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Bispherical Constrained Lens Antennas

JAGANMOHAN B. L. RAO

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A spherical constrained lens having two surfaces (a pickup surface and a radiating surface) of equal radii has been known to provide multiple beams in all planes. This lens is equivalent in performance to a spherical reflector antenna but has an advantages of generating multiple beams with true time delay and no feed blockage, since the feed surface is behind the pickup and radiating surfaces. An investigation has been made of a generalized bispherical constrained lens, with the radii of its two surfaces being allowed to vary. This additional degree of freedom allows flexibility in designing a multiple-beam antenna. It is shown that, for a specified radiating aperture and allowable

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maximum phase error, a lens with a smaller focal length F (or, equivalently, axial distance F between the feed and pickup surfaces), than that of the equal-radius lens can be obtained by increasing the radius of curvature of the radiating surface, resulting in a more compact lens.

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BISPHERICAL CONSTRAINED LENS ANTENNAS

INTRODUCTION

The suitability of a spherical reflector for use as a wide-angle scanning antenna is well established [1, 2]. However its usefulness as a simultaneous-multiple-beam antenna is limited due to blockage by the feed structure. McFarland and Ajioka [3] reported on a spherical constrained lens, consisting of a feed surface behind two surfaces of equal radii, which can be used as a multiple-beam antenna with true time delay and no feed blockage. For the present report an investigation has been made of a generalized bispherical constrained lens, with the radii of its two surfaces being allowed to vary. This additional degree of freedom allows flexibility in designing a multiple-beam antenna.

For a specified radiating aperture and specified radii of lens surfaces, an optimum feed location which minimizes the maximum phase excursion on the aperture will be determined analytically. The effect of different antenna parameters on the aperture phase errors will be considered in detail. For a specified radiating aperture and allowable maximum phase error, it will be shown that a lens with smaller focal length F (axial distance F between the feed and pickup surfaces) than that of the equal-radius lens can be obtained by increasing the radius of curvature of the radiating surface, resulting in a more compact lens.

BISPHERICAL CONSTRAINED LENS

The geometry of a bispherical constrained lens is shown in Fig. 1. The lens consists of a spherical feed surface (S_1) and spherical pickup and radiating surfaces $(S_2$ and $S_3)$ of radii R and R_0 respectively. Since the antenna should be spherically symmetric for wide-angle scanning, the antenna elements on S_2 and S_3 have one-to-one correspondence and are connected by transmission lines of equal length. Each radiator on the feed surface illuminates a different portion of the antenna and corresponds to a separate beam direction; beams can be generated singly or in any combination.

Phase Errors

A feed element is assumed to be at a point P, at a distance F from the surface S_2 $(F = \overline{OP})$. Consider a typical ray path $PQGE$, where Q is an arbitrary point on the inner lens surface and can be specified by X or θ (θ = angle OCQ), which are related as $X = R \sin \theta$. Similarly, G is the corresponding point on the outer surface S_3, which is also specified by the angle θ (since, to make the antenna spherically symmetric, the angle $BE'G$ is also made equal to θ). The point G is at a radial distance of $R_0 \sin \theta$ from the lens axis. The total path length from the point F to the aperture plane at E is
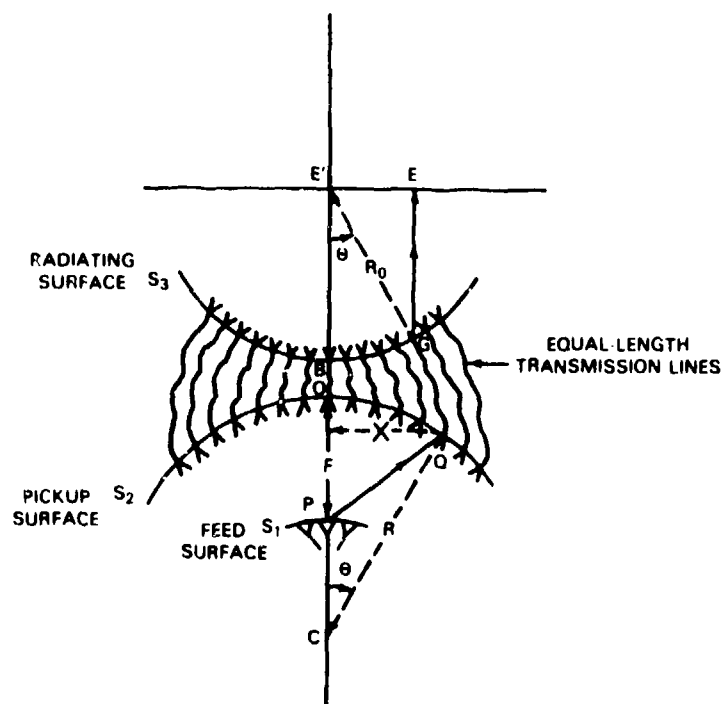
Fig. 1 — Geometry of a bispherical constrained lens

