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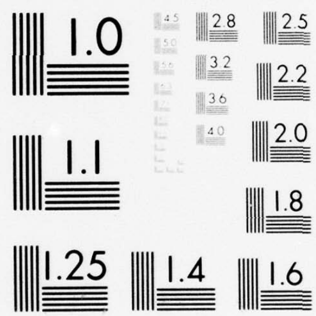
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AFAPL-TR-78-25

SOUND TRANSMISSION THROUGH DUCTS

UNIVERSITY OF DAYTON
SCHOOL OF ENGINEERING
DAYTON, OHIO 45469

MAY 1978

TECHNICAL REPORT AFAPL-TR-78-25
FINAL REPORT FOR PERIOD
8 FEBRUARY 1974 - 31 SEPTEMBER 1977

AIR FORCE AERO PROPULSION LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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This report covers analytic studies describing the propagation of sound waves in acoustically lined ducts. These ducts are typically found in quieted high-bypass turbine engine nacelles. Fortran IV computer programs were written for a CDC 6600 computer to predict maximum exponential attenuation of a least naturally attenuated mode in rectangular, annular, and cylindrical ducts either uniform or sheared flow. Additional programs were written to design optimum perforated plate liners.

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FOREWORD

This report summarizes the work done under Air Force Aero Propulsion Laboratory Contract No. F33615-74-C-2030 during the period of 8 February 1974 to 31 September 1977. It contains the analytical work describing the propagation of sound waves in the acoustically lined ducts typically found in quieted high-bypass turbojet engine nacelles. It also contains computer programs developed to predict the effects of acoustic lining in the ducts. The work was done by Dr. John J. Schauer, Mr. Eugene P. Hoffman, Mr. Marvin E. Himes, and Mr. Robert W. Guyton of the University of Dayton under Project Engineers Lt. Craig A. Lyon and Lt. Robert M. McGregor of the Aero Propulsion Laboratory, AFAPL/TBC, Wright-Patterson Air Force Base, Ohio 45433.

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SECTION I
INTRODUCTION AND SUMMARY

Recent Air Force regulation 80-36 states that Air Force tankers and transports should meet the commercial noise regulations (FAR 36). Since the dominant noise source of the high bypass turbojet engines typical for these applications is the fan, it is logical that the investigation of the duct linings which keep the fan and core noise from escaping via the inlet and exhaust ducts should receive attention. There are several reasons that this attention should be focused into the Air Force itself, rather than scattered throughout the commercial aircraft industry.

First, is the need for sophisticated, standardized prediction procedure for evaluating competitive aircraft designs on an acoustical basis. The need exists because of the differences in the many existing prediction programs currently in use by the aircraft industry. A comparison of several of these prediction programs will be a part of this report. The differences in prediction techniques between competitors would create a difference in predicted noise even for identical designs. It is important that competitive designs be evaluated after being put through the same acoustic prediction procedure.

A second reason that attention to Air Force aircraft acoustics should be focused in the Air Force is that a clear difference exists between the Air Force aircraft mission and the civilian commercial aircraft mission. This difference may make a different set of design parameters feasible for the two types of aircraft. That is, the Air Force aircraft may require fans producing higher frequency noise than their commercial counterparts or different bypass ratios. Air Force aircraft may also require more durable linings or different maintenance schedules. A prediction program developed primarily for commercial aircraft and checked out over a range of commercially feasible parameters might be of doubtful validity over a different range of parameters associated with designs meeting the Air Force mission. For this reason too, the Air Force needs its own prediction procedure, checked out over a range of parameters compatible with the Air Force mission.

A third reason for focusing Air Force attention on duct acoustics is that the area is rapidly developing via involved analytical techniques. This means that in order to follow the recent developments, a familiarity with the underlying analytical background and the physical principles involved is virtually a necessity. The familiarity can come only by a consistent, long term commitment in this area.

For these three reasons then, the Air Force contracted with the University of Dayton over the past several years to maintain an effort in the field of sound transmission through ducts. The effort was divided into three phases. Phase I consisted of the investigation of duct acoustic theory and computational procedures. Phase II consisted of assisting AFAPL in integrating Phase I work and programs from other sources into a performance/noise trade procedure. Phase III consisted of assisting AFAPL in applying the performance/noise trade procedure to selected aircraft.

1. PHASE I - DUCT ACOUSTIC RESULTS

Phase I results may be summarized under several headings as follows:

1) A least attenuated mode program was developed which iterates to find the acoustical lining impedance and subsequent design which gives a maximum exponential attenuation to the mode which is attenuated the least. This program handles any combination of 1) no flow, uniform flow, or sheared flow, with 2) rectangular, annular, or cylindrical ducts, for 3) one or two soft walls. This program was the basis for the study reported in AIAA paper number 75-129 [1].

2) A separate version of the iterating program mentioned in part (a) has been developed as a major working tool for modal analysis of constant cross-section ducts. This version is capable of finding all the modes for a given duct lining configuration but it is not entirely automatic as is version (a). This version was developed as an aid to a higher order angular mode study by Hoffman [2]. It is designed for interactive use from a time sharing terminal. The interactive procedure is somewhat routine except for initial guesses of modal characteristics and interpretation of the resultant modal characteristics. Experience or guidance is necessary for successful use of this version. This version produces input necessary for studies as reported in reference 3.

3) An analytical study was completed on the orthogonality of the duct acoustic modes for no flow, uniform flow, and sheared flow which results in isolation of the expansion coefficients for modes traveling in both directions. The uniform flow theory is implemented in a computer program for sound propagation in a segmented lining duct. Both the uniform flow and the sheared flow isolation of the model coefficients should have an important impact on the analysis of large systems of ducts as might exist in an idealization of a complete engine nacelle configuration. This study is introduced in reference 3 and detailed in this report for the first time.

4) Studies of double layer linings and single layer linings were conducted. These studies resulted in computer programs which design a perforated plate lining to correspond to given acoustic impedances. Other versions of these programs do the problem of finding the acoustic impedance of a given perforated plate type liner, single or double thickness. The double layer work is contained in a report by Hoffman [4]. His work includes a program for obtaining the local optimum lining resistances and backing and space depth to approximate simutaneously given acoustic impedances at two frequencies.

2. PHASE II - NOISE/PERFORMANCE RESULTS

1) A comparison of the attenuation predictions of several existing prediction procedures is given. These procedures include two semi-empirical procedures, least attenuated mode, plane wave techniques, finite difference on segmented lining, and equal energy in the propagating modes procedures. Two of these are based on Phase I efforts and one on an associated effort conducted by AFFDL/FBRD.

2) After several discussions with AFAPL and Design group personnel, a decision to develop a performance/noise trade procedure based on an existing noise prediction program BOEING, was made. BOEING was to be modified depending on the results of 1) above to reflect the work done in Phase I. This program was to be used in conjunction with existing performance programs in use by the Design group. The only missing link between the two programs was a need for friction factor information about acoustic linings by the Design group.

3) A study of available material concerning the surface roughness of acoustic lining materials was conducted and a computer program and report[5] written to fulfill the need of the Design group.

3. PHASE III - VERIFICATION RESULTS

The Advanced Medium STOL Transport (AMST) was selected for the initial application of a performance/noise trade procedure by AFAPL, Design, and University of Dayton personnel. Several engines were considered including the CF6-50, JT8D-17, and JT8D-209 refan. Consideration of the baseline aircraft and grown nacelle configurations will be continued in a follow-on effort under Contract F33615-78-C-2016.

SECTION II

LINING OPTIMIZATION - LEAST ATTENUATED MODE

Early in this contractual effort, emphasis was placed on obtaining an automated lining design program. The effects of a sheared velocity profile on the optimum lining was to be investigated in detail. As a consequence of this emphasis, a computer program was developed which required a minimum of input data and which iterated as required to obtain the optimum lining acoustical characteristics. The lining acoustical characteristics were then converted into an actual lining design by the use of information in the available literature regarding lining parameters. A brief excerpt from our published work [1]. will clarify the technical effort involved in this program. The following pertains to the cylindrical duct portion of the program.

The optimum duct wall problem was approached via the governing partial differential equation for sound propagation in a duct with a constant thickness boundary layer as derived by Mungur and Plumblee [6]. This partial differential is separated in the three space coordinates and time. The resultant ordinary differential equations are solved in time, in the axial coordinate, and in one cross-duct coordinate (angular variation). But in the other cross-duct coordinate (radial variation) the differential equation takes the form

$$\frac{d^2P}{dX^2} + \left(\frac{1}{X} + \frac{2k_z}{(k-k_zM)}\frac{dM}{dX}\right)\frac{dP}{dX} + H^2\left[k^2 - k_z^2(1-M^2) - 2Mk\,k_z - \frac{m^2}{H^2X^2}\right]P = O$$

where

P is the complex fluctuating pressure in the X direction

X is the normalized coordinate across the duct (radial)

M is the local Mach number

H is the duct height (radius)

k is a constant proportional to frequency

k_z is the complex axial propagation constant

m is the mode number of the pressure fluctuation in the other cross-duct coordinate (angular)

with the boundary conditions involving the wall impedance, Z, as

$$\frac{Z}{\rho c} = \pm \left. \frac{ikP}{\frac{\partial P}{\partial X}} \right|_{wall}$$

where

ρ is the fluid density

c is the speed of sound

This equation with its boundary conditions was solved as an eigenvalue problem by Mungur and Plumblee and by Eversman [7]. The present study uses an approach similar to that developed by Cremer [8] for the no flow case. Values of the imaginary part of k_z, the propagation constant, are proportional to the attenuation of the sound pressure down the duct. Picking an imaginary part of k_z and varying the real part using a numerical integration across the duct produces curves of constant attenuation in the wall impedance plane.

The curves form loops which reduce to a point as the attenuation is increased as shown by Cremer. A computerized procedure to home in on this point was developed to obtain the lining where the "least attenuated mode" has the highest attenuation. This point corresponds to the lining where the first and second radial modes are the same, a double mode point, and the lining parameters here define the "optimum" lining.

The boundary layer profile enters into the calculation in both the dM/dX term and the M terms which are functions of X. Because the steep velocity gradient at the wall causes the term involving M' to dominate at the wall, the boundary layer model used was the "law of the wall" as shown, for example, in Kays' [9]. In addition 1/7 power profiles and 1/7 power profiles modified in the one percent of the profile nearest the wall were tested in the differential equation. The "law of the wall" profile was used in the parameter studies illustrated. The wall shear was computed from the Blausius relationship for smooth walls as given by Kays.

The automatic convergence of this program has been verified over the frequency range where 2Hf/c is between 0.4 and 6.0. Here f is the frequency. This range is validated for rectangular, cylindrical, and annular ducts with one or two soft walls. It also holds for various shear layer thickness and mean flow Mach numbers of ± 0.5. The program and typical input and results for an annular duct are shown in Appendix B.

SECTION III
INTERACTIVE MODAL ANALYSIS

The major tool developed for a modal analysis of a given lining is this interactive version of OPTSHE. It is called a version of OPTSHE because it uses the same subroutines to set up the shear layer, to solve the governing partial differential equations, and to establish the governing equations. It's use of these subroutines, however, requires that the pair consisting of the modal attenuation and phase velocity be input rather than output as in the optimizing version. The flow configuration and duct configuration are input exactly as in OPTSHE. An input of modal characteristics results in an output of the lining acoustical properties which would give those characteristics. An iteration procedure is then utalized to permit the determination of a pair of modal characteristics corresponding to given lining acoustical properties. This procedure may be repeated to give other pairs of modal characteristics for this same lining. The pairs define the various modes which correspond to the given lining. In this manner angular modes or radial modes may be established. The modal results can be used in an "equal energy in the propagating modes" analysis as in Section VI of this report. The modal results can also be used as input to the modal analysis of multisectioned liners as in Section IV of this report.

This interactive version of OPTSHE is listed in Appendix C along with a typical terminal run. Notice that the input is requested by the program and that there is an open format for the input.

SECTION IV
ORTHOGONALITY STUDY AND ISOLATION OF MODAL COEFFICIENTS

1. UNIFORM FLOW

a. Introduction and Summary

An orthogonality condition, as first given by Zorumski [10], is derived for parallel uniform flow acoustic modes in rectangular, cylindrical or annular ducts with soft walls. The orthogonality condition permits the expansion of arbitrary input pressure and velocity distributions in terms of summations involving the duct modes, the mode eigenvalues, and uniquely determined coefficients. The orthogonality permits the coefficient of, for example, the first radial mode to be determined without the other radial modes or their eigenvalues being known. The orthogonality condition is demonstrated with example problems in a cylindrical duct and the coefficients examined near a double mode point for a plane wave input.

In Appendix D. 1 a d'Alembert type solution is assumed and is shown to be completely compatible with the solution presented in this section. The d'Alembert solution may be clearer when interpreting acoustic waves to be traveling in both directions. It is also shown in the appendix that isolation of the acoustic model coefficients can easily be adapted to multi-segmented duct liner theory.

Because the radial eigenvalue is contained in the boundary condition, uniform flow modes in a soft walled duct are not orthogonal in the usual sense as previously stated for example by Rice [11] for cylindrical ducts and Tester [12] for rectangular ducts. Although this does not prevent an approximate expression of arbitrary pressure and velocity distribution in terms of an expansion involving a finite number of the duct modes, as done by Rice, it does make the process approximate and laborious. This paper shows that the coefficients may be determined easily and uniquely using only the mode and eigenvalue whose coefficient is being determined. The method was mentioned by Lansing and Zorumski [13] and some results for a plane wave can be found there. The purpose of this paper is to present the method for an arbitrary pressure and pressure gradient in a manner which can be understood by practicing duct acoustic personnel.

The derivation presented here was suggested by Dr. K. G. Guderley and results in the development of the orthogonality condition without the use of the terms "adjoint" and "inner product." Dr. D. W. Quinn, also of Wright-Patterson Air Force Base AFFDL-FYS, arrived at the same results using the adjoint differential equation. Dr. Guderley continued to provide guidance at points of difficulty throughout the paper. This investigation was carried out under contract to the Air Force Aero Propulsion Laboratory (AFAPL) at WPAFB. The work was done at the Applied Mathematics Group of AFFDL (FBRD) at WPAFB and at the University of Dayton.

b. Orthogonality Condition

The derivation of the orthogonality condition will be done in detail for cylindrical ducts. It extends easily to the case of annular and rectangular ducts and the results will also be indicated for these duct types.

(1) Cylindrical Ducts

Starting with the well known Bessel's differential equation which results from a separation of variables approach to the governing partial differential equations for the sound propagation in a cylindrical duct with uniform flow, as shown for example in Rice [14], we can write

$$p(r, \theta, z, t) = P \exp[i(\omega - k_z z)] \cos m\theta \tag{1}$$

where

$$P'' + \frac{1}{x} P' + (kR)^2 \left[1 - \lambda^2(1-M^2) - 2M\lambda - \left(\frac{m}{kRx}\right)^2\right] P = 0 \tag{2}$$

and

P is the complex fluctuating pressure variation with x; $P = P(x)$

x is the normalized radial coordinate; r/R

R is the duct radius

M is the flow Mach number (constant)

k is a constant proportional to frequency; $k = \omega/c$

λ is k_z/k, the eigenvalue for (2)

ω is the circular frequency

c is the speed of sound

m is the angular mode number

and primes denote differentiation with respect to x. Multiplying Equation 2 by x, condensing the first two terms, and adding a subscript to P to denote that the ℓ^{th} radial mode also satisfies Equation 2 with $\lambda = \lambda_\ell$ we have

$$(xP_\ell')' + (kR)^2 \left\{\left[1 - \left(\frac{m}{kRx}\right)^2\right] x - (1-M^2)\lambda_\ell^2 x - 2M\lambda_\ell x\right\} P_\ell = 0 \tag{3}$$

with the boundary condition representing continuity of partical displacement at the outer wall as

$$P_\ell'(1) = - ikRA(1 - M\lambda_\ell)^2 P_\ell \tag{4}$$

where A is the specific acoustic admittance of the outer wall and from Eversman's [2] work for a cylindrical duct

$$P_\ell'(0) = 0 \tag{5}$$

The n^{th} radial mode satisfies the same equations and boundary conditions

$$(xP_n')' + (kR)^2 \left\{\left[1 - \left(\frac{m}{kRx}\right)^2\right] x - (1-M^2)\lambda_n^2 x - 2M\lambda_n x\right\} P_n = 0 \tag{6}$$

$$P_n'(1) = - ikRA(1-M\lambda_n)^2 P_n \tag{7}$$

$$P_n'(0) = 0 \tag{8}$$

Multiplying Equation 3 by P_n and integrating across the duct gives

$$\int_0^1 (xP_\ell')'P_n dx + (kR)^2 \int_0^1 P_\ell \left\{\left[1 - \left(\frac{m}{kRx}\right)^2\right] x - (1-M^2)\lambda_\ell^2 x - 2M\lambda_\ell x\right\} P_n dx = 0 \tag{9}$$

Integrating the first term by parts

$$\int_0^1 (xP_\ell')'P_n dx = \left[xP_\ell'P_n\right]_0^1 - \int_0^1 P_\ell' x P_n' dx \tag{10}$$

and integrating again by parts shows

$$\int_0^1 (xP_\ell')'P_n dx = \left[xP_\ell'P_n - xP_\ell P_n'\right]_0^1 + \int_0^1 P_\ell (xP_n')'dx \tag{10}$$

Substituting this into Equation 9 with the boundary conditions of Equations 4, 5, 7, and 8 gives

$$\int_0^1 P_\ell (xP_n')'dx + (kR)^2 \int_0^1 P_\ell \left\{\left[1 - \left(\frac{m}{kRx}\right)^2\right] x - (1-M^2)\lambda_\ell^2 x - 2M\lambda_\ell x\right\} P_n dx$$

$$+ P_\ell(1)P_n(1)ikRA(1-M\lambda_n)^2 - P_\ell(1)P_n(1)ikRA(1 - M\lambda_\ell)^2 = 0 \tag{11}$$

Nor multiplying Equation 6 by P_ℓ and integrating across the duct gives

$$\int_0^1 P_\ell (xP_n')'dx + (kR)^2 \int_0^1 P_\ell \left\{\left[1 - \left(\frac{m}{kRx}\right)^2\right] x - (1 - M^2)\lambda_n^2 x - 2M\lambda_n x\right\} P_n dx = 0 \tag{12}$$

Substracting Equation 12 from Equation 11 leaves

$$\int_0^1 \left[(1 - M^2)(\lambda_n^2 - \lambda_\ell^2) + 2M(\lambda_n - \lambda_\ell)\right] xP_\ell P_n dx +$$

$$P_\ell(1)P_n(1)\frac{iA}{kR} \left[(1-M\lambda_n)^2 - (1-M\lambda_\ell)^2\right] = 0 \tag{13}$$

Factoring $(\lambda_n^2 - \lambda_\ell^2)$ into $(\lambda_n - \lambda_\ell)(\lambda_n + \lambda_\ell)$ and simplifying the boundary condition term in a similar manner we find that

$$(\lambda_n - \lambda_\ell)\left\{ \int_0^1 (\lambda_n + \lambda_\ell - f)xP_\ell P_n dx + P_\ell(1)P_n(1)\frac{iAf}{kR} \right\} = 0 \tag{14}$$

$$f \equiv M^2(\lambda_n + \lambda_\ell) - 2M$$

If $\lambda_n \neq \lambda_\ell$ then the quantity in the braces in Equation 14 must be zero, or

$$(\lambda_n + \lambda_\ell - f) \int_0^1 xP_\ell P_n dx + P_\ell(1)P_n(1)\frac{iAf}{kR} = 0 \ ; \ \ell \neq n \tag{15}$$

is our orthogonality relationship for a cylindrical duct. We will refer to the first term in Equation 15 as the integral term and the second as the boundary condition term. In examining orthogonality in our example problems we will divide Equation 15 by $(\lambda_n + \lambda_\ell - f)$ and will refer to the resultant first term as the isolated integral and the resultant second term as the modified boundary condition term.

(2) Annular Ducts - Orthogonality Condition

With the boundary condition on the inner wall of an annular duct expressed as

$$P'(x_i) = ikRA(1 - M\lambda)^2 P(x_i) \tag{16}$$

we find that the orthogonality relationship for annular ducts is the same as for cylindrical ducts except for the boundary condition term which expands to become

$$\frac{if}{kR_o}\left[A_o P_\ell(1)P_n(1) + A_i x_i P_\ell(x_i)P_n(x_i)\right] \tag{17}$$

where the $(\ )_o$ subscript refers to outer wall quantities and $(\ )_i$ refers to quantities evaluated at the inner wall and the lower limit on the integral is x_i.

(3) Rectangular Ducts - Orthogonality Condition

Considering only the simplest case of three walls hard and one wall soft in a rectangular duct the *separation of variables* solution of the governing partial differential equation as shown by Rice becomes

$$p(x, y, z, t) = P(x)\exp[i(\omega t - k_z z)] \cos m2\pi y/w$$

with

$$P'' + (kH)^2 \left[1 - \lambda^2(1 - M^2) - 2M\lambda - \left(\frac{m2\pi}{kHw}\right)^2\right] P = 0$$

$$P'(0) = 0$$

$$P(0) = 1$$

$$P'(1) = - ikHA(1 - M\lambda)^2 P(1)$$

where H is the duct height and w is the width between the hard walled faces. Manipulating the modal solutions of these equations as before we find

$$(\lambda_n + \lambda_\ell - f) \int_0^1 P_\ell P_n dx + P_\ell(1)P_n(1)\frac{iAf}{kH} = 0 \; ; \; \ell \neq n \qquad (18)$$

as the orthogonality relationship for a rectangular duct.

c. Coefficient Isolation

(1) Cylindrical Ducts

The usefulness of the orthogonality relationships derived is dependent on whether or not they can be exploited to isolate the coefficients of expansions of arbitrary initial conditions in terms of the radial modes. We will take as initial conditions

$$P(x, z = 0) \equiv g = \sum_{\ell=1}^{\infty} b_\ell P_\ell \qquad (19)$$

and

$$\frac{\partial P}{\partial z}(x, z = 0) \equiv h \qquad (20)$$

where g and h are our arbitrary initial conditions. This breaks with tradition since the usual initial conditions are given in terms of pressure and axial velocity as used by, for example, Lansing and Zorumski [4.3] instead of pressure and axial pressure gradient as used here. The axial momentum equation, however, shows that if the axial pressure gradient is continuous then the axial velocity must also be continuous. For our separation of variables type solution

$$\frac{\partial P}{\partial z} = - i\lambda kP \tag{21}$$

we can see from Equations 19, 20, and 21 that

$$\frac{ih}{k} = \sum_{\ell=1}^{\infty} b_\ell \lambda_\ell P_\ell \tag{22}$$

In order to isolate the b_ℓ's we can substitute Equations 19 and 22 into Equation 15 by (a) multiplying Equation 19 by $P_n(1-M_2)\lambda_n xdx$, (b) multiplying Equation 19 by $P_n 2Mxdx$, and (c) multiplying Equation 22 by $P_n(1-M^2)xdx$, integrating these three from zero to one and adding to form the integral term in Equation 15 with its coefficient on the right hand side of the resultant equation. To this we add the boundary condition term

$$\Sigma b_\ell P_n(1)P_\ell(1)\frac{iA}{kR}[M^2(\lambda_n + \lambda_\ell) - 2M] \tag{23}$$

to both sides. The right hand side is then a summation of terms each of which corresponds exactly to Equation 15 which means that it will be zero except when $\ell = n$. That is, the right hand side will be

$$b_n \left\{ [2(1 - M^2)\lambda_n + 2M] \int_0^1 xP_n^2 dx + 2P_n^2(1)\frac{iA}{kR}(M^2\lambda_n - M) \right\} \tag{24}$$

and the corresponding left hand side will be

$$[(1 - M^2)\lambda_n + 2M] \int_0^1 xgP_n dx + (1 - M^2) \int_0^1 x \frac{ih}{k} P_n dx$$

$$+ \frac{iA}{kR} P_n(1) \sum_{\ell=1}^{\infty} b_\ell P_\ell(1) [M^2\lambda_n + M^2\lambda_\ell - 2M] \qquad (25)$$

The summations in the left hand side involving the boundary condition on the modes may be simplified since these sums must be the boundary condition on g and h from Equations 19 and 22. This permits the last term in Equation 25 to be written as

$$\frac{iA}{kR} P_n (1) \left[(M^2\lambda_n - 2M) g(1) + M^2 \frac{ih(1)}{k} \right] \qquad (26)$$

Combining Equations 24, 25, and 26 and solving for b_n gives

$$b_n = \frac{(\lambda_n - f_n) \int_0^1 xgP_n dx + \frac{(1 - M^2)i}{k} \int_0^1 xhP_n dx + \frac{iAP_n(1)}{kR} \left[f_n g(1) + \frac{M^2 i}{k} h(1) \right]}{(2\lambda_n - f_2) \int_0^1 xP_n^{\,2} dx + \frac{iA}{kR} P_n^{\,2}(1) f_2} \qquad (27)$$

where

$$f_n \equiv M^2\lambda_n - 2M$$

$$f_2 \equiv 2M^2\lambda_n - 2M$$

(2) Annular Ducts

Using Equation 17 in Equation 15 with Equations 19 and 22, the expansion coefficients for an annular duct may be written in the same form as Equation 27 except that the boundary condition terms change in both the numerator and the denominator. The boundary condition term in the numerator of Equation 27 becomes

$$\frac{i}{kR_o}\left\{A_oP_n(1)\left[f_ng(1)+\frac{M^2i}{k}h(1)\right]+A_ix_iP_n(x_i)\left[f_ng(x_i)+\frac{M^2i}{k}h(x_i)\right]\right. \tag{28}$$

and the boundary condition term in the denominator of Equation 27 becomes

$$\frac{if_2}{kR_o}\left[A_oP_n^{\,2}(1)+A_ix_iP_n^{\,2}(x_i)\right] \tag{29}$$

and the lower limit on the integral is x_i for an annular duct.

(3) Rectangular Ducts

The the three hard and one soft walled rectangular duct we use Equation 18 with Equations 19 and 22 to obtain exactly the same results as for the cylindrical duct except that the x is missing from inside the integrals and R is replaced by H in Equation 27.

d. Example Problems

Five example problems in cylindrical ducts are presented to illustrate the method and permit a directed discussion of the orthogonality conditions. The modal coefficients b_i's, shown in Table 1 are computed for modes which are normalized to a value of 1 + 0i on the duct centerline for Cases 1, 2, 4, and 5 which are axisymmetric (zero angular mode) examples. Case 3, which is for an angular mode of three, has the radial modes computed by expanding as shown by Eversman for x less than .001 and then normalizing the maximum value of the real part of the resulting radial pressure distribution to 1.0. A plane wave of magnitude 1 + 0i at z = 0 is the initial condition used in all cases. This results in g = ih/k = 1.0 for g and h in Equations 19 and 22. The b_i's are then computed from Equation 27.

TABLE 1

CYLINDRICAL DUCT WITH UNIFORM FLOW, MACH NO. = -.5

	Case 1	Case 2	Case 3	Case 4	Case 5
kR	0.8π	4π	4π	0.8π	0.8π
A	.3013 + .4029i	.08112 + 03384i	.06084 + .02538i	.2234 + .3006i	.2257 + .3020i
k_{z_1}/k	1.3307 - .6066i	1.979 - .01045i	1.943 - .04988i	1.391 - 1.070i	1.450 - 1.072i
k_{z_2}/k	1.389 - 1.688i	1.962 - .05746i	1.789 - .02499i	1.576 - 1.070i	1.509 - 1.072i
k_{z_3}/k	.8785 - 3.2556i	1.849 - .02875i	1.509 - .02577i	.8520 - 3.159i	.8521 - 3.161i
b_1	1.243 + 1.366i	1.312 - .073i	.309 + .429i	4.978 - 1.689i	14.161 - 5.395i
b_2	-.127 - .686i	-.570 - .264i	.466 - .131i	-4.218 + 2.450i	-14.770 + 7.982i
b_3	-.160 + .146i	-.176 + .482i	.229 + .082i	-.151 + .098i	-.151 + .099i
Remarks	4/3 opt.	4/3 opt.	Ang Mode = 3	near opt.	very near opt.

(1) Single Mode Points

Cases 1 and 2 are examples of single mode points. That is, the wall admittance has been selected for Cases 1 and 2 at about 4/3 of the "optimum" admittance. For this purpose "optimum" means the admittance at which the first and second radial modes coalesce for zero angular mode. At 4/3 of this admittance the radial modes are well separated for our examples. The coefficients of these two cases are typical of single mode points. That is the first coefficient is of the same order of magnitude as the initial conditions which are of magnitude one. And the higher order radial mode coefficients are of smaller magnitude. These first two cases were chosen to give a spread in the reduced frequency, kR. This was of interest particularly in examining the orthogonality of the modes in each case because the boundary condition terms of Equation 15 are inversely proportional to kR and the "optimum" admittance goes down as kR goes up giving a total effect about inversely proportional to kR squared. This would mean that as kR gets large the integral of $xP_\ell P_n$ would get small, approaching the "usual" orthogonality. Checking orthogonality, the integration of $xP_\ell P_n$ was performed numerically and the modified boundary condition term added to this. For the accuracy used in these computations the sum of the integral plus the modified boundary condition term was less than 0.3% of the integral for all three combinations of $\ell \neq n$ in Case 1. Case 2, where the integral itself is much smaller, the sum of the integral plus the modified boundary condition term was less than 10% of the value of the integral, $\ell \neq n$.

Case 3 is for an angular mode of three. Here, with the accuracy used, the numerical integration plus the modified boundary condition term was less than 4% of the integral, $\ell \neq n$. The coefficient of the first mode for Case 3 is significantly smaller than for Cases 1 and 2. This is expected since the higher order radial modes must build up the plane wave near the duct centerline where all the modes start with zero magnitude and zero slope.

(2) Near Double Mode Points

Cases 4 and 5 illustrate the anticipated behavior as a double mode point is approached. The first two radial modes for Case 4 have the imaginary parts of k_z/k the same while the real parts differ by about ten percent. Case 5 is almost identical but the real parts differ by only four percent. Notice that coefficients of the first two modes, b_1 and b_2 both get large and are of opposite sign. The situation is analogous to Tester's [15] investigation of double mode points in rectangular ducts with no flow. The large coefficients which arise can be explained, intuitively at least, by considering the function space (initial pressure and pressure gradient distribution) obtained by linear combinations of its basis functions (the radial modes) as analogous to points in a two dimensional space (a plane) obtained by linear combinations of basis vectors (any two nonparallel vectors in the plane). The mode shapes of the first two modes approaching each other corresponds to two unit vectors in the plane becoming almost parallel. In order to get a point in the plane not in the direction of the almost parallel vectors by a linear combination of these vectors the two coefficients of the vectors must be very large and of opposite sign. This causes the difference between the vectors to be amplified while the parallel component of the vectors tends to cancel. This corresponds to the given initial conditions which must be formed by linear combination of the radial modes. When two modes become almost identical their coefficients becoming large and of opposite sign corresponds to the difference between the two modes being amplified to create a portion of the initial condition corresponding to the difference between the two modes. The similarities between the modes are not similarly amplified because of the difference in sign between the coefficients. Even as the modes approach each other they still satisfy the orthogonality condition (Equation 15) as was demonstrated numerically for Cases 4 and 5.

As the modes approach each other in shape their propagation in the z direction approaches the exponential times $A + Bz$

(A and B constants) behavior shown by Tester. This behavior can be seen in the present context and notation by considering the form of the first two terms in the infinite series (Equation 19). The series takes the form

$$P(x, z) = b_1 P_1(x) e^{-ik_{z_1} z} + b_2 P_2(x) e^{-ik_{z_2} z} + \ . \ . \ . \tag{30}$$

Near a double mode point $P_1(x)$ is almost identical with $P_2(x)$ and k_{z1} is almost equal to k_{z_2}. Defining at any fixed value of x, for example $x = x_1$,

$$P_2(x_1) = P_1(x_1) + \Delta P \tag{31}$$

where ΔP is a small difference and

$$k_{z_2} = k_{z_1} + \varepsilon \tag{32}$$

where ε is also small, permits the writing of Equation 30 as

$$P(x_1, z) = b_1 P_1(x_1) e^{-ik_{z_1} z} + b_2 [P_1(x_1) + \Delta P] e^{-ik_{z_1} z} e^{-i\varepsilon z} + \ldots \tag{33}$$

The last exponential can be expanded in the usual exponential series, the higher order terms in εz neglected and the $\Delta P \varepsilon$ product assumed negligible to give

$$P(x_1, z) \approx b_1 P_1(x_1) e^{-ik_{z_1} z} + b_2 P_1(x_1) e^{-ik_{z_1} z} + b_2(\Delta P - P_1(x_1) i\varepsilon z) e^{-k_{z_1} z} + \ . \ . \ . \tag{34}$$

This rearranges to give the form

$$P(x_1, z) = \left\{ (b_1 + b_2) P_1(x_1) + b_2 [\Delta P - i P_1(x_1) \varepsilon z] \right\} e^{ik_{z_1} z} + \ . \ . \ . \tag{35}$$

if x_1 is chosen at the centerline in the axisymmetric case where both modes are normalized to 1.0, then ΔP is zero and

$$P(0, z) + [(b_1 + b_2) - i b_2 \varepsilon z] \, P_1(0) e^{ik_{z_1} z} + \ . \ . \ . \tag{36}$$

While both b_1 and b_2 are large, they are of opposite sign and both the $b_1 + b_2$ constant term and the $ib_2\epsilon$ linear term coefficient are of reasonable size. This is the exponential times A + Bz behavior near a double mode point also shown by Tester.

e. Discussion

The uses of an orthogonality condition such as that derived here are several. First, it can be used to find the amounts of the first or first few radial modes present in an arbitrary radial initial condition without finding or using the higher order modes. This is an advantage because often the higher order modes are attenuated so rapidly that they are of little importance when studying the sound propagation. Second, as shown by Lansing and Zorumski, orthogonality can be used in a mode matching type of analysis at a discontinuity in duct lining. Third, an orthogonality condition for the radial mode may be combined with an orthogonal expansion in the theta direction in a double expansion to give a capability for finding the amounts of the various radial and angular modes present in an initial pressure distribution which is a function of both radial and angular position.

Completeness of the radial eigenfunctions has been assumed.

f. Summary

Expansion coefficients have been isolated for uniform flow with the help of the orthogonality condition derived. Example problems have been presented to illustrate the effects of being near a double mode point.

2. SHEARED FLOW

a. Introduction

A method for isolating the acoustic modal coefficients is presented for a softwalled cylindrical duct containing a sheared parallel flow. The linear acoustic equations for the sheared parallel flow are cast into the form of a linear eigenvalue problem with associated eigenvectors and by utilizing the definition and properties of the complex inner product an adjoint eigenvalue problem is formed so that the usual definition of orthogonality exists between the acoustic modal vectors and the adjoint modal vectors. The modal coefficients can be isolated by using the orthogonality property. An alternate approximate technique using the Galerkin method is also presented as a means of isolating the sheared flow modal coefficients.

The partial differential Equation 37 which governs the linear acoustic wave notion for a sheared parallel flow contained in a cylindrical duct (Figure 1) is a derived by Mungur and Plumblee [6].

$$\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} + \frac{2M(r)}{c}\frac{\partial^2 P}{\partial z \partial t} + M^2(r)\frac{\partial^2 P}{\partial z^2} - 2\rho c\frac{\partial M(r)}{\partial r}\frac{\partial v_r}{\partial z} - \nabla^2 P = 0 \qquad (37)$$

where ∇^2 is the Laplacian operator in cylindrical coordinates

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

and the Mach number $M(r)$ depends only on the radial coordinate. It can be shown that, if the fluctuating quantities $p(r, z, \theta, t)$ and $v(r, z, \theta, t)$ are proportional to $e^{ik(ct-\lambda z)}$ cos $n\theta$, the radial momentum relationship becomes

$$v_r(r) = \frac{iP'(r)}{\rho ck(1-M(r)\lambda)} \qquad (38)$$

and the partial differential Equation 37 reduces to the following ordinary differential equation and wall boundary condition.

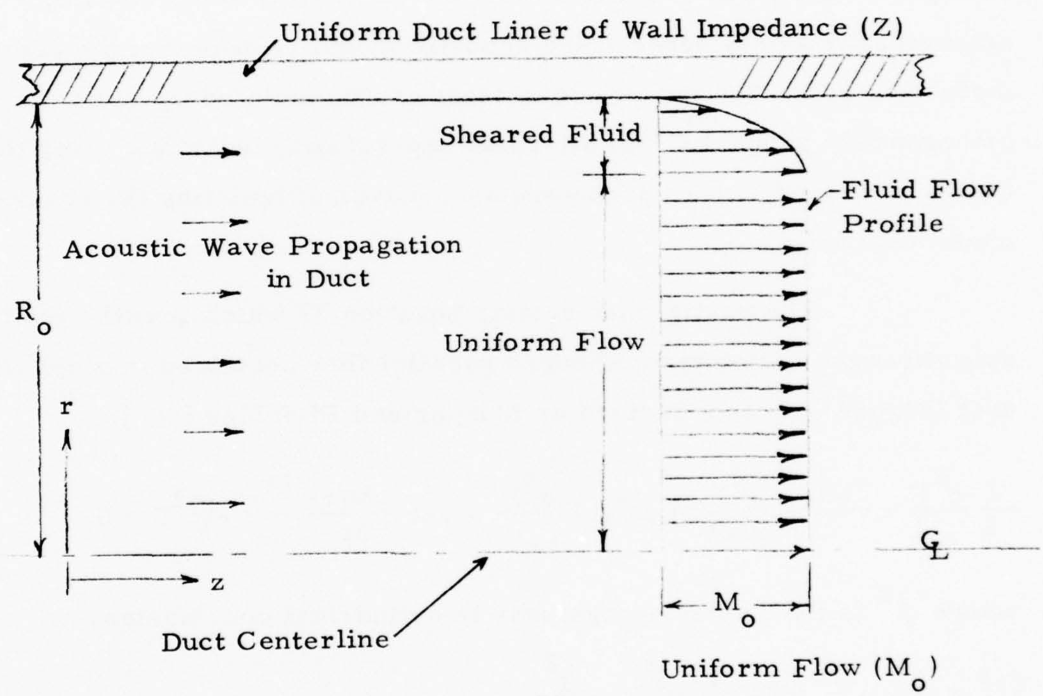


Figure 1. Sheared Flow in a Cylindrical Duct with Acoustic Propagation

$$P''(r) + \left(\frac{1}{r} + \frac{2\lambda M'(r)}{(1-M(r)\lambda)}\right) P'(r) + k^2 \left[(1-M(r)\lambda)^2 - \lambda^2 - \frac{m^2}{(kr)^2}\right] P(r) = 0 \qquad (39)$$

and the no=slip wall boundary condition is

$$Z_{wall} \equiv \left(\frac{P}{v_r}\right)\Bigg|_{wall} = -ik\rho c \left.\frac{P(r)}{P'(r)}\right|_{wall} \qquad (40)$$

The governing ordinary differential equations for a uniform flow and for a sheared parallel flow are identical except for the additional term of $\frac{2\lambda M'}{1-M\lambda} P'$ in Equation 39 for the sheared flow making the uniform flow orthogonality condition derived earlier in this report not applicable to the sheared flow case. Therefore a different orthogonality condition was developed in order to isolate the sheared flow modal coefficients.

Schauer and Hoffman [3] presented in general an adjoint method of solving for the modal coefficients in a sheared flow. This method will be presented in detail on the following pages. Kraft and Wells [16] have used an adjoint function to find modal coefficients, but only in the case of a uniform flow where the acoustic modes are assumed to be traveling in one direction. It is anticipated that the sheared flow adjoint solution will converge to the uniform flow adjoint solution as the sheared region of the fluid, the boundary layer, goes to zero.

As an alternate technique it is shown that the Galerkin method as applied by Unruh and Eversman [17] and by Yurkovich [18] to an attenuating duct can be extended so that approximate sheared flow modal coefficients can be found.

b. Analysis Adjoint Method

As in uniform flow it is assumed that the radial acoustic pressure distribution g(r) can be expanded as a linear combination of the modal pressure distributions $P_\ell(r)$

$$g(r) = \sum_\ell b_\ell P_\ell(r) \qquad (41)$$

where $P_\ell(r)$ satisfies the sheared flow differential Equation 39 and the wall boundary condition, Equation 40. The radial distribution of the axial pressure gradient H(r) is

$$H(r) = -ik \sum_\ell b_\ell \lambda_\ell P_\ell \tag{42}$$

and λ_ℓ is the axial eigenvalue of Equation 39. The radial acoustic velocity v(r), is

$$v(r) = \sum_\ell b_\ell v_\ell(r) \tag{43}$$

where $v_\ell(r)$ is the modal radial acoustic velocity as given by Equation 38. Equations 41, 42, and 43 can be represented by one vector equation

$$\vec{q}(r) \equiv \begin{Bmatrix} g(r) \\ \frac{iH(r)}{k} \\ v(r) \end{Bmatrix} = \sum_\ell b_\ell \vec{y}_\ell(r) \tag{44}$$

where $\vec{y}_\ell$ is defined by

$$\vec{y}_\ell \equiv \begin{Bmatrix} P_\ell \\ \lambda_\ell P_\ell \\ v_\ell \end{Bmatrix} \tag{45}$$

It is pointed out that the modal vector components of $\vec{y}_\ell(r)$ are functions of the cross duct coordinate, r.

A linear eigenvalue problem can be formed by defining [A] to be a matrix operator such that

$$[A]\vec{y}(r) = \lambda\vec{y}(r)$$

where λ is again an axial eigenvalue of Equation 39. The form of [A] can be found by manipulating the sheared flow acoustic differential Equation 39 and the radial momentum Equation 38. The differential Equation 39 is rearranged to get

$$\frac{1}{k^2(1-M^2)} \left\{P'' + \frac{1}{r}P' + \frac{2\lambda M'}{1-M\lambda} P' + k^2(1-2M\lambda - \frac{m^2}{(kr)^2})P\right\} = \lambda^2 P \tag{46}$$

From the radial momentum relationship Equation 38

$$\frac{P'}{1-M\lambda} = -i\rho ckv$$

substituting for $\frac{P'}{1-M\lambda}$ in Equation 46 gives

$$\frac{1}{k^2(1-M^2)}\left\{P'' + \frac{1}{r}P' + k^2(1-2M\lambda-\frac{m^2}{(kr)^2})P - 2i\rho ckM'\lambda v\right\} = \lambda^2 \qquad (47)$$

By solving for λ from the radial momentum Equation 38 and then multiplying by v it can be shown that

$$\lambda v = \frac{1}{M}(v - \frac{iP'}{\rho ckM}) \qquad (48)$$

After substituting the expression for λv from Equation 48 into the modified differential Equation 47 the final form of the transformed differential equation becomes

$$\frac{1}{k^2(1-M^2)}\left\{P'' + \frac{1}{r}P' + k^2(1-2M\lambda - \frac{m^2}{(kr)^2})P - 2i\rho ck\frac{M'}{M}(v - \frac{iP'}{\rho ck})\right\} = \lambda^2 P \quad (49)$$

It can be verified that the matrix [A] defined below

$$[A] \equiv \begin{bmatrix} 0 & 1(\ ) & 0 \\ \dfrac{(\ )''+(\frac{1}{r}-\frac{2M'}{M})(\ )'+(k^2-\frac{m^2}{r^2})(\ )}{k^2(1-M^2)} & \dfrac{-2M}{(1-M^2}(\ ) & \dfrac{-2i\rho ckM'}{k^2(1-M^2)M}(\ ) \\ \dfrac{-i(\ )'}{\rho ckM} & 0 & \dfrac{1}{M}(\ ) \end{bmatrix} \qquad (50)$$

and the relation $A\vec{y} = \lambda\vec{y}$ represents the set of equations

$$\left\{\begin{array}{l} \lambda P = \lambda P \\ \text{transformed differential Equation 49} \\ \text{radial momentum relation 12} \end{array}\right\}$$

where $\vec{y}$ has been defined previously as $\vec{y} = \begin{Bmatrix} P \\ \lambda P \\ v \end{Bmatrix}$.

A complex inner product is now defined over the interval of interest such that

$$< \vec{\alpha}(r), \vec{\beta}(r) > = \int_{o}^{r} \vec{\alpha}(r) \cdot \overline{\vec{\beta}(r)} \, dr \tag{51}$$

where ($\overline{\quad}$) represents the complex conjugate and ($\cdot$) represents the vector product in the usual sense. (See Nomizu [19] for details on inner product spaces.) The definition of the adjoint [A*] of the operator [A] is given below.

$$<[A]\vec{y}, \vec{z}> = <\vec{y}, [A^*]\vec{z}> \tag{52}$$

and the vector $\vec{z} = \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix}$ is the adjoint vector.

By using the properties and definition of the complex inner product it can be shown that if λ is an eigenvalue of Equation 53.

$$[A]\vec{y} = \lambda \vec{y} \tag{53}$$

λ must then be an eigenvalue of adjoint Equation 54.

$$[A^*]\vec{z} = \overline{\lambda} \vec{z} \tag{54}$$

and $\vec{z}_\ell$ is the adjoint eigenvector corresponding to $\overline{\lambda}_\ell$. If the inner product $<A\vec{y}_\ell, \vec{z}_k>$ is formed and $\vec{y}_\ell$ is the ℓ^{th} eigenvector of Equation 53 and $\vec{z}_k$ is the k^{th} adjoint eigenvector of Equation 54, it can be shown that

$$(\lambda_\ell - \lambda_k) <\vec{y}_\ell, \vec{z}_k> = 0 \qquad \text{and} \qquad <\vec{y}_\ell, \vec{z}_k> = 0 \tag{55}$$

Hence the adjoint eigenvector is always orthogonal for $\ell \neq k$ to the eigenvector of Equation 53 when the eigenvalues are different.

