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Vortex-Excited Unsteady Forces on Resonantly Vibrating, Bluff Structures

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NAVAL RESEARCH LABORATORY
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ERRATA

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p. 11 — Equation 16(b) should read

$$\varphi_1 = \arctan\left(\frac{W\ C_R - C_L}{C_R + W\ C_L}\right).$$

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This report considers recent experimental findings pertaining to the vortex-excited oscillations of bluff cylinders, and compares various approaches to measuring the resultant fluid forces. As one example, measurements of the structural damping and crossflow response amplitudes were made for several systems over a representative range of wind tunnel flow speeds. The fluid dynamic damping or reaction in-phase with the cylinder's velocity also was measured for different incident flow speeds over a range which included the locking-on of the vortex shedding to the resonant cylinder vibrations. A mathematical model is proposed in connection with these measurements, and the components

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20. Abstract (Continued)

of the excitation or lift force on a vibrating structure can be obtained from the model once the structural damping and fluid reaction forces are known.

In a second approach, the fluid forces were measured on a cylinder which was forced to oscillate in water at different frequencies and crossflow amplitudes over a range of incident flow speeds. Both the steady and unsteady forces are altered significantly in the locking-on regime, with large amplifications in both the lift and drag. These vortex-excited forces are then incorporated into the mathematical model for self-excited, resonant vibrations and the complementary nature of the two measurement schemes is discussed. The results obtained from the present study are suitable in general for application not only to the vortex-excited vibrations of rigid cylinders, but also to other flexible bluff structures and cables in air and in water.

CONTENTS

VORTEX-EXCITED UNSTEADY FORCES ON RESONANTLY VIBRATING, BLUFF STRUCTURES

INTRODUCTION

When one of the characteristic frequencies of a bluff, or unstreamlined, structure is near the frequency at which vortices are naturally shed, resonant self-excited vibrations often occur if the structure is lightly damped. There is also a range of frequencies near this so-called Strouhal frequency of shedding where forced vibrations of a bluff body cause the vortex frequency to be captured by, or to lock onto, the body frequency. This means that the body and wake oscillations have the same frequency and that the Strouhal frequency of vortex shedding from a stationary body is suppressed. This locking-on, or wake capture as it is often called, causes vortex-excited oscillations to occur over a range of flow speeds. The steady and unsteady forces which act on a structure are amplified as a result of these vibrations, and the changes in these forces are closely coupled to changes in the flow field near the body.

The purpose of this report is to consider recent experimental findings pertaining to the vortex-excited oscillations of bluff cylinders, and to compare various approaches to measuring the fluid forces. As one example, measurements of the structural damping and crossflow response amplitudes were made for several systems over a

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representative range of wind tunnel flow speeds. The fluid dynamic damping, or reaction in-phase with the cylinder's velocity, also was measured for different incident flow speeds over a range which included the locking-on of the vortex shedding to the resonant cylinder vibrations. A mathematical model is proposed in connection with these measurements, and the components of the excitation or lift force on a vibrating structure can be obtained from the model once the structural damping and fluid reaction forces are known.

In another approach, the fluid forces were measured on a cylinder which was forced to oscillate in water at different frequencies and crossflow amplitudes over a range of incident flow speeds. Both the steady and unsteady forces are altered significantly in the locking-on regime, with large amplification of the lift and drag. These vortex-induced forces are then incorporated into the mathematical model for self-excited, resonant vibrations and the complementary nature of the two measurement methods is discussed. The results obtained from the present study are suitable in general for application not only to the vortex-excited vibrations of rigid cylinders, but also to other flexible bluff structures and cables in air and in water.

RELATED INVESTIGATIONS

A number of papers and reports which consider various aspects of the problem of vortex-excited oscillations and their forced-body analogues have recently appeared, as have the proceedings of a recent symposium on flow-induced structural vibrations [1]. The topics discussed at this symposium ranged from basic studies of vortex

formation and generation, to large-scale field studies of the vibrations of marine structures, and to architectural aerodynamics. For instance, Griffin & Ramberg have examined on a relatively small scale the effects of cylinder oscillations on wake similarity [2], vortex circulation and steady drag in the wakes of circular cylinders at normal and yawed incidence [3, 4, 5]. King, Prosser & Johns [6] have reported the results of laboratory-scale experiments in which they investigated both in-line and crossflow vibrations of flexible cylindrical piles in steady water currents. King [7] has also compared model experiments such as those just mentioned with full-scale experiments on offshore structures [8], and has extended his investigations to include the effects of incident flows at angles other than normal to the axis of a flexible cylinder [9].

The complexities involved in a complete formulation of the equations of fluid motion past stationary and vibrating bluff bodies still preclude a solution from first principles, and so alternative and less complicated methods have been sought for engineering applications. A formulation which couples an equation for the structural motion with a nonlinear fluid oscillator for the fluctuating lift was first proposed by Bishop & Hassan [10] and formulated with a van der Pol oscillator for the lift by Hartlen & Currie [11]. Bishop & Hassan were the first to observe that the process of locking-on involves rapid phase shifting of the unsteady fluid forces relative to the vibration displacement, and that this phase transformation is accompanied by large amplifications of both the lift and drag forces. Soon after, a modified form of this original concept which more

accurately predicted the resonant response of an elastically-mounted cylinder was proposed and compared with experiments by Skop & Griffin [12] and Griffin, Skop & Koopmann [13]. Iwan & Blevins [14] also have proposed an oscillator formulation which is based upon the integral momentum equation for a system composed of a bluff cylinder in a uniform, steady flow. These so-called non-linear or "wake-oscillator" models have been generalized by Skop & Griffin [15], Blevins & Burton [16] and Iwan [17] to include the vortex-excited oscillations of elastic structural elements. The formulations of Skop & Griffin and Iwan were arrived at independently but are identical and yield the same results for a variety of cylindrical structures such as flexible cantilevers, taut cables and pivoted,rigid members in air and water.

A wake-oscillator type of formulation recently was applied by Blake [18] to the problem of predicting the vortex-excited response of blunt-based hydrofoil struts. Good agreement was obtained for the predicted and measured modal displacement amplitudes of a cantilever strut under varying structural damping and flow conditions. This successful extension of the nonlinear oscillator concept to the case of the vortex-excited bending vibrations of hydrofoils gives further evidence of the potential importance of such methods in engineering practice.

Most recently the unsteady lift forces on vibrating cylinders were measured by Griffin & Koopmann [19] in a wind tunnel, and by Mercier [20] and Sarpkaya [21] in water tunnels. Further evidence was given in these experiments of the highly nonlinear dependence

between the various fluid dynamic force components and the amplitude and frequency of the vortex-excited and forced, resonant vibrations.

UNSTEADY FORCES ON RESONANTLY VIBRATING, BLUFF STRUCTURES

The equation of motion for a resonantly vibrating, cylindrical structure, excited by vortex shedding, can be written in the form

$$\ddot{y} + 2\zeta_s \dot{y} + y = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 \left(C_L - C_R\right) \tag{1}$$

where $(\bullet) = \frac{d}{d\tau}$. Here $y = y/D$, $\tau = \omega_n t$, $\mu = \frac{\rho D^2}{8\pi^2 S^2 M}$ and ζ_S is the structural damping ratio. The force coefficients are

$$\text{Lift:} \quad C_L = \frac{\bar{F}_L}{\frac{1}{2}\rho V^2 D} = C_L \sin(\alpha\tau + \varphi), \alpha = \frac{\omega}{\omega_n} , \tag{2a}$$

$$\text{Reaction:} \quad C_R = \frac{\bar{F}_R}{\frac{1}{2}\rho V^2 D} = C_R \sin(\alpha\tau + \varphi_1), \alpha = \frac{\omega}{\omega_n} , \tag{2b}$$

as defined by Griffin and Koopmann [19], where φ and φ_1 are the phase angles between the lift and the displacement and the reaction and the acceleration, respectively. The two forces represented by C_L and C_R are orthogonal, as will be shown later. An analogous formulation of the equation of motion recently has been introduced by Blake, Maga & Finkelstein [22], who investigated the vortex-excited oscillations of propellors and hydrofoils.

If the displacement y is written as

$$y = Y \sin \alpha\tau, \quad \alpha = \frac{\omega}{\omega_n} \tag{3}$$

then the equation of motion separates into

$$\sin \alpha\tau: \; -\alpha^2 Y + Y - \mu\left(\frac{\omega_s}{\omega_n}\right)^2 \left[C_L \cos \varphi - C_R \cos \varphi_1\right] = 0 \tag{4a}$$

and

$$\cos \alpha\tau: \; 2\zeta_s Y - \mu \left(\frac{\omega_s}{\omega_n}\right)^2 \left[C_L \sin \varphi - C_R \sin \varphi_1\right] = 0 \tag{4b}$$

when the coefficients of $\sin \alpha\tau$ and $\cos \alpha\tau$ are grouped appropriately. The various force components are identified as follows

STRUCTURAL			FLUID	
INERTIA	STIFFNESS		INERTIA	ADDED MASS
$-\alpha^2 Y$	$+ \quad Y =$		$\mu\left(\frac{\omega_s}{\omega_n}\right)^2$	$\left[C_L \cos \varphi - C_R \cos \varphi_1\right]$
STRUCTURAL DAMPING			FLUID EXCITATION	FLUID DAMPING
$+ 2\zeta_s Y$	$= \mu \left(\frac{\omega_s}{\omega_n}\right)^2$		$\left[C_L \sin \varphi\right.$	$\left. - C_R \sin \varphi_1\right]$.

