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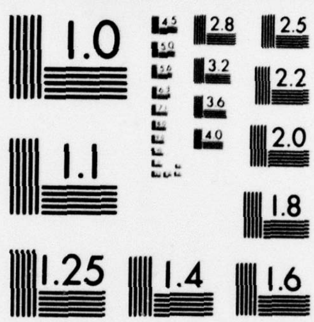

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Introduction

Our research has been focused on three goals. The first goal was to find ways of describing various thinnings of point processes that arise for example in the debugging of computer programs. Our work on this lead us to studies of extreme values of stochastic processes and the convergence of intergrals. Our results in these areas are described in Sections I-III.

The second goal was to determine optimal policies for dynamically controlling certain queueing processes. In order to do this, we had to answer some basic questions in dynamic programming concerning monotone optimal policies and the equivalence of continuous and discrete time models. We discuss this in Section IV.

Our third goal, as we describe in Section V, was to continue developing the theory and applications of semi-stationary processes. The emphasis here was on semi-stationary clearing processes which are useful for modeling input-output systems such as batch service queues and inventory systems.

I. Thinnings of Point Processes and Random Measures

Most of our research during the last 4 years has been on thinnings of random measures. A random measure refers to randomly located masses or points in a region, and thinning of it means that some of this mass is randomly deleted. A point process is a special random measure having only unit masses. Such thinnings arise in debugging computer software packages, filtering particles from a solution, deleting bad data, and the wearing of physical surfaces such as ball bearings or tires. Over the last two decades about twenty papers have been written on thinnings of point processes. These studies focused on the "locations of points" of the processes and consequently they appear to be very different mathematically. We took a different approach in [1] and described thinnings by looking at the number of points deposited and thinned in various "regions". In doing so, we found that essentially all of the thinnings studied to date were compositions of random measures. This idea enabled us to unify and significantly extend this newly developing theory of thinning.

Our results in [1] are basically convergence theorems for various compositions of measures. These answer questions such as: (1) What is the end result when one applies a particular thinning to a random measure?

(2) When is the end result a Poisson, Compound Poisson or infinitely divisible random measure and what are their defining parameters? (3) How does the randomness of the initial masses interact with the randomness of the thinning operation? (4) What does it mean to thin in a continuous manner such as the wearing or erosion of physical objects or land?

In analyzing thinnings we found that inverses of random measures played a key role. We proved some basic properties of such inverses which are also useful in other contexts.

Our second thinning paper [2] is a study of interchangeable thinnings. This means that the probabilistic nature of the thinning is invariant under rearrangement of the masses. This is more natural that independent identically distributed thinnings for many applications. We study both discrete and continuous thinnings. Here we view thinnings as special cases of compounding where mass can be increased or decreased. Several results that unify and extend earlier results by Renyi and Kallenberg on independent thinnings are proved. This paper contains several results we documented earlier on thinnings of point processes over time. In these contexts thinning is done over a long time period and the resulting point processes is analyzed by looking at the lifetime of the points under the thinning.

If the lifetime distributions have certain types of tails, then the thinned process after a long time converges to a Poisson or infinitely divisible point process or such a process with randomized parameters. One may also be interested in the residual lives of the points after a long period. This tells how much more thinning time is needed to completely thin the points. We also show how to analyze such residual random measures.

The last part of [2] is a study of sequential thinnings of points on the real line. Here a single thinning operation over the whole line deletes points by a rule such that the number of points retained between deleted points are independent and identically distributed. A sequence of these thinning operations are performed and the thinned process tends to be a renewal process whose inter-point distribution is a certain limiting distribution that arises in the theory of branching processes. Mathematically this analysis is a study of the convergence of a large number of compositions of random measures as in [1]. This is of interest by itself: it may be of use in other contexts.

Just recently we started analyzing thinnings of cluster point processes. Namely cluster centers are randomly placed in a region and each center gives rise to a point process. The total system of points is a cluster point process. A typical thinning may delete a cluster center, and hence its related point process. Also points may be deleted individually. It appears that our various approaches for analyzing thinnings can handle these more complex systems. We will document this next year.

II. Extreme Values of Stochastic Processes

Some problems concerning extreme values of stochastic processes are actually thinning problems. For example, consider a discrete time stochastic process and let $N_n(t)$ be the number of times that this process exceeds the value n in time t. As n increases the process $N_n(t)$ becomes thinner (contains fewer points). Similarly let $M_n(t)$ be the total amount of these exceedances above the level n. This $M_n(t)$ is a random measure which also becomes thinner as n increases.

We study the thinning of such exceedance measures in [3] when the basic stochastic process is a recurrent Markov chain, a regenerative process or a semi-stationary process. We present necessary and sufficient conditions for the convergence of random measures related to exceedances.

In particular we show that high level exceedances of certain random walks and queueing processes are compound Poisson processes. This extends some earlier work by Lindvall. The results here are also useful for describing thinnings of point processes over time where the initial points or the thinning mechanism has certain Markovian or regenerative dependencies.

One problem in thinning over time is to find out how long it may take to eliminate all, or maybe 90 percent, of the points. This can be described by the convergence of the maximum or k-th maximum of a large number of random variables. Namely, the variables are the lifetimes of the points under the thinning operation. During the last year we have obtained several results on extremes of independent and dependent variables. One significant result is a necessary and sufficient condition for extreme values of independent infinitesimal processes to converge. This is analogous to the general functional central limit theorem for sums of independent infinitesimal variables. We are still studying the many ramifications of this result and its extension to multidimensional settings where there are many applications. We hope to document our results in this area next year.