Equation 55 is the usual definition of orthogonality. Assuming a modal expansion exists, let $\vec{q}(r)$ be the generalized vector which fully describes the conditions at any plane in the duct under consideration.

$$\vec{q}(r) = \begin{Bmatrix} q_1(r) \\ q_2(r) \\ q_3(r) \end{Bmatrix} = \sum_{\ell} b_{\ell} \vec{y}_{\ell}(r) \tag{56}$$

The components of q (r) are the following

$q_1(r) = g(r)$ radial pressure distribution

$q_2(r) = \frac{ih(r)}{k}$ radial distribution of the axial pressure gradient

$q_3(r) = v_r(r)$ radial acoustic velocity

When the inner product between the k^{th} adjoint modal vector $\vec{z}_k$ and Equation 56 is formed, all terms except one in the summation are zero from the orthogonality condition, Equation 56. Hence,

$$< \vec{q}(r), \vec{z}_k(r) > = < b_k \vec{y}_k, \vec{z}_k >$$

and solving for b_k

$$b_k = \frac{< \vec{q}(r), \vec{z}_k(r) >}{< \vec{y}_k(r), z_k(r) >} \tag{57}$$

Thus if the adjoint modal vectors can be generated, the sheared flow modal coefficients, b_k, can be found. The eigenvalues and eigenvectors of Equation 42 may be found by solving the differential Equation 39 with the associated boundary condition (4), thus giving $P_{\ell}(r)$, λ_{ℓ}. Equation 38 is used to find $v_{\ell}(r)$.

c. Construction of Adjoint Matrix [A*]

The only restriction placed on [A*] is that it must satisfy Equation 52.

$$< [A]\vec{y}, \vec{z} > = < \vec{y}, [A^*]\vec{z} >$$

Let $[\alpha_{iJ}] = [A]$ and $[\alpha^*_{iJ}] = [A^*]$ i, J = 1, 2, 3 then Equation 52 becomes

$$\int_o^r (\alpha_{iJ} y_J)\, \overline{Z}_i \, dr = \int_o^r y_J \, \overline{(\alpha^*_{Ji}\, Z_i)} \, dr \tag{58}$$

where the definition of the inner product, Equation 51, has been used, and the repeated index (unless the index is underlined) means summation over that index. Hence

$$\alpha_{iJ} y_J \overline{Z}_i = \left\{ \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} \right\}^T \begin{Bmatrix} \overline{Z}_1 \\ \overline{Z}_2 \\ \overline{Z}_3 \end{Bmatrix}$$

and

$$y_J \overline{\alpha^*_{Ji} Z_i} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix}^T \begin{bmatrix} \overline{\alpha}^*_{11} & \overline{\alpha}^*_{12} & \overline{\alpha}^*_{13} \\ \overline{\alpha}^*_{21} & \overline{\alpha}^*_{22} & \overline{\alpha}^*_{23} \\ \overline{\alpha}^*_{31} & \overline{\alpha}^*_{32} & \overline{\alpha}^*_{33} \end{bmatrix} \begin{Bmatrix} \overline{Z}_1 \\ \overline{Z}_2 \\ \overline{Z}_3 \end{Bmatrix}$$

The operator $\alpha_{\underline{iJ}}$ on $y_{\underline{J}}$ can be transposed to $\overline{Z}_{\underline{i}}$ by performing integrations by parts where necessary. Thus

$$\int_o^r (\alpha_{\underline{iJ}} y_{\underline{J}})\, \overline{Z}_{\underline{i}} \, dr = \int_o^r y_{\underline{J}} \, (\alpha^T_{\underline{iJ}}\, \overline{Z}_{\underline{i}}) \, dr + [BC(y_{\underline{J}} \overline{Z}_{\underline{i}})]\; \alpha^T_{\underline{iJ}}$$

and (59)

$$\int_o^r (\alpha_{iJ} y_J)\, \overline{Z}_i \, dr = \int_o^r y_J \, (\alpha^T_{iJ}\, \overline{Z}_i) \, dr + [BC(y_J \overline{Z}_i)]\; \alpha^T_{iJ}$$

Use Equations 58 and 59 to get Equation 60.

$$\int_o^r y_J \, \overline{(\alpha^*_{Ji}\, Z_i)} \, dr = \int_o^r y_J \, (\alpha^T_{iJ}\, \overline{Z}_i) \, dr + [BC(y_J \overline{Z}_i)]\; \alpha^T_{iJ} \tag{60}$$

If the Boundary terms collectively are made to be zero at the end-points then

$$\int_0^r y_J \overline{(\alpha^*_{Ji} Z_i)}\, dr = \int_0^r y_J (\alpha^T_{iJ} \overline{Z}_i)\, dr \tag{61}$$

By letting $\overline{\alpha^*_{Ji}} = \alpha^T_{iJ}$ Equation 61 is automatically satisfied. The construction of $[A^*]$ is complete.

$[A^*] = [\alpha^*_{iJ}] = [\overline{\alpha^T_{Ji}}]$

The system of equations represented by $[A^*]\,\vec{Z}(r) = \overline{\lambda}\,\vec{Z}(r)$ can now be reduced to a single differential in $Z_2(r)$. Since λ is known the radial distribution of $Z_2(r)$ can be found using an appropriate numerical integration scheme or Runga-Kutta technique, and finally the sheared flow modal coefficients may be isolated.

d. Approximate Method of Isolating the Modal Coefficients (Galerkin Method)

As in the previous analysis on the adjoint method it is assumed that the radial distribution of the Pressure, g(r), and the radial distribution of the axial Pressure graduent, H(r), may be expanded in a series of sheared flow modes, P_ℓ, which satisfy the differential equation (3). Hence

$$g(r) = \sum_\ell b_\ell P_\ell = P(r)$$

$$H(r) = -ik \sum_\ell \lambda_\ell b_\ell P_\ell = \frac{\partial P(r)}{\partial Z}$$

and the differential operator L is defined below

$$L[P] \equiv P'' + \left(\frac{1}{r} + \frac{2\lambda M^1}{P-M\lambda}\right) P' + k^2\left[(1-\lambda M)^2 - \lambda^2 - \frac{m^2}{(kr)^2}\right] P = 0 \tag{39}$$

The Garerkin method as applied by Unruh and Eversman [17] assumes that each sheared radial pressure distribution, P, may be expanded in a series of the no flow eigenfunctions satisfying the wall condition as

$$P = \sum_{n=1}^{N} C_m R_{mn}(k_n r) \qquad (62)$$

where $R_{mn}(k_n r)$ is the n^{th} eigenfunction for the no flow case and m is the angular mode number. Thus each sheared flow mode, P, may be made up of a linear combination of the no flow modes, $R_{mn}(k_n r)$. After having substituted Equation 62 into the differential Equation 39 an error, $\epsilon(r)$, resulted.

$$L \left[\sum_{n=1}^{N} C_n R_{mn}(k_n r) \right] = \epsilon(r) \qquad (63)$$

Equation 63 is subsequently multiplied by $(1-M\lambda)$ to remove this term from the denominator to give

$$(1-M\lambda) L \left[\sum_{n=1}^{N} C_n R_{mn} \right] = (1-M\lambda) \epsilon(r) \qquad (64)$$

Equation 64 is now multiplied by rR_{mp} and integrated over the radius of the duct to give

$$\int_0^{R_o} (1-M\lambda) L \left[\sum_n C_n R_{mn} \right] R_{mp} \, rdr = \int_0^{R_o} (1-M\lambda) \epsilon(r) R_{mp} \, rdr \qquad (65)$$

If the error $\epsilon(r)$ is forced to be orthogonal to R_{mp} with weighting $(1-M\lambda)r$ over the duct radius, then Equation 65 represents a set of homogenious equations in C_n and λ. It can be shown [18] that Equation 65 can be cast into the form of a linear eigenvalue problem such that

$$[B]\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \lambda \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \qquad (66)$$

where

$$x_1 = \{C_n\} \qquad x_2 = \{\lambda C_n\} \qquad x_3 = \{\lambda^2 C_n\}$$

and the Matrix B is composed of known numerical quantities which come from carrying out the integration of Equation 65.

$$\int_o^{R_o} (1-M\lambda)\, L\,[\Sigma_n C_n R_{mn}]\, R_{mp}\, rdr = 0 \qquad (67)$$

Since Matrix B is known, the eigenvalues, λ, and corresponding eigenvectors can be found. Essentially Unruh and Eversman [17] and Yurkovich [18] found the sheared flow eigenvalues and the no flow modal coefficients, C_n, which were used to build up each sheared flow modal pressure distribution, P. Hence

$$P_{\ell\,\text{sheared}} = \sum_{n=1}^{N} C_{n\ell} R_{mn} \qquad (68)$$

To find approximate values of the sheared flow modal coefficients, b_ℓ, Equation 68 is substituted into the expressions for g(r) and H(r). Thus

$$g(r) = \sum_{\ell=1}^{2N} b_\ell \left(\sum_{n=1}^{N} C_{n\ell} R_{mn} \right) \qquad (69)$$

and

$$H(r) = -ik \sum_{\ell=1}^{2N} \lambda_\ell b_\ell \left(\sum_{n=1}^{N} C_{n\ell} R_{mn} \right) \qquad (70)$$

It is known that the no flow modal pressures R_{mn} are orthogonal to each other with weighting r. Thus

$$\left\langle R_{mn}, R_{mJ} \right\rangle \equiv \int_0^{R_o} r\, R_{mn}\, R_{mJ}\, dr = 0 \qquad \text{for } J \neq n \text{ and } \langle\ \ \rangle$$

means inner product. After forming the inner product of Equations 69 and 70 with R_{mJ} and using the no flow orthogonality, the following equation results

$$\left\langle g(r), R_{mJ} \right\rangle = \sum_{\ell=1}^{2N} b_\ell\, C_{J\ell} \left\langle R_{mJ}, R_{mJ} \right\rangle \qquad J = 1 \ldots N$$

and (71)

$$\left\langle \frac{iH(r)}{k}, R_{mJ} \right\rangle = \sum_{\ell=1}^{2N} \lambda_\ell\, b_\ell\, C_{J\ell} \left\langle R_{mJ}, R_{mJ} \right\rangle \qquad J = 1 \ldots N$$

Equations 71 represent a set of 2N linear equations in 2N unknowns, b_ℓ. Equations 71 can be written in compact matrix notation as

$$\begin{Bmatrix} \left\{ \left\langle g, R_{mJ} \right\rangle \right\} & J=1 \ldots N \\ \left\{ \left\langle \frac{iH}{k}, R_{mJ} \right\rangle \right\} & J=1 \ldots N \end{Bmatrix} = \begin{bmatrix} [A] \\ [B] \end{bmatrix} \left\{ b_\ell \right\} \qquad (72)$$

where [A] and [B] are Nx 2N matrices with

$$[A] = [C_{J\ell} \left\langle R_{mJ}, R_{mJ} \right\rangle]$$

and $\qquad J = 1 \ldots N, \quad \ell = 1 \ldots 2N$

$$[B] = [\lambda_\ell\, C_{J\ell} \left\langle R_{mJ}, R_{mJ} \right\rangle]$$

The components of [A] and [B] are known from the previous solution of the eigenvalue problem. Premultiplying by the inverse of the Matrix in Equation 72 will then give expressions for the sheared flow modal coefficient, b_ℓ.

$$b_\ell = \begin{bmatrix} [A] \\ \\ [B] \end{bmatrix}^{-1} \begin{Bmatrix} \left\langle\left\langle g, R_{mJ} \right\rangle\right\rangle_{J=1..N} \\ \\ \left\langle\left\langle \frac{iH}{k}, R_{mJ} \right\rangle\right\rangle_{J=1..N} \end{Bmatrix} \tag{73}$$

e. Discussion

Methods for evaluating the acoustic modal coefficients have been developed for the case of a cylindrical duct with a sheared mean flow. Extension of these methods to include rectangular and annular ducts with sheared flow is straightforward, although generating the adjoint vector in the adjoint method will become more complex because of the additional sheared flow region near the second wall.

The adjoint method of evaluating the modal coefficients is exact and allows each modal value to be evaluated separately, but it requires that the axial eigenvalues and radial pressure distributions for the sheared flow to be known. The Galerkin Method gives approximate values for the modal coefficients and requires only the no flow modal pressure distributions as input, but, as pointed out by Unruh and Eversman [17] and Yurkovich [18], there are limitations on the application of the method.

As stated, axial eigenvalues are needed to implement the exact adjoint method of evaluating the modal coefficients. Usually each eigenvalue is associated with a "forward" or a "backward" traveling wave and it is generally accepted that there are an infinite number of axial eigenvalues corresponding to each direction. This interpretation works out well in a duct with a uniform flow where only two arbitrary conditions ($P(r)$, and $u(r)$ or $\frac{\partial P}{\partial z}(r)$) may be specified across the duct. Each infinite set of uniform flow eigenvalues, corresponding to either the forward or the backward direction may be thought of as generating an arbitrary function; hence $P(r)$ or $\frac{\partial P}{\partial z}(r)$, $(u(r))$. Thus, two infinite sets of eigenvalues were necessary. In a duct with sheared flow the situation is a little different.

If the duct has a uniform flow region, then only two conditions ($P(r)$, $\frac{\partial P}{\partial z}(r)$ or $u(r)$) may be specified in that region; but in the sheared flow region, since the partial differential equation is third order in the axial direction (z), a third condition must be specified in that region. That third condition can be $v(r)$ (transverse acoustic velocity), but the $v(r)$ must be compatible with the wall boundary condition (P/v) and the uniform flow interface with the sheared flow. Thus $v(r)$ is arbitrary in the sheared flow region, but not in the uniform flow region. Difficulty now arises in trying to explain three arbitrary functions (namely $P(r)$, $\frac{\partial P}{\partial z}(r)$, $v(r)$) with only two infinite sets of eignefunctions in the sheared flow region. Further investigation into this apparent problem is continuing.

The eigenvalues which were discarded and called invalid in the Galerkin Method [17] are being looked at more closely and the criteria for calling these eigenvalues invalid is being reviewed. It is apparent also that the approximate method (Galerkin's) of evaluating the modal coefficients is not quite compatible with the adjoint method since the former neglects the radial acoustic velocity in the sheared region entirely. It is believed that this discrepancy will be resolved when the problem in the sheared flow region is solved.

When the modal coefficients are found, the conditions everywhere in the duct will be known. The next logical step would be to analyze the effects of sheared flow on a multisegmented liner's performance as fone by Zorumski [10] for a uniform flow.

The only added feature of the sheared flow would be making the radial acoustic velocity in the sheared region continuous across the impedance discontinuity. Some preliminary results are that the acoustic pressure and the radial acoustic velocity must both be zero at the wall at the discontinuity. The actual effects of the sheared flow on the liners performance remains as a major area in which work is to be done.

f. Summary

Two methods are developed for finding the sheared flow acoustic modal coefficients. An exact method uses an adjoint solution to isolate the coefficients; an approximate technique makes use of Galerkin's Method in finding the coefficients. Several problems must be resolved before either method can be used to analyze the performance of multi-segmented liners.

SECTION V
LINER ANALYSIS

Several programs have been developed to permit the design of liners with a given acoustic impedance and to ascertain the acoustic impedance of liners of a given design. The perforated plate lining programs LINPART and LINFREQ are based on papers by Nelsen, et al [23]. These programs are incorporated as subroutines where needed in our major programs. Listings of these programs are in Appendix E.

A design program for two layer liners based on a random walk technique for finding local optimums is detailed in Hoffman's [4] work.

SECTION VI
COMPARISON OF AVAILABLE DUCT ATTENUATION PREDICTIONS

A comparison of attenuation prediction techniques was conducted. Techniques compared were 1) Least Attenuated Mode attenuation as computed using OPTSHE, the lining optimization program developed under Phase I of this contract, 2) Plane Wave Propagation from either Rice [11] or Quinn [24], with optimum uniform or optimum three-section liners respectively, 3) Equal Energy in each propagating mode with a uniform liner as computed using the interactive version of OPTSHE, and 4) Two semi-emperical approaches, DUCTEC, a Pratt & Whitney program, and BOEING, a Boeing program based on a report by Dunn and Peart [25].

Comparison of Peak *Attenuation per* L/H versus Eta (Rf/c) is shown in Figure 2 for the no flow, L/H = 2, cylindrical duct. Results for the finite difference method (3-sectional liner) were direct output of program DUCT with optimum impedence as input. DUCTECH and Least Attenuated Mode results were also direct outputs of programs DUCTECH and OPTSHE.

Equal Energy results were indirectly the output of program OPTSHE. At a particular eta, energy loss for each propagating mode was determined by OPTSHE output using the optimum impedence for the least attenuated mode as the optimum impedence for the combined modes. The individual energies were then summed and equalized over all propagating modes by dividing total energy by the number of propagating modes. The equalized energy loss was then used to calculate sound level attenuation per propagating mode. Data for the Equal Energy curve is shown in Table G. 1, Appendix G.

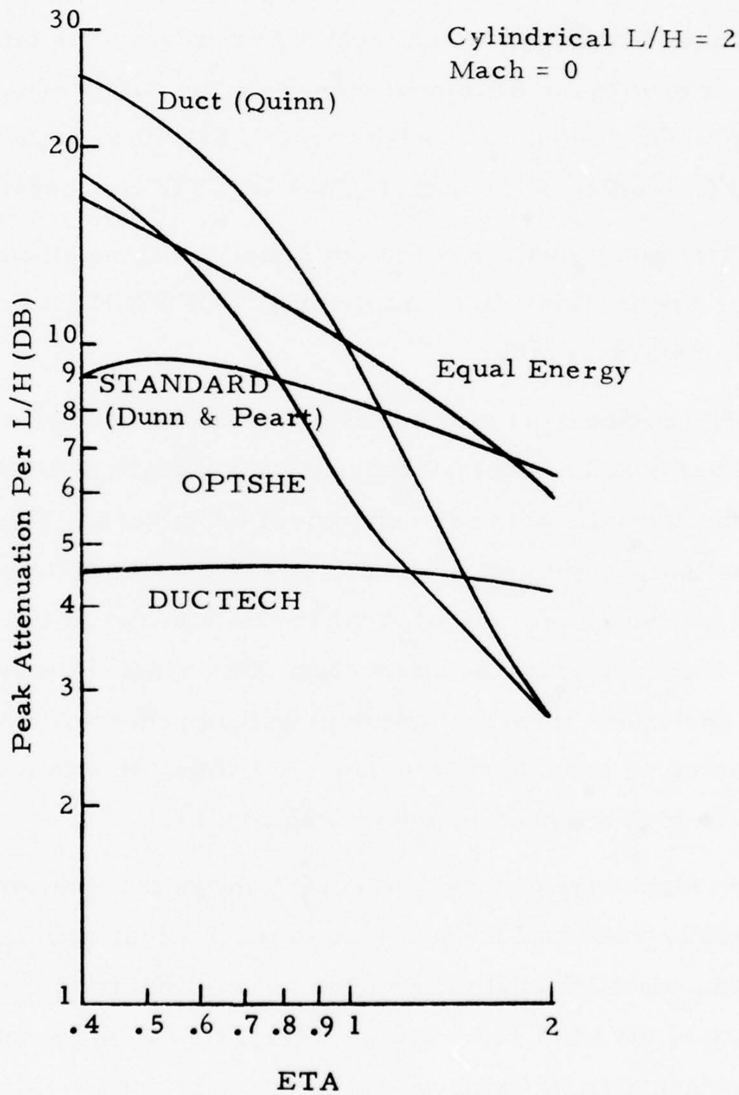


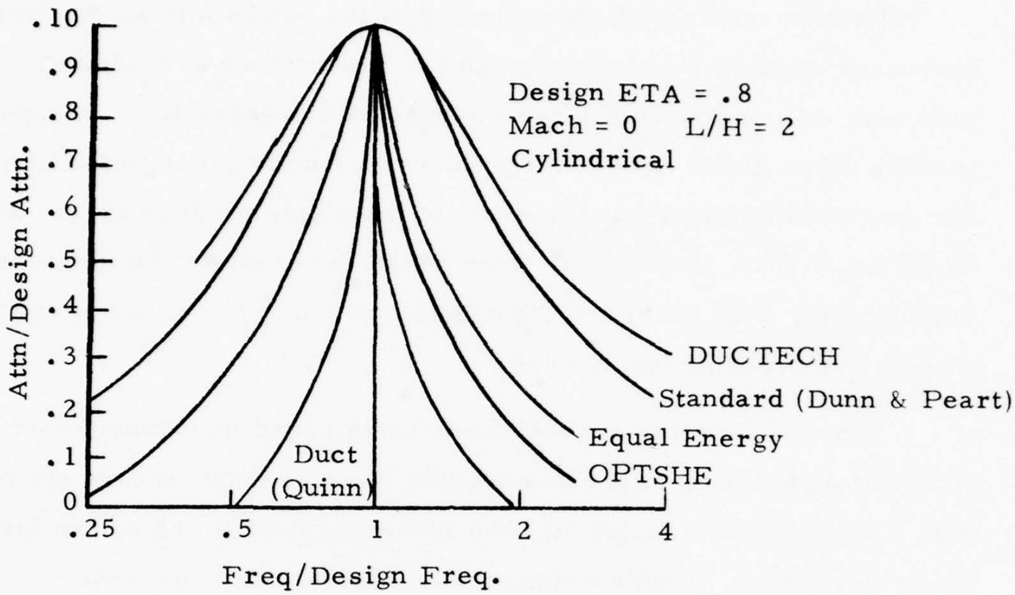
Figure 2. Comparison Peak Attenuation vs. ETA

Lining Attenuation Spectra comparisons for the no flow, L/H = 2, cylindrical duct at design Eta's of .8 and 2.9 are shown in Figure 3a and 3b, respectively. Wall designs for the Least Attenuated Mode, Finite Difference (single liner), and Equal Energy methods at each design Eta were performed through program LINPAR with corresponding optimum wall impedences as input. For frequencies different than the design frequency, wall impedences were the output of program LINFREQ with optimum design parameters from LINPAR as input. Spectra by the DUCTECH method were output of the DUCTECH program.

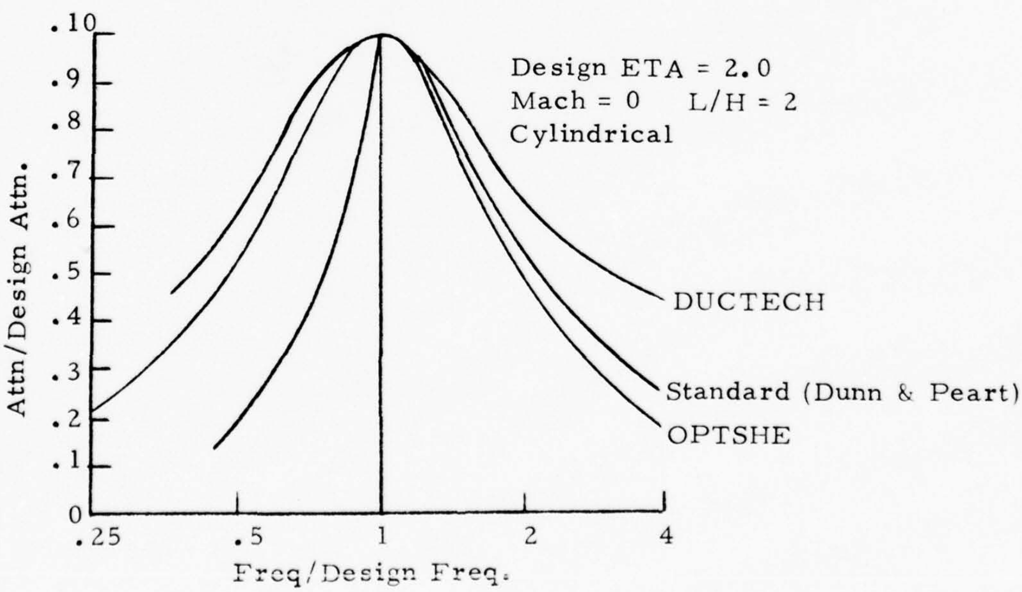
Data and impedences for the Equal Energy method at design Eta = .8 are in Table G. 1, Appendix G. OPTSHE data can be read directly from this table.

The comparison of the peak attenuations in Figure 3a shows some interesting trends. At low frequency the analytical attenuation peaks were substantially above the empirical techniques. This would seem to indicate that, while high attenuations are theoretically possible, the narrow bandwidth of the analytical methods shown in the low frequency half of Figure 3 make the attenuation of an actual liner very sensitive to any deviations from the optimum wall impedance. That is, practical manufacturing considerations make it difficult to attain the analytically predicted high attenuation at low frequencies.

At higher frequencies, Figure 3 shows that the band width of the analytically predicted attenuations is much wider than it is at low frequencies, although still not as wide as the emperical band widths. This widening of the band width can be interpreted to mean that the sensitivity of the attenuation to lining wall impedance is donw. Consequently, manufacturing tolerances would not be expected to have the devastating effects that they had at low frequencies. This means that the actual linings should approach the theoretical attenuation.



(a)



(b)

Figure 3. Comparison Lining Attenuation Spectra

Simutaneously with the widening of the band width as the frequency increases another phenomena occurs. The number of modes that propagate with only reasonably large attenuation increases too. It appears that the slope of the equal energy curve is much nearer to the slope of the empirical curves that the slope of the single mode curves. This would mean that, in the ducts from which the empirical approaches derived their data base, the distribution of energy over the propagating modes was probably fairly even.

The equal energy curves shown were based on a single duct lining. A higher attenuation might be expected from an equal energy approach with a multisegment lining like the three segment lining shown for a plane wave input. Studies along these lines are in progress.

SECTION VII

ACOUSTIC LINING SURFACE ROUGHNESS

Evaluation of the effects of acoustical linings on aircraft performance requires an accounting for the possible changes in weight, size and surface roughnesses caused by the acoustical treatment. Methods of accounting for weight changes and size changes are inherent in most design procedures. The design procedures, however, usually evaluate losses caused by skin friction based on a smooth surface assumption. The results presented here permit the smooth surface capability to be extended to acoustical type rough surfaces with minimum impact on the established design procedures. The extention to rough surfaces follows a form adapted from Pratt and Whitney [26] using data obtained by Boeing [27]. A more complete description of the development is in Schauer [5].

The correlation presented is a very simplified version of the Boeing [27] technique for computing the effects of surface roughness on skin friction coefficient. Its advantage is that it does not necessarily depend on other computer programs. That is, it fits well with standard design methods for smooth surfaces. The Boeing data on which the correlation is based is relatively independent of Mach number when put into the form $(C_{FR} - C_F)/C_F$. Although the data is based on average skin friction coefficient for an entire plate, it should apply to local values of skin friction coefficient as well. This follows from the usual relationship for turbulent flow over a smooth flat plate where the local friction coefficient is simply a constant times the average friction coefficient.

SECTION VIII

DISCUSSION AND RECOMMENDATIONS

1. PHASE I - SOUND TRANSMISSION PREDICTION

Schematically the problem of sound transmission prediction is of the form shown in Figure 4 consisting of (1) a definition of the acoustic source, (2) the attenuating effects of ducting, and (3) radiation to the far field from the ducting ends. All three of these areas deserve consideration.

First let's consider the acoustic source. The work of Lordi and Homicz [28] is certainly a step in the right direction. They are predicting analytically the radial distribution of pressure and axial velocities at a plane behind a fan. This is just the information required as input to duct attenuation programs. The prediction, however, involves an assumption of the duct around the fan and also the duct extending from the plane. Consequently, there is a coupling of the two problems, that of predicting the input to the duct and that of designing the duct to have a minimal output. From our viewpoint, the problem of how the source changes as the duct changes has not been adequately specified. This is perhaps of more importance than the prediction of the initial conditions analytically because in an actual design the initial conditions can be measured with a known ducting as reported by Kraft [20]. An analytical, or empirical, specification of how the initial conditions change with ducting is desirable since an analytical ducting design is the objective of sound transmission prediction work. It is recommended that future work consider this coupling of the initial conditions with the duct design.

Now let's consider the problem of radiation from the duct ends, leaving the duct attenuation consideration until last. It is not difficult to use the analysis of Tyler and Sofrin [29] to find the far field radiation

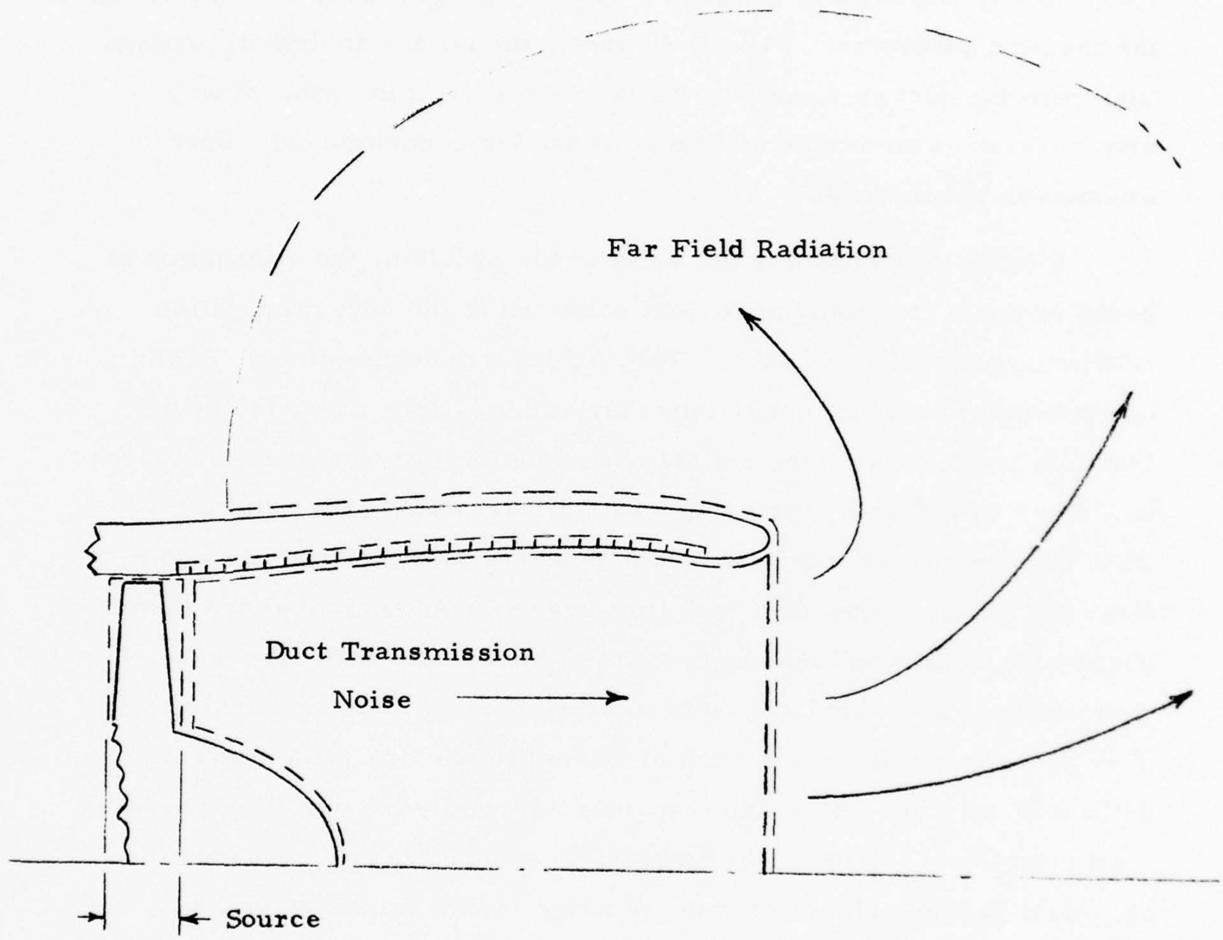


Figure 4. Schematic of Aircraft Noise Problem

associated with the pressure fluctuations at the duct exit plane. The only strong assumptions involved are neglecting the effects of the mean flow velocity outside the exit on the acoustic propagation, and neglecting the forward quadrants. With these assumptions, the analytical propagation from the duct exit plane to the far field is well in hand. A way around these assumptions will be discussed in connection with duct attenuation predictions.

Finally let's consider the heart of the problem, the attenuation of sound in the duct. Many analytical solutions to the duct propagation problem are readily available. They all involve assumptions. From our viewpoint the most noticeable shortcomings are, first, the best treatment of the exit plane reflections neglects flow, and second, there is a clear weakness in analyzing a varable area duct with a mean flow. That the flow causes significant effects on the lining design is evident from the constant area duct with flow results. An analytical and numerical procedure seems clear at this time which would eliminate these shortcomings and simultaneously improve the exit radiation analysis. This procedure follows the work of Quinn [30] in analyzing variable ducts with no flow. Here Quinn maps conformally the varable area into a rectangular area, using the mapping to change the no flow acoustic equations in the variable area coordinates to new equations in the rectangular coordinates. The new equations in rectangular coordinates in the rectangular region are solved by finite difference procedures. A preliminary study indicates it is feasible to map the flow acoustic equations in a similar manner. Further, the study shows it seems feasible to map a portion of the region outside the duct exit into the rectangular region too. This eliminates the difficulties at the exit plane and also incorporates mean flow into the radiation analysis in the region close to the exit where it is most significant. Our recommendation, then, is that this promising approach be emphasized in future work.

Still considering the attenuation of sound in a duct, another approach which is promising is the utilization of the sheared flow modal coefficient isolation developed in Section IV into an operating attenuation program in a multisection duct. That is, the sheared flow orthogonality now must be utilized in a manner analogus to the utilization of the uniform flow orthogonality in the multisection duct program CO.

In conclusion, the present status of Phase I - Sound Transmission Prediction is adequate to support the initial version of Phases II and III - Development and Implimentation of a Performance/Noise Trade Methodology. Improvement in prediction is possible and desirable to permit improvement in the initial methodology.

2. PHASE II - PERFORMANCE/NOISE TRADE METHODOLOGY

The general scheme developed by AFAPL for performing Performance/Noise Trades seems to be completely satisfactory at this time. The sound transmission work developed in Phase I has not been inserted directly into the noise analysis portion of the methodology. The comparison studies of Section VI show how the noise analysis would be changed by inclusion of an analytical duct analysis instead of the emperical duct analysis presently incorporated in the noise analysis.

The Phase I technology developed here fits into the Performance/Noise Methodology in two ways. First, it provides a possible correction to the empirical noise analysis. Second, and more important, it provides the design information necessary to assess the pressure recovery effects, weight and size penalties associated with the noise improvement. This degree of integration between the Phase I work and the Performance/Noise Trade Methodology seems adequate at the present time. It is recommended that the methodology be used in its present form to gain experience before changes are incorporated.

3. PHASE III - TRADE STUDY VERIFICATION

The Performance/Noise Trade procedure is in the process of being applied to the Air Force AMST by AFAPL and ASD personnel. A comparison of the predicted noise and performance with the actual noise and performance cannot be made at this time. Preparation for correction of differences between actual and predicted data is represented by both the capabilities developed under Phase I and the comparisons of prediction techniques of Section VI. Progress in the application of the Performance/Noise Methodology might be facilitated by giving future contractors more responsibility in this area.

4. SUMMARY OF RECOMMENDATIONS

Future work might be guided by the following recommendations:

1) The sheared flow orthogonality should be developed from its present status as a theoretical development to an operational program.

2) The variable area duct with flow analysis should receive attention because it promises to circumvent several analytical difficulties in the current prediction methods.

3) The Performance/Noise Trade Methodology should be implimented in its present form before changes are made.

4) Future contractors might be given more responsibility in the application of the Performance/Noise Trade Methodology.

APPENDIX A

PROGRAM OPTSHE

A. 1 PROGRAM LISTING

```
      PROGRAM OPTSHE (INPUT, OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION X(50),Y(50),S(50),SEOLD(50)
      DIMENSION NDRL(20),DRLD(20)
      COMPLEX KZ
      REAL K,NU
      COMMON /BLK1/ FM,ETA,PI
      COMMON /BLKA/ SSE,TTF,DSH,H,SM
      COMMON /BLKB/ IDUCT,ROUT,K,KZ
      COMMON /BLKC/ NSOFT,XSTART,NIT
      COMMON /BLKD/ HA,DSHA,RM
      COMMON /BLKE/ US,NU,AL,BL,C
C  THE FOLLOWING DATA ARE THE DRILL NUMBERS AND THEIR DIAMETERS FOR
C  SELECTING POSSIBLE OPTIMUM LININGS
      DATA (NDRL(I),I=1,7) /60,54,50,46,41,35,24/
      DATA (DRLD(I),I=1,7) /.0400,.0550,.0700,.0810,.0960,.1100,.1520/
C  A LINING COVER SHEET OF THE FOLLOWING THICKNESS IN INCHES WILL BE
C  USED FOR CALCULATING POSSIBLE OPTIMUM LININGS
      DATA TH /.040/
C************************************************************************
C  IDUCT= 1          RECTANGULAR DUCT
C  IDUCT= 0          CYLINDRICAL DUCT OR ANNULAR DUCT WITH ONLY THE OUTSIDE
C                    WALL SOFT
C  IDUCT=-1          ANNULAR DUCT WITH A SOFT INSIDE WALL OR TWO SOFT WALLS
C  NSOFT=1 OR 2      GIVES THE NUMBER OF SOFT WALLS
C  INTRP=0          NORMAL RUN
C  INTRP=1          COMPLETE DIAGNOSTIC TO BE OUTPUT
C************************************************************************
      PI = 3.1415926536
    1 READ (5,100) FM,FREQ,DSH,H,ROUT,IDUCT,NSOFT,INTRP
C
C  INPUT PARAMETERS
C  FM=MACH NO. (+ EXHAUST, - INLET)
C  FREQ=FREQUENCY WITH WHICH THE LINING IS OPTIMIZED-HZ
C  DSH=SHEAR LAYER THICKNESS/H
C  H=DUCT HEIGHT FOR A RECTANGULAR DUCT-INCHES
C   =RADIUS FOR A CYLINDRICAL DUCT-INCHES
C   =ROUT-RIN FOR AN ANNULAR DUCT-INCHES
C  TEMP=AVERAGE DUCT TEMPERATURE-RANKINE
C  PRESS=AVERAGE DUCT PRESSURE-PSI
C  RHO=AVERAGE DUCT DENSITY-LBS MASS/FT**3
C
      IF (EOF(5).NE.0.) GO TO 22
  100 FORMAT (5F10.5,3I10)
      READ (5,110) TEMP,PRESS,RHO
```

```
110 FORMAT (5F10.5)
    WRITE (6,100)
100 FORMAT (1H1,30X,*COMPUTATION OF OPTIMUM DUCT LININGS*//)
    NCON=0
    ICON=0
    NIT=0
    NITM=6
    RSIN=+.
    IF (IDUCT) 33,3,2
  2 WRITE (6,104) NSOFT
104 FORMAT (1X,*RECTANGULAR DUCT WITH*I2* SOFT WALL(S)*)
    GO TO 4
  3 RI=ROUT-H
    IF (RI.GE..01) WRITE (6,105) ROUT,RI
105 FORMAT (1X,F7.4,* INCHES OD/2*,F12.4,* INCHES ID/2 FOR AN ANNULAR
   +DUCT WITH ONLY THE OUTER WALL SOFT*)
    IF (RI.LT..01) WRITE (6,109) ROUT
109 FORMAT (1X,*CYLINDRICAL DUCT WITH A RADIUS OF*F7.3* INCHES*)
    GO TO 4
 33 RI=ROUT-H
    IF (NSOFT-1) 26,26,27
 26 WRITE (6,106) ROUT,RI
106 FORMAT (* *F7.4,* INCHES OD/2*,F12.4,* INCHES ID/2 FOR AN ANNULAR
   XDUCT WITH ONLY THE INNER WALL SOFT*)
    GO TO 4
 27 WRITE (6,107) ROUT,RI
107 FORMAT (* *F7.4,* INCHES OD/2*,F12.4,* INCHES ID/2 FOR AN ANNULAR
   XDUCT WITH BOTH INNER AND OUTER WALLS SOFT*)
  4 CONTINUE
    WRITE (6,103) H,DSH
103 FORMAT (1X,*H=*,F5.4,* INCHES*/,1X,*D/H=*,F6.4* DELTA/H*)
    IF ((NSOFT.EQ.2).AND.(IDUCT.EQ.1)) H=H/2.
    IF ((NSOFT.EQ.2).AND.(IDUCT.EQ.1)) DSH=2.*DSH
    C=49.02*12.*SQRT(TEM )
    K=2.*PI*FREQ/C
    ETA=2.*H*FREQ/C
    WRITE (6,101) FM,ETA
101 FORMAT  (1X,F5.3,8H MACH NO,10X,F5.3,8H FREQ PAR)
    IF ((IDUCT.EQ.0).AND.((ROUT-H).EQ.0.)) GO TO 64
    IF (IDUCT.EQ.-1) GO TO 65
    SEA=.22/ETA
    SEL=.2/SQRT(ETA)
    SEH=.27/(ETA*ETA)
    IF (ETA.LT..7) SEH=.39/ETA
    IF ((FM.GT.0.).AND.(ETA.LT.1.)) SEA=SEA-(SEA-SEL)*2.*FM
    IF ((FM.LT.0.).AND.(ETA.LT.1.)) SEA=SEA-FM*2.*(SEH-SEA)
    IF (IDUCT) 65,64,65
 65 ETAP=ETA
    P=.5-.15*(1.-RI/ROUT)
    RM=H*P+RI
    IF (NSOFT.EQ.2) ETAP=ETA*(RM-RI)/(ROUT-RI)
    SEA=.22/ETAP
    IF (ETAP.GT.1.) GO TO 66
    SEL=(.22/SQRT(ETAP))*(RI/ROUT)**.2
    IF (NSOFT.EQ.2) SEL=SEL*(ROUT/RM)**.2
```

```
      SEH=.27/(ETAP*ETAP)
      IF (ETAP.LT..7) SEH=.38/ETAP
      IF(ETAP.LT..4) SEH=1.0
      IF (FM.LT.0.) SEA=SEA-(SEH-SEA)*2.*FM
      IF (FM.GT.0.) SEA=SEA-(SEA-SEL)*2.*FM
      GO TO 65
   64 SED=.47*ETA**(-1.15)
      IF (ETA.LT..5) SED=.7*ETA**(-.25)
      SEL=.34*ETA**(-.64)
      SEH=.55/(ETA*ETA)
      IF (ETA.LT..8) SEH=.65/ETA
      IF (ETA.LT..5) SEH=.92*ETA**(-.38)
      IF ((FM.GT.0.).AND.(ETA.LT.2.)) SED=SED-(SED-SEL)*2.*FM
      IF ((FM.LT.0.).AND.(ETA.LT.1.2)) SED=SED-(SEH-SED)*2.*FM
      IF (RI.EQ.0.) SEA=SED
      IF (RI.EQ.0.) GO TO 65
      SEA=SED-1.035*(SED-SEA)*(1.-EXP(-3.358*RI/ROUT))
   66 SEL=SEU=SEA
      SEL=SEU=SEA
      IF (IDUCT) 61,62,63
   63 TE=ETA+.15/ETA
      IF (FM) 53,54,54
   53 TEH=.85
      IF (ETA.GT..5) TEH=1.75*ETA**1.15
      IF (ETA.GE.2.5) TEH=2.*ETA
      FME=SQRT(1.+2.*FM)
      TE=TEH-(TEH-TE)*FME
      TEF=TE*.95*(1.-EXP(-ETA*ETA))
      IF (ETA.GT.4.) TEF=TEF*(1.+.02*(1.-EXP(-(ETA-4.)*(ETA-3.))))
      TEL=TE*(1.1+EXP(-ETA*ETA))
      IF (ETA.GT.4.) TEL=TEL*(1.-.07*(1.-EXP(-(ETA-4.)*(ETA-3.))))
      IF (ETA.LT.1.) TEL=1.47*TE/ETA
      GO TO 55
   54 FME=1.-2.*FM
      IF (ETA.LT.2.) FME=FME*FME
      TEH=.55*ETA**1.3
      IF (ETA.GT.1.5) TEH=.6*ETA**1.065
      TED=.45*ETA**1.3
      IF ((IDUCT.EQ.0).AND.(TED.LT.TEH)) TEH=TED+(RI/ROUT)*(TEH-TED)
      TE=TEH+(TE-TEH)*FME
      TEF=TE*.95*(1.-EXP(-TE*TE))
      IF (IDUCT.EQ.1.AND.ETA.LT.1.5) TEF=.5*TEF
      IF ((IDUCT.EQ.0).AND.(ETA.LT..7).AND.(FM.GT..1)) TEF=.2*TEF
      TEL=TE*(1.1+EXP(-ETA*ETA))
      IF (ETA.LT.1.) TEL =1.47*TE/ETA
      GO TO 55
   62 IF (RI.GT.(ROUT/2.)) GO TO 63
   67 TE=ETA
      IF (ETA.LT.2.5) TE=.73*ETA**1.35
      IF (ETA.LT.1.) TE=.73*ETA
      IF (ETA.LT..6) TE=.45
      IF (FM) 53,54,54
   61 IF (NSOFT.EQ.1) GO TO 67
      TE=ETAP
      IF (ETAP.LT.2.5) TE=.73*ETAP**1.35
      IF (ETAP.LT.1.) TE=.73*ETAP
```

```
      IF (ETAP.LT..5) TE=.45
      IF (FM.GE.0.) 74,73
   74 FME=1.-2.*FM
      IF (ETAP.LT.2.) FME=FME*FME
      TEH=.55*ETAP**1.3
      IF (ETAP.GT.1.5) TEH=.5*ETAP**1.065
      TEC=.45*ETAP**1.3
      IF (TEC.LT.TEH) TEH=TEC+(KI/KM)*(TEH-TEC)
      TE=TEH+(TE-TEH)*FME
      TEF=TE*.95*(1.-EXP(-TE*TE))
      IF (ETAP.LT..7.AND.FM.GT..1) TEF=.2*TEF
      TEL=TE*(1.1+EXP(-TE*TE))
      IF (ETAP.LT.1.) TEL=1.47*TE/ETAP
      GO TO 55
   73 TEH=.35
      IF (ETAP.GT..5) TEH=1.75*ETAP**1.15
      IF (ETAP.GT.2.5) TEH=2.*ETAP
      FME=SQRT(1.+2.*FM)
      TE=TEH-(TEH-TE)*FME
      U=(TE+ETAP)/2.
      TEF=TE*.96*(1.-EXP(-U*U))
      IF (ETAP.GT.4.) TEF=TEF*(1.+.02*(1.-EXP(-(ETAP-4.)*(ETAP-3.))))
      TEL=TE*(1.1+EXP(-TE*TE))
      IF (ETAP.GT.4.) TEL=TEL*(1.-.07*(1.-EXP(-(ETAP-4.)*(ETAP-3.))))
      IF (ETAP.LT.1.) TEL=1.47*TE/ETAP
   55 CONTINUE
C
C   DIVIDE THE INTERVAL IN HALF 15 TIMES
C    AND CHECK EACH TIME FOR A LOOP TO SEE WHICH HALF TO KEEP
C
      IF ((FM.EQ.0).OR.(USH.EQ.0.)) GO TO 51
      NU=144.*(.279+.001909*TEMP-2.568E-7*TEMP*TEMP)/(RHO*1.E5)
C   NU IS KINETIC VICOSITY IN UNITS OF INCHES**2/SEC
      US=SQRT(.0223*C*C*FM*FM*((C*ABS(FM)*USH*H/NU)**(-.25)))
      AL=5.*NU/(H*US)
      BL=6.*AL
   51 CONTINUE
      SKIP=0.
      NOK=0
      NOL=0
      NC=0
      NLOOP=0
      NCUSP=0
      IST=0
      JJJ=15
      IF (IOUCT.EQ.-1.AND.NSOFT.EQ.2) JJJ=5
      DO 15 J=1,JJJ
      SE=(SEL+SEU)/2.
   18 NP=20
      IF (NOK.EQ.0.AND.NCUSP.EQ.0) NP=40
      IF (NIT.GE.2) NP=20
      IF (ETA.LT.2.) NP=20
      IF (NCUSP.GT.10) GO TO 34
      DELN=(ALOG(TEL)-ALOG(TEF))/(NP-1)
      DO 20 I=1,NP
      TE=EXP(ALOG(TEF)+(I-1)*DELN)
```

```
      IW=0
      CALL SHESET (SE,TE,RS,XS,IW)
      X(I) = RS
      Y(I) = XS
   20 CONTINUE
      CALL LOOP3 (X,Y,NP,LOOP,I,N)
      IF(INTRP.EQ.1) WRITE (6,102)
  102 FORMAT (/16H (R STAR,X STAR))
      IF(INTRP.EQ.1) WRITE (6,501) (X(IW),Y(IW),IW=1,NP)
  501 FORMAT (3(1X,5(*(*F7.4,*,*F8.4,*)*3X)/))
      IF(INTRP.EQ.1) WRITE (6,502) SE,SEL,SEU,TEF,TEL,LOOP
  502 FORMAT (1X,*SE,SEL,SEU,TEF,TEL,LOOP*5F12.6,I10)
      IF (LOOP.GE.1) 13,14
   13 NLOOP = NLOOP+1
      NCK=NCK+1
      IF (SKIP.EQ.1.) GO TO 19
      SELA=SEL
   19 SKIP=0.
      SEL=SE
      B=TEF
      IF (N.LT.NP) TEL=EXP(ALOG(B)+(N+1)*DELN)
      IF (I.GE.3) TEF=EXP(ALOG(B)+(I-2)*DELN)
      IF (NIT.EQ.1) GO TO 75
      IF (NCON.EQ.(NIT-1)) ICON=1
      IF (NLOOP.EQ.2) NCON=NIT
      IF (NIT.GE.2.AND.NCK.EQ.2.AND.ICON.EQ.0) GO TO 75
      IF (NIT.GE.2.AND.NCK.EQ.4) GO TO 75
      IF (NOL.EQ.0.AND.NCUSP.EQ.0.AND.NIT.GT.0) SEU=1.02*SE
      IF (NOL.EQ.0.AND.NCUSP.EQ.0.AND.NIT.EQ.0) SEU=1.1*SE
      IF (NOL.EQ.0.AND.NLOOP.GT.3) SEU=(1.+.03*NLOOP)*SE
      IF (NLOOP-2) 15,16,17
C
C  IF WE LOOP TWICE CHECK UB, IF LOOP AGAIN CHANGE UB
C
   16 SE=SEU
      GO TO 18
   17 SEU=SEU+.1*(SEU-SELA)
      IF (NLOOP.GT.5) SELA=.9*SELA
      SKIP=1.
      SE=SEU
      GO TO 18
   14 NOL=NOL+1
      IST=IST+1
      NLOOP = 0
      IF (NCK) 49,49,37
   49 IF((ETA/NSOFT).LT.2..OR.NIT.GE.2) 46,36
   37 SEU=SE
      GO TO 15
   36 V=.02/(1.+.5*NIT)
      VV=.025/(1.+.5*NIT)
      NC=NC+1
      SEOLD(NC)=SE
      SE=(1.+(-1)**NC*V*NC)*SEA
      IF (NC.GE.7) SE=(1.+(-1)**NC*VV*NC)*SEA
      IF (NC.EQ.14) SE=1.45*SEA
      IF (NC.EQ.16) SE=1.7*SEA
      IF (NC.EQ.18) SE=2.0*SEA
      IF (NC.EQ.20) SE=2.5*SEA
```

```
      SEU=SE*(1.+.05/(.5+NIT))
      SEL=SE*(1.-.05/(.5+NIT))
      IF(NC.GE.3) SEU=SEUOLD(NC-1)
      IF(NC.GE.3) SEL=SELOLD(NC-1)
      IF (NC.GT.20) GO TO 34
      GO TO 18
   45 IUP=1
      IF (IDUCT.EQ.-1.AND.MOD(NIT,2).EQ.0.AND.(ETA/NSOFT).LT..85) IUP=0
      IF(IDUCT.EQ.1.AND.ETA.LT.1.5) IUP=0
      IF(IDUCT.EQ.-1.AND.MOD(NIT,2).EQ.1.AND.ETAP.LT..4) IUP=1
      IF(IUP.EQ.1) CALL INSP (Y,X,NP,ICUSP,NOL,IM,XM)
      IF(IUP.EQ.0) CALL INSP (X,Y,NP,ICUSP,NOL,IM,XM)
      IF (ICUSP.EQ.1) GO TO 48
      S(IST)=SE
      IF (IM.EQ.1.OR.IM.EQ.NP) NOL=NOL-1
      IF(IUP.EQ.0.AND.XM.GT.(1.2*RSIN)) GO TO 36
      IF (NCUSP.GE.1.OR.NIT.GE.2) GO TO 47
      GO TO 36
   47 IF (IM.EQ.1) TEF=EXP(ALOG(TEF)-5.*DELN)
      IF (IM.EQ.1) GO TO 18
      IF (IM.EQ.NP) TEL=EXP(ALOG(TEL)+5.*DELN)
      IF (IM.EQ.NP) GO TO 18
      IF (NCUSP.EQ.0) GO TO 36
      GO TO 44
   48 NOLM=IST-1
      IF (SE.GT.S(1)) GO TO 39
      SEL=SE
      SEU=S(1)
      IF (NOL-2) 41,41,42
   42 DO 40 M=2,NOLM
   40 IF (S(M).LT.SEU.AND.S(M).GT.SEL) SEU=S(M)
      GO TO 41
   39 SEU=SE
      SEL=S(1)
      IF (NOL-2) 41,41,43
   43 DO 50 M=2,NOLM
   50 IF (S(M).GT.SEL.AND.S(M).LT.SEU) SEL=S(M)
   41 NOL=0
      IST=0
      IF (IDUCT.EQ.-1.AND.ETA.LT..5.AND.FM.LT.-.1) GO TO 72
      IF (NIT.GT.0.AND.ETA.LT.4.) GO TO 72
      IF (NCUSP.NE.0) GO TO 71
      IF (NIT.GE.2) GO TO 71
   72 IF (IM.LT.(NP-3)) TEL=EXP(ALOG(TEL)+(IM+3)*DELN)
      IF (IM.GT.3) TEF=EXP(ALOG(TEF)+(IM-3)*DELN)
   71 NCUSP=NCUSP+1
      SE=(SEL+SEU)/2.
      IF(INTRP.EQ.1) WRITE (6,504) ICUSP,SEL,SEU
  504 FORMAT (1X,*ICUSP,SEL,SEU*I10,2F15.6)
      IF (NCUSP.EQ.1.AND.NIT.EQ.1) GO TO 75
      IF (NCUSP.EQ.2.AND.NIT.GE.2) GO TO 75
      IF(NCUSP.EQ.4) GO TO 75
      GO TO 18
   44 IF (NOL.GT.1) GO TO 45
      IF (XM.GT.5.) GO TO 31
```

```
      SE=SEL
      IF ((TEL-TEF).LT..001) GO TO 75
      GO TO 18
   45 SE=SEU
      IF ((TEL-TEF).LT..001) GO TO 75
      IF (NOL.EQ.2) GO TO 18
      IF (NOL.EQ.3) SE=SEL-(SEU-SEL)/2.
      IF (NOL.EQ.4) SE=SEU+(SEU-SEL)/2.
      IF (NOL.EQ.5) SE=SEL+SEL-SEU
      IF (NOL.EQ.6) SE=SEU+SEU-SEL
      IF (NOL.GE.7) GO TO 84
      GO TO 18
   15 CONTINUE
   75 TE=(TEF+TEL)/2.
      IW=1+INTRP
      CALL SHESET (SE,TE,RS,XS,IW)
      IWR=0
      IF(NSOFT.EQ.1.OR.NIT.GE.(NIMM-2).OR.INTRP.EQ.1) IWR=1
      IF(NSOFT.EQ.1.OR.IDUCT.EQ.1) WRITE (6,303)
  303 FORMAT (/////* OPTIMUM WALL LINING*)
      IF(NSOFT.GT.1.AND.IDUCT.NE.1.AND.MOD(NIT,2).EQ.0.AND.IWR.EQ.1)
     +WRITE (6,308)
  308 FORMAT (/////* OPTIMUM INNER WALL*)
      IF(NSOFT.GT.1.AND.IDUCT.NE.1.AND.MOD(NIT,2).NE.0.AND.IWR.EQ.1)
     +WRITE (6,309)
  309 FORMAT (/////* OPTIMUM OUTER WALL*)
      IF(IWR.EQ.1) WRITE (6,304)
  304 FORMAT (/60H        SIGMA ETA           TAU ETA       OPT R STAR       OPT
     XX STAR)
      IF(IWR.EQ.1) WRITE (6,305) SE,TE,RS,XS
  305 FORMAT (4F15.6)
      DSE=(SEU-SEL)/2.
      DTE=(TEL-TEF)/2.
      IF(IWR.EQ.1) WRITE (6,306) DSE,DTE
  306 FORMAT (*     +OR-*F8.5,*     +OR-*F8.5/)
      IF (IDUCT.EQ.-1.AND.NSOFT.EQ.2) 23,24
   23 NIT=NIT+1
      IF (NIT.EQ.1) GO TO 25
      SEA=SEL=SEU=SE
      IF (NIT.GE.2.AND.NCUSP.GE.1) GO TO 83
      IF (NIT.GE.2) GO TO 85
      TEF=TE*.95*(1.-EXP(-TE*TE))
      TEL=TE*(1.04+EXP(-TE*TE))
      GO TO 34
   83 TEF=TEF-.1*(TE-TEF)
      IF ((TE-TEF).LT..004) TEF=TE-.004
      TEL=TEL+.1*(TEL-TE)
      IF ((TEL-TE).LT..004) TEL=TE+.004
      GO TO 34
   85 TEF=TEF-.5*(TE-TEF)
      SPD=.2/(1.+2.*NIT)
      IF ((TE-TEF).LT.SPD) TEF=TE-SPD
      TEL=TEL+.5*(TEL-TE)
      IF ((TEL-TE).LT.SPD) TEL=TE+SPD
```

