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RELIABILITY AND LAUNCHING POLICY FOR A SYSTEM OF SATELLITES.(U)

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RELIABILITY AND LAUNCHING POLICY FOR A SYSTEM OF SATELLITES

by

Theresa F. Klaschka

Procurement Executive, Ministry of Defence
Farnborough, Hants

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17. Abstract

This Report analyses the reliability of a system of satellites in which a number of satellites are launched at intervals. The system is considered operational if at least one satellite in orbit is functioning. Analytical methods were investigated for determining a launching policy which would optimise some suitable system reliability parameter, but only a near-optimal policy could be found. The equations derived provide expressions for the reliability and mean down time of the system within each time interval between successive launches and throughout the mission. The reliability model which was used included redundancy within the satellite, a launch success probability, and a satellite cut-off time (or end of life) due to fuel exhaustion.

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(6) RELIABILITY AND LAUNCHING POLICY FOR A SYSTEM OF SATELLITES.

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SUMMARY

This Report analyses the reliability of a system of satellites in which a number of satellites are launched at intervals. The system is considered operational if at least one satellite in orbit is functioning. Analytical methods were investigated for determining a launching policy which would optimise some suitable system reliability parameter, but only a near-optimal policy could be found. The equations derived provide expressions for the reliability and mean down time of the system within each time interval between successive launches and throughout the mission. The reliability model which was used included redundancy within the satellite, a launch success probability, and a satellite cut-off time (or end of life) due to fuel exhaustion.

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LIST OF CONTENTS

1 INTRODUCTION

Satellite reliability over a required mission time has conventionally been achieved by employing high reliability design techniques and by incorporating redundancy within the satellite. However, a situation can be reached where, in order to achieve very long mission durations, it is preferable to employ a system of redundant satellites launched at suitable intervals to obtain a high probability of continuous service throughout the mission. In this case the reliability of the satellite system, rather than the reliability of the individual satellites, must be considered. Furthermore, the timing of satellite launches affects system reliability, and therefore the launch policy should be chosen to optimise some suitable system reliability parameter. This Report describes an analytical approach to these problems, making use of simplified models and approximations where appropriate.

The assumed satellite reliability model is defined in section 2.1. The reliability of the satellite system in a general interval between two successive launches is derived in section 2.2, as a function of the separate satellite reliabilities. In section 3, the mean down time of the system in a general interval between successive launches is found in terms of the system reliability. These general results are then applied to particular cases in section 4.

The ways in which these results may be used are discussed in section 5. The mean fractional down time in each interval may be constrained to equal the mean fractional down time allowed for the mission, thus determining the launch intervals that are required. When successive launch intervals are established in this way, they settle towards a steady-state value, and equations are provided which determine this value. Some of these results are presented graphically.

The equations may also be used to evaluate the mean down times in each interval and in the whole mission, with some other criterion being used for the choice of launch dates. For example, the use of equal launch intervals might be evaluated, since this policy has several attractions.

2 RELIABILITY OF THE SATELLITE SYSTEM

2.1 The satellite reliability model

Each satellite is modelled as containing duplex redundancy at the highest (*ie* satellite) level, and its duplicate sets of equipment are assumed to have a constant failure rate λ leading to a reliability function

$$2e^{-\lambda t} - e^{-2\lambda t} \quad ,$$

where t is the time from launch. This function is thought to provide a fair representation of the reliability-time curve of a typical satellite employing some redundancy; further at that stage of the system design where these results might be employed, there would probably be little better information as to the true shape of the curve.

The probability of a successful launch is taken to be P, and the satellite is assumed to fail (cut-off) after a time t_c, because of exhaustion of its fuel or some other expendables. The satellite reliability, including launch success probability and cut-off, is therefore given by

$$\left.\begin{aligned} R_x(t) &= P\left(2e^{-\lambda t} - e^{-2\lambda t}\right) && \text{for } 0 \leqslant t < t_c \\ R_x(t) &= 0 && \text{for } t_c \leqslant t \ , \end{aligned}\right\} \quad (1)$$

where t is the time from launch of the satellite.

When considering the satellite reliability in a particular interval (the ith interval, say) between successive launches, what is required is its reliability as a function of the time t_i from the start of that interval. (The term 'interval' is used, throughout this Report, to mean the interval between two successive satellite launches.) To obtain $R_x(t_i)$ from $R_x(t)$ it should be noted that t_i and t are only the same when satellite x is launched at the start of the ith interval. Otherwise t exceeds t_i by the sum of the appropriate number of preceding interval durations, τ_{i-1}, τ_{i-2}, etc.

In this analysis, all satellites are assumed to have the same values of launch success probability P and failure rate parameter λ, and the cut-off time of each satellite is assumed to coincide with the launch of a later satellite. This latter assumption, which brings the benefit of analytical simplification, rests on the fact that if a satellite cut-off occurs without a new satellite coming available at the same time, then whenever the first satellite is needed before cut-off, system down time will result after cut-off. Thus it makes sense to provide each satellite with fuel sufficient to extend the cut-off time to the launch time of a later satellite. Four cases are treated in this Report, being the occurrence of cut-off after one, two, three or four launch intervals. This means that, after the first few launch intervals, the number of available satellites in any interval is one, two, three or four respectively. 081

2.2 System reliability as a function of satellite reliability

Let R represent the reliability of the satellite system, which is defined as being operational if at least one of the available satellites is functioning correctly. ('Available satellites' are those which have been launched and have not yet reached their cut-off time.) During interval i, the probability of system failure is the probability that all satellites available in that interval have failed. This probability is given by the product

$$1 - R(t_i) = \prod_x \left(1 - R_x(t_i)\right) \quad ,$$

where R_x is the reliability of satellite x,

t_i is the time from the start of the ith interval

and $\prod_x$ denotes the product over x.

Therefore, the system reliability during this interval is given by

$$R(t_i) = 1 - \prod_x \left(1 - R_x(t_i)\right) \quad . \tag{2}$$

For example, if there are two available satellites

$$R(t_i) = R_a(t_i) + R_b(t_i) - R_a(t_i)R_b(t_i) \quad .$$

3 MEAN SYSTEM DOWN TIME IN A GENERAL INTERVAL

It has been shown that during a general interval i, between two successive launches, the reliability of the satellite system is described by a function $R(t_i)$. The duration of the interval is τ_i, and the function $R(t_i)$ is continuous within the interval (although not at its boundaries). If the system should fail at time t_i, the down time g in the interval would be $\tau_i - t_i$. The probability of the system being operational at time t_i is $R(t_i) = R(\tau_i - g)$. This probability is the same as the probability of the down time being g or less, which is the cumulative distribution function of down
081 time $F(g)$. Therefore

$$F(g) = R(t_i)$$

$$= R(\tau_i - g) \quad . \tag{3}$$

The two functions $R(t_i)$ and $F(g)$ are illustrated in Fig 1. In general $R(0) = F(\tau_i) < 1$ and $R(\tau_i) = F(0) > 0$. The probability density function of down time $f(g)$ is defined by

$$f(g) = \frac{dF(g)}{dg} \qquad \text{for } 0 < g < \tau_i \quad . \tag{4}$$

To find the mean system down time $\bar{g}$ in an interval, use could be made of the probability density function $f(g)$, but it is more convenient to find $\bar{g}$ directly in terms of the system reliability, as follows. Referring to the third illustration in Fig 1, which shows the system failure probability $1 - R(t_i)$ as a function of time, the elemental area shaded has length equal to the down time for a satellite which fails between t_i and $t_i + dt_i$, and has height equal to the probability of the system failing between t_i and $t_i + dt_i$. Similarly, the area below the line $1 - R(0)$ has width equal to τ_i (the down time when the satellite fails at $t_i = 0$), and height equal to $1 - R(0)$ (the probability of failing at $t_i = 0$). It is therefore clear that the area under the curve $1 - R(t_i)$ is equal to the mean down time in the interval, so integrating along the t_i axis to find the same area;

$$\bar{g} = \int_0^{\tau_i} \left(1 - R(t_i)\right) dt_i$$

$$= \tau_i - \int_0^{\tau_i} R(t_i) dt_i \quad . \tag{5}$$

4 DERIVATION OF MEAN DOWN TIMES IN PARTICULAR CASES

Having obtained the results of sections 2 and 3, the derivation of mean down time $\bar{g}$ in any particular interval consists in finding the reliabilities $R_x(t_i)$ of each satellite available during the interval, obtaining the resulting system reliability using equation (2), and applying equation (5) to find $\bar{g}$. In the following subsections, $\bar{g}$ is determined for particular intervals, with cut-off assumed to be after one, two or three intervals. The steady-state value 081 of mean down time, which results when successive launches are at equal intervals, is also calculated.

4.1 First interval

In the first interval only one satellite (subscript a) is available. In this case $t = t_1$, and, using equation (1)

$$R_a(t_1) = P\left(2e^{-\lambda t_1} - e^{-2\lambda t_1}\right) .$$

It follows from equation (2) that

$$R(t_1) = R_a(t_1) ,$$

and applying equation (5)

$$\bar{g}_1 = \tau_1 - 2P\int_0^{\tau_1} e^{-\lambda t_1} dt_1 + P\int_0^{\tau_1} e^{-2\lambda t_1} dt_1$$

$$= \tau_1 - \frac{2P}{\lambda}\left(1 - e^{-\lambda\tau_1}\right) + \frac{P}{2\lambda}\left(1 - e^{-2\lambda\tau_1}\right) . \qquad (6)$$

Rewriting equation (6) in terms of the mean fractional down time x_1 and the normalized launch interval y_1, where $x_1 = \bar{g}_1/\tau_1$ and $y_1 = \lambda\tau_1$, gives

$$x_1 = 1 - \frac{P}{y_1}\left[2\left(1 - e^{-y_1}\right) - \tfrac{1}{2}\left(1 - e^{-2y_1}\right)\right] , \qquad (7)$$

which is shown graphically for various values of P in Fig 2.

