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SATELLITE NETWORKS FOR MULTIPLE COVERAGE. (U)

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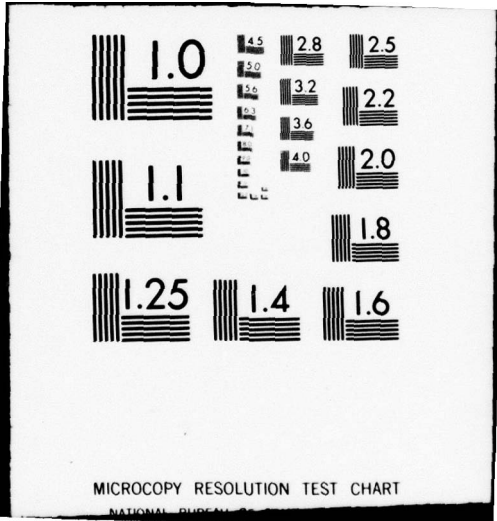
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**SATELLITE NETWORKS FOR MULTIPLE COVERAGE**

by

**MELCHOR LOUIS SUAREZ**

**B. E. M. E., VILLANOVA UNIVERSITY  
(1974)**

**SUBMITTED IN PARTIAL FULFILLMENT OF THE  
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**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**

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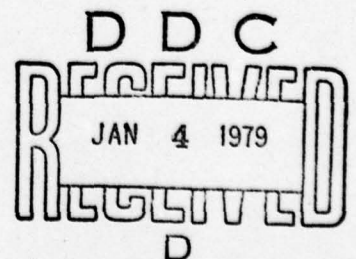
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Chairman, Departmental  
Graduate Committee



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# SATELLITE NETWORKS FOR MULTIPLE COVERAGE

by

MELCHOR LOUIS SUAREZ

Submitted to the Department of  
Aeronautics and Astronautics on  
May 25, 1977 in partial fulfillment  
of the requirements for the degree  
of Master of Science

## ABSTRACT

A constellation of satellites that gives multiple continuous coverage of the whole earth is studied here. A computer program is described which helps to find the optimal configuration. In particular, performance of a twenty-four satellite constellation is analyzed for triple coverage and the results given. Constellations where the number of orbital planes equal the number of satellites ( $N = P$ ) and constellations where  $N \neq P$  are both analyzed. Circular orbits are assumed. The computer program is relatively short using a sampling grid over a quarter of the globe. The results are approximately correct sufficient for a feasibility study.

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My sincerest thanks go also to Dr. Richard Battin for guidance in the final stages of the thesis which, as usual, brought the most toil.

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My gratitude must go also to Frank Barnes and Tom Thorvaldsen for their technical insights which were helpful and extremely timely.

J. G. Walker's work was the most useful reference for the pages that follow, so my thanks extend across the Atlantic for Farnborough, England as well as to Dr. Paul Cefola who dropped the Walker reports on my desk.

Lastly, for her very efficient and professional typing, a double thanks to Terry Suntheimer.

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## Introduction

In order to update a vehicle's state vector, single, double, triple or quadruple coverage may be desired from a constellation of artificial satellites. One important consideration to the constellation designers is the optimal configuration in terms of pure geometrical distribution. Thus, what is the best coverage  $N$  satellites can produce and what are the Keplerian elements of each satellite at some point in time?

In the literature one can find the optimal coverage for constellations of up to 15 satellites<sup>1</sup>. There is presently a need for better coverage than is afforded by these. The earth central angle between user and satellite is too large for many applications.

J. G. Walker has laid down much of the ground work for analysis of such constellations. What is needed, however, is a method, employing a computer program, to simplify and make possible analysis of systems where  $N$  is considerably greater than 15.

Constellations where each satellite is placed in a single orbital plane ( $N=P$ ) and those where equal numbers of satellites are contained in fewer planes ( $N > P$ ) are both considered here. For the  $N=P$  case, type I, the inclination is assumed the same for all orbits, the spacing in

longitude of ascending node (LAN) is made equal to  $360^\circ/N$  and the phase angle between satellites is given by  $F \times 360/N$ .  $F$  is an integer whose optimal value, along with inclination, is to be found.

For the  $N > P$  case, type II, all possibilities where  $P$  is a factor of  $N$  are considered. The increment of LAN is then  $360/P$ , the phase angle is defined the same way as for type I constellations, and the inclinations for all  $P$  planes is the same.

The 'Grader' and 'Grader 1' programs analyze these two types of constellations respectively. They take a grid of samples over half of the northern hemisphere to give an approximation of the performance of the different possible constellations. It is meant to be an aid to the systems engineer who wants to get an idea of what the constellation will look like. One thus arrives at the most probable optimal configuration, the number of orbital plane ( $P$ ), the phase angle number ( $F$ ), and the largest earth central angle ( $R_{\max}$ ) between the user and the satellite which must be tolerated to get the coverage specified.

## Section I

### A Look at Whole Earth Continuous Coverage

In 'whole earth continuous coverage' one is striving to have at any point in time, anywhere on the globe, a prescribed number of satellites 'visible'. The 'prescribed number' depends of course on the user's intent. If one desires single coverage then at least one satellite should be visible, and so on. Visibility requirements can usually be very neatly defined by a maximum allowable earth central angle between the user and the satellite. In the example chosen in this paper, we are looking for triple coverage with a maximum earth central angle of approximately 45 degrees.

Note that although the coverage requirement may be fulfilled, the type of coverage the satellites afford could be poor for navigation. A measure of the navigability of a given constellation is not included here but is being studied at this time<sup>(1)</sup>.

Results of the study are shown in Table 1. The N/P/F notation denotes the number of satellites (N), the number of orbital planes (P), and the phase angle number (F).  $R_{\max}$  represents the largest angular distance to acquire three satellites. This number should be as low as possible. The inclination ( $\delta$ ) is to a reference plane which can be, but does not have to be, the equatorial plane.  $D_{\min}$  represents

Table 1

## Summary of Triple Coverage Analysis

Constellation Code N/P/F	Minimum $R_{\max}$ (deg)	At Inclination $\delta$ (deg)	Minimum Distance Between Any Two Satellites ( $D_{\min}$ ) (deg)
24/24/14	51	53.0	17.1
24/3/1	50	53.3	12.2

the minimum angular distance between two satellites encountered with this scheme. Constellations where  $D_{\min}$  was equal to zero were discarded due to the relatively high probability of collision for circular orbits of equal altitudes. Poor navigation also results at those instances and places where this occurs. Walker also takes this approach. The assumption is made that the proper station keeping is available to keep the relative geometry constant. Similar perturbations to all the orbits may be ignored such as those shifting the longitudes of ascending nodes.

## Section II

### Finding the Optimal Configuration

Finding the optimal configuration can be long and tedious if attempted by purely deterministic approaches, especially when the number of satellites is large. There are too many variables involved as can be supported by anyone who has attempted it<sup>(2)</sup>. In order to speed up the process, a program called 'GRADER' was introduced (see Section IV and Appendix A for more details).

The program narrows the field of contenders to one or two configurations for an N-satellite constellation. It helps to pinpoint the number of orbital planes, F number, and inclination of the orbits. This is accomplished by an iterative method on the part of the user. It is not intended to give an extremely accurate optimal inclination or  $R_{\max}$ .  $D_{\min}$ , the minimum distance between two satellites, must also be found by other means. An algorithm which can be used in finding the actual value of the optimal inclination and the corresponding  $R_{\max}$  are outlined in Section III.

The way the figure of merit is given to a configuration by the Grader program will be discussed in detail in Section IV and Appendix A. Basically the program looks at a sufficiently large sampling area, checking for single, double or triple coverage at each sampling point. The number of failures in

coverage is divided by the number of samples taken to get failures per samples (FPS). To find, then, the best performance possible from a N-satellite constellation, the procedure shown below has been found helpful:

- a) One starts with  $N = P$  constellations
- b) then runs the program for the different possible F numbers at elapsed angle equal to 0. The threshold angle for visibility must be set so that the curves, as shown in Figures 1, 2, 3 to 7, do not bottom off. Inclinations can be set initially to run from 40 to 70 at increments of 5 degrees.
- c) The  $N > P$  constellations can then be checked by running the program for different values of P. At each P, the meaningful F numbers are checked also at elapsed angle set to zero and inclination varying from 40 to 70.
- d) The configurations with the lowest minima can then be checked at other elapsed angles until the inclination with the lowest maximum value is clearly defined.

For  $N = 24$  with triple coverage, the configuration which behaves the best seems to be 24/3/1. This is found from figures 1 to 7 which show that although a few configurations have very low FPS values at E.A. equal to zero, e.g. 24/6/5, 24/12/1, 24/24/8, 24/24/14, 24/24/16, at other E.A. values they have higher minimums than 24/3/1.

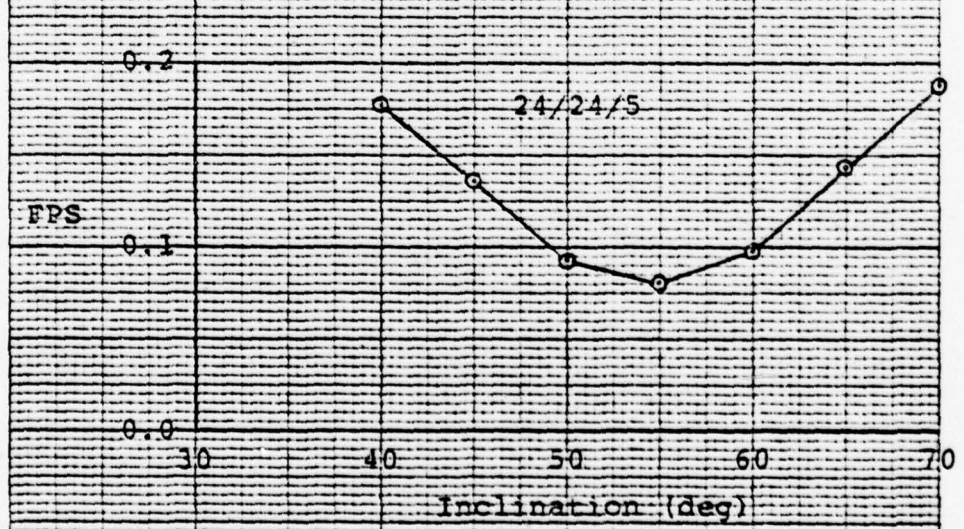
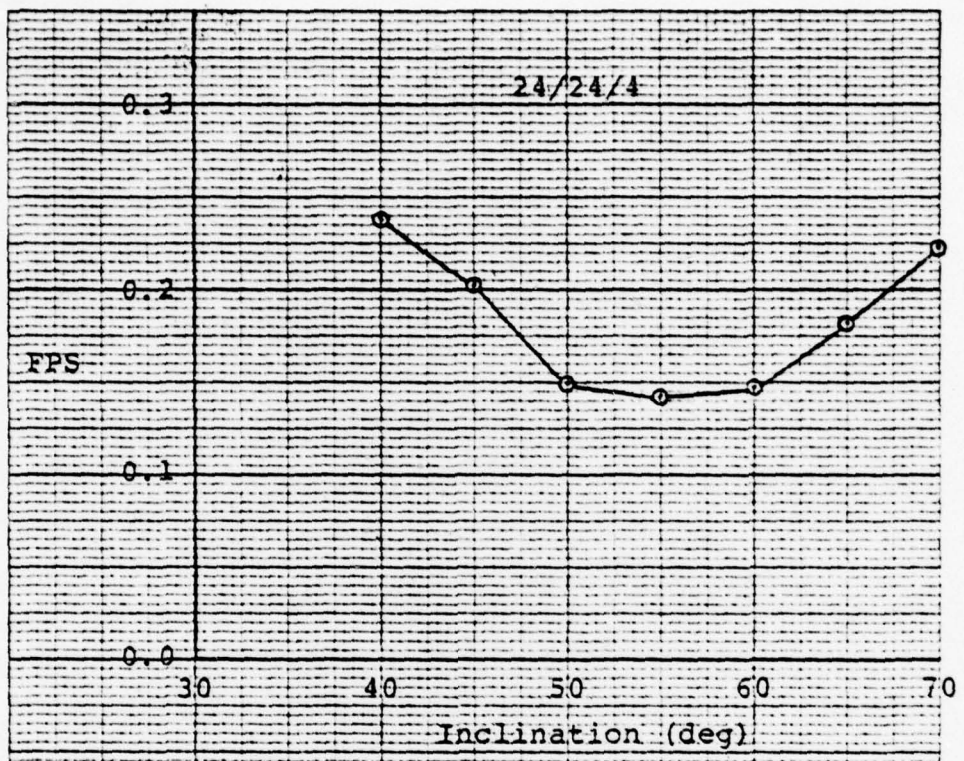


Fig.1 - Failures Per Sample vs. Inclination for different phase angle numbers at Elapsed Angle of zero. The threshold angle (alpha) is set to 45°.

