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Estimating the Spectrum of Electron Density Fluctuations from Simulations of Ionospheric Plasma Clouds

NORMAN J. ZABUSKY AND JOSEPH BLOCK

*Geophysical and Plasma Dynamics Branch
Plasma Physics Division*

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20. ABSTRACT: (Continued)

This important fact is deduced from analytical considerations of model figures with well-separated scales.

Using a combination of analytic concepts and sample numerical calculations, we also deduce that only 40% of the modes are useful for estimating the exponent p in a power-law fit to the one-dimensional spectrum of electron density fluctuations (EDF's) obtained in numerical simulations. The remaining 60% of the modes provide support for the numerical simulations and are subject to various errors.

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ESTIMATING THE SPECTRUM OF ELECTRON DENSITY FLUCTUATIONS FROM SIMULATIONS OF IONOSPHERIC PLASMA CLOUD

I. INTRODUCTION

Numerical simulations of ionospheric plasma clouds yield information for four related purposes:

- (1) To interpret past barium cloud experiments;
- (2) To predict phenomena in new experiments;
- (3) To validate algorithms used in system codes which determine the life-time of sky regions occluded by striations following a nuclear explosion; and
- (4) To predict the temporal evolution of the spectrum of electron-density fluctuations (EDF's) for use in propagation studies.

In this memorandum we study the spectrum of *simple geometric figures* that have shapes similar to those that arise in the evolution of ionospheric plasma clouds.

The evolution of large plasma clouds is at least a two-dimensional intrinsically nonlinear process. Essentially, we are studying the evolution of a deformable dielectric where each element of the dielectric is convected by a velocity proportional to the local electric field. The process is more complex if the ionosphere is compressible, if recombination chemistry processes are important and if the neutral wind varies with altitude. At present, even in the simplest cases, we must resort to computer simulations to obtain answers to the questions posed above. With our present understanding we may divide a cloud's evolutionary history into several overlapping time phases:

- (1) Steepening of an initially smooth cloud;
- (2) Distortion at an angle to the $\mathbf{E} \times \mathbf{B}$ drift velocity;
- (3) Cleavage into strips or "fingers" of ionization^{1,2,3};
- (4) Bifurcation-and-pinching of strips;

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(5) Decay of small field-aligned "rods" of ionization.

Thus, the "cascade" to smaller scale lengths is due essentially to cleavage, bifurcation and pinching.

In numerical simulations, it has been observed that the "sides" of the cleavage-produced indentations and protuberances can be very steep^{1,2,3}. Thus, almost from the beginning, there will be electron density fluctuations that affect propagation. With increasing elapsed time, more of the cloud will be affected and the amplitude of these high wave-number components will increase until some near-steady state is achieved.

In the next section we examine the spectrum of a variety of simple and composite geometric figures and conclude that the *amplitude* of the spectrum at large wavenumber is proportional to the *perimeter* of these figures. Thus, the more tortuous is the backside boundary, the larger is the amplitude.

In the final section, we examine one-dimensional discrete spectra and conclude that only an intermediate 40% of the modes can be trusted to deduce a power spectrum from a particular numerical simulation with periodic boundary conditions. The remainder of the modes are needed for the support of the computation. The lowest (long-wavelength) modes are slightly distorted because of artificially imposed boundary conditions (in our case periodic boundary conditions). The highest 40-50% of the modes are in error for several reasons. First, the spatial and temporal discretization methods for solving the nonlinear partial differential equations are usually second-order; second, artificial nonlinear damping is usually inserted if the true evolutionary processes give rise to steep structures; and finally, for a fixed-interval mesh with periodic boundary conditions, we in effect are sampling a periodic function and introducing aliasing errors that result from "folding back" modes.⁴

II. POWER SPECTRUM OF ELECTRON DENSITY FLUCTUATIONS:

ANALYTICAL CONSIDERATIONS

On the basis of the continuous and discrete one-and two-dimensional Fourier transforms

of simple geometric figures, we present insights into the parameters that control the amplitude and power-law dependence of EDF's. In particular, we show that at long wavelengths the *area* of EDF's controls the amplitude while at short wavelengths it is the *perimeter* of steep regions that controls the amplitude. The existence of a "power law" spectrum of EDF's depends upon the separation of space scales.

A. One-Dimensional Continuous Transforms

We first examine the energy spectrum of two one-dimensional rectangular functions as shown in Fig. 1. This will serve to illustrate the idea that at small k the "area" determines the amplitude of the spectrum whereas at large k the perimeter controls the amplitude.

In one-dimension the direct and inverse Fourier transform are

$$\hat{f}(k) = \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx, \quad (1)$$

and

$$f(x) = \left(\frac{1}{2\pi} \right) \int_{-\infty}^{+\infty} \hat{f}(k) e^{+ikx} dk, \quad (2)$$

and Parseval's theorem gives⁵

$$\int_{-\infty}^{+\infty} |\hat{f}|^2 dk = 2\pi \int_{-\infty}^{+\infty} f^2(x) dx. \quad (3)$$

For the function given in Fig. 1,

$$\hat{f} = 4LA \left(\frac{\sin z}{z} \right) \cos mz e^{-imz}, \quad (4)$$

where $z = kL$.

Thus, the integrated energy spectrum in the region from k_0 to ∞ is

$$\begin{aligned} IES(k_0) &= \int_{k_0}^{\infty} \hat{f} \hat{f}^* dk = 16LA^2 \int_{z_0}^{\infty} \frac{(\sin \xi \cos m\xi)^2}{\xi^2} d\xi \\ &= 16LA^2 \left\{ \frac{1}{4} \pi + \frac{1}{z} (\sin z \cos mz)^2 \right. \\ &\quad \left. - \frac{1}{4} \left[m_+ Si(2m_+ z) + m_- Si(2m_- z) \right. \right. \\ &\quad \left. \left. + 2 Si(2z) - 2m Si(2mz) \right] \right\}, \end{aligned} \quad (5)$$

where

$$m_{\pm} = m \pm 1 \quad \text{and} \quad Si(z) = \int_0^z \frac{\sin x}{x} dx. \quad (6)$$

At $k = 0$

$$IES(0) = 4\pi LA^2, \quad (7)$$

a result obtained from Parseval's theorem. Now if we take $z = n\pi > 1$ then

$$\begin{aligned} IES(n\pi) &= \frac{2LA^2}{n\pi} + O(n\pi)^{-3}, & (m = 1), \\ IES(n\pi) &= \frac{4LA^2}{n\pi} + O(n\pi)^{-3}, & (m > 1). \end{aligned} \quad (8)$$

That is, the coefficient of $(LA^2/n\pi)$ is a number indicating the total number of discontinuities of the well-separated rectangular functions. This implies that there is a redistribution of energy in k space, while the total energy $= IES(0)/\pi$, is constant. In two dimensions, we will see that the perimeter of the figures corresponds to the total number of discontinuities in one dimension.

B. Two-Dimensional Continuous Transforms

Fig. 2 shows the pill-box $f_p(x)$ and the frustum of a cone $f_f(x)$ that we consider as basic figures to represent two dimensional clouds. These figures have enough parameters to provide realistic models of EDF's and their symmetry allows an easy interpretation of formulas in asymptotic limits.

If we define the Fourier transform $\hat{f}(\mathbf{k})$ as

$$\hat{f}(\mathbf{k}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx dy f(x) e^{-i\mathbf{k} \cdot \mathbf{x}}, \quad (9)$$

$$= \int_0^{2\pi} d\phi \int_0^{\infty} r f(r) e^{ikr \cos(\theta - \phi)} dr, \quad (10)$$

where

$$\mathbf{k} \cdot \mathbf{x} = kr \cos(\theta - \phi),$$

since $k_1 = k \cos \theta$, $k_2 = k \sin \theta$, $x = r \cos \phi$ and $y = r \sin \phi$. For figures with azimuthal symmetry we may write

$$\hat{f}(k) = 2\pi \int_0^{\infty} r f(r) J_0(kr) dr. \quad (11)$$

For the pill box and the frustum of a cone we obtain, respectively

$$\hat{f}_p = A(2\pi R^2) J_1(z)/z, \quad (12)$$

$$\hat{f}_F = A 2\pi R^2 [z^{-2} (z_0 - z)^{-1}] \left[z J_0(z) - z_0 J_0(z_0) + \int_z^{z_0} J_0(z') dz' \right], \quad (13)$$

where $z = kR$, $z_0 = kR_0$. Note, that the Fourier transform of these isolated figures with azimuthal symmetry are real.

We define the one-dimensional energy spectrum $E(k)$ as

$$E(k) = k \int_0^{2\pi} \hat{f}(\mathbf{k}) \hat{f}(\mathbf{k})^* d\theta \quad (14)$$

and we obtain the asymptotic limits:

small kR

$$E_P = A^2 2\pi k [\pi R^2]^2, \quad (15a)$$

$$E_F = A^2 2\pi k \left[\frac{1}{3} \pi (R^2 + R R_0 + R_0^2) \right]^2; \quad (15b)$$

large kR

$$E_P = A^2 16\pi^2 R (k^{-2}) [\cos(kR - 3\pi/4)]^2 + O(kR)^{-3}, \quad (16a)$$

large kR and kR_0

$$E_F = A^2 16\pi^2 (R_0/\delta^2) (k^{-4}) \left[\cos \left(kR_0 - \frac{1}{4}\pi \right) - \left(\frac{R}{R_0} \right)^{1/2} \cos \left(kR - \frac{1}{4}\pi \right) \right]^2 + O \left[(kR)^{-5}, (kR_0)^{-5} \right], \quad (16b)$$

where $\delta = R_0 - R$. If $\delta \ll R_0$, Eq. (16b) can be written as

$$E_F = A^2 16\pi^2 (R_0/\delta^2) (k^{-4}) \left\{ \left(2 \sin \frac{1}{2} k\delta \right) \cos \left[kR_0 - \frac{1}{2} k\delta - \frac{3\pi}{4} \right] - \frac{1}{2} (\delta/R) \cos (kR - \pi/4) + O(\delta^2/R^2) \right\}^2 + O(kR)^{-5}. \quad (17)$$

Several features of $E(k)$ deserve comment. At small kR , the "volume" ($A^2 \times$ effective area) of the elementary figures, Eqs (15a) and (15b), is recovered as we expect from Parseval's theorem. At $(kR) \gg 1$, the pill-box function has a *modulated* power-law dependence k^{-p} with $p = 2$ and k -period $2\pi/R$. At $(kR, kR_0) \gg 1$ the function with the linear "skirts" and $\delta/R \ll 1$ has a *doubly* modulated power-law dependence with $p = 4$ and k -periods $4\pi/\delta$ and approximately $4\pi/(R_0 + R)$.

If $(\delta/R) \ll k\delta \ll 1$ then the leading term of (17) is dominant, or

$$E_F = A^2 16 \pi^2 (R_0 k^{-2}) \left\{ \cos(kR_0 - \frac{1}{2}k\delta - 3\pi/4) + O(k\delta)^2 + O(\delta/R)^2 \right\}^2. \quad (18)$$

That is, if the space scales are well-separated, there is an intermediate region where E_F has a k^{-2} dependence. Now if one sets $R = R_0$ ($\delta = 0$), we recover Eq. (16a). Note, if the linear skirts were replaced by a polynomial of degree "d", then the large- k power-law dependence would be $k^{-2(d+1)}$, that is $p = 2(d+1)$.

C. Energy Spectrum of M Pill-Boxes

If we have M azimuthally symmetric figures $f_m(\mathbf{x})$ whose centers of area are at $\mathbf{x}_m = \mathbf{e}_x x_m + \mathbf{e}_y y_m$, then the Fourier transform of the set is given by

$$f_M = \sum_{m=1}^M f_m(\mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{x}_m},$$

and the one-dimensional energy spectrum, $E_M(k) = k \int_0^{2\pi} |\hat{f}_M|^2 d\theta$, is

$$E_M(k) = 2\pi k \sum_{m=1}^M |\hat{f}_m(k)|^2 + 4\pi k \sum_{m \neq n=1}^M |f_m f_n^*| J_0(kr_{mn}), \quad (19)$$

where $r_{mn} = [(x_m - x_n)^2 + (y_m - y_n)^2]^{1/2}$ and the Bessel function J_0 arises by integrating over $\theta = \tan^{-1}(k_2/k_1)$, namely

$$\int_0^{2\pi} \exp[-ikr_{mn}(\cos \theta \cos \phi_{mn} + \sin \theta \sin \phi_{mn})] d\theta = 2\pi J_0(kr_{mn}),$$

where $\phi_{mn} = \tan^{-1}[(y_m - y_n)/(x_m - x_n)]$.

Now consider M identical pill-boxes of radius R_m . At small- k the self-interaction terms contribute $E_{M, \text{self}} = MA_m^2 2\pi^3 R_m^4 k$ and the cross-interaction terms contribute $E_{M, \text{cross}} = M(M-1) A_m^2 2\pi^3 R_m^4 k$ and thus the total is

$$E_M = A_m^2 (2\pi^3) (M^2 R_m^4) k, \quad (\text{small } kR_m). \quad (20)$$

At large- k the self-interaction terms contribute $E_{M, \text{self}} = A_m^2 16\pi^2 (MR_m) k^{-2} \cos^2 \left[kR_m - \frac{3\pi}{4} \right]$

and the cross-interaction terms contribute

$$E_{M, \text{cross}} = A_m^2 16\pi^2 (2/\pi)^{1/2} R_m^{1/2} k^{-(5/2)} \sum_{m \neq n=1}^M \sum_{n=1}^M \cos^2 \left[kR - \frac{3\pi}{4} \right] \cos \left[kr_{mn} - \frac{\pi}{4} \right].$$

This term can be neglected because it falls-off more rapidly at large- k . Hence

$$E_M = A_m^2 16 \pi^2 (M R_m) k^{-2} \cos^2(k R_m - 3\pi/4), \quad (\text{large } k R_m). \quad (21)$$

Thus, if we take one pill-box of radius R and height $A=1$ and divide it into M pill boxes of radius $R_m = R/M^{1/2}$ (that is, preserving total area) then the 1-D energy spectrum is the same at small- k

$$E_M = (2\pi^3) R^4 k, \quad (kR \ll 1)$$

but at large- k the spectrum is

$$E_M = M^{1/2} \left\{ 16 \pi^2 R k^{-2} \cos^2 \left[k R M^{-1/2} - \frac{3\pi}{4} \right] \right\}, \quad (kR \gg M^{1/2})$$

namely $M^{1/2}$ times larger. From the above considerations, one may conjecture that at large- $k\bar{R}$ the amplitude of the one-dimensional energy spectrum is proportional to the perimeter-to-area ratio of a piecewise constant figure, where $\bar{R} \propto (\bar{Q})^{1/2}$. Thus, if we were to modulate the boundary of the pill-box function with $b_m \cos m\alpha$ ($b_m < \bar{R}$ and α is the polar angle) as depicted in Fig. 2c, we would expect to find a rise in $E(k)$ at $k \simeq m/\bar{R}$ with an amplitude proportional to the total perimeter $P = \int_0^{2\pi} ds$, as shown in Fig. 2f.

D. Computational Examples of One-Dimensional Energy Spectra

We illustrate the properties of some of the formulas of the preceding section in Figs. 3-6. Fig. 3 shows the spectrum of a single pill-box of radius π , Eq. (12). For $k < 0.1$ the spectrum grows linearly in k with a slope proportional to the area. At $k \simeq 0.4 (= 0.4\pi/R)$ the spectrum is a maximum and the modulated power-law spectrum begins. Several modulations are plotted and the k^{-2} power-law dependence is obtained by plotting Eq. (12) at k intervals $\Delta k = 2\pi/R = 2$. Fig. 4a shows $E(k)$ for two touching pill-boxes, namely $r_{12} = 2\pi$, and each of radius π . The asymptotic power-law dependencies are the same as previously, but the small- k region is shifted up by a factor of 4 while the large- k region is shifted up by a factor of 2, consistent with Eqs. (20) and (21), respectively. The maximum occurs at $k \simeq 0.25$ down from 0.4. The slanted arrow points to an indentation which is the first sign of interference effects. Fig. 4b, for $M = 2$, $r_{12} = 5\pi$ and $R = \pi$, shows the same asymptotic behavior as in Fig. 4a. However, the

maximum region is strongly modulated by interference effects. This is more pronounced in Fig. 4c for $M = 2$, $r_{12} = 10\pi$, and $R = \pi$ where the leading minimum at $k \approx 0.1$ $[= (0.8\pi)/(r_{12} - 2R)]$ is decreased by a factor of ~ 2.0 from Fig. 4b. Note also, there are now three large maxima due to interference effects. Thus, the inclusion of many *identical* pill boxes does not change the asymptotic envelopes (k^{+1} or k^{-2}) including the maxima of the modulation. The region beginning at $k_{min} = \text{Max } (r_{12} - 2R)^{-1}$ is modulated due to interference effects. If a large number of different radii were used to calculate the energy spectrum the modulation nulls would be "filled" and the asymptotic power-law dependencies would prevail.

Figs. 5a, 5b, and 5c show one-dimensional energy spectra for a frustum of a cone with inner radius $R = \pi$ and outer radii, $R_o = 1.1, 1.01 \pi$ and 1.001π , respectively. The low- k regions are identical. The transition point, k_{tr} , from a k^{-2} spectrum to a k^{-4} spectrum is taken as the intersection of the two lines drawn tangent to the maxima (see the single vertical arrow). It moves to higher k with decreasing $\delta = R_o - R$, namely $k_{tr} \delta \approx \frac{1}{2} \pi$. (Note, for $k > 8$ we have not plotted every point, but have sampled the function at equispaced intervals. In Fig. 5b, this accounts for the smooth behavior of the curve at $k \sim 50$ and the saw-tooth behavior for $k > 100$. For Fig. 5c, a similar comment applies for $k > 30$ and $k \sim 500$.) Figure 6 shows the one-dimensional energy-spectrum of two frustums $A = 1$, $R = \pi$, $R_o = 1.01\pi$ (like Fig. 5a) and $r_{12} = 10\pi$. The transition point $k_{tr} \approx 60$ is increased slightly and except for the expected increase in amplitude, the large- k regions are identical. The low- k region shows a deep null and four maxima, similar to Fig. 4c.

