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MINIMIZING A PROJECT COST WITH BOUNDS ON THE EXPECTATION AND VARIANCE OF THE DELAY TIME,

by

James E. Falk

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In this paper we discuss a problem involving a project consisting of a number of tasks, each of which must be performed in a sequential manner. Any of the tasks is subject to a potential delay of known duration beyond its scheduled starting time. The task delay times may be decreased with the addition of funding.

We seek to minimize the cost of completing the project, subject to bounds on both the expectation and variance of the total delay time. (cont'd)

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Introduction

We consider here a "project" consisting of a set of ordered "tasks" numbered 1,...,n . In this paper, we make the simplifying assumption that these tasks must be performed serially, and in the order in which they are numbered. The results contained herein, however, can be easily generalized to partially ordered tasks (see Falk [2]).

We suppose that each task j may be delayed S_j units *beyond its scheduled start time*. A delay in any task will thus result in a delay of the project completion time. The total project delay is therefore (see Rose [4]):

$$S = \max_{1 \leq j \leq n} \{S_j\} .$$

If each task will undergo a delay of S_j units with probability P_j , the expected project delay, $E(S)$, can be computed. If we further assume that each delay time S_j is a variable which can be controlled at a cost $C_j(S_j)$, we may consider the problem

$$\text{minimize} \quad \sum_{j=1}^{n} C_j(S_j)$$

$$\text{subject to} \quad E(S) \leqq e$$
$$S \geqq 0 \,,$$

where e is a given, prescribed upper bound on the project delay. This problem was solved by Falk and Rose [3], who showed that $E(S)$ was representable as the maximum of a set of linear functions of $S_1,\ldots,S_n$. Thus $E(S)$ is convex. An efficient solution procedure was then realized for infinitely constrained problems [1].

In this paper, an additional constraint of the form

$$V(S) \leqq v$$

is imposed, where $V(S)$ is the variance of the random variable S and v is a given, prescribed upper bound on this variance. It will be shown that $V(S)$ can also be realized as the maximum of a set of convex functions in a manner similar to the development of $E(S)$. It turns out that $V(S)$ is, therefore, convex. A modification of the previously developed algorithm is presented to treat the new constraint.

Background

A project consists of tasks $1,\ldots,n$, which must be performed in order. Each task may undergo a delay of S_j time units, with given probability P_j. The project delay is then

$$S = \max_{1 \leqq j \leqq n} \{S_j\} \,.$$

Let $\sigma = (\sigma_1,\ldots,\sigma_n)$ denote a permutation of the set $\{1,\ldots,n\}$ which ranks the quantities S_j, i.e.:

$$S_{\sigma_1} \geqq S_{\sigma_2} \geqq \cdots \geqq S_{\sigma_n} \,. \tag{1}$$

Then the *expected* project delay is derived to be [4]:

$$E(S) = \sum_{j=1}^{n} \left[P_{\sigma_j} \prod_{k=1}^{j-1} \left(1 - P_{\sigma_k}\right) \right] S_{\sigma_j} , \qquad (2)$$

where

$$\prod_{k=1}^{0} \left(1 - P_{\sigma_k}\right) \doteq 1 . \qquad (3)$$

Note that the expression (2) depends on the ranking of the quantities S_j . In particular, if the S_j are considered to be variables, the linear expression (2) representing $E(S)$ changes whenever the ranking of the S_j changes.

Define, for each permutation σ of $\{1,\ldots,n\}$,

$$E_\sigma(S) \doteq \sum_{j=1}^{n} \left[P_{\sigma_j} \prod_{k=1}^{j-1} \left(1 - P_{\sigma_k}\right) \right] S_{\sigma_j} , \qquad (4)$$

where (3) holds. Thus $E_\sigma(S) = E(S)$ *if* the ranking of the set $\{S_j\}$ is that prescribed by σ , i.e., *if* (1) holds. In any event, $E_\sigma(S)$ is a linear function of $S_1,\ldots,S_n$, defined for *all* $S_1,\ldots,S_n$.

It was shown in [3] that

$$E(S) = \max_{\sigma\in\Sigma} E_\sigma(S) , \qquad (5)$$

where Σ is the set of *all* permutations of the integers $1,\ldots,n$. This result is important as it establishes the convexity of E , and also allows the set $\{S: E(S) \leq e\}$ to be represented as the intersection of a finite (albeit very large) number of half spaces.

We now assume that there is given a cost function $C_j(S_j)$, representing the cost of reducing the potential delay of task j to S_j units. For applications, C_j would normally be decreasing, reflecting the fact that long delays are cheap, but reduction in such delays would require some expense, e.g., additional servers or service facilities.

