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SECONDARY INSTABILITY IN BOUNDARY-LAYER FLOWS, (U)

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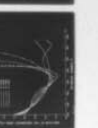
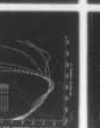
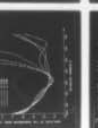
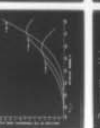
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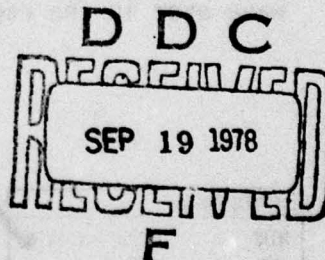
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Secondary Instability in Boundary-Layer Flows

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Abstract

The stability of a secondary Tollmien-Schlichting wave, whose wavenumber and frequency are nearly one half of those of a fundamental Tollmien-Schlichting instability wave (T-S wave), is analyzed by using the method of multiple scales. Under these conditions, the fundamental wave acts as a parametric excitor for the secondary wave. When the amplitude of the fundamental wave is small, the amplitude of the secondary wave deviates slowly from its unexcited state. However, as the amplitude of the fundamental wave increases, so does the amplitude of the secondary wave even in the regions where it is damped in the unexcited state.

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I. Introduction

Secondary instabilities in two-dimensional boundary layers over flat plates is analyzed. One of the many disturbances in the flow, a Tollmien-Schlichting instability wave (T-S wave), develops and starts to amplify at a given location on the plate. When this wave grows appreciably downstream, the flow will deviate from the Blasius flow and hence it may be unstable. When this occurs, experimental data shows that the higher harmonics and subharmonics of the principal T-S wave start to amplify faster than the T-S wave itself. It is therefore predicted that the higher harmonics and the subharmonics of the T-S wave play a big role in transition to turbulent flows. This behavior of the disturbances is demonstrated by Kachanov, Kozlov and Levchenko¹ and Veasov, Ginevsky and Karavosov² in their experimental works where they excited a known disturbance on a flat plate by a vibrating ribbon and measured the amplitudes of the disturbance spectrum at different downstream locations.

Kachanov, Kozlov and Levchenko¹, in their study of the nonlinear development of a wave in boundary layers, divided the process of the development of a disturbance into four regimes: (1) a linear and slow nonlinear development of disturbances and generation of higher harmonics, (2) an increase in the amplitude of the principal wave, a decrease in the amplitudes of its harmonics and an appearance of low frequencies and subharmonics in the spectrum, (3) an interaction of low-frequency oscillations with the principal wave and its higher harmonics, three-dimensional and rapid amplification of all spectral modes and (4) a damping of singled out harmonic modes, smoothing of the spectrum and a transition to the turbulent regime.

Klebanoff, Tidstrom and Sargent³, in their experimental study of the three dimensional nature of boundary-layer instability, observed the secondary instabilities induced by nonlinear interaction between a two-dimensional fundamental and a three-dimensional secondary wave at a position not very far away from the linear region.

In the present study, we are concerned with the linear and two-dimensional interaction between the fundamental T-S wave and its $\frac{1}{2}$ subharmonic. The wavenumber and the frequency of the subharmonic wave are nearly one half of those of the fundamental wave. The initial amplitude of the fundamental wave is varied and the corresponding amplitudes of the subharmonic wave are calculated at various downstream locations. It is found that the amplitude of the subharmonic is oscillatory in nature and is not amplified appreciably when the fundamental wave is considerably small. As the amplitude of the T-S wave grows downstream, its subharmonic grows in agreement with the experimental observations of Refs. 1 and 2. von Kerczek⁴ analyzed the stability of unsteady boundary layers which do not depend on the streamwise coordinate.

II. Problem Formulation

We consider secondary instabilities of a two-dimensional, steady, incompressible flow past a flat plate. The equations describing the motion are the unsteady, dimensionless Navier-Stokes equations:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \quad (1)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = - \frac{\partial P}{\partial x} + \frac{1}{R} \nabla^2 U, \quad (2)$$

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = - \frac{\partial P}{\partial y} + \frac{1}{R} \nabla^2 V, \quad (3)$$

$$U = V = 0 \quad \text{at } y = 0, \quad (4)$$

$$U \rightarrow U_\infty, V \rightarrow 0 \quad \text{and } P \rightarrow \rho U_\infty^2 \quad \text{as } y \rightarrow \infty, \quad (5)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Here, x and y are made dimensionless by using a reference length δ_r , the time is made dimensionless by using δ_r/U_∞ and the velocities are made dimensionless by using the freestream velocity U_∞ . The Reynolds number is defined as $R = U_\infty \delta_r / \nu$, where ν is the fluid kinematic viscosity.

A. Basic-State

We assume that each basic flow quantity is the sum of a mean flow quantity (the Blasius value) and an unsteady disturbance quantity:

$$\hat{U}(x, y) = U_0(y) + \epsilon U_1(y) \exp[i(kx - \omega t)] + cc + o(\epsilon^2), \quad (6)$$

$$\hat{V}(x, y) = \epsilon V_1(y) \exp[i(kx - \omega t)] + cc + o(\epsilon^2), \quad (7)$$

$$\hat{P}(x, y) = P_0 + \epsilon P_1(y) \exp[i(kx - \omega t)] + cc + o(\epsilon^2), \quad (8)$$

where U_0 and P_0 are mean flow quantities, ϵ is the amplitude of the T-S wave and cc stands for the complex conjugate of the preceding terms. The dimensionless frequency is represented by ω and the real part of k is the wavenumber whereas the imaginary part of k is the negative of the growth rate. von Kerczek⁴ treated the case for which Eqs. (6) - (8) are independent of x .