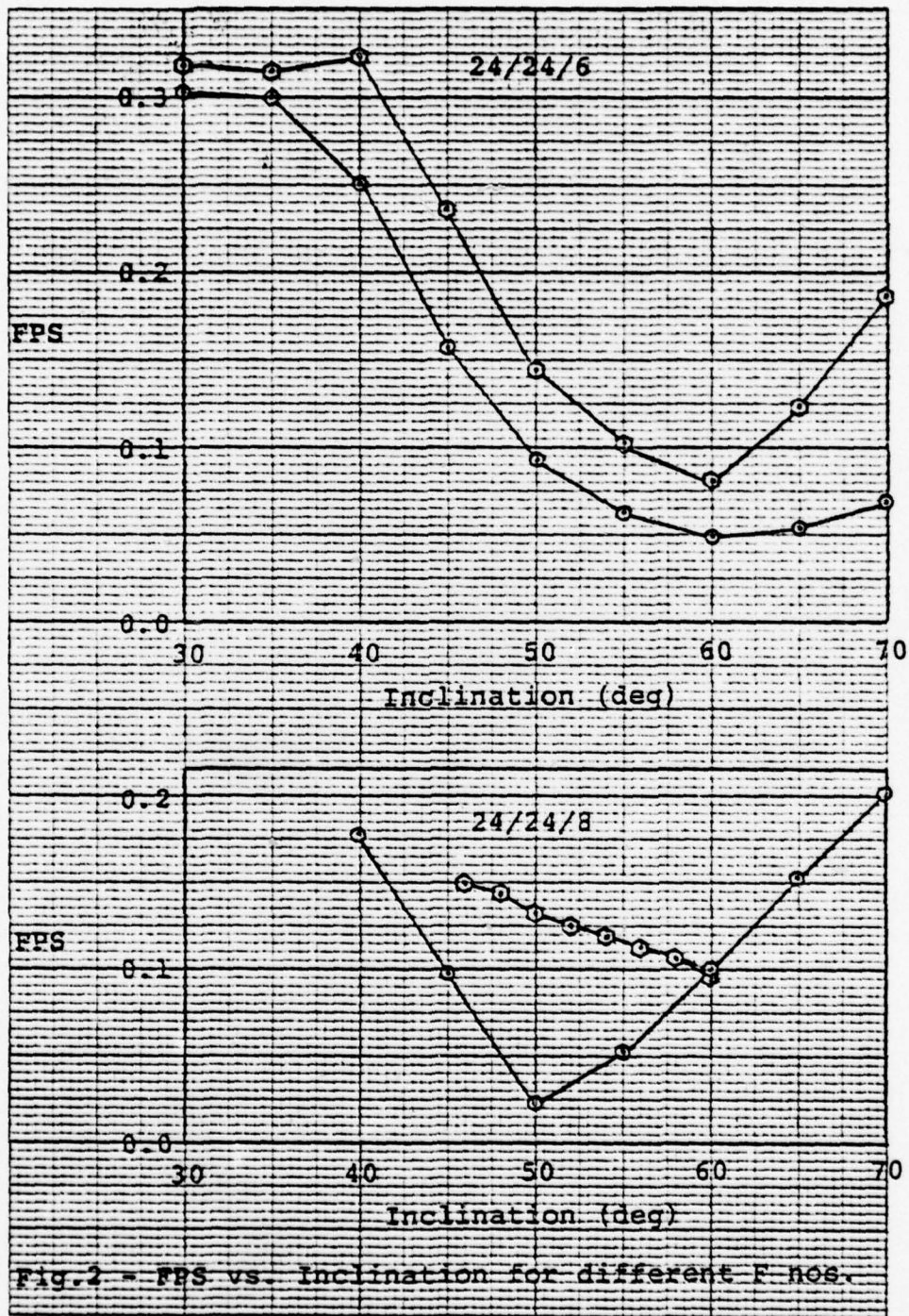


Fig.2 - FPS vs. Inclination for different F nos.

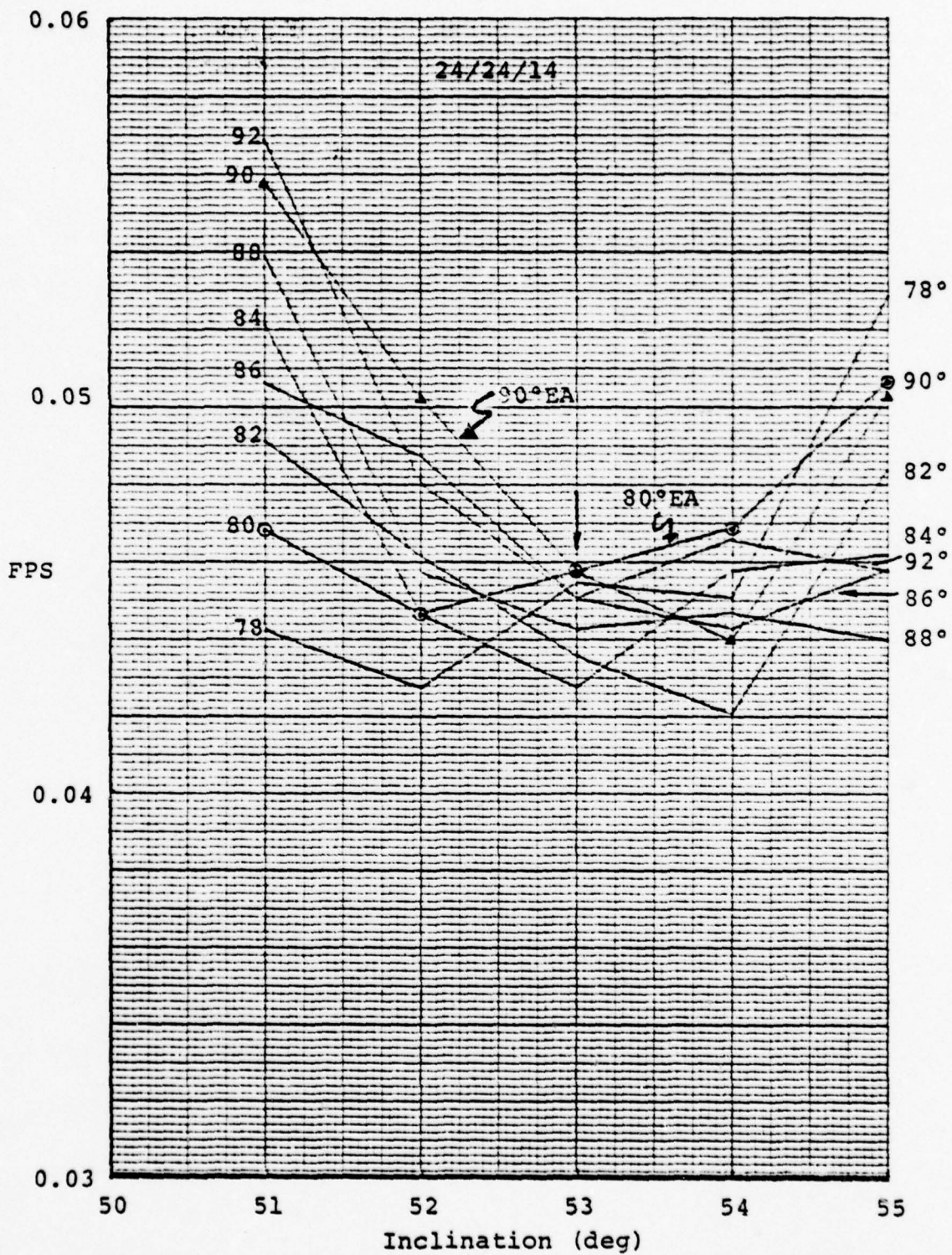


Figure 3 - FPS vs. inclination for constellation 24/24/14

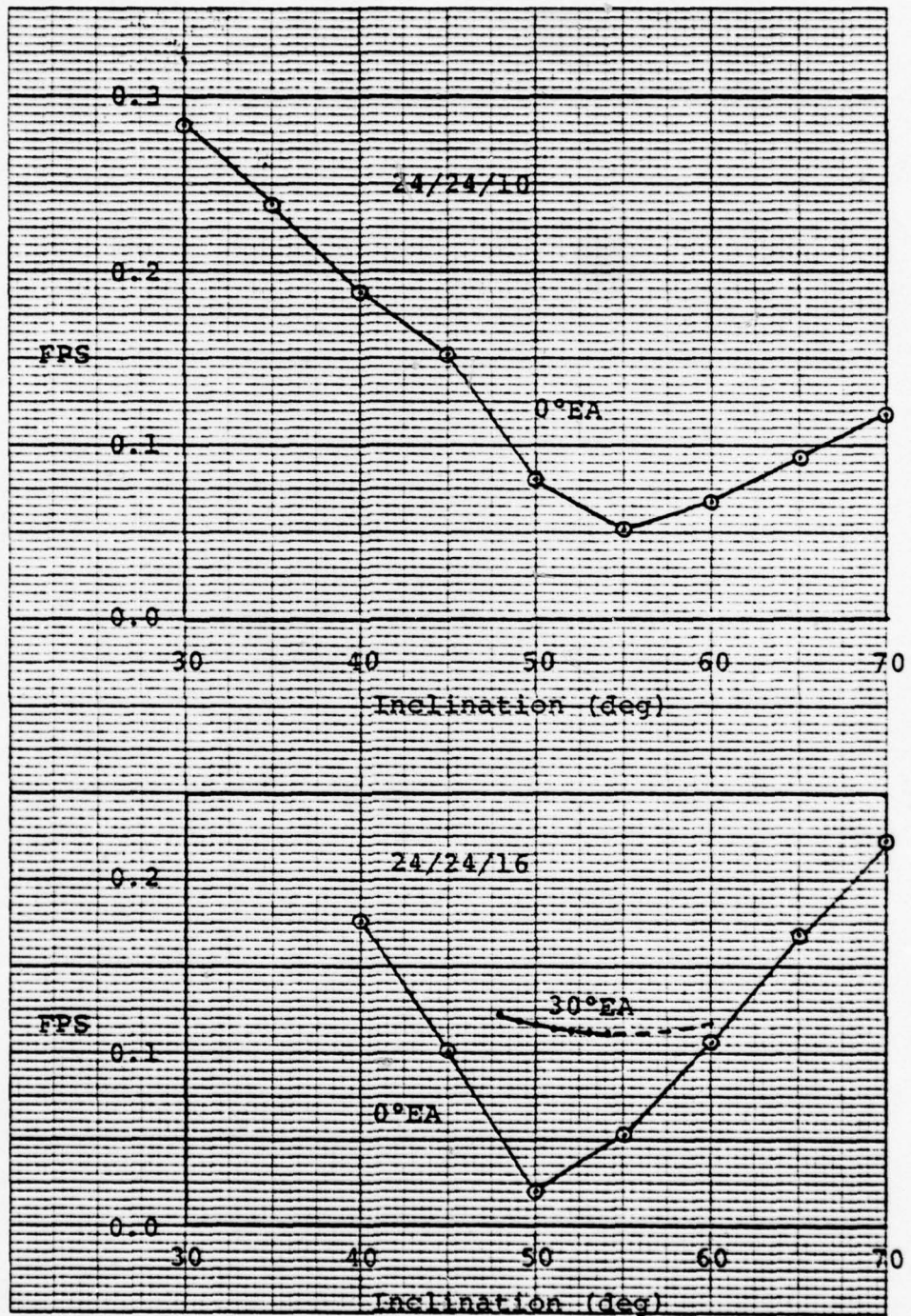


Fig.4 - FPS vs. Inclination for different F nos.

