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**THE DETECTION ASSOCIATION PROCESSOR**

**TECHNICAL REPORT NO. 6**

**VELA NETWORK EVALUATION AND AUTOMATIC PROCESSING RESEARCH**

Prepared by  
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Sponsored by  
ADVANCED RESEARCH PROJECTS AGENCY  
Nuclear Monitoring Research Office  
ARPA Program Code No. 7F10  
ARPA Order No. 2551

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Acknowledgment: This research was supported by the Advanced Research Projects Agency, Nuclear Monitoring Research Office, under Project VELA-UNIFORM, and accomplished under the technical direction of the Air Force Technical Applications Center under Contract Number F08606-77-C-0004.

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## ABSTRACT

This report concerns the design of a seismic event detection association processor. This processor consists of algorithms which automatically convert seismic signal detections from a network of stations stored centrally as a list of station detection bulletins into a corresponding event list, while discarding spurious detections, or false alarms. These algorithms process single site short-period detections as well as array detections. An automatic depth estimate is also performed. Testing via simulation indicates that the resulting event list is accurate enough so that relevant waveform data may be retrieved automatically. It is recommended that the detection association processor be further evaluated on both simulated and real networks.

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## TABLE OF CONTENTS

SECTION	TITLE	PAGE
	ABSTRACT	iii
I.	INTRODUCTION	I-1
II.	REQUIREMENTS FOR A DETECTION ASSOCIATOR	II-1
III.	ASSOCIATOR MODULES	III-1
IV.	EVALUATION AND RESULTS	IV-1
V.	RECOMMENDATIONS AND CONCLUSIONS	V-1
VI.	REFERENCES	VI-1
	APPENDIX	A-1

LIST OF FIGURES

FIGURE	TITLE	PAGE
III-1	AUTOMATED NETWORK LOCATION	III-3
III-2	COLLAPSING GRID LOCATION STRATEGY	III-5
IV-1	DAP PERFORMANCE	IV-7

LIST OF TABLES

TABLE	TITLE	PAGE
IV-1	DETECTION ASSOCIATION PROCESSOR RESULTS	IV-4
IV-2	LOCATION ERROR ANALYSIS	IV-6
IV-3	ANALYSIS OF LOCATION FAILURES	IV-8

SECTION I  
INTRODUCTION

The detection association processor, DAP, is the basic estimation and control system for an automatic seismic network. The DAP converts a stream of automatic station detection bulletins into an event list with enough accuracy to retrieve relevant waveform data. The DAP can operate on an array network, a single site network, or a mixed network. This report is primarily concerned with the documentation of those algorithms comprising the DAP.

The report consists of five sections. Section II defines in greater detail the role of the DAP in an automatic seismic network. Section III discusses the DAP operation for an array, single site, and a mixed network. In Section IV the performance of the DAP upon four days of simulated network data is analyzed. In Section V conclusions are drawn and recommendations made relevant to installing the DAP as an automatic network element.

## SECTION II

### REQUIREMENTS FOR A DETECTION ASSOCIATOR

The detection association processor carries out the function of transforming signal detections obtained independently by a network of seismic stations to seismic event detections. The location, depth, and origin time of the estimated seismic event focus are then used to edit waveform segments associated with the events by estimating arrival times of the seismic phases, particularly the P-phase.

The requirement for information supplied by the detection bulletins from single sensor seismic stations is the seismic station identification, a detection ranking parameter to rank bulletins in the order of probability of detection, the arrival time of the signal, and an estimation of the arrival time error.

Detection ranking parameter depends on a station's noise statistics. For example, if a detector sequentially measures  $m_b$ , then the bulletins may be ranked by  $m_b$ ; another possibility is the signal-to-noise ratio, S/N, of the detections; and another would be the Z-statistic. The latter normalizes a detector output, such as power in decibels, by subtracting a running estimate of the mean noise level. That difference is then divided by a running estimate of the standard deviation of the noise level. The purpose of Z is to normalize time-varying noise levels and noise standard deviations to an approximately fixed unit normal statistical distribution. Larger positive deviations of Z, the presumed unit normal statistic, are related to lower probability of a false alarm detection. The probability of the false alarm can be determined from normal statistic tables. In this report, Z-statistic estimates were used as the detection ranking parameter.

For array seismic stations, additional information must be provided. This is direction and velocity of the signal as indicated by the maximum beam output, and the corresponding estimates of the standard deviation of those quantities.

The detection bulletin list, which is continuously updated in real-time, is input data to the DAP. From comparisons of arrival time and other information obtained from detection bulletins, preliminary event location and depth estimates are generated in real-time. In reporting such locations, a time lag is required for propagating the event around the world and for DAP processing. The time lag is estimated to be between  $\frac{1}{2}$  hour and 1 hour. A time requirement for event locations within one or two hours of real-time is considered reasonable for a DAP.

The purpose of the DAP is to support those seismic analysts responsible for collecting and processing segmented event records associated with seismic events. Once the analyst obtains records of event signals, the analyst then measures accurate arrival times of selected data and other parameters to accurately determine the event focus, magnitude, and other event source parameters. Thus it is seen that the DAP is a tool which supports rapid data collection by seismic analysts. The association function of the DAP is to automatically associate crude arrival time measurements and other measurements of an automatic detector with all events detected by at least four stations. Given one-to-one association of four or more arrival times with an event, a list of approximate event locations can be generated by the DAP. The sole operational function of this preliminary seismic event list is to either automatically or interactively retrieve seismic event edit records for use by the seismic analyst who routinely performs much more meaningful evaluation and measurement of the possible event. The preliminary event location need only be accurate enough to enable the analyst to request records of some specified length assumed to be considerably greater than 15 seconds as a

representation of desired chart-period seismic phases. For this data retrieval purpose, locations need only be accurate to about  $2^{\circ}$  at teleseismic distance and  $1^{\circ}$  at regional distance.

