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A STATISTICAL APPROACH TO MEASUREMENT OF GUN TUBE WEAR.(U)

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FINAL REPORT

A STATISTICAL APPROACH TO MEASUREMENT OF GUN TUBE WEAR

Prepared for

U. S. Army Armament R & D Command
Picatinny Arsenal
Dover, New Jersey 07801

DAAK 10-77-C-0074

Prepared by

Paul H. Wirsching

MAY 1978

ENGINEERING EXPERIMENT STATION
COLLEGE OF ENGINEERING
THE UNIVERSITY OF ARIZONA
TUCSON, ARIZONA

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It is demonstrated herein that it is possible to estimate wear in a given tube using the mean and/or standard deviation of muzzle velocity from reference lot samples of size n=7 or 10 rounds for the 155 MM and 105 MM guns

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MAY 1978

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ABSTRACT

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It is demonstrated herein that it is possible to estimate wear in a given tube using the mean and/or standard deviation of muzzle velocity from reference lot samples of size n=7 or 10 rounds for the 155MM and 105MM guns respectively. Because ballistic tests can be used effectively for estimation of tube wear, a sound basis for life estimation of gun tubes can be developed.

CHAPTER I

Introduction

A normal acceptance test of a newly manufactured lot of propellant requires the firing of 77 rounds of artillery ammunition at a Government proving ground at a total cost of approximately $7,000. If this test yields data which is unuseable in determining the disposition of the test lot, i.e. the calibration rounds fail specification requirements, the test must be rerun. One of the major reasons believed responsible for obtaining unusable data is that the gun tube used in the test was worn to the point that the ballistic performance of the round has degraded below an acceptable level. The cost of replacing a worn out gun tube used in these tests, e.g. of size 155MM, is about $70,000. Clearly, gun tube wear is an important problem in ballistic acceptance testing. The general goal of this study is to investigate the physical behavior of the firing/wear process using statistical methods and some techniques of probibilistic design.

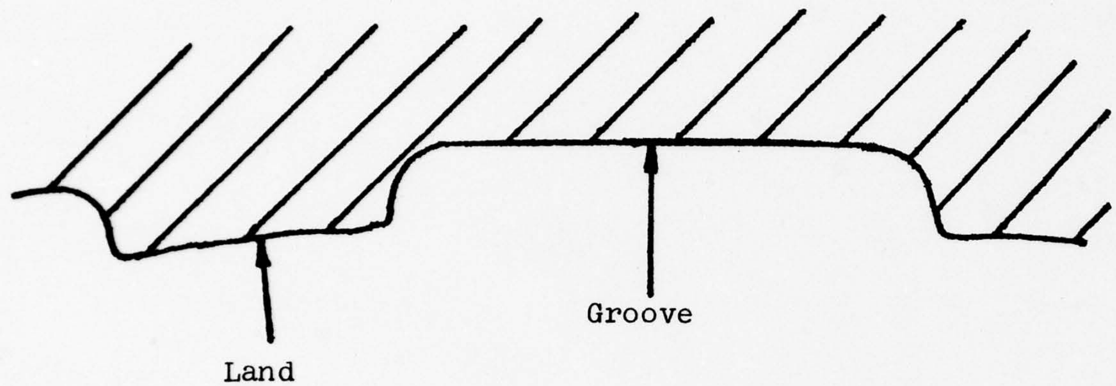
CHAPTER II

A Description of the Wear Process in Gun Tubes

A. A Description of the Gun System

In the general sense, a gun is any projectile throwing device. In the specific case considered herein, a gun is a weapon which fires at low elevation and at high velocity as opposed to a howitzer which fires at higher angles of elevation and lower velocities. Both guns and howitzers are types of artillery.

A main part of a gun is the gun tube. Shown in Figure 1 is a cross section of a typical gun tube. The muzzle is the forward end of the tube through which the projectile exits. The breech is the rear end of the tube through which the ammunition is loaded. The chamber is the portion of the tube which houses the ammunition. The rifled bore is the accelerating tube for the projectile. Rifling consists of lands and grooves which spiral down the length of the bore. As shown in Figure 1, the lands are the raised portion and the grooves are the lower portion. The rifling imparts spin to the projectile, stabilizing the projectile and thereby increasing the accuracy and carrying power of the ammunition. All gun tubes discussed herein employ spin-stablized ammunition.



Bore Dia. (Distance Across Lands)
105MM 4.134" + .002"
155MM 6.100" + .002"

FIGURE 1
Cross Section of Rifled Bore

Figure 2 shows the various sections of a typical gun tube. Upon loading, the projectile is inserted through the breech into the chamber and is aligned by the centering cylinder so that its forward section protrudes into the bore with the rotating band resting against the forcing cone. The rotating band is usually made of copper and is slightly larger than the tube inside diameter. When the round is fired, the pressure from the burning propellant causes the rotating band to engage and then be engraved by the rifling as it accelerates down the tube.

B. A Description of the Physical Process of Gun Tube Wear

The high initial resistance to motion presented by the rotating band/rifling interaction results in gases building up high starting pressure before motion is imparted to the projectile. When the gun tube is new, there is very little leakage of gas between the rotating band and the grooves of the rifling. As the tube is used, the hot gases produced by the burning of the propellant begin to erode the bore surface. Erosion occurs mainly through the formation of heat checks (surface cracks). Repeated thermal shock caused by the short duration, in the order of milliseconds, of the firing cycle results in the formation of a series of small cracks as shown in Figure 3. These cracks then erode locally increasing

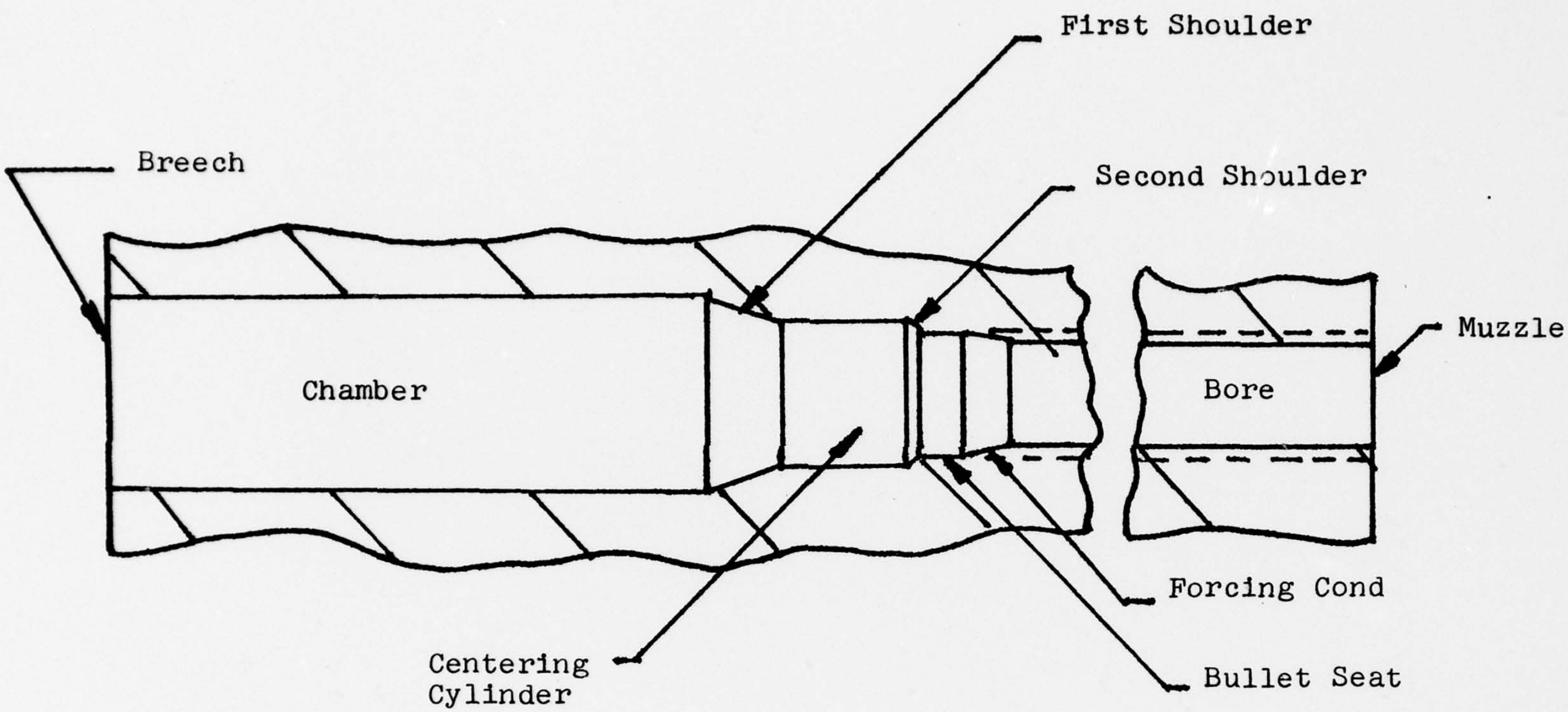
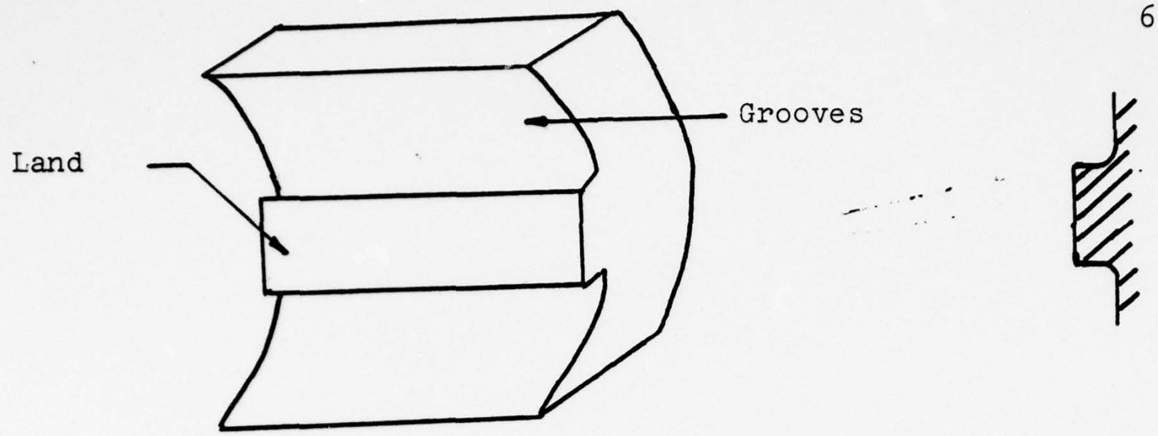
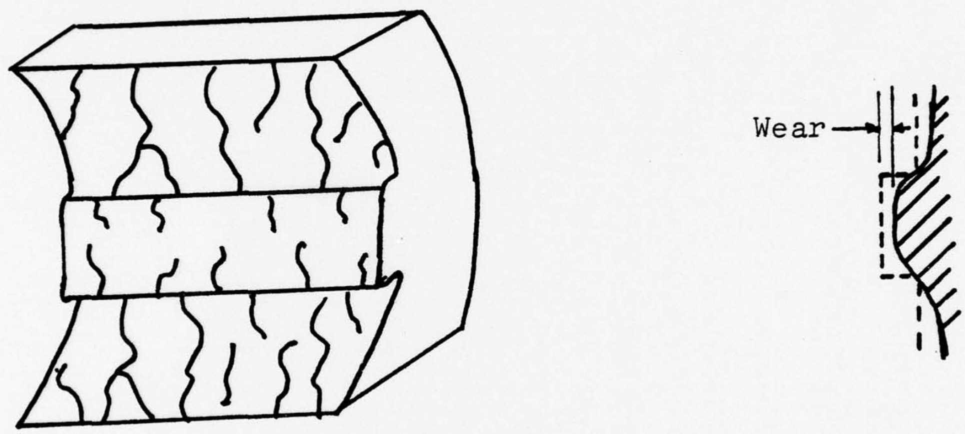


FIGURE 2
Cross Section of Typical Gun Tube

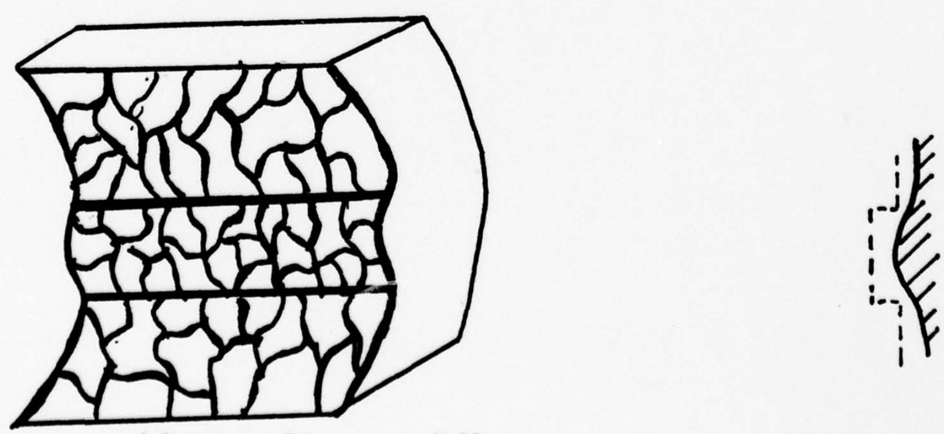


New Tube
Showing Shape of Lands and Grooves



Early Stages of Wear

Heat checks beginning to form, corners on Lands Rounding.



Advance Stages of Wear

As use continues the heat checks increase in depth and size due to gas erosion until large amounts of metal are removed.

FIGURE 3
Progressive Gun Tube Wear

in depth and size with firing. In turn the clearance between the projectile and gun tube is increased. The erosion is greatest on the surface of the lands and at or very near the origin of rifling at the breech end. Wear which occurs at the origin of rifling allows the projectile to enter further into the forcing cone. The combined effect is to reduce the effective chamber volume and lower the engraving resistance leading to lower starting pressures and increased leakage between the projectile rotating band and the rifling grooves. This leads eventually to projectile wobbling (balloting) within the gun tube. The rate of erosion is much greater with higher velocity rounds, i.e. a high velocity tank gun such as a 105MM erodes at a rate many times that of a howitzer of the same size.

C. Definition of Equivalent Full Charge Factor (EFC)

A given gun tube can use many types of ammunition. Because of variations in projectile weight, size of propelling charge, and configuration, each type of ammunition has a different affect on the rate of wear in that particular tube. To account for the effects of various ammunition, the following empirical approach is taken. For example, in the 105MM tank gun, the anti-personnel (AP) round is considered the primary round and is assigned an equivalent full charge (EFC) factor of 1.00 while the high explosive anti-tank (HEAT) round is given an EFC factor of 0.40. These factors, based on previous experience, indicate that firing two HEAT rounds has the same effect on wear as firing one AP round. This EFC factor can then be used as a common denominator for determining the round life condemnation limit of a gun tube. However, the "number of rounds fired" used herein refers to one specific type of ammunition only since the data was aquired from propellant assessment firings in which the type of ammunition used remained constant through the life of the gun tube.