III. DISCRETE FOURIER TRANSFORMS AND ENERGY SPECTRA

OF FIGURES IN PERIODIC REGIONS

A. Analytical Results in One Dimension

Most numerical simulations of ionospheric plasma clouds are done with finite-difference methods on a periodic mesh. Thus, we now consider and compare the continuous and discrete periodic Fourier transforms of the trapezoidal figures shown in Fig. 7. The continuous

transform is

$$\hat{f}(k) = \int_{-L}^L f(x) e^{-ikx} dx = A (\beta + \alpha) L \left\{ \frac{\sin z_+}{z_+} \frac{\sin z_-}{z_-} \right\}, \quad (22)$$

where $z_{\pm} = \frac{1}{2} kL(\beta \pm \alpha)$ and $k = \pi\nu/L$, $\nu = 1, 2, \dots$. The first null is at $k_{n1} = 2\pi/(\beta + \alpha)L$ and at large k we have a function proportional to k^{-2} modulated at two frequencies, the slower associated with the width of the skirts, $(\beta - \alpha)L$. This is the one-dimensional version of the frustum of a cone treated earlier. The discrete Fourier transform $\hat{F}(k)$ on an interval $-N + 1 \leq n \leq N$ is

$$\begin{aligned} \hat{F}(k) &= \sum_{n=-N+1}^{n=N} f(nh) e^{-iknh}, \\ &= A(b + a) \left\{ \frac{\sin \frac{1}{2} \theta(b-a)}{(b-a) \sin \frac{1}{2} \theta} \frac{\sin \frac{1}{2} \theta(b+a)}{(b+a) \sin \frac{1}{2} \theta} \right\}, \end{aligned} \quad (23)$$

where $\theta = hk = (\pi\nu/N)$, and $\nu = (-N+1), \dots, -1, 0, 1, \dots, N$.

That is, the discrete system has $2N$ independent modes and Parseval's theorem has the form

$$\sum_{n=-N+1}^N f^2(nh) = 2N \sum_{\nu=-N+1}^N \hat{F}(k) \hat{F}^*(k). \quad (24)$$

Note, the highest mode available to the discrete system is $k = \pi/h$ and the lowest is $k = \pi/Nh$.

The first null is at

$$\nu_{n1} = 2N/(b+a). \quad (25)$$

Since the discrete equivalent of a pill box function has $b=a+1$, the first null is at $\nu_{n1} = 2N/(2a+1)$.

Thus, the essential differences between (22) and (23) is the appearance in the denominator of $(b \pm a) \sin \frac{1}{2} \theta$ instead of $(\beta \pm \alpha) \frac{1}{2} kL$, an effect called aliasing.⁵ This effect causes an up-turn at the high end of the discrete spectrum. The error made in comparing $\hat{f}$ with $\hat{F}$ is

$$\begin{aligned} \epsilon &= 1 - \hat{f}/\hat{F} \\ \epsilon &= 1 - (z_+ z_-)/(b^2 - a^2) \left[\sin \frac{1}{2} \theta \right]^2 = 1 - \left[\frac{1}{2} \theta / \sin \frac{1}{2} \theta \right]^2, \end{aligned} \quad (26)$$

or: 147% at $\nu = N$; 62.6% at $\nu = \frac{3N}{4}$; 23% at $\nu = \frac{1}{2}N$ and 5.3% at $\nu = \frac{1}{4}N$. The errors in the

energy spectrum, $(1 - |\hat{f}/\hat{F}|^2)$, are appreciably larger. Thus, one should be cautious about using modes beyond $\nu = 0.5N$ to estimate the exponent of power-law behavior in a numerical simulation.

B. Computational Results in Two Dimensions

For two-dimensional figures on a discrete lattice, we present "integrated" or one-dimensional energy spectra and estimated power-law exponents and amplitudes. The one-dimensional spectra have "half" the information needed to reconstruct the original discrete function, since phase information is lost through averaging.

In this section we are concerned with assessing the accuracy of spectral representations of EDF's. These are presently obtained from numerical simulations of finite-difference representations of nonlinear partial differential equations. To accomplish this, we present one-dimensional energy spectra of simple figures (like those shown in Fig. 2) and also the exponents p and amplitudes G obtained by fitting $10^G k^{-p}$ to the discrete spectra for various regions of k .

We will conclude by showing that p and G should be assessed with only 40% of the available modes. A quantitative assessment of: the sensitivity of errors to the fitting range; and the ability to extrapolate this exponent to higher k values are subjects requiring further study.

1. Equations

To obtain one-dimensional spectra we first take the discrete Fourier transform by using the 2D analog of (23),

$$\hat{F} = \hat{F}(k_1, k_2) = \sum_{n,m=-N+1}^{n,m=N} f(nh, mh) e^{-i(k_1 nh + k_2 mh)}, \quad (27)$$

where we have assumed a square region with equal mesh spacings in the x and y directions and $Nh = L$. Thus, the discrete system has $(2N)^2$ independent modes in the square

$$-N + 1 \leq \nu_i \leq N, \quad (i = 1, 2), \quad (28a)$$

or

$$\frac{\pi}{h} \left(-1 + \frac{1}{N}\right) \leq k_i \leq \frac{\pi}{h} \quad (i = 1, 2). \quad (28b)$$

The one-dimensional spectra are obtained by summing over bands:

X (sum over the k_2 -direction)

$$X = X(\nu_1) = \sum_{\nu_2=-N+1}^{\nu_2=+N} \hat{F}\hat{F}^*; \quad (29a)$$

Y (sum over the k_1 -direction)

$$Y = Y(\nu_2) = \sum_{\nu_1=-N+1}^{\nu_1=+N} \hat{F}\hat{F}^*; \quad (29b)$$

C (sum over "circular-bands", $k = (k_1^2 + k_2^2)^{1/2} = \text{const.}$)

$$C(\nu) = \sum \hat{F}\hat{F}^* W; \quad (29c)$$

where $\nu = (\nu_1^2 + \nu_2^2)^{1/2}$, and W is a bilinear weighting factor and the sum is around circles of constant radius.⁶ The circular band average is commonly used in isotropic turbulence studies as it does not favor any particular direction as do the previous two averages. It is calculated from an algorithm that "area-weights" the modes of the rectangular lattice by their closeness to circular bands on the k_1, k_2 plane. These bands are bounded by circles with radius $\nu = 1/2, 3/2, \dots, \left[N + \frac{1}{2}\right]$, and the weighted contribution in bands with radius $(2M-1)/2$ and $(2M+1)/2$ is called mode "m". The weighted contribution within the circle of radius 1/2 is designated as mode zero. Note that

$$\sum_{\text{all modes}} \hat{F}\hat{F}^* = \sum_{\nu_1=-N+1}^{\nu_1=N} X(\nu_1) = \sum_{\nu_2=-N+1}^{\nu_2=N} Y(\nu_2) > \sum_{\nu=0}^{\nu=N} C(\nu). \quad (30)$$

The last inequality follows because the circular bands do not extend into farthest corners.

We have also fitted these one-dimensional spectra with the power-law function

$$H = 10^G \nu^{-p} \quad (31)$$

using a least-squares procedure over different ranges of ν . Table 1 contains summary information on the transformed figures and their one-dimensional spectra as given in Figs. 8-22. Here the amplitude $(\log_{10} X)$, $(\log_{10} Y)$ and $(\log_{10} C)$ are plotted as X , Y and O vs $\log_{10}(1+\nu)$ from $0 \leq \nu \leq N$. At the bottom of Table 1 is given the angle measured from the horizontal corresponding to exponents p .

The fitted values of p and G are designated with a * or + referring respectively to the 256^2 lattice or the 64^2 lattice, respectively. For the former, the mode ranges are (6-64), (25-64), (51-102) and (51-128), while for the latter the mode ranges are (1-16), (6-16), (6-25) and (12-32).

Fig. 8 shows the spectra for a pill box of radius 16 on a 256×256 mesh. Thus the nulls should be separated by $k\pi = \pi$ or $\nu = 8$ as the figure shows. The peaks at 14 and 22 give a straight line at 21.5° ($p=2.0$) and the peaks do not begin to deviate from the line until $70 < \nu < 86$. Aliasing errors qualitatively measured by "AL" in the figure arise rapidly for $\nu > 86$. The k^{+1} spectrum expected at low- k occurs at about mode 2. The exponent p that is determined in the first ν region ($6 \leq \nu \leq 64$) is accurate for C and X (2.06 and 1.99, respectively). However aliasing effects seem to occur in all the subsequent bands as seen by the monotonic decrease in p . The exponents obtained by fitting X (or Y) data do not show as strong an aliasing effect. The first band, where reliable p 's are obtained, correspond to 45% of the total number of modes.

Fig. 9 shows the spectra for a 15×15 square ($2b_x = 2a_x = 2b_y = 2a_y = 15$) on a 256×256 mesh. The first null is at $17 \left[= 1 + \frac{256}{15+1} \right]$ and the following nulls occur at every 16 mode numbers (17, 33, 49, etc.). A straight line drawn through the peaks $\nu = 25$ and 41 at an angle of -21.5° , very close to $p = 2$. A straight line drawn through last two peaks has a slope of -6.2° ($k^{-0.54}$) thus showing the effect of aliasing. The height AL is due to aliasing. The band average, C , shows a k^{+1} (11.31°) behavior at mode 2. Sizeable errors begin to occur at the fourth maximum of C , mode 73, (shown by the vertical arrow). Hence, to obtain the apriori exponent ($p = 2$) we have used (100%) $(73-25)/128 = 37.5\%$ of the modes. Referring to Table 1, we see that C gives a good fit for p for $6 \leq \nu \leq 102$ but the X -fits fluctuate widely from 2.0. The C data resulting from band averaging is smoothed and p is not as sensitive to the initial and final points of the fitting interval. The highest interval gives small p 's ($p = 1.32$ or 1.42) due to aliasing.

Fig. 10 shows spectra for a 31×31 square on a 256×256 mesh. The nulls are separated by 8 mode numbers. The peaks at mode 13 and 21 give an angle of -22° whereas the highest modes give an angle of -1.8° ($k^{-0.16}$). The effect of aliasing is increased at the highest modes. The p 's for C -data show the monotonic decrease due to aliasing and the p 's for X -data show larger fluctuations than previously, due to the sensitivity to the location of the fitting ranges.

Fig. 11 shows consistent results for a 15×15 square on a 64×64 lattice, a size typical of those used in present day numerical simulations. The nulls are spaced by $4 \left[= \frac{64}{15+1} \right]$ mode numbers, hence a coarse sampling. If one compares with Fig. 9, one sees approximately the same percentage of modes available for determining spectral exponents. The fit to C -data in $6 \leq \nu \leq 16$ is 2.03 comparing favorably with $p = 2.0$ in C -data of Fig. 9. However, here the X -data gives a larger p ($= 2.57$), compared to $p = 1.62$ for Fig. 9.

Fig. 12 shows results for a trapezoidal figure, that is a 14×14 square with skirts ($b_x - a_x =$) 2 units wide on each side, as depicted in top view on Fig. 7c. The first two maximum (at $\nu = 25$ and $\nu = 41$) give an angle of -23.5° ($k^{-2.17}$), and the maxima at $\nu = 73$ and 89 give an angle of -38.7° (k^{-4}). The null due to the slow modulation term $\left[\sin \frac{1}{2} \theta(b-a) \right]$ in Eq. 23 is at $\nu = 128$ and is already evident in the more rapid fall-off at the highest modes. Thus, aliasing phenomena are suppressed. This is also evident because p falls monotonically from 2.41 to 4.71 with a value of 4.13 for $51 \leq \nu \leq 102$. Hence 40% ($= 100\% \times (51/128)$) of the modes provide a reasonable estimate of the true power law $p = 4.0$. Note the X -data yield $p = 3.87$.

Fig. 13 is also for a trapezoidal figure with skirts ($b_x - a_x =$) 4 units wide on a 24×24 square giving fast-modulation nulls every 16 mode numbers and the first slow-modulation null at $\left\{ 2N/(b-a) = \right\} 64$, as indicated by the double arrow on the Fig. 13. This causes the deviation from the regular k -space oscillation apparent in the previous figures. Note, that a k^{-4} (-38.7°) region seems to be evident in the range $20 \leq \nu \leq 50$ but a k^{-2} region is not really apparent. One concludes, that a figure with skirts of 4 units on all sides of a 24×24 square

does not yield well-separated space scales and k -space regions are not well-separated. Thus, errors occur because of interference of slow and fast modulation terms. This is apparent when we compare p values with those obtained from the C -data of Fig. 12. The variation in p 's obtained from Fig. 13 is not monotonic and in the region $51 \leq \nu \leq 102$, $p = 2.12$! In other words, because of the destructive interference, the k^{-4} spectral dependence of a corresponding continuous figure is *not* obtained by a straight-forward fitting procedure.

Fig. 14 shows results similar to Fig. 13, except that the identical figure (see Table 1) is placed on a 64×64 lattice. Thus the previous discussion applies with all mode numbers reduced by a factor of 4. Note, the non-monotonic behavior of p 's with a value of $p = 4.31$ in $6 \leq \nu \leq 25$.

The next four figures (15-19) show spectra of a *rotated* rectangle with and without skirts on two different meshes. Fig. 15 is a 15×7 rectangle whose major axis is *parallel* to the x -axis. The line through the X and Y maxima is at $20^\circ(k^{-1.8})$, whereas the line through C is at $-18.5^\circ(k^{-1.67})$. Note the effects of aliasing. There is a "good" p behavior in $6 \leq \nu \leq 64$ and "poor" behavior in the next region. Aliasing is evident in the small values of p in $51 \leq \nu \leq 128$.

Fig. 16 shows the effect of rotating the 15×7 rectangle through 30° . All slopes shown are $-27.0^\circ(k^{-2.6})$, that is, larger than in Fig. 15. The lower-range p values for X data (2.29 and 2.54) and Y data (2.39 and 2.27) are consistent with lines drawn on the figures. The poor estimate of $p = 1.68$ for C -data in $25 \leq \nu \leq 64$ is to do the suprising nulls at $\nu = 33$ and 63 . (shown by the double arrows).

Fig. 17 shows the rotated 15×7 rectangle of the previous figure on a 64×64 mesh. The same comments apply.

Fig. 18 shows the above rotated rectangle with skirt of 2 units added to each side on a 64×64 mesh. The two lines drawn, have the same slope (angle $\approx -36.5^\circ$, corresponding to $k^{-3.7}$). There is no noticable k^{-2} region. The values of p in the two intermediate regions are consistent with $p = 3.7$.

The next three figures (19-21) show two 15×7 rectangles on a 256×256 mesh at varying separation along a 30° line as depicted in Fig. 7d. Fig. 19 at zero separation (corresponding to a 30×7 rotated rectangle) has an angle of 22.5° ($k^{-2.1}$), although there are near-plateau regions at -7° ($k^{-0.61}$). This is to be compared to the single 15×7 rectangle in Fig. 16 which shows larger slopes and the absence of near-plateaus. In both cases, the effect of aliasing is reduced.

For a separation of $L_s = 2$, Fig. 20, shows a larger angle, -23° (or $k^{-2.12}$) than in Fig. 17 with steep interference regions at angle of -41° ($k^{-4.35}$). Finally, on Fig. 21, for a separation of $L_s = 16$, the angle is -25.5° ($k^{-2.38}$) and again aliasing seems reduced. The fitted p values range from 2.14 to 2.37 in $6 \leq \nu \leq 64$ and anomalous values occur in the $25 \leq \nu \leq 64$ because of the interference structure observed in the figures.

To obtain Fig. 22 we have placed four rectangles on a 256×256 lattice as depicted in Fig. 7e (the values of the spectrum above an ordinate of 3.0 are suppressed.) The nulls and slow modulated pattern associated with single rectangles are not evident, as one expects. However, large fluctuations in Y and C are observed in the mid-range of ν and angles are about -17° ($k^{-1.53}$). The fitted p values substantiate this observation, showing values of $p < 2.02$ with values of between 1.21 and 1.66 in $51 \leq \nu \leq 128$. That is, interference effects enhance aliasing when no skirts are present.

Table 2 presents an overview of these detailed considerations. We have computed the mean and standard deviation of p in each band for C -data and X -data of the 11 different cases on a 256×256 lattice described in Table 1. We have done the same for the 4 different cases on a 64×64 lattice in the region $6 \leq \nu \leq 16$. The lowest region gives a reasonable $p \approx 2.3$ with the smallest standard deviation of 0.4. The third region shows larger p values (2.56 for C -data and 2.58 for X -data) than the last (2.06 and 2.30). However, the last region shows large standard deviations. This undoubtedly results from the competition between aliasing (which causes the spectrum to rise) and skirts (which causes the spectrum to fall more rapidly and additionally causes slow modulation terms to appear in the spectrum). The data for $64 \times$

64 lattice in $6 \leq \nu \leq 16$ seems to be dominated by the cases with skirts, for it falls closer to 4.0 and has a large standard deviation.

IV Conclusions

For EDF's with steep sides, a power law exponent of $p = 2.3$ with a standard deviation of 0.4 can be estimated from numerical simulations on a 256×256 lattice if an intermediate 40% of the modes are used. The remaining 60% of the modes are required to "support" the calculation. The lowest 15% of the modes are influenced by the periodic boundary conditions and the upper 45% of the modes are affected by interference effects due to skirts and errors (including numerical dissipation, dispersion and aliasing).