Using a different notation Sarpkaya [21] has expressed the measured fluid force on a vibrating cylinder as the sum of orthogonal components in the form

$$C_{TOTAL} = \frac{\bar{F}}{\frac{1}{2}\rho V^2 D} = C_{mh} \sin \alpha\tau - C_{dh} \cos \alpha\tau, \tag{5}$$

where C_{mh} is an "inertia" force and C_{dh} is a "drag" force. These

force components are related to Sarpkaya's "generalized Morison coefficients" $C_{d\ell}$ and $C_{m\ell}$ by

$$C_{dh} = \frac{8}{3\pi} C_{d\ell} \left(2\pi \frac{\bar{Y}}{D}\right)^2 V_r^{-2} \tag{6a}$$

and

$$C_{mh} = \pi^2 C_{m\ell} \left(2\pi \frac{\bar{Y}}{D}\right) V_r^{-2} \tag{6b}$$

where V_r is the "reduced velocity" $V_r = \frac{2\pi V}{\omega_n D}$.

In this case the equation of motion for the cylinder is written as

$$\ddot{y} + 2\zeta_s \dot{y} + y = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 \left[C_{mh} \sin \alpha\tau - C_{dh} \cos \alpha\tau\right]. \tag{7}$$

(The drag force C_{dh} is negative when energy is transferred to the cylinder, as is the case for resonant, vortex-excited oscillations.) If a steady-state response is assumed, then

$$y = Y \sin (\alpha \tau - \epsilon),$$

where ϵ is an undefined phase angle. This equation of motion then separates into

$$\sin \alpha\tau: \quad \alpha^2 Y \cos \epsilon + 2\zeta_s Y \sin \epsilon + Y \cos \epsilon - \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_{mh} = 0 \tag{8a}$$

and

$$\cos \alpha\tau: \quad \alpha^2 y \sin \epsilon + 2\zeta_s Y \cos \epsilon - Y \sin \epsilon + \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_{dh} = 0 \tag{8b}$$

when the coefficients of sin $\alpha\tau$ and cos $\alpha\tau$ are again grouped appropriately. If the first equation is multiplied by sin ϵ and the second by cos ϵ, then

$$2\alpha\zeta_s Y - \mu\left(\frac{\omega_s}{\omega_n}\right)^2 \left[C_{mh}\sin\epsilon - C_{dh}\cos\epsilon\right] = 0 \qquad (9a)$$

when the two results are added. Next, equation (8a) is multiplied by cos ϵ and equation (8b) by sin ϵ, to obtain

$$-\alpha^2 Y + Y - \mu\left(\frac{\omega_s}{\omega_n}\right)^2 \left[C_{mh}\cos\epsilon + C_{dh}\sin\epsilon\right] = 0 \qquad (9b)$$

when the two equations are subtracted. The terms in equations (9a) and (9b) can be arranged and labeled in the same manner as before:

STRUCTURAL INERTIA — STRUCTURAL STIFFNESS — FLUID FORCE TERMS

$$-\alpha^2 Y + Y = \mu\left(\frac{\omega_s}{\omega_n}\right)^2 \left[C_{dh}\sin\epsilon + C_{mh}\cos\epsilon\right]$$

STRUCTURAL DAMPING — FLUID FORCE TERMS

$$2\alpha\zeta_s = \mu\left(\frac{\omega_s}{\omega_n}\right)^2 \left[-C_{dh}\cos\epsilon + C_{mh}\sin\epsilon\right]$$

These equations have the same form as those formulated by Griffin & Koopmann [19], and the two approaches are in fact identical when

$$C_L \sin\varphi = -C_{dh}\cos\epsilon, \qquad \text{EXCITATION}^+ \qquad (10a)$$

$$C_L \cos\varphi = C_{dh}\sin\epsilon, \qquad \text{FLUID INERTIA} \qquad (10b)$$

$$C_R \sin\varphi_1 = -C_{mh}\sin\epsilon, \qquad \text{FLUID REACTION (DAMPING)} \qquad (10c)$$

$$C_R \cos\varphi_1 = -C_{mh}\cos\epsilon, \qquad \text{ADDED MASS} \qquad (10d)$$

+See Appendix I for a further discussion of these equations.

These relations can be used to compare results from the two approaches.

From the above system of equations it follows that $\tan \varphi_1 = \tan \epsilon$ and

$$\tan \varphi = - \frac{1}{\tan \epsilon} = - \frac{1}{\tan \varphi_1} . \qquad (11)$$

(Recall that ϵ and φ are measured from y, whereas φ_1 is measured from $\ddot{y}$.) It also follows from these equations that

$$C_L^2 = C_{dh}^2, \text{ or } |C_L| = |C_{dh}|,$$

and that

$$C_R^2 = C_{mh}^2, \text{ or } |C_R| = |C_{mh}|.$$

The phase relations between the various forces components are shown in Figure 5(b) for a typical set of conditions.

A relation between the displacement Y and the total fluid force can be obtained by squaring and adding equations (4a) and (4b) to yield

$$(1 - \alpha^2)^2 + (2 \alpha \zeta_s)^2 Y^2 = \mu^2 \left(\frac{\omega_s}{\omega_n}\right)^4 \left[C_L^2 + C_R^2 - 2 C_L C_R (\cos \varphi \cos \varphi_1 + \sin \varphi \sin \varphi_1) \right].$$

This equation simplifies to

$$\left[(1 - \alpha)^2 + (2 \zeta_s \alpha)^2\right] Y^2 = \mu^2 \left(\frac{\omega_s}{\omega_n}\right)^4 \left[C_L^2 + C_R^2\right] \qquad (12)$$

since $\tan \varphi = - \frac{1}{\tan \varphi_1}$. The phase angles φ and φ_1 can be retrieved from the equations

$$2 \zeta_s Y = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_T \left[\frac{C_L}{C_T} \sin \varphi - \frac{C_R}{C_T} \sin \varphi_1\right],$$

$$(1 - \alpha^2)\ Y = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_T \left[\frac{C_L}{C_T} \cos \varphi - \frac{C_R}{C_T} \cos \varphi_1\right],$$

where

$$C_T = \sqrt{C_L^2 + C_R^2}.$$

Also choose $\varphi_1 - \varphi = \pi/2$, so that these equations reduce to

$$2 \alpha \zeta_s\ Y = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_T\ [- \sin \beta \cos \varphi_1 - \cos \beta \sin \varphi_1]$$

and

$$(1 - \alpha^2)\ Y = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_T\ [+ \sin \beta \sin \varphi_1 - \cos \beta \cos \varphi_1]$$

where $\beta = \arctan \frac{C_L}{C_R}$. Simplifying further, one obtains the equations

$$- \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_T \sin(\varphi_1 + \beta) = 2 \alpha \zeta_s\ Y, \tag{13a}$$

$$- \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_T \cos(\varphi_1 + \beta) = (1 - \alpha^2)\ Y, \tag{13b}$$

and

$$\tan(\varphi_1 + \beta) = \frac{2 \alpha \zeta_s}{(1 - \alpha^2)} = W. \tag{13c}$$

This latter equation can be rearranged as

$$\tan \varphi_1 = \frac{W - \tan \beta}{1 + W \tan \beta}. \tag{13d}$$

If $W = \tan \gamma$, then

$$\tan \varphi_1 = \tan(\gamma - \beta) \tag{14a}$$

and

$$\varphi_1 = \gamma - \beta. \tag{14b}$$

The tangents of the various phase angles are

$$\tan \varphi_1 = \frac{-\left(C_L/C_R\right) + W}{1 + W\, C_L/C_R} = \frac{-C_L + W\, C_R}{C_R + W\, C_L} \tag{15a}$$

$$\tan \varphi = -\frac{1}{\tan \varphi_1} = \frac{C_R + W\, C_L}{C_L - W\, C_R} \tag{15b}$$

or, alternately, the angles φ and φ_1 are

$$\varphi = \text{arc tan}\left(\frac{C_R + W\, C_L}{C_L - W\, C_R}\right). \tag{16a}$$

$$\varphi_1 = \text{arc tan}\left(\frac{C_L + W\, C_R}{C_R + W\, C_L}\right). \tag{16b}$$

according to the formulation of Griffin & Koopmann [1].

