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STUDIES OF THE EARTH'S GRAVITY FOR GEODETIC PURPOSES.(U)

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STUDIES OF THE EARTH'S GRAVITY FOR GEODETIC PURPOSES

Urho A. Uotila

Richard H. Rapp

The Ohio State University
Research Foundation
Columbus, Ohio 43212

December 1978

Final Report for Period 1 July 1975 to 30 September 1978

Approved for public release; distribution unlimited.

AIR FORCE GEOPHYSICS LABORATORY
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HANSCOM AFB, MASSACHUSETTS 01731

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

14 DGS-279

19 **REPORT DOCUMENTATION PAGE**

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 18 AFGL-TR-78-0301	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 STUDIES OF THE EARTH'S GRAVITY FOR GEODETIC PURPOSES.		5. TYPE OF REPORT & PERIOD COVERED 9 Final Rep[redacted]t - 1 Jul[redacted]975-30 Sep[redacted]78.
		6. PERFORMING ORG. REPORT NUMBER Geodetic Science 279
7. AUTHOR(s) 10 Urho A. Uotila Richard H. Rapp		8. CONTRACT OR GRANT NUMBER(s) 15 F19628-76-C-0010
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Geodetic Science The Ohio State University - 1958 Neil Avenue Columbus, Ohio 43210		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62101F 16 760003AC 17 03
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Geophysics Laboratory Hanscom AFB, Massachusetts 01731 Contract Monitor: Bela Szabo/LW		12. REPORT DATE 11 Dece[redacted]978
		13. NUMBER OF PAGES 35 pages
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 33p.		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

A - Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

geodesy, gravity, gradiometry, collocation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report summarizes the work carried out from 1 July 1975 to September 1978 on an AFGL project directed towards the study of the earth's gravity field. Each report is described and put in context with the total research effort. The main areas described in the report relate to the theory and application of least squares collocation; the theory and development of covariance functions; the theory and results from using airborne gradiometry; and the use of satellite to satellite tracking data for the recovery of anomalies on the surface of the earth.

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SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

Foreword

This report was prepared by Urho A. Uotila and Richard H. Rapp, Professors, Department of Geodetic Science at The Ohio State University, under Air Force Contract No. F19628-76-C-0010, OSURF Projects No. 710334 and 710335. This is a final report of the contract covering time period July 1, 1975 to September 30, 1978. It has been administered by the Air Force Geophysics Laboratory, Air Force Systems Command, Hanscom AFB, Massachusetts with Mr. Bela Szabo, Contract Monitor.

The research done under this contract has been previously reported in twenty-two scientific and two internal reports. In the following the scientific work done under this contract will be summarized. The subheadings 1.1-1.7, and 2, are written by Uotila and 1.8-1.12, 3, 4, and 5 are written by Rapp related to research work done under OSURF projects 710334 and 710335, respectively, which have been under their supervision.

The authors wish to thank all of those who have participated in the research under the contract. Special mention should be made about the excellent contributions of Drs. Hajela, Jekeli, Kearsley, Moritz, Rapp, Rummel, Schwarz, Sjöberg, Sünkel and Tscherning to the successful completion of the research contract. The authors acknowledge the cooperation and support given and express their appreciation to the Contract Monitor, Mr. Bela Szabo, for the stimulating technical and scientific discussions.

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1. Collocation and Related Subjects

Dr. Moritz prepared six, Dr. Sjöberg two, and Drs. Rummel, Sünkel, Tscherning and Jekeli one technical report in this area. Part of the summaries reported under 1.1-1.7 are taken directly from Moritz's and Sünkel's reports without quotation marks. The titles of the reports are used as the subheadings for the presentation.

1.1 Integral Formulas and Collocation

Various aspects of the interplay between least-squares collocation and classical integral formulas were subjects of Moritz's report (1975) "Integral Formulas and Collocation."

The first three sections of the report were concerned with the proof that integral formulas, such as Stokes' and Molodensky's equations, might be considered as limiting cases of collocation for homogeneous and regularly and densely distributed data. Moritz showed that the two types of methods are indeed compatible and, what is more, they even complement each other from the point of view of practical application.

Sections 4 and 5 dealt with applications of least-squares methods to adjust continuous data, with a view to using them in Stokes' and other integral formulas.

Practical and numerical aspects were presented in the last section and the relative merits of both types of techniques and their interplay were considered. With respect to numerical computation, Moritz concluded that integral formulas and collocation techniques mutually complement each other, so that in many practical cases a judicious combination of the two procedures may be practically most convenient.

Throughout his report, the so-called spherical approximation was used. It consisted in neglecting, in equations that relate quantities of the anomalous gravity field, small terms on the order of the flattening; thereby ellipsoidal relations were formally transformed into spherical formulas. More precisely, a point on or near the earth's surface, having geodetic coordinates Φ, λ and height h above the ellipsoid, was mapped into a point that had spherical coordinates Φ, λ and height h above a sphere of radius R, $R = 6370$ km being the mean radius of the earth and Φ, λ, h being numerically the same in both cases.

The spherical approximation considerably simplified formulas, while its error was negligible in most practical cases. It underlies Stokes' and Vening Meinesz' integral formulas, as well as most solutions of Molodensky's problem; it also underlies least-squares collocation. If necessary, ellipsoidal effects could easily be taken into account by small correction terms to the spherical approximation as shown by Moritz in his earlier report (1974).

1.2 Covariance Functions in Least-Squares Collocation

Least-squares collocation depends essentially on the covariance functions used. This is especially true for accuracy studies, for which, on the other hand, collocation provides a powerful mathematical apparatus.

For instance, it is known that some analytical expressions for covariance functions lead to imaginary standard errors. What is wrong with such functions? They are not positive definite.

It is also well known that the covariance functions are responsible for the precise mathematical structure of the gravity field through covariance propagation. This implies that the basic covariance functions must be harmonic.

It appears, therefore, appropriate to elaborate, in some detail mathematical properties of covariance functions such as positive definiteness and harmonicity.

Another question is how to characterize a covariance function sufficiently well by a small number of parameters, in such a way that two different covariance functions that have these parameters in common, give approximately the same result.

To find a good analytical covariance function, one tries to represent it as a linear combination of simpler functions. It is, therefore, desirable to know the behavior of such simple models which may serve as building blocks for a global covariance function.

All these problems were considered in the first part of Moritz's report (1976a): "Covariance Functions in Least-Squares Collocation." He dealt with the mathematical structure of covariance functions. The properties of isotropy, harmonicity and positive definiteness were discussed, and Moritz suggested that a covariance function may be characterized by three essential parameters: the variance, the correlation length and a curvature parameter. He also considered some spatial covariance models (planar and spherical).

The second part of his report dealt with the following question: What happens if the "true" covariance function is unknown and least-squares collocation is, instead, performed with a "wrong" (or more precisely, non-optimal) covariance function?

How does the result change with respect to the optimal case, and what is the effect on accuracy studies?

Moritz treated the influence of covariances on the results of collocation. Formulas were developed for the standard error of collocation results when using non-optimal covariance functions, also for the case of stepwise collocation. Finally the behavior of interpolation errors with and without the additional use of horizontal gradients was studied by means of power series expansions for covariance functions and by means of Gaussian covariance functions. Moritz concluded that non-optimal covariance functions had relatively little influence on the interpolated values but a very strong effect on covariances as calculated using the conventional formulas.

