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TIME-SERIES MODELING OF RAINFALL DENSITY INFORMATION.(U)
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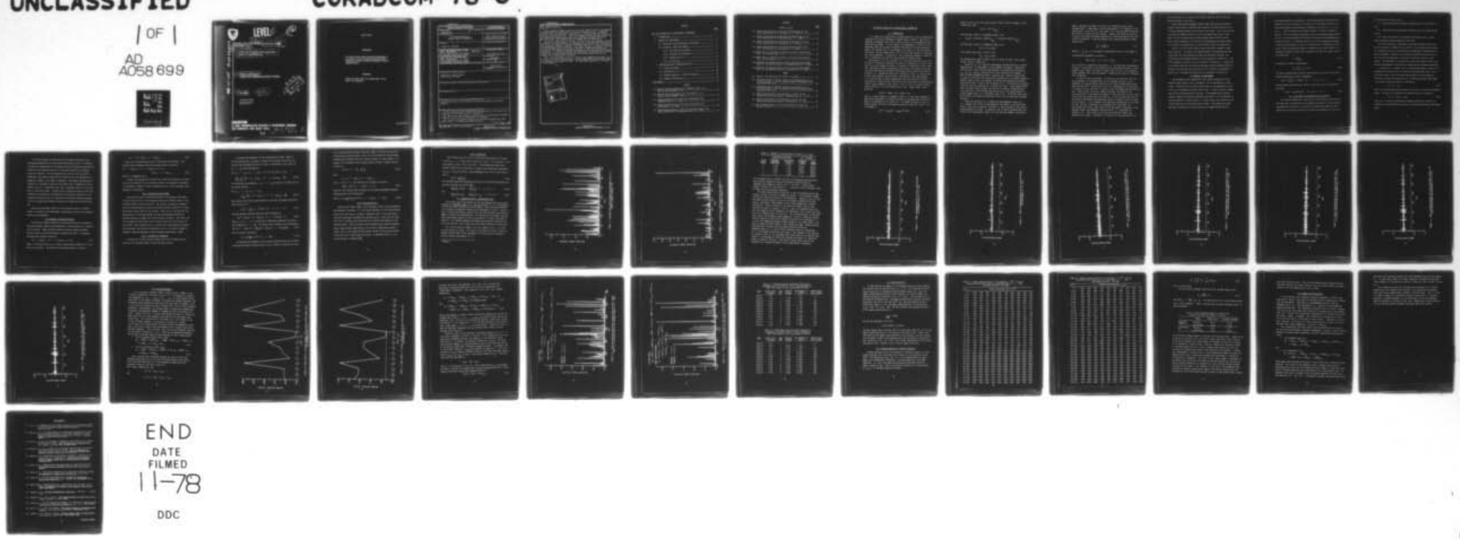

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TIME-SERIES MODELING OF RAINFALL DENSITY INFORMATION.

Richard J. D'Accardi
CENTER FOR COMMUNICATIONS SYSTEMS

July [redacted]78

38 p.

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In the literature there are basically two approaches to the prediction of climatological occurrence (rainfall) statistics:

(a) One can use large numbers of observations of climatological phenomena to compile statistics on the probability of occurrence.

(b) One can use computational models and available climatological data (past history) to calculate occurrence statistics.

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20. ABSTRACT (cont'd)

Recent modeling attempts start with very small "point" rainfall distribution functions which are transformed into specific attenuation data. The "point" rainfall rates are based upon a large history of data which does not indicate the spatial variation of rain. There are, however, indications that outage time on line-of-sight communication links can be estimated from distributions of "point" rainfall rates, ~~derived from U.S. Weather Service information~~.

Due to the random nature of rainfall, and due to the time dependence of such information, a logical approach to forecasting this phenomenon and interpreting the results with respect to systems performance seems to lie within the realm of non-stationary time-series modeling. This report presents an attempt to develop statistical models which can be used to forecast in near-real-time and to characterize the underlying stochastic processes of short-term rainfall density information. It should be mentioned that the analysis and modeling are directed towards the accurate characterization of precipitation density and not the probability of precipitation occurrence.

Due to the wavelengths involved in Line-of-Sight Communication links, i.e., 5-30 GHz, the size of raindrops have a definite dispersive and absorptive effect on propagated electromagnetic energy. The importance, therefore, of this work is self-evident.

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TIME-SERIES MODELING OF CLIMATOLOGICAL INFORMATION

1. INTRODUCTION

It is known that communications systems operating in the millimeter-wave range may be critically dependent on meteorological factors. Specifically, atmospheric phenomena such as hard, steady rainfall, and thunderstorms may cause serious outages. In this chapter, the phenomenon of rainfall will be addressed, where the accuracy of weather forecasts will not be considered. Simply, the interest here is not whether or not there will be precipitation, but rather, from the communicators' point of view, what density of precipitation can be expected if it does rain. Such information becomes an important tool in determining the need for back-up modes of communications.

Work done by Lin, [1], and Chen, [2], indicate techniques that allow the basis of long-term information for empirical conversion of rainfall rate x(t) for a given integration time T on the basis of 60-minute rainfall data. Their technique provides a procedure of obtaining a fairly small (5-minute) rain-rate distribution from NOAA climatological data on short-time rates of rainfall (i.e., thunderstorm activity). Obviously, there is a great stress on the "short-interval" point-rainfall distribution. Rice and Holmberg, [3], have provided a widely accepted model utilizing the point-rainfall distribution in obtaining the "fraction of time during which t minute average rainfall rates exceed any given value." This, in turn, is used to project adverse effects of rain on microwave radio links. Stated simply, they postulate:

$$\text{rainfall} = \text{mode 1 rain} + \text{mode 2 rain} \quad .$$

That is, rainfall density is composed of mode 1, an individual exponential mode which corresponds to a physical analysis of thunderstorms, and that of mode 2, corresponding to all other rain. The number of hours of rainy "t minute periods" for which an average surface-point (station) rainfall rate is exceeded is:

$$T_t(R) = T_{1t}q_{1t}(R) + T_{2t}q_{2t}(R) \text{ hours} \quad , \tag{1.1}$$

where T_{1t} and T_{2t} are the *average annual* rainfall totals for modes 1 and 2, respectively, and

$$q_{1t}(R) = \exp(-R/\overline{R}_{1t}),$$

the time that a rate R is exceeded by mode 1 rain,

$$q_{2t}(R) = 0.35 \exp(-.453074\, R/\overline{R}_{2t}) + 0.65 \exp(-2.857143\, R/\overline{R}_{2t}),$$

the time that a rate R is exceeded by mode 2 rain,

$$\overline{R}_{1t} = M_1/T_{1t} \text{ mm/hr},$$

the average annual mode 1 rainfall rate,

$$\overline{R}_{2t} = M_2/T_{1t} \text{ mm/hr},$$

the average annual mode 2 rainfall rate; M_1 and M_2 are mode 1 and 2 annual rainfall totals, respectively.

Dutton et al, [4], used this methodology with modifications to predict communications link performance in the European environment. Among their modifications, a more sophisticated approach in estimating the parameters, T_{1t}, T_{2t}, R_{1t}, and R_{2t}, using multiple linear regression was proposed. In both of these aforementioned works, extensive use is made of contour maps of annual precipitation with appropriate interpolation to estimate parameters. Temporal (year to year) and seasonal variations were assessed from monthly precipitation data. Similar work by Bartow, [5], is also based on long-term statistics and the assumption of a constant rain rate. These aforementioned works are widely used to predict the effects of weather on communications links operating in the 8-30 MHz band and basically yield approximations of sizable outage times (i.e., hours/year) due to rain. Sufficient data is not yet available to validate the prediction error [4] for specified confidence intervals.

Since the available data is random and time dependent, Jones, [6], [7], suggests the fitting of autoregressive (AR) models to the given time series. The critical question, of course, is identification of the order of the autoregressive process. In addition, one is always concerned about the choice of the autoregressive model over the other two commonly used models, namely, the moving averages, and mixture of autoregressive and moving averages models. Jones seems to ignore these models in his research of the subject area. He implies in a later paper, [8], that the modified Akaike's FPE criteria is useful for this purpose. However, one cannot be certain as to which of the three difference equations will best characterize the given rainfall information. Using the asymptotically unbiased estimate of mean-square error:

$$\hat{\sigma}_p^2 = \frac{S_p}{n - p - 1} \tag{1.2}$$

where $S_p = \sum_{t=1}^{n} e_t^2$, n is the number of observations, and p is the number of autoregressive parameters, he obtains:

$$FPE_p = \hat{\sigma}_p^2 \{1 + (1 + p/n)\} \sim \frac{1}{n} \chi_p^2 \quad . \tag{1.3}$$

This criterion is computed for each order, p. The order that corresponds to $\min(FPE_p)$ is equivalent to the minimum variance and autocorrelation criteria commonly used. Again, no mention is made of the utility of the integrated moving averages (IMA), and mixed models in modeling this type of climatological information.

In view of these shortcomings, it should also be noted that the time-series approach is considered here, not only because the data is time dependent, but because weather observations are usually made some distance away from the area of use and may not be fully representative of conditions at the point of use, and there is usually a time lag between observations and use. These factors, along with random occurrence, contribute to the *non-stationarity* of the underlying process in that the values of the climatological data may change during the time lag. If the non-stationarities are not properly approached, then meaningful results of analysis would be impossible to obtain. In this chapter, the time-series methodology will be shown for daily rainfall information for Greenwood Lakes, New Jersey, and Long Branch, New Jersey, taken from the 1974 New Jersey Climatological Data, National Oceanic and Atmospheric Administration, U. S. Department of Commerce, [9]. These data are the station accumulations taken from recording rain gauges. Inference will be made as to the applicability of shorter-term data, i.e., hourly, and fractions thereof, should such data become available in the future.

There exists widely acceptable models, [3], which utilize the point-rainfall distribution in obtaining the "fraction" of time during which the "t" minute average rainfall rates exceed a given value. This in turn is used to project the adverse effects of rain on microwave radio links. These models utilize long-term data including average annual rainfall totals. Thus, another advantage of the time-series methodology now becomes evident as the models so obtained rely on a much shorter history of observations.

Section 2 of this report briefly covers some basic concepts in time-series analysis. In section 3 the identification of the appropriate time-series model will be presented including the necessary filtering. The fitting and one-step-ahead forecasts will be presented in section 3.1. The "fit" obtained in section 3.1 will be checked in section 3.2. A comparison of the model order and classification obtained using the method proposed in section 2 and that obtained using Akaike's FPE criterion is presented in section 3.3. A summary and conclusions are presented in section 3.4.

2. CONCEPTS IN TIME-SERIES

Any phenomenon which changes with time, and any collection of data measuring some aspect of such a phenomenon, can be considered as a time-series. Time-series can be either deterministic or non-deterministic functions of an independent variable, usually time. In most instances, however, they will be non-deterministic functions. A non-deterministic function exhibits random or fluctuating properties and, hence, it is not possible to exactly forecast its future values; or in other words, such time-series can be described only by statistical laws or models. We assume that we may describe a time-series at a given time t by a random variable and its associated probability distribution. Thus we may describe the behavior of a time-series at all instances by an ordered set of random variables and the associated probability distributions, denoted by $\{X_t\}$ and f_{Xt} , $t = 0, \pm 1, \pm 2, \cdots$. Such an ordered set of random variables is called a stochastic process. Thus, an observed time-series x_t can be considered as one realization of an infinite ensemble of functions which may have been generated by a stochastic process. A stochastic process is said to be strictly stationary if the joint distribution of any set of observations is unaffected by shifting all times of observations ahead or backward by any integer amount k. A stationary stochastic process may be described in terms of its mean μ which is estimated by:

$$x = \frac{1}{n} \sum_{t=1}^{n} x_t \, , \tag{2.1}$$

its variance σ^2 which is estimated by:

$$s_x^2 = \frac{1}{n} \sum_{t=1}^{n} (x_t - \bar{x})^2. \tag{2.2}$$

its *sample autocovariance* function, which measures the extent to which two random variables are linearly independent:

$$c_{xx}(k) = \frac{1}{n} \sum_{t=1}^{n-k} (x_t - \bar{x})(x_{t+k} - \bar{x}) \quad k = 0, 1, \cdots, n - 1, \tag{2.3}$$

and the sample autocorrelation function, which acts like a correlation coefficient:

$$r_{xx}(k) = c_{xx}(k)/c_{xx}(0) \quad k = 0, 1, \cdots, n - 1. \tag{2.4}$$

2.1. *Stationary and Non-Stationary Time Series*

A stationary time-series is one which is in statistical equilibrium in the sense that its properties do not change with respect to time, whereas a non-stationary time-series is such that its properties change with time. Time-series occurring in practice are usually *non-stationary* in nature and can be divided into three classes:

(i) Those which exhibit stationary properties over a long period of time.

