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USE OF COVARIATES TO EXPLAIN (OR PREDICT) LIFE LENGTH. (U)
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**Use of Covariates to
Explain (or Predict) Life Length**

by

W. A. Thompson, Jr.

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The Navy has a problem involving the successive breakdowns of gas turbines. When breakdown occurs the turbines are not simply repaired but they are overhauled incorporating improved design features. They are looking for reliability growth from the first to the second, to the third, etc. installation.

How can data of this kind be analyzed? [↑]

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USE OF COVARIATES TO EXPLAIN
(OR PREDICT) LIFE LENGTH ^{1.}

W. A. Thompson, Jr.

University of Missouri-Columbia

In medical, biological and engineering research, test results often appear in the form of a life table and it is desired to explain or predict life length in terms of, or perhaps allowing for, certain explanatory variables. Thus we may wish to explain human longevity in terms of geographical location or time of remission of Leukemia patients in terms of drug administered.

The Navy has a problem involving the successive breakdowns of gas turbines. When breakdown occurs the turbines are not simply repaired but they are overhauled incorporating improved design features. They are looking for reliability growth from the first to the second, to the third, etc. installation.

How can data of this kind be analysed?

Table 1 gives a miniature set of fictitious but representative data. In the table, a + sign indicates that the particular turbine installation was still operating when the data was analysed.

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Table 1 - Life length (operating hours)

Turbine	installation		
	1st	2nd	3rd
1	643	970+	
2	860	1800	49+
3	792	1200+	
4	600	1430+	
5	1004	880	1500+

This report illustrates an analysis using the methods of Thompson (1977). Since improved design features are incorporated, the various installations of a turbine are treated as being unrelated. The data is reorganized as in Table 2 and the time axis is divided into intervals sufficiently short so that individual failures and losses occur in separate intervals.

Table 2 - Ordered life length data

failure	Turbine	Installation	Operating hours
	2	3	49+
a	4	1	600
b	1	1	643
c	3	1	792
d	2	1	860
e	5	2	880
	1	2	970+
f	5	1	1004
	3	2	1200+
	4	2	1430+
	5	3	1500+
g	2	2	1800

A logistic model is assumed for the conditional probability of surviving an interval $[t_j, t_{j+1})$ given that the turbine is operating at the beginning of the interval. Specifically the conditional survival probability for a k^{th} installation is assumed to be $[1 + \exp(\beta_k + \eta_j)]^{-1}$; $k = 1, 2, 3$. The parameters are estimated by maximum likelihood. Only differences between installation effects are estimable; we arbitrarily take $\beta_1 = 0$, measuring the installation effects as deviations from the first. It turns out that for a time interval not containing a failure the corresponding interval effect is estimated as $\hat{\eta}_j = -\infty$. Likelihood equations for the remaining parameters are prepared using the numbers at risk and surviving as given in Table 3.

The likelihood equations are:

$$\frac{5}{1+e^{\eta_a}} + \frac{5}{1+e^{\beta_2+\eta_a}} + \frac{1}{1+e^{\beta_3+\eta_a}} = 10$$

$$\frac{4}{1+e^{\eta_b}} + \frac{5}{1+e^{\beta_2+\eta_b}} + \frac{1}{1+e^{\beta_3+\eta_b}} = 9$$

$$\frac{3}{1+e^{\eta_c}} + \frac{5}{1+e^{\beta_2+\eta_c}} + \frac{1}{1+e^{\beta_3+\eta_c}} = 8$$

$$\frac{2}{1+e^{\eta_d}} + \frac{5}{1+e^{\beta_2+\eta_d}} + \frac{1}{1+e^{\beta_3+\eta_d}} = 7$$

$$\frac{1}{1+e^{\eta_e}} + \frac{5}{1+e^{\beta_2+\eta_e}} + \frac{1}{1+e^{\beta_3+\eta_e}} = 6$$

$$\frac{1}{1+e^{\eta_f}} + \frac{1}{1+e^{\beta_2+\eta_f}} + \frac{1}{1+e^{\beta_3+\eta_f}} = 4$$

$$\frac{1}{1+e^{\beta_2+\eta_g}} = 0$$

$$\frac{5}{1+e^{\beta_2+\eta_a}} + \frac{5}{1+e^{\beta_2+\eta_b}} + \frac{5}{1+e^{\beta_2+\eta_c}} + \frac{5}{1+e^{\beta_2+\eta_d}} + \frac{5}{1+e^{\beta_2+\eta_e}} + \frac{5}{1+e^{\beta_2+\eta_f}} + \frac{1}{1+e^{\beta_2+\eta_g}} = 27$$

$$\frac{1}{1+e^{\beta_3+\eta_a}} + \frac{1}{1+e^{\beta_3+\eta_b}} + \frac{1}{1+e^{\beta_3+\eta_c}} + \frac{1}{1+e^{\beta_3+\eta_d}} + \frac{1}{1+e^{\beta_3+\eta_e}} + \frac{1}{1+e^{\beta_3+\eta_f}} = 6$$

Table 3 a - Number of turbine's at risk

		failure interval						
		a	b	c	d	e	f	g
installation	1	5	4	3	2	1	1	0
	2	5	5	5	5	5	3	1
	3	1	1	1	1	1	1	0
		11	10	9	8	7	5	1

Table 3 b - Number of turbines surviving

		failure interval							
		a	b	c	d	e	f	g	
installation	1	4	3	2	1	1	0	0	11
	2	5	5	5	5	4	3	0	27
	3	1	1	1	1	1	1	0	6
		11	9	8	7	6	4	0	44

The equations are solved theoretically and then numerically, obtaining: $\hat{\beta}_1 = 0$, $\hat{\beta}_2 = 3.156$, $\hat{\beta}_3 = -\infty$, $\hat{\eta}_a = -1.449$, $\hat{\eta}_b = -1.186$, $\hat{\eta}_c = -.833$, $\hat{\eta}_d = -.306$, $\hat{\eta}_e = .590$, $\hat{\eta}_f = .910$, and $\hat{\eta}_g = \infty$. Taking the lengths of all intervals to approach 0, the estimated survival probabilities of Table 4 are obtained. Survival probabilities here are the unconditional probabilities that a turbine will last longer than an indicated number of operating hours.

Table 4 - Estimated Survival Probability's

		installation		
		1	2	3
time range	[0,600)	1.00	1.00	1.00
	[600,643)	.81	.99	1.00
	[643,792)	.62	.98	1.00
	[792,860)	.43	.96	1.00
	[860,880)	.25	.93	1.00
	[880,1004)	.09	.86	1.00
	[1004,1800)	.03	.78	1.00
	[1800, -)	.00	.00	-

Reference

Thompson, W. A. Jr. (1977) "On the Treatment of Grouped Observations in Life Studies" *Biometrics* 33, 463-470.