1.3 Least-Squares Collocation as a Gravitational Inverse Problem

In geodesy and geophysics we frequently meet with the situation that a model defined by a set of, say, N parameters is to be determined from a smaller number $n < N$ of observations.

As an example, the internal structure of the earth may be defined by a set of N parameters describing the density, the rigidity, and the compressibility of the earth as a function of depth. The n observations comprise velocities of seismic surface waves, together with the mass and the polar moment of inertia of the earth. If the model for the earth's internal structure is to be realistic, then N will be large and $n < N$.

We thus have $n < N$ equations for N unknowns, which is obviously an underdetermined problem admitting an infinite number of possible improperly posed problem. (A problem is properly posed if it has a unique solution that depends continuously on the data.)

Originally, the equations expressing the data x_i as functions of the model parameters s_r will, in general, be nonlinear:

$$x_i = f_i(s_1, s_2, \ldots, s_N), \quad i = 1, 2, \ldots, n.$$

By a suitable application of Taylor's theorem it is usually possible to approximate these equations by linear ones:

$$x_i = \sum_{r=1}^{N} a_{ir} s_r ,$$

or in matrix notation:

$$\underline{x} = \underline{A}\,\underline{s} .$$

The formal solution of this system of linear equations may be written as

$$\underline{s} = \underline{A}^{-1}\underline{x} .$$

If $\underline{A}$ were a regular square matrix, the $\underline{A}^{-1}$ would be the ordinary inverse matrix of $\underline{A}$. In our underdetermined case, however, $\underline{A}^{-1}$ must be understood in the sense of generalized matrix inverses.

At any rate, the solution of the above equations may be considered as an inversion of these equations with respect to the parameters s_r, which accounts for the name, geophysical inverse problems.

Another typical example of an "improperly posed" inverse problem is the determination of subsurface mass distributions which produce a given anomalous gravity field at the earth's surface. This problem is sometimes called an inverse problem of potential theory.

The determination of the earth's external gravitational field from geodetic gravimetric and satellite data may also be considered as an inverse problem that is mathematically quite similar to the determination of the internal structure of the earth from seismic and other data.

This geodetic inverse problem is likewise underdetermined. The external gravitational field requires for a complete description an infinite number of parameters, for instance, the set of all coefficients in the expansion of the external gravitational potential in spherical harmonics. This infinite number, $N = \infty$, of parameters is to be determined from a finite number n of observations.

Even in the seismic inverse problem it is, at least theoretically, appropriate to take $N = \infty$ if we wish to admit reasonable general functions for density, rigidity, and compressibility because it cannot be assumed a priori that such functions depend on a finite number of parameters only.

Thus, in general, the space of parameters will be infinite-dimensional rather than N-dimensional. In other words, the proper general setting for (linear) geodetic and geophysical inverse problems will be infinite-dimensional Hilbert space. This was pointed out by Krarup (1969) for the geodetic case and by Backus (1970) for geophysical inverse problems.

The geodetic inverse problem, the determination of the external gravitational field from data of different kind, is usually solved by least-squares collocation. This technique has many features in common with other geophysical inversion methods. It may, therefore, be of interest to compare these techniques and to exhibit some cross-connections. This was done by Moritz in his report (1976b): "Least-Squares Collocations as a Gravitational Inverse Problem."

He also discussed least-squares collocation from the point of view of analytically representing the external gravitational field by a linear combination of suitable simpler harmonic functions.

Moritz said that the subject of his report is purely conceptual, aiming at a better understanding of least-squares collocation by considering it in relation to other methods; no new computational formulas were derived. His paper is useful as a contribution to the present discussion on the conceptual foundations of least-squares collocation.

1.4 On the Computation of a Global Covariance Model

In his report Moritz (1976a) showed that the local behavior of a covariance function of the gravity anomaly, Δg, can be characterized quite well by means of three constants: C_0 = variance of Δg, ξ = correlation length and G_0 = variance of the horizontal gradient of Δg. In his report "On the Computation of a Global Covariance Model" (Moritz, 1977), he treated the problem of determining a global covariance function if the following data are given: the variance of gravity anomalies, the variance of second-order gradients, the correlation length and the lower degree variances. He proposed that covariance model is a linear combination of the reciprocal distance covariance function and a covariance function of logarithmic type. He also found that the correlation length is already fixed, within rather narrow limits, by the remaining data. It turned out that the gradient variance, G_0, rather than the correlation length, ξ, provides a basis for calculating a global covariance function from local covariance functions.

1.5 Statistical Foundations of Collocation

Users of least-squares collocation ask for a theory that gives an answer to practically meaningful questions: What is the accuracy of our results? Can we apply statistical testing techniques? How can we compute statistical distributions of gravity anomalies or of deflections of the vertical? A reasonable answer to these questions requires some statistical theory of the anomalous gravitational field. But is this field really a stochastic phenomenon? Such questions seem to motivate research into the statistical foundations of collocation.

Least-squares collocation has its roots in many fields:

1. Least-squares estimation;
2. Prediction theory of stochastic processes;
3. Approximation theory;
4. Functional analysis, especially the theory of Hilbert spaces with kernel functions;
5. Potential theory;
6. Inverse and improperly posed problem.

All of these "many facets of collocation" present relevant aspects which must be taken into account in a complete and balances treatment.

The relation to the theory of inverse problems is clear; our data are functionally related to the gravitational field; to determine this field from the data, we must somehow invert those functional relations. Now the gravity field requires infinitely many parameters for its full determination; the number of measurements, however, is essentially finite. Therefore, we have an improperly posed problem. To get a unique solution, we must impose additional conditions, which may have the form of a least-squares principle or of a norm in Hilbert space.

Historically, collocation has developed from least-squares prediction of gravity anomalies, which is an application of the prediction theory of stochastic processes. Hence, statistical considerations have played an essential role in collocation from the very beginning.

Also, the relation to classical least-squares adjustment has soon been noted. In fact, collocation models bear formal resemblance to conventional adjustment models. The characteristic difference, however, is the infinite number of parameters necessary to fully characterize the gravitational field. This fact furnished an essential link to stochastic processes and to infinite-dimensional Hilbert spaces.

Least-squares estimation and stochastic processes give a very convenient mathematical formalism and terminology. They also provide the basis for a statistical interpretation of the results, essential for feasibility studies.

The practical success of the statistical treatment of collocation has sometimes overshadowed its equally significant analytical aspects, especially the fact that there is a clean analytical structure underlying it. This mathematical structure is based on the harmonic character of the anomalous gravitational field and on the fact that all quantities of this field can be expressed as linear functionals of the anomalous potential. The analytical character of collocation is best brought out by approaching it from the standpoint of approximation theory, working in a Hilbert space with a kernel function.

These two aspects, the statistical and the analytical aspect, are both indispensable and mutually complement each other. In fact, evident already in the fundamental paper (Krarup, 1969), seems to be generally agreed upon, although there is some controversy on details.

A literal interpretation of the anomalous gravitational field as a stochastic process has encountered two objections. First, there is only one Earth; a probability space of many possible earths is logically unobjectable, but appears unnatural, since all realizations except one (the real Earth) are unobservable. Secondly, Lauritzen (1973) has proved that there is no ergodic Gaussian process, harmonic outside a sphere. This has sometimes been misinterpreted as a proof that no ergodic process modeling the anomalous gravity field exists at all, so that the covariance function, in principle, cannot be estimated from the data. In fact, however, the Gaussian structure enters essentially into Lauritzen's proof, and there do exist non-Gaussian ergodic processes suitable for collocation.