The 64×64 lattice does not provide enough freedom to have a large separation of space scales, that is only 13 modes ($\approx 0.4 \times 32$) can be used to estimate p . Hence, one should treat the p 's estimated on lattices of order 64×64 as tentative. However, the amplitude at the intermediate modes is probably reliable.

This conclusion suggests that a hierarchical set of numerical simulations must be made on lattices of size $\geq 150 \times 150$. Each run would have a different initial normalizing length. Hence the resulting spectra would apply to different regions of k -spaces thereby yielding the separation of scales necessary to obtain a trustworthy power spectrum.

More work is required to design such a hierarchy of simulation runs and to develop asymptotic techniques for matching the output of these runs.

ACKNOWLEDGMENTS

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5. Champeney, D.C. "Fourier Transforms and Their Physical Applications" New York: Academic Press, 1973. Provides good background information on continuous processes in one, two and three dimensions.
6. G.S. Deem provided CIREN, a code that computes band averages from a two-dimensional array of equispaced modes.

Table 1. Information Summary of Discrete Fourier Transformed Figures and Fitted Power Law Exponents and Amplitude

Fig	Type	Skirt Length	2N*	Trapezoidal Figures				n _{yx}	n _{xy}	p for various k-ranges*						G for various k-ranges					
				2b _x	2a _x	2b _y	2a _y			25-64 6-16	51-102 6-25	51-128 12-32	16-64 +1-16	25-64 6-16	51-102 6-25	51-128 12-32					
8	Circle	0	256	Radius = 16				8	8												
9	Square	0	256	15	15	15	15	16	16												
10	Square	0	256	31	31	31	31	8	8												
11	Square	0	64	15	15	15	15	4	4												
12	Square with skirts	2	256	18	14	18	14	16	16												
13	Square with skirts	4	256	20	12	20	12	16	16												
14	Square with skirts	4	64	20	12	20	12	4	4												
15	Rectangle	0	256	15	15	7	7	16	32												
16	Rectangle at 30°	0	256	15	15	7	7	16	32												
17	Rectangle at 30°	0	64	15	15	7	7	4	8												
18	Rectangle at 30° in skirts	2	64	19	15	11	7	376	711												
19	Two rectangles both at 30° separation = 0	0	256	15	15	7	7	16	32												
20	Two rectangles both at 30° separation = 2	0	256	15	15	7	7	16	32												
21	Two rectangles both at 30° separation = 16	0	256	15	15	7	7	16	32												
22	Four rectangles	0	256	-	-	-	-	-	-												

*n values correspond to angles of lines drawn on Figs. 8-22

Angle	5.71°	11.31°	16.70°	21.80°	26.57°	30.96°	35.0°	38.66°	45.0°	50.19°
p	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0

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Table 2 -- OVERVIEW: Mean and Standard Deviation of p Exponents

	DATA	CASES	$(2N)^2 = 256^2$ RANGE of ν				$(2N)^2 = 64^2$ RANGE of ν	
			6-64	25-64	51-102	51-128	CASES	6-16
MEAN	C	11	2.25	1.98	2.56	2.06	4	3.13
	X	11	2.26	2.13	2.58	2.30	4	3.49
STD.DEV	C	11	0.42	1.1	0.90	1.1	4	1.18
	X	11	0.43	1.1	0.78	1.1	4	1.03

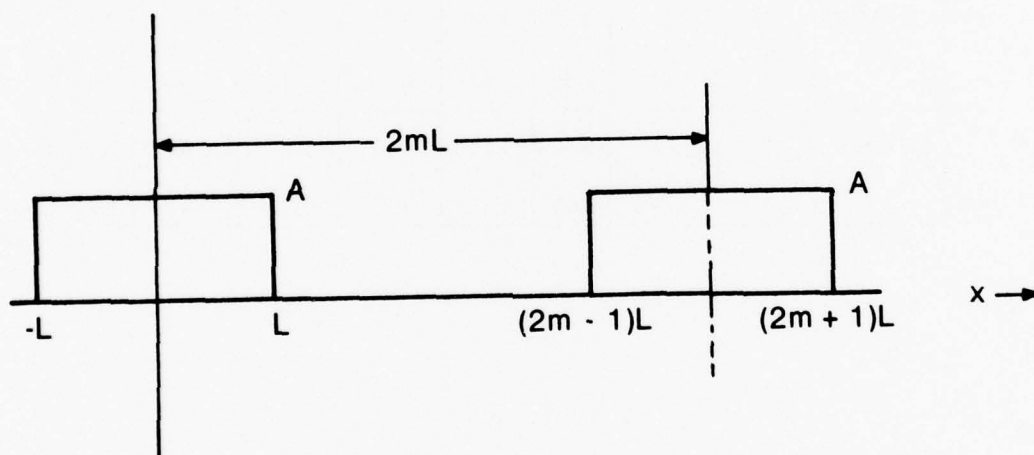


Fig. 1 — Two rectangular figures in one dimension

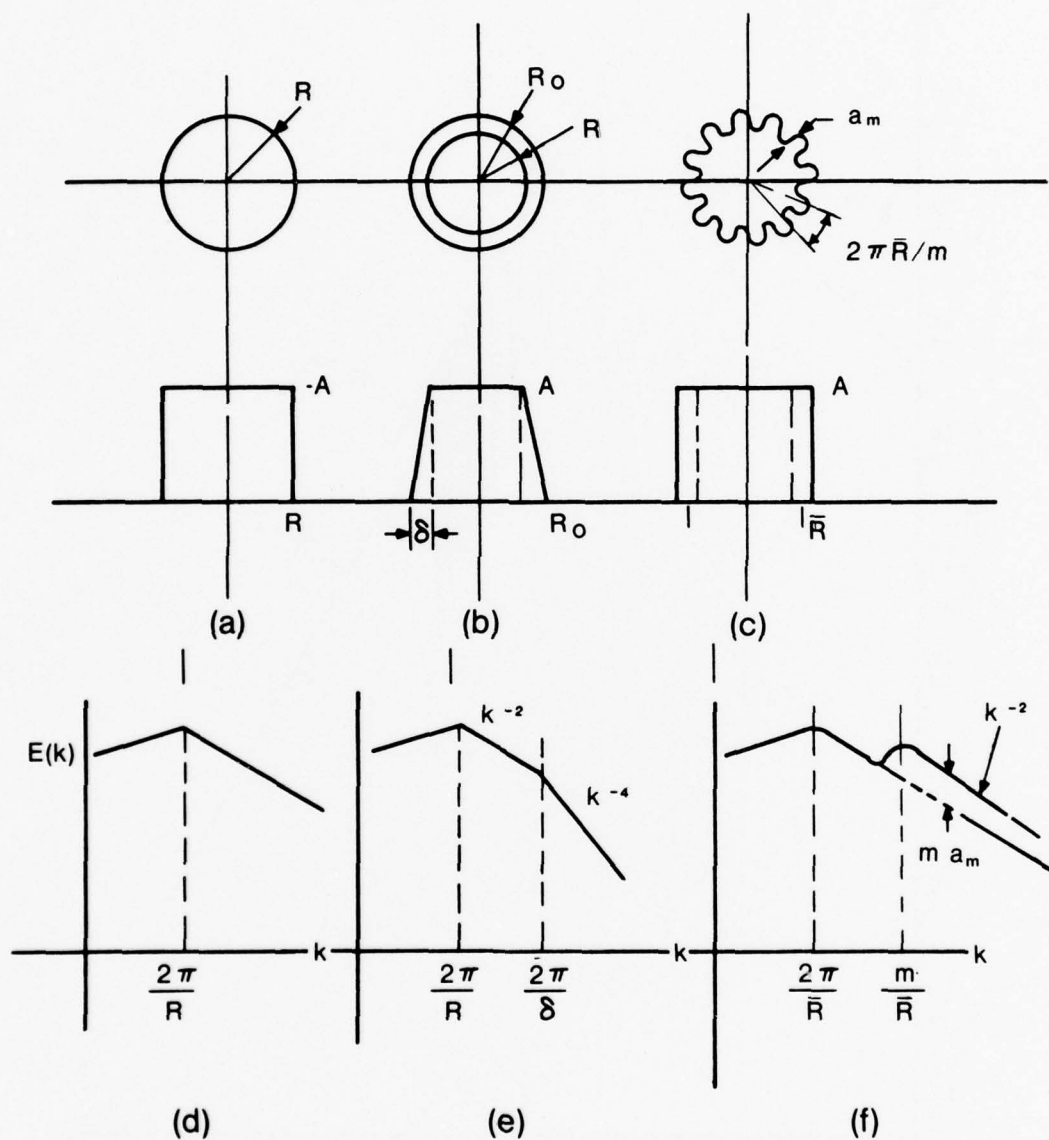


Fig. 2 — Figures with azimuthal symmetry (a,b,c) and schematic of their one-dimensional energy spectra (d,e,f). (a,d) Pill-box or cylinder, f_p ; (b,e) Frustum of a cone, f_f ; (c,f) Pill-box with azimuthal modulation at wave number m .

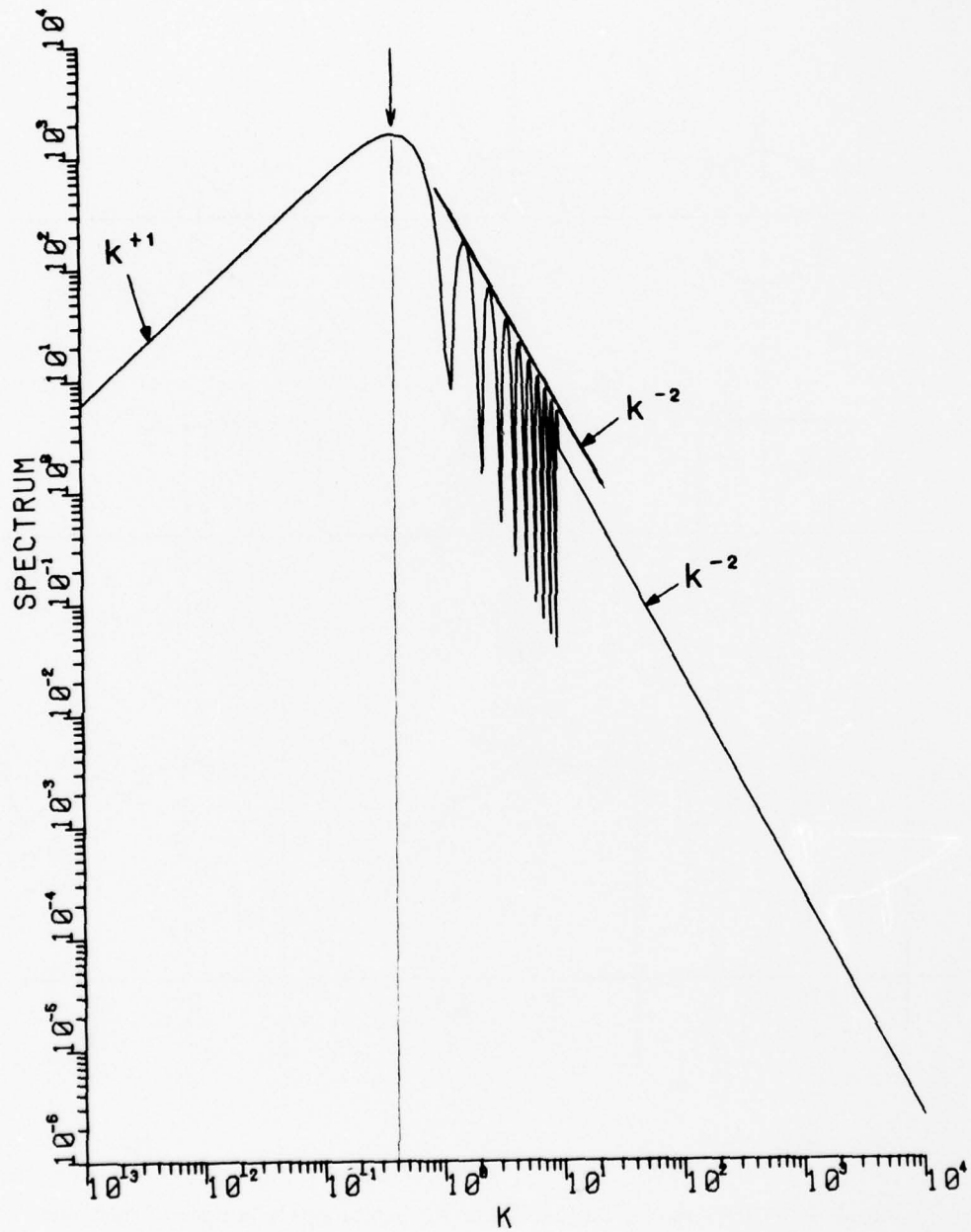
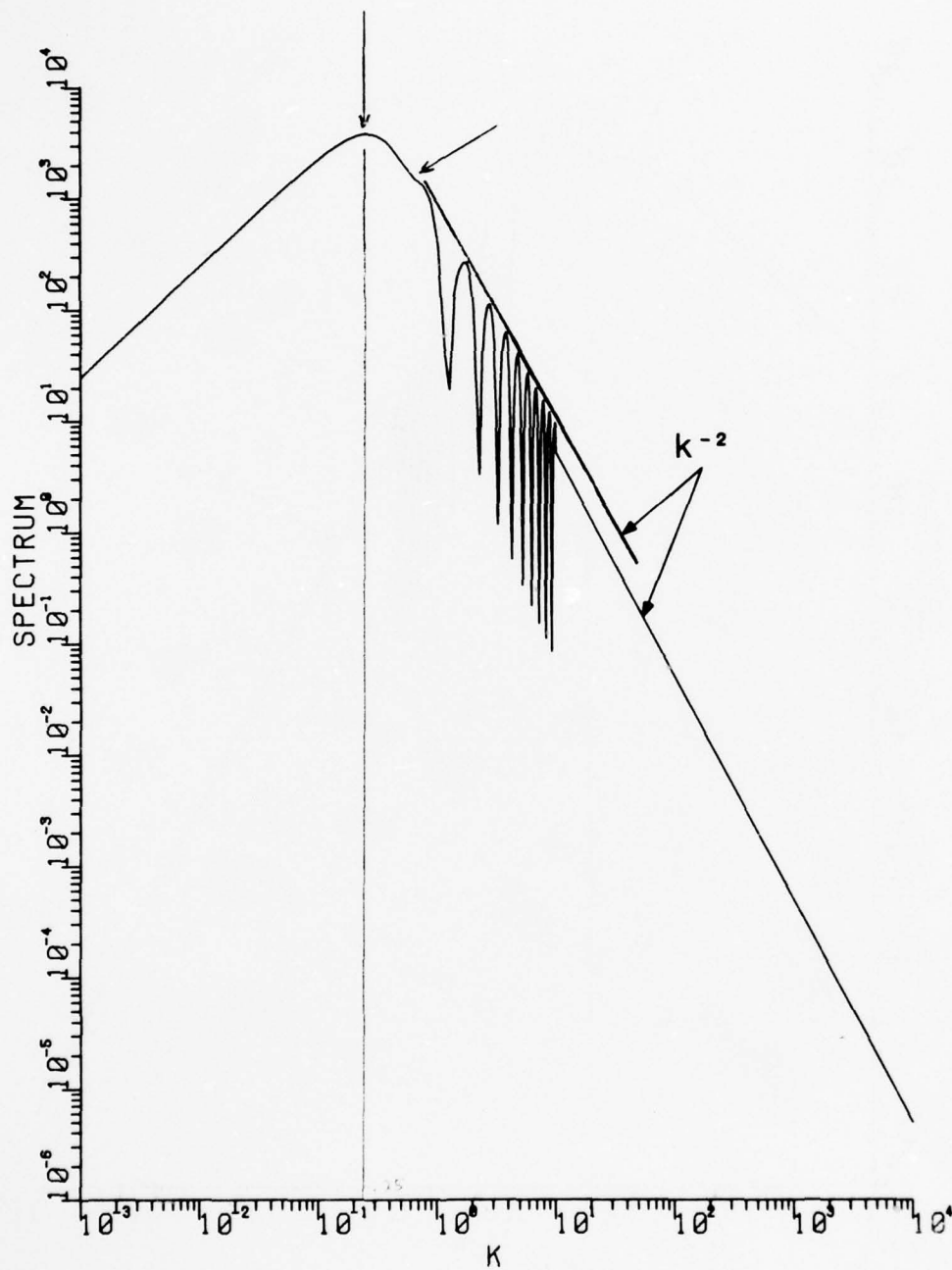
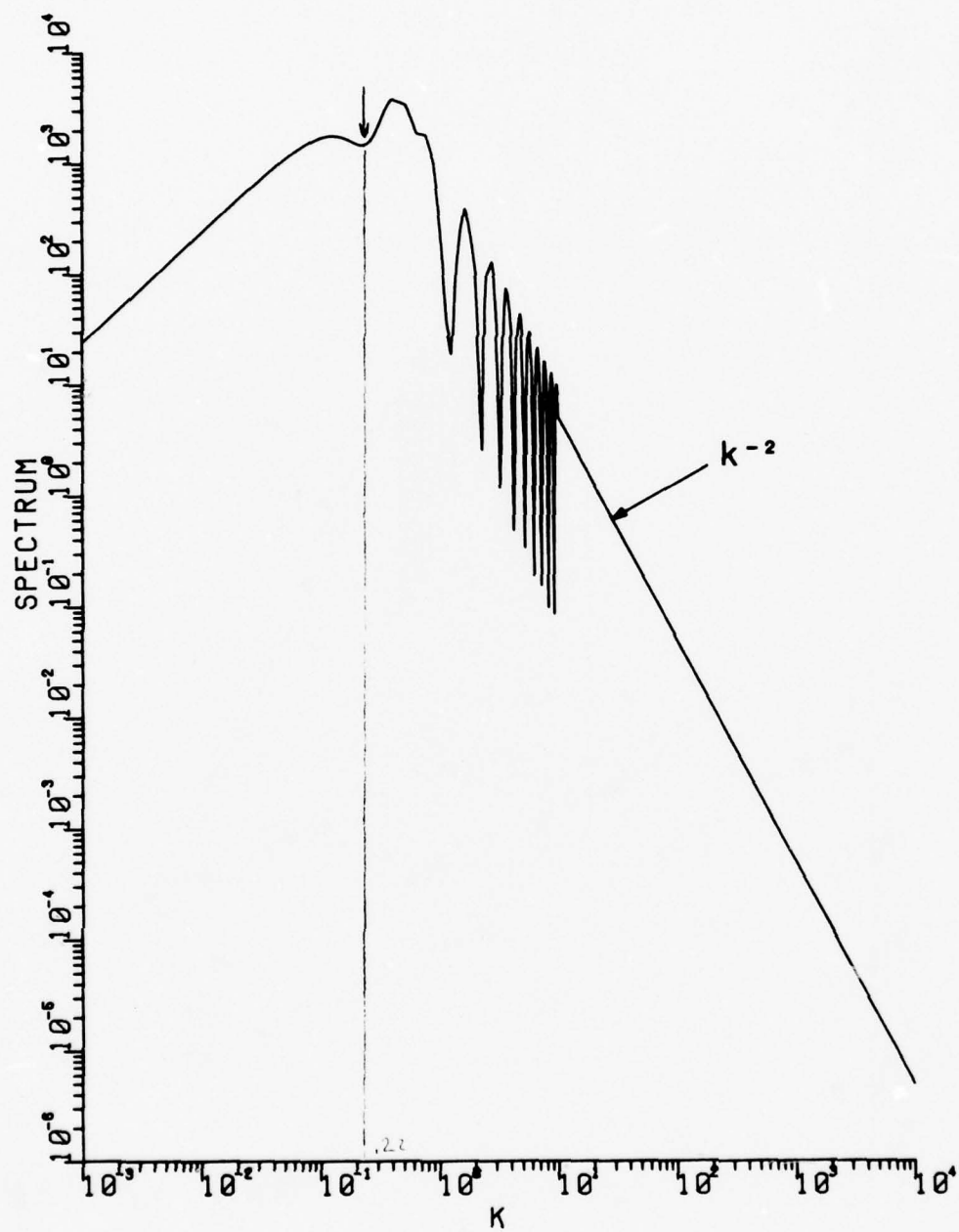


