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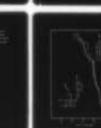
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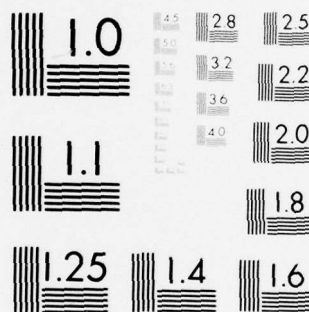
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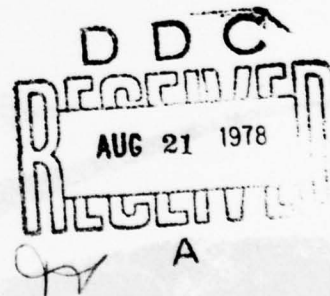
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ABSTRACT

This paper is divided into three parts. First, a review of currently available incremental theories of plasticity and creep constitutive models is given. Second, a formulation is presented for the non-isothermal, elastic-plastic-creep-large strain analysis by the finite element method. Third, results obtained with the AGGIE I computer program for several isothermal, elastic-creep and elastic-plastic analyses are presented. The present numerical results show good agreement with experimental and other numerical results.

INTRODUCTION

Although the finite element method has long been recognized as a very powerful analysis tool, its usefulness has been negated to some extent due to our inability to find suitable constitutive relations for modeling problems with combined elastic-plastic-creep-large strain behavior. This fact is brought out when one considers the wide abundance of plasticity hardening rules and creep constitutive equations which have been developed. Many of the shortcomings of the constitutive models are due to simplifying assumptions such as the uncoupling of creep, plasticity, rate and other mechanisms and the separation of inelastic strain into time dependent and

independent parts. Although early researchers were forced somewhat into these assumptions, there currently seems to be a trend in the other direction; for example, the so-called "unified theories" are attracting more and more attention.

The purpose of this paper is several fold. First, to review the classical plasticity theory and the several categories of creep constitutive models currently available. Second, to present an incremental formulation for the non-isothermal elastic-plastic-creep-large strain analysis by the finite element method. And, third, to present some numerical results for several elastic-creep and elastic-plastic problems that have been obtained with the AGGIE computer program.

INCREMENTAL THEORY OF PLASTICITY

The classical incremental theories of plasticity make use of an initial yield condition, a hardening rule, and a flow rule in characterizing the strain-hardening response of a material. Although these classical theories continue to be utilized extensively in finite element computer programs, this may be true only because more suitable models have not yet been developed. Comparison [1,2] of the models with experimental results indicates relatively good agreement in uniaxial cases under simple loading conditions. However, for biaxial and triaxial cases and situations where the loading is cyclic, when creep and plasticity interact, when the strain rates are high, etc., the results are often in disagreement with experiment. The difficulty is compounded by the fact that the hardening rules give good results for some materials but behave poorly for others.

Yield Condition

The two most widely used yield conditions are the Tresca (maximum shear stress) and von Mises (J_2 theory) conditions. For isotropic metals,

the von Mises yield condition generally provides a better description of initial yielding than does the Tresca condition. However, for rocks and soils, the Tresca condition is often used. Other yield conditions have been proposed, however, these have not found wide use because of their mathematical complexity. The von Mises yield condition is used in all work reported herein.

Flow Rule

A flow rule is used to separate the total strain increment into elastic and plastic components. The most generally accepted flow rule, termed the normality condition, states that as the stress state of a material point comes into contact with and pierces the material's yield surface, the resulting plastic strain increment is along the outward normal to the yield surface at the point of penetration. Experimental evidence has shown that the normality condition is generally valid for a wide range of materials [3].

Hardening Rule

The hardening rule provides a description of the changing size and shape of the subsequent yield surface during plastic flow. In addition to simple expansion and/or translation, experimental evidence has shown that subsequent yield surfaces may exhibit corners, general distortion, various Bauschinger effects, and dependence on prior cyclic history, strain rate and hold time to mention only a few parameters [3]. For simplicity, most finite element programs make use of hardening rules which account only for expansion and/or translation of the yield surface.

The classical isotropic hardening rule postulates that the yield surface expands uniformly during plastic deformation. In its simplest form wherein one assumes the von Mises yield condition and associated flow rule, the rate

of strain hardening may be obtained by relating a value of equivalent total plastic strain to a point on a uniaxial stress-strain curve, so that a simple tensile test is all that is necessary to determine the hardening rule parameters. The simplicity of applying the isotropic hardening rule has made it very popular in finite element plasticity analysis.

In contrast, the kinematic hardening model of Prager-Ziegler [4] proposes that the yield surface translates as a rigid shape during plastic flow; the direction of translation being given by a vector connecting the current center of the yield surface and the current stress state. This gives rise to an ideal Bauschinger effect in which the reverse yield stress is lowered by an amount equal to the prior strain hardening.

The Besseling-White (mechanical sublayer) model [5] makes use of a superposition of elastic-perfectly plastic stress states to approximate strain hardening behavior. This model is often idealized mechanically as a parallel arrangement of elastic-perfectly plastic layers whose yield stresses are adjusted to duplicate a piece-wise linearization of the uniaxial stress-strain curve (the number of layers being equal to the number of points selected on the stress-strain curve). Like the kinematic model, the mechanical sublayer model predicts a rigid translation of the yield surface.

