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A FAST-RESPONSE, PHASE-SENSITIVE DEMODULATOR AND MODULATOR CIRC--ETC(U)

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# A FAST-RESPONSE, PHASE-SENSITIVE DEMODULATOR AND MODULATOR CIRCUIT FOR THE IMPROVEMENT OF SERVO SYSTEMS


Anthony F. Magaruel


USL RESEARCH AND DEVELOPMENT REPORT NO. 293

LEVEL II



## U.S. Navy Underwater Sound Laboratory

Fort Trumbull, New London, Conn.

## SUMMARY PAGE

### THE PROBLEM

The problem was to develop circuitry which would improve the performance of carrier-type servo systems operating from one-speed synchro data.

### FINDINGS

A circuit consisting of a simple, reliable electromechanical unit with the characteristics of a fast-response, phase-sensitive demodulator and modulator was developed. Through its use, the limitations imposed on servo-system performance by synchro residual voltage and inadequate carrier-system compensation are removed. A potentially substantial improvement in the static and dynamic accuracies of carrier-type servo systems is thus provided.

### RECOMMENDATIONS

Full advantage of the circuitry can be realized only when it is associated with precision gear-train assemblies having a minimum of backlash. Where high degrees of accuracy are required, gear-train assemblies should be precision-adjusted, with antibacklash gears between the error-sensing synchro and the prime mover.

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by

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## ABSTRACT

The extensive use of synchros as data-transmission and error-sensing devices in automatic control systems suggests the need for improved circuitry to decrease the complexity of carrier-system compensation and, if possible, to increase the resolution of the synchros themselves. Past methods, while affording definite improvement in carrier-system performance, have not proved completely satisfactory because they inevitably introduce an unwanted time constant into the system. This added time constant often limits the degree of improvement that can be provided in the system. An electromechanical circuit that is a satisfactory solution to the dual problem of carrier-system compensation and synchro resolution is described. The circuit is essentially a fast-response demodulator and modulator unit that is simple to adjust and reliable in its application. Through its use, carrier systems can achieve a degree of performance hitherto unattainable.

## ADMINISTRATIVE INFORMATION

This report was originally prepared as a dissertation in partial fulfillment of the requirements for the degree of Master of Science at the University of Connecticut, 1955, under Laboratory problem number D1B2F and Navy index number NE-050961-2. The material was considered to be pertinent to the Laboratory's interest in improved data-transmission systems.


## ACKNOWLEDGMENTS

The author wishes to express his gratitude to Dr. John M. Ide, Chief Scientist, and Captain Edward J. Fahy, Commanding Officer and Director, of the U. S. Navy Underwater Sound Laboratory, for their cooperation with the University of Connecticut in arranging the graduate extension courses which made this investigation possible. The author is further indebted to Dr. Ralph J. Kochenburger, Professor of Electrical Engineering at the University of Connecticut, and to J. T. Kroenert, formerly of the Underwater Sound Laboratory, for their encouragement and assistance in carrying out this program of graduate study.

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## INTRODUCTION

In many applications, and particularly in modulated carrier-type servo systems, there is need for a phase-sensitive demodulator and modulator circuit having a fast response. It can be readily demonstrated that through the use of such a circuit the performance limits of a specific servo system can be greatly extended, and in many cases the complexity of system analysis and design may be appreciably reduced.

The analysis of a modulated carrier-type servo system in which the band width is a small percentage of the carrier frequency is generally performed with acceptable accuracy on the basis of a continuous system. In such a carrier system, the method of analysis is not altered by the inclusion of the fast-response, phase-sensitive demodulator and its associated modulator circuit. In systems where the band width approaches the usable limit of approximately 50 per cent of the carrier frequency, the analysis is carried out with increased validity through the use of the demodulator and modulator circuit by following the techniques developed by Linvill,[1] Ragazzini and Zadeh,[2] and others.[3] Here, however, some justified approximations to the functions of the demodulator and modulator circuit are made in order to reduce and simplify the mathematical analysis required to arrive at the carrier-system transfer functions.

The criteria of system performance in this discussion are based on the system velocity constant, $K_v$, and the prescribed response to a step function of position input, although other criteria could just as well have been used. In addition, the circuitry and all components are considered to be operating in their linear region. Since the description and analysis of the demodulator and modulator circuit and its application to a modulated carrier-type servo system is of prime importance in this presentation, no attempt is made to introduce the effects of various nonlinearities on the performance of the system.

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[1] William K. Linvill, "Sampled Data Control Systems Studied through Comparison of Sampling with Amplitude Modulation," *Transactions of the American Institute of Electrical Engineers*, vol. 70, 1951, pt. II, pp. 1779-1788.

[2] J. R. Ragazzini and L. A. Zadeh, "The Analysis of Sampled Data Systems," *Transactions of the A.I.E.E.*, vol. 71, 1952, pt. II, pp. 225-234.

[3] See G. W. Johnson and D. P. Lindorff, "Analysis of Sampled-Data Control Systems," *Applications and Industry*, A.I.E.E., July 1954, pp. 147-153.

## THE PROBLEM

Synchro data transmission is used extensively in instrument-type servo systems. These systems, generally classed as those requiring less than 100 watts of power drive, most often employ two-phase induction motors as prime movers. The advantages of such servo systems include the convenience of using AC amplifiers throughout and the ease and accuracy with which position information is transmitted and position error is sensed. However, there are two well-known factors which impose limitations on the performance of feedback control systems operating from one-speed synchro data: (1) synchro null residual voltage, composed mainly of quadrature voltage, which is present in all synchro data-transmission systems; and (2) adequate carrier-system compensation, which is a continuing problem. The first factor limits the amplifier voltage gain which can be introduced between the synchro control transformer and the drive motor; the second factor involves the common use of carrier-type compensating networks, which require that any gain-phase compensation be of the phase-lead type. This type of compensation, with the exception of tachometric feedback, is not always desirable, however, because of its susceptibility to higher-frequency noise components.

The problem of carrier-system compensation has been widely treated in the literature,[4] but no wholly satisfactory solution has been found. Various types of notch networks, including the bridged-T, the twin-T, and tuned RLC circuits, have been analyzed. All these networks are very restricted in their range and are subject to carrier-frequency fluctuations. In addition, many of these networks are comparatively difficult to synthesize, and they entail many algebraic and numerical computations.[5]

The process of demodulating the modulated carrier, adding noncarrier-type compensation, and, subsequendy, restoring the carrier form of transmission, which is necessary for AC amplification and motor drive, has been treated in the literature.[6] (The two usual methods of demodulation are

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[4] See H. Elmore Blanton, "Carrier Compensation for Servomechanisms," *Journal of The Franklin Institute*, vol. 250, 1950, pt. I, p. 391; pt. II, p. 525; Donald MacDonald, "Electromechanical Lead Networks for AC Servomechanisms," *Review of Scientific Instruments*, vol. 20, November 1949, p. 775.

[5] See H. M. James, N. B. Nichols, and R. S. Phillips, *Theory of Servomechanisms*, McGraw-Hill Book Co., Inc., New York, 1947, Radiation Laboratory Series, vol. 25, pp. 117-124.

[6] See Blanton and MacDonald, *op. cit.*

mean rectification and peak rectification. Both require a filter circuit to attenuate the carrier-frequency component and its harmonics. This filter inevitably introduces unwanted attenuation and phase shift of the modulation.) Although this process eliminates quadrature voltage and allows noncarrier-type compensation to be added, it can be seen that the methods employed introduce an additional time constant into the system, thereby increasing the complexity of the system analysis and design.

A more recent development involving the use of the synchronous-vibrator circuit has proved to be satisfactory in the application of carrier-system compensation, but it does not reduce the synchro null residual voltage.

Thus, if the advantages of modulated carrier systems operating from synchro data are to be fully realized, some circuitry, preferably simple, is required to remove or reduce the synchro null residual voltage and permit the unrestricted use of adequate compensation. This circuitry should not add unwanted complexities to the original system transfer function.