Fig. 3 — One-dimensional energy spectrum for a single pill box of radius $R = \pi, A = 1$.

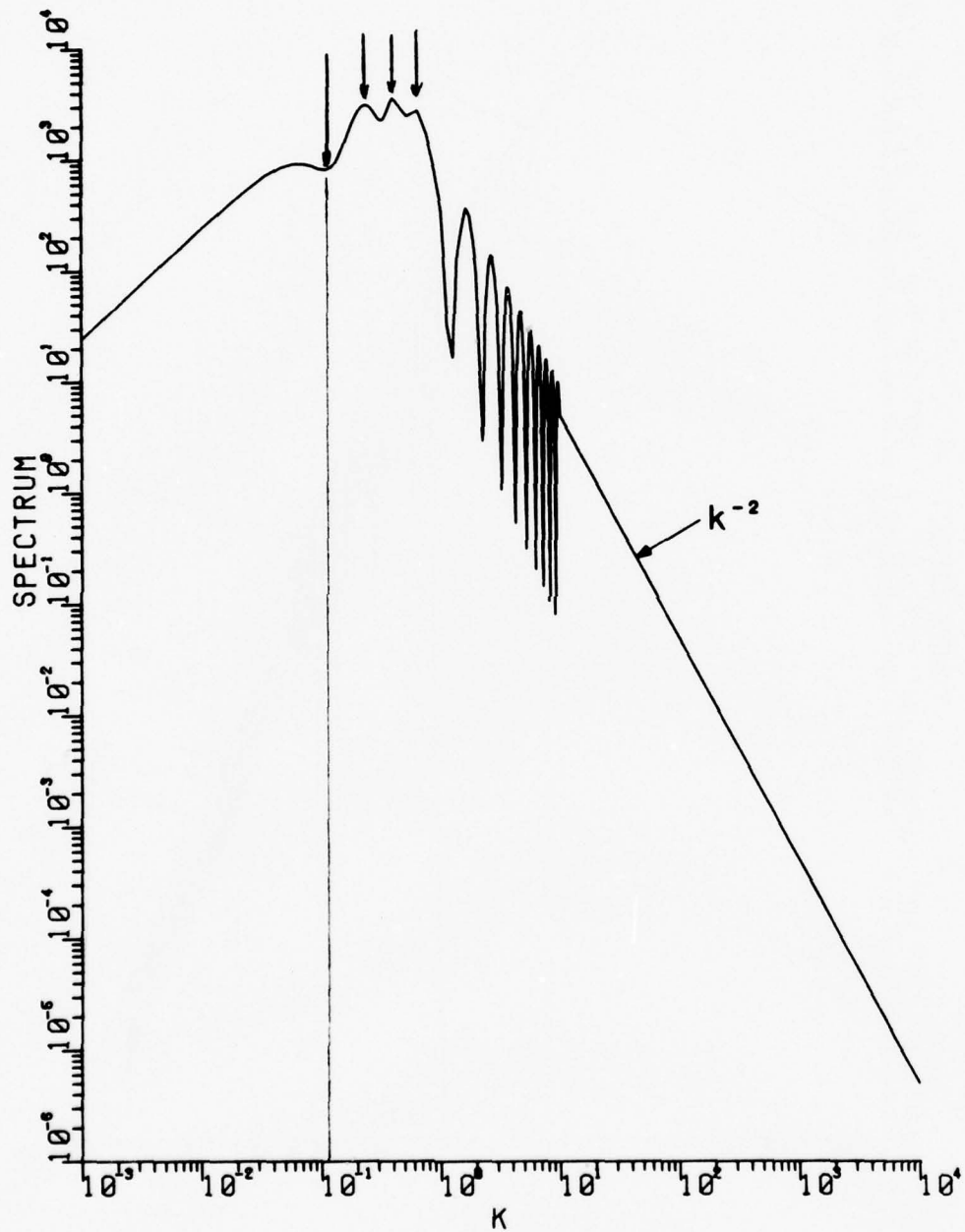


a) $r_{12} = 2\pi$;

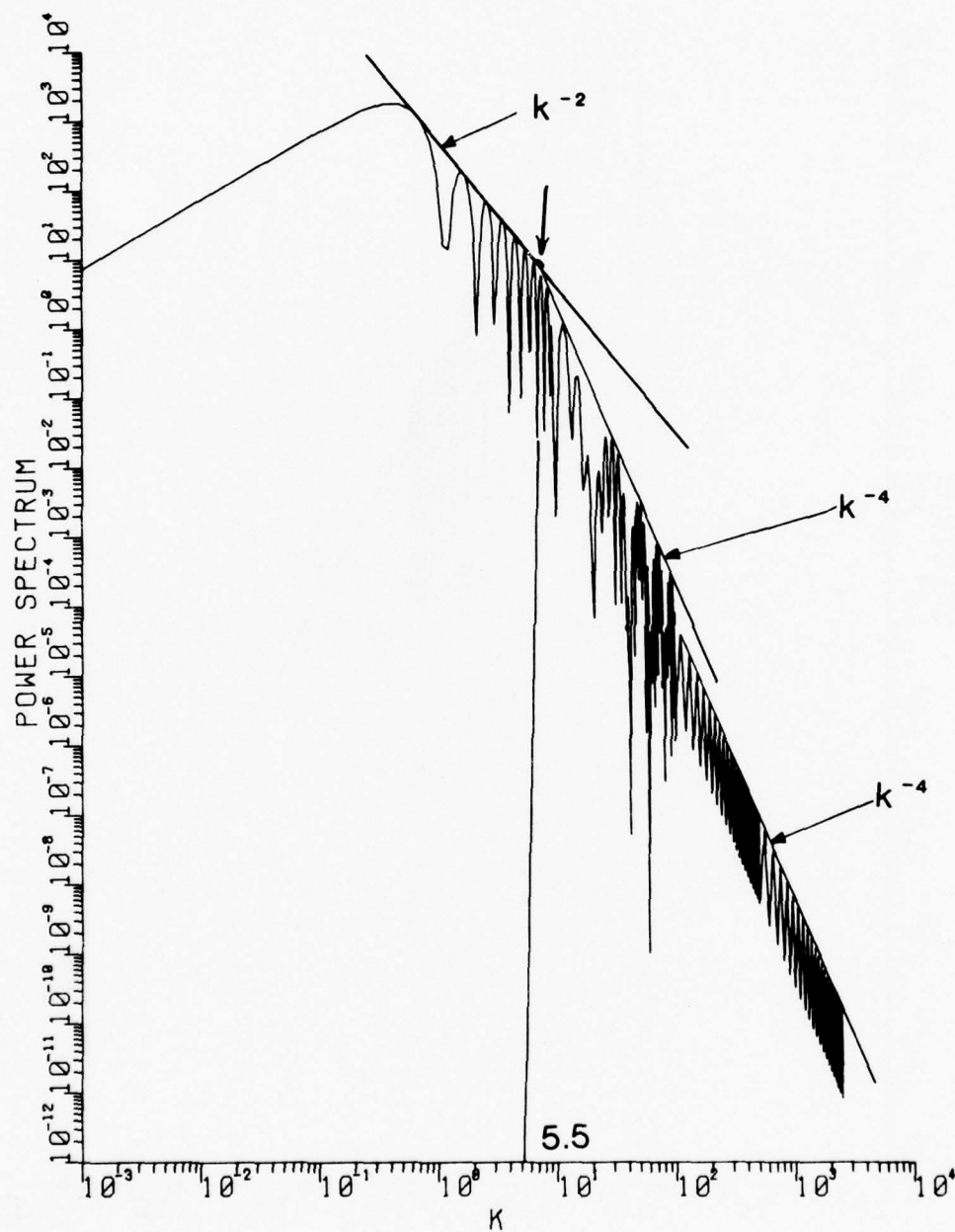
Fig. 4 — One-dimensional energy spectrum for two pill boxes, each of $R = \pi$ and $A = 1$. Distance between centers:



b) $r_{12} = 5\pi$;

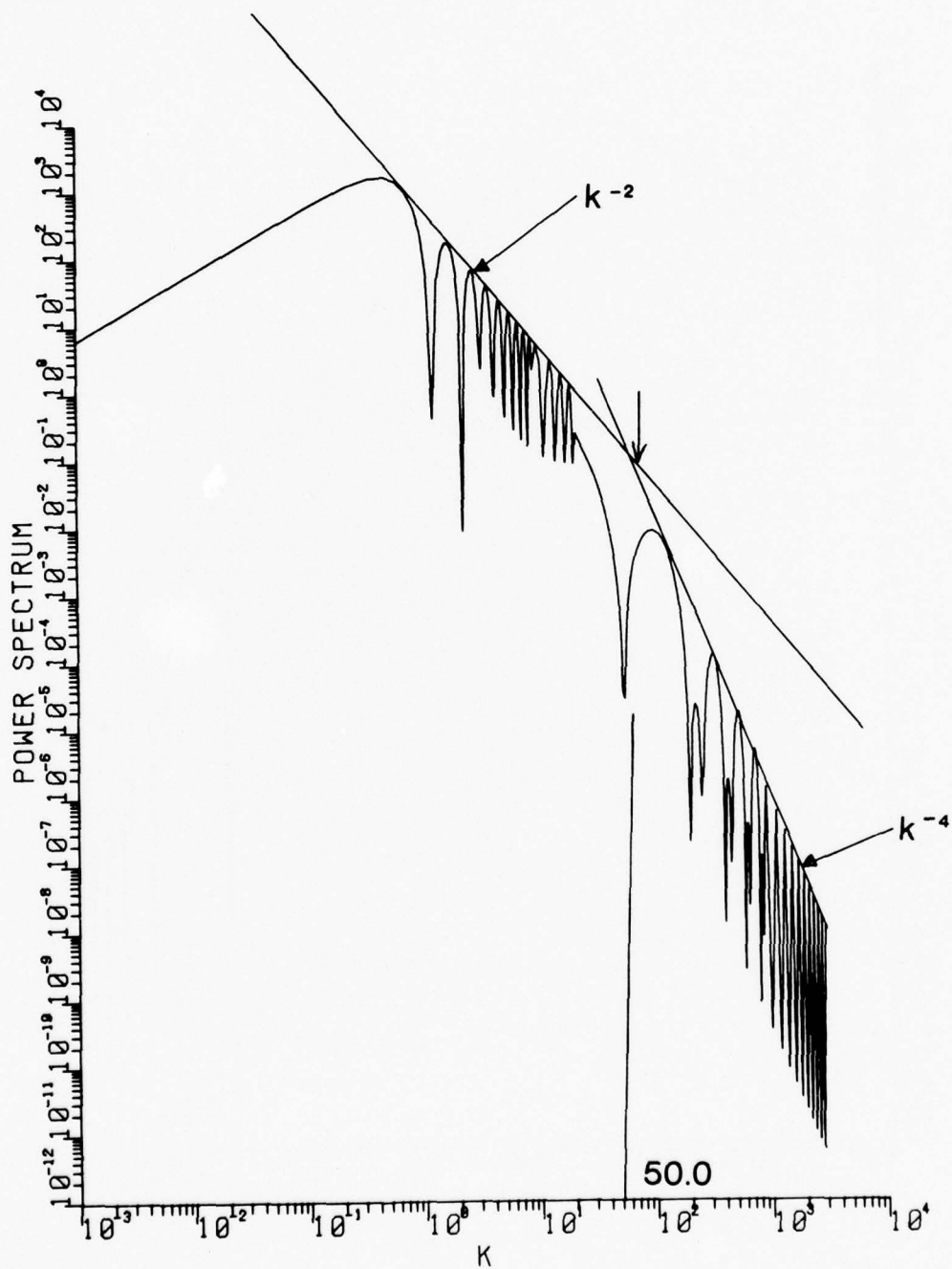


c) $r_{12} = 10\pi$.

