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AN ALGEBRAIC MODEL FOR CONTINUOUS-WAVE THERMAL-BLOOMING EFFECTS

by

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This paper presents an algebraic model for continuous-wave thermal-blooming effects that is accurate enough to well represent a large wave-optics data base, simple enough to suggest some previously unnoticed universal relationships, and complete enough to allow variation of the many physical variables of interest in systems analysis exercises.

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SECTION 1

INTRODUCTION

The purpose of this document is to provide a standard algebraic model for a continuous-wave (CW) high-energy-laser (HEL) beam on target. The beam is degraded by thermal blooming as well as numerous other effects. The algebraic model is intended for systems analysis exercises, where the numerous parameters are to be explored, making more detailed computer simulation of atmospheric propagation impractical. It is an outgrowth of efforts to fit scaling laws to a large body of data generated by detailed atmospheric propagation simulations using a finite-difference wave-optics code.*

Any such wave-optics code provides a detailed intensity profile on target, from which several different summary characteristics can be extracted. These include, for instance, peak intensity, line-of-sight beam dispersion, and beam area. The last of these further admits several definitions, including area to some percentage of total power, and area defined by a ratio of squared integral of intensity to integral of intensity squared (suggested independently by Lincoln Laboratory and Draper Laboratory researchers). If beams on target were Gaussian in shape, all such characteristics would convey equivalent information. The simplified algebraic model assumes that this is nearly the case, and speaks nominally of peak intensity on target.

Regression analysis of the results has shown that peak intensity can be correlated with an integral Ψ_h, which represents the accumulation along range (starting from the center of the aperture) of phase perturbation due to heating in a beam with absorption, scattering, convective clearing, and focusing. Figures 1, 2, and 3 show the tightness of the correlation obtained for three different beam shapes. The ordinate is the phase integral Ψ_h and the abscissa is the ratio $R = (I_U - I)/I$, where I is peak intensity and I_U is unbloomed peak intensity.

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* Data provided by D. Cordray of Naval Research Lab (NRL).

Any one of many numerically similar functional forms could be fit to the curves defined by the correlation data. This document selects one that has been found most useful because of its algebraic simplicity. Section 2 shows how the simple algebraic form leads to a number of universal relationships that are independent of almost all physical details of the propagation process. These relationships describe the variation of peak intensity as a function of power alone, with all other variables held constant. Important relationships are shown to depend only on beam shape. This conclusion has not previously been evident from other scaling laws.

Section 3 discusses the parameters appearing in Section 2, showing how each depends on actual physical variables that describe the engagement, the laser, and the atmosphere. Section 4 provides a technique for accurately evaluating the phase integral along range required in Section 3. For several common beam shapes, Section 5 discusses numerical values of regression parameters defined in Section 3. Section 6 provides a concise summary of all formulas required for systems analysis exercises.

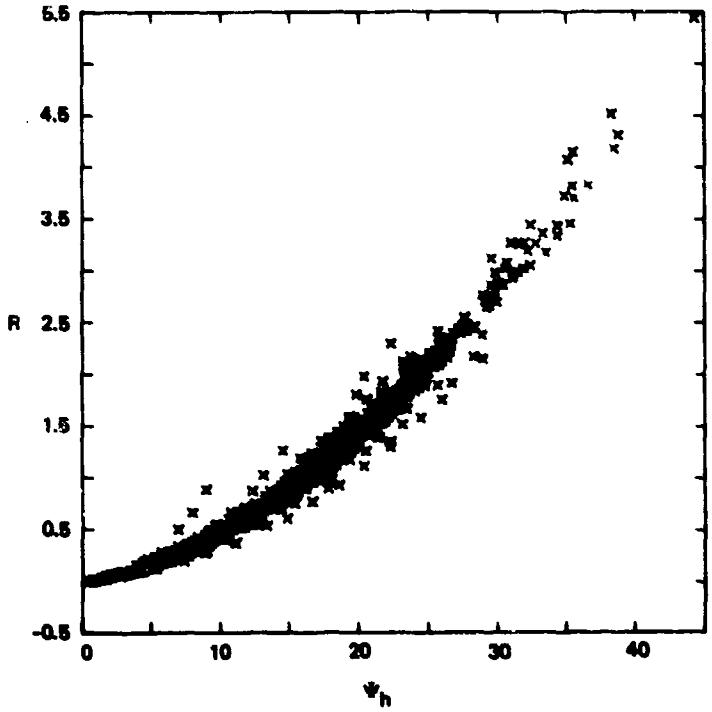

Figure 1. Correlation for infinite Gaussian beam.