The DAP is designed to merge detection bulletins generated by a world-wide network on the order of 25 array or single-sensor stations. The process must keep up, within reporting time requirements, with a highly variable rate of detection bulletin generation by the network. This is due to event swarms, false alarms due to local events or noise, and false alarms due to the coda and later phases associated with larger events. It is estimated that for a 25 station network, a maximum of about 2000 bulletins per day with at least 10 words per bulletin need to be processed. To handle this volume of information, it is expected that most of the DAP operations need to be performed in an automatic mode, possibly with quality checking and station and network performance monitoring done interactively by an analyst. In this report, designs of automated DAP's are evaluated using simulated detection bulletins generated by an earth seismicity model and a network of simulated station detection processors given by Shoup and Sax (1974).

Another requirement of the DAP is to obtain algorithms which can perform the association of signal detections to event locations for a network of single sensor stations, a network of array stations, or a mixed network of both types of stations.

### SECTION III ASSOCIATOR MODULES

There are three major associator modules. These are the Array Network Processor, Single-Site Network Processor, and a Mixed Network Processor. The Array Network Processor operates on detection bulletins from automatic detectors at array stations. The Single-Site Network Processor operates on detection bulletins from automatic detectors at sites providing only a single vertical component measurement of signals. The Mixed Network Processor operates on detections from a network of both types of stations. The sub-system functions performed by the network are as follows:

- Real-Time Bulletin Processing
  - Add new bulletins to the current stack of detection bulletins in the detection bulletin buffer
  - Maintain ranking of the detection bulletins by Z-statistic
  - Remove aged bulletins from workspace after a fixed number of failures to associate
  - Remove bulletins from workspace which are associated with previously located events.
  
- Association Processing
  - Set up one or more preliminary trial locations
  - Find detections which are consistent with the locations
  - Refine the trial locations and repeat the association tests
  - Iterate the association tests until a specified number of detections are associated e.g. four detections, and a locatable event is declared.

- Processing
  - Perform final location of a declared locatable event with arrival times from associated detection bulletins
  - Use depth grid and depth constrained locations to find best estimate of the depth.

Figure III-1 illustrates the flow of decisions and actions performed by the detection association processor. The inputs are detection bulletins from seismic stations; either arrays serviced by an array module or single-site stations serviced by a single-site module. The detection association processor operates on a fully loaded buffer of the most recently acquired detection bulletins sent by seismic station detection processors. As space becomes available in the bulletin buffer, the detection bulletin buffer is fed new bulletins from a queue. This space becomes available in several ways. One way is to fail to associate with different events some specified number of times, e. g., three times. This aging process is a critical factor in managing the workspace of the detection bulletin buffer. If the aging process is not carefully designed, bulletins will age out because the workspace is clogged with coda detections from preceding larger events. This is definitely a problem with the DAP models described in this report and this problem needs to be attended if better results are to be obtained. Another way for a detection bulletin to be discarded is to have been associated with an event already located and registered on the event list generated as an output of the system. Upon discarding a bulletin in one of these ways, a new bulletin is fed in serially from the queue and inserted into the detection bulletin buffer. The insertion is rank ordered by the magnitude of the detector output, so that the most probable event detections are further up toward the top of the stack. First, all of the array detections are tested for association with the presumed location of the top ranked detection. If the association tests are passed, the location is updated by the Kalman location algorithm. Otherwise, a counter is incremented to register the number of times the detection bulletin failed to associate.

