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DYNAMIC PLASTIC RESPONSE OF
CIRCULAR PLATES WITH TRANSVERSE
SHEAR AND ROTATORY INERTIA

by
Norman Jones
and
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Report Number 78-9

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Abstract

The response of a simply supported circular plate made from a rigid perfectly plastic material and subjected to a uniformly distributed impulsive velocity is developed herein. Plastic yielding of the material is controlled by a yield criterion which retains the transverse shear force as well as bending moments and the influence of rotatory inertia is included in the governing equations. Various equations and numerical results are presented which may be used to assess the importance of transverse shear effects and rotatory inertia for this particular problem.

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Notation

a	defined by equation (46a)
m_r, m_θ	$M_r/M_0, M_\theta/M_0$
p	lateral pressure
q	Q_r/Q_0
r, θ	polar coordinates
t	time
w	transverse displacement
$\bar{w}$	dimensionless transverse displacement (equation (10g))
z	coordinate through plate thickness (Figure 1)
H	plate thickness
I	dimensionless rotatory inertia defined by equation (46b)
I_r	$\rho H^3/12$
M_r, M_θ	radial and circumferential bending moments per unit length defined in Figure 1
M_0	magnitude of bending moment per unit length required for plastic flow of cross-section
Q_r	transverse shear force per unit length defined in Figure 1
Q_0	magnitude of Q_r required for plastic flow of cross-section
R	outside radius of plate
R_B, R_S	bending and shear energies divided by the initial kinetic energy
T	dimensionless time defined by equation (10f)

Notation (cont'd)

V_0	initial impulsive velocity
$\bar{w}$	dimensionless transverse displacement defined by equation (10g)
α	r/R
β	dimensionless radius of an axisymmetric interface
γ	transverse shear strain
κ_r, κ_θ	radial and circumferential curvature changes
μ	ρh
v	$Q_0 R / 2M_0$
ρ	density of material
σ_0	uniaxial yield stress
ψ	rotation of mid-plane due to bending
$[x]$	$x_2 - x_1$
$\dot{(\)}$	$\partial(\) / \partial t$ or $\partial(\) / \partial T$.

1. Introduction

The rigid-plastic idealisation of a ductile material considerably simplifies theoretical investigations into the dynamic response of structures subjected to large dynamic loads which cause inelastic behavior [1-4, etc.]. These analyses can give surprisingly accurate yet simple predictions for a wide range of practical problems. However, it turns out that transverse shear effects can exercise an important influence on the dynamic plastic behavior of various structural members as discussed in Reference [4].

Two recent theoretical studies on beams loaded dynamically [5,6] have examined the effect of rotatory inertia in the governing equations and the influence of transverse shear force as well as bending moment in the yield condition for a rigid perfectly plastic material. References [4] to [6] contain citations to earlier work which explore the influence of transverse shear effects on the dynamic plastic response of beams, while various yield criteria are discussed in Reference [7].

The influence of transverse shear forces on the static plastic collapse of circular plates has been examined by several authors [8-12], but no papers appear to have been published for any dynamic loading case. Moreover, the influence of rotatory inertia on the dynamic plastic response

of circular plates has not been examined, despite the fact that many authors have explored its effect for linear elastic plates [13, 14, etc.].

Reference [15] contains a review of many of the theoretical solutions on the dynamic response of circular plates which have been obtained since the publication of Reference [16]. However, the analyses were developed for plates made from rigid perfectly plastic materials which were controlled by a yield criterion relating the circumferential and radial bending moments, while the influence of transverse shear forces were disregarded. Wang [17] examined the behavior of a rigid perfectly plastic circular plate which was simply supported around the outer boundary and subjected to a uniformly distributed impulsive velocity V_0 . It may be shown that the transverse shear force in this analysis is infinitely large at the supports immediately after the start of motion. It is the purpose of the work in section 3 of this article to seek the behavior of Wang's problem when the circular plate is made from a rigid perfectly plastic material with a finite transverse shear strength. The simultaneous influence of transverse shear and rotatory inertia effects is then examined in section 4.

2. Basic Equations

The equilibrium equations for the dynamic behavior of the element of an axisymmetrically loaded circular plate shown in Figure 1 may be written in the form

$$\partial M_r / \partial r + (M_r - M_\theta) / r + Q_r = I_r \partial^2 \psi / \partial t^2 \quad (1a)$$

$$\text{and } \partial Q_r / \partial r + Q_r / r = -p + \mu \partial^2 w / \partial t^2, \quad (1b)$$

where $I_r = \rho H^3 / 12$, $\mu = \rho H$, $\partial w / \partial r = \psi + \gamma$, ψ is the rotation of lines which were originally perpendicular to the initial mid-plane ($z = 0$) due to bending and

$$\gamma = \partial w / \partial r - \psi, \quad \kappa_r = \partial \psi / \partial r, \quad \kappa_\theta = \psi / r \quad (2a-c)$$

are the transverse shear strain, radial curvature change, and circumferential curvature change, respectively.

The dynamic continuity condition across a discontinuity front, which travels from region 1 to region 2 with a velocity c in a continuum with a constant density ρ , may be written [18,19]

$$[\sigma_{i1}] = -\rho c [\partial u_i / \partial t], \quad (3)$$

where $[X] = X_2 - X_1$, and when the particle velocity ($\partial u_i / \partial t$) in region 1, which is normal to the discontinuity front, is negligible compared with c . The displacements u_i act along the x_i axes with x_1 directed from region 1 to region 2 and normal to the discontinuity front.

$$\text{Now, } x_1 = r, \quad x_2 = r\theta, \quad x_3 = z, \quad \sigma_{11} = \sigma_r, \quad \sigma_{21} = 0, \quad \sigma_{31} = \sigma_{\theta z},$$

$u_1 = -z\psi$, $u_2 = 0$, and $u_3 = w$ for the particular case of an axisymmetrically loaded circular plate with the variables defined in Figure 1 and in the Notation. Thus, if equation (3) with

$i = 1$ is multiplied by z and integrated with respect to z then

$$[M_r] = -cI_r[\dot{\psi}], \quad (4a)$$

while equation (3) with $i = 3$ when integrated with respect to z gives

$$[Q_r] = -c\mu[\dot{w}]. \quad (4b)$$

The kinematic continuity condition associated with equation (3) is [18,19]

$$[\partial u_i / \partial t] = -c[\partial u_i / \partial x_1], \quad (5)$$

which using the variables appropriate for an axisymmetrically loaded circular plate predicts

$$[\dot{\psi}] = -c[\partial \psi / \partial r] \quad (6a)^\dagger$$

$$\text{and } [\dot{w}] = -c[\partial w / \partial r]. \quad (6b)$$

3. Impulsive Loading of a Circular Plate with Transverse Shear

It was remarked in the Introduction that the transverse shear force at the simply supported edge of the impulsively loaded circular plate examined in Reference [17] is infinitely large at the start of motion. A theoretical analysis of the same problem is presented in this section but for a plate made from a rigid perfectly plastic material with a finite transverse shear strength. Plastic flow is controlled by the simplified yield criterion shown in Figure 2 which was used by Sawczuk and Duszek [8] to examine the static loading of circular plates. Q_0 and M_0 are the respective values of the transverse shear force per unit length and bending moment per unit length required for independent plastic yielding of the plate cross-section.

[†] This condition may also be obtained from the equivalent postulate $[\psi] = 0$ which was used in References [5] and [20] for beams.

3.1 Class I Plates, $0 < \nu \leq 3/2$

The dimensionless transverse velocity profile for this class of plates subjected to a uniformly distributed initial impulsive velocity V_0 is

$$\dot{\bar{w}} = \dot{\bar{W}} \quad \text{for } 0 \leq \alpha \leq 1, \quad (7)$$

which gives a circumferential shear hinge at the supports as indicated in Figure 3(b). Thus, if M_θ is assumed constant in the rigid region $0 \leq \alpha < 1$, then equations (1a,1b) with $I_r = 0$, and $p = 0$, and equation (7) give

$$\ddot{\bar{w}} = -\nu/3, \quad q(\alpha) = -\alpha, \quad (8a,b)$$

$$m_r(\alpha) = -2\nu(1-\alpha^2)/3, \quad m_\theta(\alpha) = -2\nu/3 \quad (9a,b)$$

when satisfying $q(1) = -1$, and $m_r(1) = 0$, where

$$\alpha = r/R, \quad \nu = Q_0 R / 2M_0, \quad q = Q_r / Q_0, \quad m_r = M_r / M_0,$$

$$m_\theta = M_\theta / M_0, \quad T = 12M_0 t / \mu V_0 R^2, \quad \bar{w} = 12M_0 W / \mu V_0^2 R^2,$$

$$\text{and } \dot{\bar{w}} = \dot{W} / V_0. \quad (10a-h)$$

Now, equation (8a) predicts

$$\bar{w}(T) = T - \nu T^2 / 6 \quad (11)$$

since $\dot{\bar{w}}(0) = 1$ and $\bar{w}(0) = 0$. Thus, motion ceases when

$$T_1 = 3/\nu \quad (12)$$

and the associated maximum permanent transverse displacement is

$$\bar{w}_f = 3/2\nu. \quad (13)$$

This transverse displacement is manifested as a shear slide at the supports which must not therefore become too large to avoid failure of the plate. A suitable failure criterion for engineering purposes was developed in Reference [21] for beams and may be written for the present case in the form

$$W_f \leq kH, \quad (14)$$

where $0 < k \leq 1$ and H is the plate thickness.