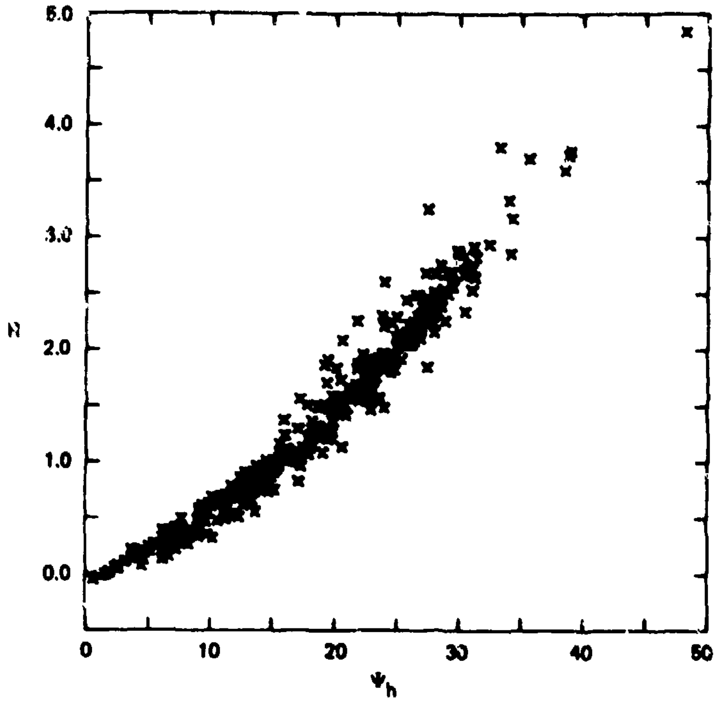

Figure 2. Correlation for $1/e^2$ truncated Gaussian beam.

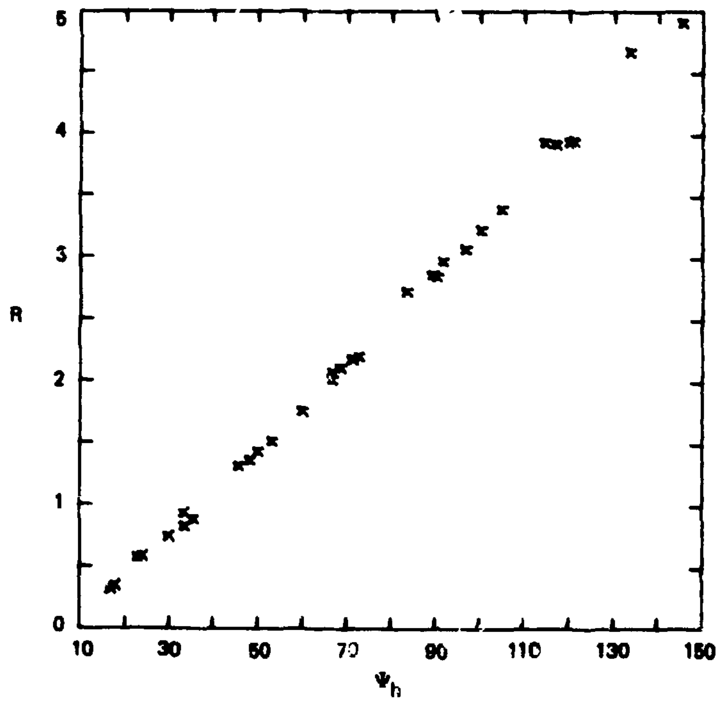

Figure 3. Correlation for uniform beam.

SECTION 2

INTENSITY DEGRADATION AS A FUNCTION OF POWER

For a class of problems differing only in time-average power (P), the ratio of bloomed to unbloomed intensity focused on target can be modeled as

$$\frac{I(P)}{I_U(P)} = \frac{\sigma_L^2}{\sigma_L^2 + \sigma_B^2}$$

In this expression, σ_L^2 is the 1/e line-of-sight beam dispersion in radians due to linear effects, including diffraction, beam quality, turbulence, and jitter. The latter two effects separate into "high" and "low" frequencies, which, respectively, do and do not impact blooming. The latter is represented in the term σ_B^2. To some extant, the form of $I(P)/I_U(P)$ expresses the familiar idea of "rss-ing" (root-sum-squaring). For a Gaussian beam, dispersive effects combine by summing variances, and the denominator $(\sigma_L^2 + \sigma_B^2)$ resembles such a sum.

The blooming term (σ_B^2) depends on power (P) in a way that can be modeled by a variety of functional forms. The choice of form is a tradeoff between simplicity and range of validity. In this report, we use the simplest form known to be valid for power levels of practical interest. This form is nominally*

$$\sigma_B^2 = C_B P^a$$

The use of P^a with $a > 1$ allows reproduction of a well-known physical phenomenon: there exists a critical power P_C such that the

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* This type of formula has also been suggested and used by F. Gebhardt and J. Wallace. In particular, Wallace suggested a = 3/2.

intensity on target is maximum. That P_C can be related to σ_L^2, C_B, and a as follows. The unbloomed intensity on target is proportional to P, so that actual intensity is proportional to $P[I(P)/I_U(P)]$, or $P\sigma_L^2/(\sigma_L^2 + C_B P^a)$. Differentiating with respect to P and setting the derivative to zero gives

$$P_C = \left[\frac{\sigma_L^2}{(a - 1)C_B}\right]^{1/a}$$

From the value of P_C, the ratio $I(P_C)/I_U(P_C)$ is readily found to be

$$\frac{I(P_C)}{I_U(P_C)} = \frac{a - 1}{a}$$

This result is interesting because it depends only on the parameter a, expressive of beam shape alone, and not on σ_L^2 or C_B, which contain much of the physics of the problem. It therefore presents a physical phenomenon that is essentially separable from other physical phenomena in the overall propagation process.

The result concerning $I(P_C)/I_U(P_C)$ indicates what to expect from adaptive phase correction for thermal blooming. Since phase correction does not change beam shape, it will move the whole curve of intensity-out versus power-in in such a way that the new peak, the old peak, and the origin lie on a straight line, as illustrated in Figure 4.