Similar expressions can be found from the formulation employed by Sarpkaya [21] and his students {see Raposo [23], Meyers [24] and Fortik [25]}. The equations of motion reduce to

$$2\,\alpha\,\zeta_s\, Y = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_T \left[\frac{C_{mh}}{C_T} \sin \epsilon - \frac{C_{dh}}{C_T} \cos \epsilon\right],$$

and

$$(1 - \alpha^2)\, Y = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 C_T \left[\frac{C_{mh}}{C_T} \cos \epsilon + \frac{C_{dh}}{C_T} \sin \epsilon\right],$$

where

$$C_T = \sqrt{C_{mh}^2 + C_{dh}^2}\,.$$

If $\sin \beta' = \frac{C_{dh}}{C_T}$, and $\cos \beta' = \frac{C_{mh}}{C_T}$, then these equations become

$$2 \alpha \zeta_s Y = \mu \left(\frac{\omega_s}{\omega_n} \right)^2 C_T \sin (\epsilon - \beta') \tag{17a}$$

$$(1 - \alpha^2) Y = \mu \left(\frac{\omega_s}{\omega_n} \right)^2 C_T \cos (\epsilon - \beta') \tag{17b}$$

which yield the result

$$\tan (\epsilon - \beta') = \frac{2 \alpha \zeta_s Y}{(1 - \alpha^2) Y} = \frac{2 \alpha \zeta_s}{(1 - \alpha^2)} = W. \tag{17c}$$

This equation can be rearranged as

$$\tan \epsilon = \frac{W + \tan \beta'}{1 - W \tan \beta'}. \tag{18}$$

If $W = \tan \gamma$ as before, then

$$\epsilon = \gamma + \beta',$$

where

$$\beta' = \text{arc tan} \left(\frac{C_{dh}}{C_{mh}} \right) \tag{19a}$$

and

$$\gamma = \text{arc tan} \left(\frac{2 \alpha \zeta_s}{1 - \alpha^2} \right). \tag{19b}$$

In order for self-excited oscillations to take place, energy must transferred to the cylinder and its mounting system from the fluid. The energy transfer $\bar{E}^+$ over a cycle of the oscillation is given by the integral

$$\bar{E} = \int_0^T \bar{F}_L \, d\bar{y} = \int_0^T \bar{F}_L \, \dot{\bar{y}} \, dt \tag{20}$$

The lift force is, in Sarpkaya's notation,

$$\bar{F}_L = -C_{dh} \, (1/2 \, \rho \, V^2 \, D \, L) \cos \alpha \tau ,$$

+ See Appendix I for a further discussion of the work done and energy dissipated by the various forces over a cycle of the cylinder motion.

and the cylinder vibration velocity is

$$\dot{y} = \alpha\, Y\, D \cos(\alpha\,\tau - \epsilon),$$

so that the equation for the energy transfer is

$$\bar{E} = \frac{1}{2}\,\rho\, V^2\, D^2\, L \int_0^{\omega_n T} (-C_{dh}) \cos\alpha\,\tau\, (Y\,\alpha) \cos(\alpha\,\tau - \epsilon)\, d\tau$$

or

$$E = \frac{\bar{E}}{\frac{1}{2}\,\rho\, V^2\, D^2\, L} = (-C_{dh})\, Y \int_0^{2\pi} \cos u \cos(u - \epsilon)\, du,$$

$$= \pi\,(-C_{dh})\, Y \cos\epsilon\,. \qquad (21a)$$

A necessary condition for self-excited oscillations to take place is

$$E = \pi\,(-C_{dh})\, Y \cos\epsilon > 0$$

or, equivalently, $\cos\epsilon > 0$ since $-C_{dh}$ is positive. This leads to the condition

$$C_{mh} - W\, C_{dh} > 0 \qquad (21b)$$

for self-excited oscillations since

$$\cos\epsilon = \frac{C_{mh} - W\, C_{dh}}{\sqrt{(C_{mh} - W\, C_{ch})^2 + (C_{dh} + W\, C_{mh})^2}}\,.$$

In the notation adopted by Griffin & Koopmann [19], the energy transfer is

$$\bar{E} = 1/2\,\rho\, V^2\, D^2\, L \int_0^{\omega_n T} C_L \sin(\alpha\,\tau + \varphi)\, (Y\alpha) \cos\alpha\tau\, d\tau$$

or

$$E = C_L\, Y \int_0^{2\pi} \sin(u + \varphi) \cos u\, du = \pi\, C_L\, Y \sin\varphi. \qquad (22)$$

Since C_L is positive, a necessary condition for self-excited oscillations is $\sin \varphi > 0$. This leads to the condition

$$C_R + W C_L > 0$$

since

$$\sin \varphi = \frac{C_R + W C_L}{\sqrt{(C_R + W C_L)^2 + (C_L - W C_R)^2}}$$

This is analogous to the condition derived between C_{mh} and C_{dh}.

UNSTEADY FLUID FORCE MEASUREMENTS

Some interesting and useful comparisons can be made between the fluid forces measured when a cylinder is forced to vibrate and the fluid forces measured when the cylinder is resonantly excited by vortex shedding. The forced cylinder measurements discussed here were made by Sarpkaya [21] and Mercier [20], and the self-excited cylinder measurements were made by Griffin & Koopmann [19] and others [26, 27, 28].

Typical examples of the measurements reported by Sarpkaya are shown in Figures 1 and 2. The measured values for the inertia coefficient C_{mh} are shown in Figure 1 as a function of the reduced velocity V_r, for a displacement amplitude of $Y = 0.50$. Of particular note is the rapid change in C_{mh} in the vicinity of $V_r = 5$. This effect corresponds to the onset of locking-on between the vortex and vibration frequencies when the characteristic frequency of the flow approaches the vibration frequency. The drag or resistance force C_{dh} is plotted in Figure 2 as a function of V_r at the same displacement amplitude, $Y = 0.50$. This component of the total fluid dynamic

force becomes negative near $V_r = 5$. The terminology employed by Sarpkaya is somewhat unwieldy and is an extension of his previous work [29] with the so-called Morison equation [30] for purely periodic flow past stationary cylinders. In effect, C_{dh} is the negative of the lift coefficient as it is usually characterized, so that the negative value of C_{dh} near $V_r = 5$ suggests a net transfer of energy to the cylinder in that region. These forced-cylinder results can be compared to the vortex-excited, transverse oscillations of structures which typically occur in the range $V_r = 4.5$ to 8 when the damping of a structure is sufficiently small [19, 26, 31].

The excitation component of the lift force $C_L \sin\Phi = -C_{dh} \cos\epsilon$ measured by Griffin & Koopmann [19] is plotted in Figure 3 for two cases. For the first case, termed System I, the cylinder reached a maximum displacement amplitude of $Y = 0.47$, while for the second case the maximum displacement was $Y = 0.27$. The conditions for these experiments are listed in Table 1. The lift force coefficients in Figure 3 were deduced from structural and fluid dynamic damping or reaction forces on freely-vibrating cylinders in a wind tunnel. A typical example of the fluid dynamic damping ratio ζ_F as a function of the reduced velocity V_r is shown in Figure 4. There is a marked decrease in the damping in the region of the locking-on, and much the same effect has been observed by Blake, Maga & Finkelstein [22] during measurements of the vortex-excited bending vibrations of blunt-based hydrofoil models.

A comparison now will be made between the force components of Sarpkaya's forced vibration experiments and the vortex-excited forces.

Consider as an example the measurements $C_{mh} = -0.5$ and $C_{dh} = -0.2$ at $V_r = 5.5$ and $Y = 0.25$, reported by Sarpkaya [21], and assume $W = 0.05$ as a typical value [see reference 19]. For this case $\epsilon = 204.7^\circ$ and

$$C_{mh} - W C_{dh} = -0.5 - 0.05 (-0.2) = - .49,$$

so that self-excited oscillations do not take place (see equation 21b). A phase diagram of the various force components is shown as Case 1 in Figure 5(a). As an additional example consider a set of conditions such that $C_{mh} = 1.6$ and $C_{dh} = -0.9$ at $V_r = 5$ and $Y = 0.5$. Assuming once again for convenience that $W = 0.05$, then $\epsilon = -26.5^\circ$ and

$$C_{mh} - W C_{dh} = 1.6 - 0.05 (-0.9) = 1.64,$$

so that self-excited oscillations are possible for these conditions. This is shown as Case 2 in Figure 5(b).

The excitation component of the lift force is defined as

$$C_L' = C_L \sin\phi = -C_{dh} \cos\epsilon \qquad (10a)$$

and is important from the standpoint of energy transfer to the vibrating structure. The measurements made by Sarpkaya can be compared to the measurements of other investigators by means of the above equation. The maximum value of the force coefficient $-C_{dh}$ in Figure 2 occurs at $V_r = 5$, and the same result is obtained at the displacement amplitudes $Y = 0.13$, 0.25 and 0.75 [21]. The several values of $-C_{dh}$ and C_{mh} are listed in Table 2 together with the related values of the force components $-C_{dh} \cos\epsilon$. For all of these cases of forced vibration the condition for self-excitation

$$C_{mh} - W C_{dh} > 0$$

is satisfied, thus assuring the possibility of the equivalent vortex-

excited oscillation. The results for $-C_{dh} \cos \epsilon$ are plotted against the peak displacement in Figure 6 together with a host of similar findings from the results obtained by other investigators. The conditions under which the experiments were performed are described in Table 3.