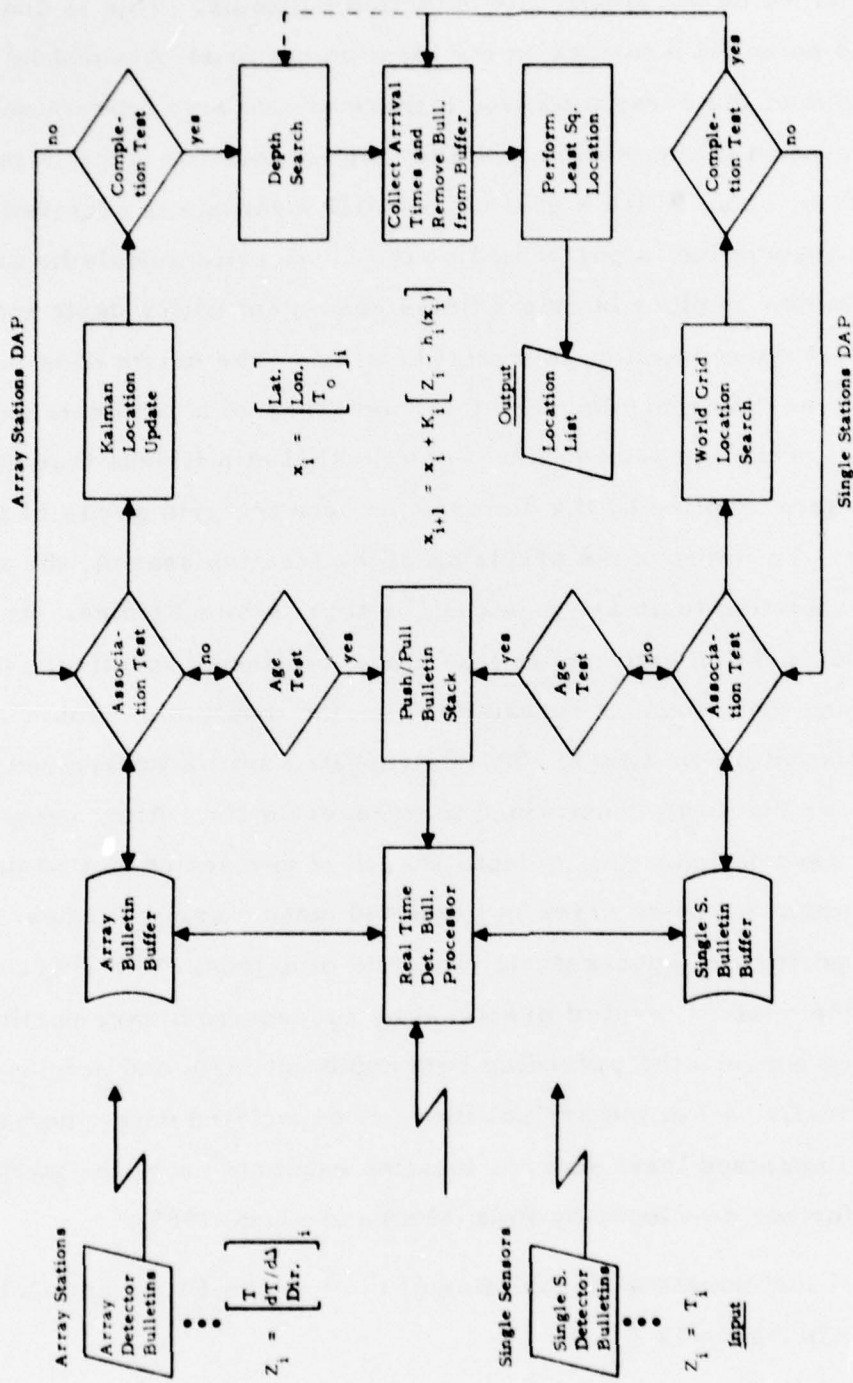
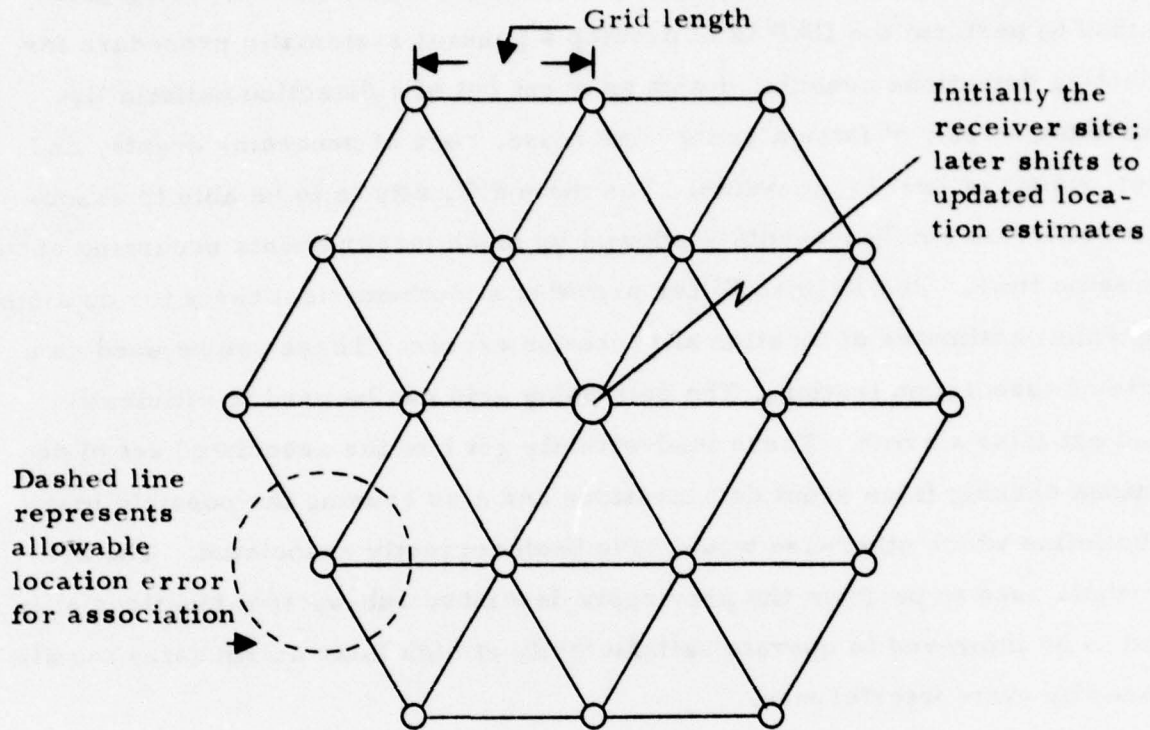