(ii) Those which are approximately stationary over very short periods of time.

(iii) Those which exhibit non-stationary properties, that is, their visual properties change continuously with time.

At present there exist techniques to analyze stationary time-series, but the techniques available for the analysis of non-stationary time-series are inadequate and do not lend themselves to meaningful interpretations of physical problems. However, one can adjust non-stationary time-series so that the existing techniques of stationary time-series analysis can be applied. The adjustment is accomplished by applying a proper filter to the observed non-stationary time-series to remove the non-stationary components.

The selection of a proper filter is accomplished through a search for a mathematical function which will transform a non-stationary series into a stationary series. One of the most used and most efficient methods of filtering is through the application of a difference equation, [13], [14]. A first-order difference equation is defined by:

$$y_t = x_t - x_{t-1}, \tag{2.5}$$

where x_t is the observed non-stationary series and y_t is the first-difference series. Similarly, a second-order difference equation is defined by:

$$w_t = x_t - 2x_{t-1} + x_{t-2}, \tag{2.6}$$

and so on. A first or second-order difference equation will usually be sufficient to transform most practically occurring non-stationary time-series, [13].

To identify whether the observed series exhibits stationary or non-stationary properties one can use certain data-analysis tools. In addition to graphical representation of the observed series the sample autocorrelation function of the observed series and a trend test applied to the observed series are important. For the observed series and its first and second differences, the sample autocorrelation functions (2.4) are computed and a trend test, Kendall's tau, [11], is performed. (The sample autocorrelation function of a stationary series has the property that is dampens out fairly rapidly, that is, it approaches zero; also, a stationary series will be such that it contains no trend). Following this procedure one obtains sufficient information to determine if the observed series exhibits stationary or non-stationary properties; and if it exhibits non-stationary properties whether a first or second-order difference equation will remove the non-stationarities.

Once we have obtained a model for the stationary series, a "backward filter" is applied to the fitted model so that future values of the observed series can be forecasted.

2.2 Parametric Time-Series Models

To be able to forecast values for an observed series we fit parametric time-series models, either an autoregressive, a moving average, or a combination of the two. These stationary stochastic models assume the process (series) remains in equilibrium about a constant mean level. The general autoregressive process is given by:

$$X_t - \mu = \alpha_1(X_{t-1} - \mu) + \cdots + \alpha_m(X_{t-m} - \mu) + Z_t, \qquad (2.7)$$

where μ is the mean of X_t, Z_t is a purely random process, [14], and m is the order of the process. The general moving average process is given by:

$$X_t - \mu = Z_t - \beta_1 Z_{t-1} - \cdots - \beta_q Z_{t-q}; \quad (2.8)$$

μ and Z_t are as defined above, and q is the order of the process. The general mixed autoregressive-moving average process is given by:

$$X_t - \mu = \alpha_1(X_{t-1} - \mu) + \cdots + \alpha_m(X_{t-m} - \mu) + Z_t$$
$$- \beta_1 Z_{t-1} - \cdots - \beta_q Z_{t-q}, \quad (2.9)$$

where q is independent of m.

We shall now consider the criterion for selecting the process, its order (which gives the best fit to an observed series), the procedure to estimate its parameters, diagnostic check of goodness-of-fit, and how the model can be employed in forecasting.

2.2.1 Selecting the Best Model

The criterion used for selecting the order of the process (that which will give the best fit) is the residual variance for different orders of the parametric models fitted to the data. The residual variances are computed and are plotted against the order; the minimum residual variance will correspond to the correct order for the process. After this has been done for the autoregressive, the moving average, and the mixed autoregressive-moving average processes we compare the minimum residual variances. The minimal one will correspond to the process (and its order) which will give the best fit to the data. When fitting a model to a given set of observations one should always consider the principle of parsimony, that is, the smallest number of parameters should be employed to obtain adequate representation, [13].

2.2.2 Estimation of Parameters

To obtain the residual variances above, one first estimates the parameters for the different orders of each individual process.

To estimate the parameters for the autoregressive process, [14], we first assume that the Z_t process is normal with zero mean and variance σ_Z^2. Then the log-likelihood function for fixed m, conditional on the values x_1, $x_2, \cdots, x_m$ can be expressed as

$$l(\mu, \alpha_1, \cdots, \alpha_m \mid x_1, \cdots, x_m) = -(n-m)(\ln\sqrt{(2\pi)} + \ln\sigma_Z)$$

$$-\frac{1}{2\sigma_Z^2}\sum_{t=m+1}^{n}\left[(x_t-\mu) - \alpha_1(x_{t-1}-\mu) - \cdots - \alpha_m(x_{t-m}-\mu)\right]^2. \qquad (2.10)$$

For estimating the parameters $\mu, \alpha_1, \cdots, \alpha_m$, we need only consider the sum of squares function

$$S(\mu, \alpha_1, \cdots, \alpha_m \mid x_1, \cdots, x_m)$$

$$= \sum_{t=m+1}^{n}\left[(x_t-\mu) - \alpha_1(x_{t-1}-\mu) - \cdots - \alpha_m(x_{t-m}-\mu)\right]^2. \qquad (2.11)$$

Now assuming that $\hat{\mu}$ may be approximated by $\bar{x}$ and that the sample autocovariance function (2.3)

$$c_{xx}(j) \simeq \sum_{t=m+1}^{n}(x_t-\hat{\mu})(x_{t-j}-\hat{\mu}) \qquad j = 1, \cdots, m, \qquad (2.12)$$

then the maximum likelihood equations may be expressed as

$$c_{xx}(j) = \hat{\alpha}_1 c_{xx}(j-1) + \hat{\alpha}_2 c_{xx}(j-2) + \cdots + \hat{\alpha}_m c_{xx}(j-m), \qquad (2.13)$$

where $j = 1, 2, \cdots, m$. Solving the m simultaneous equations one obtains the estimates $\hat{\alpha}_1, \cdots, \hat{\alpha}_m$. The residual sum of squares may be expressed as

$$S(\hat{\mu}, \hat{\alpha}_1, \cdots, \hat{\alpha}_m) \simeq (n-m)\left[c_{xx}(0) - \hat{\alpha}_1 c_{xx}(1) - \cdots - \hat{\alpha}_m c_{xx}(m)\right], \qquad (2.14)$$

and the residual variance by

$$s_Z^2 = \frac{1}{n-2m-1}\, S(\hat{\mu}, \hat{\alpha}_1, \cdots, \hat{\alpha}_m). \qquad (2.15)$$

To estimate the parameters for the moving average process and the mixed autoregressive-moving average process, we use a numerical technique to build up the log-likelihood function recursively, [14]. By varying the values of the parameters (usually between -1 and +1), we can search for the parameter estimates which minimize the sum of squares function for each process. For example, for the general moving average process the sum of squares function is given by:

$$S(\hat{\mu}, \hat{\beta}_1, \cdots, \hat{\beta}_q) = \sum_{t=q}^{n} z_t^2 , \tag{2.16}$$

where

$$z_t = x_t - \mu + \beta_1 z_{t-1} + \cdots + \beta_q z_{t-q} ,$$

and $z_t = 0$ for $t < q$. The residual sum of squares is given by

$$S_z^2(q) = S(\hat{\mu}, \hat{\beta}_1, \cdots, \hat{\beta}_q)/(n - q - 1). \tag{2.17}$$

Similarly, the residual sum of squares for the mixed autoregressive-moving average process can be expressed as

$$s_z^2(m, q) = \frac{1}{n - 2m - q - 1} S(\hat{\mu}, \hat{\alpha}_1, \cdots, \hat{\alpha}_m, \hat{\beta}_1, \cdots, \hat{\beta}_q). \tag{2.18}$$

2.2.3 Checking the Fit

Once we have fitted a model to the stationary series, we must determine the adequacy of the model. If it was found necessary to filter the observed series, the first step is to apply a "backward filter" of the same form so that the fitted model represents the observed series. Thus, with the "backward filter" inserted, the fitted model will simulate the behavior of the observed series. Then the residuals—the observed series minus the modeled series—should behave approximately like a purely random process (white noise), that is, the sample autocorrelation function (2.3) should be effectively zero for all lags except the zeroth. To determine the fit, a test for white noise is applied, [14].

2.2.4 Forecasting

After checking the fit, we can use the resulting equation to forecast a value $x_{t+\ell}$, $\ell \geq 1$, when we are currently at time t. This forecast is said to be made at origin t for a lead time ℓ. The minimum mean square error forecast, [13], for any lead time ℓ is given by the conditional expectation $E_t\ x_{t+\ell}$, of $x_{t+\ell}$ at origin t, given knowledge of all the x's up to time t; that is:

$$\hat{x}_t(\ell) = E_t\left[x_{t+1}\right]. \tag{2.19}$$

The required conditional expectation occurring in the forecasting models can be found using Box and Jenkins [13]:

$$E_t\left[x_{t+j}\right] = \hat{x}_t(j), \quad E_t\left[z_{t+j}\right] = 0 \quad j = 1, 2, \cdots \tag{2.20}$$

and

$$E_t\left[x_{t-j}\right] = x_{t-j}, \quad E_t\left[z_{t-j}\right] = z_{t-j} \quad j = 0, 1, 2, \cdots \tag{2.21}$$

3. IDENTIFICATION OF CLIMATOLOGICAL DATA

The initial step in the analyses of the two observed time series (daily rainfall for Greenwood Lakes, N. J., and Long Branch, N. J.) is to determine if they are either stationary or non-stationary. Both series were plotted in an attempt to graphically detect any non-randomness or trend. Figures 3.1 and 3.2 display the 365-day rainfall totals for both stations, and certainly appear to exhibit non-stationarities. The inference of the graphic displays were statistically tested for trend using Kendall's Tau test as detailed in section 2. For further evidence, the sample autocorrelation functions were also calculated for first, second, and third order difference filtered data.

The critical value for Kendall's Tau test, [10]* at the α = .05 level of significance is $\pm$ 1.645. The results of the tests are given in table 3.1 showing a higher order filter requirement for the series for the lesser rainfall density (Long Branch, N. J.).