Mercier [20] measured the force coefficients C_{mh} and C_{dh} on a cylinder which was forced to vibrate in water. These measurements were not nearly as extensive and precise as were those of Sarpkaya, but it is possible to <u>estimate</u> the various force components given by equations (10a to c) from Mercier's published data. The results are listed in Table 2(b) and are plotted in Figures 6 and 7. The minimum value of C_{dh} was measured once again at V_r = 5.5-6, and for all of the cases listed in Table 2(b) the condition

$$C_{mh} - W C_{dh} > 0$$

was satisfied. The estimated force coefficients $-C_{dh} \cos \epsilon$ from Mercier's measurements are in reasonably good agreement with the other results plotted in Figure 6. All of the lift force measurements in the figure exhibit the nonlinear dependence upon amplitude that is characteristic of vortex-excited oscillations. This nonlinear "wake-oscillator" was first observed by Bishop & Hassan [10], and has since been studied in considerable detail by others [11, 12, 14].

The measured vortex-excited displacements in Figure 6 are scaled by the modal response factor

$$I_i^{\frac{1}{2}} \quad |\psi_i(x)|$$

which takes account of the normal modes $\psi_i(x)$ of a flexible structure

of immersed length L. As shown by Skop & Griffin [15] and Iwan [17],

$$I_i = \frac{\int_0^L \psi_i^4 \ (x) \ dx}{\int_0^L \psi_i^2 \ (x) \ dx} \quad . \tag{23}$$

For the special case of a circular cylinder $|\psi_i(x)| = I_i = 1$, and the corresponding values for cantilevers and pivoted rods are listed in reference 31.

Several important characteristics of the lift force behavior during locking-on are clear from the results plotted in Figure 6. First, there is a maximum of the excitation component of the lift force at a peak-to-peak displacement between 0.6 and 1 diameters for all of the cases shown in the figure. Second, the maximum of the lift coefficient is approximately $C_L' = 0.5$ to 0.6 for most cases, a notable exception being Sarpkaya's result, $C_L' = 0.75$. The excitation force coefficient C_L' decreases to zero at displacement amplitudes near $Y = 1$, and this results in a limiting amplitude of vortex-excited oscillation of one to one and one-half diameters. Sarpkaya's measurements give convincing evidence of this limit, since in his water tunnel experiments with forced cylinders the sign of the force coefficient C_{dh} changed from negative to positive as the vibration displacement was increased from 0.75 diameters to 1.03 diameters. Mercier [20] also observed a change in the sign of the force coefficient C_{dh} as the vibration displacement $\bar{y}$ was increased to 1.3 diameters from the values listed in Table 2(b).

The formulations for the fluid dynamic forces due to vortex shedding introduced by Blake, Maga & Finkelstein [22] and Griffin &

Koopmann [19] are based on the hypothesis that both flow-induced lift (excitation) and reaction (damping) forces act on a resonantly vibrating, bluff structure. The fluid damping forces were measured in air by Griffin & Koopmann, see Figure 4, and in water by Blake and his colleagues, and the lift coefficient was obtained by equating the lift component $C_L \sin \varphi$ to the sum of the structural damping and fluid reaction forces as given by equation (10a).

It is possible to deduce the damping or reaction effect of the fluid from Sarpkaya's total force coefficients by means of equation (10c),

$$C_R \sin \phi_1 = -C_{mh} \sin \epsilon,$$

as a further step in comparing the two approaches described in the previous section. The results are listed in Table 2(a) and are plotted in Figure 7, and they yield further evidence that the fluid force decomposition described by equations (10a) through (10d) is an appropriate one. A comparison between the force components deduced from Sarpkaya's forced-vibration measurements of C_{mh} and C_{dh} and plotted in Figures 6 and 7 shows that the excitation and reaction forces are very nearly equal. Though Mercier's force measurements were not as extensive as those of Sarpkaya, it is possible to estimate the reaction effect of the fluid from his faired curves of C_{mh} and C_{dh}. The results are listed in Table 2(b) and plotted in Figure 7. The same reaction or damping forces component as reported by Griffin & Koopmann is plotted in Figure 7, and all of the measurements show the same general pattern of behavior even though these latter experiments were performed in a wind tunnel.

It is important to remark upon the complementary nature of the two approaches just described for measuring the vortex-excited forces on bluff bodies. For instance, the Morison-type force coefficients C_{mh} and C_{dh}, as obtained from a Fourier decomposition of the total force measured by driving the cylinder, do not give the necessary detailed picture of the various force components which act due to resonant, vortex-excited oscillations. On the other hand, it is an exceedingly difficult experimental undertaking to measure the resultant fluid dynamic force on a resonantly vibrating, self-excited bluff cylinder. Only the fluid damping, structural damping and added mass components of the total force system are conveniently measured in this latter case. Thus, in applying the results from the two approaches just discussed, it is necessary to recognize the limitations inherent in each and to take advantage of their complementary nature in solving problems associated with the vortex-excited vibrations of bluff structures.

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APPENDIX I

It is useful to consider the work done and the energy dissipated over a cycle of the cylinder's oscillation by the various forces in equations (1) and (7). The work and energy dissipation are given by the well-known equation

$$\Delta\bar{E} = \bar{W} = \int_o^T \bar{F}\, d\bar{y} = \int_o^T \bar{F}\, \frac{d\bar{y}}{dt}\, dt. \qquad (A1)$$

Employing the nomenclature introduced earlier, and evaluating this integral for the structural and fluid dynamic forces in equation (1), one obtains

$$2\zeta_s \int_o^{\omega_n T} (\dot{y})^2\, d\tau = \mu \left(\frac{\omega_s}{\omega_n}\right)^2$$

$$\left\{ \int_o^{\omega_n T} C_L \sin(\alpha\tau + \varphi)\, \dot{y}\, d\tau - \int_o^{\omega_n T} C_R \sin(\alpha\tau + \varphi_1)\, \dot{y}\, d\tau \right\}.$$

The structural inertia and stiffness terms are conservative and do no net work over a cycle of the oscillation. The structural velocity $\dot{y}$ is

$$\dot{y} = \alpha Y \cos(\alpha\tau),$$

so that equation (A1) reduces to

$$2\zeta_s \int_o^{\omega_n T} (\dot{y})^2\, d\tau = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 \left\{ C_L Y \int_o^{2\pi} \sin(u + \varphi) \cos u\, du - C_R Y \int_o^{2\pi} \sin(u + \varphi_1) \cos u\, du \right\}, \quad u = \alpha\tau$$

or

$$2\,\zeta_s \int_o^{\omega_n T} (\dot{y})^2\, d\tau = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 \left\{ \pi\, C_L\, Y \sin \varphi - \pi\, C_R\, Y \sin \varphi_1 \right\}. \tag{A2}$$

When equation (7) is evaluated in the same manner, one obtains

$$2\,\zeta_s \int_o^{\omega_n T} (\dot{y})^2\, d\tau = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 \left\{ \int_o^{\omega_n T} C_{mh} \sin \alpha\, \tau\, \dot{y}\, d\tau - \int_o^{\omega_n T} C_{dh} \cos \alpha\, \tau\, \dot{y}\, d\tau \right\}. \tag{A3}$$

From the assumed solution to equation (7),

$$\dot{y} = \alpha\, Y \cos (\alpha\, \tau - \epsilon)$$

so that equation (A3) reduces to

$$2\,\zeta_s \int_o^{\omega_n T} (\dot{y})^2\, d\tau = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 \left\{ C_{mh}\, Y \int_o^{2\pi} \sin u \cos (u - \epsilon)\, du - C_{dh}\, Y \int_o^{2\pi} \cos u \cos (u - \epsilon)\, du \right\}$$

or

$$2\,\zeta_s \int_o^{\omega_n T} (\dot{y})^2\, d\tau = \mu \left(\frac{\omega_s}{\omega_n}\right)^2 \left\{ \pi\, C_{mh}\, Y \sin \epsilon - \pi\, C_{dh}\, Y \cos \epsilon \right\}. \tag{A4}$$

A straight-forward comparison of equation (A4) with equation (A2) yields the results

$$C_L \sin \varphi = - C_{dh} \cos \epsilon \quad \text{EXCITATION (10a)}$$

$$C_R \sin \varphi_1 = - C_{mh} \sin \epsilon \quad \text{FLUID REACTION (DAMPING) (10c)}$$

as previously had been obtained. The "generalized Morison coefficient" C_{mh} from Sarpkaya's formulation [21], has a component $C_{mh} \sin \epsilon$ which acts to oppose the velocity $\dot{y}$ of the structure (since ϵ is typically negative for vortex-excited, resonant oscillations) and acts effectively as a fluid reaction force as had been hypothesized by Griffin & Koopmann [19]. Likewise, the Morison coefficient C_{dh}, which is negative during vortex-excited oscillations, as shown in Figure 2, is in effect a lift force which transfers energy to the vibrating structure through the component $-C_{dh} \cos \epsilon$. This latter force is in phase with the velocity term $\dot{y}$. The results given by equations (A2) and (A4) show that the excitation force on the cylinder is balanced by a fluid reaction force and a structural damping force. The reaction or damping effect of the fluid can be measured simply in terms of an effective fluid damping factor ζ_F as shown in Figure 4, which is taken from the wind tunnel experiments with resonantly vibrating circular cylinders reported by Griffin & Koopmann [19]. Blake, Maga & Finkelstein [22] have made analogous measurements in a water tunnel with model hydrofoils which were resonantly excited by vortex shedding.