$$d = \overline{PQ} + \overline{QG} + \overline{GE},\tag{1}$$

where

$$\overline{PQ} = \sqrt{F^2 + 2R(R - F)(1 - \cos\theta)},$$

$$\overline{OG} = \text{transmission line length} = \text{a constant},$$

and

$$\overline{GE} = R_0 \cos\theta.$$

Because the transmission line lengths $\overline{OB}$ and $\overline{QG}$ are equal, the path-length difference between an axial ray path $\overline{POBE'}$ and a nonaxial ray path $\overline{PQGE}$ is

$$\epsilon = \overline{PQ} + \overline{GE} - (\overline{PO} + \overline{BE}').\tag{2}$$

Substituting for $\overline{PQ}$ and $\overline{GE}$ from (1) and noting that $\overline{PO} = F$ and $\overline{BE}' = R_0$, one finds that the phase error in wavelengths is

$$(\epsilon/\lambda) = (R/\lambda)\sqrt{f^2 + 2(1 - f)(1 - \cos\theta)} - f - r_0(1 - \cos\theta),\tag{3}$$

where $f = F/R$, $r_0 = R_0/R$, and λ is the wavelength.

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In the case of a spherical reflector it has been noted [1] that the total phase excursion over a prescribed aperture is least when the phase error at the edge of the aperture is zero. The same is true in the present case, as the lens is spherically symmetric and the phase errors are due to spherical aberration. Using this fact, one can find an optimum feed position (distance F) for specified values of R and R_0, an optimum value for R_0 for given values of F and R, or an optimum value of R for specified values of F and R_0. In other words an optimum relationship exists between F, R, and R_0 which minimizes the maximum phase error for a specified aperture. Let the radiating aperture be specified by its diameter D or by an angle θ_a such that $D = 2R_0 \sin \theta_a$. The optimum relationship is obtained by equating the phase error, given by (3), to zero when $\theta = \theta_a$ and solving for f, and one obtains

$$f = \frac{1 - r_0^2 \sin^2 (\theta_a/2)}{1 + r_0} .$$ (4)

A special case of $r_0 = 1$ (or $R_0 = R$) corresponds to a lens discussed by McFarland and Ajioka [3]. As mentioned earlier, this is a bispherical lens that is equivalent to a spherical reflector. The optimum focal length obtained from (4) for $r_0 = 1$ is

$$F_{op} = \frac{R \cos^2 \frac{\theta_a}{2}}{2} = \frac{1}{4} \left(R + \sqrt{R^2 - R^2 \sin^2 \theta_a} \right),$$ (5)

which agrees exactly with that of a spherical reflector [1].

The optimum relationship given by (4) is shown in Fig. 2 for different values of $\sin \theta_a$. As f is increased, r_0 decreases and becomes zero for $f = 1$. As f is increased further, r_0 becomes negative. What this means is that the radiating surface is concave for $f < 1$ and convex for $f > 1$.
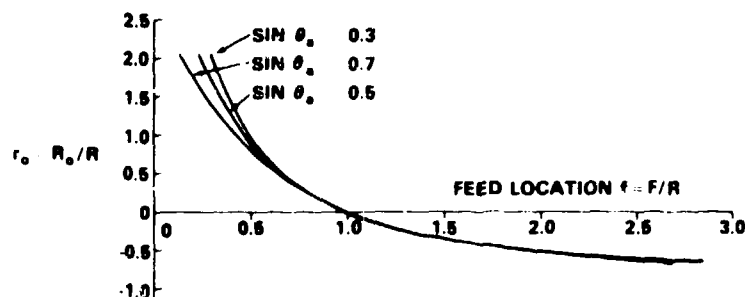

Fig. 2 — Optimum relation between r_0 and f for different values of θ_a

Figure 3 shows the path-length error (and hence phase error) normalized to the width of the radiating aperture D and for the specific value $\sin \theta_a = 0.4$ with R as a parameter. In obtaining Fig. 3, the optimum value of f for the specified $\sin \theta_a$ (or θ_a) and r_0 is first determined from (4) and substituted in (3) to determine the path-length error. For proper interpretation of the results in Fig. 3, the radiating aperture is assumed to be a constant, which means R_0 is assumed to be a constant, since $D = 2R_0 \sin \theta_a$. Different values of r_0 are obtained by changing R, the radius of curvature of the inner surface of the lens.