The problem becomes

$$\text{minimize} \quad C(S) = \sum_{j=1}^{n} C_j(S_j)$$

$$\text{subject to} \quad E_\sigma(S) \leqq e \ , \quad \text{for all } \sigma\varepsilon\Sigma \ ,$$
$$S \geqq 0 \ .$$

This is the program solved in [3]. Because of the potentially large number of constraints (if $n = 10$, $|\Sigma| = 3{,}628{,}800$), the Blankenship-Falk method for infinitely constrained problems [1] is applied. As specialized to the above problem, this algorithm becomes

Step 0: Set $k = 0$, select $\sigma^0 \varepsilon \Sigma$, set $F_0 = \{\sigma^0\}$.

Step 2: Given $F_k = \{\sigma^0,\dots,\sigma^k\}$, solve the problem

$$\text{minimize} \quad C(S)$$

$$\text{subject to} \quad E_\sigma(S) \leqq e \ , \quad \text{for all } \sigma\varepsilon F_k \ ,$$
$$S \geqq 0 \ ,$$

to get a trial solution S^k .

Step 3: Test the trial solution S^k as a possible solution of the desired problem. To do this, we generate a permutation σ^{k+1} by simply ranking the components of S^k . If $\sigma^{k+1} \varepsilon S^k$, we are done, since then $E_\sigma(S^k) \leqq 0$ for all $\sigma\varepsilon\Sigma$. Otherwise set $F_{k+1} = F_k \cup \{\sigma^{k+1}\}$ and return to Step 2.

This method will converge in a finite number of steps, provided only that $C(S)$ is lower semicontinuous. If $C(S)$ is strictly convex, we can modify the rule for updating F_k by dropping constraints which are not binding at S^k , and then keep the total number of constraints imposed on the subproblems manageable.

The Variance

For a given set of task delays ordered by

$$S_{\sigma_1} \geqq S_{\sigma_2} \geqq \dots \geqq S_{\sigma_n} \, , \tag{6}$$

the variance $V(S)$ may be written

$$\begin{aligned} V(S) &= E\left((S - E(S))^2\right) \\ &= \sum_{j=0}^{n} p_{\sigma_j}\left(S_{\sigma_j} = E(S)\right)^2 \end{aligned}$$

where $S_{\sigma_0} = 0$, $p_{\sigma_0} = \prod_{k=1}^{n} \left(1 - P_k\right)$, and

$$p_{\sigma_j} = P_{\sigma_j} \prod_{k=1}^{j-1} \left(1 - P_{\sigma_k}\right) \, , \quad (j \neq 0) \, .$$

This expression for $V(S)$ is, of course, only valid over the region defined by (6). It is easy to show that $V(S)$ is convex over that region. To simplify notation in the following theorem, we shall assume $\sigma_j = j$, i.e., σ is the identity permutation I .

<u>Theorem</u>. The function

$$V_I(S) \doteq \sum_{j=0}^{n} p_j\left(S_j - E_I(S)\right)^2$$

is convex, where $\sum_{j=0}^{n} p_j = 1$, $p_j \geqq 0$.

<u>Proof</u>. $\sum_{j=0}^{n} p_j\left(S_j - E_I(S)\right)^2 = \sum_j p_j S_j^2 - \left(\sum_j p_j S_j\right)^2$. Therefore

$$\nabla V_I(S) = 2\begin{pmatrix} p_0 S_0 \\ \vdots \\ p_n S_n \end{pmatrix} - 2\left(\sum_j p_j S_j\right)\begin{pmatrix} p_0 \\ \vdots \\ p_n \end{pmatrix} ,$$

so that

$$\nabla^2 V_I(S) = 2\left[\begin{pmatrix} p_0 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & & \\ 0 & 0 & \cdots & p_n \end{pmatrix} - \begin{pmatrix} p_0 \\ \vdots \\ p_n \end{pmatrix}^{(p_0,\ldots,p_n)}\right],$$

and this is positive semidefinite by the Cauchy-Schwartz Inequality.

Following the treatment of $E(S)$, define

$$V_\sigma(S) = E_\sigma(S^2) - \left(E_\sigma(S)\right)^2 .$$

This expression is defined for all S, but gives the correct value for $V(S)$ only when the components of S are ordered as prescribed by σ.

We may write

$$V(S) = \max_\sigma E_\sigma(S^2) - \max_\sigma \left(E_\sigma(S)\right)^2 ,$$

and, even though both of the above maxima are attained at the same σ, we may *not* write $V(S) = \max_\sigma V_\sigma(S)$. It is true, however, that

$$V(S) = \max_\sigma E_\sigma\left(\left(S - E(S)\right)^2\right) ,$$

as follows from the result (5). We now show that the function being maximized is convex in S.