Clearly, P_C marks the upper limit of power levels having practical interest. In fact, operation well below P_C may be of interest, so let us consider P at some fraction of P_C: $P = P_C/b$. Then

$$\frac{I\left(\frac{P_C}{b}\right)}{I(P_C)} = \frac{\sigma_L^2 + C_B(P_C)^a}{b\left[\sigma_L^2 + C_B\left(\frac{P_C}{b}\right)^a\right]}$$

which simplifies to

$$\frac{I\left(\frac{P_C}{b}\right)}{I(P_C)} = \frac{ab^{a-1}}{b^a(a - 1) + 1}$$

This result, too, is independent of σ_L^2 and C_B, and therefore is independent of the phenomena controlling them.

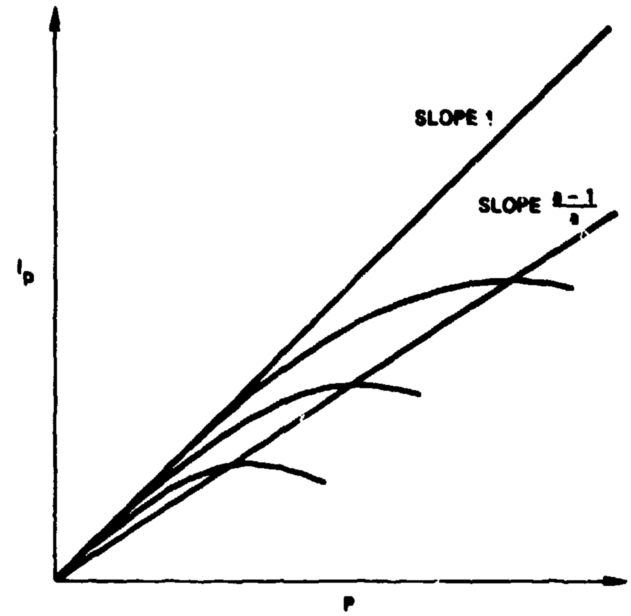

Figure 4. Adaptive correction of phase without change of beam shape.

The formula describing operation below P_C is plotted in Figure 5 for several typical values of a. For all cases, there is little point in operating at power levels above $P_C/2$, because more than 85% of the limiting peak intensity is already available at $P_C/2$.

The formulas for $I(P_C)/I_U(P_C)$ and $I(P_C/b)/I(P_C)$ can be combined to relate $I(P_C/b)$ to $I_U(P_C)$. The result is

$$\frac{I\left(\frac{P_C}{b}\right)}{I_U(P_C)} = \frac{(a - 1)b^{a-1}}{b^a(a - 1) + 1}$$

This result, too, is independent of the parameters σ_L^2 and C_B that carry most of the physics of the problem. It means essentially that if $I_U(P_C)$ is specified, I for any other condition is implicitly specified.

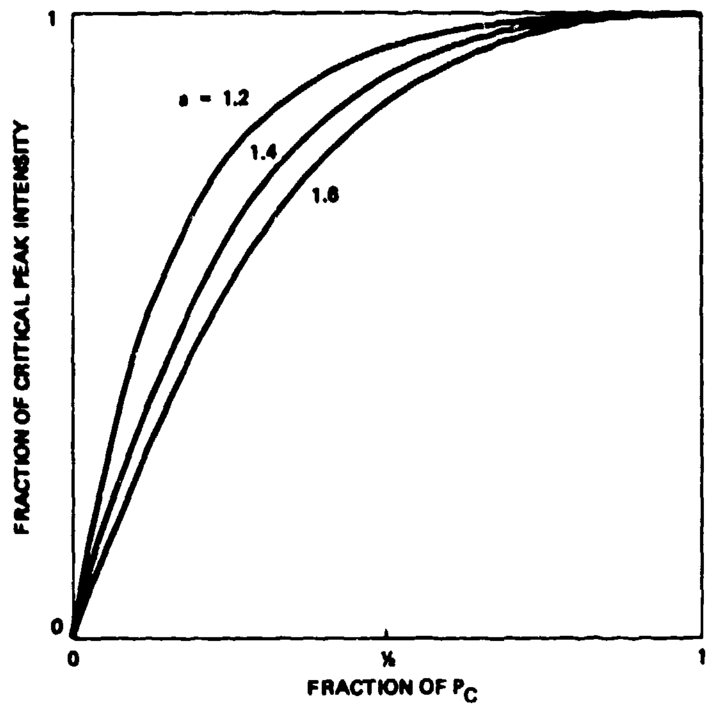

Figure 5. Operation below P_C.

The model presented in this section is most accurate in the regime of physical interest for systems modeling, namely, below the zero-wander value of P_C. This is the case because below this power there is relatively little blooming, with little attendant beam distortion and no beam breakup. There is then a one-to-one Gaussian-like correspondence between intensity I and the various dispersive σ^2 terms. Operation far above such a power level requires a more complex model, involving linear and quadratic laser power terms in σ_B^2, presented by Breaux.[(1)*]

The ratio-type results of this section are of a very general nature, and are valid regardless of the methodology used to evaluate a, σ_L^2, C_B, or $I_U(P_C)$. Specific techniques for evaluating these are provided in the Sections 3 through 5.

