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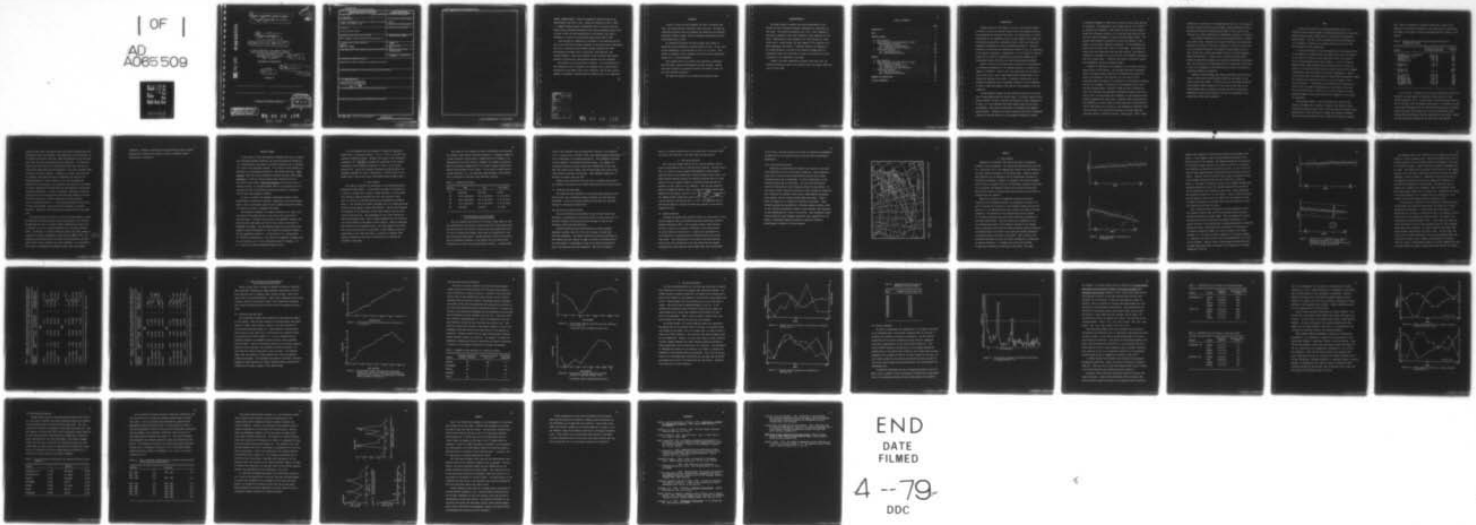
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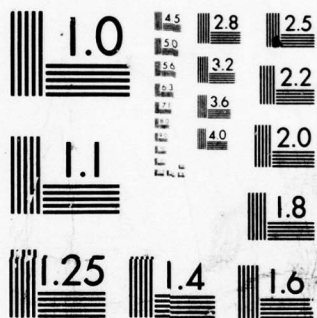
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⑥ TRENDS IN ATMOSPHERIC VISIBILITY ACROSS
THE UNITED STATES FROM 1955 TO 1972 •

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B.S. ①
LEVEL II

⑩ by
JERROLD STUART FOSTER

⑭ AFIT-CI-79-115T

A thesis submitted to the Graduate Faculty of
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FOSTER, JERROLD STUART. Trends in Atmospheric Visibility Across the United States from 1955 to 1972. (Under the direction of TED L. TSUI)

Regular daytime synoptic observations from 14 stations across the United States, excluding observations with relative humidity greater than or equal to 90%, and with precipitation or fog present, were used to determine if a trend in the visibility existed from 1955 to 1972.

Trend analysis by comparison of statistics at the beginning and the end of the period and by linear regression of the monthly mean visibilities were used to uncover any relationship between visibility and other meteorological variables such as wind speed, wind direction, and relative humidity. Time series analysis of the visibility and relative humidity by spectrum and cross-spectrum analysis was also performed.

The results of this study seem to indicate: (1) a deterioration of the visibility at all 14 stations, (2) a bimodal (six to twelve month) oscillation in the annual cycle of the visibility, and (3) that relative humidity is inversely correlated with the annual cycle of the visibility.

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BIOGRAPHY

Jerrold S. Foster was born November 28, 1949, in Brooklyn, New York. His family moved to Cincinnati, Ohio soon after. He began his elementary education there and completed his elementary and secondary education in Miami, Florida, where he graduated from Miami Coral Park Senior High School in 1967.

He received the Bachelor of Science degree with a major in Chemistry from the University of Miami, Florida in 1971. At this time he was commissioned a 2nd Lieutenant in the U. S. Air Force. From January to December 1972 he completed the Air Force Basic Meteorology Course at N. C. State University.

From January 1973 to July 1976 he was stationed at Barksdale, Air Force Base, Louisiana and Ramstein Air Base, Germany. During this time he served as a weather forecaster and weather briefer for Air Force and NATO operations.

The author is married to the former Miss Cynthia Lea Bush.

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Finally, the author expresses his thanks to his wife, Lea, for editing the manuscript and for her patience and moral support during the course of this study.

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INTRODUCTION

Growing concern for the quality of our air, not only with regard to health, but also for aircraft safety and for the aesthetic beauty of our environment has motivated this study. The concentration of various constituents of the atmosphere, such as water droplets, dust, smoke and other pollution particles, condensation nuclei, and the air molecules themselves, determines the optical visibility through it. Therefore, visibility is often used in evaluating the quality of our air, especially in the aviation operations. Consequently, it is of interest to aircraft safety and air clarity to understand how visibility changes and the relationship between visibility and other meteorological variables.

However, because of the human factor involved in the determination of visibility observations, it is sometimes difficult to evaluate the changes in visibility. That is why a very large data set, 18 years, is used in this study. Such a large sample should override most of the subjectivity inherent in the data. In addition, subjectivity can be overcome by using some sort of smoothing technique, such as averaging, in order to make each number of the data set less dependent on the preceding one.

In meteorological usage, the term visibility means both the condition of being visible and the "visual range." The latter term was introduced by Bennett in 1930 to indicate the distance at which something can be seen, i.e., the clearness with which objects stand out from their surroundings. Since then the definition of visibility has undergone many revisions. Throughout the period covered by this study "prevailing visibility" has been defined to be the greatest visibility, equaled

or exceeded throughout at least half the horizon circle, which need not be continuous. One difficulty in the analysis was the use of the "+" in reporting the visibility. This symbol was used when the observer judged the prevailing visibility to be more than seven miles and also more than twice the distance to the most distant marker visible. The observer then encoded the visibility as twice the distance to that marker, or seven miles, whichever was greater. If the visibility was greater than the coded value, then a "+" was added. This symbol was deleted from the observations when they were archived thus making it impossible to determine if, and by how much, the visibility was greater than the recorded value. Therefore, any study of visibility becomes more problematic, and may be somewhat biased.

Research in visibility, as an indicator of air quality, has been going on most of this century. One technique described by Holzworth (1960) and common to many studies was to determine the per cent frequency of occurrence of the visibility in a few (four or five) categories within the range of the visibility at the station in question. The per cent frequency is calculated at the beginning and the end of the period being studied. Holzworth (1960) used this technique and found the visibility near Sacramento, California had deteriorated from 1935 to 1956. He also related poor visibility to wind directions which brought pollutants from urban areas. Holzworth (1962) determined that the visibility at various cities was lower from 1955 to 1959 than from 1929 to 1938 because of an increase in the frequency of visibility less than seven miles. He related this decrease to the wind speed and direction relative to pollution sources. Miller et al. (1972), using

comparisons of statistics at the beginning and the end of the period and multiple regression showed that the summer, daytime visibility in Ohio, Kentucky, and Tennessee deteriorated from 1962 to 1969, and that wind direction and relative humidity were significantly related to visibility.

The southwest United States is an area of great interest in visibility studies today. Because of the large visibilities, over 100 miles, reported by stations in this area, a decrease of only 10 or 15 per cent in the prevailing visibility is considered a large change, and is of more concern than a similar percentage decrease in the more populated eastern United States. A recent study by Trijonis and Yuan (1977) showed the visibility throughout the southwest United States is decreasing. The concern over the deterioration of the visibility in this area, as well as the remainder of the United States, points to the need for continuing research of visibility.