It is useful to find the lower limit of mean fractional down time x, which is approached as the normalized launch interval y approaches zero. This gives

$$(x_1)_{min} = 1 - P . \qquad (8)$$

This result is of interest if the mean fractional down time in the interval is required to be less than this minimum, because then the only way to overcome this difficulty is to set the interval duration to zero, whence the mean fractional
081 down time is immaterial.

Equation (7) may also be written as a power series in e^{-y_1} ; *ie*

$$x_1 = 1 - \frac{P}{y_1}\left[\frac{3}{2} - 2e^{-y_1} + \frac{1}{2}e^{-2y_1}\right] . \quad (9)$$

4.2 Subsequent intervals when cut-off is after one interval

When cut-off is after one interval there is only one satellite available in any interval, that being the satellite which was launched at the start of that interval. The ith interval is therefore described by the same equations as the first, except that $t_1, \tau_1, \bar{g}_1, x_1, y_1$ become $t_i, \tau_i, \bar{g}_i, x_i, y_i$. The mean fractional down time in the ith interval is therefore

$$x_i = 1 - \frac{P}{y_i}\left[2\left(1 - e^{-y_i}\right) - \tfrac{1}{2}\left(1 - e^{-2y_i}\right)\right] , \quad (10)$$

which is again represented by the curves of Fig 2. The lower limit to x_i , which is approached as y_i tends to zero is

$$(x_i)_{min} = 1 - P . \quad (11)$$

4.3 Steady-state launch interval when cut-off is after one interval

It was stated earlier that the launch intervals approach a steady-state constant value if the mean fractional down time is constrained to be the same for all intervals. When cut-off is after one interval, the steady-state is reached immediately, since equation (10) describes all intervals and each interval is independent of the previous ones. Replacing x_i and y_i in equation (10) by x_s and y_s , the steady-state launch interval y_s is given by the implicit equation

$$x_s = 1 - \frac{P}{y_s}\left[2\left(1 - e^{-y_s}\right) - \tfrac{1}{2}\left(1 - e^{-2y_s}\right)\right] , \quad (12)$$

which is yet again represented by the curves of Fig 2.

4.4 Second interval when cut-off is after two or more intervals

In this case two satellites are available during the second interval. One, subscript a , was launched at the start of the first interval, so that 081

$t = \tau_1 + t_2$. The second, subscript b , was launched at the start of the second interval, so that $t = t_2$. Using equation (1) the two satellite reliabilities are given by

$$R_a(t_2) = P\left(2e^{-\lambda(\tau_1+t_2)} - e^{-2\lambda(\tau_1+t_2)}\right)$$

and

$$R_b(t_2) = P\left(2e^{-\lambda t_2} - e^{-2\lambda t_2}\right) ,$$

and, using equation (2),

$$\begin{aligned} R(t_2) &= R_a(t_2) + R_b(t_2) - R_a(t_2)R_b(t_2) \\ &= 2P\left(1 + e^{-\lambda\tau_1}\right)e^{-\lambda t_2} - P\left(1 + 4Pe^{-\lambda\tau_1} + e^{-2\lambda\tau_1}\right)e^{-2\lambda t_2} \\ &\qquad + 2P^2e^{-\lambda\tau_1}\left(1 + e^{-\lambda\tau_1}\right)e^{-3\lambda t_2} - P^2e^{-2\lambda\tau_1}e^{-4\lambda t_2} . \end{aligned}$$

Applying equation (5) and then substituting for $R(t_2)$ gives

$$\begin{aligned} \bar{g}_2 &= \tau_2 - \int_0^{\tau_2} R(t_2)dt_2 \\ &= \tau_2 - \frac{2P}{\lambda}\left(1 + e^{-\lambda\tau_1}\right)\left(1 - e^{-\lambda\tau_2}\right) \\ &\qquad + \frac{P}{2\lambda}\left(1 + 4Pe^{-\lambda\tau_1} + e^{-2\lambda\tau_1}\right)\left(1 - e^{-2\lambda\tau_2}\right) \\ &\qquad - \frac{2P^2}{3\lambda}e^{-\lambda\tau_1}\left(1 + e^{-\lambda\tau_1}\right)\left(1 - e^{-3\lambda\tau_2}\right) \\ &\qquad + \frac{P^2}{4\lambda}e^{-2\lambda\tau_1}\left(1 - e^{-4\lambda\tau_2}\right) . \end{aligned} \qquad (13)$$

Rewriting equation (13) in terms of the mean fractional down time $x_2 = \bar{g}_2/\tau_2$
081 and the normalized launch intervals $y_1 = \lambda\tau_1$ and $y_2 = \lambda\tau_2$ gives

$$x_2 = 1 - \frac{P}{y_2}\left[2\left(1 + e^{-y_1}\right)\left(1 - e^{-y_2}\right) - \tfrac{1}{2}\left(1 + 4Pe^{-y_1} + e^{-2y_1}\right)\left(1 - e^{-2y_2}\right)\right.$$
$$\left. + \frac{2P}{3}\left(e^{-y_1} + e^{-2y_1}\right)\left(1 - e^{-3y_2}\right) - \frac{P}{4}e^{-2y_1}\left(1 - e^{-4y_2}\right)\right] \; . \qquad \ldots\ldots (14)$$

As y_2 approaches zero, x_2 tends to its lower limit given by

$$(x_2)_{min} = (1 - P)\left(1 - 2Pe^{-y_1} + Pe^{-2y_1}\right) \; . \qquad (15)$$

If the first interval is also set to zero, then $y_1 = 0$, and $(x_2)_{min}$ reduces to

$$(x_2)_{min} = (1 - P)^2 \; . \qquad (16)$$

Equation (14) may also be written as a power series in e^{-y_2} ; *ie*

$$x_2 = 1 - \frac{P}{y_2}\left[\frac{3}{2} + \left(2 - \frac{4P}{3}\right)e^{-y_1} - \left(\frac{1}{2} - \frac{5P}{12}\right)e^{-2y_1} - 2\left(1 + e^{-y_1}\right)e^{-y_2}\right.$$
$$+ \frac{1}{2}\left(1 + 4Pe^{-y_1} + e^{-2y_1}\right)e^{-2y_2} - \frac{2P}{3}\left(e^{-y_1} + e^{-2y_1}\right)e^{-3y_2}$$
$$\left. + \frac{P}{4}e^{-2y_1}e^{-4y_2}\right] \; . \qquad \ldots\ldots (17)$$

4.5 Subsequent intervals when cut-off is after two intervals

When cut-off is after two intervals, there are two satellites available during any interval after the first, one being launched at the start of the interval, and the other at the start of the previous interval. The ith interval is therefore described by the equations in section 4.4 except that t_1, t_2, τ_1, τ_2, $\bar{g}_2$, x_2, y_1, y_2 are replaced by t_{i-1}, t_i, τ_{i-1}, τ_i, $\bar{g}_i$, x_i, y_{i-1}, y_i . Referring to equation (14), the mean fractional down time in the ith interval for $i \geqslant 2$ is given by

$$x_i = 1 - \frac{P}{y_i}\left[2\left(1 + e^{-y_{i-1}}\right)\left(1 - e^{-y_i}\right) - \tfrac{1}{2}\left(1 + 4Pe^{-y_{i-1}} + e^{-2y_{i-1}}\right)\left(1 - e^{-2y_i}\right)\right.$$
$$\left. + \frac{2P}{3}\left(e^{-y_{i-1}} + e^{-2y_{i-1}}\right)\left(1 - e^{-3y_i}\right) - \frac{P}{4}e^{-2y_{i-1}}\left(1 - e^{-4y_i}\right)\right] .$$

...... (18)

The limiting values and other equations in section 4.4 can also be generalized to the ith interval by the above transformation of variables.

4.6 Steady-state launch interval when cut-off is after two intervals

If the mean fractional down time x_i is constrained to the same value x_s for all i, and successive normalised launch intervals y_i are determined, these settle towards a steady-state value y_s. Setting $y_i = y_{i-1} = y_s$ in equation (18) gives the implicit equation for y_s

$$x_s = 1 - \frac{P}{y_s}\left[\frac{3}{2} - \frac{4P}{3}e^{-y_s} - \left(2 - \frac{5P}{12}\right)e^{-2y_s} + 2Pe^{-3y_s}\right.$$
$$\left. - \left(-\frac{1}{2} + \frac{2P}{3}\right)e^{-4y_s} - \frac{2P}{3}e^{-5y_s} + \frac{P}{4}e^{-6y_s}\right] .$$

...... (19)

This is shown graphically in Fig 3 for various values of the launch success probability P.