In his paper "Statistical Foundation of Collocation" (1978a), Moritz deals with mathematical models suitable as a basis for statistical treatment of collocation. As a preparation he discusses first the stochastic processes on the circle, because such processes are simple to understand and they exhibit already essential features of the problem.. Then he treats stochastic processes on the sphere, which are suitable as statistical models for collocation. Moritz shows that Lauritzen's theorem on the nonexistance of ergodic Gaussian stochastic process models for collocation to be essentially dependent on the Gaussian character. He gives two non-Gaussian ergodic models, one of a ginuinely probabilistic character similar to Lauritzen's model, and another based on a formal probability theory in rotation group space.

This second model gives a statistical foundation of the usual homogeneous and isotropic covariance analysis of the anomalous gravity field. The model also provides a basis for the study of the statistical distribution of quantities related to this field. It also allows a formal statistical treatment of the anomalous gravitational field which is independent of an interpretation of this field as some genuinely physical stochastic process and seems, therefore, to be preferable.

If the approach presented by Moritz is accepted, then a detailed theory of statistical distributions for geodetically relevant quantities, such as gravity anomalies, geoidal heights, and deflections of the vertical, could be developed and applied to the statistical testing of the results of least-squares collocation.

1.6 The Operational Approach to Physical Geodesy

There are essentially two possible approaches to physical geodesy (as also to other natural sciences): they might be called the model approach and the operational approach. Essentially, the first approach starts from a theory, the second from the observations. Obviously, the two approaches are closely related to the deductive method and the inductive method in the natural sciences.

In the model approach, one starts from a mathematical model or from a theory and then tries to fit this model to reality, for instance by determining the parameters of this model from observations. The classical geodetic example are the centuries-old attempts to determine the parameters of an earth ellipsoid by observations.

Perhaps the most elaborate form of this model approach is the boundary-value problem of physical geodesy in the formulation of Molodensky. It has a mathematically enormously interesting and deep theory and is practically highly significant, as the many gravimetric geoid determinations and computations of deflections of the vertical show. However, this approach has its weaknesses: the required continuous gravity coverage is practically not realizable; on the other hand, many other important data cannot be incorporated into this theory. The model selects its data.

At present we have a great number of geodetic measurements of very different types, from terrestrial angle and distance measurements to satellite data of various kinds. The question arises: how can we use and combine all these data in the best possible way? This is the operational approach.

Let us summarize. In the model approach one asks: how can I best determine my model by suitable observations? In the operational approach one asks: how can I make best use of all my observations?

As a matter of fact, the two approaches do not compete with each other; each one incorporates important aspects, and the two approaches mutually complement each other.

The operational approach to physical geodesy has come up at a relatively recent date, when a huge number of measurements of new types was available and when it turned out that the classical, especially the gravimetric approach failed to give a complete answer in view of the lack in gravity data.

In geometrical geodesy already least-squares adjustment is in the spirit of an operational approach (how can I best use all my measurements). In physical geodesy, operational methods have been known under the names "least-squares collocation," "integrated geodesy," "operational geodesy." All these methods are very similar; they all aim at an adequate treatment of the gravity field, in addition to an adjustment of measuring errors. They all use quadratic minimum principles incorporating not only the measuring errors, but also the anomalous gravity field.

Moritz (1978b) presented a systematic treatment of the operational approach in his report: "The Operational Approach to Physical Geodesy." He shows that, using contemporary mathematical techniques, a straight road leads from the non-linear observational equations to collocation with kernel functions and least-squares collocation. After linearization an improperly posed problem was obtained, to which Moritz applied three different standard methods of solution: 1) a restriction of the solution space, leading to "pure-collocation," 2) variational principles of Tichonov type, by which measuring errors can be taken into account, leading to a generalized collocation with kernel functions, and 3) a statistical approach, leading to least-squares collocation.

At the end of his report, Moritz discussed these various stages and possible alternatives. All three approaches seem to converge on collocation with kernel functions and least-squares collocation.

1.7 Approximation of Covariance Functions by Non-positive Definite Functions

During the last decade, when least-squares collocation presented itself as the data processing model in physical geodesy, the most serious argument against was the inversion of a large matrix resulting in much computer time needed for this purpose. No attention was paid to the time used for calculating the linear functionals on the covariance function because for simple problems this time is definitely inferior to the inversion time. The situation, however, changed immediately when problems were attacked which involved many and/or difficult covariance calculations. Although Rapp and Tscherning (1974) succeeded in deriving closed expressions of covariance functions for different models of anomaly degree variances, the closed expressions still consist of functions like logarithmic and trigonometric functions, which are expensive in terms of computer time.

We mention only a few kinds of application: prediction of mean gravity anomalies over rectangular blocks from point values, prediction of mean gravity anomalies over larger areas from mean gravity anomalies over smaller areas, prediction of mean gravity anomalies from satellite altimetry data, all problems involving satellite dynamics. All these applications have one common feature: it is necessary to calculate covariances by numerical integration. In case of mean gravity anomaly prediction the integration is at most twofold, in case of satellite dynamics, however, it is multifold. In the former case an explicit integration procedure can be avoided, if one replaces the rectangular area of integration by a circular one. The so-called smoothing operation is caused by an isotropic smoothing operator acting on the covariance function which itself is also isotropic. Therefore, the convolution of the smoothing operator with the covariance function corresponds to a product of the corresponding eigenvalues, which is naturally very simple. In order to obtain closed expressions for the mean gravity anomaly function, however, a further artificial assumption has to be made: the eigenvalues of the smoothing operator have to be replaced by some other values (Schwarz, 1976). In problems involving satellite dynamics as satellite-to-satellite ranging probably the only way to calculate covariances is by numerical integration over some time interval. Using exact covariances for the integration procedure is extremely time consuming.

These were the reasons why the question arose whether it is possible to use some more or less accurate approximations of the exact covariance functions; the approximating functions should be simple, easy to handle, accurate and should consume as little mass storage as possible. Sünkel (1978) studied this problem and reported his findings in the report: "Approximation of Covariance Functions by Non-Positive Definite Functions." He studied three different kinds of approximating functions, all of them being finite elements; 1) a step function, 2) a piecewise linear function and 3) a cubic-spline function.

The basic principle underlying his investigations was well known and frequently applied in many fields; the network principle: generate a net of fixed points (here grid points) and perform very accurate measurements at these points (here, calculate exact covariances); these fixed points serve as a basis for small scale measurements which can be performed using simpler apparatus (here, more or less interpolation of covariances by means of finite elements). His report was primarily devoted to the study of interpolation errors, perturbation of spectra and to the consequences of the approximation for the predicted signal and its mean square error. Sünkel concluded that because of its smoothness and its most favourable approximation properties the spline function representation of the covariance function presents itself as a very useful tool for this kind of application.

1.8 Covariance Expressions for Second and Lower Order Derivatives of the Anomalous Potential

Since Moritz (1972) discussed the theory and application of least squares collocation techniques for gravimetric geodesy problems a number of additional studies have been done to gain better insight into collocation from a theoretical point of view, and to apply least squares collocation techniques to geodetic problems of interest.