Substituting Eqs. (6) - (8) into Eqs. (1) - (3), subtracting the mean flow quantities and linearizing the resulting equations in the unsteady disturbance quantities, we obtain the following equations describing the T-S wave:

$$\mathcal{L}_1(U_1, V_1; k) \equiv DV_1 + ikU_1 = 0, \quad (9)$$

$$\mathcal{L}_2(U_1, V_1, P_1; k, \omega) \equiv i(U_0 k - \omega)U_1 + V_1 DU_0 + ikP_1 - \frac{1}{R}(D^2 - k^2)U_1 = 0, \quad (10)$$

$$\mathcal{L}_3(U_1, V_1, P_1; k, \omega) \equiv i(U_0 k - \omega)V_1 + DP_1 - \frac{1}{R}(D^2 - k^2)V_1 = 0, \quad (11)$$

$$U_1 = V_1 = 0 \quad \text{at } y = 0, \quad (12)$$

$$U_1, V_1, P_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty, \quad (13)$$

where $D \equiv \partial/\partial y$.

B. Stability Analysis

To study the stability of the basic state, we superpose small unsteady disturbances on the basic flow quantities according to

$$U(x, y, t) = \hat{U}(x, y) + \tilde{u}(x, y, t), \quad (14)$$

$$V(x, y, t) = \hat{V}(x, y) + \tilde{v}(x, y, t), \quad (15)$$

$$P(x, y, t) = \hat{P}(x, y) + \tilde{p}(x, y, t), \quad (16)$$

where $\hat{U}$, $\hat{V}$ and $\hat{P}$ represent the basic-state given by Eqs. (6) - (8) and $\tilde{u}$, $\tilde{v}$ and $\tilde{p}$ are the time dependent disturbances which are assumed to be small compared with the basic-state quantities.

Substituting Eqs. (14) - (16) into Eqs. (1) - (5), subtracting the basic-state quantities and linearizing the resulting equations in the unsteady disturbance quantities, we obtain

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0, \quad (17)$$

$$\frac{\partial \tilde{u}}{\partial t} + \hat{U} \frac{\partial \tilde{u}}{\partial x} + \tilde{u} \frac{\partial \hat{U}}{\partial x} + \hat{V} \frac{\partial \tilde{u}}{\partial y} + \tilde{v} \frac{\partial \hat{U}}{\partial y} + \frac{\partial \tilde{p}}{\partial x} - \frac{1}{R} \nabla^2 \tilde{u} = 0, \quad (18)$$

$$\frac{\partial \tilde{v}}{\partial t} + \hat{U} \frac{\partial \tilde{v}}{\partial x} + \tilde{u} \frac{\partial \hat{V}}{\partial x} + \hat{V} \frac{\partial \tilde{v}}{\partial y} + \tilde{v} \frac{\partial \hat{V}}{\partial y} + \frac{\partial \tilde{p}}{\partial y} - \frac{1}{R} \nabla^2 \tilde{v} = 0, \quad (19)$$

$$\tilde{u} = \tilde{v} = 0 \quad \text{at } y = 0, \quad (20)$$

$$\tilde{u}, \tilde{v}, \tilde{p} \rightarrow 0 \quad \text{as } y \rightarrow \infty. \quad (21)$$

To determine an approximate solution to Eqs. (17) - (21), we follow the method of multiple scales⁵ and seek a uniform expansion in the form

$$\tilde{u} = u_0(x_0, x_1, y, t) + \epsilon u_1(x_0, x_1, y, t) + o(\epsilon^2), \quad (22)$$

$$\tilde{v} = v_0(x_0, x_1, y, t) + \epsilon v_1(x_0, x_1, y, t) + o(\epsilon^2), \quad (23)$$

$$\tilde{p} = p_0(x_0, x_1, y, t) + \epsilon p_1(x_0, x_1, y, t) + o(\epsilon^2), \quad (24)$$

where $x_1 = \epsilon x_0$ is a slow scale. Substituting Eqs. (22) - (24) into Eqs. (17) - (21), using Eqs. (6) - (8) and equating the coefficients of like powers of ϵ , we obtain

Order ϵ^0

$$M_1(u_0, v_0) \equiv \frac{\partial u_0}{\partial v_0} + D v_0 = 0, \quad (25)$$

$$M_2(u_0, v_0, p_0) \equiv \frac{\partial u_0}{\partial t} + U_0 \frac{\partial u_0}{\partial x_0} + v_0 D U_0 + \frac{\partial p_0}{\partial x_0} - \frac{1}{R} (D^2 - \frac{\partial^2}{\partial x_0^2}) v_0 = 0, \quad (26)$$

$$M_3(u_0, v_0, p_0) \equiv \frac{\partial v_0}{\partial t} + U_0 \frac{\partial v_0}{\partial x_0} + D p_0 - \frac{1}{R} (D^2 - \frac{\partial^2}{\partial x_0^2}) v_0 = 0, \quad (27)$$

$$u_0 = v_0 = 0 \quad \text{at } y = 0, \quad (28)$$

$$u_0, v_0, p_0 \rightarrow 0 \quad \text{as } y \rightarrow \infty, \quad (29)$$

Order ϵ :

$$M_1(u_1, v_1) = - \frac{\partial U_0}{\partial x_1}, \quad (30)$$

$$\begin{aligned}
M_2(u_1, v_1, p_1) = & -U_0 \frac{\partial u_0}{\partial x_1} - [U_1 \exp(ikx_0 - i\omega t) + cc] \frac{\partial u_0}{\partial x_0} - iku_0 [U_1 \times \\
& \exp(ikx_0 - i\omega t) - cc] - [V_1 \exp(ikx_0 - i\omega t) + cc] Du_0 \\
& - v_0 [DU_1 \exp(ikx_0 - i\omega t) + cc] - \frac{\partial p_0}{\partial x_1} + \frac{2}{R} \frac{\partial^2 u_0}{\partial x_0 \partial x_1}, \quad (31)
\end{aligned}$$

$$\begin{aligned}
M_3(u_1, v_1, p_1) = & -U_0 \frac{\partial v_0}{\partial x_1} - [U_1 \exp(ikx_0 - i\omega t) + cc] \frac{\partial v_0}{\partial x_0} - iku_0 [V_1 \times \\
& \exp(ikx_0 - i\omega t) - cc] - [V_1 \exp(ikx_0 - i\omega t) + cc] Dv_0 \\
& - v_0 [DV_1 \exp(ikx_0 - i\omega t) + cc] + \frac{2}{R} \frac{\partial^2 v_0}{\partial x_0 \partial x_1}, \quad (32)
\end{aligned}$$

$$u_1 = v_1 = 0 \quad \text{at } y = 0, \quad (33)$$

$$u_1, v_1, p_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty. \quad (34)$$