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* Chapter 5

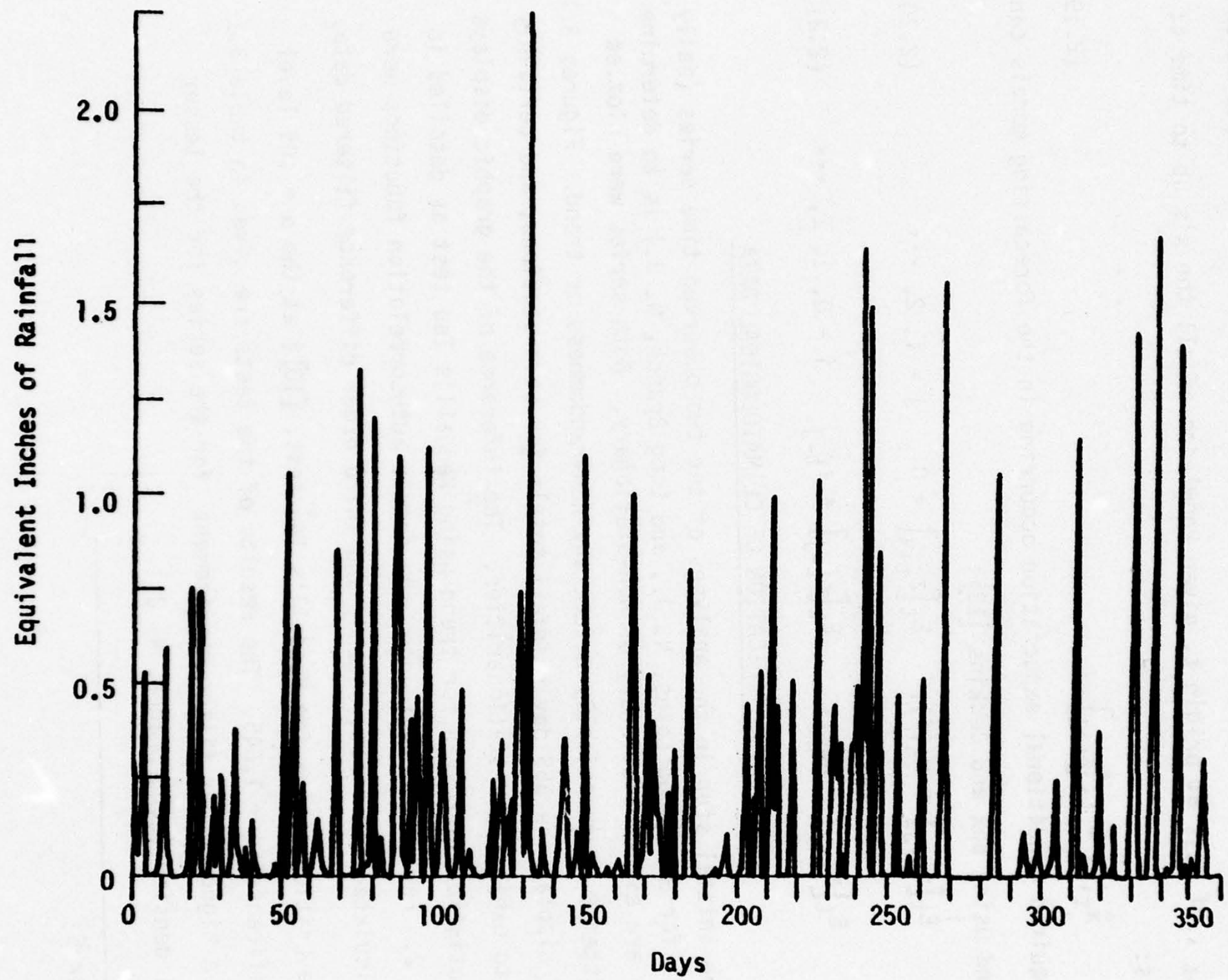

FIGURE 3.1 MEASURED DAILY PRECIPITATION FOR GREENWOOD LAKES, N. J. JANUARY THROUGH DECEMBER, 1974

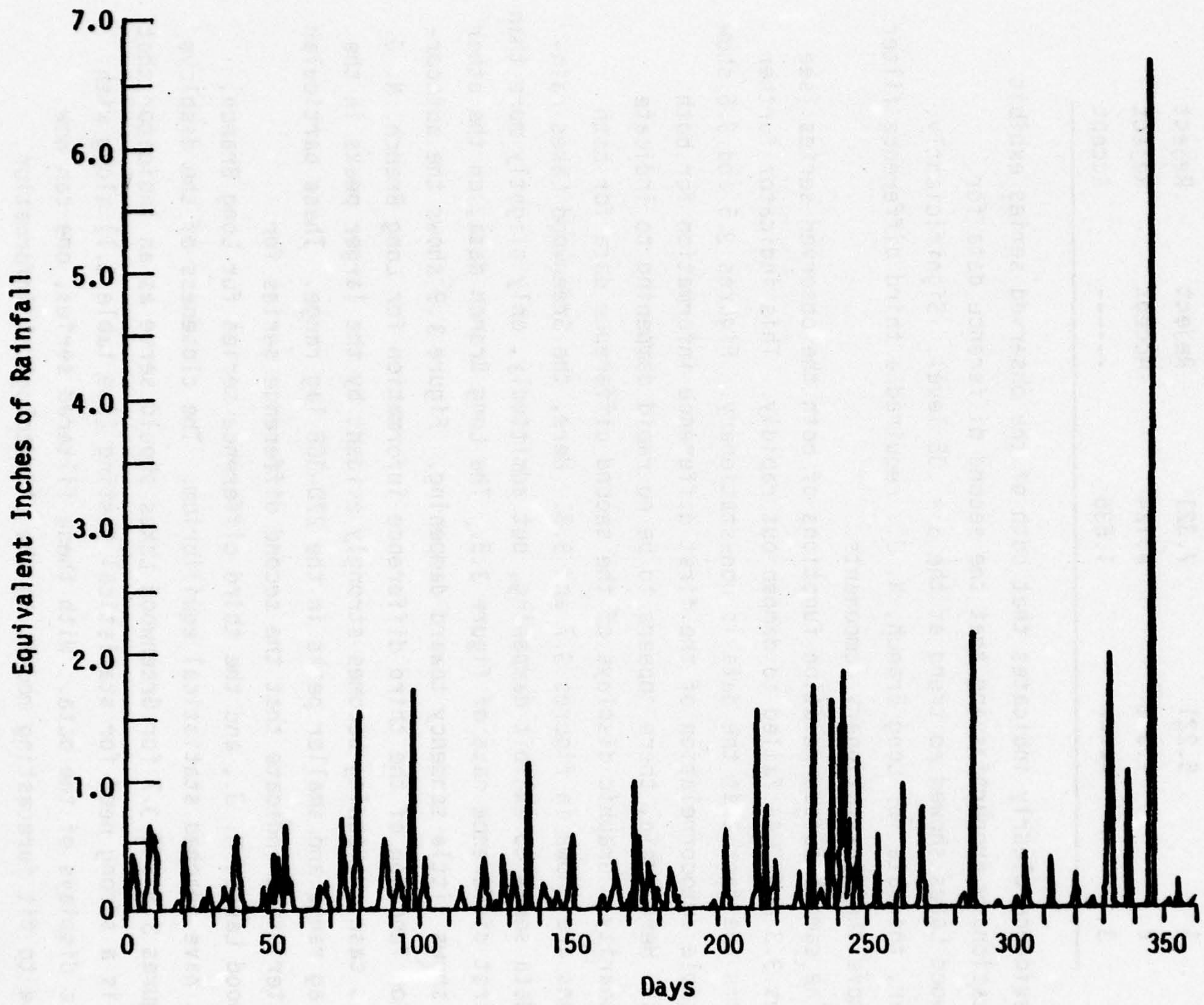

FIGURE 3.2 MEASURED DAILY PRECIPITATION FOR LONG BRANCH, N. J. - JANUARY THROUGH DECEMBER, 1974

Table 3.1 Kendall's Tau Statistics for Trend, $Z_0 = \pm 1.645$

	CALCULATED STATISTICS		H_0: NO TREND	
FILTER ORDER	GREENWOOD LAKES DATA	LONG BRANCH DATA	GREENWOOD LAKES	LONG BRANCH
0	9.158	11.316	Reject	Reject
1	5.221	7.321	Reject	Reject
2	1.532	4.124	Accept	Reject
3	-----	1.636	------	Accept

This evidence clearly indicates that both of the observed series exhibit non-stationary properties and that the second difference data for Greenwood Lakes showed no trend at the $\alpha = .05$ level. Significiantly, however, the data for Long Branch, N. J., required a third difference filter to remove the non-stationary components.

The sample autocorrelation functions of both the observed series (see figures 3.3 and 3.4) failed to dampen out rapidly. This indicator further confirms the fact that the data is non-stationary. Figures 3.5 and 3.6 show the sample autocorrelation of the first difference information for both series. Here again, there appears to be no rapid dampening to indicate stationarity. Graphic displays of the second difference data for both stations are shown in figures 3.7 and 3.8. Here, the Greenwood Lakes rainfall data seems to exhibit dampening, but admittedly, only slightly more than the first difference data of figure 3.5. The Long Branch data, on the other hand, shows little tendency toward dampening. Figure 3.9 shows the autocorrelation function of the third difference information for Long Branch, N. J. In this case, dampening becomes strongly evident by the larger peaks in the 0-50 lag range and smaller peaks in the 270-365 lag range. These particular characteristics indicate that the second difference series for Greenwood Lakes, N. J., and the third difference series for Long Branch, N. J., have reached statistical equilibrium. The closeness of the displays of figures 3.5 and 3.7 for Greenwood Lakes should serve as an indicator that there is a strong need for statistical testing (see table 3.1) along with graphic displays of the data. With these filtered series, one can now proceed to fit forecasting models to the climatological information.