4.7 Third interval when cut-off is after three or more intervals

In this case three satellites are available in the third interval. Their launch times are at the start of the first, second and third intervals, so that $t = \tau_1 + \tau_2 + t_3$ for satellite subscript a, $t = \tau_2 + t_3$ for satellite b, and $t = t_3$ for satellite c. Using equation (1), the three satellite reliabilities are given by

$$R_a(t_3) = P\left(2e^{-\lambda(\tau_1+\tau_2+t_3)} - e^{-2\lambda(\tau_1+\tau_2+t_3)}\right) ,$$

$$R_b(t_3) = P\left(2e^{-\lambda(\tau_2+t_3)} - e^{-2\lambda(\tau_2+t_3)}\right) ,$$

and

$$R_c(t_3) = P\left(2e^{-\lambda t_3} - e^{-2\lambda t_3}\right) .$$

Applying equation (2) the system reliability is given by

$$R(t_3) = R_a(t_3) + R_b(t_3) + R_c(t_3)$$
$$- R_b(t_3)R_c(t_3) - R_a(t_3)R_c(t_3) - R_b(t_3)R_c(t_3)$$
$$+ R_a(t_3)R_b(t_3)R_c(t_3) \quad ,$$

and applying equation (5)

$$\bar{g}_3 = \tau_3 - \int_0^{\tau_3} R(t_3)dt_3 \quad .$$

$\bar{g}_3$ may now be derived, as a function of τ_1, τ_2 and τ_3, in a similar manner to $\bar{g}_2$ in section 4.4. In terms of the normalized launch intervals $y_1 = \lambda\tau_1$, $y_2 = \lambda\tau_2$, $y_3 = \lambda\tau_3$, the mean fractional down time in the third interval $x_3 = \bar{g}_3/\tau_3$, when cut-off is after three or more intervals, is given by

$$x_3 = 1 - \frac{P}{y_3}\left[a_1\left(1 - e^{-y_3}\right) + a_2\left(1 - e^{-2y_3}\right) + a_3\left(1 - e^{-3y_3}\right)\right.$$
$$\left. + a_4\left(1 - e^{-4y_3}\right) + a_5\left(1 - e^{-5y_3}\right) + a_6\left(1 - e^{-6y_3}\right)\right] \quad , \qquad \text{...... (20)}$$

where $a_1 = 2\left(1 + e^{-y_2} + e^{-y_1-y_2}\right)$,

$$a_2 = -\frac{1}{2}\left(1 + 4Pe^{-y_2} + e^{-2y_2} + 4Pe^{-y_1-y_2} + 4Pe^{-y_1-2y_2} + e^{-2y_1-2y_2}\right) \quad ,$$

$$a_3 = \frac{2P}{3}\left(e^{-y_2} + e^{-2y_2} + e^{-y_1-y_2} + 4Pe^{-y_1-2y_2} + e^{-2y_1-2y_2} + e^{-y_1-3y_2}\right.$$
$$\left. + e^{-2y_1-3y_2}\right) \quad ,$$

$$a_4 = -\frac{P}{4}\left(e^{-2y_2} + 4Pe^{-y_1-2y_2} + e^{-2y_1-2y_2} + 4Pe^{-y_1-3y_2} + e^{-2y_1-4y_2}\right.$$
$$\left. + 4Pe^{-2y_1-3y_2}\right) \quad ,$$

$$a_5 = \frac{2P^2}{5}\left(e^{-y_1-3y_2} + e^{-2y_1-3y_2} + e^{-2y_1-4y_2}\right) ,$$

and $$a_6 = -\frac{P^2}{6} e^{-2y_1-4y_2} .$$

If the first two intervals are zero, then x_3 approaches its minimum value as y_3 tends to zero; *ie*

$$(x_3)_{min} = (1 - P)^3 . \tag{21}$$

4.8 Subsequent intervals when cut-off is after three intervals

As before, equation (20) can be generalised to the case of any interval after the second, when cut-off is after three intervals. The generalised equation is

$$x_i = 1 - \frac{P}{y_i}\sum_{n=1}^{6} a_n\left(1 - e^{-ny_i}\right) , \tag{22}$$

where the coefficients a_1 to a_6 are as defined in section 4.7, except that y_1 and y_2 are everywhere replaced by y_{i-2} and y_{i-1} respectively.

4.9 Steady-state launch interval when cut-off is after three intervals

The steady-state launch interval is found by setting $y_i = y_{i-1} = y_{i-2} = y_s$ in equation (22) and its associated coefficient definitions. The steady-state normalized launch interval y_s is then given by the equation

$$\begin{aligned} x_s = 1 - \frac{P}{y_s}\Bigg[& \frac{3}{2} - \frac{4}{3}Pe^{-y_s} - \frac{11}{12}Pe^{-2y_s} - \left(2 - \frac{5}{3}P^2\right)e^{-3y_s} \\ & + P\left(\frac{29}{12} - \frac{3}{5}P\right)e^{-4y_s} + P\left(\frac{4}{3} - \frac{3}{5}P\right)e^{-5y_s} + \left(\frac{1}{2} - \frac{73}{30}P^2\right)e^{-6y_s} \\ & - P\left(\frac{4}{3} - P\right)e^{-7y_s} - P\left(\frac{5}{12} - P\right)e^{-8y_s} + \frac{3}{5}P^2e^{-9y_s} \\ & + P\left(\frac{1}{4} - \frac{2}{5}P\right)e^{-10y_s} - \frac{2}{5}P^2e^{-11y_s} + \frac{P^2}{6}e^{-12y_s}\Bigg] . \end{aligned} \tag{23}$$

This is shown graphically in Fig 4 for various values of P .

081

4.10 Steady-state launch interval when cut-off is after four intervals

In a similar manner, the relationship between mean fractional down time and launch interval in the steady-state, for the case when cut-off is after four launch intervals, is

$$x_s = 1 - \frac{P}{y_s} \sum_{n=0}^{20} a_n e^{-ny_s} , \qquad (24)$$

where $a_0 = \frac{3}{2}$,

$$a_1 = -\frac{4}{3} P ,$$

$$a_2 = -\frac{11}{12} P ,$$

$$a_3 = -P\left(\frac{4}{3} - \frac{5}{3} P\right) ,$$

$$a_4 = -2 + \frac{5}{12} P + \frac{16}{15} P^2 ,$$

$$a_5 = P\left(2 + \frac{7}{15} P\right) ,$$

$$a_6 = P\left(\frac{7}{4} + \frac{7}{30} P - \frac{12}{5} P^2\right) ,$$

$$a_7 = P\left(2 - \frac{58}{15} P + \frac{14}{15} P^2\right) ,$$

$$a_8 = \frac{1}{2} - \frac{2}{3} P - \frac{61}{30} P^2 + \frac{14}{15} P^3 ,$$

$$a_9 = -P\left(\frac{2}{3} + \frac{2}{3} P - \frac{58}{105} P^2\right) ,$$

$$a_{10} = -P\left(\frac{13}{12} + \frac{P}{6} - \frac{76}{21} P^2\right) ,$$

$$a_{11} = -P\left(\frac{2}{3} - 3P + \frac{208}{105} P^2\right) ,$$

$$a_{12} = P\left(\frac{1}{4} + \frac{6}{5} P - \frac{403}{280} P^2\right) ,$$

$$a_{13} = P^2\left(\frac{1}{5} - \frac{14}{15} P\right) ,$$

$$a_{14} = P\left(\frac{1}{4} - \frac{7}{30} P - \frac{14}{15} P^2\right) ,$$

$$a_{15} = -P^2\left(\frac{4}{5} - \frac{4}{3}P\right) ,$$

$$a_{16} = -P^2\left(\frac{7}{30} - \frac{8}{21}P\right) ,$$

$$a_{17} = \frac{8}{21}P^3 ,$$

$$a_{18} = P^2\left(\frac{1}{6} - \frac{2}{7}P\right) ,$$

$$a_{19} = -\frac{2}{7}P^3 ,$$

and $$a_{20} = \frac{P^3}{8} .$$

Equation (24) is plotted in Fig 5 for various values of P.

5 RESULTS AND DISCUSSION

In section 4, equations were derived which described the reliability and down time of the modelled satellite system in any interval between two successive launches. From these equations the overall system reliability and the total mean down time for the whole mission can be found. The overall system reliability (the probability that the system operates throughout the mission) is the product of the system reliabilities at the end of each interval. The total system mean down time (throughout the mission) is simply the sum of the mean down times in each interval.

In the equations which describe the system, the durations of the intervals between successive launches are independent variables. Ideally these intervals would be chosen to optimise some mission parameter, subject to certain requirements. Possible mission parameters and/or requirements might include mission reliability, total mean down time, number of satellites available, mission duration, and launch success probability. Optimisation of any particular mission parameter does not appear to be possible analytically and furthermore it is not possible to generalize as to which mission parameters should be optimized, and which should be regarded as requirements or constraints.

The choice of launch times to give equal mean fractional down times in each interval throughout the mission was thought to provide a policy which would be near optimal for most requirements. (This was confirmed by some numerical optimisations performed using the Satellite System Replenishment Program described in
081 Ref 1.) This policy (in common with a number of others) leads to a launch

schedule in which the later launch intervals approach a steady-state constant value. The relationship between this steady-state interval and the mean fractional down time is illustrated in Figs 2 to 5. If this "equal mean fractional down time" policy is adopted, the equations derived in section 4 can be used to find the launch intervals for any particular case, provided that the launch success probability P, the mean fractional down time x, and the number of intervals before cut-off of any one satellite are specified. Four such cases are shown in Figs 6 to 9. The convergence to the steady-state value is seen in all four cases. It can also be seen that one or more of the initial launch intervals are zero. This occurs when the required mean fractional down time cannot be achieved however short the interval; a zero interval is then chosen since its mean fractional down time will then be immaterial.

