

CANONICAL CORRELATIONS WITH RESPECT TO A COMPLEX STRUCTURE

BY

STEEN A. ANDERSSON

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OFFICE OF NAVAL RESEARCH

THEODORE W. ANDERSON, PROJECT DIRECTOR

DEPARTMENT OF STATISTICS

STANFORD UNIVERSITY

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1. Introduction

Let E be a vector space of dimension $2p$ over the field of real numbers $\mathbb{R}$. Let $x_1, \dots, x_N$ ($N \geq 2p$) be identically distributed independent observations from a normal distribution with mean value 0 and unknown covariance Σ . That is, Σ is a positive definite form on the dual space E^* to E . The maximum likelihood estimator $\hat{\Sigma}$ for Σ is well-known to be given by

$$\hat{\Sigma}(x_1, \dots, x_N) = ((x^*, y^*) \rightarrow \frac{1}{N} \sum_{i=1}^N x^*(x_i) y^*(x_i) ; x^*, y^* \in E^*) .$$

The distribution of $\hat{\Sigma}$ is the Wishart distribution on the set $\rho(E^*)_{\mathbb{R}}$ of positive definite forms on E^* with N degrees of freedom and parameter $\frac{1}{N} \Sigma$. Suppose now that E is also a vector space over the field $\mathbb{C}$ of complex numbers such that the restriction to the subfield of real numbers in $\mathbb{C}$ is the original vector space structure on E . The dimension of E as a vector space over $\mathbb{C}$ is then p . The vector space E^* is then also a vector space over the complex numbers under the definition $zx^* = x^* \circ \bar{z} = (x \rightarrow x^*(\bar{z}x) ; x \in E)$, $x^* \in E^*, z \in \mathbb{C}$. The set $\rho_{\mathbb{C}}(E^*)_{\mathbb{R}} = \{\Sigma \in \rho(E^*)_{\mathbb{R}} \mid \Sigma(zx^*, y^*) = \Sigma(x^*, \bar{z} y^*), \forall x^*, y^* \in E^*, \forall z \in \mathbb{C}\}$ defines a null hypothesis in the statistical model described above. The condition $\Sigma(zx^*, y^*) = \Sigma(x^*, \bar{z} y^*)$, $\forall x^*, y^* \in E^*, \forall z \in \mathbb{C}$ is in Andersson [2] called the $\mathbb{C}$ -property and in terms of matrices it has the formulation: For every basis $e_1^*, \dots, e_p^*$ for the complex vector space E^* the matrix for a Σ with the $\mathbb{C}$ -property with respect to the basis $e_1^*, \dots, e_p^*, ie_1^*, \dots, ie_p^*$ for the real vector space E^* has the form

$$(1.1) \quad \begin{pmatrix} \Pi & F \\ -F & \Pi \end{pmatrix}$$

The statistical problem of testing $\Sigma \in \rho_{\mathbb{C}}(E^*)_r$ versus $\Sigma \in \rho(E^*)_r$ is invariant under the action of the group $GL_{\mathbb{C}}(E)$ of complex one-to-one linear mappings onto the sample and parameter space $\rho(E^*)_r$ given by

$$(1.2) \quad \begin{aligned} GL_{\mathbb{C}}(E) \times \rho(E^*)_r &\rightarrow \rho(E^*)_r \\ (f, \Sigma) &\rightarrow \Sigma \circ (f^* \times f^*) \end{aligned}$$

where f^* is the dual mapping to $f \in GL_{\mathbb{C}}(E)$. The restriction of the action to the subset $\rho_{\mathbb{C}}(E^*)_r$ is transitive. Since all tests invariant under (1.2) have a factorization through a maximal invariant function we shall find a representation of a maximal invariant function into R_+^p , describe the distribution as a density with respect to a restriction of the Lebesgue measure and state an interpretation of this representation. The matrix for a complex linear mapping of E with respect to a basis of the form $e_1, \dots, e_p, ie_1, \dots, ie_p$, where $e_1, \dots, e_p$ is a basis for the complex vector space E is of the form