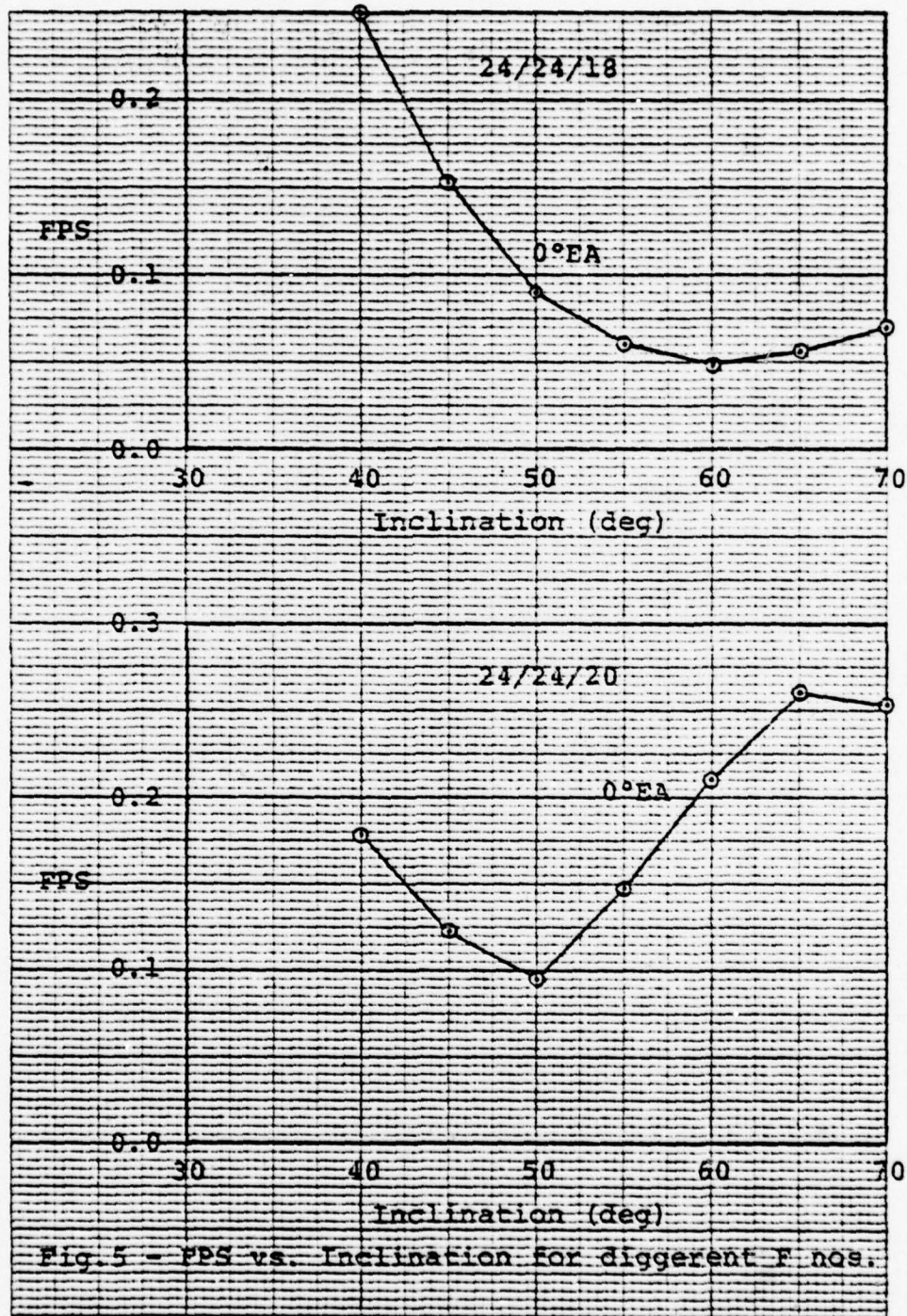
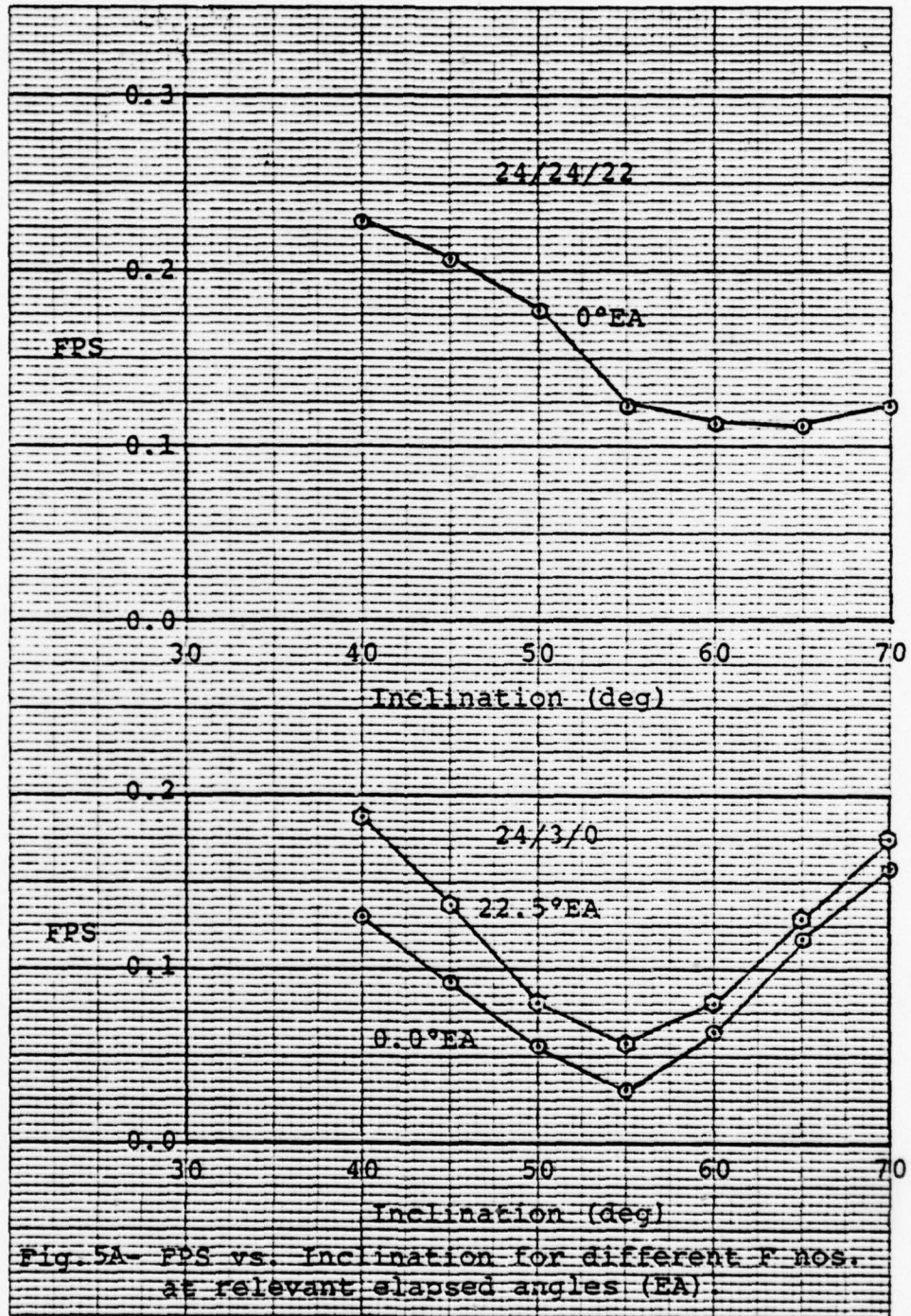


Fig. 5 - FPS vs. Inclination for diggerent F nos.



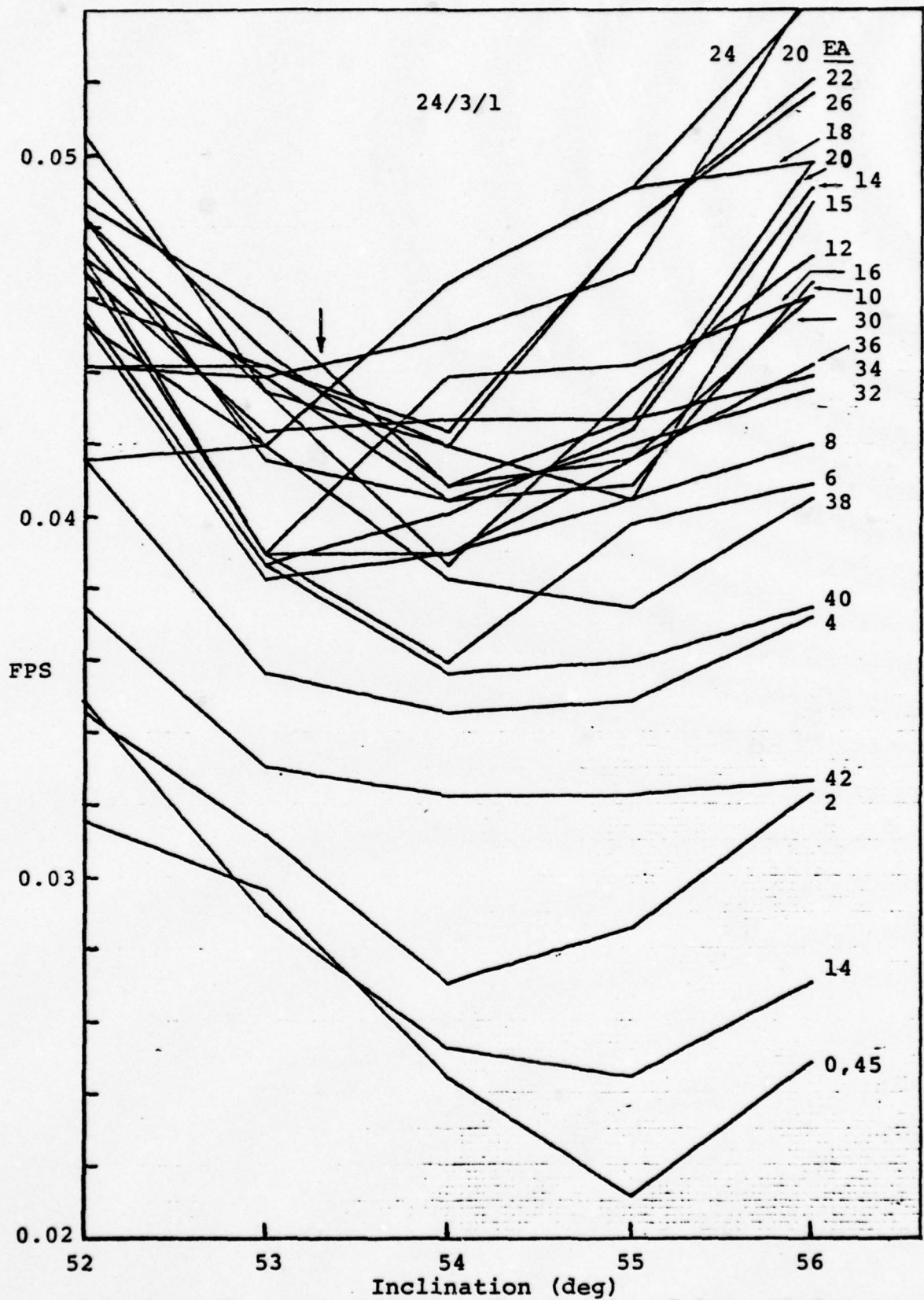


Fig.6 - FPS vs. Inclination for E.A.'s of 0 to 45 degrees.

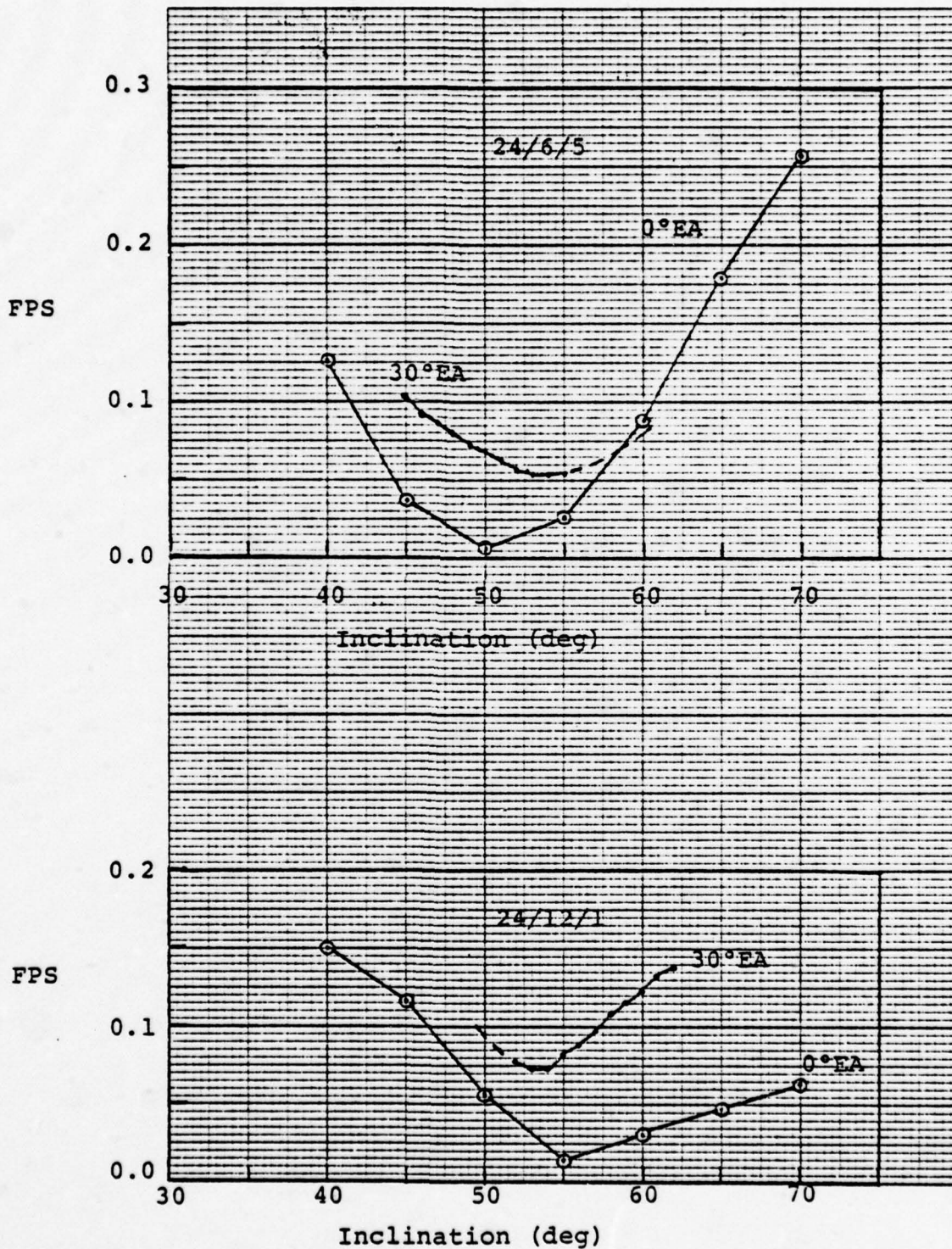


Figure 7 - FPS vs. Inclination.