The generalised stress fields given by equations (8b) and (9) are statically admissible provided $0 < \nu \leq 3/2$.

3.2 Class II Plates, $3/2 \leq \nu \leq 2$.

If $\nu \geq 3/2$, then equation (9) shows that m_θ violates the yield condition throughout a plate and m_r penetrates the yield surface in a central region. Thus, the first stage of motion for the present case is governed by the velocity profile sketched in Figure 3(c) which gives plastic bending throughout a plate with a stationary shear hinge at the supports. This phase of motion is completed when shear sliding ceases at the supports and is followed by a final stage of motion with the velocity profile illustrated in Figure 3(d).

3.2.1 First Phase of Motion, $0 \leq T \leq T_1$.

The transverse velocity profile in Figure 3(c) is

$$\dot{w}(r,t) = \dot{W}(t) + \{\dot{W}_1(t) - \dot{W}(t)\}r/R, \quad (15)$$

which predicts $\dot{\kappa}_r = 0$ and $\dot{\kappa}_\theta \leq 0$ if $\dot{W} > \dot{W}_1$ according to equations (2) with $\gamma = 0$ in the region $0 \leq \alpha < 1$. Thus, the normality

rule of plasticity requires

$$m_\theta = -1, \quad -1 \leq m_r \leq 0, \quad \text{and} \quad -1 \leq q \leq 1. \quad (16a-c)$$

Equations (15), (16a), and (1a,b) with $I_r = p = 0$ give

$$\ddot{\bar{W}}_1 = 1-\nu, \quad \ddot{\bar{W}} = \nu - 2, \quad (17a,b)$$

$$q(\alpha) = \alpha\{2(3-2\nu)\alpha + 3(\nu-2)\}/\nu,$$

$$\text{and } m_r(\alpha) = -1 - (3-2\nu)\alpha^3 - 2(\nu-2)\alpha^2, \quad (18a,b)$$

since $q(1) = -1$, $m_r(1) = 0$, and $m_r(0) = -1$. Thus,

$$\bar{W}_1 = T + (1-\nu)T^2/2, \quad \text{and} \quad \bar{W} = T + (\nu-2)T^2/2 \quad (19a,b)$$

because $\dot{\bar{W}}(0) = 1$, $\dot{\bar{W}}_1(0) = 1$, $\bar{W}(0) = 0$, and $\bar{W}_1(0) = 0$. This phase of motion terminates at

$$T_1 = 1/(\nu-1) \quad (20)$$

when $\dot{\bar{W}}_1 = 0$, and the associated shear sliding at the supports is

$$\bar{W}_1(T_1) = 1/\{2(\nu-1)\}. \quad (21)$$

The total energy dissipated due to shearing deformations is

$$R_s = \nu/\{3(\nu-1)\} \quad (22)$$

when non-dimensionalised with respect to the initial kinetic energy $\mu\pi R^2 V_0^2/2$.

It is straightforward to show that the generalised stress fields (18) are statically admissible provided $3/2 \leq \nu \leq 2$.

3.2.2 Second Phase of Motion, $T_1 \leq T \leq T_f$.

The equilibrium equations (1a,b) together with equation (15) with $\dot{W}_1 = 0$ and equations (16a-c) predict

$$\ddot{W} = -1, \quad q(\alpha) = -\alpha(3-2\alpha)/v, \quad (23a,b)$$

$$\text{and} \quad m_r(\alpha) = 2\alpha^2 - \alpha^3 - 1 \quad (23c)$$

since $m_r(1) = 0$, and $m_r(0) = -1$. Now, integrating equation (23a) and making the displacements and velocities continuous at T_1 with equations (19) gives

$$\bar{w}(\alpha, T) = (2-T/2)(1-\alpha)T + (\alpha-1/2)/(v-1). \quad (24)$$

Finally, motion ceases at

$$T_f = 2 \quad (25)$$

when $\dot{W} = 0$ and

$$\bar{w}(\alpha, T_f) = (4v-5)/\{2(v-1)\} + (3-2v)\alpha/(v-1). \quad (26)$$

The ratio of the energy dissipated in bending to that dissipated in shear is

$$R_B/R_S = 2 - 3/v. \quad (27)$$

3.3 Class III Plates, $v \geq 2$.

It is evident from equation (18b) that $\partial^2 m_r(0, T)/\partial \alpha^2 \leq 0$ when $v \geq 2$, which leads to a yield violation at the plate center. These yield violations are avoided when a plate responds with the three phases of motion indicated in Figure 4.

3.3.1 First Phase of Motion, $0 \leq T \leq T_1$.

A stationary hinge circle forms at a dimensionless radius β_1 ($\beta_1 = r_1/R$) and transverse shear sliding develops at the plate supports as shown in Figure 4(b). This transverse velocity field may be written

$$\dot{\bar{w}}(\alpha, T) = \dot{\bar{W}}(T) \quad \text{for } 0 \leq \alpha \leq \beta_1, \quad (28a)$$

$$\text{and } \dot{\bar{w}}(\alpha, T) = \dot{\bar{W}}_1(T)(\alpha - \beta_1)/(1 - \beta_1) + \dot{\bar{W}}(T)(1 - \alpha)/(1 - \beta_1), \quad \beta_1 \leq \alpha \leq 1. \quad (28b)$$

Equations (2) with $\gamma = 0$ and the flow rule of plasticity again give equations (16), which together with the equilibrium equations (1), equations (28), and $q(1) = -1$, $m_r(1) = 0$, $m_r(\beta_1) = -1$, $q(0) = 0$, $[q(\beta_1, T)] = [m_r(\beta_1, T)] = 0$ predict

$$\ddot{\bar{W}} = 0, \quad \ddot{\bar{W}}_1 = -\{(1 - \beta_1)^2(1 + \beta_1)\}^{-1}, \quad (29a, b)$$

$$q(\alpha) = 0, \quad m_\theta(\alpha) = m_r(\alpha) = -1 \quad \text{for } 0 \leq \alpha \leq \beta_1, \quad (30a-c)$$

$$\text{while } q(\alpha) = -(\alpha - \beta_1)^2(2\alpha + \beta_1)/\{\alpha(1 - \beta_1)^2(2 + \beta_1)\}, \quad m_\theta(\alpha) = -1,$$

$$\text{and } m_r(\alpha) = v(\alpha - \beta_1)^3(\alpha + \beta_1)/\{\alpha(1 - \beta_1)^2(2 + \beta_1)\} - 1 \quad \text{when } \beta_1 \leq \alpha \leq 1, \quad (31a-c)$$

$$\text{where } \beta_1 = \{(4v^2 - 8v + 1)^{1/2} - 1\}/2v. \quad (32)$$

Equations (28) and (29) with the initial conditions $\dot{\bar{W}}(0) = \dot{\bar{W}}_1(0) = 1$, and $\bar{W}(0) = \bar{W}_1(0) = 0$ give

$$\bar{W}(\alpha, T) = T, \quad 0 \leq \alpha \leq \beta_1, \quad (33a)$$

$$\text{and } \bar{W}(\alpha, T) = T - vT^2(\alpha - \beta_1)/\{2(2 + \beta_1)(1 - \beta_1)^2\}, \quad \beta_1 \leq \alpha \leq 1. \quad (33b)$$

This phase of motion terminates when $\dot{\bar{W}}_1 = 0$ which occurs at

$$T_1 = (1+\beta_1)(1-\beta_1)^2 \quad (34)$$

and the associated dimensionless transverse displacements are

$$\bar{w}(\alpha, T_1) = (1+\beta_1)(1-\beta_1)^2, \quad 0 \leq \alpha \leq \beta_1 \quad (35a)$$

$$\text{and } \bar{w}(\alpha, T_1) = (1-\beta_1^2)(1-\beta_1/2-\alpha/2), \quad \beta_1 \leq \alpha \leq 1, \quad (35b)$$

while the corresponding dimensionless energy dissipated due to transverse shear deformations is

$$R_S = (2+\beta_1)(1-\beta_1)/3. \quad (36)$$

3.3.2 Second Phase of Motion, $T_1 \leq T \leq T_2$.

No transverse shear deformations occur during this phase of motion. The transverse velocity profile illustrated in Figure 4(c) with a circumferential hinge travelling at speed $\dot{\beta}$ is given by equations (28) with $\dot{\bar{w}}_1 = 0$ and β_1 replaced by $\beta(T)$ and is similar to that used by Wang [17] during the first phase of motion of the bending only solution for a simply supported circular plate loaded impulsively. Thus, following a theoretical procedure similar to Wang [17] and matching the velocity and displacement fields at $T = T_1$ with equations (33) shows that this phase of motion ends at

$$T_2 = 1 \quad (37)$$

when $\beta = 0$. The associated transverse displacements are

$$\bar{w}(\alpha, T_2) = 1 - \alpha^2/2 - \alpha^3/2, \quad 0 \leq \alpha \leq \beta_1 \quad (38a)$$

$$\text{and } \bar{w}(\alpha, T_2) = (1-\beta_1^2)(2-\beta_1-\alpha)/2 + \beta_1(1+3\beta_1/2)(1-\alpha), \\ \beta_1 \leq \alpha \leq 1. \quad (38b)$$

It may be shown that the transverse shear force $q(\alpha, T)$ and the other generalised stresses are statically admissible.