The hardening model proposed by Mroz employs the concept of a field of surfaces of constant work hardening moduli. Each point of a piece-wise linear uniaxial stress-strain curve is represented in stress space by a surface geometrically similar to the initial yield surface but of different size. The yield surface is assumed to expand and translate within this field, contacting and pushing each surface along with it as each is encountered. Conceptually, the Mroz model is similar to the mechanical sublayer model.

Recently, Krieg [6] proposed a two surface plasticity model where the

yield surface translates and expands within an enclosing "limit surface", which also is allowed to translate and expand independently of the yield surface. The hardening modulus is then assumed to be a function of the distance between the two surfaces at the loading point. The model requires two material tests: a uniaxial tension test carried out to moderate strains and a uniaxial loading, unloading, reverse loading test. The model has not yet found wide use but would appear to be an improvement over simple isotropic and kinematic hardening. Another model which has not found wide use as of yet is the piecewise linear strainhardening theory of Hodge [7] which makes use of a yield polygon.

As experimental evidence points out, isotropic and kinematic hardening tend to bracket actual material response in many cases and, for this reason, a number of combined isotropic-kinematic models have been proposed. Most models are based on a constant ratio of expansion to translation although some results have been reported for a variable ratio based on accumulated plastic strain [8].

CREEP CONSTITUTIVE MODELS

As in plasticity theory, there exists a wide variety of constitutive equations which have been proposed for modeling creep behavior. Unlike the plasticity models, the various creep models differ significantly in mathematical form and physical basis; some are purely equation of state approaches, some are based on viscoelastic and hereditary integral methods, some are based on adhoc rules while some have a rigorous thermodynamic basis, etc. This section will provide a brief overview of several creep models (by category as listed below) which are currently available. For our purposes, the following categories are listed: 1) Phenomenological (equation-of-state) theory, 2) Memory or hereditary theory, 3) Nonlinear viscoelastic

theory, 4) subelement theory, 5) Krempl's theory, 6) Valanis' theory, and 7) Unified theory.

The most widely used means of describing creep behavior is the phenomenological (equation-of-state) creep theory. A good review of this theory is given in Refs. [9] and [10]. This representation of creep strains is similar to that used in the incremental theory of plasticity, i.e., three relationships are used: 1) uniaxial creep law (obtained from a uniaxial creep test), 2) a flow rule, and 3) a hardening law. Nickell's survey [10] of computer programs which incorporate creep strains shows that a majority of these programs use the phenomenological creep theory. References [11] and [12] show application of this theory to plane stress, plane strain, and axisymmetric problems which contain no plastic deformation. In Ref. [13], isothermal elastic-plastic-creep computer programs are developed for two-dimensional analysis. In Ref. [14], a non-isothermal elastic-plastic-creep computer program is developed for flat and curved, thick and thin shell elements of triangular and quadrilateral form. Subtle refinements to the phenomenological theory have been discussed and their importance stressed by various researchers. In Refs. [11] and [12], the importance of time step size on convergence and solution time is pointed out. Reference [15] indicates the importance of the subincremental approach as applied to creep strains. For load reversal conditions, auxiliary procedures are outlined in Ref. [16] which avoids the inconsistency which is present if the phenomenological theory is applied in the usual manner.

The memory (or hereditary) theory has a good theoretical background [17] and has been implemented into a finite element computer program by Rashid [18]. A comparison of advantages and disadvantages of the phenomenological and memory theories is given in Ref. [19]. This theory is similar to linear vis-

coelasticity, however, metals are characterized generally as nonlinear viscoelastic materials and generally do not obey the linear superposition that linear viscoelastic materials do. However, both Schapery [17] and Rashid [18] have demonstrated that the simple integral approach as used in linear viscoelasticity can be used if a reduced time replaces the physical time. Normally, creep, relaxation and/or recovery tests are required to determine the necessary material parameters.

In the nonlinear viscoelastic approach, the constitutive relation is expressed as an integral polynomial [20]. It is pointed out in Refs. [18] and [20] that although the multiple integral approach does hold promise, the experimental data (multistep creep tests) necessary to implement this theory are generally not available at this time. As of yet, no application of this theory to a production-scale finite element computer program has apparently been made.

In the subelement theory [21], the material is idealized by a number of subelements each possessing secondary creep behavior, but with different creep rates. Due to interaction of the subelements, primary and recovery behavior can be determined from this model. The necessary material constants are determined from the results of a standard creep test. Details of a finite element program utilizing the subelement method are reported in Ref. [21].

Kremp1 [22, 23] has presented a constitutive theory for modeling elastic-plastic-creep response which is based on total strain (as opposed to the conventional approach where the total strain is separated into various components). Operational definitions of aging, history dependence and rate dependence are included. A loading and unloading criteria is employed whereby the elastic response is handled as an integral part of the constitutive equation. The constitutive equation contains separation functions for

describing rate and history dependence. The history dependence is accounted for by a tensor valued function called the microstructure memory function with a discontinuous growth law which is operative at points of unloading and serves to expand the yield surface due to prior strain history. Rate dependence is modeled by time derivatives of stress in a differential constitutive model and by time dependence of the kernel in an integral form of the constitutive law. Krempl's theory has not yet found wide use in general finite element programs.