III. Solution

A. Basic-State Solution

The solutions to Eqs. (9) - (13) can be expressed as

$$U_1 = A\zeta_{11}(y), \quad (35)$$

$$V_1 = A\zeta_{12}(y), \quad (36)$$

$$P_1 = A\zeta_{13}(y), \quad (37)$$

where A is a constant and ζ_{1n} ($n = 1, 2, 3$) are the eigenfunctions of the parallel problem given by the following equations:

$$\mathcal{L}_1(\zeta_{11}, \zeta_{12}; k) = 0, \quad (38)$$

$$\mathcal{L}_2(\zeta_{11}, \zeta_{12}, \zeta_{13}; k, \omega) = 0, \quad (39)$$

$$\mathcal{L}_3(\zeta_{11}, \zeta_{12}, \zeta_{13}; k, \omega) = 0, \quad (40)$$

$$\zeta_{11} = \zeta_{12} = 0 \quad \text{at } y = 0, \quad (41)$$

$$\zeta_{11}, \zeta_{12}, \zeta_{13} \rightarrow 0 \quad \text{as } y \rightarrow \infty. \quad (42)$$

B. Zeroth-Order Solution

The solution of the zeroth-order problem given by Eqs. (25) - (29)

is taken in the form

$$u_0 = B(x_1)\zeta_{21}(y)\exp[i(Kx_0 - \Omega t)] + cc, \quad (43)$$

$$v_0 = B(x_1)\zeta_{22}(y)\exp[i(Kx_0 - \Omega t)] + cc, \quad (44)$$

$$p_0 = B(x_1)\zeta_{23}(y)\exp[i(Kx_0 - \Omega t)] + cc, \quad (45)$$

where K is the dimensionless propagation constant and Ω is the dimensionless frequency for the secondary mode. The amplitude B is still an undetermined function; it is determined by imposing the solvability condition at the next level of approximation. The quasi-parallel Orr-Sommerfeld problem is

$$\mathcal{L}_1(\zeta_{21}, \zeta_{22}; K) = 0, \quad (46)$$

$$\mathcal{L}_1(\zeta_{21}, \zeta_{22}, \zeta_{23}; K, \Omega) = 0, \quad (47)$$

$$\mathcal{L}_3(\zeta_{21}, \zeta_{22}, \zeta_{23}; K, \Omega) = 0, \quad (48)$$

$$\zeta_{21} = \zeta_{22} = 0 \quad \text{at } y = 0, \quad (49)$$

$$\zeta_{21}, \zeta_{22}, \zeta_{23} \rightarrow 0 \quad \text{as } y \rightarrow \infty. \quad (50)$$

C. First-Order Problem

Since the homogeneous parts of Eqs. (30)-(34) are the same as Eqs. (25)-(29) and since the latter equations have a nontrivial solution, the inhomogeneous Eqs. (30)-(34) have a solution if, and only if, a solvability condition is satisfied. The inhomogeneous parts in Eqs. (30)-(34) contain terms proportional to $\exp[i(kx_0 - \omega t)]$, $\exp[i(kx_0 - \omega t) - i(\bar{K}x_0 - \omega t)]$, their complex conjugates and others. Secular terms will arise at this order when $k \approx 2K$ and $\omega \approx 2\Omega$; that is, when a subharmonic resonance exists. We consider the case of perfect time resonance and introduce a detuning parameter σ for the spatial part according to

$$k = 2K + \epsilon\sigma, \quad \sigma = O(1), \quad (51)$$

$$\omega = 2\Omega. \quad (52)$$

Using Eqs. (51) and (52), we obtain

$$(kx_0 - \omega t) - (\bar{K}x_0 - \Omega t) = (Kx_0 - \Omega t) + (\sigma x_1 + 2iK_1 x_0), \quad (53)$$

where K_1 is the imaginary part and $\bar{K}$ is the complex conjugate of K .

To determine B , we seek a particular solution for the first-order problem in the form

$$u_1 = z_1(y; x_1) \exp[i(Kx_0 - \Omega t)] + cc, \quad (54)$$

$$v_1 = z_2(y; x_1) \exp[i(Kx_0 - \Omega t)] + cc, \quad (55)$$

$$p_1 = z_3(y; x) \exp[i(Kx_0 - \Omega t)] + cc. \quad (56)$$

Substituting Eqs. (35)-(37), Eqs. (43)-(45) and Eqs. (51)-(56) into Eqs. (30)-(34) and equating the coefficients of $\exp[i(Kx_0 - \Omega t)]$, we obtain

$$\mathcal{L}_1(z_1, z_2; K) = g_1, \quad (57)$$

$$\mathcal{L}_1(z_1, z_2, z_3; K, \Omega) = g_2, \quad (58)$$

$$\mathcal{L}_3(z_1, z_2, z_3; K, \Omega) = g_3, \quad (59)$$

$$z_1 = z_2 = 0 \quad \text{at } y = 0, \quad (60)$$

$$z_1, z_2, z_3 \rightarrow 0 \quad \text{as } y \rightarrow \infty, \quad (61)$$

where g_1 , g_2 and g_3 are defined in Appendix A.

