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A NOTE ON THE MEASUREMENTS OF QUANTIZED AREAS AND BOUNDARIES. (U)
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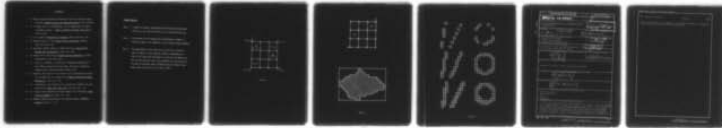
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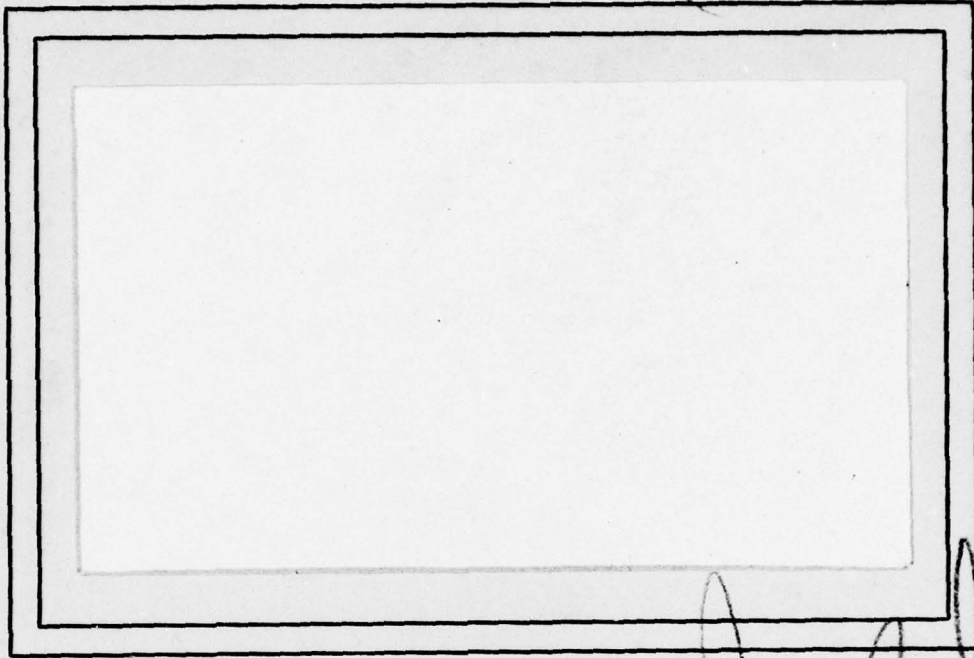


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A NOTE ON THE MEASUREMENTS OF
QUANTIZED AREAS AND BOUNDARIES

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ABSTRACT

Pixels in quantized pictures are considered as cells in a grid, but a pixel may be represented by a lattice point placed at its center. It is argued that the perimeter of an area is its grid point boundary, not its lattice point boundary, the former always being 4 units greater than the latter, whatever the shape of the area. Use of the grid point boundary gives satisfactory values of perimeter, P , area, A , and compactness, P^2/A , with a minimum value of 16 for P^2/A . However, quantized drawn lines are most conveniently represented by the line joining lattice points, if an empirical procedure is adopted to remove ambiguities arising from the quantization.

P_squared

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1. THE TECHNOLOGY OF PICTURE QUANTIZATION

Innumerable methods of quantizing pictures all have in common that the quantized picture is divided into elements, pixels, to each of which is assigned a level of intensity and color specified by one or more integers. Discussion in this note is restricted to square pixels, each of the same size, arranged in a rectangular array and used to display a one-bit, i.e. black-and-white, picture. The discussion could, however, be extended to multitoneal pictures and to pixels arranged in a hexagonal array or with other geometric variations.