The endochronic theory of Valanis [24] and the "unified" theories [25-27] represent significant departures from the classical approaches for handling elastic-plastic-creep behavior. Valanis develops his endochronic theory from continuum and thermodynamic concepts but avoids the definition of a yield surface. The unified theories treat the inelastic strain as a unified quantity which is not separable into time dependent and time independent parts. This inseparability has been observed experimentally, especially at high temperatures, but has been largely ignored for computational simplicity. Although the unified theories have not yet found their way into general finite element programs, these authors are of the opinion that this approach will ultimately provide much more reliable constitutive models.

INCREMENTAL EQUATIONS OF EQUILIBRIUM

In this section, an incremental finite element formulation for the non-isothermal, elastic-plastic-creep-large strain problem is developed. One may begin with the equation of equilibrium written in terms of the 2nd Piola-Kirchhoff stress

$$\frac{\partial}{\partial a_j} [S_{jk}(\delta_{ik} + \frac{\partial u_i}{\partial a_k})] + \rho_0 F_{0i} = 0 \quad (1)$$

where a_j and u_i are Lagrangian coordinates and displacements, respectively,

S_{jk} is the 2nd Piola-Kirchhoff stress tensor, ρ_0 is undeformed density and F_{oi} is the body force per unit undeformed volume per unit mass. Applying the virtual work principle at time $t + \Delta t$ yields

$$\int_{V_0} S_{ij}^{t+\Delta t} \delta E_{ij}^{t+\Delta t} dV = \delta R^{t+\Delta t} \quad (2)$$

where

$$\delta R^{t+\Delta t} = \int_S T_i^{t+\Delta t} \delta u_i^{t+\Delta t} dS + \int_{V_0} \rho_0 F_{oi}^{t+\Delta t} \delta u_i^{t+\Delta t} dV \quad (3)$$

and $\delta E_{ij}^{t+\Delta t}$ is the variation of the Green-Lagrange strains at time $t+\Delta t$ and $T_i^{t+\Delta t}$ are surface tractions at time $t+\Delta t$ applied to the deformed surface S . Equation (2) may be put into incremental form by writing

$$\begin{aligned} S_{ij}^{t+\Delta t} &= S_{ij}^t + \Delta S_{ij} \\ E_{ij}^{t+\Delta t} &= E_{ij}^t + \Delta E_{ij} \end{aligned} \quad (4)$$

where S_{ij}^t and E_{ij}^t are stresses and strains at time t and ΔS_{ij} and ΔE_{ij} are increments of stress and strain, respectively. The strain increment may be decomposed into components which are linear and nonlinear in the displacement increments

$$\Delta E_{ij} = \Delta E_{ij}^L + \Delta E_{ij}^{NL} \quad (5)$$

where

$$\begin{aligned} \Delta E_{ij}^L &= 1/2 \left(\frac{\partial \Delta u_i}{\partial a_j} + \frac{\partial \Delta u_j}{\partial a_i} + \frac{\partial \Delta u_k}{\partial a_i} \frac{\partial u_k^t}{\partial a_j} + \frac{\partial u_k^t}{\partial a_i} \frac{\partial \Delta u_k}{\partial a_j} \right) \\ \Delta E_{ij}^{NL} &= 1/2 \left(\frac{\partial \Delta u_k}{\partial a_i} \frac{\partial \Delta u_k}{\partial a_j} \right) \end{aligned} \quad (6)$$

Substituting Eqs. (4) and (5) into (2) yields

$$\int_{V_0} S_{ij}^t \delta \Delta E_{ij}^L dV + \int_{V_0} S_{ij}^t \delta \Delta E_{ij}^{NL} dV + \int_{V_0} \Delta S_{ij} \delta (\Delta E_{ij}^L + \Delta E_{ij}^{NL}) dV = \delta R^{t+\Delta t} \quad (7)$$

The stress increment may be decomposed into two components, one which is dependent upon total strain and one which is independent of strain (i.e., creep, thermal, etc.):

$$\Delta S_{ij} = D_{ijkl} \Delta E_{kl} + \Delta P_{ij} \quad (8)$$

where D_{ijkl} is the usual effective tangent modulus and ΔP_{ij} is a stress increment due to strain independent phenomena (as is usually assumed in creep). Substituting Eq. (8) into Eq. (7), making use of Eq. (5), and neglecting terms which would be nonlinear in displacement increments, yields the following:

$$\begin{aligned} & \int_{V_0} \Delta E_{kl}^L D_{ijkl} \delta(\Delta E_{ij}^L) dV + \int_{V_0} (S_{ij}^t + \Delta P_{ij}) \delta(\Delta E_{ij}^{NL}) dV \\ & = - \int_{V_0} (S_{ij}^t + \Delta P_{ij}) \delta(\Delta E_{ij}^L) dV + \delta R^{t+\Delta t} \end{aligned} \quad (9)$$

The term ΔP_{ij} may be interpreted as the change in stress required to account for the creep and thermal strains. Equation (9) takes on the following form when put into matrix form

$$[M]\{\ddot{q}^{t+\Delta t}\} + ([K_L^t] + [K_{NL}^t])\{\Delta q\} = \{R^{t+\Delta t}\} - \{F^t\} \quad (10)$$

where $[M]$ is the mass matrix, $[K_L^t]$ and $[K_{NL}^t]$ are "linear" and "nonlinear" stiffness matrices, $\{R^{t+\Delta t}\}$ is a vector of forces due to externally applied loads, $\{F^t\}$ is a vector of forces due to internal stress, and $\{\Delta q\}$ is the increment of the nodal displacements. Complete details of the derivation of the quantities in Eq. (10), without creep, may be found in Ref. [28].