Three points regarding time saving measures in the search can be made at this point. First, one does not need to vary the elapsed angle until all the F numbers have been analyzed at an EA equal to zero. Only those F values which look promising need to be investigated further. If a few values of F show promise, then the node angle through which the satellites must travel before the geometry repeats itself can be a good guide for the next move in the search. The smaller the repetition angle, the better. An idea of the repetition angle can be had by looking at just the nodes of each satellite and seeing when a satellite will move to the equatorial position, etc. Looking at 24/24/14, we see that with a phase angle of:

$$\begin{aligned} \Delta\omega &= F \times 360/N \\ &= 14 \times 360/24 \\ &= 210 \text{ deg,} \end{aligned}$$

the longitude of ascending nodes and node angles are:

Sat No.	$\Omega$	$\omega$	Sat No.	$\Omega$	$\omega$
0	0	0	6	90	180
1	15	210	7	105	30
2	30	60	8	120	240
3	45	270	9	135	90
4	60	120	10	150	300
5	75	330	11	165	150

Sat. No.	$\Omega$	$\omega$	Sat. No.	$\Omega$	$\omega$
12	180	0	18	270	180
13	195	210	19	285	30
14	210	60	20	300	240
15	225	270	21	315	90
16	240	120	22	330	300
17	255	330	23	345	150

Observing the node angles of satellites 5 and 17, at  $330^\circ$ , as well as 10 and 22 at  $300^\circ$ , and 7 and 19 at  $30^\circ$ , one can be fairly certain that the overall geometry will repeat itself every 30 degrees.

Second, as mentioned before, some phase angle numbers cause satellites to coincide periodically. The odd numbered F's in the 24/24/F constellations were found to have this characteristic. The F's versus inclination graph for 24/24/5 is shown in Figure 1. The indication is that of high failure rates as would be expected.

Thirdly, some F numbers are unlikely candidates for optimality judging by the inherently poor distribution which they offer. For example 24/24/0 means that all satellites will have the same node angle, thus they will all be at the equator initially and move upwards at the same rate. This is obviously poor for global coverage and need not be ana-

lyzed further. F numbers such as 1 and 23 can also be envisioned as rather poor but may still be checked quickly for completeness.

Up to this point only integer numbers have been used for F. Justification for this can be found in references 2 and 3. They have found that fractional values of F can be discarded. One may note that if a fractional value were used, a certain discontinuity would result in the distribution.

The different configurations were initially checked from 40 to 70 degrees in inclination at increments of 5 degrees. This, as opposed to finer plotting, gives a fairly good idea of their performance while saving much computer time. Inclinations below 40 degrees or above 70 were not checked due to the indications of the curves as well as references 3 and 4.

Figures 3 and 6 show the results found from constellations 24/24/14 and 24/3/1 which were the two most promising ones. Note that for 24/24/14, relative to the sampling grid, there is repetition after 360 degrees of elapsed angle at which time the satellite numbered 0 is back at the equator. Therefore, every 10 degrees of elapsed angle was checked up to 360 and increments of 2 degrees were used in the area of interest. Only elapsed angles of 78 to 92 are shown for clarity. All other elapsed angles produced curves with FPS values below those shown.

Constellation 24/3/1, which contains 8 satellites per orbital plane, was only checked at elapsed angles between 0 and 45 degrees since relative to the grid pattern this is the repetition angle. Note that this is confirmed by Figure 6 which shows that the curves for E.A. of 0 and 45 fall directly on top of each other. The fact that there are 8 satellites in each orbital plane at forty five degrees apart means that a satellite at the equator, for instance, will be replaced by one moving towards the same point in the equator after forty-five degrees.

The value of the largest angular distance to acquire three satellites ( $R_{\max}$ ) can be found in one of two ways. The first way is discussed in Section III and can be used when the number of satellites is relatively low (below 24 at least). It involves drawing the constellation being considered on a globe of some type and then finding the satellites surrounding the points of worst coverage. Using a closed form expression for  $R_{\max}$ , a search is then made for the best inclination and so forth. The method is tedious and long. It can be computerized, however the runs tend to be very long and expensive due to the number of satellite combinations which could be involved in producing the worst case. This has been done by Walker<sup>4</sup>. The program, written in FORTRAN, is approximately 700 lines long.

The second method, employed in this work, is to take the Grader programs and for the prime candidates, say con-

stellations 24/24/14 and 24/3/1, to raise the visibility threshold angle ( $\alpha$ ). Runs are then made to determine what value of  $\alpha$  gives no failures per sample (FPS = 0). This  $\alpha$  value is a good approximation of  $R_{\max}$ . Shown below are outputs of runs made for the constellation 24/3/1, Figure 8. The results can be found in Table 1 of Section I. In this case  $R_{\max}$  is approximately 50 degrees.

```

24/ 3/ 1
ELAPSED ANGLE   = 20.00 (DEG)  <=====
THRESHOLD ANGLE = 49.   (DEG)

```

INC	52.99	FAIL/SAMPLE	0.0014
INC	53.49	FAIL/SAMPLE	0.0011
INC	53.99	FAIL/SAMPLE	0.0011
INC	54.49	FAIL/SAMPLE	0.0007
INC	54.99	FAIL/SAMPLE	0.0003

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687

```

24/ 3/ 1
ELAPSED ANGLE   = 20.00 (DEG)  <=====
THRESHOLD ANGLE = 50.   (DEG)

```

INC	52.99	FAIL/SAMPLE	0.0000
INC	53.49	FAIL/SAMPLE	0.0000
INC	53.99	FAIL/SAMPLE	0.0000
INC	54.49	FAIL/SAMPLE	0.0000
INC	54.99	FAIL/SAMPLE	0.0000

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687

Figure 8 - These outputs show how a threshold angle of 50° causes all failures in coverage to become zero at both 20 and 36 elapsed angles.

---

24/ 3/ 1  
ELAPSED ANGLE = 36.00 (DEG) <=====  
THRESHOLD ANGLE = 49. (DEG)

---

INC	52.99	FAIL/SAMPLE	0.0014
INC	53.49	FAIL/SAMPLE	0.0011
INC	53.99	FAIL/SAMPLE	0.0007
INC	54.49	FAIL/SAMPLE	0.0003
INC	54.99	FAIL/SAMPLE	0.0003

---

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687

---

---

24/ 3/ 1  
ELAPSED ANGLE = 36.00 (DEG) <=====  
THRESHOLD ANGLE = 50. (DEG)

---

INC	52.99	FAIL/SAMPLE	0.0000
INC	53.49	FAIL/SAMPLE	0.0000
INC	53.99	FAIL/SAMPLE	0.0000
INC	54.49	FAIL/SAMPLE	0.0000
INC	54.99	FAIL/SAMPLE	0.0000

---

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687

---

Figure 8 (continued) Note how there are no failures in coverage when the threshold angle is increased from 49 to 50 degrees.

### Section III

#### Finding the Point of Worst Coverage

The method mentioned at the end of Section II and employed by Walker to find the point of worst coverage will be explained below. He gives a brief explanation of it in reference 2. The following comments might serve as a help to the reader since the approach will be covered in greater detail with some added modifications. This method was not used, however, in finding  $R_{\max}$  as the author had initially intended. It was found to be somewhat cumbersome. In the end it does yield as precise a value as is desired. For the specific task at hand however, an approximate value is all that is required.

The point of worst coverage for a constellation is of interest because it is used as the measuring stick for performance. It represents the greatest angular distance ( $R_{\max}$ ) from the local vertical to a third subsatellite point (in the case of triple coverage) anywhere on the globe and at any elapsed angle of motion.

One might think that the point of worst coverage is the one with the greatest angular radius to three equidistant subsatellite points. Looking more closely, however, one finds that moving the user position closer to one of the

subsatellite points means that a circle of radius 'R' will no longer contain three subsatellite points. See Figure 9. Thus, the worst point is still to be found. We are assuming here a somewhat useful distribution where the satellites are not clustered.

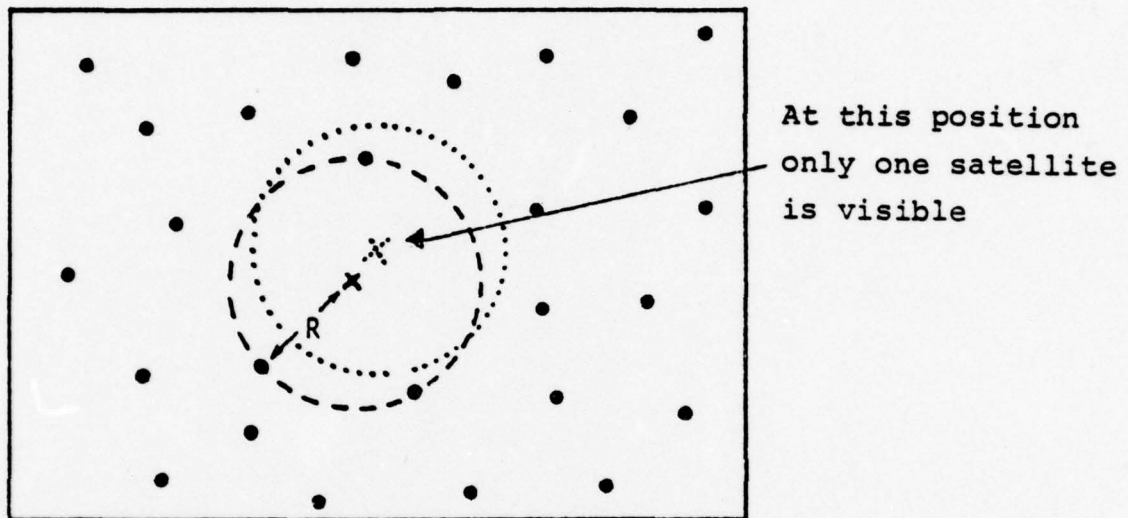


Fig. 9 The dotted circle shows worse coverage than the dashed circle.

For a true worst case position, then, one must usually find a point where three or more subsatellite points lie on the circumference of a circle with radius  $R_{\max}$  and two subsatellite points lie within that radius. If there are less than three subsatellite points on the circumference, then there is a good chance that this is not the worst point. See Figure 10. Although in Figure 10 the user can see five satellites, if his angular visibility was only  $R - \epsilon$ , he would only see two satellites, where  $\epsilon$  is much smaller than  $R$ .

Thus  $R$  and not  $R - \epsilon$  is required to have triple coverage. Note that for double coverage the user needs one subsatellite point within the circle and so on.

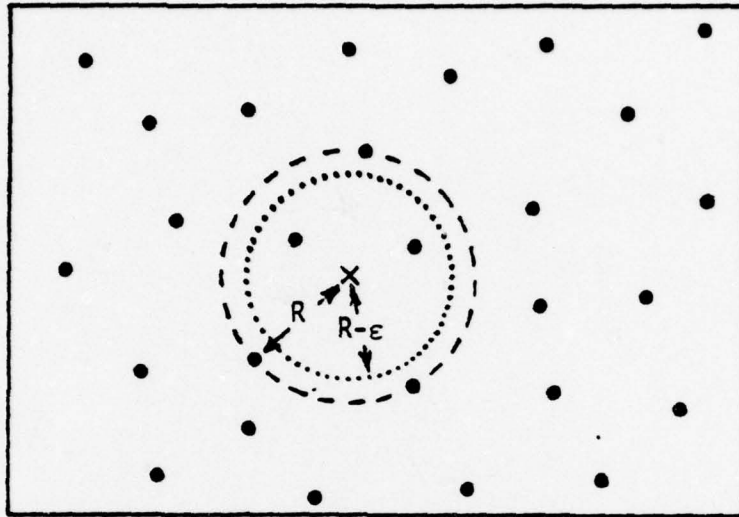


Fig. 10 Typical point of worst coverage at center of circle.

For a given constellation, then, one must first map the satellites (or subsatellite points) on a sphere. See Appendix B for a program which gives each individual satellite's latitude and longitude from the constellation parameters  $N/P/F$ , inclination, and elapsed angular motion. Then find the critical points which give maximum  $R$  values. In the case of 24/24/14, the critical points were thought to occur at 0 and 15 degrees elapsed angle. Although this was later found to be incorrect, these elapsed angles will be used as example. At 0 degrees, the satellites producing maximum  $R$  are shown in Figure 11, and at 15 degrees, in Figure 12.

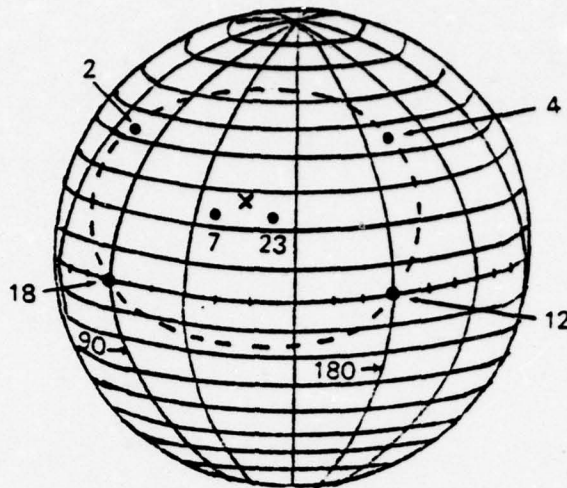


Figure 11 - Satellites making up what was thought to be the worst case of 24/24/14 at 0° E.A., 53° inclination.

Thus after the program 'Grader' has placed one in the vicinity of the optimal inclination, a graphical analysis to find the satellites making up the worst cases is performed. One can then generate a plot such as in Figure 13. The program used for this is explained in Appendix C. An even finer plot can be made to pinpoint the most favored inclination as shown in Figure 14 for this example, that inclination was 52.33°

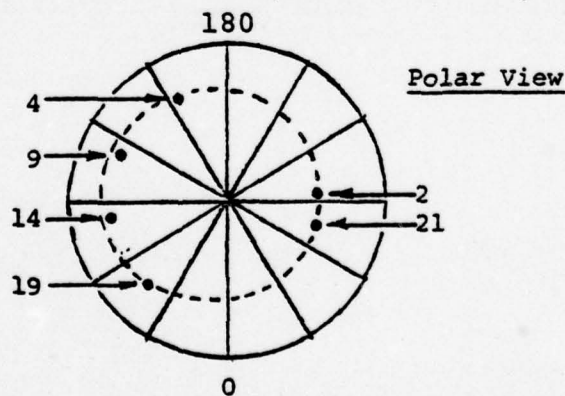


Figure 12 - Thought to be worst case satellites. Elapsed angle is 15 degrees.