To obtain the solvability condition, we multiply Eqs. (57)-(59) by ζ_{21}^* , ζ_{22}^* and ζ_{23}^* , respectively, where the ζ^* 's are the solutions of the adjoint homogeneous problem, add the equations and integrate the resulting equation by parts from $y = 0$ to $y = \infty$. The adjoint problem corresponding to the eigenvalue K is

$$iK\zeta_{22}^* - D\zeta_{23}^* = 0, \quad (62)$$

$$i(U_0K - \Omega)\zeta_{23}^* + \zeta_{22}^*DU_0 - D\zeta_{21}^* - \frac{1}{R}(D^2 - K^2)\zeta_{23}^* = 0, \quad (63)$$

$$i(U_0K - \Omega)\zeta_{22}^* + iK\zeta_{21}^* - \frac{1}{R}(D^2 - K^2)\zeta_{22}^* = 0, \quad (64)$$

$$\zeta_{22}^* = \zeta_{23}^* = 0 \quad \text{at } y = 0, \quad (65)$$

$$\zeta_{21}^*, \zeta_{22}^*, \zeta_{23}^* \rightarrow 0 \quad \text{as } y \rightarrow \infty. \quad (66)$$

Then, the solvability condition can be expressed as

$$\int_0^\infty (g_1\zeta_{21}^* + g_2\zeta_{22}^* + g_3\zeta_{23}^*)dy = 0. \quad (67)$$

Substituting for g_1 , g_2 and g_3 from Appendix A into Eq. (67), we obtain the following differential equation for the evolution of B :

$$\frac{dB}{dx_1} = \frac{f_2}{f_1} AB \exp(i\alpha x_1 - 2K_1 x_0), \quad (68)$$

where f_1 and f_2 are given in quadratures in terms of ζ_{2n} , ζ_{2n}^* , k and K and they are defined in Appendix B. To account for the parallel growth rate, we let

$$b = B \exp(-K_1 x) \quad (69)$$

and substitute Eq. (69) into Eq. (68) to obtain

$$\frac{db}{dx_0} = -K_i b + \frac{f_2}{f_1} \epsilon A b \exp[(i\epsilon\sigma - 2K_i)x_0]. \quad (70)$$

So far, the flow is assumed to be parallel. To account for the nonparallel effects only, we follow Nayfeh, Saric and Mook⁶ and Saric and Nayfeh⁷ and obtain

$$\frac{db}{dx_0} = \left(\frac{f_3}{f_1} - K_i\right)b, \quad (71)$$

where f_3 is defined in Appendix B. Hence, to account for the combined effects of the subharmonic resonance and nonparallelism, we have

$$\frac{db}{dx} = \left(\frac{f_3}{f_1} - K_i\right)b + \frac{f_2}{f_1} \epsilon A b \exp(i\epsilon\sigma - 2K_i)x] \quad (72)$$

Note that when the nonparallel effects are neglected (i.e., $f_3 = 0$) Eq. (72) reduces to Eq. (70). Moreover, when the effects of subharmonic resonance are neglected, (i.e., $f_2 = 0$) Eq. (72) reduces to Eq. (71). Thus, Eq. (72) describes the nonparallel spatial growth of the amplitude of the subharmonic mode for different initial amplitudes of the fundamental mode.

IV. Computational Procedure

A. Solutions of the basic-state and the zeroth-order problems

The procedure for the solutions of the basic state and the zeroth-order problem, given by Eqs. (38)-(42) and Eqs. (46)-(50), respectively, are the same. Therefore, we will here explain the methodology only for the basic-state problem.

Equations 38-42 can be expressed as a system of first-order differential equations; that is,

$$\frac{dz}{dy} = Gz, \quad (73)$$

where z is a 4×1 matrix with the elements

$$z_1 = \zeta_{11}(y), \quad z_2 = D\zeta_{11}(y), \quad z_3 = \zeta_{12}(y), \quad z_4 = \zeta_{13}(y), \quad (74)$$

and the elements of the 4×4 G matrix are given in Appendix C.

To determine starting solutions for the integration of Eqs. (73), we assume that $U_0 = 1$, $DU_0 = 0$ and $D^2U_0 = 0$ at $y = y_0$ with y_0 being any value of y larger than the boundary layer thickness. The matrix G then has constant coefficients at $y = y_0$ and Eqs. (73) have solutions of the form

$$z_i = \sum_{j=1}^4 c_{ij} \exp(\lambda_j y) \quad \text{for } i = 1, 2, 3, 4, \quad (75)$$

where the c_{ij} are constants, the λ 's are the solutions of

$$|G - \lambda I| = 0, \quad (76)$$

and I is the identity matrix. Equation (76) has the roots

$$\lambda_{1,2} = \pm k, \quad \lambda_{3,4} = \pm [k^2 + i(k - \omega)R]^{\frac{1}{2}}. \quad (77)$$

Two of these roots have positive real parts and make the solution grow exponentially as $y \rightarrow \infty$ and must be discarded according to the boundary conditions. This leaves two linear independent solutions that decay exponentially with y . To use SUPORT, we express the boundary conditions at $y = y_0$ in the form

$$Qz = 0 \quad \text{at } y = y_0, \quad (78)$$

where Q is a 2×4 matrix with constant coefficients and given in Appendix C.

The eigenvalues are not known a priori and must be determined along with the eigenfunctions. For given values of ω and R , we guess a value for k , generate the matrix G at $y = y_0$ and integrate the system of equations from $y = y_0$ to $y = 0$. If the guessed value of k does not satisfy the boundary conditions at $y = 0$, k is incremented by using a Newton-Raphson scheme and the procedure is repeated until the boundary conditions are satisfied. Integration is done by using a technique developed by Scott and Watts⁸. This technique orthonormalizes the solution of the set of equations whenever a loss of independence is detected.

