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PRELIMINARY CRITERIA FOR OPTIMIZING THE COST EFFECTIVENESS OF SYSTEM IMPROVEMENTS TO ENHANCE SURVIVABILITY

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The Lanchester law of combat has been used to develop a figure of merit for survivability. The economic principle of marginal utility has been applied to demonstrate a proposed method of evaluating the cost-effectiveness of various possible survivability enhancing improvements.

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INTRODUCTION

In the AMSAA Survivability Primer (ref. 1), survivability is defined as having four main ingredients:

Detectability
Hitability
Vulnerability
Repairability

The aspect of survivability that is emphasized in the AMSAA primer is the design techniques needed and used to enhance the survivability of a given weapon system. The primer makes no attempt to consider the factors involved in determining whether it is economically viable to make a given system change. However, experienced system designers are well aware of the tension between the various aspects of system effectiveness but often a "seat-of-the-pants" adjustment is the only technique available to make these trade-offs.

Recently, Dr. Frank Grubbs of ARRADCOM's Ballistic Research Laboratory prepared a DARCOM manual (ref. 2) on the evaluation of weapon systems which includes a substantial chapter on survivability. This chapter discusses the reasons why survivability can best be evaluated as a part of weapon systems effectiveness. To do this, Dr. Grubbs defines kill rate as:

Kill Rate = rate of fire X probability of hit X probability of kill given a hit

Given this definition, kill rate becomes a fundamental parameter in a wide variety of situations. Several chapters of the manual discuss these situations as summarized below:

- Effectiveness of a Weapon for Stochastic Duels (Chapter 17)
- Target Coverage and Target Damage Studies (Chapter 21)
- Combat Theory (Chapters 28, 29)
- Weapon Equivalence Studies (Chapter 30)
- Optimal Firing Studies (Chapter 31)
- Weapon Allocation Problems (Chapter 32)

The Weapons Systems Analysis Manual goes on to say that "SURVIVABILITY IS AN ELEMENT OF SYSTEMS ANALYSIS."

The point of view adopted in this report is that the evaluation of surviability cannot be separated from the evaluation of effectiveness of the weapon system.

A case in point is the well known situation of the machine gun in WW1. Because it was so effective when used to protect the approaches to the trenches, it became the principal target of all other weapons. This was the reason that the machine gun nest was celebrated in both fact and fiction as the focal point of every attack until silenced. A major reason for the use of extensive artillery fire preceding infantry attacks was to reduce the effectiveness of the machine gun.

To make the machine gun more survivable, all possible efforts were made to provide protective earthwork fortifications. Furthermore, supporting fields of fire and positions on commanding terrain features were chosen both to increase its killing effectiveness and to increase its survivability. The mere fact that it was so effective (and thus drew enemy fire) decreased the length of time that it (or its crew) survived.

This example is only one of many that can be found to support the conclusion that survivability of a specific weapon system can only be evaluated in terms of a specific military environment, and includes its own effectiveness as a parameter. The mathematical arguments which follow provide a quantifiable analysis of this conclusion.

Measures of Survivability

A payoff function for survivability could be given as the number of items of a weapon system "killed" by the enemy per unit time, less the number of items which are restored to effective action. This statement is extremely difficult to quantify on a theoretical or empirical basis, although many people have attempted to do so (ref. 2). Many factors must be considered (refs. 2,3). To enhance the understanding of the problem, various sketchy or schematic analytic models are used in the following discussion. The theoretical results from these models will be considered, beginning with the elementary Lanchester Square Law and proceeding from there to more complex assumptions.

The basic idea behind all these models is best expressed in terms of kill rate in the terminology of the DARCOM manual. The probability of success of blue is roughly proportional to the ratio of blue kill rate to the total kill rate. Let the probability of blue winning be [P(B)]. It is related to the kill rates of blue and red [KR(BR) and KR(RB)] as follows:

$$P(B) = \frac{KR(BR)}{KR(BR) + KR(RB)} \qquad (1)$$

Lanchester Square Law

As the simplest approach, the Lanchester Square Law may be expressed as follows: the number killed (of blue) is proportional to the number (n) of the red systems present. This may be written as the product: Bn.

This model incorporates the design of the blue weapons only through the value of B, the factor of proportionality. But B also depends on the effectiveness of the red weapons, which is under the control of the red designers. The red designer can increase the number of blue systems killed by increasing either B or n or both. The decision as which to do may be in economic terms or in terms of the cost of red lives.