We now present a summary of the determination of D_{ijkl} and ΔP_{ij} for kinematic hardening for the non-isothermal case. We assume a yield

surface can be expressed by

$$F = f - K^2 = 0 \quad (11)$$

For kinematic hardening, we write

$$f = 1/2(S'_{ij} - \alpha'_{ij})(S'_{ij} - \alpha'_{ij}) \quad (12)$$

and S'_{ij} and α'_{ij} are deviatoric components of the 2nd Piola-Kirchhoff stress and yield surface center, respectively. The yield surface size parameter, K , is taken to be a function of temperature, T , such that

$$K^2 = 1/3[\sigma_y(T)]^2 \quad (13)$$

where σ_y is the temperature-dependent initial (or subsequent) yield stress.

In general, one can write

$$F(S_{ij}, \alpha_{ij}, E_{ij}^P, T) = 0 \quad (14)$$

where E_{ij}^P is the plastic component of strain.

For a stress point to remain on the yield surface during plastic flow requires that

$$\dot{F} = \frac{\partial F}{\partial S_{ij}} (\dot{S}_{ij} - \dot{\alpha}_{ij}) - \frac{\partial(K^2)}{\partial T} \dot{T} = 0 \quad (15)$$

where the dot denotes rate. Considering Eq. (14), one can also write

$$dF = \frac{\partial F}{\partial S_{ij}} dS_{ij} + \frac{\partial F}{\partial E_{ij}^P} dE_{ij}^P + \frac{\partial F}{\partial T} dT \quad (16)$$

and note that $dF = 0$ for a stress point to remain on the yield surface.

It is assumed that the plastic strain rate (flow rule) can be expressed as

$$\dot{E}_{ij}^P = \lambda \frac{\partial F}{\partial S_{ij}} \dot{F} \quad (17)$$

where λ is a scalar parameter to be determined.

From the normality condition,

$$\dot{\alpha}_{ij} \frac{\partial F}{\partial S_{ij}} = c \dot{E}_{ij}^P \frac{\partial F}{\partial S_{ij}} \quad (18)$$

where c is a scalar (hardening modulus) to be determined from a uniaxial stress-strain curve.

Substituting Eqs. (17) and (18) into (15) yields

$$\frac{\partial f}{\partial S_{ij}} [\dot{S}_{ij} - c \lambda \frac{\partial F}{\partial S_{ij}} (\frac{\partial F}{\partial S_{kl}} \dot{S}_{kl} + \frac{\partial F}{\partial T} \dot{T})] - \frac{\partial(K^2)}{\partial T} \dot{T} = 0 \quad (19)$$

For small elastic strains, the decomposition of strains is assumed

$$\dot{S}_{ij} = E_{ijmn} (\dot{E}_{mn} - \dot{E}_{mn}^P - \dot{E}_{mn}^C) \quad (20)$$

where E_{ijmn} is the elastic constitutive tensor and E_{mn}^C are the creep strain components. Substituting Eq. (17) into (20), combining the results with Eq. (19) and solving for λ , one obtains:

$$\lambda = \frac{E_{ijmn} \frac{\partial f}{\partial S_{ij}} (\dot{E}_{mn} - \dot{E}_{mn}^C) - \frac{\partial(K^2)}{\partial T} \dot{T}}{\beta} \quad (21)$$

where

$$\beta = \left[c \frac{\partial f}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + E_{pqrs} \frac{\partial f}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}} \right] \left(\frac{\partial F}{\partial S_{kl}} \dot{S}_{kl} - \frac{\partial(K^2)}{\partial T} \dot{T} \right) \quad (22)$$

Substituting Eq. (21) into (19) and the result into (20) and then comparing the final result to Eq. (8), one can show that the instantaneous modulus tensor is given by

$$D_{ijmn} = E_{ijmn} - \frac{E_{ijvw} \frac{\partial f}{\partial S_{vw}} \frac{\partial F}{\partial S_{tu}} E_{tumn} \dot{T}}{\beta} \quad (23)$$

and the rate of change of stress due to creep strains, temperature changes, etc. is given by

$$\dot{P}_{ij} = -D_{ijmn} \dot{E}_{mn}^C + \frac{\frac{\partial(K^2)}{\partial T} \dot{T} \frac{\partial F}{\partial S_{ij}} \dot{T}}{\beta} \quad (24)$$

Although the above expressions are given in rate form, the conversion to incremental form is obvious.

COMPUTER PROGRAM

The finite element computer program, AGGIE I, is based on the incremental formulation outlined above. The program is capable of handling two- and three-dimensional geometries utilizing isoparametric solid and shell elements. The plasticity theory contained in the program is based on the von Mises yield condition, the associated flow rule, and several optional hardening rules (isotropic, kinematic, combined isotropic-kinematic, or the mechanical sublayer model). At present, the program contains the equation-of-state creep model with auxiliary hardening and load reversal rules recommended in Ref. [16]. The creep strain model may be specified in either functional form or in terms of a creep strain vs. time data matrix.