## PROPOSED SOLUTION

### Qualitative Analysis

The proposed solution to the two problems of quadrature-voltage elimination and carrier-system compensation consists of a comparatively simple electromechanical circuit having the properties of a combined fast-response, phase-sensitive demodulator and modulator unit. A schematic diagram of the circuit and its application to a synchro-data system are shown in Fig. 1.

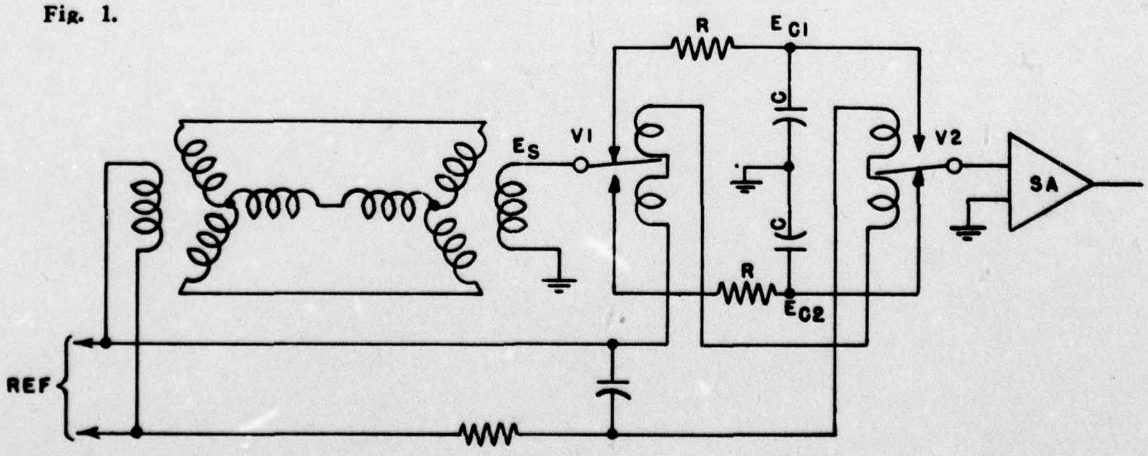


Fig. 1 - Schematic Diagram of Basic Application of the Demodulator and Modulator Circuit

The synchronous vibrators, V1 and V2, are the break-before-make type, the time between break and make corresponding to a very small percentage of the carrier period, < 5 per cent. They are so connected that the synchro-error voltage, $E_s$, is applied to the RC networks during alternate half cycles of the carrier frequency and is transferred to the input of the servo amplifier during successive alternate half cycles. In effect, the second vibrator, V2, samples and holds, for approximate half-cycle periods, the signals processed via the RC networks during the preceding half-cycle periods by the first vibrator, V1.

The functions of the vibrators and their associated RC networks are shown in Fig. 2. The

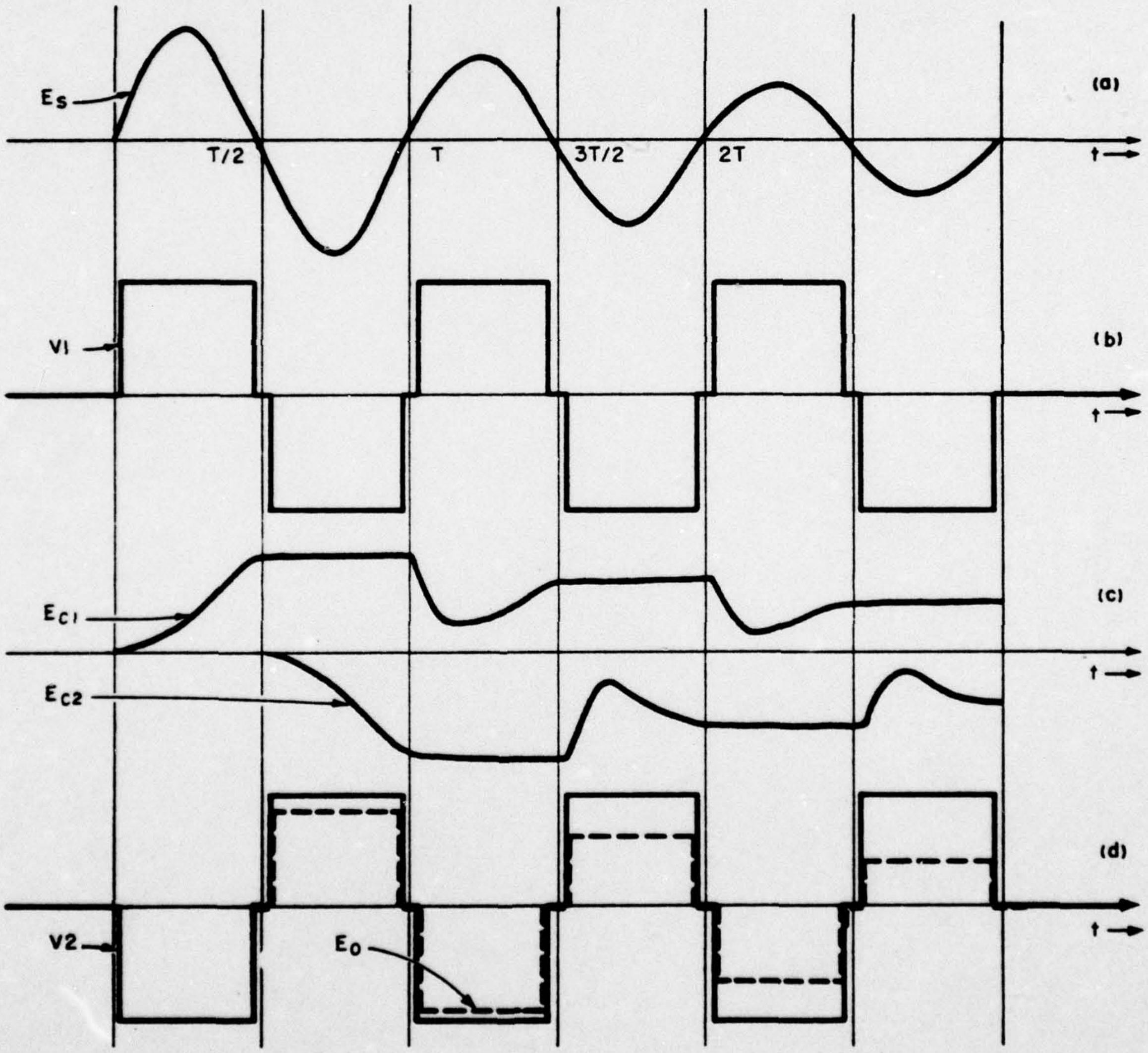


Fig. 2 - Time Function of the Synchronous-Vibrator Circuit

modulated carrier voltage, $E_s$, is given as a function of time in Fig. 2A. For purposes of simplicity, vibrator V1 is considered to be synchronized with $E_s$ so that the upper and lower contacts make and break at the times indicated in Fig. 2B. In order to determine the voltages $E_{c1}$ and $E_{c2}$ appearing across the capacitors, a transient solution is made, although it will be shown that a steady-state solution is adequate. The transient solution provides the integrating time required to ensure that the voltages $E_{c1}$ and $E_{c2}$, at the time the contactors of vibrator V1 break, are a maximum for each half cycle of input-error voltage. Voltages $E_{c1}$ and $E_{c2}$ are given as functions of time in Fig. 2C for the applied voltage $E_s$ of Fig. 2A and for a correctly chosen integrating time. Represented in Fig. 2D are the contact periods of vibrator V2 (solid line) and the resultant voltage $E_o$ (dotted line), which is applied to the input of the servo amplifier. Thus, vibrator V1 and its associated RC networks effectively demodulate each half cycle of input-signal voltage and leave a measured value of this voltage across the capacitors until the next contact periods. Vibrator V2 samples and holds each last measured value for an approximate half-cycle period and, at the same time, modulates this voltage at the carrier frequency. The modulated carrier system is thereby transformed to a true sampled-data-and-hold system.