D. Wear Measurement (A Definition of the "Life Remaining" Concept)

The current Military method used to determine the amount of erosion which has taken place in a gun tube is to measure the increase in diameter of the bore and compare it with an "established limit" (based on experience with the type of tube). This limit is the point at which the tube diameter has increased to a point where it is considered to be unsafe or is unable to perform its intended mission. This established limit corresponds to 0% life remaining for the gun tube. Life remaining is based on the amount of wear, i.e. increase in bore diameter, that has taken place. A new tube has 100% life remaining. Figure 4 shows the effect of the number of rounds fired on life remaining for various EFC factors.

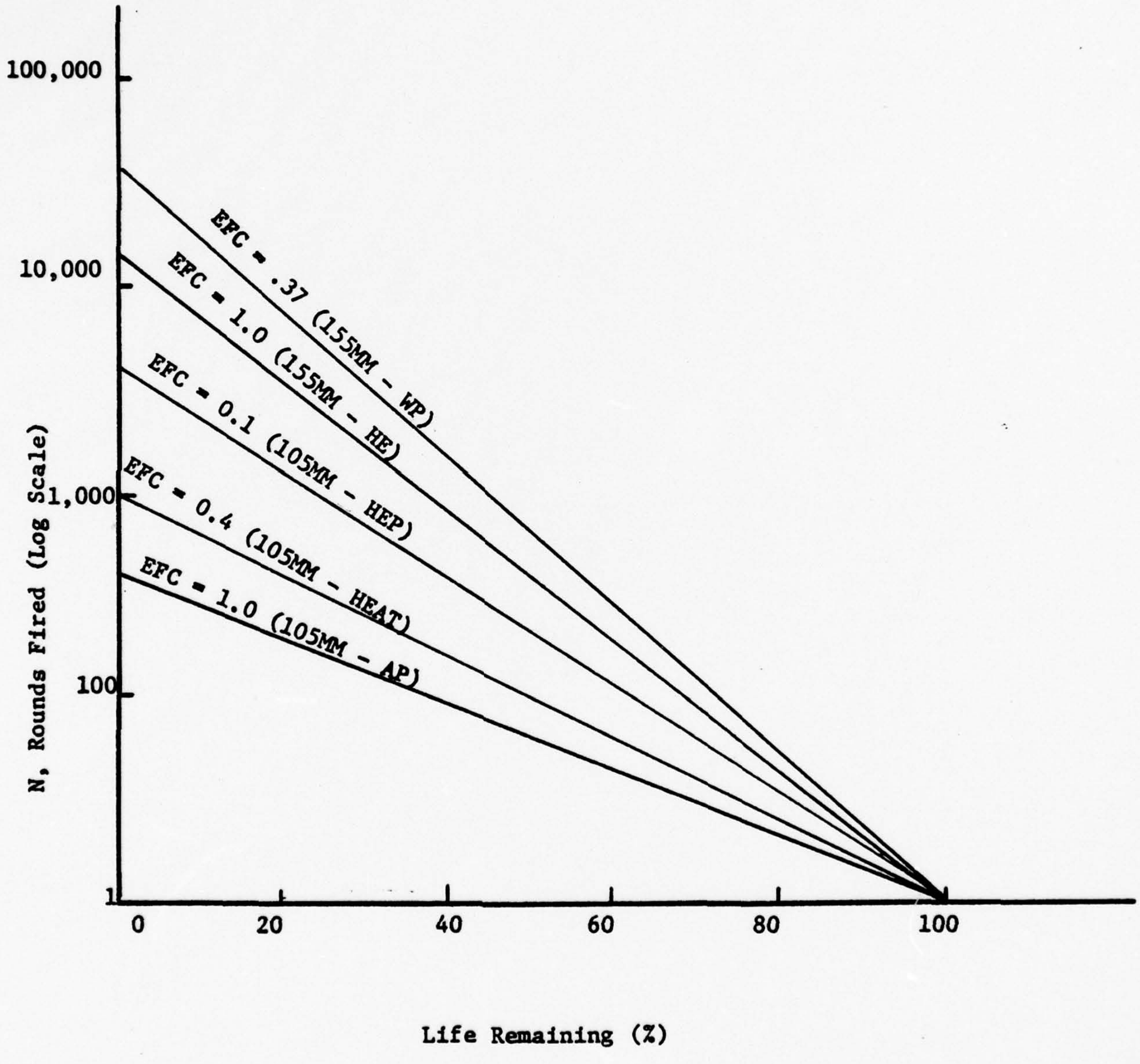


FIGURE 4

Effect of Rounds Fired on Life Remaining

The tube wear measurement is made using a pullover gage as shown in Figure 5 and is taken at the origin of rifling. The location of the wear measurement for the gun tubes considered in this report are:

Tube	Distance from Breech (in.)
105MM	25.25
155MM	30.00

E. Sources of Error in Ballistic Testing

Propellant ballistic testing is performed outside at a Government proving ground firing range. The gun tube to be used in the test is placed on a suitable gun tube mount (depending on bore), consisting of a supporting structure and recoil mechanism. Solenoid velocity coils are then located 50 feet in front of the gun (see Figure 6). The coils are located 50 feet from the muzzle to prevent damage from gun blast. This distance has only a small effect on the velocity measurements since: (1) Drag is reduced since the projectile is being followed by hot gases and (2) All tests are run at the same distance, therefore cancelling out its effect. The velocity coils sense when the projectile (magnetized prior to test) passes through. The time elapsed for the projectile to travel between coils is used to calculate the velocity.

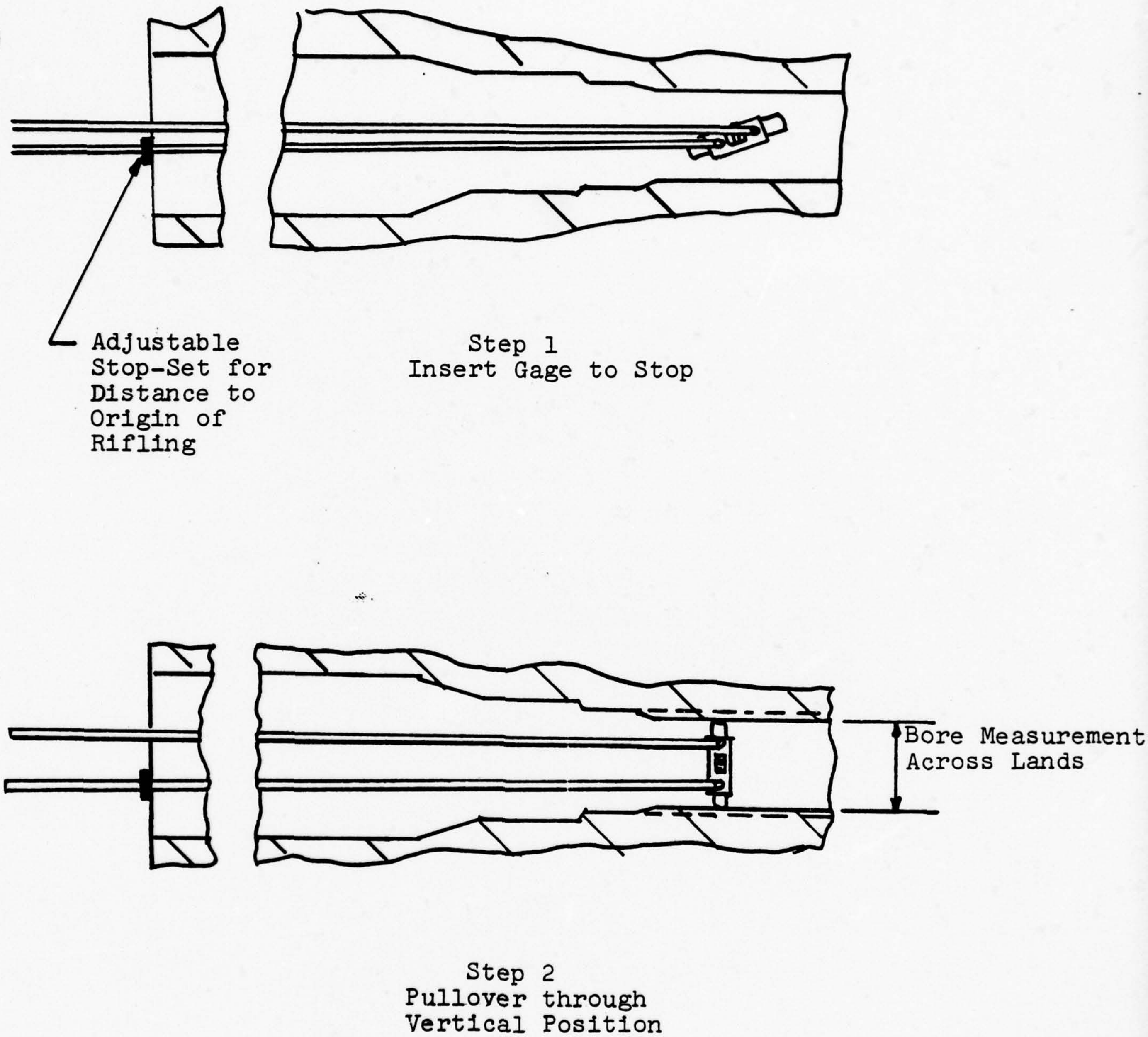


FIGURE 5

Wear Measurement Using Pullover Gage

Following is a listing of major sources of error other than normal measuring equipment error, inherent in ballistic testing.

1. Weather Conditions

The temperature and humidity at the time of test affect the flight of the round as well as the operation of the tube. When the gun tube is cold, the velocities are lower than when hot. Warming rounds are fired prior to start of test. However, on severly cold days, enough rounds may not be fired to bring the tube up to operating temperature. Therefore, the first one or two rounds fired may have a lower velocity than the remainder of the rounds. The operating temperature itself varies with ambient temperature, i.e., the longer the mean time between rounds becomes the greater the effect of temperature on velocity.

2. Delays Between Rounds

Occasionally, during the course of a ballistic test, a delay occurs due to malfunctions of the test equipment or set-up. This repair down time allows the tube to cool, affecting the velocity. Specifications call for warmer rounds to be fired if the down time exceeds a specified time as follows:

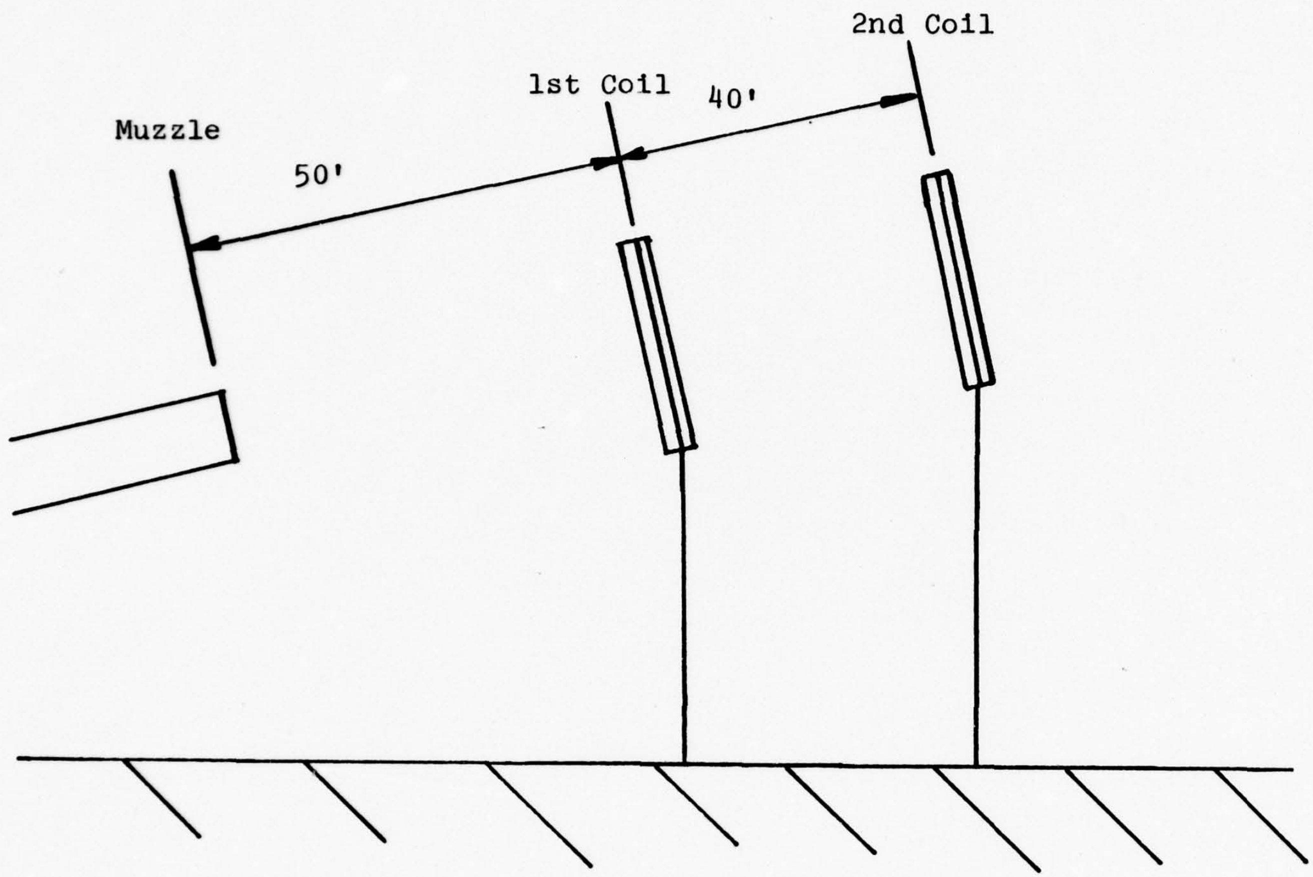


FIGURE 6
Velocity Coil Location

10 to 30 minutes - 1 warmer

30 to 60 minutes - 2 warmers

over 60 mintues - repeat initial conditioning

The normal time between rounds is three to five minutes. Therefore, in cold weather the extra down time will have an effect on the velocity.

3. Human Factors

As mentioned previously, the mean time between rounds is three to five minutes. This time varies with each crew. A less experienced crew may have both a high mean time between rounds and a higher variance in cycle time.

Other human factors involved are such things as loading accuracy (variation in the amount of propellant loaded into each cartridge), normal errors in reading test equipment gages, and accuracy in positioning the velocity coils. The positioning of the velocity coils is an important factor since occasionally a rotating band breaks and flies free of the projectile after leaving the gun tube and hits one of the coils. It then has to be repaired or replaced and relocated and may not be in exactly the same position as before.

4. Mounts

Another source of error in ballistic testing is the mount. Each mount at a firing site at a proving ground has its own recoil mechanism. The exact contribution of the variation in recoil between mounts is unknown. The extreme case would be the comparison of a fixed mount, one in which the rear supports are anchored to the ground (such as a towed howitzer mount) and a tracked vehicle mounted gun tube in which recoil is absorbed not only through the tank recoil mechanism but also through the suspension system of the tank. The comparison of the howitzer and tank mount is the extreme but significant variation may also exist between individual fixed mounts and individual tracked vehicle mounts.

F. Specific Goals and Possible Applications

1. To explore the relationship between tube wear and muzzle velocity. Can wear measurements be used to estimate muzzle velocity, and vice versa?

2. To quantify the relationship between degradation in ballistic performance and wear.

3. To quantify the magnitude of the random experimental error in propellant acceptance tests, and thereby measure the quality of current procedures. How much random experimental error is present? Are lots of "bad" propellant being accepted and vice versa?

4. To define rules, or criteria, for replacing a gun tube on the basis of muzzle velocity data. This criteria would have a probability basis.