The number of pixels in an array is clearly enumerable. Write $\{X_i, Y_j\}$ as the identification of the pixel in the i -th column and j -th row where $1 \leq i \leq m$ and $1 \leq j \leq n$ if the pixels are in an $m \times n$ array. As the result of quantizing a picture, each pixel will be either black or white. This situation is quantified by giving each pixel the value of 0 or 1, i.e. writing $\{X_i, Y_j\} = 0$ if pixel $\{X_i, Y_j\}$ is black and $\{X_i, Y_j\} = 1$ if it is white. The device which assigns the value 0 or 1 to each pixel will work according to some rules: each pixel is associated in a determinable way with an area of the original unquantized picture and is assigned a value according to the color and saturation of that area. Let pixel $\{X_i, Y_j\}$ be associated with an area $\xi_{i,j}$ in the original picture. Then the value assigned to $\{X_i, Y_j\}$ will be discrete (0 or 1) while the value associated with $\xi_{i,j}$ is continuous. In the simplest case of the original picture itself being composed only of black or white areas, values could depend on the proportions of whiteness in areas so that the value associated with the area $\xi_{i,j}$ would be 0 if it were all black, 1 if it were all white and some intermediate value if a black-white boundary crossed the area. If the value associated with area $\xi_{i,j}$ is α , i.e. if the white fraction of $\xi_{i,j}$ is α , the assignment rule takes the form $\{X_i, Y_j\} = 1$ if $\alpha \geq \tau$ else $\{X_i, Y_j\} = 0$

where the quantity τ is the threshold for assigning to pixels the values 0 or 1. It is also to be noted that $\{X_i, Y_j\}$ is not a place or a point; it is merely an identifier or label for an element in a numerical array. In practice, however, it often happens that the pixel array is displayed on a VDU or printer and in such a situation it is convenient to use the label $\{X_i, Y_j\}$ for the physical manifestation of the pixel, the black or white area defined $i - 1 \leq x \leq i, j - 1 \leq y \leq j$ where x and y are variables giving distances along the co-ordinate axes in the usual way. But the pixel labels must not be confused with the geometric variables.

2. MEASUREMENT OF AREA

If each pixel is of unit area, the area of the whole array is mn . The total white area in the array is $\sum_{i=1}^m \sum_{j=1}^n \{X_i, Y_j\}$. However, this method of double summation is often inconvenient for measuring areas and investigators have sought to use more practicable methods such as the simple mensuration formulae, e.g. base x height for the area of a rectangle. Kulpa (1) and Sankar and Krishnamurthy (2) have proposed measuring area by using Pick's theorem (3) which states that the area A of a polygon, all the vertices of which lie on a grid, is given by $A = \frac{1}{2}b + i - 1$ where b is the number of boundary points of the polygon which are also grid points and i is the number of grid points in the interior of the polygon.

Consider the polygon with the pixels $\{X_1, Y_1\}, \{X_3, Y_1\}, \{X_3, Y_3\}$ and $\{X_1, Y_3\}$ at its vertices. This is a square with three pixels along each side. There are 8 pixels in the boundary of this square and 1 in the interior and with $b = 8$ and $i = 1$, Pick's theorem gives an area of 4. Yet obviously the area of the square is 9. The apparent contradiction

comes from using identifiers of pixels as the locations of points. Although a square can be unambiguously specified by referring to the pixels which constitute its corners, a pair of pixels do not (in the absence of further elaborate definition) define an open line. It is possible to associate a line with one or more pixels - namely, the line forming the perimeter of the area defined by the pixels; but this line cannot be independently specified by the labels which specify the pixels.

Let (X_i, Y_j) be a point which is considered to represent the pixel $\{X_1, Y_1\}$. This point (X_i, Y_j) could be placed anywhere within the area of the pixel; let it be placed in the center of the pixel as in Fig. 1, which shows the square defined by the pixels represented by the points (X_1, Y_1) , (X_3, Y_1) , (X_3, Y_3) and (X_1, Y_3) . Whereas the square defined by the pixels $\{X_1, Y_1\}$, etc. has an area of 9 units and a perimeter of 12 units, that defined by the points (X_1, Y_1) , etc. has an area of 4 units and a perimeter of 8 units. The same would be true if each point (X_i, Y_j) were placed elsewhere within the pixel it represents, e.g. in the bottom left-hand corner. This example exemplifies the errors in measurements of area and perimeter which arise if areas (or the labels of areas) are confused with points (or the labels of points).