$$(1.3) \quad \begin{pmatrix} A & B \\ -B & A \end{pmatrix}$$

The expression $\Sigma \circ (f^* \times f^*)$ from (1.2) in matrix formulation becomes

$$(1.4) \quad \begin{pmatrix} A & B \\ -B & A \end{pmatrix} \begin{pmatrix} \Pi_{11} & \Pi_{12} \\ \Pi'_{12} & \Pi_{22} \end{pmatrix} \begin{pmatrix} A' & -B' \\ B' & A' \end{pmatrix}$$

with respect to the dual basis $e_1^*, \dots, e_p^*, ie_1^*, \dots, ie_p^*$ in E^* to $e_1, \dots, e_p, ie_1, \dots, ie_p$ in E .

2. Representation of the maximal invariant

2.1. Lemma. Let Π be a positive definite form on the $\mathbb{R}$ -space E . Then there exists a basis $e_1, \dots, e_p$ for the $\mathbb{C}$ -space F such that the $2p \times 2p$ real matrix for Π with respect to $e_1, \dots, e_p, ie_1, \dots, ie_p$ has the form

$$(2.1) \quad \begin{Bmatrix} I & D_\lambda \\ D_\lambda & I \end{Bmatrix}$$

where I is the $p \times p$ identity matrix and

$$(2.2) \quad D_\lambda = \text{diag}(\lambda_1, \dots, \lambda_p) \quad \text{with} \quad 1 > \lambda_1 \geq \dots \geq \lambda_p \geq 0.$$

Furthermore, the matrix D_λ is uniquely determined by Π ; and if $\lambda_1 > \lambda_2 > \dots > \lambda_p > 0$, then Π also determines the basis $e_1, \dots, e_p$ uniquely up to the sign of each basis vector.

Proof: Let $e'_1, \dots, e'_p$ be a basis for the $\mathbb{C}$ -space E and let

$$\begin{Bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi'_{12} & \Pi_{22} \end{Bmatrix}$$

be the $2p \times 2p$ real matrix for Π with respect to $e'_1, \dots, e'_p, ie'_1, \dots, ie'_p$. The assertion is then that there exists a nonsingular complex $p \times p$ matrix $Z_1 = A + iB$ such that

$$(2.3) \quad \begin{Bmatrix} A' & B' \\ -B' & A' \end{Bmatrix} \begin{Bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi'_{12} & \Pi_{22} \end{Bmatrix} \begin{Bmatrix} A & -B \\ B & A \end{Bmatrix} = \begin{Bmatrix} I & D_\lambda \\ D_\lambda & I \end{Bmatrix}$$

and that D_λ is unique; and in the case where $\lambda_1 > \dots > \lambda_p > 0$, the columns of Z are unique up to multiplication with ± 1 .

The equation (2.3) is equivalent to the complex matrix equations

$$\begin{aligned} \bar{Z}'_1 \left(\frac{1}{2}(\Pi_{11} + \Pi_{22}) + i \frac{1}{2}(\Pi'_{12} - \Pi_{12}) \right) Z_1 &= I \\ Z'_1 \left(\frac{1}{2}(\Pi'_{12} + \Pi_{12}) + i \frac{1}{2}(\Pi_{11} - \Pi_{22}) \right) Z &= D_\lambda \end{aligned} \quad (2.4)$$

If we define $Z = Z_1^{-1}$ and

$$\begin{aligned} \Phi &= \frac{1}{2}(\Pi_{11} + \Pi_{22}) + i \frac{1}{2}(\Pi'_{12} - \Pi_{12}) \\ \Psi &= \frac{1}{2}(\Pi'_{12} + \Pi_{12}) + i \frac{1}{2}(\Pi_{11} - \Pi_{22}) \end{aligned} \quad (2.5)$$

then (2.4) becomes

$$\begin{aligned} \Phi &= \bar{Z}' Z \\ \Psi &= Z' D_\lambda Z \end{aligned} \quad (2.6)$$