5. To define a performance based criteria for a replacement program of gun tubes of a given type.

CHAPTER III

Statistical Analysis of Wear Data

A. General Comments and Definitions of Terms

The data analyzed in studying the effects of gun tube wear was taken from artillery propellant acceptance tests performed at Government proving grounds. These tests provide useful data on wear estimation since: (1) only one type of round, i.e. AP, HEAT, etc., is fired from a particular gun tube throughout its wear life. This eliminates the problem of different types of rounds having different effects on the rate of wear in a gun tube, see Section IIC, Equivalent Full Charge. (2) The calibration rounds fired are made up of specially selected and tested components and can therefore be considered to give repeatable results from test to test.

Data on various ballistic parameters was extracted from Proving Ground Firing Records and tabulated. A sample of the data recorded is shown in Table 1.

155MM HOWITZER, TUBE NO. 14982, N = 7

TUBE ROUND NO.	GAGE READING (in)	PERCENT LIFE REM.	$\overline{V}$ (FPS)	S_V (FPS)
444	.001	95	1842	2.7
504	.001	95	1850	3.5
779	.003	90	1851	2.3
1019	.004	90	1857	1.6
1400	.005	85	1847	2.3
1865	.005	85	1860	2.5
2226	.006	80	1853	4.2
2450	.006	80	1856	2.8
2679	.006	80	1855	2.0
2849	.006	80	1855	2.9
3062	.007	70	1856	3.7
3108	.007	70	1855	2.3
3169	.007	70	1854	3.2
3276	.007	65	1845	3.4
3503	.007	65	1857	2.4
4096	.008	60	1839	2.1
4337	.010	55	1852	2.7
4520	.010	55	1844	3.6
4587	.010	55	1834	4.1

TABLE 1

SAMPLE DATA

Define:

μ_o = nominal mean muzzle velocity

1850 ft/s for 155MM gun

3850 ft/s for 105MM gun

$V(N)$ = stochastic process denoting the muzzle velocity associated with the number of rounds fired, N.

V_i = an observation of V.

n = sample size of calibration rounds. (typically n = 7 for 155MM firings and n = 10 for 105MM firings).

μ_V = the mean of V corresponding to life N.

σ_V^2 = the variance of V corresponding to life N.

$\bar{V}$ = the sample mean muzzle velocity.

$$\bar{V} = \frac{1}{n} \sum_{i=1}^{n} V_i \qquad (1)$$

s_V^2 = the sample variance.

$$s_V^2 = \frac{1}{n-1} \sum_{i=1}^{n} (V_i - \bar{V})^2 \qquad (2)$$

$\overline{V}$ and s_V, evaluated at a given N, are estimates of μ_V and σ_V respectively. In the life of a tube there are assumed to be k calibration rounds, so that the ballistic history of a tube would consist of a sequence of k values of V and s_V.

It is assumed that n is sufficiently small relative to the total life so that the sample of the calibration rounds, V_i (i = 1, n) is a sequence of independent and identically distributed random variables.

B. 155MM

1. Muzzle Velocity as a Function of Number of Rounds Muzzle velocity tends to decrease as the number of rounds, N, fired from the tube increases and can be considered a random process. In order to visualize this relationship, the non-dimentional statistics

$$U = \left(1 - \frac{\overline{V}}{\mu_o}\right) 10^3 \qquad (3)$$

and

$$S = \left(\frac{s_V}{\mu_o}\right) 10^3 \qquad (4)$$

were plotted against tube round number,N individually in Figures 7, 8, 9 and 10 and jointly in Figure 11 for tube serial numbers 14742 and 14982. On the basis of a visual examination of the data, it was assumed that U and S are linear functions of N. Least squares linear regressions were performed on the data to obtain the "best fit" lines shown. Relationships of the form

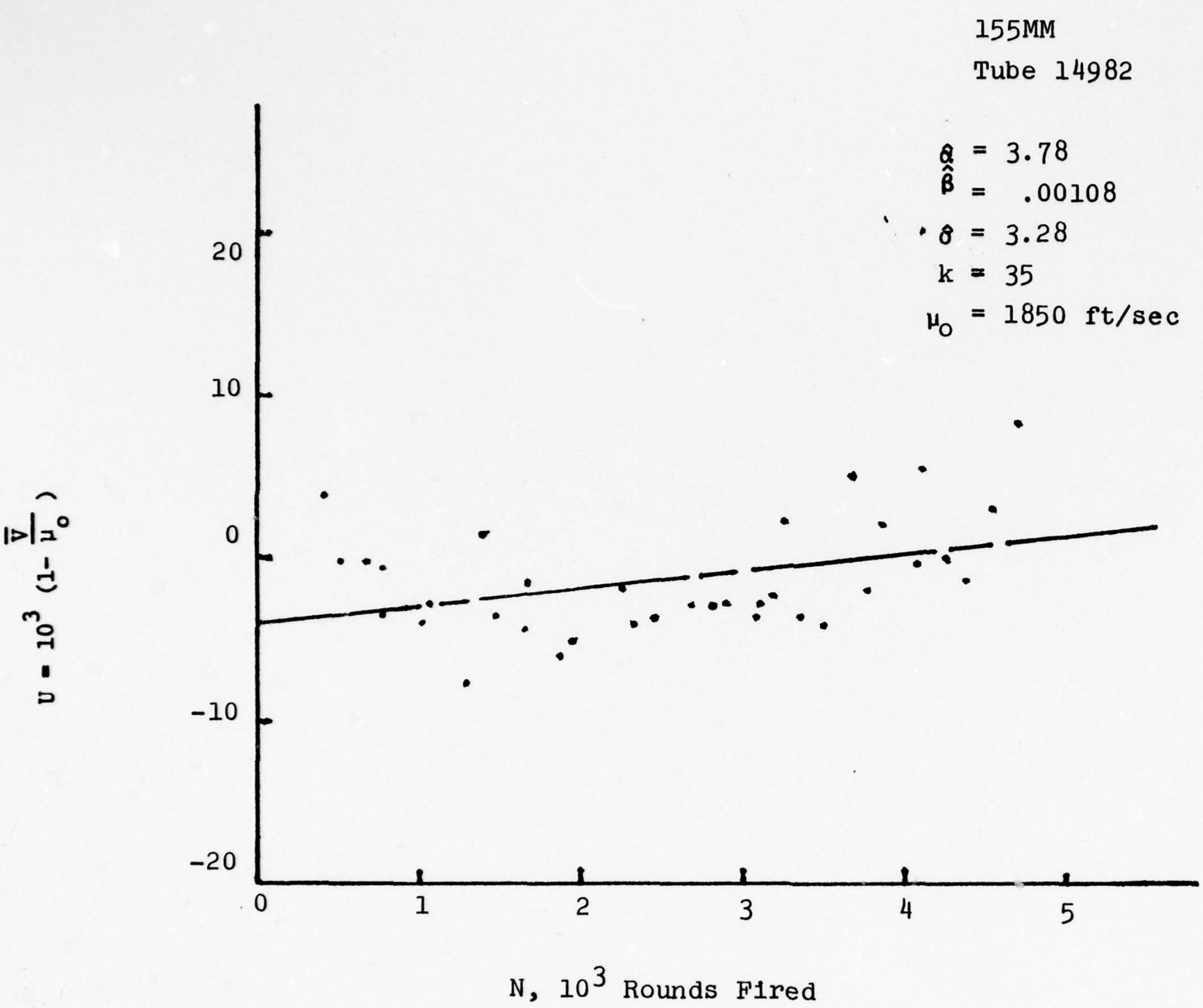

155MM
Tube 14982
$\hat{\alpha}$ = 3.78
$\hat{\beta}$ = .00108
$\hat{\sigma}$ = 3.28
k = 35
μ_o = 1850 ft/sec
20
10
0
-10
-20
$U = 10^3 (1 - \frac{V}{\mu_o})$
0
1
2
3
4
5
N, 10^3 Rounds Fired

FIGURE 7

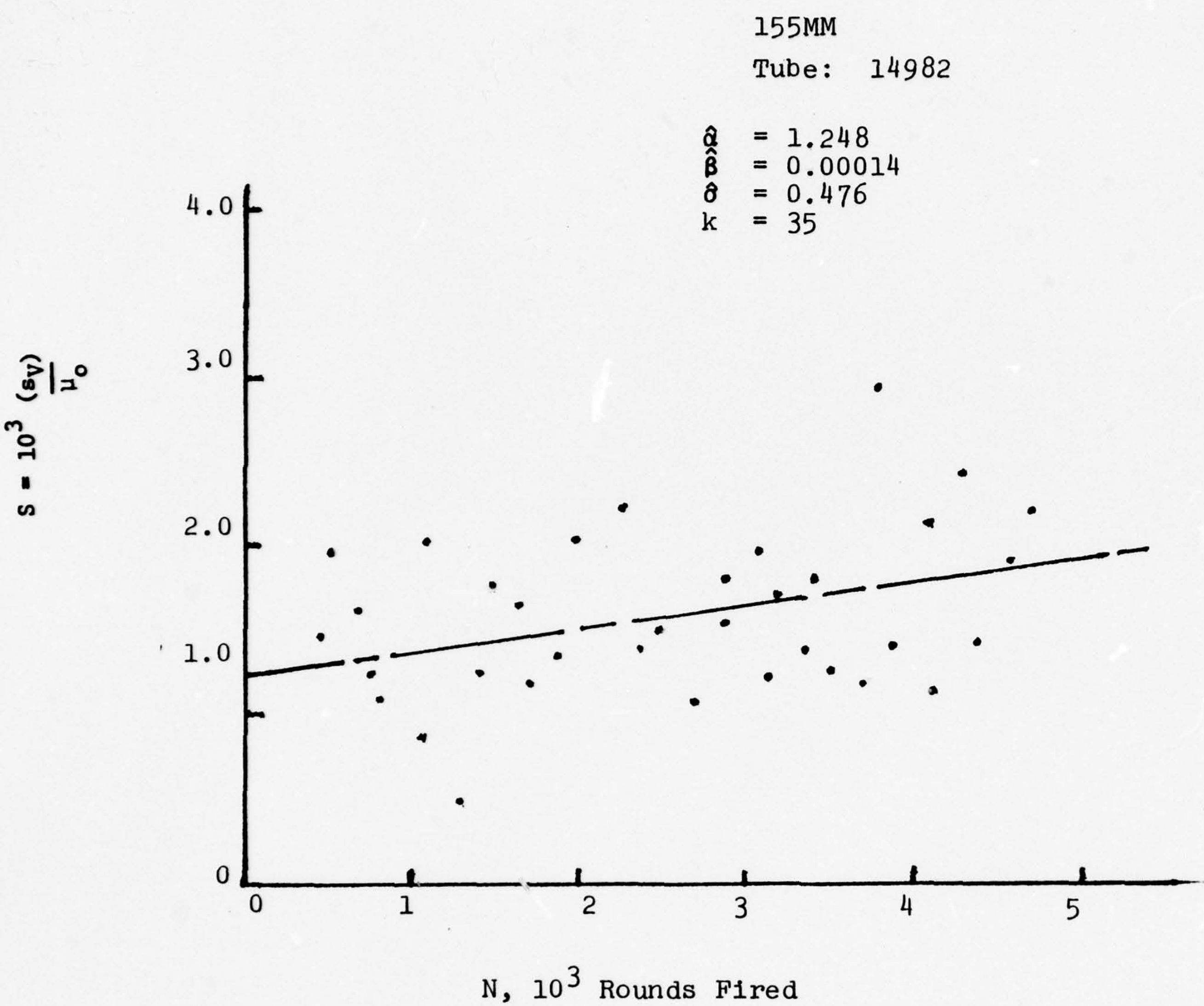

155MM
Tube: 14982
$\hat{\alpha}$ = 1.248
$\hat{\beta}$ = 0.00014
$\hat{\sigma}$ = 0.476
k = 35
$S = 10^3 \frac{(s_V)}{\mu_o}$
4.0
3.0
2.0
1.0
0
0
1
2
3
4
5
N, 10^3 Rounds Fired

FIGURE 8

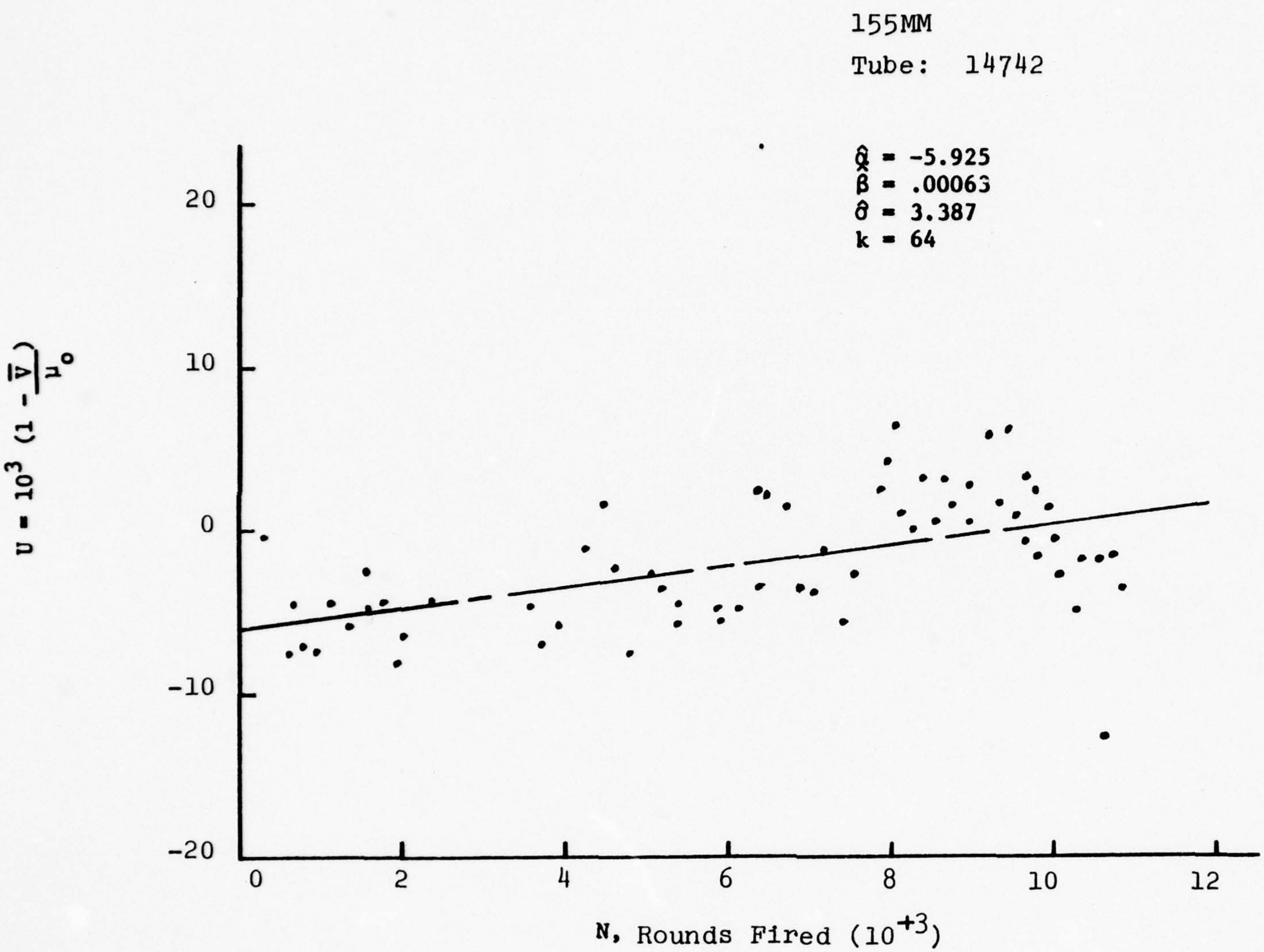

155MM
Tube: 14742
α̂ = -5.925
β = .00063
σ̂ = 3.387
k = 64
20
10
0
-10
-20
U = 10³ (1 - V̄/u₀)
0
2
4
6
8
10
12
N, Rounds Fired (10⁺³)