A point on the radiating aperture can be specified by the radial distance $R_0 \sin \theta$ from the lens axis or simply by an angle θ. In Fig. 3 the angle θ is plotted along the horizontal axis. It may be noted from Fig. 3 that for a given radiating aperture the phase errors will decrease as the radius of curvature R of the inner surface is increased.
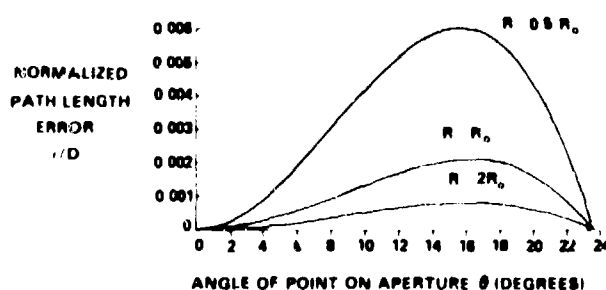

Fig. 3 — Normalized phase errors over a given radiating aperture D ($D = 2R_0 \sin \theta_a$, with $\theta_a = \sin^{-1} 0.4 = 23.5°$) for different values of R, plotted as a function of θ, where θ specifies the radial distance $R_0 \sin \theta$ from the lens axis

Maximum Phase Error on the Aperture

In any antenna design a constraint is usually imposed on allowable maximum phase error. Hence it is of interest to find the maximum phase error on the aperture for specified lens parameters F, R, and R_0.

In (3) a point on the aperture is specified by an angle θ. To find the maximum phase error on the aperture, one should first find the angle $\theta = \theta_m$ at which the phase error is maximum and then substitute that value in (3). To find the value of $\theta = \theta_m$ for which ϵ is maximum, one has to take a derivative of ϵ with respect to θ and equate it to zero:

$$\frac{de}{d\theta}\Bigg|_{\theta = \theta_m} = R\left(\frac{1 - f}{\sqrt{f^2 + 2(1 - f)(1 - \cos \vartheta_m)}} - r_0\right) \sin \vartheta_m = 0.$$

The solution $\theta_m = 0$ actually corresponds to a minimum phase error at the center of the aperture. The other solution is the proper answer, which can be shown to be

$$\cos \theta_m = 1 - \frac{(1 - f)}{2r_0^2} + \frac{f^2}{2(1 - f)}. \tag{6}$$

Substituting $\cos \theta_m$ for $\cos \theta$ in (3) and simplifying, one obtains the maximum phase error on the aperture as

$$\epsilon_{max} = R\left[\frac{1 - f}{2r_0} - f + \frac{f^2 r_0}{2(1 - f)}\right]. \tag{7}$$

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For any specified value of θ_a, r_0 can be eliminated from (7) by using the optimum relation given by (4). Then ϵ_{max} will be a function of focal length f only. Figure 4 shows the maximum phase error normalized to the width of the radiating aperture as a function of f for different values of normalized aperture diameter $D/2R_0$ ($= \sin \theta_a$). The special case of $r_0 = 1$ is indicated by the dashed line. For a specified radiating aperture and a maximum allowable phase error, Fig. 4 shows that a number of solutions are possible with different combinations of feed location f and $\sin \theta_a$, including the special case of r_0 corresponding to a spherical reflector. Compared to this special case, the phase errors can be made smaller by increasing f (decreasing r_0 or increasing R), and the phase errors will be larger for smaller f for a specified $\sin \theta_a$. These observations are true for $f < 1$.
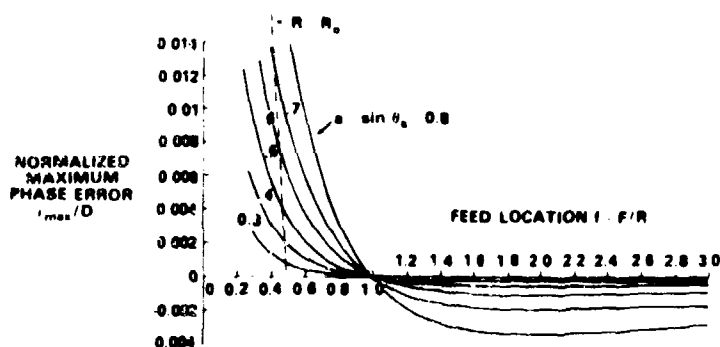