TABLE 1

THE LIFT COEFFICIENTS OF VIBRATING CIRCULAR CYLINDERS;

FROM GRIFFIN & KOOPMANN [19].

System	Damping coefficient, R_S (kg/sec)	Damping ratio, ζ_S	Mass ratio, μ	Reduced damping, $S_G = \zeta_S/\mu$	Response parameter, W^{++}
I	5×10^{-5}	6.8×10^{-4}	8.6×10^{-3}	0.079	0.068
II^{+}		3.3×10^{-4}	2.4×10^{-3}	0.14	0.033
III^{+}		1.6×10^{-4}	5.2×10^{-4}	0.31	0.016

$^{+}$Obtained by assuming R_S = constant for all three systems. R_S for System I was measured directly in vacuo; see reference 1.

$^{++}W = \dfrac{2\alpha\zeta_S}{1-\alpha^2}$, see equation (13c); data shown above correspond to the maximum amplitude $\bar{Y}_{MAX}$ for each cylinder.

TABLE 2 (a)

HYDRODYNAMIC FORCES ON CIRCULAR CYLINDERS;

FROM SARPKAYA [21].

Reduced Velocity, $V_r = 5.$

Displacement, $Y = \bar{Y}/D$	Inertia Coefficient,++ C_{mh}	Drag Coefficient, C_{dh}	Phase Angle, $\beta = \arctan\left[C_{dh}/C_{mh}\right]$	Fluid Force Components, Equation (10)+ Excitation, $-C_{dh}\cos\varepsilon$	Damping, $-C_{mh}\sin\varepsilon$	Inertia, $C_{dh}\sin\varepsilon$
0.13	0.4	-0.2	-26.6°	0.18	0.18	0.089
0.25	0.4	-0.35	-41.2°	0.26	0.26	0.23
0.50	1.0 to 2.2	-0.9	-42° to 22.3°	0.67 to 0.83	0.67 to 0.83	0.60 to 0.34
0.75	1.0 to 2.6	-0.6	-28.8° to -11.9°	0.48 to 0.54	0.48 to 0.54	0.26 to 0.11

+ $\varepsilon = \beta + \gamma$ and $\gamma = \arctan W$, where $W = \dfrac{2\alpha\zeta_s}{1-\alpha^2}$, $\alpha = \dfrac{\omega}{\omega_n}$. In addition,

$W = 0.05$ is assumed (See Table 1).

++ The inertia coefficient is evaluated here in coordinates appropriate to a cylinder vibrating in a quiescent fluid or, equivalently, a fluid in uniform, steady motion.

TABLE 2(b)

ESTIMATED HYDRODYNAMIC FORCES ON CIRCULAR CYLINDERS;

FROM MERCIER [20]

Reduced velocity, V_r = 5.5-6

Displacement $Y = \bar{Y}/D$	Inertia Coefficient, C_{mh}	Drag Coefficient, C_{dh}	Phase angle, $\beta = \arctan\left(\frac{C_{dh}}{C_{mh}}\right)$	Fluid Force Components, Equation (10)+ Excitation, $-C_{dh}\cos\epsilon$	Damping, $-C_{mh}\sin\epsilon$	Inertia, $C_{dh}\sin\epsilon$
0.3	0.31	-0.32	-45.9°	0.23	0.21	0.22
0.5	0.51 to 1.23	-0.59	-49.1° to -25.6°	0.41 to 0.54	0.37 to 0.48	0.43 to 0.23
1.0	1.64	-0.13	-4.5°	0.13	0.047	0.0037

+ $\epsilon = \beta + \gamma$ and γ = arc tan W, where $W = \frac{2\alpha\zeta_s}{1-\alpha^2}$, $\alpha = \frac{\omega}{\omega_n}$. In addition, W = 0.05 is assumed once again (See Table C1).

TABLE 3

THE LIFT COEFFICIENTS ON VIBRATING BLUFF CYLINDERS;
DESCRIPTION OF THE DATA IN FIGURE 6

Symbol	Type of cylinder	Medium	Cylinder material	Investigator(s)
▲	Flexible cantilever	Water	PVC	King
■			PVC	
			Aluminum	
			Stainless steel	
●	Pivoted rigid cylinder	Water and Air	Brass	Vickery & Watkins
+	Spring-mounted rigid cylinder	Air	Aluminum tubing	Griffin & Koopmann
■	Rigid cylinder, forced oscillations	Water	Stainless steel	Mercier
o	Rigid cylinder, forced oscillations	Water	Aluminum tubing	Sarpkaya
Δ	Flexible cantilever	Air	Aluminum	Hartlen, Baines & Currie

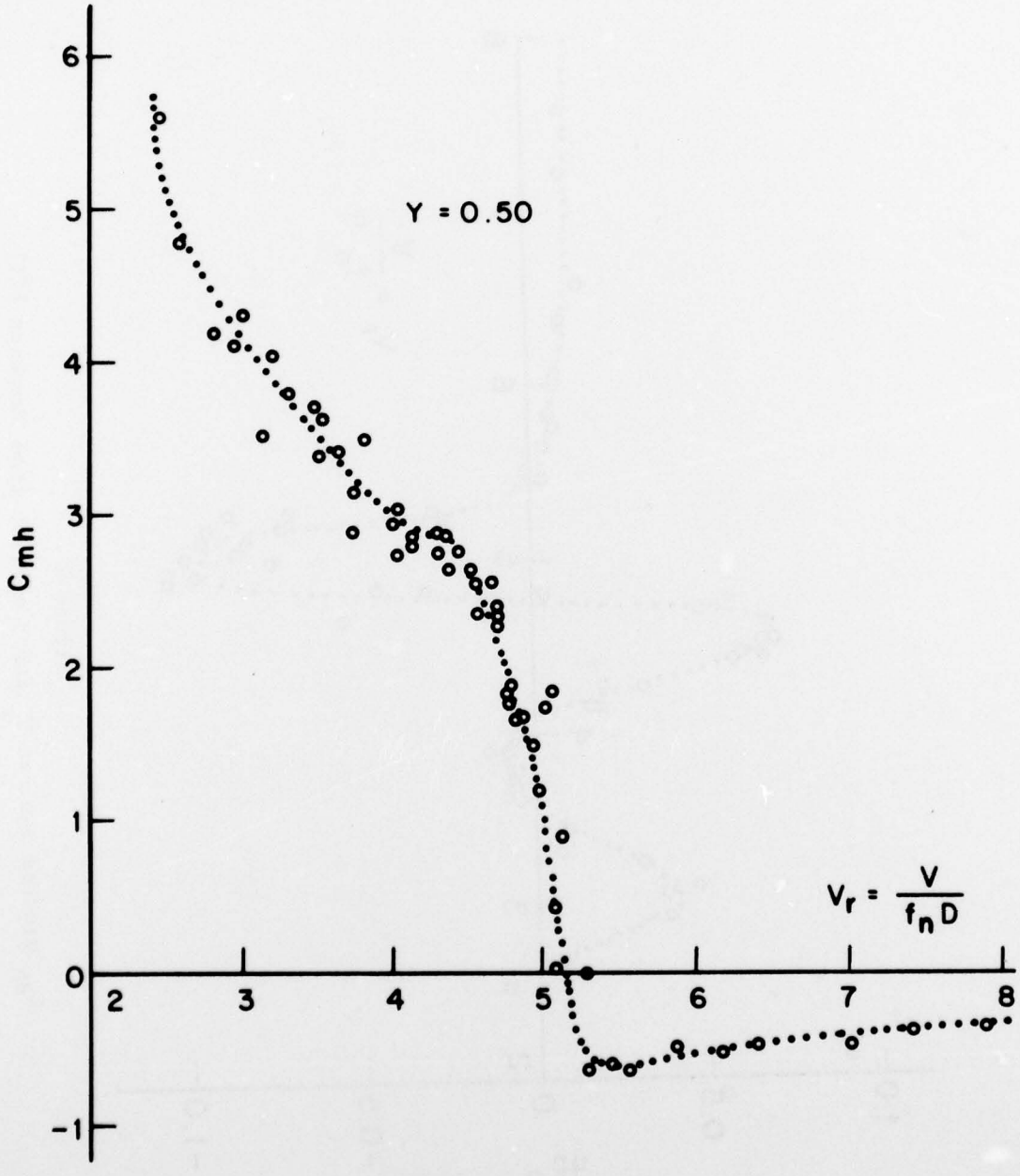


Figure 1

C_{mh} plotted against V_r for Y = 0.50: from Sarpkaya [21].

