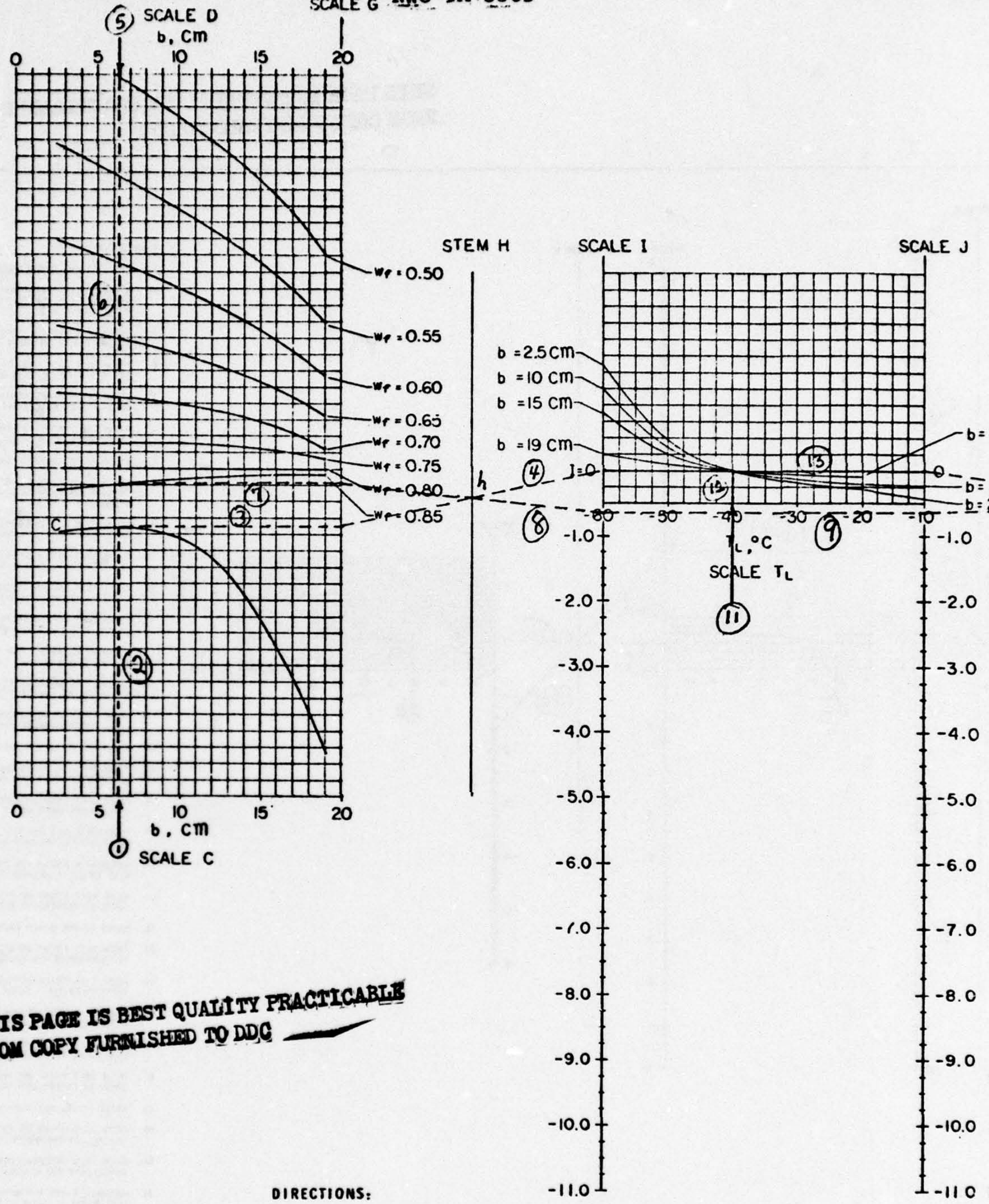
b	= 6.35 cm
T_L	= −40°C
δ	= 0.008 (FROM CHART 2)
B	= -8.2
t_2	= 183°C
t_1	= 1980°C
$E(1)-E_0$	= 2.41

RESULT:

W	= 1.15 MPa

STRUCTURAL DESIGN NOMOGRAPH
FOR THERMAL CYCLING
CHART 3 - VISCOELASTIC COMPONENT

2



REQUIRED PARAMETERS

b = 6.35 cm
w_f = 0.85
T_L = −40°C
V = 2.40 MPa
W = 1.15 MPa
S = V·W = 3.55 MPa

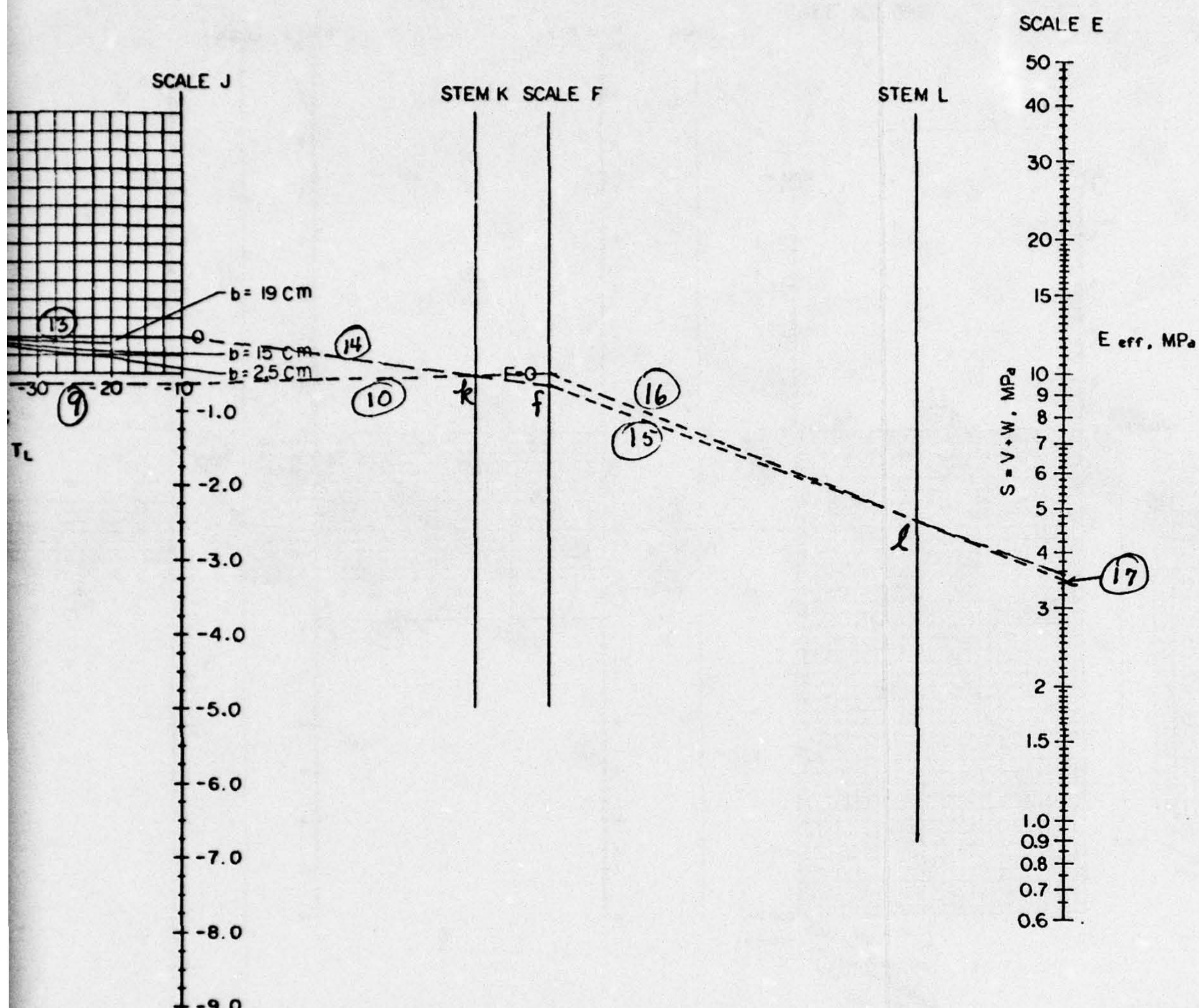
RESULTS

E_{eff} = 3.45

Figure 17.

DIRECTIONS:

1. ENTER AT THE GIVEN VALUE OF b ON THE C SCALE.
2. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS CURVE C.
3. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE G SCALE AT c.
4. CONNECT POINT c AND THE POINT MARKED I-0 ON THE I SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION h OF THE LINE ON STEM H. THIS POINT WILL BE USED AGAIN IN STEP 8.
5. ENTER AT THE GIVEN VALUE OF b ON THE D SCALE.
6. DRAW A VERTICAL LINE DOWNWARD UNTIL IT INTERSECTS THE SPECIFIED W_f CURVE.
7. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE G SCALE AT g.
8. CONNECT POINTS g AND h (SEE STEP 4) WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE I SCALE.
9. THIS INTERSECTI
IDENTICAL VALUE
10. CONNECT THIS PO
MARK THE INTERS
BE USED AGAIN I
11. ENTER AT THE LOW
12. DRAW A VERTICAL
b CURVE.
13. FROM THIS INTERS
THE J SCALE AT J
14. CONNECT POINTS
THIS LINE UNTIL
15. CONNECT POINT f
WITH A STRAIGHT
STEM L.
16. FROM F-0 ON THE
THIS LINE UNTIL
17. THIS INTERSECTIO
IS REQUIRED FOR
STEP 4 OF CHART



RVE C.

L IT INTERSECTS

SCALE WITH A
ON STEM H.

THE SPECIFIED

L IT INTERSECTS

LINE. EXTEND

9. THIS INTERSECTION ON THE I SCALE IS PROJECTED HORIZONTALLY TO ITS IDENTICAL VALUE ON THE J SCALE.

10. CONNECT THIS POINT ON THE J SCALE WITH POINT F-0 ON THE F SCALE. MARK THE INTERSECTION k OF THE LINE ON STEM K. THIS POINT WILL BE USED AGAIN IN STEP 14.

11. ENTER AT THE LOWER TEMPERATURE T_L ON THE T_L SCALE.

12. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE INTERPOLATED b CURVE.

13. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE J SCALE AT j.

14. CONNECT POINTS j AND k (SEE STEP 10) WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE F SCALE AT f.

15. CONNECT POINT f AND THE GIVEN VALUE OF S(-V+W) ON THE E SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION ℓ OF THIS LINE ON THE STEM L.

16. FROM F-0 ON THE F SCALE EXTEND A LINE THROUGH POINT ℓ. CONTINUE THIS LINE UNTIL IT INTERSECTS THE E SCALE.

17. THIS INTERSECTION PROVIDES THE VALUE OF THE E_{eff} PARAMETER, WHICH IS REQUIRED FOR THE ANALYSES OF CHART 6. USE THIS QUANTITY IN STEP 4 OF CHART 6.