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* Superscript numerals refer to similarly numbered items in the List of References.

SECTION 3

PHYSICAL MODEL

The model for degradation of intensity focused on target as a function of power has only three parameters (a, σ_L^2, and C_B), and requires only $I_U(P_C)$ to specify intensity under any conditions. A small amount of data for a given class of problems differing only in P will suffice to determine all the required quantities. But extrapolating the results to any other class of problems requires some kind of physical model. The exponent a can be assumed to be independent of many physical variables, so the more pressing problems are to admit defocus, and to model σ_L^2, C_B, and $I_U(P_C)$.

The physical variables affecting HEL propagation are designated in Table 1. In cases where the propagation path extends through significantly different altitudes, path averaging of parameters is required. (See Reference 1, p. 59 for applicable techniques regarding C_n^2.)

These parameters can be summarized in terms of four dimensionless numbers:

Absorption number $\quad N_A = \alpha Z_t$

Slew number* $\quad N_S = \dfrac{U_t}{U_w}$

Fresnel number $\quad N_F = \dfrac{2\pi R_m^2}{\lambda Z_t}$

Distortion number*† $\quad N_D = -2\sqrt{2}\left(\dfrac{\partial n}{\partial T}\right)\left(\dfrac{P Z_t N_A}{R_m^3 U_w C_p \rho_\infty n_\infty}\right)$

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* Defined appropriate to this model.

† Subscript ∞ means unperturbed natural value.

Table 1. Physical variables affecting HEL propagation.

Parameter	Variable	Units
<u>Engagement</u>		
Target range	Z_t	meters
Transverse target velocity	U_t	meters/second
<u>Laser</u>		
Aperture radius	R_m	meters
Time average power	P	Joules/second
Wavelength	λ	meters
Beam quality	M	no dimension
Beam shape		no dimension, not a scalar
<u>Atmosphere</u>		
Wind velocity	U_w	meters/second
Turbulence	C_n^2	$\text{meters}^{-2/3}$
Absorption	α	meters^{-1}
Scattering	σ	meters^{-1}
Index of refraction	n	no dimension
Temperature	T	degrees
Refraction gradient	$\partial n/\partial T$	degrees^{-1}
Density	ρ	$\text{kilogram-meters}^{-3}$
Heat capacity	C_p	Joules/degree-kilogram

There are also effective beam qualities N_Q and N'_Q, which represent the actual spreading of the beam due to various effects. These effects include beam shape and the blooming itself, and so N_Q and N'_Q must be defined later. A procedure is provided in Section 6.

The σ_L^2 is 1/e radial beam dispersion due to linear effects. It includes a number of contributors, distinguished by different subscripts: diffraction and beam quality (D), high-frequency beam distortion and motion due to turbulence (T) and jitter (J), low-frequency beam wander (W) due to turbulence, jitter, and pointer-tracker effects. Thus, we have

$$\sigma_L^2 = \sigma_D^2 + \sigma_T^2 + \sigma_J^2 + \sigma_W^2$$

The formula for the diffraction contributor is

$$\sigma_D^2 = 0.5\left(\frac{m'M\lambda}{2R_m}\right)^2$$

where m' is characteristic of beam shape. Requiring $\sqrt{2}\,\sigma_D$ to be the 63% beam radius on target makes m' equal to $2/\pi$ for an infinite Gaussian beam, equal to 0.9166 for a $1/e^2$ truncated Gaussian beam, and equal to 0.9202 for a uniform beam (see Reference 2).

The turbulence contributor refers specifically to the short-term high-frequency part of total turbulence. It has been found by Breaux[3] that for a large variety of beam shapes, total turbulence is well represented by

$$\sigma_{TT}^2 = \left(\frac{\sigma_D}{M}\right)^2\left(\frac{D_e}{r_0}\right)^2$$

where r_0 is the Fried coherence length for wave number $k = 2\pi/\lambda$

$$r_0 = 2.10\left[1.45k^2 \int_0^{Z_t} C_n^2(Z)\left(\frac{Z_t - Z}{Z_t}\right)^{5/3} dZ\right]^{-3/5}$$

and D_e is an effective aperture size, appropriate to the beam shape. For infinite Gaussian, truncated Gaussian, or uniform beams, respectively, it is

$$D_e^2 = 8R_m^2,\ 3.7R_m^2,\ 4R_m^2$$

The short-term part (σ_T^2) is smaller, varying between the values

$$\sigma_T^2 = 0.182\left(\frac{\sigma_D}{M}\right)^2\left(\frac{D_e}{r_0}\right)^2$$

for $D_e/r_0 < 3$ and

$$\sigma_T^2 = \left(\frac{\sigma_D}{M}\right)^2\left[\left(\frac{D_e}{r_0}\right)^2 - 1.18\left(\frac{D_e}{r_0}\right)^{5/3}\right]$$

for $D_e/r_0 > 3$.

To ensure model validity, the high-frequency turbulence should be limited to values small enough to cause no speckling, say $\sigma_T < 2\sigma_D$. The difference ($\sigma_{TT}^2 - \sigma_T^2$) may or may not appear in the wander term σ_W^2, depending on the particular hardware implementation being modeled. Additional jitter and wander contributors arise from the particular hardware considered, and must be set by the user.