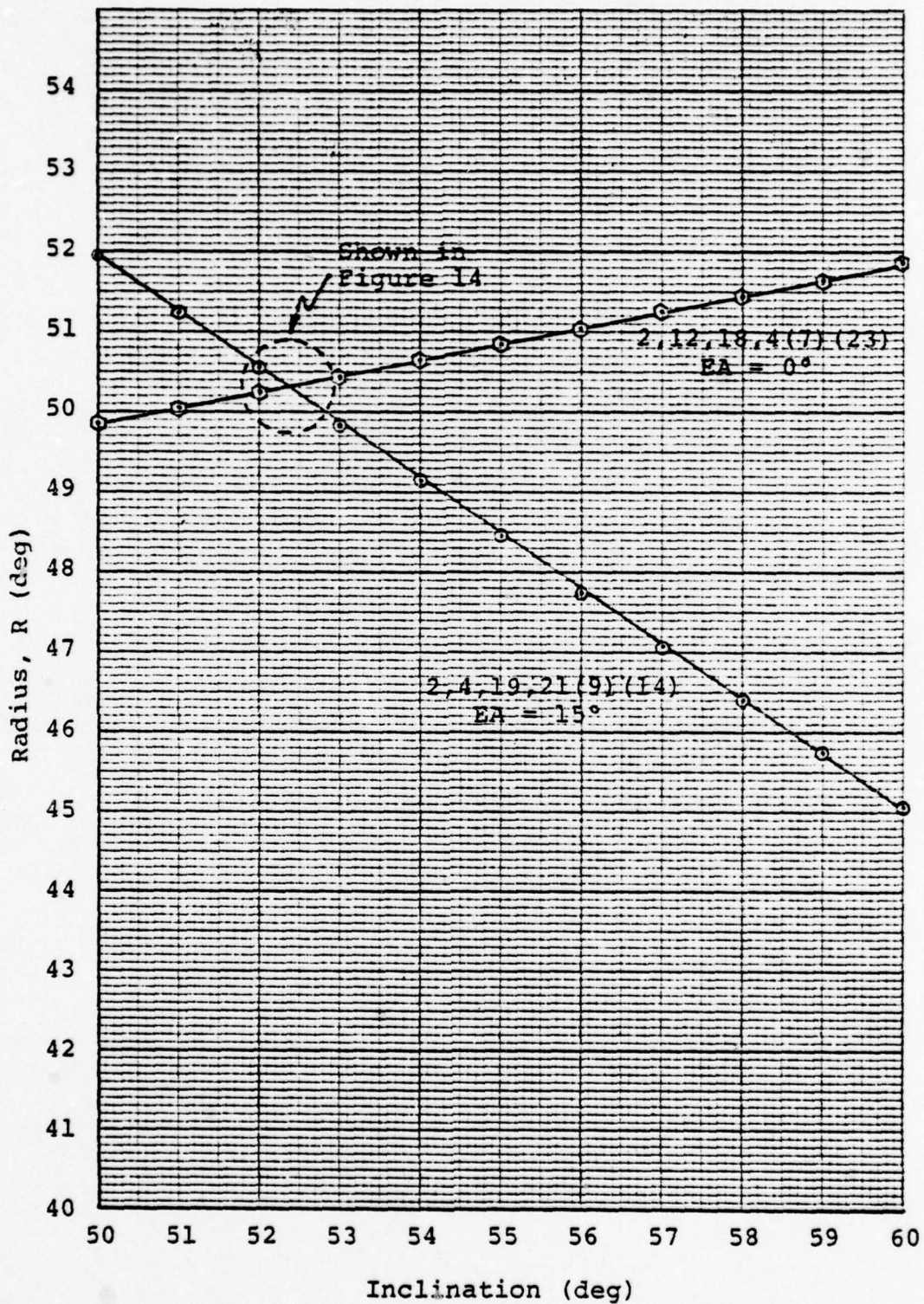


Figure 13 - A plot of R vs. inclination for what were thought to be the worst case elapsed angles of Constellation 24/24/14.

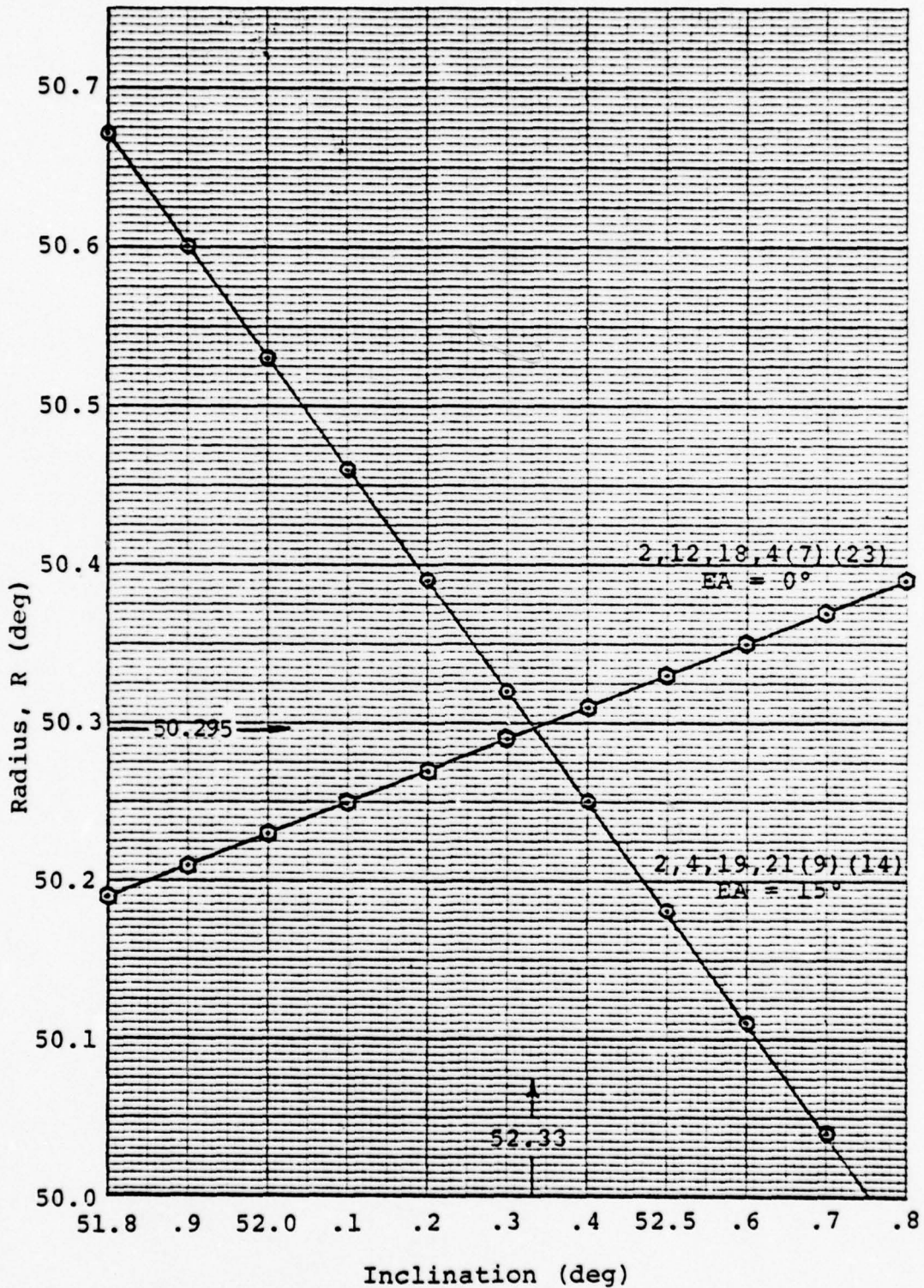


Figure 14 - Blow up of critical area for more accurate determination of optimal inclination for the example at hand.

The exception to the rule of finding the worst coverage is shown below in Figure 15. In this case there are no sub-satellite points within the circle with radius  $R_{\max}$ . However there are enough satellites on the circumference that if the user moves in any direction he still has triple coverage.

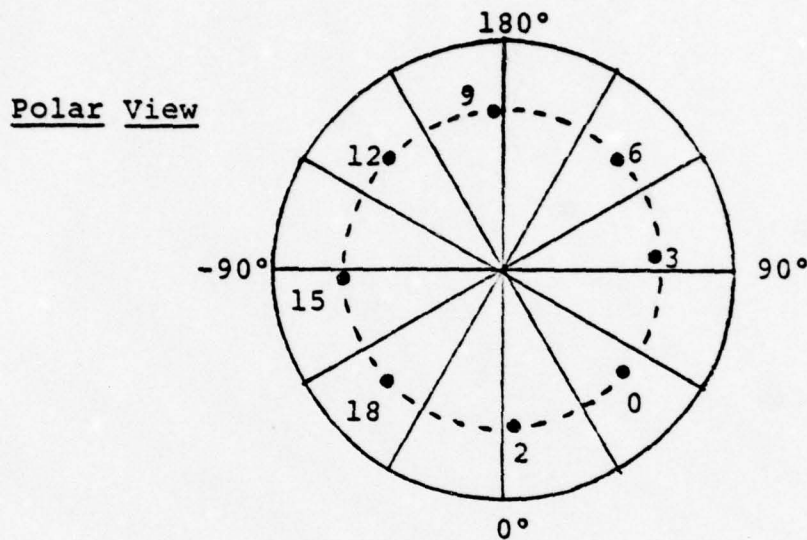


Figure 15 - Worst case for 24/24/8. An exception to the rule. E.A. is  $60^\circ$ .

In general, care must be taken not to miss areas of poor coverage when doing this kind of analysis by hand. The author had access to a slate-covered sphere with permanent latitude and longitude lines. This proved handy indeed. For measuring angular distances an ordinary compass was used with chalk on one side attached by means of rubber bands and a pencil strapped to the other end with the eraser acting as the pivot. Accuracies of  $\pm 1^\circ$  or better were found when the graphical value of  $R_{\max}$  was compared to the computed value.

## Section IV

### The 'Grader' Program

The basic purpose of this program is to provide a quick tool for checking the performance of a constellation. It can help circumvent hours of tedious three dimensional plotting of twenty or so orbits for many possible phase angles and varying inclinations. The program has been called 'Grader' since it gives a figure of merit or grade to the candidate constellations.

It is a simple program which places a position vector in  $2^\circ$  steps in the latitudinal direction. It stops at each latitude and steps the position vector in the longitudinal direction. At each sample point it takes the dot product of this vector and subsatellite position vectors. The subsatellite vectors are arrived at from the corresponding longitude of ascending node (LAN) and nodal information of each satellite. A transformation is performed to get the information from this mode to rectangular coordinates for the purpose of taking the dot product. Dot products, instead of the corresponding angle, are then compared to a predetermined value. If the dot product is greater than this threshold value, the satellite is regarded as 'visible' to the vehicle (or user). A counter keeps track of the number of visible satellites. Whenever the total number of visible at a particular position is less than three, a triple coverage failure is noted.

At the end of all the sampling the total number of failures is divided by the total number of samples. Thus one arrives at the failure rate or failures per samples.

In order to save computer time, the analysis of coverage is done over a section of the globe and not over its entirety. Angles of  $90^\circ$  to  $0^\circ$  latitude and 0 to  $180^\circ$  in longitude were chosen. All the configurations encountered exhibited symmetry about the equator and also about a plane cutting perpendicular to the equator. This is the basis for the decision to look at only half of the northern hemisphere. In any case, half of the northern hemisphere is a large enough area for the accuracy desired in a preliminary study such as this.

Below is a sample output of the Grader for the 24/24/14 constellation at  $80^\circ$  elapsed angular motion. It is plotted in Figure 3.

```
24/24/14
-----
ELAPSED ANGLE = 80.00 (DEG) <=====
THRESHOLD ANGLE = 45.00 (DEG) (ALPHA)
INC 50.99 | FAIL/SAMPLE 0.0468
INC 51.99 | FAIL/SAMPLE 0.0446
INC 52.99 | FAIL/SAMPLE 0.0457
INC 53.99 | FAIL/SAMPLE 0.0468
INC 54.99 | FAIL/SAMPLE 0.0506
-----
TOTAL NO. OF SAMPLES PER INCL. POINT = 2687
```

The Grader 1 program is almost identical to Grader only it handles cases where there are more than one satellite per orbit ( $N > P$ ) such as 24/3/1. The two programs could have been made into one rather easily. Shown in Figure 17 is a sample output of the Grader 1 program.

```

24/ 3/ 1
ELAPSED ANGLE = 40.00 (DEG) <=====
THRESHOLD ANGLE = 45. (DEG)

INC 30.99 | FAIL/SAMPLE 0.0521
INC 31.99 | FAIL/SAMPLE 0.0467
INC 32.99 | FAIL/SAMPLE 0.0337
INC 33.99 | FAIL/SAMPLE 0.0357
INC 34.99 | FAIL/SAMPLE 0.0360
INC 35.99 | FAIL/SAMPLE 0.0375
INC 36.99 | FAIL/SAMPLE 0.0445
INC 37.99 | FAIL/SAMPLE 0.0502

TOTAL NO. OF SAMPLES PER INCL. POINT = 2537

```

Figure 17 - Sample Output of the Grader-1 Program.

