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THE OUTPUT OF A TRUE CORRELATOR WITH NOISY INPUTS.(U)
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U. S. Navy Underwater Sound Laboratory
Fort Trumbull, New London, Connecticut

(6) **THE OUTPUT OF A TRUE CORRELATOR WITH NOISY INPUTS.**

by

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Consider the true correlator of the figure with noisy inputs:

$$r_1(t) = s(t) + n_1(t)$$

(9) Technical memo.

$$r_2(t) = s(t) + n_2(t)$$

where $s(t)$ is a signal common to both inputs and $n_1(t)$, $n_2(t)$ are noise waveforms.

$$\begin{array}{l} r_1(t) = s(t) + n_1(t) \rightarrow \\ r_2(t) = s(t) + n_2(t) \rightarrow \end{array} \boxed{\text{True Correlator}} \longrightarrow \rho(\tau)$$

Figure

(12) 3p.

LEVEL II

The output of the correlator is

$$(1) \quad \rho(\tau) = \frac{\int_{-\infty}^{\infty} r_1(t)\, r_2(t+\tau)\, dt}{\left[\int_{-\infty}^{\infty} r_1^2(t)\, dt \cdot \int_{-\infty}^{\infty} r_2^2(t)\, dt\right]^{\frac{1}{2}}}$$

Assume the signal is not correlated with either noise and let

$$S = \int_{-\infty}^{\infty} s^2(t)\, dt = \text{signal power}$$

$$N_i = \int_{-\infty}^{\infty} n_i^2(t)\, dt = i^{th} \text{ noise power, } i = 1,2$$

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Then

(2) $$\rho(t) = \frac{\int_{-\infty}^{\infty} (s(T) + n_1(T)) \, (s(t+T) + n_2(t+T))\,dT}{\left[\int_{-\infty}^{\infty} (s(t) + n_1(t)^2 dt \cdot \int_{-\infty}^{\infty} (s(t) + n_1(t))^2 dt\right]^{\frac{1}{2}}}$$

Evaluation of the integrals gives

(3) $$\rho(t) = \frac{S\,\rho_{ss}(t) + (N_1 N_2)^{\frac{1}{2}}\,\rho_{n_1 n_2}(t)}{\left[(S+N_1)\ (S+N_2)\right]^{\frac{1}{2}}}$$

which we can write as

(4) $$\rho(t) = \frac{(S/N_1)^{\frac{1}{2}}\,(S/N_2)^{\frac{1}{2}}\,\rho_{ss}(t) + \rho_{n_1 n_2}(t)}{\left[(1 + S/N_1)\ (1 + S/N_2)\right]^{\frac{1}{2}}}$$

or

(5) $$\rho(t) = \frac{\rho_{ss}(t) + (N_1/S)^{\frac{1}{2}}\,(N_2/S)^{\frac{1}{2}}\,\rho_{n_1 n_2}(t)}{\left[(1 + N_1/S)\ (1 + N_2/S)\right]^{\frac{1}{2}}}.$$

If both signal-to-noise ratios are large ($S/N_i >> 1$ or $N_i/S << 1$, $i = 1,2$) we can use the approximation, ignoring terms higher than the first power,

(6) $$\rho(t) \simeq \left[1 - \tfrac{1}{2}(N_1/S + N_2/S)\right]\rho_{ss}(t) + (N_1/S)^{\frac{1}{2}}\,(N_2/S)^{\frac{1}{2}}\ n_1 n_2(t),$$

if both signal-to-noise ratios are small ($S/N_i << 1$, $i = 1,2$), then the approximation becomes

(7) $$\rho(t) = (S/N_1)^{\frac{1}{2}}\,(S/N_2)^{\frac{1}{2}}\,\rho_{ss}(t) + \left[1 - \tfrac{1}{2}(S/N_1 + S/N_2)\right]\rho_{n_1 n_2}(t).$$

If one signal-to-noise ratio is high and the other is low (say $S/N_1 \gg 1, S/N_2 << 1$) then

(8) $$\rho(t) = \frac{(S/N_2)^{\frac{1}{2}}\,\rho_{ss}(t) + (N_1/S)^{\frac{1}{2}}\,\rho_{n_1 n_2}(t)}{\left[(1 + N_1/S)\ (1 + S/N_2)\right]^{\frac{1}{2}}}$$

give the approximation, ignoring powers higher than the first,

(9) $$\rho(t) \approx (S/N_2)^{\frac{1}{2}} \rho_{ss}(t) + (N_1/S)^{\frac{1}{2}} \rho_{n_1 n_2}(t).$$

Hence, for high signal-to-noise ratios, the correlator output $\rho(\tau)$ approximates the signal autocorrelation $\rho_{ss}(\tau)$, for low signal-to-noise ratios $\rho(\tau)$ follows the noise correlation $\rho_{n_1 n_2}(\tau)$, and in the mixed case, the correlator output is mixed. In fact, if both signal-to-noise ratios are unity (0db), the true correlator output is just the arithmetic mean of the signal and noise correlations

(10) $$\rho(t) = \frac{\rho_{ss}(t) + \rho_{n_1 n_2}(t)}{2} .$$

From the above calculation, we see that the correlator output will decrease in amplitude in the presence of noise, even if the noise cross-correlation is zero. If the noise correlation does not vanish, then this will appear as a bias on the true correlator output.