The integrating time RC, which is required to provide maximum voltages $E_{c1}$ and $E_{c2}$ for an input voltage $E_s \sin \omega t$, is identical with the time required to provide minimum voltages $E_{c1}$ and $E_{c2}$ for an input voltage $E_s \cos \omega t$. Thus, the quadrature voltage is eliminated by the proper selection of the RC time constant. Since the integrating time required to supply these voltages is approximately one tenth of the carrier period, the time constant added to the servo system can be considered negligible. Furthermore, noncarrier-type compensation can now be introduced, since demodulation occurs within the synchronous-vibrator circuit.

In actual practice, the vibrators are not synchronized with the carrier as shown in Fig. 2 but are phased either to lead or to lag the reference voltage by $\pi/2$ radians. With the reference field of the two-phase induction motor connected directly to the reference line voltage, the control field voltage is thus placed in the proper phase for motor operation. Since the synchro-error voltage generally leads the reference voltage by a phase angle of $\pi/6$ radians, Fig. 3 is a more accurate representation of the circuit function. As a result of this phasing procedure, the time constant required to produce maximum voltages $E_{c1}$ and $E_{c2}$ is reduced, and the carrier peak voltage is

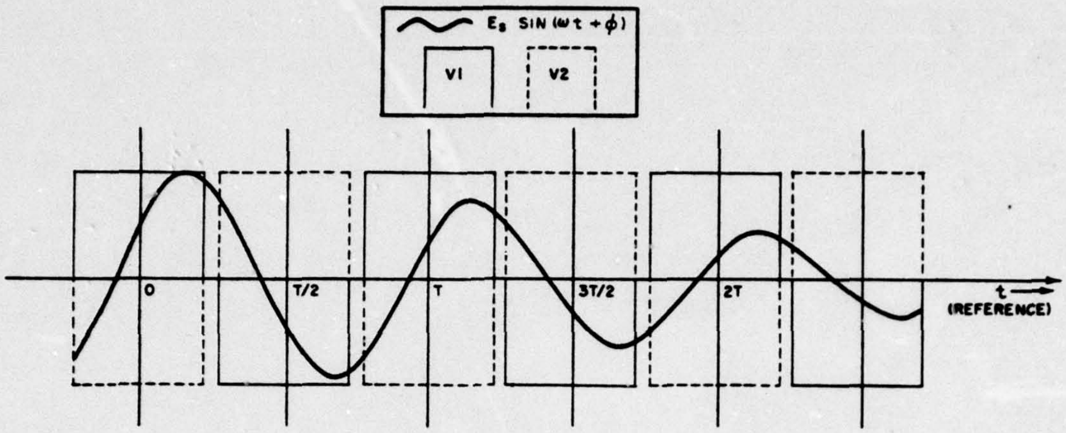


Fig. 3 - Time Function of the Synchronous-Vibrator Circuit after Adjustment-of-Phase Correction

almost immediately transferred to the contactor of vibrator V2. The time delay associated with carrier peak value and actual sampling is thus diminished to a negligible value.

## Mathematical Analysis

In the determination of the circuit transfer function, it is necessary to treat the half-cycle periods of one polarity only, since the operation of the synchronous-vibrator circuit is symmetrical. In order to analyze the function of synchronous vibrator V1, it is convenient to determine first the proper time constant required to produce maximum signal-voltage output and zero quadrature-voltage output at the instant of contactor break. Figure 4A shows the carrier-signal input voltage $E_s \sin(\omega t + \phi)$ and the quadrature component during one contact period. Figure 4B is the equivalent simple circuit.

The output voltage $E_c$ in terms of $e_s$ during the first contact period can be written as:

$$E_c = \frac{e_s}{j\omega RC + 1},$$

or, expressed in Laplace transforms, as:

$$E_c(S) = \frac{E_s}{r} \frac{S \sin \phi + \omega \cos \phi}{(S^2 + \omega^2)(S + 1/r)},$$

where

$$\tau = RC$$

and

$$S = \text{Laplace operator}.$$

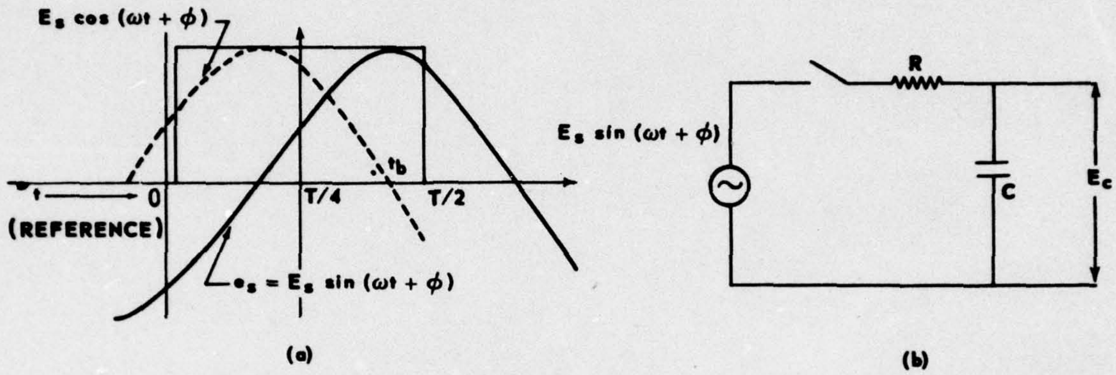


Fig. 4 - Time Response of One-Half Carrier Period and Its Equivalent Circuit

The instantaneous value of $E_c$ is obtained from the inverse transform:

$$E_c(t) = E_s \left[ \frac{\omega\tau \cos\phi - \sin\phi}{1 + \omega^2\tau^2} \epsilon^{-t/\tau} + \frac{1}{\sqrt{1 + \omega^2\tau^2}} \sin(\omega t + \phi - \psi) \right], \quad (1)$$

where

$$\psi = \tan^{-1} \omega\tau.$$

During the second and successive contact periods, the instantaneous value of $E_c$ is given by:

$$E_c(t) = E_s \left[ \frac{\omega\tau \cos\phi - \sin\phi}{1 + \omega^2\tau^2} \epsilon^{-t/\tau} + \frac{1}{\sqrt{1 + \omega^2\tau^2}} \sin(\omega t + \phi - \psi) \right] + k\epsilon^{-t/\tau}, \quad (2)$$

where k is the voltage remaining across the capacitor at each preceding break time.

To determine the value of $\tau$, which will produce $E_{c_{max}}$ at the end of the first contact period, the time derivative of Eq. (1) is equated to zero:

$$(\omega\tau \cos\phi - \sin\phi)\,\epsilon^{-t_b/\tau} = \omega\tau\sqrt{1+\omega^2\tau^2}\,\cos(\omega t_b + \phi - \psi)\,.$$

This equation can be solved graphically to obtain a solution for $\tau$ for a given break time, $t_b$. However, Fig. 4A indicates that for the conditions shown: $\omega t_b + \phi - \psi \cong \pi/2$. It is also evident that $\tau << t_b$. Therefore, a good approximation to the solution for $\tau$ is obtained from the steady-state portion of the time solution for $E_c$ given in Eqs. (1) and (2), as follows:

$$E_{c_{ss}} = E_s \frac{1}{\sqrt{1+\omega^2\tau^2}} \sin(\omega t_b + \phi - \psi), \tag{3}$$

which, of course, is a maximum when $(\omega t_b + \phi - \psi) = \pi/2$. Therefore, since $\omega t_b$ and $\phi$ are known, and $\psi = \tan^{-1}\omega\tau$, the solution for $\tau$ is readily found:

$$\tau = 1/\omega \tan(\omega t_b + \phi - \pi/2)\,. \tag{4}$$

The use of the steady-state solution for obtaining $\tau$ is also justified by the following observation. The time response of the network during the second and successive contact periods includes the term $k\epsilon^{-t_b/\tau}$. When actual values are substituted for $t_b/\tau$, this transient approaches a negligible value $(0.0004\,E_s)$ at each successive break time, indicating that the peak value of the carrier has been reached at the end of the first contact period. Also, it should be obvious from Eq. (3) that when $(\omega t_b + \phi - \psi) = \pi/2$ the quadrature component must produce zero voltage across the capacitor at the break time.