Another feature of the Grader program provided to save computer time decides which satellites can be taken out of the dot product part of the program. If one is analyzing half of the northern hemisphere, then there will be satellites which will never be visible to any of the position vectors. A quick survey at the beginning of the program is thus made to see if any satellites have too great a negative 'y' component or negative 'z' component thus making them ineffective to the area in question\*. This is discussed in detail in Appendix A.

---

\* Suggested by Robert White - C.S.Draper Laboratory

## Appendix A

### The 'Grader' Program in Detail

The 'Grader' program was written in the MAC language which lends itself well to vector operations. However, conversion to a more common language should be straightforward.

The input is shown in Figure A-1. It consists of:

- a) 'longdelt' values (seldom change)
- one card { b) inclination values { initial  
          {                          increment  
          {                          final
- c) N -- number of satellites
- d) ALPHA -- threshold angle of visibility (deg)
- one card { e) F -- phase angle number (0,1,2,3,..., N-1)
- f) EA -- elapsed angle (deg)
- one card { g) F --
- h) EA --

LONGDELT values are the incremental arcs in longitude through which the vehicle position vector is stepped. For example at 88° latitude, the increment in longitude for a net 2° motion is 1.0479 radians (or 60.04 degrees). At 86° latitude, .50588 radians (or 28.98 degrees) is the proper longitudinal step size and so on. A latitude of 90° does not have a corresponding step size since the calculations need only be done once. This is taken care of by the program. How the numbers were calculated will be explained later.

```

* COLLECT MLS1580.GRADER
* HEADING NETWORK 24/24/14 AND 24/24/5.
* GO
R *****
R * LAST REVISION TO PROGRAM: MAY 13, 1977
R *****
R LONGDELT VALUES (45)
  1.04797767291
  5.05885909053E-01
  3.35553816957E-01
  2.51494055867E-01
  2.01367776409E-01 ← Note 1
  1.68093488520E-01
  1.44415920071E-01
  1.26724659301E-01
  1.13019683190E-01
  1.02103263616E-01
  9.32142490412E-02
  8.58457373712E-02
  7.96471013844E-02
  7.43680877799E-02
  6.98253150938E-02
  6.58813537021E-02
  6.24311991331E-02
  5.93932596731E-02
  5.67031931536E-02
  5.43095946742E-02
  5.21709177248E-02
  5.02532359542E-02
  4.85285895684E-02
  4.69737457550E-02
  4.55692571675E-02
  4.42987382150E-02
  4.31483027015E-02
  4.21061225043E-02
  4.11620781126E-02
  4.03074796417E-02
  3.95348424624E-02
  3.88377055657E-02
  3.82104836699E-02
  3.76483462053E-02
  3.71471178957E-02
  3.67031968401E-02
  3.63134869029E-02
  3.59753419082E-02
  3.56865196697E-02
  3.54451443050E-02
  3.52496756117E-02
  3.50988845493E-02
  3.49918340816E-02
  3.49278648186E-02
  3.49065850398E-02
R   INCIN, INCDELT, INCFIN, N, ALPHA
R   S1,          1,          55,  24, 45
R   F,EA
  14.78
  14.80
  14.82
  14.84
  14.86
  14.88
  14.90
  14.92
  0.0
  40,          5,          70,  24, 45
  5.0

```

Figure A-1 - Listing of input for the Grader program.

INCIN, INCDELTA, INCFIN represent the initial inclination, inclination increment, and final inclination respectively. Suggested values are: 40, 5, 70 (coarse plotting) and 50, 1, 60 (fine plotting).

F, EA, representing F value or phase angle value and the elapsed angle, are in the inner most loop with a read statement at the beginning of the loop. One can have any combination whatsoever. The elapsed angle (EA) is the angular motion of all the satellites in their respective orbital planes.

#### Alpha

Alpha is the threshold angle. This earth central angle between user and satellite defines visibility for the program. If a vehicle position vector is found at any angle greater than this amount it is rendered nonvisible. For 24/24/14, 45° was found to be a good number.

It is important to choose an alpha value that is less than the expected performance of the constellation so that the percentage of failed samples will not be zero. It is equally important to use this threshold value for all the runs for a given N value so that a meaningful comparison can be made.

#### More on Longdelt

The value of this 2° net longitudinal movement, longdelt, was put in to compensate for the fact that a degree of longitude at the equator and a degree of longitude near the north

pole represent radically different angular values on the surface of the globe. If one were to disregard this, one would find that a disproportionate number of samples would be taken near the north polar area.

Let us represent the vehicle position vector in rectangular coordinates. To do this a transformation vector is needed to transform from  $\lambda, L$  to  $x, y, z$ . This then is

$$\hat{p}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} \cos \lambda_0 \cdot \cos L_0 \\ \cos \lambda_0 \cdot \sin L_0 \\ \sin \lambda_0 \end{bmatrix}$$

where  $L_0 = 0$ .

Then another position vector, two degrees away from  $\hat{p}_0$  can be defined as

$$\hat{p}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos \lambda_1 \cos L_1 \\ \cos \lambda_1 \sin L_1 \\ \sin \lambda_1 \end{bmatrix}$$

where  $\begin{cases} \lambda_1 = \lambda_0 \\ L_1 = ? \end{cases}$

Since they are both unit vectors

$$\cos \theta = \hat{p}_0 \cdot \hat{p}_1$$

$$\begin{aligned} \text{or } \cos 2^\circ &= \cos \ell_0 \cos L_0 \cos \ell_1 \cos L_1 \\ &+ \cos \ell_0 \sin L_0 \cos \ell_1 \sin L_1 \\ &+ \sin \ell_0 \sin \ell_1. \end{aligned}$$

Substituting  $\ell_0$  for  $\ell_1$  and 0 for  $L_0$  yields:

$$\cos 2^\circ = \cos \ell_0 \cos \ell_0 \cos L_1 + \sin \ell_0 \sin \ell_0$$

$$\text{so that } \cos L_1 = \frac{\cos 2^\circ - \sin^2 \ell_0}{\cos^2 \ell_0}$$

$$L_1 = \cos^{-1} \left[ \frac{\cos 2^\circ - \sin^2 \ell_0}{\cos^2 \ell_0} \right]$$

For  $80^\circ$  latitude ( $\ell_0$ ),

$$L_1 = \cos^{-1} \left[ \frac{\cos 2^\circ - \sin^2 80^\circ}{\cos^2 80^\circ} \right]$$

$$= 11.54^\circ \text{ (.2014 radians, Note 1 in Figure A-1).}$$

Thus at  $80^\circ$  latitude, the vehicle position vector is placed at 11.54 degree increments starting with  $0^\circ$ ,  $11.54^\circ$ ,  $23.08^\circ$ ,...

At each latitude then, a corresponding incremental step in longitude is made. This gives a close enough approximation to the desired homogeneous sampling pattern.

OUTPUT

The output is straightforward, a sample of which is shown again below for convenience. The N/P/F along with elapsed angle describe the constellation uniquely once an inclination is chosen. The 'threshold angle' is simply alpha as defined above. The 'total number of samples per inclination point' is the number used to calculate the 'FAIL/SAMPLE' number. It is not computed in each run (since it remains the same for this grid) but was found once by means of a counter in the program. The total number of failures is divided by the total number of samples to get F/S.

-----  
 NETWORK 24/24/14  
 -----

R \*\*\*\*\*  
 R \* LAST REVISION TO PROGRAM: MAY 12, 1977  
 R \*\*\*\*\*  
 R LONCDELT VALUES (43)  
 R INCL, INCDLT, INCLIN, N, ALPHA  
 R 51, 1, 55, 20, 45  
 R F.S.A  
 -----

24/24/14  
 ELAPSED ANGLE = 73.11 (DEG) <=====  
 THRESHOLD ANGLE = 45.00 (DEG) (ALPHA)  
 INC 51.99 | FAIL/SAMPLE 0.0442  
 INC 51.99 | FAIL/SAMPLE 0.0427  
 INC 52.99 | FAIL/SAMPLE 0.0454  
 -----  
 INC 53.99 | FAIL/SAMPLE 0.0481  
 INC 54.99 | FAIL/SAMPLE 0.0529

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687

### Disregarding Satellites Unreachable to the Sampling Area

The procedure of disregarding satellites which cannot be 'seen' by the sampling position vectors is simple and, as mentioned in the main test, can save computer time. What is done here is to look at the components of the satellite position vector and see if either the y component or the z component is sufficiently negative to render it non-visible. Assuming that the sampling area is the one used in this study, then

- a) if  $RR_j < -\sin(\alpha)$ ,  
b) or if  $RR_k < -\sin(\alpha)$  } the satellite is not visible  
to the sampling area.

where  $RR_j$  = satellite y component  
 $RR_k$  = satellite z component  
 $\alpha$  = visibility threshold angle setting.

See notes 1 and 2 of the program, Figure A-2.

### Transformation of the Satellite Position Vector from Keplerian Elements to Rectangular Coordinates

In order to see whether a satellite is within a specific angle to a sample point, the Keplerian elements are transformed to rectangular coordinates on a unit sphere. The sample point, which is in latitude, longitude coordinates is then also transformed to rectangular coordinates. The scalar product of the two can then be taken and the angle checked.

```

*000010 PRODMAC      MLS1580. GRADER
M000020 INDEX A,I,J,K,M
M000025 DIMENSION (CO,25), (SO,25)
M000030 DIMENSION (COCU,25), (SOCU,25), (SDSU,25), (CDSU,25), (SU,25)
M000040 DIMENSION (RR,25X3), (LNGDELTR,46), (CL,46), (SL,46)
R000050 *****
R000060 ***** GRADER.MAC *****
R000070 * - ANALYZES THE COVERAGE PRODUCED BY N SATELLITES OVER A SPECIF-
R000080 * IED PORTION OF THE GLOBE:
R000090 * 2 DEGREE LATTITUDE INCREMENT,
R000100 * AND 2 DEGREE INC IN LONGITUDINAL DIRECTION (NOT IN LONG)
R000110 * THE SATELLITE NETWORK IS POSTULATED BY SUAREZ.
R000120 * LONGMIN = MINIMUM LONGITUDE DESIRED. 0 OR 90 SUGGESTED.
R000130 * LONGMAX = MAXIMUM LONGITUDE DESIRED. 90 OR 180 SUGGESTED.
R000140 *
R000150 * - THIS PROGRAM ASSUMES CIRCULAR UNPERTURBED ORBITS.
R000160 * ALPHA = MAXIMUM EARTH CENTRAL ANGLE BETWEEN VEHICLE AND THE SATEL-
R000170 * LITE WHICH IT IS TRYING TO SEE. (DEG.)
R000180 * IT SHOULD BE SET TO A VALUE LESS THAN THAT EXPECTED FOR
R000190 * SUCCESSFUL TRIPLE COVERAGE.
R000200 * OMEGA = LONGITUDE OF ASCENDING NODE (DEG).
R000210 * U = NODAL ANGLE OF A SATELLITE IN ITS ORBIT (DEG).
R000220 * RR = PSEUDO COMPONENTS OF SATELLITE POSITION VECTOR IN X,Y,Z
R000230 * INERTIAL FRAME
R000240 * P = THE POSITION VECTOR OF THE VEHICLE (PO,P1,P2).
R000250 * INC = INCLINATION OF ORBIT (DEG).
R000260 * THETA = THE ANGLE BETWEEN SATELLITE AND VEHICLE.
R000270 * INCIN = INITIAL INCLINATION (DEG).
R000280 * INCDELTA = INCLINATION INCREMENT (DEG).
R000290 * INCFINE = FINAL INCLINATION FOR THE LOOP. (DEG).
R000300 * 2687 = NO. OF POSITIONS THE VEHICLE IS PLACED IN FOR SAMPLING.
R000310 * N = NO. OF SATELLITES.
R000320 *****
M000321 BEGIN CL = 0, SL = 1
S000322 45
M000323 DO TO END11 FOR M = 44 (-1) 0
M000324 LATR = M 0.034907
M000325 CL = COS(LATR), SL = SIN(LATR)
S000326 M
M000327 END11 READ LNGDELTR M
S000328

```

} Note 6

Figure A-2 - Listing of the Grader program for cases where N=P. "R" on the far left represents a comment.

```

M000330 READ1
M000340 READ INCIN, INCDEL, INCFIN, N, ALPHA
M000350 INCINR = INCIN DFGTORAD
M000360 INCDELTR = INCDEL DEGTRAD
M000370 INCFINR = INCFIN DEGTRAD - 0.001
M000380 ALPHA = ALPHA DEGTRAD
M000390 CT = COS(2 DEGTRAD)
M000400 CA = COS(ALPHAR)
M000410 SA = SIN(ALPHAR)
M000420 LANDELT = 2 PI/N
M000430 DO TO END02 FOR I=0(1)(N-1)
M000440 OMEGAR = I LANDELT
M000450 CO = CUS (OMEGAR), SO = SIN(OMEGAR)
M000500 READ F,EA
M000510 IF F = 0, GO TO READ1
M000520 EAR = EA DEGTRAD
M000530 NODEL = F LANDELT
M000540 DO TO FND00 FOR I = 0(1)(N-1)
M000550 UR = EAR + I NODEL

M000550 SU = SIN(UR)
M000560 CU = COS(UR)
M000570 COCU = CO CU, SOCU = SO CU, SOSU = SU CU, COSU = CO SU
M000580 PRINT FORMAT 15,N,N,F,EA,ALPHA
M000590 FORMAT 15
M000600 $$$/$/$/$
M000610 ELAPSED ANGLE = $$$,$$ (DEG) <=====
M000620 THRESHOLD ANGLE = $$$,$$ (DEG) (ALPHA)
M000630 -----
M000640 DO TO FND0 FOR INCR = INCINR(INCDELTR)INCFINR
M000650 M=0, NP=N, FAILD = 0
M000660 CI = COS(INCR)
M000670 SI = SIN(INCR)
M000680 } Note 4
M000690 } Note 3
M000700 } Note 2
M000710 } Note 1
M000720 } Note 0
M000730 } Note -1
M000740 } Note -2
M000750 } Note -3