FIGURE 9

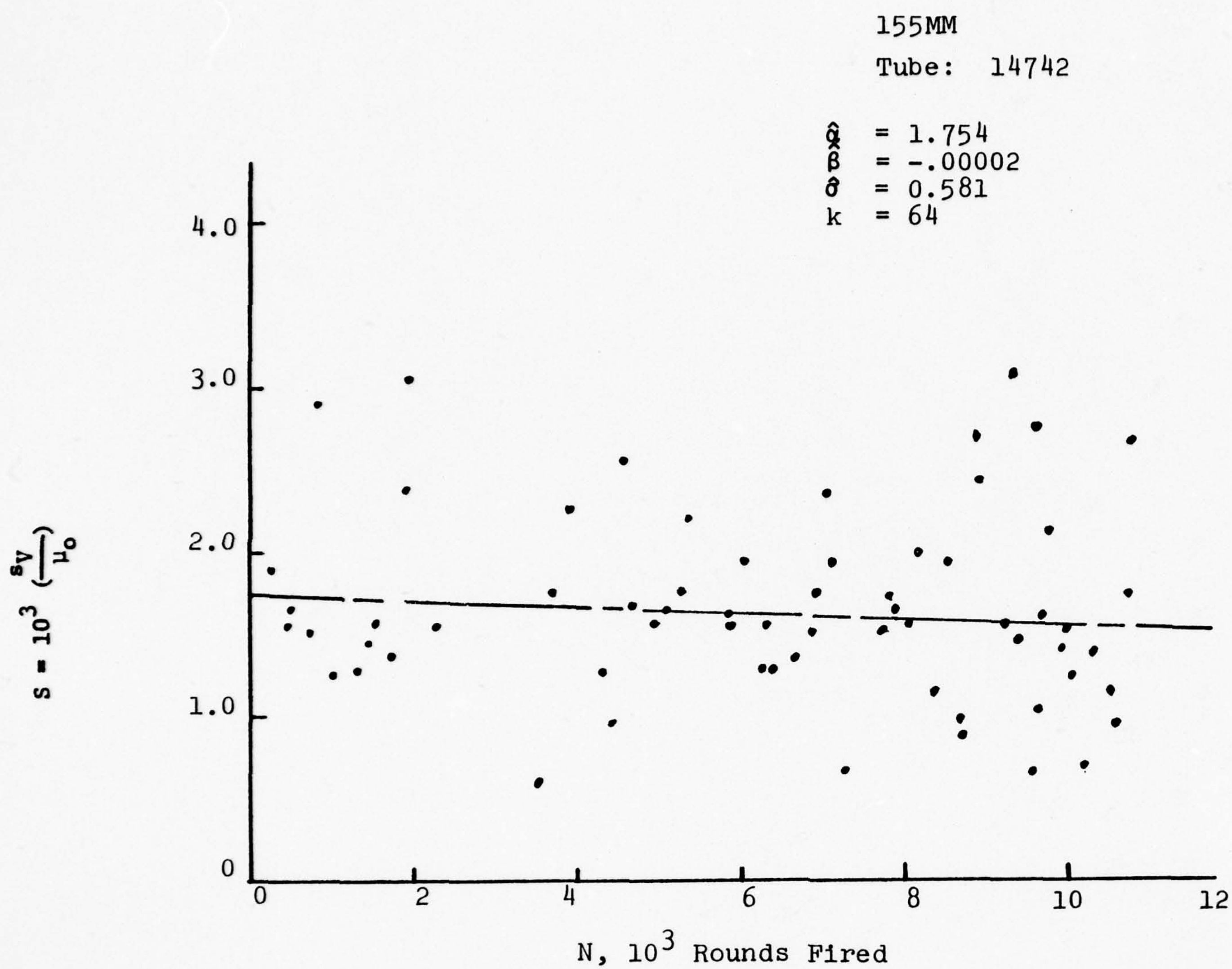

155MM
Tube: 14742
$\hat{\alpha}$ = 1.754
$\hat{\beta}$ = -.00002
$\hat{\sigma}$ = 0.581
k = 64
4.0
3.0
2.0
1.0
0
$S = 10^3 \left(\frac{s_V}{\mu_o}\right)$
0
2
4
6
8
10
12
N, 10^3 Rounds Fired

FIGURE 10

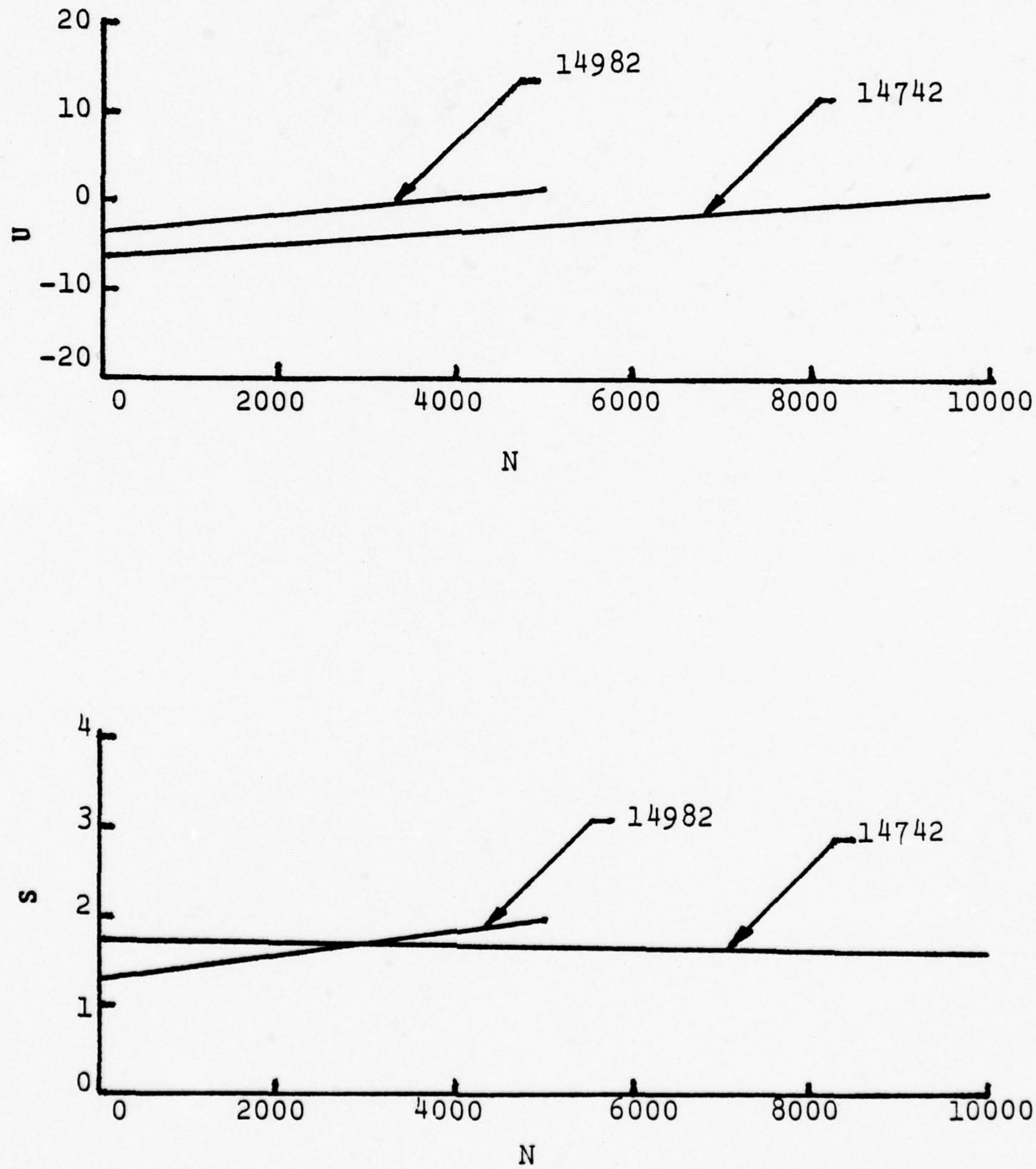


FIGURE 11 - MUZZLE VELOCITY
MEAN AND STANDARD DEVIATION AS A FUNCTION OF ROUNDS

$$Y = \alpha + \beta N \tag{5}$$

are assumed, where Y = U or S. The data consists of (N_i, Y_i), i=1, k where k is the total number of points.

Estimates of α and β from the data are,

$$\hat{\beta} = \sum_{i=1}^{k} (N_i - \overline{N})(Y_i - \overline{Y}) / \sum_{i=1}^{k} (N_i - \overline{N}) \tag{6}$$

$$\hat{\alpha} = \overline{Y} - \hat{\beta}\ \overline{N} \tag{7}$$

where $\overline{N}$ and $\overline{Y}$ are the sample means of N and Y respectively. The variance of Y given N is

$$\hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^{k} [Y_i - (\hat{\alpha} + \hat{\beta} N_i)]^2 \tag{8}$$

2 = 2. Wear as a Function of Muzzle Velocity The amount of wear, W, increases with N and can again be considered a random process. The statistics U and S were plotted against W individually in Figures 12, 13, and 14 and jointly in Figure 16 for tube serial numbers 14742 and 14982. It is assumed that wear is zero at time zero, and that the values of $\overline{V}$ and s_v at time zero are the values obtained from the regression analysis described in the above section. The estimate of β, $\hat{\beta}$, becomes

$$\hat{\beta} = (\overline{Y} - \hat{\alpha})/\overline{N} \tag{9}$$

where $\hat{\alpha}$ is specified. The values of $\hat{\alpha}$ and $\hat{\beta}$ are shown on Figures 12, 13,14 and 15.