SOME NUMERICAL RESULTS

This section presents some numerical results obtained with the AGGIE I computer program. Problems involving primarily large strain and elastic-plastic behavior have been previously reported in Refs. [2] and [8] and consequently, problems involving creep response are emphasized here. Both of these references contain experimental-numerical comparison studies of several work hardening rules and the interested reader is directed to these for details.

Figures 1 and 2 present creep strain vs. time results for a uniaxial test specimen subjected to an isothermal, monotonic and load reversal condition, respectively. The experimental results were obtained by ORNL [29]. The material in Fig. 1 is type 304 stainless steel at 1200 °F, ORNL preliminary heat no. 8043813. The effective creep strain equation was given by a 5,000 hour law:

$$\bar{\epsilon}_c = A(1 - e^{-rt}) + kt \quad (25)$$

where

$$\begin{aligned} A(\bar{\sigma}) &= 2.33 \times 10^{-6} \bar{\sigma}^{3.083} \\ r(\bar{\sigma}) &= 1.354 \times 10^{-3} \exp(0.129 \bar{\sigma}) \\ k(\bar{\sigma}) &= 7.91 \times 10^{-11} [\sinh(0.1932 \bar{\sigma})]^{4.0} \end{aligned} \quad (26)$$

and $\bar{\sigma}$ is in ksi and t in hours. The material in Fig. 2 is type 304 stainless steel at 1100 °F, heat no. 9T2796. The creep equation parameters are given by

$$\begin{aligned} A(\bar{\sigma}) &= 5.436 \times 10^{-5} \bar{\sigma}^{1.843} \\ r(\bar{\sigma}) &= 5.929 \times 10^{-5} \exp(0.2029 \bar{\sigma}) \\ k(\bar{\sigma}) &= 6.73 \times 10^{-9} [\sinh(0.1479 \bar{\sigma})]^{3.0} \end{aligned} \quad (27)$$

In both cases, the results obtained with the present computer program are seen to be in reasonable agreement with the experimental results. It should be noted that these results were obtained using strain-hardening based on total creep strain.

Figure 3 presents the isothermal, elastic-steady creep response for an infinitely long, thick-walled cylinder subjected to a constant internal pressure of 365 psi. The infinitely long cylinder has an inner radius of 0.16 inches and an outer radius of 0.25 inches. The material is assumed to have a Young's modulus of 20×10^6 psi and a Poisson's ratio of 0.499. The creep law is given by

$$\bar{\epsilon}_c = 6.4 \times 10^{-18} \bar{\sigma}^{4.4} \quad (28)$$

The present results agree exactly with those of Greenbaum and Rubinstein [30] and with an analytical solution reported in Ref. [30].

Figure 4 presents the isothermal, elastic-creep response for the same infinitely long, thick walled cylindrical geometry described above subjected to a constant internal pressure of 3,650 psi. The material is type 304 stainless steel, heat no. 9T2796, at a temperature of 1100 °F. Young's modulus

at this temperature is approximately 21.71×10^6 psi and Poisson's ratio is 0.3. The creep law parameters are the same as that given for the test shown in Fig. 2. The finite element model consisted of seven 6-node axisymmetric, isoparametric elements (with the plane strain condition imposed through boundary conditions). The effective stress ($\bar{\sigma}$) results shown are for numerical integration points approximately 0.002 inches from the inner and outer surfaces. The present results are compared to those obtained with the CREEP-PLAST computer program [31]. The discrepancy in the two solutions may be due to modeling differences although we cannot be sure.

Figure 5 presents deflection vs. load results for a simply supported beam with a shear load applied at the center. The beam (B9) is 25 inches long, 2 inches high and 1 inch wide and supported such that the effective length is 24 inches. The material is type 304 stainless steel (heat 9T2796) with a Young's modulus of 21.71×10^6 psi, Poisson's ratio of 0.3, and a yield stress of 9,000 psi at 1100°F. The beam was subjected to a load of 2,000 pounds, this load was then held constant for 312 hours, and then increased to 2,250 pounds. The finite element model consisted of five 8-node isoparametric elements and the kinematic hardening model was used to represent the strain hardening response. The time-independent, elastic-plastic computer results are compared to experimental results [32] in Fig. 5. It is seen that the results agree quite well up to the 312 hour hold period. During the hold period, the material hardens up and the results do not agree after the load is increased (since the time independent plasticity theory does not account for this).

FUTURE WORK

The work which has been reported herein has been based on incremental plasticity theory and an equation-of-state creep model. The authors are

currently evaluating the various creep constitutive models by developing uniaxial and biaxial computer programs for each and comparing numerical results with experimental results. Although we have not yet studied in detail the unified creep-plasticity formulations, it would appear that they have great promise. In the present paper, we have not included any numerical results for the non-isothermal or combined creep-plasticity problem; however, we will present such results in the near future.