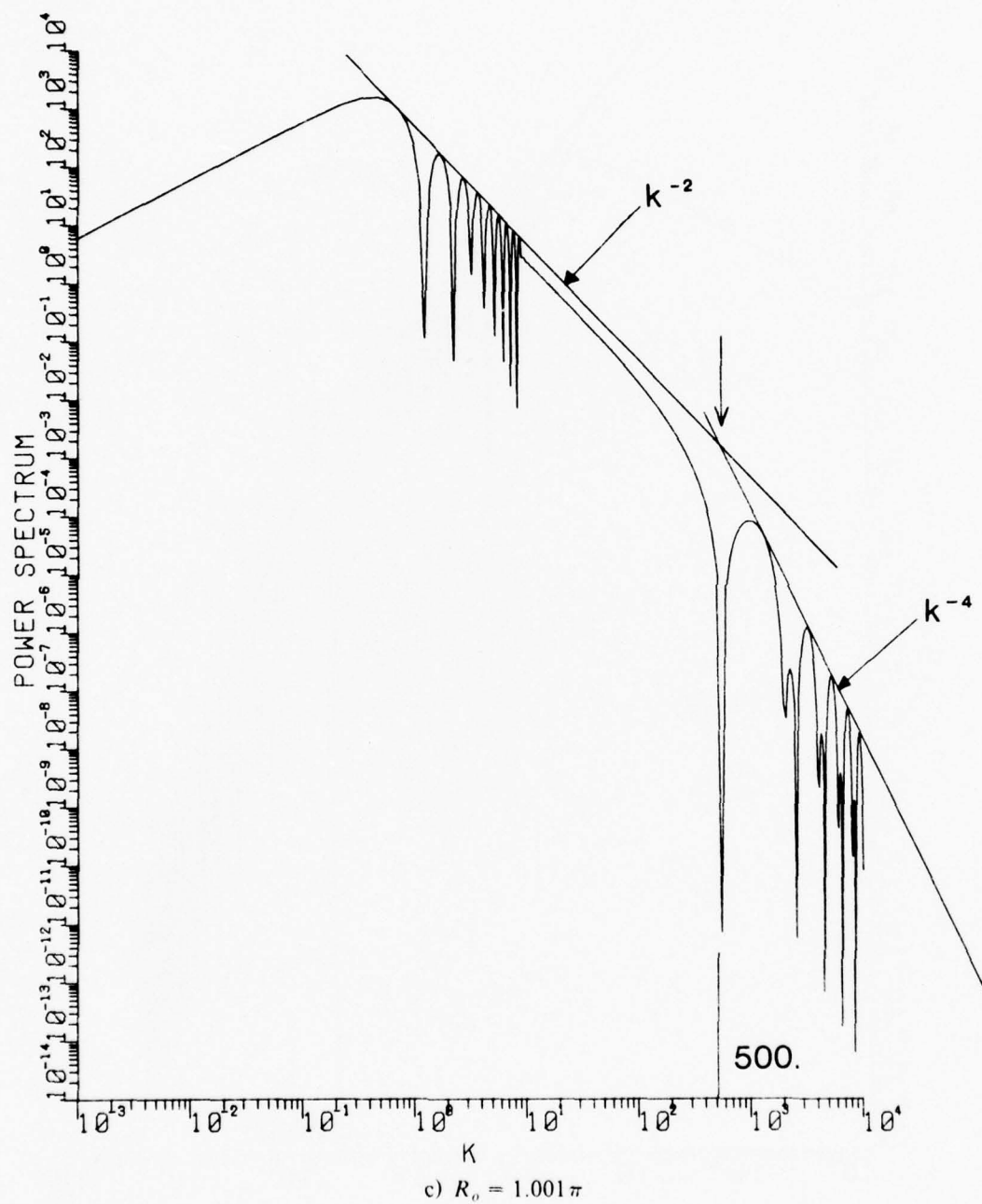


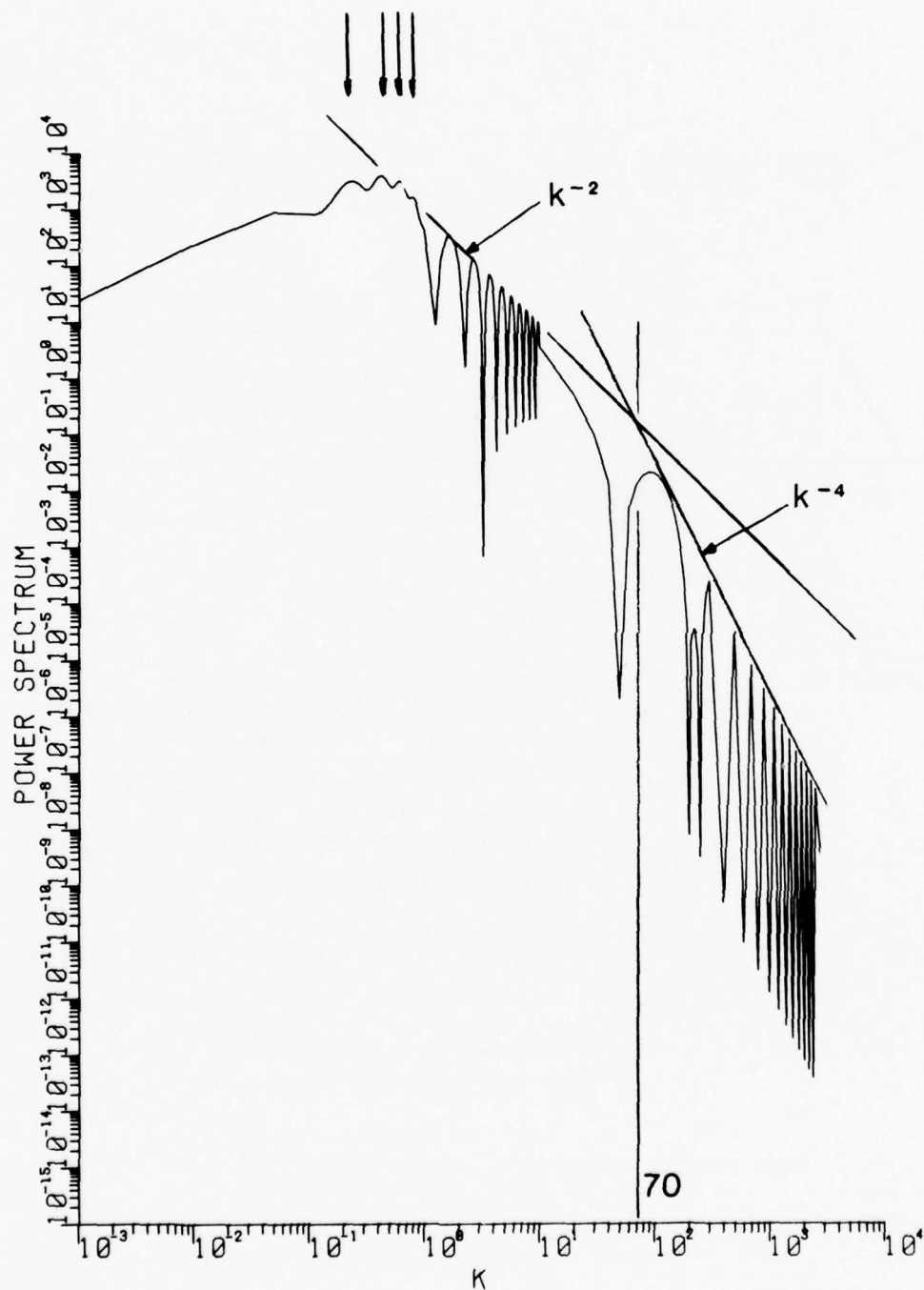
a) $R_o = 1.1\pi$;

Fig. 5 — One-dimensional energy spectrum for a frustum of a cone with $A = 1$ and $R = \pi$.



b) $R_o = 1.01\pi$,





POWER SPECTRUM OF TWO FRUSTUMS, $R_S = (1.01)R, (10)R$ APART, $R = \pi$

Fig. 6 — One-dimensional energy spectrum for two frustums of cones. $A = 1$,

$R = \pi, R_o = 1.01\pi$ and $r_{12} = 10\pi$.

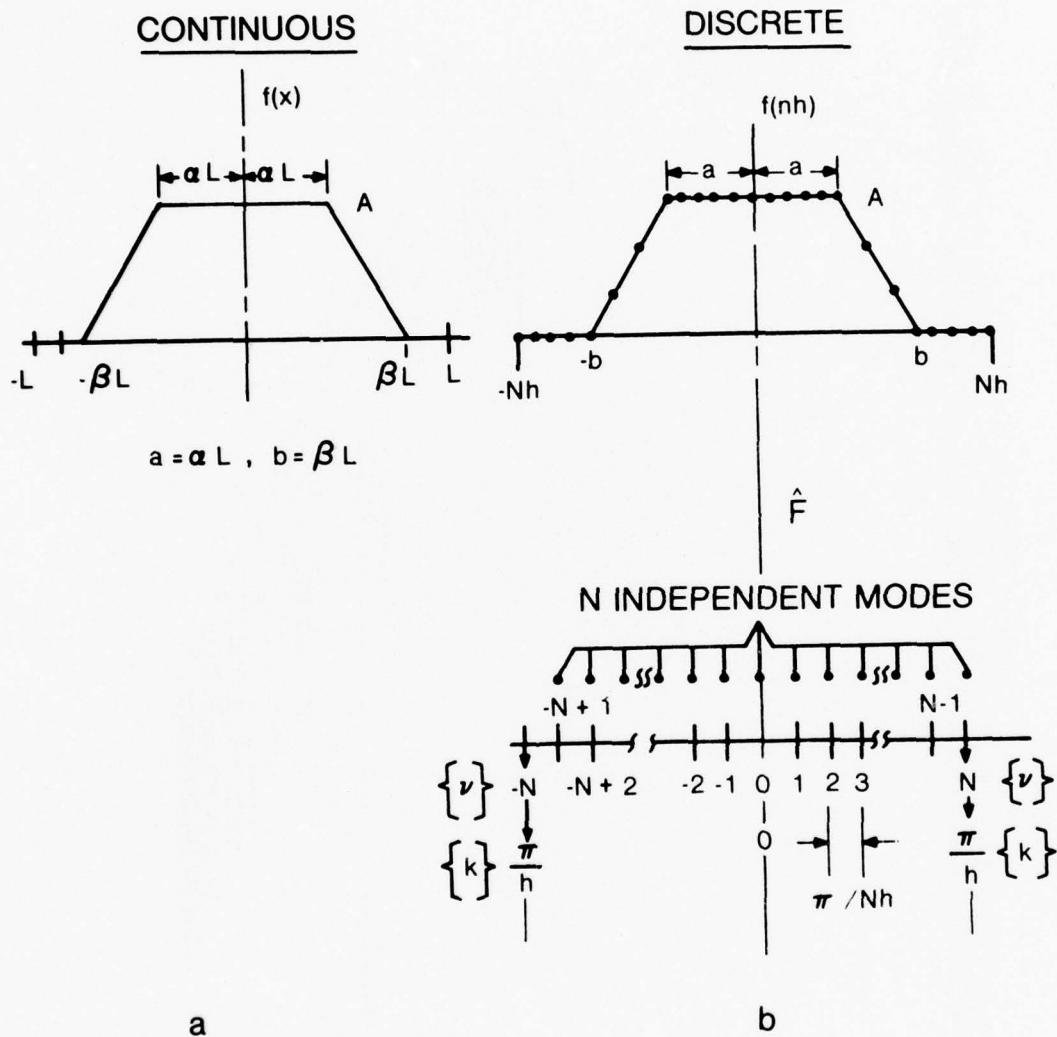


Fig. 7 — Schematics for continuous and discrete Fourier transforms in one and two dimensions.

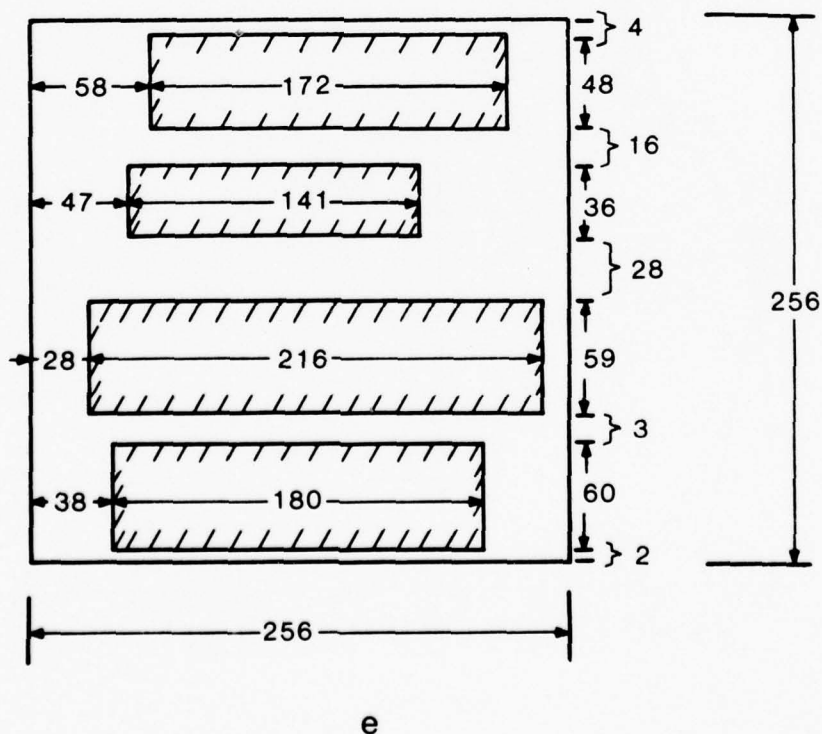
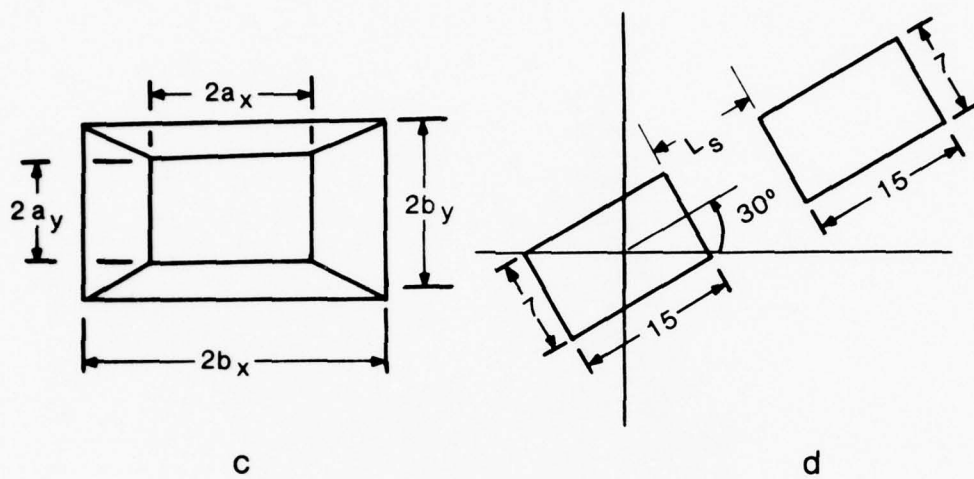


Fig. 7 - Schematics for continuous and discrete Fourier transforms in one and two dimensions.

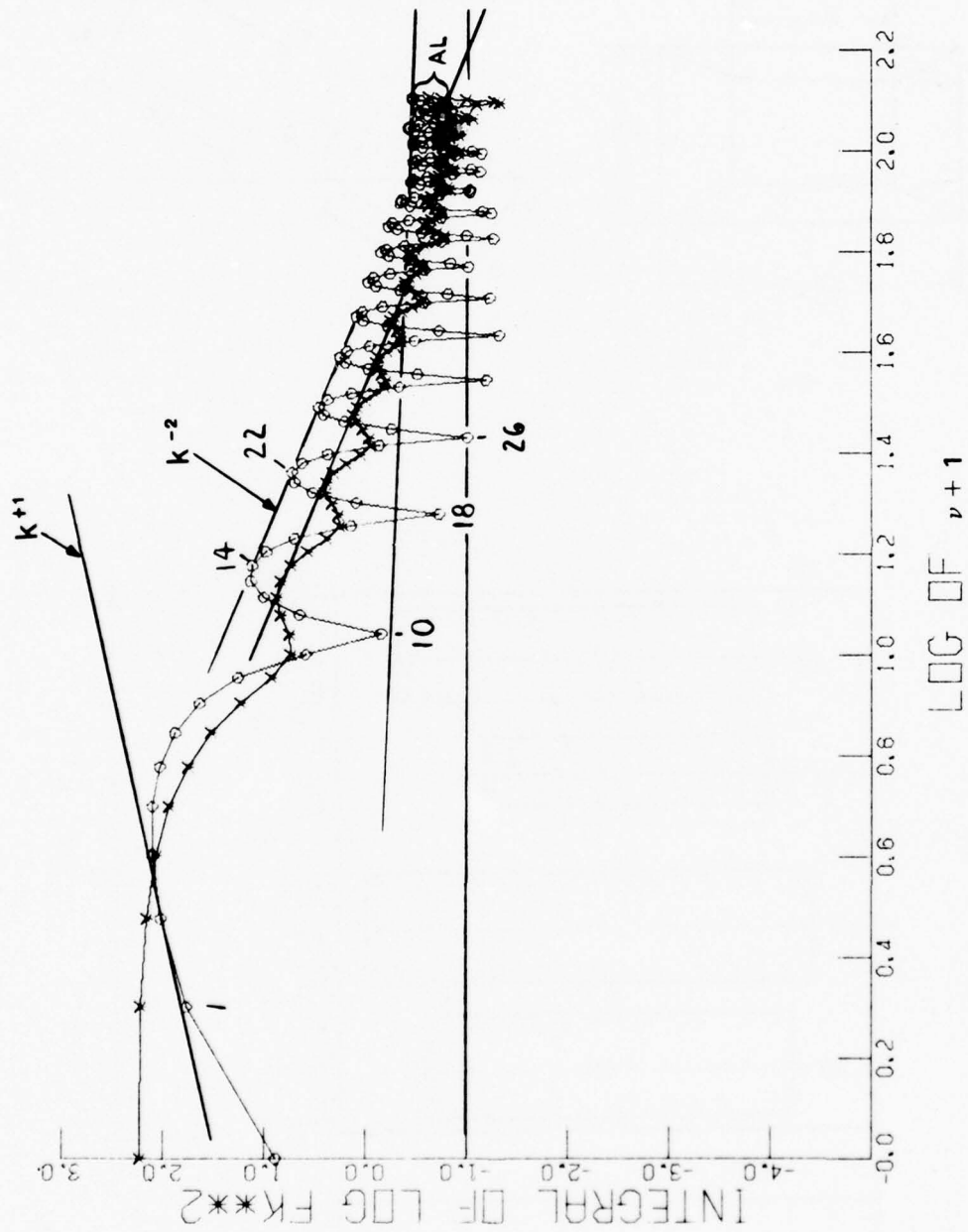


Fig. 8 — Discrete one-dimensional energy spectra ($X = Y$ and O) for a circular pill box. $A = 1$, $R = 16$ and $(2N)^2 = 256^2$.