Since Φ respectively Ψ is the matrix for a positive definite hermitian form respectively symmetric form on the $\mathbb{C}$ -space E , it follows from [3] that we can find a complex $p \times p$ diagonal matrix D and a complex nonsingular $p \times p$ matrix Y such that

$$\begin{aligned} \Phi &= \bar{Y}' Y \\ \Psi &= Y' D Y \end{aligned} \quad (2.7)$$

By permutation we can obtain that the diagonal elements $d_1, \dots, d_p$ of D have the property $|d_1| \geq |d_2| \geq \dots \geq |d_p|$. If we then multiply

the v 'th row of Y with $\exp[-i\theta_v/2]$, where $d_v = |d_v| \exp[i\theta_v]$, $v = 1, \dots, p$, and call this new matrix for Z , we obtain (2.6) with $\lambda_v = |d_v|$, $v = 1, \dots, p$. Since Π is positive definite, we have $1 > \lambda_1 > \dots \geq \lambda_p \geq 0$. The uniqueness follows from a rather elementary examination of the proof in [3] or from direct matrix calculation. Since every matrix of the form (2.1) with $1 > \lambda_1 \geq \dots \geq \lambda_p \geq 0$ is positive definite it follows from Lemma (2.1) that the mapping from $P(E^*)_r$ onto $\Omega = \{(\lambda_1, \dots, \lambda_p) \in \mathbb{R}_+^p \mid 1 > \lambda_1 \geq \dots \geq \lambda_p \geq 0\}$ determined from Lemma 2.1 is a maximal invariant function.

3. Canonical correlations with respect to a complex structure.

Interpretation.

It follows from Lemma 2.1 that there exists a basis $e_1, \dots, e_p$ for the $\mathbb{C}$ -space E such that the $2p \times 2p$ matrix for Σ with respect to $e_1^*, \dots, e_p^*, ie_1^*, \dots, ie_p^*$ has the form (2.1). In (2.1) D_λ is unique; and if $\lambda_1 > \dots > \lambda_p > 0$, the basis $e_1^*, \dots, e_p^*$ for the $\mathbb{C}$ -space E^* is unique up to a sign for each element. λ_j is called the j -th theoretical canonical correlation of Σ with respect to the complex structure, and e_j^* is called the j -th theoretical canonical linear form of Σ with respect to the complex structure $j = 1, \dots, p$. Let $x \in E^*$ have coordinates $(\alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_p)$ with respect to $e_1^*, \dots, e_p^*, ie_1^*, \dots, ie_p^*$. Then

$$(3.1) \quad \Sigma(x^*, x^*) = \sum_i \alpha_i^2 + \sum_i \beta_i^2 + 2 \sum_i \lambda_i \alpha_i \beta_i$$

$$(3.2) \quad \Sigma(ix^*, ix^*) = \sum_i \alpha_i^2 + \sum_i \beta_i^2 - 2 \sum_i \lambda_i \alpha_i \beta_i$$

$$(3.3) \quad \Sigma(x^*, ix^*) = \sum_i \lambda_i (\alpha_i^2 - \beta_i^2) .$$

Consider the problem of maximizing $\Sigma(x^*, ix^*)$ under the conditions $\Sigma(x^*, x^*) = \Sigma(ix^*, ix^*) = 1$. This is equivalent to maximizing

$$(3.4) \quad \sum_i \lambda_i (\alpha_i^2 - \beta_i^2)$$

subject to the conditions

$$(3.5) \quad \sum_i \alpha_i^2 + \sum_i \beta_i^2 = 1 \quad \text{and} \quad \sum_i \lambda_i \alpha_i \beta_i = 0 .$$