The boundary of the square in Fig. 1 is defined by the points (x_0, y_0) , (x_3, y_0) , (x_3, y_3) and (x_0, y_3) . Since area is specified by pixels which are unit cells of the grid, it is convenient to refer to points such as (x_i, y_j) as grid points, to be distinguished from points such as (X_i, Y_j) which are referred to as lattice points. This nomenclature allows the situation to be described without ambiguity: the perimeter of a quantized area is its grid point boundary and the size of an area is given by the

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number of lattice points (i.e. the number of pixels) contained within it (see below).

3. CONSEQUENCES OF DISTINGUISHING GRID AND LATTICE POINTS

It is first to be noted that the grid points and the lattice points define two interleaved networks, both of which are square nets in the case under consideration. Furthermore, each unit cell of the grid (i.e. each pixel $\{X_i, Y_j\}$) contains just one lattice point (i.e. the point (X_i, Y_j)) and each unit cell of the lattice (e.g. the cell defined by the points (X_i, Y_j) , (X_{i+1}, Y_j) , (X_{i+1}, Y_{j+1}) and (X_i, Y_{j+1})) contains just one grid point (viz. the point (x_i, y_j)). The following arguments apply only to simply-connected regions, i.e. areas containing no holes; they can, however, be readily extended to encompass multiply connected regions (cf. (2)). Consider such a region R consisting of pixels with the value 1, all other pixels in the $m \times n$ array having the value of 0.

(i) The grid boundary of R encloses all the lattice points which represent the pixels constituting R , and no other lattice points. This follows immediately from the definitions of the terms. Formally, if any lattice point representing one of the pixels within R is itself outside the boundary then that pixel is outside the boundary and the boundary is false, i.e. is not the true perimeter of R . And if, within the boundary, there is a latticepoint representing a pixel not contained in R , then that pixel is a hole in R which is impossible since R is simply connected and has no holes.

(ii) The area of R is the area within the grid boundary. This follows from (i) since each lattice point represents the area of its corresponding pixel.

(iii) The lattice boundary (namely, the boundary formed by the lattice points representing all the pixels the edges or vertices of which contribute to the grid boundary) encloses all the grid points enclosed by the grid boundary and no other grid points. For if any grid point enclosed by the grid boundary were outside the lattice boundary it would be associated with a pixel which contributed to the grid boundary but was not represented by a lattice point in the lattice boundary, which contradicts the definition; from (i) there can be no grid point within the grid boundary which is not associated with a pixel contained in \underline{R} ; and also from (i) no grid boundary point can lie within the lattice boundary.

(iv) The area within the lattice boundary is g_i , the number of grid points enclosed by the grid boundary. This conclusion follows from the arguments of (iii) and (ii) if the representation is inverted so that a grid point (x_i, y_j) is taken to represent the area of the corresponding unit cell of the lattice, specified by the points (X_i, Y_j) , (X_{i+1}, Y_j) , (X_{i+1}, Y_{j+1}) and (X_i, Y_{j+1}) .

(v) The number of grid boundary points, g_b , and the number of lattice boundary points, l_b , are related by $g_b = l_b + 4$. Let the number of internal lattice points be l_i and, as before, the number of internal grid points be g_i . By Pick's theorem and (ii), the area of \underline{R} is $l_b + l_i = \frac{1}{2}g_b + g_i - 1$. And by Pick's theorem and (iv), the area within the lattice boundary is $g_i = \frac{1}{2}l_b + l_i - 1$. On eliminating g_i between the two equations, l_i drops out and it follows that $g_b = l_b + 4$. The conclusion holds whatever the shape of the area, but in areas only one pixel wide, the lattice point boundary turns back on itself and encloses zero area.