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84 CONTINUE
   IF(MOD(N,2).NE.0) RSIN=RS
   IF(RSIN.LT.0.) RSIN=.1
   IF (NIT.GT.(NITM-2)) 24,51
28 ETAP=ETA*(ROUT-RM)/H
   SEA=.22/ETAP
   SEL=(.22/SQRT(ETAP))*(RI/ROUT)**.2
   IF (NSOFT.EQ.2) SEL=SEL*(ROUT/RM)**.2
   SEH=.27/(ETAP*ETAP)
   IF (ETAP.LT..7) SEH=.39/ETAP
   IF (FM.LT.0.) SEA=SEA-(SEH-SEA)*2.*FM
   IF (FM.GT.0.) SEA=SEA-(SEA-SEL)*2.*FM
   SEC=.47*ETAP**(-1.15)
   IF (ETAP.LT..6) SEC=.7*ETAP**(-.25)
   SEL=.34*ETAP**(-.64)
   SEH=.55/(ETAP*ETAP)
   IF (ETAP.LT..8) SEH=.65/ETAP
   IF(ETAP.LT..5) SEH=.82*ETAP**(-.38)
   IF (FM.GT.0..AND.ETAP.LT.2.) SEC=SEC-(SEC-SEL)*2.*FM
   IF (FM.LT.0..AND.ETAP.LT.1.2) SEC=SEC-(SEH-SEC)*2.*FM
   SE2=SEC-1.035*(SEC-SEA)*(1.-EXP(-3.35*R1/ROUT))
   SEA=SEL=SEU=SE2+SE
   SEA=SEA*(R1/ROUT)**.2
   IF(ETA.LT.1.6) SEA=(1.+.19*R1/ROUT)*SEA
   SE=SEL=SEU=SEA
   TE=TE*H/(RM-R1)
   TEF=TE*.97*(1.-EXP(-ETA))
   TEL=TE*(1.03+EXP(-ETA))
   IF(INTRP.EQ.1) WRITE (6,308)
   GO TO 51
24 DBPH=20.*PI*SE/ALOG(10.)
   DBPIN=DBPH/H
   IF ((NSOFT.EQ.2).AND.(IDUCT.EQ.1)) DBPH=2.*DBPH
   WRITE (6,307) DBPH,DBPIN
307 FORMAT (* ATTENUATION PER DUCT HEIGHT =*,F5.2,* DB PER H*/* ATTENU
  UATION PER INCH =*,F5.3,* DB PER INCH*)
   WRITE (6,311)
311 FORMAT (* EACH OF THE FOLLOWING LININGS IS OPTIMUM FOR THE GIVEN
  XINPUT CONDITIONS.   SEVERAL ARE GIVEN TO PERMIT A SELECTION BY THE
  X DESIGNER.*//)
   RES=ETA*RS
   REA=ETA*XS
   WRITE (6,411) FREQ,TEMP,PRESS,RES,REA
411 FORMAT (7X,*INPUT INCLUDES*/7X,*FREQ =*F6.0* HZ*/7X,*TEMP =*F5.0*
  +RANKINE*/7X,*PRESS=*F5.1* PSI*//7X,*NORMALIZED LINING RESISTANCE=
  +*F5.4/7X,*NORMALIZED LINING REACTANCE =*F6.4//)
   WRITE (6,312)
312 FORMAT (6X, * FACE SHEET        DRILL NO.        DRILL DIA      FRACTIO
  ONAL   CORE BACKING*)
   WRITE (6,313)
313 FORMAT (6X, * THICK - IN          STD              INCHES         OPEN A
  AREA   DEPTH,INCHES*)
   DO 30 I=1,7
   CALL DL (RES,REA,TEMP,PRESS,FREQ,FM,TH,DRLD(I),IDUCT,NIT,RI,ROUT,K
  +,OA,DP)
   WRITE (6,316) TH,NDRL(I),DRLD(I),OA,DP
```

```
310 FORMAT (1X,F15.4,I15,3F15.4)
 30 CONTINUE
    IF (NIT.EQ.(NITM-1)) 51,35
 34 WRITE (6,503)
503 FORMAT (* THE ITERATIONS ON SIGMA ETA VALUES SHOW NO SIGN OF CONVE
   ERGING AFTER 20 ITERATIONS*)
    GO TO 35
 31 WRITE (6,505)
505 FORMAT (* XM IS GT 5. AT SE=(SEU+SEL)/2.*)
 35 GO TO 1
 22 CONTINUE
    END
    SUBROUTINE SHESET (SE,TE,RS,XS,IW)
    COMMON /BLK1/ FM,ETA,PI
    COMMON /BLKA/ SSE,TTE,DSH,H,SM
    COMMON /BLKB/ IDUCT,ROUT,K,KZ
    COMMON /BLKC/ NSOFT,XSTART,NIT
    COMMON /BLKD/ HA,DSHA,RM
    COMMON /BLKE/ US,NU,AL,BL,C
    COMMON /BLKF/ VM,RHS
    COMPLEX AI,ZETAS,KZ,P,PP,RHS
    REAL K,NU
    DIMENSION Y(4),YL(4)
    LOGICAL MODE
    EXTERNAL DERIV
    MODE=.FALSE.
    SSE=SE
    TTE=TE
    AI=CMPLX(0.,1.)
    N=4
    ACC=1.E-7
    IF (IDUCT) 21,22,23
 23 XSTART=0.
    SDX=1.
    GO TO 24
 22 XSTART=ROUT/H-1.
    SDX=1.
    GO TO 24
 21 IF (MOD(NIT,2).EQ.0) XSTART=ROUT/H
    IF (MOD(NIT,2).NE.0) GO TO 22
    SDX=-1.
    IF ((NSOFT.EQ.2).AND.(NIT.EQ.0)) 26,24
 26 RI=ROUT-H
    HA=RM-RI
    XSTART=RM/HA
    X=XSTART
    SDX=-1.
    DSHA=DSH*H/HA
    IF ((DSH.NE.0.).AND.(FM.NE.0.)) ALA=5.*NU/(HA*US)
    IF ((DSH.NE.0.).AND.(FM.NE.0.)) BLA=6.*ALA
    Y(1)=1.
    Y(2)=Y(3)=Y(4)=0.
    GO TO 80
 24 X=XSTART
    IF (NIT.GT.0) GO TO 27
    Y(1)=1.
```

```
      Y(2)=Y(3)=Y(4)=0.
      GO TO 80
   27 Y(1)=YL(1)
      Y(2)=YL(2)
      Y(3)=YL(3)
      Y(4)=YL(4)
      GO TO 81
   80 IF(IW.EQ.2) WRITE (6,100)
  100 FORMAT (/*                                PRESSURE                                PRESS GRAD
     1                MACH                    X                SM                          RE RHS
     2IM RHS *)
  101 FORMAT (1X,I4,9F14.7)
      DX=SDX*2.E-4
      J=1
      CALL SET (N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      IF(IW.EQ.2) WRITE (6,101) J,Y,VM,X,SM,RHS
   82 CALL STEP (N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      J=J+1
      IF (J.GT.1000) GO TO 31
      IF(IW.EQ.2.AND.MOD(J,20).EQ.0) WRITE (6,101) J,Y,VM,X,SM,RHS
      IF ((1.-ABS(X-XSTART)).GT.ABS(DX)) GO TO 82
      DX=SDX*(1.-ABS(X-XSTART))
      CALL SET (N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      CALL STEP (N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      IF (IDUCT.EQ.-1.AND.NSOFT.EQ.2.AND.NIT.EQ.0) Y(3)=Y(3)*H/HA
      IF (IDUCT.EQ.-1.AND.NSOFT.EQ.2.AND.NIT.EQ.0) Y(4)=Y(4)*H/HA
      IF(IW.EQ.2) WRITE (6,101) J,Y,VM,X,SM,RHS
   31 IF(IW.EQ.0) GO TO 3
      IF (IDUCT.NE.-1.OR.NSOFT.NE.2) GO TO 3
      YL(1)=Y(1)
      YL(2)=Y(2)
      YL(3)=Y(3)
      YL(4)=Y(4)
    3 P=CMPLX(Y(1),Y(2))
      PP=CMPLX(Y(3),Y(4))
      ZETAS=-AI*PI*P/PP
      IF (DSH.EQ.0.) ZETAS=ZETAS*(1.-FM*KZ/K)**2
      IF ((IDUCT.EQ.-1).AND.(MOD(NIT,2).EQ.0)) ZETAS=-ZETAS
      RS=REAL(ZETAS)
      XS=AIMAG(ZETAS)
      RETURN
      END
      SUBROUTINE DERIV (N,X,Y,DY)
      COMMON /BLK1/ FM,ETA,PI
      COMMON /BLKA/ SE,TE,DSH,H,SM
      COMMON /BLKB/ IDUCT,RDUT,K,KZ
      COMMON /BLKC/ NSOFT,XSTART,NIT
      COMMON /BLKD/ HA,DSHA,RM
      COMMON /BLKE/ US,NU,AL,BL,C
      COMMON /BLKF/ VM,RHS
      COMPLEX A,B,RHS,P,PP,KZ
      REAL K,NU
      DIMENSION Y(N),DY(N)
      AKZ=K*TE/ETA
      BKZ=-K*SE/ETA
```

```
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) AKZ=H*AKZ/HA
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) BKZ=H*BKZ/HA
      KZ=CMPLX(AKZ,BKZ)
      IF ((FM.EQ.0.).OR.(DSH.EQ.0.)) GO TO 12
      SIGN=1.
      IF (FM.LT.0.) SIGN=-1.
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) 31,32
   31 DB=X-(XSTART-1.)
      IF (DB.GE.DSHA) GO TO 12
      ALA=5.*NU/(HA*US)
      BLA=5.*ALA
      YP=DB*HA*US/NU
      Q=.407*YP
      IF (YP.GT.5) Q=.407*(-3.05+5.*ALOG(YP))
      IF (YP.GT.30) Q=.407*(5.5+2.5*ALOG(YP))
      SQ=1./.407+.1*(EXP(Q)-1.-Q-Q*Q/2.-Q*Q*Q/6.)
      SM=SIGN*HA*US*US/(U*NU*.407*SQ)
      E=1.-DB/DSHA
      IF (E.LT..1) SM=SM*10.*E
      GO TO 17
   32 DA=ABS(X-XSTART)
      DB=(XSTART+1.-X)
      IF (DB.GT.1.) DB=X-(XSTART-1.)
      IF ((DA.GE.DSH).AND.(DB.GE.DSH)) GO TO 12
      IF ((DA.LT.DSH).AND.(NSOFT.EQ.2).AND.(IDUCT.EQ.1)) GO TO 12
      IF ((DA.LT.DSH).AND.(XSTART.EQ.0.).AND.(IDUCT.EQ.0)) GO TO 12
      YP=DA*H*US/NU
      IF (DB.LT.DA) YP=DB*YP/DA
      Q=.407*YP
      IF (YP.GT.5) Q=.407*(-3.05+5.*ALOG(YP))
      IF (YP.GT.30) Q=.407*(5.5+2.5*ALOG(YP))
      SQ=1./.407+.1*(EXP(Q)-1.-Q-Q*Q/2.-Q*Q*Q/6.)
      SM=SIGN*H*US*US/(U*NU*.407*SQ)
      E=1.-DA/DSH
      IF (DB.LT.DA) E=1.-DB/DSH
      IF (E.LT..1) SM=SM*10.*E
      IF (DB.LT.DA) SM=-SM
      IF ((IDUCT.EQ.-1).AND.(MOD(NIT,2).EQ.0)) SM=-SM
      GO TO 17
   12 SM=0.
   17 CONTINUE
      CALL MACH (FM,VM,DA,DB,ALA,BLA)
      A=2*KZ*SM/(K-VM*KZ)
      IF (IDUCT) 22,22,21
   22 IF (X.GT.0.) A=A+1./X
   21 B=H*H*(K*K-KZ*KZ*(1.-VM*VM)-2.*VM*K*KZ)
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) B=HA*HA*B/(H*H)
      P =CMPLX(Y(1),Y(2))
      PP=CMPLX(Y(3),Y(4))
      RHS=-A*PP-B*P
      DY(1)=Y(3)
      DY(2)=Y(4)
      DY(3)=REAL(RHS)
      DY(4)=AIMAG(RHS)
      RETURN
      END
      SUBROUTINE MACH (FM,VM,DA,DB,ALA,BLA)
```

```
      COMMON /BLKA/ SE,TE,DSH,H,SM
      COMMON /BLKB/ IDUCT,ROUT,K,KZ
      COMMON /BLKC/ NSOFT,XSTART,NIT
      COMMON /BLKD/ HA,DSHA,RM
      COMMON /BLKE/ US,NU,AL,BL,C
      REAL K,NU
      IF (DSH) 1,1,2
    1 VM=FM
      GO TO 10
    2 SIGN=1.
      IF (FM.LT.0.) SIGN=-1.
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) 3,4
    3 IF (DB.GE.DSHA) GO TO 1
      YP=DB*HA*US/NU
      IF (DB.LT.BLA) GO TO 5
      VM=SIGN*US*(5.5+2.5*ALOG(YP))/C
      GO TO 9
    5 IF (DB.LT.ALA) GO TO 6
      VM=SIGN*US*(-3.05+5.*ALOG(YP))/C
      GO TO 9
    6 VM=SIGN*US*YP/C
      GO TO 9
    4 IF ((DA.GE.DSH).AND.(DB.GE.DSH)) GO TO 1
      IF ((NSOFT.EQ.2).AND.(DA.LT.DSH).AND.(IDUCT.EQ.1)) GO TO 1
      IF ((XSTART.EQ.0.).AND.(IDUCT.EQ.0).AND.(DA.LT.DB)) GO TO 1
      Y=DA
      IF (DB.LT.DA) Y=DB
      YP=Y*H*US/NU
      IF (Y.LT.BL) GO TO 7
      VM=SIGN*US*(5.5+2.5*ALOG(YP))/C
      GO TO 9
    7 IF (Y.LT.AL) GO TO 8
      VM=SIGN*US*(-3.05+5.*ALOG(YP))/C
      GO TO 9
    8 VM=SIGN*US*YP/C
    9 L=0
      IF (IDUCT.EQ.-1.AND.NSOFT.EQ.2.AND.NIT.EQ.0) 11,12
   11 E=1.-DB/DSHA
      IF (E.LT..1) VM=VM+(1.-10.*E)*(FM-VM)
      GO TO 10
   12 E=1.-DA/DSH
      IF (DB.LT.DA) E=1.-DB/DSH
      IF (E.LT..1) VM=VM+(1.-10.*E)*(FM-VM)
   10 RETURN
      END
      SUBROUTINE LOOP3 (X,Y,NP,M,IO,NO)
      DIMENSION X(50),Y(50),S(50)
      JJ=NP-1
      DO 1 J=1,JJ
      S(J) = (Y(J+1)-Y(J))/(X(J+1)-X(J))
    1 CONTINUE
      M=0
      II=NP-3
      DO 2 I=1,II
      J = I+2
      DO 3 N=J,JJ
```

```
      LOOP=0
      Z = (Y(N)-Y(I)+S(I)*X(I)-S(N)*X(N))/(S(I)-S(N))
      IF (((X(I).LE.Z.AND.X(I+1).GE.Z).OR.(X(I).GE.Z.AND.X(I+1).LE.Z))
     X.AND.((X(N).LE.Z.AND.X(N+1).GE.Z).OR.(X(N).GE.Z.AND.X(N+1).LE.Z)))
     X   LOOP=1
      IF (LOOP.EQ.1) GO TO 4
      GO TO 3
    4 M=M+1
      IF (M.GT.1) GO TO 5
      IC=I
      NC=N
      GO TO 3
    5 IF ((N-I).LT.(NC-IC)) 6,3
    6 IC=I
      NC=N
    3 CONTINUE
    2 CONTINUE
      RETURN
      END
      SUBROUTINE INSP (X,Y,NP,ICUSP,N,IM,XM)
      DIMENSION X(NP),Y(NP),Z(2),W(2),MX(5),MM(5),D(5)
    1 DO 2 J=1,5
      MX(J)=MM(J)=1
    2 D(J)=10.
      II=JJ=MX(1)=MM(1)=1
      DS=10.
      IF(X(2).LE.X(1)) DS=(X(1)-X(2))**2+(Y(1)-Y(2))**2
      DO 5 J=2,NP
    4 IF(X(J).LT.X(J-1)) GO TO 6
      IF(J.NE.2.AND.MM(JJ).EQ.(J-1)) JJ=JJ+1
      MX(II)=J
      D(II)=(X(J)-X(J-1))**2+(Y(J)-Y(J-1))**2
      GO TO 5
    6 IF (J.NE.2.AND.MX(II).EQ.(J-1)) II=II+1
      MM(JJ)=J
    5 CONTINUE
      NN=1
      DMIN=D(1)
      DO 7 J=2,5
      IF(D(J).LT.DMIN) NN=J
    7 IF(D(J).LT.DMIN) DMIN=D(J)
      IF(DS.GE.DMIN) GO TO 8
      IM=1
      GO TO 40
    8 IM=MX(NN)
      XM=X(IM)
      IF(IM.EQ.NP) GO TO 40
      XL=AMAX1(X(IM-1),X(IM+1))
      IF (XM.LT.0.) GO TO 11
      XPA=.99*XM
      IF (XL.GT.0.) XLA=XM-.01*(XM-XL)
      IF (XL.LE.0.) XLA=0.
      XA=AMAX1(XPA,XLA)
      GO TO 12
   11 XPA=1.01*XM
      XLA=XM-.01*(XM-XL)
      XA=AMAX1(XPA,XLA)
   12 CONTINUE
```

```
      Z(1)=X(IM-1)
      Z(2)=X(IM)
      W(1)=Y(IM-1)
      W(2)=Y(IM)
      YA1=ATKN(Z,W,2,1,XA)
      IF(YA1.GT.100.) GO TO 40
      Z(1)=X(IM+1)
      W(1)=Y(IM+1)
      YB1=ATKN(Z,W,2,1,XA)
      IF (YB1.GT.100.) GO TO 40
      IF (YB1.GE.YA1) IJP=1
      IF (YB1.LT.YA1) IJP=0
      IF (N.EQ.1) IUP1=IUP
      IF (IUP.NE.IJP1) ICUSP=1
      IF (IUP.EQ.IJP1) ICUSP=0
      RETURN
   40 ICUSP=0
      RETURN
      END
      SUBROUTINE CUSP (X,Y,NP,ICUSP,N,IM,XM)
      DIMENSION X(NP),Y(NP),Z(50),W(50)
      IM=1
      XM=X(1)
      DO 10 J=2,NP
      IF (X(J).GT.XM) IM=J
   10 IF (X(J).GT.XM) XM=X(J)
      IF (XM.GT.5..OR.IM.EQ.1.OR.IM.EQ.NP) GO TO 40
      XL=AMAX1(X(1),X(NP))
      IF (XM.LT.0.) GO TO 11
      XPA=.99*XM
      IF (XL.GT.0.) XLA=XM-.01*(XM-XL)
      IF (XL.LE.0.) XLA=0.
      XA=AMAX1(XPA,XLA)
      GO TO 12
   11 XPA=1.01*XM
      XLA=XM-.01*(XM-XL)
      XA=AMAX1(XPA,XLA)
   12 CONTINUE
      XA=.90*XM
      DO 20 J=1,IM
      Z(IM+1-J)=X(J)
   20 W(IM+1-J)=Y(J)
      YA1=ATKN(Z,W,IM,1,XA)
      IF (YA1.GT.100.) GO TO 40
      NM=NP-IM+1
      DO 30 J=1,NM
      Z(J)=X(IM+J-1)
   30 W(J)=Y(IM+J-1)
      YB1=ATKN(Z,W,NM,1,XA)
      IF (YB1.GT.100.) GO TO 40
      IF (YB1.GE.YA1) IJP=1
      IF (YB1.LT.YA1) IJP=0
      IF (N.EQ.1) IUP1=IUP
      IF (IJP.NE.IJP1) ICUSP=1
      IF (IJP.EQ.IJP1) ICUSP=0
```

```
      RETURN
   40 ICUSP=0
      RETURN
      END
      FUNCTION ATKN(X,Y,N,K,XB)
      DIMENSION X(300),Y(300)
      COMMON XX(300),YY(300)
      K1=K+1
      IF(X(N)-X(1)) 100,10,10
10    IF(XB-X(1))20,20,30
20    LL=0
      GO TO 200
30    IF(X(N)-XB)40,40,50
40    LL=N-K1
      GO TO 200
50    LL=1
      LU=N
60    IF(LU-LL-1) 180,180,70
70    LI=(LL+LU)/2
      IF(X(LI)-XB) 80,80,90
80    LL=LI
      GO TO 60
90    LU=LI
      GO TO 60
100   IF(XB-X(1)) 120,20,20
120   IF(X(N)-XB) 130,40,40
130   LL=1
      LU=N
140   IF(LU-LL-1) 180,180,150
150   LI=(LL+LU)/2
      IF(X(LI)-XB) 160,170,170
160   LU=LI
      GO TO 140
170   LL=LI
      GO TO 140
180   LL=LL-(K1+1)/2
      IF(LL) 20,200,190
190   IF(LL+K1-N) 200,200,40
200   DO 210 I=1,K1
      I1=LL+I
      XX(I)=X(I1)-XB
210   YY(I)= Y(I1)
      DO 220 I=1,K
      DO 220 J=I,K
      P=XX(J+1)-XX(I)
      IF (P.EQ.0.) GO TO 240
  220 YY(J+1)=(1./P)*(YY(I)*XX(J+1)-YY(J+1)*XX(I))
      ATKN=YY(K1)
      RETURN
  240 ATKN=999.
      RETURN
      SUBROUTINE CL (R,X,T,P,F,FM,TH,D,IDUCT,NIT,RI,ROUT,K,DA,DP)
      REAL M,K
      M=ABS(FM)
      TR=T/519.
```

```
      PR=P/14.7
      A=.0762*TR*TR*TH/(PR*(TR+.415))
      B=.0000374*(TR**.75)*SQRT(F)/SQRT(PR*(TR+.415))
      OA=(A+B*(1.+TH/D)+.1085*M)/(R+B)
      IF(OA.LT.0.) GO TO 1
      EX=EXP(-8.65*M*M-.5192*M)
      C=.000469*F*(TH+.85*D*(1.-.7*SQRT(OA))*EX)/SQRT(TR)
      DP=ATAN(1./(C/OA-X))/K
      IF(IDUCT.EQ.1) GO TO 2
      E=5.E-7
      IF(IDUCT.EQ.0) GO TO 3
      IF(NIT.GT.1.AND.MOD(NIT,2).EQ.0) GO TO 3
      DP=DP*RI/(RI-DP/2.)
      XP=X-C/OA
      RS=K*RI
      CALL BESS (RS,1,1,E,BL,AL)
      CALL BESS (RS,0,1,E,EL,DL)
      AL=XP*AL
      BL=XP*BL
      AP=(EL+BL)/(DL+AL)
      XS=K*(RI-DP)
      NN=0
    4 CALL BESS (XS,1,1,E,AX,BX)
      CALL BESS (XS,0,1,E,CX,DX)
      DE=(AP*BX-AX)/(AP*(DX-BX/XS)-CX+AX/XS)
      XSN=XS-DE
      IF(ABS(XSN-XS).LT.0.001) GO TO 5
      XS=XSN
      NN=NN+1
      IF(NN.LT.10) GO TO 4
    5 DP=RI-XSN/K
      GO TO 2
    3 XP=X-C/OA
      DP=DP*ROUT/(ROUT+DP/2.)
      RS=K*ROUT
      CALL BESS (RS,1,1,E,BL,AL)
      CALL BESS (RS,0,1,E,DL,EL)
      AL=XP*AL
      BL=XP*BL
      AP=(BL-DL)/(AL-EL)
      XS=K*(ROUT+DP)
      NN=0
    6 CALL BESS (XS,1,1,E,AX,BX)
      CALL BESS (XS,0,1,E,CX,DX)
      DE=(AP*BX-AX)/(AP*(DX-BX/XS)-CX+AX/XS)
      XSN=XS-DE
      IF(ABS(XSN-XS).LT.0.001) GO TO 7
      XS=XSN
      NN=NN+1
      IF(NN.LT.10) GO TO 6
    7 DP=XSN/K-ROUT
      GO TO 2
    1 DP=999.
    2 RETURN
      END
```

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```
C
C     COMPUTES BESSEL FUNCTIONS
C     J0(X)           IF NN=0 KK=0
C     J1(X)           IF NN=1 KK=0
C     J0(X),Y0(X)  IF NN=0 KK=1
C     J1(X),Y1(X)  IF NN=1 KK=1
C     E=ACCURACY
C
      SUBROUTINE BESS(X,NN,KK,E,Y1,Y2)
      COMMON/CFC/ C(15),D(15),C1(15),D1(15),C2(15),D2(15),C3(15),D3(15)
      DATA  C,D,C1,D1           /1.0E0,-1.07208251953125E-5,-2.88027914
     A465172E-3,-2.32845131502204 E-2,-9.04575114137013 E-2,-2.452751623
     B331013 E-1,-5.55162401579686 E-1,-1.06419349102279 E0,-1.92304242000
     C2979 E0,-3.17399384201 87 E0,-4.95394321891922 E0,-7.3943965118938
     D1 E0,-1.06414712035636 E1,-1.48558951379629 E1,-2.02130078191578 E
     E1,7.03125E-4,5.19521984415029 E-2,2.03503080053406 E-1,4.475351925
     F65856 E-1,7.93177634454767 E-1,1.23745905920051 E0,1.78042682191 55
     G8 E0,2.42206019141725 E0,3.16237469420385 E0,4.00137386789395 E0,4
     H.93306019604771 E0,5.97543534316542 E0,7.11050059950930 E0,8.34425
     I59273 4063 E0,9.67570037477811 E0,1.0E0,-1.47354125975562 E-4,-5.51
     J710054749374 E-3,-3.44056671293060 E-2,-1.19703729312153 E-1,-3.03
     K908275315734 E-1,-6.04125155303842 E-1,-1.25207012841032 E0,-2.194
     L060325850773 E0,-3.55605929754501 E0,-5.49856781297308 E0,-8.126813
     M31631461 E0,-1.16005503463665 E1,-1.60840445298517 E1,-2.175642262
     N70039 E1,5.859375E-3,5.07160551564345 E-2,2.53793042221047 E-1,5.2
     O5436578722598 E-1,8.35715238793175 E-1,1.30465451905885 E0,1.93225
     P56955115 E0,2.59855350886123 E0,3.36353445994587 E0,4.22719701432
     Q150 E0,5.18954318791611 E0,6.25058936266157 E0,7.41032156793225 E0
     R,8.66374559100747 E0,1.00255520474147 E1/
      DATA C2,D2,C3,D3/ 1.0E0,1.57928466796875 E-5,-2.44381083312872 E-3
     A,-2.13606553455783 E-2,-6.74772615793729 E-2,-2.43041074174527 E-1
     B,-5.46937480621760 E-1,-1.07238696130943 E0,-1.90864421882 48 E0,-
     C3.15269771849344 E0,-4.92675756393717 E0,-7.35055714080549 E0,-1.0
     D600201883280 85 E1,-1.48063893153652 E1,-2.01544589357849 E1,-1.1718
     E75E-3,4.50835597826057 E-2,1.92179768196320 E-1,4.38309004023673 E
     F-1,7.83295223635590 E-1,1.22707123961156 E0,1.75960453320143 E0,2.
     G41067731972933 E0,3.15087816569130 E0,3.98959992540356 E0,4.927037
     H46293770 E0,5.96318713929023 E0,7.09364622813375 E0,8.33161264213 1
     I91 E0,9.66388474813034 E0,3.0E0,-6.65435791015625 E-5,-4.931858675
     J48867 E-3,-3.27735233246136 E-2,-1.16427130443485 E-1,-3.033525086
     K28447 E-1,-6.55627315168281 E-1,-1.24994451493528 E0,-2.177611100 7
     L9763 E0,-3.54454735335622 E0,-5.47120537308531 E0,-8.0929737653329
     M8 E0,-1.15593674179525 E1,-1.60348373363787 E1,-2.15983655306451 E
     N1,2.734375E-3,7.41127859149277 E-2,2.45564251843836 E-1,5.15653670
     O334203 E-1,8.85301630297341 E-1,1.35473897252989 E0,1.921935369381
     P49 E0,2.56767200432531 E0,3.35253753367135 E0,4.21592524683140 E0,
     Q5.17501291313 2522 E0,6.23684574180062 E0,7.39837227534312 E0,8.6566
     R0658234390 E0,1.00113546978 7135 E1/
      PI=3.14159265358979
      IF(X-7.0E0)16,20,20
16    ONE=1.0E0
      K=KK
      N=NN
      Z=X/2.0
      G=.577215664901532
```

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```
1       IF(N) 3,3,4
3       S=ONE
        U=ONE
        EM=ONE
5       R=Z/EM
        U=-U*R*R
        S=S+U
        EM=EM+ONE
        IF( ABS(U)-E)6,5,5
6       Y1=S
        IF(K)2,8,2
8       RETURN
4       S=ONE
        U=ONE
        EM=ONE
7       R=(Z*Z)/(EM*(EM+ONE))
        U=-U*R
        S=S+U
        EM=EM+ONE
        IF( ABS(U)-E)18,7,7
18      Y1=S*Z
        F=Y1
        IF(K)2,10,9
10      RETURN
2       S=Y1*(G+ALOG(Z))+Z*Z
        U=Z*Z
        EN=2.
        Q=ONE
11      R=(Z/EN)*(Z/EN)*(ONE+(ONE/(EN*Q)))
        U=-U*R
        S=S+U
        Q=Q+(ONE/EN)
        EN=EN+ONE
        IF( ABS(U)-E)12,11,11
12      Y2=S*(2./PI)
        IF(N)14,13,13
13      RETURN
9       N=-1
        K=-1
        GO TO 3
14      Y2=(F*Y2-(ONE/(PI*Z)))/Y1
        Y1=F
15      RETURN
20      K=KK
        N=NN
        SQ= SQRT (.5)
        CX= COS (X)
        SX= SIN (X)
        CZ=SQ*(CX+SX)
        SZ=SQ*(SX-CX)
        X1=X*X
        R= SQRT(2.0   /(PI*X))*0.01
        Y=X*0.1
        Z=Y*Y
        PO=X1*EVAL(C,D,E,15,7)
```

```
      QO=-(X/8.0  )*EVAL(C1,D1,E,15,Z)
      IF(K) 23,21,23
21    IF(N) 25,22,25
22    Y1=R*(PO*CZ-QO*SZ)
      GO TO 15
23    IF(N) 26,24,26
24    Y1=R*(PO*CZ-QO*SZ)
      Y2=R*(PO*SZ+QO*CZ)
      GO TO 15
25    P11=X1*EVAL(C2,D2,E,15,Z)
      Q1=(X/8.0  )*EVAL(C3,D3,E,15,Z)
      Y1=R*(Q1*SZ-P11*CZ)
      GO TO 15
26    P11=X1*EVAL(C2,D2,E,15,Z)
      Q1=(X/8.0  )*EVAL(C3,D3,E,15,Z)
      Y1=R*(P11*SZ+Q1*CZ)
      Y2=R*(Q1*SZ-P11*CZ)
      GO TO 15
      END
C     SUBROUTINE TO EVALUATE CONTINUED FRACTION
                          FUNCTION EVAL(A,B,E,N,Z)
      DIMENSION A(15),B(15)
      ONE=1.0
      C=Z+B(1)
      EVAL=A(1)/C
      W=EVAL
      T=ONE
      DO 3 I=2,N
      D=Z+B(I)
      R=A(I)/(C*D)
      S=ONE/(ONE+R*T)
      V=S-ONE
      T=S
      W=W*V
      EVAL=EVAL+W
      C=D
      IF( ABS(W)-E) 10,10,3
3     CONTINUE
10    RETURN
      END
      SUBROUTINE LINPAR (R,X,T,P,F,FM,TH,D,DA,DP,CEE)
      REAL M
      M=ABS(FM)
      TR=T/519.
      PR=P/14.7
      A=.0762*TR*TR*TH/(PR*(TR+.415))
      B=.000374*(TR**.75)*SQRT(F)/SQRT(PR*(TR+.415))
      DA=(A+B*(1.+TH/D)+.1085*M)/(R+B)
      IF (DA.LT.0.) GO TO 1
      EX=EXP(-8.65*M*M-.0192*M)
      C=.000489*F*(TH+.85*D*(1.-.7*SQRT(DA))*EX)/SQRT(TR)
      DP=(CEE/(6.283185*F))*ATAN(1./(C/DA-X))
      GO TO 2
    1 DP=999.
    2 RETURN
      END
```

```
C     FOR CDC 6600 USE*****CALL STEP MUST BE EXECUTED WITH PARAMETERS
C     AS ILLUSTRATED ON CARD DFEQ.327
      SUBROUTINE SET(N,X,Y,DX,F,DIS,MODE,DXMAX,DXMIN)
C     DFEQ. CONSISTS OF TWO SUBROUTINE SUBPROGRAMS, SET AND STEP, TO
C     SOLVE THE INITIAL VALUE PROBLEM
C                   DY/DX = F(X,Y), Y(X0) = Y0
C     WHERE Y IS AN N-VECTOR WITH REAL COMPONENTS.
C
C
C     CALL SET(N,X,Y,DX,F,DIS,MODE,DXMAX,DXMIN),
C     WHERE
C         N IS AN INTGER CONSTANT OR VARIABLE,
C         X IS A REAL VARIABLE, THE INDEPENDENT VARIABLE,
C         Y IS A REAL VARIABLE N-ARRAY, THE DEPENDENT VARIABLE,
C         DX IS A REAL CONSTANT OR VARIABLE, THE CURRENT STEP SIZE,
C         F IS THE NAME OF SUBROUTINE SUBPROGRAM OF THE FORM F(N,X,Y,DY)
C           WHICH PROVIDES SUBROUTINE SET WITH DY/DX,
C           DY IS A REAL VARIABLE N-ARRAY, DY/DX,
C         DIS IS A REAL VARIABLE OR CONSTANT WHICH PROVIDES A TOLERANCE
C             FOR THE ADAMS-MOULTON LOCAL ERROR CHECK IN THE VARIABLE
C             STEP SIZE MODE,
C         MODE IS A LOGICAL VARIABLE OR CONSTANT WHICH WHEN TRUE (FALSE)
C              INITIATES THE FIXED (VARIABLE) STEP SIZE MODE,
C         DXMAX (DXMIN) IS A REAL VARIABLE OR CONSTANT GIVING UPPER
C              (LOWER) BOUNDS FOR ABS(DX) IN THE VARIABLE STEP MODE,
C     INITIALIZES NECESSAY VARIABLES PRIOR TO ANY INTEGRATIONS.
C
C     CALL STEP(N,X,Y,DX,F,DIS,MODE,DXMAX,DXMIN) THEN INTEGRATES FROM X
C     TO X+DX.  UPON EXIT FROM STEP, X,
C     Y, AND DX WILL HAVE BEEN SET TO THEIR NEW VALUES. THUS TO
C     INTEGRATE OVER SUCCEEDING STEPS, CALL STEP(N,X,Y,DX,F,DIS,MODE,
C     DXMAX,DXMIN) IS EXECUTED FOR EACH STEP DESIRED.
C
C     THE SUBROUTINE SUBPROGRAM F MUST BE INCLUDED IN ANY JOB
C     CALLING SET-STEP. IT IS PROVIDED BY THE USER AND MAY HAVE ANY
C     LEGAL SUBROUTINE NAME. THE VARIABLES CORRESPONDING TO Y AND DY
C     MUST BE DIMENSIONED APPROPRIATELY THEREIN.
C
      DIMENSION Y(N)
C
C
C     THE VALUE OF NMAX AND THE DIMENSIONS ASSIGNED TO THE FIRST POSITIO
C     OF THE VARIABLES  D  AND  W  IN THE FOLLOWING DIMENSION STATE-
C     MENT SHOULD BE IDENTICAL.
C
      DIMENSION D(90,5), W(90,5)
C
      EXTERNAL F
      LOGICAL MODE,CODE
      DATA NMAX/90/
      DATA ERF/100./
      DATA DXF/2./
      M=N
      IF (M.LT.1.OR.M.GT.NMAX) GO TO 20
      RA=DIS
```

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```
      CODE=MODE                                                         DFEQ.05
      HMAX =DXMAX                                                       DFEQ.05
      HMIN=DXMIN                                                        DFEQ.05
      RB=ERF                                                            DFEQ.05
      BETA=DXF                                                          DFEQ.06
      K=0                                                               DFEQ.06
      K2=0                                                              DFEQ.06
      DO 10 I=1,M                                                       DFEQ.06
10    W(I,1)=Y(I)                                                       DFEQ.06
      CALL F(M,X,Y,D(1,5))                                              DFEQ.06
      RETURN                                                            DFEQ.06
20    PRINT 30                                                          DFEQ.06
30    FORMAT(55H1SUBROUTINE SET HAS ARGUMENT OUT-OF-RANGE. SEE LISTING.)DFEQ.06
      CALL SYSTEM(200,1_ )                                              DFEQ.06
      RETURN                                                            DFEQ.07
      ENTRY STEP                                                        DFEQ.07
40    XC=X                                                              DFEQ.07
      H=DX                                                              DFEQ.07
      IF (K.NE.0) IF (K-2) 50,50,110                                    DFEQ.07
      XP=XC                                                             DFEQ.07
      DO 45 I=1,M                                                       DFEQ.07
45    W(I,5)=W(I,1)                                                     DFEQ.07
50    K1=4-K                                                            DFEQ.07
      DO 70 I=1,M                                                       DFEQ.07
      DO 60 J=K1,4                                                      DFEQ.08
60    D(I,J)=D(I,J+1)                                                   DFEQ.08
      W(I,2)=H*D(I,4)                                                   DFEQ.08
      W(I,1)=W(I,1)+.5*W(I,2)                                           DFEQ.08
70    Y(I)=W(I,1)                                                       DFEQ.08
      X=XC+.5*H                                                         DFEQ.08
      CALL F(M,X,Y,D(1,5))                                              DFEQ.08
      DO 80 I=1,M                                                       DFEQ.08
      W(I,3)=H*D(I,5)                                                   DFEQ.08
      W(I,1)=W(I,1)+.5*(W(I,3)-W(I,2))                                  DFEQ.08
80    Y(I)=W(I,1)                                                       DFEQ.09
      CALL F(M,X,Y,D(1,5))                                              DFEQ.09
      DO 90 I=1,M                                                       DFEQ.09
      W(I,4)=H*D(I,5)                                                   DFEQ.09
      W(I,1)=W(I,1)+W(I,4)-.5*W(I,3)                                    DFEQ.09
90    Y(I)=W(I,1)                                                       DFEQ.09
      X=XC+H                                                            DFEQ.09
      CALL F(M,X,Y,D(1,5))                                              DFEQ.09
      DO 100 I=1,M                                                      DFEQ.09
      W(I,1)=W(I,1)-W(I,4)+.16666666666666667*(W(I,2)+2.*(W(I,3)+W(I,4))+HDFEQ.09
     1*D(I,5))                                                          DFEQ.10
100   Y(I)=W(I,1)                                                       DFEQ.10
      K=K+1                                                             DFEQ.10
      K1=K                                                              DFEQ.10
      CALL F(M,X,Y,D(1,5))                                              DFEQ.10
      RETURN                                                            DFEQ.10
110   DO 130 I=1,M                                                      DFEQ.10
      W(I,2)=W(I,1)                                                     DFEQ.10
      DO 120 J=1,4                                                      DFEQ.10
120   D(I,J)=D(I,J+1)                                                   DFEQ.10
      W(I,3)=W(I,2)+.41666666666666667E-1*H*(55.*D(I,4)-59.*D(I,3)+37.*D(IDFEQ.11
```

```
     1,2)-9.*D(I,1))
 130  Y(I)=W(I,3)
      X=XC+H
      CALL F(M,X,Y,D(1,5))
      DO 140 I=1,M
      W(I,1)=W(I,2)+.4166666666666667E-1*H*(9.*D(I,5)+19.*D(I,4)
     1-5.*D(I,3)+D(I,2))
 140  Y(I)=W(I,1)
      CALL F(M,X,Y,D(1,5))
      IF (CODE) RETURN
      ERR=0.
      DO 150 I=1,M
 150  ERR=AMAX1(ERR,ABS(W(I,1)-W(I,3))/(14.*AMAX1(ABS(W(I,1)),1.)))
      IF (ERR.GE.RA) IF (HMIN-ABS(H)) 160,155,155
      K1=0
      IF (R3*ERR.LT.RA) IF (HMAX-ABS(H)) 155,155,210
 155  K2=0
      RETURN
 160  IF (K1.NE.3) GO TO 180
      DO 170 I=1,M
      W(I,1)=W(I,5)
 170  D(I,5)=D(I,1)
      X=XP
      GO TO 200
 180  DO 190 I=1,M
      W(I,1)=W(I,2)
 190  D(I,5)=D(I,4)
      X=XC
 200  K=0
      K2=0
      DX=SIGN(AMAX1(ABS(H)/BETA,HMIN),H)
      GO TO 40
 210  K2=K2+1
      IF (K2.LT.6) RETURN
      K=0
      DX=SIGN(AMIN1(ABS(H)*BETA,HMAX),H)
      K2=0
      RETURN
      END
```

A.2 OPTSHE INPUT - TEST CASE

Card 1

Column	1	11	21	31	41	59	70	80
	.5	5000.	0.	4.	5.	-1	2	1

Card 2

Column	1	11	21
	540.	14.7	.0750

Input Data is defined on comment cards at start of OPTSHE. Read and Formats Statements are on the first page of the program listing.