It can, therefore, be concluded that a good approximation to the transfer function representing $E_c(t)$ vs. $E_s(t)$ is unity, including a small time delay which, for all practical purposes, can be neglected. What is wanted, however, is $E_c(t)$ vs. $e(t)$, where $e(t)$ is the modulating voltage.

It is obvious that values of $e(t)$ occurring at the carrier nulls have no effect on the modulated output.[7] In fact, where peak demodulation is employed, the only values of $e(t)$ which have any effect on the system output are those which exist at the carrier peaks. Therefore, a modulated carrier system can, in a sense, be approximated by a sampled-data system having a sampling rate which is twice the rate of the carrier frequency. Sampling of the carrier peaks is, in effect,

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[7] See Linvill, *op. cit.*

sampling of e(t). The samples, or measured values of the carrier peaks, are not, however, applied to the servo amplifier until they are sampled by the contactors of vibrator V2. The true sampler, so far as the servo system is concerned, is vibrator V2. Vibrator V1 measures and holds the values of e(t) existing at the carrier peaks; these values are then sampled and transferred to the servo drive by vibrator V2. It is hardly an approximation, therefore, to conclude that the synchronous-vibrator network can be represented as a true sampled-data device which samples and holds the last measured value of the modulation for approximately each half cycle. In other words, the insertion of the vibrator network produces little or no change in the theoretical gain and usable band-width characteristics of the modulated carrier system. The actual circuit functions and their equivalent are shown in Fig. 5. Since the delay time between

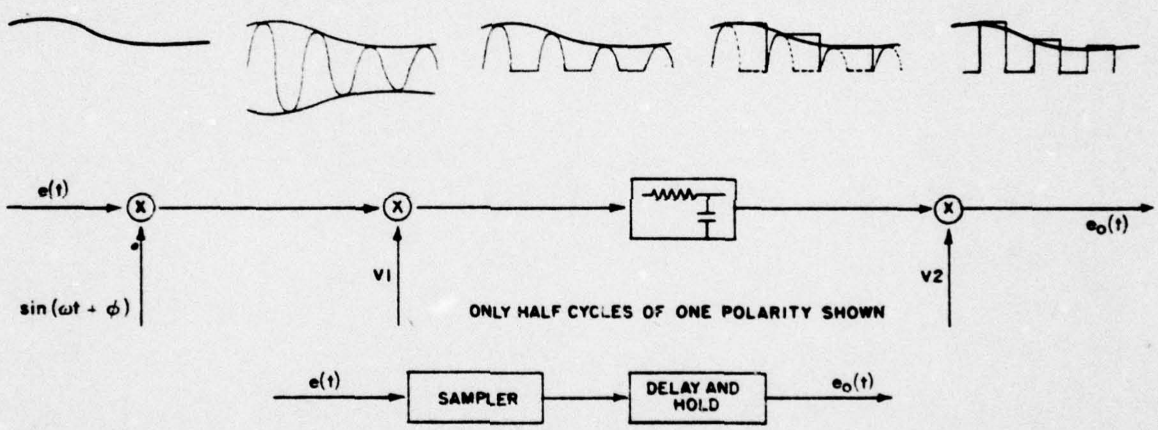


Fig. 5 - Circuit Transfer Function $e_o(t)/e(t)$ and Its Equivalent Circuit

carrier peaks and sampling can be considered negligible, established techniques for analyzing sampled-data systems can be employed in determining the system response.

Where system-performance specifications cannot be realized because of the limitations imposed by the synchro null residual voltage alone, insertion of the synchronous-vibrator network, as described above, removes this limitation. Where other limitations are encountered, compensation networks are required. Noncarrier-type compensation is very effectively introduced within the synchronous-vibrator network, as shown in Fig. 6. (Half cycles of only one polarity are shown.)

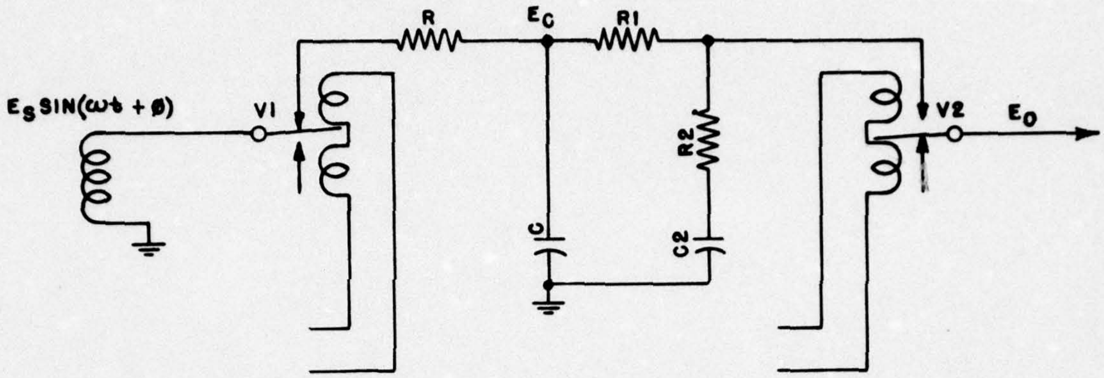


Fig. 6 - Schematic Diagram Showing Addition of Phase-Lag Compensation

Here, $R_1R_2C_2$ is recognized as a phase-lag compensation network, and RC is the integrating time discussed previously. Peak rectification of the carrier frequency is provided by RC, and $R_1R_2C_2$ is selected to give the proper phase shift at the low end of the modulating frequency. Therefore,

$$(R_1 + R_2)C_2 >> RC.$$

It is possible to choose a value of C so that at the carrier frequency:

$$R_1 + R_2 >> 1/\omega C \text{ (greater than } 10 \times 1/\omega C).$$

Thus, the two networks can be considered in cascade, *i.e.*, as though they were isolated from each other. The transfer function of the lag network is given by:

$$E_o/E_c = \frac{1 + j\omega\alpha T}{1 + j\omega T},$$

where

$$T = (R_1 + R_2)C_2$$

and

$$\alpha = \frac{R_2}{R_1 + R_2}.$$

At the carrier frequency,

$$\omega T,\ \omega\alpha T >>> 1,$$

therefore

$$E_o/E_c \cong \frac{R_2}{R_1 + R_2} \quad (\text{at } \omega = \omega_c).$$

Addition of the compensation-lag network introduces no phase shift at the carrier frequency and, therefore, does not alter the basic function of the synchronous-vibrator network. It does, however, introduce a carrier attenuation equal to $R_2/(R_1+R_2)$ and a droop-type hold circuit having a droop time equal to $(R_1 + R_2)CC_2/(C+C_2)$. Neither of these effects are of serious consequence. The attenuation is easily made up in the AC amplifier, while the droop rate is made small by proper selection of $R_1$ and $R_2$.

Phase-lead compensation networks, though less seldom used, are analyzed in the same manner. Figure 7 shows the vibrator network with added phase-lead compensation. As in Fig. 6, RC is