Because of these findings, the present visibility study was undertaken to verify whether deteriorating visibility was common to many station across the country. If this is true, are there some meteorological elements and/or combination of them that are the cause of the deteriorating trend in visibility, and what are they? Using linear regression and time series analysis, what sort of information can we obtain about these other parameters?

DATA

All data were acquired from the National Climatic Center in Asheville, North Carolina through the Meteorology Division of the Environmental Protection Agency (EPA) located at the Research Triangle Park, North Carolina. The data, stored on magnetic tape for easy adaptation to the computer, consisted of all regular airways observations that were archived for 1948 through 1972.

To insure the fidelity of the representation of the characteristics of the regions of the United States included in this study, the records for at least two stations in each region were chosen to be analyzed and to be checked against each other (Table 1). In all, 14 National Weather Service (NWS) reporting stations were selected, the main criterion for the selection being that the airport location had not changed significantly during the period of time under investigation. Table 1 is sub-divided into four groups based on climate and/or geographical region. Four stations were selected in group 1, two each in groups 2 and 3, and six in group 4. The analysis of the visibility at these stations does not give a detailed representation of the visibility trend over the entire United States. However, the visibility trend in the more populated regions of the United States can be suggested if the results are consistent.

The data were edited to cover the period from 1 January 1955 through 31 December 1972. However, a change in the number of daily observations taken was made in 1965. Thus, the data set contains hourly observations of each station from 1 January 1955 through 31 December 1964 and 3-hourly observations from 1 January 1965 through 31 December

1972. Since the analysis of visibility trends was in terms of the frequency of occurrence with respect to the total number of observations, the change in the number of daily observations had little impact on the results.

Table 1. Stations used in this study and grouped according to geographical region.

Station	Distance and direction of airport from city	Elevation (ft)
1. Washington, D. C. (National)	3 mi. S	65
Raleigh-Durham, N. C.	10 mi. NW	441
Atlanta, Ga.	7 mi. SW	1034
Birmingham, Ala.	6 mi. NE	630
2. Houston, Tx.	7 mi. SE	662
Austin, Tx.	3 mi. E	621
3. Tucson, Az.	-	2555
Phoenix, Az.	3 mi. E	1107
4. St. Louis, Mo.	13 mi. NW	564
Peoria, Ill.	6 mi. SW	662
Milwaukee, Wis.	7 mi. S	663
Ft. Wayne, Ind.	5 mi. S.	828
Cleveland, Ohio	10 mi. SW	805
Youngstown, Ohio	8 mi. N	1186

It was decided to examine only the daytime observations because the visibility at night depends on different factors and may be less reliable (Houghton, 1945). In other words, the daytime observations may not be consistent with the nighttime observations. In addition, in order to avoid any overlapping with the nighttime observations due to seasonal sunrise and sunset time differences, only those observations taken between 0900 and 1600 local time were considered for this study. Hourly data were used when available and the 0900, 1200, and 1500 GMT

(Greenwich Mean Time) observations were used during the period when only three-hourly data were available. This means that for those stations in the eastern and central time zones, eight observations per day were used until 1965, and three observations per day thereafter. For Tucson and Phoenix there were eight hourly observations until 1965, and only two three-hourly observations per day thereafter. The actual variables taken from each observation included: visibility (in miles), wind direction (in degrees), wind speed (in knots), temperature and dew point (in degrees Fahrenheit), relative humidity (in %) and the weather and obstruction-to-vision group. Wind direction was reported to the nearest of sixteen compass points through 1964 and then reported to the nearest ten degrees thereafter. In order to maintain a continuity in the wind direction data, the thirty-six wind directions were changed to the sixteen points used prior to 1965 by using the conversion formula; $(11.25 + \text{Direction}) / 22.5 + 1$. Calm winds were deleted from the wind direction data because a calm wind does not represent any specific direction. Therefore, calm winds were also deleted from the wind speed data.

The data base was then edited to delete those observations in which the naturally occurring aerosols, fog and precipitation, were reported. Precipitation may occur at various relative humidities but natural fog is unlikely to occur at relative humidities less than 90% (Holzworth, 1959). In addition, at relative humidities greater than 95%, changes in visibility are nearly independent of particle size (Hanel, 1971). By eliminating those observations with high relative humidities, the most severe cases of poor visibility have been eliminated. It is the aim of this study to investigate the visibility trends under fair weather

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INTERESTING
particle size
distribution

conditions. Therefore, observations with precipitation and/or relative humidity 90% or greater were deleted in order to eliminate natural obstructions to visibility.

ANALYSIS SCHEME

In the course of this study numerous techniques were used to examine the relationship between visibility and other meteorological parameters, i.e., wind direction, wind speed, and relative humidity, and to determine if a trend in the visibility existed. If so, does the trend indicate an improving or a deteriorating visibility? The methods used were: Trend Analysis, using the method developed by Holzworth and Maga (1960); Mean Visibility, to find the relation between mean visibility and wind direction and wind speed; Time Series Analysis, both spectrum and co-^{cross?}spectrum analysis, to determine whether there is any periodicity in the visibility as well as correlations between the periods of visibility and the other meteorological variables.

In order to identify the ensemble relationships between certain meteorological variables and visibility, regression by the least-squares method, which statistically gives the most probable correlation between two variates, was used to accomplish this task.

The regression technique is a common statistical tool, hence, only an outline of the method and how it was used in this study will be described below. Visibility was treated as the dependent variable, and wind direction, wind speed, and time of day or time of year were the independent variables. The least-squares method written in matrix form assumes a linear relationship, $Y = XB$, between the dependent variable Y and the independent variable X . In this study, Y is a column matrix of n visibility observations, X is a matrix of independent variables with $(n \times m)$ elements, and B is the coefficient matrix of m elements. If $m = 2$ there is a linear relationship between y and x .

Y can be separated into the estimated \hat{Y} through the regression equation and ϵ , the error function. $Y = \hat{Y} + \epsilon = X\hat{B} + \epsilon$, where \hat{B} is the estimated coefficient matrix. Through \hat{B} the square of the correlation coefficient, $R^2 = S_{XY}^2 / S_X^2 S_Y^2$, represents the percentage of the variance contribution of the regression equation, $Y = X\hat{B} + \epsilon$, to the total variance of Y. S_{XY} is the estimated covariance and S_X^2 and S_Y^2 are the estimated variances of X and Y, respectively. If R^2 is close to 1 it implies that a very strong linear relationship exists between X and Y.

1. Trend Analysis

The trend in visibility was determined by the method developed by Holzworth and Maga (1960). In this method the range of visibility at a station is divided into a number of class intervals. For this study the visibility range was divided into five class intervals, which were the same for all stations except Tucson and Phoenix, as shown in Table 2. The intervals are chosen arbitrarily but in a fashion producing relatively large samples in each class interval. The per cent frequency of occurrence of the visibility in each class interval was then calculated by month and year. The percentages for each class interval are then plotted against time. Then, a linear regression line is fitted for each class interval by the method of least squares. Using the beginning and end points of the regression line, a per cent change in the visibility in each class interval is determined. These changes are directly proportional to the trend line slope. A positive or negative slope indicates whether the occurrence of visibility in an interval has increased or decreased.

From these per cent changes the trend is determined by calculating the resultant shift between visibility intervals. A frequency change in a given visibility interval must be compensated for by changes in the frequencies of the other intervals. However, the change is continuous. When the visibility changes from 30 to 5 miles it passes through intermediate values between 30 and 5 miles. If the trend is toward deteriorating visibility, i.e., the visibility range decreases, the resultant shift is detected in each next lower visibility interval.

Table 2. Visibility intervals.