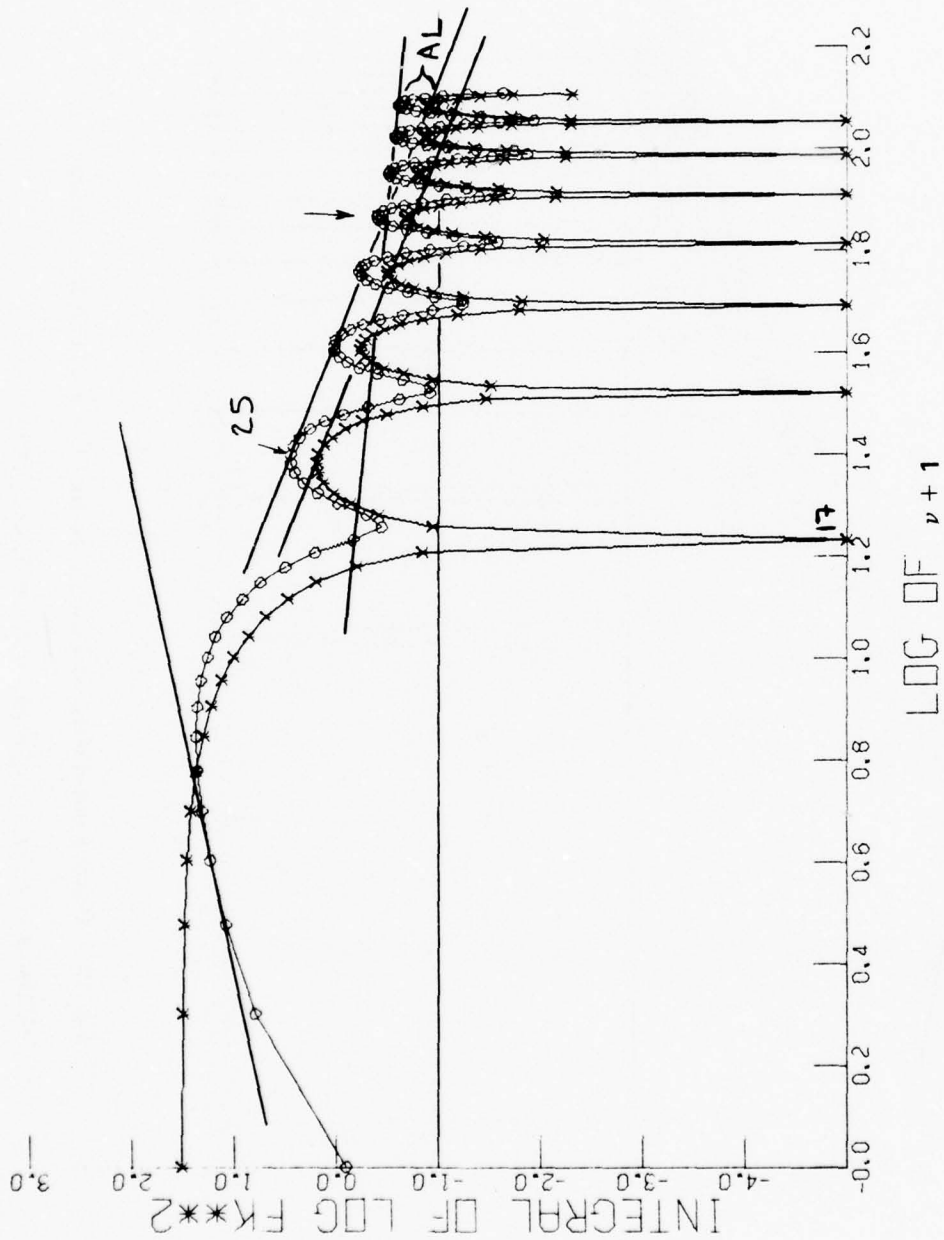


Fig. 9 — Discrete one-dimensional energy spectra ($X = Y$ and O) for a square pill-box. $A = 1$, $2a_x = 2a_y = 2b_x = 2b_y = 15$ and $(2N)^2 = 256^2$.

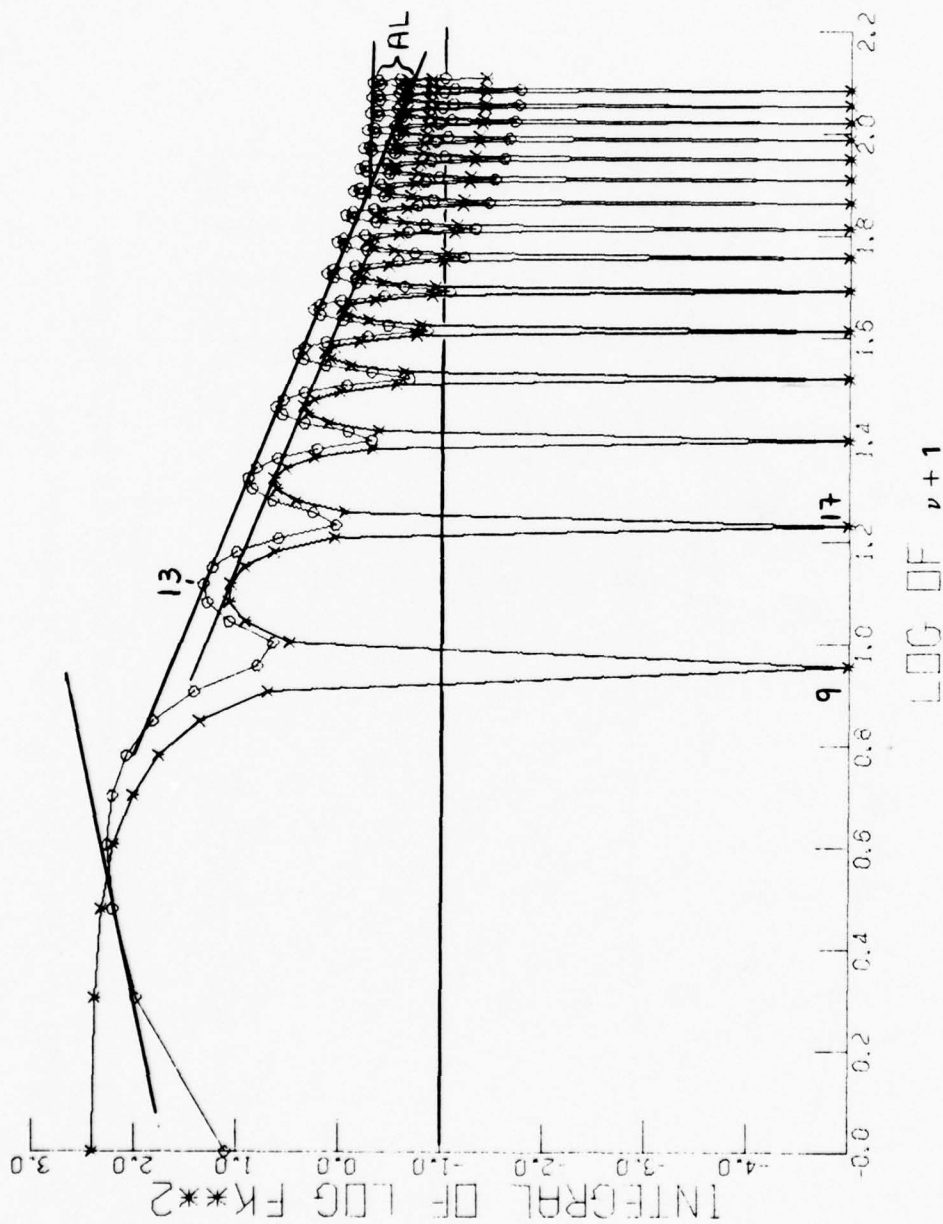


Fig. 10 — Discrete one-dimensional energy spectra ($X = Y$ and O) for a square pill-box $A = 1$, $2a_x = 2a_y = 2b_x = 2b_y = 31$ and $(2N)^2 = 256^2$.

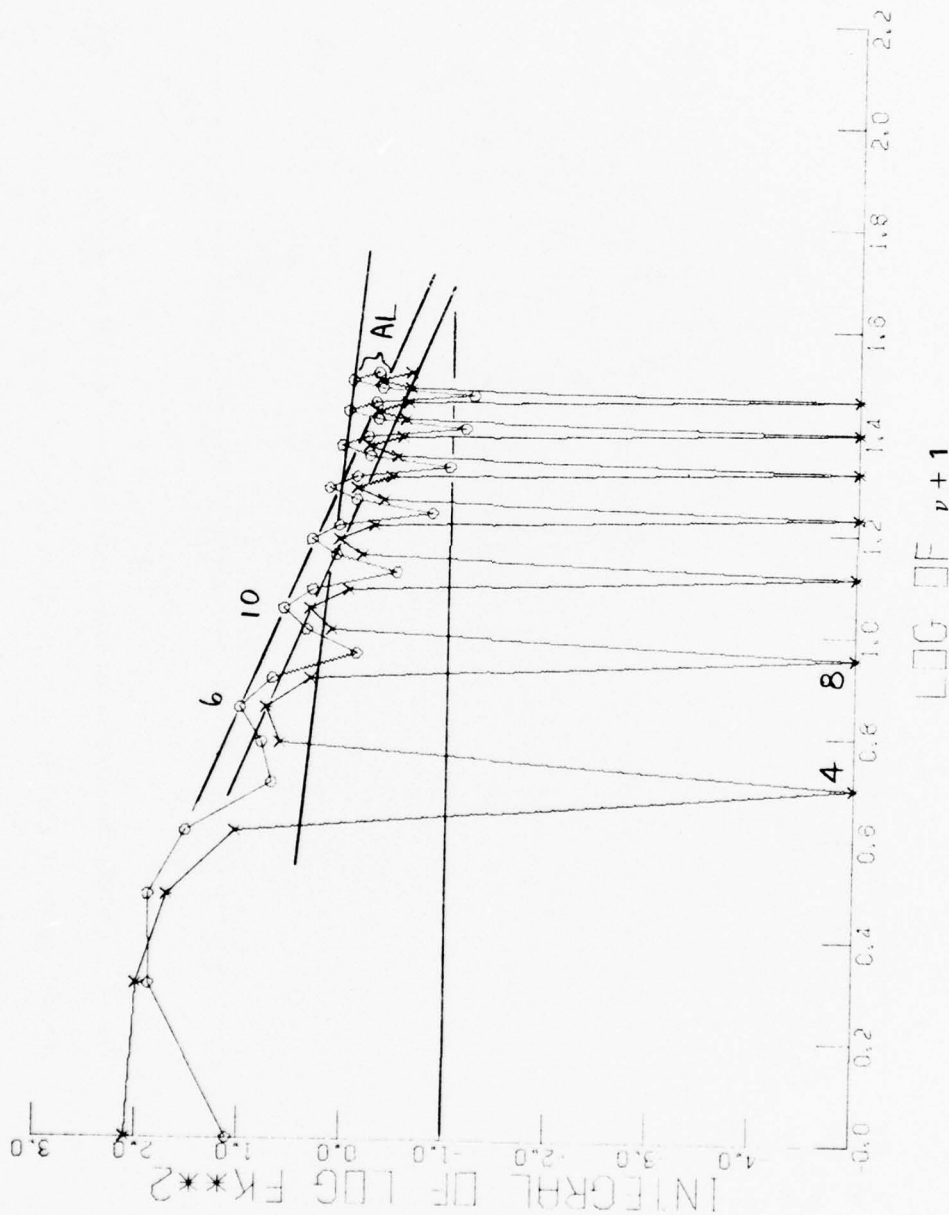


Fig. 11 — Discrete one-dimensional energy spectra ($X = Y$ and O) for a square pill box. $A = 1$, $2a_x = 2a_y = 2b_x = 2b_y = 15$ and $(2N)^2 = 64^2$.

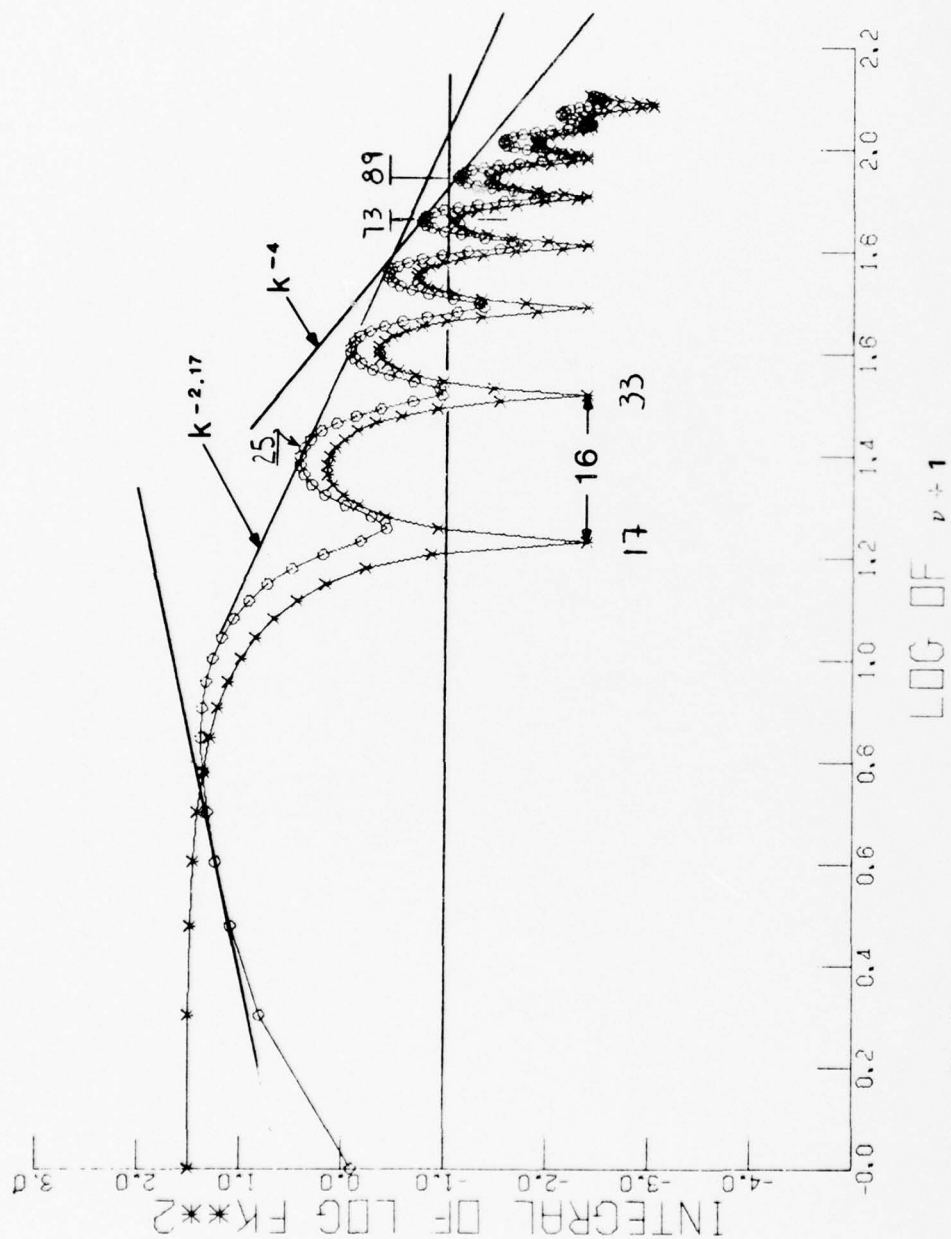


Fig. 12 — Discrete one-dimensional energy spectra ($X = x$ and O) for the tra-
pezoidal figure of Fig. 7c. $A = 1$, $2a_x = 2a_y = 14$, $2b_x = 2b_y = 18$ and
 $(2N)^2 = 256^2$.

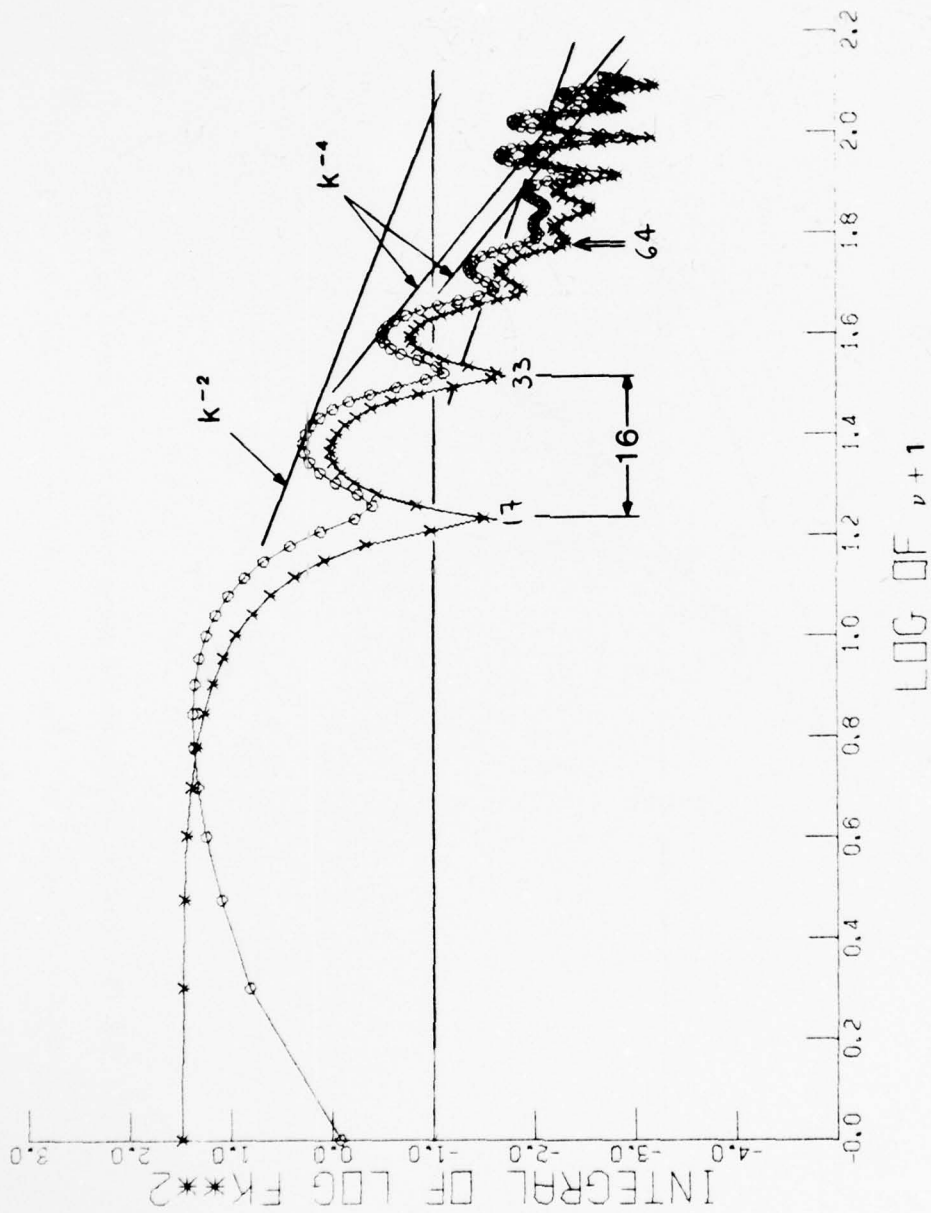


Fig. 13 — Discrete one-dimensional energy spectra ($X = Y$ and O) for the tra-
pezoidal figure of Fig. 7c. $A = 1$, $2a_1 = 2a_2 = 12$, $2b_1 = 2b_2 = 20$, and
 $(2N)^2 = 256^2$.