3.3.3 Third Phase of Motion, $T_2 \leq T \leq T_f$.

Again no transverse shear deformations develop during this final phase of motion which is governed by the transverse displacement profile in Figure 4(d). Thus, the theoretical procedure for this phase of motion is similar to that employed by Wang [17] for the final phase of motion in the bending only case and is also similar to the second phase of motion in section 3.2.2 for class II plates.

It may be shown that motion finally ceases when

$$T_f = 2, \quad (39)$$

and the final deflection profile is

$$\bar{w}(\alpha, T_f) = (1-\alpha)(\alpha^2 + 2\alpha + 3)/2 \quad \text{for } 0 \leq \alpha \leq \beta_1, \quad (40a)$$

$$\text{and } \bar{w}(\alpha, T_f) = (1-\alpha)(1 + 2\beta_1 + 3\beta_1^2)/2 + (1-\beta_1^2)(2 - \beta_1 - \alpha)/2$$

$$\text{when } \beta_1 \leq \alpha \leq 1. \quad (40b)$$

The ratio of energy dissipated in bending to that dissipated in shear is

$$R_B/R_S = (1+\beta_1+\beta_1^2)/(2-\beta_1-\beta_1^2), \quad (41)$$

where β_1 is given by equation (32).

4. Impulsive Loading of a Circular Plate with Transverse Shear and Rotatory Inertia

4.1 Plates with $0 < \nu \leq 3/2$.

It is evident that the transverse velocity field illustrated in Figure 3(b) and used to describe the behavior of the class I simply supported circular plates in section 3.1 does not involve any rotation of the plate elements. Thus, $\psi = 0$ and the rotatory inertia term in equation (1a) is zero even when $I_r \neq 0$. The theoretical analysis in section 3.1 therefore remains valid for the case when transverse shear and rotatory inertia effects are retained in the basic equations.

4.2 Plates with $\nu \geq 3/2$.

It may be shown that the transverse velocity fields illustrated in Figures 3(c,d) and 4 do not give statically admissible solutions when the influence of rotatory inertia is retained in equation (1a). For example, it may be shown that the solution of the equilibrium equations (1a,b) with the velocity field illustrated in Figure 3(c) gives a yield violation near the plate center since $m_r = -1$ and $\partial m_r / \partial \alpha < 1$ at $\alpha = 0$. It turns out that in order to satisfy the kinematic and static requirements, plastic hinges do not develop in a plate, a circumstance which was also found in Reference [5] for beams.

If $M_\theta = M_r = -M_0$ and $|Q_r| < Q_0$ throughout a plastic zone in a circular plate with $I_r \neq 0$, then equations (1a,b) give

$$\partial^2 \ddot{w} / \partial r^2 + r^{-1} \partial \ddot{w} / \partial r - \mu \ddot{w} / I_r = 0. \quad (42)$$

If $w(r, t)$ is written using the separation of variables, then the spatial dependence of w is governed by a modified Bessel equation of zero order. Thus,

$$\ddot{w} = C_1(t) I_0 \{ (\mu / I_r)^{1/2} r \} \quad (43)$$

when disregarding the usual $K_0 \{ (\mu / I_r)^{1/2} r \}$ term to avoid a singularity at $r = 0$ and where $C_1(t)$ is an arbitrary function of time, and $I_0 \{ (\mu / I_r)^{1/2} r \}$ is a modified Bessel function of the first kind of order zero. Equation (43) therefore leads to a velocity field in the plastic zone

$$\dot{w} = C(t) I_0 \{ (\mu / I_r)^{1/2} r \} + D(r), \quad (44)$$

where $C(t)$ and $D(r)$ are found from the initial conditions and the boundary conditions at the interface.

The response of a simply supported circular plate which is subjected to a uniformly distributed impulsive velocity V_0 consists of the two phases of motion illustrated in Figure 5.

4.2.1 First Phase of Motion, $0 \leq T \leq T_1$.

The transverse velocity profile illustrated in Figure 5(b), which has a central zone governed by equation (44) with a stationary axisymmetric interface at $\alpha = \beta_1$ and a stationary shear hinge at the supports ($\alpha=1$), may be written in the form

$$\dot{\tilde{w}}(\alpha, T) = 1 + \{ \dot{\tilde{w}}(T) - 1 \} I_0(a\alpha) / I_0(a\beta_1), \quad 0 \leq \alpha \leq \beta_1, \quad (45a)$$

$$\text{and } \dot{\bar{w}}(\alpha, T) = \dot{\bar{w}}_1(T)(\alpha - \beta_1)/(1 - \beta_1) + \dot{\bar{w}}(T)(1 - \alpha)/(1 - \beta_1),$$

$$\beta_1 \leq \alpha \leq 1, \quad (45b)$$

since $\dot{\bar{w}}(0, 0) = 1$ and $\dot{\bar{w}}(0) = 1$, and where

$$a^2 = 6/I, \text{ and } I = 6I_r/\mu R^2. \quad (46a, b)$$

Equations (45) give $[\dot{\bar{w}}(\beta_1, T)] = 0$ and $\partial \dot{\bar{w}}(0, T)/\partial \alpha = 0$. Furthermore, $\dot{\gamma} = 0$, $\dot{\kappa}_r \leq 0$ and $\dot{\kappa}_\theta \leq 0$ in the central plastic zone ($0 \leq \alpha \leq \beta_1$) with $\dot{\bar{w}} \leq 1$ which is consistent with the normality requirements of plasticity associated with the portion $m_\theta(\alpha, T) = m_r(\alpha, T) = -1$ and $|q(\alpha, T)| < 1$ of the yield surface in Figure 2, while in the outer region $\beta_1 \leq \alpha \leq 1$, $\dot{\gamma} = 0$, $\dot{\kappa}_r = 0$, and $\dot{\kappa}_\theta \leq 0$ if $\dot{\bar{w}}_1 \leq \dot{\bar{w}}$ and therefore $m_\theta(\alpha, T) = -1$, $-1 \leq m_r(\alpha, T) \leq 0$ and $|q(\alpha, T)| < 1$.

Now, it may be shown when substituting the above generalized stresses and velocity fields (45) into the equilibrium equations (1a, b) and when ensuring $q(0, T) = 0$, $q(1, T) = -1$, $m_r(1, T) = 0$, $[m_r(\beta_1, T)] = 0$, and $[\partial m_r(\beta_1, T)/\partial \alpha] = 0^*$ that

*It may be shown when using equation (1a) for the present case with $m_\theta = -1$ for $0 \leq \alpha \leq 1$ with β_1 time-independent

that $[q(\beta_1, T)] = 0$ may be replaced by the requirement

$$[\partial m_r(\beta_1, T)/\partial \alpha] = 0 \text{ provided } [\partial^2 \psi(\beta_1, T)/\partial T^2] = 0.$$

$$q(\alpha) = [\sqrt{6I}/v] \ddot{\bar{W}}_1(a\alpha) / I_0(a\beta_1),$$

$$m_r(\alpha) = m_\theta(\alpha) = -1, \quad \text{for } 0 \leq \alpha \leq \beta_1, \quad (47a-c)^\dagger$$

$$\text{and } q(\alpha) = (1-\alpha) \{ (3\beta_1 + 3\alpha\beta_1 - 2\alpha^2 - 2\alpha - 2) \ddot{\bar{W}}_1 - \\ - (1-\alpha)(1+2\alpha) \ddot{\bar{W}} \} / \{ v\alpha(1-\beta_1) \} - 1/\alpha,$$

$$m_r(\alpha) = -(1-\alpha)^2 \{ (3-4\beta_1 - 2\alpha\beta_1 + 2\alpha + \alpha^2) \ddot{\bar{W}}_1 + \\ + (1-\alpha^2) \ddot{\bar{W}} \} / \{ \alpha(1-\beta_1) \} - I(1-\alpha^2) (\ddot{\bar{W}}_1 - \ddot{\bar{W}}) / \{ \alpha(1-\beta_1) \} - \\ - (1-\alpha)(2v-1)/\alpha, \quad m_\theta(\alpha) = -1, \quad \text{when } \beta_1 \leq \alpha \leq 1, \quad (48a-c)$$

$$\text{where } \ddot{\bar{W}} = -[(1-\beta_1)^2 \{ 2 + \beta_1 - v(1-\beta_1)^2 \} + I\{\beta_1 + v(1-\beta_1)^2\}] / \Omega, \quad (49a)$$

$$\ddot{\bar{W}}_1 = [(1-\beta_1)^2 (1+2\beta_1) - v(1-\beta_1)^3 (1+3\beta_1) - I\{\beta_1 + v(1-\beta_1)^2\}] / \Omega, \quad (49b)$$

$$\text{and } \Omega = (1-\beta_1)^2 \{ (1-\beta_1)^2 (1+4\beta_1 + \beta_1^2) + I(3+2\beta_1 + \beta_1^2) \}. \quad (49c)$$