aerojet solid propulsion company
SACRAMENTO, CALIFORNIA

STRUCTURAL DESIGN NOMOGRAPH FOR THERMAL CYCLING

CHART 4 - TOTAL EFFECTIVE MODULUS

6-24-77

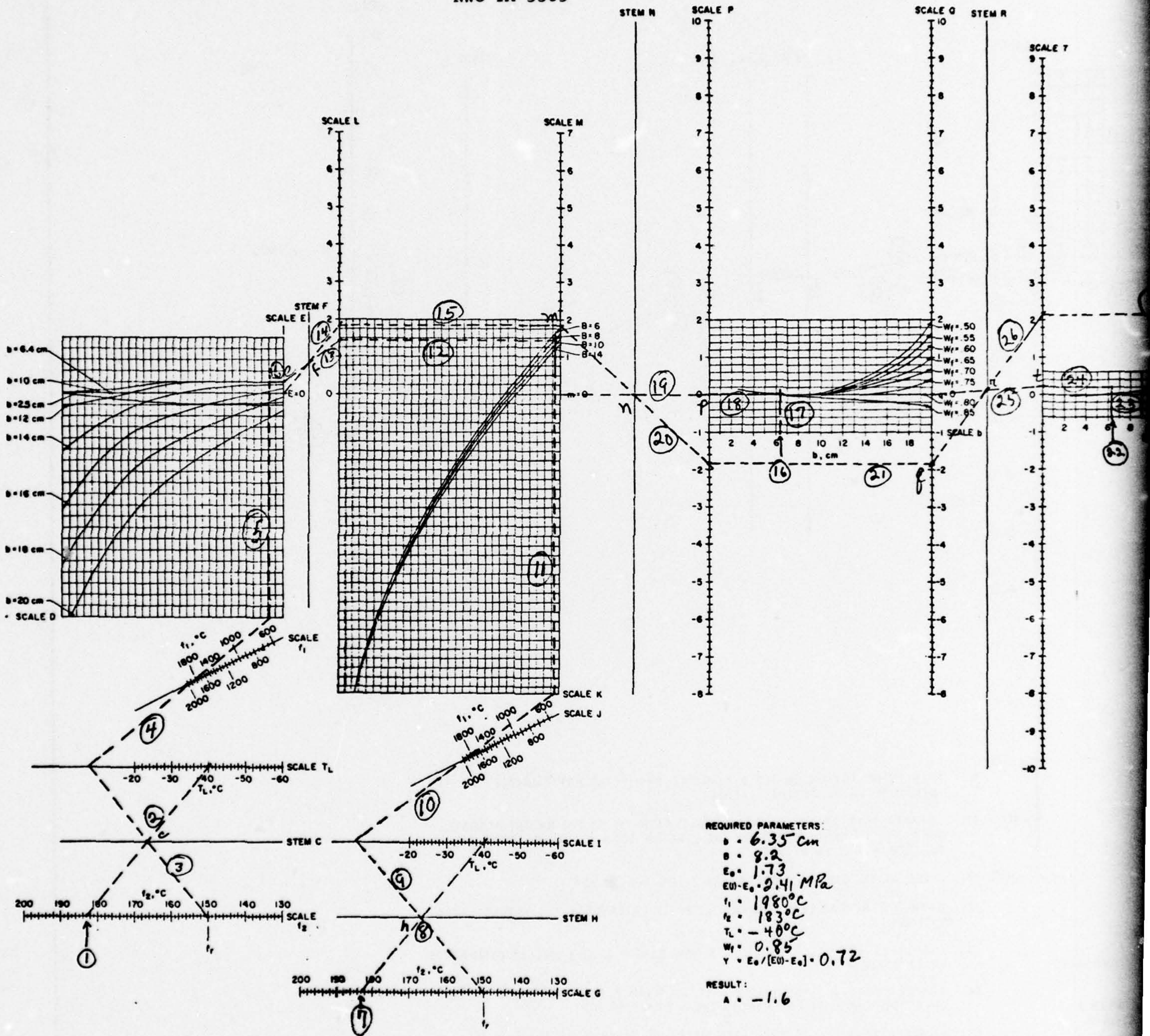
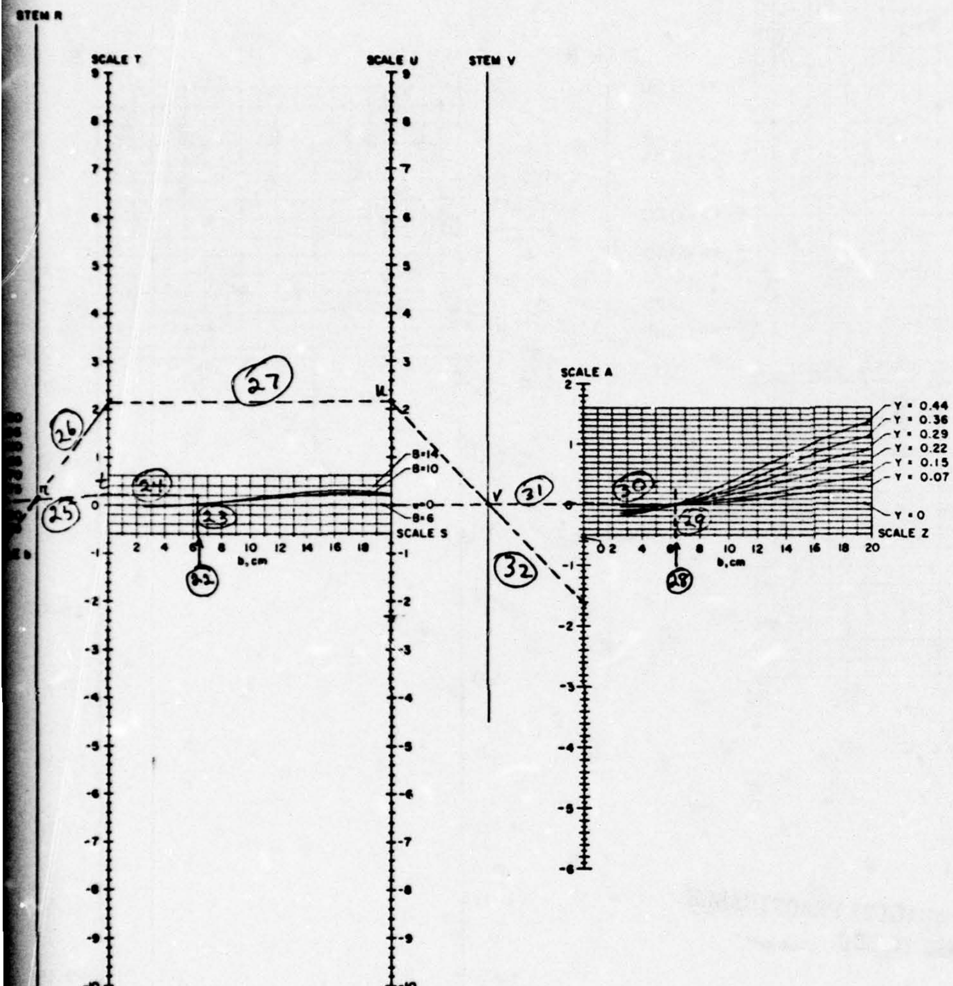

STEM N
SCALE P
SCALE Q
STEM R
SCALE T
SCALE L
SCALE M
STEM F
SCALE E
SCALE D
SCALE f_1
SCALE K
SCALE J
SCALE T_L
STEM C
SCALE I
STEM H
SCALE f_2
SCALE G
SCALE b
b, cm
REQUIRED PARAMETERS:
b = 6.35 cm
B = 8.2
E_0 = 1.73
E(t)-E_0 = 2.41 MPa
f_1 = 1980°C
f_2 = 183°C
T_L = −40°C
W_f = 0.85
Y = E_0/[E(t)-E_0] = 0.72
RESULT:
A = −1.6

Figure 18.

1

THIS PAGE IS BEST QUALITY PRACTICABLE FROM COPY FURNISHED TO DDC



DIRECTIONS:

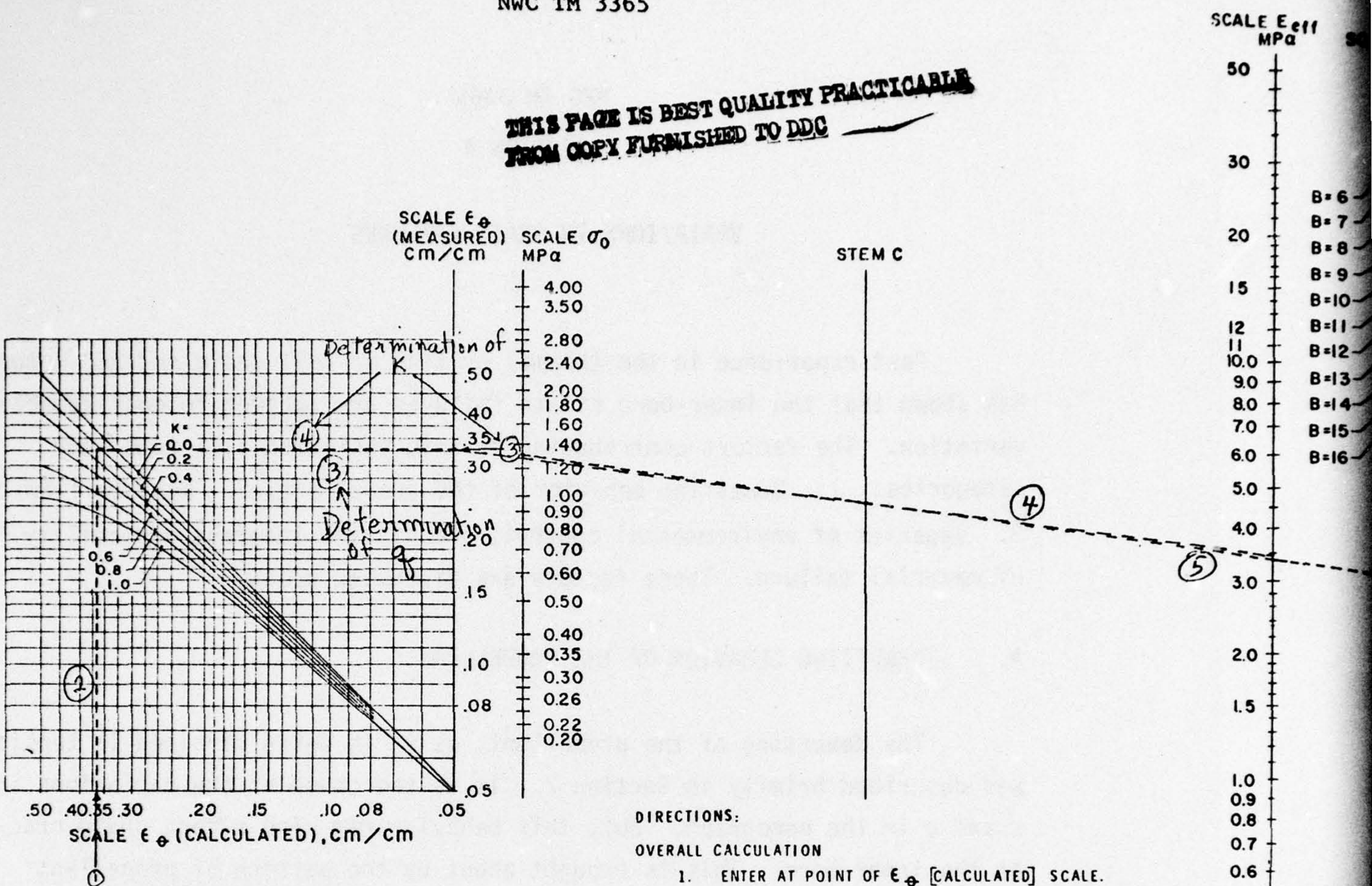
1. ENTER AT THE GIVEN VALUE OF f_2 ON SCALE f_2.
2. CONNECT THIS POINT ON SCALE f_2 AND THE GIVEN POINT OF T_L ON THE T_L SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH STEM C AT c.
3. CONNECT c AND THE POINT f_r ON THE f_2 SCALE WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE T_L SCALE.
4. FROM THIS LAST INTERSECTION DRAW A CONNECTING STRAIGHT LINE THROUGH THE GIVEN VALUE OF f_1 ON SCALE f_1. EXTEND THIS LINE UP TO THE D SCALE.
5. FROM THIS INTERSECTION ON THE D SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN b CURVE.
6. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE E SCALE AT e.
7. ENTER AT THE GIVEN VALUE OF f_2 ON SCALE G.
8. CONNECT THIS POINT ON SCALE G AND THE GIVEN POINT OF T_L ON THE I SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH STEM H AT h.
9. CONNECT h AND THE POINT f_r^* ON THE G SCALE WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE I SCALE.
10. FROM THIS LAST INTERSECTION DRAW A CONNECTING STRAIGHT LINE THROUGH THE GIVEN VALUE OF f_1 ON SCALE J. EXTEND THIS LINE UP TO THE K SCALE.
11. FROM THIS INTERSECTION ON THE K SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN B CURVE.
12. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE L SCALE AT ℓ.
13. CONNECT ℓ AND E-0 (ON THE E SCALE) WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM F AT f.
14. CONNECT INTERSECTIONS e AND f WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE L.
15. FROM THIS POINT ON THE L SCALE DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE M SCALE AT m.
16. ENTER AT THE GIVEN VALUE OF b ON THE b SCALE.
17. FROM THIS POINT ON THE b SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN W_f CURVE.
18. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE P SCALE AT p.
19. CONNECT p AND M-0 (ON THE M SCALE) WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM N AT n.
20. CONNECT n AND m (ON THE M SCALE) WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE P.
21. FROM THIS POINT ON THE P SCALE DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE Q SCALE AT q.
22. ENTER AT THE GIVEN VALUE OF b ON THE S SCALE.
23. FROM THIS POINT ON THE S SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN B CURVE.
24. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE T SCALE AT t.
25. CONNECT t AND Q-0 (ON THE Q SCALE) WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM R AT r.
26. CONNECT r AND q (ON THE Q SCALE) WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE T.
27. FROM THIS POINT ON THE T SCALE DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE U SCALE AT u.
28. ENTER AT THE GIVEN VALUE OF b ON THE Z SCALE.
29. FROM THIS POINT ON THE Z SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN Y CURVE.
30. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE A SCALE AT a.
31. CONNECT a AND V-0 WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM V AT v.
32. CONNECT v AND u (ON THE U SCALE) WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE A.

THIS LAST INTERSECTION WITH SCALE A GIVES THE REQUIRED VALUE OF A. THIS QUANTITY IS TO BE USED IN STEP 9 ON CHART 6.

STRUCTURAL DESIGN NOMOGRAPH FOR THERMAL CYCLING

CHART 5 - DAMAGE ANALYSIS

7-12-77



SCHEMATIC EXAMPLE

REQUIRED PARAMETERS:

SUBCALCULATION I

ϵ_θ (CALC) = 0.379 cm/cm
ϵ_θ (MEAS) = 0.33 cm/cm
K = 0.25

SUBCALCULATION II

ϵ_θ (CALC) = 0.379 cm/cm
K = 0.25 [SEE SUBCALCULATION I]
σ_0 = 1.313 MPa
E_{eff} = 3.45 MPa [FROM CHART 4]
B = 8.2
$\bar{N}_g$ = 15.9 CYCLES
A = −1.6 [FROM CHART 5]

RESULT:

q = −0.95

DIRECTIONS:

OVERALL CALCULATION

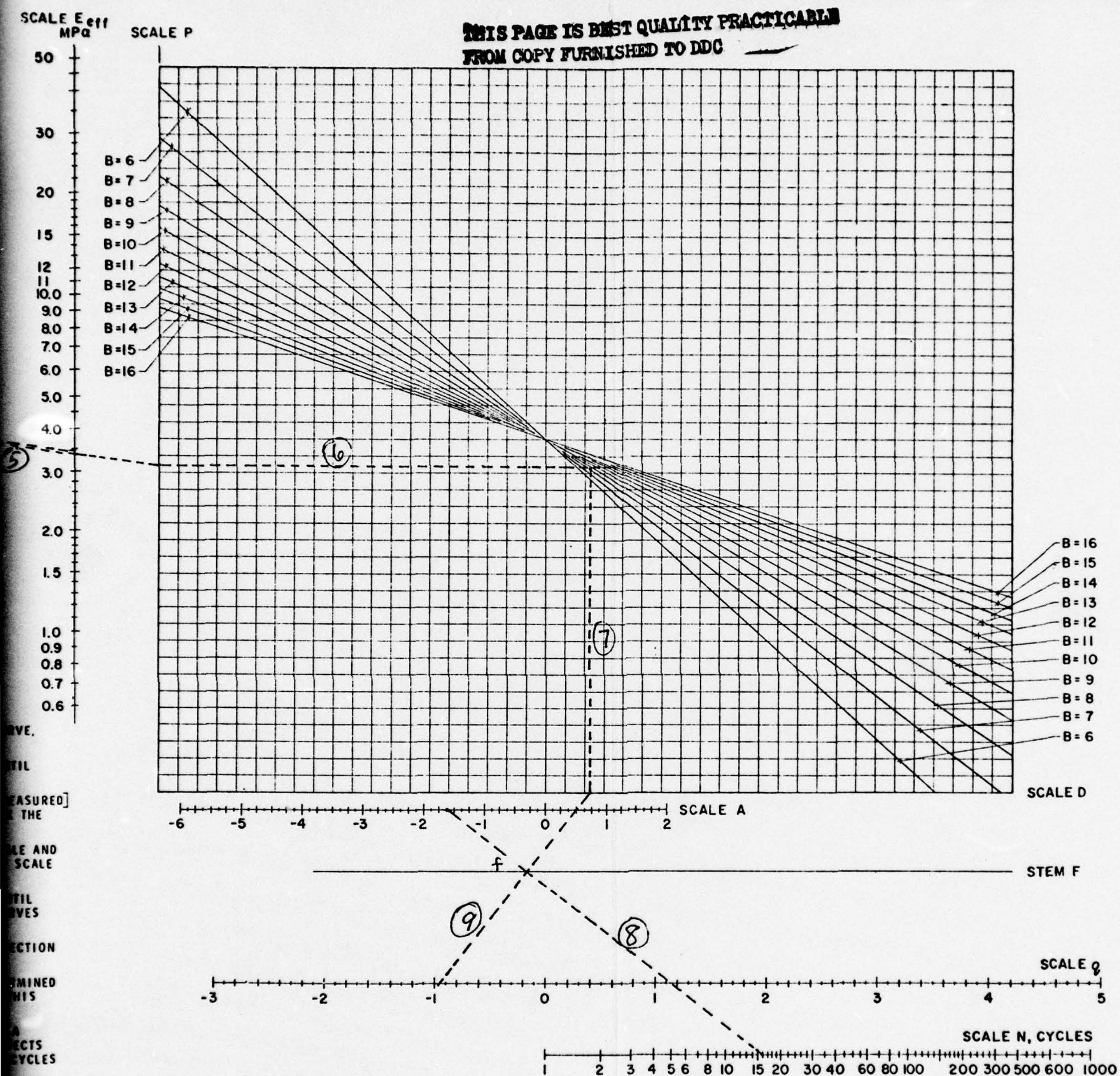
1. ENTER AT POINT OF ϵ_θ [CALCULATED] SCALE.
2. DRAW A VERTICAL LINE UNTIL IT INTERSECTS THE K CURVE, HAVING THE K VALUE FROM SUBCALCULATION I.
3. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE ϵ_θ [MEASURED] SCALE.
4. DRAW LINE BETWEEN THIS INTERSECTION ON THE ϵ_θ [MEASURED] SCALE AND THE GIVEN VALUE ON THE E_{eff} SCALE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM C AT c.
5. DRAW A LINE BETWEEN c AND THE VALUE OF THE σ_0 SCALE AND EXTEND LINE TO THE RIGHT UNTIL IT INTERSECTS THE P SCALE AT p.
6. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE GIVEN B CURVE [INTERPOLATED CURVES SHOULD BE DRAWN IN ADVANCE].
7. DRAW A VERTICAL LINE DOWNWARD FROM THE B INTERSECTION UNTIL IT INTERSECTS THE SCALE AT POINT d.
8. CONNECT d AND THE GIVEN VALUE ON THE q SCALE DETERMINED IN SUBCALCULATION II. MARK THE INTERSECTION OF THIS LINE WITH THE STEM F AT f.
9. DRAW A LINE BETWEEN f AND THE GIVEN VALUE ON THE A SCALE. EXTEND THIS LINE DOWNWARD UNTIL IT INTERSECTS THE N SCALE. THIS INTERSECTION IS THE NUMBER OF CYCLES TO FAILURE, N.

SUBCALCULATION I - EMPIRICAL DETERMINATION OF K.

THE GIVEN QUANTITIES ARE FOR A STRAIN EVALUATION CYLINDER AND INCLUDE: ϵ_θ [CALCULATED] AND ϵ_θ [MEASURED].

1. ENTER AT THE GIVEN POINT ON ϵ_θ [CALCULATED] SCALE.
2. DRAW A VERTICAL LINE UPWARD.
3. ENTER AT THE GIVEN POINT OF ϵ_θ [MEASURED] SCALE.
4. DRAW A HORIZONTAL LINE TO THE LEFT.
5. THE INTERSECTION OF THE HORIZONTAL AND VERTICAL LINES DEFINES THE K CURVE APPROPRIATE TO THE GIVEN PROPELLANT FORMULATION. THIS CURVE MAY FALL BETWEEN THOSE DRAWN SO AN INTERPOLATED CURVE SHOULD BE DRAWN FOR THE LATER CALCULATIONS.

Figure 19.



SUBCALCULATION II - EMPIRICAL DETERMINATION OF q.

THE GIVEN QUANTITIES ARE THE SAME AS REQUIRED IN THE OVERALL CALCULATION (EXCEPT FOR q) PLUS THE GEOMETRIC MEAN NUMBER OF CYCLES TO FAILURE, N, OF THREE OR MORE STRAIN EVALUATION CYLINDERS.

1-7 STEPS 1 THROUGH 7 OF THE OVERALL CALCULATION ARE PERFORMED AS DESCRIBED, BUT STEPS 8 AND 9 ARE REVERSED.

8. ENTER THE VALUE OF A ON THE A SCALE AND THE GEOMETRIC MEAN OF OBSERVED CYCLES TO FAILURE ON THE N SCALE AND CONNECT THEM WITH A STRAIGHT LINE. MARK THE INTERSECTION OF STEM F AT f.

9. CONNECT POINT f ON THE STEM F AND THE INTERSECTION d ON THE D SCALE (SEE STEP 7). EXTEND THE LINE DOWNWARD, THE INTERSECTION WITH THE q SCALE GIVES THE EMPIRICAL VALUE OF q.

aerojet solid propulsion company
SACRAMENTO, CALIFORNIA

STRUCTURAL DESIGN NOMOGRAPH
FOR THERMAL CYCLING
CHART 6 - FINAL CALCULATIONS

5-16-77

SECTION 4

VARIATIONS IN GRAIN FAILURES

Past experience in the thermal cycling of full scale and SEC motors has shown that the inner-bore strain failures are subject to considerable variation. The factors contributing to this variation fall into four categories: 1. Dewetting behavior of the propellant; 2. surface flaws; 3. vagaries of environmental control; and 4. the inherent variability of material failure. These factors are discussed below.

A. DEWETTING BEHAVIOR OF THE PROPELLANT

The dewetting of the propellant, as it is being strained in tension, was described briefly in Section 2. It is the cause of the variations in K and q in the nomograph. But, this behavior may also affect grain cracking at the inner-bore. This is brought about by the pattern of propellant dewetting, which greatly affects its notch, or flaw, sensitivity.

A propellant which dewets in local bands will tend to accentuate a flaw or notch; thus accelerating the inner-bore cracking of a grain.

Some propellants dewet uniformly through the material converting it to a sponge. This soft spongy material acts to reduce flaw, or notch, sensitivity. Hence, grain inner-bore cracking rates are reduced.

All propellants range in their dewetting between the two limiting behaviors described above. But, as a simple rule, those propellants exhibiting the largest values of K and q (in the nomographic analysis) may be associated with the thinnest bands of localized dewetting (high flaw sensitivity).

Please note: Negative values of q indicate a softening of the propellant. The positive values, indicate that the propellant has become harder.

The smallest values of K and q (q becoming more negative) are expected to correlate with an overall spongy propellant and reduced flaw sensitivity.

B. SURFACE FLAWS

Past experience has shown that inner-bore cracks can be initiated by seemingly insignificant flaws; i.e., (1) a flick of red burning rate catalyst that was about .03 in. x .03 in. x .06 in., (2) a ridge left in the propellant where two parts of a casting mandrel were mated to within 0.005 in., (3) accidentally made surface scratches that were thought to be about 0.002 in. deep; and (4) near-surface casting bubbles.

It is essential, then, that every crack be examined to detect its probable origin, and to determine if the motor behavior is an outlier because of an unusual flaw.

C. VAGARIES OF ENVIRONMENTAL CONTROL

This testing is more sensitive to variations in the temperature environment than to all of the other test variables. Hence, it is critical that the temperature be controlled within as narrow limits as possible. A limit control of better than ± 1°C is recommended.

The next most important parameter is atmospheric moisture. Care must be taken not to expose a cold grain to the atmosphere. A test for accidental moisture exposure is a change in the location of the bore crack away from the region of highest strains. Usually, the presence of air moisture will interact with the grain to cause failure initiation at a point that is about one-third the length of the motor from the exposed end. The high strain region is usually at the mid-point.

D. STATISTICS OF MATERIAL FAILURE

When tested for fatigue, solid propellant failures usually follow a Weibull statistical distribution. The Weibull distribution is usually hard to characterize accurately, so we use a normal logarithmic distribution for convenience. From either of these distributions we can generate the required statistical inferences.

The most important statistical inference is the prediction of the "expected" range of motor failures for a set of n motors. That is, the prediction of the number of cycles-to-failure for the first motor of the set, N_1, to the last (or nth) motor of the set, N_n. The expected range, then, is N_1 to N_n.

The range prediction is made in terms of the expected first failure in the set of n motors. Figure 20 contains plots of the ratio $N_1/\overline{N}$ versus the motor sample size n at various levels of the propellant log-normal standard deviation, $\sigma(\log t_f)$. The use of the curve requires: 1. The nomographic prediction of $\overline{N}$, which is taken as the value of the prediction (N); 2. the motor sample size, n, for which the prediction is to be made; and 3. an estimate of $\sigma(\log t_f)$, $\hat{\sigma}(\log t_f)$. The curve is entered at n and $\hat{\sigma}(\log t_f)$ and the expected ratio $N_1/\overline{N}$ is read directly. The expected value of N_1 is obtained by the relation

$$N_1 = (N_1/\overline{N}) \times N \qquad (16)$$

The expected value of N_n is based upon the logarithmic nature of the statistical distribution and is approximated by

$$N_n \cong N/(N_1/\overline{N}) \qquad (17)$$

Thus, both N_1 and N_n can be approximated.

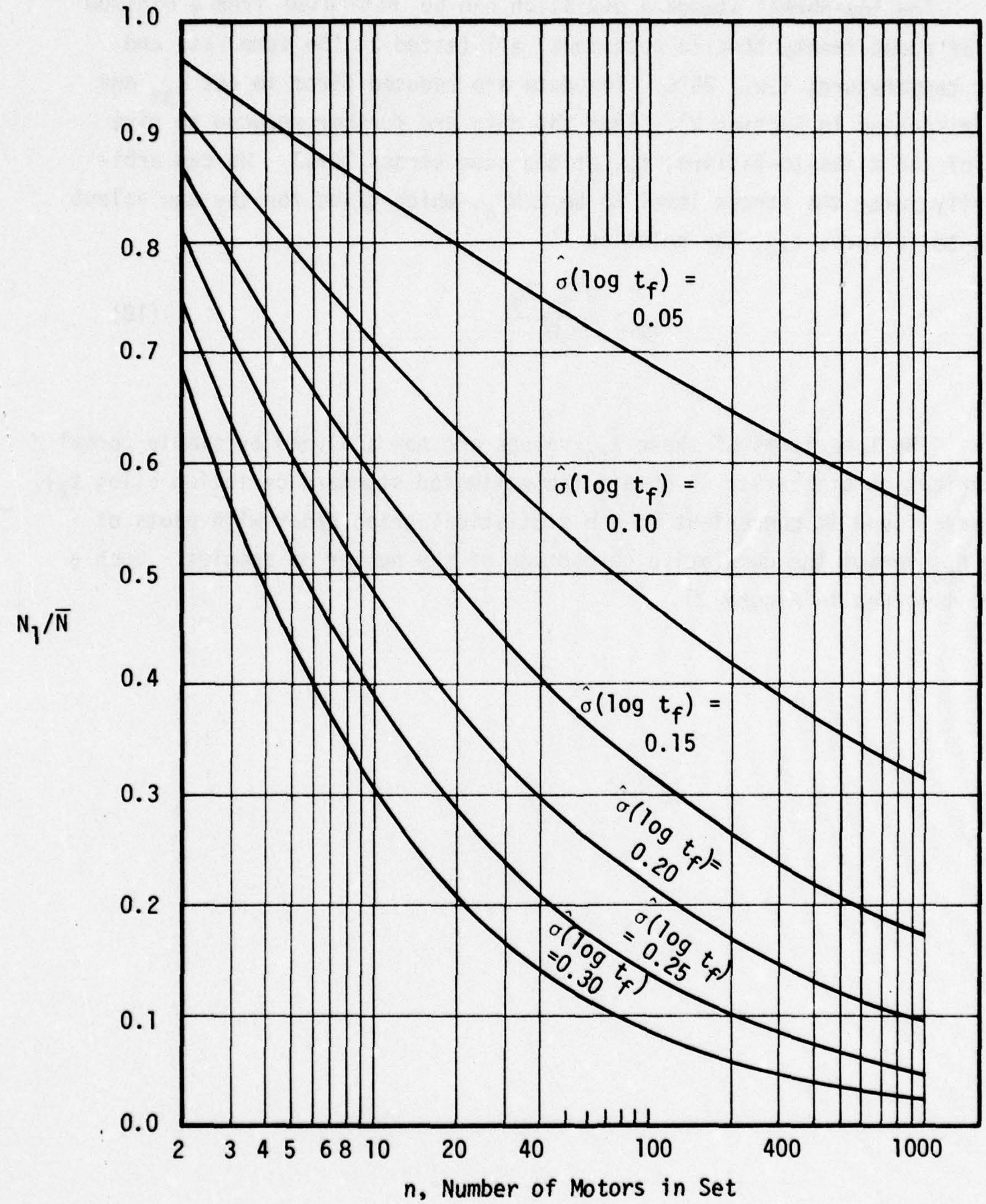


FIGURE 20. EXPECTED NUMBER OF CYCLES TO FIRST FAILURE VERSUS MOTOR SAMPLE SIZE

The log-normal standard deviation can be estimated from a minimum set of about twenty tensile specimens, all tested at the same rate and test temperature; i.e., 25°C. The data are reduced first to get σ_{tm} and t_f (discussed in Section 2). Then the data are further reduced to give all of the times-to-failure, t_f, at the same stress level. We can arbitrarily choose the stress level to be 2 MP_a, which gives for the equivalent time-to-failure, t_{fe}, the relation

$$t_{fe} = \frac{\sigma^B_{tm} \ t_f}{2^B} \tag{18}$$

The logarithms of these t_{fe} values are now analyzed by simple normal distribution statistics to obtain the estimated standard deviation $\hat{\sigma}(\log t_f)$. We have found it convenient to use statistical graph paper with plots of log t_{fe} versus the cumulative percentage of the number of samples. Such a plot is given in Figure 21.