The four examples shown in Figs 6 to 9 also demonstrate the separate effects of altering the three parameters, launch success probability P (Figs 7 and 8), mean fractional down time x (Figs 8 and 9), and the number of intervals before cut-off (Figs 6 and 7).

By summing the launch intervals determined in a particular case, the relationship may be found between normalized mission duration λT_{k+1} (the date when the $(k + 1)$th satellite would have been launched, if there had been one), and mean fractional down time x, for a given number of satellites k, and launch success probability P. This is shown in Fig 10 for $P = 0.9$. (It should be noted that discontinuities of slope occur at $x = 0.01$ and $x = 0.1$, due to changes in the number of zero intervals occurring there. Fig 10 only shows the curve between these two values of x.) For example, if a mission duration of $T_{k+1} = 10$ years is required, and the satellite equipment failure rate before redundancy application is $\lambda = 0.2$, then the normalized mission duration requirement is $\lambda T_{k+1} = 2$. Reading from Fig 10, it can be seen that, if seven satellites are used, a mean fractional down time $x = 0.0125$ can be achieved; for $k = 6$ satellites, $x = 0.0177$ can be achieved, and for $k = 5$ satellites, $x = 0.0275$ can be achieved. Alternatively, if the failure rate is $\lambda = 0.2$, and a mean fractional down time $x = 0.01$ is required, and seven satellites are available, then Fig 10 shows that $\lambda T_{k+1} = 1.7$, from which it follows that a mission duration of 8.5 years can be achieved.

For any particular normalized mission duration λT_{k+1}, it is possible to find the relationship between achieved mean fractional down time and number of satellites k, for a given launch success probability P. The case where $\lambda T_{k+1} = 2.46$ (which could result from $T_{k+1} = 10$ years and $\lambda = 0.246$ year^{-1} 081 for example) is shown in Fig 11. This shows that for P = 0.9, ten satellites are required to achieve less than 1% mean down time, but that only seven satellites are required to achieve less than 2% mean down time.

6 CONCLUSIONS

A mathematical model of the reliability of a system of satellites has been defined. Equations have been derived which describe the reliability and mean down time of the system in any interval between successive launches, and the method has been indicated by which system reliability and mean down time for the whole mission can be determined. A suboptimal launching policy has been defined, and the launching intervals which result from that policy have been derived. It has been shown that the launching intervals settle towards a steady-state constant value. Relationships between various parameters relating to the reliability of the system have been illustrated; these relationships and the equations describing the reliability aspects of the system, provide the design tools required during the early stages of system definition.

NOTATION

a_n	coefficients defined in text (sections 4.7 and 4.10)
$f(g)$	probability density function of down time, in a given interval
$F(g)$	cumulative distribution function of down time, in a given interval
g	down time, in a given interval
$\bar{g}$	mean (expected) down time, in a given interval
k	number of satellites launched during the mission
P	probability of a successful launch
$R(t_i)$	reliability of the satellite system in the ith interval
$R_x(t)$, $R_x(t_i)$	reliability of satellite x
t	time from launch (refers to a particular satellite)
t_i	time from start of ith interval
t_c	time from launch to cut-off of a particular satellite
T_{k+1}	mission duration (date $(k + 1)$th satellite would be launched if there were one)
x	mean fractional down time in an interval or over whole mission ($x = \bar{g}/\tau$ in a given interval)
y	(normalized) interval between two successive satellite launches ($y = \lambda\tau$)
λ	failure rate of the satellite equipment before redundancy application
τ	duration of interval between two successive satellite launches

SUBSCRIPT NOTATION

Number subscripts or i or s refer to intervals (i denoting the general case, and s denoting the steady-state).

Letter subscripts or x refer to satellites (x denoting the general case).

Subscripts are omitted except where ambiguity might arise.

REFERENCE

No.	Author	Title, etc
1	G.J. Davison J.I. Thomas	The satellite system replenishment program. RAE Technical Report 77114 (1977)

Reports quoted are not necessarily available to members of the public or to commercial organisations.

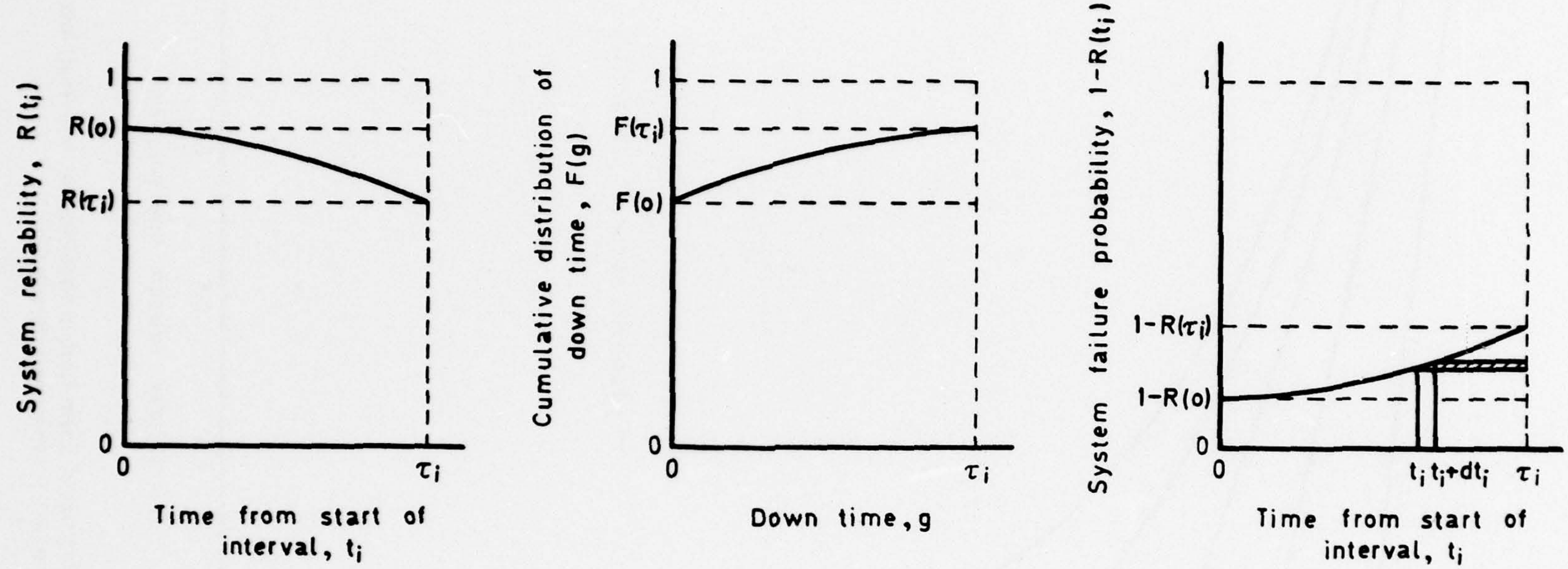

Fig 1 System reliability, down-time distribution and system failure probability, in a general interval i