Interval	PNX	TUC	All others
F1	<10 miles	<12 miles	<1 mile
F2	>10 to <20 miles	>12 to <30 miles	>1 to <4 miles
F3	>20 to <30 miles	>30 to <50 miles	>4 to <7 miles
F4	>30 to <40 miles	>50 to <60 miles	>7 to <12 miles
F5	>40 miles	>60 miles	>12 miles

2. Mean Visibility and Relationships to other Meteorological Variables

Over the eighteen year period of this study a large amount of data was considered for each station, between 35,040 and 37,960 observations. The data then had to be edited to include only those observations without fog or precipitation present, which meant the possibility of some days without any observations. Considering this as well as the results from the regression analyses, it was apparent that the monthly mean visibility was a better basic meteorological variable. A monthly mean

would be less dependent upon the subjectivity inherent to each observation. As pointed out by Stringer (1972), most meteorological observations are not independent of preceding observations. This dependence decreases with the length of time between successive events. For example, the correlation between successive monthly mean visibilities is much less than that between hourly values, and the annual means have little correlation between successive observations. Since seasonal changes are of interest the monthly means are used. REALLY?

Monthly mean visibilities were then used to determine both persistent seasonal variations and any obvious variations over the entire period.

(i) Visibility and Wind Speed

By finding the mean visibility for each wind speed between one and fifteen knots, the relationship between visibility and wind speed was determined. A plot of the mean visibility versus speed was then made, showing the relationship between them.

(ii) Visibility and Wind Direction

The mean visibility was determined for each of the sixteen wind directions, as described in the previous section on Data, and a plot of the mean visibility versus wind direction was made to show how the visibility varied with wind direction.

Visibility/wind direction associations were further examined employing a monthly wind rose of per cent frequency of eight wind direction categories. The sixteen reported and converted directions in the original data were reduced to eight directions by shifting half of the total number of observations in each of the deleted directions to each of the adjacent direction categories. This will determine if

there is a favored direction and one can relate that to the mean visibility versus the direction of the urban areas from the airport.

3. Time Series Analysis

One of the most useful methods used to find the dominant oscillations in meteorological data is Fourier-series analysis. The time series, $f(t)$, is a series of data arranged chronologically, usually equally spaced in time, as the monthly mean visibility series in this study. The Fourier-series analysis ascribes the observed variation shown by a time series, $f(t)$, to an infinite number of oscillations with discrete frequencies, or $F(n)$, where n is the frequency. This was done using the method of Brigham and Morrow (1967), by taking the Fourier-series representation of the periodic function, $f(t) = \sum_{n=-\infty}^{\infty} F(n)e^{\frac{jn2\pi t}{T}}$, where $F(n)$ is the complex Fourier coefficient, given by $F(n) = \frac{1}{T} \int_{-T/2}^{T/2} f(t)e^{-\frac{jn2\pi t}{T}} dt$. T is the period of the function and $j = (-1)^{\frac{1}{2}}$. The equations for $f(t)$ and $F(n)$ are known as a Fourier-transform pair and $F(n)$ is the Fourier transform of $f(t)$.

(i) Spectrum Analysis

Treating the monthly mean visibility values as a time series, a Fast Fourier Transform (FFT) was performed using the Cooley-Tukey procedure (Brigham and Morrow, 1967). This procedure yields a power spectrum which is a graph of the variance contribution of the ordered array of periods within the time series. It depicts the distribution of the probability of oscillations with differing periods in the given time series. The interpretation of the peaks in the power spectrum would be that the oscillations in the time series with the periods associated with those peaks are more probable than other oscillations.

In this study, the power spectrum is utilized to identify the predominant periodicities in the visibility series, which may have meteorological significance.

(ii) Cross Spectrum Analysis

Frequently two meteorological variables are related at some particular frequencies but not at any other frequencies. Simple regression would tend to cancel out these relationships and give some average correlation which may be misleading. In these circumstances a cross spectrum analysis can be very useful. The cross spectrum analysis reveals the correlation of each frequency between two time series. In this study a cross correlation function and cross correlation coefficient were calculated for stations within similar climatological and geographical groups, and for stations between these four groups. Figure 1 shows the scheme used to compare the visibility time series. Cross spectrum analysis was performed and a coherence squared function, which is the correlation coefficient squared at each frequency, was calculated via FFT computations for a few of the stations. This information showed to what extent the major harmonic frequencies were responsible for the degree of correlation among the stations or among the different meteorological variables of the same station.

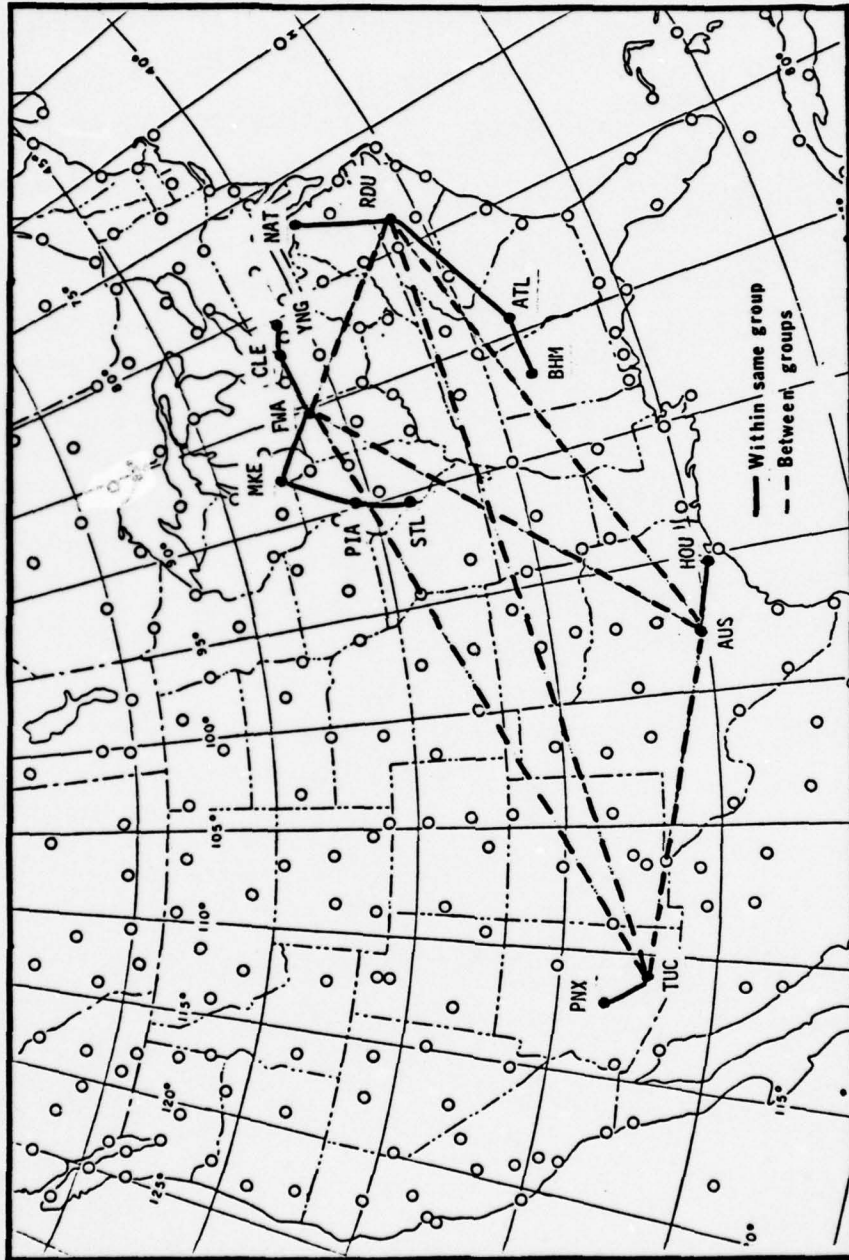


Figure 1. Scheme used for matching stations in the cross spectra analysis described in the text.

RESULTS

1. Trend Analysis

Regression of visibility with time of year gave no significant evidence of any relationship. The correlation coefficient was less than 0.3. This result led to our changing from reliance upon and emphasis on the basic visibility parameter to the monthly mean. Using the monthly mean visibility in the regression did not significantly increase the correlation coefficient. However, it appeared to be a better variable since it does not depend as much on the individual taking the observation and it decreases the dependence between two successive observations. Over an entire month the subjectivity within the data is less likely to interfere with the analysis.