Thus, equations (49) with $\dot{\bar{W}}_1(0) = \dot{\bar{W}}(0) = 1$ predict

$$\dot{\bar{W}} = 1 - [(1-\beta_1)^2 \{ 2 + \beta_1 - v(1-\beta_1)^2 \} + I\{\beta_1 + v(1-\beta_1)^2\}] T / \Omega \quad (50a)$$

$$\text{and } \dot{\bar{W}}_1 = 1 - [v(1-\beta_1)^3 (1+3\beta_1) - (1-\beta_1)^2 (1+2\beta_1) + \\ + I\{\beta_1 + v(1-\beta_1)^2\}] T / \Omega, \quad (50b)$$

so that motion ceases at

$$T_1 = \Omega [I\{\beta_1 + v(1-\beta_1)^2\} - (1-\beta_1)^2 \{ 1 + 2\beta_1 - v(1-\beta_1)(1+3\beta_1) \}]^{-1} \quad (51)$$

when $\dot{\bar{W}}_1 = 0$, and the associated dimensionless transverse displacement at the supports is

$^\dagger I_0(\)$ and $I_1(\)$ are modified Bessel functions of the first kind of orders zero and one, respectively.

$$\bar{W}_1(T_1) = \Omega[2I\{\beta_1 + v(1-\beta_1)^2\} - 2(1-\beta_1)^2\{1+2\beta_1 - v(1-\beta_1)(1+3\beta_1)\}]^{-1}. \quad (52)$$

It was remarked previously that the flow rule of plasticity requires $\dot{\bar{W}}_1 - \dot{\bar{W}} \leq 0$ and $\dot{\bar{W}} \leq 1$ which leads to the restriction

$$3(1+\beta_1)/\{2(1-\beta_1)(1+2\beta_1)\} \leq v \leq \{(1-\beta_1)^2(2+\beta_1) + I\beta_1\}/\{(1-\beta_1)^2(1-\beta_1^2 - \eta)\}. \quad (53)$$

The location of the stationary interface between the two plastic zones at $\alpha = \beta_1$ is obtained from the requirement that

$$\begin{aligned} [\partial^2 \psi(\beta_1, T)/\partial t^2] &= 0^*, \text{ or } I_1(a\beta_1)/I_0(a\beta_1) = \\ &= a^{-1} (1-\beta_1) \{2v(1-\beta_1)(1+2\beta_1) - 3(1+\beta_1)\} [(1-\beta_1)^2 \{2+\beta_1 - v(1-\beta_1)^2\} + \\ &\quad + I\{\beta_1 + v(1-\beta_1)^2\}]^{-1}. \end{aligned} \quad (54)$$

This equation may be evaluated numerically to predict the position of the interface β_1 as shown in Figure 6. It turns out that the inequality (53) is satisfied up to at least $v = 50$ when the calculations were terminated.

4.2.2 Second Phase of Motion, $T_1 \leq T \leq T_f$.

The transverse velocity is zero at the supports and the dimensionless radius β of the central plastic zone decreases with time during the second phase of motion which is governed

*It was remarked in a previous footnote that the requirement

$$[q(\beta_1, T)] = 0 \text{ may be replaced by } [\partial m_r(\beta_1, T)/\partial \alpha] = 0 \text{ provided } [\partial^2 \psi(\beta_1, T)/\partial T^2] = 0.$$

by the transverse velocity profile in Figure 5(c) which is described by equations (45) with $\dot{\bar{w}}_1 = 0$ and β_1 replaced by $\beta(T)$. This velocity profile gives $[\dot{\bar{w}}(\beta, T)] = 0$ and therefore $[q(\beta, T)] = 0$ is required according to equation (4b). Furthermore, if $[m_r(\beta, T)] = 0$, then from equation (4a), $[\dot{\psi}(\beta, T)] = 0$, which leads to the expression

$$\dot{\bar{w}} = a(1-\beta)I_1(a\beta)/\{I_0(a\beta) + a(1-\beta)I_1(a\beta)\}. \quad (55)$$

Thus, the equilibrium equations (1a,b) with $q(0, T) = 0$, $m_r(0, T) = m_\theta(0, T)$, $[m_r(\beta, T)] = 0$, $[q(\beta, T)] = 0$, and $m_r(1, T) = 0$ gives

$$q(\alpha, T) = (\sqrt{6I}/v)(1-\beta)\{I_1(a\alpha)/I_0(a\beta)\}(d/dT)\{\dot{\bar{w}}/(1-\beta)\},$$

$$m_r(\alpha, T) = m_\theta(\alpha, T) = -1 \quad \text{for } 0 \leq \alpha \leq \beta, \quad (56a-c)$$

and

$$q(\alpha, T) = \{4(\alpha^3 - \beta^3) - 6(\alpha^2 - \beta^2) - 12b\beta(1-\beta)\}[2v\alpha(1-\beta)\{(1-\beta)^2(1+3\beta) + I(1+\beta) + 12b\beta(1-\beta)\}]^{-1},$$

$$m_r(\alpha, T) = (1-\alpha)\{\beta(1+\beta-\beta^2) - \alpha(1+\alpha-\alpha^2) - I(\alpha-\beta)\}\{\alpha(1-\beta)^3(1+3\beta) + \alpha I(1-\beta^2) + 12ab\beta(1-\beta)^2\}^{-1} - \beta(1-\alpha)/\{\alpha(1-\beta)\},$$

$$\text{and } m_\theta(\alpha, T) = -1, \quad \text{when } \beta \leq \alpha \leq 1, \quad (57a-c)$$

$$\text{where } b = I_1(a\beta)/\{aI_0(a\beta)\}, \quad (58a)$$

$$\text{and } (d/dT)\{\dot{\bar{w}}/(1-\beta)\} = -\{(1-\beta)^3(1+3\beta) + I(1-\beta^2) + 12b\beta(1-\beta)^2\}^{-1}. \quad (58b)$$

Equations (55) and (58b) may be solved to give the velocity of propagation ($\dot{\beta}$) of the interface at $\alpha=\beta$

$$\dot{\beta} = -\beta \{1 + a^2 b(1-\beta)\}^2 [a^2 b(1+c\beta) \{(1-\beta)^3(1+3\beta) + I(1-\beta^2) + 12b\beta(1-\beta)^2\}]^{-1}, \quad (59a)$$

where

$$c = aI_2(a\beta)/I_1(a\beta), \quad (59b)$$

and $I_2(\)$ is a modified Bessel function of the first kind of order two.

It is evident from equation (55) that when $\beta = 0$ and $T = T_f$ then $\dot{\bar{w}} = 0$ and the motion of the plate ceases. The duration of the second phase of motion may be obtained numerically from the expression

$$T_f - T_1 = \int_{\beta(T_1)}^0 d\beta/\dot{\beta} \quad (60)$$

according to equation (59a), where $\beta(T_1)$ is calculated from equation (54). It turns out that a numerical evaluation of equation (60) up to $v = 25$ when the calculations were terminated gives a total duration of response $T_f = 2$.

The maximum permanent transverse displacement at $\alpha = 0$ when $T = T_f$ may be evaluated numerically from the expression

$$\bar{w}(0, T_f) = \bar{w}(0, T_1) + \int_{\beta(T_1)}^0 \dot{\bar{w}}(0, T) d\beta/\dot{\beta}, \quad (61a)$$

$$\text{where} \quad \dot{\bar{w}}(0, T) = 1 + (\dot{\bar{w}}-1)/I_0(a\beta) \quad (61b)$$

from equation (45a) (with β_1 replaced by $\beta(T)$), and

$$\begin{aligned} \bar{w}(0, T_1) = T_1 - [(1-\beta_1)^2 \{2+\beta_1 - \nu(1-\beta_1^2)\} + \\ + I\{\beta_1 + \nu(1-\beta_1)^2\}] T_1^2 / \{2\Omega I_0(a\beta_1)\} \end{aligned} \quad (61c)$$

according to the integral of equation (45a) with $\alpha = 0$ and where T_1 is given by equation (51).

5. Discussion

It may be shown that the theoretical analyses presented in sections 3 and 4 are kinematically and statically admissible and therefore exact within the setting of classical plasticity for the yield surface in Figure 2. The amount of shear sliding at the plate supports in these analyses should satisfy the criterion represented by equation (14) as discussed in Reference [21]. In addition, the material is assumed to be strain rate insensitive, and in order to remain consistent with an infinitesimal theory the difference between the maximum transverse displacements at the plate center and the transverse shear sliding at the supports should be less than the plate thickness, approximately.