One aspect of this work is the development of consistent covariance functions. One step in this direction was described in Tscherning and Rapp (1974). In this paper an anomaly degree variance model was developed that was fitted to different types of information that could be related to the anomaly degree variances. In that paper the following model was chosen:

$$c_\ell = \frac{A(\ell-1)}{(\ell-2)(\ell+B)} \tag{1}$$

where $A = 425.28$ mgal^2, $B = 24$, and a parameter s (used in the application of c_ℓ), $s = 0.999617$. In that report a number of covariance and cross covariance functions were derived involving anomalies, deflections of the vertical, and geoid undulations. All these quanties were considered to be global covariance functions based on a single anomaly degree variance model.

What was lacking from the above study were the covariances involving a number of derivatives that could be related to gravity gradients. To extend the earlier work Tscherning (1976) developed an extended set of covariance function equations for use with several different anomaly degree variance models. More specifically Tscherning (ibid.) chose to model the potential degree variances which refer to the disturbing potential. Thus if $\mathrm{cov}(T_P, T_Q)$ is the covariance between the disturbing potential at points P and Q he writes:

$$\begin{aligned}\mathrm{cov}(T_P, T_Q) = & \sum_{\ell=0}^{n} \overset{c}{\sigma}_\ell(T,T)\, s^{\ell+1} P_\ell(t) \\ & + \sum_{\ell=0}^{\infty} \overset{m}{\sigma}_\ell(T,T)\, s^{\ell+1} P_\ell(t)\end{aligned} \tag{2}$$

where: $\overset{m}{\sigma}_\ell(T,T)$ are the model potential degree variances and $\overset{c}{\sigma}_\ell(T,T)$ are corrections to the model potential degree variances to degree n. We have:

$$s = R_b^2/(r_P \cdot r_Q) \quad \text{and} \quad t = \cos\psi \tag{3}$$

where R_b is the radius of the Bjerhammer sphere, r is a geocentric distance and ψ is the arc between the two points. Tscherning (ibid.) writes the $\sigma(T,T)$ values in the following general form:

$$\overset{m}{\sigma}_{\ell,i} = A_i \prod_{j=0}^{i} (\ell + k_j)^{-1} \tag{4}$$

where i (1, 2, or 3) is a model number and A_i is a constant in units of $(m/sec)^4$. The σ_ℓ values are related to the c_ℓ values by (Tscherning and Rapp, ibid.)

$$\sigma_\ell = \frac{R_b^2}{(\ell - 1)^2} c_\ell \qquad (5)$$

Using (4) and (5) with (1) Tscherning gave: $k_0 = -2$, $k_1 = -1$, $k_2 = B$, $A = A_i\, 10^{10}/R_b^2$ with R_b in meters.

At this point Tscherning (1976) developed the necessary equations to evaluate the covariances and auto covariances for the following quantities: (1) the height anomaly; (2) the negative gravity disturbance r; (3) the gravity anomaly (Δg); (4) the radial component of the gradient of Δg; (5) the second order radial derivative of T; (6),(7) the latitude and longitude components of the deflection of the vertical; (8),(9) the derivatives in the northern and eastern direction of Δg; (10),(11) the derivatives of the gravity disturbance in the northern and eastern direction; (12)-(14) the second order derivatives of T in the northern, in mixed northern and eastern, and in the eastern direction. Specific equations for these quantities can be found in Tscherning (ibid., p. 2 and 3).

To implement these equations Tscherning devised a subroutine called COVAX that was given in the report. A few corrections to the text and the computer program have been made since its original publication. Such corrections can be obtained directly from Tscherning.

1.9 Two Models for the Degree Variances of Global Covariance Functions

In carrying out the computations with COVAX, Tscherning found that the variance of the vertical gradient of the gravity anomaly was about $7000E^2$ which implies a horizontal anomaly gradient (C_0) variance of $3500E^2$. Moritz (1977) showed that this high variance implies a correlation length (ξ) smaller than found in practice. He then postulated a form of an anomaly covariance function that implied an anomaly degree variance model that would avoid the apparent problems of the Tscherning-Rapp model when dealing with gradient covariance functions. Jekeli (1978) proceeded to investigate the Moritz suggestion and attempted to provide new numerical estimates of the anomaly degree variance models.

Jekeli (ibid.) expressed the anomaly covariance function, $C(P,Q)$, in the following form:

$$C(P,Q) = \sum_{\ell=2}^{\infty} c_\ell \left(\frac{R_c^2}{r_P r_Q}\right)^{n+2} P_\ell(\cos\psi) \qquad (6)$$

where R_c is the radius of a sphere to which the c_ℓ values are referred and c_ℓ is given by:

$$c_\ell = \alpha_1 \frac{\ell - 1}{(\ell + A)} \sigma_1^{\ell+2} + \alpha_2 \frac{\ell - 1}{(\ell - 2)(\ell + B)} \sigma_2^{\ell+2}, \quad \ell \geq 3 \qquad (7)$$

In this expression the parameters to be estimated are α_1, α_2, σ_1, and σ_2. It was noted that if $\alpha_1 = 0$, equation (7) reduces to the Tscherning-Rapp model. The values of A and B are not easily found through adjustment procedures; rather a trial and error technique is chosen.

Jekeli then proposed to determine the parameters of the model by using the following data; anomaly degree variances to degree 52 from Rapp (1977) or to degree 20 from the GEM 9 solution; point anomaly variance; the vertical gradient variance; and mean anomaly variances for 1° and 5° blocks. Special consideration was given to using a high gradient variance ($7000E^2$) and a low gradient variance ($400E^2$) to see the effect on the adjusted model.

Many different solutions were made by Jekeli to obtain solutions for the two component ($\alpha_1, \alpha_2 \neq 0$) model and a one component ($\alpha_1 = 0, \alpha_2 \neq 0$) model. Using the GEM 9 degree variances to degree 20 the results of Jekeli's model fits are shown in Table 1 (from Jekeli, p. 53). The best fit to the data is found to be with the two component model with the low gradient variance. The one component model does not give a good point anomaly variance when a low gradient varance is used. At the lower degree ($\ell \sim 20$) the c_ℓ value from one model can be twice that of another model. At degree 60 this difference has decreased significantly; the magnitude of c_ℓ being on the order of 5 mgal^2 for all models. These values are somewhat high when compared to recent (Rapp 1978) determinations showing values more on the order of 3 mgal^2.

1.10 A Model Comparison in Least-Squares Collocation

In addition to considering improved anomaly degree variance models needed for covariance function computation, several studies were carried out to obtain a better understanding of some theoretical problems in least squares collocation. One such study was carried out by Rummel (1976). Here Rummel pointed out that two models of least squares collocation are in current use. The first he refers to as the model of Method One which is:

$$\ell = Ax + s' + n \tag{8}$$

where:
- ℓ is a vector of observations;
- x is a vector of unknown parameters;
- A is the coefficient matrix;
- s' is the random signal part of ℓ;
- n is a random signal or noise.

The estimation of the signal at an arbitrary point is then given by (Rummel, ibid)

$$s = C_{ss'}(C_{s's'} + C_{nn})^{-1}(\ell - Ax) \tag{9}$$

where $C_{s's'}$ is the covariance matrix between the "observed" quantities and $C_{ss'}$ is the covariance matrix between the signal (s) to be estimated and the "observed" signal. Model One is the model originally suggested by Moritz (1972).

Table 1. Parameters of Anomaly Degree Variance Models (from Jekeli, 1978).