The dispersion due to blooming is to be combined with the linear dispersions simplified by σ_D^2, σ_T^2, σ_J^2, and σ_W^2. We proceed now to consider the blooming term

$$\sigma_B^2 = C_B P^a$$

The physical phenomenon that causes σ_B^2 is accumulation along range of phase due to heating, which increases with power (P), and decreases, to some extent, with that portion of σ_L^2 that actually experiences the blooming, namely $\sigma_L^2 - \sigma_W^2$. It has been found possible to define a measure Ψ_h for heating phase such that the following expanded expression well represents σ_B^2

$$\sigma_B^2 = C_B'(\sigma_L^2 - \sigma_W^2)(\Psi_h)^a$$

where C_B' is a dimensionless coefficient that depends on beam shape. Clearly

$$C_B = C_B'(\sigma_L^2 - \sigma_W^2)\left(\frac{\Psi_h}{P}\right)^a$$

The form of Ψ_h that makes the above representation of σ_B^2 possible is proportional to $N_D N_F / N_Q'$. The phase integral (Ψ_h) also has an additional factor to make it into an integral along range that takes account of extinction, clearing, and focusing of the beam. Its evaluation is the subject of Section 4.

Next, $I_U(P_C)$ is easily estimated by considering total extinction ($\varepsilon = \alpha + \sigma$), beam shape and quality, and the spreading due to linear effects. The estimate is

$$I_U(P_C) = \frac{P_C \exp(-\varepsilon Z_t)}{2\pi Z_t^2 \sigma_L^2 B}$$

Here, B is a beam-shape factor, which can be roughly interpreted as the ratio of average intensity to "peak" intensity in the focal plane. For a Gaussian beam, it is exactly 1/2, and for any realistic beam shape, it is quite close to 1/2.

SECTION 4

PHASE INTEGRAL

In Section 3, the problem of evaluating C_B was reduced to the problem of evaluating the phase integral Ψ_h. Many integration procedures have been investigated,[1,4,5] and most can be made numerically adequate. The following is straightforward to explain. Let

$$\Psi_h = \int_0^1 \Psi_h(z)\, dz$$

where

$$z = \frac{Z}{Z_t}$$

$$\Psi_h(z) = \frac{N_D N_F \exp(-\varepsilon Z) \int dt\ I\left[x_0(z,t),\ y_0(z,t)\right]}{N_Q'\left[\frac{R_s(z)}{R_m}\right]^2 \left(\frac{2R_m}{U_w}\right)\left(\frac{P}{\pi R_m^2}\right)}$$

Here, $\exp(-\varepsilon Z)$ represents extinction. The variable $R_s(z)$ represents spot radius at z, approximated by rss-ing focus and diffraction effects.

$$R_s(z) = R_m\left[(1 - z)^2 + \left(\frac{2zN_Q}{N_F}\right)^2\right]^{1/2}$$

The term $2R_m/U_w$ represents the clearing time at the aperture, and provides a normalization for the time integration. $P/\pi R_m^2$ provides a normalization for the intensity, which depends on time (t) through x_0 and y_0, which are both 0 for $t = 0$. For a beam slewed in the x direction

$$y_0(z,t) = 0$$

but $x_0(z,t)$ varies with the local clearing velocity. Assuming x_0, y_0 nondimensionalized by aperture radius R_m

$$x_0(z,t) = \frac{-tU_\infty(z)}{R_s(z)}$$

with $U_\infty(z)$ representing that velocity, approximated by

$$U_\infty(z) = U_W(1 + zN_S)$$

As an example, consider a Gaussian beam with amplitude

$$A_0(x_0, y_0) = \exp(-x_0^2 - y_0^2)$$

For the Gaussian profile, the normalized intensity

$$\frac{I\left[x_0(z,t),\ y_0(z,t)\right]}{\left(P/\pi R_m^2\right)} = 2\exp\left[-2\left(\frac{tU_\infty(z)}{R_s(z)}\right)^2\right]$$

The time integral of normalized intensity is

$$\left[\frac{2R_s(z)}{U_\infty(z)}\right]\left(\frac{U_W}{2R_m}\right)\sqrt{\frac{\pi}{2}}\left(\frac{1}{2}\right)$$

Absorbing the $\sqrt{\pi/2}$ (1/2) in the overall C_B' relating to beam shape, and substituting for $U_\infty(z)$ and $R_s(z)$, leaves the required z integral to be just

$$\Psi_h = \int_0^1 \frac{N_D N_F \exp(-\varepsilon Z)\ dz}{N_Q'(1 + zN_S)\left[(1 - z)^2 + \left(\frac{2zN_Q}{N_F}\right)^2\right]^{1/2}}$$

The integral does not appear to be amenable to direct analytic evaluation, so the options are numerical integration or analytic approximation. Typically, numerical integration is difficult because small steps are required to handle the rapid variation of the integrand. Therefore, analytic approximation is preferred. Probably the fewest approximations are required if integration by parts is employed. In that case we find

$$\Psi_h = \left[\exp(-\varepsilon Z)\,\Psi_h'\right]_0^1 + \varepsilon Z_t \int_0^1 \exp(-\varepsilon Z)\,\Psi_h'\, dz$$