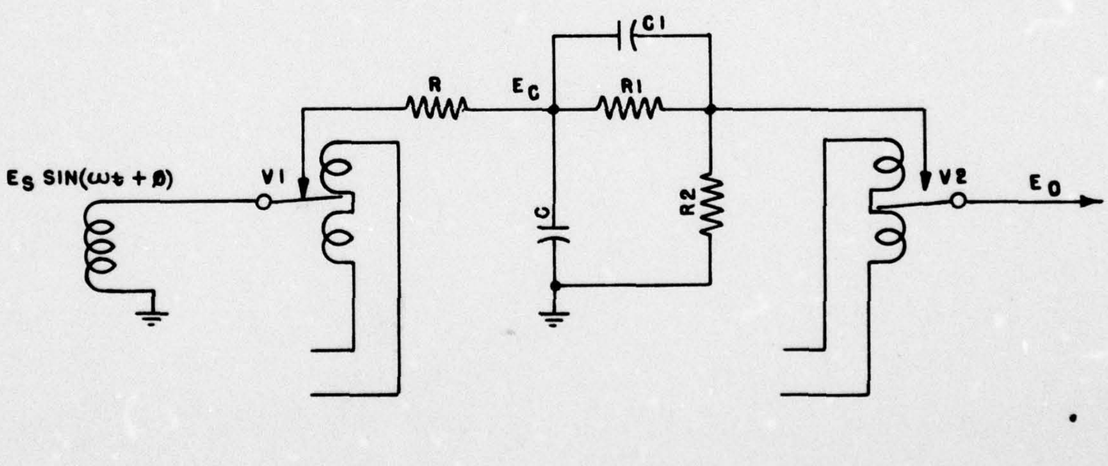


Fig. 7 - Schematic Diagram Showing Addition of Phase-Lead Compensation

the integrating time at the carrier frequency and $R_1R_2C_1$ is the phase-lead network. In order that the two networks can be considered in cascade, it is necessary that at the carrier frequency:

$$(R_1 + R_2)\,\frac{1 + j\omega\alpha T}{1 + j\omega T} >> 1/\omega C,$$

where

$$T = R_1C_1$$

and

$$\alpha = \frac{R_2}{R_1 + R_2}.$$

This condition is always possible and yields the same results as the phase-lag network, *i.e.*, no change in the basic function of the synchronous-vibrator network.

The function of the synchronous-vibrator network is best described by an example of its application in a typical position-control system operating from synchro data. Figure 8 is a block diagram

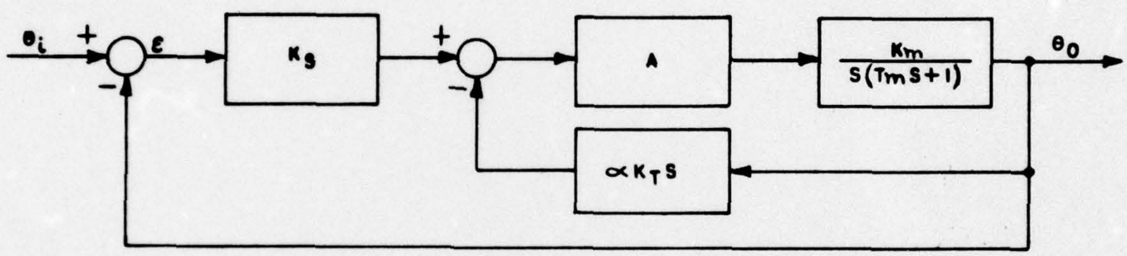


Fig. 8 - Block Diagram of Typical Position-Control System

of a single time-constant control system incorporating tachometric feedback and utilizing one-speed, 60-cycle synchro data transmission. The control motor is a two-phase induction type, and it is directly connected to a tachometer. The output shaft must be capable of following an input shaft rotation of 10 rpm, and the error in electrical angular alignment of the shafts must not exceed 0.5 degree. The maximum overshoot to a step function of position input must not exceed 25 per cent. The motor is connected to the load through a 200:1 gear reduction ratio so that it may operate at 55 per cent of synchronous speed during normal operation.

The transfer function, on the basis of a noncarrier-type system, is given by:

$$G(S) = \frac{K_v}{S(TS + 1)},$$

where

$$K_v = \frac{AK_sK_m}{1 + \alpha AK_TK_m} \text{ sec.}^{-1} \tag{5}$$

and

$$T = \frac{T_m}{1 + \alpha A K_T K_m} \text{ sec.} \tag{6}$$

Nominal values for this type of instrument control system are:

$T_m$ = 0.1 sec. (time constant of motor-load combination),

$K_m = 0.015 \dfrac{\text{rad/sec.}}{\text{volt}}$ (motor constant referred to output),

$K_T = 6 \dfrac{\text{volt}}{\text{rad/sec.}}$ (tachometer constant referred to output),

$K_s = 57 \dfrac{\text{volt}}{\text{rad}}$ (synchro control transformer constant),

A = servo amplifier gain $\dfrac{\text{volt}}{\text{volt}}$.

The value of amplifier gain A and the feedback factor $(1 + \alpha A K_T K_m)$ required to provide the desired performance are now determined. Since the synchro unit itself contains a maximum electrical position error of approximately 0.25 degree, the velocity error must be held to a maximum of 0.25 degree in order to meet the specification of a maximum error of 0.5 degree at 10 rpm. The velocity constant of the system, $K_v$, must therefore be:

$$K_v = \frac{\dot{\theta}_o}{\epsilon} = \frac{60}{0.25} = 240 \text{ sec.}^{-1}.$$

The maximum overshoot to a step function of position input is determined from the calculated time response,[8] as follows:

$$\frac{\theta_o}{\theta_i}(t) = 1 - \frac{2\sqrt{K_v T}}{\sqrt{4K_v T - 1}} \epsilon^{-t/2T} \sin\left[\frac{t}{2T}\sqrt{4K_v T - 1} + \tan^{-1}\sqrt{4K_v T - 1}\right]. \tag{7}$$

The time derivative of Eq. (7) is equated to zero, yielding:

$$\frac{\theta_o}{\theta_i}(t)_{max} = 1 + \epsilon^{\frac{-\pi}{\sqrt{4K_v T - 1}}}. \tag{8}$$

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[8] See W. R. Ahrendt and J. F. Taplin, *Automatic Feedback Control*, McGraw-Hill Book Co., Inc., New York, 1951.

Equation (8) is solved for $\theta_o/\theta_i(t)_{max} = 1.25$, giving:

$$K_v T = 1.5$$

and

$$T = 0.00625 \text{ sec.}$$

Substituting this value of T in Eq. (6) and combining it with Eq. (5), we obtain:

$$(1 + \alpha A K_T K_m) = 16$$

and

$$A = \frac{(16)(240)}{(0.015)(57)} = 4500 .$$

It is evident that even if it were theoretically possible to obtain a velocity constant of 240 sec.$^{-1}$ from the uncompensated, 60-cycle carrier system, the amplifier gain required to provide such a degree of follow-up accuracy would be impossible to use because of the appearance of the synchro residual voltage (approximately 0.06 volt) at the system nulls. Elimination of the synchro residual voltage, or what amounts to an increase in resolution of the error-sensing device, is needed.

In order to determine the maximum obtainable $K_v$ of the single time-constant, 60-cycle carrier system, without compensation and with the specified degree of stability, an analysis is made on the basis of a sampled-data system. As pointed out above, a carrier system can be approximated by a sampled-data system having a sampling rate which is twice the rate of the carrier frequency. The new block diagram appears as Fig. 9, with the transfer function, at sampling instants only, given as:

$$G_q(S) = \frac{\theta_q}{\epsilon}(S) = \sum_{n=-\infty}^{n=+\infty} G'\left(S + j\,\frac{2\pi n}{T}\right), \tag{9}$$

where

$$G'(S) = \frac{1 - \epsilon^{-TS}}{TS}\, G(S)$$

and

T = time of sampling interval (½ carrier period).

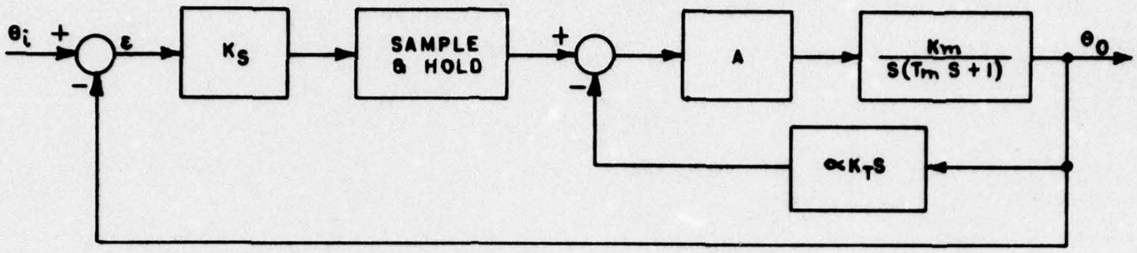


Fig. 9 - Block Diagram of Typical Position-Control System after Addition of the Demodulator and Modulator Circuit

Substituting $j\omega$ for S in Eq. (9), we obtain:

$$G_q(j\omega) = \sum_{n=-\infty}^{n=+\infty} G'\left[j\left(\omega + \frac{2\pi n}{T}\right)\right].$$

It is now necessary to compute $G(j\omega)$ for various frequencies and $G_q(j\omega)$ for various values of n. The transfer locus on the G-plane, when plotted for these specific values of $\omega$ and n, indicates a maximum obtainable $K_v$ of 146 sec.$^{-1}$. Therefore, in addition to the removal of the synchro residual voltage, compensation must be introduced if the performance specifications are to be realized. Since the maximum usable band width of approximately one half the carrier frequency has been reached ($\tau = 0.00625$ sec.), extension of the system band width through the use of phase-lead networks cannot appreciably improve the system performance. Provision must, therefore, be made for the elimination of the synchro residual voltage and for the introduction of a suitable phase-lag compensation network.