Model		init. value	stand. dev.		Model Parameters		adj. value	Variances implied by the Model plus the value of the c_2-term						
		$G_{OH} \pm$ s.d. E^2	(C_0) $(\mathrm{mgal})^2$	$(\bar{C}_0)$ $(\mathrm{mgal})^2$	α_1, α_2 $(\mathrm{mgal})^2$	σ_1, σ_2	c_3, c_{20} $(\mathrm{mgal})^2$	C_0 $(\mathrm{mgal})^2$	G_{OH} E^2	L_0 m^2	$\bar{C}_0(1^\circ)$ $(\mathrm{mgal})^2$	$\bar{C}_0(5^\circ)$ $(\mathrm{mgal})^2$	$\mathrm{RMS}(\bar{C}_0)$ $(\mathrm{mgal})^2$	$\mathrm{RMS}(c_n)$ $(\mathrm{mgal})^2$
2L	A=100 B= 20	200± 10	25	3	18.3906 658.6132	.9943667 .9048949	35.1 4.5	1828.7	200.3	919.4	862.44	251.78	5.2	2.74
2H	A=140 B= 10	3500 ± 100	25	3	14.0908 160.6701	.9939083 .9997595	24.9 7.1	1870.2	3526.	779.4	856.95	275.48	12.2	4.68
1L	B= 30	200± 1	25	3	491.1365	.9982959	29.8 10.4	1287.4	199.6	888.5	817.82	357.28	76.2	6.32
1H	B= 30	3500 ± 20	25	3	454.2862	.9996025	27.5 9.6	1802.6	3504.	852.1	820.86	339.39	63.9	5.83
TR*	B= 24	-	-	-	425.28	.999617	31.5 10.2	1795.0	3542.	926.1	848.00	367.32	77.1	6.28

* The parameters of Model TR had been determined by Tscherning and Rapp (1974).

Model 2L: two-component model (3.23) with an observed value of G_{OH} = 200 E^2

Model 2H: two-component model (3.23) with an observed value of G_{OH} = 3500 E^2

Model 1L: single component model (3.12) with an observed value of G_{OH} = 200 E^2

Model 1H: single component model (3.12) with an observed value of G_{OH} = 3500 E^2

A second model for use in least squares collocation was suggested by Moritz and Schwarz (1973). This model is called Model Two by Rummel and is written in the following form:

$$\ell = A x + R s + n \tag{10}$$

where s is the desired signal and appears directly in the model. The values of s, x, and n in (10) are not necessarily the same as the values of in (8) and (9). The solution for a signal is given by (Rummel):

$$s = C_{ss} \mathbf{R}^T (R C_{ss} R^T + C_{nn})^{-1} (\ell - A x) \tag{11}$$

It is this form of the collocation solution that is often reduced to one involving an inversion of a matrix whose size is equal to that of the signals being estimated, and not one requiring the inversion of a matrix whose size corresponds to the number of observations used in the process.

Rummel clearly shows that, in general, the results from (9) and (11) will not be the same. The only case of equivalence will be when we have:

$$s' = R s \tag{12}$$

In most gravimetric applications R will have to be a matrix transforming an infinite set of signals s (such as gravity anomalies) into s' (such as a geoid undulation). In practice this could only be done in an approximate way. Thus Rummel proved that the two models used in least squares collocation are different. Only in special cases should Model Two be preferred with most applications being done using Model One.

1.11 Potential Coefficient Determinations from 10° Terrestrial Gravity Data By Means of Collocation

A specific application of Model One was carried out by Sjöberg (1978a). He considered the estimation of potential coefficients from mean gravity anomaly data given in 416 10° equal area anomaly blocks. In this case equation (9) takes the following form:

$$\begin{Bmatrix} \bar{C}_{\ell m} \\ \bar{S}_{\ell m} \end{Bmatrix} = \begin{Bmatrix} c_C^T \\ c_S^T \end{Bmatrix} (C + D)^{-1} \underline{\Delta g} \tag{13}$$

where $\bar{C}$, $\bar{S}$ are the predicted fully normalized potential coefficients; C is the covariance matrix between the mean anomalies and c_C, c_S are the cross covariances between the potential coefficients and the mean anomalies; D is the error covariance matrix of the anomalies. Instead of computing the mean covariance functions by numerical integration Sjöberg (ibid) developed several different procedures including the use of the smoothing operator of Pellinen, β_n, and the evaluation of the point covariance functions at a certain elevation above the mean sphere.

Sjöberg carried out three different solutions: 1) a least squares collocation solution considering the mean elevations of the anomaly blocks; 2) a least squares collocation solution setting the elevations to zero; and 3) a collocation solution setting the elevations to zero and the noise matrix D to zero. These solutions were then compared to the potential coefficients found from the usual numerical integration procedure, to the GEM 9 potential coefficients, and to coefficients computed from 5° mean anomalies (Rapp, 1977a). Some results in terms of differences are summarized in Table 2.

Table 2. Comparison of Potential Coefficients From Least Squares Collocation and Other Sets (data from Sjöberg, 1978a)

	Percentage Difference (%)	RMS Undulation Difference (m)	RMS Anomaly Difference (mgals)
GEM 9, # 251	68	8.7	7.2
GEM 9, Int. 10°	73	8.8	7.6
GEM 9, Coll. 1	68	9.2	7.2
GEM 9, Coll. 2	68	8.8	7.2
GEM 9, Coll. 3	71	9.9	7.5
# 251, Int. 10°	32	2.0	3.6
# 251, Coll. 1	30	2.8	3.3
# 251, Coll. 2	30	3.0	3.3
Int. 10°, Coll. 1	13	2.4	1.6
Int. 10°, Coll. 2	14	2.6	1.7
Int. 10°, Coll. 3	15	3.2	1.7
Coll. 1, Coll. 2	3	2.1	0.7

From this data several things can be seen:

1. The coefficients from the 5° solution agree better with the GEM 9 coefficients than the 10° solutions.

2. The effect of including the elevations in the the collocation solution slightly degrades the comparisons with GEM 9.

3. The best agreement with the coefficients from the 10° anomalies takes place when the D matrix is included with the collocation solution.

4. The collocation solutions agree better with the GEM 9 coefficients than do the coefficients found from the usual integration procedures.

Sjöberg (ibid) also considered the computational effort in carrying out these computations. He found that 19 seconds of computer time were needed for the usual integration (or summation) procedure while 561 seconds were needed for the collocation 2 solution. Although the coefficients from the collocation solution may be slightly better than the summation the tremendous increase in computational effort does not justify the use of least squares collocation for the estimation of potential coefficients from a global set of gravity anomalies.