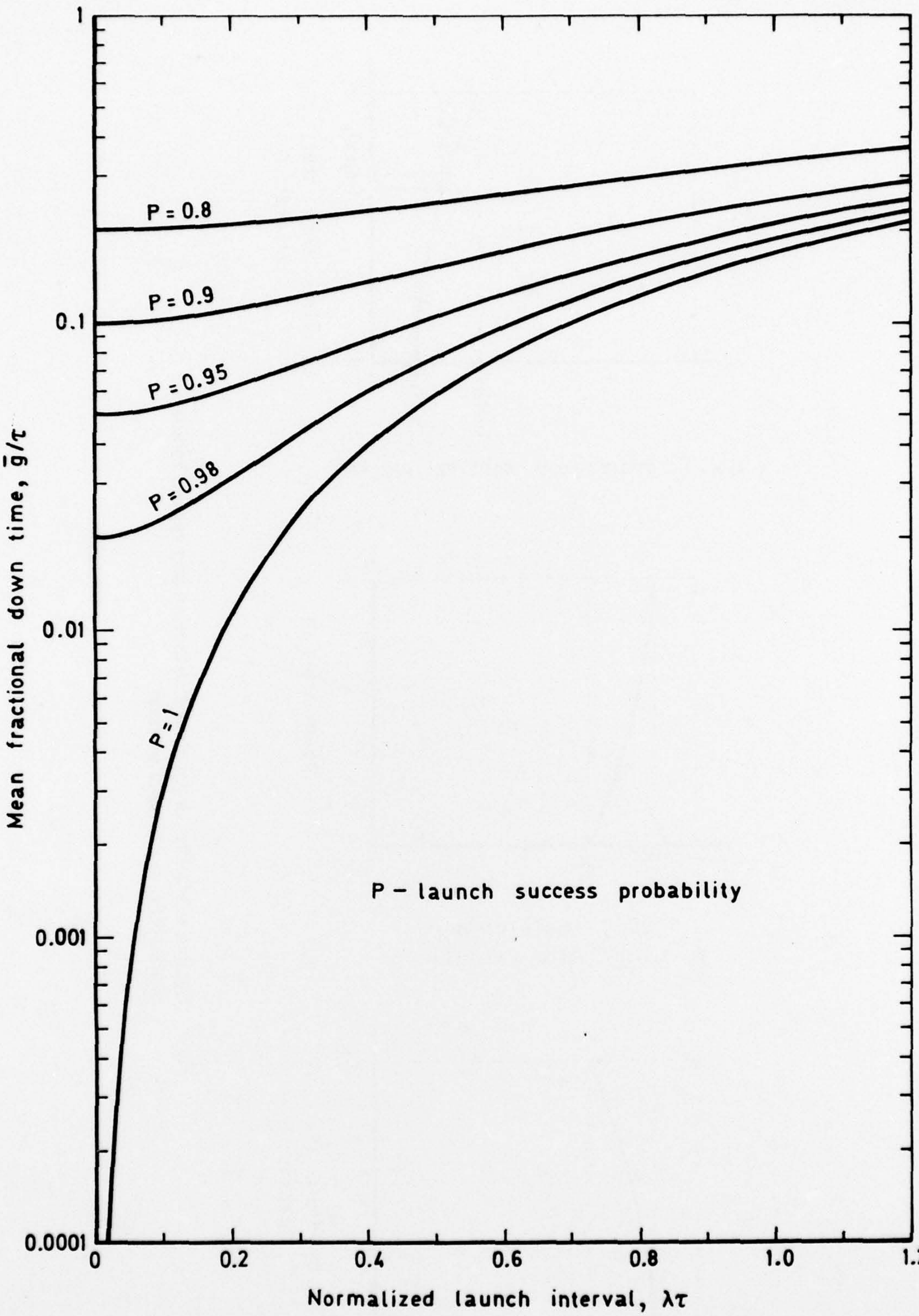

Fig 2 Mean fractional down time in first interval, and in all intervals when cut off is after one interval

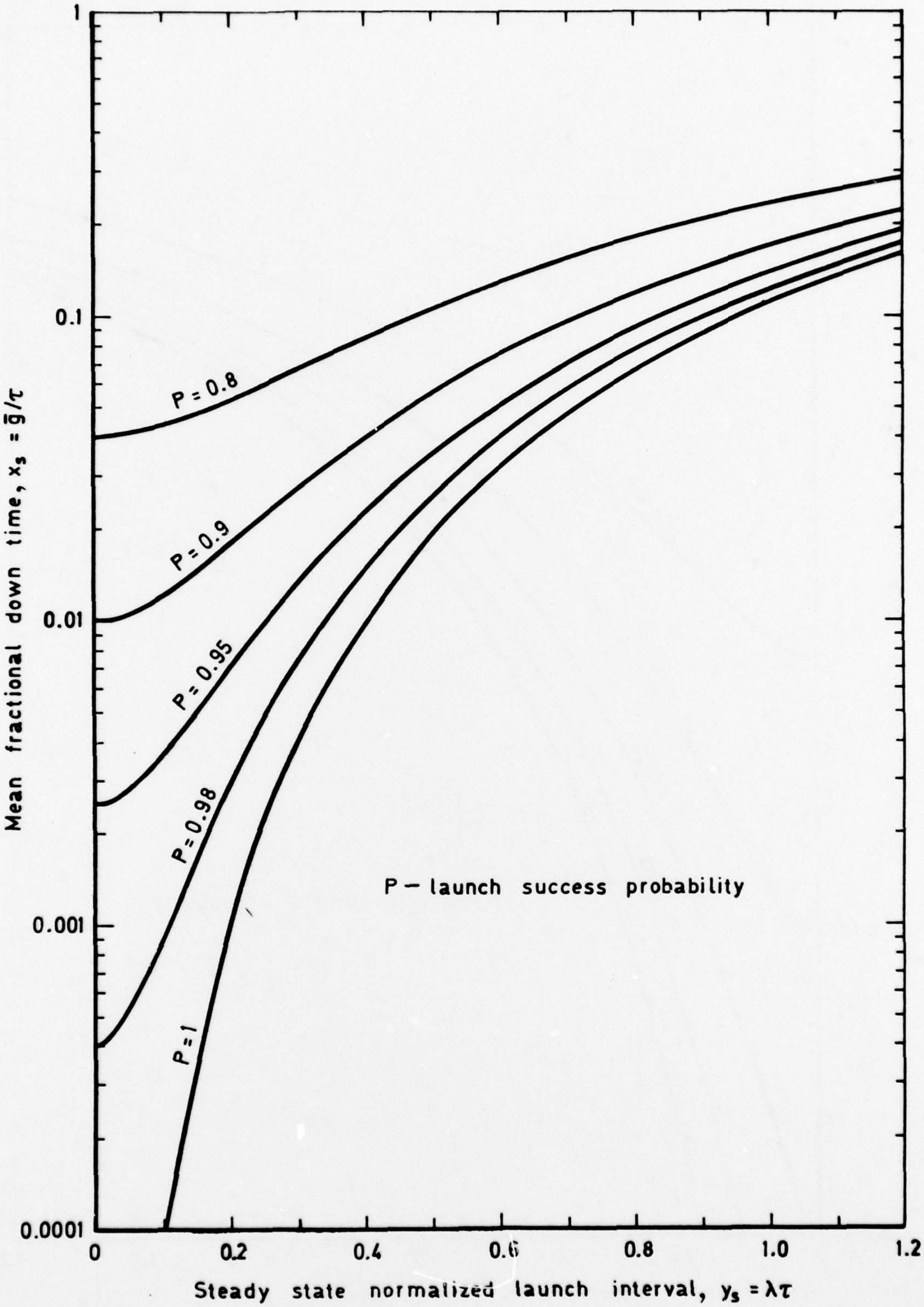

Fig 3 Variation of mean fractional down time with steady-state normalised launch interval when cut off is after two intervals

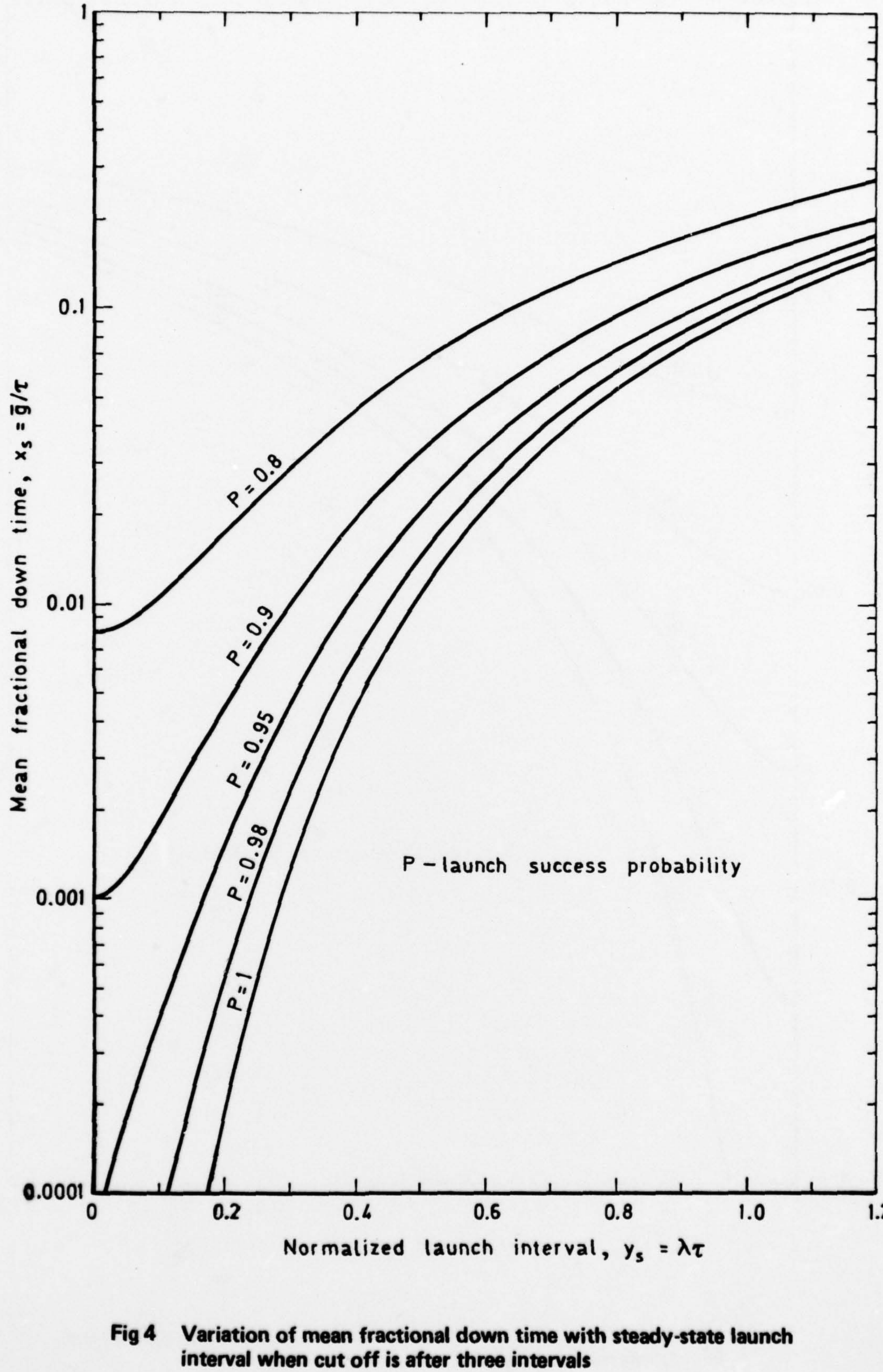

Fig 4 Variation of mean fractional down time with steady-state launch interval when cut off is after three intervals