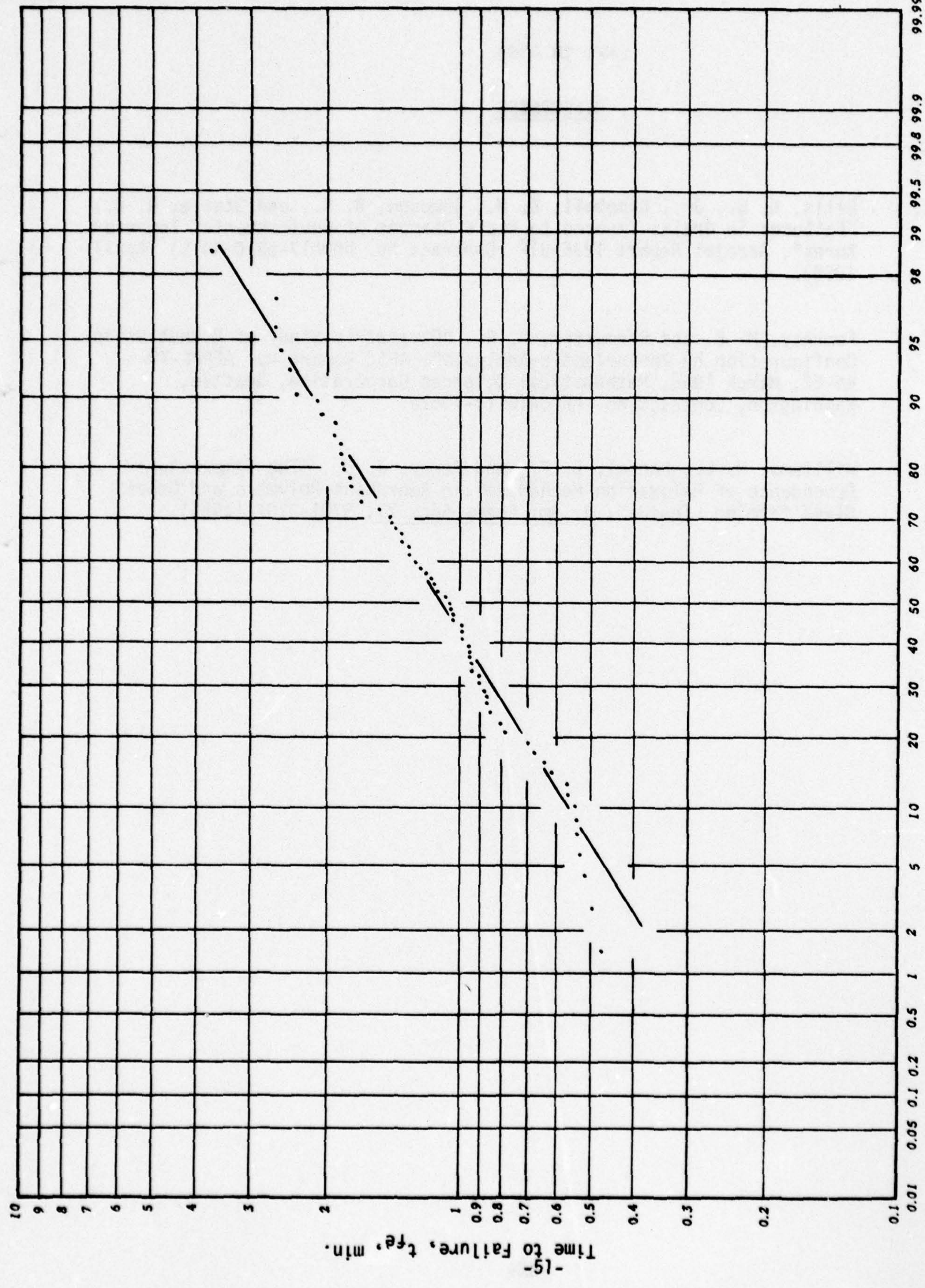


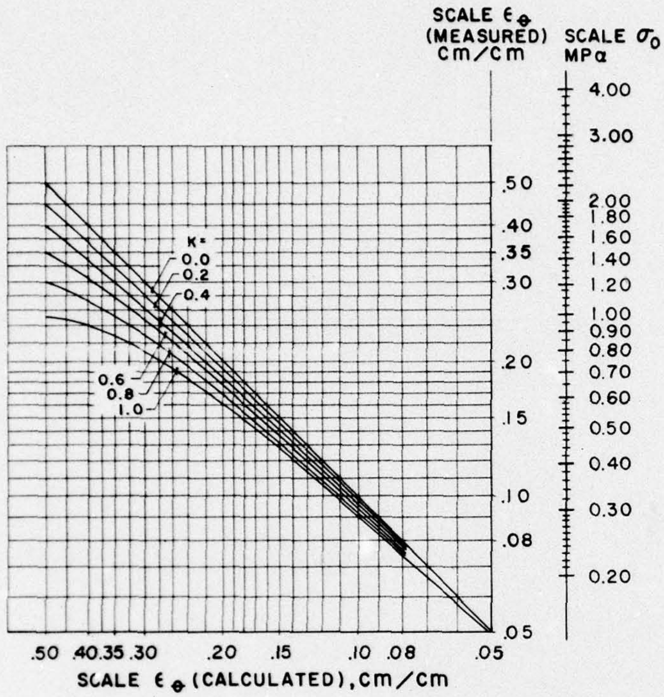
FIGURE 21. TYPICAL LOGARITHMIC DISTRIBUTION OF PROPELLANT TIME-TO-FAILURE DATA

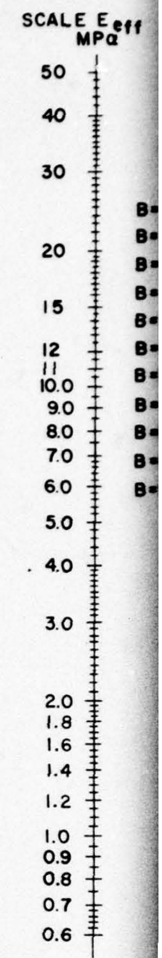
REFERENCES

1. Bills, K. W., Jr., Campbell, D. M., Sampson, R. C., and Steele, R. D., "Failures in Grains Exposed to Rapid Changes of Environmental Temperatures", Aerojet Report 1236-81F (Contract No. N00017-68-C-4415) (April 1969).

2. Fourney, M. E. and Parmerter, R. R., "Parametric Study of Rocket Grain Configuration by Photoelastic Analysis", AFSC Report No. AFRPL-TR-66-52, March 1966, Mathematical Sciences Corporation, Seattle, Washington, Contract No. AF 04(611)-1052a.

3. Williams, M. L., Landel, R. F., and Ferry, J. D., "The Temperature Dependence of Relaxation Mechanisms in Amorphous Polymers and Other Glass Forming Liquids", J. Am. Chem. Soc. 77, 3701-3707 (1955).





REQUIRED PARAMETERS:

ϵ_θ (CALC) = ____ cm/cm

K = _____ [SEE SUBCALCULATION I]

ϵ_θ (CALC) = ____ cm/cm

ϵ_θ (MEAS) = ____ cm/cm

σ_0 = _____ MPa

E_{eff} = _____ MPa [FROM CHART 4]

B = _____

ϱ = _____ [SEE SUBCALCULATION II]

$\bar{N}_g$ = _____ CYCLES

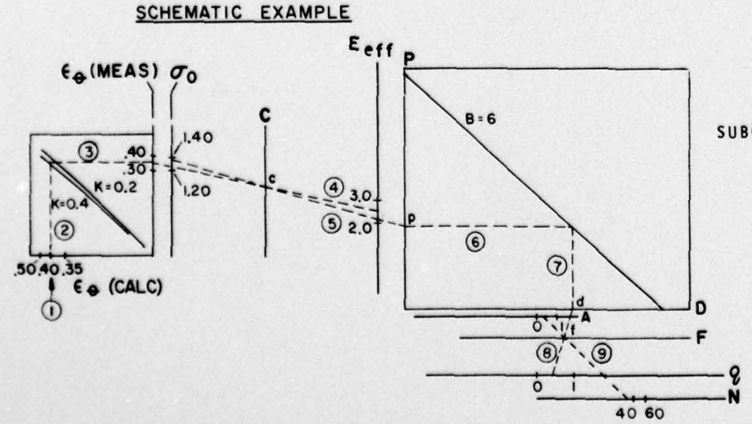
A = _____ [FROM CHART 5]

RESULT:

ϵ_θ (MEAS) = ____ cm/cm

N = ____ CYCLES

SCHEMATIC EXAMPLE



DIRECTIONS:

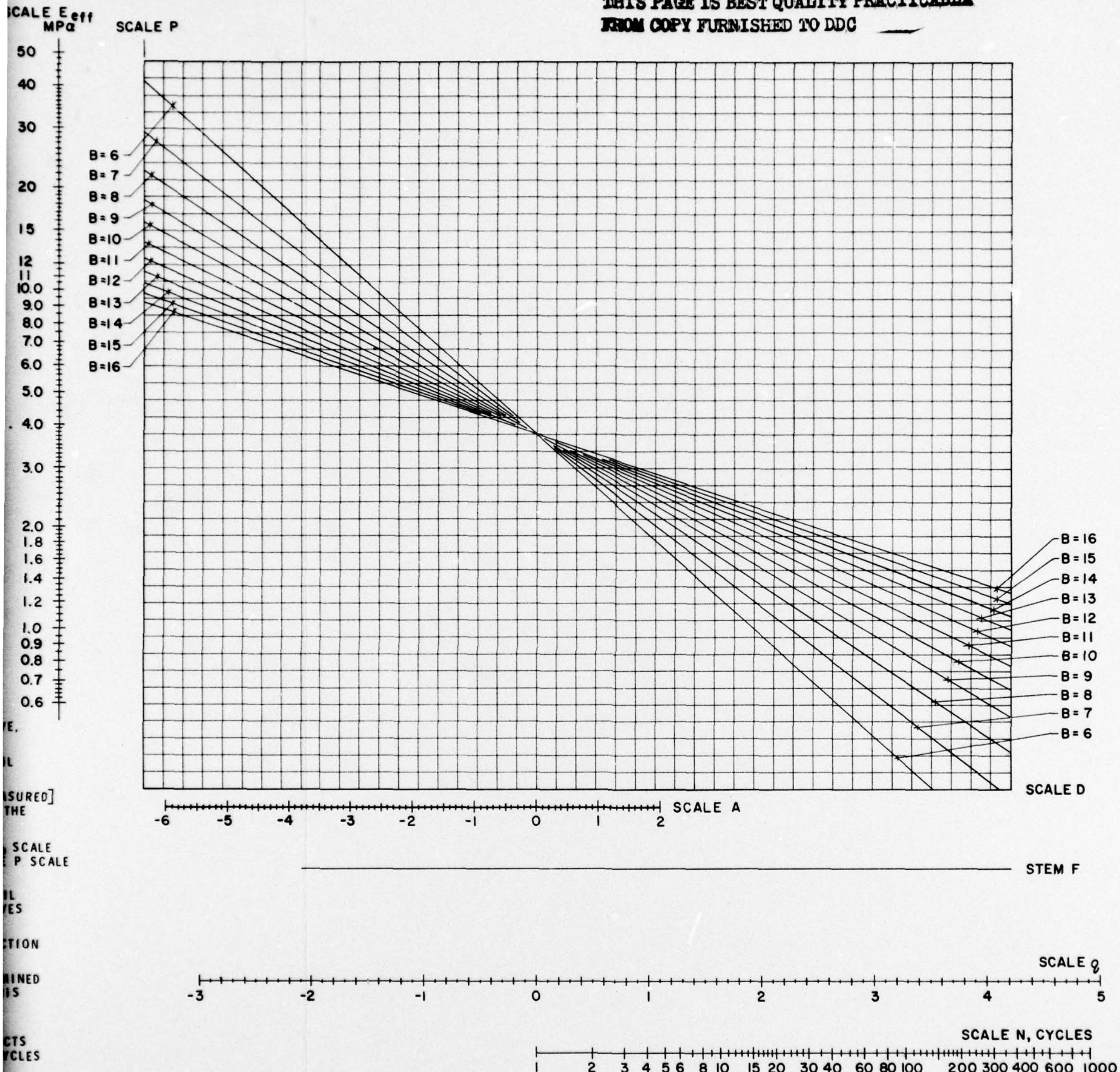
OVERALL CALCULATION

1. ENTER AT POINT OF ϵ_θ [CALCULATED] SCALE.
2. DRAW A VERTICAL LINE UNTIL IT INTERSECTS THE K CURVE, HAVING THE K VALUE FROM SUBCALCULATION I.
3. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE ϵ_θ [MEASURED] SCALE.
4. DRAW LINE BETWEEN THIS INTERSECTION ON THE ϵ_θ [MEASURED] SCALE AND THE GIVEN VALUE ON THE E_{eff} SCALE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM C AT c.
5. DRAW A LINE BETWEEN c AND THE VALUE OF σ_0 ON THE σ_0 SCALE AND EXTEND LINE TO THE RIGHT UNTIL IT INTERSECTS THE P SCALE AT p.
6. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE GIVEN B CURVE [INTERPOLATED CURVES SHOULD BE DRAWN IN ADVANCE].
7. DRAW A VERTICAL LINE DOWNWARD FROM THE B INTERSECTION UNTIL IT INTERSECTS THE D SCALE AT POINT d.
8. CONNECT d AND THE GIVEN VALUE ON THE ϱ SCALE DETERMINED IN SUBCALCULATION II. MARK THE INTERSECTION OF THIS LINE WITH THE STEM F AT f.
9. DRAW A LINE BETWEEN f AND THE GIVEN VALUE ON THE A SCALE. EXTEND THIS LINE DOWNWARD UNTIL IT INTERSECTS THE N SCALE. THIS INTERSECTION IS THE NUMBER OF CYCLES TO FAILURE, N.

SUBCALCULATION I - EMPIRICAL DETERMINATION OF K.

THE GIVEN QUANTITIES ARE FOR A STRAIN EVALUATION CYLINDER AND INCLUDE: ϵ_θ [CALCULATED] AND ϵ_θ [MEASURED].

1. ENTER AT THE GIVEN POINT ON ϵ_θ [CALCULATED] SCALE.
2. DRAW A VERTICAL LINE UPWARD.
3. ENTER AT THE GIVEN POINT OF ϵ_θ [MEASURED] SCALE.
4. DRAW A HORIZONTAL LINE TO THE LEFT.
5. THE INTERSECTION OF THE HORIZONTAL AND VERTICAL LINES DEFINES THE K CURVE APPROPRIATE TO THE GIVEN PROPELLANT FORMULATION. THIS CURVE MAY FALL BETWEEN THOSE DRAWN SO AN INTERPOLATED CURVE SHOULD BE DRAWN FOR THE LATER CALCULATIONS.