FIGURE III-1  
AUTOMATED NETWORK LOCATION

After completing the association tests of all of the array stations, tests are performed on the single-site detection bulletins. This is done by centering a grid of potential locations on the location estimate obtained by the preceding operations of the array module. If there are no array detection bulletins, then the location of the station with the largest detector value is taken as an initial location, about which a grid of potential locations is centered. See Figure III-2. Association is performed by the single-site module by searching for the maximum number of origin times consistent with a depth location at each grid point corresponding to a trial location. The origin time is said to be consistent if the difference between the origin time of a candidate detection bulletin and that of the top ranked 'key' detection bulletins is less than the expected timing error implied by the distance between the grid points of potential event locations. To improve the precision of the location search, the single-site module association tests are repeated, in toto, several times. At each stage, the output location estimate is taken as a new initial location. The grid size is halved and the search is repeated to find the maximum number of associates with consistent origin times. This is repeated until a prescribed precision is attained for the depth constrained location estimate. After completing the single-site association tests, a depth search is performed to find the focal depth with minimum variance error in predicted station arrival times. This search is also performed successively on a grid of depths. The search is carried out to the desired level of precision by successive binary partitions of the grid distance between the preceding best depth estimate and neighboring grid points. Finally, all of the arrival times of associated detection bulletins are used for a linearized least squares location estimate using the method of Geiger (1910), further developed by Bolt (1960) and Flinn (1965).

A mathematical description of the Kalman Filter and Geiger's method is given in Appendix A.

This grid is centered on the prior estimate of the event location



This figure has to be projected on the surface of a sphere to really see it

FIGURE III-2  
COLLAPSING GRID LOCATION STRATEGY

The rationale for using the Kalman Filter or collapsing grid method to perform the DAP is to develop a general systematic procedure for selecting detections associated with an event out of a detection bulletin list consisting mostly of false alarms from noise, coda of preceding events, and later phases of preceding events. The main difficulty is to be able to associate and locate smaller events shadowed by much larger events occurring about the same time. The Kalman Filter provides a mathematical basis for obtaining sequential estimates of location and location errors. These can be used as a basis of association testing. The collapsing grid can be used to efficiently weed out false alarms. These inadvertently get into the associated set of detections causing false event declarations and also causing the possible loss of bulletins which otherwise would have been correctly associated. The algorithms used to perform the previously described sub-system functions still need to be improved to operate satisfactorily at high false alarm rates mostly caused by event interference.

## SECTION IV EVALUATION AND RESULTS

The DAP was tested on four days of simulated network data. This test used the network simulator, which has been reported upon earlier (Shoup and Sax, 1974). The network simulator is a model of earth seismicity attached to a network of simulated automatic detectors. The time period included 100 simulated events. The array, single-site, and mixed modes were tested. Since the network simulator assumes an array network, the single-site simulation required that the array detection bulletins be reduced to equivalent single-site detection bulletins. This was achieved by having the DAP ignore the array derived data, i. e., azimuth and  $dT/d\Delta$ , for those stations specified as single-sites.

The simulated detectors generated false detection bulletins at a rate of 12 per day. Interference problems such as large timing errors, and detection errors caused by signal coda and later phases were also simulated.

The reason for using a false alarm rate from ambient noise of 12 per day was that Shoup and Sax (1974) in a previous report on the DAP found that this was an optimum false alarm rate for operating automatic detectors. Higher false alarm rates of detectors degraded performance by tending to saturate the bulletin buffer. Lower false alarm rates made it more difficult to obtain the four bulletins required to locate smaller events. Obviously, as the DAP algorithms are improved, then operation at higher false alarm rates becomes feasible. The interaction between station detector threshold strategy and DAP performance is a very important factor in the overall detection performance of world-wide seismic networks and needs more careful investigation.

Before examining results, it is useful to look at where things can go wrong. Numerous bulletins originating from, but not correctly associated with a large event can prevent overshadowed smaller events from attaining top ranked status in the bulletin buffer. In the configuration shown in Figure III-1, events occurring simultaneously must be processed serially. Out of such multiple occurrences, the smaller events can be missed; even with a very large number of bulletins reporting.

For example, if several events go off at about the same time, the largest event detections will be at the top of the Z-ranked workspace of the bulletin buffer. After that event is associated and located, all bulletins with small arrival time errors are pulled out of the workspace. Possibly a substantial number of later phases and coda detections will still be left in the workspace. Some of these will graduate to the top of the workspace and attempts will be made to associate them with other detections in the workspace. Obviously since they are not P-waves of an event, they are unlikely to associate. They tend to clog the bulletin buffer during which period of time, good P-wave detection bulletins will be aged out. One way to avoid this problem is to provide the capability of performing association tests on many key detections in parallel. If a detection ages out on trying to associate with a key detection at one level, it is pushed down a level and tries to associate with a key detection at the next level. This could possibly prevent large false detections from clogging the workspace. The DAP strategies considered in this report do not adequately handle this problem and therefore, a substantial number of small events following larger events failed to be located; in fact, one event with 15 good detections was missed possibly for this reason.