(vi) Grid boundary points and lattice boundary points are always 4-way connected for areas on a square grid. The connectedness of the points of

the grid boundary arises from the nature of the grid. The connectedness of lattice boundary points arises because areas which touch only at one or more vertices are separate and distinct, so that two lattice boundary points which are separated by $\sqrt{2}$ units of distance represent pixels which either are from two distinct areas or else have a common neighbor represented by a lattice point which is also in the lattice boundary.

4. THE COMPACTNESS OF AREAS

The dimensionless quantity P^2/A , the square of the perimeter divided by the area, is the most frequently used measure of the compactness of areas in the Euclidean plane (4) and its minimum value is 4π when the area is a circle. Rosenfeld (5) has shown that certain definitions of perimeter can lead, in digitized objects, to values of P^2/A less than 4π , while Sankar and Krishnamurthy (2) have pointed to a way of defining both perimeter and area of digitized objects so that $P^2/A > 4\pi$ always, although they have not shown that their measure has a definite minimum value.

If the perimeter of an area is taken as its grid-point boundary, then for any area, of whatever shape, the length of the perimeter, P , is not less than the perimeter of the rectangle, with sides parallel to the co-ordinate axes, which just contains the area; and if the area has no re-entrant features P equals the perimeter of the circumscribed rectangle (Fig.2). For a perimeter of given length, P^2/A is a minimum for maximum A , namely the area of the rectangle itself. It then follows that if areas are considered in conjunction with the perimeter given by the grid point boundary, the minimum value of P^2/A is 16 when the area is a square oriented parallel to the co-ordinate axes.

It is of interest to note that if a circle of radius r (where r is an

integer) is centered at a grid point and then quantized, the argument just given shows that the grid-point boundary of the quantized area is of length $8r$ so that, for large r , as A approximates to πr^2 (6), P^2/A tends to $64/\pi=20.4$. The boundary length must be multiplied by $\pi/4$ to give the true value of the original curve, i.e. the circumference of the circle.

Multiplication of the digital length of an arbitrarily shaped boundary by this factor is a method recommended for estimating the length of a smooth curve from its digitized representation (7).

5. THE QUANTIZATION OF LINES

The previous discussion has been concerned with the results of quantizing an area and the significant line involved - the perimeter of the area - has been considered as the boundary of the quantized area. Slightly different considerations apply if the picture being quantized is a drawn line rather than an area or a computer-generated line, although a drawn line is inevitably a long but very narrow area. Suppose that the line is white on a black ground. The questions arise: what is the minimum width of the drawn line if the quantization is to produce an unbroken line; and how is the quantization to be interpreted?

Let the width of the line be ω and assume that the area $\xi_{i,j}$ of the picture, which is quantized as the pixel $\{X_i, Y_j\}$, is isomorphous with the pixel, i.e. is a square with sides of length λ . Assume also that there is neither gap nor overlap between adjacent picture areas, e.g. between $\xi_{i,j}$ and $\xi_{i+1,j}$, which are quantized as adjacent pixels. Suppose, as before, that $\{X_i, Y_j\}$ is given the value 1 if a fraction α of the square $\xi_{i,j}$ is white and if $0 \leq \alpha \leq 1$. (The case of quantizing the line and ascribing many bits, i.e. many gray levels, to the pixels has been considered by