SUBCALCULATION II - EMPIRICAL DETERMINATION OF q

THE GIVEN QUANTITIES ARE THE SAME AS REQUIRED IN THE OVERALL CALCULATION (EXCEPT FOR q) PLUS THE GEOMETRIC MEAN NUMBER OF CYCLES TO FAILURE, N, OF THREE OR MORE STRAIN EVALUATION CYLINDERS.

1-7 STEPS 1 THROUGH 7 OF THE OVERALL CALCULATION ARE PERFORMED AS DESCRIBED, BUT STEPS 8 AND 9 ARE REVERSED.

8. ENTER THE VALUE OF A ON THE A SCALE AND THE GEOMETRIC MEAN OF OBSERVED CYCLES TO FAILURE ON THE N SCALE AND CONNECT THEM WITH A STRAIGHT LINE. MARK THE INTERSECTION OF STEM F AT f.

9. CONNECT POINT f ON THE STEM F AND THE INTERSECTION d ON THE D SCALE (SEE STEP 7). EXTEND THE LINE DOWNWARD, THE INTERSECTION WITH THE q SCALE GIVES THE EMPIRICAL VALUE OF q.

aerojet solid propulsion company
SACRAMENTO, CALIFORNIA

STRUCTURAL DESIGN NOMOGRAPH
FOR THERMAL CYCLING
CHART 6 - FINAL CALCULATIONS

5-16-77

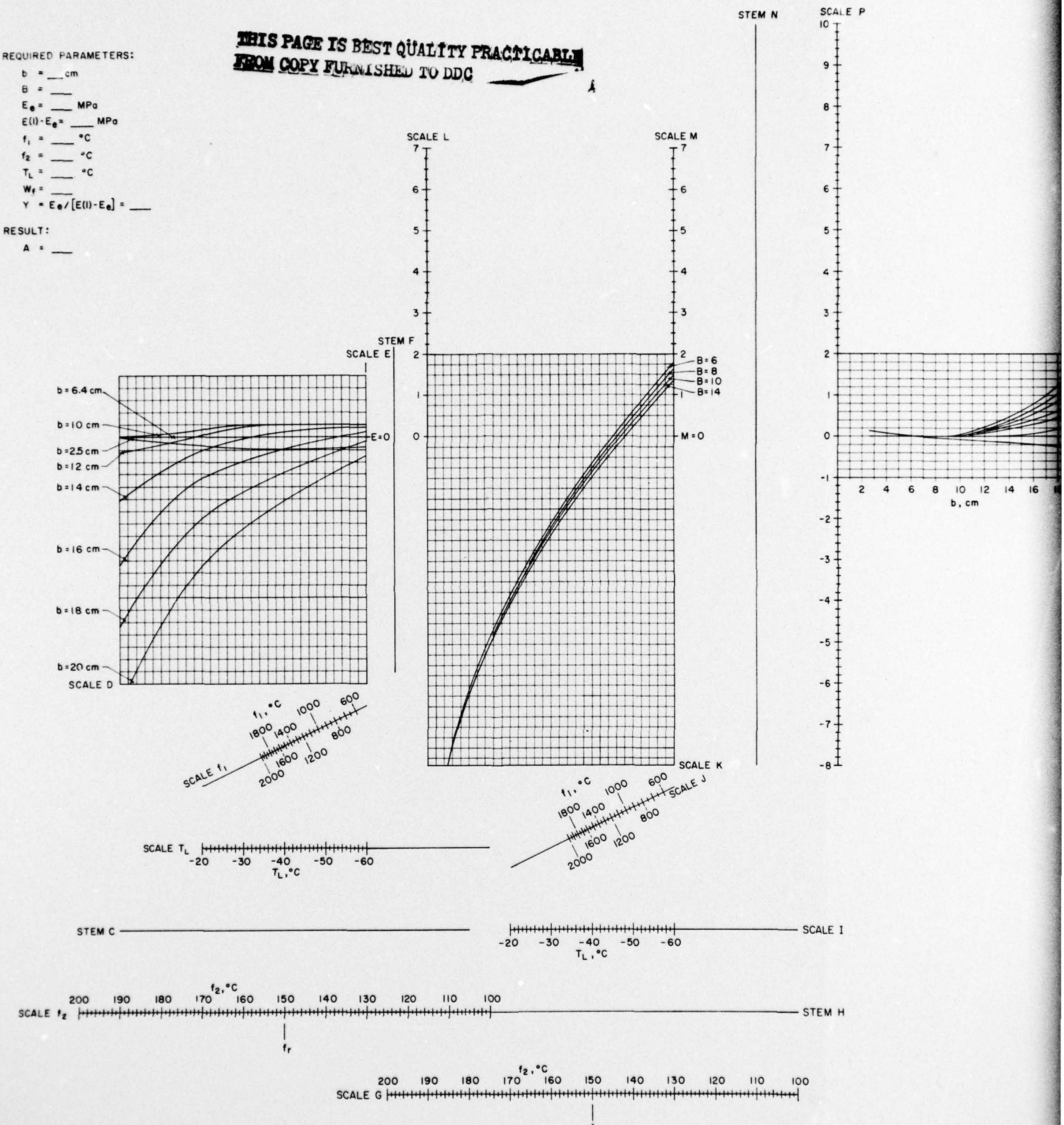
2

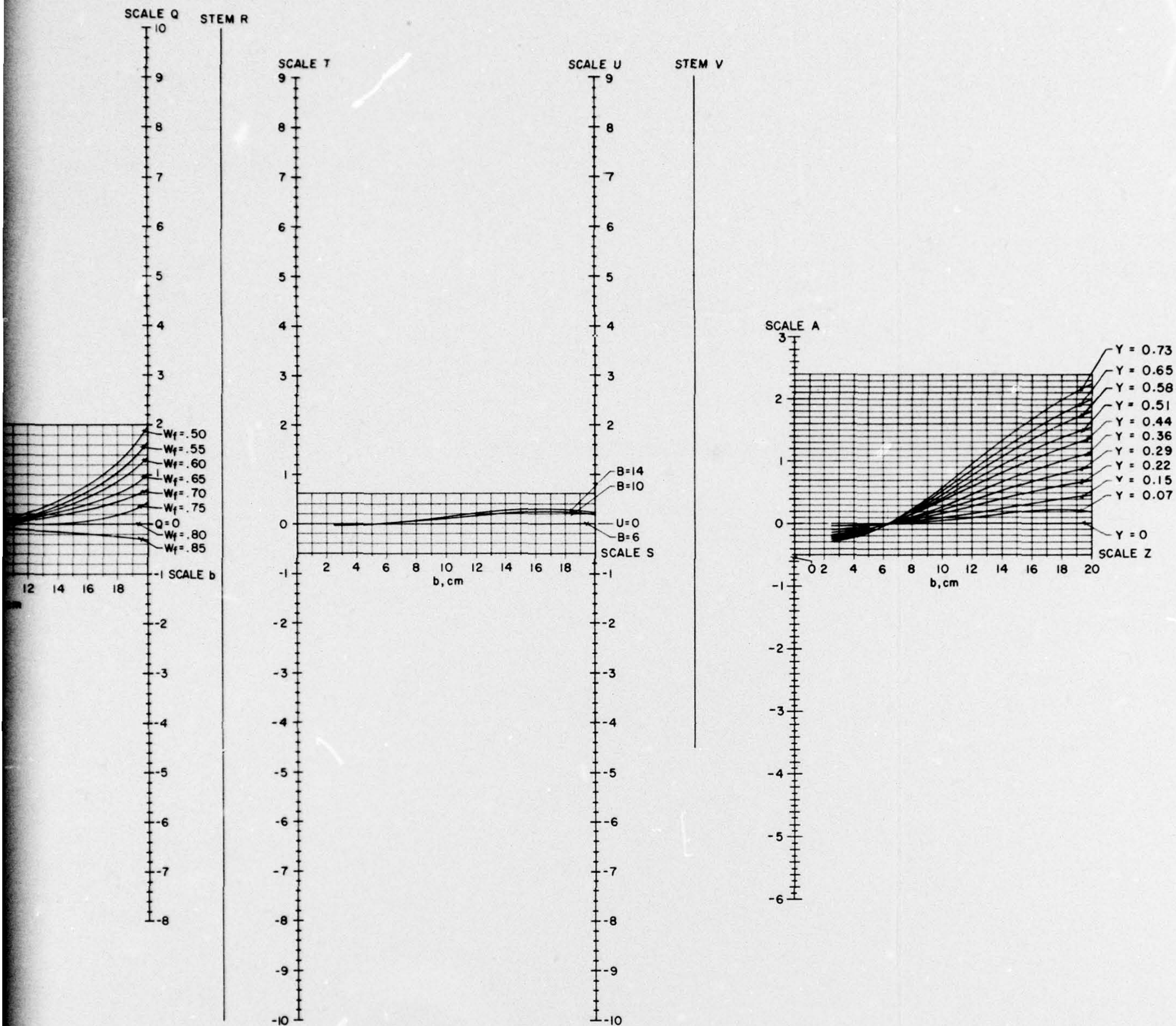
REQUIRED PARAMETERS:

- b = ___ cm
- B = ___
- E_e = ___ MPa
- $E(1) - E_e$ = ___ MPa
- f_1 = ___ °C
- f_2 = ___ °C
- T_L = ___ °C
- W_f = ___
- $Y = E_e / [E(1) - E_e]$ = ___

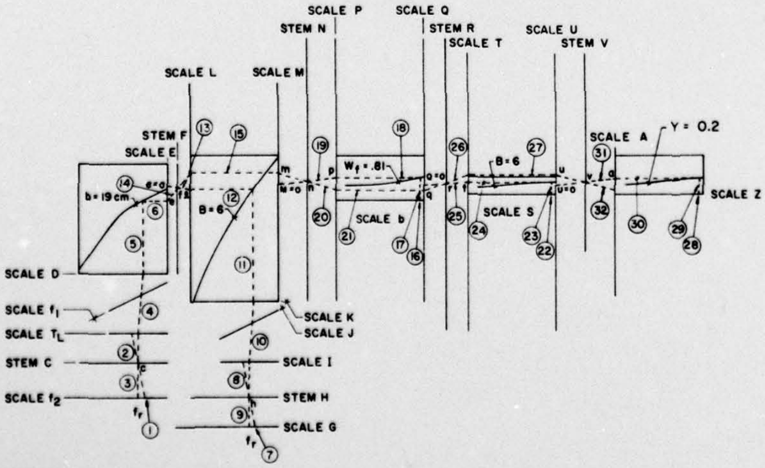
RESULT:

- A = ___





SCHEMATIC EXAMPLE

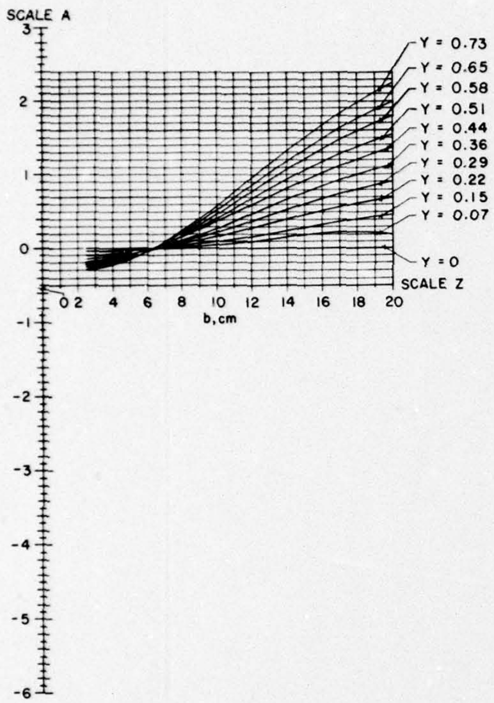


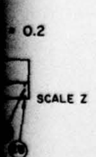
DIRECTIONS:

1. ENTER AT THE
2. CONNECT THIS SCALE WITH A WITH STEM C
3. CONNECT c AN EXTEND THIS L
4. FROM THIS LA THE GIVEN VA
5. FROM THIS IN UNTIL IT INTE
6. FROM THIS IN INTERSECTS T
7. ENTER AT THE
8. CONNECT THIS SCALE WITH A WITH STEM H
9. CONNECT h AN THIS LINE UNT
10. FROM THIS LA THE GIVEN VA
11. FROM THIS IN UNTIL IT INTE
12. FROM THIS IN INTERSECTS TH
13. CONNECT ℓ AN INTERSECTION
14. CONNECT INTER UNTIL IT INTE
15. FROM THIS PO UNTIL IT INTE
16. ENTER AT THE
17. FROM THIS PO INTERSECTS TH
18. FROM THIS IN INTERSECTS TH
19. CONNECT p AN INTERSECTION
20. CONNECT n AN LINE UNTIL IT
21. FROM THIS PO UNTIL IT INTE
22. ENTER AT THE
23. FROM THIS PO IT INTERSECTS
24. FROM THIS IN INTERSECTS TH
25. CONNECT t AND INTERSECTION
26. CONNECT r AN LINE UNTIL IT
27. FROM THIS PO UNTIL IT INTE
28. ENTER AT THE
29. FROM THIS PO INTERSECTS TH
30. FROM THIS IN INTERSECTS TH
31. CONNECT a AN LINE WITH THE
32. CONNECT v AN LINE UNTIL IT

THIS LAST INTERSEC QUANTITY IS TO BE

V





DIRECTIONS:

1. ENTER AT THE GIVEN VALUE OF f_2 ON SCALE f_2.
2. CONNECT THIS POINT ON SCALE f_2 AND THE GIVEN POINT OF T_L ON THE T_L SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH STEM C AT c.
3. CONNECT c AND THE POINT f_r ON THE f_2 SCALE WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE T_L SCALE.
4. FROM THIS LAST INTERSECTION DRAW A CONNECTING STRAIGHT LINE THROUGH THE GIVEN VALUE OF f_1 ON SCALE f_1. EXTEND THIS LINE UP TO THE D SCALE.
5. FROM THIS INTERSECTION ON THE D SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN b CURVE.
6. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE E SCALE AT e.
7. ENTER AT THE GIVEN VALUE OF f_2 ON SCALE G.
8. CONNECT THIS POINT ON SCALE G AND THE GIVEN POINT OF T_L ON THE I SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH STEM H AT h.
9. CONNECT h AND THE POINT f_r ON THE G SCALE WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE I SCALE.
10. FROM THIS LAST INTERSECTION DRAW A CONNECTING STRAIGHT LINE THROUGH THE GIVEN VALUE OF f_1 ON SCALE J. EXTEND THIS LINE UP TO THE K SCALE.
11. FROM THIS INTERSECTION ON THE K SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN B CURVE.
12. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE L SCALE AT ℓ.
13. CONNECT ℓ AND E=0 (ON THE E SCALE) WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM F AT f.
14. CONNECT INTERSECTIONS e AND f WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE L.
15. FROM THIS POINT ON THE L SCALE DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE M SCALE AT m.
16. ENTER AT THE GIVEN VALUE OF b ON THE b SCALE.
17. FROM THIS POINT ON THE b SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN W_f CURVE.
18. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE P SCALE AT p.
19. CONNECT p AND M=0 (ON THE M SCALE) WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM N AT n.
20. CONNECT n AND m (ON THE M SCALE) WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE P.
21. FROM THIS POINT ON THE P SCALE DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE Q SCALE AT q.
22. ENTER AT THE GIVEN VALUE OF b ON THE S SCALE.
23. FROM THIS POINT ON THE S SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN B CURVE.
24. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE T SCALE AT t.
25. CONNECT t AND Q=0 (ON THE Q SCALE) WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM R AT r.
26. CONNECT r AND q (ON THE Q SCALE) WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE T.
27. FROM THIS POINT ON THE T SCALE DRAW A HORIZONTAL LINE TO THE RIGHT UNTIL IT INTERSECTS THE U SCALE AT u.
28. ENTER AT THE GIVEN VALUE OF b ON THE Z SCALE.
29. FROM THIS POINT ON THE Z SCALE DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN Y CURVE.
30. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT INTERSECTS THE A SCALE AT a.
31. CONNECT a AND U=0 WITH A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM V AT v.
32. CONNECT v AND u (ON THE U SCALE) WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS SCALE A.

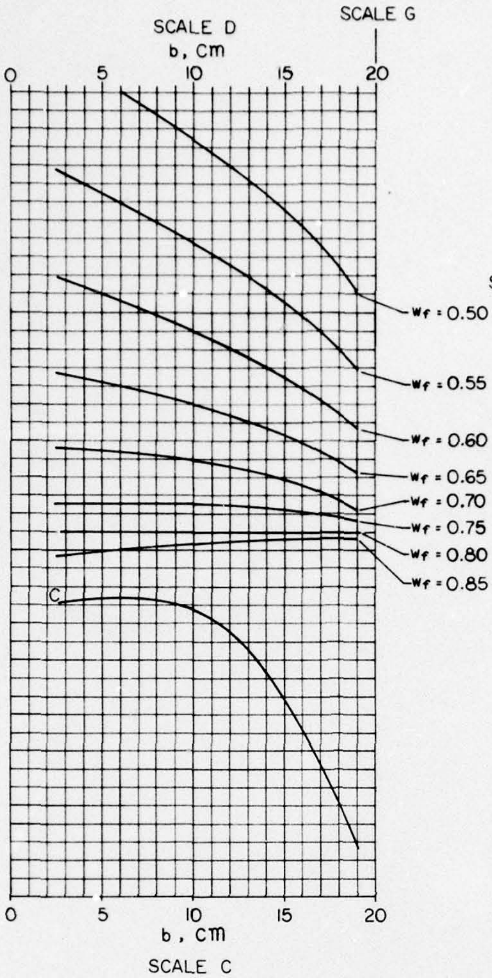
THIS LAST INTERSECTION WITH SCALE A GIVES THE REQUIRED VALUE OF A. THIS QUANTITY IS TO BE USED IN STEP 9 ON CHART 6.

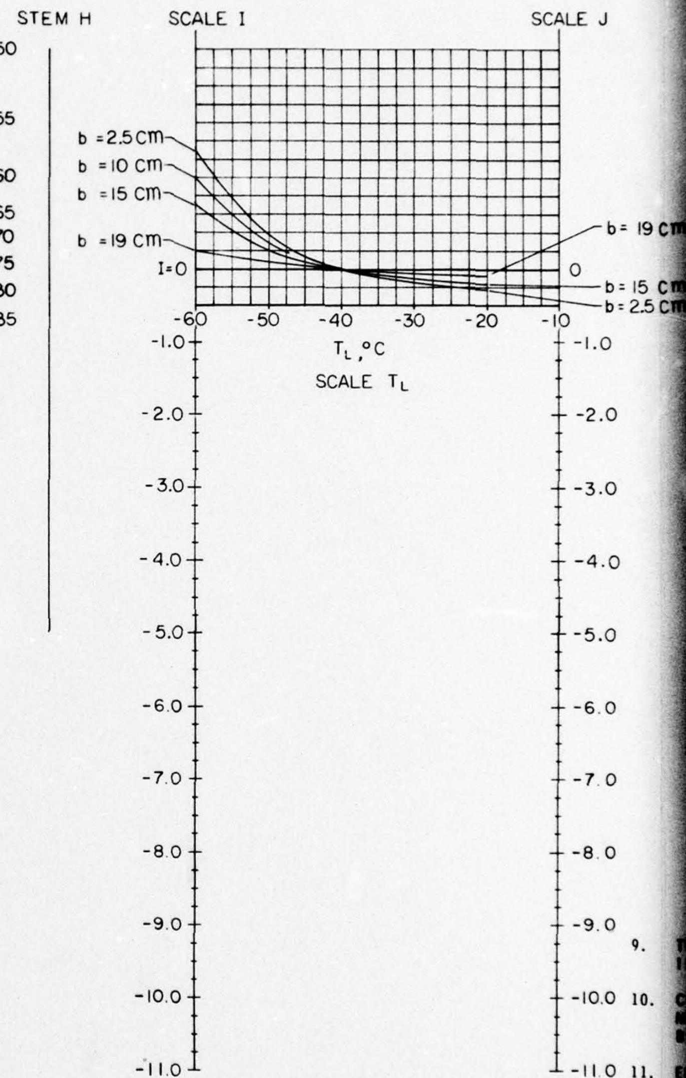
aerojet solid propulsion company
SACRAMENTO, CALIFORNIA

STRUCTURAL DESIGN NOMOGRAPH
FOR THERMAL CYCLING

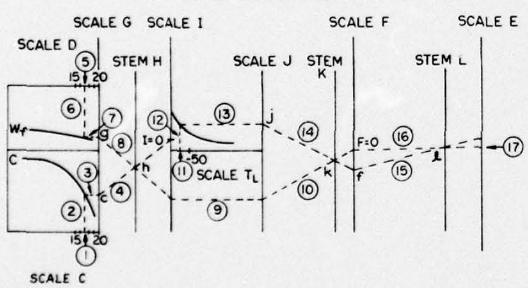
CHART 5 - DAMAGE ANALYSIS

7-12-77





SCHEMATIC EXAMPLE



REQUIRED PARAMETERS

b = ___ cm
w_f = ___
T_L = ___ °C
V = ___ MPa
W = ___ MPa
S = V + W = ___ MPa

RESULTS

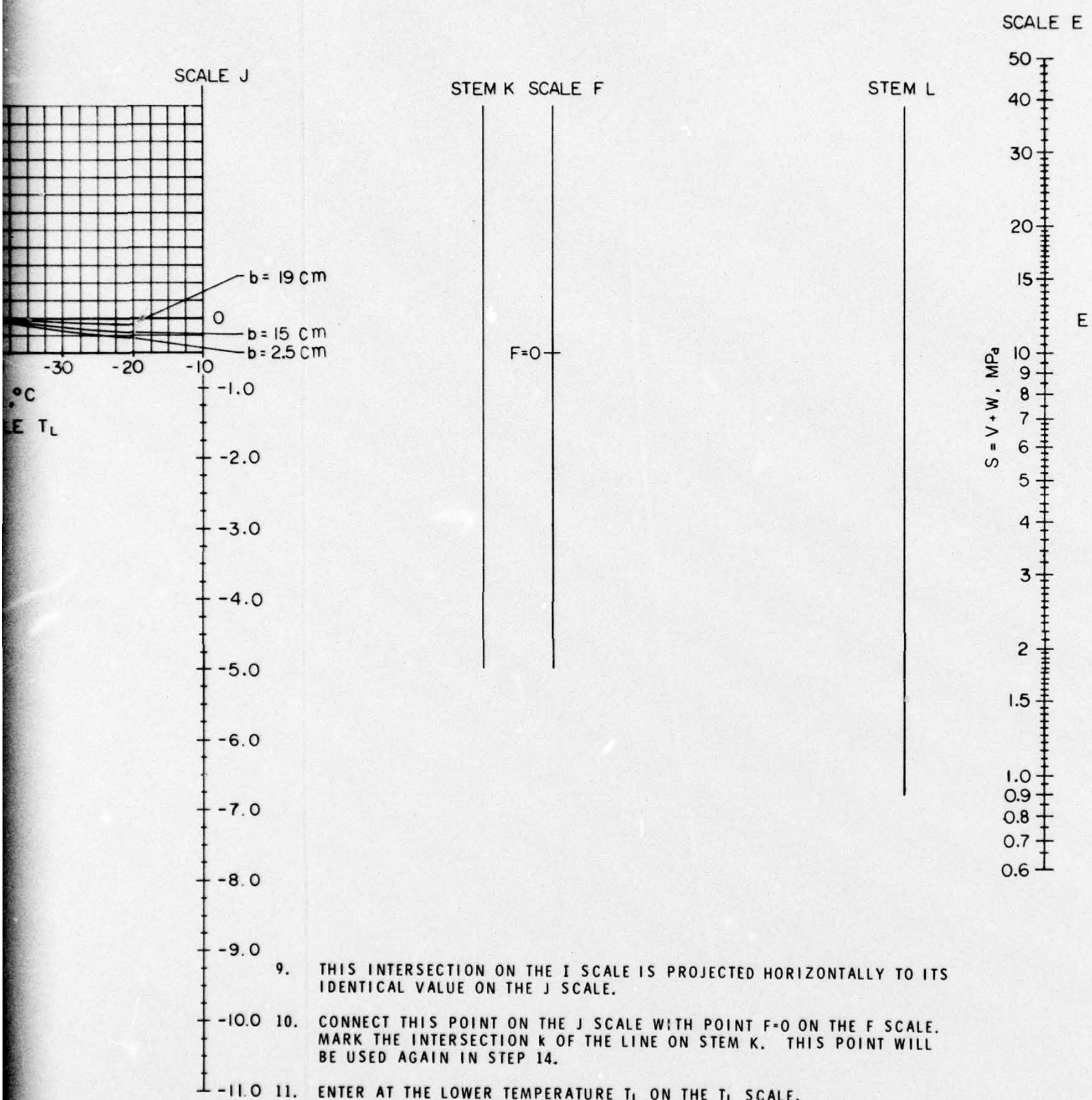
E_{eff} = ___ MPa

DIRECTIONS:

1. ENTER AT THE GIVEN VALUE OF b ON THE C SCALE.
2. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS CURVE C.
3. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE G SCALE AT c.
4. CONNECT POINT c AND THE POINT MARKED I=0 ON THE I SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION h OF THE LINE ON STEM H. THIS POINT WILL BE USED AGAIN IN STEP 8.
5. ENTER AT THE GIVEN VALUE OF b ON THE D SCALE.
6. DRAW A VERTICAL LINE DOWNWARD UNTIL IT INTERSECTS THE SPECIFIED w_f CURVE.
7. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE G SCALE AT g.
8. CONNECT POINTS g AND h (SEE STEP 4) WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE I SCALE.

9.
10.
11.
12.
13.
14.
15.
16.
17.

1



CURVE C.

NTIL IT INTERSECTS

I SCALE WITH A
NE ON STEM H.

TS THE SPECIFIED

NTIL IT INTERSECTS

HT LINE. EXTEND

9. THIS INTERSECTION ON THE I SCALE IS PROJECTED HORIZONTALLY TO ITS IDENTICAL VALUE ON THE J SCALE.

10. CONNECT THIS POINT ON THE J SCALE WITH POINT F=0 ON THE F SCALE. MARK THE INTERSECTION k OF THE LINE ON STEM K. THIS POINT WILL BE USED AGAIN IN STEP 14.

11. ENTER AT THE LOWER TEMPERATURE T_L ON THE T_L SCALE.

12. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE INTERPOLATED b CURVE.

13. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE J SCALE AT j.

14. CONNECT POINTS j AND k (SEE STEP 10) WITH A STRAIGHT LINE. EXTEND THIS LINE UNTIL IT INTERSECTS THE F SCALE AT f.

15. CONNECT POINT f AND THE GIVEN VALUE OF S(=V+W) ON THE E SCALE WITH A STRAIGHT LINE. MARK THE INTERSECTION ℓ OF THIS LINE ON THE STEM L.

16. FROM F=0 ON THE F SCALE EXTEND A LINE THROUGH POINT ℓ. CONTINUE THIS LINE UNTIL IT INTERSECTS THE E SCALE.

17. THIS INTERSECTION PROVIDES THE VALUE OF THE E_{eff} PARAMETER, WHICH IS REQUIRED FOR THE ANALYSES OF CHART 6. USE THIS QUANTITY IN STEP 4 OF CHART 6.