155MM

Tube: 14982

$\hat{\alpha} = -3.779$
$\hat{\beta} = 457.8$
$\hat{r} = .318$

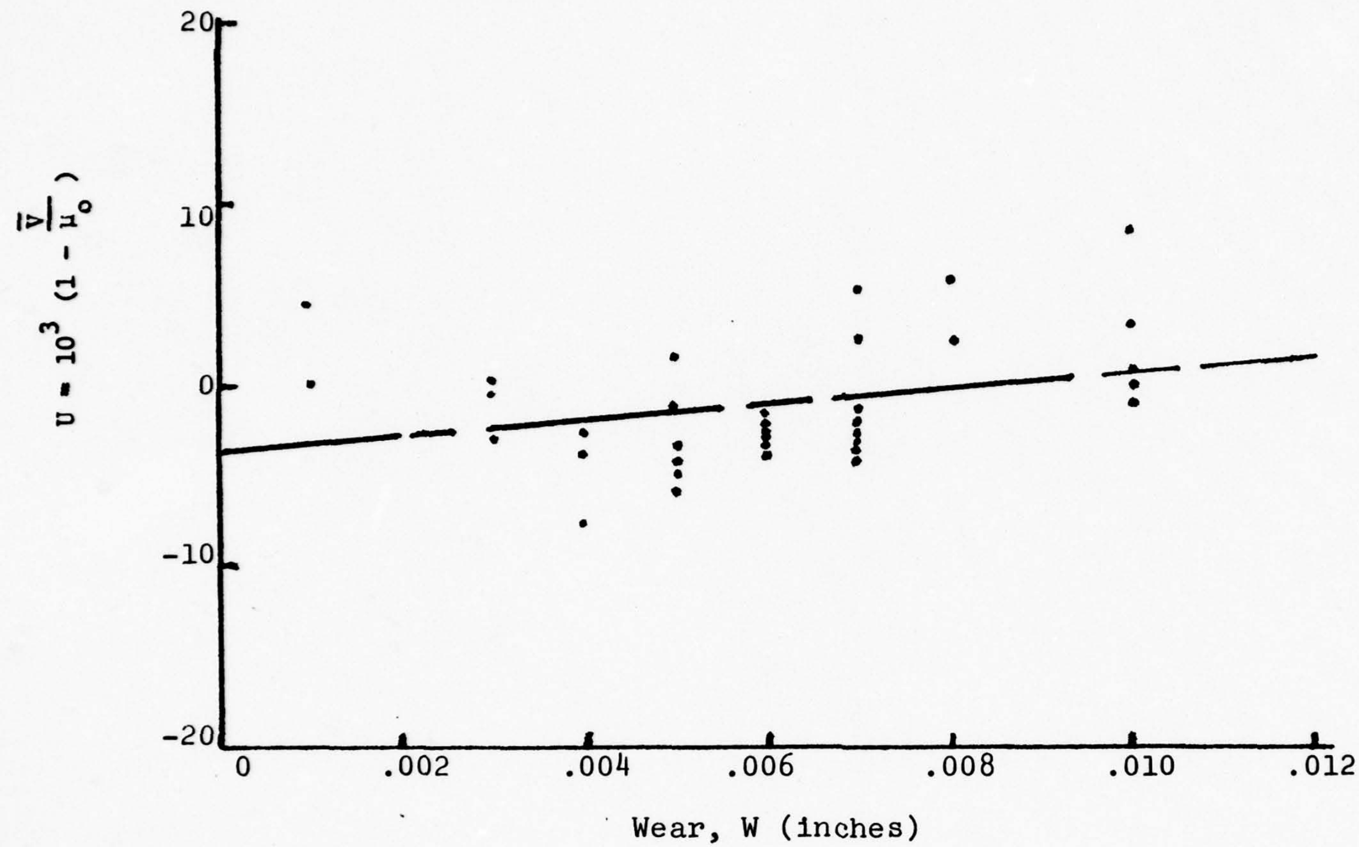


FIGURE 12

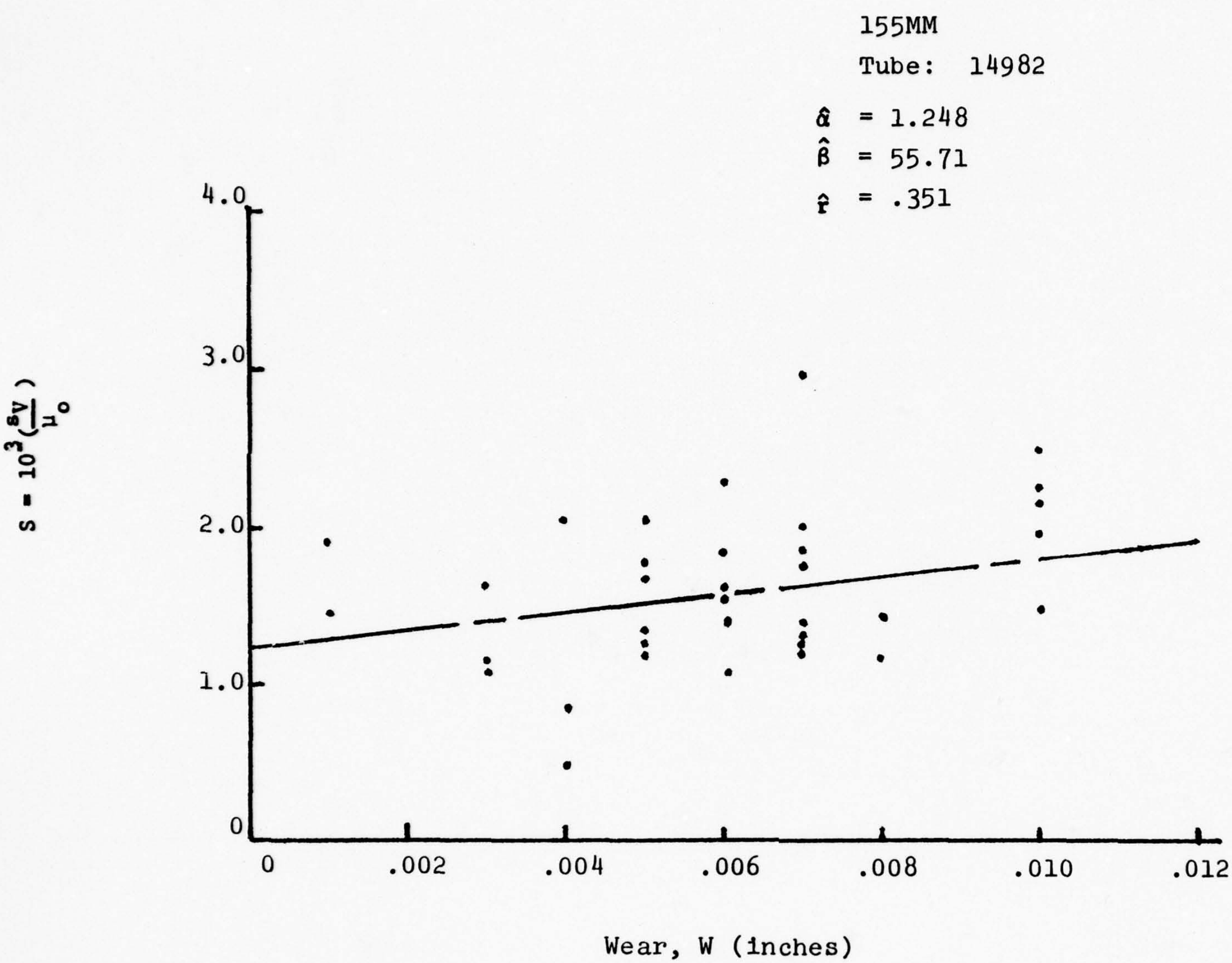

155MM
Tube: 14982
$\hat{\alpha}$ = 1.248
$\hat{\beta}$ = 55.71
$\hat{r}$ = .351
$S = 10^3 (\frac{s_V}{\mu_o})$
4.0
3.0
2.0
1.0
0
.002
.004
.006
.008
.010
.012
Wear, W (inches)

FIGURE 13

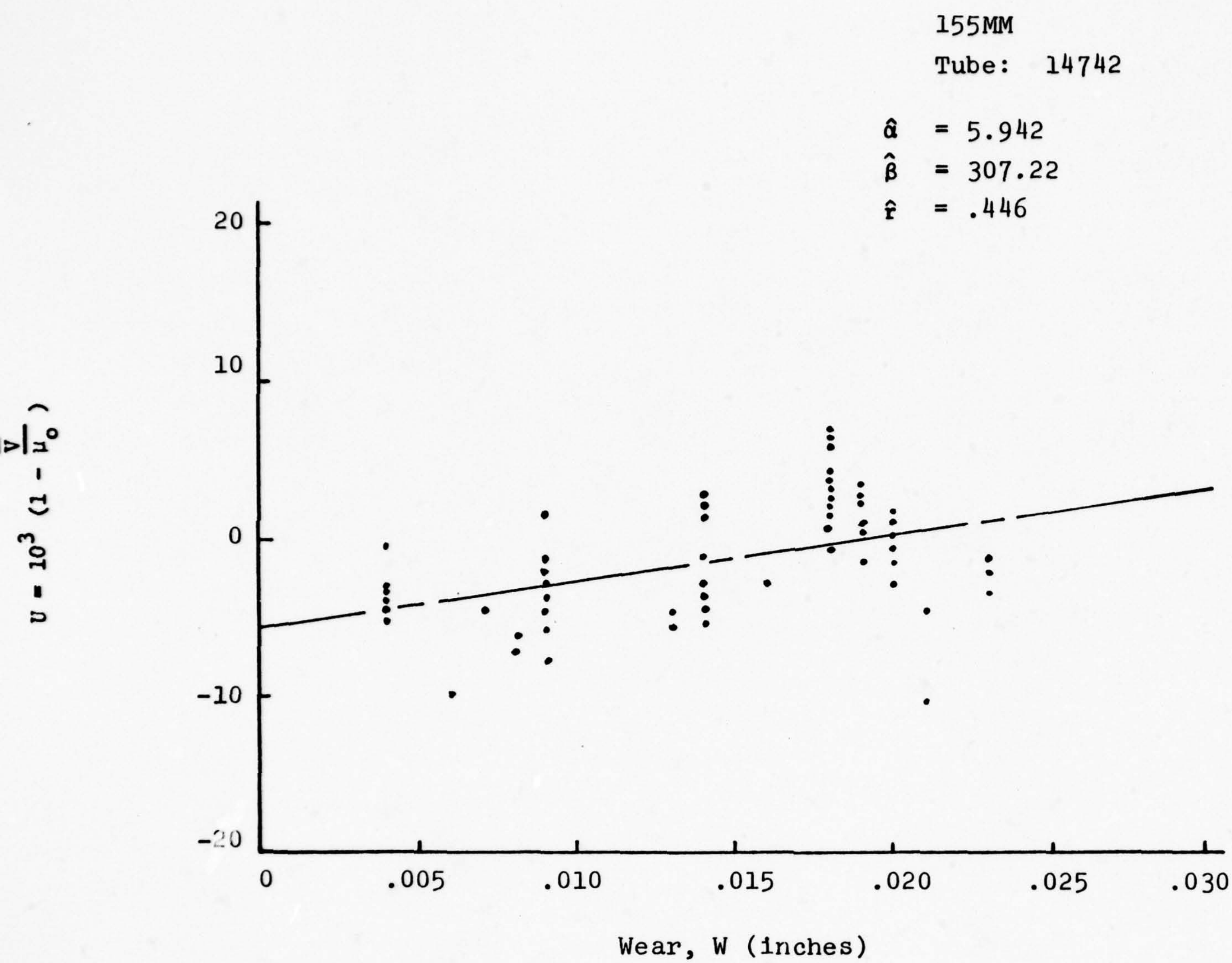

155MM
Tube: 14742
$\hat{\alpha}$ = 5.942
$\hat{\beta}$ = 307.22
$\hat{r}$ = .446
$U = 10^3 (1 - \frac{\bar{V}}{\mu_o})$
20
10
0
-10
-20
0
.005
.010
.015
.020
.025
.030
Wear, W (inches)

FIGURE 14

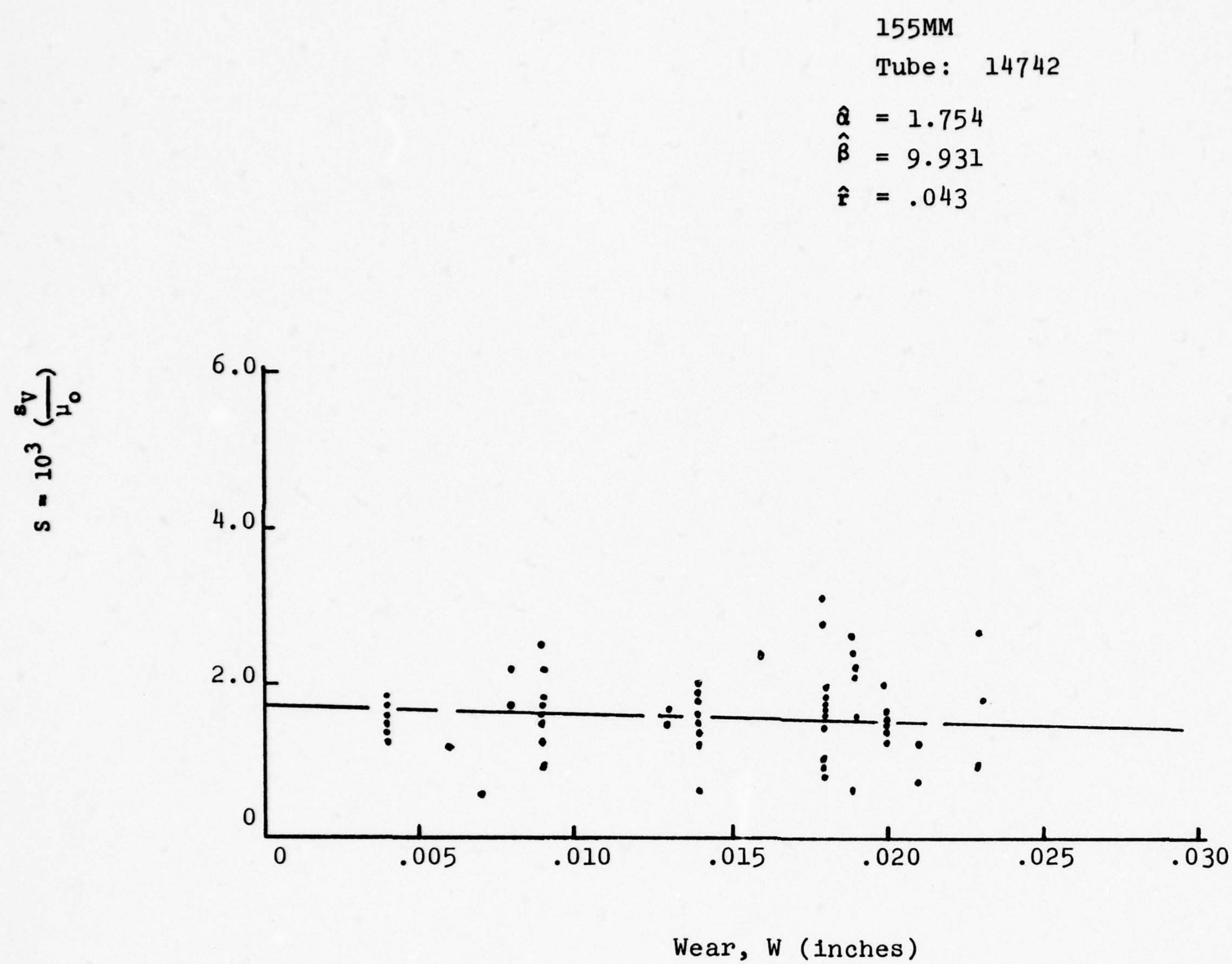

155MM
Tube: 14742
$\hat{\alpha}$ = 1.754
$\hat{\beta}$ = 9.931
$\hat{r}$ = .043
$S = 10^3 \left(\frac{s_V}{\mu_o}\right)$
6.0
4.0
2.0
0
0
.005
.010
.015
.020
.025
.030
Wear, W (inches)

FIGURE 15

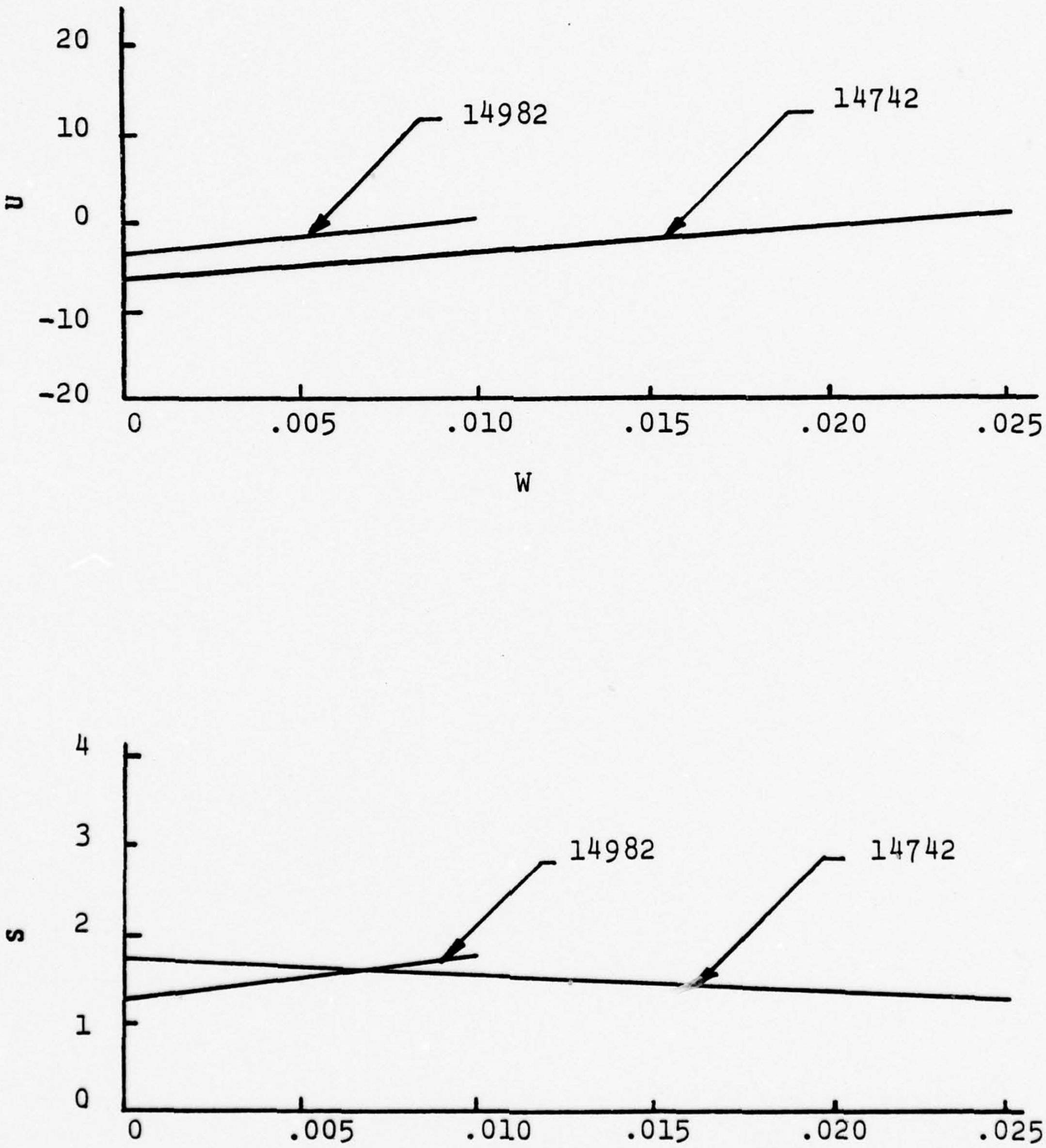


FIGURE 16 - MUZZLE VELOCITY

MEAN AND STANDARD DEVIATION AS A FUNCTION OF WEAR

3. <u>Correlation Between Y and W</u> Both Y and W are random variables and observation of Figure 12 through 15 suggests a linear relationship between the two. A measure of the relationship is the correlation coefficient $\hat{r}$ which is estimated by

$$\hat{r} = \frac{\sum_{i=1}^{k} (W_i - \overline{W})(Y_o - \overline{Y})}{[\sum_{i=1}^{k} (W_i - \overline{W})^2 \sum_{i=1}^{k} (Y_i - \overline{Y})^2]^{1/2}} \qquad (10)$$

The calculated values of these parameters are shown on Figures 12 through 15 for each gun tube. The computer printouts listing the actual data entered are shown in the Appendix.