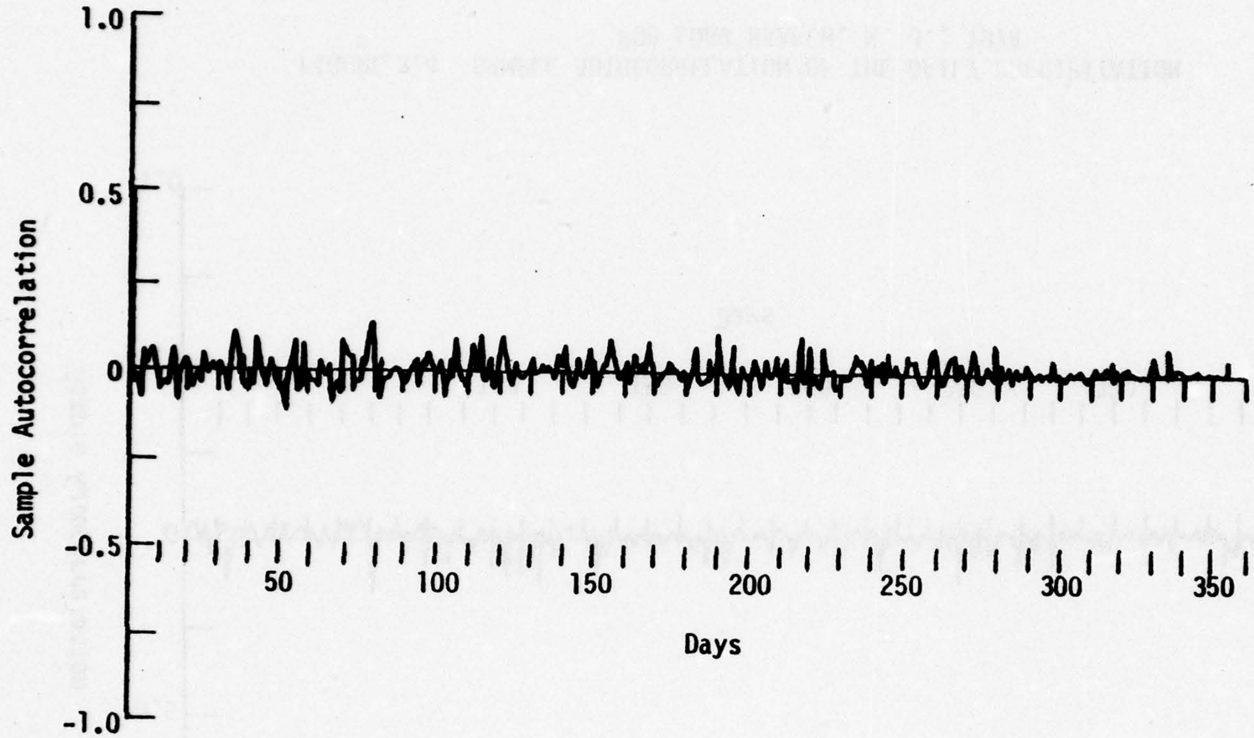

FIGURE 3.3 SAMPLE AUTOCORRELATION OF THE DAILY PRECIPITATION FOR GREENWOOD LAKES, N. J., 1974

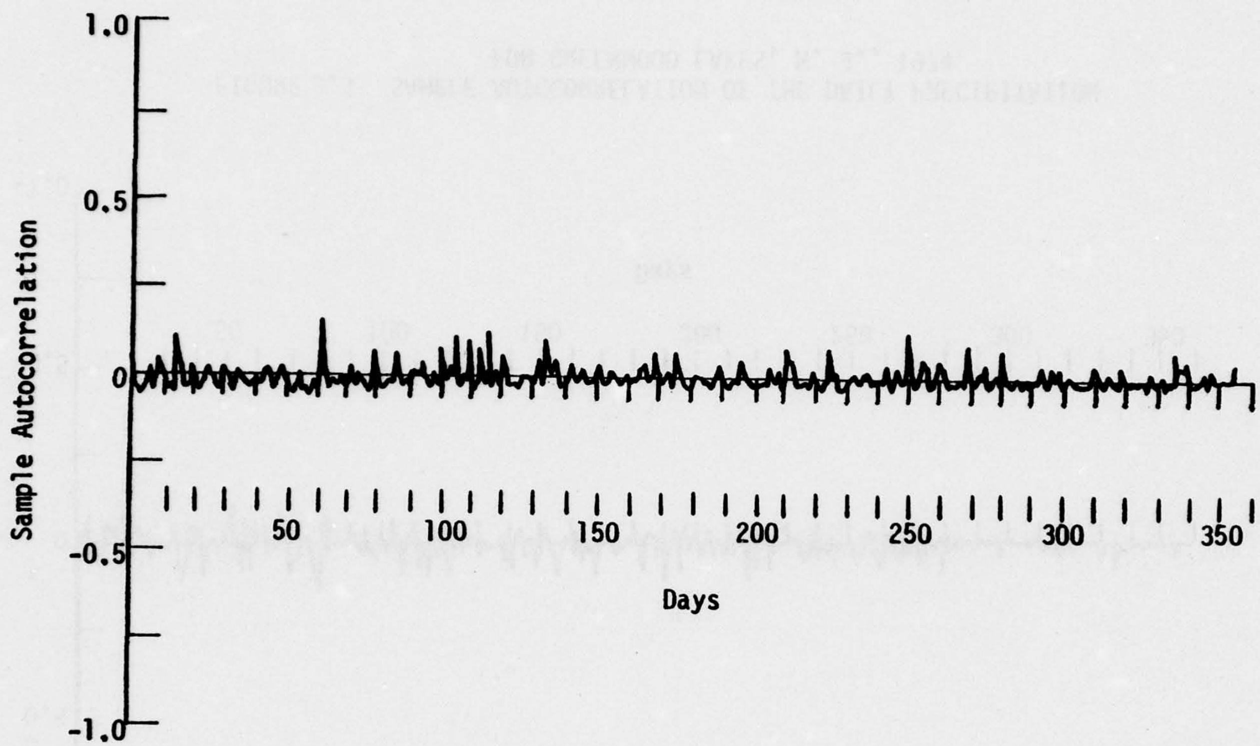

FIGURE 3.4 SAMPLE AUTOCORRELATION OF THE DAILY PRECIPITATION FOR LONG BRANCH, N. J., 1974

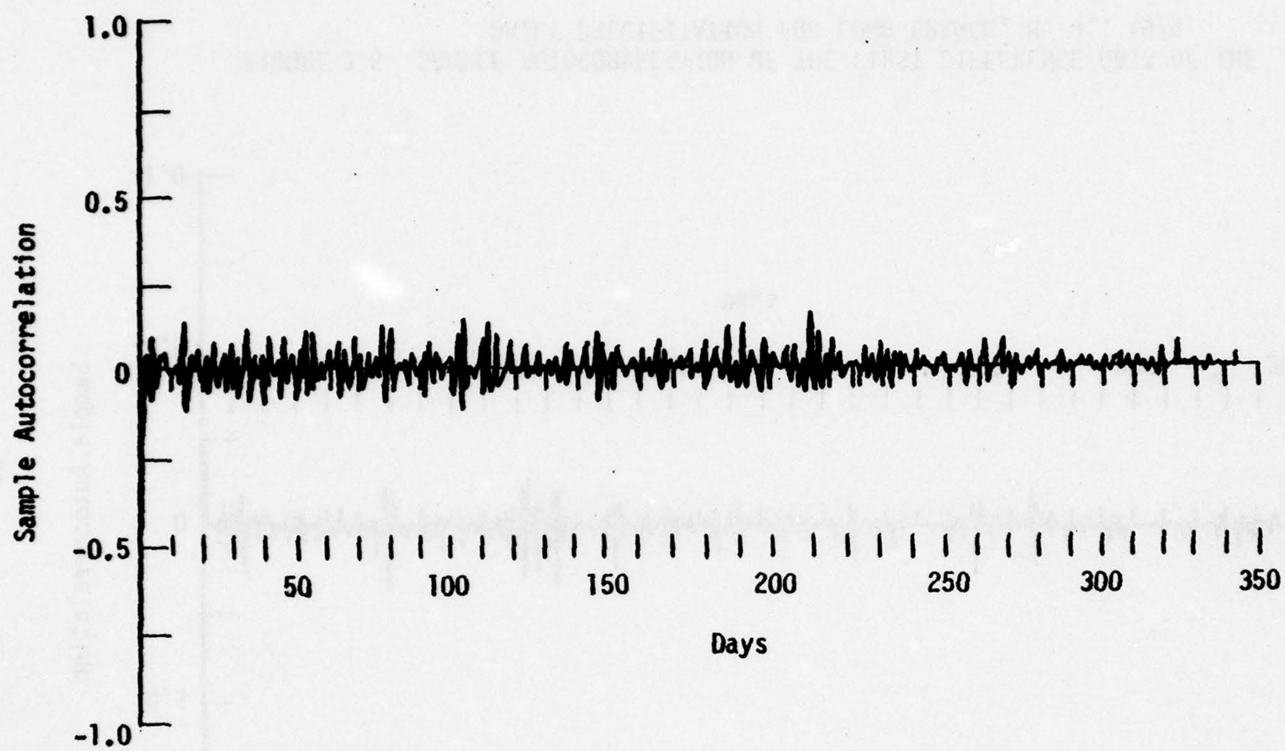

FIGURE 3.5 SAMPLE AUTOCORRELATION OF THE FIRST DIFFERENCE DATA OF THE DAILY PRECIPITATION FOR GREENWOOD LAKES, N. J., 1974

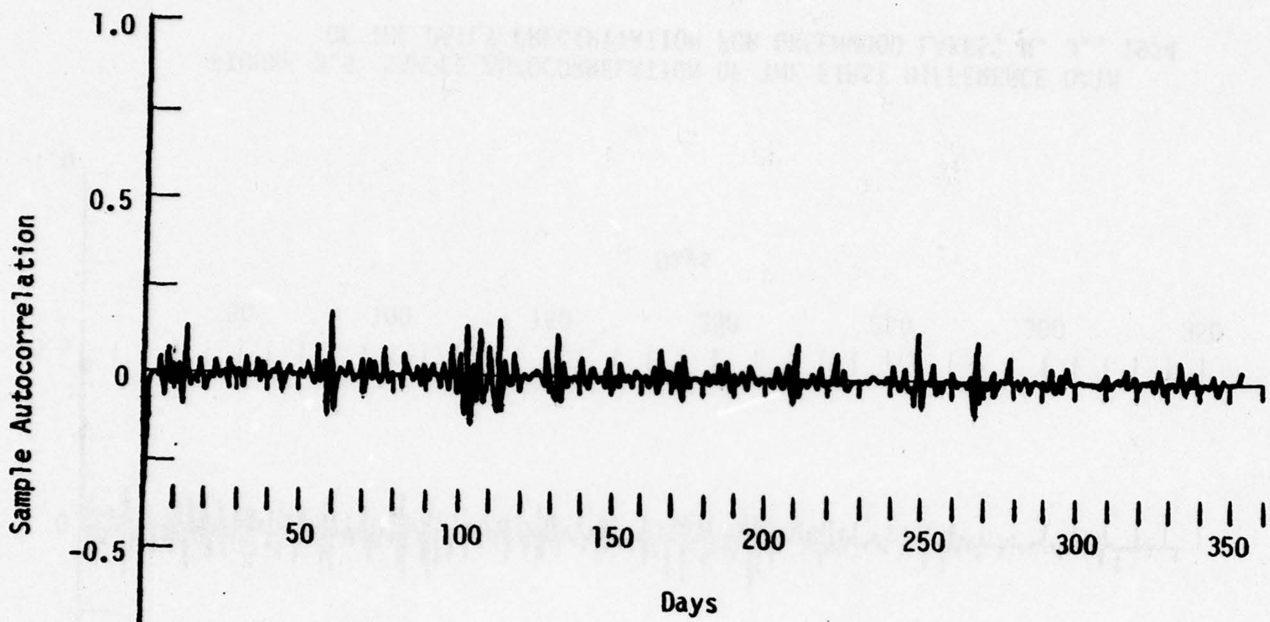

FIGURE 3.6 SAMPLE AUTOCORRELATION OF THE FIRST DIFFERENCE DATA OF THE DAILY PRECIPITATION FOR LONG BRANCH, N. J., 1974

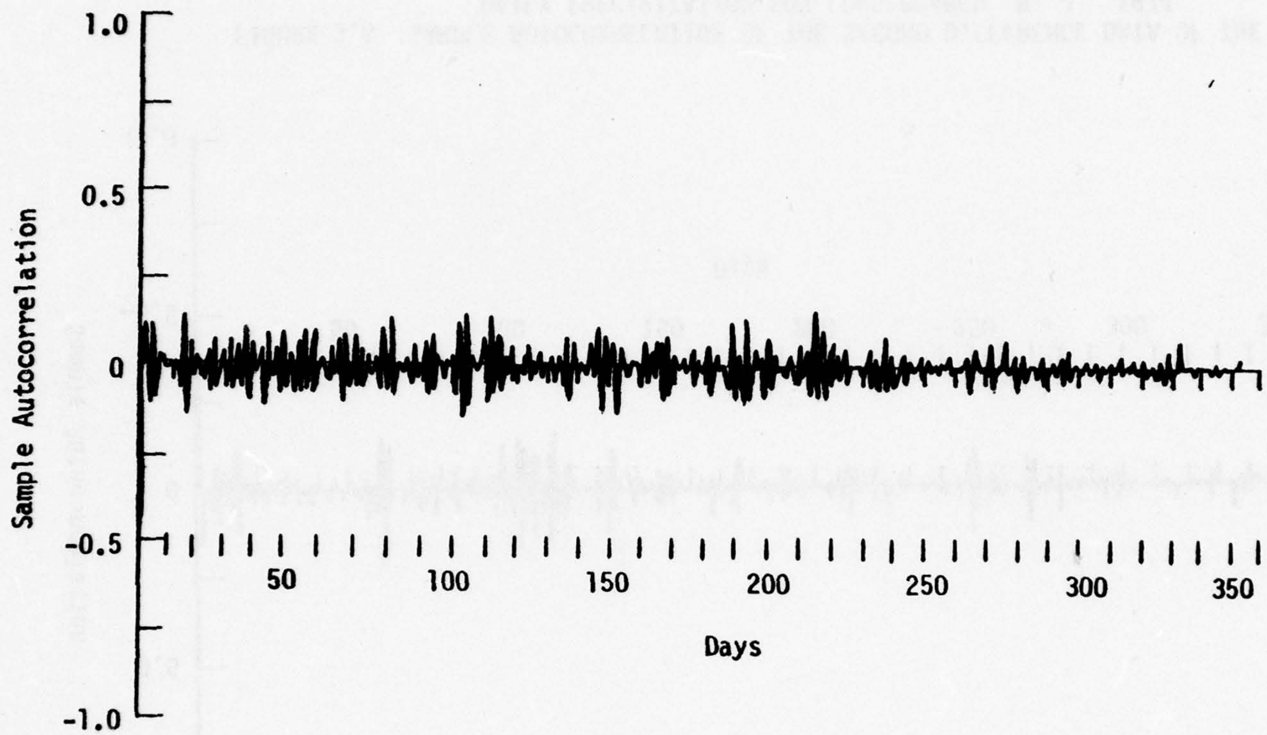

FIGURE 3.7 SAMPLE AUTOCORRELATION OF THE SECOND DIFFERENCE DATA OF THE DAILY PRECIPITATION FOR GREENWOOD LAKES, N. J., 1974