The theoretical analysis in section 3 with $I = 0$ and a finite transverse shear strength ($\nu < \infty$) is compared in Figures 7 and 8 with the theoretical predictions of Wang [17] which retains neither transverse shear ($\nu = \infty$) nor rotatory inertia ($I = 0$) effects. Incidentally, the various equations in section 3 with $\nu \rightarrow \infty$ reduce to the corresponding theoretical predictions in Reference [17]. It is evident from Figures 7 and

8 that transverse shear effects play an important role when ν is small, as expected. However, the results in Figures 7 and 8 with $I = 0$ and $\nu > 5$, approximately, are similar to those of Wang, although Figure 9 reveals that a significant portion of the initial kinetic energy is dissipated through shearing deformations at the supports for larger values of ν . The theoretical solution in Reference [8] for a simply supported circular plate subjected to a uniformly distributed static pressure indicates that transverse shear effects do not influence the static collapse behavior for the yield surface in Figure 2 when $\nu \geq 3/2$. Thus, the present study demonstrates that transverse shear effects are more important for the dynamic case than for the corresponding static problem as also found in Reference [20] for beams and discussed in References [4] and [5]. It should be noted that $\nu = R/H$ for the particular case of a circular plate having a solid homogeneous cross-section with $Q_0 = \sigma_0 H/2$ and $M_0 = \sigma_0 H^2/4$. On the other hand, if a circular plate is constructed with a sandwich cross-section, then an inner core of thickness h and a shear yield stress τ_0 supports a maximum transverse shear force $Q_0 = \tau_0 h$ (per unit length), while thin exterior sheets of thickness t can independently carry a maximum bending moment $M_0 = \sigma_0 t(h + t)$, where σ_0 is the corresponding tensile yield stress. In this circumstance $\nu = Q_0 R/2M_0$ gives

$$\nu = \left(\frac{R}{H} \frac{\tau_0}{\sigma_0/2} \right) \left\{ \frac{h/H}{1 - (h/H)^2} \right\}$$

when $H = h + 2t$. Thus, a sandwich plate with $2R/H = 15$, $\sigma_o/2\tau_o = 8$, and $h/H = 0.735$ (e.g., a 0.5 in. thick core with 0.1 in. sheets gives $h/H = 0.714$) gives $\nu = 1.5$ for which transverse shear effects are very important according to the results in Figure 7.

It is evident from Figure 7 that the inclusion of rotatory inertia in the governing equations and the retention of transverse shear as well as bending effects in the yield criterion leads to an increase in the permanent transverse shear sliding at the plate supports and a decrease in the maximum final transverse displacement which occurs at the plate center. However, the inclusion of I gives rise to respective changes in these quantities of approximately 11.5% and 14.2% at most. Thus, the simpler theoretical analysis in section 3 with $I = 0$ would probably suffice for most practical purposes. If greater accuracy is required, then it is only necessary to include I for circular plates with $1.5 \leq \nu \leq 4$, approximately.

The duration of response $T_f = 3/\nu$ is independent of rotatory inertia effects when $\nu \leq 3/2$. Furthermore, $T_f = 2$ is independent of both I and ν when $\nu \geq 3/2$.

It turns out that the theoretical analysis for the impulsively loaded simply supported circular plate presented herein has many features in common with the corresponding theoretical solution for an impulsively loaded simply supported beam which was discussed in References [5] and [22]. A beam with $I = 0$ has three classes of motion $\nu \leq 1$, $1 \leq \nu \leq 1.5$, and $\nu \geq 1.5$ and transverse velocity profiles associated with

each of these regions are similar to those in Figures 3 and 4 here for the three classes of plate behavior examined in section 3. Two classes of behavior occur for impulsively loaded simply supported beams with $v \leq 1$ and $v \geq 1$ and $I \neq 0$ [5]. The corresponding transverse velocity profiles are similar to those found in section 4 here.

6. Conclusions

A theoretical solution for an impulsively loaded circular plate made from a rigid perfectly plastic material has been developed when the transverse shear force as well as bending moments are retained in the yield condition and the influence of rotatory inertia is included in the governing equations. Transverse shear effects are important for small values of $v(Q_0 R/2M_0)$, as expected, while rotatory inertia can further decrease the maximum permanent transverse displacement up to about 14 per cent when $v > 1.5$. Thus, the simple theoretical analysis with $I = 0$ in section 3 should suffice for most practical purposes, except possibly for circular plates with $1.5 \leq v \leq 4$, approximately, when greater accuracy is required.

Acknowledgements

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Titles of Figures

Figure 1. Element of a Circular Plate.

Figure 2. Yield Surface.

Figure 3. (a) Impulsive Loaded Circular Plate, (b) Velocity Profile for Class I Plates, (c) Velocity Profile for the First Phase of Motion for Class II Plates, (d) Velocity Profile for the Second Phase of Motion for Class II Plates.

Figure 4. (a) Impulsively Loaded Circular Plate with $v \geq 2$, (b), (c), (d) are the Dimensionless Velocity Profiles for the First, Second, and Third Phases of Motion for Class III Plates.

Figure 5. (a) Impulsively Loaded Circular Plate with $v \geq 3/2$ and $I_r \neq 0$. (b), (c), Velocity Profiles for the First and Second Phases of Motion.

Figure 6. Variation of β_1 with v , where $I = 1/2 v^2$ for a circular plate with a solid cross-section.

_____ equation (32), -----equation (54).

Figure 7. Variation of Maximum Permanent Transverse Displacements at Plate Center ($\bar{W}_f$) and Plate Supports ($\bar{W}_1$).

Figure 8. Permanent Deformed Profiles of Circular Plates ($0 \leq \alpha \leq 1$).

Figure 9. Proportion of Initial Kinetic Energy Absorbed Due to Shearing (R_S) and Bending (R_B) Deformations.