```

Figure A-2 (continued) The Grader program.

```

R000760 ----- SATELLITE POSITION LOOP -----
M000770 DO TO END1 FOR A = 0(1)(N-1)
M000780 I = 3 M
M000790 RR = COCU - SCSU CI
S000800 I A
M000810 J = I + I A
M000820 RR = 50CU + CCSU CI A
S000830 J A
M000840 K = I + 2 A
M000850 RR = SU SI A
S000860 K A
M000870 IF RR < -SA, NP=NP-1,GO TO END1
M000880 J
M000890 IF RR < -SA, NP=NP-1,GO TO FND1
M000900 K
S000910 M = M+1
M000920 NIL = 0
R000930 FND1 ----- FND SAT. POSITION LOOP -----
M000940 NPP = 3 (NP-1)
R000950 ----- LATITUDE LOOP -----
M000960 DO TO FND43 FOR M = 45(-1)0
M000970 P = SL M
S000980 2 DO TO END43 FOR LONGR = 0(LONGDELTR )J.1416 M
M000990 P = CL COS(LONGR)
S001000 0 M
M001010 P = CL SIN(LONGR) M
S001020 1 M
M001030 ZUM = 0
M001040 DO TO ENDS FOR I = 0(3)(NPP)
M001050 DOT = RR . P
M001060 I
M001070 IF DOT > CA, ZUM = ZUM + 1
M001080 IF ZUM = 3, GO TO END43
M001090 NIL = 0
M001100 IF ZUM < 3, FAILD=FAILD + 1
M001110 NIL = 0
M001120 ENDS
M001130 END43
R001140 ----- END LONGITUDE AND LATITUDE LOOPS-----

```

Note 5

Note 1

Note 7

Note 8

Figure A-2 (continued) The Grader program.

```

M001150
M001160
M001170
M001180
M001190
M001200 ENDO
M001210
M001220
E001230
M001240
M001250
M001260
M001270

FPS = FAILD/2647
INC = INCR RADTODEG
PRINT FORMAT 62. INC. FPS
FORMAT 62
INC $$$.$$ | FAIL/SAMPLE 1.1111
NIL = 0
PRINT FORMAT 80
FORMAT 80

TOTAL NO. OF SAMPLES PER INCL. POINT = 2687
PRINT SKIP
GO TO READ2
START AT BEGIN

```

Figure A-2 (continued) The Grader program.

The transformation of the satellite elements then can be done by using a simplified version of the R matrix shown below\*

$$\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \underline{R} \begin{bmatrix} q_\xi \\ q_\eta \\ q_\zeta \end{bmatrix}$$

The figure shown shows the vector components. The R matrix is:

$$\begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ u_1 & u_2 & u_3 \end{pmatrix}$$

$$\begin{aligned} l_1 &= \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \\ l_2 &= -\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i \\ l_3 &= \sin \Omega \sin i \\ m_1 &= \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i \\ m_2 &= -\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i \\ m_3 &= -\cos \Omega \sin i \\ n_1 &= \sin \omega \sin i \\ n_2 &= \cos \omega \sin i \\ n_3 &= \cos i \end{aligned}$$

---

\* Battin, Astronautical Guidance, p. 18

However, since one is considering simply a position on the unit sphere, the vector components  $q_\eta$  and  $q_\zeta$  can be set to zero and  $q_\xi$  set of 1. Thus

$$\begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \begin{bmatrix} l_1 \\ m_1 \\ n_1 \end{bmatrix}$$

Notes 3, 4 and 5 on the program source show how  $RR_i$ ,  $RR_j$ , and  $RR_k$  are derived corresponding to  $l_1$ ,  $m_1$ , and  $n_1$  respectively.

Transformation of the User Position Vector to Rectangular Coordinates

Since latitude and longitude angles lend themselves readily to a grid sampling pattern on a unit sphere, they are employed here. After the corresponding increment is made, they are transformed to rectangular coordinates.

The transformation is then,

$$\begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(\text{lat}) \cos(\text{long}) \\ \cos(\text{lat}) \sin(\text{long}) \\ \sin(\text{lat}) \end{bmatrix}$$

This corresponds to notes 6, 7, and 8 of the program listing.

Other features which can save computer time have been incorporated and will be discussed in Appendix A. By incorporating all these, the author was able to cut the cost per run by more than 60%.

Figure A-3 shows a listing of the Grader-1 program. Only the part which is different is shown for brevity. The rest is exactly the same for both programs.

```

*000010 PRODMAC      MLS1580,GRADER1
*000020 INDEX A,I,J,K,M,0
*000030 DIMENSION (COCU,24),(SOSU,24),(SOSU,24),(COSU,24),(SU,24)
*000040 DIMENSION (RR,24X3),(LONGDELTR,46),(CL,46),(SL,46)
*****
*000050 GRADER1,MAC
*000060 - ANALYZES THE COVERGAE PRODUCED BY N SATELLITES OVER A SPECIF-
*000070 IED PORTION OF THE GLOBE;
*000080 2 DEGREE LATITUDE INCREMENT,
*000090 AND 2 DEGREE INC IN LONGITUDINAL DIRECTION (NOT IN LONG)
*000100
*000110
*000120
*000130
*000140 - THIS PROGRAM ASSUMES CIRCULAR UNPERTURBED ORBITS.
*000150 ALPHA = MAXIMUM EARTH CENTRAL ANGLE BETWEEN VEHICLE AND THE SATEL-
*000160 LITE WHICH IT IS TRYING TO SEE. (DEG.)
*000170 = IT SHOULD BE SET TO A VALUE LESS THAN THAT EXPECTED FOR
*000180 SUCCESSFUL TRIPLE COVERAGE.
*000190 OMEGA = LONGITUDE OF ASCENDING NODE (DEG).
*000200 U = NODAL ANGLE OF A SATELLITE IN ITS ORBIT (DEG).
*000210 RR = PSEUDO COMPONENTS OF SATELLITE POSITION VECTOR IN X,Y,Z
*000220 INERTIAL FRAME
*000230 P = THE POSITION VECTOR OF THE VEHICLE (P0,P1,P2).
*000240 INC' = INCLINATION OF ORBIT (DEG).
*000250 THETA = THE ANGLE BETWEEN SATELLITE AND VEHICLE.
*000260 INCIN = INITIAL INCLINATION (DEG).
*000270 INCDELTA = INCLINATION INCREMENT (DEG).
*000280 INCFIN = FINAL INCLINATION FOR THE LOOP.(DEG).
*000290 2687 = NO. OF POSITIONS THE VEHICLE IS PLACED IN FOR SAMPLING.
*000300 N = NO. OF SATELLITES.
*****
*000310 BEGIN      CL =0,      SL =1
*000320 45
*000330 DO TO END11 FOR M = 44(-1)0
*000340 LATR = M 0.034907
*000350 CL = COS(LATR), SL = SIN(LATR)
*000360 M
*000370 END11      READ LONGDELTR
*000380 M

```

Figure A-3 - Listing of the Grader-1 program for constellations where N>P. Only the first part is listed. The remainder is the same as is found in Grader.

```

M000390 READ1 READ INCIN, INCDEL, INCFIN, N, ALPHA
M000400 INCINR = INCIN DEGTORAD
M000410 INCDELTR = INCDEL DEGTORAD
M000420 INCFINR = INCFIN DEGTORAD - 0.001
M000430 ALPHA = ALPHA DEGTORAD
M000440 CT = COS(2 DEGTORAD)
M000450 CA = COS(ALPHAR)
M000460 SA = SIN(ALPHAR)
M000470 READ2 READ O, F, EA
M000480 IF O<1, GO TO READ1
M000490 RUN = RUN + 1
M000500 PAR = EA DEGTORAD
M000510 PHASE = F 2 PI/N
M000520 NPO = N/O
M000530 LANDEL = 2 PI/C
M000540 NODEDEL = 2 PI/NPO
M000550 -----
M000560 I=0
M000570 IF RUN = 1, PRINT FORMAT 12
M000580 FORMAT 12
M000590 L, A, N, MODE (RADIANS) FOR RUN ONE ONLY.
M000600 DO TO END00 FOR ORBIT = 0(1)(0-1)
M000610 OMEGAR = LANDEL ORBIT
M000620 CO = CC5 (OMEGAR), SO = SIN (OMEGAR)
M000630 DO TO END00 FOR J = 0(1)(NPO-1)
M000640 UR = PAR + ORBIT PHASE + J NODEDEL
M000650 IF RUN=1, PRINT FORMAT 14, OMEGAR, UR
M000660 FORMAT 14
M000670 11.111 | 11.1111
M000680 SU = SIN(UR), CU = COS(UR)
M000690 C0CU = C0 CU, S0CU = S0 CU, S0SU = S0 SU, C0SU = C0 SU
M000700 I = I + 1
M000710 PRINT SKIP
M000720 PRINT FORMAT 15, L, O, F, EA, ALPHA, SP3
M000730 FORMAT 15
M000740 11/11/11
M000750 FLAPSED ANGLE = 11.11 (DEG)
M000760 THRESHOLD ANGLE = 11. (DEG)
M000770 <=====
M000780 -----

```

Figure A-3 (continued) The Grader-1 program.

## Appendix B

### Network Orbit Plotting Program and Derivation

After one has an idea of a suitable inclination for a given configuration of  $N$  satellites, the question of plotting the network and finding the point of worst coverage arises. After looking at different ways of representing the different ways of representing the different orbits and the satellites within those orbits, this method was found to be the best to obtain the quickest spherical plot. From the plot one can then find the trouble areas and the resulting maximum radius ( $R_{\max}$ ) for triple coverage.

The program deals with both types of constellations (studied in this paper), the type where  $N = P$  and the type where  $N > P$ , where  $P$  is the number of orbits. The  $N > P$  case will be used as an example here. Basically there are four steps to the program:

- 1) Input describing the constellation and run,
- 2) the computation of LAN and node for each satellite,
- 3) the transformation to rectangular coordinates,
- 4) and the transformation to latitude and longitude.

Figure B-1 shows the source programs. It was developed on a Hewlett-Packard 9820 desk calculator. Figure B-2 shows the output of a run used in the analysis of the 24/3/1 con-

```

0: FXD 2; ENT "MONTH.DATE",R81
1: PRT "      MONTH.DATE",R81; PRT "      1977"
2: FXD 0; ENT "SATS",A
3: ENT "PLANES",R3
4: ENT "PHASE NO.(F)",R6; R6*360/a- R0
5: PRT "..... N/P/F.....",A,R3,R6
6: FXD 1; ENT "INCLINATION",B; PRT "INCL.",B
7: ENT "NO.OF PTS",R20; PRT "NO.OF PTS",R20
8: ENT "PLOT ANGLE INCR.",R21; PRT "PLOT ANGLE INCR.",R21
9: 360/A→C; IF A>R3; 360/R3→C; a/R3→R7
10: COS B→R1; SIN B→R2; 0→R4→R10→R110
11: "10"; SPC 1; PRT "SAT.NO.-----",R10; 0→R100
12: IF A>R3; GTO "13"
13: R10*C→R11; R10*R0→R12; GTO "15"
14: "13"; R4*C→R11; R4+R0*360/R7→R12
15: "15"; R12-360→R13; IF 0>R13; GTO "17"
16: R13→R12; GTO "15:"
17: "17"; PRT "LAN,NODE,R11,R12
18: PRT "LONG,LAT"
19: "20"; COS R11*COS R12-SIN R11*SIN R12*R1→R31
20: IF R31=0; .00001→R31
21: SIN R11*CIS R12+COS R11*SIN R12*R1→R32
22: SIN R12*R2→R33
23: ATN (R32/R31)→R41
24: IF 0>R31; R41+180→R41
25: IF 0>R41; 360+R41→R41
26: PRT R41; PRT ASN R33
27: R12+R21→R12; R100+1→R100
28: IF R20>R100; GTO "20"
29: R110+1→R110; IF R110>R7-1; 0→R110; R4+1→R4
30: R10+1→R10; IF ≤R10 A-1; GTO "10"
31: SPC 7; END

```

Figure B-1 - Program giving latitude and longitude in time of multisatellite constellations.