B. Solution of the adjoint problem

The solution procedure is exactly the same as for the solution of the basic-state problem. The coefficients of the z matrix are

$$z_1 = \zeta_{22}^*(y), \quad z_2 = D\zeta_{22}^*(y), \quad z_3 = \zeta_{23}^*(y), \quad z_4 = \zeta_{21}^*(y), \quad (79)$$

and the adjoint problem has the same eigenvalues as the zeroth-order problem.

C. Solution of the solvability condition

The calculations are repeated at different x locations to evaluate f_1 , f_2 , f_3 , k and K for a given frequency along the x -axis. A fourth-order fixed step size Runge-Kutta integration scheme is used to solve Eq. (72) to find the amplitude of the subharmonic mode for different initial amplitudes of the T-S mode.

V. Numerical Results and Concluding Remarks

Computations were performed for three different dimensionless frequencies of the subharmonic wave, $F = 44 \times 10^{-6}$, 52×10^{-6} and 60×10^{-6} , by using different values of ϵ , where $F = \Omega/R = \Omega^* \nu/U_\infty^2$ with Ω being the dimensionless boundary-layer frequency and Ω^* being the dimensional frequency of the subharmonic wave. Here, ϵ along with the amplitude function A , which is not a constant for nonparallel flows, determines the amplitude of the fundamental wave. The amplitude function A can be calculated by using the usual nonparallel theory^{6,7} and the calculated values of A for different frequencies are shown in Fig. 1. If A is normalized at the initial location, ϵ then represents the initial amplitude of the fundamental wave as a fraction of the mean flow. The calculations are started at the first neutral stability point of the fundamental wave and continued further downstream well into the unstable region of the subharmonic mode. The amplitude of the subharmonic mode, $b(x)$, is also normalized at the initial starting point.

Figures 2, 3 and 4 illustrate the variations of $b(x)$ as functions of R and ϵ for three different values of F . The solid curves are for the case of no interaction between the fundamental and the subharmonic waves. The point where the solid curve has a minimum is the first neutral stability point of the subharmonic mode.

The results show that the presence of the fundamental mode causes the amplitude of the subharmonic mode to oscillate before reaching its unstable region. As the subharmonic wave approaches its unstable region, it starts to amplify at a faster rate. Finally, it breaks away from the oscillatory growth region and amplifies considerably. The amplification of the subharmonic mode increases with increasing values of ϵ .

The amplification rate of the subharmonic wave can be much faster than that of the fundamental wave. However, as the amplitude of the subharmonic builds up, one can no longer neglect its effect on the fundamental wave and a nonlinear analysis is necessary to account for the interaction.

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APPENDIX A

$$g_1 = - \frac{dB}{dx_1} \zeta_{21} \quad (A1)$$

$$g_2 = - U_0 \frac{dB}{dx_1} \zeta_{21} - \frac{dB}{dx_1} \zeta_{23} + \frac{2iK}{R} \frac{dB}{dx_1} \zeta_{21} - [i(k - R)\zeta_{11} \bar{\zeta}_{21} + \zeta_{12} D\bar{\zeta}_{21} + D\zeta_{11} \bar{\zeta}_{22}] A \bar{B} \exp(i\sigma x_1 - 2K_i x_0) \quad (A2)$$

$$g_3 = - U_0 \frac{dB}{dx_1} \zeta_{22} + \frac{2iK}{R} \frac{dB}{dx_1} \zeta_{22} - [-R\zeta_{11} \bar{\zeta}_{22} + ik\zeta_{12} \bar{\zeta}_{21} + \zeta_{12} D\bar{\zeta}_{22} + D\zeta_{12} \bar{\zeta}_{22}] A \bar{B} \exp(i\sigma x_1 - 2K_i x_0) \quad (A3)$$

APPENDIX B

$$f_1 = \int_0^{\infty} [-\zeta_{21}\zeta_{21}^* - (U_0\zeta_{21} + \zeta_{23})\zeta_{22}^* - U_0\zeta_{22}\zeta_{23}^* + \frac{2iK}{R}(\zeta_{21}\zeta_{22}^* + \zeta_{22}\zeta_{23}^*)]dy \quad (B1)$$

$$f_2 = \int_0^{\infty} [i(k - R)\zeta_{11}\bar{\zeta}_{21} + \zeta_{12}D\bar{\zeta}_{21} + D\zeta_{11}\bar{\zeta}_{22}]\zeta_{22}^* + [-iR\zeta_{11}\bar{\zeta}_{22} + ik\zeta_{12}\bar{\zeta}_{21} + \zeta_{12}D\bar{\zeta}_{22} + D\zeta_{12}\bar{\zeta}_{22}]\zeta_{23}^* dy \quad (B2)$$

$$f_3 = \epsilon_2 \int_0^{\infty} [\frac{\partial \zeta_{21}}{\partial x_2} \zeta_{21}^* + \zeta_{22}^*(U_0 \frac{\partial \zeta_{21}}{\partial x_2} + \frac{\partial U_0}{\partial x_2} \zeta_{21} + V_0 D\zeta_{21} + \frac{\partial \zeta_{23}}{\partial x_2}) + (U_0 \frac{\partial \zeta_{22}}{\partial x_2} + V_0 D\zeta_{22} + \zeta_{22}DV_0)\zeta_{23}^* - \frac{2iK}{R}(\frac{\partial \zeta_{21}}{\partial x_2} \zeta_{22}^* + \frac{\partial \zeta_{22}}{\partial x_2} \zeta_{23}^*) - \frac{2i}{R} \frac{dK}{dx_2} (\zeta_{21}\zeta_{22}^* + \zeta_{22}\zeta_{23}^*)]dy \quad (B3)$$

where $x_2 = \epsilon_2 x_0$ and $\epsilon_2 = R^{-1}$ expressing the slight nonparallelism of the flow.