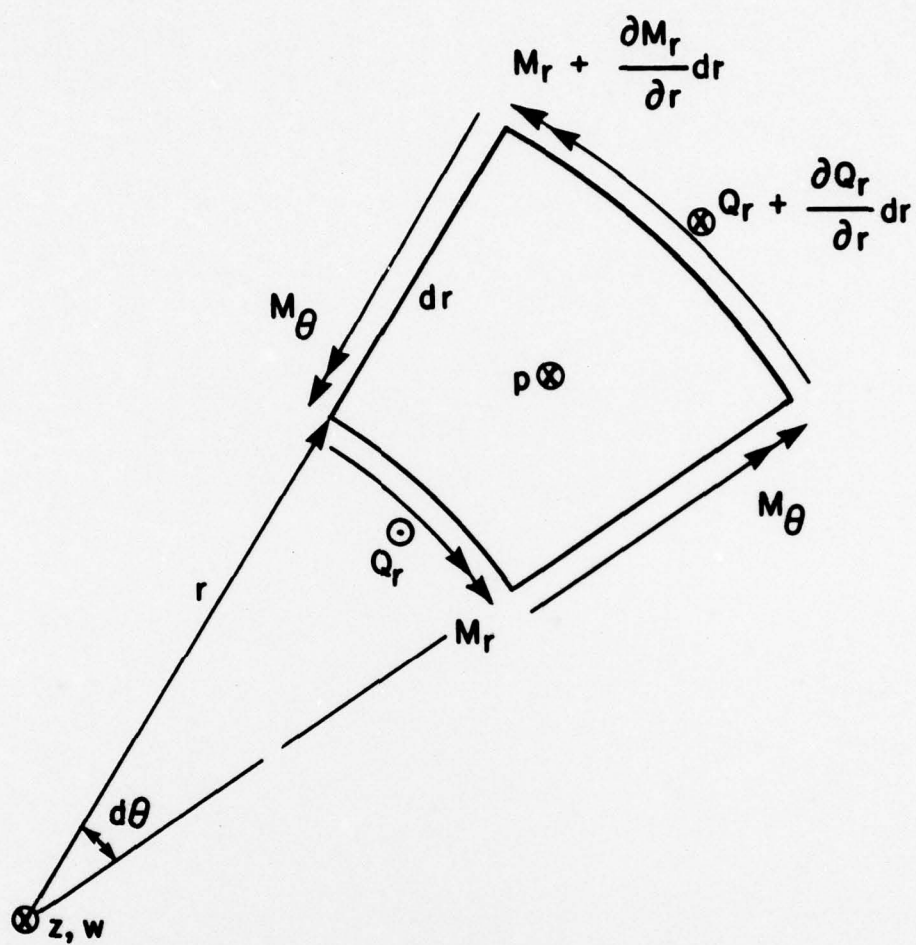


FIGURE 1

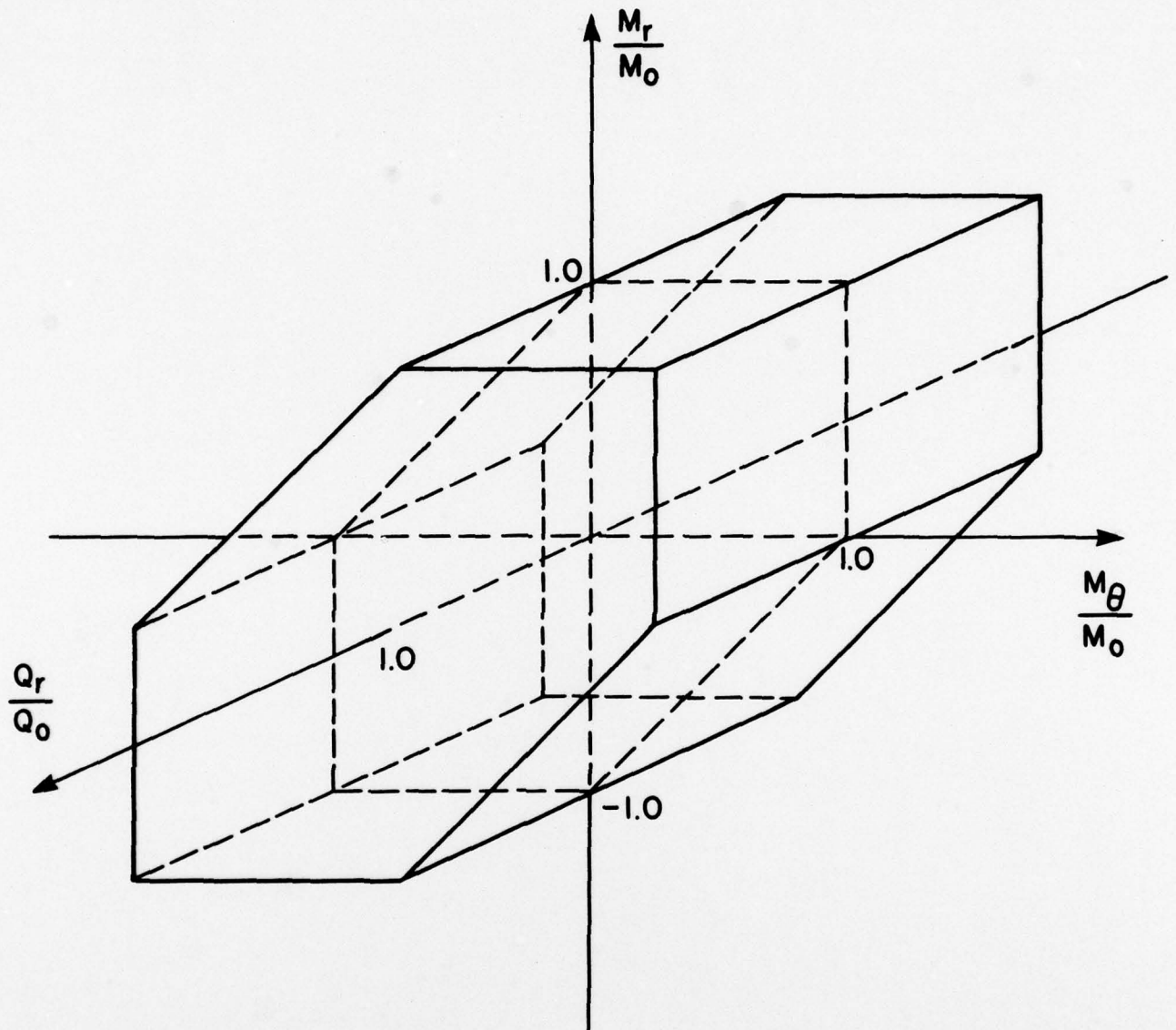


FIGURE 2

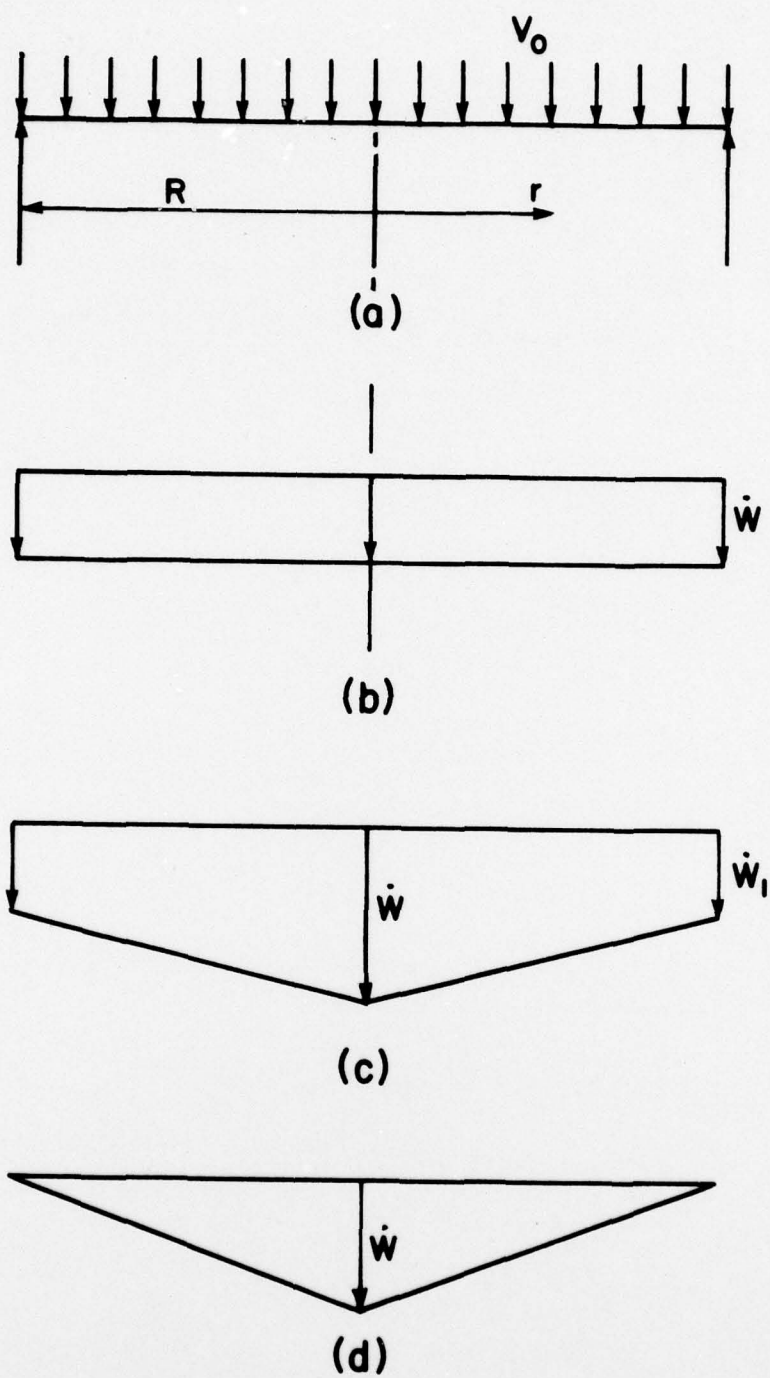
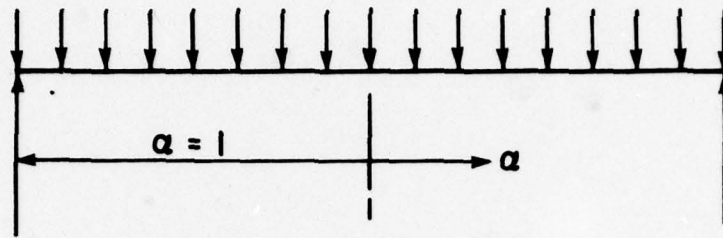
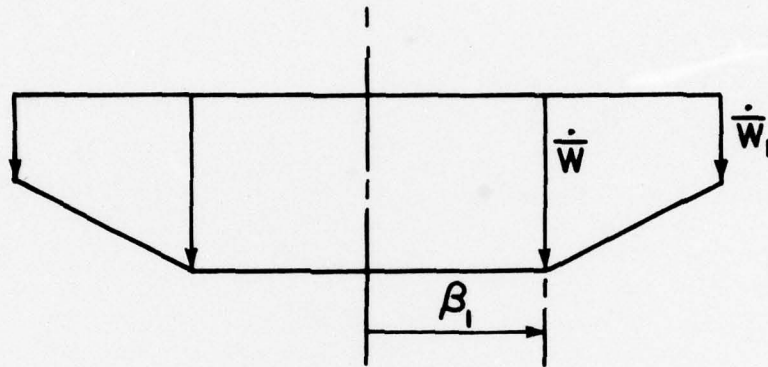