A.3 TYPICAL OUTPUT - FOR INPUT GIVEN

The following 12 pages of print-out are typical output for input given.

```
 5.0000 INCHES OD/2        1.0000 INCHES ID/2 FOR AN ANNULAR DUCT WITH BOTH INNER AND OUTER WALLS SOFT
H=4.0000 INCHES
D/H=0.0000 DELTA/H
 .500 MACH NO            2.926 FREQ PAR

                 PRESSURE                    PRESS GRAD                   MACH           X             SM            RE RHS       IM RHS
 20       .9938007    -.0000498       .0643443      .0160705       .5000000     1.6658430     0.0000000    -10.3361659   -2.5951801
 40       .9930175    -.0017419       .3825685      .0953221       .5000000     1.6354430     0.0000000    -10.5255727   -2.6037369
 60       .8350264    -.0400039      1.8491035      .4347151       .5000000     1.4946430     0.0000000     -9.9911024   -2.0371143
 80       .0801535    -.1883548      3.8030165      .5811995       .5000000     1.2386430     0.0000000     -4.3920742    1.2746316
100      -.9397416    -.2489259      3.7443181     -.2823331       .5000000      .9826430     0.0000000      5.2805632    5.2977110
120     -1.6120672     .0221118      1.0737375    -1.8399149       .5000000      .7266430     0.0000000     15.2714621    6.5431461

(R STAR,X STAR)
(  .7693,    .3222)  (  .7514,  -.0031)  (  .7403,  -.0286)  (  .7259,  -.0540)  (  .7111,  -.0790)
(  .6923,   -.1033)  (  .6724,  -.1265)  (  .6497,  -.1483)  (  .6249,  -.1684)  (  .5933,  -.1863)
(  .5701,   -.2020)  (  .5408,  -.2151)  (  .5107,  -.2255)  (  .4892,  -.2372)  (  .4437,  -.2381)
(  .4195,   -.2404)  (  .3901,  -.2402)  (  .3616,  -.2376)  (  .3345,  -.2329)  (  .3098,  -.2263)
(  .2848,   -.2181)  (  .2626,  -.2085)  (  .2423,  -.1978)  (  .2241,  -.1863)  (  .2078,  -.1742)
(  .1933,   -.1619)  (  .1819,  -.1496)  (  .1722,  -.1373)  (  .1647,  -.1270)  (  .1532,  -.1180)
(  .1551,   -.1115)  (  .1511,  -.1081)  (  .1457,  -.1079)  (  .1370,  -.1095)  (  .1245,  -.1105)
(  .1095,   -.1089)  (  .0940,  -.1041)  (  .0796,  -.0966)  (  .0671,  -.0874)  (  .0555,  -.0776)

SE,SEL,SEU,TEF,TEL,LOOP     .202102     .202102     .202102     .139121    1.017039      0

                 PRESSURE                    PRESS GRAD                   MACH           X             SM            RE RHS       IM RHS
 20       .9938009    -.0000488       .0642693      .0157492       .5000000     1.6658430     0.0000000    -10.3841479   -2.5442577
 40       .9930256    -.0017071       .3821253      .0934151       .5000000     1.6354430     0.0000000    -10.5133175   -2.5575833
 60       .8352225    -.0392063      1.8458435      .4260787       .5000000     1.4946430     0.0000000     -9.9775300   -1.9973511
 80       .0813834    -.1346761      3.8022555      .5703658       .5000000     1.2386430     0.0000000     -4.3801137    1.2440876
100      -.9359100    -.2444456      3.7377381     -.2741510       .5000000      .9826430     0.0000000      5.2700143    5.1839813
120     -1.6083249     .0203042      1.0824113    -1.8481940       .5000000      .7266430     0.0000000     15.2040293    6.4117826

(R STAR,X STAR)
(  .7743,    .0176)  (  .7654,  -.0087)  (  .7536,  -.0351)  (  .7394,  -.0613)  (  .7227,  -.0871)
(  .7035,   -.1121)  (  .6920,  -.1358)  (  .6581,  -.1580)  (  .6321,  -.1783)  (  .6044,  -.1964)
(  .5751,   -.2120)  (  .5446,  -.2249)  (  .5135,  -.2350)  (  .4820,  -.2423)  (  .4506,  -.2467)
(  .4197,   -.2485)  (  .3990,  -.2476)  (  .3607,  -.2444)  (  .3331,  -.2390)  (  .3071,  -.2317)
(  .2823,   -.2229)  (  .2606,  -.2127)  (  .2402,  -.2014)  (  .2219,  -.1894)  (  .2058,  -.1768)
(  .1918,   -.1639)  (  .1800,  -.1512)  (  .1705,  -.1390)  (  .1633,  -.1279)  (  .1532,  -.1186)
(  .1544,   -.1113)  (  .1510,  -.1085)  (  .1458,  -.1087)  (  .1372,  -.1106)  (  .1245,  -.1120)
(  .1092,   -.1105)  (  .0934,  -.1055)  (  .0789,  -.0977)  (  .0663,  -.0884)  (  .0557,  -.0783)

SE,SEL,SEU,TEF,TEL,LOOP     .198050     .178254     .217865     .139121    1.017039      0

(R STAR,X STAR)
(  .7327,    .0305)  (  .7248,   .0071)  (  .7149,  -.0167)  (  .7029,  -.0405)  (  .6838,  -.0640)
(  .6724,   -.0371)  (  .6539,  -.1092)  (  .6332,  -.1302)  (  .6105,  -.1497)  (  .5852,  -.1674)
(  .5692,   -.1830)  (  .5330,  -.1964)  (  .5049,  -.2073)  (  .4763,  -.2158)  (  .4474,  -.2216)
(  .4197,   -.2250)  (  .3905,  -.2259)  (  .3631,  -.2245)  (  .3368,  -.2211)  (  .3118,  -.2157)
(  .2882,   -.2087)  (  .2654,  -.2003)  (  .2463,  -.1907)  (  .2281,  -.1802)  (  .2118,  -.1691)
(  .1975,   -.1577)  (  .1854,  -.1453)  (  .1754,  -.1358)  (  .1674,  -.1252)  (  .1612,  -.1167)
(  .1563,   -.1104)  (  .1516,  -.1070)  (  .1455,  -.1063)  (  .1367,  -.1073)  (  .1245,  -.1078)
(  .1101,   -.1060)  (  .0951,  -.1014)  (  .0811,  -.0942)  (  .0686,  -.0855)  (  .0530,  -.0761)

SE,SEL,SEU,TEF,TEL,LOOP     .210136     .169158     .231205     .139121    1.017039      0

(R STAR,X STAR)
(  .8057,    .0074)  (  .7950,  -.0203)  (  .7817,  -.0492)  (  .7657,  -.0773)  (  .7471,  -.1047)
(  .7257,   -.1310)  (  .7017,  -.1559)  (  .6754,  -.1783)  (  .6458,  -.1937)  (  .6155,  -.2179)
(  .5867,   -.2333)  (  .5519,  -.2457)  (  .5186,  -.2551)  (  .4851,  -.2613)  (  .4520,  -.2646)
(  .4195,   -.2650)  (  .3882,  -.2628)  (  .3593,  -.2581)  (  .3299,  -.2513)  (  .3033,  -.2427)
(  .2787,   -.2325)  (  .2561,  -.2211)  (  .2357,  -.2085)  (  .2174,  -.1954)  (  .2013,  -.1818)
(  .1875,   -.1680)  (  .1761,  -.1544)  (  .1670,  -.1413)  (  .1696,  -.1295)  (  .1550,  -.1196)
(  .1532,   -.1120)  (  .1506,  -.1094)  (  .1462,  -.1101)  (  .1377,  -.1129)  (  .1246,  -.1149)
( [illegible] )  ( [illegible] )  ( [illegible] )  ( [illegible] )  ( [illegible], -.1799)
```

(R STAR,X STAR)

(.7071,	.0382)	(.7000,	.0162)	(.6912,	-.0060)	(.6804,	-.0284)	(.6677,	-.0506)
(.6523,	-.0724)	(.6361,	-.0936)	(.6174,	-.1137)	(.5967,	-.1326)	(.5743,	-.1500)
(.5503,	-.1555)	(.5250,	-.1790)	(.4988,	-.1903)	(.4719,	-.1993)	(.4446,	-.2060)
(.4173,	-.2102)	(.3904,	-.2122)	(.3640,	-.2119)	(.3385,	-.2096)	(.3142,	-.2053)
(.2912,	-.1995)	(.2697,	-.1922)	(.2498,	-.1836)	(.2317,	-.1742)	(.2154,	-.1640)
(.2011,	-.1535)	(.1887,	-.1429)	(.1783,	-.1325)	(.1699,	-.1232)	(.1631,	-.1152)
(.1575,	-.1092)	(.1521,	-.1057)	(.1455,	-.1047)	(.1365,	-.1050)	(.1247,	-.1051)
(.1147,	-.1032)	(.0962,	-.0987)	(.0824,	-.0920)	(.0701,	-.0835)	(.0535,	-.0746)

SE,SEL,SEU,TEF,TEL,LOOP .218270 .195443 .240097 .139121 1.017039 0

(R STAR,X STAR)

(.8331,	-.0041)	(.8269,	-.0347)	(.8118,	-.0652)	(.7933,	-.0953)	(.7728,	-.1244)
(.7483,	-.1522)	(.7223,	-.1782)	(.6931,	-.2020)	(.6617,	-.2231)	(.6286,	-.2413)
(.5941,	-.2563)	(.5587,	-.2680)	(.5231,	-.2764)	(.4876,	-.2815)	(.4526,	-.2834)
(.4187,	-.2823)	(.3861,	-.2765)	(.3551,	-.2723)	(.3260,	-.2640)	(.2989,	-.2539)
(.2740,	-.2423)	(.2512,	-.2295)	(.2307,	-.2158)	(.2124,	-.2015)	(.1965,	-.1868)
(.1838,	-.1720)	(.1719,	-.1574)	(.1633,	-.1435)	(.1573,	-.1309)	(.1537,	-.1203)
(.1519,	-.1131)	(.1504,	-.1101)	(.1468,	-.1114)	(.1384,	-.1152)	(.1247,	-.1180)
(.1073,	-.1168)	(.0909,	-.1112)	(.0757,	-.1025)	(.0629,	-.0921)	(.0525,	-.0813)

SE,SEL,SEU,TEF,TEL,LOOP .151832 .153703 .202102 .139121 1.017039 0

(R STAR,X STAR)

(.6832,	.0449)	(.6768,	.0243)	(.6689,	.0035)	(.6592,	-.0175)	(.6478,	-.0385)
(.6344,	-.0592)	(.6192,	-.0793)	(.6020,	-.0987)	(.5831,	-.1169)	(.5625,	-.1339)
(.5403,	-.1493)	(.5169,	-.1628)	(.4924,	-.1744)	(.4671,	-.1839)	(.4414,	-.1911)
(.4155,	-.1962)	(.3897,	-.1990)	(.3644,	-.1997)	(.3398,	-.1984)	(.3152,	-.1952)
(.2937,	-.1904)	(.2726,	-.1841)	(.2530,	-.1766)	(.2350,	-.1681)	(.2138,	-.1589)
(.2013,	-.1492)	(.1918,	-.1394)	(.1811,	-.1299)	(.1722,	-.1211)	(.1649,	-.1135)
(.1545,	-.1078)	(.1526,	-.1043)	(.1456,	-.1029)	(.1354,	-.1028)	(.1248,	-.1025)
(.1113,	-.1003)	(.0972,	-.0962)	(.0837,	-.0897)	(.0715,	-.0818)	(.0639,	-.0730)

SE,SEL,SEU,TEF,TEL,LOOP .226355 .202102 .243990 .139121 1.017039 0

(R STAR,X STAR)

(.8935,	-.0249)	(.8806,	-.0596)	(.8622,	-.0933)	(.8404,	-.1273)	(.8151,	-.1593)
(.7855,	-.1694)	(.7550,	-.2171)	(.7208,	-.2418)	(.6845,	-.2632)	(.5455,	-.2810)
(.6014,	-.2951)	(.5579,	-.3053)	(.5264,	-.3117)	(.4836,	-.3144)	(.4519,	-.3138)
(.4155,	-.3101)	(.3812,	-.3036)	(.3469,	-.2947)	(.3168,	-.2838)	(.2910,	-.2712)
(.2635,	-.2573)	(.2427,	-.2424)	(.2222,	-.2267)	(.2042,	-.2106)	(.1836,	-.1942)
(.1755,	-.1778)	(.1650,	-.1618)	(.1571,	-.1465)	(.1521,	-.1326)	(.1439,	-.1210)
(.1499,	-.1132)	(.1503,	-.1105)	(.1461,	-.1132)	(.1399,	-.1189)	(.1252,	-.1230)
(.1069,	-.1221)	(.0837,	-.1159)	(.0730,	-.1062)	(.0601,	-.0950)	(.0438,	-.0835)

SE,SEL,SEU,TEF,TEL,LOOP .169564 .152607 .202102 .139121 1.017039 0

(R STAR,X STAR)

(.6461,	.0541)	(.6428,	.0357)	(.6361,	.0168)	(.6279,	-.0023)	(.6181,	-.0214)
(.6057,	-.0404)	(.5935,	-.0591)	(.5787,	-.0772)	(.5622,	-.0945)	(.5441,	-.1108)
(.5245,	-.1257)	(.5037,	-.1392)	(.4818,	-.1510)	(.4589,	-.1610)	(.4355,	-.1690)
(.4115,	-.1751)	(.3878,	-.1791)	(.3641,	-.1811)	(.3409,	-.1813)	(.3184,	-.1796)
(.2973,	-.1765)	(.2764,	-.1716)	(.2573,	-.1656)	(.2396,	-.1585)	(.2235,	-.1507)
(.2090,	-.1425)	(.1952,	-.1337)	(.1851,	-.1253)	(.1756,	-.1174)	(.1675,	-.1106)
(.1594,	-.1055)	(.1535,	-.1018)	(.1459,	-.1000)	(.1365,	-.0993)	(.1252,	-.0985)
(.1123,	-.0963)	(.0988,	-.0922)	(.0857,	-.0862)	(.0737,	-.0788)	(.0631,	-.0706)

SE,SEL,SEU,TEF,TEL,LOOP .239289 .202102 .263215 .139121 1.017039 0

```
(R STAR,X STAR)
( .9452,  -.0439)  ( .9257,  -.0820)  ( .9041,  -.1195)  ( .8787,  -.1558)  ( .9435,  -.1902)
( .8167,  -.2221)  ( .7808,  -.2503)  ( .7422,  -.2762)  ( .7015,  -.2975)  ( .6533,  -.3146)
( .6194,  -.3275)  ( .5734,  -.3361)  ( .5309,  -.3405)  ( .4835,  -.3411)  ( .4437,  -.3382)
( .4119,  -.3321)  ( .3761,  -.3233)  ( .3428,  -.3121)  ( .3120,  -.2991)  ( .2839,  -.2845)
( .2594,  -.2638)  ( .2354,  -.2521)  ( .2151,  -.2343)  ( .1973,  -.2174)  ( .1820,  -.1997)
( .1533,  -.1821)  ( .1593,  -.1649)  ( .1521,  -.1485)  ( .1479,  -.1336)  ( .1458,  -.1211)
( .1413,  -.1128)  ( .1505,  -.1103)  ( .1496,  -.1143)  ( .1415,  -.1219)  ( .1257,  -.1272)
( .1000,  -.1263)  ( .0869,  -.1195)  ( .0708,  -.1091)  ( .0579,  -.0972)  ( .0477,  -.0851)

SE,SEL,SEJ,TEF,TEL,LOOP     .160257     .144240     .202102     .139121    1.017039      1

(R STAR,X STAR)
( .1570,  -.1310)  ( .1556,  -.1274)  ( .1545,  -.1240)  ( .1535,  -.1209)  ( .1529,  -.1181)
( .1523,  -.1157)  ( .1519,  -.1137)  ( .1515,  -.1121)  ( .1512,  -.1109)  ( .1537,  -.1103)
( .1501,  -.1101)  ( .1492,  -.1103)  ( .1479,  -.1103)  ( .1462,  -.1119)  ( .1440,  -.1131)
( .1412,  -.1144)  ( .1379,  -.1157)  ( .1340,  -.1168)  ( .1296,  -.1177)  ( .1248,  -.1183)
(
SE,SEL,SEJ,TEF,TEL,LOOP     .181185     .160267     .202102     .580312     .788090      0

(R STAR,X STAR)
( .1525,  -.1325)  ( .1516,  -.1285)  ( .1509,  -.1249)  ( .1503,  -.1215)  ( .1501,  -.1185)
( .1500,  -.1160)  ( .1500,  -.1138)  ( .1502,  -.1122)  ( .1503,  -.1111)  ( .1504,  -.1105)
( .1512,  -.1106)  ( .1498,  -.1111)  ( .1489,  -.1121)  ( .1474,  -.1136)  ( .1454,  -.1153)
( .1425,  -.1171)  ( .1392,  -.1183)  ( .1350,  -.1205)  ( .1303,  -.1217)  ( .1251,  -.1225)
(
SE,SEL,SEJ,TEF,TEL,LOOP     .170726     .160257     .181185     .580312     .788090      0

(R STAR,X STAR)
( .1503,  -.1331)  ( .1495,  -.1290)  ( .1493,  -.1252)  ( .1486,  -.1217)  ( .1486,  -.1186)
( .1437,  -.1159)  ( .1491,  -.1137)  ( .1495,  -.1120)  ( .1499,  -.1110)  ( .1513,  -.1105)
( .1504,  -.1106)  ( .1502,  -.1114)  ( .1495,  -.1126)  ( .1482,  -.1143)  ( .1452,  -.1163)
( .1435,  -.1185)  ( .1400,  -.1205)  ( .1357,  -.1224)  ( .1308,  -.1238)  ( .1254,  -.1248)
(
SE,SEL,SEJ,TEF,TEL,LOOP     .155436     .150257     .170726     .580312     .788090      1

(R STAR,X STAR)
( .1494,  -.1159)  ( .1494,  -.1148)  ( .1495,  -.1139)  ( .1497,  -.1130)  ( .1498,  -.1123)
( .1493,  -.1117)  ( .1501,  -.1112)  ( .1502,  -.1108)  ( .1503,  -.1106)  ( .1503,  -.1105)
( .1503,  -.1105)  ( .1503,  -.1107)  ( .1501,  -.1110)  ( .1499,  -.1114)  ( .1495,  -.1119)
( .1491,  -.1125)  ( .1485,  -.1133)  ( .1477,  -.1140)  ( .1468,  -.1149)  ( .1458,  -.1158)
(
SE,SEL,SEJ,TEF,TEL,LOOP     .158111     .165435     .170725     .628983     .727107      1

(R STAR,X STAR)
( .1502,  -.1117)  ( .1503,  -.1115)  ( .1503,  -.1113)  ( .1503,  -.1111)  ( .1503,  -.1109)
( .1503,  -.1108)  ( .1504,  -.1107)  ( .1504,  -.1106)  ( .1504,  -.1105)  ( .1504,  -.1105)
( .1503,  -.1105)  ( .1503,  -.1105)  ( .1503,  -.1105)  ( .1502,  -.1105)  ( .1502,  -.1106)
( .1501,  -.1107)  ( .1500,  -.1108)  ( .1499,  -.1109)  ( .1498,  -.1111)  ( .1496,  -.1112)
(
SE,SEL,SEJ,TEF,TEL,LOOP     .170726     .158111     .170726     .653442     .694570      0

(R STAR,X STAR)
( .1501,  -.1117)  ( .1501,  -.1115)  ( .1502,  -.1113)  ( .1502,  -.1111)  ( .1502,  -.1109)
( .1503,  -.1108)  ( .1503,  -.1107)  ( .1503,  -.1105)  ( .1503,  -.1105)  ( .1503,  -.1105)
( .1503,  -.1105)  ( .1503,  -.1105)  ( .1503,  -.1105)  ( .1503,  -.1106)  ( .1502,  -.1106)
( .1502,  -.1107)  ( .1501,  -.1109)  ( .1500,  -.1110)  ( .1499,  -.1111)  ( .1498,  -.1113)
(
SE,SEL,SEJ,TEF,TEL,LOOP     .169419     .158111     .170726     .653442     .694570      0
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```
(R STAR,X STAR)
( .1500,  -.1117)   ( .1501,  -.1115)   ( .1501,  -.1113)   ( .1502,  -.1111)   ( .1512,  -.1109)
( .1502,  -.1108)   ( .1503,  -.1107)   ( .1503,  -.1105)   ( .1503,  -.1105)   ( .1513,  -.1105)
( .1503,  -.1105)   ( .1503,  -.1105)   ( .1503,  -.1105)   ( .1503,  -.1106)   ( .1512,  -.1107)
( .1502,  -.1108)   ( .1501,  -.1109)   ( .1500,  -.1110)   ( .1499,  -.1112)   ( .1498,  -.1114)
(
SE,SEL,SEU,TEF,TEL,LOOP    .168765    .168111    .169419    .653442    .694570      0

(R STAR,X STAR)
( .1500,  -.1117)   ( .1500,  -.1114)   ( .1501,  -.1112)   ( .1501,  -.1111)   ( .1512,  -.1109)
( .1502,  -.1108)   ( .1503,  -.1107)   ( .1503,  -.1105)   ( .1503,  -.1105)   ( .1513,  -.1105)
( .1503,  -.1105)   ( .1503,  -.1105)   ( .1503,  -.1105)   ( .1503,  -.1106)   ( .1513,  -.1107)
( .1502,  -.1108)   ( .1501,  -.1109)   ( .1501,  -.1110)   ( .1500,  -.1112)   ( .1498,  -.1114)
(
SE,SEL,SEU,TEF,TEL,LOOP    .168438    .168111    .168765    .653442    .694570      1

             PRESSURE                  PRESS GRAD                MACH          X            SM           RE RHS       IM RHS
 20     .9939753   -.0000673    .0079745     .0217144     .5000000   1.6658430   0.0000000   -1.2887475   -3.5087412
 40     .9931330   -.0023582    .0475985     .1293067     .5000000   1.6354430   0.0000000   -1.3199923   -3.5688858
 60     .9735686   -.0567350    .2515995     .6480913     .5000000   1.4946430   0.0000000   -1.6229173   -3.7817333
 80     .6073102   -.7618695   1.6135262    2.3897040    .5000000   1.0338430   0.0000000   -5.0043902   -3.4582520
100    -.3578068  -1.7628125   4.3722505    3.2775589    .5000000    .6882430   0.0000000  -12.0560134   -1.2484847

   SIGMA ETA      TAU ETA     OPT R STAR    OPT X STAR
    .168501       .574862      .150336       -.110486
 +OR- .000163  +OR- .010839

THE ABOVE VALUES ARE FOR THE INNER WALL OF THE ANNULAR DUCT

             PRESSURE                  PRESS GRAD                MACH          X            SM           RE RHS       IM RHS
 20    -.3553971  -1.7611272  11.8552818    8.8597650    .5000000    .2562000   0.0000000  -68.7005911    -.8663309
 40    -.0249617  -1.4330615   9.9365757    8.7483631    .5000000    .2866000   0.0000000  -58.0816895   -6.3479662
 60     .7656133   -.6110587   5.0330982    7.2047201    .5000000    .3354000   0.0000000  -34.6923641  -20.7141578
 80    1.1732851    .1145789   1.6331178    3.9756403    .5000000    .5234000   0.0000000  -19.9789551  -28.1300597
100    1.1911811    .3537041  -1.1800305   -1.2158231    .5000000    .7154000   0.0000000  -11.5215981  -23.0877250
120     .5200239   -.4576182  -4.0789622   -4.0637495    .5000000    .9714000   0.0000000  -11.3801761    3.1790458
140    -.8253993  -1.0655854  -5.9141501     .4723139    .5000000   1.2274000   0.0000000     .9717419   29.7478208

(R STAR,X STAR)
( .3503,  -.1501)   ( .3394,  -.1469)   ( .3304,  -.1432)   ( .3236,  -.1396)   ( .3139,  -.1365)
( .3159,  -.1342)   ( .3144,  -.1331)   ( .3139,  -.1335)   ( .3140,  -.1356)   ( .3143,  -.1394)
( .3141,  -.1450)   ( .3131,  -.1521)   ( .3108,  -.1606)   ( .3068,  -.1699)   ( .3010,  -.1796)
( .2933,  -.1892)   ( .2837,  -.1982)   ( .2727,  -.2061)   ( .2604,  -.2127)   ( .2473,  -.2178)
(
SE,SEL,SEU,TEF,TEL,LOOP    .296326    .295925    .295926   1.676773   2.063069      0

             PRESSURE                  PRESS GRAD                MACH          X            SM           RE RHS       IM RHS
 20    -.3553899  -1.7511294  11.8535630    8.8590654    .5000000    .2562000   0.0000000  -68.3414669    -.9696813
 40    -.0247330  -1.4331197   9.9484159    9.7457815    .5000000    .2866000   0.0000000  -57.8105375   -6.3727491
 60     .7692080   -.6111955   5.0603955    7.2031613    .5000000    .3954000   0.0000000  -34.6516585  -20.6090789
 80    1.1731222    .1158152   1.6533474    3.9336279    .5000000    .5234000   0.0000000  -20.0950317  -28.0244927
100    1.2059171    .3769311  -1.1089549   -1.0372216    .5000000    .7090000   0.0000000  -11.6913696  -23.5070155
120     .5529215   -.4198020  -3.9951081   -4.0742549    .5000000    .9650000   0.0000000  -11.2602758    2.2011225
140    -.7724725  -1.0636322  -5.8720583     .2073539    .5000000   1.2210000   0.0000000     .3635605   28.9046563
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```
(R STAR,X STAR)
(  .3502,  -.1528)   ( .3386,  -.1491)   ( .3295,  -.1443)   ( .3228,  -.1408)   ( .3132,  -.1372)
(  .3155,  -.1346)   ( .3143,  -.1333)   ( .3141,  -.1335)   ( .3146,  -.1355)   ( .3152,  -.1393)
(  .3154,  -.1450)   ( .3147,  -.1524)   ( .3125,  -.1512)   ( .3097,  -.1708)   ( .3028,  -.1809)
(  .2950,  -.1908)   ( .2853,  -.2001)   ( .2740,  -.2082)   ( .2614,  -.2150)   ( .2480,  -.2201)
(
SE,SEL,SEU,TEF,TEL,LOOP     .292967     .283201     .302733     1.676773     2.063069     0

(R STAR,X STAR)
(  .3518,  -.1446)   ( .3407,  -.1425)   ( .3317,  -.1399)   ( .3249,  -.1372)   ( .3139,  -.1348)
(  .3145,  -.1332)   ( .3144,  -.1326)   ( .3134,  -.1333)   ( .3129,  -.1355)   ( .3124,  -.1393)
(  .3117,  -.1447)   ( .3102,  -.1514)   ( .3075,  -.1594)   ( .3033,  -.1681)   ( .2975,  -.1771)
(  .2843,  -.1861)   ( .2807,  -.1945)   ( .2701,  -.2020)   ( .2584,  -.2083)   ( .2458,  -.2132)
(
SE,SEL,SEU,TEF,TEL,LOOP     .304844     .294683     .315005     1.676773     2.063069     0

(R STAR,X STAR)
(  .3485,  -.1585)   ( .3366,  -.1535)   ( .3275,  -.1482)   ( .3210,  -.1431)   ( .3158,  -.1386)
(  .3145,  -.1353)   ( .3139,  -.1334)   ( .3145,  -.1332)   ( .3157,  -.1351)   ( .3171,  -.1390)
(  .3180,  -.1450)   ( .3179,  -.1528)   ( .3162,  -.1622)   ( .3126,  -.1726)   ( .3057,  -.1835)
(  .2987,  -.1941)   ( .2886,  -.2040)   ( .2768,  -.2127)   ( .2636,  -.2197)   ( .2436,  -.2251)
(
SE,SEL,SEU,TEF,TEL,LOOP     .285049     .275547     .296926     1.676773     2.063069     1

             PRESSURE                  PRESS GRAD                  MACH          X           SM            RE RHS        IM RHS
 20    -.3554578   -1.7013612   11.8373182    8.7849629     .5000000    .2562000    0.0000000    -71.4640704   -12.6051314
 40    -.0254764   -1.5005267    9.8713135    8.3621875     .5000000    .2866000    0.0000000    -59.4652421   -15.1718264
 60     .7650926    -.6359796    5.1828220    6.3028145     .5000000    .3954000    0.0000000    -31.2919922   -22.0287360
 80    1.2214354    -.0817240    2.2422193    3.2280989     .5000000    .5234000    0.0000000    -16.6577735   -25.1767180
100    1.3915543     .1262943     .5269522     .0534083     .5000000    .6514000    0.0000000    -11.3583783   -23.7228195
120    1.1426215    -.5175691   -2.6763459   -4.4501532     .5000000    .9074000    0.0000000    -15.7572365    -8.8859767
140    -.1358552   -1.6638724   -7.4818583   -3.2625528     .5000000   1.1634000    0.0000000    -19.2405014    20.0561106

   SIGMA ETA        TAU ETA       OPT R STAR      OPT X STAR
    .230988        1.302190        .314115        -.133192
+OR- .005333  +OR- .088422

THE ABOVE VALUES ARE FOR THE OUTER WALL OF THE ANNULAR DUCT

             PRESSURE                  PRESS GRAD                  MACH          X           SM            RE RHS        IM RHS
 20    -.7963592   -1.8437311   -8.9463030   -1.3059411     .5000000   1.2438000    0.0000000     -8.6954933    43.5974052
 40    -.5289719   -1.7904911   -8.6271627   -2.5708177     .5000000   1.2134000    0.0000000    -12.1394195    39.5310394
 60     .5277052   -1.1096777   -5.1979499   -6.5429095     .5000000   1.0726000    0.0000000    -20.1232675    15.9435716
 80    1.4935178     .5854753   -1.6811052   -4.8499410     .5000000    .8166000    0.0000000    -13.1756117   -26.8638912
100    1.5321687     .6656828    1.5348440    4.9459719     .5000000    .5606000    0.0000000    -17.3107234   -43.5608951
120     .6748097   -1.1343830    8.0106593   12.6410733     .5000000    .3558000    0.0000000    -52.1333263   -26.5936346

(R STAR,X STAR)
(  .1418,  -.1487)   ( .1412,  -.1427)   ( .1410,  -.1372)   ( .1411,  -.1321)   ( .1414,  -.1277)
(  .1417,  -.1240)   ( .1421,  -.1211)   ( .1422,  -.1193)   ( .1420,  -.1187)   ( .1415,  -.1197)
(  .1407,  -.1226)   ( .1400,  -.1277)   ( .1401,  -.1349)   ( .1418,  -.1436)   ( .1452,  -.1525)
(  .1495,  -.1603)   ( .1539,  -.1665)   ( .1573,  -.1713)   ( .1592,  -.1749)   ( .1536,  -.1777)
(
SE,SEL,SEU,TEF,TEL,LOOP     .290988     .290988     .290983     1.659557     1.934823     1

(R STAR,X STAR)
(  .1432,  -.1485)   ( .1426,  -.1441)   ( .1423,  -.1400)   ( .1421,  -.1361)   ( .1421,  -.1324)
(  .1422,  -.1291)   ( .1423,  -.1261)   ( .1425,  -.1236)   ( .1426,  -.1214)   ( .1425,  -.1198)
(  .1423,  -.1188)   ( .1419,  -.1185)   ( .1413,  -.1192)   ( .1404,  -.1209)   ( .1395,  -.1237)
(  .1357,  -.1279)   ( .1384,  -.1333)   ( .1390,  -.1397)   ( .1407,  -.1456)   ( .1434,  -.1533)
(
SE,SEL,SEU,TEF,TEL,LOOP     .293897     .290333     .295807     1.659557     1.861191     0
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(R STAR,X STAR)
(  .1425,  -.1480)   (  .1420,  -.1442)   (  .1417,  -.1400)   (  .1416,  -.1361)   (  .1416,  -.1324)
(  .1417,  -.1291)   (  .1419,  -.1261)   (  .1421,  -.1235)   (  .1423,  -.1214)   (  .1424,  -.1198)
(  .1423,  -.1189)   (  .1420,  -.1187)   (  .1414,  -.1193)   (  .1407,  -.1210)   (  .1339,  -.1238)
(  .1393,  -.1279)   (  .1392,  -.1333)   (  .1399,  -.1396)   (  .1416,  -.1463)   (  .1443,  -.1529)
(
SE,SEL,SEU,TEF,TEL,LOOP     .292442     .290998     .293897     1.669557     1.861181          1

                 PRESSURE                      PRESS GRAD                   MACH           X            SM            RE RHS        IM RHS
 20      -.7954671    -1.8499181    -8.9117834    -1.2455777      .5000000    1.2438000    0.0000000    -14.1136233    33.8447843
 40      -.5324311    -1.7970259    -8.4478615    -2.2138392      .5000000    1.2134000    0.0000000    -16.3136455    29.8318631
 60       .4750278    -1.2532143    -5.7782529    -5.0570618      .5000000    1.0726000    0.0000000    -19.9852000    10.5764360
 80      1.3624140      .0423850    -1.5118471    -3.8502322      .5000000     .8166000    0.0000000    -12.1777348   -17.7487686
100      1.4015950      .2629189     1.2259341     2.6614474      .5000000     .5606000    0.0000000    -13.0770991   -30.2257930
120       .8573304     -.7515738     5.3227135     8.1886433      .5000000     .3750000    0.0000000    -35.5931730   -27.5682400

     SIGMA ETA          TAU ETA      OPT R STAR      OPT X STAR
      .293170          1.770706        .142224        -.118714
  +OR- .000727     +OR- .101149

THE ABOVE VALUES ARE FOR THE INNER WALL OF THE ANNULAR DUCT

                 PRESSURE                      PRESS GRAD                   MACH           X            SM            RE RHS        IM RHS
 20      -.0980002    -1.9047111    11.5342154    11.1970095      .5000000     .2562000    0.0000000    -72.6704717    -6.0181209
 40       .2238716    -1.5630030     9.4999181    10.3253662      .5000000     .2866000    0.0000000    -61.6354523   -11.7685510
 60       .9473713     -.4840277     4.3073083     8.7145500      .5000000     .3954000    0.0000000    -36.3821093   -27.5271301
 80      1.2533954      .3766824      .8318080     4.5684620      .5000000     .5234000    0.0000000    -19.5529896   -35.3006391
100      1.1153295      .5967193    -1.8394772    -2.0870047      .5000000     .7282000    0.0000000     -9.4070283   -25.7454124
120       .3414810     -.4461803    -4.2134723    -4.5018325      .5000000     .9842000    0.0000000     -9.1099887     7.8572187
140      -.9385104    -1.0196301    -5.1324105     1.2188445      .5000000    1.2402000    0.0000000      6.1633785    32.9210322

(R STAR,X STAR)
(  .4143,  -.1478)   (  .3972,  -.1513)   (  .3816,  -.1526)   (  .3678,  -.1522)   (  .3558,  -.1507)
(  .3455,  -.1482)   (  .3371,  -.1453)   (  .3304,  -.1423)   (  .3252,  -.1394)   (  .3214,  -.1370)
(  .3189,  -.1351)   (  .3173,  -.1342)   (  .3166,  -.1342)   (  .3165,  -.1354)   (  .3156,  -.1377)
(  .3147,  -.1414)   (  .3165,  -.1463)   (  .3157,  -.1523)   (  .3140,  -.1592)   (  .3112,  -.1669)
(
SE,SEL,SEU,TEF,TEL,LOOP     .293170     .293170     .293170     1.618983     1.922429          0

                 PRESSURE                      PRESS GRAD                   MACH           X            SM            RE RHS        IM RHS
 20      -.0979957    -1.9047119    11.5356792    11.1967872      .5000000     .2562000    0.0000000    -72.4411888    -6.0487681
 40       .2210167    -1.5680227     9.5074625    10.3248859      .5000000     .2866000    0.0000000    -61.4679185   -11.7573537
 60       .9439552     -.4338532     4.3232798     8.7195819      .5000000     .3954000    0.0000000    -36.3802803   -27.4549937
 80      1.2566960      .3780852      .8408049     4.5822397      .5000000     .5234000    0.0000000    -19.6454311   -35.2452928
100      1.1135496      .6015957    -1.8473973    -2.0695267      .5000000     .7282000    0.0000000     -9.4449707   -25.7671570
120       .3435153     -.4385906    -4.2133059    -4.6036595      .5000000     .9842000    0.0000000     -8.9809818     7.7234584
140      -.9312096    -1.0183003    -5.1932704     1.1683697      .5000000    1.2402000    0.0000000      6.1367325    32.6960672

(R STAR,X STAR)
(  .4130,  -.1507)   (  .3975,  -.1539)   (  .3817,  -.1549)   (  .3676,  -.1543)   (  .3554,  -.1524)
(  .3451,  -.1497)   (  .3366,  -.1465)   (  .3299,  -.1432)   (  .3247,  -.1401)   (  .3210,  -.1374)
(  .3185,  -.1354)   (  .3172,  -.1343)   (  .3167,  -.1342)   (  .3167,  -.1353)   (  .3171,  -.1377)
(  .3173,  -.1414)   (  .3173,  -.1463)   (  .3166,  -.1524)   (  .3150,  -.1595)   (  .3123,  -.1673)
(
SE,SEL,SEU,TEF,TEL,LOOP     .290825     .286570     .294973     1.618983     1.922429          0
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(R STAR,X STAR)
( .4127,  -.1423)  ( .3904,  -.1462)  ( .3814,  -.1481)  ( .3680,  -.1483)  ( .3553,  -.1473)
( .3463,  -.1454)  ( .3379,  -.1431)  ( .3312,  -.1405)  ( .3259,  -.1381)  ( .3220,  -.1360)
( .3192,  -.1345)  ( .3174,  -.1338)  ( .3164,  -.1340)  ( .3150,  -.1353)  ( .3158,  -.1377)
( .3155,  -.1413)  ( .3150,  -.1461)  ( .3140,  -.1519)  ( .3121,  -.1586)  ( .3031,  -.1659)
(
SE,SEL,SEU,TEF,TEL,LOOP     .297861     .293170     .302115     1.618983     1.922429        0

(R STAR,X STAR)
( .4154,  -.1567)  ( .3981,  -.1593)  ( .3815,  -.1597)  ( .3671,  -.1584)  ( .3546,  -.1559)
( .3441,  -.1525)  ( .3356,  -.1488)  ( .3288,  -.1450)  ( .3238,  -.1414)  ( .3203,  -.1383)
( .3191,  -.1359)  ( .3170,  -.1345)  ( .3168,  -.1342)  ( .3172,  -.1352)  ( .3179,  -.1376)
( .3185,  -.1413)  ( .3189,  -.1464)  ( .3185,  -.1527)  ( .3171,  -.1501)  ( .3145,  -.1682)
(
SE,SEL,SEU,TEF,TEL,LOOP     .236134     .282046     .293170     1.618983     1.922429        0

(R STAR,X STAR)
( .4110,  -.1370)  ( .3954,  -.1413)  ( .3810,  -.1437)  ( .3681,  -.1444)  ( .3557,  -.1439)
( .3449,  -.1426)  ( .3386,  -.1408)  ( .3319,  -.1387)  ( .3265,  -.1367)  ( .3224,  -.1350)
( .3135,  -.1338)  ( .3175,  -.1334)  ( .3162,  -.1338)  ( .3154,  -.1352)  ( .3149,  -.1377)
( .3144,  -.1412)  ( .3136,  -.1458)  ( .3123,  -.1515)  ( .3102,  -.1579)  ( .3071,  -.1650)
(
SE,SEL,SEU,TEF,TEL,LOOP     .302551     .293170     .305874     1.618983     1.922429        0

(R STAR,X STAR)
( .4175,  -.1630)  ( .3984,  -.1650)  ( .3813,  -.1647)  ( .3654,  -.1627)  ( .3536,  -.1595)
( .3479,  -.1555)  ( .3343,  -.1511)  ( .3276,  -.1467)  ( .3227,  -.1426)  ( .3134,  -.1391)
( .3175,  -.1364)  ( .3157,  -.1346)  ( .3169,  -.1341)  ( .3177,  -.1350)  ( .3198,  -.1373)
( .3193,  -.1411)  ( .3205,  -.1463)  ( .3204,  -.1529)  ( .3193,  -.1606)  ( .3159,  -.1691)
(
SE,SEL,SEU,TEF,TEL,LOOP     .281443     .277423     .293170     1.618983     1.922429        1

(R STAR,X STAR)
( .3240,  -.1411)  ( .3224,  -.1397)  ( .3209,  -.1385)  ( .3198,  -.1374)  ( .3138,  -.1364)
( .3141,  -.1356)  ( .3175,  -.1350)  ( .3171,  -.1345)  ( .3169,  -.1343)  ( .3158,  -.1342)
( .3159,  -.1343)  ( .3159,  -.1347)  ( .3171,  -.1353)  ( .3174,  -.1362)  ( .3176,  -.1373)
( .3173,  -.1386)  ( .3181,  -.1402)  ( .3183,  -.1420)  ( .3184,  -.1441)  ( .3135,  -.1464)
(
SE,SEL,SEU,TEF,TEL,LOOP     .287307     .281443     .293170     1.740428     1.870984        0

(R STAR,X STAR)
( .3234,  -.1419)  ( .3218,  -.1404)  ( .3204,  -.1391)  ( .3193,  -.1379)  ( .3134,  -.1368)
( .3177,  -.1353)  ( .3172,  -.1352)  ( .3169,  -.1346)  ( .3168,  -.1343)  ( .3158,  -.1342)
( .3159,  -.1343)  ( .3171,  -.1346)  ( .3174,  -.1352)  ( .3178,  -.1361)  ( .3131,  -.1372)
( .3155,  -.1385)  ( .3188,  -.1401)  ( .3191,  -.1413)  ( .3193,  -.1440)  ( .3135,  -.1464)
(
SE,SEL,SEU,TEF,TEL,LOOP     .234375     .281443     .287307     1.740428     1.870984        1
```