A test of the hypotheses that W and Y are linearly correlated consists of comparing the null hypothesis,

$$H_o : \rho = 0 \qquad (11)$$

against the alternate hypothesis,

$$H_1 : \rho \neq 0 \qquad (12)$$

at the 100α% level of significance. The null hypothesis can be rejected when

$$\frac{|\hat{r}| (k - 2)^{1/2}}{(1 - \hat{r}^2)^{1/2}} > t_{1 - \alpha/2; k - 2} \qquad (13)$$

where:

$t_{1-\alpha/2;k-2}$ = the value of Student's t distribution with k-2 degrees of freedom. α = the level of significance.

Example

For tube 14982, Fig. 6, sample calculations are as follows:

For $\hat{r} = .397$; $k = 35$; $\alpha = .10$

$$\frac{|\hat{r}|\,(k-2)^{1/2}}{(1-\hat{r}^2)^{1/2}} > t_{1-\alpha/2;\,k-2} \qquad (14)$$

$$\frac{(.318)\,(33)^{1/2}}{\{1-(.318)^2\}^{1/2}} = 1.927 > t_{1-\frac{.1}{2};\,33} = 2.0345 \qquad (15)$$

Therefore, reject the null hypothesis that $\rho = 0$. An approximate 90% confidence interval can be established about ρ based on n and $\hat{r}$ by referring to Fig. 14.6 of Ref. 10. The 90% confidence limits are

$$P\,(.04 \leq P \leq .55) = .90 \qquad (16)$$

A complete tabulation of correlation coefficient results is shown in Table 2.

4. Investigation of the Source of Scatter in $\bar{V}$

Considerable scatter seems to exist in $\bar{V}$ and s_V (Figures 1 and 2). The source of this randomness may be (a) statistical, because of the small sample, or (b) physical because of random experimental error. Following is a discussion of a method for identifying the contribution of each source.

Tube	Process, Y Stat.	Sample Coorelation Coefficient $\hat{r}$	$\frac{\lvert\hat{r}\rvert\sqrt{k-2}}{\sqrt{1-\hat{r}^2}}$	$t_{1-\alpha/2,\,k-2}$	Accept/Reject H_o	Confidence Limits on ρ
14982	U	.318	1.927	1.692	A	{.04, .55}
	S	.351	2.153	1.692	A	{.08, .66}
14742	U	.446	3.729	1.673	A	{.26, .70}
	S	.043	.322	1.673	R	{.11, .56}

TABLE 2

Results of Coorelation Analysis Between Y and N

Consider data from a reference lot, V_i; i = 1, n, and the sample mean $\overline{V}$. The mean and variance of $\overline{V}$ are,

$$\mu_{\overline{V}} = E(\overline{V}) = \mu \qquad (17)$$

$$\sigma_{\overline{V}} = Var(\overline{V}) = \sigma_v^2 / n \qquad (18)$$

One might ordinarily expect V_i to have a normal distribution due to the natural randomness in the physical process. Furthermore, the central limit theorem is working. For any life N, assume

$$\overline{V} \sim N(\mu_v, \sigma_v / \sqrt{n}) \qquad (19)$$

Figure 17 illustrates the data available and defines U, S, and the mean values, obtained by regression analysis, μ_U and μ_s which are functions of N.

σ_V can be estimated by

$$s_V \overset{\sim}{=} \mu_o \quad \mu_s \qquad (20)$$

and therefore

$$s_{\overline{V}} = \mu_o \quad \mu_s / \sqrt{n} \qquad (21)$$

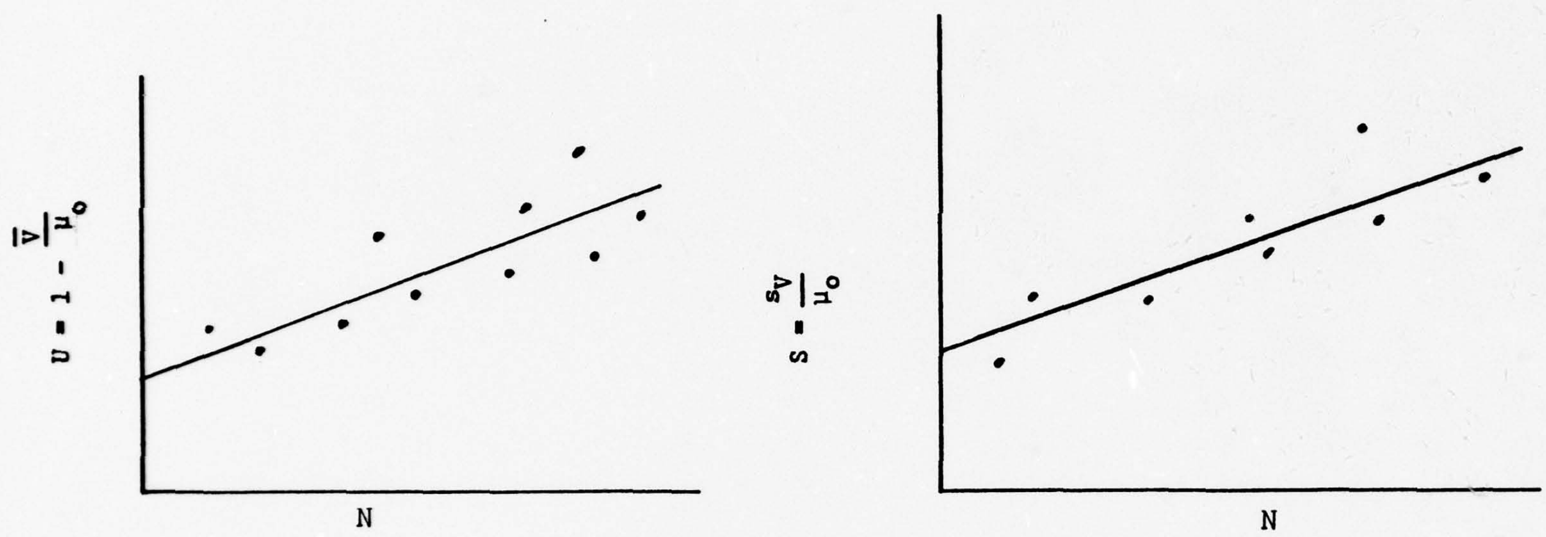


FIGURE 17

The Form of the Available Data

The anticipated statistical scatter in $\overline{V}$ could be estimated by $s_{\overline{V}}$. The actual scatter observed is described as follows.

An estimate of the variance of $U = (1 - \overline{V}/\mu_o)$ can be made by

$$s_u^2 = \frac{1}{k-2} \sum_{i=1}^{n} d_i^2 \tag{22}$$

The least squares analysis gives s_U. (e.g., $\hat{s}$ in Figure 1). Note that,

$$\text{Var}\,(1 - \overline{V} / \mu_o) = \frac{1}{\mu_o^2} \text{Var}\,(\overline{V}) \tag{23}$$

s_U^2 is an estimate for Var (U).

So that $\sigma_{\overline{V}}$ can be estimated by,

$$s_* = \mu_o \; s_U \tag{24}$$

If the scatter in $\overline{V}$ is due only to statistical effects, then,

$$s_* \cong s_{\overline{V}} \tag{25}$$

Ordinarily one expects random experimental error so that,

$$s_* > s_{\overline{V}} \tag{26}$$

Example, Substituting the data for tube 14982, the first estimate becomes,

$$s_{\bar{V}} = \frac{\mu_o \quad \mu_S}{\sqrt{n}} = \frac{(1850)\ (1.9 \times 10^{-3})}{\sqrt{7}} = 1.33 \qquad (27)$$

and the second estimate becomes,

$$s_* = \mu_o\ s_U = (1850)\ (3.28201 \times 10^{-3}) = 6.07 \qquad (28)$$

Comparing,

$$s_* = 6.07 > s_{\bar{V}} = 1.33 \qquad (29)$$

Repeating the above procedure for tube 14742, the results are,

$$s_* = 4.93 > s_{\bar{V}} = 2.24 \qquad (30)$$

These results indicate that the scatter in V represented by s_* is greater than that expected from statistical effects alone.

5. The Product Model for Describing Experimental and Statistical Error:

A model used in probabilistic design to separate the statistical and experimental effects is proposed as follows. Let,

$$\bar{V} = \xi\,\eta \qquad (31)$$

where ξ is a random variable

$$\xi \sim (1, \frac{\sigma_V}{\mu_V \sqrt{n}}) \qquad (32)$$

which accounts for statistical scatter, and η is a random variable

(33)

$$\eta \sim (\mu_V, \sigma_\eta) \tag{33}$$

which accounts for experimental errors. η and ξ are assumed to be independent. The mean and variance of $\overline{V}$ are respectively,

$$E(\overline{V}) = \mu_{\overline{V}} = \mu_V \tag{34}$$

$$\sigma_{\overline{V}}^2 = \mu_\eta^2 \mu_\xi^2 \left(\frac{\sigma_\eta^2}{\mu_\eta^2} + \frac{\sigma_\xi^2}{\mu_\xi^2} + \frac{\sigma_\eta^2 \sigma_\xi^2}{\mu_\eta^2 \mu_\xi^2} \right) \tag{35}$$

Dividng by μ_η, μ_ξ

$$C_{\overline{V}}^2 = C_\eta^2 + C_\xi^2 + C_\eta^2 C_\xi^2 \tag{36}$$

where C is the coefficient of variation.

For C_η and C_ξ small,

$$C_\eta^2 C_\xi^2 << C_\eta^2 + C_\xi^2 \tag{37}$$

and

$$C_{\overline{V}}^2 = C_\eta^2 + C_\xi^2 \tag{38}$$

where,

$$C_{\overline{V}}^2 = \frac{\sigma_{\overline{V}}^2}{\mu_V^2} \tag{39}$$

and

$$C_\xi^2 = \frac{\sigma_V^2}{\eta} \tag{40}$$

Estimate

$$\frac{\sigma_V^2}{\mu_V^2 N} \text{ by } \frac{S_V^2}{\mu_V^2 N} = \frac{S_{\overline{V}}^2}{\mu_V^2}, \quad \sigma_{\overline{V}}^2 \text{ by } s_*^2$$

and μ_V by $\overline{V}$. Then solve equation 38 for $C\eta$, a measure of the contribution of experimental error to the scatter in $\overline{V}$.

$$C_\eta^2 = \frac{s_*^2}{\overline{V}^2} - \frac{s_{\overline{V}}^2}{\overline{V}^2} \tag{41}$$

Example, for tube 14982 $s_* = 6.07$, $s_{\overline{V}} = 1.33$ and $\overline{V} = 3852$.

Substituting in equation 41,

$$C_\eta^2 = \left(\frac{6.07}{3852}\right)^2 - \left(\frac{1.33}{3852}\right)^2 = 2.36 \text{ X } 10^{-6} \tag{42}$$

$$C_\eta = .00154 \tag{43}$$

and therefore,

$$\sigma_\eta = (.00154)\ (1852) = 2.85 \tag{44}$$

Repeating for tube 14742 where $s_* = 4.93$, $s_{\overline{V}} = 2.24$, and $\overline{V} = 1853$,

$$C_\eta = .00114 \tag{45}$$

and

$$\sigma_\eta = 2.11 \tag{46}$$

6. <u>Examination of Scatter in s_v</u> The scatter in s_v shown in Figure 6 was examined as follows: Assuming that the individual velocities, V_i are distributed normally with standard deviation σ. Then, at any N, and for any interval ΔN in which the distribution of V does not change,

$$s_v^2 = [1/(n-1)] \sum_{i=1}^{n} (V_i - \bar{V})^2 \tag{47}$$

has

$$(n-1)\, s_v^2 / \sigma^2 \sim \chi^2 (n-1) \tag{48}$$

$\chi^2 (n-1)$ is the chi-squared random variable with (n-1) degrees of freedom where n is the lot size. Consider the statistic,

$$Y_i = \frac{(n-1)\,(s_{v_i}^2)}{\sigma_i^2} \tag{49}$$

s_{v_i} is the sample standard deviation and σ_i is the estimate of σ_v corresponding to a given N. Note that in the linear regression analysis to determine μ_s it is assumed that s_v for a given N has a normal distribution. This assumption would not generally be considered valid, but here it is assumed that a reasonable approximation for σ_i is

$$\sigma_i = \mu_o \mu_S \tag{50}$$

If the variation is s_{v_i} is due to statistical scatter alone,

$$Y_i \sim \chi^2 (n-1) \quad (51)$$

with

$$\mu_Y = n-1 \quad (52)$$

and

$$\sigma_Y = \sqrt{2(n-1)} \quad (53)$$

Consider the estimate of (μ_Y, σ_Y) from the data actually observed.

$$\bar{Y} = \frac{1}{k} \sum_{i-1}^{k} Y_i \quad (54)$$

$$s_Y^2 = \frac{1}{k-1} \sum_{i=1}^{k} (Y_i - \bar{Y})^2 \quad (55)$$

Example, For the 155MM data, the sample size, n =7 and from eq 43.

$$\mu_Y = 6 \quad (56)$$

and

$$\sigma_Y = 3.46 \quad (57)$$

Consider the first sample for tube 14982, n=7, s_{v_i} = 2.7 fps; and σ_1 = 2.405 fps (S = 1.3 as taken from regression line of Figure 8 for N = 444).

$$Y_1 = \frac{(7-1)\ (2.7)^2}{(2.405)^2} = 7.53 \tag{58}$$

All other Y_i, i=1, k are calculated in the same manner. The mean of Y is

$$\overline{Y} = \frac{1}{k} \sum_{i=1}^{k} Y_i = 6.53 \tag{59}$$

where k = total number of samples taken = 35, and,

$$s_Y^2 = \frac{1}{k-1} \sum_{i=1}^{k} (Y_i - \overline{Y})^2 = 13.54 \tag{60}$$

The corresponding results for tube 14742 are $\overline{Y} = 7.12$ and $s_Y^2 = 24.70$. Therefore, if there are other than statistical factors influencing the scatter in ballistic data, then we expect $\overline{Y} >$ n-1 and $s_Y^2 > 2\ (n-1)$. Comparing for tube 14742, μ_Y, the expected mean if the variation in s_{v_i} is due to statistical scatter alone, to $\overline{Y}$, the mean estimated from the data actually observed: $\overline{Y} = 6.53$ is slightly higher than $\mu_Y = 6$. Similarly comparing the variances, $s_{\overline{Y}} = 3.68$ is higher than $\sigma_Y = 3.46$. For tube 14742, $\overline{Y} = 7.12 > \mu_Y = 6$ and $s_Y = 4.97 > \sigma Y = 3.46$.