When time is used as the independent variable and the mean visibility is the dependent variable, the regression technique can be used to identify the variations of the visibility longer than the data period, i.e., the regression line can be taken as the trend of the mean visibility. The deviation of each January mean from the average of all January means was taken and similar deviations calculated for the other eleven months. These were plotted and regressed on time. If a deteriorating trend in the visibility existed, it was thought that the deviations would show the trend with mainly positive deviations occurring during the first half of the period and negative deviations occurring during the last half of the period. Such a trend in the deviations was shown in quite a few cases, although not as pronounced as expected (Figure 2). A straight line of best fit was drawn through these deviations in each half of the period. The lines

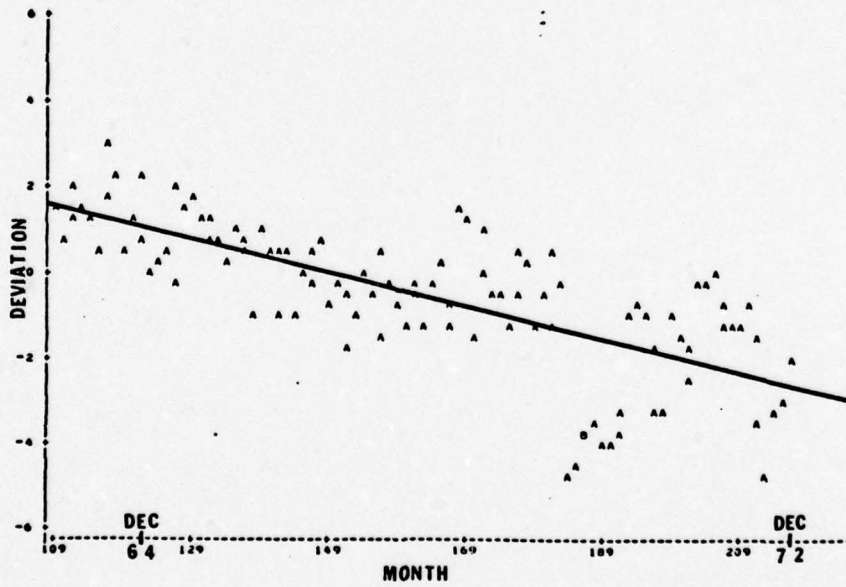
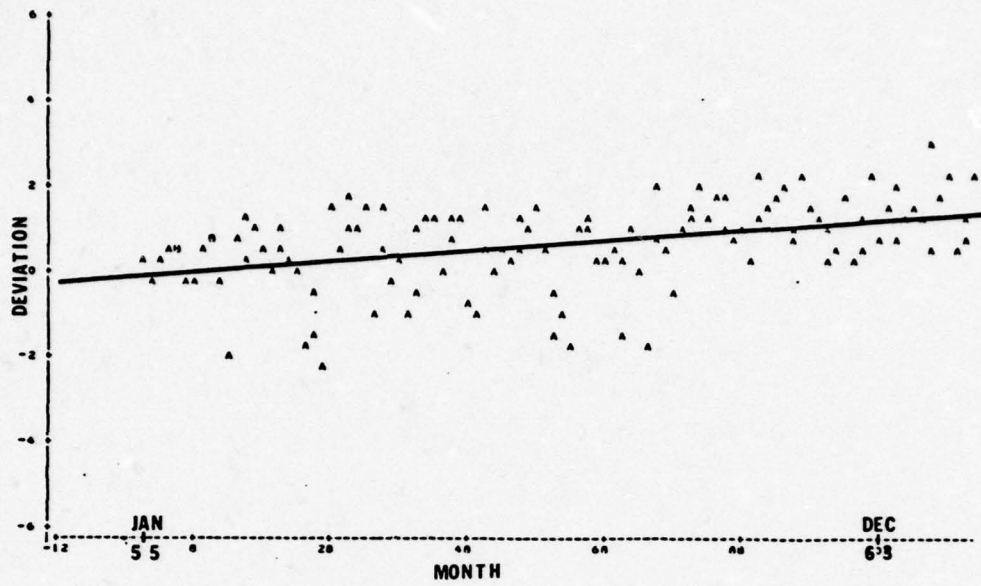


Figure 2. Trend determined by deviations for Birmingham, Al.

showed a small negative or positive slope during the first half of the period. A steep negative slope occurred during the last half of the period for most of the stations. The negative slopes over the last half of the period probably were not due to the use of the "+" in reporting the visibility, as described in an earlier section. If the "+" were the cause of these negative slopes, these declines would not have been continuous. Instead, the plot would have had a group of deviations outside of two standard deviations from the line of best fit, as shown in Figure 3. The group of deviations outside of two standard deviations (2σ) represents the period of April 1970 through February 1971, when 90% of all the observations were archived as seven miles. These observations were probably recorded as 7+ by the observer. Since a great variability from one month to the next has been observed in the visibility trends, no definite quantitative conclusions about long term visibility trends can be made from this data set alone.

More conclusive results indicating an existing downward trend in the visibility between 1955 and 1972 were obtained using the method developed by Holzworth and Maga (1960). When one examines the results obtained using the class intervals with monthly frequencies, it is obvious that most of the stations had a deteriorating trend in visibility during this period. Most months show a decrease in the frequency of the highest visibility category, F5, and an increase in the lowest category, F2. The lowest visibility category is F2, because category F1, visibility less than one mile, had too few observations to be used in the procedure. However, some of these changes are small and there are many months which show an increase in the frequency of the higher categories, F4 and F5.

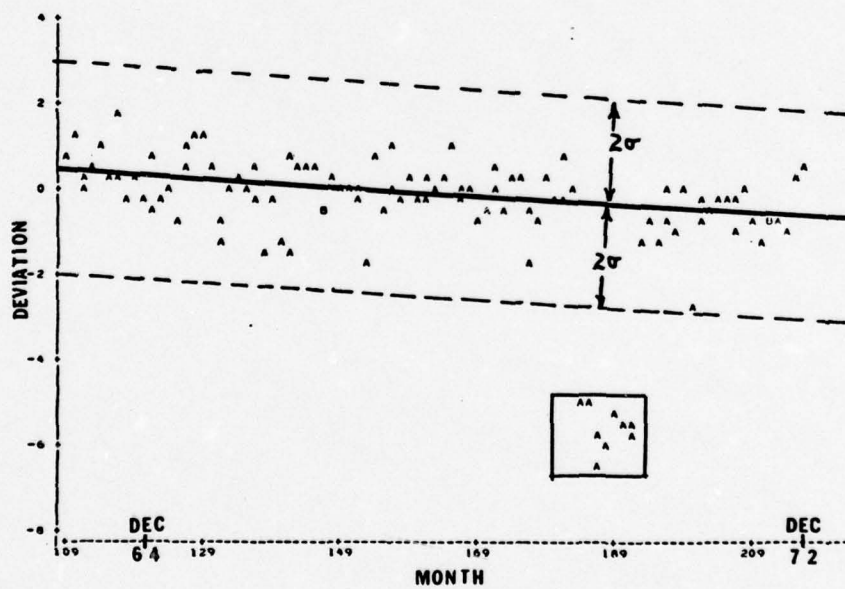
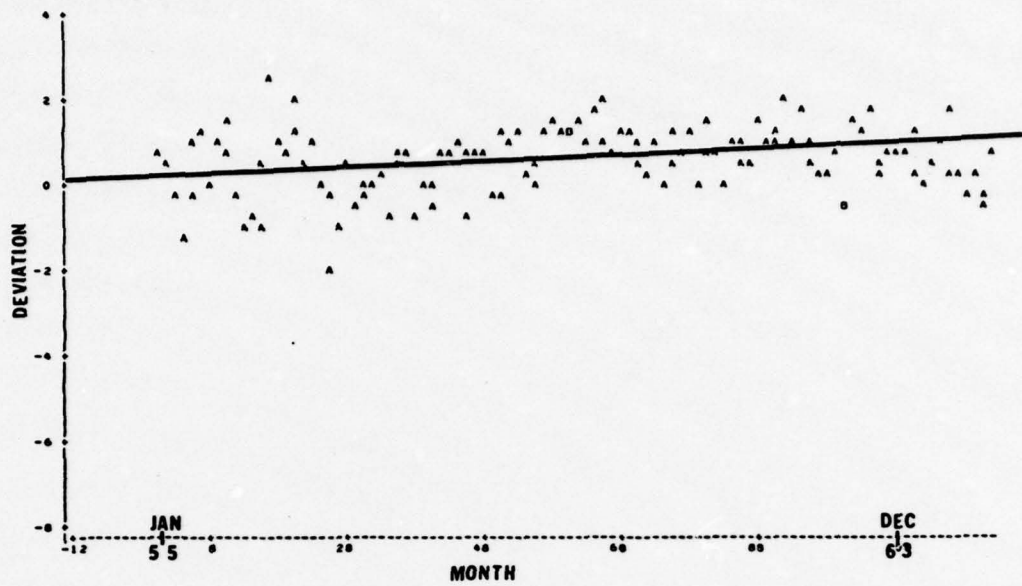


Figure 3. Effect of "+" on determining the trend by deviations for Austin, Tx. Group of deviations outside of 2σ represent the months, April 1970 through February 1971.