	PRESSURE		PRESS GRAD		MACH	X	SM	RE RHS	IM RHS
20	-.0930422	-1.9050790	11.5214389	11.0795034	.5000000	.2562000	0.0000000	-74.3310635	-24.4113720
40	.2202016	-1.5736500	3.4871475	10.3252658	.5000000	.2966000	0.0000000	-60.1816962	-25.2006200
60	.3655378	-.6122204	4.7539015	7.3962182	.5000000	.3954000	0.0000000	-30.5359380	-28.5019782
80	1.3735712	.0952451	2.0047410	3.5324067	.5000000	.5234000	0.0000000	-14.8203253	-29.6932766
100	1.5335799	.3212858	.5285184	-.0471639	.5000000	.6514000	0.0000000	-9.6191700	-27.1937110
120	1.3381117	-.4141898	-2.3836278	-5.2035292	.5000000	.9010000	0.0000000	-16.0599689	-11.7154000
140	.0856649	-1.8313532	-7.7239189	-4.5445495	.5000000	1.1570000	0.0000000	-23.1891407	19.1605674

```
   SIGMA ETA        TAU ETA       OPT R STAR     OPT X STAR
    .285841         1.801421        .316905        -.134195
 +OR- .001465    +OR- .034286

THE ABOVE VALUES ARE FOR THE OUTER WALL OF THE ANNULAR DUCT

                PRESSURE                      PRESS GRAD                    MACH          X             SM            RE RHS        IM RHS
 20      -.6835238    -2.1376206    -9.5540493    -2.3791577       .5000000     1.2438000     0.0000000    -17.0604376    39.5716067
 40      -.3833347    -2.0477099    -9.0957442    -3.5126510       .5000000     1.2134000     0.0000000    -19.5570055    34.9603728
 60       .6815946    -1.2812739    -5.9434871    -6.8345724       .5000000     1.0726000     0.0000000    -23.1194302    12.0645015
 80      1.5478790      .4362381    -1.3558169    -5.0557059       .5000000      .8166000     0.0000000    -11.2552843   -23.5061974
100      1.6107286      .7162837      .8230570     3.6036245       .5000000      .5506000     0.0000000    -10.8167912   -40.6746169
120      1.2202459     -.5226875     4.3341393    10.7158771       .5000000      .3878000     0.0000000    -35.0025070   -40.0049406
140       .2722427    -2.2099014    11.4065283    15.5538933       .5000000      .2598000     0.0000000    -81.8334645   -35.9615341

(R STAR,X STAR)
(  .1393,  -.1203)   (  .1402,  -.1186)   (  .1405,  -.1171)   (  .1407,  -.1159)   (  .1418,  -.1150)
(  .1403,  -.1144)   (  .1409,  -.1142)   (  .1407,  -.1144)   (  .1404,  -.1150)   (  .1411,  -.1162)
(  .1395,  -.1179)   (  .1392,  -.1202)   (  .1389,  -.1232)   (  .1387,  -.1267)   (  .1339,  -.1309)
(  .1394,  -.1354)   (  .1405,  -.1403)   (  .1420,  -.1452)   (  .1440,  -.1500)   (  .1452,  -.1545)
(
SE,SEL,SEU,TEF,TEL,LOOP     .285841     .285841     .285841     1.734754     1.868087        0

                PRESSURE                      PRESS GRAD                    MACH          X             SM            RE RHS        IM RHS
 20      -.6645196    -2.1376224    -9.6554003    -2.3785990       .5000000     1.2438000     0.0000000    -16.8425458    39.4844249
 40      -.3831891    -2.0477646    -9.1035730    -3.5038605       .5000000     1.2134000     0.0000000    -19.3433429    34.9002000
 60       .6847430    -1.2819955    -5.9819383    -6.8296833       .5000000     1.0726000     0.0000000    -23.0018055    12.0808546
 80      1.5600227      .4348873    -1.3805357    -5.0553547       .5000000      .8166000     0.0000000    -11.4333440   -23.5143291
100      1.6211604      .7141775      .8658402     3.6065659       .5000000      .5606000     0.0000000    -11.1218533   -40.6414294
120      1.2183772     -.5241000     4.4713383    10.7000422       .5000000      .3878000     0.0000000    -35.1702288   -39.8037962
140       .2539585    -2.2071772    11.5013281    15.5012123       .5000000      .2598000     0.0000000    -81.7610641   -35.5770953

(R STAR,X STAR)
(  .1394,  -.1202)   (  .1397,  -.1185)   (  .1401,  -.1170)   (  .1404,  -.1159)   (  .1416,  -.1150)
(  .1403,  -.1144)   (  .1408,  -.1142)   (  .1408,  -.1144)   (  .1407,  -.1151)   (  .1414,  -.1163)
(  .1402,  -.1180)   (  .1399,  -.1203)   (  .1397,  -.1232)   (  .1397,  -.1267)   (  .1400,  -.1308)
(  .1407,  -.1352)   (  .1418,  -.1399)   (  .1433,  -.1447)   (  .1452,  -.1495)   (  .1474,  -.1539)
(
SE,SEL,SEU,TEF,TEL,LOOP     .283935     .280730     .287090     1.734754     1.868087        0
ICUSP,SEL,SEU         1          .283935       .285841

(R STAR,X STAR)
(  .1407,  -.1150)   (  .1408,  -.1148)   (  .1408,  -.1146)   (  .1408,  -.1144)   (  .1409,  -.1143)
(  .1409,  -.1142)   (  .1409,  -.1142)   (  .1408,  -.1142)   (  .1408,  -.1142)   (  .1408,  -.1143)
(  .1407,  -.1145)   (  .1407,  -.1146)   (  .1406,  -.1143)   (  .1405,  -.1151)   (  .1404,  -.1155)
(  .1403,  -.1159)   (  .1402,  -.1163)   (  .1401,  -.1169)   (  .1400,  -.1173)   (  .1399,  -.1179)
(
SE,SEL,SEU,TEF,TEL,LOOP     .284888     .283935     .285841     1.762009     1.803698        0

(R STAR,X STAR)
(  .1405,  -.1150)   (  .1407,  -.1147)   (  .1407,  -.1145)   (  .1408,  -.1144)   (  .1408,  -.1143)
(  .1408,  -.1142)   (  .1408,  -.1142)   (  .1408,  -.1142)   (  .1408,  -.1143)   (  .1408,  -.1143)
(  .1408,  -.1145)   (  .1408,  -.1147)   (  .1407,  -.1143)   (  .1407,  -.1152)   (  .1406,  -.1155)
(  .1405,  -.1159)   (  .1404,  -.1163)   (  .1404,  -.1163)   (  .1403,  -.1174)   (  .1402,  -.1180)
(
SE,SEL,SEU,TEF,TEL,LOOP     .283935     .283935     .285841     1.762009     1.803698        0
ICUSP,SEL,SEU         1          .283935       .284888
```

	PRESSURE		PRESS GRAD		MACH	X	SM	RE RHS	IM RHS
20	-.6685399	-2.1377159	-9.6425441	-2.3484195	.5000000	1.2438000	0.0000000	-18.8504707	34.6159754
40	-.3844523	-2.0510186	-9.0391389	-3.3325308	.5000000	1.2134000	0.0000000	-20.7474769	30.1129055
60	.6675388	-1.3530325	-5.8611073	-6.0912507	.5000000	1.0726000	0.0000000	-22.7183061	9.2710591
80	1.5219021	.1492687	-1.3001779	-4.3898058	.5000000	.8166000	0.0000000	-11.8932416	-20.0654701
100	1.5379306	.4244637	1.1327052	2.8150654	.5000000	.5606000	0.0000000	-11.3528420	-33.5792574
120	1.0697343	-.6298820	4.7415507	8.9540220	.5000000	.3814000	0.0000000	-33.4451885	-33.7175500
140	.0953412	-2.0455985	11.4142111	13.1238725	.5000000	.2534000	0.0000000	-78.1461590	-32.0124873

SIGMA ETA	TAU ETA	OPT R STAR	OPT X STAR
.284411	1.777267	.140843	-.114205
+OR- .000475	+OR- .006562		

THE ABOVE VALUES ARE FOR THE INNER WALL OF THE ANNULAR DUCT
ATTENUATION PER DUCT HEIGHT = 7.76 DB PER H
ATTENUATION PER INCH = 1.940 DB PER INCH

EACH OF THE FOLLOWING LININGS IS OPTIMUM FOR THE GIVEN INPUT CONDITIONS. SEVERAL ARE GIVEN TO PERMIT A SELECTION BY THE DESIGNER.

FACE SHEET THICK - IN	DRILL NO. STD	DRILL DIA INCHES	FRACTIONAL OPEN AREA	CORE BACKING DEPTH, INCHES
.0300	60	.0400	.1445	.3791
.0300	54	.0550	.1434	.3752
.0300	50	.0700	.1428	.3717
.0300	46	.0810	.1425	.3693
.0300	41	.0960	.1422	.3662
.0300	35	.1100	.1413	.3634
.0300	24	.1520	.1415	.3553

```
                  PRESSURE                      PRESS GRAD                  MACH         X           SM           RE RHS        IM RHS
 20      .1259973   -2.0039614     11.1977867     13.0393053      .5000000     .2562000    0.0000000    -76.5527489   -31.1909062
 40      .4344461   -1.6269398      9.1029795     12.0832041      .5000000     .2866000    0.0000000    -61.9431901   -31.4188171
 60     1.1316285    -.5023997      4.2683024      8.5355334      .5000000     .3954000    0.0000000    -30.8858250   -33.8935536
 80     1.4806842     .3085122      1.5407767      4.1226663      .5000000     .5234000    0.0000000    -14.0995291   -34.4058763
100     1.5853247     .5578342       .0753624      -.4809053      .5000000     .6642000    0.0000000     -8.4930714   -30.1368307
120     1.2779280    -.3727965     -2.8428522     -5.9115522      .5000000     .9202000    0.0000000    -16.5508798    -9.5711906
140     -.0959063   -1.8499003     -8.1072063     -4.1414913      .5000000    1.1762000    0.0000000    -21.0538145    24.9937791

(R STAR,X STAR)
(  .3212,   -.1371)   (  .3211,   -.1370)   (  .3210,   -.1369)   (  .3209,   -.1368)   (  .3208,   -.1367)
(  .3207,   -.1366)   (  .3206,   -.1365)   (  .3205,   -.1365)   (  .3205,   -.1364)   (  .3204,   -.1363)
(  .3203,   -.1362)   (  .3202,   -.1362)   (  .3202,   -.1361)   (  .3201,   -.1360)   (  .3200,   -.1360)
(  .3200,   -.1359)   (  .3199,   -.1358)   (  .3199,   -.1358)   (  .3198,   -.1357)   (  .3198,   -.1357)
(
SE,SEL,SEU,TEF,TEL,LOOP     .284411      .284411      .284411     1.770049     1.784485         0
                  PRESSURE                      PRESS GRAD                  MACH         X           SM           RE RHS        IM RHS
 20      .1259973   -2.0089614     11.1977867     13.0393053      .5000000     .2562000    0.0000000    -76.5527489   -31.1909062
 40      .4344461   -1.6269838      9.1029795     12.0892041      .5000000     .2866000    0.0000000    -61.9431901   -31.4188171
 60     1.1316285    -.5023997      4.2683024      8.5355384      .5000000     .3954000    0.0000000    -30.8858250   -33.8935536
 80     1.4806842     .3085122      1.5407767      4.1226663      .5000000     .5234000    0.0000000    -14.0995291   -34.4058763
100     1.5853247     .5578342       .0753624      -.4809053      .5000000     .6642000    0.0000000     -8.4930714   -30.1368307
120     1.2779280    -.3727965     -2.8428522     -5.9115522      .5000000     .9202000    0.0000000    -16.5508798    -9.5711906
140     -.0959063   -1.8499003     -8.1072063     -4.1414913      .5000000    1.1762000    0.0000000    -21.0538145    24.9997791

(R STAR,X STAR)
(  .3212,   -.1371)   (  .3210,   -.1370)   (  .3209,   -.1359)   (  .3208,   -.1367)   (  .3207,   -.1366)
(  .3205,   -.1365)   (  .3205,   -.1364)   (  .3204,   -.1363)   (  .3203,   -.1362)   (  .3202,   -.1361)
(  .3201,   -.1361)   (  .3200,   -.1360)   (  .3200,   -.1359)   (  .3199,   -.1358)   (  .3198,   -.1357)
(  .3198,   -.1357)   (  .3197,   -.1356)   (  .3196,   -.1355)   (  .3196,   -.1355)   (  .3195,   -.1354)
(
SE,SEL,SEU,TEF,TEL,LOOP     .284411      .284411      .284411     1.770049     1.788304         0

(R STAR,X STAR)
(  .3212,   -.1371)   (  .3210,   -.1369)   (  .3209,   -.1368)   (  .3207,   -.1367)   (  .3206,   -.1365)
(  .3205,   -.1364)   (  .3203,   -.1363)   (  .3202,   -.1362)   (  .3201,   -.1360)   (  .3200,   -.1359)
(  .3199,   -.1358)   (  .3198,   -.1357)   (  .3197,   -.1357)   (  .3197,   -.1356)   (  .3196,   -.1355)
(  .3195,   -.1354)   (  .3195,   -.1354)   (  .3194,   -.1353)   (  .3193,   -.1353)   (  .3193,   -.1352)
(
SE,SEL,SEU,TEF,TEL,LOOP     .284411      .284411      .284411     1.770049     1.793133         0

(R STAR,X STAR)
(  .3212,   -.1371)   (  .3210,   -.1369)   (  .3208,   -.1367)   (  .3206,   -.1366)   (  .3205,   -.1364)
(  .3203,   -.1362)   (  .3202,   -.1361)   (  .3200,   -.1360)   (  .3199,   -.1358)   (  .3198,   -.1357)
(  .3197,   -.1356)   (  .3196,   -.1355)   (  .3195,   -.1354)   (  .3194,   -.1353)   (  .3194,   -.1353)
(  .3193,   -.1352)   (  .3192,   -.1352)   (  .3192,   -.1351)   (  .3192,   -.1351)   (  .3191,   -.1351)
(
SE,SEL,SEU,TEF,TEL,LOOP     .284411      .284411      .284411     1.770049     1.799265         0

(R STAR,X STAR)
(  .3212,   -.1371)   (  .3209,   -.1369)   (  .3207,   -.1366)   (  .3205,   -.1364)   (  .3203,   -.1362)
(  .3201,   -.1360)   (  .3200,   -.1359)   (  .3198,   -.1357)   (  .3197,   -.1356)   (  .3196,   -.1355)
(  .3195,   -.1354)   (  .3194,   -.1353)   (  .3193,   -.1352)   (  .3192,   -.1352)   (  .3192,   -.1351)
(  .3191,   -.1351)   (  .3191,   -.1351)   (  .3191,   -.1351)   (  .3191,   -.1351)   (  .3191,   -.1352)
(
SE,SEL,SEU,TEF,TEL,LOOP     .284411      .284411      .284411     1.770049     1.807034         0
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```
(R STAR,X STAR)
(  .3200,  -.1380)   (  .3198,  -.1377)   (  .3196,  -.1374)   (  .3195,  -.1371)   (  .3134,  -.1369)
(  .3133,  -.1366)   (  .3192,  -.1364)   (  .3191,  -.1362)   (  .3130,  -.1360)   (  .3130,  -.1358)
(  .3139,  -.1356)   (  .3189,  -.1355)   (  .3189,  -.1354)   (  .3189,  -.1353)   (  .3130,  -.1352)
(  .3130,  -.1351)   (  .3190,  -.1351)   (  .3191,  -.1350)   (  .3192,  -.1350)   (  .3132,  -.1350)
(
SE,SEL,SEU,TEF,TEL,LOOP     .276285     .273774     .284411     1.770049     1.807034        0
ICUSP,SEL,SEU           1        .276285        .284411

(R STAR,X STAR)
(  .3131,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3131,  -.1351)
(  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3131,  -.1351)
(  .3131,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3131,  -.1351)
(  .3191,  -.1351)   (  .3191,  -.1351)   (  .3192,  -.1351)   (  .3192,  -.1352)   (  .3132,  -.1352)
(
SE,SEL,SEU,TEF,TEL,LOOP     .280348     .276235     .284411     1.801143     1.807034        0

(R STAR,X STAR)
(  .3190,  -.1351)   (  .3190,  -.1350)   (  .3191,  -.1350)   (  .3191,  -.1350)   (  .3131,  -.1350)
(  .3191,  -.1350)   (  .3191,  -.1350)   (  .3191,  -.1350)   (  .3191,  -.1350)   (  .3131,  -.1350)
(  .3131,  -.1350)   (  .3131,  -.1350)   (  .3191,  -.1350)   (  .3192,  -.1350)   (  .3132,  -.1350)
(  .3192,  -.1350)   (  .3192,  -.1350)   (  .3192,  -.1350)   (  .3192,  -.1350)   (  .3132,  -.1350)
(
SE,SEL,SEU,TEF,TEL,LOOP     .276285     .276235     .284411     1.801143     1.807034        0

(R STAR,X STAR)
(  .3131,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3131,  -.1351)
(  .3131,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3131,  -.1351)
(  .3131,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1352)   (  .3131,  -.1352)
(  .3191,  -.1352)   (  .3191,  -.1352)   (  .3191,  -.1352)   (  .3191,  -.1352)   (  .3131,  -.1352)
(
SE,SEL,SEU,TEF,TEL,LOOP     .284411     .276235     .284411     1.801143     1.807034        0

(R STAR,X STAR)
(  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3131,  -.1351)
(  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3131,  -.1351)   (  .3131,  -.1351)
(  .3131,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3191,  -.1351)   (  .3131,  -.1351)
(  .3191,  -.1352)   (  .3191,  -.1352)   (  .3191,  -.1352)   (  .3191,  -.1352)   (  .3131,  -.1352)
(
SE,SEL,SEU,TEF,TEL,LOOP     .284411     .276235     .284411     1.799536     1.807034        0
ICUSP,SEL,SEU           1        .280348        .284411

              PRESSURE                    PRESS GRAD                  MACH          X            SM           RE RHS        IM RHS
  20     .1253993    -2.0090286    11.1935387    13.0178607    .5000000     .2562000    0.0000000    -75.3730622   -34.5334746
  40     .4345313    -1.6291010     9.1165743    11.9810204    .5000000     .2866000    0.0000000    -61.3103795   -33.8106844
  60    1.1387569     -.5253028     4.3913855     8.3022097    .5000000     .3954000    0.0000000    -29.6984195   -34.0811592
  80    1.5130268      .2377737     1.8054875     3.3429773    .5000000     .5234000    0.0000000    -13.1613568   -33.6051585
 100    1.6553139      .4943199      .5333793     -.1744243    .5000000     .6514000    0.0000000     -8.1325941   -30.2026155
 120    1.4925731     -.3545185    -2.2233245    -5.9391415    .5000000     .9010000    0.0000000    -16.6519935   -13.5956183
 140     .2303999    -2.0030866    -8.0835844    -5.5328965    .5000000    1.1570000    0.0000000    -26.5341858    19.3805809
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SIGMA ETA	TAU ETA	OPT R STAR	OPT X STAR
.282380	1.800769	.319098	-.135118
+OR- .002032	+OR- .001173		

THE ABOVE VALUES ARE FOR THE OUTER WALL OF THE ANNULAR DUCT
ATTENUATION PER DUCT HEIGHT = 7.71 DB PER H
ATTENUATION PER INCH = 1.926 DB PER INCH

EACH OF THE FOLLOWING LININGS IS OPTIMUM FOR THE GIVEN INPUT CONDITIONS. SEVERAL ARE GIVEN TO PERMIT A SELECTION BY THE DESIGNER.

FACE SHEET THICK - IN	DRILL NO. STD	DRILL DIA INCHES	FRACTIONAL OPEN AREA	CORE BACKING DEPTH, INCHES
.0300	60	.0400	.0640	.2493
.0300	54	.0550	.0635	.2445
.0300	50	.0700	.0632	.2403
.0300	46	.0810	.0631	.2375
.0300	41	.0960	.0629	.2337
.0300	35	.1100	.0628	.2304
.0300	24	.1520	.0627	.2212

APPENDIX B

INTERACTIVE VERSION OF PROGRAM OPTSHE

B. 1 PROGRAM LISTING

```
      PROGRAM OPTSHE(TAPE7,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON/BLK1/FM,ETA,PI,AM
      COMMON/BLKA/SSF,TTF,DSH,H,SM
      COMMON/BLKB/IDUCT,ROUT,K,KZ
      COMMON/BLKC/NSOFT,XSTART,NIT
      COMMON/BLKD/HA,DSHA,RM
      COMMON/BLKE/US,NU,AL,BL,C
      COMMON/BLKH/ACC,JLIM
      DIMENSION X(50),Y(50)
      COMPLEX KZ
      REAL K,NU
      WRITE(6,135)
      READ(5,*) JP
      IF(JP.NE.1) GO TO 77
      WRITE(6,100)
      READ(5,*) FREQ,AM,ROUT,TEMP
      PI=3.1416
      C=49.02*12*SQRT(TEMP)
      H=ROUT
      K=2.*PI*FREQ/C
      ETA=2.*H*FREQ/C
      WRITE(6,120)
      READ(5,*) FM,DSH
      WRITE(6,130)
      READ(5,*) IDUCT,NSOFT,NIT,IW
      WRITE(6,140)
      READ(5,*) ACC,JLIM
      WRITE(6,150) FREQ,AM,ROUT,TEMP
      WRITE(6,160) FM,DSH,ETA
      WRITE(6,170) IDUCT,NSOFT,NIT,IW,JLIM,ACC
      IF(FM.EQ.0. .OR. DSH.EQ.0.) GO TO 16
      WRITE(6,180)
      READ(5,*) PRESS,RHO
      NU=144.*(.279+.001939*TEMP-2.568E-7*TEMP*TEMP)/(RHO*1.E5)
      US=SQRT(.0223*C*C*FM*FM*((C*ABS(FM)*DSH*H/NU)**(-.25)))
      AL=5.*NU/(H*US)
      BL=6.*AL
16    CONTINUE
      WRITE(6,190)
      READ(5,*)LLL
      IF(LLL.EQ.4) CALL EIGEN
      IF(LLL.EQ.3) WRITE(6,230)
      IF(LLL.EQ.3) READ(5,*) FM,DSH,AM,FREQ,JLIM
      IF(LLL.EQ.3) WRITE(6,150) FREQ,AM,ROUT,TEMP
      IF(LLL.EQ.3) WRITE(6,160) FM,DSH,ETA
      IF(LLL.NE.1 .AND. LLL.NE.3 .AND. LLL.NE.4) GO TO 700
      IF(LLL.EQ.4 .OR. LLL.EQ.3) GO TO 16
```

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```
      WRITE(6,200)
      READ(5,*) SE,TE
      WRITE(6,210)
      READ(5,*) STEP
      DO 17 I=1,10
      CALL SHESET(SE,TE,RS.XS,IW)
      WRITE(6,250) SE,TE,RS,XS
      TE=TE+STEP
17    CONTINUE
      GO TO 16
700   IF(LLL.NE.2) STOP
      WRITE(6,200)
      READ(5,*) SE,TE
      CALL SHESET(SE,TE,RS,XS,1)
       WRITE(6,250) SE,TE,RS,XS
      GO TO 16
77    STOP
135   FORMAT(1X,*ENTER 1*)
100   FORMAT(1X,*ENTER  FREQ,AM,ROUT,TEMP*)
120   FORMAT(1X,*ENTER  FM,DSH*)
130   FORMAT(1X,*ENTER  IDUCT,NSOFT,NIT,IW*)
140   FORMAT(1X,*ENTER ACC,JLIM*)
150   FORMAT(1X,*FREQ=*,F8.2,3X,*AM=*,F5.2,3X,*ROUT=*,F6.3,
     23X,*TEMP=*,F8.3)
160   FORMAT(1X,*FM=*,F8.2,5X,*DSH=*,F6.5,3X,*ETA=*,F8.5)
170   FORMAT(1X,*IDUCT=*,I2,3X,*NSOFT=*,I2,3X,*NIT=*,I4,3X,
     3*IW=*,I2,3X,*JLIM=*,I4,3X,*ACC=*,E6.1)
180   FORMAT(1X,*ENTER PRESS,RHO*)
190   FORMAT(1X,*ENTER 0,1,2,3,4*)
200   FORMAT(1X,*ENTER SE,TE*)
210   FORMAT(1X,*ENTER STEP*)
230   FORMAT(1X,*ENTER FM,DSH,AM,FREQ,JLIM*)
250   FORMAT(1X,*SE=*,F10.8,4X,*TE=*,F12.8,4X,*RS=*,F12.8,
     24X,*XS=*,F13.8)
      END
      SUBROUTINE EIGEN
      DIMENSION TE(3),RS(3),XS(3)
      WRITE(6,100)
      READ(5,*) SE,TE
      WRITE(6,120)
      READ(5,*) RS,XS
      WRITE(6,140)
      READ(5,*)RSD,XSD
      DELRS=RS(3)-RS(1)
      DELXS=XS(3)-XS(1)
      DELTE=TE(3)-TE(1)
      DERRS=DELRS/DELTE
      DERXS=DELXS/DELTE
      R=RSD-RS(2)
      X=XSD-XS(2)
      DENOM=DERRS*DERRS+DERXS*DERXS
      DELTE=(R*DERRS+X*DERXS)/DENOM
      DELSE=-(X*DERRS-R*DERXS)/DENOM
      TEN=TE(2)+DELTE
      SEN=SE+DELSE
      WRITE(6,455) TEN,SEN
100   FORMAT(1X,*ENTER SE, TE'S *)
120   FORMAT(1X,*ENTER RS'S,  XS'S *)
```

```
140   FORMAT(1X,*ENTER RSD, XSD *)
455   FORMAT(1X,*TEN= *,F12.8,6X,*SEN= *,F12.8)
      RETURN
      END
      SUBROUTINE SHESET (SE,TE,RS,XS,IW)
      COMMON/BLK1/FM,ETA,PT,AM
      COMMON /BLKA/ SSE,TTE,DSH,H,SM
      COMMON /BLKB/ IDUCT,ROUT,K,KZ
      COMMON /BLKC/ NSCET,XSTART,NIT
      COMMON /BLKD/ HA,DSHA,RM
      COMMON /BLKE/ US,NU,AL,BL,C
      COMMON /BLKF/ VM,RHS
      COMMON/BLKH/ACC,JLIM
      COMPLEX AI,ZETAS,KZ,C,PP,RHS
      REAL K,NU
      DIMENSION Y(4),YL(4)
      LOGICAL MODE
      EXTERNAL DERIV
      MODE=.FALSE.
      SSE=SE
      TTE=TE
      AI=CMPLX(0.,1.)
      N=4
      IF (IDUCT) 21,22,23
   23 XSTART=0.
      SDX=1.
      GO TO 24
   22 XSTART=ROUT/H-1.
      SDX=1.
      GO TO 24
   21 IF (MOD(NIT,2).EQ.0) XSTART=ROUT/H
      IF (MOD(NIT,2).NE.0) GO TO 22
      SDX=-1.
      IF ((NSCET.EQ.2).AND.(NIT.EQ.0)) 26,24
   26 RI=ROUT-H
      HA=RM-RI
      XSTART=RM/HA
      X=XSTART
      SDX=-1.
      DSHA=DSH*H/HA
      IF ((DSH.NE.0.).AND.(FM.NE.0.)) ALA=5.*NU/(HA*US)
      IF ((DSH.NE.0.).AND.(FM.NE.0.)) BLA=6.*ALA
      Y(1)=1.
      Y(2)=Y(3)=Y(4)=0.
      GO TO 80
   24 X=XSTART
      IF(AM.GT..0.AND.XSTART.EQ..0.AND.IDUCT.EQ.0) X=.001
      IF (NIT.GT.0) GO TO 27
      Y(1)=1.
      CC=1.0
      IF(X.EQ..001) Y(1)=CC*X**AM
      Y(2)=Y(3)=Y(4)=0.
      IF(X.EQ..001) Y(3)=CC*AM*X**(AM-1.0)
      GO TO 80
   27 Y(1)=YL(1)
      Y(2)=YL(2)
      Y(3)=YL(3)
      Y(4)=YL(4)
```

```
      GO TO 82
80    IF(IW.GE.1) WRITE(7,100)
  100 FORMAT (/*                                    PRESSURE                               PRESS GRAD
     1                MACH                          X                SM                     RE RHS
     2IM RHS *)
  101 FORMAT (1X,I4,9F14.7)
      DX=SDX*2.E-4
      J=1
      A=0.0
      CALL SET (N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      IF(IW.GE.1) WRITE(7,101)J,Y,VM,X,SM,RHS
   82 CALL STEP (N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      J=J+1
      IF(J.GT.JLIM) WRITE(6,728) J,X
728   FORMAT(1X,*DID NOT REACH WALL IN *,I4,* STEPS,X= *,F7.6)
      IF(J.GT.JLIM) GO TO 31
      IF(IW.GE.1.AND.(MOD(J,20).EQ.0.OR.J.EQ.2.OR.(ABS(DX).GT..005.AND.
     +MOD(J,5).EQ.0).OR.(A.LT.0..AND.MOD(J,5).EQ.0)))
     +WRITE(7,101)J,Y,VM,X,SM,RHS
      B=1.-ABS(X-XSTART)
      A=B-DSH
      IF(A.GT.ABS(DX)) GO TO 82
      IF(A.LT.2.E-6) GO TO 84
      DX=SDX*A
      CALL SET(N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      CALL STEP(N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      IF(IW.GE.1) WRITE(7,101) J,Y,VM,X,SM,RHS
84    IF(ABS(DX).LE.(DSH/19.).AND.B.GT.ABS(DX)) GO TO 82
      IF(B.LT.ABS(DX).AND.B.GT.2.E-6) GO TO 86
      IF(B.LE.2.E-6) GO TO 88
      DX=SDX*DSH/20.
      CALL SET(N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      GO TO 82
86    DX=SDX*B
      DX=SDX*(1.-ABS(X-XSTART))
      CALL SET(N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
      CALL STEP(N,X,Y,DX,DERIV,ACC,MODE,.05,2.E-6)
88    CONTINUE
      IF(IDUCT.EQ.-1.AND.NSOFT.EQ.2.AND.NIT.EQ.0) Y(3)=Y(3)*H/HA
      IF(IDUCT.EQ.-1.AND.NSOFT.EQ.2.AND.NIT.EQ.0) Y(4)=Y(4)*H/HA
      IF(IW.GE.1) WRITE(7,101) J,Y,VM,X,SM,RHS
   31 IF(IW.EQ.0) GO TO 3
      IF (IDUCT.NE.-1.OR.NSOFT.NE.2) GO TO 3
      YL(1)=Y(1)
      YL(2)=Y(2)
      YL(3)=Y(3)
      YL(4)=Y(4)
    3 P=CMPLX(Y(1),Y(2))
      PP=CMPLX(Y(3),Y(4))
      ZETAS=-AI*PI*P/PP
      IF (DSH.EQ.0.) ZETAS=ZETAS*(1.-FM*KZ/K)**2
      IF ((IDUCT.EQ.-1).AND.(MOD(NIT,2).EQ.0)) ZETAS=-ZETAS
      RS=REAL(ZETAS)
      XS=AIMAG(ZETAS)
      RETURN
      END
```

```
      SUBROUTINE DERIV (N,X,Y,DY)
      COMMON/BLK1/FM,ETA,PI,AM
      COMMON /BLKA/ SE,TE,DSH,H,SM
      COMMON /BLKB/ IDUCT,FOUT,K,KZ
      COMMON /BLKC/ NSOFT,XSTART,NIT
      COMMON /BLKD/ HA,DSHA,RM
      COMMON /BLKE/ US,NU,AL,BL,C
      COMMON /BLKF/ VM,RHS
      COMPLEX A,B,RHS,P,DP,KZ
      REAL K,NU
      DIMENSION Y(N),DY(N)
      AKZ=K*TE/ETA
      BKZ=-K*SE/ETA
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) AKZ=H*AKZ/HA
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) BKZ=H*BKZ/HA
      KZ=CMPLX(AKZ,BKZ)
      IF ((FM.EQ.0.).OR.(DSH.EQ.0.)) GO TO 12
      SIGN=1.
      IF (FM.LT.0.) SIGN=-1.
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) 31,32
31    DB=X-(XSTART-1.)
      IF (DB.GE.DSHA) GO TO 12
      ALA=5.*NU/(HA*US)
      BLA=6.*ALA
      YP=DB*HA*US/NU
      Q=.407*YP
      IF (YP.GT.5) Q=.407*(-3.05+5.*ALOG(YP))
      IF (YP.GT.30) Q=.407*(5.5+2.5*ALOG(YP))
      SQ=1./.407+.1*(EXP(Q)-1.-Q-Q*Q/2.-Q*Q*Q/6.)
      SM=SIGN*HA*US*US/(C*NU*.407*SQ)
      E=1.-DB/DSHA
      IF (E.LT..1) SM=SM*10.*E
      GO TO 17
32    DA=ABS(X-XSTART)
      DB=(XSTART+1.-X)
      IF (DB.GT.1.) DB=X-(XSTART-1.)
      IF ((DA.GE.DSH).AND.(DB.GE.DSH)) GO TO 12
      IF ((DA.LT.DSH).AND.(NSOFT.EQ.2).AND.(IDUCT.EQ.1)) GO TO 12
      IF ((DA.LT.DSH).AND.(XSTART.EQ.0.).AND.(IDUCT.EQ.0)) GO TO 12
      YP=DA*H*US/NU
      IF (DB.LT.DA) YP=DB*YP/DA
      Q=.407*YP
      IF (YP.GT.5) Q=.407*(-3.05+5.*ALOG(YP))
      IF (YP.GT.30) Q=.407*(5.5+2.5*ALOG(YP))
      SQ=1./.407+.1*(EXP(Q)-1.-Q-Q*Q/2.-Q*Q*Q/6.)
      SM=SIGN*H*US*US/(C*NU*.407*SQ)
      E=1.-DA/DSH
      IF (DB.LT.DA) E=1.-DB/DSH
      IF (E.LT..1) SM=SM*10.*E
      IF (DB.LT.DA) SM=-SM
      IF ((IDUCT.EQ.-1).AND.(MOD(NIT,2).EQ.0)) SM=-SM
      GO TO 17
12    SM=0.
17    CONTINUE
```

AD-A057 803 DAYTON UNIV OHIO SCHOOL OF ENGINEERING F/G 20/1
SOUND TRANSMISSION THROUGH DUCTS.(U)
MAY 78 J J SCHAUER, E P HOFFMAN, R W GUYTON F33615-74-C-2030
UNCLASSIFIED UDSE-TR-78-03 AFAPL-TR-78-25 NL