<u>Theorem</u>. $E_\sigma\left(\left(S - E(S)\right)^2\right)$ is convex for all S.

<u>Proof</u>. $E_\sigma\left(\left(S - E(S)\right)^2\right) = \sum_{j=0}^{n} p_{\sigma_j}\left(S_{\sigma_j} - E(S)\right)^2$, where $p_{\sigma_0} = \prod_{k=1}^{n} (1 - P_k)$ and $p_{\sigma_j} = P_{\sigma_j} \prod_{k=1}^{j-1} \left(1 - P_{\sigma_k}\right)$. Then

$$E_\sigma\left((S - E(S))^2\right) = \sum_{j=0}^{n} p_{\sigma_j}\left(S_{\sigma_j}^2 - 2S_{\sigma_j}E(S) + E^2(S)\right)$$

$$= E_\sigma(S^2) - 2E_\sigma(S)E(S) + E^2(S)$$

$$= E_\sigma(S^2) + \left(E(S) - E_\sigma(S)\right)^2 - E_\sigma^2(S) .$$

Now $E(S) - E_\sigma(S)$ is a convex, nonnegative function of S. Therefore, its square is convex. Also, $E_\sigma(S^2) - E_\sigma^2(S)$ is convex by the previous theorem. The result is immediate.

It follows immediately that $V(S)$ is convex.

The Problem

We now address a natural generalization of the problem outlined in the introduction by adding a restriction on the variance of the project delay time. The problem becomes

$$\left.\begin{array}{rl} \text{minimize} & C(S) \\ \text{subject to} & E(S) \leqq e \\ & V(S) \leqq v \\ & S \geqq 0 \end{array}\right\} \text{Problem P,}$$

where e and v are given upper bounds on the expectation and variance of the project delay time.

Unfortunately, imposition of the family of constraints

$$E_\sigma\left(S - E(S)\right)^2 \leqq v , \quad \text{for all } \sigma$$

appears difficult to work with. We therefore consider a related problem

$$\left.\begin{array}{rl} \text{minimize} & C(S) \\ \text{subject to} & E_\sigma(S) \leqq e \\ & V_\sigma(S) \leqq v \\ & S \geqq 0 \end{array}\right\} \text{Problem P'.}$$

The feasible region of Problem P' is contained in the feasible region of Problem P. This follows since

$$V_\sigma(S) \leq v \ , \quad \text{for all } \sigma$$

implies

$$V_{\bar{\sigma}}(S) \leq v \ ,$$

for that $\bar{\sigma}$ agreeing with the order on S .

The solution of Problem P' follows the algorithm prescribed in the preceding section. Step 3 is implemented in precisely the same manner as before, i.e., a trial solution S^t is checked as the actual solution by ranking its components to get σ^t . If both $E_\sigma t(S^t) \leq e$ and $V_\sigma t(S^t) \leq v$, we are done. Otherwise we add σ^t to the set F_k , thus forming F_{k+1} , and continue.

Note that there is some chance that a solution of P' is not a solution of P, since P is a relaxation of P'. If $C(S)$ is convex, any local solution of P is global. Therefore, a solution of P' can be checked for local (and thus global) optimality of P. Furthermore, it is possible that the solution of P' is sufficient for a decision maker. The variance is simply a measure of distance from the mean, but so is the function $\overline{V}(S) = \max\{V_\sigma(S) : \sigma \in \Sigma\}$. Furthermore, in all of the problems which we solved, $V(S) = \overline{V}(S)$ at a solution.

<u>Example</u>

$$\text{minimize} \quad C(S) = -5S_1 - 10S_2 - 2S_3 + 138$$

$$\text{subject to} \quad E(S) \leq 6$$

$$V(S) \leq 3$$

$$R \begin{cases} 0 \leq S_1 \leq 10 & P_1 = 0.4 \\ 0 \leq S_2 \leq 8 & P_2 = 0.5 \\ 0 \leq S_3 \leq 4 & P_3 = 0.8 \end{cases}$$

We select (arbitrarily) $\sigma^0 = (1,2,3)$, and form the problem

$$\begin{aligned} \text{minimize} \quad & C(S) \\ \text{subject to} \quad & 0.4S_1 + 0.3S_2 + 0.24S_3 \leqq 6 \\ & 0.4S_1^2 + 0.3S_2^2 + 0.24S_3^2 \\ & - (0.4S_1 + 0.3S_2 + 0.24S_3)^2 \leqq 3 \\ & S \in R \, . \end{aligned}$$