If one wished to use a "percentage" model (i.e., what fraction of friendly systems survive each combat situation), then the fraction remaining can be approximated as: [one minus (the above kill rated divided by the number of friendly items in combat)] plus the fraction repaired as follows:

$$1 - Bn/m + (P_r/m) \int_o^t Bn\,dt \qquad (2)$$

P_r is the probability of repair of an item, for example a tank which has been "killed." This formula reflects the truism that if a large number of tanks are killed because of an overwhelming force ratio (i.e., n/m is large), the only way to fight is to repair those tanks put out of action. This would require a tank design (and a supporting logistics system) so that P_r is large. The logistics system design must, therefore, take into account those units of the system most likely to be damaged in combat.

Those units, and the equipment needed to repair or replace them, are probably not the same as those which must be replaced during the peacetime life cycle of the major item involved. Thus the supply of replacement/spare parts and the training of personnel needed to make these repairs quickly in the field are objects of intense study by the logistics community at this time.

The question is: Is the rapid repair and return to combat a viable concept?

Arguing against this policy is the "principle of war" called pursuit. When used by an aggressor, pursuit indicates that a commander should take measures which would prevent the recovery and repair of most damaged vehicles.

A potential enemy strategy (or, for that matter, our strategy against an enemy) is to prevent the recovery and repair of our damaged vehicles by conducting mopping up operations using "scorched earth" procedures and other follow-up methods which prevent the exploitation of the damaged materiel in any meaningful manner. If a damaged vehicle comes within his control, even for a short period of time, the enemy could use a Molotov cocktail or a special incendiary bomb against it. These measures would be very effective in preventing retrieval and repair of a combat vehicle no matter what the state of the conflict. Thus, relying on the repair of a significant number of systems as an important concept appears to place our combat forces at a serious disadvantage.

In this connection we note from equation 1 that the larger the ratio Bn/m becomes, the more tanks there are that can be repaired. On the other hand, the larger the ratio, the greater is the likelihood that red will force blue to retreat, and blue will lose the opportunity to repair immobile vehicles. Because of the retrograde movement, it is more likely that the damaged vehicles will be captured and/or destroyed by the enemy. As a result, in a typical European scenario, payoff on combat "repairability" is likely to be small except for those tanks and other vehicles which do not suffer a mobility kill. In any event, it does not deserve a primary emphasis for combat.

On the other hand, for noncombat operations, repairability as a part of the integrated logistics system (ILS) has a large payoff. The reason for the difference is clear; the noncombat failures and the subsequent repairs occur on vehicles which remain in our possession as long as they can move or be moved.

Search and Destroy Model

Returning to the question of combat models, up to this point in our discussion we have considered an extremely simple model because only simple models can be discussed in a general way. To include "detectability" and "hitability" in any adequate way in the analysis, we must proceed to a model which is more complex but yet simple enough to illustrate the problems--the search and destroy model.

In this model, which is the next level of complexity, the mode of delivery by the weapon is assumed to be "indirect fire." The problem requires that we define the number of weapons detected and the number of those detected which are destroyed.

Following the BDM notation in equation 3 as far as possible, we define an attrition coefficient which is the rate at which a given weapon system destroys a particular enemy system.

Let m_i be the number of the blue systems of the "ith" type. Let A_{ji} be the blue attrition coefficient. Then $A_{ji}\, m_i$ is the number of red systems of the "jth" type destroyed by the blue systems of the "ith" type.

To calculate A_{ji}, one must calculate the number of red systems found as a result of the target acquisition system operation. To do this, the fraction of the "jth" red system found per unit time by the "ith" blue system is defined as α_{ij}, and the number is n_j. The product, $\alpha_{ij}\, n_j$, is the number wanted: A_{ji}.

This number may vary due to both terrain and to the combat situation, i.e., whether blue is defending, attacking, or in a "meeting" engagement. The next step Is to estimate the number of red systems destroyed.

Let Q_{ji} be the probability that an attack by the "ith" blue system is successful in killing one red system of the "jth" type.

Then P_{ji}, which equals $(1-Q_{ji})$, is the probability of survival of one attack, and $(1-(P_{ji})^k)$ is the probability that the particular red system of the "jth" type is killed by blue in "k" attacks.

Now to find the rate of killing, we multiply the number found, $\alpha_{ij}\, n_j$, by the fraction killed as follows:

$$\alpha_{ij}\, n_j\, [1 - (P_{ji})^{k}]$$

The number of attacks, k, may be assumed to be proportional to the number m_i of the blue "ith" system present on the battlefield and within range. Let the factor of proportionality be γ_{ji} and the result becomes

$$R_{ji} = \alpha_{ij}\, n_j\, [1 - (P_{ji})^{\gamma_{ji}\, m_i}]$$

[Temporarily we adopt R_1 as a symbol standing for the rate at which the "ith" blue system kills the "jth" red system. R_2 is the rate for red against blue.]