1.12 A Comparison of Bjerhammar's Method and Collocation in Physical Geodesy

Another study was conducted by Sjöberg (1978b) on the relationship between least squares collocation and the Bjerhammar theory. The Bjerhammar theory postulates a set of fictitious gravity anomalies, Δg^*, located on the surface of the Bjerhammar sphere (which is internal to all the masses). These anomalies are related to the observed anomalies (Δg) through the Poisson integral. In principal the Δg^* values can be found from solving:

$$\Delta g = A \, \Delta g^* \tag{14}$$

where the elements of A are determined from the elements of the Poisson integral. The solution of (14) depends on the number of given anomalies and the number of anomalies to be found. A similar equation to (14) for anomaly prediction can be written using least squares collocation procedures:

$$\Delta g_i = C_i \, (C + D)^{-1} \, \Delta g \tag{15}$$

where the elements of C_i and C depend on a spatial covariance function. Sjöberg then examines various solutions of (14) writing a general form as:

$$\Delta g^* = Q A^T \, (A Q A^T)^{-1} \, \Delta g \tag{16}$$

where Q is a weight matrix. If the number N of the Δg^* values approaches infinity Sjöberg shows that in this case the solution of (16) and the resultant prediction of new anomalies from the original set can be written as:

$$\Delta g_i = C_i \, C^{-1} \, \Delta g \tag{17}$$

where the elements of the C matrix are computed from:

$$C_{ij} = \sum_{n=0}^{\infty} c_n^* \left(\frac{r_B^2}{r_i r_j} \right)^{n+2} P_n (\cos \psi_{ij}) \tag{18}$$

Here the c_n^* are values uniquely associated with a minimum norm process of a generalized Bjerhammar method. These c_n^* values are analogous to the anomaly degree variances (c_n) of the least squares collocation technique. Thus for each type of degree variance in a collocation solution there is a corresponding minimum norm solution in the generalized Bjerhammar theory (Sjöberg, ibid, p. 9).

Sjoberg (ibid) continues this discussion for a number of different cases, such as including noise in the Bjerhammar theory, and an analysis of a reflexive prediction process suggested by Bjerhammar in 1974. These studies showed that for a special case of $c_n^* = c_n = 2n + 1$ the concept of reflexive prediction was less sensitive to changes of the radius of the Bjerhammar sphere than was collocation.

In the second method structural differences between the gravitational and inertial fields are used to separate the two effects. The differences show up in the second and higher order gradients of the force fields. Therefore, additional measurements are necessary in this case. Moritz (1967) has shown that for an aircraft with inertial stabilization the second derivatives of the force field do not contain inertial disturbances, so that purely gravitational second-order gradients can be measured. They are used to obtain the gravitational force vector by integrating along the flight path. It should be noted that, in contrast to the first method, a rigorous separation of gravitation and inertia is possible in this case, and that from a theoretical point of view this approach is preferable. The practical difficulties originate in the design of instruments accurate enough to make an application feasible. Advances in instrument development have been rapid during the last years and a gradiometer with an accuracy of a few Eotvos may be available in the near future. Therefore, the capabilities of an airborne gradiometer system was studied by Schwarz (1976) in his report: "Geodetic Accuracies Obtainable from Measurements of First and Second Order Gravitational Graditents."

The accuracy study of Schwarz was performed using the method of least-squares collocation. There were three reasons why this approach was especially suited for the problem. First, it allowed the combination of heterogeneous data in a consistent way. This was very important because geoidal heights, gravity anomalies, and different second-order gradients were used as measurements. They must be evaluated in such a way that their common origin from the same anomalous gravity field was part of the system. In least-squares collocation this is achieved by describing the statistical structure of the field by a covariance function. Second, mean gravity values at ground level must be estimated using point values on a profile in flying altitude and additional information on ground. This involved interpolation between profiles, downward continuation, combination of different quantities, and estimation of mean values. All these steps could be united in a single step procedure in the collocation method. This was impossible when using the corresponding integral formulas. Third, different assumptions on the structure of the gravity field and on the accuracy of the measurements must be investigated. Again this was simple with the collocation method because it only involved a change of the fundamental covariance function or of the error variances.

A detailed analysis of interpolation, downward continuation and mean value determinations was given by Schwarz. The influence of measureing errors was considered and the effects were discussed in connection with the stability problem. Finally, the contribution of accurate satellite altimeter to a combined accelerometer-gradiometer system was taken into account.

The results of his study showed that a system of this kind could significantly contribute to our knowledge of the anomalous gravity field if second-order gravitational gradients could be measured with an accuracy of a few Eotvos.

Sjoberg (ibid) found that the prediction results and the stability of the two methods would be the same if the collocation procedure was applied with half the depth to the Bjerhammar sphere used in the reflexive prediction. This condition was based only on the analysis of one degree varinace model with additional analysis being needed.

2. Analyses of Airborne Measurements

Dr. Schwarz was doing studies on airborne and satellite measurements and their application for determination of mean anomalies. He prepared two reports and did extensive computations for a test area. Lenny Krieg was assisting him in the computations for the test area. In the following Schwarz's work is summarized based on his formal and informal reports. Some parts of his reports are quoted here without quotation marks.

2.1 Geodetic Accuracies Obtainable from Measurements of First and Second Order Gravitational Gradients

Experiments with airborne gravimeters have been performed over the last 17 years. Although the equipment designed for this objective has a high degree of sophistication, the results obtained so far are not accurate enough for geodetic purposes. The reason lies in the complicated structure of the force field acting on a moving gravimeter. Gravimeters are basically accelerometers and they measure the resultant of gravitational and inertial forces. If they are used as stationary instruments, as in most terrestrial applications, the only inertial force acting on the gravimeter is the centrifugal force. Therefore, the output of the instrument is the combined effect of gravitational attraction and centrifugal force, i.e. gravity. The situation is more complicated in a moving gravimeter. The Coriolis force has to be taken into account and, more important, irregular accelerations of the base will strongly influence the result of the measurements. Such undesirable inertial forces are especially strong in a moving aircraft and there is no way to rigorously separate the gravitational part from the inertial part by using gravimeter measurements only. Therefore, additional information is necessary to extract the gravitational effect.

Two methods have been proposed to reach this goal. In the first one information on the frequency behaviour of the different forces is used to separate gravity and disturbing accelerations by statistical filtering techniques. Meissl (1970) has investigated this approach using the theory of stochastic processes and certain assumptions on the power spectra of the force fields involved. He concludes that it is most difficult to separate gravitation and inertia in the medium frequency range with half wavelength between 30 and 150 km. This is only possible if detailed information on the two spectra is available which usually will not be the case. The high frequencies can be blocked by a low pass filter, the low frequencies can be improved by regularly updating altitude and position. The remaining errors will, however, be of a size which will not allow a useful geodetic application of the filtered data. The findings from a probabilistic error analysis were confirmed by results obtained by Szabo and Anthony (1971) in an analysis of actual measurements.

2.2 Simulation Study of Airborne Gradiometry

In his previous report Schwarz (1976) studied the accuracy of airborne gradiometry and he drew some conclusions about optimal point configurations and data combinations. His report (1977): "Simulation Study of Airborne Gradiometry" supplements some of the previous investigations. These simulation studies displayed the behaviour of individual experiments only and were therefore not meant for checking results of an accuracy study. The interest of a simulation study was in the domain of operational realization and optimal performance.

In order to get an operational program for airborne gradiometry the most important problem to cope with was the efficient handling of large amounts of data. The proposed measuring system will produce about 250 observations per profile and degree. In order to cover a $20^\circ \times 25^\circ$ area with profiles spaced at 1° we have to treat 130 000 measurements. For mean gravity values below $1^\circ \times 1^\circ$ we have to use 20' spacings and the above number of measurements will triple. It was shown by Schwarz (1977) how the number of observations can be reduced without significantly impairing the accuracy of the results. For an operational program, however, it is necessary to use all information available. Not so much to increase accuracy but to make results more reliable. Therefore, Schwarz incorporated both viewpoints in the program.

The simulation studies presented by Schwarz showed that least-squares collocation offers an adequate model to estimate gravity anomalies from airborne gradiometer measurements. The procedure is simple numerically and allows to handle large amounts of data with regular requirements on core strage and small demands on computer time.