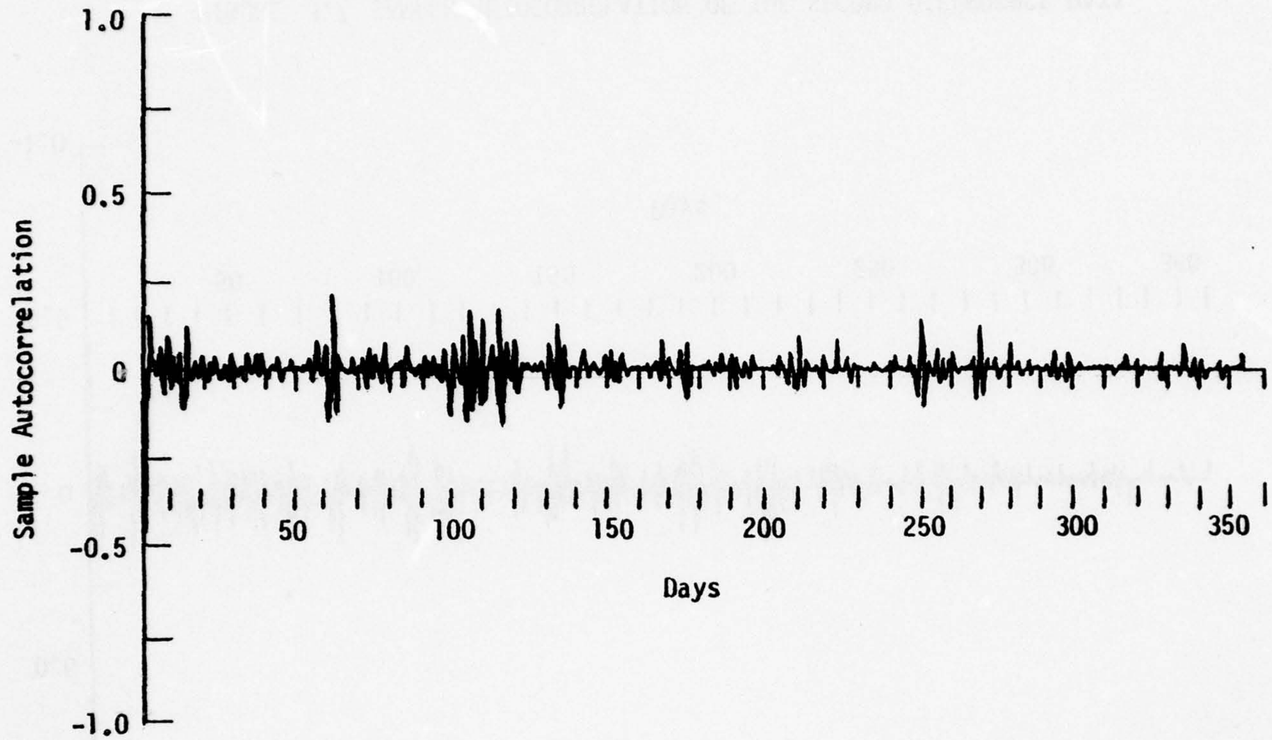

FIGURE 3.8 SAMPLE AUTOCORRELATION OF THE SECOND DIFFERENCE DATA OF THE DAILY PRECIPITATION FOR LONG BRANCH, N. J., 1974

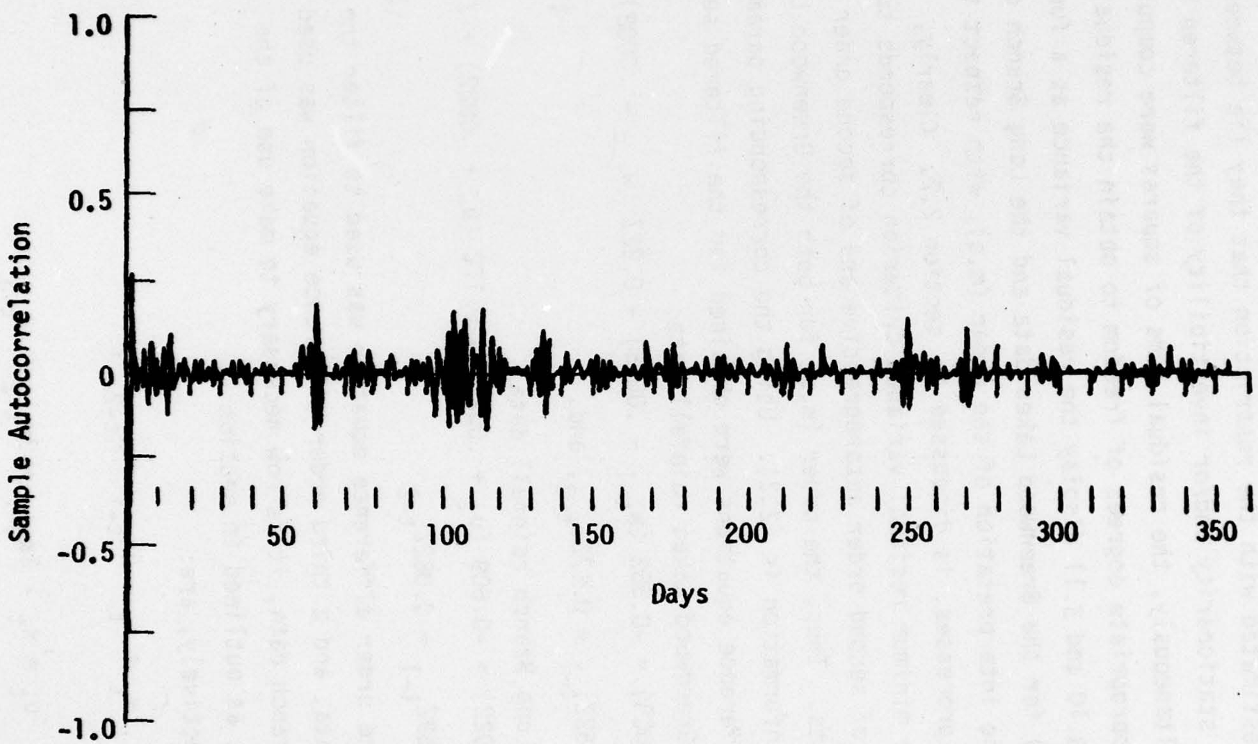

FIGURE 3.9 SAMPLE AUTOCORRELATION OF THE THIRD DIFFERENCE DATA OF THE DAILY PRECIPITATION FOR LONG BRANCH, N. J., 1974

3.1 Fitting the Models

To fit stationary stochastic models, either AR, MA, or ARMA, to the filtered information as outlined in section 2, it is necessary to estimate the parameters, $\alpha_1, \alpha_2, \ldots, \alpha_m, \beta_1, \beta_2, \ldots, \beta_q$, for each process and for each order of the process considered. Following the proposed procedure, the parameters were estimated with the restriction that they lie between -1 and +1 to insure the stationarity and/or invertibility of the filtered stochastic processes. Simultaneously, the residual sums of squares were computed and divided by the appropriate degrees of freedom to obtain the residual variances. Figures 3.10 and 3.11 display the residual variance as a function of model order (m,q) for the Greenwood Lakes data and the Long Branch data, respectively. The interpretation of the order (m,q), with respect to the AR, MA, and ARMA processes, is discussed in section 2.2. Clearly, in both cases the minimum residual variance criterion corresponds to mixed model consisting of second order autoregressive and of second order moving averages components. Thus, the order (m,q) for both the Greenwood Lakes and the Long Branch information is (2,2). Using the corresponding parameters, the following difference equations were obtained for the filtered series:

a. for the Greenwood Lakes rainfall data:

$$(w_t - .0008) = -0.582\,(w_{t-1} - .0008) - 0.227\,(w_{t-2} - .0008) + Z_t + 0.762Z_{t-1} + 0.172Z_{t-2}, \text{ and,} \tag{3.1}$$

b. for the Long Branch rainfall data:

$$(u_t + .0022) = -0.609\,(u_t + .0022) - 0.172\,(u_t + .0022) + Z_t + 0.769Z_{t-1} + 0.082Z_{t-2} \tag{3.2}$$

Since a second order difference equation was used to filter the Greenwood Lakes data, and a third order difference equation was used to filter the Long Branch data, it is now necessary to make use of the *backwards filters*, as outlined in section 2. The filters, respectively, are:

$$w_t = x_t - 2x_{t-1} + x_{t-2}\ ,$$

and

$$u_t = x_t - 3x_{t-1} + 3x_{t-2} - x_{t-3}\ .$$

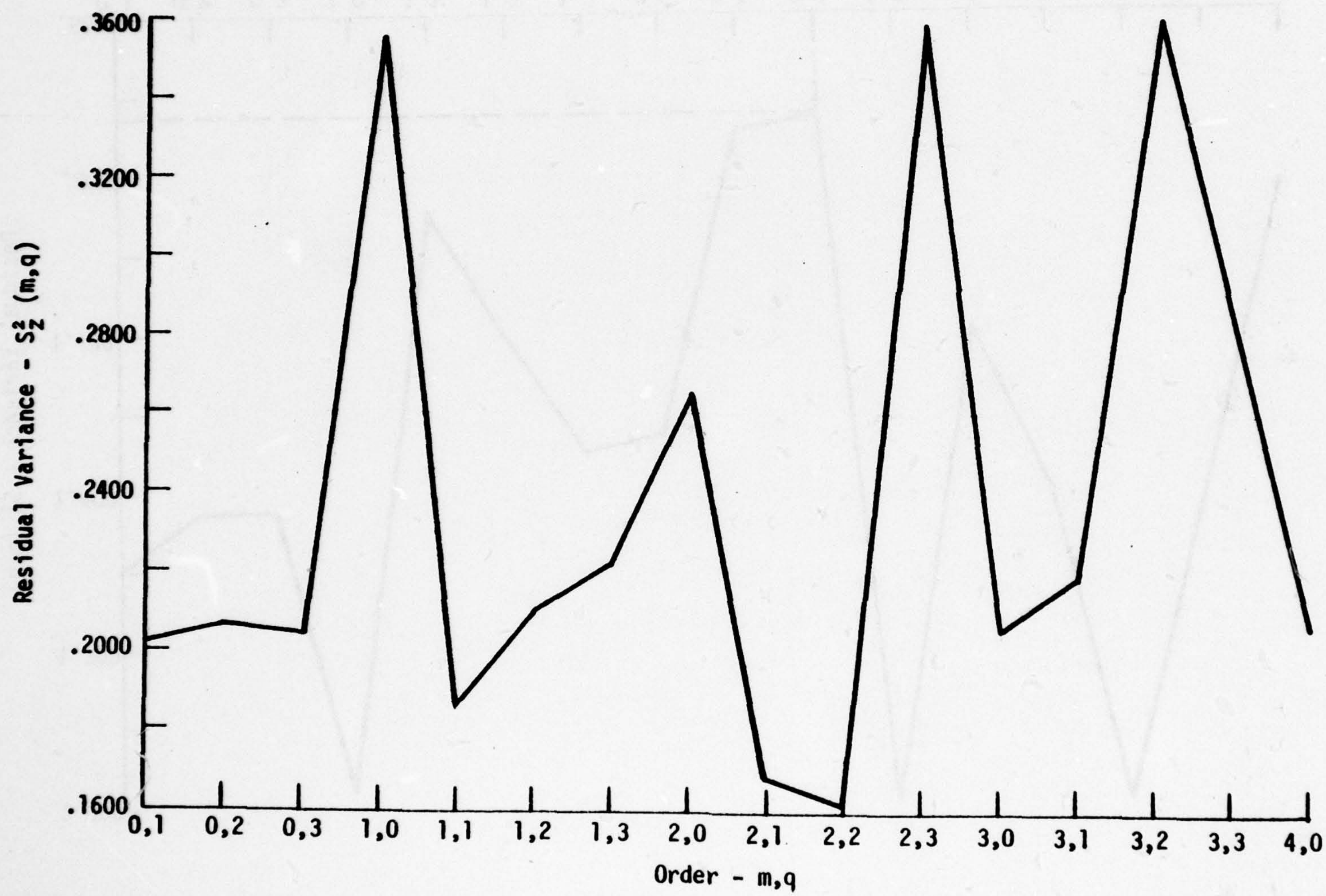

FIGURE 3.10 MODEL ORDER vs. RESIDUAL VARIANCE FOR THE DAILY PRECIPITATION, 1974, FOR GREENWOOD LAKES, N. J.

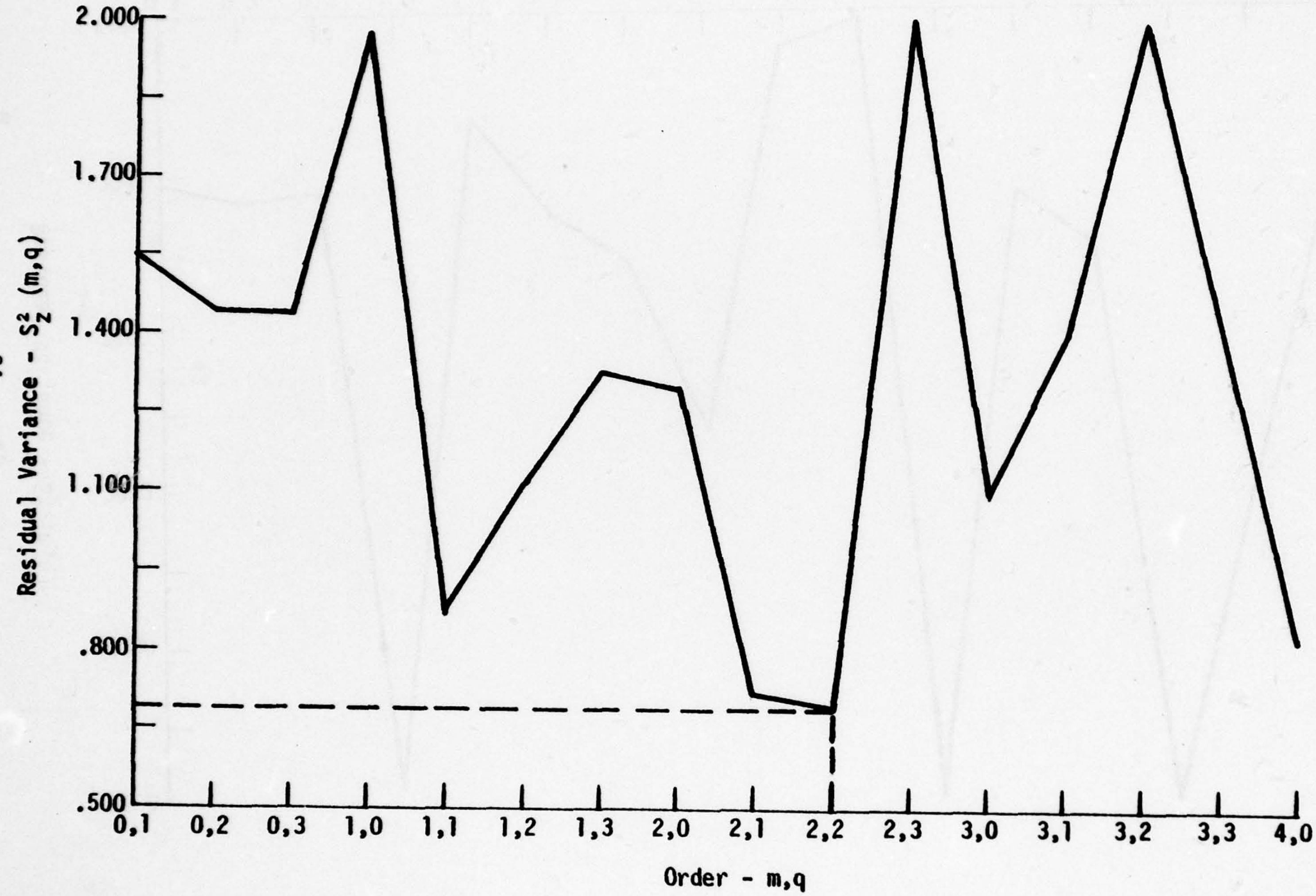

FIGURE 3.11 MODEL ORDER vs. RESIDUAL VARIANCE FOR THE DAILY PRECIPITATION, 1974 FOR LONG BRANCH, N. J.

We insert the filters into equation (3.1) and (3.2) to obtain the appropriate forecasting models that will be used to characterize the climatological information. Thus, equations (3.1) and (3.2) become, respectively:

$$\hat{x}_t = 1.418x_{t-1} - 0.063x_{t-2} - 0.128x_{t-3} - 0.227x_{t-4} + .0015 + Z_t + 0.762Z_{t-1} + 0.172Z_{t-2} \text{ for Greenwood Lakes,} \quad (3.3)$$

and

$$\hat{x}_t = 2.391x_{t-1} - 1.345x_{t-2} - 0.311x_{t-3} + 0.093x_{t-4} + 0.172x_{t-5} - .0039 + Z_t + 0.769Z_{t-1} + 0.082Z_{t-2} \text{ for Long Branch.} \quad (3.4)$$

Setting the unknown Z_t's equal to their conditional expectations of zero and assuming the values $x_{t-1}, x_{t-2}, \ldots, x_{t-m+d}$, have been realized, one can use equations (3.3) and (3.4) to simulate the observed climatological series. In addition, if t is replaced by $t + \ell$ in the above equations, one can forecast ℓ steps ahead, $\ell = 1, 2, \ldots, L$, for both series. Figures 3.12 and 3.13 show the simulated information for both stations which clearly fits the observed series very well.

Tables 3.2 and 3.3, p. 28 show the ℓ step ahead forecasts (up to $\ell = 11$ lead times in advance) at origin $t = 142$ and $t = 144$, respectively, with the associated confidence intervals, and updating of the forecasted values. Ordinarily, as ℓ increases, the forecasts become less accurate. The short-term accuracy, however, can be maintained by *updating* the forecasted values of the series as additional information becomes available. For example, the $t = 142$ origin forecast of x_{144} may be updated to become the $t = 143$ origin forecast of x_{144} by adding a constant multiple of the one-step ahead forecast error, $\theta_\ell Z_{t+1} = \theta_1 Z_{143}$, to the $t = 142$ origin forecast of x_{144}. The forecast error for this case is,

$$Z_{143} = x_{143} - \hat{x}_{143} \ ,$$

and $\theta_\ell = \theta_1$ is given by $\theta_1 = \phi_1 - \hat{\beta}_1$. This is done when $x_{t+1} = x_{143}$ becomes available. The basis for updating the original forecasted values for ℓ steps ahead as additional observations become available is:

$$\hat{x}_{t+1}(\ell) = \hat{x}_t(\ell + 1) + \theta_\ell z_{t+1} \ . \quad (3.5)$$

FILTERED MODEL:

$$(w_t - .0008) = -0.582\,(w_{t-1} - .0008) - 0.227\,(w_{t-2} - .0008) + Z_t + 0.762\,Z_{t-1} + 0.172\,Z_{t-2}$$

ARMA FORECASTING MODEL:

$$\hat{x}_t = 1.418\,x_{t-1} - 0.063\,x_{t-2} - 0.128\,x_{t-3} - 0.227\,x_{t-4} + .0015 + Z_t + 0.762\,Z_{t-1} + 0.172\,Z_{t-2}$$

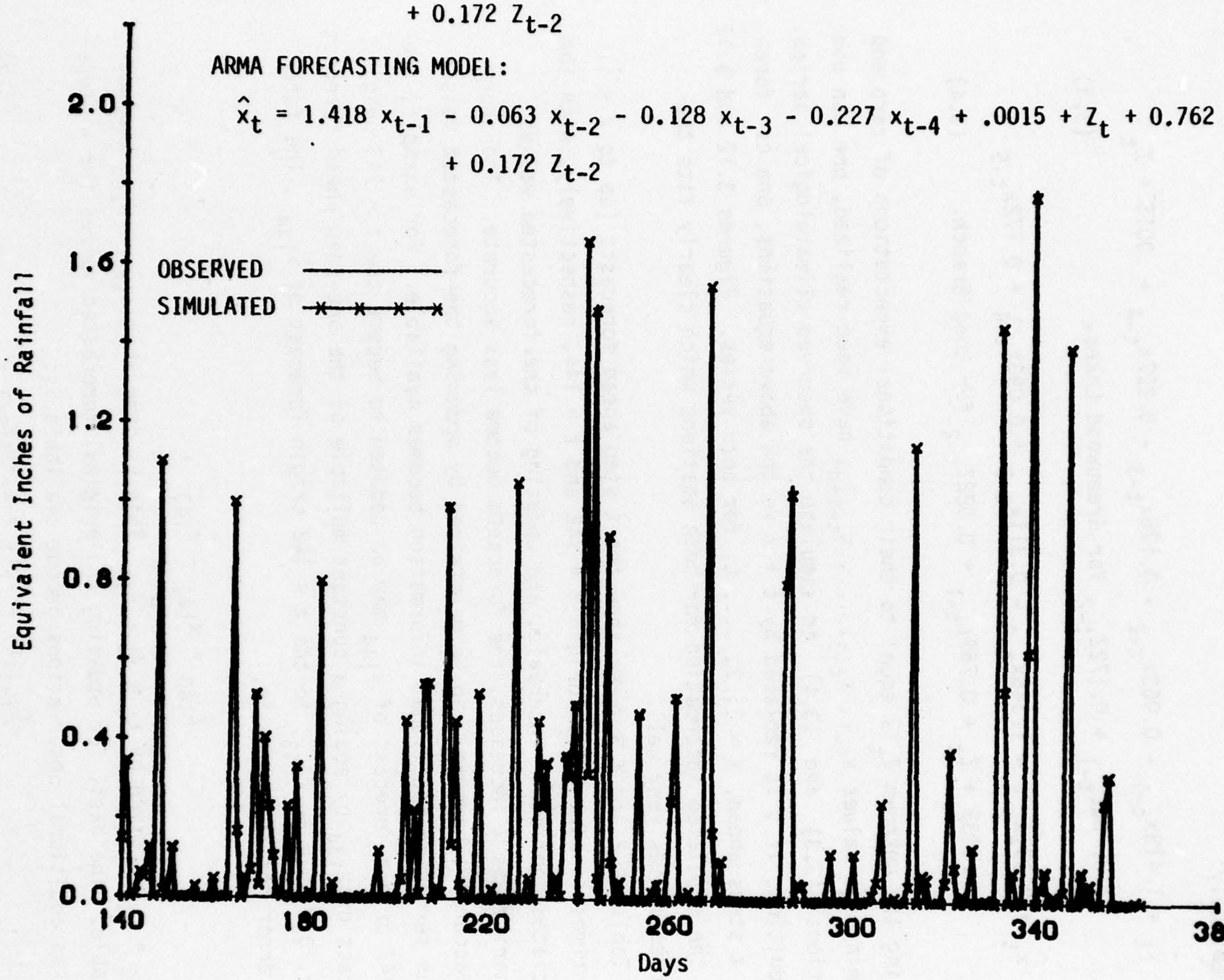

FIGURE 3.12 SIMULATED PRECIPITATION DENSITY SERIES USING THE MIXED (ARMA) MODEL vs. THE OBSERVED SERIES FOR GREENWOOD LAKES, N. J., 1974