As a figure of merit for survivability analysis we may take the ratio of these two rates, say R_1/R_2. It is the blue object to make this ratio as large as possible, and the red object to make it small.

$$R_1/R_2 = \frac{\alpha_{ij}\, n_j\, [(1 - (P_{ji})^{\gamma_{ji} m_i}]}{\hat{\alpha}_{ji}\, m_i\, [(1 - (\hat{P}_{ij})^{\hat{\gamma}_{ij} n_j}]}$$

The expression for R_1 can be further simplified under the assumption that the number of kills per attack is small and thus the expression $(P_{ji})^k$ can be simplified as follows:

$$(P_{ji})^k = (1-Q_{ji})^k = 1 - kQ_{ji} + \frac{k(k-1)}{2}\ (Q_{ji})^2 + \cdots - \cdots$$

$$\simeq 1 - kQ_{ji}. = 1 - (\gamma_{ji} m_i)\ Q_{ji}$$

On substituting this in the expression for R_1/R_2 it becomes:

$$\frac{R_1}{R_2} = \frac{\alpha_{ij}\, n_j\, \gamma_{ji}\, m_i\, Q_{ji}}{\hat{\alpha}_{ji}\, m_i\, \hat{\gamma}_{ij}\, n_j\, Q_{ij}} = \frac{\alpha_{ij}\, \gamma_{ji}\, Q_{ji}}{\hat{\alpha}_{ji}\, \hat{\gamma}_{ij}\, \hat{Q}_{ij}}$$

This ratio obviously depends on the characteristics of both the weapon and the target. Thus, the accuracy of the weapon fire control system, the time of flight of the round, the time to prepare to fire, the design of the lethal mechanism (i.e., k.e., or chemical effects)--all of the characteristics related to the weapon must be evaluated in terms of the target hitability (speed and maneuverability) and vulnerability.

Thus, blue has control by design of all factors of the ratio R_1/R_2, except for the factor $\hat{\gamma}_{ij}$ which relates to the range and speed of reaction of the red system.

Remember at this point that ratio R_1/R_2 used above applies only to a duel between the "ith" system of blue and the "jth" system of red. It is quite obvious that the general expression, when summed over all systems, does not reduce to such a relatively simple number. However, the discussion of the role of survivability in system design is simplified by this assumption. Since it remains complex even when this assumption is made, the assumption will be continued for the purpose of discussion.

We now multiply R_1/R_2 by $\hat{\gamma}_{ij}$ to define a payoff function P_1 which is to be optimized under economic constraints:

$$P_1 = \hat{\gamma}_{ij} R_1/R_2 = \frac{\alpha_{ij} \gamma_{ji} Q_{ji}}{\hat{\alpha}_{ji} \hat{Q}_{ij}}$$

Economics of Survivability

If we assume that the economic principle of marginal utility (ref. 4) may be applied to weapon system optimization, we may choose a strategy for determining the trade-off's or the optimum utilization of improvement funds.

To apply the principle of marginal utility, we start with the payoff function P_1, for blue as defined previously. The derivative of P_1 with respect to cost is obtained from this expression in order to proceed with the application of the principle. In accord with the principle, the characteristic of the system which has the highest relative value, i.e., gives the greatest increase in effectiveness (survivability) per dollar, would be that chosen to be improved. If that particular improvement were to be made and a decision is pending to further improve the survivability, the derivatives are evaluated at the new point and the indicated improvement made. The details of this procedure follow.

If we take the payoff function for the U.S. Forces to be given as:

$$\frac{\alpha_{12}\,\gamma_{21}\,Q_{21}}{\hat{\alpha}_{12}\,\hat{Q}_{12}} = P_1$$

Then let us take the logarithmic derivative with respect to costs in dollars as follows:

$$\frac{d\ln P_1}{dC} = \frac{\partial \ln P_1}{\partial \alpha_{12}}\frac{\partial \alpha_{12}}{\partial C} + \frac{\partial \ln P_1}{\partial \gamma_{21}}\frac{\partial \gamma_{21}}{\partial C} + \frac{\partial \ln P_1}{\partial \hat{\alpha}_{12}}\frac{\partial \hat{\alpha}_{12}}{\partial C} +$$

$$\frac{\partial \ln P_1}{\partial Q_{21}}\frac{\partial Q_{21}}{\partial C} + \frac{\partial \ln P_1}{\partial \hat{Q}_{12}}\frac{\partial \hat{Q}_{12}}{\partial C}$$