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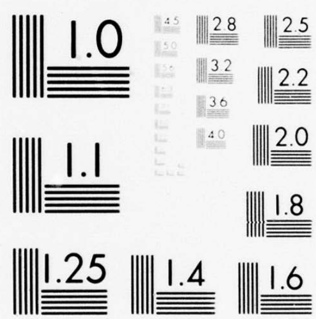
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2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0

```
      CALL MACH (FM,VM,DA,DB,ALA,BLA)
      A=2*KZ*SM/(K-VM*KZ)
      IF (IDUCT) 22,22,21
   22 IF (X.GT.0.) A=A+1./X
   21 B=H*H*(K*K-KZ*KZ*(1.-VM*VM)-2.*VM*K*KZ)
      IF(X.GT..0) B=B-(AM/X)**2
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) B=HA*HA*B/(H*H)
      P =CMPLX(Y(1),Y(2))
      PP=CMPLX(Y(3),Y(4))
      RHS=-A*PP-B*P
      DY(1)=Y(3)
      DY(2)=Y(4)
      DY(3)=REAL(RHS)
      DY(4)=AIMAG(RHS)
      RETURN
      END
      SUBROUTINE MACH (FM,VM,DA,DB,ALA,BLA)
      COMMON /BLKA/ SE,TE,DSH,H,SM
      COMMON /BLKB/ IDUCT,ROUT,K,KZ
      COMMON /BLKC/ NSOFT,XSTART,NIT
      COMMON /BLKD/ HA,DSHA,RM
      COMMON /BLKE/ US,NU,AL,BL,C
      REAL K,NU
      IF (DSH) 1,1,2
    1 VM=FM
      GO TO 10
    2 SIGN=1.
      IF (FM.LT.0.) SIGN=-1.
      IF ((IDUCT.EQ.-1).AND.(NSOFT.EQ.2).AND.(NIT.EQ.0)) 3,4
    3 IF (DB.GE.DSHA) GO TO 1
      YP=DB*HA*US/NU
      IF (DB.LT.BLA) GO TO 5
      VM=SIGN*US*(5.5+2.5*ALOG(YP))/C
      GO TO 9
    5 IF (DB.LT.ALA) GO TO 6
      VM=SIGN*US*(-3.05+5.*ALOG(YP))/C
      GO TO 9
    6 VM=SIGN*US*YP/C
      GO TO 9
    4 IF ((DA.GE.DSH).AND.(DB.GE.DSH)) GO TO 1
      IF ((NSOFT.EQ.2).AND.(DA.LT.DSH).AND.(IDUCT.EQ.1)) GO TO 1
      IF ((XSTART.EQ.0.).AND.(IDUCT.EQ.0).AND.(DA.LT.DB)) GO TO 1
      Y=DA
      IF (DB.LT.DA) Y=DB
      YP=Y*H*US/NU
      IF (Y.LT.BL) GO TO 7
      VM=SIGN*US*(5.5+2.5*ALOG(YP))/C
      GO TO 9
    7 IF (Y.LT.AL) GO TO 8
      VM=SIGN*US*(-3.05+5.*ALOG(YP))/C
      GO TO 9
    8 VM=SIGN*US*YP/C
    9 L=0
      IF (IDUCT.EQ.-1.AND.NSOFT.EQ.2.AND.NIT.EQ.0) 11,12
```

```
11 E=1.-DB/DSHA
   IF (E.LT..1) VM=VM+(1.-10.*E)*(FM-VM)
   GO TO 10
12 E=1.-DA/DSH
   IF (DB.LT.DA) E=1.-DB/DSH
   IF (E.LT..1) VM=VM+(1.-10.*E)*(FM-VM)
10 RETURN
   END
   SUBROUTINE LOOP3 (X,Y,NP,M,IC,NC)
   DIMENSION X(50),Y(50),S(50)
   JJ=NP-1
   DO 1 J=1,JJ
   S(J) = (Y(J+1)-Y(J))/(X(J+1)-X(J))
 1 CONTINUE
   M=0
   II=NP-3
   DO 2 I=1,II
   J = I+2
   DO 3 N=J,JJ
   LOOP=0
   Z = (Y(N)-Y(I)+S(I)*X(I)-S(N)*X(N))/(S(I)-S(N))
   IF (((X(I).LE.Z.AND.X(I+1).GE.Z).OR.(X(I).GE.Z.AND.X(I+1).LE.Z))
  X.AND.((X(N).LE.Z.AND.X(N+1).GE.Z).OR.(X(N).GE.Z.AND.X(N+1).LE.Z)))
  X    LOOP=1
   IF (LOOP.EQ.1) GO TO 4
   GO TO 3
 4 M=M+1
   IF (M.GT.1) GO TO 5
   IC=I
   NC=N
   GO TO 3
 5 IF ((N-I).LT.(NC-IC)) 6,3
 6 IC=I
   NC=N
 3 CONTINUE
 2 CONTINUE
   RETURN
   END
   SUBROUTINE INSP (X,Y,NP,ICUSP,N,IM,X1)
   DIMENSION X(NP),Y(NP),Z(2),W(2),MX(5),MM(5),D(5)
 1 DO 2 J=1,5
   MX(J)=MM(J)=1
 2 D(J)=10.
   II=JJ=MX(1)=MM(1)=1
   DS=10.
   IF(X(2).LE.X(1)) DS=(X(1)-X(2))**2+(Y(1)-Y(2))**2
   DO 5 J=2,NP
 4 IF(X(J).LT.X(J-1)) GO TO 6
   IF(J.NE.2.AND.MM(JJ).EQ.(J-1)) JJ=JJ+1
   MX(II)=J
   D(II)=(X(J)-X(J-1))**2+(Y(J)-Y(J-1))**2
   GO TO 5
 6 IF (J.NE.2.AND.MX(II).EQ.(J-1)) II=II+1
   MM(JJ)=J
 5 CONTINUE
```

```
      NN=1
      DMIN=D(1)
      DO 7 J=2,5
      IF(D(J).LT.DMIN) NN=J
    7 IF(D(J).LT.DMIN) DMIN=D(J)
      IF(DS.GE.DMIN) GO TO 8
      IM=1
      GO TO 40
    8 IM=MX(NN)
      XM=X(IM)
      IF(IM.EQ.NP) GO TO 40
      XL=AMAX1(X(IM-1),X(IM+1))
      IF (XM.LT.0.) GO TO 11
      XPA=.99*XM
      IF (XL.GT.0.) XLA=XM-.01*(XM-XL)
      IF (XL.LE.0.) XLA=0.
      XA=AMAX1(XPA,XLA)
      GO TO 12
   11 XPA=1.01*XM
      XLA=XM-.01*(XM-XL)
      XA=AMAX1(XPA,XLA)
   12 CONTINUE
      Z(1)=X(IM-1)
      Z(2)=X(IM)
      W(1)=Y(IM-1)
      W(2)=Y(IM)
      YA1=ATKN(Z,W,2,1,XA)
      IF(YA1.GT.100.) GO TO 40
      Z(1)=X(IM+1)
      W(1)=Y(IM+1)
      YB1=ATKN(Z,W,2,1,XA)
      IF (YB1.GT.100.) GO TO 40
      IF (YB1.GE.YA1) IUP=1
      IF (YB1.LT.YA1) IUP=0
      IF (N.EQ.1) IUP1=IUP
      IF (IUP.NE.IUP1) ICUSP=1
      IF (IUP.EQ.IUP1) ICUSP=0
      RETURN
   40 ICUSP=0
      RETURN
      END
      SUBROUTINE CUSP (X,Y,NP,ICUSP,N,IM,XM)
      DIMENSION X(NP),Y(NP),Z(50),W(50)
      IM=1
      XM=X(1)
      DO 10 J=2,NP
      IF (X(J).GT.XM) IM=J
   10 IF (X(J).GT.XM) XM=X(J)
      IF (XM.GT.5..OR.IM.EQ.1.OR.IM.EQ.NP) GO TO 40
      XL=AMAX1(X(1),X(NP))
      IF (XM.LT.0.) GO TO 11
      XPA=.99*XM
      IF (XL.GT.0.) XLA=XM-.01*(XM-XL)
      IF (XL.LE.0.) XLA=0.
      XA=AMAX1(XPA,XLA)
```

```
      GO TO 12
   11 XPA=1.01*XM
      XLA=XM-.01*(XM-XL)
      XA=AMAX1(XPA,XLA)
   12 CONTINUE
      XA=.90*XM
      DO 20 J=1,IM
      Z(IM+1-J)=X(J)
   20 W(IM+1-J)=Y(J)
      YA1=ATKN(Z,W,IM,1,XA)
      IF (YA1.GT.100.) GO TO 40
      NM=NP-IM+1
      DO 30 J=1,NM
      Z(J)=X(IM+J-1)
   30 W(J)=Y(IM+J-1)
      YB1=ATKN(Z,W,NM,1,XA)
      IF (YB1.GT.100.) GO TO 40
      IF (YB1.GE.YA1) IUP=1
      IF (YB1.LT.YA1) IUP=0
      IF (N.EQ.1) IUP1=IUP
      IF (IUP.NE.IUP1) ICUSP=1
      IF (IUP.EQ.IUP1) ICUSP=0
      RETURN
   40 ICUSP=0
      RETURN
      END
      FUNCTION ATKN(X,Y,N,K,XB)
      DIMENSION X(300),Y(300)
      COMMON XX(300),YY(300)
      K1=K+1
      IF(X(N)-X(1)) 100,10,10
10    IF(XB-X(1))20,20,30
20    LL=0
      GO TO 200
30    IF(X(N)-XB)40,40,50
40    LL=N-K1
      GO TO 200
50    LL=1
      LU=N
60    IF(LU-LL-1) 180,180,70
70    LI=(LL+LU)/2
      IF(X(LI)-XB) 80,80,90
80    LL=LI
      GO TO 60
90    LU=LI
      GO TO 60
100   IF(XB-X(1)) 120,20,20
120   IF(X(N)-XB) 130,40,40
130   LL=1
      LU=N
140   IF(LU-LL-1) 180,180,150
150   LI=(LL+LU)/2
      IF(X(LI)-XB) 160,170,170
160   LU=LI
```

```
      GO TO 140
  170 LL=LI
      GO TO 140
  180 LL=LL-(K1+1)/2
      IF(LL) 20,200,190
  190 IF(LL+K1-N) 200,200,40
  200 DO 210 I=1,K1
      I1=LL+I
      XX(I)=X(I1)-XB
  210 YY(I)= Y(I1)
      DO 220 I=1,K
      DO 220 J=I,K
      P=XX(J+1)-XX(I)
      IF (P.EQ.0.) GO TO 240
  220 YY(J+1)=(1./P)*(YY(I)*XX(J+1)-YY(J+1)*XX(I))
      ATKN=YY(K1)
      RETURN
  240 ATKN=999.
      RETURN
      SUBROUTINE CL (P,X,T,F,E,FM,TH,D,IDUCT,NIT,RI,ROUT,K,OA,DP)
      REAL M,K
      M=ABS(FM)
      TR=T/519.
      PR=P/14.7
      A=.0762*TR*TR*TH/(PR*(TR+.416))
      B=.0000374*(TR**.75)*SQRT(F)/SQRT(PR*(TR+.416))
      OA=(A+B*(1.+TH/D)+.1085*M)/(1+B)
      IF(OA.LT.0.) GO TO 1
      EX=EXP(-8.65*M*M-.8122*M)
      C=.000469*F*(TH+.85*D*(1.-.7*SQRT(OA))*EX)/SQRT(TR)
      DP=ATAN(1./(C/OA-X))/K
      IF(IDUCT.EQ.1) GO TO 2
      E=5.E-7
      IF(IDUCT.EQ.0) GO TO 3
      IF(NIT.GT.1.AND.MOD(NIT,2).EQ.0) GO TO 3
      DP=DP*RI/(RI-DP/2.)
      XP=X-C/OA
      RS=K*RI
      CALL BESS (RS,1,1,E,BL,AL)
      CALL BESS (RS,0,1,E,FL,DL)
      AL=XP*AL
      BL=XP*BL
      AP=(FL+BL)/(DL+AL)
      XS=K*(RI-DP)
      NN=0
    4 CALL BESS (XS,1,1,E,AX,BX)
      CALL BESS (XS,0,1,E,CX,DX)
      DE=(AP*BX-AX)/(AP*(DX-BX/XS)-CX+AX/XS)
      XSN=XS-DE
      IF(ABS(XSN-XS).LT.0.001) GO TO 5
      XS=XSN
      NN=NN+1
      IF(NN.LT.10) GO TO 4
    5 DP=RI-XSN/K
```

```
      GO TO 2
    3 XP=X-C/OA
      DP=DP*ROUT/(ROUT+DP/2.)
      RS=K*ROUT
      CALL BESS (RS,1,1,E,BL,AL)
      CALL BESS (RS,0,1,E,DL,EL)
      AL=XP*AL
      BL=XP*BL
      AP=(BL-DL)/(AL-EL)
      XS=K*(ROUT+DP)
      NN=0
    6 CALL BESS (XS,1,1,E,AX,BX)
      CALL BESS (XS,0,1,E,CX,DX)
      DE=(AP*BX-AX)/(AP*(DX-BX/XS)-CX+AX/XS)
      XSN=XS-DE
      IF(ABS(XSN-XS).LT.0.001) GO TO 7
      XS=XSN
      NN=NN+1
      IF(NN.LT.10) GO TO 6
    7 DP=XSN/K-ROUT
      GO TO 2
    1 DP=999.
    2 RETURN
      END
C
C     COMPUTES BESSEL FUNCTIONS
C     J0(X)         IF NN=0 KK=0
C     J1(X)         IF NN=1 KK=0
C     J0(X),Y0(X)  IF NN=0 KK=1
C     J1(X),Y1(X)  IF NN=1 KK=1
C     E=ACCURACY
C
      SUBROUTINE BESS(X,NN,KK,E,Y1,Y2)
      COMMON/CFS/ C(15),D(15),C1(15),D1(15),C2(15),D2(15),C3(15),D3(15)
      DATA  C,D,C1,D1                /1.0E0,-1.07208251953125E-5,-2.88027914
     A465172E-3,-2.32548131802204 E-2,-9.04675114137019 E-2,-2.482751629
     B91013 E-1,-5.55162461579686 E-1,-1.08419349102279 E0,-1.923042420
     C2979 E0,-3.17399388420187 E0,-4.95394321891922 E0,-7.394396611893
     D1 E0,-1.06414712085636 E1,-1.48556951879629 E1,-2.02130078191578 E
     E1,7.03125E-4,5.19621984415029 E-2,2.00513080053400 E-1,4.475351926
     F85868 E-1,7.93177634454767 E-1,1.23746935920051 E0,1.780426821916
     G9 E0,2.42206019141729 E0,3.16237469420305 E0,4.00137388789996 E0,4
     H.93906019504771 E0,5.97543534316542 E0,7.11050059960930 E0,8.34425
     I692794063 E0,9.67670507477811 E0,1.0E0,-1.47354125976562 E-4,-5.51
     J71005474937 4 E-3,-3.44056671293060 E-2,-1.19703729312153 E-1,-3.08
     K908275315734 E-1,-6.64125165303642 E-1,-1.26207012841032 E0,-2.194
     L06925850773 E0,-3.56605929754501 E0,-5.49858781297308 E0,-8.126813
     M31631461 E0,-1.16005053463665 E1,-1.50840445298517 E1,-2.175642262
     N70039 E1,5.859375E-3,8.07160651584343 E-2,2.53793042221047 E-1,5.2
     O5436578722598 E-1,8.95715236793175 E-1,1.36465451905888 E0,1.93226
     P669598118 E0,2.59655360868123 E0,3.35353445994687 E0,4.22719701432
     Q160 E0,5.18954818791911 E0,6.25058936266157 E0,7.41032156793225 E0
     R,8.66874559100747 E0,1.00258620474147 E1/
```

```
      DATA C2,D2,C3,D3/ 1.0E0,1.57928466796875 E-5,-2.44381083312872 E-3
     A,-2.18806583455783 E-2,-8.74772818793729 E-2,-2.43041074174527 E-1
     B,-5.46997406021760 E-1,-1.07238696135943 E0,-1.90686442188248 E0,-
     C3.15269771849344 E0,-4.92676756393717 E0,-7.36056714080549 E0,-1.0
     D6002016828086 E1,-1.46063893153652 E1,-2.01544589357849 E1,-1.1718
     E75E-3,4.50835597826087 E-2,1.92179768195320 E-1,4.38309004023673 E
     F-1,7.83295223635590 E-1,1.22707123981156 E0,1.76960463320143 E0,2.
     G41087731972993 E0,3.15087818669130 E0,3.98959932540356 E0,4.927037
     H46293770 E0,5.96318713929023 E0,7.09834622813375 E0,8.331612642131
     I91 E0,9.66388474813034 E0,3.0E0,-6.65435791015625 E-5,-4.93186867
     J48867 E-3,-3.27735233246138 E-2,-1.16427130443488 E-1,-3.033525086
     K28447 E-1,-6.55627715168281 E-1,-1.24994451493528 E0,-2.1775111007
     L9703 E0,-3.54454775335622 E0,-5.47128637308531 E0,-8.0929737653329
     M8 E0,-1.15593674179525 E1,-1.60348373383787 E1,-2.16983655306451 E
     N1,2.734375E-3,7.41127859149277 E-2,2.45864251843836 E-1,5.16653870
     O334203 E-1,8.86301830297341 E-1,1.35473897252939 E0,1.92193536938l
     P49 E0,2.58787206492691 E0,3.35253763367135 E0,4.21592524683140 E0,
     Q5.17802913182522 E0,6.23884574180982 E0,7.39837227534312 E0,8.6566
     R0658284390 E0,1.00135469787135 E1/
      PI=3.14159265358979
      IF(X-7.0E0)16,20,20
 16   ONE=1.0E0
      K=KK
      N=NN
      Z=X/2.0
      G=.577215664901532
 1    IF(N)3,3,4
 3    S=ONE
      U=ONE
      EM=ONE
 5    R=Z/EM
      U=-U*R*R
      S=S+U
      EM=EM+ONE
      IF( ABS(U)-E)6,5,5
 6    Y1=S
      IF(K)2,8,2
 8    RETURN
 4    S=ONE
      U=ONE
      EM=ONE
 7    R=(Z*Z)/(EM*(EM+ONE))
      U=-U*R
      S=S+U
      EM=EM+ONE
      IF( ABS(U)-E)18,7,7
 18   Y1=S*Z
      F=Y1
      IF(K)2,10,9
 10   RETURN
 2    S=Y1*(G+ALOG(Z))+Z*Z
      U=Z*Z
      EN=2.
      Q=ONE
```

```
11    R=(Z/EN)*(Z/EN)*(ONE+(ONE/(EN*Q)))
      U=-U*R
      S=S+U
      Q=Q+(ONE/EN)
      EN=EN+ONE
      IF( ABS(U)-E)12,11,11
12    Y2=S*(2./PI)
      IF(N)14,13,13
13    RETURN
9     N=-1
      K=-1
      GO TO 3
14    Y2=(F*Y2-(ONE/(PI*Z)))/Y1
      Y1=F
15    RETURN
20    K=KK
      N=NN
      SQ= SQRT (.5)
      CX= COS (X)
      SX= SIN (X)
      CZ=SQ*(CX+SX)
      SZ=SQ*(SX-CX)
      X1=X*X
      R= SQRT(2.0  /(PI*X))*0.01
      Y=X*0.1
      Z=Y*Y
      P0=X1*EVAL(C,D,E,15,Z)
      Q0=-(X/8.0  )*EVAL(C1,D1,E,15,Z)
      IF(K)23,21,23
21    IF(N)25,22,25
22    Y1=R*(P0*CZ-Q0*SZ)
      GO TO 15
23    IF(N)26,24,26
24    Y1=R*(P0*CZ-Q0*SZ)
      Y2=R*(P0*SZ+Q0*CZ)
      GO TO 15
25    P11=X1*EVAL(C2,D2,E,15,Z)
      Q1=(X/8.0  )*EVAL(C3,D3,E,15,Z)
      Y1=R*(Q1*SZ-P11*CZ)
      GO TO 15
26    P11=X1*EVAL(C2,D2,E,15,Z)
      Q1=(X/8.0  )*EVAL(C3,D3,E,15,Z)
      Y1=R*(P11*SZ+Q1*CZ)
      Y2=R*(Q1*SZ-P11*CZ)
      GO TO 15
      END
C     SUBROUTINE TO EVALUATE CONTINUED FRACTION
                        FUNCTION EVAL(A,B,E,N,Z)
      DIMENSION A(15),B(15)
      ONE=1.0
      C=Z+B(1)
      EVAL=A(1)/C
      W=EVAL
      T=ONE
```

```
      DO 3 I=2,N
      D=Z+B(I)
      R=A(I)/(C*D)
      S=ONE/(ONE+R*T)
      V=S-ONE
      T=S
      W=W*V
      EVAL=EVAL+W
      C=D
      IF( ABS(W)-E)10,10,3
3     CONTINUE
10    RETURN
      END
      SUBROUTINE LINPAR (R,X,T,P,F,FM,TH,D,OA,DP,CEE)
      REAL M
      M=ABS(FM)
      TR=T/519.
      PR=P/14.7
      A=.0762*TR*TR*TH/(PR*(TR+.416))
      B=.0005374*(TR**.75)*SQRT(F)/SQRT(PR*(TR+.416))
      OA=(A+B*(1.+TH/D)+.1085*M)/(R+B)
      IF (OA.LT.0.) GO TO 1
      EX=EXP(-8.65*M*M-.8192*M)
      C=.000469*F*(TH+.85*D*(1.-.7*SQRT(OA))*EX)/SQRT(TR)
      DP=(CEE/(6.283185*F))*ATAN(1./(C/OA-X))
      GO TO 2
    1 DP=999.
    2 RETURN
      END
C     FOR CDC 6600 USE*****CALL STEP MUST BE EXECUTED WITH PARAMETERS    DFEQ.001
C     AS ILLUSTRATED ON CARD DFEQ.027                                    DFEQ.002
      SUBROUTINE SET(N,X,Y,DX,F,DIS,MODE,DXMAX,DXMIN)                    DFEQ.003
C     DFEQ. CONSISTS OF TWO SUBROUTINE SUBPROGRAMS, SET AND STEP, TO     DFEQ.004
C     SOLVE THE INITIAL VALUE PROBLEM                                    DFEQ.005
C                       DY/DX = F(X,Y),  Y(X0) = Y0                      DFEQ.006
C     WHERE Y IS AN N-VECTOR WITH REAL COMPONENTS.                       DFEQ.007
C                                                                        DFEQ.008
C                                                                        DFEQ.009
C     CALL SET(N,X,Y,DX,F,DIS,MODE,DXMAX,DXMIN),                         DFEQ.010
C     WHERE                                                              DFEQ.011
C          N IS AN INTGER CONSTANT OR VARIABLE,                          DFEQ.012
C          X IS A REAL VARIABLE, THE INDEPENDENT VARIABLE,               DFEQ.013
C          Y IS A REAL VARIABLE N-ARRAY, THE DEPENDENT VARIABLE,         DFEQ.014
C         DX IS A REAL CONSTANT OR VARIABLE, THE CURRENT STEP SIZE,      DFEQ.015
C          F IS THE NAME OF SUBROUTINE SUBPROGRAM OF THE FORM F(N,X,Y,DY)DFEQ.016
C            WHICH PROVIDES SUBROUTINE SET WITH DY/DX,                   DFEQ.017
C           DY IS A REAL VARIABLE N-ARRAY, DY/DX,                        DFEQ.018
C        DIS IS A REAL VARIABLE OR CONSTANT WHICH PROVIDES A TOLERANCE   DFEQ.019
C            FOR THE ADAMS-MOULTON LOCAL ERROR CHECK IN THE VARIABLE     DFEQ.020
C            STEP SIZE MODE,                                             DFEQ.021
C       MODE IS A LOGICAL VARIABLE OR CONSTANT WHICH WHEN TRUE (FALSE)   DFEQ.022
C              INITIATES THE FIXED (VARIABLE) STEP SIZE MODE,            DFEQ.023
C       DXMAX (DXMIN) IS A REAL VARIABLE OR CONSTANT GIVING UPPER        DFEQ.024
C               (LOWER) BOUNDS FOR ABS(DX) IN THE VARIABLE STEP MODE,    DFEQ.025
```

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```
C     INITIALIZES NECESSAY VARIABLES PRIOR TO ANY INTEGRATIONS.
C
C     CALL STEP(N,X,Y,DX,F,DIS,MODE,DXMAX,DXMIN) THEN INTEGRATES FROM X
C     TO X+DX.  UPON EXIT FROM STEP, X,
C     Y, AND DX WILL HAVE BEEN SET TO THEIR NEW VALUES. THUS TO
C     INTEGRATE OVER SUCCEEDING STEPS, CALL STEP(N,X,Y,DX,F,DIS,MODE,
C     DXMAX,DXMIN) IS EXECUTED FOR EACH STEP DESIRED.
C
C     THE SUBROUTINE SUBPROGRAM F MUST BE INCLUDED IN ANY JOB
C     CALLING SET-STEP. IT IS PROVIDED BY THE USER AND MAY HAVE ANY
C     LEGAL SUBROUTINE NAME. THE VARIABLES CORRESPONDING TO Y AND DY
C     MUST BE DIMENSIONED APPROPRIATELY THERIN.
C
      DIMENSION Y(N)
C
C
C     THE VALUE OF NMAX AND THE DIMENSIONS ASSIGNED TO THE FIRST POSITIO
C     OF THE VARIABLES  D  AND  W  IN THE FOLLOWING DIMENSION STATE-
C     MENT SHOULD BE IDENTICAL.
C
      DIMENSION D(90,5), W(90,5)
C
      EXTERNAL F
      LOGICAL MODE,CODE
      DATA NMAX/90/
      DATA ERF/100./
      DATA DXF/2./
      M=N
      IF (M.LT.1.OR.M.GT.NMAX) GO TO 20
      RA=DIS
      CODE=MODE
      HMAX=DXMAX
      HMIN=DXMIN
      RB=ERF
      BETA=DXF
      K=0
      K2=0
      DO 10 I=1,M
   10 W(I,1)=Y(I)
      CALL F(M,X,Y,D(1,5))
      RETURN
   20 PRINT 30
   30 FORMAT(55H1SUBROUTINE SET HAS ARGUMENT OUT-OF-RANGE. SEE LISTING.)
      CALL SYSTEM(200,1L )
      RETURN
      ENTRY STEP
   40 XC=X
      H=DX
      IF (K.NE.0) IF (K-2) 50,50,110
      XP=XC
      DO 45 I=1,M
   45 W(I,5)=W(I,1)
   50 K1=4-K
      DO 70 I=1,M
```

```
      GO TO 200
180   DO 190 I=1,M
      W(I,1)=W(I,2)
190   D(I,5)=D(I,4)
      X=XC
200   K=0
      K2=0
      DX=SIGN(AMAX1(ABS(H)/BETA,HMIN),H)
      GO TO 40
210   K2=K2+1
      IF (K2.LT.6) RETURN
      K=0
      DX=SIGN(AMIN1(ABS(H)*BETA,HMAX),H)
      K2=0
      RETURN
      END
```

```
***********          JJSCDES    //// END  OF LIST ////
```

```
      DO 60 J=K1,4
60    D(I,J)=D(I,J+1)
      W(I,2)=H*D(I,4)
      W(I,1)=W(I,1)+.5*W(I,2)
70    Y(I)=W(I,1)
      X=XO+.5*H
      CALL F(M,X,Y,D(1,5))
      DO 80 I=1,M
      W(I,3)=H*D(I,5)
      W(I,1)=W(I,1)+.5*(W(I,3)-W(I,2))
80    Y(I)=W(I,1)
      CALL F(M,X,Y,D(1,5))
      DO 90 I=1,M
      W(I,4)=H*D(I,5)
      W(I,1)=W(I,1)+W(I,4)-.5*W(I,3)
90    Y(I)=W(I,1)
      X=XO+H
      CALL F(M,X,Y,D(1,5))
      DO 100 I=1,M
      W(I,1)=W(I,1)-W(I,4)+.1666666666666667*(W(I,2)+2.*(W(I,3)+W(I,4))+H
     1*D(I,5))
100   Y(I)=W(I,1)
      K=K+1
      K1=K
      CALL F(M,X,Y,D(1,5))
      RETURN
110   DO 130 I=1,M
      W(I,2)=W(I,1)
      DO 120 J=1,4
120   D(I,J)=D(I,J+1)
      W(I,3)=W(I,2)+.4166666666666667E-1*H*(55.*D(I,4)-59.*D(I,3)+37.*D(I
     1,2)-9.*D(I,1))
130   Y(I)=W(I,3)
      X=XO+H
      CALL F(M,X,Y,D(1,5))
      DO 140 I=1,M
      W(I,1)=W(I,2)+.4166666666666667E-1*H*(9.*D(I,5)+19.*D(I,4)
     1-5.*D(I,3)+D(I,2))
140   Y(I)=W(I,1)
      CALL F(M,X,Y,D(1,5))
      IF (CODE) RETURN
      ERR=0.
      DO 150 I=1,M
150   ERR=AMAX1(ERR,ABS(W(I,1)-W(I,3))/(14.*AMAX1(ABS(W(I,1)),1.)))
      IF (ERR.GE.RA) IF (HMIN-ABS(H)) 160,155,155
      K1=0
      IF (RB*ERR.LT.RA) IF (HMAX-ABS(H)) 155,155,210
155   K2=0
      RETURN
160   IF (K1.NE.3) GO TO 180
      DO 170 I=1,M
      W(I,1)=W(I,5)
170   D(I,5)=D(I,1)
      X=XP
```

B.2 OPERATING INSTRUCTIONS AND TYPICAL INTERACTIVE RUN

OPERATING INSTRUCTIONS FOR TERMINAL VERSION OF PROGRAM "OPTSHE"

Program has Free Format input

ENTER 1	Program runs only if "1" is entered.
ENTER FREQ, AM, ROUT, TEMP	Frequency, Angular mode, duct height (radius) in inches, temperature in degrees Rankine are entered.
ENTER FM, DSH	Mach Number, Shear Layer thickness as % of duct height areeentered; Dsh = 0 is for uniform flow.
ENTER IDUCT, NSOFT, NIT, IW	Iduct is type of duct; Iduct = 0 for cylindrical, Iduct = 1 for rectangular. NSOFT = number of soft walls; NIT = 0, IW = 0.
ENTER ACC, JLIM	ACC = Accuracy of Numical Integration, JLIM = Maximum number of steps across duct.
ENTER PRESS, RHO	Pressure in psi, density in lbm/ft^3.
ENTER 0, 1, 2, 3, 4	0 - Terminates Program; 1 - Varies TE; 2 - Uses one SE, TE; 3 - Not usable; 4 - Uses Newton-Raphson iteration to find SE, TE for a given RS, XS. RS + iXS = Normalized impedance/ETA For Option "4" above, input must be for one "SE" and three "TE"'s.
ENTER RSD, XSD	RSD = desired Resistance for option 4. XSD = desired Reactance for option 4.

Program gives: TEN = New guess value for TE

SEN = New guess value for SE

The following is a sample run of the terminal version of "OPTSHE."

```
ENTER 11
ENTER  FREQ,AM,ROUT,TEMP   683.5,1.,40.,540.
ENTER  FM,DSH    -.5,0.
ENTER  IDUCT,NSOFT,NIT,IW  0,1,0,0
ENTER ACC,JLIM   1.E-06,1000
FREQ=  683.50    AM= 1.00   ROUT=40.000   TEMP= 540.000
FM=     -.50     DSH=.00000   ETA= 4.00016
IDUCT= 0   NSOFT= 1   NIT=   0   IW= 0   JLIM=1000   ACC=.1E-05
ENTER 0,1,2,3,4  2
ENTER SE,TE       .009,7.8
SE=-.00900000    TE=  7.80000000    RS=  -.25702228     XS=   -.34148768
ENTER 0,1,2,3,4  0
      STOP
      .190 CP SECONDS EXECUTION TIME
..ATTACH,X,HOF,CY=4
..X
ENTER 11
ENTER  FREQ,AM,ROUT,TEMP   683.5,1.,40.,540.
ENTER  FM,DSH    -.5,0.
ENTER  IDUCT,NSOFT,NIT,IW  0,1,0,0
ENTER ACC,JLIM   1.E-06,1000
FREQ=  683.50    AM= 1.00   ROUT=40.000   TEMP= 540.000
FM=     -.50     DSH=.00000   ETA= 4.00016
IDUCT= 0   NSOFT= 1   NIT=   0   IW= 0   JLIM=1000   ACC=.1E-05
ENTER 0,1,2,3,4  1
ENTER SE,TE       .009,7.9
ENTER STEP        -.01
SE= .00900000    TE=  7.90000000    RS=  1.26041850     XS=   5.27827982
SE= .00900000    TE=  7.89000000    RS=   .93494678     XS=   4.12101154
SE= .00900000    TE=  7.88000000    RS=   .72679291     XS=   3.23253290
SE= .00900000    TE=  7.87000000    RS=   .58664406     XS=   2.52370288
SE= .00900000    TE=  7.86000000    RS=   .48869678     XS=   1.93961848
SE= .00900000    TE=  7.85000000    RS=   .41844558     XS=   1.44461841
SE= .00900000    TE=  7.84000000    RS=   .36727461     XS=   1.01442753
SE= .00900000    TE=  7.83000000    RS=   .32985189     XS=    .63179144
SE= .00900000    TE=  7.82000000    RS=   .30278764     XS=    .28391594
SE= .00900000    TE=  7.81000000    RS=   .28390764     XS=   -.03911463
ENTER 0,1,2,3,4  1
ENTER SE,TE       .009,7.8
ENTER STEP        -.01
SE= .00900000    TE=  7.80000000    RS=   .27184691     XS=   -.34536930
SE= .00900000    TE=  7.79000000    RS=   .26582237     XS=   -.64178914
SE= .00900000    TE=  7.78000000    RS=   .26551477     XS=   -.93473310
SE= .00900000    TE=  7.77000000    RS=   .27102752     XS=  -1.23044324
SE= .00900000    TE=  7.76000000    RS=   .28291231     XS=  -1.53550163
SE= .00900000    TE=  7.75000000    RS=   .30227014     XS=  -1.85735015
SE= .00900000    TE=  7.74000000    RS=   .33095911     XS=  -2.20496336
SE= .00900000    TE=  7.73000000    RS=   .37197856     XS=  -2.58981070
SE= .00900000    TE=  7.72000000    RS=   .43017421     XS=  -3.02734119
SE= .00900000    TE=  7.71000000    RS=   .51355996     XS=  -3.53936459
ENTER 0,1,2,3,4  4
ENTER SE, TE'S  .009,7.8,7.79,7.78
ENTER RS'S,  XS'S.27184691,.26582237,.26551477,-.345,-.642,
-.9347
ENTER RSD, XSD  .5,-.4
TEN=  7.79829189       SEN=    .01685323
```