Klaasman (8)). Then if $\omega/\lambda < \tau$ the line will not appear in the quantized picture when it is running parallel to either x or y axes; and from consideration of diagonal lines it is found that if $\omega/\lambda < \sqrt{2} (1 - (1 - \tau)^{\frac{1}{2}})$ the line will not appear in the quantized picture, whatever its direction. However, if the quantized line is to have no gaps in it, special cases must be allowed for. If the line is nearly parallel to the y-axis, say, and if it covers the boundary between the two regions $\xi_{i,j}$ and $\xi_{i+1,j}$, it is necessary to have $\omega/\lambda \geq 2\tau$ for at least one of the pixels $\{X_i, Y_j\}$ or $\{X_{i+1}, Y_j\}$ to be set at the value of 1; and if the boundary runs exactly along the center of the line, both of these pixels will be set at 1. The general case of arbitrary τ and a line inclined at an arbitrary angle is very complicated, but for the present purposes it suffices to consider the usual practical situation of $\tau = \frac{1}{2}$ for which $\{X_i, Y_j\} = 1$ only if at least half the area of $\xi_{i,j}$ is white. The preceding argument then requires $\omega \geq \lambda$. A line of width λ inclined to the co-ordinate axes will sometimes set at the value 1 successive pixels which are 4-way connected but sometimes ones which are diagonally placed, i.e. only 8-way connected. If it were to be required that all related pixels of a quantized line should be 4-way connected, it would be necessary to take $\omega \geq \lambda \sqrt{2}$, although in this case the line would frequently be digitized as two pixels thick (Fig.3).

Now, a drawn line of finite thickness is often interpreted as a geometrical line, infinitesimally thin. If the drawn line could be quantized as a single string of pixels, there would be no difficulty in using the line connecting the center points (X_i, Y_j) of the pixels as the digital version of the geometrical line which the drawn line represents. However, the examination above of the quantization of drawn lines shows

that in practice the quantization will in most cases include ambiguities. Although these ambiguities are no greater than those included in the physical thickness of the drawn line, they are likely to have more serious effects since the purpose of quantization is often to provide data for numerical analysis of a kind for which the drawn line cannot be used. The simplest procedure for dealing with the quantization is to take as the digitized drawn line the line joining the centers of the pixels and to resolve the ambiguities by a systematic empirical procedure: in the case of a closed curve, using the outermost layer of pixels and in the case of open curves taking (say) the leftmost-uppermost possibility when a choice is presented (see Fig. 3). This would provide a line the successive points of which might be 8-way connected, so that the line is different in character from the boundary of an area, the points of which are necessarily only 4-way connected. The two lines have the further distinction that they could not be superposed: the digitized drawn line is represented by lattice points while the edge or boundary of an area is delineated in grid points.

6. SUMMARY AND CONCLUSIONS

The distinction between grid points and lattice points is a valid one (cf.(9)) and its use removes some difficulties and paradoxes which have crept into discussion of quantized pictures. An area can be quantized only as an array of area elements (pixels), not as an array of points, although a pixel may be represented by a lattice point. A pixel, and so an area comprising many pixels, is bounded by grid elements: in some accounts, e.g. (10), this is recognized; in some, e.g. (1) which refers to

a boundary defined by raster points, it is unclear whether the boundary is taken to be on the grid or on the lattice; and in some cases, e.g. (2), the boundary is unequivocally taken to be defined by lattice points, which is an erroneous procedure according to the present argument. It is of note that the grid elements which form the boundary or perimeter of an area may be expressed as direction vectors so that the perimeter can be described by a chain of such vectors and these vectors are then specified by a 4-way coding scheme (7).

Use of the grid-point boundary as perimeter of an area also provides a measure of compactness, P^2/A , which does not violate the conclusions of Euclidean geometry and which has a minimum value, 16, for a square area oriented parallel to the co-ordinate axes.

Although drawn lines are narrow strips of area, the possibility of meeting ambiguities in quantized drawn lines can be avoided by using a procedure different from that used with perimeters and taking a line defined by lattice points instead of a line defined by grid points. This kind of line, which may be expressed as a chain of vectors specified by 4-way or 8-way coding schemes, has been extensively considered by Freeman (11). The extent to which the quantized line approximates the true position of the drawn line is considered by Klaasman (8).