The sampled-data analysis given above could have been carried out somewhat more simply by the following method.

A sampled-data system operates as an open-loop system except at sampling instants. Therefore, the output response of the system, which is assumed to have a hold circuit, to a step function of position input can be written as:

$$\theta_o(S) = \left[1 - \epsilon^{-TS}\theta_T - \epsilon^{-2TS}\theta_{2T} - \epsilon^{-3TS}\theta_{3T} - \dots\right]\frac{G(S)}{S}, \qquad (10)$$

where $\theta_{nT}$ = the angle through which the output has moved during the $n^{th}$ sampling interval and T = the time of the sampling interval.

Equation (10) follows from the fact that during the first sampling period the output response for that period is:

$$\theta_{o1}(S) = \frac{1 - \epsilon^{-TS}}{S} G(S).$$

During the second sampling period, the step applied becomes $(1 - \theta_T)$, where $\theta_T$ is the angle through which the output moves during the first sampling period. The output response for the second sampling period is then given by:

$$\theta_{o2}(S) = \frac{\epsilon^{-TS} - \epsilon^{-2TS}}{S} (1 - \theta_T) G(S).$$

The output responses during successive sampling periods are determined in the same manner and, when all responses are combined, the summation yields Eq. (10). (The principle of superposition is justified because of the assumption of linear operation throughout.)

The time response of $G(S)/S$, where $G(S) = K_v/S(TS + 1)$, is given by:

$$\theta_o'(t) = K_v\left[t - T\left(1 - \epsilon^{-t/T}\right)\right].$$

The angle through which the output moves during the first sampling period is then:

$$\theta_T = K_v\left[t - T\left(1 - \epsilon^{-t/T}\right)\right]\Bigg|_{t=T}. \tag{11}$$

The angle through which the output moves during two sampling periods is:

$$\theta_{2T} = K_v\left[t - T\left(1 - \epsilon^{-t/T}\right)\right]\Bigg|_{t=2T} - \theta_T K_v\left[t - T\left(1 - \epsilon^{-t/T}\right)\right]\Bigg|_{t=T}. \tag{12}$$

Investigation shows that for $1 < T/T < 2.5$, the maximum overshoot to a step function of position input occurs during the second sampling period.[9] A solution of Eq. (12), with $\theta_{2T} = 1.25$, yields the maximum obtainable $K_v$ for a maximum overshoot of 25 per cent. In the example which is being discussed,

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[9] See G. V. Lago and J. G. Truxal, "Design of Sampled Data Feedback Systems," *Applications and Industry,* A.I.E.E., November 1954, pp. 247-253.

$$T = 0.00625 \text{ sec.}$$

$$T = 0.00833 \text{ sec. (½ carrier period)}$$

$$\theta_T = K_v\,[0.00833 - 0.00625\,(1 - \epsilon^{-1.33})] = 0.00373\,K_v$$

$$\theta_{2T} = K_v\,[0.0167 - 0.00625\,(1 - \epsilon^{-2.66})] - [0.00373\,K_v]^2$$

$$13.9 \times 10^{-6} K_v^2 - 1.05 \times 10^{-2} K_v + 1.25 = 0$$

and

$$K_{v_{max.}} = 146 \text{ sec.}^{-1}.$$

This result agrees with the result obtained by the frequency-response method of analysis and involves much less numerical computation. The method described above is practical and convenient to use, since it is almost always possible, with instrument-type servo systems, to obtain the ratio T/T within the limits stated above through the use of tachometric feedback or phase-lead compensation. Where the system time constant, T, is much greater than the sampling period, it is obvious that the system analysis is best performed on a noncarrier-type basis.

Introduction of the synchronous-vibrator circuit converts the carrier system to a true sampled-data system and makes the sampled-data analysis given above more valid. The synchro residual voltage is eliminated, as previously described, and it is now necessary to design only a proper phase-lag network, which will allow the system specifications to be met, namely, $K_v = 240$ sec.$^{-1}$ and the maximum overshoot to a step function of position input $\leq$ 25 per cent.

Figure 10 shows the plot of asymptotic gain vs. frequency for $K_v = 146$ sec.$^{-1}$ (43.28 db), with the zero-db crossover appearing at $\omega = 140$, which is the plot indicated by the analysis for the sampled-data system carried out above. Also shown in Fig. 10 is the asymptotic plot of gain vs. frequency indicated in the analysis made on the basis of a noncarrier-type system. This curve, plotted for $K_v = 240$ sec.$^{-1}$ (47.6 db), shows the zero-db crossover appearing at $\omega = 200$. In both cases, the gains indicated provide a degree of stability consistent with a maximum overshoot of 25 per cent to a step function of position input. In order that the sampled-data system have a velocity constant equal to 240 sec.$^{-1}$, a phase-lag network is introduced as shown, having its break frequencies located at $\omega = 5$ and $\omega = 10$. It is now possible to introduce the amplifier gain required to yield $K_v = 240$ sec.$^{-1}$. The amplifier gain was found previously to be 4500, but since a carrier attenuation of 2 is introduced into the synchronous-vibrator circuit by the phase-lag network, the gain must be increased to 9000.