```
ENTER 0,1,2,3,4   1
ENTER SE,TE       .01685,7.8
ENTER STEP        -.01
SE= .01685000     TE=  7.80000000     RS=    .50517324     XS=    -.35536146
SE= .01685000     TE=  7.79000000     RS=    .49409280     XS=    -.64531823
SE= .01685000     TE=  7.78000000     RS=    .49345344     XS=    -.93191788
SE= .01685000     TE=  7.77000000     RS=    .50343322     XS=   -1.22093685
SE= .01685000     TE=  7.76000000     RS=    .52500344     XS=   -1.51842697
SE= .01685000     TE=  7.75000000     RS=    .56009915     XS=   -1.83114296
SE= .01685000     TE=  7.74000000     RS=    .61196698     XS=   -2.16707439
SE= .01685000     TE=  7.73000000     RS=    .68579902     XS=   -2.53616664
SE= .01685000     TE=  7.72000000     RS=    .78987267     XS=   -2.95135629
SE= .01685000     TE=  7.71000000     RS=    .93765053     XS=   -3.43011309
ENTER 0,1,2,3,4   4
ENTER SE, TE'S    .01685,7.8,7.79,7.78
ENTER RS'S,  XS'S.505173,.494092,.493453,-.35536,-.64532,-.931918
ENTER RSD, XSD    .5,-.4
TEN=   7.79851046        SEN=     .01688194
ENTER 0,1,2,3,4   2
ENTER SE,TE       .01688194,7.79851046
SE= .01688194     TE=  7.79851046     RS=    .50378065     XS=    -.39903830
ENTER 0,1,2,3,4   1
ENTER SE,TE       .01688194,7.8
ENTER STEP        -.001
SE= .01688194     TE=  7.80000000     RS=    .50611084     XS=    -.35541398
SE= .01688194     TE=  7.79900000     RS=    .50451982     XS=    -.38472238
SE= .01688194     TE=  7.79800000     RS=    .50303754     XS=    -.41394482
SE= .01688194     TE=  7.79700000     RS=    .50166318     XS=    -.44308723
SE= .01688194     TE=  7.79600000     RS=    .50039598     XS=    -.47215555
SE= .01688194     TE=  7.79500000     RS=    .49923526     XS=    -.50115566
SE= .01688194     TE=  7.79400000     RS=    .49818042     XS=    -.53009341
SE= .01688194     TE=  7.79300000     RS=    .49723093     XS=    -.55897462
SE= .01688194     TE=  7.79200000     RS=    .49638635     XS=    -.58780509
SE= .01688194     TE=  7.79100000     RS=    .49564629     XS=    -.61659058
ENTER 0,1,2,3,4   4
ENTER SE, TE'S    .01688194,7.799,7.798,7.797
ENTER RS'S,  XS'S.50451982,.50303754,.50166318,-.384722,-.413944,
-.443087
ENTER RSD, XSD    .5,-.4
TEN=   7.79847160        SEN=     .01675477
ENTER 0,1,2,3,4   2
ENTER SE,TE       .01675477,7.79847160
SE= .01675477     TE=  7.79847160     RS=    .50000633     XS=    -.39998641
ENTER 0,1,2,3,4   0
      STOP
      3.547 CP SECONDS EXECUTION TIME
```

APPENDIX C

D'ALEMBERT TYPE SOLUTION

Isolation of modal coefficients for forward and backward traveling acoustic waves in a constant area, solf-walled circular duct containing a uniform flow.

C.1 NOTATION

z	axial coordinate
P	complex fluctuating pressure
r	radial coordinate
R_o	duct wall radius
h	normalization parameter $h = R_o$ for cylindrical duct
y	normalized radial coordinate $y = \frac{r}{h} = \frac{r}{R_o}$ for cylindrical duct
M	mean flow mach number
k	a constant proportional to frequency $k = \frac{2\pi f}{c} = \frac{w}{c}$
w	circular frequency
C	speed of sound
k_z	complex axial propagation constant
λ	normalized axial propagation constant; eigenvalue of Equation 2; $\lambda = \frac{k_z}{k}$
n	the angular (spinning) mode number
A	normalized wall admittance; $A = \frac{\rho c}{Z}$
Z	wall impedance; $Z = \frac{P_w}{v_w}$
v_w	radial acoustic velocity in wall liner
b_e	modal coefficient

Primes denote differentation with respect to the argument.

<u>Superscripts</u>

$+$ represents modes and modal constants associated with acoustic waves propagating in the same direction as the orientation of the axial coordinate frame (forward traveling)

- represents modes and modal constants associated with acoustic waves propagating in the opposite direction as the orientation of the axial coordinate frame (backward traveling).

C.2 INTRODUCTION

Derivation of the cross-orthogonality condition for the cylindrical duct and isolation of the modal coefficients for forward traveling modes will be presented in detail on the following pages. The method of solution can be extended to rectangular and annular ducts. The orthogonality condition for modes traveling in the same direction will be presented here again for comparison to the cross-orthogonality condition.

C.3 GENERAL EQUATIONS (Linear Acoustics)

The governing partial differential equation for a constant area cylindrical duct with a uniform flow is known to be (from variables separable type solution)

$$-\frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} - \frac{2M}{C}\frac{\partial P}{\partial z \partial t} + (1-M^2)\frac{\partial^2 P}{\partial Z^2} + \frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} - \left(\frac{n^2}{r^2}\right) P = 0 \qquad (1)$$

where the angular dependency has been assumed to be of form $P(\theta) \alpha \cos(n\theta)$. The usual method of solving the above equation is to assume $P(r,z,t) \alpha P(r) \exp(i(wt-k_z z))$ [4.8], where $k_z = k\lambda$; substitution of $P(r,Z,t)$ into 1 reduces it to Bessels differential equation:

$$P''(r) + \frac{1}{r}P'(r) + k^2\left[1-\lambda^2(1-M^2)-2M\lambda-\frac{n^2}{(kr)^2}\right] P(r) = 0 \qquad (2)$$

in which $P(r)$ must satisfy the boundary condition at the wall, namely

$$\frac{P'(r)}{P(r)} = (1-M\lambda)^2 A \qquad \text{[continuity of particle displacement]} \qquad (3)$$

C.4 MODAL SOLUTION

Assume that the pressure in the duct has a modal solution consisting of forward and backward traveling modes: (refer to Figure C.1) hence,

$$P(r,z,t) = \sum_{\ell=1}^{\infty} b_\ell^+ P_\ell^+ e^{ik(ct-\lambda_\ell^+ z)} + b_\ell^- P_\ell^- e^{ik(ct+\lambda_\ell^- z)} \tag{4}$$

where $P(r,z,t)$ satisfies the partial differential Equation 1 and the wall boundary condition. Substitution of Equation 4 into the partial differential Equation 1 yields the following:

$$\Bigg[k^2 \Sigma \left\{ b_\ell^+ P_\ell^+ e^{ik(ct-\lambda_\ell^+ z)} + b_\ell^- P_\ell^- e^{ik(ct+\lambda_\ell^- z)} \right\} - 2Mk^2 \Sigma \left\{ \lambda_\ell^+ b_\ell^+ P_\ell^+ e^{ik(ct-\lambda_\ell^+ z)} - \lambda_\ell^- b_\ell^- P_\ell^- e^{ik(ct+\lambda_\ell^- z)} \right\}$$
$$- (1-M^2)k^2 \Sigma \left\{ \lambda_\ell^{+2} b_\ell^+ P_\ell^+ e^{ik(ct-\lambda_\ell^+ z)} + \lambda_\ell^{-2} b_\ell^- P_\ell^- e^{ik(ct+\lambda_\ell^- z)} \right\} + \Sigma \Bigg\{ b_\ell^+ \left(P_\ell^{+''} + \frac{1}{r} P_\ell^{+'} - \frac{n^2}{r^2} P_\ell^+ \right) e^{ik(ct-\lambda_\ell^+ z)}$$
$$+ b_\ell^- \left(P_\ell^{-''} + \frac{1}{r} P_\ell^{-'} - \frac{n^2}{r^2} P_\ell^- \right) e^{ik(ct+\lambda z)} \Bigg\} \Bigg] = 0 \tag{5}$$

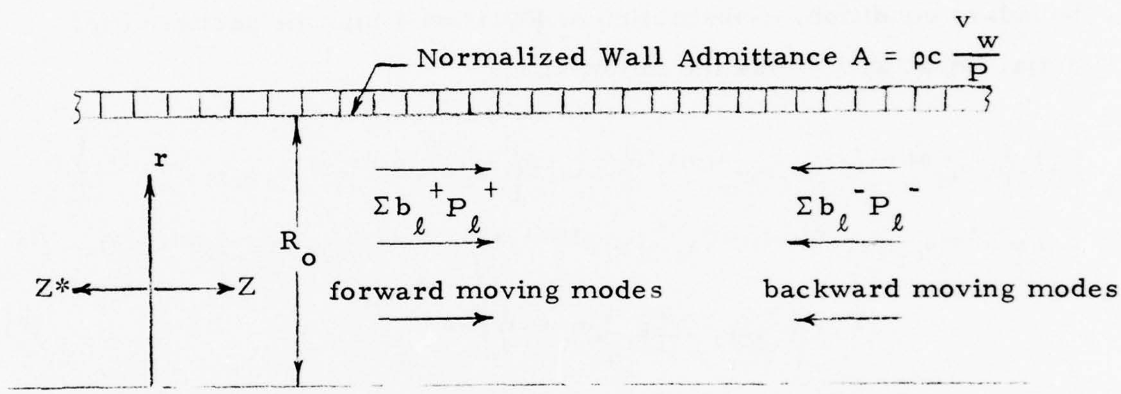
Now group +'s together, -'s together in Equation 5 to give

$$\Sigma b_\ell^+ \left[k^2 P_\ell^+ - 2Mk^2 \lambda^+ P_\ell^+ - (1-M^2)k^2 \lambda^{+2} + P_\ell^{+''} + \frac{1}{r} P_\ell^{+'} - \frac{n^2}{r^2} P \right] e^{ik(ct-\lambda_\ell^+ z)}$$
$$+ \Sigma b_\ell^- \left[k^2 P_\ell^- + 2Mk^2 \lambda_\ell^- P_\ell^- - (1-M^2)k^2 \lambda_\ell^{-2} + P_\ell^{-''} + \frac{1}{r} P_\ell^{-'} - \frac{n^2}{r^2} P \right] e^{ik(ct+\lambda_\ell^- z)} = 0 \tag{6}$$

Equation 6 is rearranged to get Equation 7

$$\Sigma b_\ell^+ \left[P_\ell^{+''} + \frac{1}{r} P_\ell^{+'} + \left(k^2 (1-(1-M^2)\lambda^{+2}) - 2Mk^2 \lambda^+ - \frac{n^2}{r^2} \right) P \right] e^{i(wt-k\lambda^+ z)}$$
$$+ \Sigma b_\ell^- \left[P_\ell^{-''} + \frac{1}{r} P_\ell^{-'} + \left(k^2 (1-(1-M^2)\lambda_\ell^{-2}) + 2Mk^2 \lambda_\ell^- - \frac{n^2}{r^2} \right) P \right] e^{i(wt+k\lambda^- z)} = 0 \tag{7}$$

The differential equation associated with the b_ℓ^+ coefficient is just the governing acoustic differential equation for each mode propagating in a forward direction (Equation 1)



v_w = radial acoustic velocity at wall

$$y = \frac{r}{R_o} = \frac{r}{h}$$

$$\Delta z = -\Delta z^* \qquad \frac{\partial}{\partial z} = -\frac{\partial}{\partial z^*} \qquad \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial z^{*2}}$$

Figure C.1 The d'Alembert Problem

$$\therefore\; P_\ell^{+''} + \frac{1}{r} P_\ell^{+'} + \left[k^2(1-(1-M^2)\lambda_\ell^{+2}) - 2Mk^2\lambda_\ell^{+} - \frac{n^2}{r^2} \right] P = 0 \qquad (8)$$

and satisfying the boundary condition $iA = \dfrac{-P_\ell^{+}(h)'}{k(1-M\lambda_\ell^{+})^2 P_\ell^{+}(h)}$. (9)

The differential equation associated with the b_ℓ^- coefficient is the governing acoustic differential equation for each mode propagating in a backward direction (in Z* direction (Figure 1), M is negative W.R.T. Z* direction)

$$P_\ell^-(r)'' + \frac{1}{r} P_\ell^-(r)' + \left[k^2(1-(1-M^2)\lambda_\ell^{-2} + 2Mk^2\lambda_\ell^{-} - \frac{n^2}{r^2} \right] P_\ell(r) = 0 \qquad (10)$$

and satisfying the boundary condition $iA = \dfrac{-P_\ell^{-1}(h)}{k(1+M\lambda_\ell^{-})^2 P_\ell^{-}(h)}$. (11)

(M is assumed to be positive in Equation 7).

Modes traveling in the same direction must satisfy Equation 8 and boundary condition 9, or Equation 10 and boundary condition 11. By multiplying Equation 8 by $rP_K^+ dr$ and integrating over the duct cross-section yields the following

$$\int_o^h rP_K^+ \left[P_\ell^{+''} + \frac{1}{r} P_\ell^{+'} + \left(k^2(1-(1-M^2)\lambda_\ell^{+2}) - 2Mk^2\lambda_\ell^{+} - \frac{n^2}{r^2} \right) P_\ell^+ \right] dr =$$

$$\int_o^h rP_K^+ (0) dr = 0 \qquad (12)$$

After integrating by parts $rP_K^+ P_\ell^{+''}$ and $P_K^+ P_\ell^{+'}$ in Equation 12

$$\left[P_K^+ r P_\ell^{+'} \right]_o^h - \int_o^h r P_\ell^{+'} P_K^{+'} dr + \int_o^h rP_K^+ \left(k^2(1-(1-M^2)\lambda_\ell^{+2}) - 2Mk^2\lambda_\ell^{+} - \frac{n^2}{r^2} \right) P_\ell^+ dr = 0$$

again integrating by parts $\int rP_\ell^{+'} P_K^{+'} dr$ to get Equation 13

$$0=hP_K^{+}(h)P_\ell^{+}(h)\left[\frac{P_\ell^{+'}(h)}{P_\ell^{+}(h)} - \frac{P_K^{+'}(h)}{P_K(h)}\right] + \int_o^h rP_\ell^{+}\left[P_K^{+''}+\frac{1}{r}P_K^{+'} + \right.$$

$$\left.\left(k^2(1-(1-M^2)\lambda_\ell^{+2}) -2Mk^2\lambda_\ell^{+}-\frac{n^2}{r^2}\right)P_K^{+}\right] dr = 0 \qquad (13)$$

But P_K^{+} also satisfies Equation 8 and boundary condition 9, hence

$$P_K^{+''}+\frac{1}{r}P_K^{+'}+\left[k^2(1-(1-M^2)\lambda_K^{+2})-2Mk^2\lambda_K^{+}-\frac{n^2}{r^2}\right]P_K^{+}=0 \qquad (14)$$

Multiple Equation 14 by $rP_\ell^{+}dr$ and integrate over duct radius to get the following:

$$\int_o^h rP_\ell^{+}\left[P_K^{+''}+\frac{1}{r}P_K^{+'}+\left[k^2(1-(1-M^2)\lambda_K^{+2})-2Mk^2\lambda_K^{+}-\frac{n^2}{r^2}\right]P_K^{+}\right]dr =$$

$$\int_o^h rP_\ell^{+}(0)dr=0 \qquad (15)$$

Subtracting Equation 15 from Equation 13 gives the following relationship:

$$0=hP_k^{+}(h)P_\ell^{+}(h)\left[\frac{P_\ell^{+'}(h)}{P_\ell^{+}(h)} - \frac{P_K^{+}(h)}{P_K^{+}(h)}\right] + \int_o^h rP_K^{+}P_\ell^{+}\left[k^2(1-M^2)(\lambda_K^{+2}-\lambda_\ell^{+2})+\right.$$

$$\left.2Mk^2(\lambda_K^{+}-\lambda_\ell^{+})\right] dr = 0 \qquad (16)$$

from boundary condition at wall, Equation 9, it is seen that

$$\frac{P_\ell^{+'}(h)}{P_\ell^{+}(h)} = -iAk(1-M\lambda_\ell^{+})^2 \quad \text{and} \quad \frac{P_K^{+'}(h)}{P_K^{+}(h)} = -iAk(1-M\lambda_K^{+})^2$$

Substituting above expressions into Equation 16 gives Equation 17:

$$0=hP_K^{+}(h)P_\ell^{+}(h)(-iAk)\left[2M(\lambda_K^{+}-\lambda_\ell^{+})-M^2(\lambda_K^{+2}-\lambda_\ell^{+2})\right] +$$

$$\int_o^h rP_K^{+}P_\ell^{+}\left[k^2(1-M^2)(\lambda_K^{+2}-\lambda_\ell^{+2})+2Mk^2(\lambda_K^{+}-\lambda_\ell^{-})\right]dr \qquad (17)$$

factor out $(\lambda_K^+ - \lambda_\ell^+)$

$$(\lambda_K^+ - \lambda_\ell^+)\left\{\left[k^2(1-M^2)(\lambda_K^+ + \lambda_\ell^+) + 2Mk^2\right]\int_o^h (rP_K^+ P_\ell^+)dr - iAkhP_K^+(h)P_\ell^+(h)\left[2M - M^2(\lambda_K^+ + \lambda_\ell^+)\right]\right\} = 0 \qquad (18)$$

Equation 18 is the general orthogonality condition between modes traveling with the flow; if $\lambda_K^+ \neq \lambda_\ell^+$ then

$$\int_o^h rP_K^+ P_\ell^+ dr + \frac{iAkhP_K^+(h)P_\ell^+(h)[-2M + M^2(\lambda_K^+ + \lambda_\ell^+)]}{k^2(1-M^2)(\lambda_K^+ + \lambda_\ell^+) + 2Mk^2} = 0 \qquad (19)$$

For modes traveling against the flow, the orthogonality condition is again Equation 18 with M replaced by -M

$\therefore$ if $\lambda_K^- \neq \lambda_\ell^-$ then

$$\int_o^h rP_K^- P_\ell^- dr + \frac{iAkhP_K^-(h)P_\ell^-(h)[+2M + M^2(\lambda_K^- + \lambda_\ell^-)]}{k^2[(1-M^2)(\lambda_K^- + \lambda_\ell^-) - 2M]} = 0 \qquad (20)$$

Equations 19 and 20 involve relationships between modes traveling in the same direction. To get a relationship between modes traveling in opposite directions, again start with the governing acoustic differential equation.

For positive traveling modes:

$$P_\ell^{+''} + \frac{1}{r}P_\ell^{+'} + \left[k^2(1-(1-M^2)\lambda_\ell^{+2}) - 2Mk^2\lambda_\ell^+ - \frac{n^2}{r^2}\right]P_\ell^+ = 0 \qquad (21)$$

For negative traveling modes:

$$P_K^{-''} + \frac{1}{r}P_K^{-'} + \left[k^2(1-(1-M^2)\lambda_K^{-2}) + 2Mk^2\lambda_K^- - \frac{n^2}{r^2}\right]P_K^- = 0 \qquad (22)$$

Multiply Equation 21 by $rP_K^{-}dr$ and integrate over duct radius, giving

$$\int_0^h rP_K^{-}\left[P_\ell^{+''}+\frac{1}{r}P_\ell^{+'}+[k^2(1-(1-M^2)\lambda_\ell^{+2})-2Mk^2\lambda_\ell^{+}-\frac{n^2}{r^2}]P_\ell^{+}\right]dr=\int_0^h rP_K^{-}(0)dr=0$$

Integrate $\int_0^h rP_K^{-}(P_\ell^{+''}+\frac{1}{r}P_\ell^{+'})dr$ by parts to give

$$\int_0^h rP_K^{-}(P_\ell^{+''}+\frac{1}{r}P_\ell^{+'})dr=\int_0^h P_K^{-}\frac{d(rP_\ell^{+'})}{dr}dr=[P_K^{-}rP_\ell^{+}]_0^h-\int_0^h rP_\ell^{+'}P_K^{-'}dr$$

integrate by parts again to get the following

$$\int_0^h rP_K^{-}(P_\ell^{+''}+\frac{1}{r}P_\ell^{+'})dr=[rP_K^{-}P_\ell^{+'}]_0^h-[rP_K^{-'}P_\ell^{+}]_0^h+\int_0^h rP_K^{-}P_\ell^{+''}dr=$$

$$h[P_K^{-}(h)P_\ell^{+'}(h)-P_K^{-'}(h)P_\ell^{+}(h)]+\int_0^h rP_K^{-}P_\ell^{+''}dr$$

Substitute above expression into Equation 21 to get

$$\int_0^h rP_K^{-}[L(P_\ell^{+})]dr=hP_K^{-}(h)P_\ell^{+}(h)\left[\frac{P_\ell^{+'}(h)}{P_\ell^{+}(h)}-\frac{P_K^{-'}(h)}{P_K^{-}(h)}\right]+$$

$$\int_0^h rP_\ell^{+}\left[P_K^{-''}+\frac{1}{r}P_K^{-'}+[k^2(1-(1-M^2)\lambda_\ell^{+2})-2Mk^2\lambda_\ell^{+}-\frac{n^2}{r^2}]P_K^{-}\right]dr=0$$

The R.H.S. of above equation is

$$hP_K^{-}(h)P_\ell^{+}(h)\left[\frac{P_\ell^{+'}(h)}{P_\ell^{+}(h)}-\frac{P_K^{-'}(h)}{P_K^{-}(h)}\right]+\int_0^h rP_\ell^{+}\Big[P_K^{-''}+\frac{1}{r}P_K^{-'}+$$

$$[k^2(1-(1-M^2)\lambda_\ell^{+2})-2Mk^2\lambda_\ell^{+}-\frac{n^2}{r^2}]P_K^{-}\Big]dr=0 \qquad (23)$$

Now multiply L.H.S. of Equation 22 by $rP_\ell^{+}dr$ and integrate over duct radius to get Equation 24.

$$\int_o^h rP_\ell^+ \left[P_K^{-''} + \frac{1}{r} P_K^{-'} + [k^2(1-(1-M^2)\lambda_K^{-2}) + 2Mk^2\lambda_K^- - \frac{n^2}{r^2}] P_K^- \right] dr = 0 \qquad (24)$$

Subtract Equation 24 from Equation 23 to get Equation 25.

$$hP_K^-(h)P_\ell^+(h) \left[\frac{P_\ell^{+'}(h)}{P_\ell^+(h)} - \frac{P_K^{-'}(h)}{P_K^-(h)} \right] + \int_o^h rP_K^- P_\ell^+ \left[k^2 \left((1-M^2)(\lambda_K^{-2} - \lambda_\ell^{+2}) - 2M(\lambda_K^- + \lambda_\ell^+) \right) \right] dr = 0 \qquad (25)$$

Substituting boundary conditions of Equations 9 and 10 into Equation 25 gives Equation 26.

$$hP_K^-(h)P_\ell^+(h)(-iAk) \left[(1-M\lambda_\ell^+)^2 - (1+M\lambda_K^-)^2 \right] + \int_o^h rP_K^- P_\ell^+ k^2 \left((1-M^2)(\lambda_K^- - \lambda_\ell^+) - 2M \right) (\lambda_K^- + \lambda_\ell^+) dr = 0 \qquad (26)$$

Further simplifying Equation 26 will give the expression below.

$$(\lambda_\ell^+ + \lambda_L^-) \left\{ k^2 \left[(1-M^2)(\lambda_K^- - \lambda_\ell^+) - 2M \right] \int_o^h rP_K^- P_\ell^+ dr + iAkhP_K^-(h)P_\ell^+(h) \left[M^2(\lambda_K^- - \lambda_\ell^+) + 2M \right] \right\} = 0$$

Since in general $(\lambda_K^- + \lambda_\ell^+)$ is never zero, the term inside the brackets must always be zero.

$$k^2 \left[(1-M^2)(\lambda_K^- - \lambda_\ell^+) - 2M \right] \int_o^h rP_K^- P_\ell^+ dr + iAkh \left(M^2(\lambda_K^- - \lambda_\ell^+) + 2M \right) P_K^-(h)P_\ell^+(h) = 0 \qquad (27)$$

Two relationships now exist; one between modes traveling in opposite directions (Equation 27), and one between modes traveling in the same direction (Equations 19 and 20). It must be shown now that using these relationships the modal coefficients can be isolated independently of one another, given an initial pressure and axial pressure gradient distribution.

C.5 ISOLATION OF MODAL COEFFICIENTS

The problem now is to utilize Equations 19, 20 and 27 to isolate the modal coefficients involved in specifying the pressure $g(r) = \sum_\ell (b_\ell^+ P_\ell^+ + b_\ell^- P_\ell^-)$ and the axial pressure gradient $\frac{\partial P}{\partial z} = H(r)$, at some arbitrary z_o, taken to be zero for simplicity.

$$\therefore\ g = \sum_\ell (b_\ell^+ P_\ell^+ + b_\ell^- P_\ell^-) \Big|_{=z_o=0}$$

and

$$H = \frac{\partial P}{\partial z}\Big|_{z=z_o=0} = -ik\Sigma(\lambda_\ell^+ b_\ell^+ P_\ell^+ - \lambda_\ell^- b_\ell^- P_\ell^-)$$

No Flow

For the no flow condition, the modal pressure distribution for the ℓ^{th} forward and ℓ^{th} backward traveling waves are the same; the orthogonality condition is just $\int_o^h rP_\ell P_n dx=0$ (superscripts are neglected on modal pressures). To solve for the coefficients b_ℓ^+, b_ℓ^-, the inner product is formed between g and ℓ^{th} mode, and h and ℓ^{th} mode, to give two equations in two unknowns. Solution is straightforward for b_ℓ^+, b_ℓ^-, (conventional orthogonality exists).

Uniform Flow

For uniform flow, first express

$$\Sigma b_\ell^+ P_\ell^+ = g - \Sigma b_\ell^- P_\ell^- \tag{28}$$

and

$$\Sigma \lambda_\ell^+ b_\ell^+ P_\ell^+ = \frac{iH}{k} + \Sigma \lambda_\ell^- b_\ell^- P_\ell^- \tag{29}$$

Restating the orthogonality condition for forward traveling modes Equation 19

$$k^2 \left[(1-M^2)(\lambda_n^+ + \lambda_\ell^+) \right] 2M \int_o^h rP_n^+ P_\ell^+ + iAhkP_n^+(h)P_\ell^+(h) \left[M^2(\lambda_n^+ + \lambda_\ell^+) - 2M \right] = 0 \tag{30}$$

By multiplying Equation 28 by $[(1-M^2)\lambda_n^+ + 2M]P_n^+ rdr$ and integrating over duct height h results in the following:

$$\sum_\ell \left\{ \left[(1-M^2)\lambda_n^+ + 2M \right] \int_o^h P_n^+ P_\ell^+ r \right\} = \left[\int_o^h grP_n^+ dr - \sum_\ell b_\ell^- \int_o^h P_\ell^- rP_n^+ dr \right]$$

$$\left[(1-M^2)\lambda_n^+ + 2M \right] \tag{31}$$

The above L. H. S. of Equation 30 inside braces looks like part of the term associated with the integral term in Equation 19 (the orthogonality condition for forward traveling modes). To complete the integral term in Equation 19, $(1-M^2)\lambda_\ell^+ \int_o^h rP_\ell^+ P_h^+ dr$ must be added to L. H. S. of Equation 31 for each (ℓ); to do this operation, Equation 29 is used (pressure gradient equation). Multiplying Equation 29 by $(1-M^2)rP_h^+ dr$ and integrating over the duct height yields Equation 32.

$$\sum_\ell b_\ell^+ \left\{ (1-M^2)\lambda_\ell^+ \int_o^h rP_\ell^+ P_n^+ dr \right\} = \left\{ \int_o^h \left[\frac{iH}{k} + \sum_\ell \lambda_\ell^- b_\ell^- P_\ell^- \right] rP_n^+ dr \right\} (1-M^2) \tag{32}$$

Adding Equation 31 and 32 forms the integral term on L. H. S. of Equation 19.

$$\sum_\ell b_\ell^+ \left\{ \left[(1-M^2)(\lambda_n^+ + \lambda_\ell^+) + 2M \right] \int_o^h P_n^+ P_\ell^+ rdr \right\} = \left[(1-M^2)\lambda_n^+ + 2M \right] \int_o^h$$

$$\left[gP_n^+ - \Sigma b_\ell^- P_\ell^- P_h^+ \right] rdr + (1-M^2) \int_o^h \left[\frac{iHP_n^+}{k} + \Sigma b_\ell^- P_\ell^- P_n^+ \right] rdr \tag{33}$$

To make the L. H. S. of Equation 30 reduce to a single term, (when $\ell = n$), $\frac{iAh}{k} P_n^+(h) P_\ell^+(h)[M^2(\lambda_n^+ + \lambda_\ell^+) - 2M]$ has to be added inside the brackets, thus giving the orthogonality condition for forward traveling modes as in Equation 19 for all terms except $\ell = n$. Adding $\frac{iAh}{k} P_n^+(h) P_\ell^+(h)[M^2(\lambda_n^+ + \lambda_\ell^+) - 2M]$ inside the braces of L. H. S. of Equation 33 is equivalent to adding

$$\sum_\ell b_\ell^+ \frac{iAh}{k} P_n^+(h) P_\ell^+(h) \left[M^2(\lambda_n^+ + \lambda_\ell^+) - 2M \right]$$

to both sides. Carrying out this operation then allows the reduction of L. H. S. to one term, hence, when $\ell = n$ (since all others are zero by Equation 19) resulting in Equation 34.

$$b_n^+ \left\{ \left[(1-M^2) 2\lambda_n^+ + 2M \right] \int_o^h r P_n^+ P_n^+ dr + \frac{iAh}{k} P_n^{+2}(h) \left[M^2(2\lambda_n^+) - 2M \right] \right\} =$$

$$\left[(1-M^2)\lambda_n^+ + 2M \right] \int_o^h \left[g P_n^+ - \sum_\ell b_\ell^- P_\ell^- P_h^+ \right] r dr + (1-M^2) \int_o^h \frac{iH}{k} P_n^+ + \sum_\ell \lambda_\ell^- b_\ell^- P_\ell^- P_h^+ \, r dr + \tag{34}$$

$$\frac{iAh}{k} P_n^+(1) \sum_\ell b_\ell^+ \left[M^2(\lambda_n^+ + \lambda_\ell^+) - 2M \right] P_\ell^+(h)$$

At this point it looks as if whatever is gained on L. H. S. is made more complicated on the right hand side of Equation 34, the second orthogonality condition (for modes moving in opposite direction) will allow the reduction of Equation 34 to a simple relation. The orthogonality condition for modes traveling in opposite directions is Equation 27.

$$\left[(1-M^2)(\lambda_n^- - \lambda_\ell^+) - 2M \right] \int_o^h r P_n^- P_\ell^+ dr + \frac{iAh}{k} \left[(M^2(\lambda_n^- - \lambda_\ell^+) + 2M \right] P_n^-(h) P_\ell^+(h) = 0 \tag{27}$$

Now group summations $(\sum_\ell)$ on R. H. S. of Equation 34 together giving Equation 35.

$$
\begin{aligned}
\text{L.H.S. }(34) = {} & \left[(1-M^2)\lambda_n^{\,+}+2M\right]\int_o^h gP_n^{\,+}\,r\,dr + (1-M^2)\frac{i}{k}\int_o^h HP_n^{\,+}\,r\,dr \,+ \\
& \sum_\ell b_\ell^{\,-}\left\{\left[(1-M^2)(\lambda_\ell^{\,-}-\lambda_n^{\,+})-2M\right]\int_o^h P_n^{\,+}P_\ell^{\,-}\,r\,dr\right\} + \sum_\ell b_\ell^{\,+}\frac{iAh}{k}\left[M^2(\lambda_n^{\,+}+\lambda_\ell^{\,+})-2M\right]P_n^{\,+}(h)P_\ell^{\,+}(h)
\end{aligned}
\tag{35}
$$

The middle term of the R.H.S. of the above equation is the integral term associated with the orthoganality condition of Equation 27; by adding and subtracting the proper boundary condition terms to the R.H.S. of the above equation, Equation 27 can be formed, thus eliminating these terms.

Hence, most add $P_n^{\,+}(h)P_\ell^{\,-}(h)\frac{iAh}{k}[M^2(\lambda_\ell^{\,-}-\lambda_n^{\,+})+2M]$ within the brackets of Equation 35, subtract outside the brackets to get Equation 36

$$
\begin{aligned}
\text{L.H.S. }(34) = {} & \left[(1-M^2)\lambda_n^{\,+}+2M\right]\int_o^h gP_n^{\,+}\,r\,dr + (1-M^2)\frac{i}{k}\int_o^h HP_n^{\,+}\,r\,dr \,+ \\
& \sum_\ell b_\ell^{\,-}\left\{P_n^{\,+}(h)P_\ell^{\,-}(h)\frac{iAh}{k}\left[M^2(\lambda_\ell^{\,-}-\lambda_n^{\,+})+2M\right] + \left[(1-M^2)(\lambda_\ell^{\,-}-\lambda_n^{\,+})-2M\right]\int_o^h P_n^{\,+}P_\ell^{\,-}\,r\,dr\right\} \\
& \frac{iAh}{k} + \sum_\ell\left\{b_\ell^{\,+}\left[M^2(\lambda_n^{\,+}+\lambda_\ell^{\,+})-2M\right]P_n^{\,+}(h)P_\ell^{\,+}(h) - b_\ell^{\,-}\left[M^2(\lambda_\ell^{\,-}-\lambda_n^{\,+})+2M\right]P_n^{\,+}(h)P_\ell^{\,-}(h)\right\}
\end{aligned}
\tag{36}
$$

Since the middle term of Equation 36 is just the cross-orthoganality condition, (Equation 27), the middle term drops out completely. Also the summation of the boundary condition terms is over ℓ, hence, $P_n^{\,+}(h)$ can be pulled outside the summation. Hence,

$$\text{L.H.S. }(34) = \left[(1-M^2)\lambda_n^+ + 2M\right]\int_o^h gP_n^+ r\,dr + (1-M^2)\frac{i}{k}\int_o^h HP_n^+ r\,dr +$$

$$\frac{iAh}{k}P_n^+(h)\left\{\sum_\ell\left[b_\ell^+P_\ell^+(h)+b_\ell^-P_\ell^-(h)\right](M^2\lambda_n^+ - 2M) +\right.$$

$$\left.\sum_\ell M^2\left[\lambda_\ell^+ b_\ell^+ P_\ell^+ - \lambda_\ell^- b_\ell^- P_\ell^-(h)\right]\right\} \tag{37}$$

but $g(h) = \sum_\ell (b_\ell^+P_\ell^+(h)+b_\ell^-P_\ell^-(h))$ --- pressure at wall

and $\frac{i}{k}H(h) = \sum_\ell(\lambda_\ell^+b_\ell^+P_\ell^+ - \lambda_\ell^-b_\ell^-P_\ell^-)$ --- $\frac{i}{k}\frac{\partial P}{\partial Z}\Big/_{\text{wall}}$

Therefore Equation 37 reduces to the final form, Equation 38.

$$b_n^+\left\{\left[(1-M^2)2\lambda_n^+ + 2M\right]\int_o^h rP_n^+P_n^+ dr + \frac{iAh}{k}P_n^{+2}(h)\left[M^2(2\lambda_n^+)-2M\right]\right\} =$$

$$\left[(1-M^2)\lambda_n^+ + 2M\right]\int_o^h gP_n^+ r\,dr + (1-M^2)\frac{i}{k}\int_o^h HP_n^+ r\,dr +$$

$$\frac{iAh}{k}P_n^+(h)\left[(M^2\lambda_n^+ - 2M)g(h) + \frac{iM^2}{k}H(h)\right] \tag{38}$$

if r is normalized, $y = \frac{r}{h}$, then

$$b_n^+\quad\left[(1-M^2)2\lambda_n^+ + 2M\right]\int_o^1 yP_n^+P_n^+ dy + \frac{iA}{hk}P_n^{+2}(1)\left[M^2(2\lambda_n^+)-2M\right] =$$

$$\left[(1-M^2)\lambda_n^+ + 2M\right]\int_o^1 gP_n^+ y\,dy + \frac{(1-M^2)i}{k}\int_o^1 HP_n^+ y\,dy +$$

$$\frac{iA}{hk}P_n^+(1)\left[(M^2\lambda_n^+ - 2M)g(1) + \frac{iM^2}{k}H(1)\right] \tag{39}$$

The relation for b_n^- can be arrived at using similar manipulations that were used to get b_n^+. The result is stated below.

$$b_n^- \left\{\left[(1-M^2)2\lambda_n^- - 2M\right]\int_o^1 yP_n^- P_n^- dy + \frac{iA}{hk} P_n^{-2}(1)\left[M^2(2\lambda_n^-)+2M\right]\right\} =$$

$$\left[(1-M^2)\lambda_n^- - 2M\right]\int_o^1 gP_n^- ydy - (1-M^2)\frac{i}{k}\int_o^1 HP_n^- ydy +$$

$$\frac{iA}{hk} P_n^-(1)\left[(M^2\lambda_n^- + 2M)g(1) - \frac{iM^2}{k} H(1)\right] \tag{40}$$

The solutions for b_n^+, b_n^- are given below.

$$b_n^+ = \frac{\left[(1-M^2)\lambda_n^+ + 2M\right]\int_o^1 gP_n^+ ydy + (1-M^2)\frac{i}{k}\int_o^1 HP_n^+ ydy + \frac{iA}{hk} P_n^+(1)\left[g(1)[M^2\lambda_n^+ - 2M] + \frac{iH(1)}{k} M^2\right]}{2\left\{\left[(1-M^2)\lambda_n^+ + M\right]\int_o^1 P_n^+ P_n^+ ydy + \frac{iA}{hk} P_n^+(1) P_n^+(1)\left[M^2\lambda_n^+ - M\right]\right\}} \tag{41}$$

and

$$b_n^- = \frac{\left[(1-M^2)\lambda_n^- - 2M\right]\int_o^1 gP_n^- ydy - (1-M^2)\frac{i}{k}\int_o^1 HP_n^- ydy + \frac{iA}{hk} P_n^-(1)\left[g(1)(M^2\lambda_n^- + 2M) - \frac{iH(1)}{k} M^2\right]}{2\left\{\left[(1-M^2)\lambda_n^- - M\right]\int_o^1 P_n^- P_n^- ydy + \frac{iA}{hk} P_n^-(1) P_n^-(1)\left[M^2\lambda_n^- + M\right]\right\}} \tag{42}$$

where M is the Mach number relative to the z axial coordinate of Figure 1.

C.6 DISCUSSION

The significance of Equations 41 and 42 is that the modal coefficients can be found independently of one another instead of solving a truncated system of linear equations as done by Zorumski [10]. Kraft and Wells [16] developed an adjoint function which was othoganal to all modes, except one, which were traveling in the same direction. This adjoint function allowed each modal coefficient to be evaluated in terms of the pressure distribution only. Kraft [20] did not generalize the adjoint method to include both forward and backward traveling modes, but relied on Zorumski's technique in determining the modal coefficients, with the adjoint method being invoked to assure that the modal expansion is valid. The only problem with the adjoint method is that the pressure alone is not sufficient to completely specify the coefficients in a duct where reflected waves exist; hence that method must be modified.

To match an arbitrary pressure distribution and an arbitrary axial pressure gradient distribution both sets of modal coefficients are necessary. If the b_ℓ^-'s are all zero (no reflected waves) then the pressure and pressure gradient cannot both be arbitrary; if the pressure distribution is specified, then the b_ℓ^+'s are completely determined [17].

One may think that this method of isolating the modal coefficients which requires the axial pressure gradient instead of the axial acoustic velocity is limited if the acoustic velocity is specified. To circumvent this apparent problem, one need only imagine that the duct walls are rigid or hard so that the wall boundary condition is the radial acoustic velocity is zero at the wall. In that case the modal pressures are again orthogonal in the usual sense, and the hard wall modal coefficients are easily computed. Having computed the coefficients, one may then compute the axial pressure gradient which is needed in the solution of the soft-wall modal coefficients. Hence, the apparent problem is solved; namely, converting the axial acoustic velocity to axial acoustic pressure gradient.

One thing to be pointed out is that the expression, Equation 41, for the modal coefficients for the forward moving modes is the same as derived in [3]. The expression, Equation 47, for the modal coefficients of the backward moving modes is the same as Equation 41, except that M is replaced by -M, and H(r) is replaced by -H(r). The reasoning for this is that the backward moving mode "see's" a flow and axial pressure gradient of opposite sign as that for a forward moving mode.

The method of isolating the modal coefficients assumes that all the axial Eigenvalues, $\lambda_\ell^{\pm}$'s, and radial pressure distributions are readily obtainable if need be, although usually only the first few modes are necessary to get a good approximation to the initial pressure and axial pressure gradient profiles. A further consequence of the orthoganality conditions derived previously is that any axial Eigenvalue, $\lambda_\ell^{\pm}$, of Equation 2 which satisfies the wall boundary condition must be considered as an admissable mode. Hence, any "strange" modes [12] and [20] must be included in the set of basis functions.

This method of isolating the modal coefficients is easily adapted to multi-sectioned duct theory [10], Figure C.2, where finite discontinuities in duck wall impedances occur. If constraints are put on the modal coefficients, such as terminating the duct so that no backward moving modes exist (b_ℓ^-'s are zero at exit), simple linear equations can be developed to iterate to the correct values of the modal coefficients at the duct inlet that will meet the exit condition. Once the coefficients at the duct inlet are known, one can proceed down the duct to each wall impedance discontinuity and, by matching the pressure and axial pressure gradient, proceed past the discontinuities to the duct termination. Using an acoustic energy expression [21] the energy flux at any plane in the duct may be computed to determine the overall attenuation up to that point. The theory is used in Program CO detailed in Appendix E.

C.7 SUMMARY OF UNIFORM FLOW ORTHOGANALITY

An orthogonality condition for modes traveling in opposite directions was derived. Using this cross-orthoganality condition and one previously derived [3], an equation was developed which allows each modal coefficient to be evaluated independent of all other modes when the pressure and axial pressure gradient are known. The method can be used to predict the overall attenuation in a duct with several different wall impedances.

$$P(t, r, z) = \sum_{\ell} b_{\ell}^{+} P_{\ell}^{+}(r) e^{ik(ct - \lambda_{\ell}^{+} z)} + b_{\ell}^{-} P_{\ell}^{-}(r) e^{ik(ct + \lambda_{\ell}^{-} z)}$$

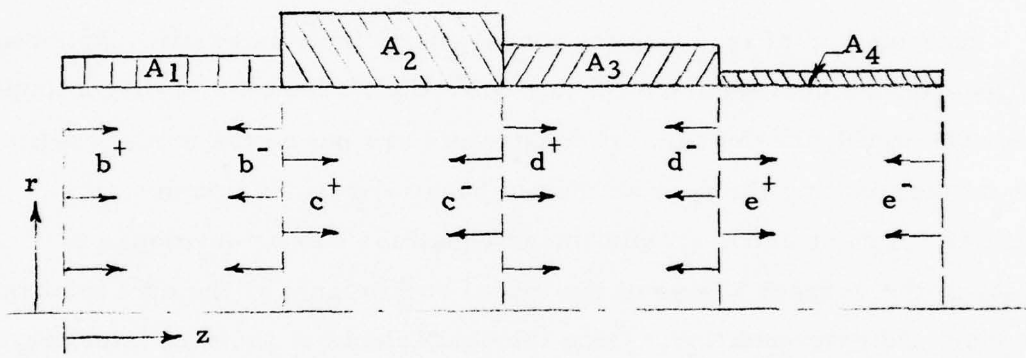


Figure C.2 Multisection Duct with Finite Discontinuities in Wall Admittance and a Uniform Parallel Flow

APPENDIX D

CONSTRUCTION OF SHEARED FLOW ADJOINT EQUATIONS

$[A] \equiv [\alpha_{iJ}]$ i, J=1, 2, 3 then from Equation 14

$$\alpha_{11}=0 \quad \alpha_{12}=1 \quad \alpha_{13}=0 \quad \alpha_{21}=(\;)''+(\frac{1}{r}-\frac{2M'}{M})(\;)'+(k^2-\frac{m^2}{r^2})(\;)$$

$$\alpha_{22}=-\frac{2M}{(1-M^2)}(\;) \quad \alpha_{23}=\frac{-2i\rho ckM'}{k^2(1-M^2)M}(\;) \quad \alpha_{31}=\frac{-i(\;)'}{\rho ckM} \quad \alpha_{32}=0$$

$$\alpha_{33}=\frac{1}{M}(\;)$$

The notation ( ) means "operating on" and primes denote differentiation with respect to the argument. To determine the adjoint operator α^*_{iJ}, α^T_{iJ} must first be found. From Equation 23 it is apparent that if α_{iJ} is not a differential operator then $\alpha^T_{iJ}=\alpha_{iJ}$. Thus

$$\alpha^T_{11}=0 \quad \alpha^T_{12}=1(\;) \quad \alpha^T_{13}=0 \quad \alpha^T_{22}=\frac{-2M}{(1-M^2)}(\;) \quad \alpha^T_{23}=\frac{-2i\rho ckM'}{k^2(1-M^2)M}(\;)$$

$$\alpha^T_{32}=0 \quad \alpha^T_{33}=\frac{1}{M}(\;)$$

Only α^T_{21} and α^T_{31} are now unknown.

From the definition of the transpose, Equation 23

$$\int_o^{R_o}(\alpha_{iJ}y_J)\overline{Z}_i dr = \int_o^{R_o} y_J(\alpha_{iJ}{}^T\overline{Z}_i)dr + [BC] \qquad (D.1)$$

Putting α_{21} into the above

$$\int_o^{R_o}\left[\frac{y_1''+(\frac{1}{r}-\frac{2M'}{M})y_1'+(k^2-\frac{m^2}{r^2})y_1}{k^2(1-M^2)}\right]\overline{Z}_2 dr = \int_o^{R_o} y_1(\alpha_{21}{}^T\overline{Z}_2)dr + [BC] \qquad (D.2)$$

By performing integrations by parts on the above equation it can be shown that

$$\int_0^{R_o}\left[\frac{y_1''+(\frac{1}{r}-\frac{2M'}{M})y_1'+(k^2-\frac{m^2}{r^2})y_1}{k^2(1-M^2)}\right]Z_2dr=\int_0^{R_o}y_1\left[\left(\frac{\overline{Z}_2}{k^2(1-M^2)}\right)''-\left(\frac{(\frac{1}{r}-\frac{2M'}{M})\overline{Z}_2}{k^2(1-M^2)}\right)'+\frac{(k^2-\frac{m^2}{r^2})}{k^2(1-M^2)}\overline{Z}_2\right]dr+$$
$$\left[\frac{y_1'\overline{Z}_2}{k^2(1-M^2)}-y_1\left(\frac{\overline{Z}_2}{k^2(1-M^2)}\right)'+y_1\left(\frac{\frac{1}{r}-\frac{2M'}{M}}{k^2(1-M^2)}\right)\overline{Z}_2\right]_0^{R_o} \quad (D.3)$$

From Equation D. 3 and D. 2 it can be seen that

$$\alpha_{21}^T=\left(\left(\frac{(\ )}{k^2(1-M^2)}\right)''-\left(\frac{(\frac{1}{r}-\frac{2M'}{M})(\ )}{k^2(1-M^2)}\right)'+\frac{(k^2-\frac{m^2}{r^2})}{k^2(1-M^2)}(\ )\right)$$

To find α_{31}^T put α_{31} into Equation D. 1 to get Equation D. 4.

$$\int_0^{R_o}(\alpha_{31}y_1)\overline{Z}_3dr=\int_0^{R_o}y_1(\alpha_{31}^T\overline{Z}_3)dr+[BC] \quad (D.4)$$

Substituting for α_{31} in the above equation,

$$\int_0^{R_o}\frac{-iy_1'}{\rho ckM}Z_3dr=\int_0^{R_o}y_1(\alpha_{31}^TZ_3)dr+[BC] \quad (D.5)$$

Integrate the left hand side of the above quation by parts to get Equation D. 6.

$$\int_0^{R_o}y_1\left(\frac{i\overline{Z}_3}{\rho ckM}\right)'dr+\left[\frac{-iy_1\overline{Z}_3}{\rho ckM}\right]_0^{R_o}=\int_0^{R_o}y_1(\alpha_{31}^T\overline{Z}_3)dr+[BC] \quad (D.6)$$

thus

$$\alpha_{31}^T=\left(\frac{i(\ )}{\rho ckM}\right)'$$

It is now possible to write out the matrix for [A*].

$$\alpha_{iJ}^*=\overline{\alpha_{Ji}^T} \qquad \alpha_{11}^*=0 \qquad \alpha_{12}^*=\left[\left(\frac{(\ )}{1-M^2}\right)''-\left(\frac{(\frac{1}{r}-\frac{2M'}{M})(\ )}{(1-M^2)}\right)'+\frac{(k^2-\frac{m^2}{r^2})(\ )}{(1-M^2)}\right]\frac{1}{k^2}$$

$$\alpha^*_{13} = \left(\frac{-i(\ )}{\rho ckM}\right)' = 0 \qquad \alpha^*_{21} = 1(\ ) \qquad \alpha^*_{22} = \frac{-2M}{(1-M^2)}(\ ) \qquad \alpha^*_{23} = 0 \qquad \alpha^*_{31} = 0$$

$$\alpha^*_{32} = \frac{2i\rho ckM'}{k^2(1-M^2)M}(\ ) \qquad \alpha^*_{33} = \frac{1}{M}(\ )$$

The adjoint eigenvalue problem is

$$A^*\vec{Z} = \bar{\lambda}\vec{Z} \tag{D. 7}$$

Writing out the equations of D. 7 gives the following equations.

$$\frac{1}{k^2}\left[\left(\frac{Z_2}{(1-M^2)}\right)'' - \left(\frac{(\frac{1}{r} - \frac{2M'}{M})Z_2}{(1-M^2)}\right)' + \frac{(k^2 - \frac{m^2}{r^2})}{(1-M^2)}Z_2\right] - \left[\left(\frac{iZ_3}{\rho ckM}\right)'\right] = \bar{\lambda} Z_1 \tag{D. 8}$$

$$Z_1 - \frac{2M}{(1-M^2)} Z_2 = \bar{\lambda} Z_2 \tag{D. 9}$$

$$\frac{2i\rho ckM' Z_2}{k^2(1-M^2)M} + \frac{Z_3}{M} = \bar{\lambda} Z_3 \tag{D. 10}$$

Solving for Z_1 and Z_3 in Equations D. 9 and D. 10 and substituting into Equation D.8 reduces the set of equations to one equation in Z_2.

$$Z_1 = (\bar{\lambda} + \frac{2M}{1-M^2})Z_2, \qquad Z_3 = \frac{-2i\rho ckM' Z_2}{k^2(1-M^2)(1-M\bar{\lambda})}, \qquad \text{and}$$

$$\left(\frac{Z_2}{k^2(1-M^2)}\right)'' - \left(\frac{(\frac{1}{r} - \frac{2M'}{M})Z_2}{k^2(1-M^2)}\right)' + \frac{(k^2 - \frac{m^2}{r^2})}{k^2(1-M^2)} - \left(\frac{2M' Z_2}{k^2(1-M^2)(1-M\bar{\lambda})M}\right) -$$

$$\left(\bar{\lambda}^2 + \frac{2M\bar{\lambda}}{1-M^2}\right) Z_2 = 0 \tag{D. 11}$$

Equations D.11 show that $Z_1(r)$ and $Z_3(r)$ can be expressed in terms of $Z_2(r)$ alone and that boundary conditions on $Z_2(r)$ are needed to uniquely determine the radial distribution of $Z_2(r)$ to within an arbitrary constant.

The boundary condition terms came from Equations D. 3 and D. 6 and are set equal to zero collectively.

$$\left[\frac{y_1'\overline{Z}_2}{k^2(1-M^2)} - y_1\left(\frac{\overline{Z}_2}{k^2(1-M^2)}\right)' + y_1\frac{(\frac{1}{r}-\frac{2M'}{M})\overline{Z}_2}{k^2(1-M^2)}\right]_o^{R_o} + \left[\frac{-iy_1\overline{Z}_3}{\rho ckM}\right]_o^{R_o} = 0 \qquad \text{(D. 12)}$$

But $\vec{y} = \begin{Bmatrix} P \\ \lambda P \\ v \end{Bmatrix}$

so that $y_1 = P$ and Equation D. 12 becomes

$$\left[\frac{P'\overline{Z}_2}{k^2(1-M^2)} - P\left(\frac{\overline{Z}_2}{k^2(1-M^2)}\right)' + P\frac{(\frac{1}{r}-\frac{2M'}{M})\overline{Z}_2}{k^2(1-M^2)} - \frac{iP\overline{Z}_3}{\rho ckM}\right]_o^{R_o} = 0 \qquad \text{(D. 13)}$$

Two conditions on $Z_2(r)$ are needed to make the differential equation of D. 11 a complete problem.

If the boundary condition terms of Equation D.13 are set equal to zero at r=0 and at $r=R_o$ and the substitution for Z_3 is made, then the necessary two boundary conditions on Z_2 will be specified. For a cylindrical duct it may be assumed that the sheared portion of the flow is confined to within a small distance δ from the duct wall so that $M'(r) = 0$ for $r \leq R_o - \delta$ and the resulting two boundary conditions are at r = 0

$$P'(0)\overline{Z}_2(0) - P(0)\overline{Z}_2'(0) + \lim_{r\to 0}\left(\frac{P(r)\overline{Z}_2(r)}{r}\right) = 0 \qquad \text{(D. 14)}$$

and at $r = R_o$ (remembering that $M \to 0$ at the duct wall)

$$P'(R_o)\overline{Z}_2(R_o) - P(R_o)\overline{Z}_2'(R_o) + P(R_o)\overline{Z}_2(R_o)\left[\frac{1}{R_o} + 2M'(R_o)\lambda\right] = 0 \qquad \text{(D. 15)}$$

The form of the adjoint differential equation of D. 11 in the non-sheared portion of the fluid is (for $M'(r)=0$)

$$\overline{Z}_2'' - \frac{\overline{Z}_2'}{r} + k^2[1-2M\lambda - (1-M^2)\lambda^2 + \frac{1-m^2}{(kr)^2}]\overline{Z}_2 = 0 \qquad \text{(D. 16)}$$

where the complex conjugate has been taken. By letting $\overline{Z}_2 = rF$ Equation D. 16 can be transformed into Bessel's differential equation in F.

$$F'' + \frac{F'}{r} + k^2[1-2M\lambda - (1-M^2)\lambda^2 - \frac{m^2}{(kr)^2}]F = 0 \qquad \text{(D. 17)}$$

which is the same differential equation for P(r) in the uniform flow portion of the fluid. If $(r\overline{Z}_2)$ is required to be finite at r=0, then F may be composed only of Bessel Functions of the first kind,

$$\overline{Z}_2 = rJ_m(\alpha r) \quad \text{where} \quad \alpha = k^2(1-2M\lambda - (1-M^2)\lambda^2)$$

Thus for a cylindrical duct the adjoint can easily be found in the uniform flow section and it remains only to solve the adjoint differential Equation (Equation 11D) through the boundary layer at the duct wall.

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APPENDIX E

PROGRAM CO

```
      PROGRAM CO(INPUT=164/80,OUTPUT=164/80,TAPE5=INPUT,TAPE6=OUTPUT
C     THIS PROGRAM CALCULATES THE MODAL COEFFICIENTS AT
C     INTERFACES OF CHANGING ACOUSTIC IMPEDANCE FOR CYLINDRICAL
C     DUCTS
C
      COMPLEX B(10,2,10),LAM(10,2,10),ALPHA(10,2,10),A(10),I
      COMPLEX TOT1,TOT2,TOT3,TOT4,TOT5,TOT6,ZETA(10)
      COMPLEX PRESS,PGRAD,BFJ,PART(10,10),X1(10)
      DIMENSION IP(10)
      REAL LENGTH,LH1(10),LHTOTAL,DIST(12)
      REAL M,K,SE(10,2,10),TE(10,2,10),LH(10),LHH(10)
      INTEGER S,T,Z,ZZ,AM
      DATA (SE(1,1,Z), Z=1,3)/.00463418,.00356115,.00474047/
      DATA (SE(1,1,Z), Z=4,6)/.007555887,.01972692,2.72958570/
      DATA (TE(1,1,Z), Z=1,3)/2.68660591,2.49601728,2.02622534/
      DATA (TE(1,1,Z), Z=4,6)/1.17018950,-.44900763,-2.70879486/
      DATA (SE(1,2,Z), Z=1,3)/.23470328,.00545436,.01776125/
      DATA (SE(1,2,Z), Z=4,6)/.0270159,.04341887,2.75535781/
      DATA (TE(1,2,Z), Z=1,3)/6.56106018,7.90046629,7.46994469/
      DATA (TE(1,2,Z), Z=4,6)/6.63357129,5.01567345,2.75901951/
      DATA (SE(2,1,Z), Z=1,3)/.00022709,.00019883,.00026028/
      DATA (SE(2,1,Z), Z=4,6)/.000415708,.00110952,2.78251522/
      DATA (TE(2,1,Z), Z=1,3)/2.67947629,2.490009,2.018339/
      DATA (TE(2,1,Z), Z=4,6)/1.16260343,-.48236131,-2.71082085/
      DATA (SE(2,2,Z), Z=1,3)/.00509038,.00060155,.00130394/
      DATA (SE(2,2,Z), Z=4,6)/.00170124,.00249697,2.78395652/
      DATA (TE(2,2,Z), Z=1,3)/6.27638483,7.8656835,7.43375475/
      DATA (TE(2,2,Z), Z=4,6)/6.5860194,4.34066305,2.71348187/
      DO 70 Z=1,6
      SE(3,1,Z)=SE(2,1,Z)
      SE(3,2,Z)=SE(2,2,Z)
      TE(3,1,Z)=TE(2,1,Z)
      TE(3,2,Z)=TE(2,2,Z)
      SE(4,1,Z)=SE(1,1,Z)
      SE(4,2,Z)=SE(1,2,Z)
      TE(4,1,Z)=TE(1,1,Z)
      TE(4,2,Z)=TE(1,2,Z)
      SE(5,1,Z)=SE(1,1,Z)
      SE(5,2,Z)=SE(1,2,Z)
      TE(5,1,Z)=TE(1,1,Z)
      TE(5,2,Z)=TE(1,2,Z)
70    CONTINUE
      ZETA(1)=CMPLX(.4,-2.)
      ZETA(2)=CMPLX(.05,-3.1)
      ZETA(3)=ZETA(2)
      ZETA(4)=CMPLX(.4,-2.)
      ZETA(5)=ZETA(4)
      I=CMPLX(0.,1.)
      WRITE(6,5)
```

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C
C       ETA=2*K*F/C (A FREQUENCY PARAMETER)
C       AM IS THE ANGULAR MODE
C       ROUT IS THE OUTER RADIUS OF THE DUCT
C       FM IS THE FLOW MACH NUMBER
C
5       FORMAT(1X,*ENTER ETA,AM,ROUT,FM   *)
        READ(5,*) ETA,AM,H,M
        WRITE(6,7)
7       FORMAT(1X,*ENTER THE NUMBER OF EIGENVALUES   *)
        READ(5,*) J
        WRITE(6,8)
8       FORMAT(1X,*ENTER THE NUMBER OF INTERFACES   *)
        READ(5,*) NI
        T=NI-1
        IF(T.EQ.0) GO TO 4
        DO 15 Z=1,T
        WRITE(6,16) Z
C
C       LH(I) IS THE RATIO OF THE LENGTH IN THE AXIAL DIRECTION OF
C       SEGMENT I TO THE OUTER RADIUS
C
16      FORMAT(1X,*ENTER LH(*,I1,*)   *)
        READ(5,*) LHH(Z)
        JJ=2*Z
15      LH(JJ)=LHH(Z)
        GO TO 18
4       LH(2)=0.
18      LAST=2*NI
        DO 32 Z=1,J
        DO 33 ZZ=1,J
        B(1,1,ZZ)=B(1,2,ZZ)=CMPLX(0.,0.)
        PART(Z,ZZ)=CMPLX(0.,0.)
33      CONTINUE
32      CONTINUE
C
C       INITIALIZE THE INPUT PRESSURE DISTRIBUTION TO BE THE
C       FIRST FORWARD PROPAGATING MODE
C
        B(1,1,1)=CMPLX(1.,0.)
        H=H/12.
        R=H
        PI=3.1415926
        K=PI*ETA/H
        WRITE(6,9)
9       FORMAT(1X,*LIST EIGENVALUES? 1-YES, 0-NO   *)
        READ(5,*) IANS
C
C       CALCULATE THE RADIAL AND AXIAL EIGENVALUES
C
        DO 3 KK=1,LAST
        A(KK)=1./(ETA*ZETA(KK))
        DO 2 Z=1,J
        DO 1 S=1,2
```

```
      LAM(KK,S,Z)=CMPLX(TE(KK,S,Z),-SE(KK,S,Z))/ETA
      ALPHA(KK,S,Z)=1.0-LAM(KK,S,Z)**2*(1.0-M**2)
      ALPHA(KK,S,Z)=ALPHA(KK,S,Z)-2.0*M*(-1.0)**(S+1)*LAM(KK,S,Z)
      ALPHA(KK,S,Z)=(K*R)**2*ALPHA(KK,S,Z)
      ALPHA(KK,S,Z)=CSQRT(ALPHA(KK,S,Z))
      IF(IANS.NE.1) GO TO 100
      WRITE(6,17) KK,S,Z,ALPHA(KK,S,Z)
17    FORMAT(1X,*ALPHA(*,I1,*,*,I1,*,*,I1,*)=*,2F11.7)
      WRITE(6,11) KK,S,Z,LAM(KK,S,Z)
11    FORMAT(1X,*LAM(*,I1,*,*,I1,*,*,I1,*)=*,2F11.7)
100   CONTINUE
1     CONTINUE
2     CONTINUE
      IF(IANS.NE.1) GO TO 3
      WRITE(6,25)
      WRITE(6,25)
25    FORMAT(1X,* *)
3     CONTINUE
21    CONTINUE
      DO 50 KK=1,NI
      NN=2*KK
      LL=2*KK-1
      DO 10 N=1,J
      S=1
20    CONTINUE
      CALL MATCH(KK,N,S,J,AM,M,K,LAM,ALPHA,H,A,B)
      IF(NN.EQ.LAST) GO TO 22
      CALL TRANS(NN,S,N,K,LAM,H,LH,B)
22    S=S+1
      M=-M
      IF(S.EQ.2) GO TO 20
10    CONTINUE
50    CONTINUE
      WRITE(6,25)
      WRITE(6,25)
C
C     PRINT THE MODAL COEFFICIENTS
C
      DO 95 KK=1,NI
      NN=2*KK
      DO 30 N=1,J
      WRITE(6,40) NN-1,1,N,B(NN-1,1,N),NN-1,2,N,B(NN-1,2,N)
40    FORMAT(1X,*B(*,I1,*,*,I1,*,*,I1,*)=*,E12.6,1X,E12.6,
     +1X,*B(*,I1,*,*,I1,*,*,I1,*)=*,E12.6,1X,E12.6)
30    CONTINUE
      WRITE(6,25)
      WRITE(6,25)
      DO 60 N=1,J
      WRITE(6,40) NN,1,N,B(NN,1,N),NN,2,N,B(NN,2,N)
60    CONTINUE
      WRITE(6,25)
      WRITE(6,25)
95    CONTINUE
90    WRITE(6,25)
      WRITE(6,25)
      WRITE(6,35)
```

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```
35    FORMAT(1X,*DO YOU WANT A PRESSURE DISTRIBUTION? 1-YES, 0-NO  *)
      READ(5,*) IANS
      IF(IANS.NE.1) GO TO 65
      WRITE(6,45)
45    FORMAT(1X,*ENTER THE SECTION NUMBER TO BE INVESTIGATED  *)
      READ(5,*) LL
      WRITE(6,85)
85    FORMAT(1X,1X,*X*,16X,*PRESSURE*,20X,*PRESSURE GRADIENT*)
      DO 55 Z=1,11
      X=0.0+0.1*(Z-1)
      CALL PRES(J,LL,ETA,X,LAM,ALPHA,AM,B,PRESS,PGRAD)
      WRITE(6,80) X,PRESS,PGRAD
80    FORMAT(1X,F3.1,5X,E13.7,2X,E13.7,6X,E13.7,2X,E13.7)
55    CONTINUE
      GO TO 90
65    CONTINUE
      STOP
      END
C
C     SUBROUTINE "MATCH" CALCULATES THE MODAL COEFFICIENTS
C
      SUBROUTINE MATCH(KK,N,S,J,AM,M,K,LAM,ALPHA,H,A,B)
      COMPLEX BF1,BF2,BF3,BF4,BF5,BF6,BF7,BF9,BF10,BF11
      COMPLEX B(10,2,10),LAM(10,2,10),ALPHA(10,2,10),A(10)
      COMPLEX TOT1,TOT2,TOT3,TOT4,TOT5,TOT6,I,BFJ
      INTEGER S,AM
      REAL K,M
      NN=2*KK
      LL=2*KK-1
      I=CMPLX(0.,1.)
      CALL INT(KK,N,S,J,K,AM,B,LAM,ALPHA,TOT1,TOT2)
      BF1=((1.0-M**2)*LAM(NN,S,N)+2.0*M)*TOT1
      BF2=(1.0-M**2)*(-1.0)**(S+1)*I/K*TOT2
      TOT5=BFJ(ALPHA(NN,S,N),AM)
      BF3=I*A(NN)*TOT5/(K*H)
      CALL G(1.,J,LL,ALPHA,AM,B,TOT3)
      BF4=(M**2*LAM(NN,S,N)-2.0*M)*TOT3
      CALL H1(1.,J,LL,ALPHA,LAM,K,AM,B,TOT4)
      BF4=BF4+M**2*(-1.0)**(S+1)*I/K*TOT4
      BF5=BF3*BF4
      BF6=BF1+BF2+BF5
      BF7=2.0*LAM(NN,S,N)*(1.0-M**2)+2.0*M
      CALL INT2(KK,S,N,B,LAM,AM,ALPHA,TOT6)
      BF9=BF7*TOT6
      BF10=I*A(NN)*TOT5**2
      BF10=BF10*(2.0*M**2*LAM(NN,S,N)-2.0*M)/(K*H)
      BF11=BF9+BF10
      B(NN,S,N)=BF6/BF11
      RETURN
      END
C
C     SUBROUTINE "TRANS" TRANSFERS THE MODAL COEFFICIENTS AT
C     THE BEGINNING OF A LINING SEGMENT TO THE NEXT CHANGE
C     IN ACOUSTIC IMPEDANCE
C
```

```
      SUBROUTINE TRANS(NN,S,N,K,LAM,H,LH,B)
      COMPLEX LAM(10,2,10),B(10,2,10),I
      REAL K,LH(10)
      INTEGER S
      I=CMPLX(0.,1.)
      JJ=NN+1
      B(JJ,S,N)=B(NN,S,N)*CEXP((-1.0)**S*I*K*LAM(NN,S,N)*H*LH(NN))
      RETURN
      END
C
C     SUBROUTINE "INT" CALCULATES THE INTEGRAL OF THE PRESSURE
C     DISTRIBUTION MULTIPLIED BY THE N TH MODE OF PRESSURE
C
      SUBROUTINE INT(KK,N,S,J,K,AM,B,LAM,ALPHA,TOT1,TOT2)
      REAL K
      INTEGER S,AM,AM1
      COMPLEX TOT1,TOT2
      COMPLEX T1,T2,T3,T4,T5,T6,SUM1,SUM2,SUM3,SUM4
      COMPLEX ALPHA(10,2,10),LAM(10,2,10),B(10,2,10),BFJ,I
      I=CMPLX(0.,1.)
      NN=2*KK
      LL=2*KK-1
      SUM1=CMPLX(0.,0.)
      SUM2=CMPLX(0.,0.)
      SUM3=CMPLX(0.,0.)
      SUM4=CMPLX(0.,0.)
      AM1=AM+1
      DO 10 L=1,J
      T1=ALPHA(LL,1,L)*BFJ(ALPHA(LL,1,L),AM1)*BFJ(ALPHA(NN,S,N),AM)
      T2=ALPHA(NN,S,N)*BFJ(ALPHA(LL,1,L),AM)*BFJ(ALPHA(NN,S,N),AM1)
      T5=(ALPHA(LL,1,L))**2-(ALPHA(NN,S,N))**2
      SUM1=SUM1+B(LL,1,L)*(T1-T2)/T5
      SUM2=SUM2+B(LL,1,L)*LAM(LL,1,L)*(T1-T2)/T5
      T3=ALPHA(LL,2,L)*BFJ(ALPHA(LL,2,L),AM1)*BFJ(ALPHA(NN,S,N),AM)
      T4=ALPHA(NN,S,N)*BFJ(ALPHA(LL,2,L),AM)*BFJ(ALPHA(NN,S,N),AM1)
      T6=(ALPHA(LL,2,L))**2-(ALPHA(NN,S,N))**2
      SUM3=SUM3+B(LL,2,L)*(T3-T4)/T6
      SUM4=SUM4+B(LL,2,L)*LAM(LL,2,L)*(T3-T4)/T6
10    CONTINUE
      TOT1=SUM1+SUM3
      TOT2=(-I)*K*(SUM2-SUM4)
      RETURN
C     SUBROUTINE "G" CALCULATES THE PRESSURE AT A POINT FOR A
C
C     GIVEN NORMALIZED DISTANCE, X, FROM THE CENTER OF THE DUCT
C
      END
```