MONTH, DATE	SAT. NO. -----	SAT. NO. -----
5.02	2.0	5.0
1977	LAN, NODE	LAN, NODE
..... N/P/F .....	0.0	0.0
24	90.0	225.0
3	LONG, LAT	LONG, LAT
1	90.0	211.0
INCL. .	53.0	-34.4
NO. OF PTS	106.0	220.7
PLOT ANGLE INCR.	51.9	-40.9
	121.2	232.2
	48.6	-46.4
	133.0	246.0
	43.0	-50.5
SAT. NO. -----	144.4	261.7
0.0	37.7	-52.7
LAN, NODE		
0.0	SAT. NO. -----	SAT. NO. -----
0.0	3.0	6.0
LONG, LAT	LAN, NODE	LAN, NODE
0.0	0.0	0.0
0.0	135.0	270.0
6.1	LONG, LAT	LONG, LAT
8.0	149.0	270.0
12.4	34.4	-53.0
Note B-1 { 15.9	157.1	286.3
19.2	27.3	-51.9
23.5	164.3	301.2
26.0	19.7	-48.0
30.9	170.0	313.0
	11.9	-43.0
SAT. NO. -----	177.0	324.4
1.0	4.0	-37.7
LAN, NODE		
0.0	SAT. NO. -----	SAT. NO. -----
45.0	4.0	7.0
LONG, LAT	LAN, NODE	LAN, NODE
31.0	0.0	0.0
34.4	180.0	315.0
40.7	LONG, LAT	LONG, LAT
40.9	180.0	329.0
52.2	0.0	-34.4
46.4	186.1	337.1
66.0	-0.0	-27.3
50.5	192.4	344.3
81.7	-15.9	-19.7
52.7	199.2	350.0
	-23.5	-11.9
	206.0	357.0
	-30.9	-4.0

Figure B-2 Part of the output for plot of 24/3/1 on a globe.

stellation. Let us first look at the output closely to understand the purpose of the program. Then the algorithm which gives that output will be discussed.

### The Output

Month-Date - 5.02 - 1977: the program's first question to the user is the date. The number 5.02 was typed in referring to May 2.

N/P/F - 24,3,1: This refers to the constellation 24/3/1.

INCL. - 53.0: The inclination is requested by the program and then printed.

NO. OF PTS. - 5.0: The number of points to be plotted for each satellite can be expressed also as the number of nodal positions a satellite will inhabit in its orbit.

PLOT ANGLE INCR. - 10.0: The plot angle increment refers to the increment in nodal angle for these positions. In this case, therefore, we are looking at node angles of 0,10,20,30, and 40°.

SAT. NO. - 0.0: This refers to the satellite number. Zero is used because it makes the programming simpler.

LAN, NODE - 0.0 - 0.0: These two numbers represent the longitude of ascending node and the node angle of the n-th satellite.

LONG, LAT - 0.0, 0.0; 6.1, 8.0;...: This is the desired output and it is the longitude and latitude of the satellite at the node angles requested. See note B-1.

One is then ready to plot.

The following is a description of the variables found in the program.

<u>HP Notation</u>	<u>Regular Notation</u>	<u>Description</u>
A	N	the number of satellites
B	i	inclination (this assumed the same for all the planes)
C	$\Delta\Omega$	increment in longitude g ascending node from plane to plane
RO	$\Delta\omega$	increment in node angle ( $\Delta\omega = F \times 360/N$ )
R1	$\cos i$	
R2	$\sin i$	
R3	P	number of planes in which the satellites are distributed
R4		the number of individual plane (0,1,..., P - 1)
R6	F	the phase angle number
R7	NPO	number of satellites per orbit NPO = N/0
R8	$\phi$	phase angle between a given satellite and the satellite in the next adjacent longitude of ascending node, $\phi = F \times 360/0$
R10	i	satellite number
R11	$\Omega_i$	longitude of ascending node of ith satellite
R12	$\omega_i$	node of the ith satellite

<u>HP Notation</u>	<u>Regular Notation</u>	<u>Description</u>
R13		dummy variable for R12
R20		the number of points to be plotted per satellite
R21		increment through which all the satellites are to be moved in their respective orbits
R31	x	x component of the ith satellite
R32	y	y component of the ith satellite
R33	z	z component of the ith satellite
R41	l	longitude of the ith satellite
R81		month.date in one variable
R110	j	jth satellite of a given orbital plane when $N > P$ (0,1,2,..., NPO - 1)

## Appendix C

### Program for Plotting R vs. Inclination For a Given Group of Satellites

Once the satellites surrounding the worst coverage points have been identified as in Section III, the program for plotting R versus inclination is very helpful. This was shown in Section III with the example of the constellation 24/24/14 where it was used to produce Figures 13 and 14. Although it was not used in obtaining the final results given in Table 1 (p. 9), for better accuracy something similar might be employed.

Listed below are the steps in the program originally written for a large desk calculator (Hewlett-Packard 9820).

```
1:
PRT "SAT NOS"
" "
1:
FXD 0:ENT "SAT N
OS?";R8,R9,R10;
PRT "    SAT NOS
....";R8,R9,R10F
2:
FXD 2:ENT "MONTH
.DATE?";R6;PRT "
    DATE";R6;
PRT "
1977"
3:
ENT "LAN;NODE? S
AT A";R21,R41;
PRT "LAN;NODE-SA
T A";R21,R41F
4:
ENT "LAN;NODE-SA
T B";R22,R42;
PRT "SAT B";R22,
R42F
5:
ENT "LAN;NODE SA
T C";R23,R43;
PRT "SAT C";R23,
R43F
6:
ENT "INITIAL INC
?";B;ENT "INC.DE
LT?";C;ENT "FINA
L INC?";R7F
7:
PRT "IN.;DELT;&F
INAL INC";PRT "I
NCLINATION";B,C,
R7F
```

```

8:
ENT "ELAPSED NODE
E?" ; R13 ; PRT "ELA
PSED NODE" ; R13 F
9:
0+A ; SPC 1 ; PRT "L
ONGS, LATS" ; B+R1 F
10:
"20" ; R(21+A)+R11
; R(41+A)+R13+R12
; COS R11+COS R12
-SIN R11 SIN R12*
COS R1+R(31+A) F
11:
IF R(31+A)=0 ; 1E-
9+R(31+A) F
12:
SIN R11+COS R12+
COS R11+R(71+A) F
13:
SIN R12+R(91+A) F
14:
ATN (R(71+A)/R(3
1+A)) ; R(51+A) F
15:
IF 0>R(31+A) ; R(5
1+A)+180 ; R(51+A)
F
16:
IF 0>R(51+A) ; 360
+R(51+A) ; R(51+A)
F
17:
PRT R(51+A) ; ASN
R(91+A)+R(61+A) ;
PRT R(61+A) F
18:
A+1+A ; IF A<2 ;
GTO "20" F

```

```

19:
"-----" F
20:
ACS (R32R33+R72R
73+R92R93)+R2 F
21:
ACS (R31R33+R71R
73+R91R93)+R14 F
22:
ACS (R31R32+R71R
72+R91R92)+R15 F
23:
ACS ((COS R2-
COS R14+R(51+A)
/(SIN R14+R(51+A)
SIN R15)))+R3 F
24:
ACS ((COS R14-
COS R2+R(51+A)
/(SIN R2+R(51+A)
SIN R15)))+R4 F
25:
ACS ((COS R15-
COS R2+R(51+A)
/(SIN R2+R(51+A)
SIN R14)))+R5 F
26:
ATN (TAN (R2/2)*
1/COS (.5(R4+R5-
R3))) ; R105 F
27:
PRT "FOR INC.=..
...." ; R1 ; PRT "RA
DIUS = ....." ; R
105 ; SPC 2 F
28:
R1+C+R1 ; IF R1<R7
; 0+A ; GTO "20" F
29:
SPC 6 F

```

Below is an explanation of the variables found in the program. The units are in degrees unless otherwise specified.

Program Variable	Corresponding Variable	Comments
A		counter (no units)
B	$i_0$	initial inclination
C	$\Delta i$	increment in inclination
R1	$i$	inclination
R2	$a$	side a of the spherical triangle
R3	A	angle A opposite to side a on spherical triangle
R4	B	angle B opposite to side b on spherical triangle
R5	C	angle C opposite to side c on spherical triangle
R6		month.date of the run
R7	$i_f$	final inclination
R11	$\Omega_i$	longitude of ascending node for the i-th satellite
R12	$\omega_i$	node angle of the i-th satellite
R13	EA	elapsed node under investigation
R14	$b$	side b of the spherical triangle
R15	$c$	side c of the spherical triangle
R21	$\Omega_A$	longitude of ascending node of satellite A
R22	$\Omega_B$	longitude of ascending node of satellite B
R23	$\Omega_C$	longitude of ascending node of satellite C
R31	$x_A$	component in the x direction of satellite A (length units)

Program Variable	Corresponding Variable	Comments
R32	$x_B$	component in the x direction of satellite B (length units)
R33	$x_C$	component in the x direction of satellite C (length units)
R41	$\omega_A$	node angle of satellite A at 0 E.A.
R42	$\omega_B$	node angle of satellite B at 0 E.A.
R43	$\omega_C$	node angle of satellite C at 0 E.A.
R51	$l_A$	longitude of satellite A
R52	$l_B$	longitude of satellite B
R53	$l_C$	longitude of satellite C
R61	$L_A$	latitude of satellite A
R62	$L_B$	latitude of satellite B
R63	$L_C$	latitude of satellite C
R71	$y_A$	component in the y direction of satellite A (length units)
R72	$y_B$	component in the y direction of satellite B (length units)
R73	$y_C$	component in the y direction of satellite C (length units)
R91	$z_A$	component in the z direction of satellite A (length units)
R92	$z_B$	component in the z direction of satellite B (length units)
R93	$z_C$	component in the z direction of satellite C (length units)
R105	R	the final resulting radius of the circumcircle containing satellites A, B, and C on its circumference

The Input

The input to the program is shown as typed by the computer in Figure C-2.

----	STP----		
	SAT NOS.....	2	96.94
		4	48.65
		19	207.82
	DATE	4.05	33.33
		1977	317.18
	LAN,NODE-SAT A	30.00	33.33
		60.00	FOR INC.=.....
	SAT B	60.00	51.00
		120.00	RADIUS = .....
	SAT C	285.00	51.24
		30.00	
	IN., DELT, & FINAL	50.00	96.48
	INCLINATION	1.00	49.57
		60.00	208.38
	ELAPSED NODE	15.00	33.86
			316.62
			33.86
			FOR INC.=.....
			52.00
			RADIUS = .....
			50.53
<hr/>			
	LONGS, LATS		
		97.37	96.00
		47.73	50.48
		207.27	208.96
OUTPUT		32.80	34.38
		317.73	316.04
		32.80	34.38
	FOR INC.=.....	50.00	FOR INC.=.....
			53.00
	RADIUS = .....		RADIUS = .....
		51.95	49.83

Figure C-2 - Sample Input and Output of the R vs. Inclination Program. It gives the radius (R) of the circumcircle containing satellites 2, 4, and 19 on its circumference for varying inclinations. It also gives the corresponding longitudes and latitudes. See graph on Figure 13.

95.49  
 51.39  
 209.55  
 34.89  
 315.45  
 34.89  
 FOR INC.=.....  
 54.00  
 RADIUS = .....  
 49.14

93.18  
 55.00  
 212.08  
 36.85  
 312.92  
 36.85  
 FOR INC.=.....  
 58.00  
 RADIUS = .....  
 46.40

94.96  
 52.30  
 210.16  
 35.40  
 314.84  
 35.40  
 FOR INC.=.....  
 55.00  
 RADIUS = .....  
 48.45

92.51  
 55.89  
 212.75  
 37.31  
 312.25  
 37.31  
 FOR INC.=.....  
 59.00  
 RADIUS = .....  
 45.73

94.40  
 53.21  
 210.79  
 35.89  
 314.21  
 35.89  
 FOR INC.=.....  
 56.00  
 RADIUS = .....  
 47.76

91.81  
 56.77  
 213.43  
 37.76  
 311.57  
 37.76  
 FOR INC.=.....  
 60.00  
 RADIUS = .....  
 45.06

93.80  
 54.11  
 211.43  
 36.37  
 313.57  
 36.37  
 FOR INC.=.....  
 57.00  
 RADIUS = .....  
 47.08

Figure C-2 (continued)

Input #1 (2,4,19) - the numbers of the satellites comprising this spherical triangle as taken from Appendix B. (only for reference purposes).

Input #2 (4.05) - this is the month and the date of the run.

Input #3 (30,60) - the longitude of ascending node ( $\Omega$ ) and node angle ( $\omega$ ) of satellite A as found in Appendix B.

Input #4 (60,120) - the same as input #3 but for satellite B.

Input #5 (285,30) - the same as input #3 but for satellite C.

Input #6 (50,1,60) - these are the initial inclinations of the satellite orbits, the increment, and the final inclination.

Input #7 (15) - the elapsed node of all the satellites in their orbits.

#### The output

A sample output is listed above taken from the 24/24/14 constellation. The satellites involved are 2, 4, 19, 21 (9) (14) as listed in Figure 14. The three used for the program are 2, 4, 19. Only three of the four satellites are required to define the circle whose radius is being sought. One must be sure that the fact that there were four satellites equidistant from a given point is not coincidental only to the particular inclination at which it was analyzed. In other words it might be that satellites 2, 4, 19 and 21 do not all remain on the circumcircle as the inclination is varied. This was checked in the above example.

Essentially lines 10 through 13 compute the rectangular coordinates of the satellites from their Keplerian elements as was done in the Grader program.

Lines 14 through 18 compute the longitude and latitudes of each satellite at each inclination for reference.

Lines 20 through 22 compute the angular length of the sides which make up the spherical triangle--sides a, b, and c. This is done by taking the dot product of the corresponding satellite position vectors and then taking the arccosine to find the earth central angle between them. This angle is then the arc comprising one side of the spherical angle.

Lines 23 to 25 employ a trigonometric formula to give the angles of the spherical triangle once the three sides are done. The formula\* states that:

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} .$$

This in turn is also used to produce angles B and C.

Line 26 gives the radius of the circumcircle (R) which contains the three satellites in its circumference. This handy formula<sup>4</sup> is:

$$\tan R = \tan \frac{1}{2} \cdot \sec \frac{1}{2}(B + C - A).$$

---

\* Taken from standard Math table.

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