APPENDIX C

$$g_{11} = 0, \quad g_{12} = 1, \quad g_{13} = 0, \quad g_{14} = 0 \quad (C1)$$

$$g_{21} = i(U_0 k - \omega)R + k^2, \quad g_{22} = 0, \quad g_{23} = R \frac{dU_0}{dy}, \quad g_{24} = ikR \quad (C2)$$

$$g_{31} = -ik, \quad g_{32} = g_{33} = g_{34} = 0 \quad (C3)$$

$$g_{41} = 0, \quad g_{42} = -ik/R, \quad g_{43} = -[i(U_0 k - \omega) + k^2/R], \quad g_{44} = 0 \quad (C4)$$

Q is a 2x4 matrix consisting of the last two rows of the matrix B^{-1} .

The matrix B has the elements:

$$b_{11} = b_{12} = b_{13} = b_{14} = 1 \quad (C5)$$

$$b_{21} = -k, \quad b_{22} = \tilde{k}, \quad b_{23} = k, \quad b_{24} = -\tilde{k} \quad (C6)$$

$$b_{31} = i, \quad b_{32} = ik/\tilde{k}, \quad b_{33} = -i, \quad b_{34} = -ik/\tilde{k} \quad (C7)$$

$$b_{41} = (\omega/k - 1), \quad b_{42} = 0, \quad b_{43} = (\omega/k - 1), \quad b_{44} = 0 \quad (C8)$$

where

$$\tilde{k} = [k^2 + i(k - \omega)R]^{\frac{1}{2}} \quad (C9)$$

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Figure Captions

Figure 1. Amplitude of the fundamental mode at different frequencies.

Figure 2. Amplitude of the subharmonic mode for different initial amplitudes of the fundamental mode at $F = 44 \times 10^{-6}$.

Figure 3. Amplitude of the subharmonic mode for different initial amplitudes of the fundamental mode at $F = 52 \times 10^{-6}$.

Figure 4. Amplitude of the subharmonic mode for different initial amplitudes of the fundamental mode at $F = 60 \times 10^{-6}$.