III. Convergence of Integrals

Our study of thinnings lead us to study the convergence in distribution of random integrals of the form

$$\xi_n Y_n = \int_X Y_n(t)d\xi_n(t),$$

where ξ_n is a random measure on a space X and Y_n is a random function, and the (ξ_n, Y_n) has certain asymptotic properties. The ξ_n in our contexts was usually an infinite random measure (e.g., a Poisson process), and so these integrals could not be analyzed by the standard theory for finite measures.

In [4] we study the convergence of nonrandom and random integrals. The basic results are necessary and sufficient conditions for the convergence of nonrandom intergral $\mu_n f_n = \int f_n(x)d\mu_n(x)$ where μ_n and f_n converge in some sense. These are analogous to the classical result that if nonnegative f_n converge to f in μ-measure then $\int f_n d\mu \to \int f d\mu$ if and only if the f_n are uniformly μ-integrable. We present similar results for the convergence of measures ν_n defined by

$$\nu_n(A\times B) = \int_A k_n(x,B)d\mu_n(x) \quad \text{for } A\times B \subset X\times Y$$

where μ_n is a measure on X and k_n is a kernel from X to Y. We then show how these results apply to random integrals and measures. A major application pertains to the convergence of mixtures of probabilities that is a key to describing extremes of exchangeable random variables.

IV. Optimal Control of Random Walks, Birth and Death Processes and Queues

The first result in queueing control appeared a decade ago. Since then there have been many ad hoc studies of queueing control problems, but a nice unified theory has not yet been developed. Two reasons for this are (1) the state space of these models is often infinite, and (2) a slight change in the cost function of a model may result in a drastic change in its mathematical analysis. One of our research goals was to solve some basic queueing control problems and to unify part of the theory dealing with monotone optimal policies.

The major problem we address in [7] is a follows. Consider a single sewer queueing system with Poisson arrivals and exponential service times (an M/M/1 queue) whose arrival and service rates are controllable. At each customer arrival or departure the number of customers in the system is observed. Based on this number an arrival and service rate are selected from a prescribed set. Costs are charged for using these rates and for holding customers in the system. The aim is to find a policy for successively selecting the arrival and service rates so as to minimize the expected discounted or average cost of the system.

Under some natural conditions on the costs there should be an optimal policy which increases the service rate and decreases the arrival rate as the number of waiting customers increases. The following questions arise:

(1) What are natural conditions on the costs that lead to such monotone optimal policies?

(2) How can one prove the existence of a monotone optimal policy?

(3) Knowing that a monotone optimal policy exists, how can it be computed especially for an infinite or large state space?

We answer these questions in [7].

A major theme in queueing and inventory control problems is to establish the existence of monotone optimal policies. This reduces the search for an optimal policy to the subclass of monotone policies, and the computation of these policies becomes easier. In [5] we present general conditions under which Markov decision processes have monotone optimal policies. We illustrate this for a random walk problem and for a multimachine replacement problem. Some of the techniques in [7] are also extended and used in our queueing analysis in [5]. We found that essentially all of the queueing and inventory control models that have been studied are controlled random walks or Markov chains with monotone matrices. That is why they have monotone optimal policies. These are a very small class of decision processes. It seems like other subclasses of decision processes should also have monotone optimal policies, but we have not been able to find conditions to identify such subclasses.

A key step in our queueing control paper [5] was to show that a continuous time queueing process is equivalent to a discrete time random walk. This enabled us to analyze an ostensibly more complex continuous time process in terms of a simpler discrete time process. Such an equivalence also holds for general continuous and discrete time processes as we discuss in [6]. This study clarifies some earlier work on the equivalence.

V. Semi-Stationary Clearing Processes

Many input-output systems such as batch services queues, dams, inventories, and dispatching of men and equipment can be viewed as a clearing process. Such a process describes the net quantity in a system which receives an exogeneous random input over time and has an output mechanism that intermittently clears random quantities from the system. A semistationary clearing process is strictly stationary over its clearing epochs.

We study these processes in [8]. We describe their asymptotic distribution and show how it arises in the limit of certain functionals. An asymptotic distribution is different from a limiting distribution, but it has some similiar properties. We identify clearing processes whose asymptotic distribution is uniform. One example is a modulo c clearing process in which an amount c is removed whenever that amount is exceeded. For this example the Palm probability of the process must be used rather than the usual probability to obtain a uniform distribution. This subtle use of the Palm probability gives a partial answer to an anomaly in a classical economic lot size inventory model. We also present several limit theorems for clearing processes including a functional central limit law and law of the iterated algorithm.

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The research has been focused on three goals. The first goal was to find ways of describing various thinnings of point processes that arise for example in the debugging of computer programs. Work on this lead to studies of extreme values of stochastic processes and the convergence of intergrals.

The second goal was to determine optimal policies for dynamically controlling certain quequeing processes. In order to do this, some basic questions had to be answered in dynamic programming concerning monotone optimal policies.

20. Abstract continued.

and the equivalence of continuous and discrete time models.

The third goal, was to continue developing the theory applications of semi-stationary processes. The emphasis here was on semi-stationary clearing processes which are useful for modeling input-output systems such as batch service queues and inventory systems.