If we suppose that $\lambda_1 > \lambda_2 > \dots > \lambda_p > 0$, we get by using Lagrange's multipliers that the maximum point is achieved at $\alpha_1 = \pm 1$, $\alpha_2 = \dots = \alpha_p = \beta_1 = \dots = \beta_p = 0$, and the maximum value is λ_1 . By induction it follows that $\pm e_j^*$ are the only linear forms uncorrelated with $e_1^*, \dots, e_{j-1}^*$ for which $\Sigma(e_j^*, e_j^*) = \Sigma(ie_j^*, ie_j^*) = 1$ and $\Sigma(e_j^*, ie_j^*)$ is maximal. The maximum values are λ_j , $j = 1, \dots, p$.

The canonical correlations $\lambda_1, \dots, \lambda_p$ with respect to the complex structure can be found as the positive roots of the equation

$$(3.6) \quad \left| \begin{pmatrix} \Sigma'_{12} + \Sigma_{12} & \Sigma_{22} - \Sigma_{11} \\ \Sigma_{22} - \Sigma_{11} & -\Sigma'_{12} - \Sigma_{12} \end{pmatrix} - \lambda \begin{pmatrix} \Sigma_{11} + \Sigma_{22} & \Sigma'_{12} - \Sigma_{12} \\ \Sigma_{12} - \Sigma'_{12} & \Sigma_{11} + \Sigma_{22} \end{pmatrix} \right| = 0$$

where

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma'_{12} & \Sigma_{22} \end{pmatrix} ,$$

with respect to a basis of the form $f_1^*, \dots, f_p^*, if_1^*, \dots, if_p^*$.

4. The distribution of the empirical canonical correlations with respect to a complex structure.

The estimator $\hat{\Sigma}(x_1, \dots, x_N)$ for Σ in the observations point $(x_1, \dots, x_N)$ is given in the introduction. Suppose that $\Sigma \in \mathcal{P}_{\mathbb{C}}(E^*)_r$ and let $e_1^*, \dots, e_p^*$ be a basis for E^* such that the $2p \times 2p$ matrix for Σ with respect to the basis $e_1^*, \dots, e_p^*, ie_1^*, \dots, ie_p^*$ is the $2p \times 2p$ identity matrix. The distribution of $\hat{\Sigma}$ in terms of matrices is a Wishart distribution with a representation as a density with respect to the restriction of the Lebesgue measure to all positive definite $2p \times 2p$ matrices $\mathcal{P}(\mathbb{R}^{2p})_r$ as follows

$$(4.1) \quad c \cdot |\det \theta|^{(N-2p-1)/2} \exp\{-\frac{1}{2} \operatorname{tr}(\theta)\} d\theta, \theta \in \mathcal{P}(\mathbb{R}^{2p}) \quad .$$

The canonical correlations and linear forms (with respect to the complex structure) of $\hat{\Sigma}(x_1, \dots, x_N)$ is called the empirical canonical correlations and linear forms with respect to the complex structure.

The classical theory of canonical correlations is due to Hotelling [4].

We shall find the distribution of these. If we define Φ and Ψ from the $2p \times 2p$ real matrix θ , as in formula (2.5), we have a one-to-one and onto mapping between $\mathcal{P}(\mathbb{R}^{2p})_r$ and $\mathcal{P}(\mathbb{C}^p)_r \times \mathcal{S}(\mathbb{C}^p)$, where $\mathcal{P}(\mathbb{C}^p)_r$ respectively $\mathcal{S}(\mathbb{C}^p)$ denotes the set of positive definite hermitian respectively symmetric $p \times p$ complex matrices, with Jacobian 1. Furthermore, (2.6) defines a one-to-one mapping from $GL_+(\mathbb{C}^p) \times \Omega$ into $\mathcal{P}(\mathbb{C}^p)_r \times \mathcal{S}(\mathbb{C}^p)$, where $GL_+(\mathbb{C}^p)$ is the subset of all nonsingular $p \times p$ complex matrices with a positive real part in the first row and $\Omega = \{(\lambda_1, \dots, \lambda_p) \in \mathbb{R}^p \mid 1 > \lambda_1 > \dots > \lambda_p > 0\}$.