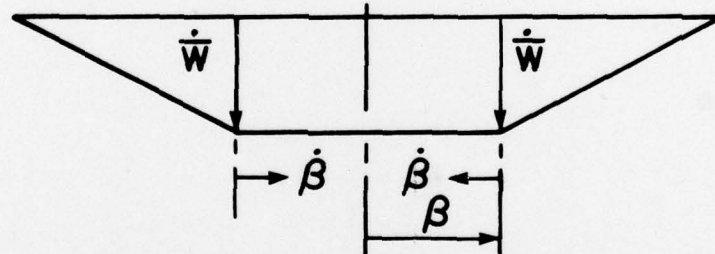
FIGURE 3



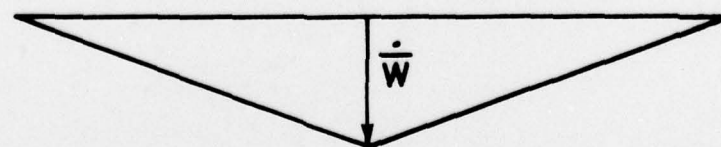
(a)



(b)



(c)



(d)

FIGURE 4

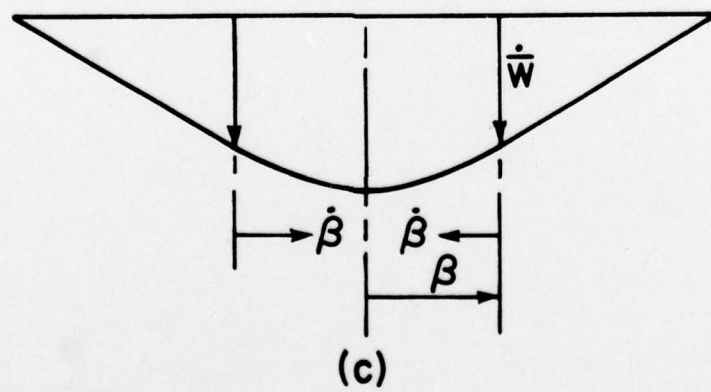
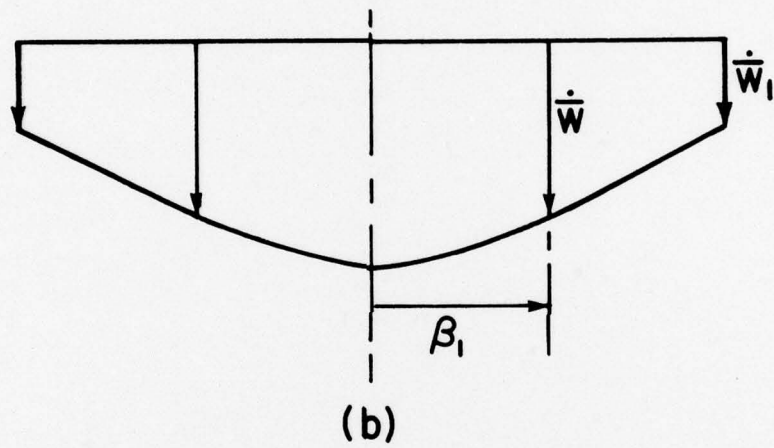
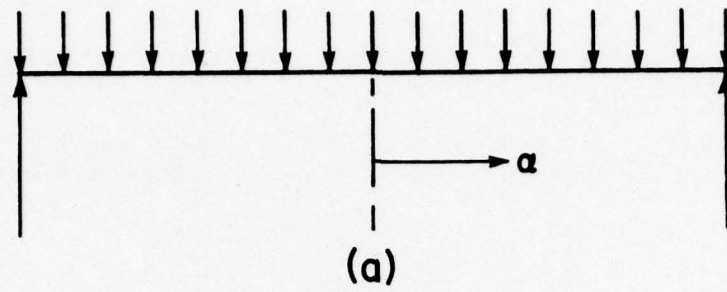