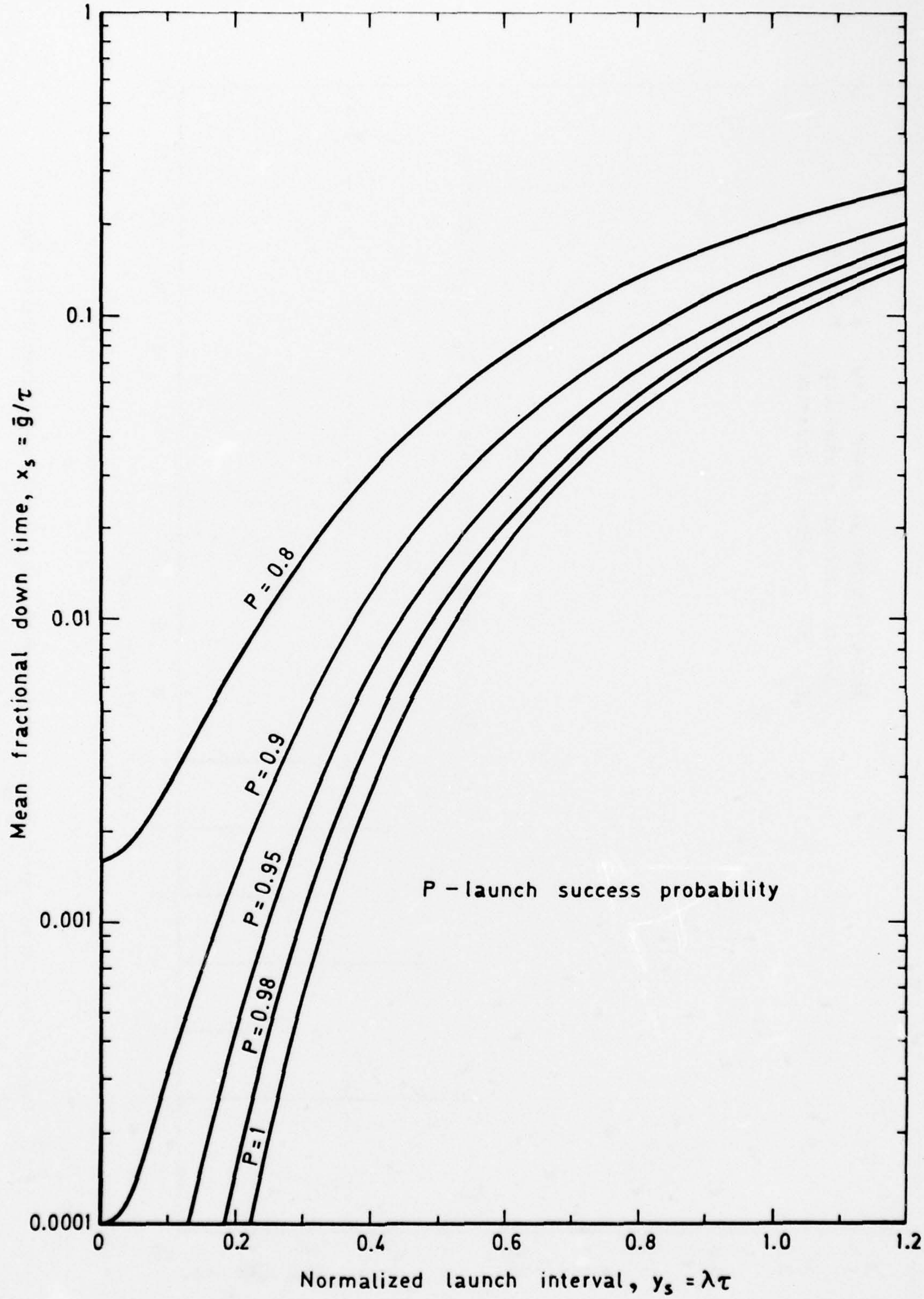

Fig 5 Variation of mean fractional down time with steady-state launch interval when cut off is after four intervals

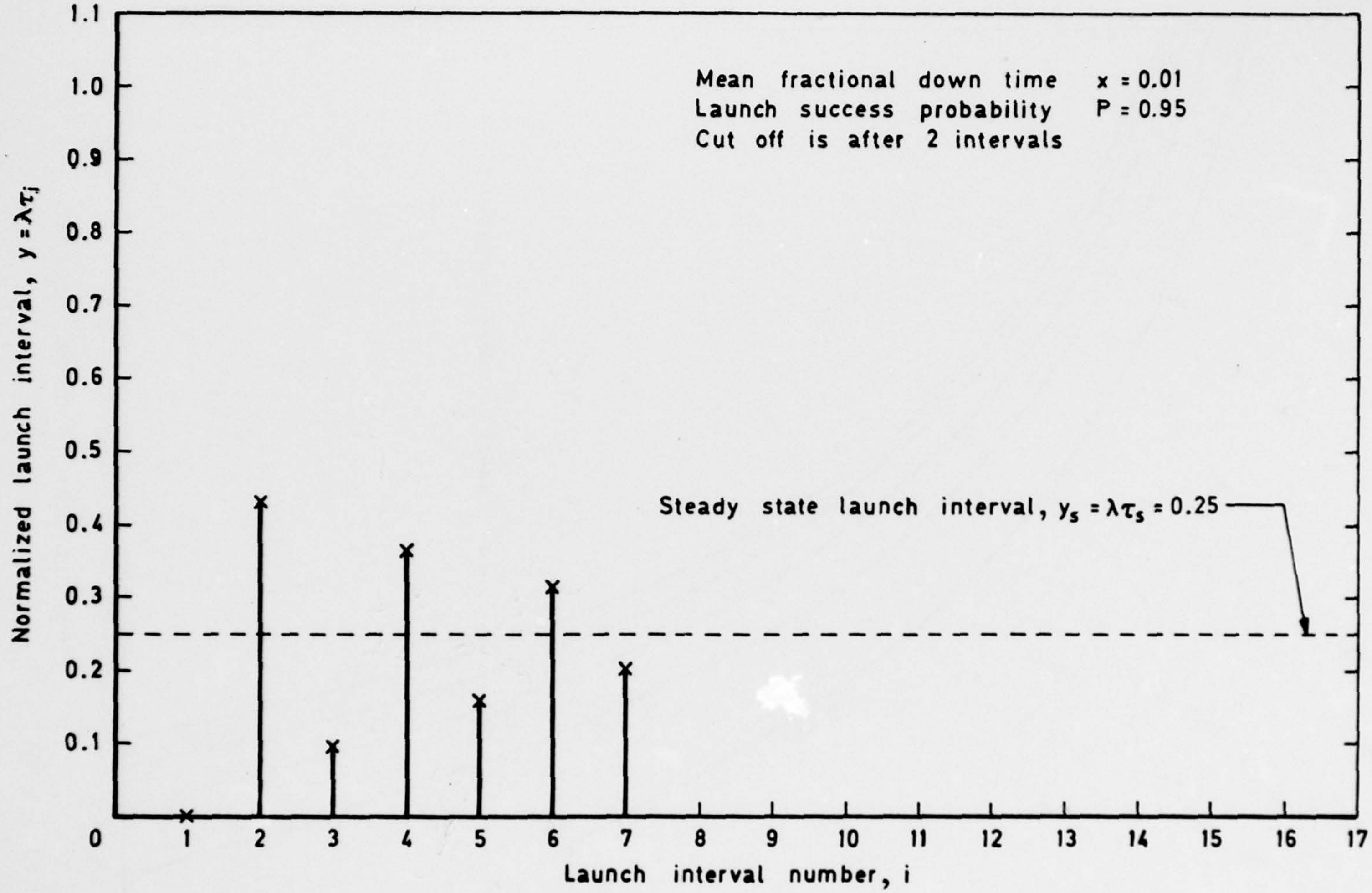

Fig 6 Intervals between successive satellite launches which maintain the mean fractional down time at 1% for all intervals, for P = 0.95 and cut off after two intervals