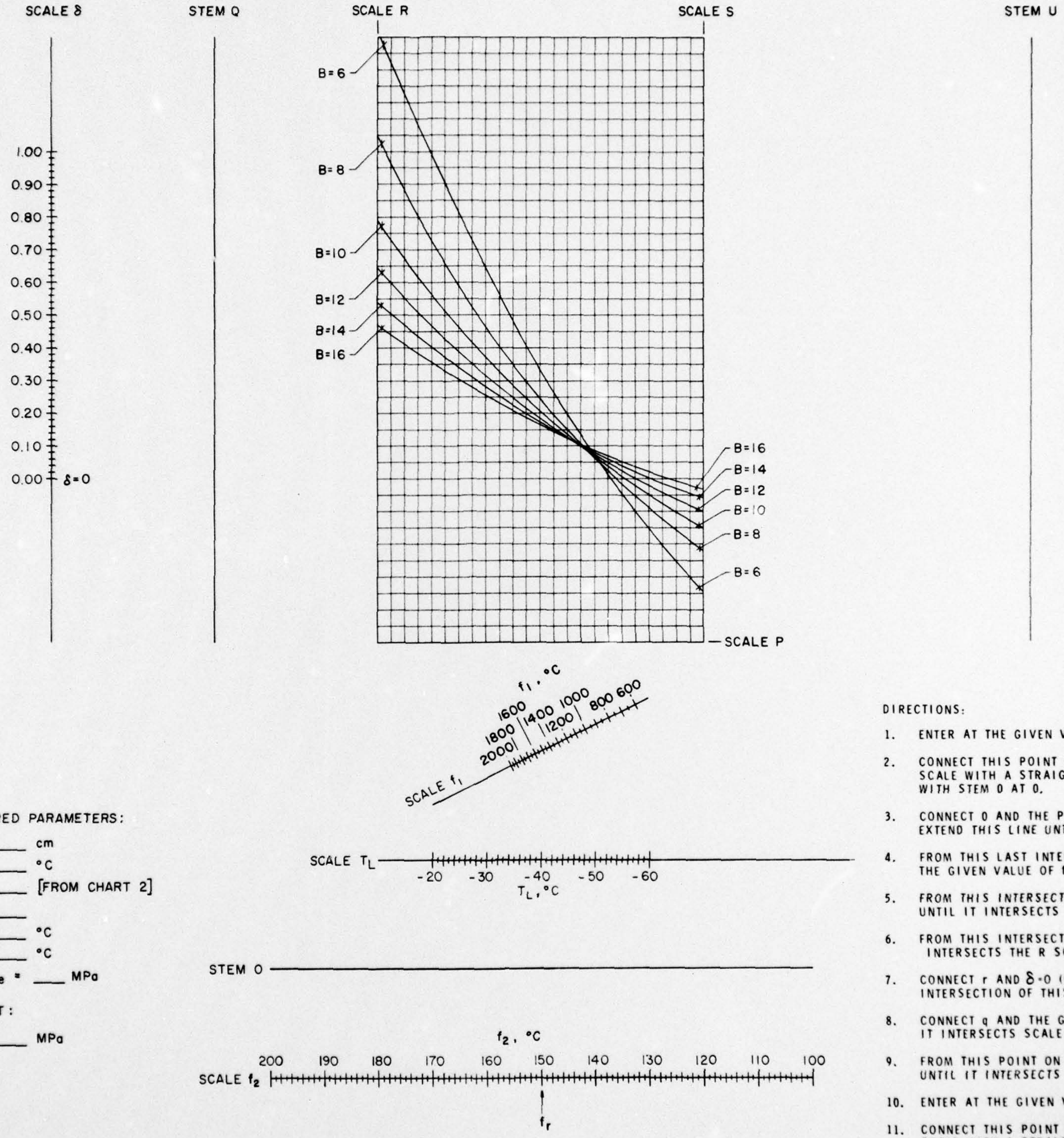
aerojet solid propulsion company
SACRAMENTO, CALIFORNIA

STRUCTURAL DESIGN NOMOGRAPH FOR THERMAL CYCLING

CHART 4 - TOTAL EFFECTIVE MODULUS

6-24-77

2



REQUIRED PARAMETERS:

b = ____ cm

T_L = ____ °C

δ = ____ [FROM CHART 2]

B = ____

f_2 = ____ °C

f_1 = ____ °C

$E(l) - E_e$ = ____ MPa

RESULT:

W = ____ MPa

DIRECTIONS:

1. ENTER AT THE GIVEN VALUE

2. CONNECT THIS POINT ON SC SCALE WITH A STRAIGHT LI WITH STEM 0 AT 0.

3. CONNECT 0 AND THE POINT EXTEND THIS LINE UNTIL IT

4. FROM THIS LAST INTERSECT THE GIVEN VALUE OF f_1 ON

5. FROM THIS INTERSECTION O UNTIL IT INTERSECTS THE G

6. FROM THIS INTERSECTION INTERSECTS THE R SCALE

7. CONNECT r AND δ-0 (ON TH INTERSECTION OF THIS LIN

8. CONNECT q AND THE GIVEN IT INTERSECTS SCALE R.

9. FROM THIS POINT ON THE UNTIL IT INTERSECTS THE

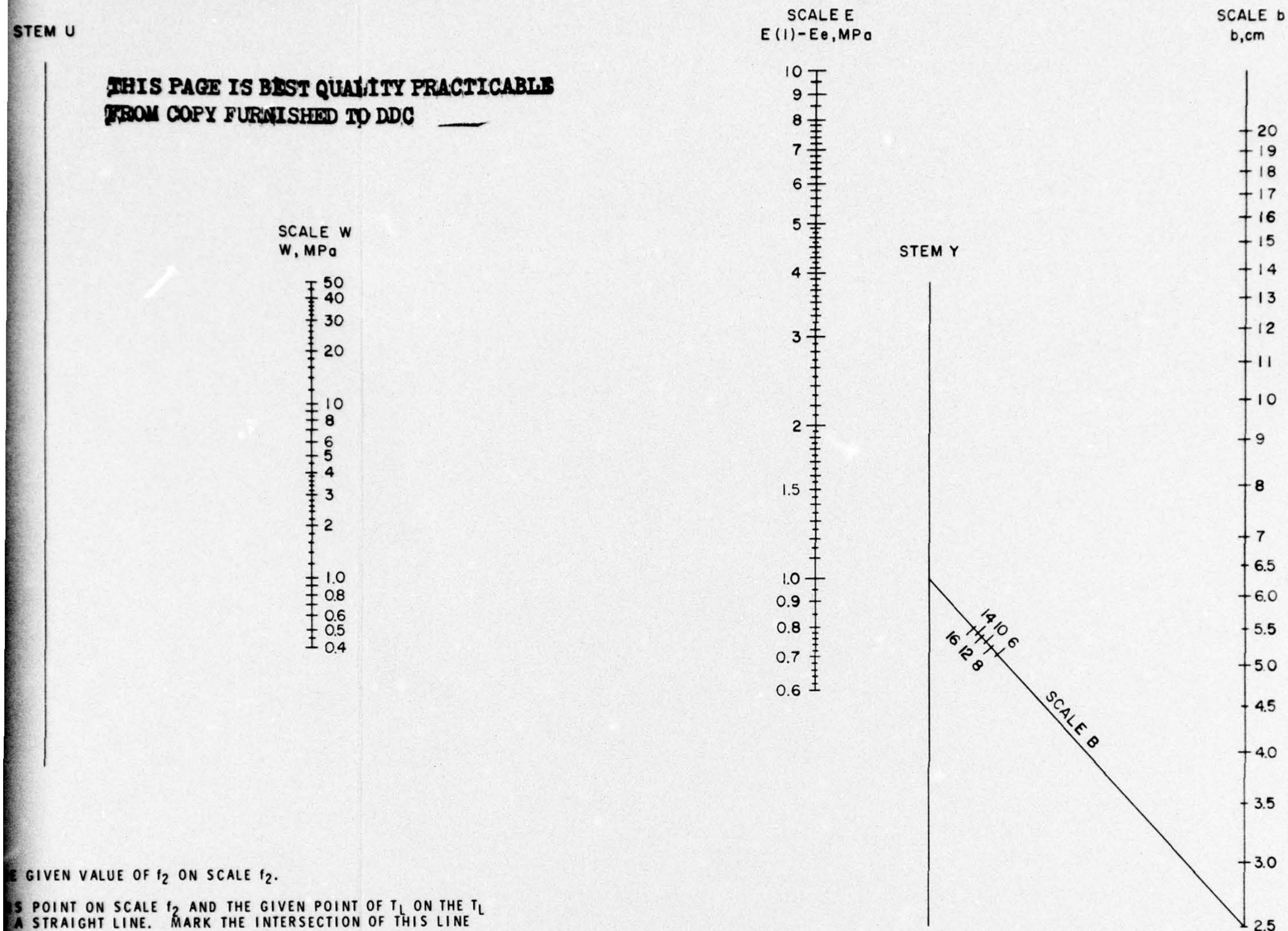
10. ENTER AT THE GIVEN VALUE

11. CONNECT THIS POINT ON S SCALE WITH A STRAIGHT L IT INTERSECTS STEM Y AT

12. CONNECT THE POINTS s AN INTERSECTION OF THIS LIN

13. CONNECT u AND THE GIVEN STRAIGHT LINE.

14. THE INTERSECTION OF THI VALUE OF W. THIS QUANT CHART 4.



E GIVEN VALUE OF f_2 ON SCALE f_2.

S POINT ON SCALE f_2 AND THE GIVEN POINT OF T_L ON THE T_L A STRAIGHT LINE. MARK THE INTERSECTION OF THIS LINE AT 0.

ND THE POINT f_r ON THE f_2 SCALE WITH A STRAIGHT LINE. LINE UNTIL IT INTERSECTS THE T_L SCALE.

AST INTERSECTION DRAW A CONNECTING STRAIGHT LINE THROUGH ALUE OF f_1 ON SCALE f_1. EXTEND THIS LINE UP TO THE P SCALE.

NTERSECTION ON THE P SCALE DRAW A VERTICAL LINE UPWARD ERSECTS THE GIVEN B CURVE.

NTERSECTION DRAW A HORIZONTAL LINE TO THE LEFT UNTIL IT THE R SCALE AT r.

ND δ-O (ON THE δ SCALE) WITH A STRAIGHT LINE. MARK THE N OF THIS LINE WITH THE STEM Q AT q.

ND THE GIVEN VALUE ON THE δ SCALE. EXTEND THE LINE UNTIL S SCALE R.

OINT ON THE R SCALE DRAW A HORIZONTAL LINE TO THE RIGHT ERSECTS THE S SCALE AT s.

GIVEN VALUE OF b ON THE b SCALE.

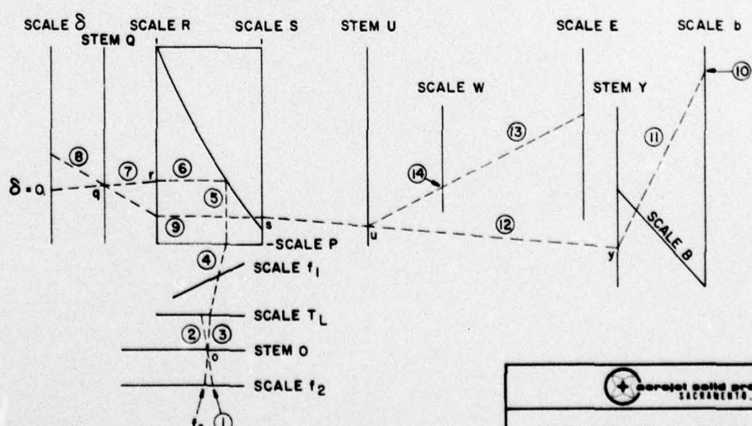
S POINT ON SCALE b AND THE GIVEN VALUE OF B ON THE B A STRAIGHT LINE. EXTEND THIS LINE TO THE LEFT UNTIL S STEM Y AT y.

POINTS s AND y WITH A STRAIGHT LINE. MARK THE N OF THIS LINE WITH THE STEM U AT u.

ND THE GIVEN VALUE OF $E(1)-E_e$ ON THE E SCALE WITH A NE.