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Figure Captions

- Fig. 1. A square of pixels, illustrating the distinction between grid points (x_i, y_j) , lattice points (X_i, Y_j) and pixels $\{\underline{X}_i, \underline{Y}_j\}$.
- Fig. 2. The perimeter of the shaded area, consisting of a number of pixels, is equal to the perimeter of the circumscribing rectangle.
- Fig. 3. The quantization of two lines and a circle when a square of side of length λ in the original picture is registered as a pixel if at least half the square is covered by the drawn line. The two lines and the circle (not centered on a grid point) are the same in the three cases illustrated and the widths of the drawn lines are (a) $3\lambda/4$, (b) λ and (c) $\lambda\sqrt{2}$.

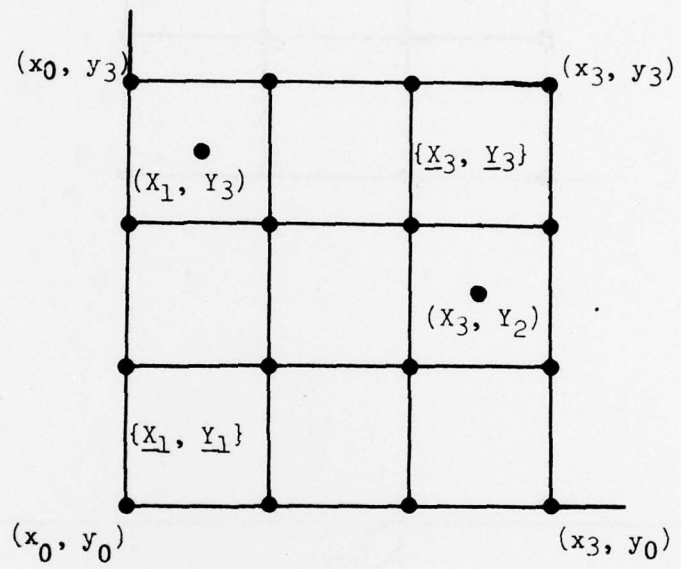


Figure 1

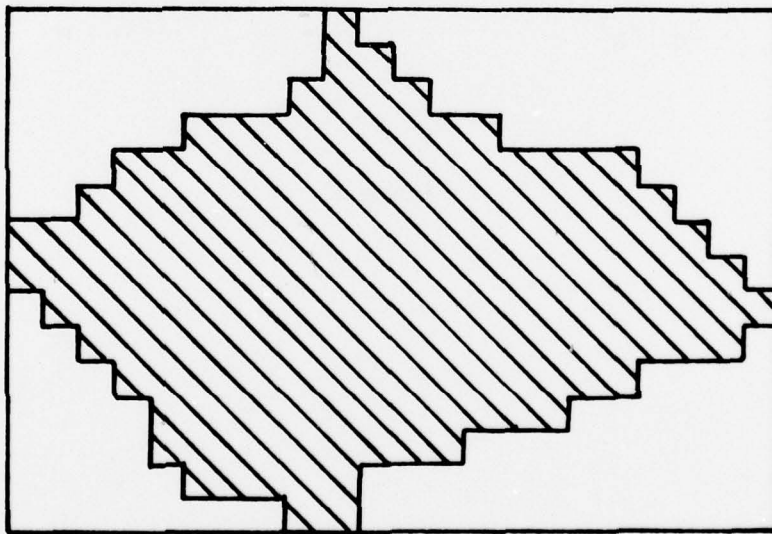
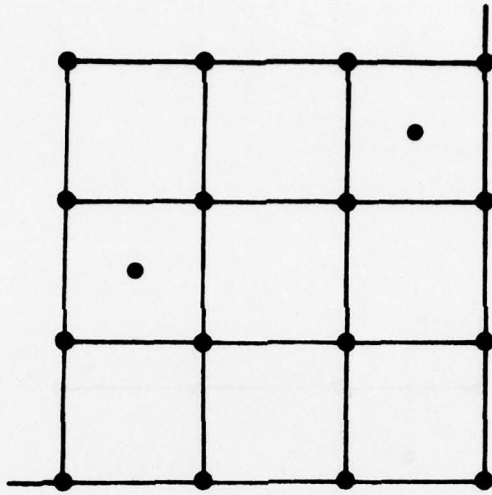