Let us now assume for illustration that the partial derivatives [$\partial\alpha_{12}/\partial C$, $\partial\gamma/\partial C$ etc.] are "constants" whose value may be obtained. [If this is not valid, we have the option of saying that for a finite increment $\Delta\alpha_{12}$, there is a given incremental cost ΔC.]

$$\frac{d\ln P_1}{dC} = \frac{1}{a_{21}}\frac{\partial \alpha_{21}}{\partial C} + \frac{1}{\gamma_{21}}\frac{\partial \gamma_{21}}{\partial C}$$

$$+ \frac{1}{Q_{21}}\frac{\partial Q_{21}}{\partial C} - \frac{1}{\hat{\alpha}_{12}}\frac{\partial \hat{\alpha}_{12}}{\partial C} - \frac{1}{\hat{Q}_{12}}\frac{\partial \hat{Q}_{12}}{\partial C}$$

We then evaluate each of the ratios $\frac{1}{\alpha_{12}}\frac{\partial \alpha_{12}}{\partial C}$; $\frac{1}{\gamma_{21}}\frac{\partial \gamma_{21}}{\partial C}$ etc. and rank order them in by putting the largest ratio first, then the next largest, etc. Assume $\frac{1}{\alpha_{21}}\frac{\partial \alpha_{21}}{\partial C}$ is first and $\frac{1}{\hat{\gamma}_{21}}\frac{\partial \gamma_{21}}{\partial C}$ is second.

We now spend the money available to improve α_{21} and we continue spending money ($\Delta\$$) up to the point where improving α_{12} is equal in cost-effectiveness to improving γ_{21}. This may be expressed mathematically as

$$\frac{1}{\alpha_{21}(o) + \frac{\partial \alpha_{21}}{\partial C} \Delta\$_{\alpha}} \quad \frac{\partial \alpha_{21}}{\partial C} = \frac{1}{\gamma_{21}} \quad \frac{\partial \gamma_{21}}{\partial C}$$

On solving this for $\Delta\$$, we find that:

$$\Delta\$_{\alpha} = \frac{\gamma_{21}(o)}{\frac{\partial \gamma_{21}}{\partial C}} - \frac{\alpha_{21}(o)}{\frac{\partial \alpha_{21}}{\partial C}}$$

Next, both γ_1 and α_{21} are improved up to the point where:

$$\Delta\$_{\gamma} = \frac{Q_{21}}{\frac{\partial Q_{21}}{\partial C}} - \frac{\gamma_{21}(o)}{\frac{\partial \gamma_{21}}{\partial C}}$$

At this point, funds would then be provided for all three (α, γ, Q). and so on. This procedure will give maximum "utility" for each dollar spent. No linear programming is required to obtain the spending strategy.

Survivability in the Life Cycle of Development

In the foregoing simplified economic analysis, the responsibility of the system manager to spend the initial acquisition funds to optimize the effectiveness-survivability is clear. It is also clear that an economic scheme may be derived to determine the optimum use of funds to improve an existing system.

What may not be so clear is how Research and Exploratory Development funds should be used and what should be the role of the ARRADCOM laboratories in developing the technology of survivability for gun-related systems.

Of course, the laboratories will support the work of the P.M.'s in survivability; but their function in supporting a survivability technology is the primary question and is not the same as supporting the P.M.'s.

Clearly, ARRADCOM's gun-oriented laboratories must strive to increase the range, the probability of hit and kill, and the ability to detect enemy targets. They also need to know the detectability of friendly weapon systems by enemy weapon systems, the probability of being hit, and, if hit, the vulnerability of a given item. We conclude that their mission is to provide technology which will reduce the cost to achieve a given effectiveness. To do this they must know or develop the knowledge of the derivatives, i.e., they must know or determine the dollar cost for an increment in effectiveness due to a change in each factor in the survivability models.

In addition, in some cases, increasing the effectiveness of one characteristic may well decrease another. For example, reducing the smoke and/or flash may reduce the range more than it decreases the detectability and thus reduce the combined effectiveness/survivability figure of merit. Thus the technology base must include the interaction terms.

The technology of weapons which leads to increased lethality and range in a smaller package also affects the detectability and vulnerability, for it allows the system to be made smaller and thus less detectable, less killable, but also more maneuverable (thus less hitable).

The conclusion is that the emphasis for ARRADCOM's Large Caliber and Fire Control and Small Caliber Weapon Systems Laboratories should be on the offensive part of system survivability and on the data base used to analyze a projected survivability related improvement.

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