C. 105MM

Data from the propellant acceptance test firings of nine separate gun tubes used on the 105MM, M68 gun was analyzed. When several sample functions, or realizations, of the wear process

		Tube No. 14742	Tube No. 14982
$s_{\overline{V}}$ Sample Standard Deviation in $\overline{V}$ due to Statistical Scatter		2.24	1.33
s_* Sample Standard Deviation in $\overline{V}$ actually Observed		4.93	6.07
$E(\chi^2)$	Statistical	6.00	6.00
	Actually Observed	7.12	6.53
$\sqrt{V(\chi^2)}$	Statistical	3.46	3.46
	Actually Observed	4.97	3.68
C_η, Coefficient of variation of η, a measure of the random experimental error as determined by scatter in $\overline{V}$.		.00114	.00154
C_ξ, Coefficient of variation of ξ, a measure of statistical scatter.		.000581	.000345

TABLE 3

A Summary of the Statistical Analysis for the 155MM Data are available, a statistical composite may be formulated using fundamentals of random process theory. Such a composite may be employed to make decisions regarding a class of gun tubes. U(N) and S(N) can be considered random processes where

$$U\ (N) = \frac{(1 - \bar{V}\ (N))\ 10^3}{\mu_o} \tag{61}$$

and

$$S\ (N) = \frac{(s_V\ (N))10^3}{\mu_o} \tag{62}$$

The record of each tube is a realization of the process, denoted as $U_i(N)$ and $S_i(N)$; all of the curves i = 1, j represent an ensemble. Figures 18 and 19 show U and S observed for j = 9 tubes of 105MM.

A complete description of any process Y(N) is the joint pdf of $Y(N_i)$ i=1,...k for any k. Here, however, we will consider only the first two moments of Y, i.e., $_Y$ (N) and $_Y$ (N).

From the ensembles of U and S shown in Figures 18 and 19, we can estimate U, U, S, S by the following

$$\bar{U}\ (N) = \frac{1}{j} \sum_{i=1}^{j} U_i\ (N) \tag{63}$$

$$s^2_{\bar{U}}\ (N) = \frac{1}{j-1} \sum_{i=1}^{j} [U_i\ (N) - \bar{U}\ (N)]^2 \tag{64}$$

and for S

$$\bar{S}\ (N) = \frac{1}{j} \sum_{i=1}^{j} S_i\ (N) \tag{65}$$

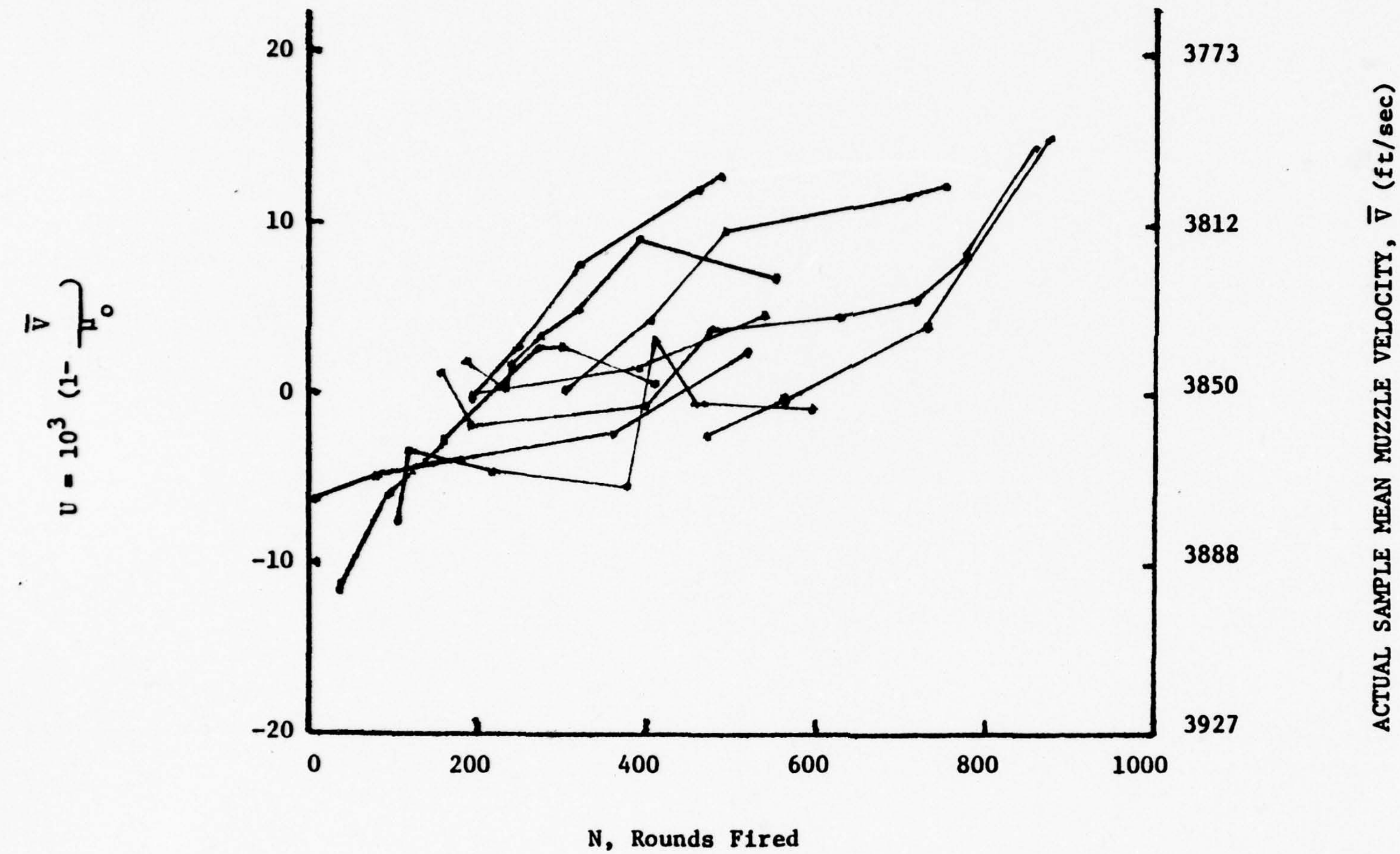


FIGURE 18

MEAN MUZZLE VELOCITY AS A FUNCTION OF ROUNDS

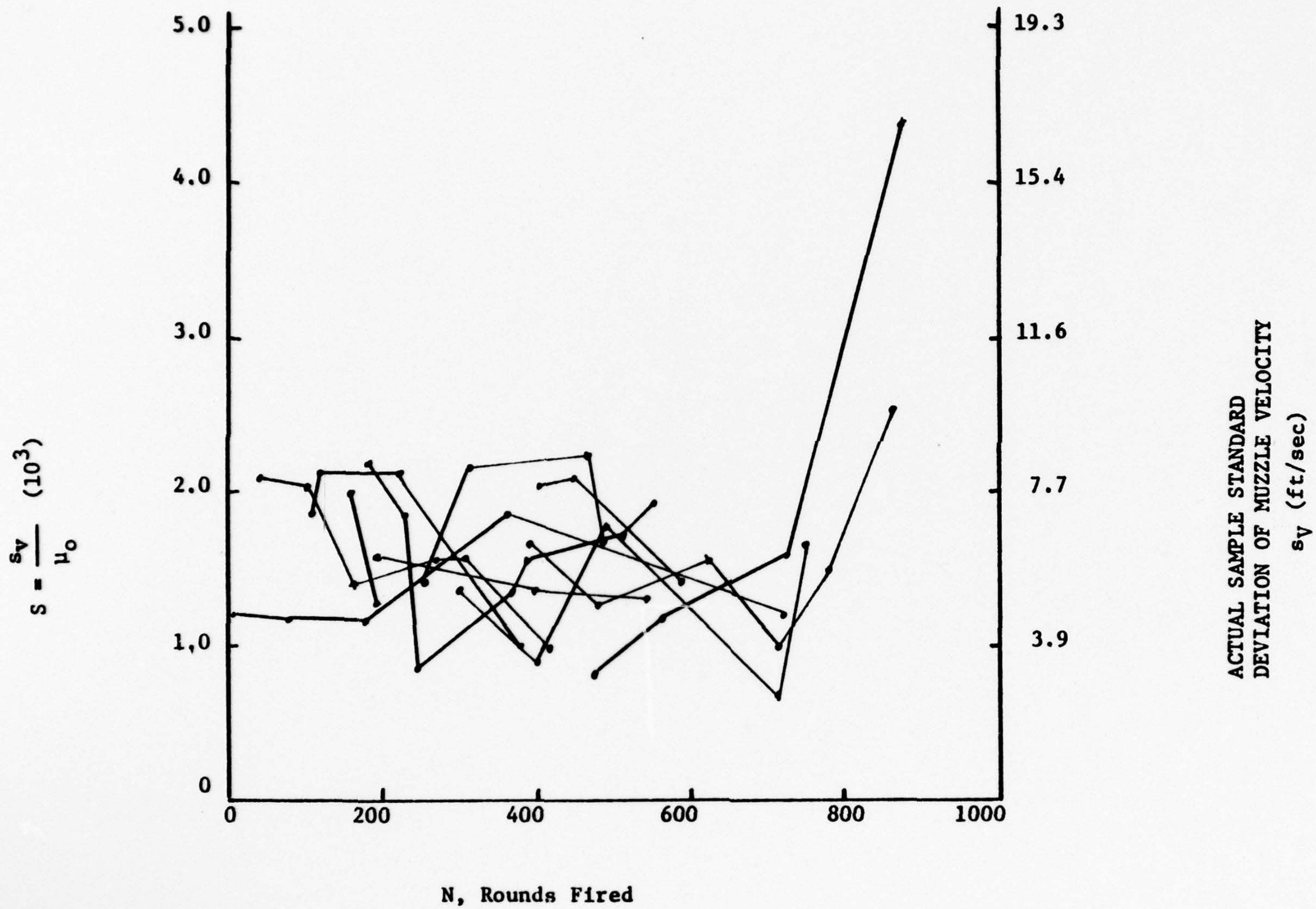


FIGURE 19

MUZZLE VELOCITY STANDARD DEVIATION AS A FUNCTION OF ROUNDS

$$s_S^2 (N) = \frac{1}{j-1} \sum_{i=1}^{j} [S_i (N) - \overline{S} (N)]^2 \qquad (66)$$

Curves of these estimates are shown in Figures 20 and 21.

A composite plot for all gun tubes was then made of the number of rounds fired versus wear measurements (see Figure 22). This curve indicates that in general, the wear rate is fairly constant with the rate at the extremes (new and old tube) slightly less than the middle. These results are opposite to what is expected as indicated by the dotted line on the Figure 22.

D. Criteria for Rejection and Replacement of a Gun Tube:

A reference calibration lot of ammunition is established to give a known velocity, μo , when fired from a new (life remaining = 100%) gun tube. As the tube wears (N increases) the difference between μ_o and V (the reference calibration mean muzzle velocity) is applied as a correction factor to V_T, the mean test round muzzle velocity This correction factor plays an important part in assessing the velocity capability of the propellant under test.

Therefore, it is the sample standard deviation s_V that is of prime concern in establishing gun tube rejection criteria because a small value of s_V indicates more confidence in having obtained the correct value of $\overline{V}$, and thereby an accurate correction factor.