The downward trend in the visibility was very apparent in the class intervals of yearly frequency data. These results showed that of the 14 stations analyzed, 13 had a downward trend in the frequency of the highest visibility category. Table 3 shows these results for Milwaukee and Ft. Wayne. Only Phoenix had an increase in the frequency of the highest category, but it did show a significant decrease in the next two highest categories, F3 and F4 (Table 4). Significance tests show that 70% of the regression lines with a decrease in category F5 visibility were statistically significant at the 10% level. The same was true for category F2. Looking at all categories, 67% of all regression lines were statistically significant at the 10% level. Considering the results of the trend analysis, it is concluded that the visibility at all 14 stations deteriorated during the period from 1955 to 1972.

A recent study of visibility in the southwest by Trijonis and Yuan (1977) showed that, during the same period, the visibility at Tucson and Phoenix had decreased. At first glance this would lead one to believe that the decrease in visibility was due to an increase in particulate matter in the ambient air (Cleeves, 1972). This seems to be contradicted by the EPA (December 1973). It was found that during the period from 1960 to 1971 the annual geometric mean for total suspended particulate matter decreased. The same is shown for the annual mean sulfur dioxide concentrations from 1964 to 1971. The EPA report also states that the sulfate percentage of the total particulate matter has increased from 1964 to 1970. However, when drawing any conclusion about the cause of deteriorating visibility, one must remember that these pollution studies were made in urban areas rather than at the airports.

Table 3. Schematic shift with time (based on least-squares linear regression) of the visibility from one interval to another using yearly percent frequencies.

Visibility Interval	Predicted Value 1955	Predicted Value (%) 1972	Net change (%) 1955 to 1972	Resultant Shift (%) between intervals
<u>MKE</u>				
<1 mile	-	-	- 0.05	0 = - 0.05
>1 to <4 miles	2.43	7.703	+ 5.273*	- 5.273 = 0
>4 to <7 miles	5.211	19.231	-14.02*	-19.243 = + 5.223
>7 to <12 miles	17.086	31.133	+14.047*	-33.29 = -19.243
>12 miles	75.231	41.941	-33.29*	0 = -33.29
<u>FWA</u>				
<1 mile	-	-	- 0.02	0 = - 0.02
>1 to <4 miles	0.526	11.786	+11.21*	-11.21 = 0
>4 to <7 miles	8.588	19.184	+10.596*	-21.786 = -11.19
>7 to <12 miles	31.956	24.425	- 7.531	-14.255 = -21.786
>12 miles	58.851	44.596	-14.255*	0 = -14.255

* Statistically significant at the 10% significance level using the F-test.

Table 4. Schematic shift with time (based on least-squares linear regression) of the visibility from one interval to another using yearly per cent frequencies.

Visibility interval	Predicted Value 1955	Predicted Value (%) 1972	Net change (%) 1955 to 1972	+ Resultant shift (%) between intervals
<u>PNX</u>				
<10 miles	2.136	2.904	+ 0.768	+ - 0.768 = 0
>10 to <20 miles	11.230	16.193	+ 4.963*	+ - 5.731 = - 0.768
>20 to <30 miles	31.743	24.124	- 7.619*	+ 0 = - 5.731
>30 to <40 miles	41.019	26.446	-14.573*	+ - 1.888 = -16.461
>40 miles	13.872	30.333	+16.461*	+ -16.461 = 0
<u>TUC</u>				
<12 miles	0.254	0.386	+ 0.132	+ - 0.132 = 0
>12 to <30 miles	2.655	4.990	+ 2.335*	+ - 2.467 = - 0.132
>30 to <50 miles	8.292	62.973	+54.681*	+ -57.148 = - 2.467
>50 to <60 miles	31.360	41.543	+10.183	+ -67.331 = -57.148
>60 miles	57.440	-9.891	-67.331*	+ 0 = -67.331

* Statistically significant at the 10% significance level using the F-test

2. Mean Visibility and Relationships to Other Meteorological Variables

Miller, et al. (1972), by means of analysis of variance, indicated that significant differences in summer daylight visibilities in the 0-6 mile range were due to southerly winds, station location, time of day, time of year, and relative humidity. These results suggested that the mean visibility should be calculated at each of the sixteen wind directions and at each wind speed less than 15 knots, but not including calm conditions.

(i) Visibility and Wind Speed

The relationship between mean visibility and wind speed was similar at each station. When the mean visibility is plotted against wind speed, there is a steady, almost linear, increase in the mean visibility as the wind speed increases (Figure 4). This continues to speeds greater than 15 knots in most cases, but at some stations the visibility starts to decrease after 12 knots (Figure 5). It can be reasoned that the visibility improves as atmospheric mixing increases, which increases with wind speed. However, when surface particles are eventually airborne because of the strong wind, the visibility will then be reduced. In this study this critical wind speed is above 12 knots. For different shape, size, and density of loose sand and dirt, there are different critical wind speeds. At some higher wind speed the visibility increases slightly as these particles are rapidly dispersed without obscuring the visibility to as great a degree as with lighter winds.

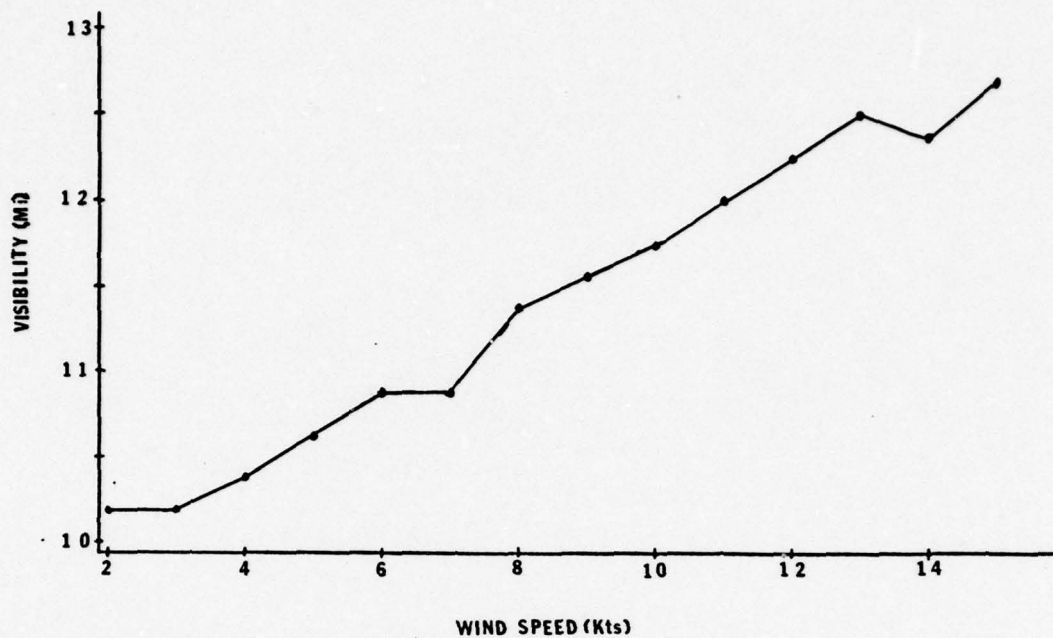


Figure 4. Relationship between visibility and wind speed for St. Louis, Mo.