Figure 2

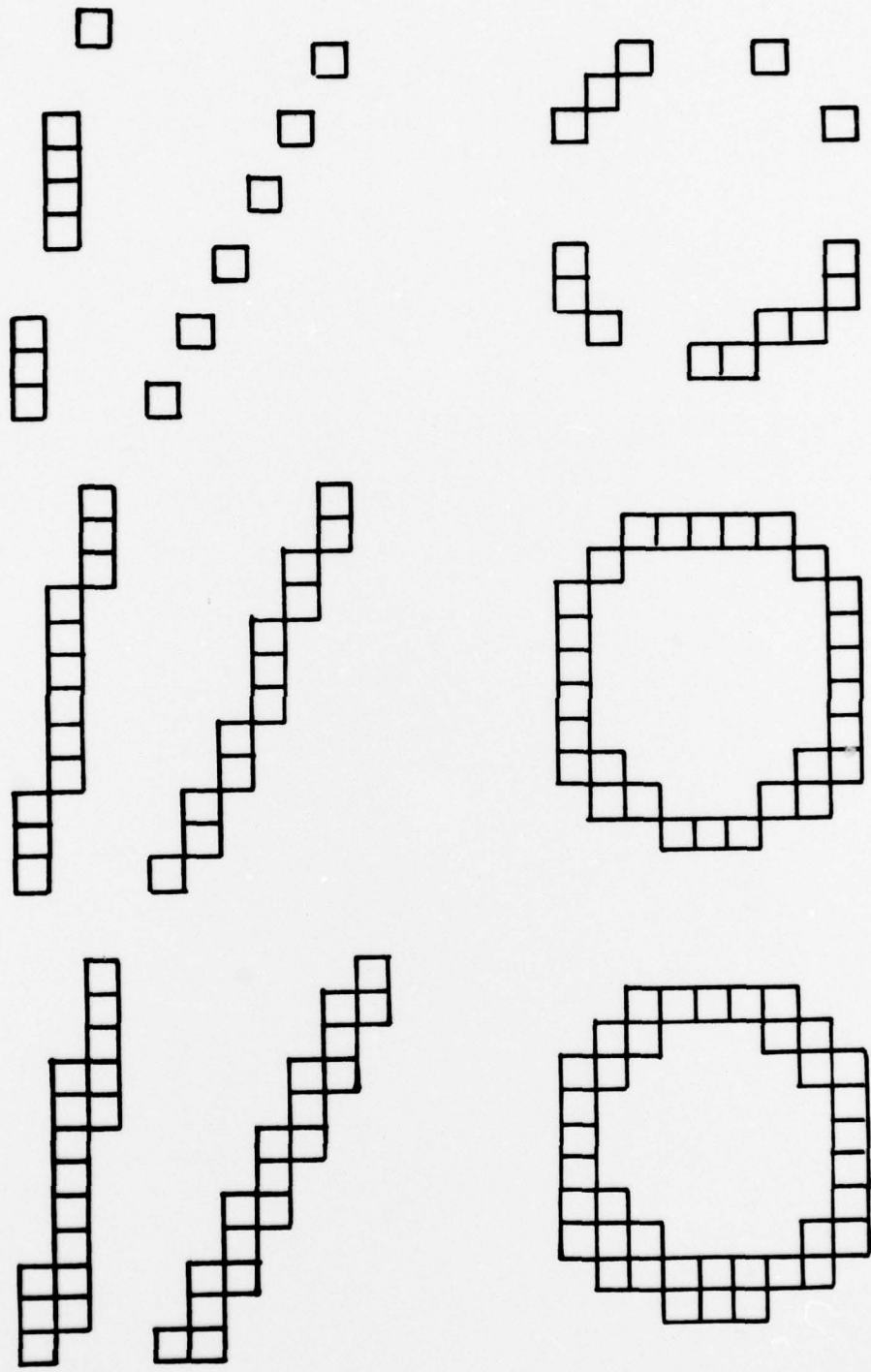


Figure 3

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