The complementary to the image (which is an open set) of this mapping has Lebesgue measure 0; and therefore from our distribution point of view, we can forget this. To find the Jacobian of this mapping defined by (2.6), we proceed as in [1]. The method is due to Hsu [5].

We have

$$(4.2) \quad \begin{aligned} d\Phi &= (d\bar{Z}')Z + \bar{Z}'(dZ) \\ d\Psi &= (dZ')\Lambda Z + Z'(d\Lambda)Z + Z'\Lambda(dZ) \end{aligned}$$

and we shall find the absolute value of the determinant of the linear mapping $(dZ, d\Lambda) \rightarrow (d\Phi, d\Psi)$ defined by (4.2). This is a composition of

$$\begin{aligned} (a) \quad \begin{pmatrix} dZ \\ d\Lambda \end{pmatrix} &\rightarrow \begin{pmatrix} (dZ)Z^{-1} \\ d\Lambda \end{pmatrix} = \begin{pmatrix} dW \\ d\Lambda \end{pmatrix}, \\ (b) \quad \begin{pmatrix} dW \\ d\Lambda \end{pmatrix} &\rightarrow \begin{pmatrix} d\bar{W}' + dW \\ dW'\Lambda + d\Lambda + \Lambda dW \end{pmatrix} = \begin{pmatrix} dY \\ dX \end{pmatrix}, \\ (c) \quad \begin{pmatrix} dY \\ dX \end{pmatrix} &\rightarrow \begin{pmatrix} \bar{Z}' dYZ \\ Z' dXZ \end{pmatrix} = \begin{pmatrix} d\Phi \\ d\Psi \end{pmatrix}. \end{aligned}$$

The Jacobians are $|\det Z|^{-2p}$, $|\det Z|^{2(2p+2)}$ respectively

$c_1 \prod_{i=1}^p \lambda_i \prod_{i < j} (\lambda_i^2 - \lambda_j^2)$ for (a), (c) respective (b). Since $\text{tr}(\Theta) = 2 \text{tr}(\Phi) = 2 \text{tr}(\bar{Z}'Z)$ and $|\det \Theta| = |\det \bar{Z}'Z|^2 \prod_{i=1}^p (1 - \lambda_i^2)$

(4.1) is transformed to the distribution

$$(4.3) \quad c_2 \cdot |\det \bar{Z}'Z|^{N-p} \exp\left\{-\frac{1}{2} \text{tr}(\bar{Z}'Z)\right\} \prod_{i=1}^p \lambda_i (1 - \lambda_i^2)^{(N-2p-1)/2} \prod_{i < j} (\lambda_i^2 - \lambda_j^2) dZ \bigotimes_{i=1}^p d\lambda_i$$

on $GL_+(\mathbb{C}^p) \times \Omega$. Integrating over $Z \in GL_+(\mathbb{C}^p)$, we get the distribution of $f_1 = \lambda_1^2, \dots, f_p = \lambda_p^2$:

$$(4.4) \quad c_3 \prod_{i=1}^p (1 - f_i)^{(N-2p-1)/2} \prod_{i < j} (f_i - f_j) df_1, \dots, df_p$$

on $\Omega = \{(f_1, \dots, f_p) \in \mathbb{R}^p \mid 1 > f_1 > \dots > f_p > 0\}$. Formula (13) in [1], p. 324, for $p_1 = p-1$ and $p_2 = p$ gives the normings constant c_3 , namely,

$$(4.5) \quad c_3 = \prod_{i=1}^{p-1} \frac{\Gamma(\frac{1}{2}(N-i))}{\Gamma(\frac{1}{2}(N-p-i))\Gamma(\frac{1}{2}(p-i))\Gamma(\frac{1}{2}(p+1-i))}.$$

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