The indefinite integral

$$\Psi_h' = \int \left(\frac{N_D N_F\, dz}{N_Q'}\right) \Bigg/ (1 + zN_S)\left[(1 - z)^2 + \left(\frac{2zN_Q}{N_F}\right)^2\right]^{1/2}$$

is available in an integral table by Klerer and Grossman.[6] Naturally, the expression contains an integration constant. In principle, this constant needs to be reset in such a way as to nullify the remainder term

$$\varepsilon Z_t \int_0^1 \exp(-\varepsilon Z)\,\Psi_h'\, dz$$

This can be accomplished at least approximately by using $\Psi_h'(z) - \Psi_h'(1/2)$ in place of $\Psi_h'(z)$. This makes the expression for Ψ_h quite complicated. Utility is greatly served by reducing it to the case of small attenuation εZ_t, where the integration constant has no impact, and Ψ_h reduces to

$$\Psi_h \approx \left[\frac{N_D N_F}{N_Q' A}\right] \exp\left[\frac{-\varepsilon Z_t}{2}\right] \ln\left|\frac{(N_S + 1 + A)(N_S + 1)}{-B + N_S + 1 + AC}\right|$$

where

$$A = [(N_S + 1)^2 + C^2]^{1/2}$$

$$B = N_S + 1 + C^2$$

$$C = \frac{2N_Q}{N_F}$$

The condition for this result to be valid can be stated as

$$1 - \exp(-\varepsilon Z_t)\Psi_h'\left(\frac{1}{2}\right) \ll -\Psi_h'(0) + \exp(-\varepsilon Z_t)\Psi_h'(1)$$

SECTION 5

CORRELATION DATA

Extensive simulations of atmospheric propagation have been performed at NRL using a finite-difference wave-optics code, and these provide the data base that establishes a strong correlation of the form suggested in this report relating intensity on target to the parameter Ψ_h. There is very little scatter, and that which does exist may be attributed to one of two factors:

(1) Earlier correlation studies by Zeiders[7] established that the spot size in the integrand denominator of Ψ_h should not be just the ideal diffraction-limited value, mainly because of the iterative effect of blooming upon itself. To account somewhat for this in a way that maintains reasonable simplicity, a free constant (m) scaling the diffraction spot was introduced and evaluated by minimizing residuals. Actually, such scaling must depend to some extent or physical variables, especially power. The idea of iteration to admit variable m has been tried, but so far has not been sufficiently successful to justify the effort.

(2) The raw data generated were not all of the same form. Data for the infinite Gaussian beam comprised peak intensities, whereas those for the truncated Gaussian comprised $1/e^2$ area, and those for the uniform beam comprised the more complex area measure of the form

$$A = \frac{\left[\int I \, dx \, dy\right]^2}{\int I^2 \, dx \, dy}$$

The latter functional (A) is currently thought to lead to the tightest correlations.

The results of analyzing three beam shapes—infinite Gaussian, $1/e^2$ truncated Gaussian, and uniform beam—are presented in Tables 2 through 4 and plotted in Figures 6 through 8. Supposing that N_D and N_F are fixed and σ_W^2 equals zero, let us compare the three beam types. We recall that C_B is proportional to $C_B'\sigma_L^2$ and that

$$P_C = \left[\frac{\sigma_L^2}{(a - 1)C_B}\right]^{1/a}$$

and

$$\frac{I(P_C)}{I_U(P_C)} = \frac{a - 1}{a}$$

The results tend to follow intuitively understandable patterns. First, the parameter m affects the tightness of the correlation much less in the case of the Gaussian beams than in the case of the uniform beam. This is to be expected because blooming in the Gaussian case is driven mainly by gradients near the aperture, whereas for the uniform beam such gradients are minimized and blooming is driven by beam shape nearer the focus, which is described by m. Secondly, the uniform beam has the smallest a and C_B'/σ_L^2, resulting in by far the largest P_C. The large P_C is to be expected because, with less gradient near the aperture, there is less lensing effect even at high power. The small a means, however, that the P_C region is very broad and $I(P_C)/I_U(P_C)$ is very small. The two Gaussian beams have anomalously different C_B', a, and m, but they turn out to have similar small P_C values. But because a is larger for these cases, $I(P_C)/I_U(P_C)$ is somewhat larger.

The actual peak intensity on target at P_C depends on P_C, on $I(P_C)/I_U(P_C) = (a-1)/a$, and on the diffraction-limited spot radius r_s

$$I(P_C) = \frac{P_C}{\pi r_s^2}\left(\frac{a - 1}{a}\right)$$

The spot radius for the three beam shapes, respectively, is proportional to $2/\pi$, 0.9166, and 0.9202, with the result that $I(P_C)$ for the truncated Gaussian is only half that of the infinite Gaussian. The value for the uniform beam is nearly three times what it is for the truncated Gaussian, but at the price of nearly six times the power (which may nevertheless be acceptable).