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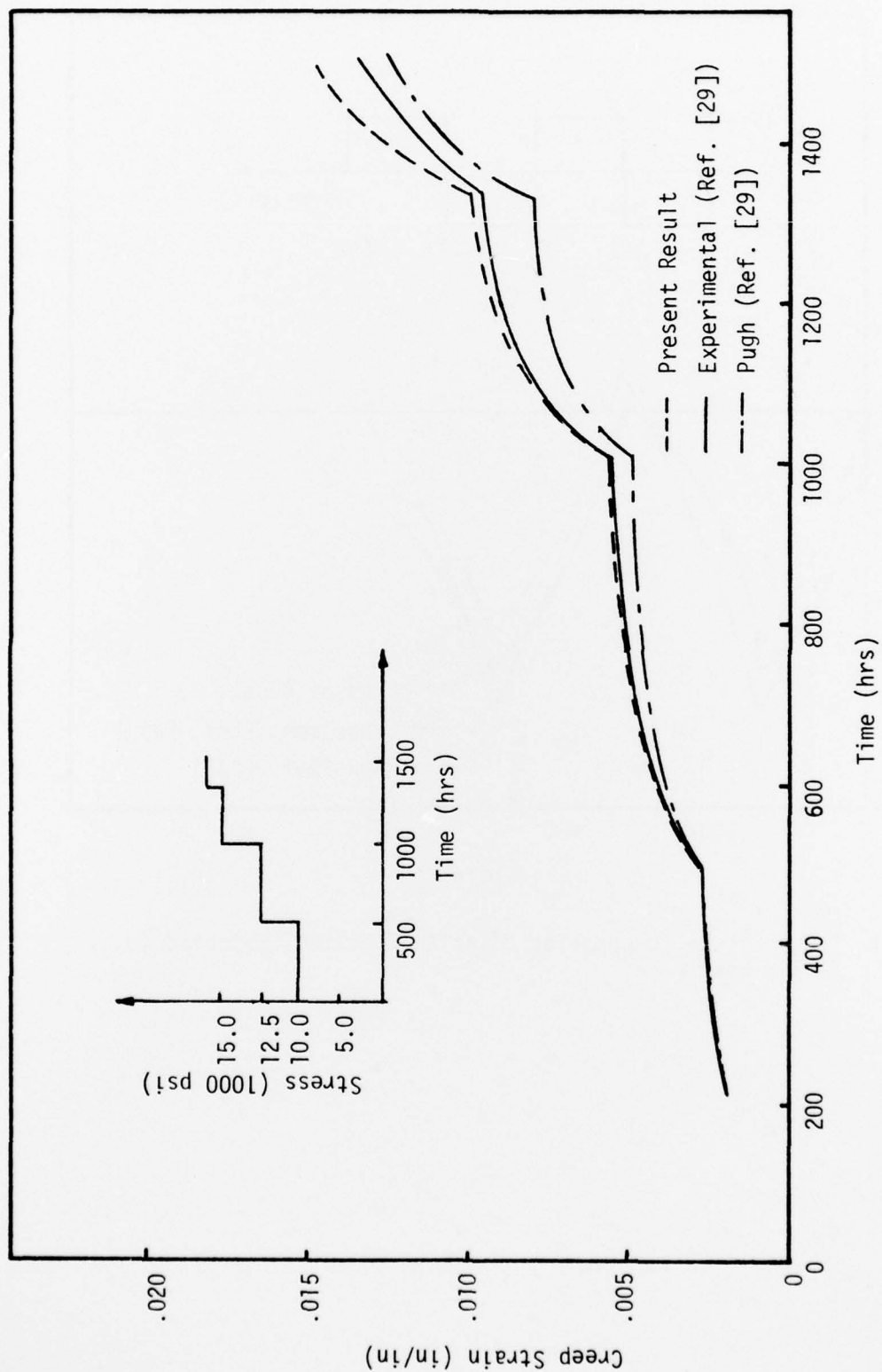


Fig. 1 Creep Strain Response of Uniaxial Specimen Subjected to Monotonically Increasing Load