Fig. 4 — Maximum phase error as a function of feed location for different apertures $D/2R_0$ ($= \sin \theta_a$)

For $f > 1$ (negative r_0) the magnitude of the phase error increases with f and reaches a maximum at $f = 2$ (Appendix) and decreases slowly as f is increased further for any given $\sin \theta_a$. Since the phase errors are smaller for $f > 1$, it may appear to be the proper choice in designing a lens as a multiple-beam antenna. But choosing $f > 1$ results in large effective F/D, which means that the feed elements must be more directive and the lens less practical, since the size of the lens increases with F/D for a given D. Therefore we will not dwell much on the lenses with $f > 1$ except for the special case of $f = 2$, which lens has some unique features making it more practical for certain special purposes as discussed in Ref. 4. This special case has come to be known as the R-KR lens [5]. Even though the phase errors are maximum for $f = 2$ in the range $1 < f < \infty$, as shown in the appendix, the errors are tolerable for many practical cases and the lens has a symmetry which allows placing the feed and pickup elements on the same spherical surface but on the opposite sides. This allows the use of inactive pickup elements as feed elements, eliminating the need for an additional set of feed elements. In the case of a two-dimensional lens, it has circular symmetry which allows 360° scanning [4].

Since the details concerning the R-KR lens are fairly well documented, we will not spend much time discussing it. However, it may be of interest that the optimum r_0 is $(1/2) \cos^2 (\theta_a/4)$ for $f = 2$, as shown in the appendix, which agrees with the result derived in a patent by Thies [5]. Insufficient design data are given in Ref. 4 except for a statement that $1/r_0 \approx 1.9$ for $\theta_a = 60°$. The value obtained by using this approximate relationship results in 50% greater maximum phase error than results from the value obtained by using the correct relationship. In addition the maximum phase error on the aperture

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for an R-KR lens is given in the appendix. This result is not available in the existing literature and should make the R-KR lens simpler to design.

DESIGN EXAMPLES

The main purpose of this report is to show that the additional degree of freedom that is introduced allows the designer to have a number of options available for specified requirements on scanning range and the allowable phase errors. It is beyond the scope of this report to pinpoint the option which is more suitable or practical for specified requirements. However some general observations should help the designer in choosing the antenna parameters which make it more suitable for his requirements.

One observation, made earlier, is that for a given radiating aperture the phase errors can be reduced by increasing the radius of curvature of the inner lens surface. Figure 5 shows that for a specified radiating aperture and specified phase errors, increasing the radius of curvature (or decreasing θ_a) of the radiating surface results in a smaller feed structure and smaller effective F/D ratio, which makes it more practical and compact. In Fig. 5, four lens geometries, each with different θ_a ($\sin \theta_a$ = 0.3, 0.4, 0.5, and 0.7) are illustrated. The radiating aperture D and the maximum allowable phase errors are the same for all the four cases. The maximum allowable phase error is assumed to be $0.002D$ (which corresponds to a phase error of 0.12λ for an aperture of 60 wavelengths, or for a beamwidth of about 1 degree). The corresponding values of f are determined using Fig. 4. The fourth case in Fig. 5 corresponds to a R-KR lens (f = 2). When f and θ_a are known, the corresponding values of r_0 are determined using (4) or Fig. 2. With note of the relationship $D = 2R_0 \sin \theta_a$, all the lens dimensions are normalized to the radiating aperture D and are drawn to scale in Fig. 5. Points C_1, C_2, C_3, and C_4 are the centers of curvature of the four inner surfaces.
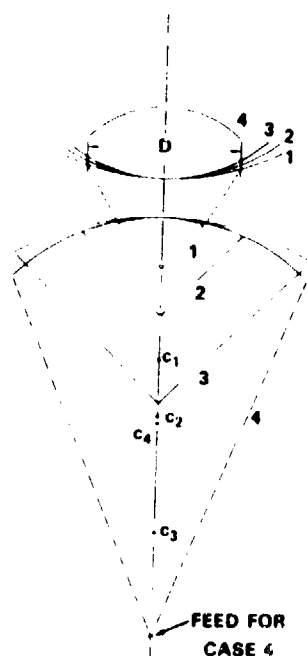