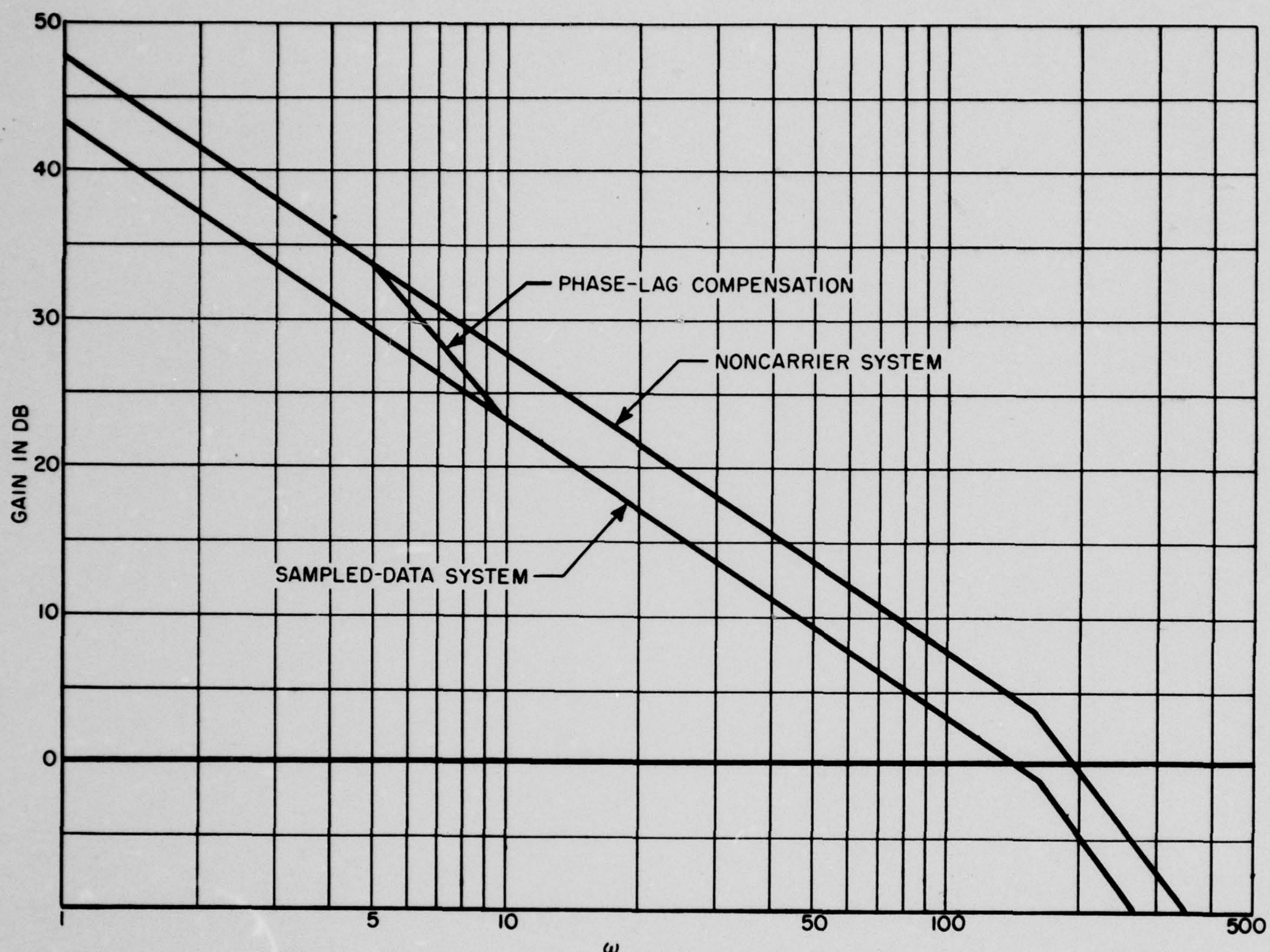



Fig. 10 - Phase-Gain Plot of the Noncarrier and Sampled-Data Systems with Phase-Lag Compensation

The numerical solution for the constants of the synchronous-vibrator circuit follow. As shown in Fig. 4A, and from the discussion of the circuit function, the period of contact of vibrator V1 is approximately one half the carrier period or, for the vibrators actually used, the period corresponding to 172 degrees. Therefore, in Eq. (4), $\omega t_b = 172$ degrees. The phase lead through the synchro generator and the synchro-control transformer was measured as 30 degrees. When coupled with a phase lag of 90 degrees introduced in the contactor periods, this lead yields $\phi = 60$ degrees (all phase angles are measured with respect to the reference phase). Therefore:

$$\tau = \frac{1}{377} \tan(172 - 60 - 90)$$

$$= 0.0011 \text{ sec.}$$

The time constants of the phase-lag network from Fig. 10 are 0.2 and 0.1 second. A suitable choice of component values is, therefore:

$$R = 11{,}000\ \Omega$$

$$C = 0.1\ \mu f$$

$$R_1 = R_2 = 200{,}000\ \Omega$$

$$C_2 = 0.5\ \mu f.$$

To complete the system design, the value of $\alpha$ in Eqs. (5) and (6) is determined from:

$$1 + \alpha A K_T K_m = 16$$

$$\alpha = \frac{15}{A K_T K_m} = 0.037.$$

Figure 11 is a schematic diagram of the final design of the system.

After proper care was taken in the phasing of the synchronous vibrators and in the precision assembly of the gear train, performance tests and measurements of the servo system were made. These tests indicated reasonable agreement with the performance predicted by the system analysis and design. The maximum overshoot to a step function of position input averaged 25 per cent for various amplitudes of steps (0.1 to 1.0 degrees), as shown in Fig. 12. The slope of output velocity as a function of error, Fig. 13, indicates a velocity constant $K_v = 250$ sec.$^{-1}$. At an output velocity of 10 rpm, CW and CCW, smooth operation was observed with a measured error of 0.24 degree. The response time was measured at $\cong 0.035$ second. At the system nulls, the

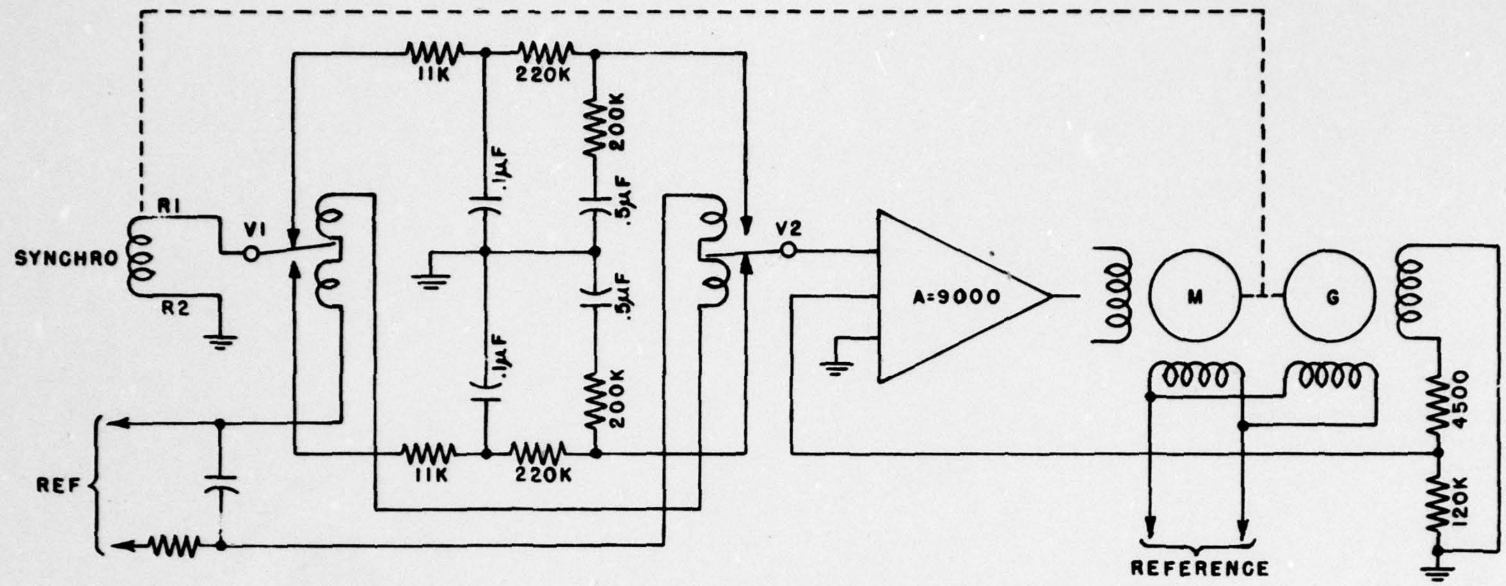



Fig. 11 - Schematic Diagram of Final System with the Demodulator and Modulator Circuit