Table 2. Infinite Gaussian: $I(P)/I_U(P) = (1 + C_B' \psi_h^a)^{-1}$.

	rms Error	Maximum Error	C_B'	a	m
	0.142	0.949	0.011851	1.5029	1.0
	0.109	0.667	0.010796	1.5891	1.5
	0.102	0.614	0.010612	1.6189	1.75
Chosen fit	$\boxed{0.101}$	$\boxed{0.565}$	$\boxed{0.010590}$	$\boxed{1.6419}$	$\boxed{2.0}$
	0.112	0.607	0.010919	1.6715	2.5

$P_C = 20.903 \qquad I(P_C)/I_U(P_C) = 0.39035 \qquad I(P_C) = 20.163$

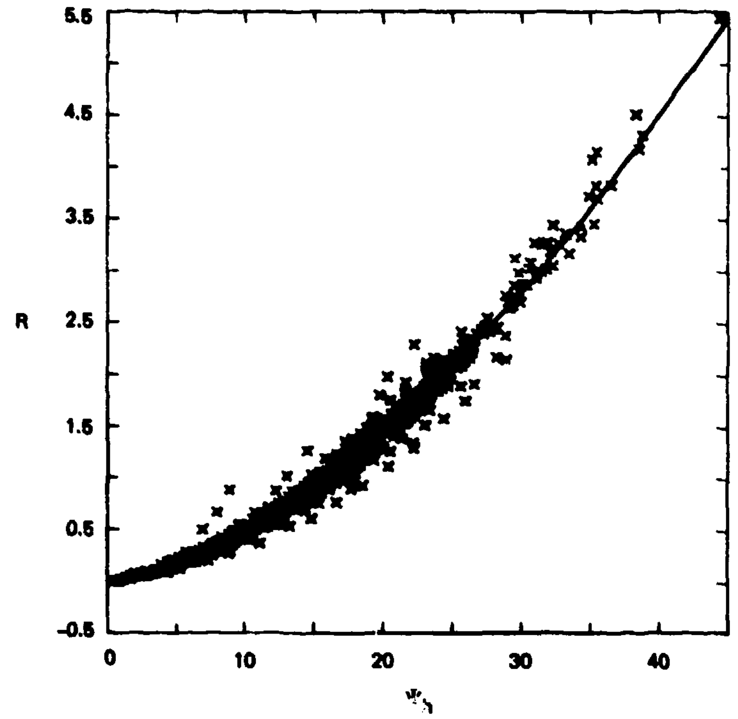

Figure 6. Functional fit for infinite Gaussian beam.

Table 3. $1/e^2$ truncated Gaussian: $I(P)/I_U(P) = (1 + C_B' \Psi_h^a)^{-1}$.

	rms Error	Maximum Error	C_B'	a	m
	0.170	0.981	0.018600	1.4323	3.5
Chosen fit	0.169	0.924	0.019023	1.4509	4.0
	0.172	0.874	0.019630	1.4646	4.5
	0.178	0.828	0.020398	1.4743	5.0
	0.187	0.788	0.021313	1.4805	5.5
	0.198	0.752	0.022366	1.4838	6.0

$P_C = 27.780$ $\quad I(P_C)/I_U(P_C) = 0.31077$ $\quad I(P_C) = 10.276$

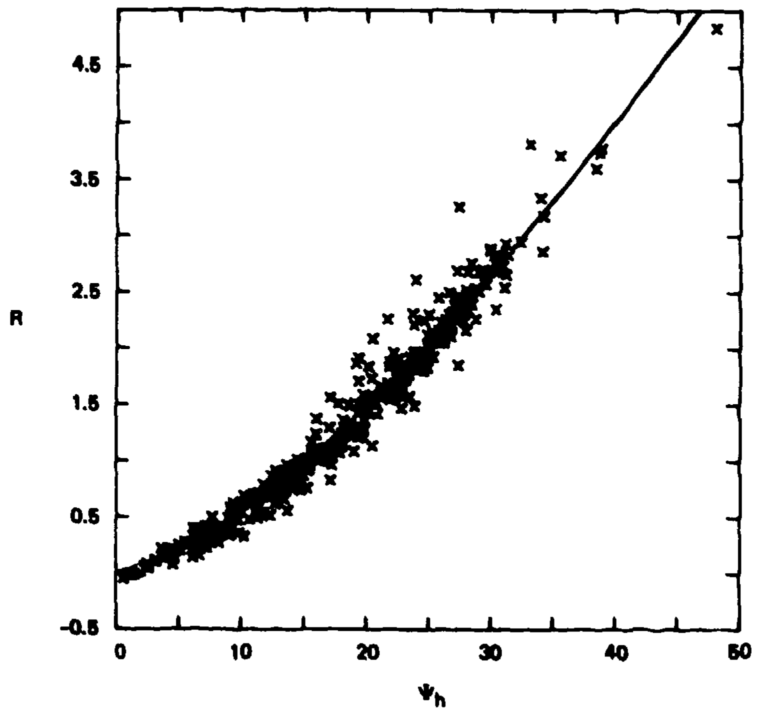

Figure 7. Functional fit for $1/e^2$ truncated Gaussian beam.