Decision errors in association tests can cause large errors in event locations obtained by successive use of the Kalman Filter. The operation of the Kalman Filter is a bootstrap operation. In association testing, since the probability of a coincidental overlapping of an event location by the confidence

region of a small array location error ellipse is on the order of 0.01, there is a small but yet significant probability for a false alarm to be accepted into a set of associated detections. Since the false alarm is likely to have a large arrival time error, any subsequent location is more likely to exceed the two degree requirement for location accuracy than would otherwise be the case. Presently, there is no means of extracting these erroneously associated bulletins. This can cause missed associations and substantial errors in the final location. The single-site association method using exhaustive search can recover from such association errors. Erroneously associated bulletins can be eliminated as the grid length is successively halved. The single-site association test may occasionally miss bulletins by incomplete coverage of all of the space required to capture all possible event locations. If necessary, this could be prevented through overlapping the event coverage of single-site association tests, by increasing the threshold of the origin time consistency test.

Table IV-1 shows the results of locating the events. The most significant information is given by the success or failure to locate, given that four or more accurate detection bulletins reported. The best results were given by the single-site associator. Successes are defined as location errors less than 200 kilometers and origin time errors less than 15 seconds for those events of which four or more stations detected the arrival time of P-waves. Failures are defined as errors exceeding those limits or failures to associate for those events of which four or more stations detected the arrival time of P-wave signals. Thus, worse results were obtained with an array network despite the availability of the additional array measurement information on array detection bulletins. This clearly indicates serious problems with the association test of the array module. The importance of not missing proper associations is demonstrated by the mixed network results. The association tests rejection threshold were set for error ellipses greater than  $1\sigma$ ,  $2\sigma$ , and  $3\sigma$  (the single-site and array-site networks used  $3\sigma$  association tests,

TABLE IV-1  
DETECTION ASSOCIATION PROCESSOR RESULTS

	Single-Site	Array	Mixed Network		
			1 $\sigma$ Rejection	2 $\sigma$ Rejection	3 $\sigma$ Rejection
Total No. of Events	128	100	100	102	102
Successes	40	30	12	26	29
Failures	17	17	35	24	21
Interfered Events*	29	21	21	22	22
Events Detected At Less Than 4 Stations	42	32	32	30	30

\* Interfered events are failures of the automatic detector to properly time events. Large timing errors were caused by misidentification of the P phase due to interference with other event phases. Most of these large timing errors could easily be prevented by improving the logic of the simulated automatic detector used for this analysis.

where  $\sigma$  is defined as the mean standard deviation of the error ellipse). Note the catastrophic drop in the success ratio for the  $2\sigma$  and  $1\sigma$  rejection thresholds. This is due to not discarding larger detections associated with located events.

The success ratio for the mixed network was less than array-site network and the single-site network. This suggests the possibility that the interface between the array module and single-site module may not be properly configured.

Table IV-2 shows that the detection association processors location errors appear to be consistent with correct association and hypocenter location. An interesting observation is that the origin time error and depth error standard deviation were halved for the array module. Looking back at Figure III-1, note that the depth grid search was performed by the array module but not by the single-site module. The dotted line indicates that the depth search was not yet implemented into the single-site module. Thus it appears that a depth grid search for minimum arrival time errors can lead to large reduction in depth and origin time estimation error. Since we might otherwise have not seen this result, it was perhaps fortunate that the depth search was not put into the single-site module.

Figure IV-1 shows the results obtained for the events analyzed by the single-site module. Note the large number of failures with four or more bulletins at magnitudes from 4.3 to 4.9. This suggests, strongly, serious problems with real-time processor servicing the bulletin buffer. Some possible sources of these problems were previously mentioned. By going back to the bulletin list and checking these failures, most were found to be preceded by larger events. The seriousness of this problem is further shown by Table IV-3. Note that events were missed with as many as 15 valid detection bulletins. Assuming that the 4 and 5 bulletin cases of failing to locate the event are as expected; those events with failed location estimates, given 6 or more bulletins, appear to be anomalous. On that basis, it is estimated that 75% of the errors could be eliminated by modifying the real-time

TABLE IV-2  
LOCATION ERROR ANALYSIS

	Single-Site	Array	Mixed Network		
			1 $\sigma$ Rejection	2 $\sigma$ Rejection	3 $\sigma$ Rejection
$\frac{1}{N} \Sigma (e_{lat}^2 + e_{lon}^2)$ (km <sup>2</sup> )	152.06	214.7	224.01	139.32	139.06
$\mu_T$ (sec)	1.08	-0.54	0.16	1.72	-0.94
$\sigma_T$ (sec)	6.61	3.03	20.85	9.61	4.84
$\mu_h$ (km)	13.86	-4.69	35.96	12.55	-0.87
$\sigma_h$ (km)	60.73	36.57	158.63	77.37	46.28
95% Error Ellipse (km)	21.35	25.4	25.9	20.4	20.4

- $\frac{1}{N} \Sigma (e_{lat}^2 + e_{lon}^2)$  = variance of location errors in kilometers  
 $\mu_T$  = mean timing error  
 $\sigma_T$  = standard deviation of origin time  
 $\mu_h$  = mean depth error  
 $\sigma_h$  = standard deviation of depth error