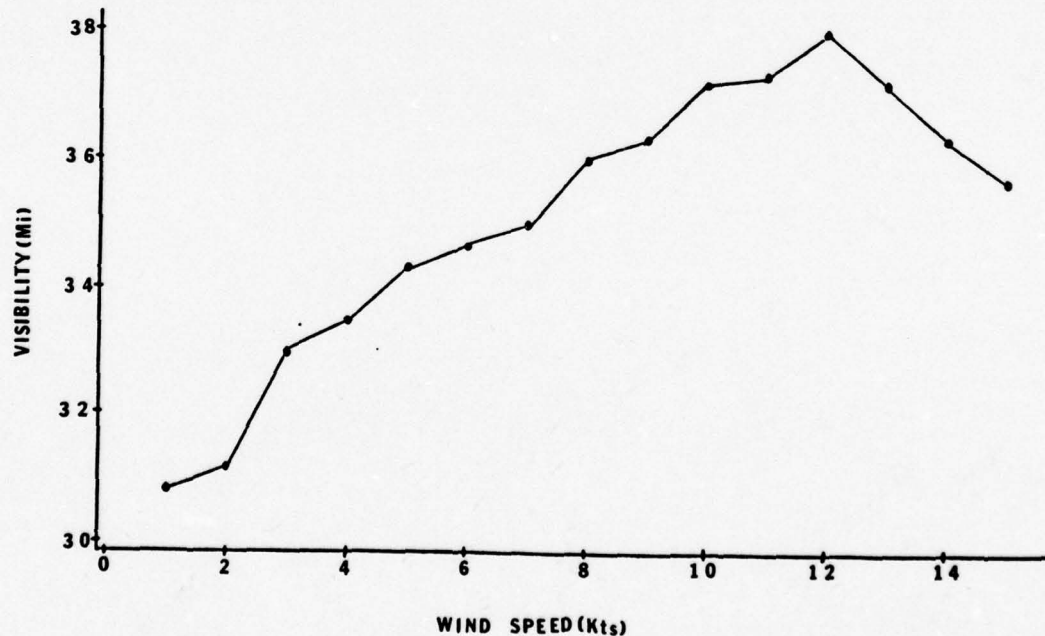


Figure 5. Relationship between visibility and wind speed showing the decrease in visibility as dust and dirt particles begin to reduce the visibility as the wind speed increases at Phoenix, Az.

(ii) Visibility and Wind Direction

The plots of the mean visibility for each wind direction were similar in that each station showed a few directions most related to lower visibilities, as shown in Figure 6. Analysis of the wind roses and the plots of mean visibility by wind direction for each station indicates that five stations, Atlanta, Birmingham, Phoenix, Milwaukee, and Austin, showed that the predominant wind direction is often one of the directions associated with poor visibility (Table 5). Of these five stations, only Atlanta and Birmingham had the predominant wind direction and the direction of worst visibility from the city. There were three other stations, Raleigh-Durham, St. Louis, and Youngstown, where the wind direction causing the worst visibility was from the city. This shows that neither the location of the airport relative to the city nor predominant wind direction are the prime factors in the reduction of visibility. Different months with the same prevailing wind direction exhibit different effects on visibility. For example, in Atlanta the predominant southeasterly winds are associated with lowest visibilities in August, but with highest visibilities in October and November.

Table 5. Summary of wind direction vs. visibility results.

Station	Direction related to poorest visibility	Direction of airport from city	Predominant direction
Atlanta	NE	SSW	N
Birmingham	SW	NE	SW
Phoenix	SE	E	SE
Milwaukee	S	S	S
Austin	SE	E	S

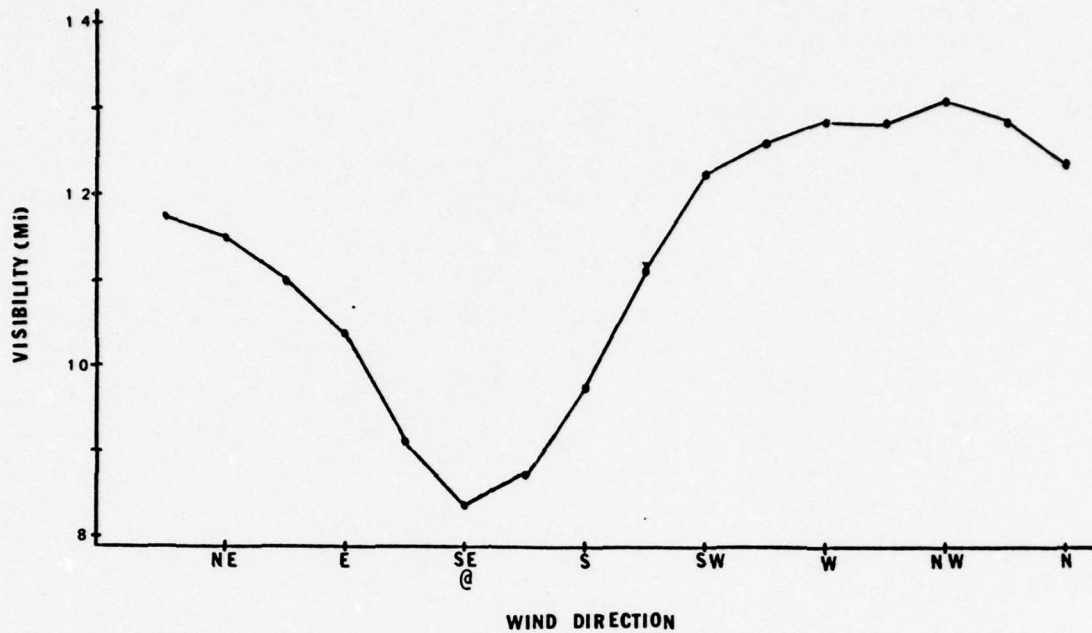


Figure 6a. Relationship between visibility and wind directions at St. Louis, Mo.

@ indicates wind is blowing from the city.

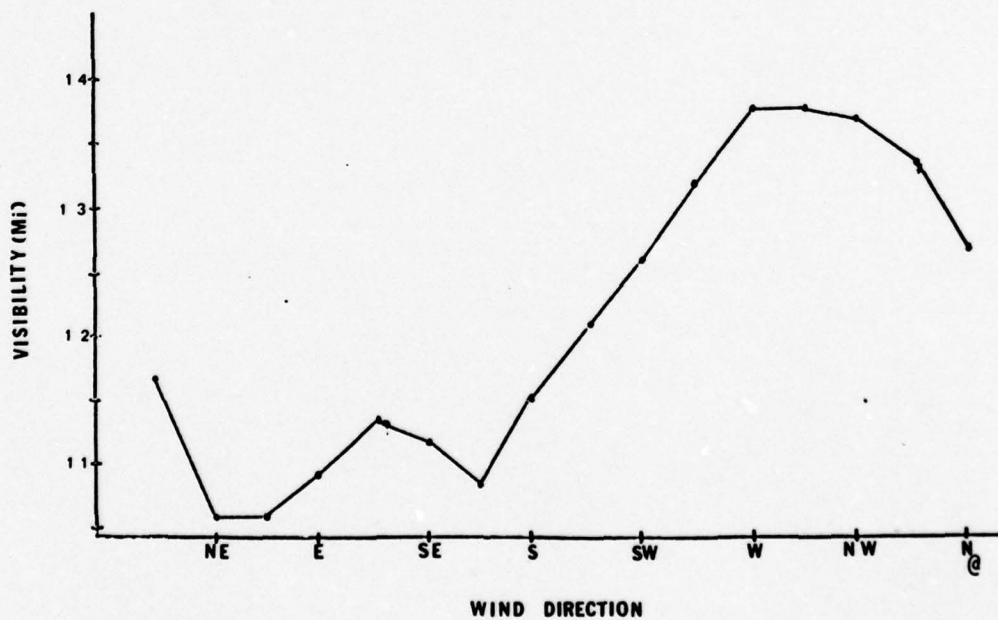


Figure 6b. Relationship between visibility and wind direction at National Airport (D.C.)