The deviations of the estimated gravity anomalies from their true values agreed well with the estimates obtained from corresponding accuracy studies (Schwarz 1976). It should be noted, however, that correlated errors in the measurements will strongly influence the accuracy of the results. A second-order Markov sequence was used by Schwarz to model the error process along the profiles. Depending on the size of the correlations and the variance of the process, the mean-square errors would more than double as compared to the uncorrelated case. Similarly, a bias in the data would impair the accuracy of the results considerable.

The simulation of gravimetric quantities from a point mass model was considered in the spectral domain and conclusions were drawn with respect to the resulting fields. In order to represent adequately regional variations of the gravity field as well as the local behaviour of the second-order gradients the medium and the high frequency part of the spectrum must be modeled equally well. This can only be achieved by using several planes of generating masses at different depths. Schwarz concluded that in many cases a model with two planes may already be sufficient.

2.3 Other Studies

Using the results of the above studies, Schwarz computed mean gravity anomalies from gradiometer and altimeter data for the selected test area which was 25° in latitude times 30° in longitude. The results of these extensive computations as well as the corresponding computer programs were communicated to AFGL.

Schwarz also did studies in combination of satellite derived harmonic coefficients and terrestrial mean gravity anomalies by least-squares collocation. The results were communicated informally to AFGL.

3. Satellite to Satellite Tracking Research

During the period of the contract an evolutionary research development has taken place in the analysis of satellite to satellite tracking data for use in the recovery of mean gravity anomalies at the surface of the earth. This work is described in three reports which are briefly discussed in the following.

The first report in this area under the contract was that of Rummel, Hajela, and Rapp (1976). This report first examined the theory where one satellite is tracked by another such that a range rate between a relay satellite (such as ATS-6) and a close satellite (such as Geos-3) could be determined. This range rate data can be used to determine the line of sight acceleration between the relay and the close satellite. A number of different geometries were considered to relate this line of sight acceleration to the gradient of the disturbing potential. For one approximation the following least square collocation solution was suggested:

$$\Delta g(Q) = \underline{C}(Q)_{\Delta g\, T_{rs}} (\underline{\underline{C}}_{T_{rj} T_{rj}} + \underline{\underline{D}}_{ji})^{-1} (\ddot{R}_{RC}^{RES} / \cos \beta)_i \tag{19}$$

where: Δg is the predicted anomaly;

$C(Q)$ is the covariance between the anomaly being predicted and the radial gravity disturbance at the close satellite observation point j;

$C_{T_{rj} T_{rj}}$ is the auto covariance matrix between the radial gravity disturbance components;

D_{jc} is the noise matrix

$\ddot{R}_{RC}^{RES}$ is the residual line of sight acceleration with respect to a reference field to which Δg will be referred;

β is the angle between the line of sight acceleration at the close satellite and the direction of the radial gravity disturbance.

Equation (19) is an approximation in the sense that only the radial component of the gravity disturbance is being considered.

The next step was a series of numerical simulation studies designed to test equation (19). To do this orbits were generated (using Geodyne) in a reference field and in a higher degree field desig... d to reflect reality. Residuals were formed and predictions using (19) were carried out using covariances obtained from COVAX (Tscherning, 1976). Predictions were made for 10° and 5° equal area anomalies with different data point intervals and different values for the elements of the D matrix. In this work it was estimated that with the geometry of the Geos-3, ATS-6 satellite situation, 10° anomalies could be recovered to an accuracy (s.d.) of about ± 4 mgals and 5° anomalies to an accuracy (s.d.) of about ± 11 mgals.

Additional tests indicated that the results were quite sensitive to errors in the initial orbital elements and the values of the elements in the D matrix.

It was clear, however, that the suggested procedure did work in a simulation mode and that it would be worthwhile to proceed to the analysis of real data. Such an analysis is described in the report of Hajela (1977).

In the research carried out by Hajela (ibid) actual Geos-3, ATS-6 range rate data provided by NASA was analyzed. The data supplied was in the ATSR format and was reformatted for entry into the Geodyn program. Five passes of Geos-3 in the area φ: 15° to 30°, and λ: 275° to 295° were selected for processing using initial state vectors supplied, in part, by NASA. After the best set of initial state vectors were found (with the data available) the range rate residuals were fitted to, and filtered by using precise continuous cubic splines to fit the range-rate data in the least squares sense. Hajela (ibid) found that the optimum spacing of the nodes where the adjacent cubic splines met was every 60 seconds in fitting range-rate observations at 10 second intervals. After the spline fit was made, the residual accelerations ($\ddot{R}$) are found by the differentiation of the spline function. Using this process a smoothed set of accelerations were obtained for parts of the five passes of Geos-3 across the area of interest.

Hajela (ibid) used this data to recover light 5° equal area anomalies. In doing this recovery a number of different variables were considered including the spacing of the nodes in the spline fitting, various accuracy estimates for use in the D matrix of equation (19), use of various arc combinations, and consideration of initial epoch vector errors.

The recovered anomalies were compared to ground truth data available from terrestrial sources. The root mean square discrepancy between the recovered anomalies and the ground truth values was ±8 mgals while the average predicted standard deviation was ±12 mgals. It was felt that these results were quite satisfactory and that the proposed method was a workable technique for gravity anomaly recovery.

After the above study it became clear that improved modeling could be done for the anomaly recovery. In addition a number of questions remained unanswered concerning the estimation process. The main problem in the modelling area was in relating the line of sight accelerations to the surface gravity anomalies. In the previous reports various techniques for evaluating the covariances were described. However the application was not as direct as one might want. To improve the situation Rummel and Rapp (1977) outlined the equations needed to work directly with line of sight accelerations instead of radial gravity disturbances. Hajela (1978) implemented these equations and carried out new tests with real world data from the ATS-6, Geos-3 satellites.

In this most recent study Hajela (ibid) examined a number of different topics. One case considered was the removal of a linear trend in the residual accelerations caused by errors in the initial state vectors. This was attempted by introducing a systematic part (AX) to the original collocation model. Sample solutions were made solving for a linear trend. However Hajela (ibid) found that a considerable portion of the signal was also removed from the residuals. Thus this technique was not used further.

Hajela also looked at the correlation between the predicted anomalies from the collocation solution using the data from the previous solution. He first made tests for the recovery of the 5° equal area anomalies. He found for the eight adjacent anomalies the largest correlation coefficient was -0.09 indicating negligible correlation. A second test was made for the recovery of 2.°5 anomalies. The largest correlation found for a 30 second data interval was 0.1 which increased to 0.3 when a 4 minute interval was used. The biggest change (from the 5° solution) was an increase in the standard deviation from ±10 mgals for the 5° anomalies to ±15 mgals for the 2.°5 anomalies.

The main purpose of the Hajela (1978) report was the implementation of the line of sight acceleration method. Hajela gives the specific equations for the rigorous implementation of this method. He then uses this method with real data based on improved satellite orbits. Data from a number of arcs was provided to us by Jim Marsh from NASA. Hajela used the new technique for 5° anomaly recovery. The recovered anomalies in eight blocks were compared to corresponding terrestrial anomalies and anomalies recently derived from Geos-3 altimeter data. The root mean square difference between the SST derived anomaly and the altimeter derived anomaly was ±7 mgals with an estimated standard deviation of each SST anomaly being approximately ±6 mgals.