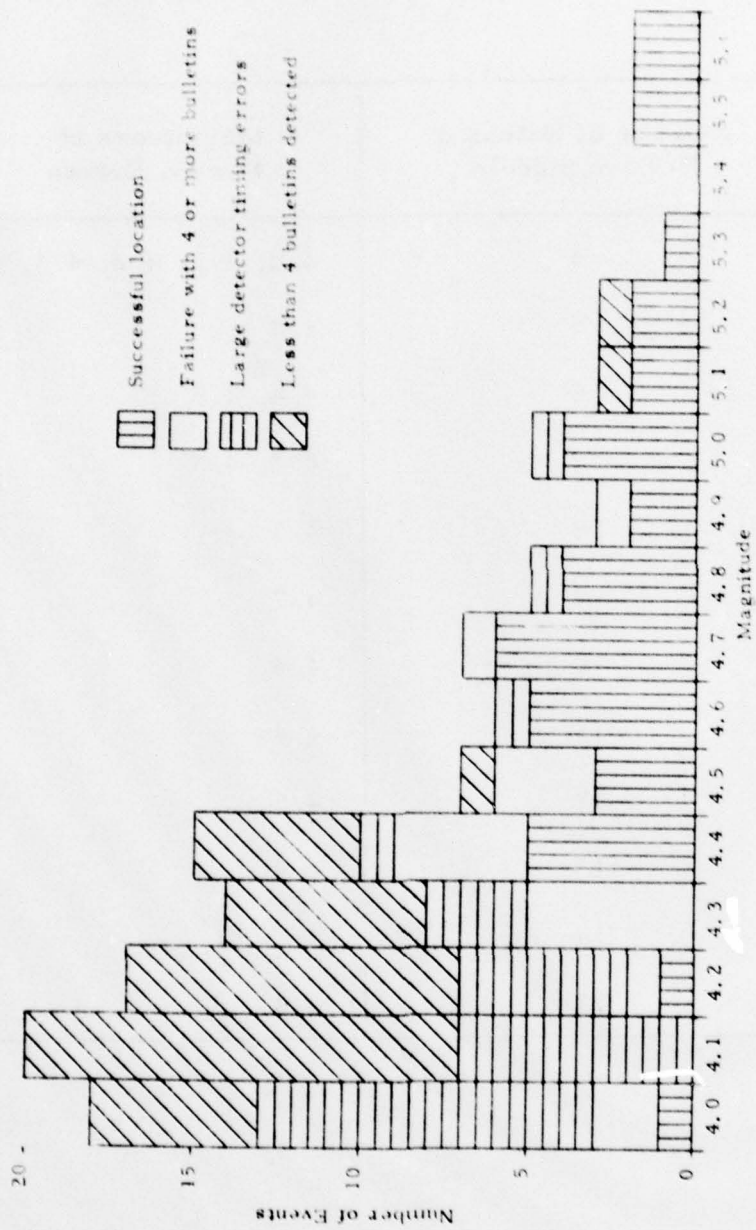


FIGURE IV-1  
DAP PERFORMANCE

TABLE IV-3  
ANALYSIS OF LOCATION FAILURES

Number of Detected P-Wave Signals	Magnitudes of Missed Events
4	4.0, 4.0, 4.2, 4.3, 4.4
5	4.3
6	4.4, 4.5
7	4.3, 4.5
8	4.3
9	4.7
10	4.4
11	4.4
12	4.5
13	4.9
14	
15	4.3

processor which services the bulletin buffer. It is conjectured that such a modification could result in a success to failure ratio of about 10 to 1.

## SECTION V

### RECOMMENDATIONS AND CONCLUSIONS

The detection association processor described in this report indicated a capability of associating and locating about 70% of the events with four or more satisfactory P-wave detections. This was done on a completely automated basis. It can handle the detection association and location function for single sensor networks, array networks, or mixed networks containing both types of stations. The accuracy of the locations appeared to be satisfactory and comparable with the visual linearized and iterative least squares hypocenter determination. In fact, a depth grid search for least squares arrival times produced depth determinations which were almost a factor of two more accurate than those hypocenter determinations that were unconstrained with respect to depth of focus.

The results clearly indicated that more work is needed to improve the detection association processor. The main discrepancy is the lack of a satisfactory criteria for removing aged and redundant detections from the workspace of the processor. For example, later phases should be identified and removed from the detection association processor workspace as soon as possible after an event location is declared. Also, the process by which detections are aged and discarded must guarantee that the detection association tests be performed on a sufficient number of key detections to associate and separate several events occurring simultaneously over a reasonably wide range of magnitude.

The association process needs to be improved. The initialization of key detections which are association tested against other detections needs to be handled as a parallel process rather than a serial process. In

other words, several events should be associated simultaneously rather than one at a time. Also, the tests used to perform the association of detections need to be improved. Time difference constraints should be imposed on all possible pairs of association candidates based on the time-distance relationship for P-waves. Other criteria might be considered in selecting key detections other than the magnitude of the detector output. For example, one such criteria is the time elapsed at a station from the previous larger magnitude detection; another, arrival times earlier than the current key detection. The first time criteria assures a lower probability of missed associations due to preceding larger events; the second, that older detections be tried as key detections before being timed out.

The suggested modifications of the automatic detection association processor should build upon other association processors and location procedures now in use, as described by Der (1977), Flinn (1965), and Bolt (1960). Although the performance shown here is less than that of existing association processors which are 80% to 90% efficient in associating and locating events compared to the 70% efficiency demonstrated here, it is probable that performance much closer to 100% efficiency might be possible after working out obvious faults in the program.

SECTION VI  
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## APPENDIX A

A mathematical description of the Kalman Filter and Geiger's method is given to specify the detailed methodology of the DAP algorithms used in this report.