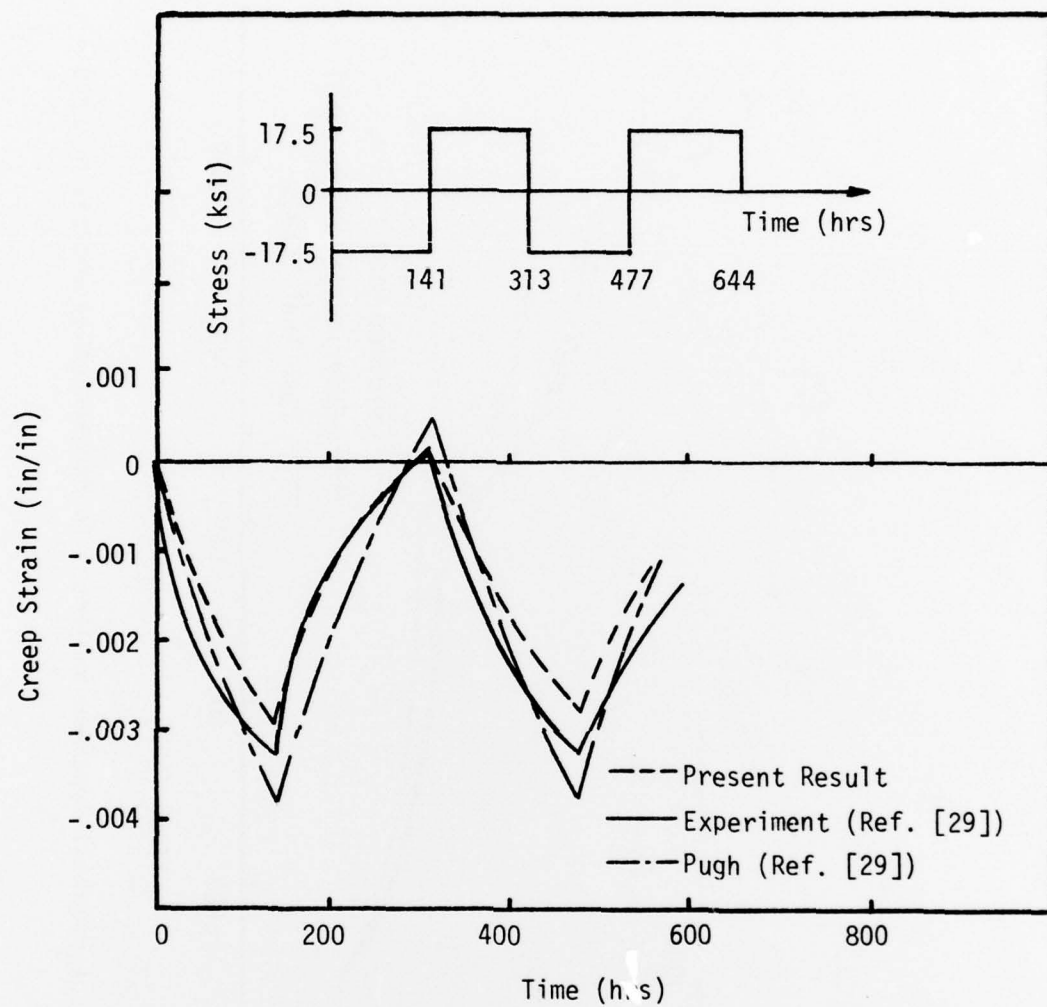


Fig. 2 Creep Strain Response of Uniaxial Specimen Subjected to Cyclic Load

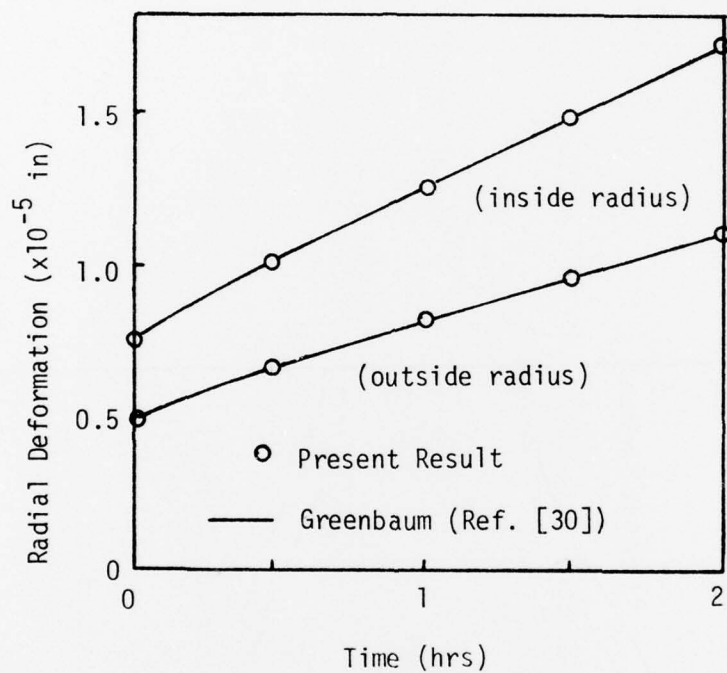


Fig. 3 Inside and Outside Radial Deformation of a Thick-Walled Cylinder Undergoing Elastic, Steady Creep