These new tests indicate that the proposed method is a valid technique for anomaly recovery from SST data. We now need more such data for further testing of the method.

4. Convergence Problems

Despite the general feeling that the convergence of the spherical harmonic expansions to finite degree, on the surface of the earth, is not a problem, we have carried out several studies to improve our understanding of the problem.

One such study, Rapp (1977b), described and tested a method for computing gravity anomalies at the surface of the earth from spherical harmonic expansion in such a way as to avoid convergence questions. The method is simple, being basically a two step process. The first step takes the spherical harmonic expansion of the disturbing potential and then evaluates it at points, or in compoartments, on a sphere that surrounds the mass of the earth. (The mass of the atmosphere was not considered in these computations.) The anomalies are then downward continued to the surface of the earth using least squares collocation techniques. To test this idea, 5° anomalies were used with their value computed at the surface of the earth in two ways. First the anomalies were computed on the bounding sphere. A downward continuation correction was applied to obtain a surface anomaly that was compared to the terrestrial data where a mean square difference of 91 mgal^2

was found. Then the potential coefficient anomalies were computed directly at the surface where a mean square difference of 109 mgal^2 was found. In this case the theoretically more correct procedure gave the better result.

The concept of the enclosing sphere computation also provides a convenient formulation of the problem of computing potential coefficients from surface gravity anomalies. This procedure can be represented in the following form:

$$C = C_A + \Delta C_I + \Delta C_{II} \tag{20}$$

where: C is the rigorous potential coefficient;
C_A is the approximate potential coefficient derived from the usual summation formula applied to uncorrected anomaly data;
ΔC_I is a correction dependent on the radius of the bounding sphere, a mean earth radius, the equatorial radius and the C_A values;
ΔC_{II} is a correction term computed by applying the usual summation formula to the upward continuation correction term.

Studies were made of the magnitude of ΔC_I and ΔC_{II} expressed as a percentage of the expected magnitudes of the total coefficients. Up to degree 12 the total correction was less than 2%. At degree 40 the total correction had reached 7%. These figures indicated the corrections terms are small and negligible with the current data accuracy. Additional investigations to higher degrees are needed using $1^\circ \times 1^\circ$ anomalies.

A completely different view of this problem was taken by Sjöberg (1977a). In the first part of his study Sjöberg constructed a simple example to show that the spherical harmonic expansion of the earth's gravitational potential is divergent at the surface of the earth. The error caused by evaluating a divergent series but including only a finite number of terms was considered in terms of an analytic continuation error and a truncation error. The percentage error in the disturbing potential at degree 60 could reach 20% when an improper downward continuation procedure was applied to a sphere with a point mass located at a height of 20 km above the sphere.

Sjöberg (ibid) extended his special case to that of the real earth containing topography. He computed errors in the potential caused by neglecting topography and compared his results to previous results found by Cook and Levallois. Specific equations for computing corrections terms due to the topography were derived and applied for a spherical harmonic expansion of the disturbing potential and the gravity anomaly. These equations were applied to obtain global error estimates and to obtain specific error estimates using $5^\circ \times 5^\circ$ mean elevations. Errors on the order of ± 1 meter were found for geoid undulations and up to 70 mgals for the anomalies. This large anomaly error is unrealistic so that there is some concern on the correctness of some of the equations or data used by Sjöberg (ibid). It may be that smaller blocks (say $1^\circ \times 1^\circ$) are needed to adequately represent the true situation. Thus additional work is called for in this area.

5. Other Areas of Research

As part of our continuing interest in estimating accuracies of gravimetric quantities Sjöberg (1977b) considered the estimation of the accuracies of the computation of the deflection of the vertical. In this estimation Sjöberg (ibid) considered that the deflections would be computed in three components: 1) a contribution from a set of potential coefficients; 2) a contribution from $1^\circ \times 1^\circ$ anomalies in an area surrounding the computation point; and 3) a contribution from detailed data in the immediate vicinity of the computation point. Since potential coefficients are known to a relatively low degree it was also necessary to consider a truncation error. Also, the use of $1^\circ \times 1^\circ$ anomalies implies that certain higher frequency information will be lost in the solution so that this effect must be considered.

For each of these effects or contributions, Sjöberg (ibid) derived approximate error equations. He applied these equations with the GEM 7 potential coefficients and existing $1^\circ \times 1^\circ$ mean anomaly information. Specific accuracy estimates due to the various error sources were made for four points having various accuracies of $1^\circ \times 1^\circ$ anomalies surrounding them. Considering all error sources with the local data surrounding the point taken out to 1° from the point, Sjöberg (ibid) estimated the error in the total deflection to range from $\pm 1.5''$ to $\pm 2.3''$ with a cap to 20° of surrounding $1^\circ \times 1^\circ$ data.

In another area Kearsley (1977) examined the prediction of mean anomalies at sea using collocation techniques applied to selected geophysical phenomena. In this paper the author reviewed some of the factors that influence anomalies at sea including bottom topography (or ocean depth), and the age of the crust in the area of interest.

In considering these dependencies, Kearsley (ibid) developed auto covariance functions for crustal age, the cross covariance function for anomalies and crustal age, auto covariance functions for depth and a cross covariance function for anomalies and depth, all using data in $1^\circ \times 1^\circ$ blocks. This information was used to predict a number of $1^\circ \times 1^\circ$ anomalies which were then compared to their known values. The root mean square difference found was ± 11.5 mgals which compares favorably with the predicted standard deviations of about ± 11 mgals. Kearsley (ibid) suggests that all the data and covariances could be combined in one general prediction process.

In the past few years great progress has been made in improving our knowledge of the earth's gravity field. Potential coefficient determinations from satellite data alone have been extended to degree 20 with many additional resonant type terms. In a number of new areas terrestrial data has become available. And of most significant importance the Geos-3 altimeter data has enabled the determination of a large number of $1^\circ \times 1^\circ$ mean anomalies. Rapp (1978) suggests and implements a procedure to combine all this data in a rigorous least squares adjustment process. The technique was applied to the GEM 9 potential coefficient set taken to degree 12 field on an almost global set of $1^\circ \times 1^\circ$ anomalies. Specifically this anomaly field contained 50650 values, 28176 of which were derived from Geos-3 altimeter data. The remaining values needed to form a global set were taken to be zero with a standard deviation of ±30 mgals. The result of the adjustment was an adjusted set of potential coefficients to degree 12 and an adjusted global set of 1° x 1° anomalies. These adjusted anomalies were developed into potential coefficients to degree 60. These coefficients must agree exactly with the coefficients found in the actual adjustment process. Higher degree solutions are possible, both in the adjustment process, and in the determination of the potential coefficients from the adjusted anomalies. The main advantage of the technique used in Rapp (ibid) is that it provides a rigorous combination of the data to obtain a consistent set of potential coefficients and 1° x 1° anomalies.

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7. List of Scientific Personnel

Supervisors and Principal Investigators

Urho A. Uotila
Richard H. Rapp

Research Associates and Consultants

D.P. Hajela
William Kearsley
Helmut Moritz
Reiner Rummel
Klaus-Peter Schwarz
Lars Sjöberg
Hans Sünkel
C.C. Tscherning

Graduate Research Associates

Joshua Greenfield
Christopher Jekeli
D. Jeyanandan
Lenny A. Krieg
Joseph C. Loon
K. Katsambalos