Table 4. Uniform beam: $I(P)/I_U(P) = (1 + C'_B \Psi_h^a)^{-1}$.

	rms Error	Maximum Error	C'_B	a	m
	0.092	0.204	0.014732	1.1380	0.5
	0.066	0.157	0.014395	0.1614	0.75
Chosen fit	0.063	0.139	0.014264	1.1777	1.0
	0.093	0.255	0.014335	1.1986	1.5
	0.112	0.317	0.014474	1.2054	1.75
	0.130	0.374	0.014660	1.2106	2.0

$P_C = 160.09$ $I(P_C)/I_U(P_C) = 0.15089$ $I(P_C) = 28.526$

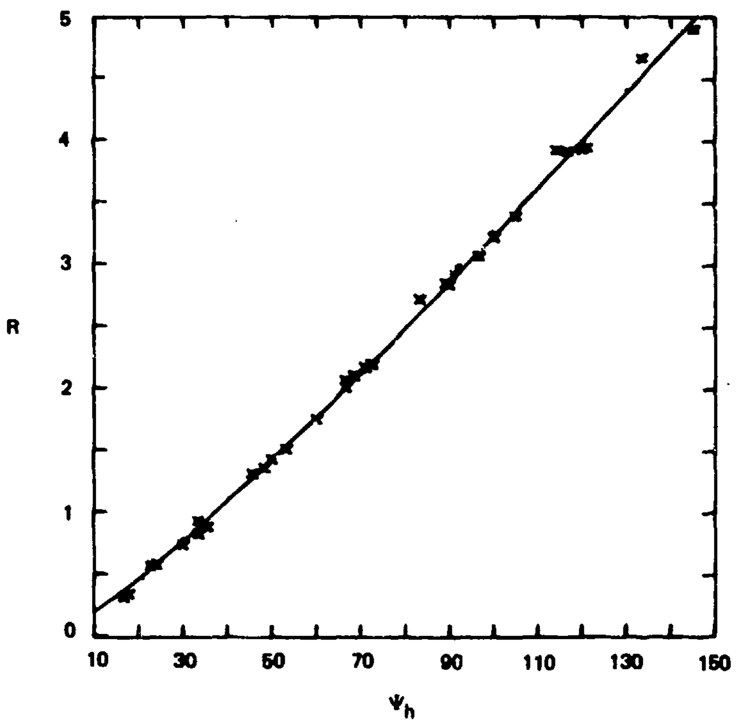

Figure 8. Functional fit for uniform beam.

SECTION 6

SUMMARY OF FORMULAS FOR SYSTEMS ANALYSIS

Suppose that one wishes to calculate peak intensity on target for a given set of conditions. The required steps are:

(1) Choose beam shape and set C_B', a, m, m', and m".

	C_B'	a	m	m'	m"
Infinite Gaussian	0.010590	1.6419	2.0	0.6366	1
Truncated Gaussian	0.028727	1.3715	4.0	0.9166	0.8893
Uniform Beam	0.014264	1.1777	1.0	0.9202	1.124

(2) Evaluate nondimensional numbers.

Absorption number $N_A = \alpha Z_t$

Slew number $N_S = \dfrac{U_t}{U_w}$

Fresnel number $N_F = \dfrac{2\pi R_m^2}{\lambda Z_t}$

Distortion number $N_D \approx (2.333 \times 10^{-9}) \dfrac{P Z_t N_A}{R_m^3 U_w}$

(3) Evaluate linear dispersions.

$$\sigma_{D0}^2 = 0.5\left(\frac{\lambda}{\pi R_m}\right)^2$$

$$\sigma_D^2 = 0.5\left(\frac{m'M\lambda}{2R_m}\right)^2$$

$$k = \frac{2\pi}{\lambda}$$

$$r_0 = 3.017(k^2C_n^2z_t)^{-3/5}$$

$D_e^2 = 8R_m^2$, $3.7R_m^2$, or $4R_m^2$ for infinite Gaussian, $1/e^2$ truncated Gaussian, or uniform beam, respectively

$$\sigma_{TT} = \left(\frac{\sigma_D}{M}\right)^2\left(\frac{D_e}{r_0}\right)^2$$

$$\sigma_T^2 = 0.182\left(\frac{\sigma_D}{M}\right)^2\left(\frac{D_e}{r_0}\right)^2 \quad \text{for } \frac{D_e}{r_0} < 3$$

$$\sigma_T^2 = \left(\frac{\sigma_D}{M}\right)^2\left[\left(\frac{D_e}{r_0}\right)^2 - 1.18\left(\frac{D_e}{r_0}\right)^{5/3}\right] \quad \text{for } \frac{D_e}{r_0} > 3$$

σ_J^2 appropriate to hardware

σ_W^2 appropriate to hardware

$$\sigma_L^2 = \sigma_D^2 + \sigma_T^2 + \sigma_J^2 + \sigma_W^2$$

(4) Evaluate effective beam qualities.

$$N_Q = \frac{(m^2\sigma_D^2 + \sigma_T^2 + \sigma_J^2)^{1/2}}{\sigma_{D0}m''}$$

$$N_Q' = \frac{(\sigma_L^2 - \sigma_W^2)^{1/2}}{(\sigma_D/M)}$$

(5) Evaluate blooming dispersion.

$$\varepsilon = \alpha + \sigma$$

$$C = \frac{2N_Q}{N_F}$$

$$B = N_S + 1 + C^2$$

$$A = [(N_S + 1)^2 + C^2]^{1/2}$$

$$\Psi_h \simeq \left[\frac{N_D N_F}{N_Q' A}\right] \exp\left[\frac{-\varepsilon Z_t}{2}\right] \ln\left|\frac{(N_S + 1 + A)(N_S + 1)}{-B + N_S + 1 + AC}\right|$$

$$\sigma_B^2 = C_B'(\sigma_L^2 - \sigma_W^2)(\Psi_h)^a$$

(6) Evaluate peak intensity.

$$I(P) = \frac{P \exp(-\varepsilon Z_t)}{\pi Z_t^2(\sigma_L^2 + \sigma_B^2)}$$

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