CTION OF THIS LAST LINE WITH SCALE W GIVES THE REQUIRED THIS QUANTITY PLUS V FROM CHART 1 GIVES S ON

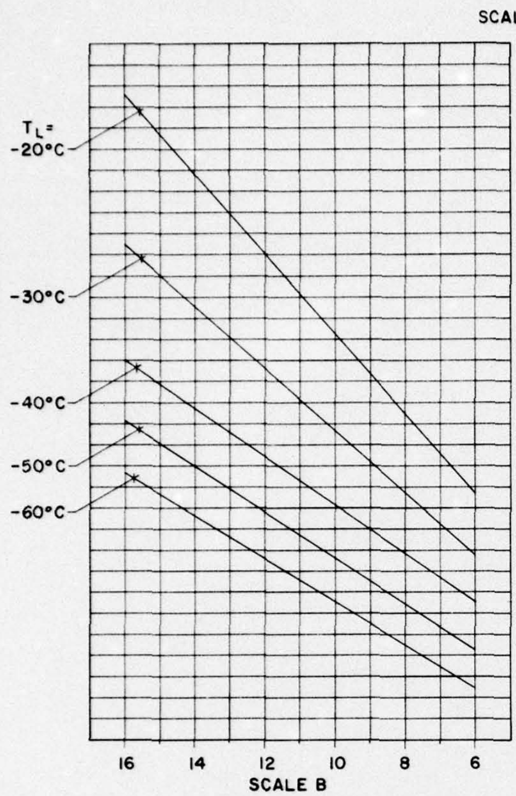
SCHEMATIC EXAMPLE

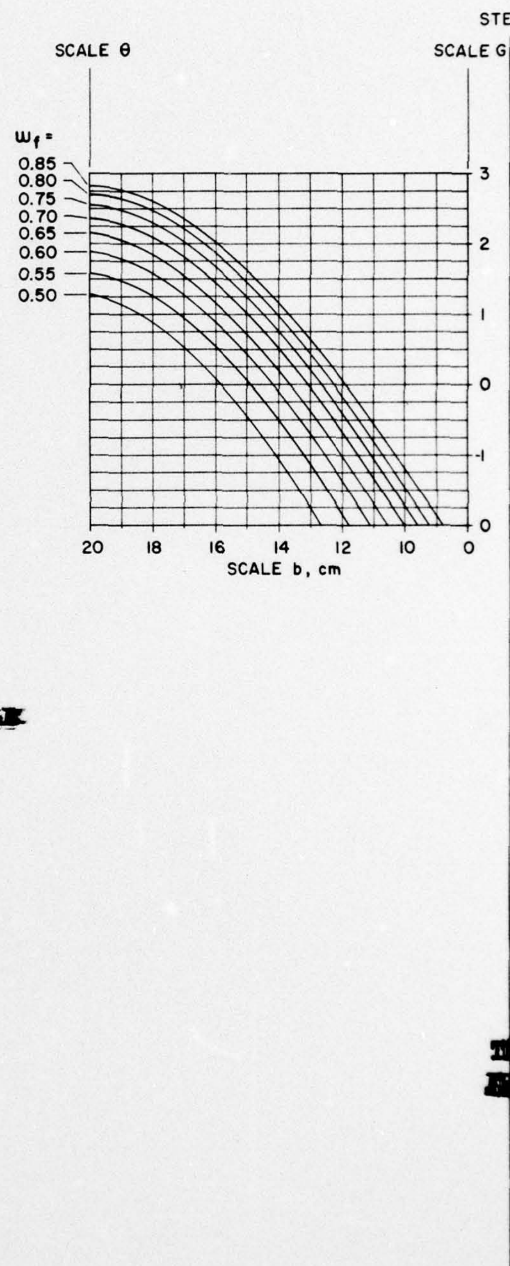


aerojet solid propulsion company
SACRAMENTO, CALIFORNIA

STRUCTURAL DESIGN NOMOGRAPH
FOR THERMAL CYCLING
CHART 3 - VISCOELASTIC COMPONENT

6-23-77





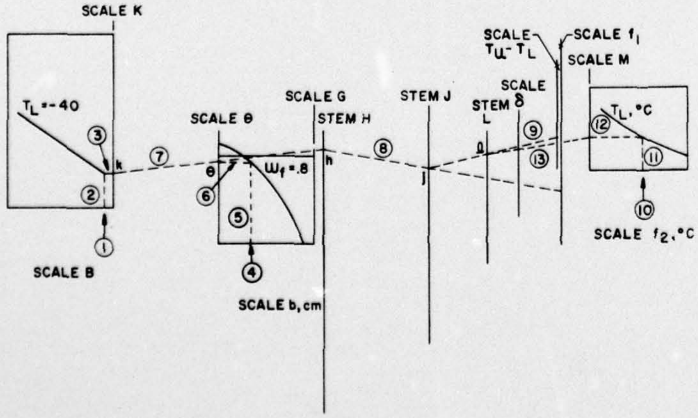
REQUIRED PARAMETERS

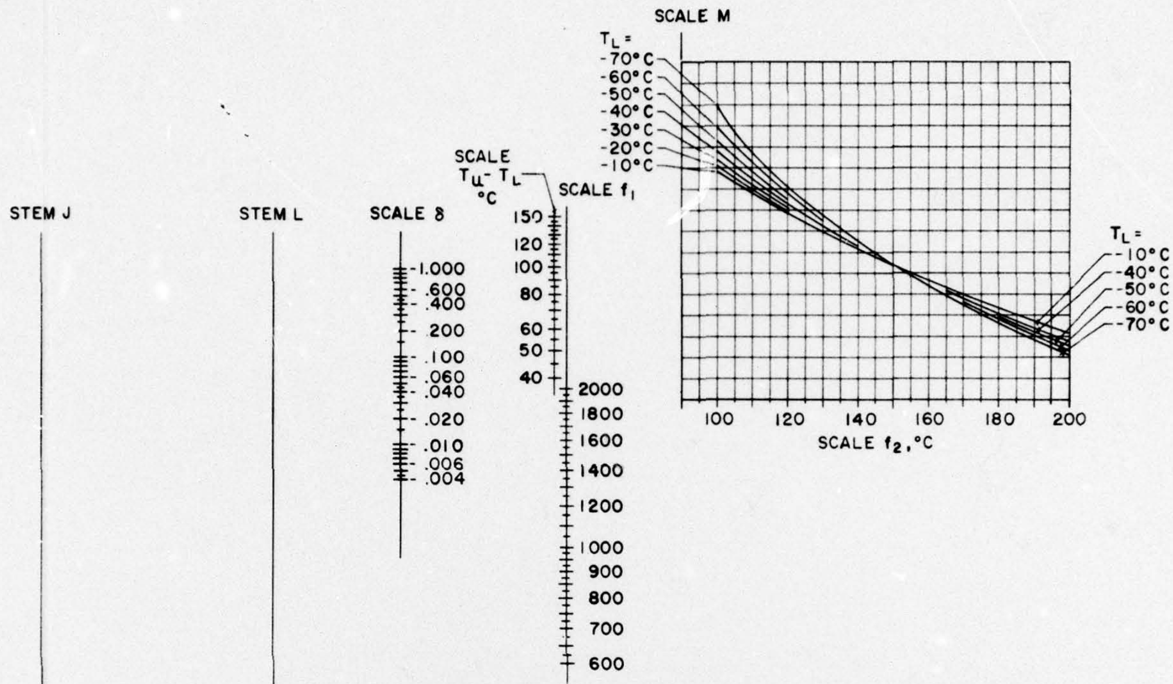
b	=	___ cm
ω_f	=	___
B	=	___
f_1	=	___ °C
f_2	=	___ °C
T_L	=	___ °C
$T_u - T_L$	=	___ °C

RESULT:

δ	=	___

SCHEMATIC EXAMPLE





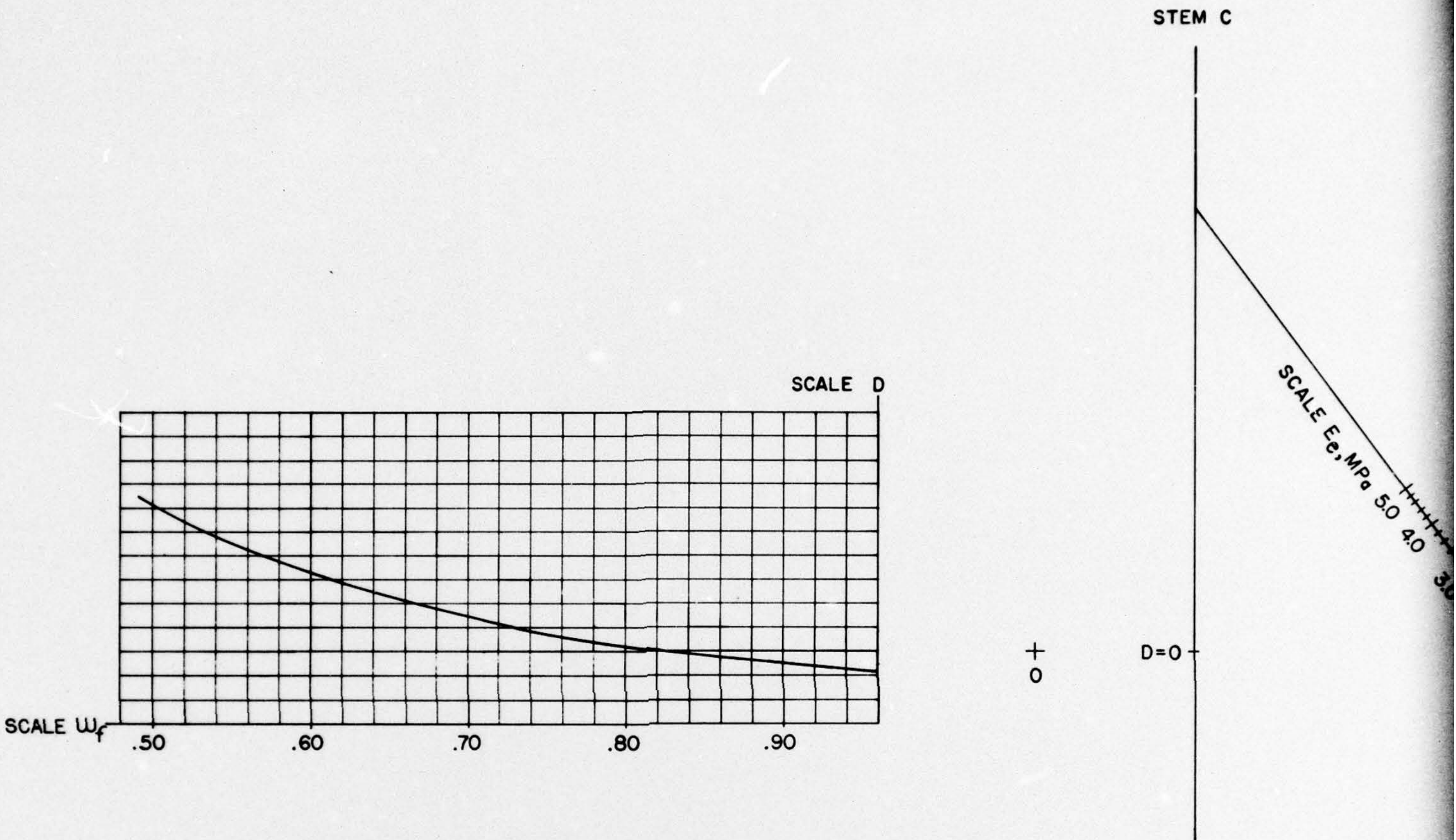
DIRECTIONS: IF THE MOTOR RADIUS, b, IS LESS THAN 9 cm THIS CHART MAY BE OMITTED AND THE ENTRY ON CHART 3 WILL BE $\delta = 0$.

1. ENTER AT THE GIVE VALUE OF B ON SCALE B.
2. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN T_L CURVE.
3. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE K SCALE. MARK THIS INTERSECTION k.
4. ENTER AT THE GIVEN VALUE OF b ON SCALE b.
5. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE GIVEN ω_f CURVE.
6. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE θ SCALE. MARK THIS INTERSECTION θ.
7. CONNECT POINTS k AND θ WITH A STRAIGHT LINE. EXTEND THE LINE UNTIL IT INTERSECTS STEM H AT h.
8. CONNECT h AND THE GIVEN VALUE ON THE f_1 SCALE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM J AT j.
9. DRAW A LINE BETWEEN j AND THE GIVEN VALUE ON THE $T_U - T_L$ SCALE. MARK THE INTERSECTION OF THIS LINE WITH THE STEM L AT ℓ.
10. ENTER AT GIVEN VALUE OF f_2 ON SCALE f_2.
11. DRAW A VERTICAL LINE UPWARD UNTIL IT INTERSECTS THE T_L CURVE.
12. FROM THIS INTERSECTION DRAW A HORIZONTAL LINE UNTIL IT INTERSECTS THE M SCALE
13. DRAW A LINE BETWEEN THIS INTERSECTION ON THE M SCALE AND THE POINT ℓ ON STEM L.

THE INTERSECTION OF THIS LINE WITH THE δ SCALE GIVES THE REQUIRED VALUE OF δ THAT IS TO BE USED IN CHART 3, STEP 4.

aerojet solid propulsion company
SACRAMENTO, CALIFORNIA

STRUCTURAL DESIGN NOMOGRAPH
FOR THERMAL CYCLING
CHART 2- TEMPERATURE DIFFERENTIAL



REQUIRED PARAMETERS:

ω_f = ___

E_e = ___ MPa

RESULT:

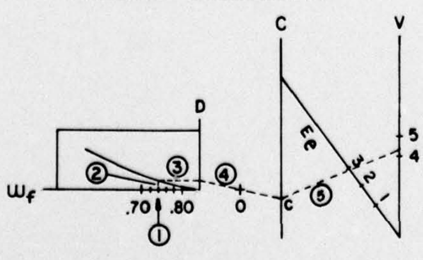
V = ___ MPa

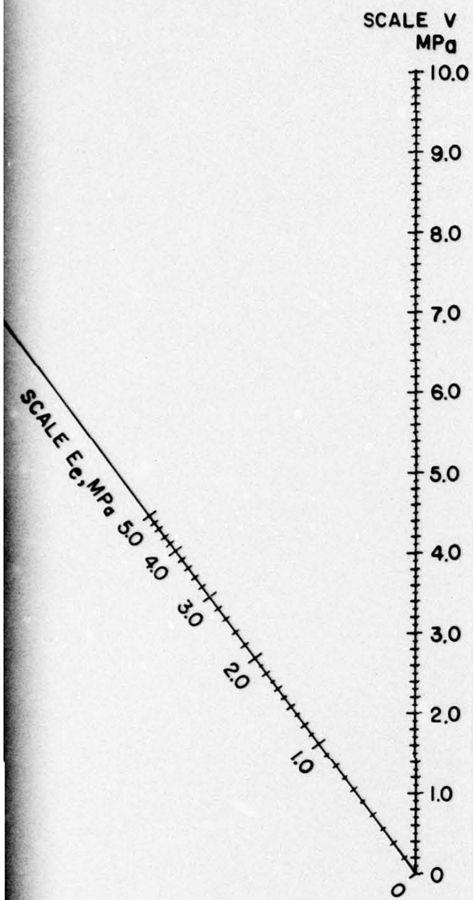
THIS PAGE IS B
FROM COPY FUR

DIRECTIONS:

1. ENTER AT GIVEN VALU
2. DRAW A VERTICAL LI
3. FROM THIS INTERSECT
UNTIL IT INTERSECTS
4. CONNECT THIS INTERS
POINT "O" WITH A ST
SECTS THE STEM C A
   - IF THE VALUE OF
THEN STEP 4 MA
MARKED D=0 ON
5. FROM THE INTERSECT
GIVEN VALUE ON THE
6. THIS INTERSECTION
ANALYSES OF CHART

SCHEMATIC EXAMPLE





IS PAGE IS BEST QUALITY PRACTICABLE
OM COPY FURNISHED TO DDC

ONS:

ER AT GIVEN VALUE ON ω_f SCALE.

W A VERTICAL LINE UNTIL IT INTERSECTS THE CURVE.

M THIS INTERSECTION DRAW A HORIZONTAL LINE PARALLEL TO THE GRID IL IT INTERSECTS THE D SCALE.

NECT THIS INTERSECTION ON THE D SCALE AND THE CENTER OF THE CROSS AT NT "O" WITH A STRAIGHT LINE. EXTEND THIS STRAIGHT LINE UNTIL IT INTER- TS THE STEM C AT POINT c.

IF THE VALUE OF D IS ZERO OR NEGLIGIBLE (CLOSE TO OR AT THE BASE LINE) THEN STEP 4 MAY BE BY-PASSED AND STEP 5 WOULD BEGIN AT THE POINT MARKED D=0 ON THE STEM C

M THE INTERSECTION POINT c ON THE STEM C EXTENT A LINE THROUGH THE EN VALUE ON THE E_e SCALE UNTIL IT INTERSECTS THE V SCALE.

S INTERSECTION PROVIDES THE VALUE OF V WHICH IS REQUIRED FOR THE LYSES OF CHART 4. NOTE THIS QUANTITY ON THAT CHART.

aerojet solid propulsion company
SACRAMENTO, CALIFORNIA

STRUCTURAL DESIGN NOMOGRAPH
FOR THERMAL CYCLING
CHART 1 – ELASTIC COMPONENT