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APPROXIMATIONS TO THE INVERSE CUMULATIVE NORMAL FUNCTION FOR US--ETC(U)
JUL 78 B W SCHMEISER
OREM-78007
N00014-77-C-0425
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Technical Report, OREM-78007

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APPROXIMATIONS TO THE INVERSE CUMULATIVE
NORMAL FUNCTION FOR USE ON HAND CALCULATORS.

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(12) 5p.

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Jul 1978

This research was supported in part by
Office of Naval Research Contract N00014-77-C-0425 NR 277-236.

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Intended for use on a pocket calculator, Page (1977) gives an approximation to the cumulative normal function

$$F(x) = \int_{-\infty}^x (2\pi)^{-1/2} \exp(-u^2/2) du$$

and its inverse $x = F^{-1}(p)$. Given here are two other approximations to the inverse function which are easier to apply.. (It should be noted that more accurate, but also more complicated, approximations are well known. See, for example, Abramowitz and Stegun (1972), equations 26.2.22 and 26.2.23.)

Given a fractile $p \geq 1/2$, Page's approximation to x , denoted x' , is calculated in a three step procedure:

1. Set $y = \ln\{p/(1 - p)\}/1.59577$
2. Set $u = [\{y + (y^2 + 3.31316)^{1/2}\}/.08943]^{1/3}$
3. Set $x' = u - (.134145u)^{-1}$.

For $p < 1/2$, $1 - p$ is used and the sign of x' is reversed.

A simpler approximation, based on results from Ramberg and Schmeiser (1972), is

$$x^* = [p^{.135} - (1 - p)^{.135}]/.1975$$

which is valid for any fractile p .

Additional accuracy for large values of x may be obtained using

$$\hat{x} = .2 + [p^{.14} - (1 - p)^{.09}]/.1596$$

where $p > 1/2$. For $p < 1/2$, $1 - p$ is used and the sign of $\hat{x}$ is reversed.

Table 1 compares the accuracy of the approximations x' , x^* , and $\hat{x}$. x' and x^* exhibit similar behavior, both being least accurate in the tail. x' is more accurate than x^* for most values of x , but for many applications the simpler x^* may be suitable. The most consistent accuracy is obtained with $\hat{x}$, which has maximum error of about .015 and which is also simpler than x' to calculate.

Table 1
Comparison of the inverse cumulative normal
function approximations x' , x^* , and $\hat{x}$.

x	$F(x)$	x'	x^*	$\hat{x}$
0	.5	.0000	.0	-.0005
.1	.5398278	.1000	.0993	.1046
.3	.6179114	.3000	.2981	.3113
.5	.6914625	.5001	.4972	.5138
1.0	.8413447	1.0006*	.9975	1.0070
1.5	.9331928	1.5011	1.5024	1.4940
2.0	.9772499	1.9991	2.0093	1.9880
2.5	.9937903	2.4901	2.5093	2.4943
3.0	.9986501	2.9693	2.9873	3.0075
3.5	.9997674	3.4332	3.4266	3.5145
4.0	.9999683	3.8800	3.8128	3.9992

*The value .9939 shown in Page (1977) is incorrect.

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unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER OREM 78007	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Approximations to the Inverse Cumulative Normal Function for Use on Hand Calculators.		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Bruce W. Schmeiser		8. CONTRACT OR GRANT NUMBER(s) N00014 - 77-C-0425
9. PERFORMING ORGANIZATION NAME AND ADDRESS Southern Methodist University Dallas, Texas 75275		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 277-236
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Analysis Program - Code 431 Office of Naval Research Arlington, VA 22217		12. REPORT DATE August 1978
		13. NUMBER OF PAGES 4
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Distribution of this document is unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Normal distribution Inverse distribution function		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Two approximations to the inverse distribution function of the normal distributions are given. Both approximations dominate, in terms of accuracy and ease of implementation, a method recently proposed in the literature.		

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