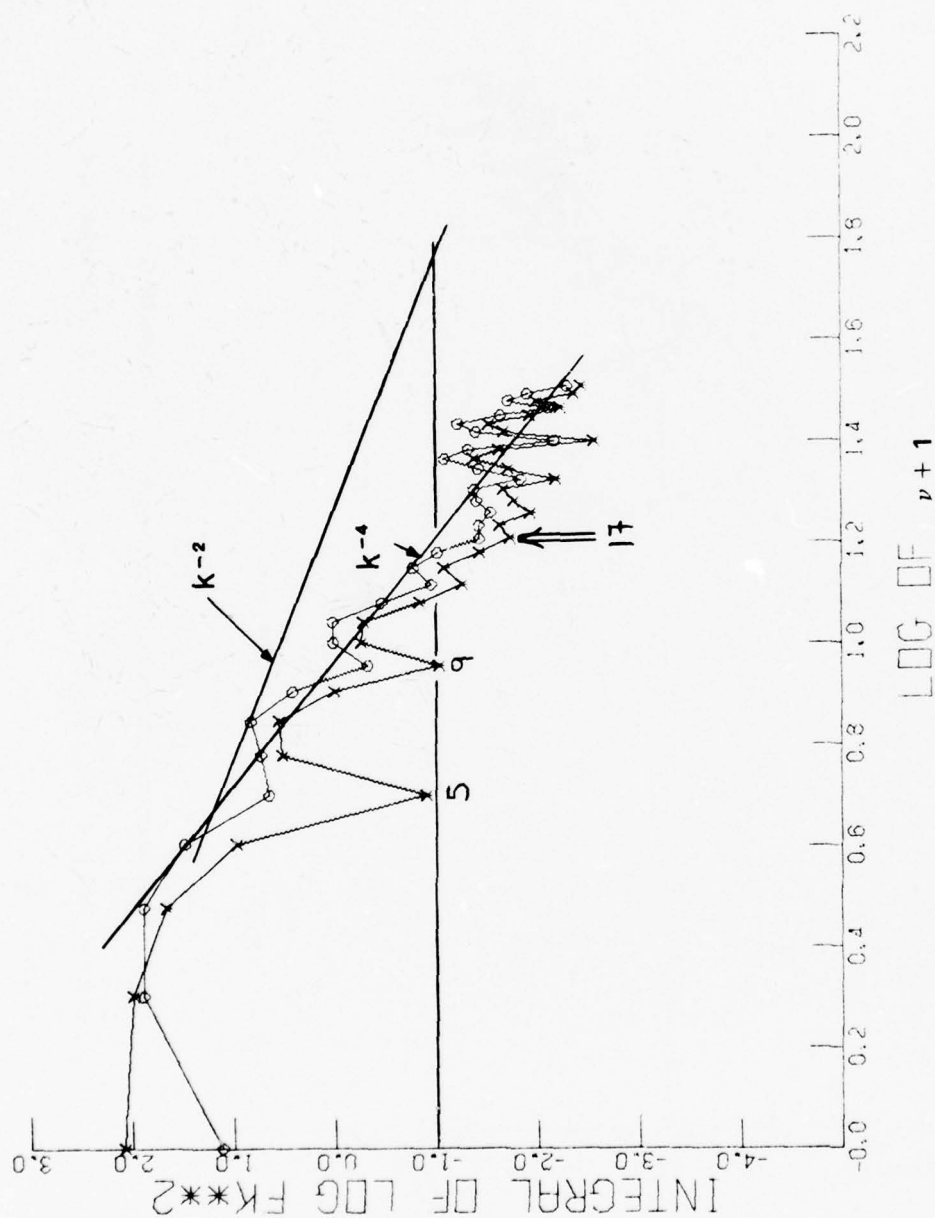


Fig. 14 — Discrete one-dimensional energy spectra ($X = Y$ and O) for the tra-
pezoidal figure of Fig. 7c. $A = 1$, $2a_x = 2a_y = 12$,
 $2b_x = 2b_y = 20$, and $(2N)^2 = 64^2$.

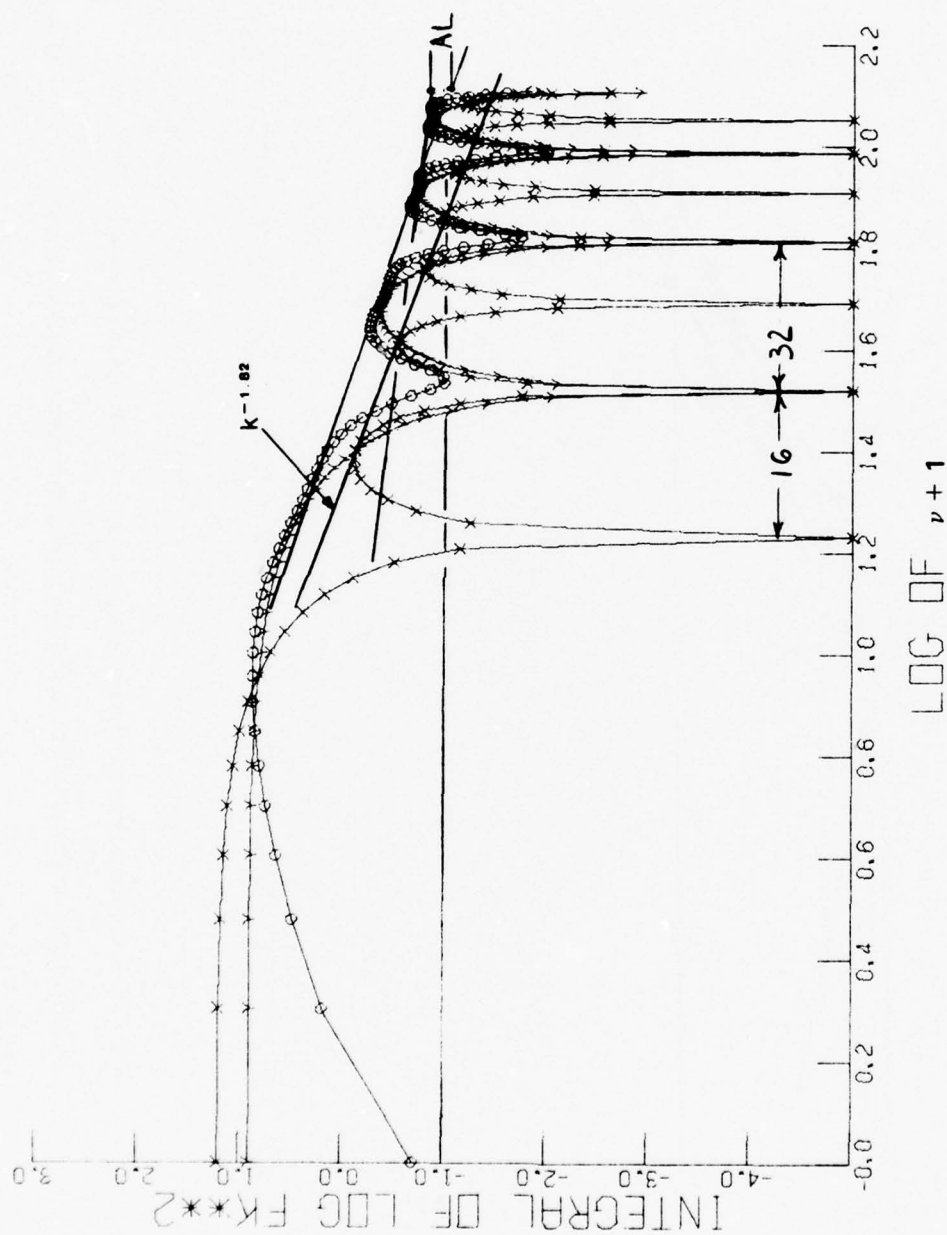


Fig. 15 — Discrete one-dimensional energy spectra. ($X = \gamma$ and O) for a rectangular figure of Fig. 7c. $A = 1$; $2a_x = 2b_x = 15$; $2a_y = 2b_y = 7$ and $(2N)^2 = 256^2$.

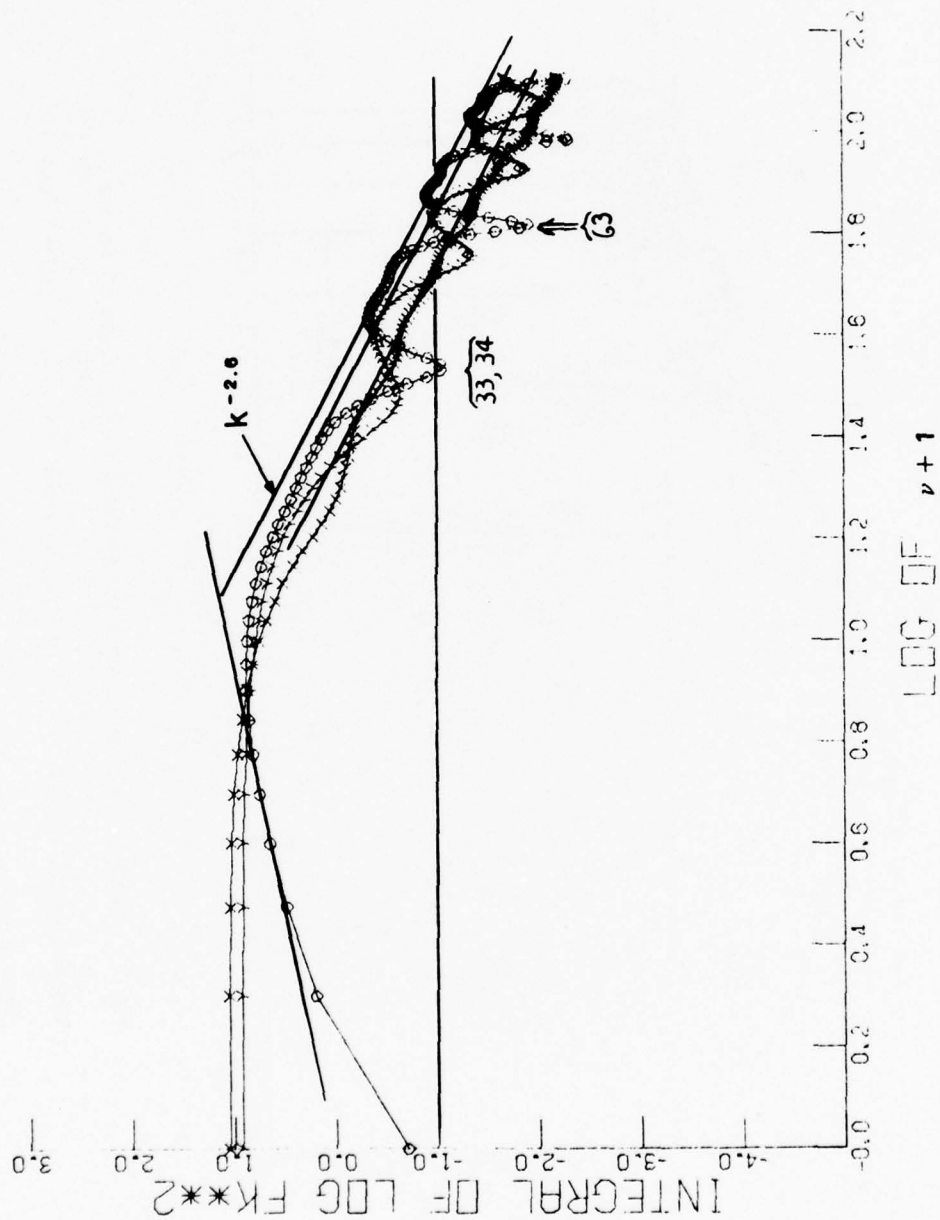


Fig. 16 — Discrete one-dimensional energy spectra (X, Y and O) for a rectangular figure rotated at 30° to the x-axis. Same dimensions as Fig. 15 and $(2N)^2 = 256^2$.

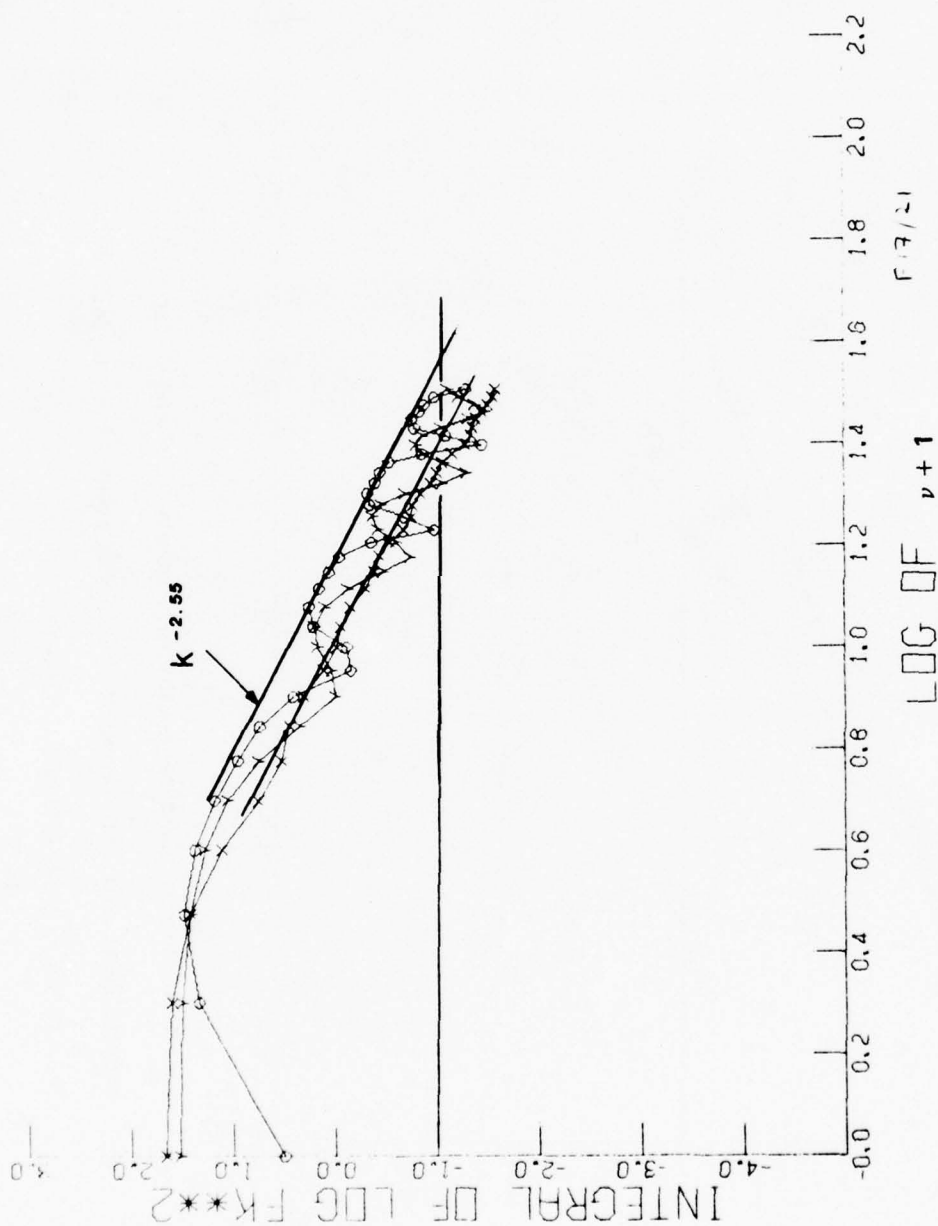


Fig. 17 — Discrete one-dimensional energy spectra (X, Y and O) for a rectangular figure rotated at 30° to the x-axis. $2a_x = 2b_x = 15$, $2a_y = 2b_y = 7$ and $(2N)^2 = 64^2$.