### Kalman Filter

Suppose there is a non-linear system

$$Z = h(x) + V \quad (A-1)$$

where

$Z$  = ( $p \times 1$ ) measurement variable matrix,

$x$  = ( $n \times 1$ ) state variable matrix,

$h(x)$  = ( $p \times 1$ ) non-linear function matrix, and

$V$  = ( $p \times 1$ ) measurement error matrix.

Define

$$E(VV^T) = R, \text{ a } (p \times p) \text{ positive matrix,} \quad (A-2)$$

and

$$E \left[ (x - \bar{x})(x - \bar{x})^T \right] = M, \text{ (n \times n) matrix, error co-} \quad (A-3) \\ \text{variance before measurement}$$

where

$\bar{x}$  = the predicted state variables before measurement.

We can find a controlling function  $J(x)$  which is often called the performance criteria, or the performance index.

$$J(x) = \frac{1}{2} \left\{ (x - \bar{x})^T M^{-1} (x - \bar{x}) + [z - h(x)]^T R^{-1} [z - h(x)] \right\}. \quad (A-4)$$

Minimizing  $J(x)$  with respect to  $x$  obtains an estimate of  $x$

$$\hat{x} = \bar{x} + K(\bar{x}) [z - h(\bar{x})] \quad (A-5)$$

where

$$K(\bar{x}) = PG^T R^{-1}, \text{ a } (n \times p) \text{ Kalman Filter matrix,} \quad (A-6)$$

$$G(\bar{x}) = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\bar{x}}, \text{ (p \times n) matrix,} \quad (A-7)$$

$$\begin{aligned} P &= E \left[ (\hat{x} - x) (\hat{x} - x)^T \right], \text{ (n \times n) matrix, error} \\ &\quad \text{covariance after measurement,} \\ &= M - MG^T [R + GMG^T]^{-1} GM. \end{aligned} \quad (A-8)$$

In our system, we have three state variables and three measurement variables. Their relationships are given below:

$$\left. \begin{aligned} Z_1 &= \cos(\Delta) \\ Z_2 &= \sin(\Delta) \sin(Az) \\ Z_3 &= T_A \end{aligned} \right\} \quad (A-9)$$

$$\left. \begin{aligned} h_1(x) &= \sin(x_E) \sin(x_S) + \cos(x_E) \cos(x_S) \cos(y_E - y_S) \\ h_2(x) &= \cos(x_E) \sin(y_E - y_S), \text{ and} \\ h_3(x) &= T_o + T \end{aligned} \right\} \quad (A-10)$$

where

- $\Delta$  = epicentral distance between the event and the station,
- $Az$  = the azimuth angle of the observer,
- $x_E$  = the latitude of the event,
- $y_E$  = the longitude of the event,
- $x_S$  = the latitude of the station,
- $y_S$  = the longitude of the station,
- $T_o$  = the origin time of the event,
- $T$  = the travel time of the event, and
- $T_A$  = the arrival time of the event.

The R Matrix:

R is a (p x p) square matrix. It is obtained from measurements, and is usually a diagonal matrix independent of state variables. It plays an important role in iterative computations of the event location.

Differentiating equation (A-9) we have

$$V_1 = dZ_1 = -\sin(\Delta) \frac{\partial \Delta}{\partial p} dp, \text{ and} \quad (\text{A-11})$$

$$V_2 = dZ_2 = \cos(\Delta)\sin(Az) \frac{\partial \Delta}{\partial p} dp + \sin(\Delta)\cos(Az)d(Az), \quad (\text{A-12})$$

where

$$p = \frac{dT}{d\Delta}.$$

The error of  $V_3$  includes a measurement error  $\epsilon(T_A)$  which was set equal to 4 sec, and an error due to imperfect location. Therefore,

$$V_3 = 4 + \frac{\partial T}{\partial \Delta} \frac{\partial \Delta}{\partial p} dp. \quad (\text{A-13})$$

The change in p in equations (A-11) and (A-12) is identical and implicit from the definition of  $Z_1$  and  $Z_2$ . But since the arrival time errors,  $dZ_3$ , are

uncorrelated with  $dZ_1$  and  $dZ_2$ , the change in  $p$  is uncorrelated in equations (A-11), (A-13), and equations (A-12) and (A-13).

The expected operator  $E$  is a linear operator. If  $R = E \left[ (x - \bar{x}) (x - \bar{x})^T \right]$ , it operates on each term of the matrix. Therefore,

$$\begin{aligned}
 R_{11} &= E(V_1 V_1) = \left[ \sin(\Delta) \frac{\partial \Delta}{\partial p} dp \right]^2, \\
 R_{12} &= E(V_1 V_2) = -\frac{1}{2} \sin(2\Delta) \sin(Az) \left[ \frac{\partial \Delta}{\partial p} dp \right]^2, \\
 R_{13} &= E(V_1 V_3) = 0, \\
 R_{21} &= E(V_2 V_1) = R_{12}, \\
 R_{22} &= E(V_2 V_2) = \left[ \cos(\Delta) \sin(Az) \frac{\partial \Delta}{\partial p} dp \right]^2, \\
 &\quad + \left[ \sin(\Delta) \cos(Az) d(Az) \right]^2, \\
 R_{23} &= E(V_2 V_3) = 0, \\
 R_{31} &= E(V_3 V_1) = 0, \\
 R_{32} &= E(V_3 V_2) = 0, \text{ and} \\
 R_{33} &= E(V_3 V_3) = 16 + \left[ \frac{\partial T}{\partial \Delta} \frac{\partial \Delta}{\partial p} dp \right]^2.
 \end{aligned}$$