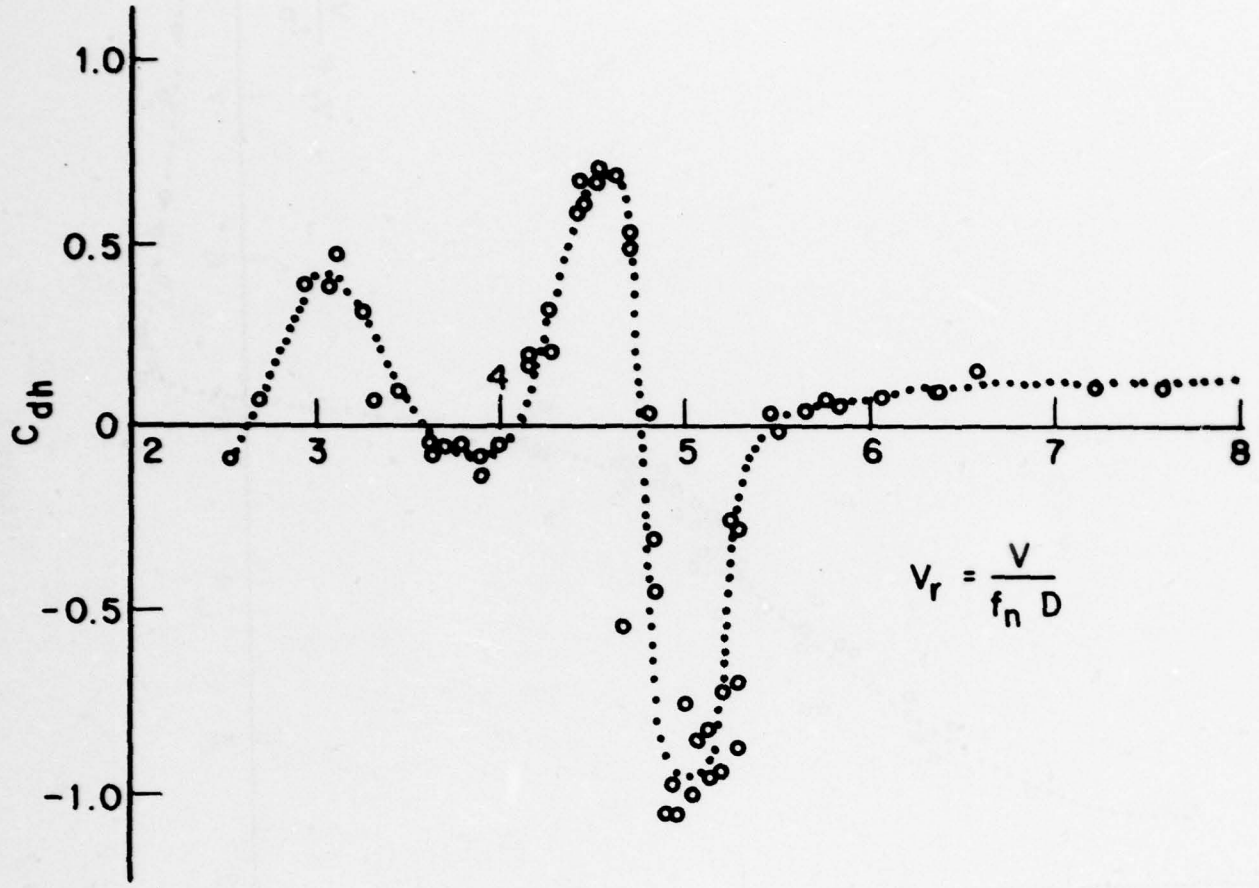


Figure 2

C_{dh} plotted against V_r for Y = 0.50; from Sarpkaya [21].

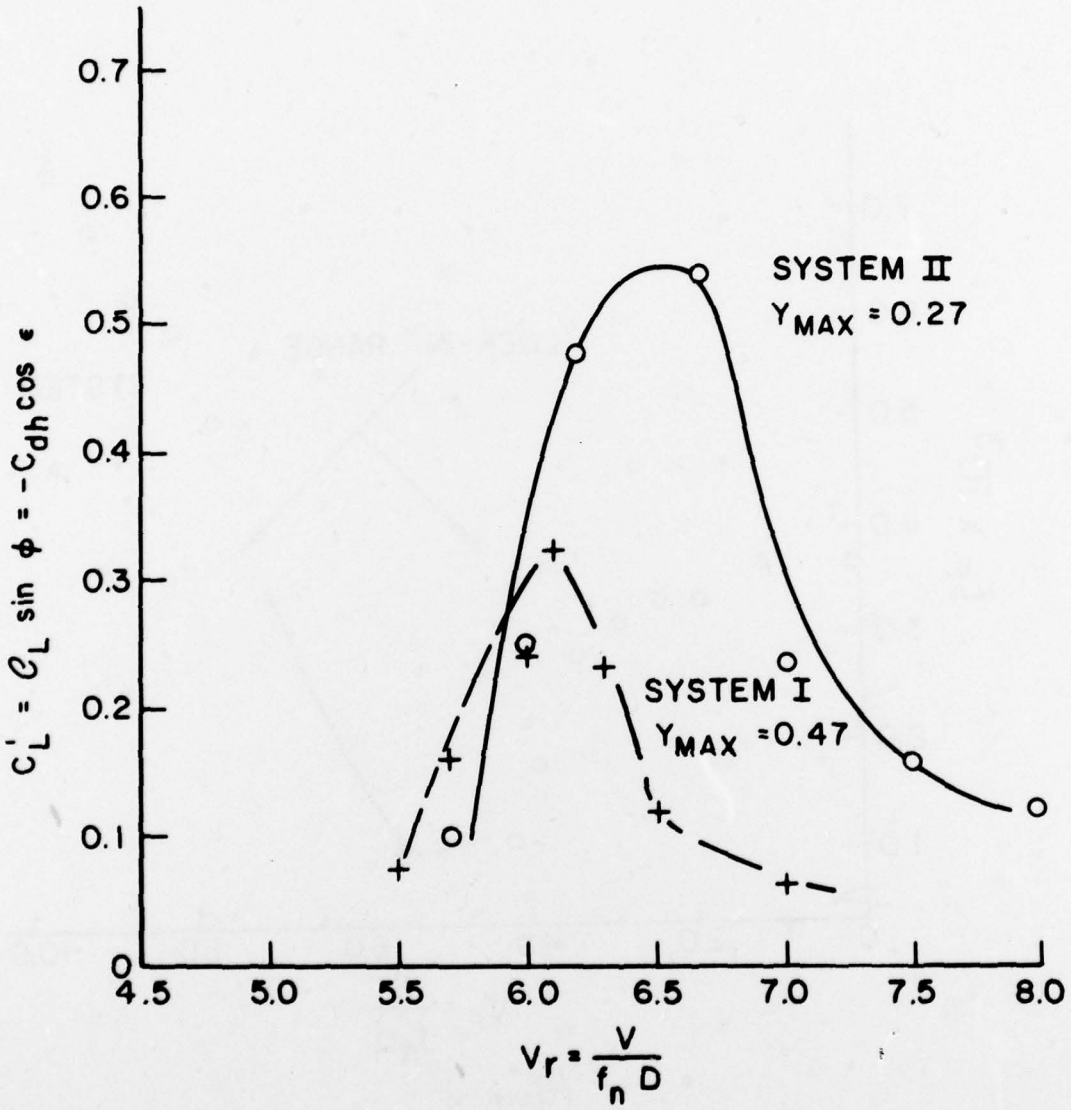


Figure 3

$C_L \sin\varphi$ plotted against V_r for two cylinders (see Table 1) resonantly excited by vortex shedding; from Griffin & Koopmann [19].