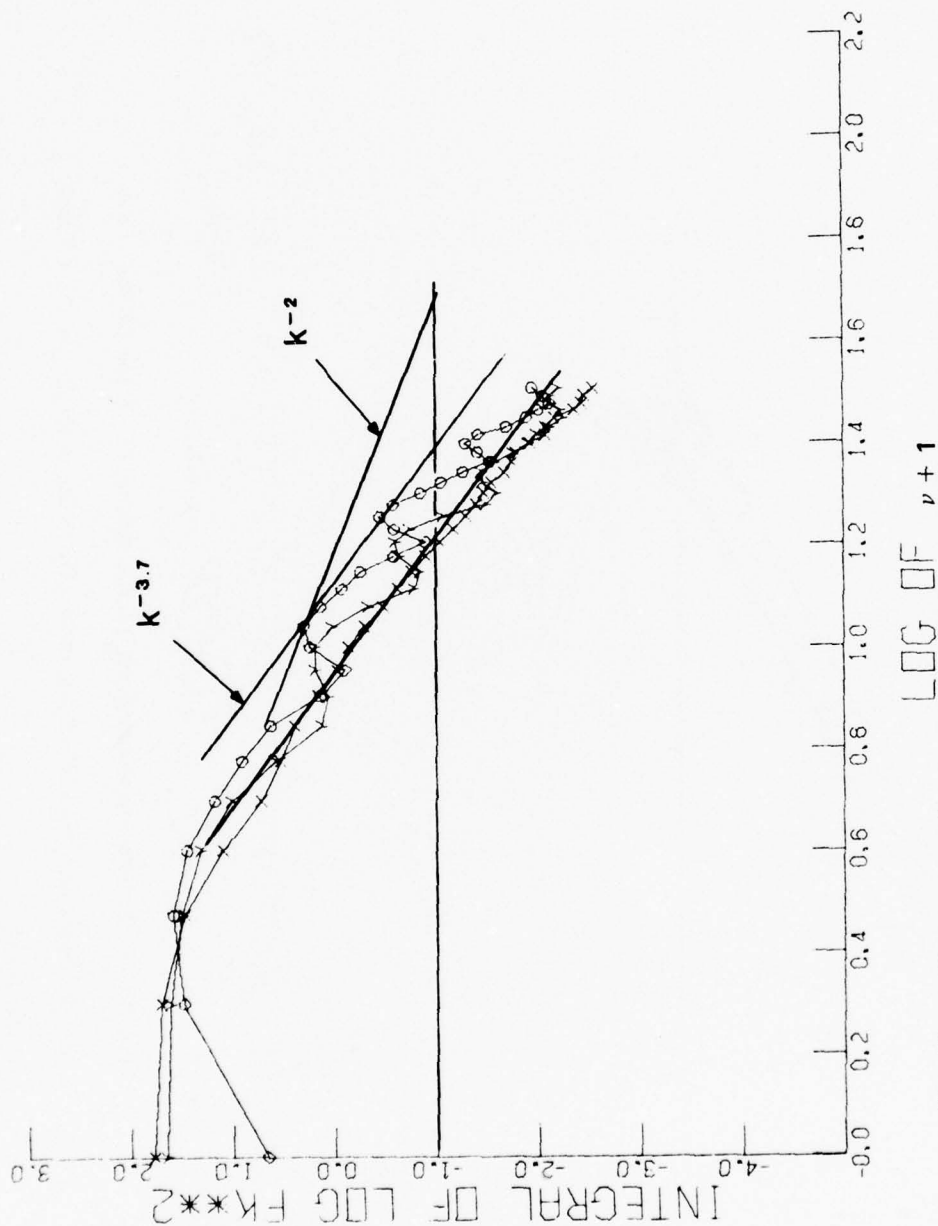


Fig. 18 — Discrete one-dimensional energy spectra (X , Y and O) for a triangular figure (Fig. 7c) rotated at 30° to the x -axis. $A = 1$, $2a_1 = 15$, $2b_1 = 19$, $2a_2 = 7$, $2b_2 = 11$ and $(2N)^2 = 64^2$.

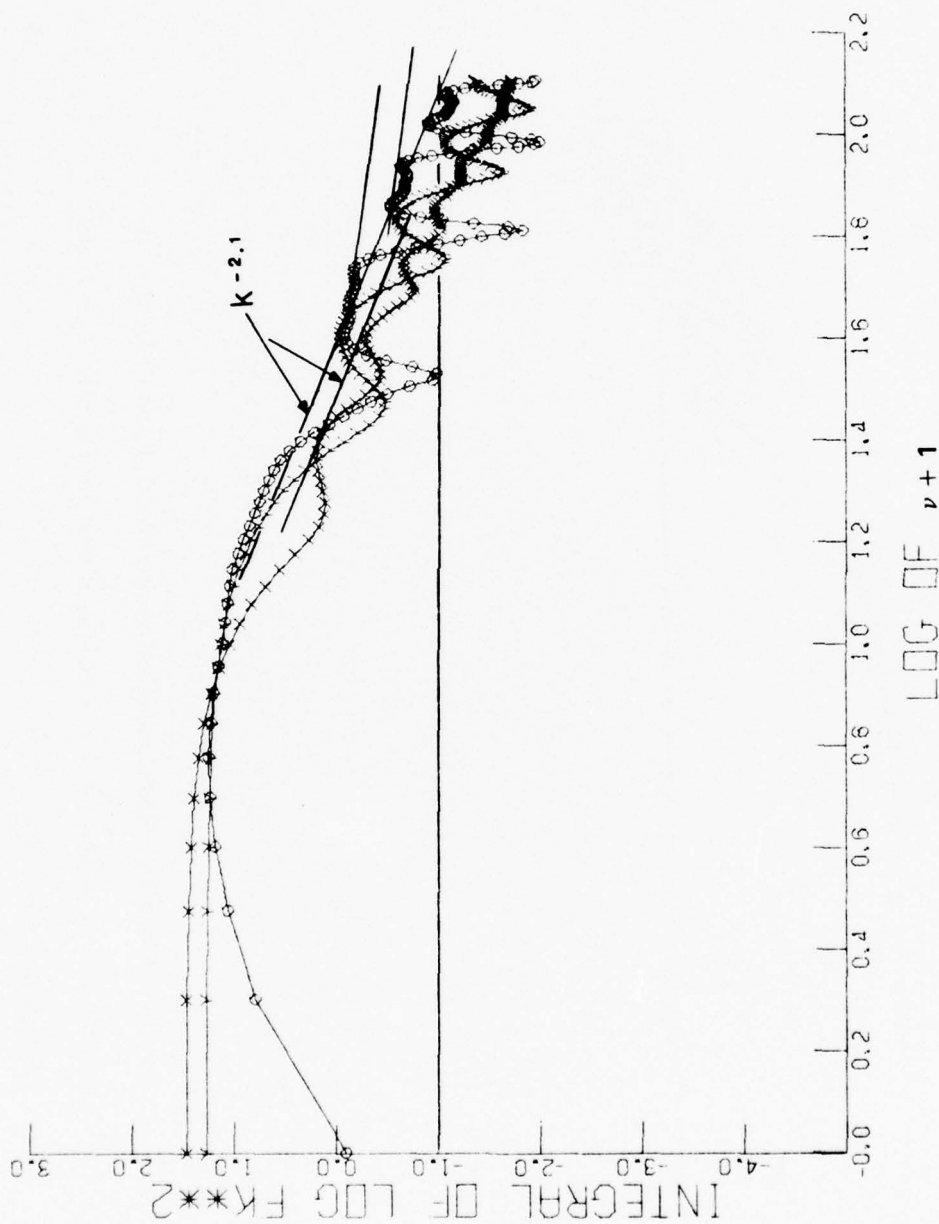


Fig. 19 — Discrete one-dimensional energy spectra (X , Y and O) for two rectangular figures at 30° to the x -axis as shown in Fig. 7d. $L_y = 0$ and $(2N)^2 = 256^2$.

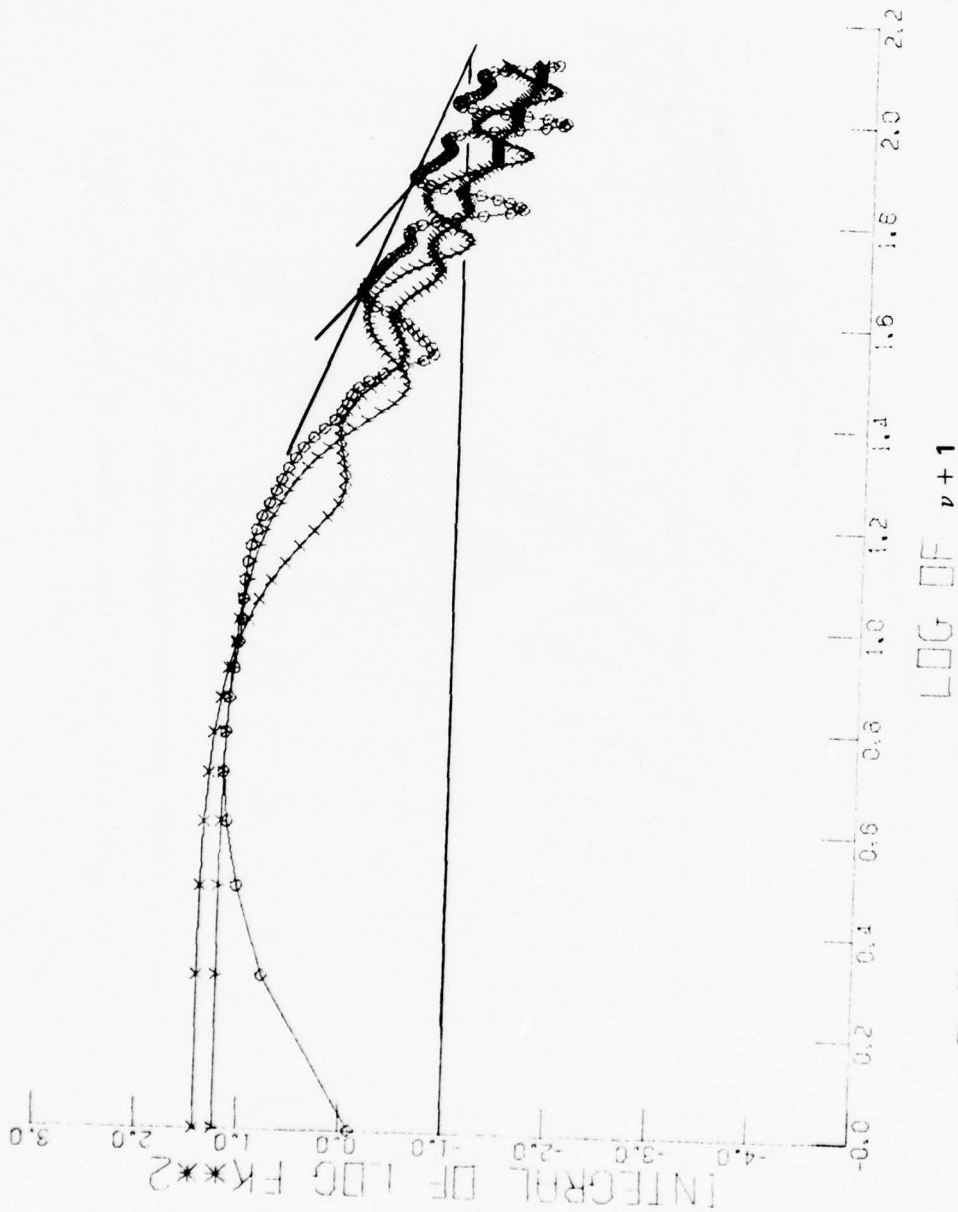


Fig. 20 — Discrete one-dimensional energy spectra (X , Y and O) for two rectangular figures at 30° to the x -axis as shown in Fig. 7d. $L_y = 2$ and $(2N)^2 = 256^2$.

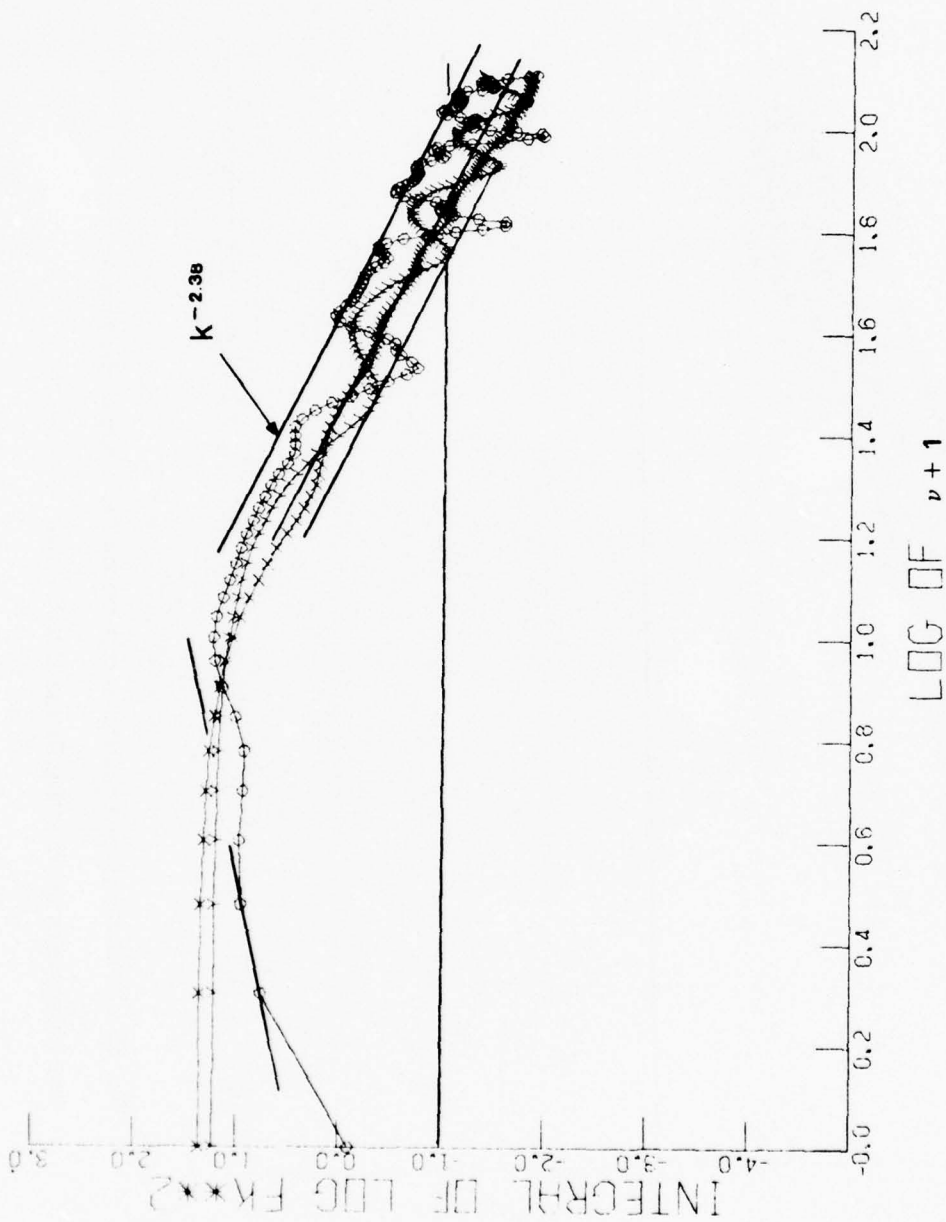


Fig. 21 — Discrete one-dimensional energy spectra (X , Y and O) for two rectangular figures at 30° to the x -axis as shown in Fig. 7d. $L_y = 16$ and $(2N)^2 = 256^2$.

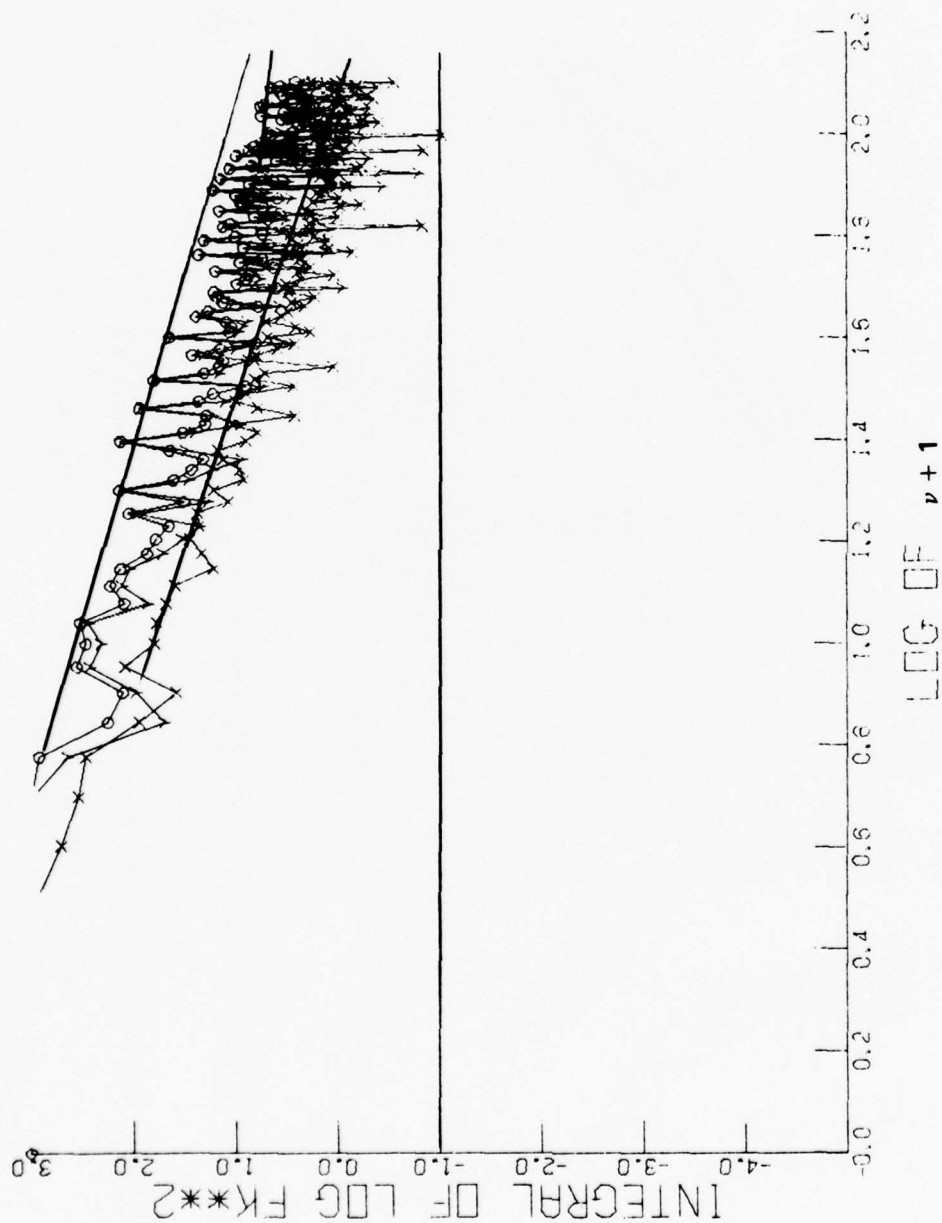


Fig. 22 — Discrete one-dimensional energy spectra (X, Y and O) for four rectangular figures as shown in Fig. 7e and $(2N)^2 = 256^2$.

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