The G Matrix:

$G(x)$  is a  $(p \times n)$  matrix. Each element can be derived from equation (A-10), so,

$$\begin{aligned}
 G_{11} &= \frac{\partial h_1(\bar{x})}{\partial x_E} = \cos(\bar{x}_E) \sin(x_S) \\
 &\quad - \sin(\bar{x}_E) \cos(x_S) \cos(\bar{y}_E - y_S), \\
 G_{12} &= \frac{\partial h_2(\bar{x})}{\partial x_E} = -\sin(\bar{x}_E) \sin(\bar{y}_E - y_S),
 \end{aligned}$$

$$\begin{aligned}
G_{13} &= \frac{\partial h_3(\bar{x})}{\partial x_E} = \frac{\partial T}{\partial \Delta} \frac{\partial \Delta}{\partial x_E} \\
&= -G_{11} \frac{\partial T}{\partial \Delta} / \sqrt{1 - \cos^2 \Delta} \\
G_{21} &= \frac{\partial h_1(\bar{x})}{\partial y_E} = -\cos(\bar{x}_E) \cos(x_S) \sin(\bar{y}_E - y_S) , \\
G_{22} &= \frac{\partial h_2(\bar{x})}{\partial y_E} = \cos(\bar{x}_E) \cos(\bar{y}_E - y_S) , \\
G_{23} &= \frac{\partial h_3(\bar{x})}{\partial y_E} = \frac{\partial h_3(\bar{x})}{\partial x_E} \frac{\partial h_1(\bar{x})}{\partial y_E} / \frac{\partial h_1(\bar{x})}{\partial x_E} \\
&= G_{13} \cdot G_{21} / G_{11} , \\
G_{31} &= \frac{\partial h_1(\bar{x})}{\partial T_o} = 0 , \\
G_{32} &= \frac{\partial h_2(\bar{x})}{\partial T_o} = 0 , \text{ and} \\
G_{33} &= \frac{\partial h_3(\bar{x})}{\partial T_o} = 1 .
\end{aligned}$$

Given an initial estimate  $\bar{x}$  in equation (A-5) we can get a modified estimate  $\hat{x}$ . Following the procedure iteratively until the correction term  $K(\bar{x}) [Z-h(\bar{x})]$  becomes insignificant, a final estimate is then obtained.

### Geiger's Method

Suppose there is a non-linear system

$$T_A = h(x) + v \quad (A-14)$$

where

$$T_A = \text{the arrival time, a measurement variable,}$$

$x$  = the state variables, representing the latitude ( $x_E$ ), longitude ( $y_E$ ), origin time ( $T_0$ ), and depth ( $z$ ) of an event, and

$$h(x) = T_0 + T(\Delta, z) , \quad (A-15)$$

where

$\Delta$  = the epicentral distance between the event and the observer, and

$V$  = the measurement error.

Linearizing  $h(x)$  about an initial estimate  $x = \bar{x}$  we have

$$T_A = h(\bar{x}) + \left. \frac{\partial h(x)}{\partial x} \right|_{x=\bar{x}} (x - \bar{x}) + 0 (|x - \bar{x}|^2) + V . \quad (A-16)$$

For small  $|x - \bar{x}|$ ,

$$T_A \approx h(\bar{x}) + \frac{\partial h(\bar{x})}{\partial x} (x - \bar{x}) + V , \quad (A-17)$$

or

$$R = C\Delta \quad (A-18)$$

where

$$R = T_A - h(\bar{x}) - V ,$$

$$C = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\bar{x}} , \text{ and} \quad (A-19)$$

$$\Delta = x - \bar{x} .$$

Suppose there are  $n$  stations or  $n$  measurements, then  $R$  is an  $(n \times 1)$  row vector;  $C$ , an  $(n \times 4)$  matrix; and  $\Delta$ , a  $(4 \times 1)$  row vector. At each station,  $C$  has four elements and their expressions are:

$$\begin{aligned}
C_{i1} &= \left( \frac{\partial h(\bar{x})}{\partial x_E} \right)_i = \left( \frac{\partial T}{\partial \Delta} \frac{\partial \Delta}{\partial x_E} \right)_i, \quad i=1, 2, \dots, n; \\
C_{i2} &= \left( \frac{\partial h(\bar{x})}{\partial y_E} \right)_i = \left( \frac{\partial T}{\partial \Delta} \frac{\partial \Delta}{\partial y_E} \right)_i, \quad i=1, 2, \dots, n; \\
C_{i3} &= \left( \frac{\partial h(\bar{x})}{\partial T_o} \right)_i = 1, \quad i=1, 2, \dots, n; \text{ and} \\
C_{i4} &= \left( \frac{\partial h(\bar{x})}{\partial z} \right)_i = \left( \frac{\partial T}{\partial z} \right)_i, \quad i=1, 2, \dots, n.
\end{aligned}
\tag{A-20}$$

Minimizing the norm of error

$$\| e \|^2 = (R - C\Delta)^T (R - C\Delta)$$

we have

$$\hat{\Delta} = (C^T C)^{-1} C^T R$$

i. e.,

$$\hat{x} = \bar{x} + (C^T C)^{-1} C^T R.
\tag{A-21}$$

This calculation can be continued iteratively until the term  $(C^T C)^{-1} C^T R$  becomes insignificant.