FILTERED MODEL:

$$(u_t + .0022) = -0.609\,(u_{t-1} + .0022) - 0.172\,(u_t + .0022) + Z_t + 0.769\,Z_{t-1} + 0.0827\,Z_{t-2}$$

ARMA FORECASTING MODEL:

$$\hat{x}_t = 2.391\,X_{t-1} - 1.345\,X_{t-2} - 0.311\,X_{t-3} + 0.093\,X_{t-4} + 0.172\,X_{t-5} - .0039 + Z_t + 0.769\,Z_{t-1} + 0.082\,Z_{t-2}$$

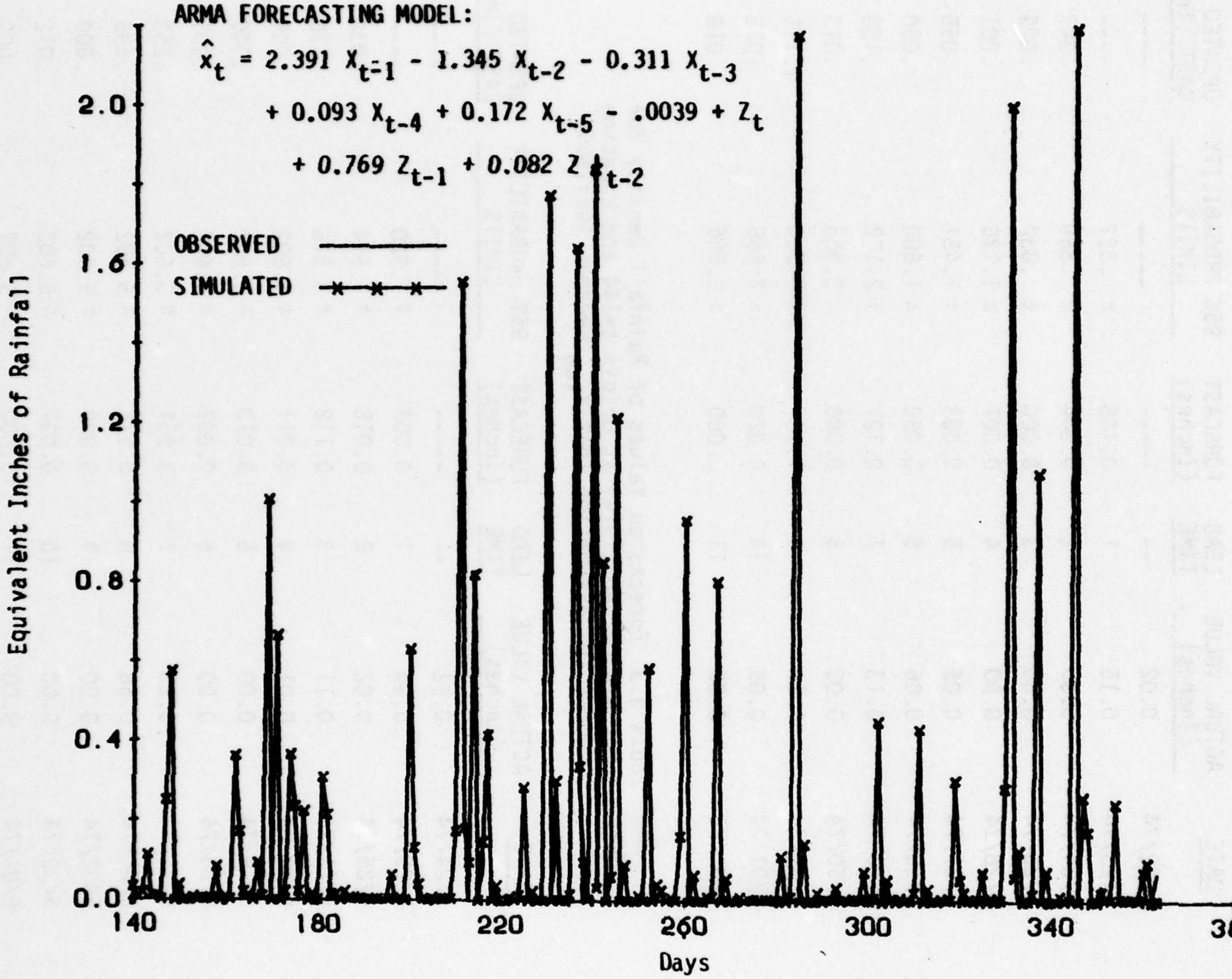

FIGURE 3.13 SIMULATED PRECIPITATION DENSITY SERIES USING THE MIXED (ARMA) MODEL vs. THE OBSERVED SERIES FOR LONG BRANCH, N.J., 1974

Table 3.2 Forecasted Values of Rainfall Density for Greenwood Lakes, NJ (1974) at Origin t=142 and Updating Under the Assumption That x_{143} Becomes Available

DATE	ACTUAL VALUE (Inches)	LEAD TIME	FORECAST (Inches)	95% PROBABILITY LIMITS	UPDATED FORECAST (Inches)
5/22/74	0.02	--	-----	-----	----
5/23/74	0.15	1	0.148	± .327	----
5/24/74	0.35	2	0.344	± .569	.347
5/25/74	0.00	3	0.000	± .832	.005
5/26/74	0.00	4	0.007	± 1.128	.001
5/27/74	0.05	5	0.063	± 1.451	.055
5/28/74	0.06	6	0.050	± 1.800	.060
5/29/74	0.13	7	0.127	± 2.172	.138
5/30/74	0.00	8	0.000	± 2.566	.013
5/31/74	1.10	9	1.101	± 2.566	1.115
6/01/74	0.05	10	0.020	± 2.566	.036
6/02/74	0.00	11	0.000	± 2.566	.018

Table 3.3 Forecasted Values of Rainfall Density for Long Branch, NJ (1974) at Origin t=144 and Updating Under the Assumption That x_{145} Becomes Available

DATE	ACTUAL VALUE (Inches)	LEAD TIME	FORECAST (Inches)	95% PROBABILITY LIMITS	UPDATED FORECAST (Inches)
5/24/74	0.02	--	-----	-----	----
5/25/74	0.00	1	0.007	± .920	----
5/26/74	0.02	2	0.018	± 1.974	.030
5/27/74	0.11	3	0.112	± 3.516	.156
5/28/74	0.01	4	0.011	± 5.602	.009
5/29/74	0.00	5	0.010	± 5.602	.000
5/30/74	0.00	6	0.002	± 5.602	.001
5/31/74	0.25	7	0.251	± 5.602	.253
6/01/74	0.58	8	0.575	± 5.602	.595
6/02/74	0.00	9	0.000	± 5.602	.000
6/03/74	0.00	10	0.037	± 5.602	.012
6/04/74	0.00	11	0.000	± 5.602	.000

3.2 Checking the Fit

To check how well the models fit the observed series for both stations, the residuals were calculated using $z_t = x_t - \hat{x}_t$. The simulated (one-step ahead forecast) $\hat{x}_t$ is subtracted from the original series, $\hat{x}_t$. Next, the sample autocorrelation function of the resiudals, for lags of 0-364, was computed according to equations (2.3) and (2.4). For the models to fit the observed information well, the sample autocorrelation function should be effectively zero for all but the zeroth lag. When the observed series is sufficiently large ($n > 50$), the sample autocorrelation of the residuals $r_{zz}(k) \sim N(0, {}^1/n)$, [11]. Thus, for both stations, the standard deviation of the sample autocorrelation is:

$$\frac{1}{\sqrt{365}} = 0.0523$$

and the 95% confidence limits are:

$$\underline{+}1.96\ (0.0523) = \underline{+}0.1025\ .$$

One would expect that, at the 5% level of significance, 365(.05) or 19 of the sample autocorrelations will lie *outside* of the above limits. Hence, from the results of the sample autocorrelation of the residuals, one can conclude that the models fitted to the Greenwood Lakes and Long Branch climatological data, equations (3.3) and (3.4), give a good representation of the values realized for 1974. Tables 3.4 and 3.5 verify these conclusions.

3.3 Proposed Approach vs. Akaike's Approach

Another approach to the classification of time-series models is introduced by Akaike, [12], which uses the criterion of final prediction error (FPE). This method was used, [8], [6], in the determination of "best" models for climatological information. Akaike assumed that an AR model (equation 2.7) has a zero mean and observations acquired from points equally spaced in time. Using the Yule-Walker equations with the estimate of x_{t-m}, $t > m$ the mean square of residuals,

TABLE 3.4 Sample Autocorrelation of the Residuals, $r_{zz}(k)$, for the Simulated Precipitation Density, Greenwood Lakes, N. J. (1974)
Confidence Interval = $\pm$0.1025

Lag K	Sample Autocorrelation, $r_{zz}(k)$								
1-10	1.000	-.149	-.408	.016	-.002	.039	-.064	.108	.037	-.061
11-20	-.009	.009	.052	-.110	-.048	.179	-.054	-.046	.009	.029
21-30	.063	-.053	-.011	-.026	-.050	.056	.008	.076	-.003	-.100
31-40	.073	-.014	.003	-.080	-.030	.182	-.036	-.045	-.032	.054
41-50	-.009	-.131	.139	.039	-.073	.008	-.026	.081	-.013	-.031
51-60	.045	-.095	-.061	.116	.095	-.061	-.087	.043	.017	-.031
61-70	.019	-.025	.016	.026	.025	-.004	-.061	.075	-.045	-.124
71-80	.097	.086	.009	-.106	.050	.070	-.130	-.055	.074	.127
81-90	-.028	-.102	.054	-.001	-.072	.031	.083	-.029	-.101	.056
91-100	.042	-.027	.006	.032	-.105	.076	.028	-.039	.059	-.066
101-110	-.024	.057	.018	-.048	-.023	-.036	.097	.039	-.053	-.011
111-120	.011	-.044	-.018	.070	.002	-.035	.019	.035	.047	-.089
121-130	.032	.147	.012	-.092	-.052	.020	.026	.009	.020	-.039
131-140	.014	-.004	.006	.011	-.033	-.022	.004	.079	.009	.083
141-150	-.005	.041	.007	-.028	.002	.027	.008	-.057	-.024	.098
151-160	-.011	-.045	.013	-.002	-.024	-.001	.057	.001	-.028	-.025
161-170	.007	.036	.026	-.024	.028	.045	-.063	-.025	.066	.002
171-180	-.011	-.015	-.006	.024	.007	.068	.029	-.041	.027	.009
181-190	-.003	-.014	-.044	.017	.074	.013	-.040	.011	.014	-.058
191-200	-.003	.096	-.024	-.035	.007	.004	.001	-.032	.004	.046
201-210	-.004	-.021	.001	.005	.007	.011	.009	.008	-.002	-.003
211-220	-.004	.004	.002	-.003	-.002	-.001	.001	-.001	.001	.001
221-230	-.001	-.001	.001	-.001	.0003	.0004	.0004	.0003	.0003	.0002
231-240	.0001	.0009	-.0003	-.0003	-.0003	-.0001	-.0002	-.0003	-.0003	-.0003
241-250	.0002	-.0001	-.0001	-.0002	.0001	.0002	-.0001	.0009	-.0007	-.0009
251-260	.0001	-.0008	-.0004	-.0003	-.0003	-.0004	.0002	-.0002	-.0004	-.0003
261-270	-.0002	-.0003	-.0003	-.0002	-.0001	-.0002	-.0004	-.0003	-.0003	-.0003
271-280	-.0002	.0002	.0002	-.0005	-.0003	-.0002	-.0003	-.0002	-.0002	-.0002
281-290	-.0002	-.0002	-.0002	-.0002	-.0002	-.0002	-.0002	-.0002	.0001	.0003
291-300	-.0001	-.0004	-.0002	-.0002	-.0002	-.0002	-.0002	-.0002	-.0001	-.0001
301-310	-.0002	-.0001	-.0001	-.0001	-.0001	-.0002	-.0001	-.0001	-.0001	-.0001
311-320	.0001	.0002	.0001	.0001	.0001	.0001	.0002	.0001	.0004	.0001
321-330	-.0002	-.0001	.0001	.0001	.0000	.0000	-.0001	-.0001	-.0001	-.0001
331-340	.0000	-.0001	-.0001	.0001	.0001	.0003	.0004	.0002	-.0002	-.0001
341-350	-.0001	.0000	.0005	.0002	-.0005	-.0002	-.0001	.0001	.0001	.0001