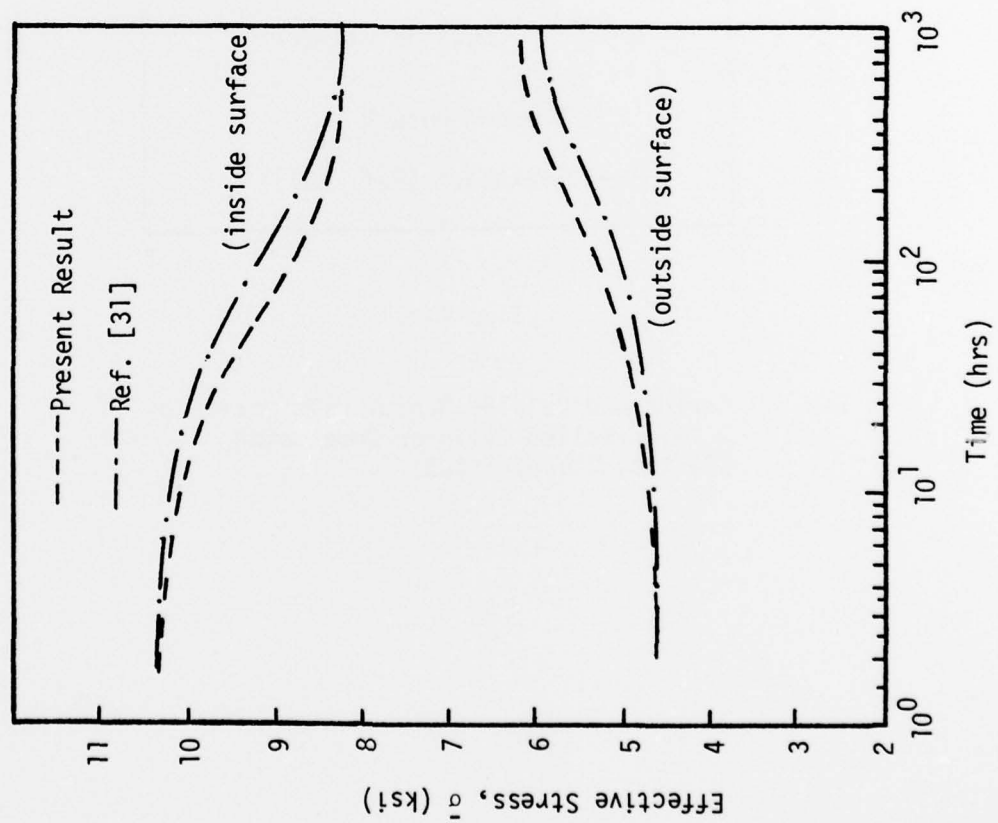


Fig. 4 Elastic-Creep Analysis of an Axisymmetric, Plane Strain, Thick-Walled Cylinder Subjected to Constant Internal Pressure

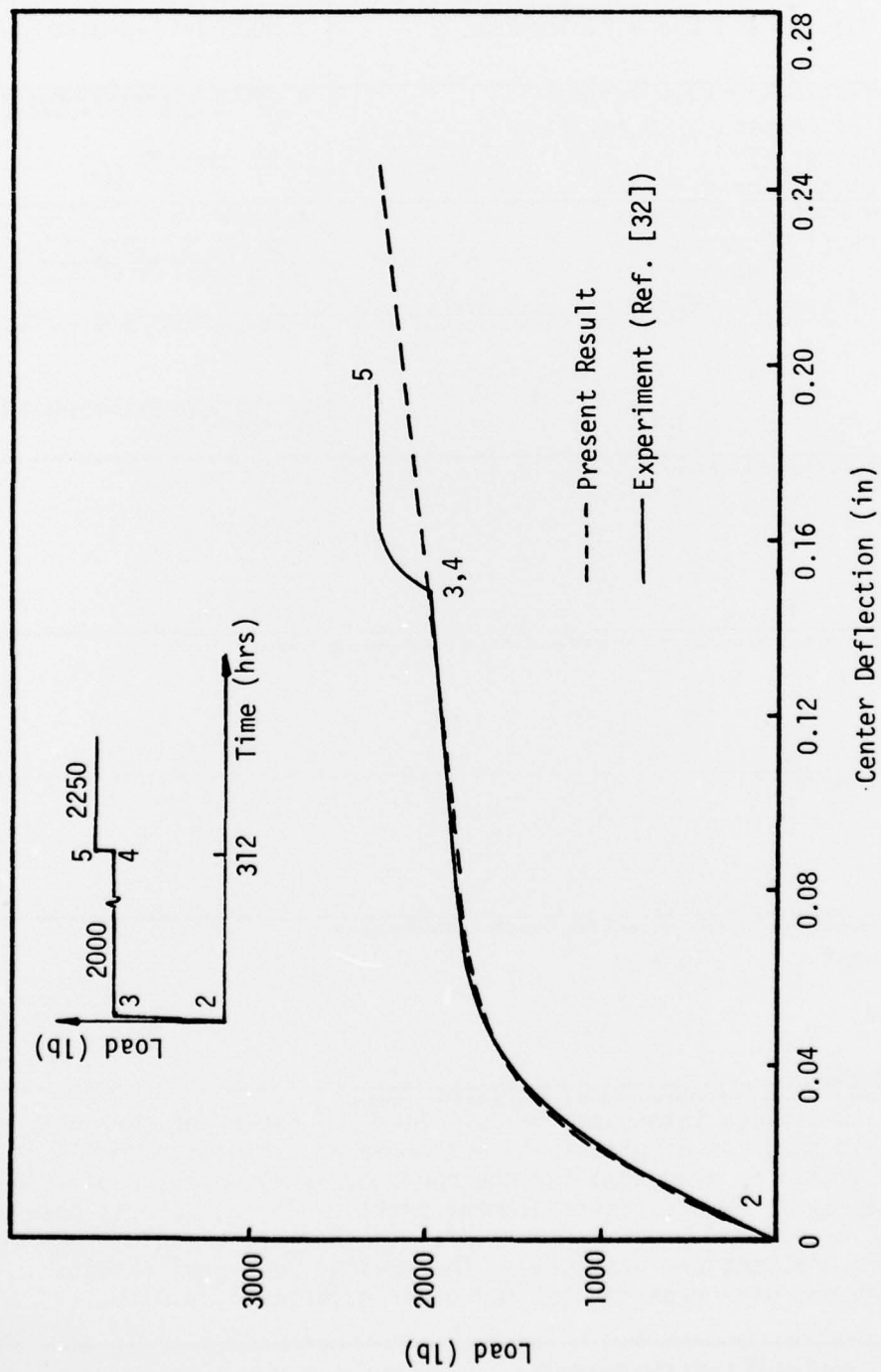


Fig. 5 Elastic-Plastic Analysis of Simply Supported Beam (B9) Subjected to Central Point Load

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