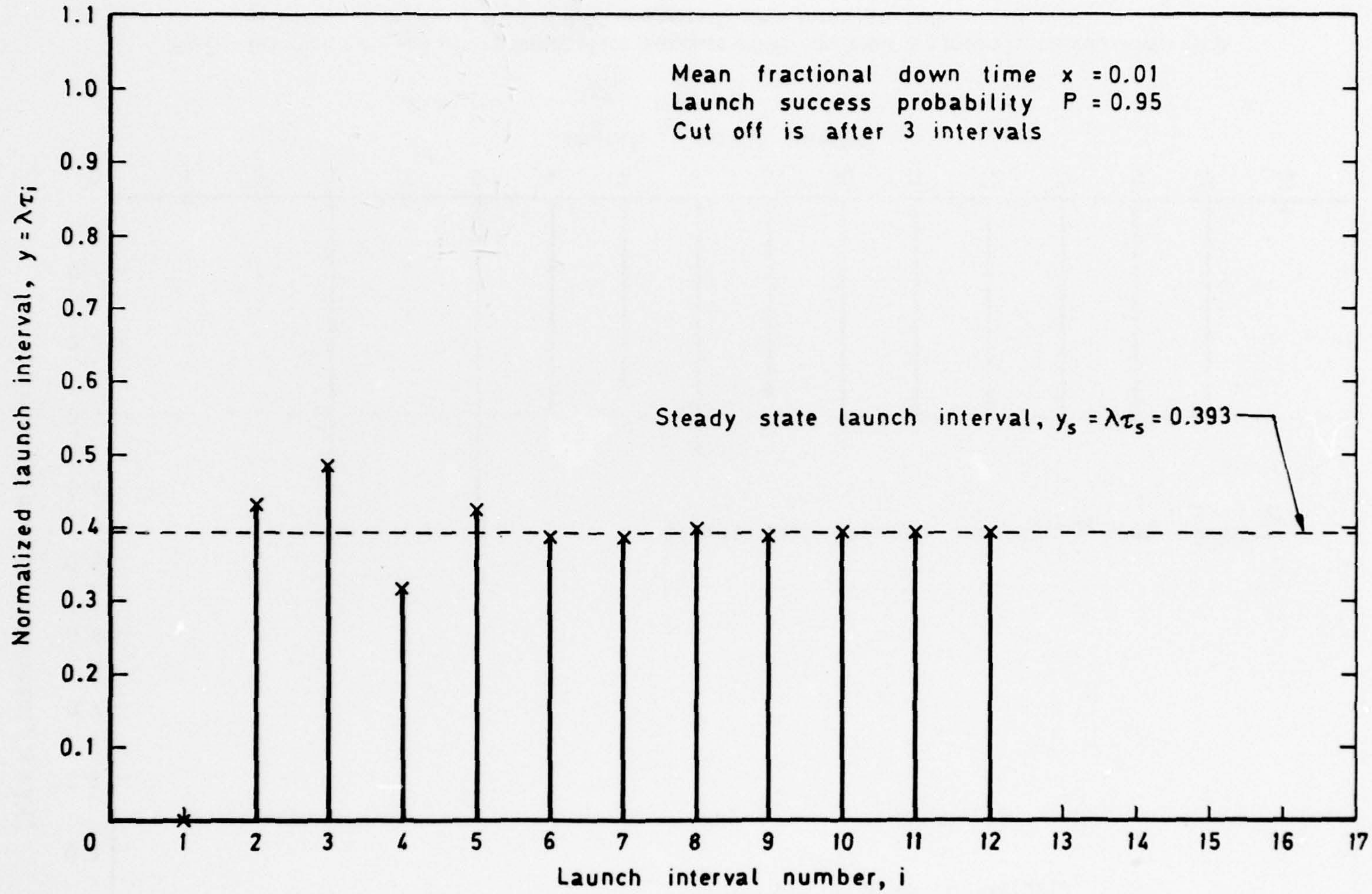

Fig 7 **Intervals between successive satellite launches which maintain the mean fractional down time at 1% for all intervals, for P = 0.95 and cut off after three intervals**

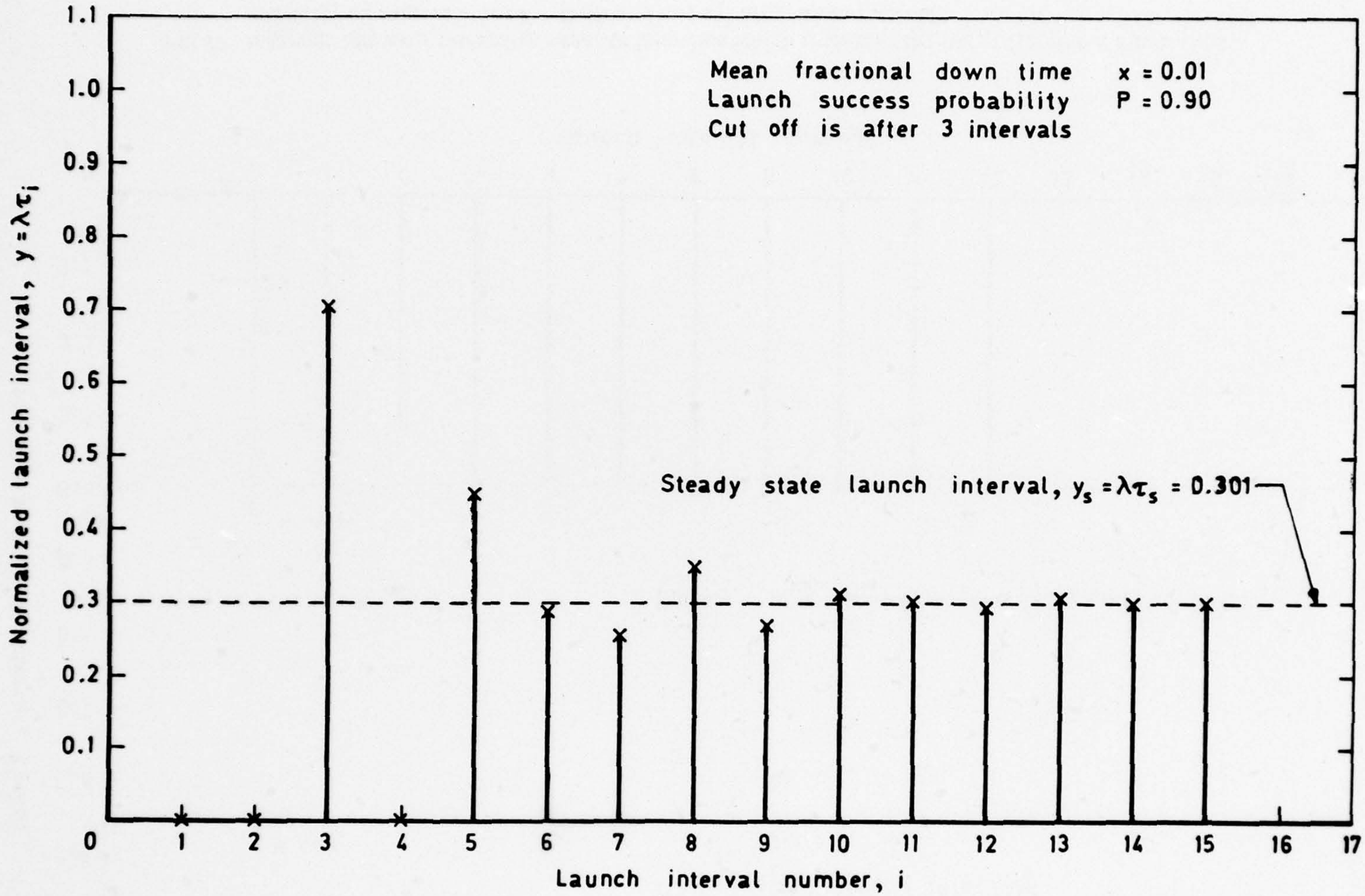

Fig 8 Intervals between successive satellite launches which maintain the mean fractional down time at 1% for all intervals, for P = 0.90 and cut off after three intervals