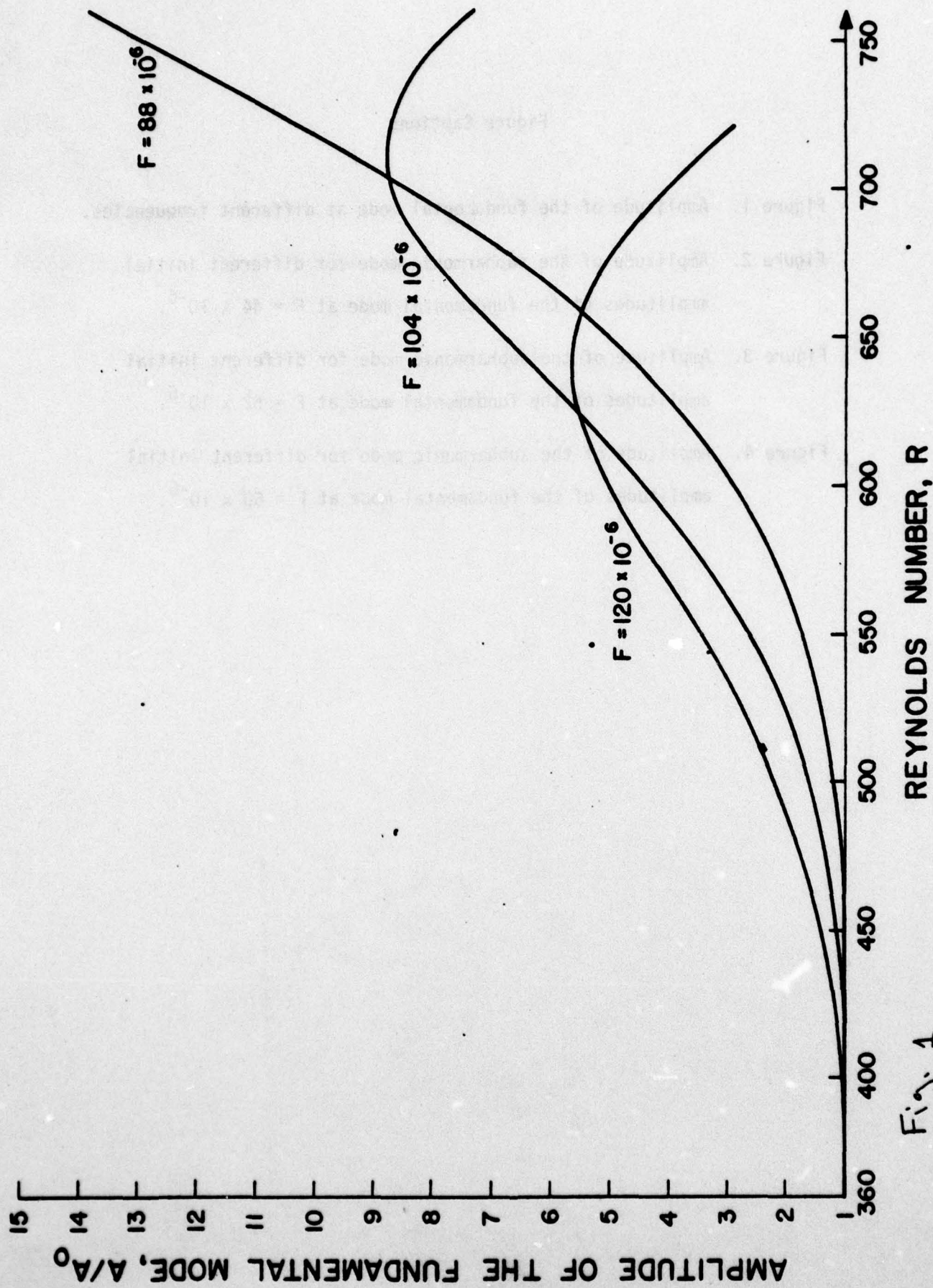


Fig. 1

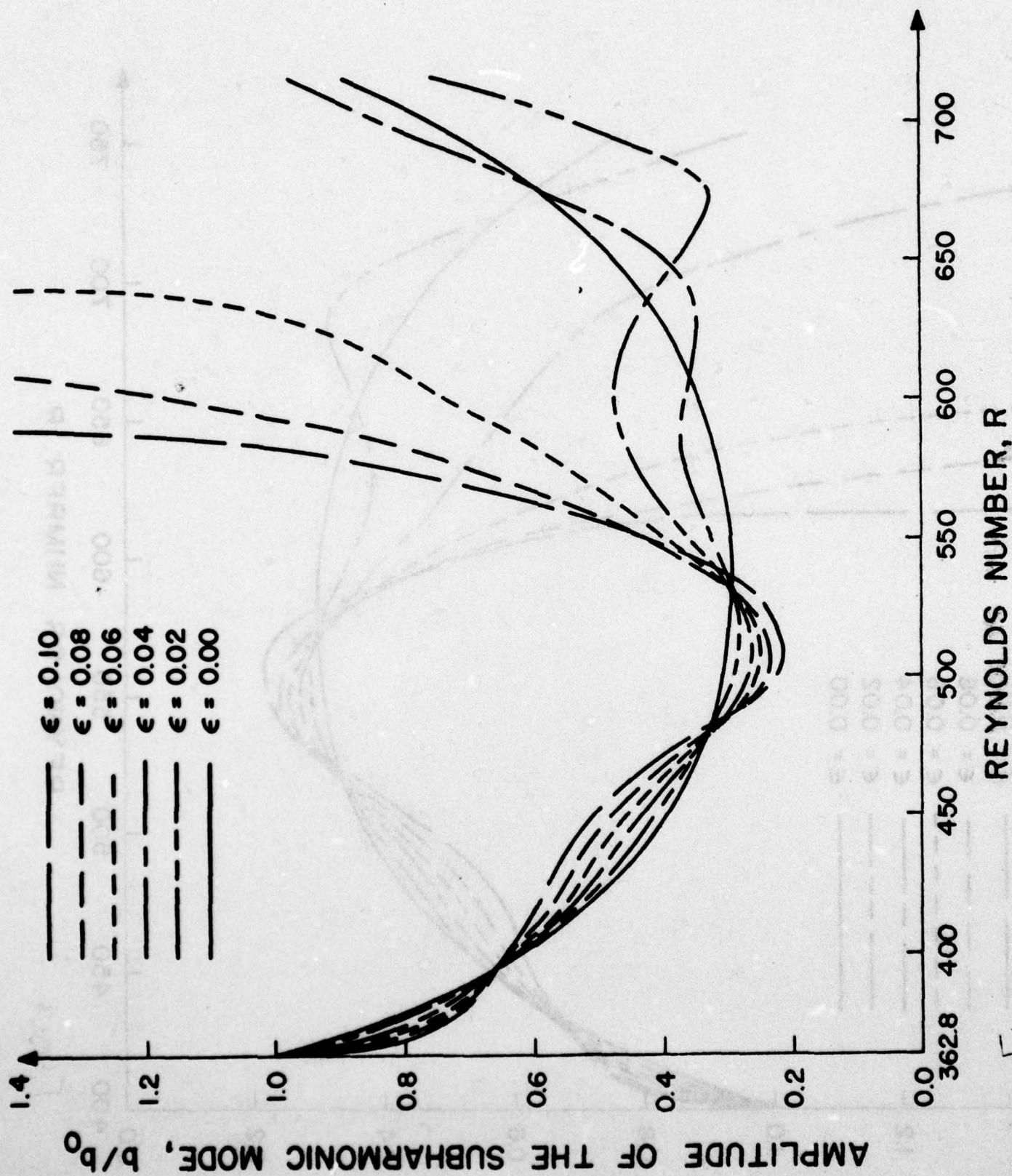


Fig. 2

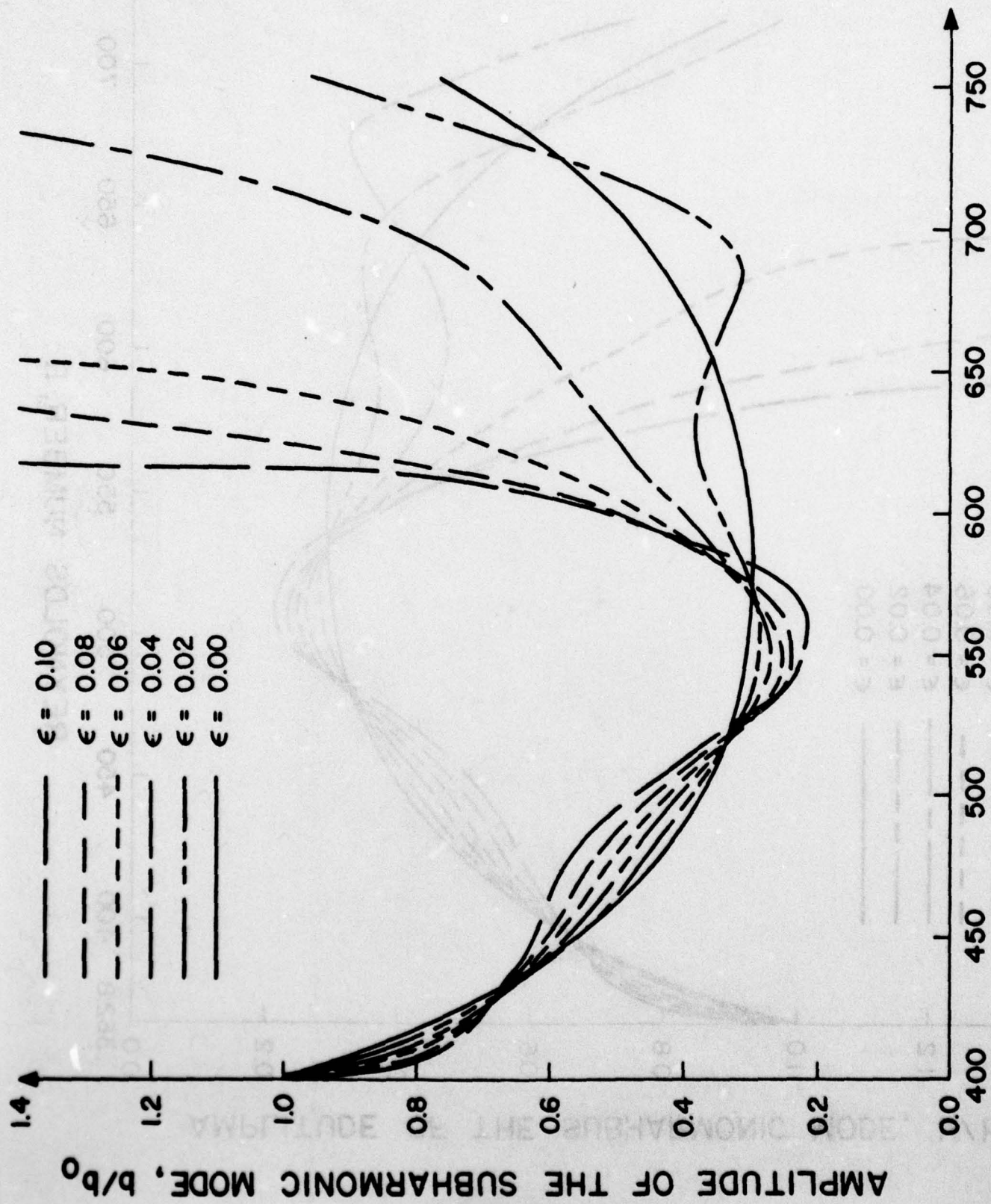


Fig. 3