```
      SUBROUTINE G(X,J,LL,ALPHA,AM,B,TOT3)
      COMPLEX B(10,2,10),ALPHA(10,2,10)
      COMPLEX FU,FU1,FU2,SUM3,TOT3,BFJ
      INTEGER AM
      SUM3=CMPLX(0.,0.)
      DO 10 L=1,J
      FU1=B(LL,1,L)*BFJ(ALPHA(LL,1,L)*X,AM)
      FU2=B(LL,2,L)*BFJ(ALPHA(LL,2,L)*X,AM)
      FU=FU1+FU2
      SUM3=SUM3+FU
10    CONTINUE
      TOT3=SUM3
      RETURN
      END
C
C     SUBROUTINE "H" CALCULATES THE PRESSURE GRADIENT AT A POINT
C     FOR A GIVEN NORMALIZED DISTANCE, X, FROM THE CENTER
C     OF THE DUCT
C
      SUBROUTINE H1(X,J,LL,ALPHA,LAM,K,AM,B,TOT4)
      COMPLEX B(10,2,10),ALPHA(10,2,10),LAM(10,2,10)
      COMPLEX SUM4,TOT4,FU,FU1,FU2,I,BFJ
      REAL K
      INTEGER AM
      I=CMPLX(0.,1.)
      SUM4=CMPLX(0.,0.)
      DO 10 L=1,J
      FU1=B(LL,1,L)*LAM(LL,1,L)*BFJ(ALPHA(LL,1,L)*X,AM)
      FU2=B(LL,2,L)*LAM(LL,2,L)*BFJ(ALPHA(LL,2,L)*X,AM)
      FU=FU1-FU2
      SUM4=SUM4+FU
10    CONTINUE
      TOT4=SUM4*K*(-I)
      RETURN
      END
C
C     SUBROUTINE "INT2" CALCULATES THE INTEGRAL OF THE SQUARE
C     OF THE N TH MODAL PRESSURE DISTRIBUTION
C
      SUBROUTINE INT2(KK,S,N,B,LAM,AM,ALPHA,TOT6)
      COMPLEX B(10,2,10),LAM(10,2,10),ALPHA(10,2,10)
      COMPLEX FU1,FU2,TOT6,BFJ
      INTEGER S,AM,AM1,AM2
      AM1=AM+1
      AM2=AM-1
      NN=2*KK
      LL=2*KK-1
      IF(AM.EQ.0) GO TO 10
      FU1=(BFJ(ALPHA(NN,S,N),AM))**2
      FU2=(BFJ(ALPHA(NN,S,N),AM2))*(BFJ(ALPHA(NN,S,N),AM1))
      TOT6=0.5*(FU1-FU2)
      RETURN
10    FU1=(BFJ(ALPHA(NN,S,N),1))**2
      FU2=(BFJ(ALPHA(NN,S,N),0))**2
      TOT6=0.5*(FU1+FU2)
      RETURN
      END
```

```
      COMPLEX FUNCTION BFJ(Z,N)
C
C     BFJ(Z,N) IS J SUB N OF Z (BESSEL FUNCTION)
C
      COMPLEX AI,V
      COMPLEX SV,SU,GN,UN,VN
      COMPLEX T,Z,ZZ,Y,G,A,B,S,PP,ONE
      IF(CABS(Z).EQ.0.) GO TO 77
      AI=(0.,1.)
      PI=3.1415927
      V=-AI*Z
      ONE=(1.,0.)
      ZRO=0.
      E=1.E-10
      PP=(2.506628274,0.)
      XN=FLOAT(N)
      IF(CABS(V)/10.-1.) 21,21,31
21    ZZ=(.5,0.)*V
      Y=ZZ*ZZ
      FAC2=1.
      IF(N) 41,42
41    DO 43 I=1,N
43    FAC2=FAC2*FLOAT(I)
42    G=ONE/CMPLX(FAC2,ZRO)
      A=G
      X=0.
51    X=X+1.
      A=A*Y/CMPLX(X,ZRO)/CMPLX(XN+X,ZRO)
      G=G+A
      U=CABS(A)-E
      IF(U) 23,23,51
23    G=(ZZ**N)*G
      GO TO 93
31    U=4.*XN*XN
      SV=(0.,0.)
      SU=ONE
      K=1
      GN=Z-PI/4.-N*PI/2.
      B=(.125,0.)/Z
      VN=(U-1.)*B
      UN=-VN*(U-9.)/2.*B
      SV=SV+VN
      SU=SU+UN
      DO 33 K=2,6
      XK=FLOAT(K)
      VN=UN*(U-(4.*XK-3.)**2)*B/(2.*XK-1.)
      UN=-VN*(U-(4.*XK-1.)**2)*B/(2.*XK)
      SV=SV+VN
33    SU=SU+UN
      BFJ=SQRT(2./PI)/CSQRT(Z)*(SU*CCOS(GN)-SV*CSIN(GN))
      RETURN
```

```
99    BFJ=CEXP(N*PI*AI/2.)*G
      IF(REAL(Z).LT.0..AND.AIMAG(Z).GT.0.) GO TO 88
      RETURN
88    BFJ=CEXP(-3.*N*PI*AI/2.)*G
      RETURN
77    BFJ=(0.,0.)
      IF(N.EQ.0) BFJ=(1.,0.)
      RETURN
      END
      SUBROUTINE PRES(J,LL,ETA,X,LAM,ALPHA,AM,B,PRESS,PGRAD)
      COMPLEX LAM(10,2,10),ALPHA(10,2,10),B(10,2,10),BFJ
      COMPLEX FU,FU1,FU2,FU3,SUM1,SUM2,PRESS,PGRAD
      INTEGER AM
      SUM1=CMPLX(0.,0.)
      SUM2=CMPLX(0.,0.)
      DO 10 L=1,J
      FU1=B(LL,1,L)*BFJ(ALPHA(LL,1,L)*X,AM)
      FU2=B(LL,2,L)*BFJ(ALPHA(LL,2,L)*X,AM)
      FU=FU1+FU2
      FU3=LAM(LL,1,L)*FU1-LAM(LL,2,L)*FU2
      SUM1=SUM1+FU
      SUM2=SUM2+FU3
10    CONTINUE
      PRESS=SUM1
      PGRAD=SUM2
      RETURN
      END
```

```
 RUN,FTN

@       4.039 CP SECONDS COMPILATION TIME
@ENTER ETA,AM,ROUT,FM  4.,0,40,.5

@ENTER THE NUMBER OF EIGENVALUES  5

ENTER THE NUMBER OF INTERFACES  2

ENTER LH(1)  .75

LIST EIGENVALUES?  1-YES, 0-NO  1

ALPHA(1,1,1)=     .1452562  1.2642084
LAM(1,1,1)=     .6716515  -.0011585
ALPHA(1,2,1)=     1.4721282   6.9562253
LAM(1,2,1)=     2.1402650   -.0586758
ALPHA(1,1,2)=     3.6414120   .0375828
LAM(1,1,2)=     .6240043   -.0008953
ALPHA(1,2,2)=    2.7971369    .0755394
LAM(1,2,2)=     1.9750117   -.0013636
ALPHA(1,1,3)=     6.8943332   .0238853
LAM(1.1,3)=     .5065563   -.0011851
ALPHA(1,2,3)=     6.3074688   .1002319
LAM(1,2,3)=     1.8674862   -.0044453
ALPHA(1,1,4)=     10.0646391   .0213496
LAM(1,1,4)=     .2937974   -.0018890
ALPHA(1,2,4)=     9.6995186   .0836030
LAM(1,2,4)=     1.6583928   -.0069040
ALPHA(1,1,5)=     13.1966252   .0245387
LAM(1,1.5)=     -.1122519   -.0049317
ALPHA(1,2,5)=     13.0278470   .0580338
LAM(1,2,5)=     1.2539184   -.0108705


ALPHA(2,1.1)=     .0089302   1.0063306
LAM(2,1,1)=     .6698691   -.0000568
ALPHA(2,2,1)=     .0709937   4.7320683
LAM(2,2,1)=     2.0690962   -.0020226
ALPHA(2,1,2)=     2.7036910   .0020492
LAM(2,1,2)=     .6225023   -.0000497
ALPHA(2,2,2)=     2.9856230   .0077840
LAM(2,2,2)=     1.9714709   -.0001504
ALPHA(2,1,3)=     6.9338692   .0013018
LAM(2,1,3)=     .5045848   -.0000651
ALPHA(2,2,3)=     6.5065811   .0070716
LAM(2,2,3)=     1.8584387   -.0003260
ALPHA(2,1,4)=     10.1000377   .0011667
LAM(2,1,4)=     .2906509   -.0001039
ALPHA(2,2,4)=     9.8480543   .0050087
LAM(2,2,4)=     1.6459005   -.0004253
```

```
ALPHA(2,1,5)=    13.2376073   .0013552
LAM(2,1,5)=     0.1205903   -.0002774
ALPHA(2,2,5)=    13.1250769   .0032051
LAM(2,2,5)=     1.2352158   -.0006247

ALPHA(3,1,1)=     .0089302   1.0063306
LAM(3,1,1)=     .6698691   -.0000568
ALPHA(3,2,1)=     .0709937   4.7320683
LAM(3,2,1)=     2.0690962   -.0020226
ALPHA(3,1,2)=     3.7036910   .0020492
LAM(3,1,2)=     .6225023   -.0000497
ALPHA(3,2,2)=     2.9856230   .0077840
LAM(3,2,2)=     1.9714709   -.0001504
ALPHA(3,1,3)=     6.9338692   .0013018
LAM(3,1,3)=     .5045848   -.0000651
ALPHA(3,2,3)=     6.5065811   .0070716
LAM(3,2,3)=     1.8584387   -.0003260
ALPHA(3,1,4)=    10.1000377   .0011667
LAM(3,1,4)=     .2906509   -.0001039
ALPHA(3,2,4)=     9.8480543   .0050087
LAM(3,2,4)=     1.6459005   -.0004253
ALPHA(3,1,5)=    13.2376073   .0013552
LAM(3,1,5)=     -.1205903   -.0002774
ALPHA(3,2,5)=    13.1250769   .0032051
LAM(3,2,5)=     1.2352158   -.0006247

ALPHA(4,1,1)=     .1452562   1.2642084
LAM(4,1,1)=     .6716515   -.0011585
ALPHA(4,2,1)=     1.4721282   6.9562253
LAM(4,2,1)=     2.1402650   -.0586758
ALPHA(4,1,2)=     3.6414120   .0375828
LAM(4,1,2)=     .6240043   -.0008953
ALPHA(4,2,2)=     2.7971369   .0755394
LAM(4,2,2)=     1.9750117   -.0013636
ALPHA(4,1,3)=     6.8943332   .0238853
LAM(4,1,3)=     .5065563   -.0011851
ALPHA(4,2,3)=     6.3074688   .1002319
LAM(4,2,3)=     1.8674862   -.0044453
ALPHA(4,1,4)=    10.0646391   .0213496
LAM(4,1,4)=     .2937974   -.0018890
ALPHA(4,2,4)=     9.6995186   .0836030
LAM(4,2,4)=     1.6583928   -.0069040
ALPHA(4,1,5)=    13.1966252   .0245387
LAM(4,1,5)=     -.1122519   -.0049317
ALPHA(4,2,5)=    13.0278470   .0580338
LAM(4,2,5)=     1.2539184   -.0108705
```

```
B(1,1,1)=  .100000E+01  0.           B(1,2,1)=0.            0.
B(1,1,2)=0.             0.           B(1,2,2)=0.            0.
B(1,1,3)=0.             0.           B(1,2,3)=0.            0.
B(1,1,4)=0.             0.           B(1,2,4)=0.            0.
B(1,1,5)=0.             0.           B(1,2,5)=0.            0.


B(2,1,1)=  .107224E+01  -.461873E-01 B(2,2,1)=-.158089E-03  .964180E-04
B(2,1,2)= -.989596E-01   .643348E-01 B(2,2,2)= .882838E-03 -.668608E-03
B(2,1,3)=  .426578E-01  -.287741E-01 B(2,2,3)=-.229084E-02  .162667E-02
B(2,1,4)= -.272913E-01   .186203E-01 B(2,2,4)= .409883E-02 -.288730E-02
B(2,1,5)=  .258513E-01  -.177915E-01 B(2,2,5)=-.103703E-01  .724852E-02


B(3,1,1)=  .106979E+01  -.784815E-01 B(3,2,1)=-.187724E-03 -.195088E-04
B(3,1,2)= -.116468E+00   .188210E-01 B(3,2,2)= .674454E-03 -.880359E-03
B(3,1,3)=  .305712E-01   .413497E-01 B(3,2,3)= .104757E-02  .261634E-02
B(3,1,4)=  .323711E-01  -.644275E-02 B(3,2,4)=-.347298E-02  .364385E-02
B(3,1,5)=  .269463E-01   .159246E-01 B(3,2,5)=-.458417E-03  .127189E-01


B(4,1,1)=  .999555E+00  -.293236E-01 B(4,2,1)= .223289E-04 -.712663E-05
B(4,1,2)= -.183774E-01  -.499105E-01 B(4,2,2)= .100928E-02 -.362893E-03
B(4,1,3)= -.132040E-01   .715555E-01 B(4,2,3)= .427187E-02  .947539E-03
B(4,1,4)=  .603931E-01  -.249638E-01 B(4,2,4)=-.843627E-02  .623144E-02
B(4,1,5)=  .299753E-03   .329887E-01 B(4,2,5)= .108228E-01  .599971E-02


1-CONTINUE, 0-STOP    1

ENTER DELTA .0001

@LIST PARTIALS?  1-YES, 0-NO  0


B(1,1,1)=  .195197E+01   .390004E-01 B(1,2,1)= .183606E-01 -.150698E-01
B(1,1,2)=  .101560E+01  -.717458E+01 B(1,2,2)=-.261584E+01  .513490E+01
B(1,1,3)= -.194688E+01   .274143E+00 B(1,2,3)= .266783E+01  .295531E+01
B(1,1,4)=  .174610E+00   .568140E+01 B(1,2,4)=-.264145E+00 -.689511E+01


B(2,1,1)=  .207401E+01  -.134046E+00 B(2,2,1)=-.673036E-01 -.114168E+00
B(2,1,2)=  .764204E+00  -.708526E+01 B(2,2,2)=-.215084E+01  .588187E+01
B(2,1,3)= -.173913E+01   .454197E+00 B(2,2,3)= .210592E+01  .183519E+01
B(2,1,4)=  .658230E-01   .557937E+01 B(2,2,4)=-.756867E-01 -.643189E+01


B(3,1,1)=  .206791E+01  -.196467E+00 B(3,2,1)= .159754E-01 -.134132E+00
B(3,1,2)=  .356203E+01  -.616841E+01 B(3,2,2)=-.511752E+00  .625076E+01
B(3,1,3)= -.528574E+00  -.171684E+01 B(3,2,3)= .228472E+01 -.162203E+01
B(3,1,4)=  .212173E+01  -.515470E+01 B(3,2,4)= .133037E+01  .631966E+01
```

```
B(4,1,1)=  .191084E+01  -.272673E-01  B(4,2,1)=  .284520E-01  -.888894E-02
B(4,1,2)=  .380326E+01  -.622155E+01  B(4,2,2)= -.788153E+00   .554465E+01
B(4,1,3)= -.655030E+00  -.187940E+01  B(4,2,3)=  .256603E+01  -.606630E+00
B(4,1,4)=  .224428E+01  -.491479E+01  B(4,2,4)=  .993865E+00   .522715E+01

1-CONTINUE,  0-STOP  0

DO YOU WANT A PRESSURE DISTRIBUTION?  1-YES,  0-NO  1

ENTER THE SECTION NUMBER TO BE INVESTIGATED  1

 X                   PRESSURE                         PRESSURE GRADIENT

0.0      .1001500E+01    .2842171E-13         .1625843E+01    -.6829814E+01
 .1      .1001167E+01   -.1183152E-01         .1995747E+01    -.8885828E+01
 .2      .1008851E+01   -.2407052E-01         .2970781E+01    -.1328979E+02
 .3      .1036947E+01   -.6095731E-02         .4203197E+01    -.1612265E+02
 .4      .1078984E+01    .1972273E-01         .5266381E+01    -.1434730E+02
 .5      .1112009E+01   -.5990515E-02         .5794504E+01    -.8126704E+01
 .6      .1130982E+01   -.8468392E-01         .5580465E+01    -.6354339E+00
 .7      .1176481E+01   -.1119220E+00         .4642866E+01     .4433282E+01
 .8      .1298269E+01   -.3316751E-02         .3304502E+01     .5684916E+01
 .9      .1427513E+01    .4335035E-01         .2308622E+01     .5022769E+01
1.0      .1180064E+01   -.6320837E+00         .2981733E+01     .6209906E+01

DO YOU WANT A PRESSURE DISTRIBUTION?  1-YES,  0-NO  1

ENTER THE SECTION NUMBER TO BE INVESTIGATED  2

 X                   PRESSURE                         PRESSURE GRADIENT

0.0      .9769849E+00   -.1474205E-01         .1598795E+01    -.6833459E+01
 .1      .9869171E+00   -.2050817E-01         .1972721E+01    -.8891780E+01
 .2      .1012398E+01   -.2459882E-01         .2964705E+01    -.1329070E+02
 .3      .1043196E+01   -.1122503E-01         .4227094E+01    -.1609951E+02
 .4      .1070392E+01    .2719755E-02         .5308967E+01    -.1430485E+02
 .5      .1094738E+01   -.1871691E-01         .5812463E+01    -.8110914E+01
 .6      .1129986E+01   -.7082811E-01         .5543096E+01    -.6821453E+00
 .7      .1194598E+01   -.8516080E-01         .4601613E+01     .4378078E+01
 .8      .1289856E+01   -.1720694E-01         .3379988E+01     .5739820E+01
 .9      .1370750E+01   -.5267412E-02         .2459718E+01     .5129538E+01
1.0      .1322296E+01   -.4511934E+00         .2454775E+01     .5722503E+01

DO YOU WANT A PRESSURE DISTRIBUTION?  1-YES,  0-NO  0

      STOP
      35.564 CP SECONDS EXECUTION TIME
..
```

APPENDIX F

LINER ANALYSIS PROGRAMS

F.1 LINPART INPUT

R = normalized lining resistance - dimensionless

X = normalized lining reactance - dimensionless

T = temperature in ^{o}R

P = duct mean pressure in psi

F = frequency in Hz

M = duct Mach number

RHO = duct mean density in lbm/ft^3

TH = face sheet thickness in inches

D = face sheet perforation hole diameters in inches

F.2 LINPART OUTPUT

OA = fractional open area of face sheet (.063 means 6.3% open)

DP = depth of backing cavity in inches

Program LINFREQ uses the same nomenclature as LINPART.

```
      PROGRAM LINPART(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL M
5     WRITE(6,200)
      READ(5,*) R,X,T,P,F,FM,RHO
6     WRITE(6,230)
      READ(5,*) TH,D
      IF(TH.EQ.0.) STOP
      CEE=12*SQRT(1.4*32.174*144*P/RHO)
      M=ABS(FM)
      TR=T/519.
      PR=P/14.7
      A=.0762*TR*TR*TH/(PR*(TR+.416))
      B=.0000374*(TR**.75)*SQRT(F)/SQRT(PR*(TR+.416))
      OA=(A+B*(1.+TH/D)+.1085*M)/(R+B)
      IF(OA.LT.0.) GO TO 1
      EX=EXP(-8.65*M*M-.8192*M)
      C=.000469*F*(TH+.85*D*(1.-.7*SQRT(OA))*EX)/SQRT(TR)
      DP=(CEE/(6.283185*F))*ATAN(1./(C/OA-X))
      GO TO 2
1     DP=999
2     WRITE(6,255) TH,D,OA,DP
      WRITE(6,232)
      GO TO 6
200   FORMAT(1X,*ENTER R,X,T,P,F,FM,RHO *)
230   FORMAT(1X,*ENTER TH,D *)
232   FORMAT(1X,55(1H*),/)
255   FORMAT(1X,*TH=*,F8.4,*  DIA=*,F8.4,*  OA=*,F9.6,*  DP=*
     +,F9.4,*  INCHES*)
      END
```

```
      PROGRAM LINFREQ(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL M
      PI=3.1416
      WRITE(6,400)
      READ(5,*) T,P,FM,TH,D,OA,DP,RHO
      CIN=12*SQRT(1.4*32.174*144*P/RHO)
      M=ABS(FM)
      TR=T/519.
      PR=P/14.7
6     WRITE(6,401)
      READ(5,*) FREQ
      IF(FREQ.EQ.0.)STOP
      F=FREQ
      A=.0762*TR*TR*TH/(PR*(TR+.416))
      B=.0000374*(TR**.75)*SQRT(F)/SQRT(PR*(TR+.416))
      EX=EXP(-8.65*M*M-.8192*M)
      C=.000469*F*(TH+.85*D*(1.-.7*SQRT(OA))*EX)/SQRT(TR)
      R=1/OA*(A+B*(TH/D+(1-OA))+.1085*M)
      X=C/OA-1/TAN(2*PI*F*DP/CIN)
      WRITE(6,501) R,X,FREQ
      GO TO 6
501   FORMAT(1X,*R=*,F10.6,*  X=*,F10.6,*  FREQ=*,F10.1,/)
400   FORMAT(1X,*ENTER TEMP,P,FM,TH,D,OA,DP,RHO*)
401   FORMAT(1X,*ENTER FREQ*)
      END
```

APPENDIX G

COMPARISON OF ATTENUATION PREDICTIONS

Table G.1 Lining Attenuation Spectrum
Equal Energy $\eta_{design} = 0.8$

η	f	AM	RM	SE	TE	Comb. Attn.
.4	136.7	0	0	.020	.860	
		1	0	.0338	.674	1.4 DB
.6	205.0	0	0	.108	1.325	
		0	1	.147	.481	
		1	0	.139	1.236	
		1	1	1.086	.0719	
		2	0	.1925	1.014	8.7 DB
.8	273.4	0	0	.284	1.381	
		0	1	1.470	.212	
		1	0	.394	1.471	
		1	1	.555	.604	
		2	0	.512	1.347	
		2	1	1.310	.265	
		3	0	.689	1.135	22.6 DB
1.0	341.7	0	0	.242	1.433	
		0	1	1.204	.323	
		0	2	2.611	.1476	
		1	0	.137	1.737	
		1	1	.425	.925	
		2	0	.240	1.515	
		2	1	1.064	.397	
		3	0	.398	1.196	
		3	1	1.728	.258	
		4	0	.730	.794	15.4 DB
1.2	410.1	0	0	.0991	1.936	
		0	1	.373	.627	
		0	2	2.290	.107	
		1	0	.0633	2.199	
		1	1	.146	1.534	
		1	2	1.473	.165	
		2	0	.103	2.015	
		2	1	.294	.847	
		2	2	2.202	.115	
		3	0	.155	1.754	
		3	1	1.184	.229	
		4	0	.242	1.377	
		4	1	1.903	.153	
		5	0	.488	.802	10.0 DB

Conditions: Cylindrical duct $\eta = hf/c$
L/H=2 Mach = 0 h = 40"

Optimum impedence/2η = (.8900-.3821 i) at η_{design}

Design from linpar: Thickness = .05" Diameter = .015"
Open area = .0036 Depth = 2.71"

Table G. 2 Peak Attention vs. Eta (Equal Energy)

η	f	AM	RM	SE	TE	Comb. Attn.
.4	136.7	0	0	.6604	.585	
		1	0	.7335	.791	37.6 DB
.8	273.4	0	0	.2835	1.381	
		0	1	1.470	.2118	
		1	0	.3944	1.4707	
		1	1	.5552	.6039	
		2	0	.5119	1.347	
		2	1	1.310	.2650	
		3	0	.6885	1.135	22.6 DB
2.	683.5	0	0	.0979	3.909	
		0	1	.0925	3.366	
		0	2	.1270	2.411	
		0	3	1.329	.229	
		0	4	3.351	.0906	
		1	0	.1475	3.933	
		1	1	.1046	2.988	
		1	2	.1970	1.560	
		1	3	2.475	.123	
		2	0	.178	3.876	
		2	1	.128	2.498	
		2	2	1.169	.269	
		2	3	3.288	.094	
		3	0	.207	3.781	
		3	1	.181	1.814	
		3	2	2.300	.138	
		3	3	3.974	.0786	
		4	0	.237	3.650	
		4	1	.138	2.765	
		4	2	3.090	.105	
		4	3	4.589	.0691	
		5	0	.2698	3.480	
		5	1	.176	2.276	
		5	2	3.753	.0883	
		5	3	5.159	.0625	
		6	0	.309	3.267	
		6	1	.265	1.579	
		6	2	4.348	.0779	
		7	0	.358	3.002	
		7	1	.893	.489	
		7	2	4.898	.0707	
		8	0	.426	2.675	
		8	1	1.960	.2316	11.7 DB

Conditions: Cylindrical duct $\eta = hf/c$
L/H = 2 Mach = 0 h = 40"

Optimum impedence/2η = (.8900-.3821 i) for all η at Mach = 0

$$E_o = \frac{1}{n} \sum_{i=1}^{n} e^{-4\pi SEi} \qquad \text{Attn} = 10 \log_{10} E_o$$

Table G.3 Data for Lining Attn. Spectra
Ductech Method

$\frac{ETA}{ETA_D}$	Cylindrical L/H = 2 ETA_{design} = .8			Cylindrical L/H = 2 ETA_{design} = 2.4		
	$ATTN/ATTN_{peak}$			$ATTN/ATTN_{peak}$		
	M = 0	M = -.5	M = .5	M = 0	M = -.5	M = .5
.40	.374	.338	.380	.414	.452	.380
.50	.570	.545	.573	.596	.623	.573
.63	.746	.732	.749	.762	.777	.749
.79	.946	.943	.946	.948	.952	.946
1.00	1.000	1.000	1.000	1.000	1.000	1.000
1.25	.946	.943	.946	.948	.952	.946
1.59	.746	.732	.749	.762	.777	.749
2.00	.570	.545	.573	.596	.623	.573
2.52	.452	.421	.457	.487	.521	.457
3.18	.374	.339	.380	.414	.452	.380
4.00	.335	.298	.342	.378	.419	.342
4.62	.307	.267	.313	--	--	--

Table G.4 Data for Lining Attn. Spectra Least Attn. Mode Theory

FREQ (HZ)	ETA/ETA$_0$	RESISTANCE*			REACTANCE*			SE			TE			ATTN/DESIGN ATTN		
		M=0	M=.2	M=-.5	M=0	M=.2	M=-.5	M=0	M=.2	M=-.5	M=0	M=.2	M=-.5	M=0	M=.2	M=-.5
54.2	.4	1.96	1.93	4.96	-8.71	-4.75	-18.3	—	.034	.121	—	.368	.906	—	.062	.14
68.4	.5	1.60	1.54	3.94	-5.03	-2.71	-11.4	.045	.062	.242	.541	.468	1.13	.068	.113	.28
86.1	.63	1.32	1.23	3.14	-2.74	-1.43	-7.10	.102	—	.443	.699	—	1.32	.154	—	.52
108.5	.79	1.09	.977	2.49	-1.29	-.608	-4.37	.267	.257	.660	.875	.707	1.41	.405	.467	.77
136.7	1.0	.890	.780	1.98	-.382	-.053	-2.65	.660	.550	.857	.585	.461	1.19	1.00	1.00	1.00
172.2	1.26	.746	.623	1.58	.210	.359	-1.57	.146	.121	.074	.780	.649	2.25	.221	.220	.08
217.0	1.59	.620	.498	1.26	.590	.812	-.887	.060	.041	—	1.12	.941	—	.090	.074	—
273.4	2.0	.518	.400	1.00	.852	2.77	-.455	—	—	.037	—	—	3.00	—	—	.04
344.5	2.52	.434	.319	.798	1.07	-.758	-.178	—	—	.021	—	—	3.88	—	—	.02

DESIGN ETA = 0.4 CYLINDRICAL DUCT

L/H = 2

RESISTANCE* = RESISTANCE / 2(ETA)

REACTANCE* = REACTANCE / 2(ETA)

Table G.5 Data for Lining Attn. Spectra Least Attn. Mode Theory

FREQ (HZ)	ETA/ETA_D	RESISTANCE*			REACTANCE*			SE			TE			ATTN/DESIGN ATTN		
		M=0	M=.5	M=-.5	M=0	M=.5	M=-.5	M=0	M=.5	M=-.5	M=0	M=.5	M=-.5	M=0	M=.5	M=-.5
162.7	.4	1.86	1.12	8.43	-12.9	-2.14	-12.7	—	—	.070	—	—	1.98	—	—	.42
205.0	.5	1.55	.889	6.71	-7.39	-1.26	-7.83	.0085	.050	.091	1.24	.866	1.67	.049	.290	.55
258.3	.63	1.29	.708	5.34	-3.43	-.714	-4.76	.029	.095	.097	1.57	.585	2.55	.167	.556	.58
325.5	.79	1.08	.564	4.26	-1.73	-.364	-2.82	.077	.109	.115	1.99	.973	3.48	.450	.636	.69
410.1	1.0	.890	.449	3.39	-.382	-.141	-1.60	.172	.171	.167	2.25	1.47	4.64	1.00	1.00	1.00
516.7	1.26	.764	.358	2.71	.523	.007	-.834	.028	.050	.056	2.96	1.94	5.95	.165	.310	.33
651.0	1.59	.646	.286	2.16	1.07	.116	-.350	.012	.025	.030	3.77	2.48	7.54	.070	.147	.18
820.2	2.0	.548	.228	1.72	1.42	.227	-.045	—	—	.018	—	—	9.54	—	—	.11
1033	2.52	.466	.182	1.38	1.63	.577	.147	—	—	—	—	—	—	—	—	—

DESIGN ETA = 1.2   CYLINDRICAL DUCT

L/H = 2

RESISTANCE* = RESISTANCE / 2(ETA)

REACTANCE* = REACTANCE / 2(ETA)

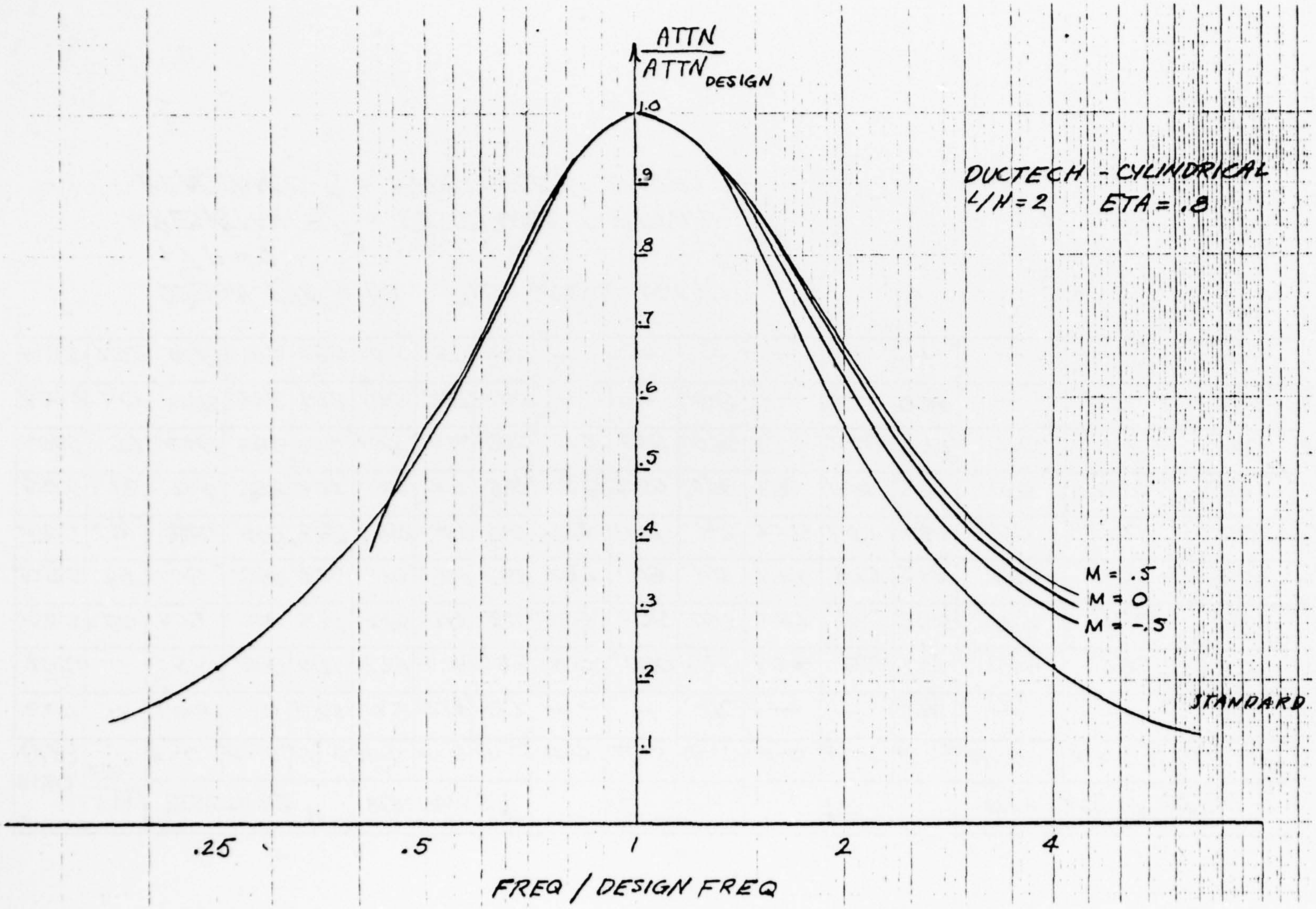

Figure G.1 Effect of Mach No. on Lining Attn. Spectra

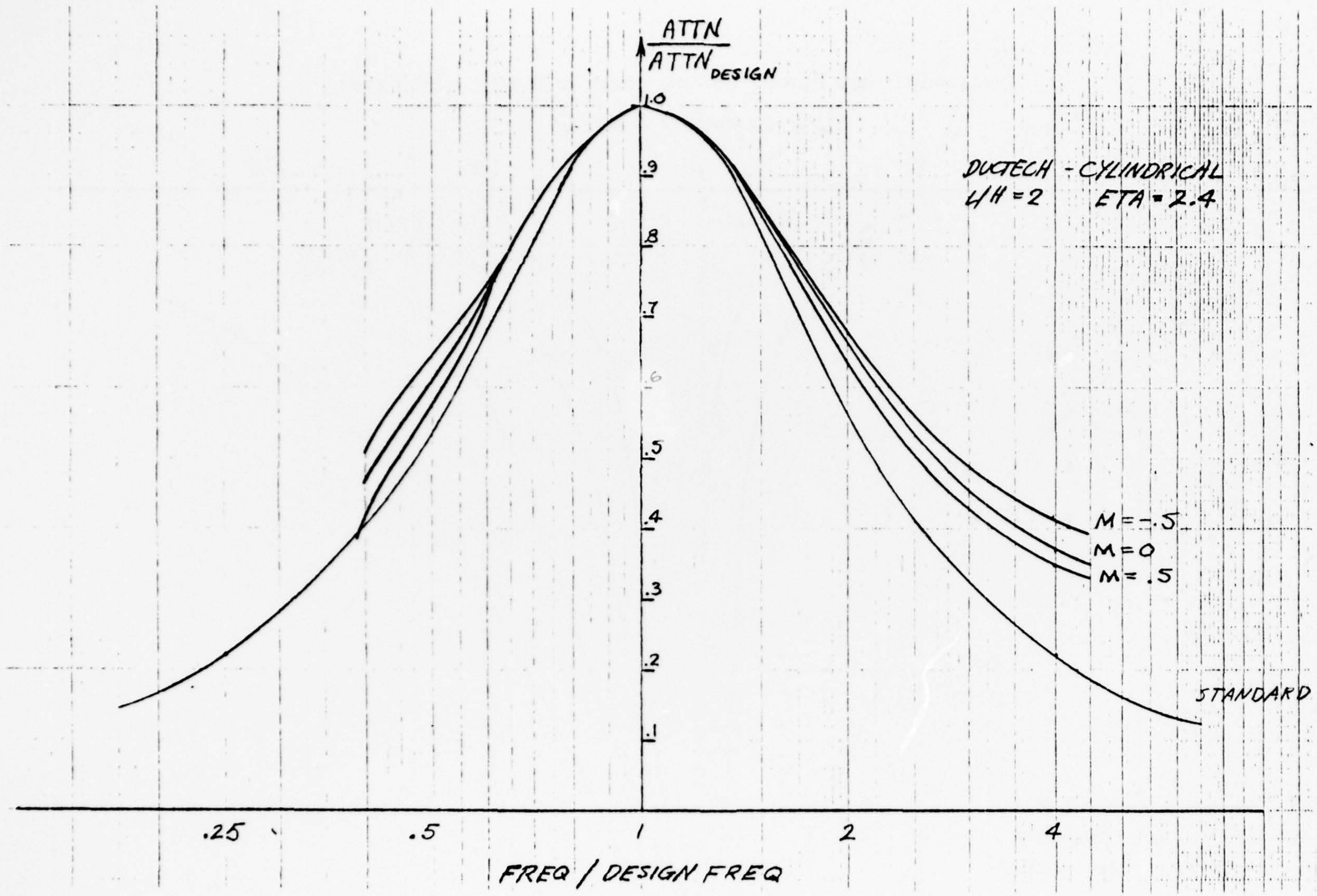

Figure G.2 Effect of Mach No. on Lining Attn. Spectra

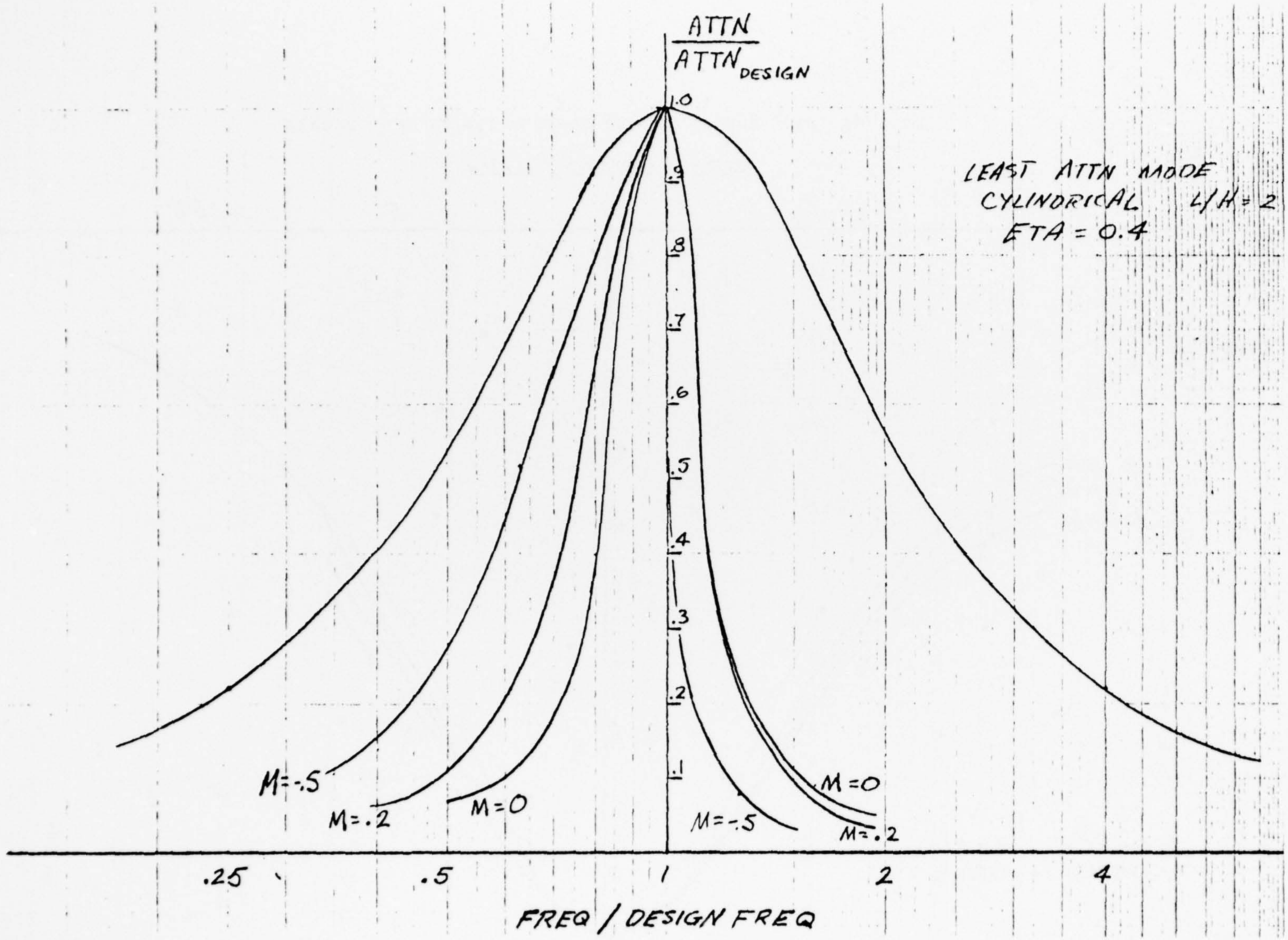

Figure G. 3 Effect of Mach No. on Lining Attn. Spectra

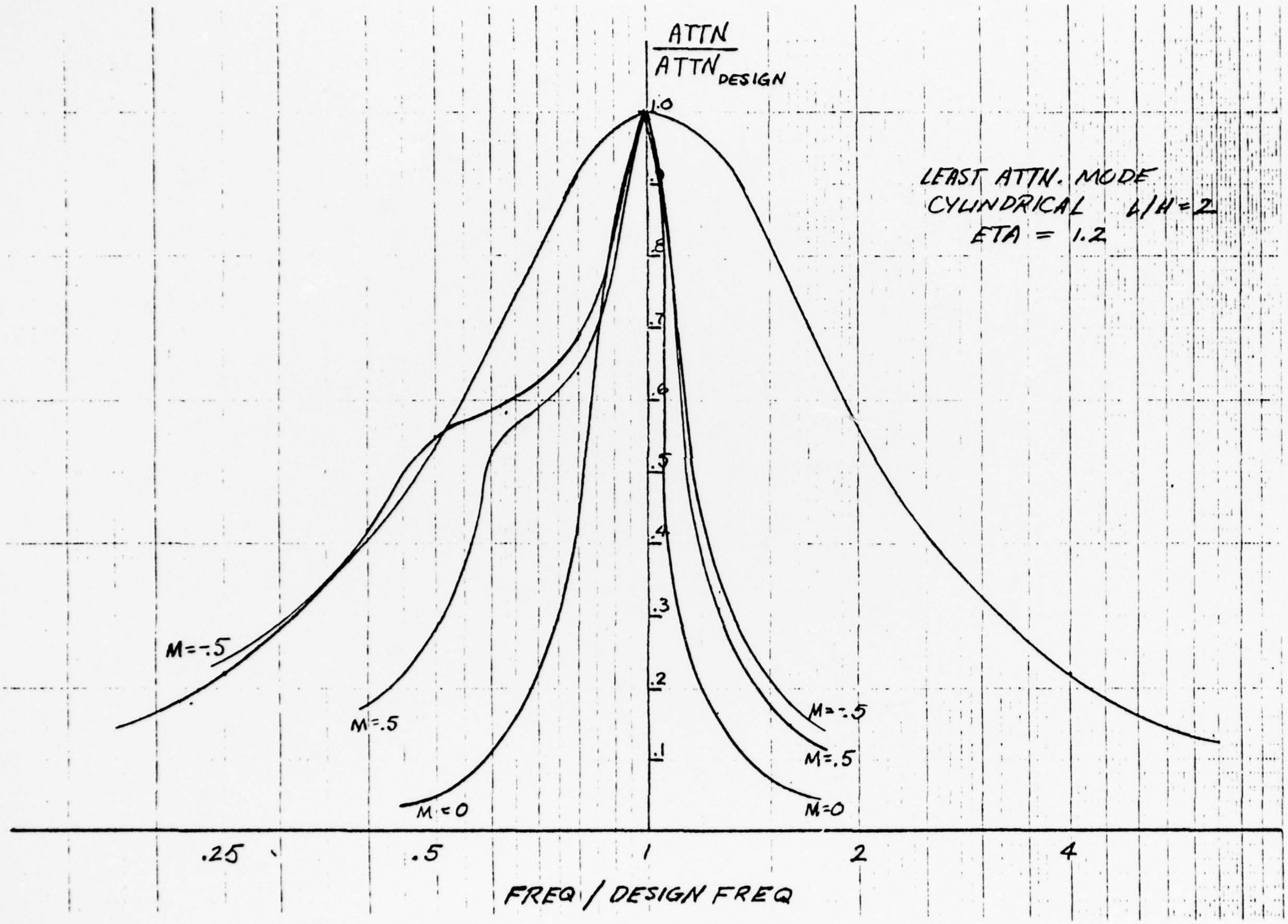

Figure G.4 Effect of Mach No. on Lining Attn. Spectra

APPENDIX H

FORTRAN SUBROUTINE FOR SKIN FRICTION COEFFICIENT OF ACOUSTIC SURFACES

```
      SUBROUTINE FRIC(I,CF,CFR,OA,RED,UINF,NU)
      REAL NU
C       I IS CODE FOR SURFACE
C    C        I=1  STAGGERED HOLES-PERFORATED PLATE
C    C        I=2  IN LINE HOLES-PERFORATED PLATE
C    C        I=3 BRUNSCOUSTIC PLATE-KS=.0015 IN
C    C        I=4  BRUNSWICK FELT METAL-KS=.0?7 IN
C    C        I=5  LAMINATE OF POLYMIDE CLOTH-KS=.006 IN
C    C CF IS SMOOTH PLATE COEFFICIENT OF FRICTION
C    C CFR IS ROUGH PLATE COEFFICIENT OF FRICTION
C    C OA IS OPEN AREA IN PERCENT  (3?% IS 3..)
C    C RED IS REYNOLDS NUMBER BASED ON HOLE DIAMETER
C    C OA AND RED INPUT ONLY FOR PERFORATED PLATES
C    UINF IS FREE STREAM VEL IN FT/SEC
C    NU IS KIN VISCOUSITY IN FT**2/SEC
C    DEFAULT VALUE OF UINF/NU IS 7.E5
      IF(UINF.EQ.0.) UINF=700.
      IF(NU.EQ.0.) NU=1.E-3
      IF(I.EQ.1) F=(1.59*OA**.5-2.31)*RED*1.E-5
      IF(I.EQ.2) F=(1.62*OA**.5-4.16)*RED*1.E-5
      IF(I.EQ.3) F=5.25E-8*UINF/NU
      IF(I.EQ.4) F=2.45E-7*UINF/NU
      IF(I.EQ.5) F=2.1?E-7*UINF/NU
      CFR=CF*(1.+F)
      RETURN
      END
```

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