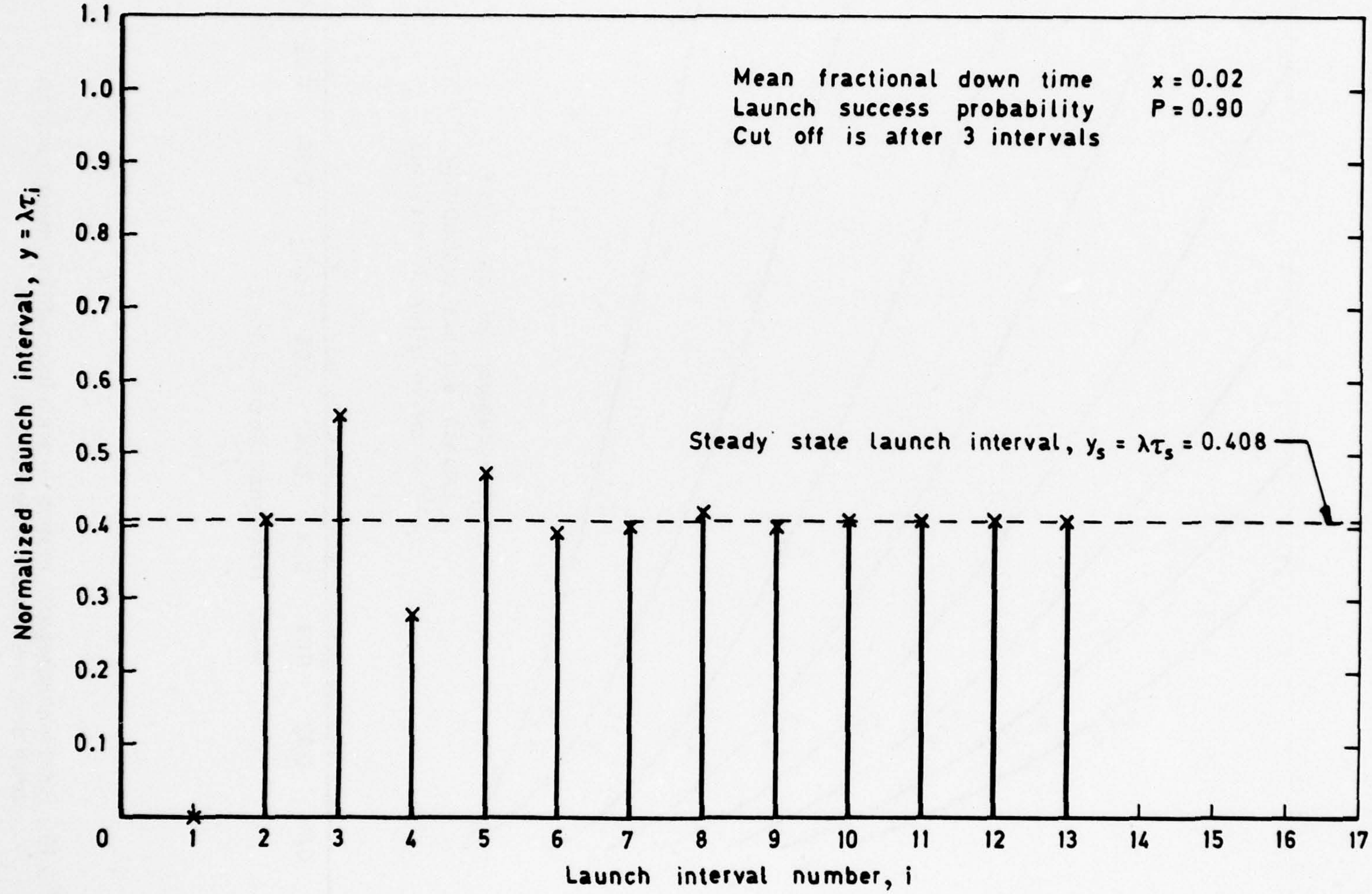

Fig 9 Intervals between successive satellite launches which maintain the mean fractional down time at 2% for all intervals, for P = 0.90 and cut off after three intervals

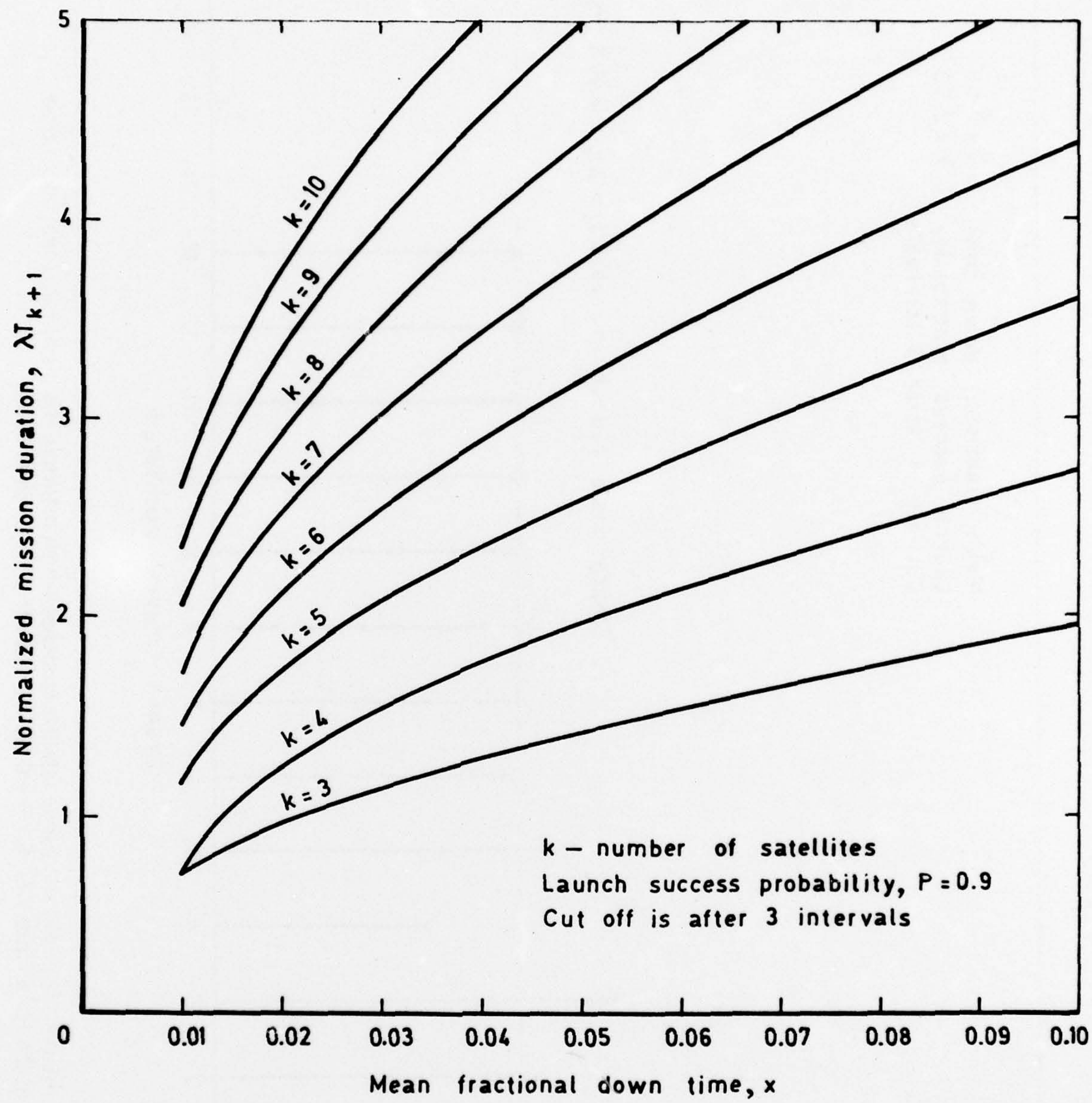

Fig 10 Relationship between mission duration (normalized), mean fractional down time, and number of satellites, for P = 0.90

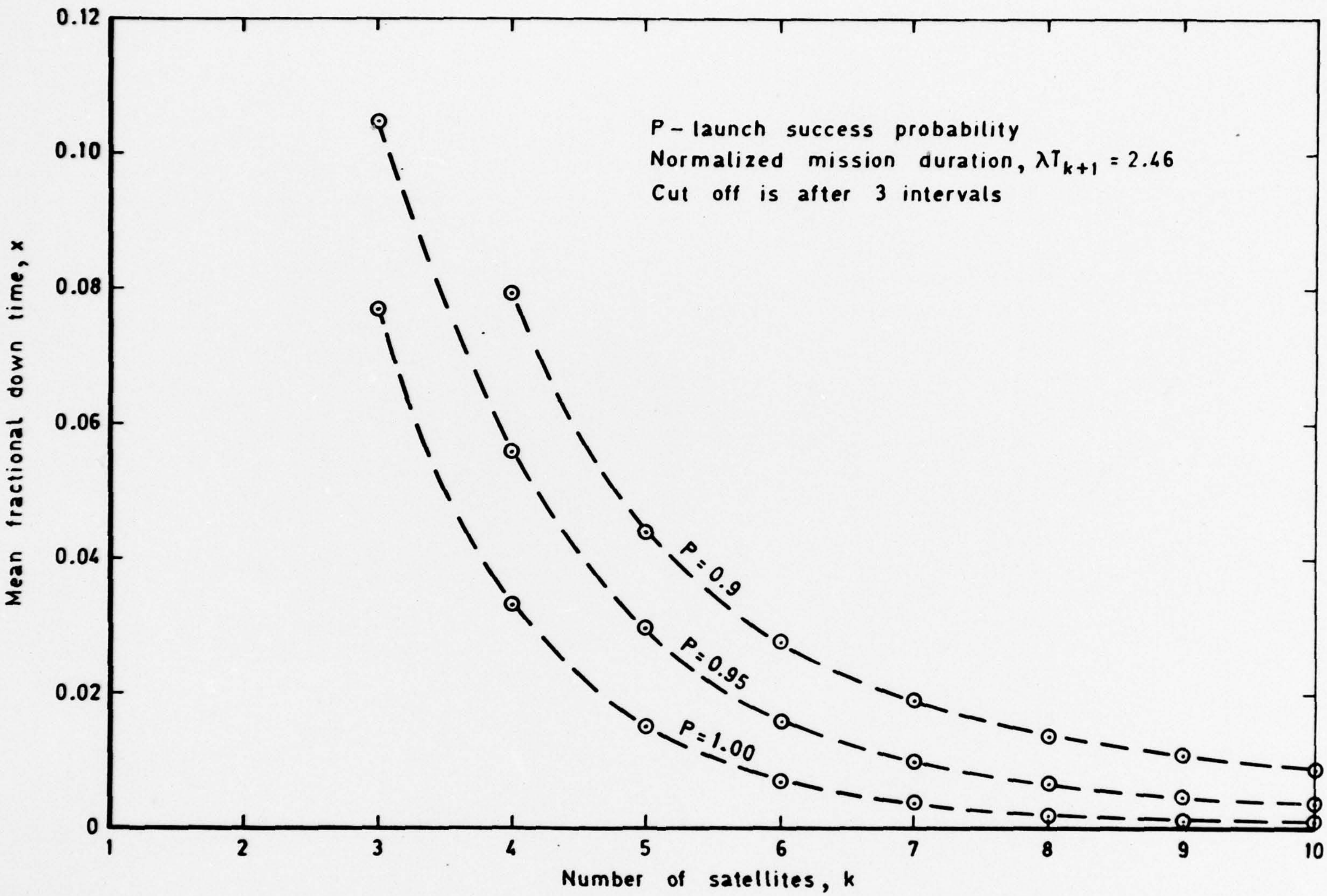

Fig 11 Mean fractional down time as a function of number of satellites and launch success probability, for normalized mission duration $\lambda T_{k+1} = 2.46$