TABLE 3.5 Sample Autocorrelation of the Residuals, $r_{zz}(k)$, for the Simulated Precipitation Density, Long Branch, N. J. (1974)
Confidence Interval = ± 0.1025

Lag K	Sample Autocorrelation, $r_{zz}(k)$								
1-10	1.000	-.249	-.246	.156	.036	.055	.115	-.046	.090	.074
11-20	.034	.039	.086	-.097	.125	.149	-.028	.015	.039	.069
21-30	.061	-.001	.077	.019	.049	.031	.061	.048	-.007	.074
31-40	.029	.058	.037	-.019	.087	.046	.015	.053	-.002	.070
41-50	.039	.003	.054	.005	.099	.005	-.019	.076	.050	.013
51-60	.027	.034	.011	.045	.058	-.017	.068	.001	.040	.073
61-70	-.145	.186	.049	-.040	.034	.017	.021	.057	.014	-.008
71-80	.039	.065	-.044	.067	.019	.009	.030	-.029	.067	.050
81-90	-.016	.004	.039	.041	-.063	.060	.086	-.053	.015	.047
91-100	-.032	.067	-.011	.004	.027	.052	-.064	.070	.038	-.114
101-110	.088	.087	-.065	.027	-.073	.108	.041	-.029	-.106	.126
111-120	.049	-.071	.025	.031	-.116	.142	-.003	-.001	-.033	-.024
121-130	.080	-.023	-.012	.024	-.017	.0003	.034	-.049	.041	.010
131-140	-.076	.098	-.020	-.101	.090	.010	-.027	-.006	-.013	.002
141-150	.002	-.011	.015	-.019	-.039	.043	.004	-.026	-.015	.004
151-160	.005	-.011	-.008	-.005	.003	-.015	.0008	.007	-.022	-.005
161-170	.016	-.029	.006	-.012	.006	.001	-.008	-.037	.037	-.017
171-180	-.032	.039	-.016	-.034	.031	-.054	.043	-.0005	-.023	-.008
181-190	.006	-.017	-.025	.036	-.005	-.023	-.008	.002	-.007	.001
191-200	-.007	-.010	-.002	-.006	.004	-.002	-.043	.016	.019	-.013
201-210	-.017	-.011	.004	-.0005	-.003	-.012	-.011	.006	-.002	-.011
211-220	-.009	-.006	-.005	-.005	-.007	-.004	-.005	-.007	-.004	-.005
221-230	-.006	-.005	-.005	-.006	-.006	-.006	-.005	-.005	-.006	-.005
231-240	-.005	-.005	-.005	-.007	-.003	-.004	-.005	-.004	-.005	-.006
241-250	-.004	-.004	-.005	-.007	-.003	-.005	-.004	-.004	-.006	-.003
251-260	-.004	-.005	-.004	-.005	-.004	-.005	-.004	-.004	-.004	-.004
261-270	-.004	-.004	-.004	-.005	-.003	-.003	-.004	-.004	-.004	-.004
271-280	-.005	-.003	-.003	-.004	-.004	-.004	-.004	-.004	-.004	-.003
281-290	-.004	-.003	-.003	-.003	-.004	-.003	-.003	-.006	-.001	-.002
291-300	-.004	-.003	-.003	-.003	-.003	-.003	-.003	-.003	-.003	-.003
301-310	-.003	-.003	-.003	-.003	-.003	-.003	-.002	-.002	-.003	-.002
311-320	-.002	-.002	-.002	-.002	-.003	-.002	-.002	-.002	-.002	-.002
321-330	-.002	-.002	-.002	-.002	-.002	-.002	-.002	-.002	-.002	-.002
331-340	-.002	-.002	-.002	-.002	-.004	.0002	.0006	.002	.001	.002
341-350	-.002	-.0005	-.0009	-.002	-.001	-.001	-.001	-.001	-.009	-.0004

$$R_p = \frac{1}{N} \sum_{t=1}^{N} (x_t - \sum_{m=1}^{P} \alpha m x_{t-m}) \quad (3.6)$$

are to be minimized.

The FPE criterion depends upon the use of the mean square error:

$$S_p = \frac{N}{N-1-P} R_p \quad (3.7)$$

and $(FPE)_p = 1 + \frac{P+1}{N}, S_p \sim \chi_p^2$. The smallest FPE will give the best estimation of the parameters. Table 3.6 below shows the best parameter order for both approaches.

Table 3.6 Best Parameter Order for the Minimum Residual Variance and Akaike's Criteria

	MODEL	ORDER (m,q)	RESIDUAL VARIANCE
Greenwood Lakes, NJ	Proposed	(2,2)	0.163
	Akaike(FPE)	(8,0)	0.158
Long Branch, NJ	Proposed	(2,2)	0.717
	Akaike(FPE)	(5,0)	0.679

The difference between methods is obvious from the above table. Based on the principle of parsimony, [13], one should reject the Akaike classifications as having a substantially larger number of parameters with which to reckon. It is noteworthy to mention that in Akaike's method, the unbiased estimate of the mean square error will usually produce good predictions, but the variance will become quite large in the analysis of the spectrum. Further, this same method is confined only to the autoregressive models and does not address the utility of the moving averages and mixed models in analyzing climatological information. By virtue of the larger number of parameters of the Akaike classification, the number of previous observations required to begin accurate forecasting is substantially less with the minimum residual variance classification. For instance, in the case of the Greenwood Lakes information, Akaike's method would require ten previous observations for forecasting, while the minimum residual variance classification would require only four previous values. The fact that fewer previous observations are required, implies that a more practical near-real-time forecasting scheme is possible from the computational point-of-view.

3.4 Summary and Conclusions

In this section, the procedural approach developed in section 2 and exercised in section 3 was used to characterize 1974 climatological information for New Jersey. Specifically, time-dependent rainfall data for Greenwood Lakes, N.J., and Long Branch, N.J., acquired from the NOAA was modeled and analyzed. The information consisted of daily rainfall accumulation for the two sites taken from recording rain gauges during the calendar year 1974. The two sites chosen were representative of northern and central New Jersey climate. In general, the climatological characteristics were fairly uniform within a 15 Km radius of each station. Therefore, the procedural approach in analyzing this type of data would be relevant for use by the Army intelligence community in a tactical situation.

The climatological data were shown to be non-stationary realizations, and following the procedural approach recommended in section 2, the following stochastic processes were formulated as the most appropriate characterizations:

i. for Greenwood Lakes, N.J.:

$$\hat{x}_t = 1.418x_{t-1} - 0.063x_{t-2} - 0.128x_{t-3} - 0.227x_{t-4} + 0.0015 + Z_t + 0.762Z_{t-1} + 0.172Z_{t-2}$$

and

ii. for Long Branch, N.J.:

$$\hat{x}_t = 2.391x_{t-1} - 1.345x_{t-2} - 0.311x_{t-3} + 0.093x_{t-4} + 0.172x_{t-5} - 0.0039 + Z_t + 0.769Z_{t-1} + 0.082Z_{t-2} .$$

These models were selected on the basis of the critierion of minimum residual variance. The results of the diagnostic check, through simulating the observed series, show this to be appropriate with respect to identifying the actual difference equations which characterize the climatological data. As expected, the analysis yielded similar difference equations for both sites (see equations 3.1 and 3.2). Furthermore, we have structured tables that show short and long-term forecasts for both Greenwood Lakes and Long Branch, N.J. *In these cases,* ARMA (2,2) models adequately characterize the underlying process as illustrated by figures 3.12 and 3.13, and in tables 3.2 and 3.3.

The information gained from the modeling and analysis will provide communicators and communications planners with a mechanism to determine short-term future communications outages in the GHz range. This, in turn, will enable planners to incorporate suitable alternatives in a tactical situation. Systems designers, on the other hand, can use the proposed modeling technique to simulate "worst case" outages due to heavy precipitation and thereby determine adequate equipment operational margins.

BIBLIOGRAPHY

1. Lin, S. H., "Dependence of Rain-Rate Distribution on Rain-Gauge Integration Time", The Bell System Technical Journal, Vol. 55, No. 1, (January 1976).

2. Chen, W. Y. S., "A Simple Method for Estimating Five-Minute Point Rain-Rate Distributions Based on Available Climatological Data," The Bell System Technical Journal, Vol. 55, No. 1, (January 1976).

3. Rice, P. L., and N. R. Holmberg, "Cumulative Time Statistics of Surface-Point Rainfall Rates", IEEE Transactions on Communications, Vol. Com-21, pp. 1131-1136, (October 1973).

4. Dutton, E. J. et al, "Prediction of European Rainfall and Link Performance Coefficients at 8 to 30 GHz", ITS, US Department of Commerce Technical Report No. ACC-ACO-16-74, (August 1974).

5. Bartow, J. E., "Prediction of the Effects of Weather on K-Band Air-to-Ground Data Link Transmissions," Research and Development Technical Report #ECOM-4386, US Army Electronics Command, (January 1976).

6. Jones, R. H., "Identification and Autoregressive Spectrum Estimation," IEEE Transactions on Automatic Control, Vol. AC-19, No. 6, (1974).

7. Jones, R. H., "Statistical Meteorology and Time Series Analysis," NTIS, US Department of Commerce, No. AD-A015-917, (July 1975).

8. Jones, R. H., "Fitting Autoregressions," Journal of the American Statistical Association, Vol. 70, No. 351, (September 1975).

9. NOAA (1974), "Climatological Data - New Jersey," Vol. 79, Nos. 1-12, NOAA, US Department of Commerce, Environmental Data Service, (Jan - Dec 1974).

10. Conover, W. J., Practical Nonparametric Statistics. New York: J. Wiley, 1971.

11. Kendall, M. G., and A. Stuart. The Advanced Theory of Statistics, Vol. 3. London, England: Griffen, 1966.

12. Akaike, H., "Fitting Autoregressive Models for Prediction," Annals of the Institute of Statistical Mathematics, Vol. 21, p. 243, (1969).

13. Box, G. E. P., and G. M. Jenkins, Time Series Analysis, Forecasting, and Control. San Francisco, California: Holden-Day, 1970.

14. Jenkins, G. M., and D. G. Watts. Spectral Analysis and its Application. San Francisco, California: Holden-Day, 1968.