Fig. 5 — Geometries of lenses for a specified D and a specified ϵ_{max}/D = 0.002 and for (case 1) $\sin \theta_a$ = 0.3, (case 2) $\sin \theta_a$ = 0.4, (case 3) $\sin \theta_a$ = 0.5, (case 4) $\sin \theta_a$ = 0.7

F om Fig. 5 it is evident that case 1, with the smallest θ_a, results in a lens which has the smallest effective F/D and is the most compact, possibly making it more practical However two other considerations will also have an influence on choosing θ_a. The first is that θ_a cannot be made smaller than that value which makes $r_0 > 2$. In such a case, if the interelement spacing on the inner lens surface is $\lambda/2$, the spacing on the radiating surface will be greater than a wavelength, which may result in grating lobes. The second consideration is that multiple beams are produced by illuminating a different area on the lens for different beams, meaning that for a given coverage of multiple beams the radiating aperture will be larger for smaller θ_a. These conflicting considerations should be taken into account in any practical design.

As illustrated in Fig. 5, for a specified phase error and radiating aperture, making θ_a larger necessitates an increase in the size of the inner lens surface and an increase in F/D, resulting in a lens which is less practical. The lens with a larger θ_a can be made more compact if the space between the inner lens surface and the feed surface is filled with dielectric material, as demonstrated in two-dimensional R-KR lens design [4] and as illustrated in Fig. 6 for a case of $f < 1$. The example shown in Fig. 6 corresponds to case 3 of Fig. 5 (sin $\theta_a = 0.5$, and, by using a relative dielectric constant of $n = 1/r_0 = 1.96$, the physical aperture of the radiating surface and pickup surface are made equal. The physical length between the feeds and the inner lens surface is also decreased by a factor of n. Comparing the case 3 of Fig. 5 with that of Fig. 6, it may be noted that the lens shown in Fig. 6 is more compact. However dielectric loading increases antenna weight. Therefore this approach may be practical only when the weight of the antenna is not a main concern.
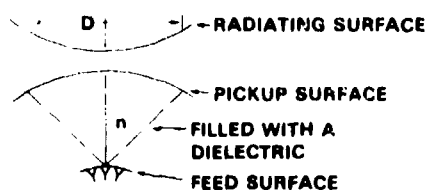

Fig. 6 — Geometry of a dielectric-loaded lens

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Appendix

VALUES OF f AND r_0 FOR MAXIMUM PHASE ERRORS WHEN $f > 1$

Figure 4 shows the maximum phase error normalized to the radiating aperture as a function of focal length f for different values of $\sin \theta_a$. For $f > 1$ the maximum phase error on the aperture increases first with f and reaches a negative peak and decreases as f is increased further. The purpose of this appendix is to find the value of f for which the maximum phase error on the aperture reaches a negative peak.

From (7) the maximum phase error normalized to the radiating aperture is

$$\frac{e_{max}}{D} = \frac{1}{2 \sin \theta_a r_0} \left[\frac{1 - f}{2 r_0} - f + \frac{f^2 r_0}{2(1 - f)} \right]. \tag{A1}$$

For any specified aperture radius ($\sin \theta_a$) there is an optimum relationship between r_0 and f as given by Eq. (4). Use of that equation can elimate either f or r_0 from (A1). Eliminating f is found to be more convenient. The value of $r_0 = r_{0m}$ for which e_{max}/D reaches a negative peak can be found by eliminating f from (A1) and taking a derivative of ϵ_{max}/D with respect to r_0 and equating it to zero. After considerable algebriac and trigonometric manipulation, it can be shown that

$$r_{0m} = -\frac{1}{2} \cos^2 \left(\frac{\theta_a}{4} \right). \tag{A2}$$

Substituting this r_{0m} value for r_0 in (4), one can obtain the value $f = f_m$ for which the ϵ_{max}/D reaches a negative peak, which is

$$f_m = 2. \tag{A3}$$

The corresponding phase error can be shown to be

$$\epsilon_{max}/D \Big|_{f=2} = - [\sin^4(\theta_a/4)] / \sin \theta_a. \tag{A4}$$