The solution of this problem is $S^0 = (5.9584, 6.8779, 4)$. This cannot be the desired solution, as the ranking it implies is $\sigma^1 = (2,1,3)$. We therefore impose the additional constraints

$$\begin{aligned} & 0.2S_1 + 0.5S_2 + 0.24S_3 \leqq 6 \\ & 0.2S_1^2 + 0.5S_2^2 + 0.24S_3^2 \\ & - (0.2S_1 + 0.55S_2 + 0.24S_3)^2 \leqq 3 \, , \end{aligned}$$

and obtain a solution $S^1 = (6.4601, 6.4601, 4)$ with value 33.0985. Since the ranking of S^1 is either (2,1,3) or (1,2,3), both of which have already been imposed, we are done.

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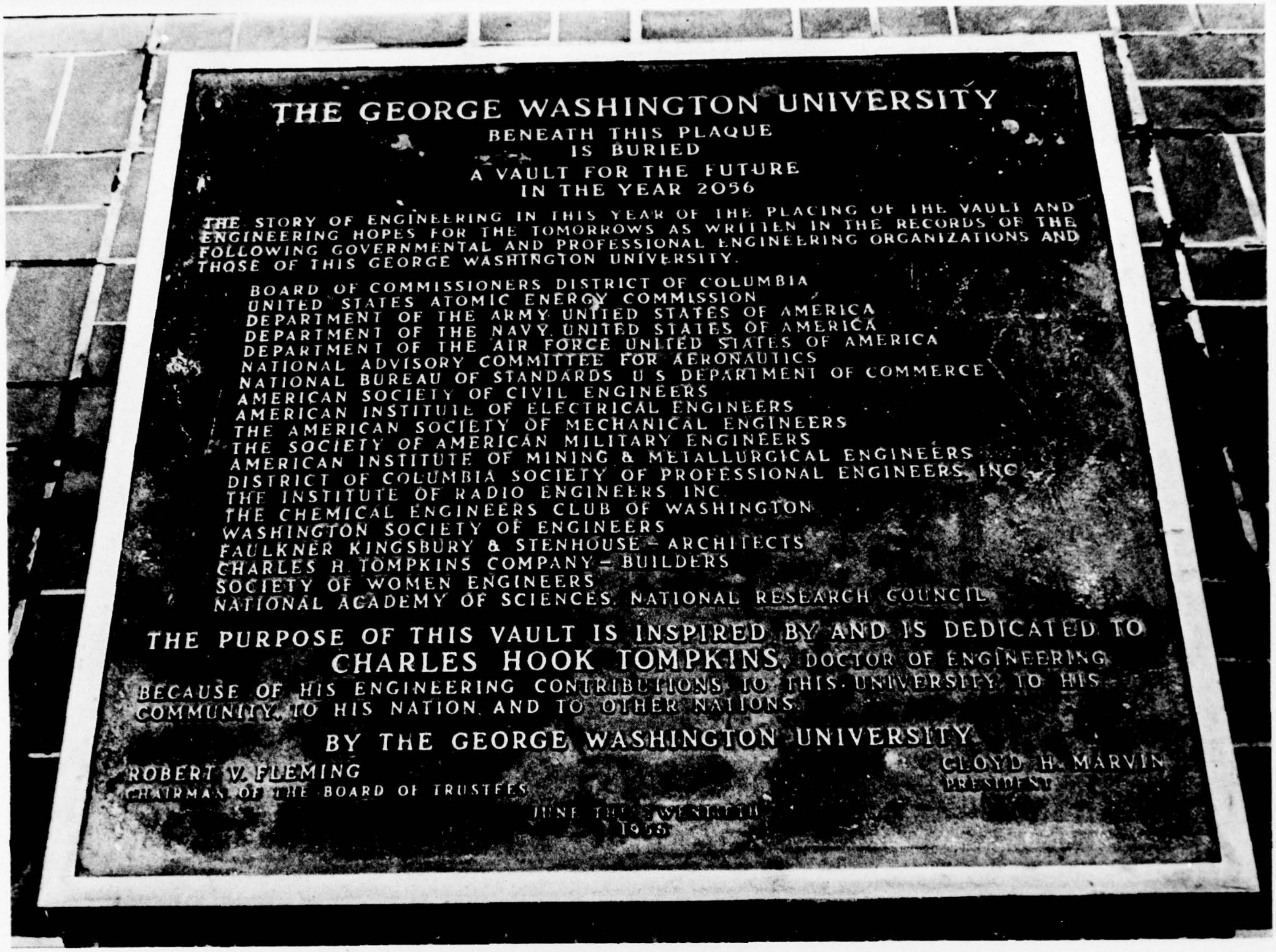

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