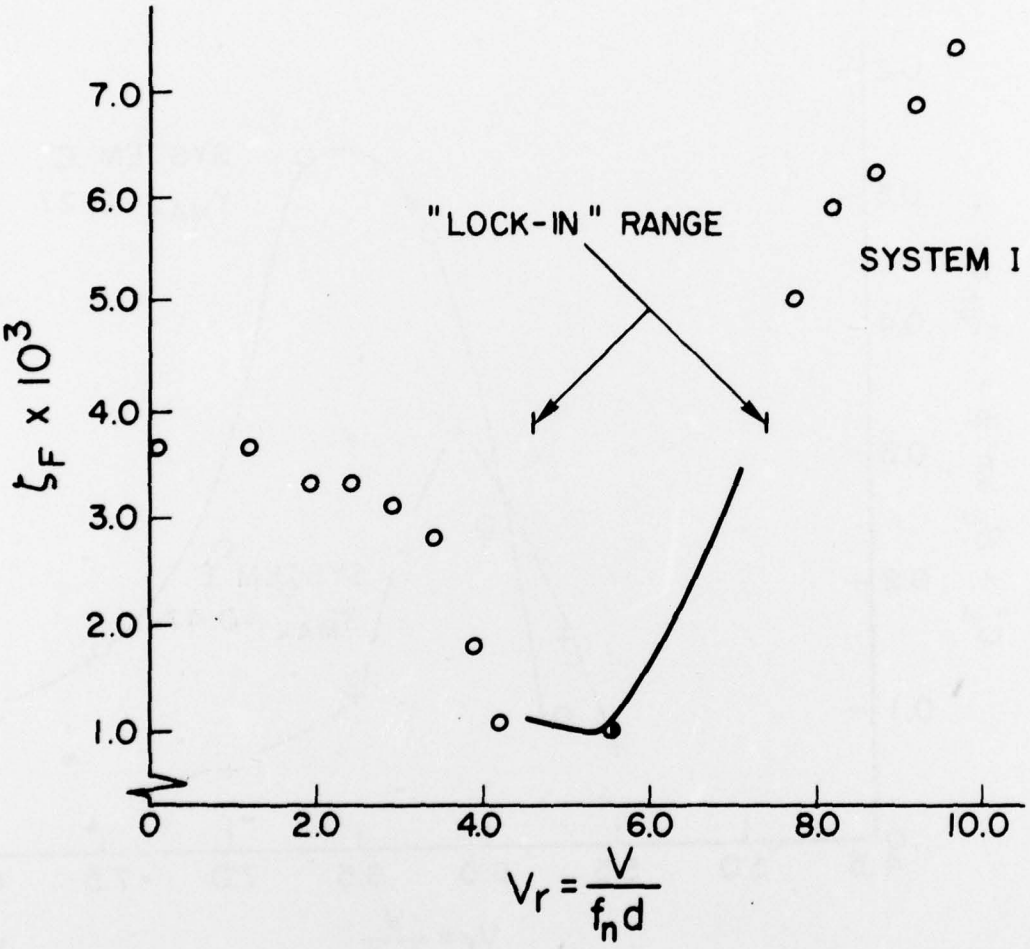
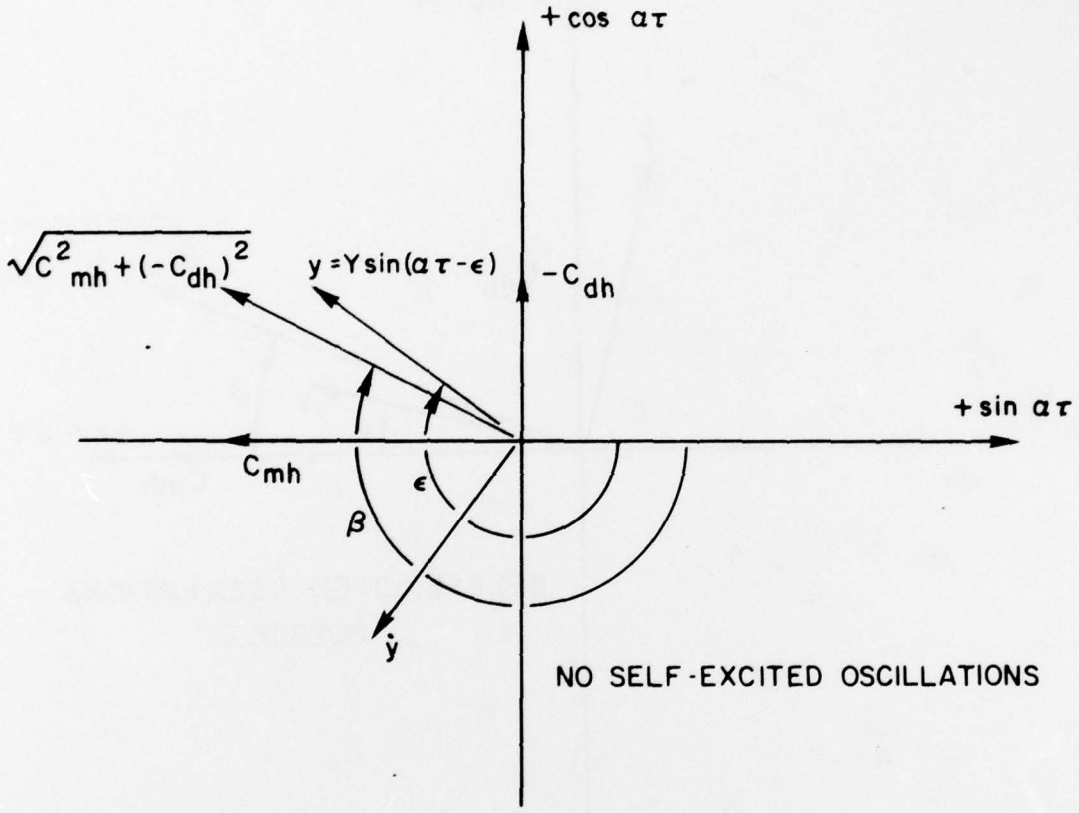


Figure 4

The fluid damping ratio ζ_F plotted against V_r; from Griffin & Koopmann [19].



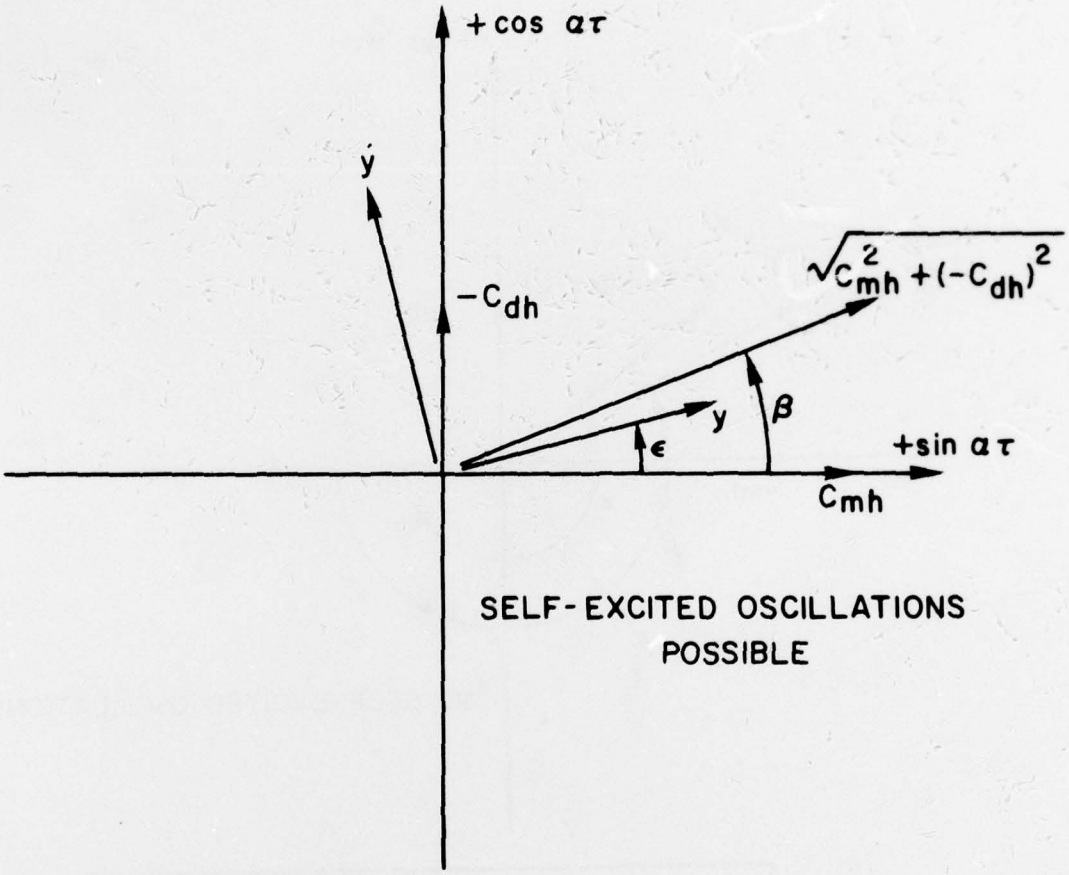
CASE 1: $Y = 0.25$, $V_r = 5.5$ $\begin{cases} C_{mh} = -0.5 \\ C_{dh} = -0.2 \end{cases}$

$\varepsilon = +204.7°$ $(W = 0.05)$

$\beta = \text{ARC TAN}\,(C_{dh}/C_{mh}) = +201.8°$

Figure 5(a)

Phase distribution of the flow-induced forces C_{mh} and C_{dh}, and the displacement y; measured forces from Sarpkaya [21].