FIGURE 5

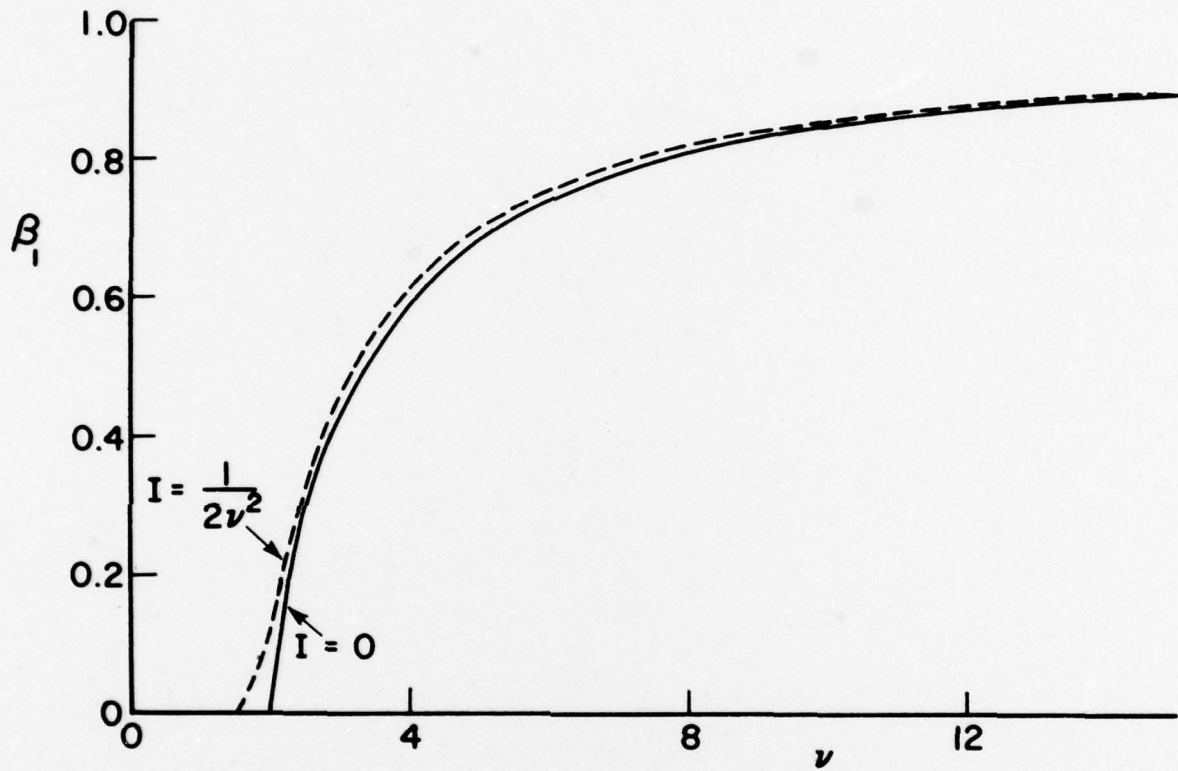


FIGURE 6

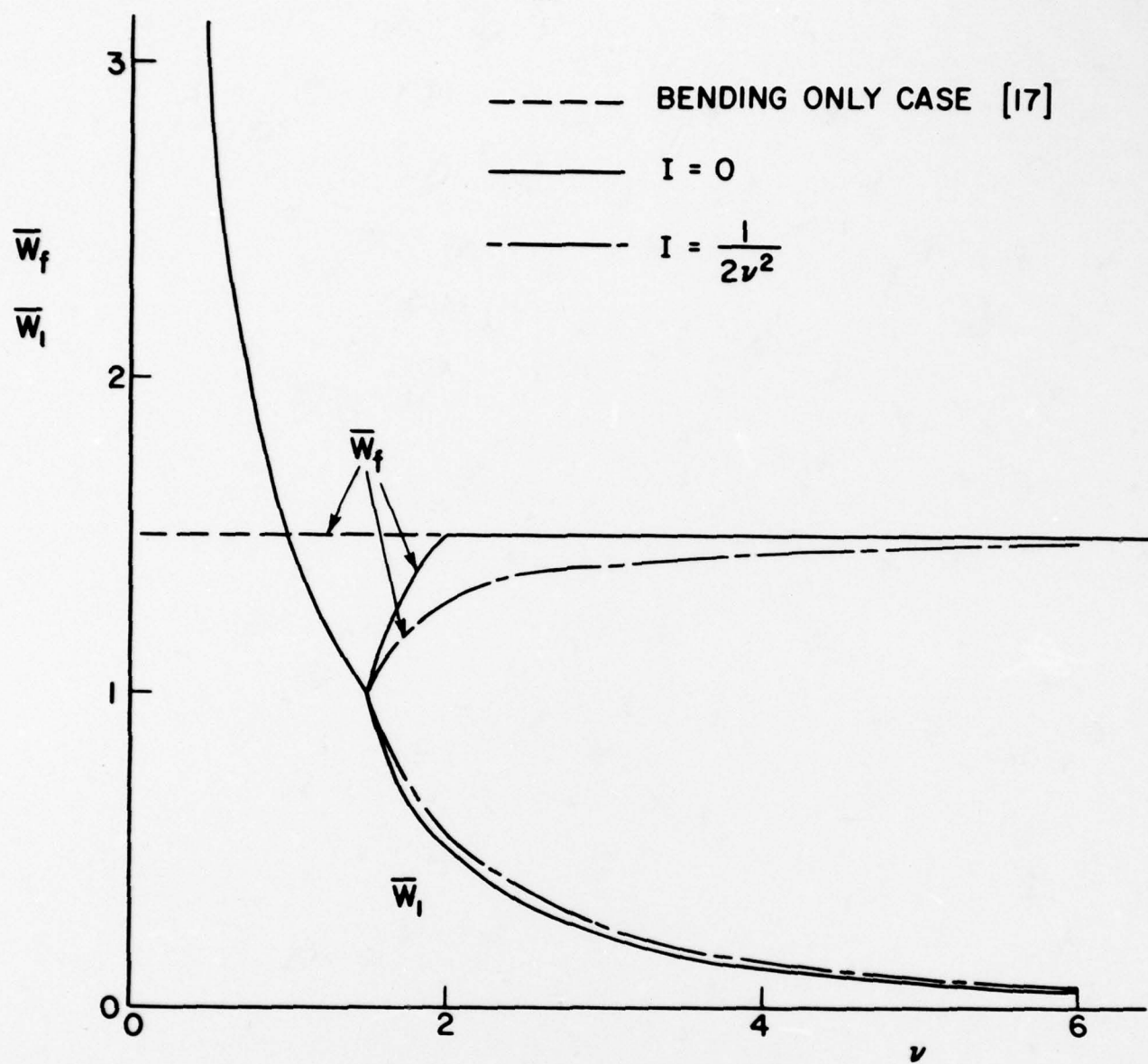


FIGURE 7

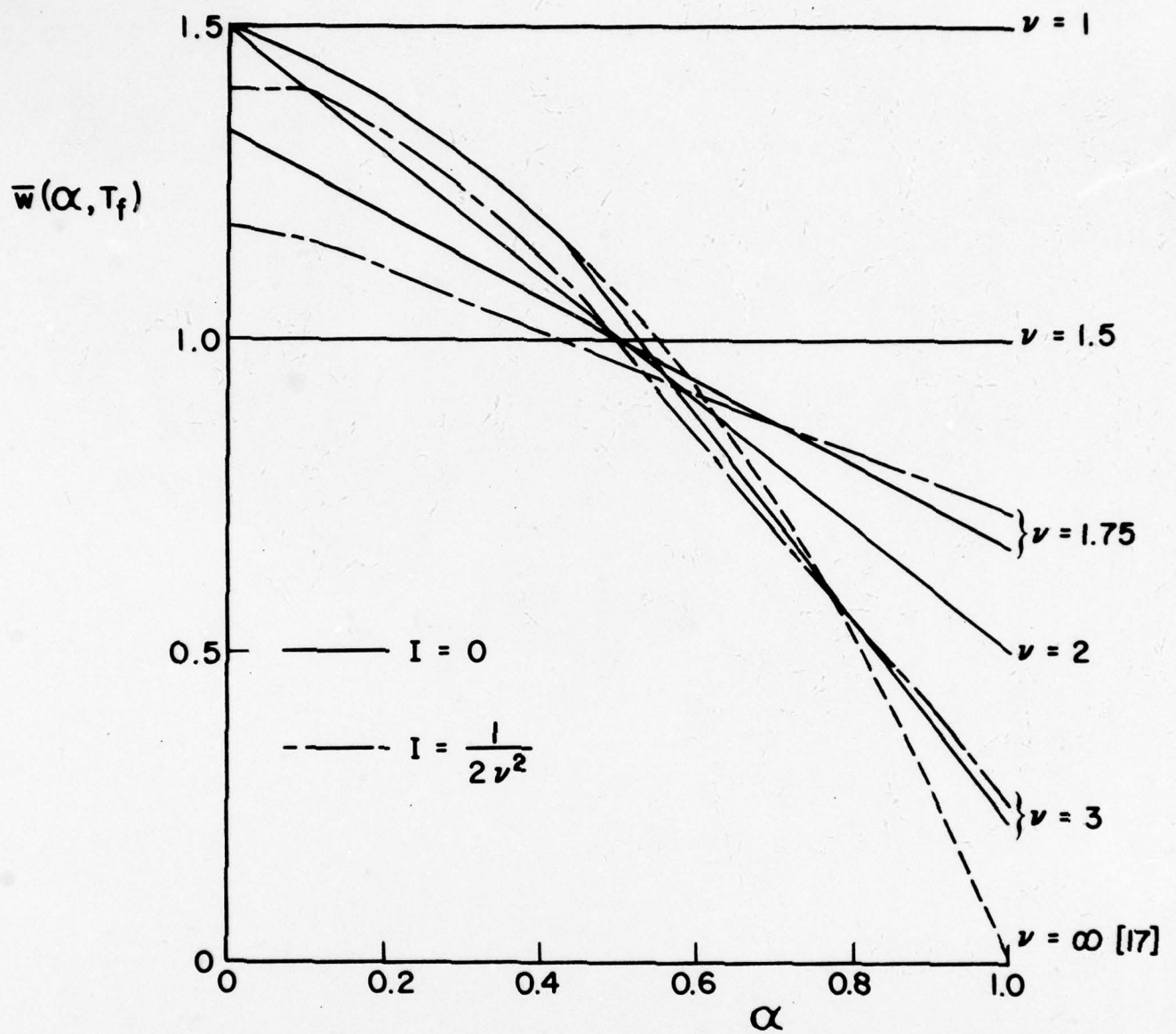


FIGURE 8

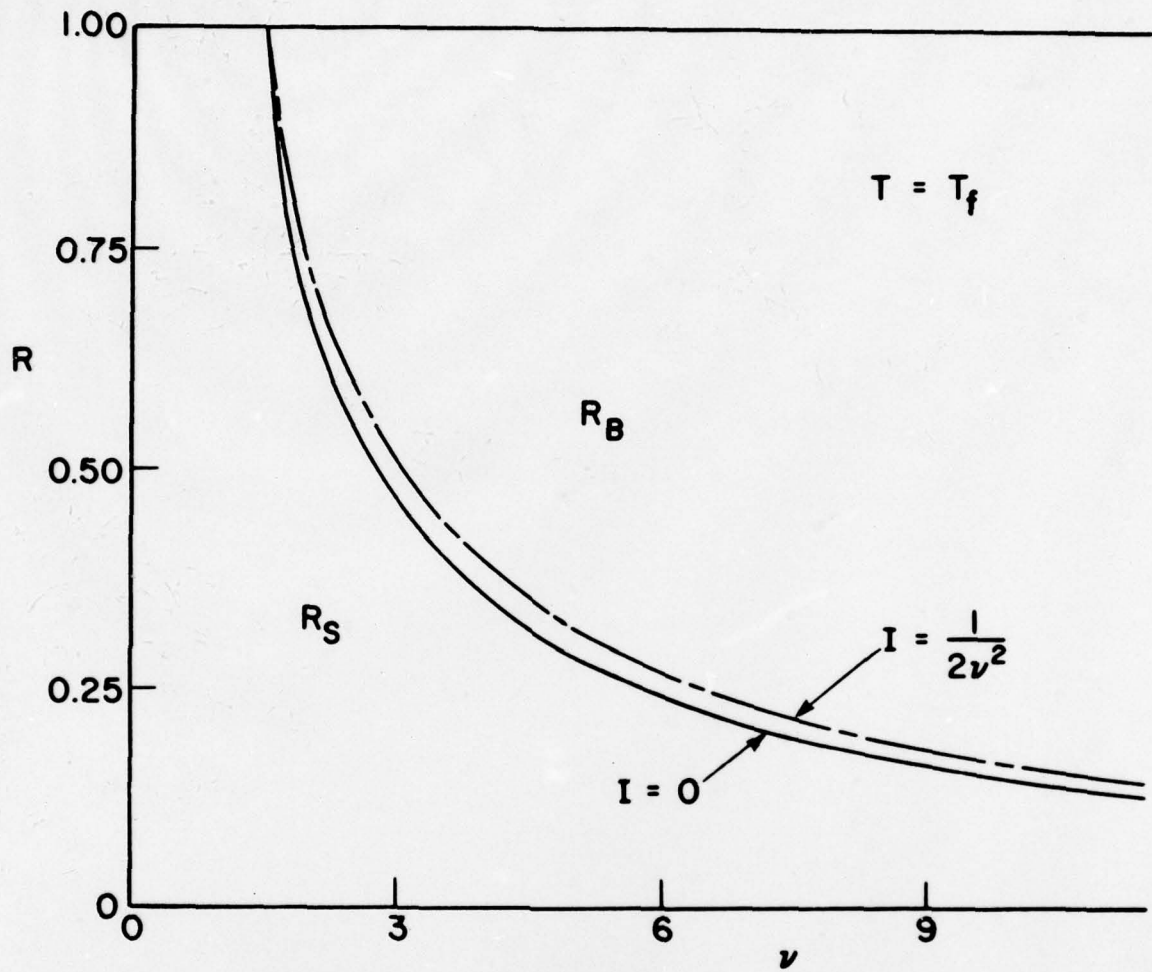


FIGURE 9

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presented which may be used to assess the importance of transverse shear effects and rotatory inertia for this particular problem.

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