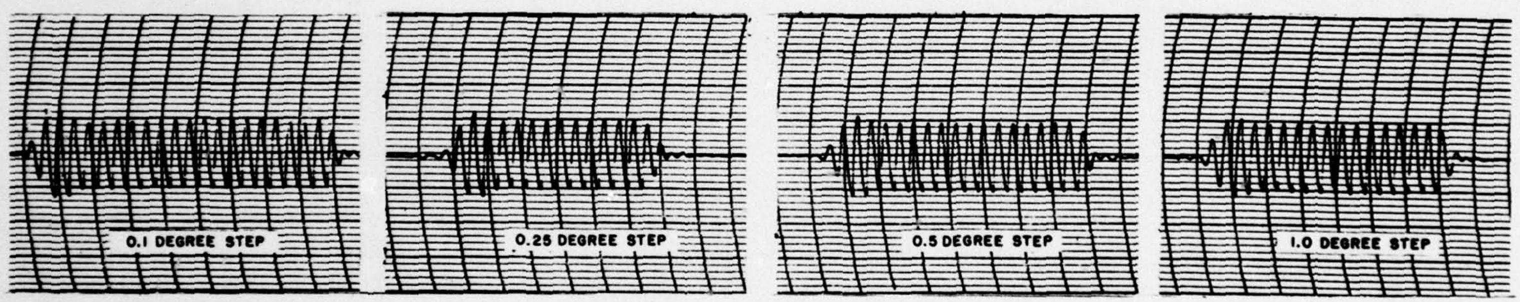
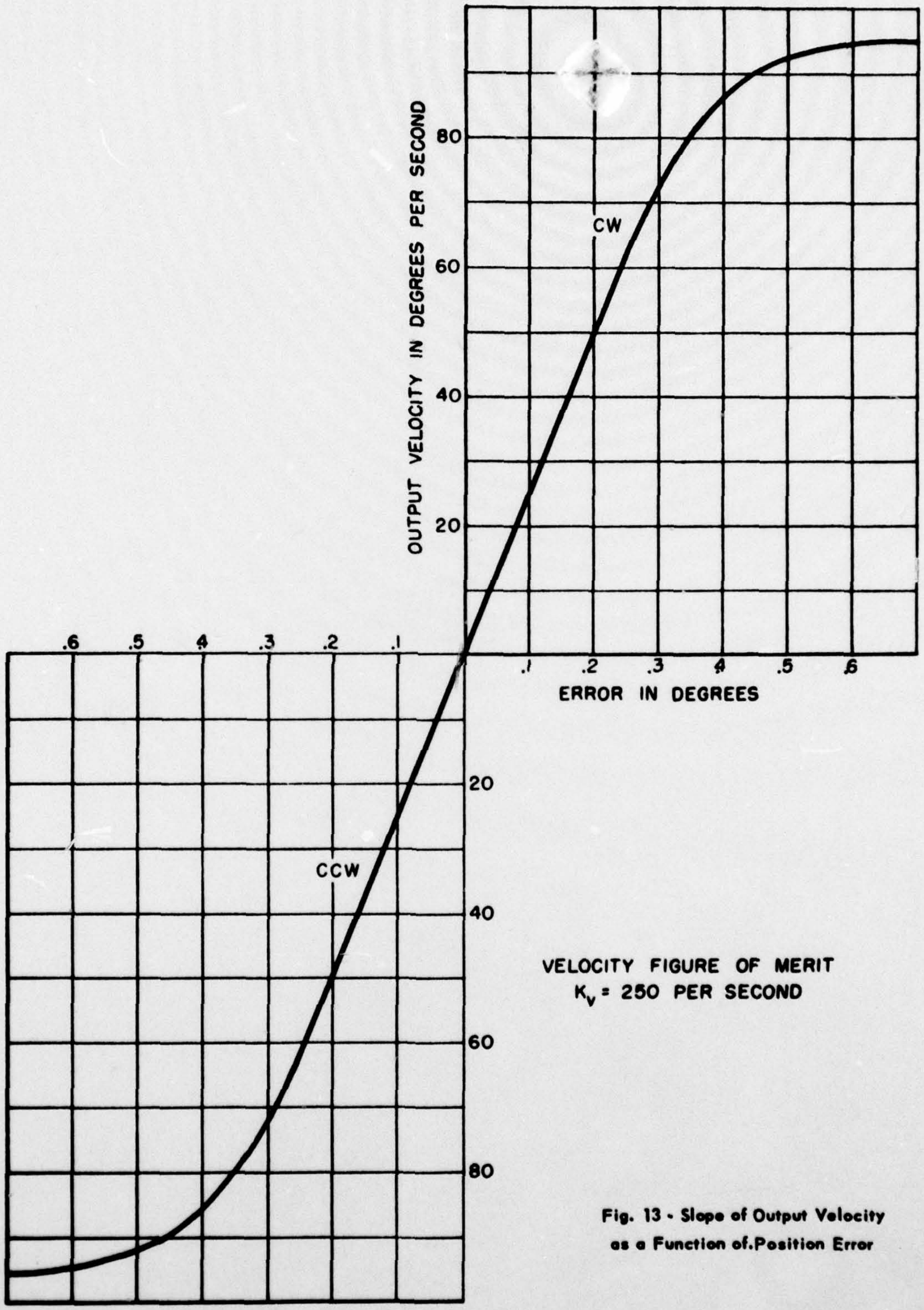



Fig. 12 - System Responses to Step Functions of Position Input




VELOCITY FIGURE OF MERIT
$K_v$ = 250 PER SECOND

Fig. 13 - Slope of Output Velocity as a Function of Position Error

voltage at the motor-control field terminals did not exceed 7 volts at any output shaft setting. It should be observed here that this voltage is almost wholly due to the tachometer residual voltage (approximately 0.02 volt), which when amplified by the factor $aA = 333$ must yield 6.66 volts across the motor terminals at the system null. In systems where greater percentages of tachometric feedback are required, the feedback can be routed through the synchronous-vibrator circuit. Since the tachometer must be phased with the synchro-control transformer for proper operation, its residual voltage will also be eliminated.

## CONCLUSIONS

It has been shown that the synchronous-vibrator demodulator and modulator circuit can be used to great advantage in the design and analysis of one-speed synchro data-transmission systems. The circuit is comparatively easy to adjust and provides a degree of performance previously unattainable without the addition of complex circuitry. The vibrators are extremely reliable and, when operated within their stated tolerances, often outlast vacuum tubes. Life tests performed on four synchronous-vibrator units indicate a life span greater than 20,000 hours of continuous operation.

Although the application of the synchronous-vibrator circuit to control systems using AC motors was described, the circuit is not restricted to AC motors. A simple rearrangement of the circuit, as shown in Fig. 14, provides a fast-response, full-wave, phase-sensitive detector for the operation

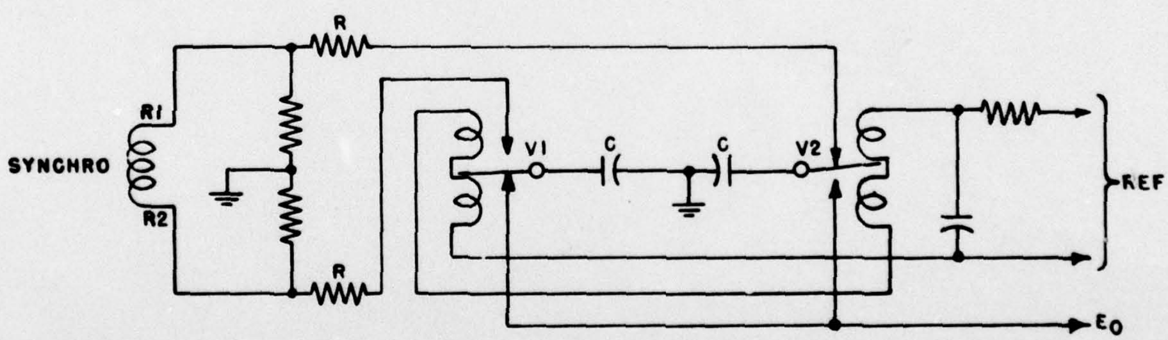



Fig. 14 - Circuit Arrangement for DC Motor Drive

of DC motors. The elimination of quadrature voltage and the addition of a negligible time delay is retained in the circuit.

It should also be noted here that, since random noise over a period of time has an average value approaching zero, the synchronous-vibrator circuit operates equally well in the elimination of random noise.

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## ABSTRACT CARD LAYOUT

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Research and Development Report No. 293.
A FAST-RESPONSE, PHASE-SENSITIVE DEMODULATOR AND MODULATOR CIRCUIT FOR THE IMPROVEMENT OF SERVO SYSTEMS, by Anthony F. Magaraci. 5 October 1955. i-vi + 23 p. UNCLASSIFIED

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II. NE-050961-2

provement in carrier-system performance, have not proved completely satisfactory because they inevitably introduce an unwanted time constant into the system. This added time constant often limits the degree of improvement that can be provided in the system. An electromechanical circuit that is a satisfactory solution to the dual problem of carrier-system compensation and synchro resolution is described. The circuit is essentially a fast-response demodulator and modulator unit that is simple to adjust and reliable in its application. Through its use, carrier systems can achieve a degree of performance hitherto unattainable.

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