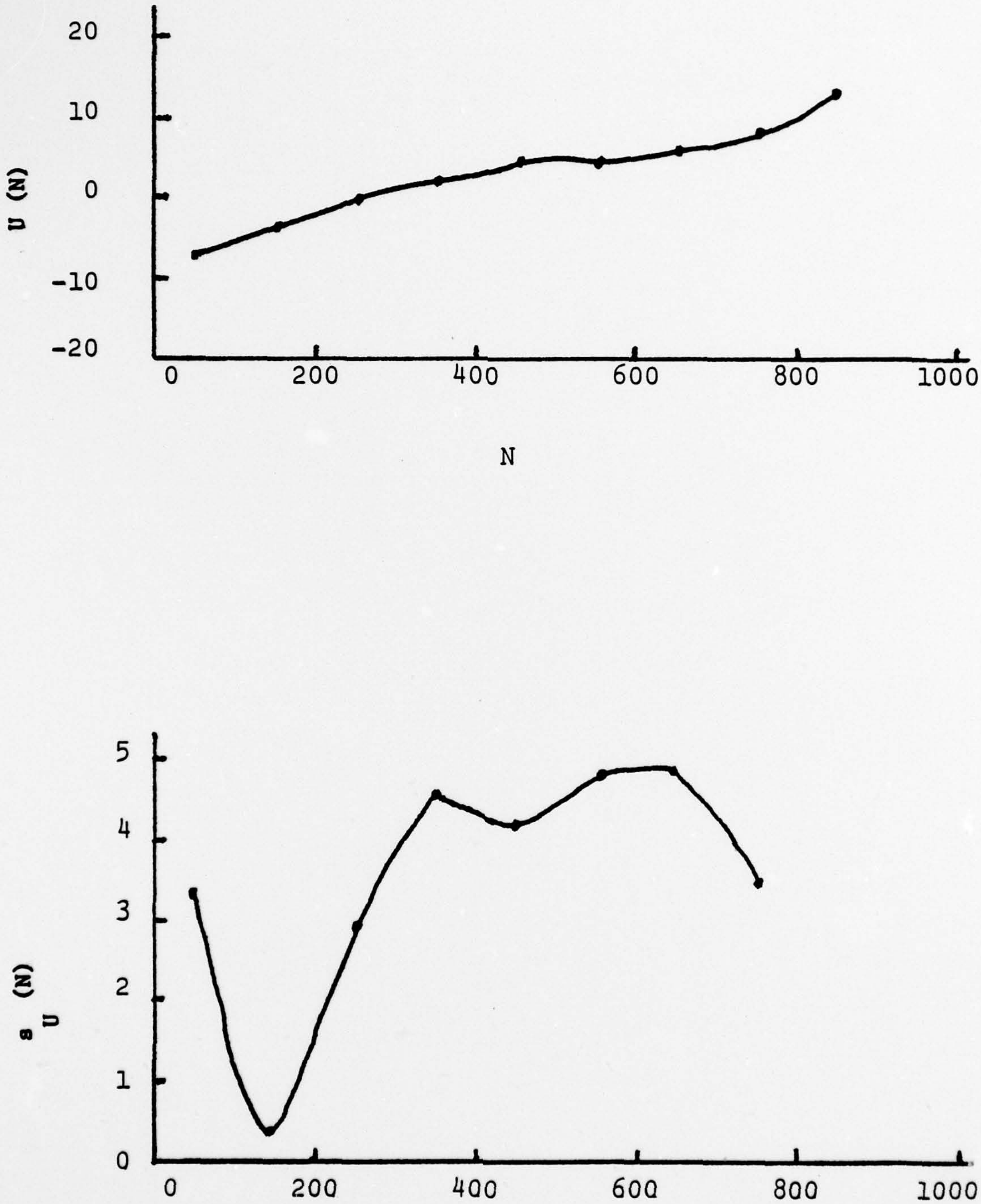


FIGURE 20

Ensemble of Curves for U

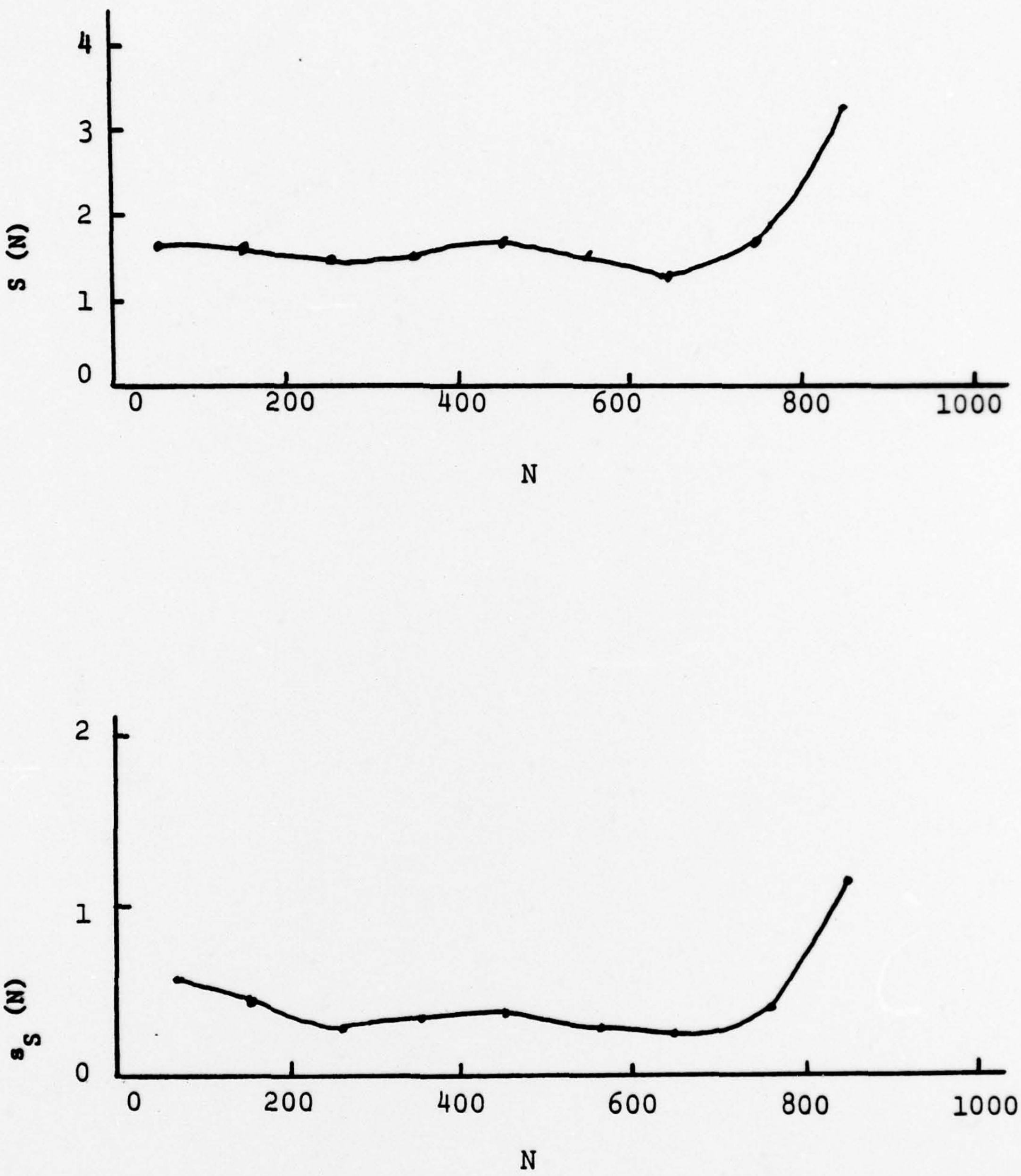


FIGURE 21
Ensemble of Curves for S

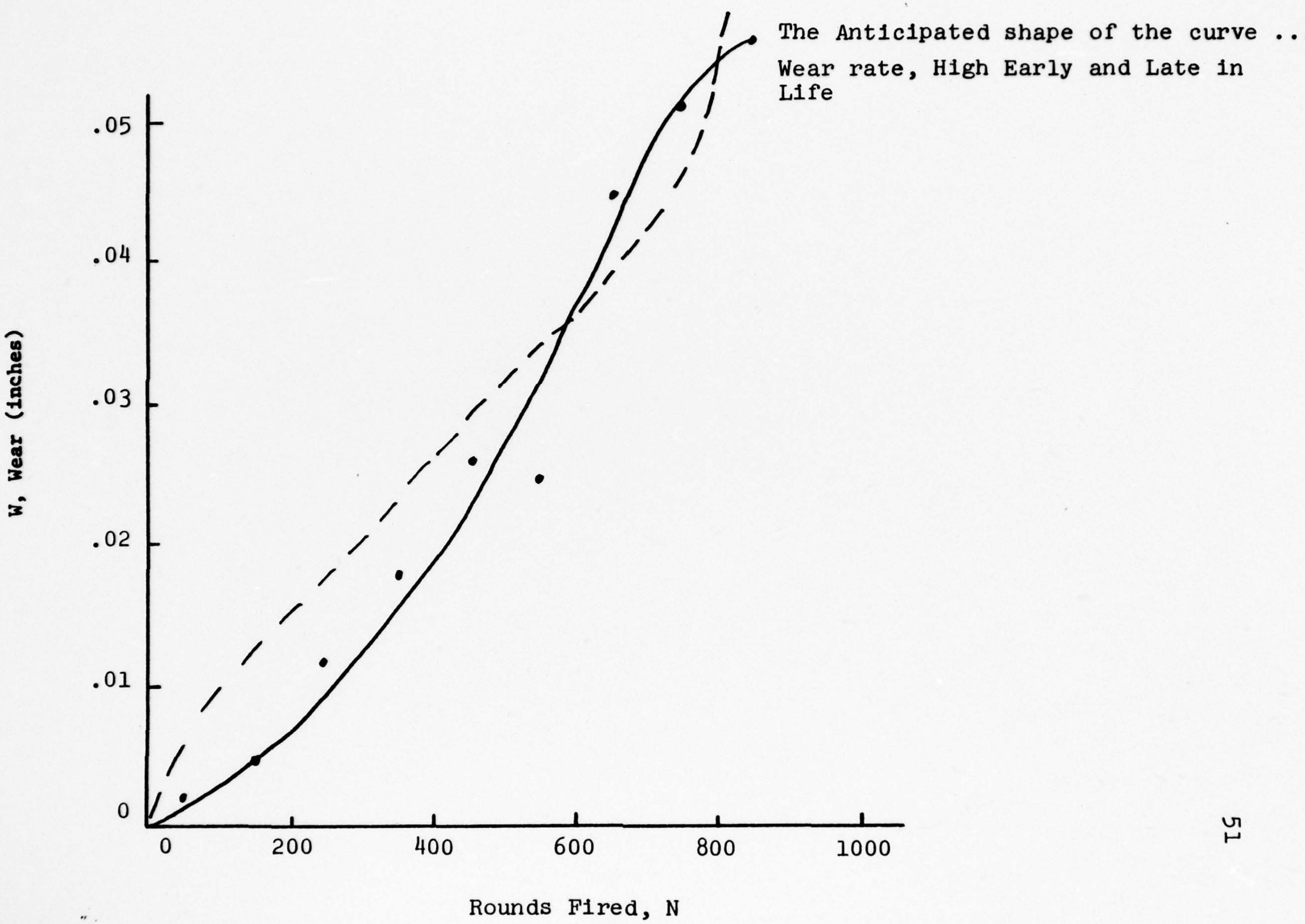


FIGURE 22
Average Wear Rate of 105MM

CHAPTER IV

Conclusions

Propellant specifications contain a muzzle velocity standard deviation requirement, σ_o, based on the type of ammunition fired.

For data analyzed herein

$$\sigma_o = 5.0 \text{ fps for 155MM and } 7.8 \text{ fps for 105MM} \qquad (67)$$

Now

$$S\ (N) = \frac{10^3 s_V}{\mu_o} \qquad (68)$$

using the values of

$$\mu_o = 1850 \text{ fps for 155MM and } 3850 \text{ fps for 105MM} \qquad (69)$$

It follows that

$$S\ (N) = 2.7 \text{ fps for 155MM and } 2.0 \text{ fps for 105MM} \qquad (70)$$

For the 105MM, the point at which S(N) = 2.0 (shown on Figure 21) cross the S(N) curve corresponding to N = 760 rounds. It is therefore recommended that, based on the data evaluated herein, a 105MM gun tube not be used for acceptance testing beyond N = 760 rounds.

Referring to Table 2 for the 155MM data, it can be seen that sample correlation coefficients are all low and in one case the hypothesis that W and Y are correlated is rejected. A possible explanation for this may be by reviewing the results presented in Table 3. For both tubes, the observed sample standard deviation, s_* was much higher than the sample standard deviation due to statistical influence, $s_{\bar{v}}$.

The values of the mean and standard deviation of χ^{*2} observed were also higher in all cases than the statistical. In addition, the coefficient of variation of η, C_η, is large in relation to the coefficient of variation of ξ indicating that a large part of the standard deviation observed is due to random experimental error of the kinds outlined in Section II E of this document.

Another indication of the large amount of experimental error involved is shown in Figure 11. The curve of the standard deviation of tube 14742 has a slight negative slope. The only reasonable explanation for this is that the contribution of the random experimental error masks the true relationship.

In general, it appears that the experimental errors associated with ballistic testing have a large effect on the data obtained as witnessed by the 155MM data. Since the 105MM data is tested in a similar manner but has a much higher muzzle velocity, an even more pronounced effect is expected.

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APPENDIX

Computer Printouts

SAMPLE SIZE = 35

X(I)	Y(I)
.6710E+03	0.
.7790E+03	-.3230E+01
.7790E+03	-.5400E+00
.5040E+03	0.
.4440E+03	.4320E+01
.2226E+04	-.1620E+01
.1926E+04	-.4870E+01
.1965E+04	-.5410E+01
.2679E+04	-.2700E+01
.1477E+04	-.3230E+01
.1668E+04	-.1080E+01
.1668E+04	-.4320E+01
.1400E+04	.1620E+01
.1293E+04	-.7570E+01
.1068E+04	-.2700E+01
.1019E+04	-.3780E+01
.4687E+04	.8650E+01
.4520E+04	.3240E+01
.4276E+04	.5400E+00
.4337E+04	-.1080E+01
.4031E+04	0.
.3849E+04	.2700E+01
.3751E+04	-.1620E+01
.3691E+04	.5410E+01
.4092E+04	.5950E+01
.3503E+04	-.3780E+01
.3321E+04	-.3230E+01
.3276E+04	.2700E+01
.3169E+04	-.2160E+01
.3108E+04	-.2700E+01
.3062E+04	-.3230E+01
.2849E+04	-.2700E+01
.2450E+04	-.3230E+01
.2343E+04	-.3780E+01
.2849E+04	-.2700E+01

```
SUM Y -.36130E+02   SUM YSQ  .45912E+03   SUM X  .88630E+05   SUM XSQ  .28082E+09

MEAN X  .25323E+04   MEAN Y -.10323E+01   SUM XY -.30321E+05
ARGUMENT NEGATIVE
FRROR NUMBER   39   DETECTED BY SQRT


ALPHA =  -3.77947

BETA =     .00108

SIGMA =   3.28201


SIGMA A =   1.24170

SIGMA B=          I

SIGMA X =1287.78297

CORRELATION COEFFICIENT =     .39664
```

X(I)	Y(I)	YEX
671.00000	0.00000	-3.05152
779.00000	-3.23000	-2.93436
779.00000	-.54000	-2.93436
504.00000	0.00000	-3.23269
444.00000	4.32000	-3.29779
2226.00000	-1.62000	-1.36456
1926.00000	-4.87000	-1.69002
1965.00000	-5.41000	-1.75620
2679.00000	-2.70000	-.87312
1477.00000	-3.23000	-2.17712
1668.00000	-1.08000	-1.96992
1668.00000	-4.32000	-1.96992
1400.00000	1.62000	-2.26066
1293.00000	-7.57000	-2.37674
1068.00000	-2.70000	-2.62083
1019.00000	-3.78000	-2.67399
4687.00000	8.65000	1.30528
4520.00000	3.24000	1.12411
4276.00000	.54000	.85940
4337.00000	-1.08000	.92558
4031.00000	0.00000	.59361
3849.00000	2.70000	.39617
3751.00000	-1.62000	.28985
3691.00000	5.41000	.22476
4092.00000	5.95000	.65979
3503.00000	-3.78000	.02080
3321.00000	-3.23000	-.17664
3276.00000	2.70000	-.22546
3169.00000	-2.16000	-.34154
3108.00000	-2.70000	-.40772
3062.00000	-3.23000	-.45762
2849.00000	-2.70000	-.68869
2450.00000	-3.23000	-1.12155
2343.00000	-3.78000	-1.23763
2849.00000	-2.70000	-.68869

SAMPLE SIZE = 35

X(I)	Y(I)
.6710E+03	.1620E+01
.7790E+03	.9200E+00
.7790E+03	.1240E+01
.5040E+03	.1890E+01
.4440E+03	.1470E+01
.2226E+04	.2270E+01
.1926E+04	.2040E+01
.1865E+04	.1340E+01
.2679E+04	.1080E+01
.1477E+04	.1780E+01
.1668E+04	.1190E+01
.1668E+04	.1670E+01
.1400E+04	.1250E+01
.1293E+04	.4800E+00
.1068E+04	.2050E+01
.1019E+04	.8600E+00
.4687E+04	.2240E+01
.4520E+04	.1950E+01
.4276E+04	.2490E+01
.4337E+04	.1460E+01
.4031E+04	.2160E+01
.3849E+04	.1410E+01
.3751E+04	.2970E+01
.3691E+04	.1200E+01
.4092E+04	.1140E+01
.3503E+04	.1290E+01
.3321E+04	.1400E+01
.3276E+04	.1840E+01
.3169E+04	.1730E+01
.3108E+04	.1240E+01
.3062E+04	.1990E+01
.2849E+04	.1830E+01
.2450E+04	.1510E+01
.2343E+04	.1400E+01
.2849E+04	.1560E+01

```
SUM Y  .55960E+02   SUM YSQ  .98015E+02   SUM X  .88630E+05   SUM XSQ  .28082E+09

MEAN X  .25323E+04   MEAN Y  .15989E+01   SUM XY  .14951E+06

ARGUMENT NEGATIVE
ERROR NUMBER    39    DETECTED BY SQRT

ALPHA =    1.24842

RETA =     .00014

SIGMA =     .47556

SIGMA A =     .17992

SIGMA B=          I

SIGMA X =1287.78297

CORRELATION COEFFICIENT =     .35552
```

X(I)	Y(I)	YEX
671.00000	1.62000	1.34128
779.00000	.92000	1.35623
779.00000	1.24000	1.35623
504.00000	1.89000	1.31817
444.00000	1.47000	1.30987
2226.00000	2.27000	1.55647
1926.00000	2.04000	1.51496
1865.00000	1.34000	1.50651
2679.00000	1.08000	1.61916
1477.00000	1.78000	1.45282
1668.00000	1.19000	1.47925
1668.00000	1.67000	1.47925
1400.00000	1.25000	1.44216
1293.00000	.48000	1.42736
1068.00000	2.05000	1.39622
1019.00000	.86000	1.38944
4687.00000	2.24000	1.89704
4520.00000	1.95000	1.87393
4276.00000	2.49000	1.84016
4337.00000	1.46000	1.84860
4031.00000	2.16000	1.80626
3849.00000	1.41000	1.78107
3751.00000	2.97000	1.76751
3691.00000	1.20000	1.75921
4092.00000	1.14000	1.81470
3503.00000	1.29000	1.73319
3321.00000	1.40000	1.70800
3276.00000	1.84000	1.70178
3169.00000	1.73000	1.68697
3108.00000	1.24000	1.67853
3062.00000	1.99000	1.67216
2849.00000	1.83000	1.64269
2450.00000	1.51000	1.58747
2343.00000	1.40000	1.57266
2849.00000	1.56000	1.64269