@ indicates wind is blowing from the city.

3. Time Series Analysis

The most interesting result of this study was found when the monthly mean visibility over the entire eighteen year period was examined. The similar pattern of bimodal oscillation, two maxima and two minima with a yearly cycle (Figure 7), was apparent at every station except Phoenix and Tucson. These extremes did not necessarily occur in the same month or season. This can be seen by comparing Figures 7a and 7b. It was expected that visibility would be greatest during the dry season and least during the wet season when suspended water droplets are abundant in the atmosphere. Figure 7 does not seem to indicate any correlation between the visibility and the dry or wet seasons.

An initial concern is if these maxima and minima are significant. The general size of the standard error of the mean, the standard deviation divided by the square root of the sample size, as shown in Table 6, varies from 0.12 to 0.37 miles. This indicates that these secondary peaks are not significant. However, the same result found at twelve different stations strongly suggests that these secondary maxima and minima are indeed significant to the periodicity of the visibility. The possibility of combining the results for all stations in an effort to increase the reliability of the monthly means was considered. This could not be done since all of the monthly mean visibilities are not equal, and the maxima and minima do not occur at the same month for each station. Therefore, this would not be a valid statistic.

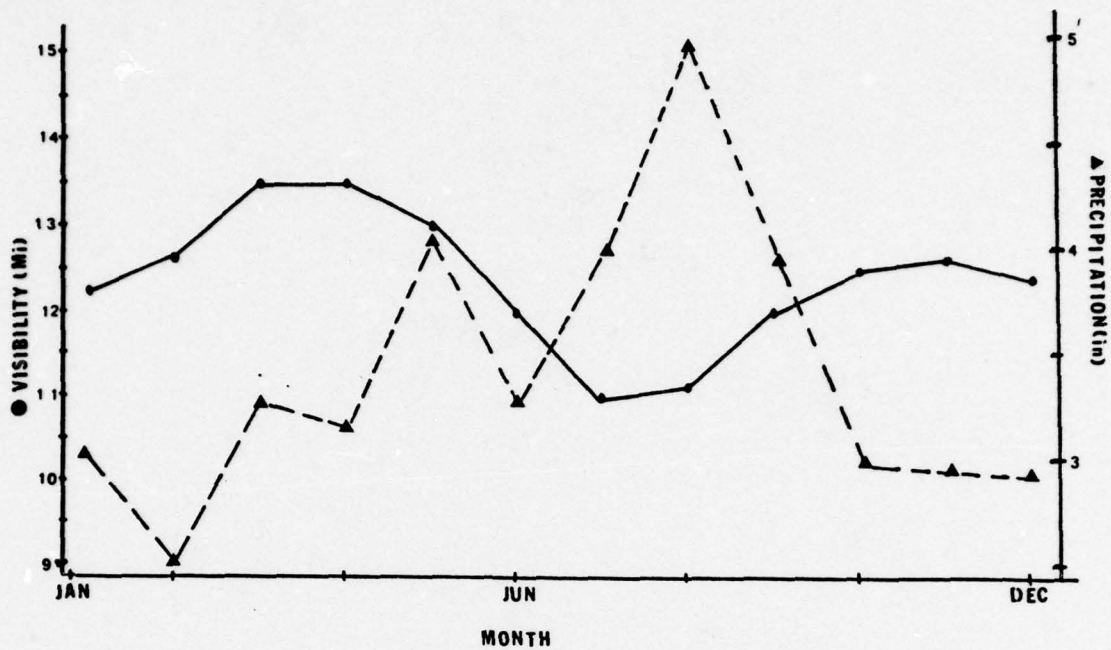


Figure 7a. Bimodal annual oscillation of visibility at National Airport (D. C.)

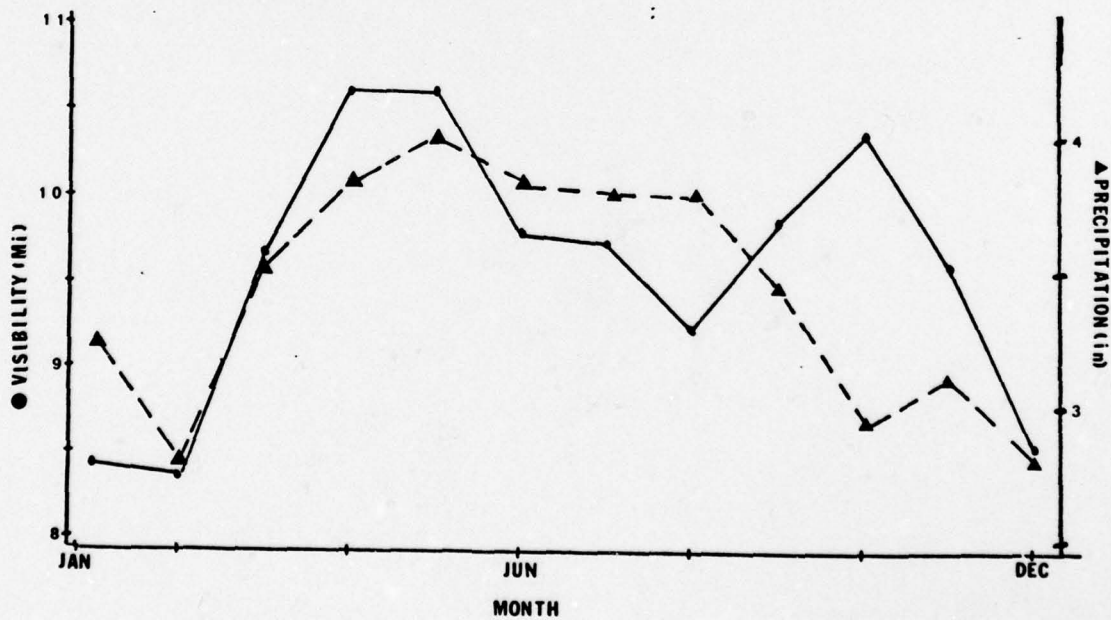


Figure 7b. Bimodal annual oscillation of visibility at Cleveland, Oh.

Table 6. Standard error of the mean for Washington National Airport.

Month	Standard error of mean (mi)
JAN	0.35
FEB	0.31
MAR	0.25
APR	0.23
MAY	0.12
JUN	0.18
JUL	0.32
AUG	0.23
SEP	0.26
OCT	0.22
NOV	0.30
DEC	0.37

(1) Spectrum Analysis

In order to substantiate the significance of the bimodal oscillation in the visibility data a fast Fourier transform (FFT) was utilized to calculate the visibility in spectral distribution. The FFT analysis showed that the time series contained two major periodic components, occurring at approximately six and twelve months (Figure 8). This result further affirms the significance of the secondary maximum and minimum found in the monthly mean visibility, even though these two major components only accounted for 35% of the total variance of the data. In other words, though the six and twelve month periods cannot account for the entire variance among the many oscillations, they are definitely the predominant ones.