CASE 2: $Y = 0.50$, $V_r = 5$ $\begin{cases} C_{mh} = 1.6 \\ C_{dh} = -0.9 \end{cases}$

$\epsilon = -26.5°$

$\beta = \text{ARC TAN}\,(C_{dh}/C_{mh}) = -29.4°$

Figure 5(b)

Phase distribution of the flow-induced forces C_{mh} and C_{dh}, and the displacement y; measured forces from Sarpkaya [21].

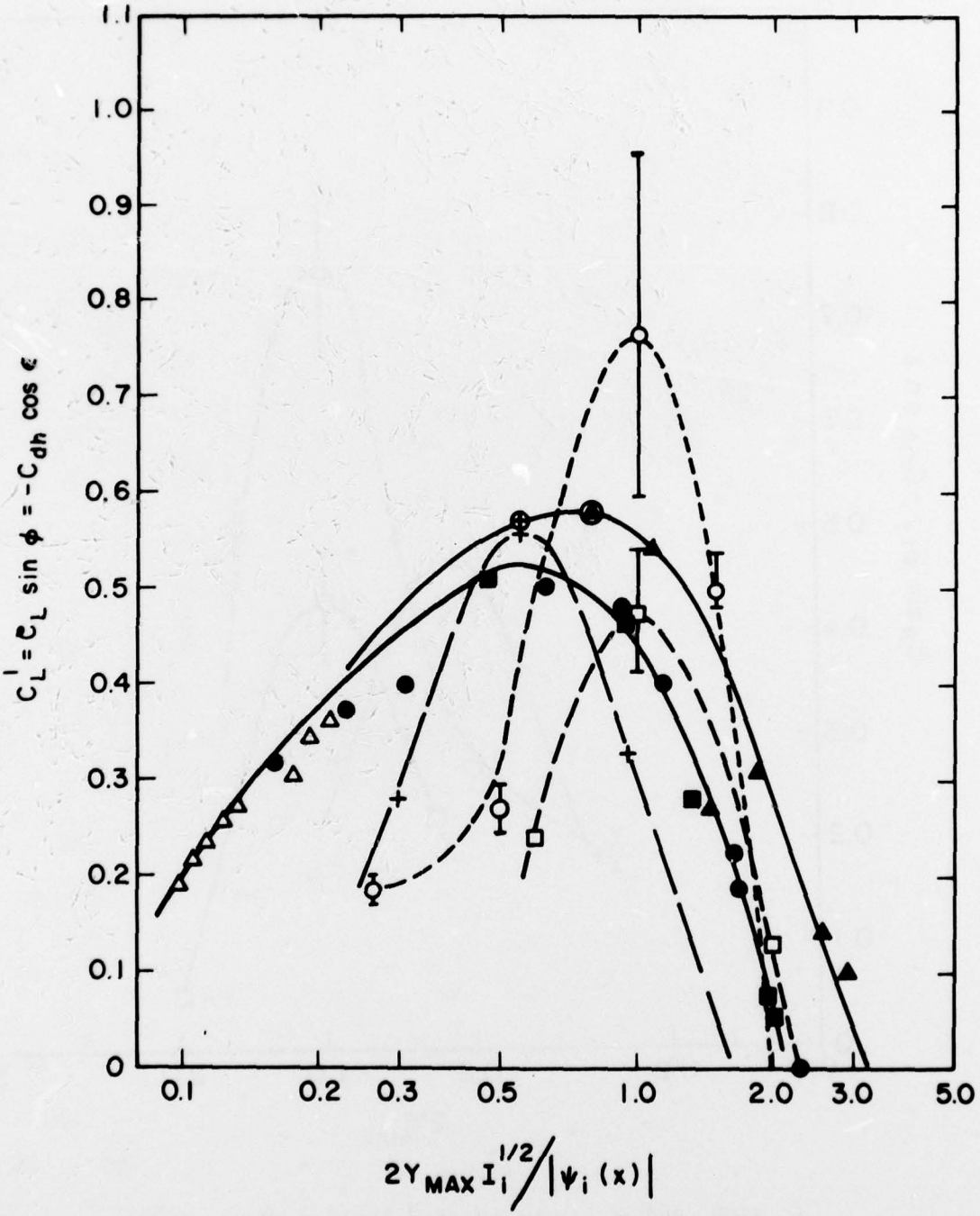


Figure 6

$C_L \sin\varphi$ and $C_{dh} \cos\epsilon$ plotted against $2Y_{MAX}$ (scaled as shown); for legend see Table 3.

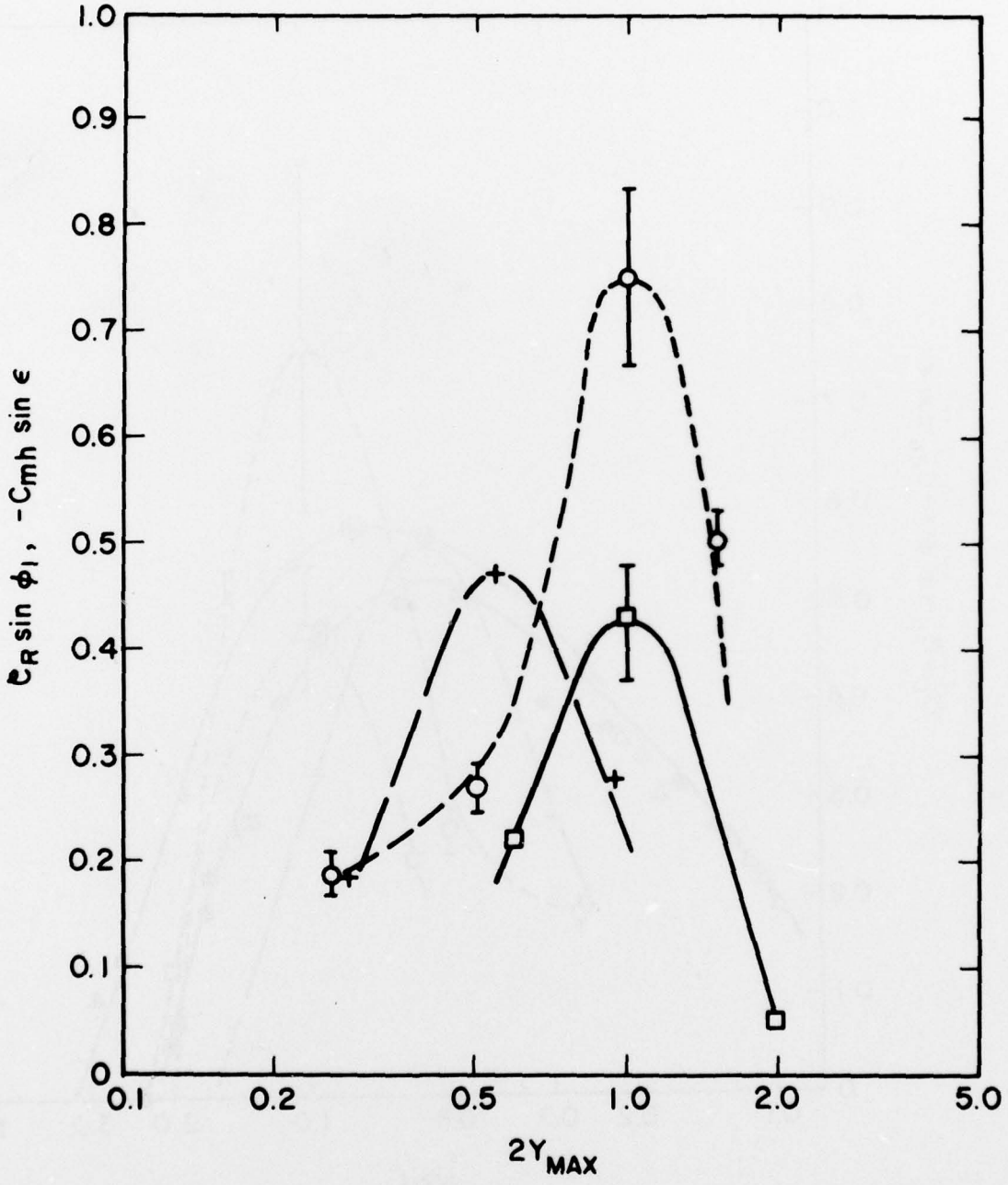


Figure 7

$C_R \sin\varphi_1$ and $-C_{mh} \sin\epsilon$ plotted against $2Y_{MAX}$; from Griffin & Koopmann [19] - -; from Sarpkaya [21] ----; from Mercier [20] ——— (all circular cylinders).