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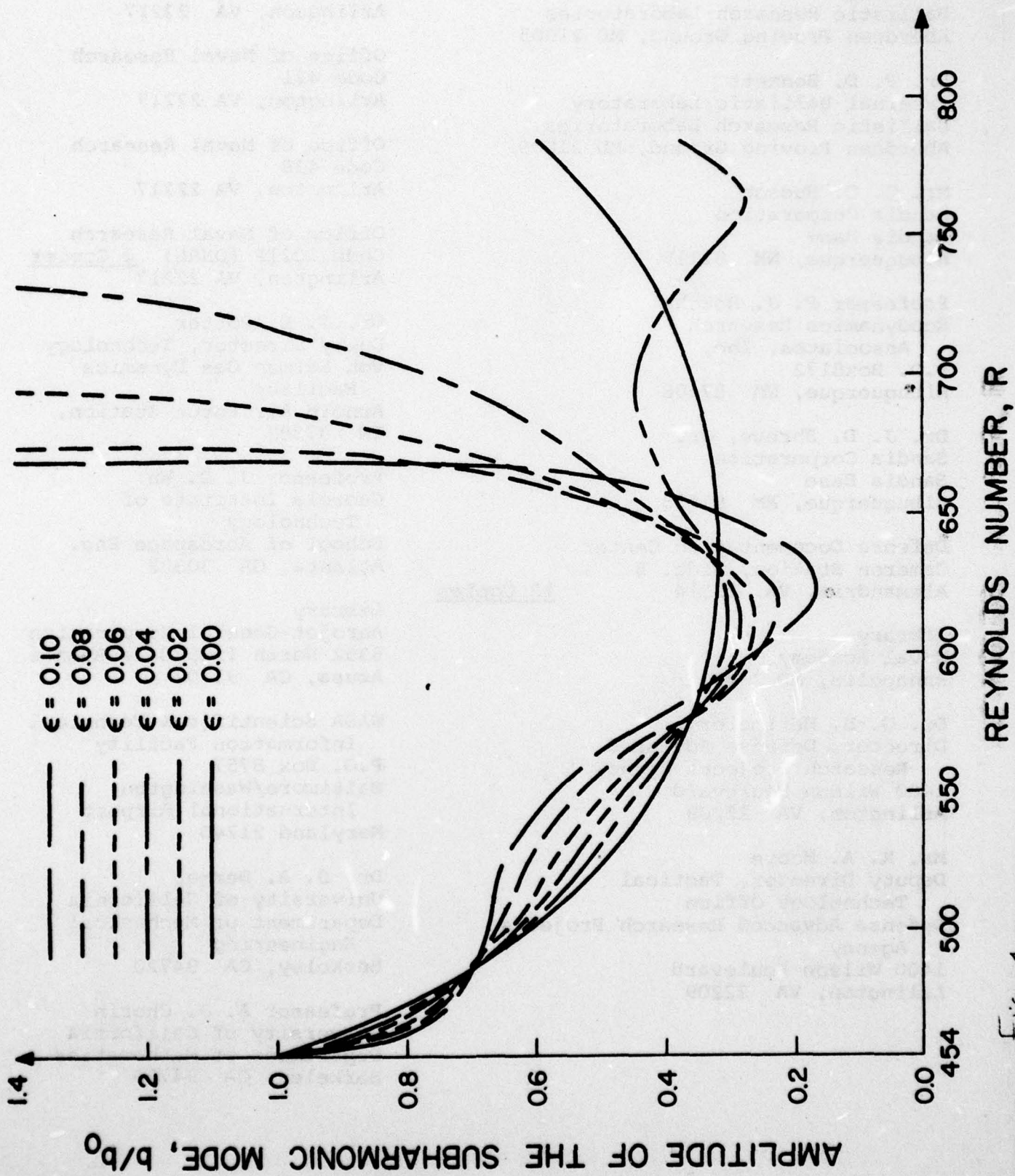


Fig. 4

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