To obtain an explanation for the two maxima and minima in the visibility cycle, a number of other meteorological variables were investigated. First, the relationship between the mean mixing height and visibility

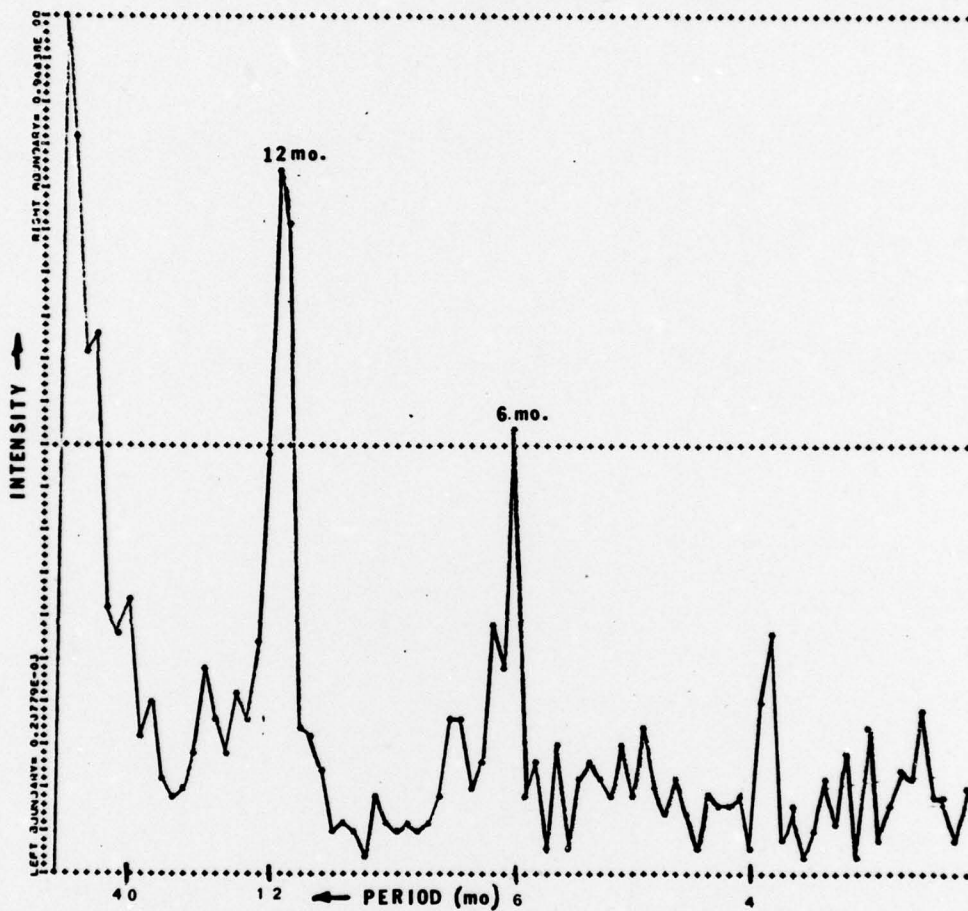


Figure 8. Fast Fourier Transform of Visibility Time Series of Raleigh-Durham, N. C.

was examined. The mixing height data were available from Mixing Heights, Wind Speeds, and Potential for Urban Air Pollution throughout the Contiguous United States (Holzworth, 1972). Mean mixing height values, excluding data collected on days when precipitation occurred, were available for the afternoon by season for the upper-air network of stations. Maps of isopleths of constant mean mixing height were also available for the continental United States. Some interpolation was required for the stations used in this project which were not upper-air stations. Since these data were available only by season, the visibility cycle had to be divided into seasons. This was done in the following manner: winter - Dec., Jan., Feb.; spring - Mar., Apr., May; summer - Jun., Jul., Aug.; autumn - Sep., Oct., Nov.

When mean mixing height values were correlated with the maxima and minima on the visibility cycle no significant relationship was found. As shown in Table 7, large values of mean mixing height do not necessarily mean good visibility. Even though the mean mixing heights do not have a high correlation with visibility, it is conceivable that visibility would be lower in stable conditions than in unstable conditions. This could also be due to the dispersive capability of the atmosphere varying during the day as suggested by Holzworth (1959, 1962).

A similar analysis was performed with visibility and seasonally averaged wind speed data. Again, no notable relationship is apparent (Table 8). This was also the case when seasonal means of daily temperature and precipitation were compared with mean visibility.

Holzworth (1959) found that visibilities tended to be lower with higher humidities. Hanel (1972) and Winkler (1973) determined that solid and liquid aerosol particles in the atmosphere reduce visibility

Table 7. Visibility and its relation to mean mixing height.

Station	Season	Visibility (miles)	Mean mixing height (meters)
Birmingham, Al.	Spring	11.0-11.3	1760
	Fall	10.3-10.5	1410
	Winter	9.8-10.0	980
	Summer	9.5- 9.7	1870
Austin, Tx.	Summer	12.8-13.2	2100
	Winter	12.5-12.8	1050
	Fall	12.2-12.4	1500
	Spring	11.4-11.6	1480

Table 8. Visibility and its relation to mean wind speed.

Station	Season	Visibility (miles)	Mean wind speed (m/sec)
Youngstown, Oh.	Spring	14.0-14.2	8.4
	Fall	12.5-12.7	6.4
	Summer	11.7-12.0	5.8
	Winter	9.5- 9.8	8.2
Cleveland, Oh.	Spring	10.5-10.7	8.5
	Fall	10.0-10.3	6.6
	Summer	9.5- 9.8	5.2
	Winter	8.3- 8.6	8.2

due to the condensation of water vapor on the particles at relative humidities up to 95%. These findings led to examination of the monthly mean relative humidity in the same manner as visibility. Relative humidity was chosen rather than specific humidity because relative humidity is a measure of the per cent saturation, indicating the growth potential of the suspended water droplets. Specific humidity measures only the concentration of the water vapor.

Figure 9 shows the annual variability of relative humidity. Comparison with the visibility shows similar form, though 180 degrees out of phase. In other words, low visibility was related to high relative humidity. As with the visibility, the relative humidity plots were similar for all twelve stations. From this, it is concluded that the visibility is inversely related to the relative humidity.

High relative humidity accompanies a high rate of condensation. Consequently, it is reasonable to assume that the growth rate of suspended water droplets is positively related to relative humidity. When the relative humidity is high, more large water droplets are present in the air, unless there are no hygroscopic condensation or ice nuclei in the atmosphere. Since the data were collected near populated urban areas, hygroscopic nuclei (e.g. sulfates) are probably plentiful in the air near the ground. Although visibility is inversely related to relative humidity, as indicated by these results, there are undoubtedly more variables involved in this process, some of which are wind, clouds, and total amount of particulate matter in the air.

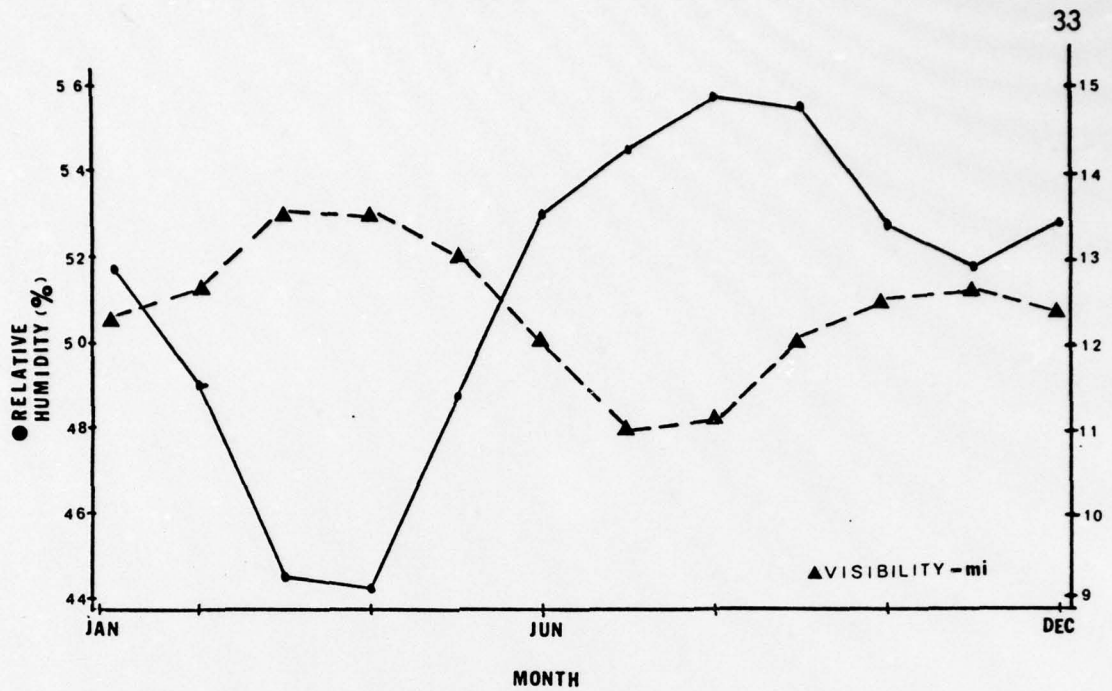


Figure 9a. Bimodal annual oscillation of relative humidity at National Airport (D.C.).

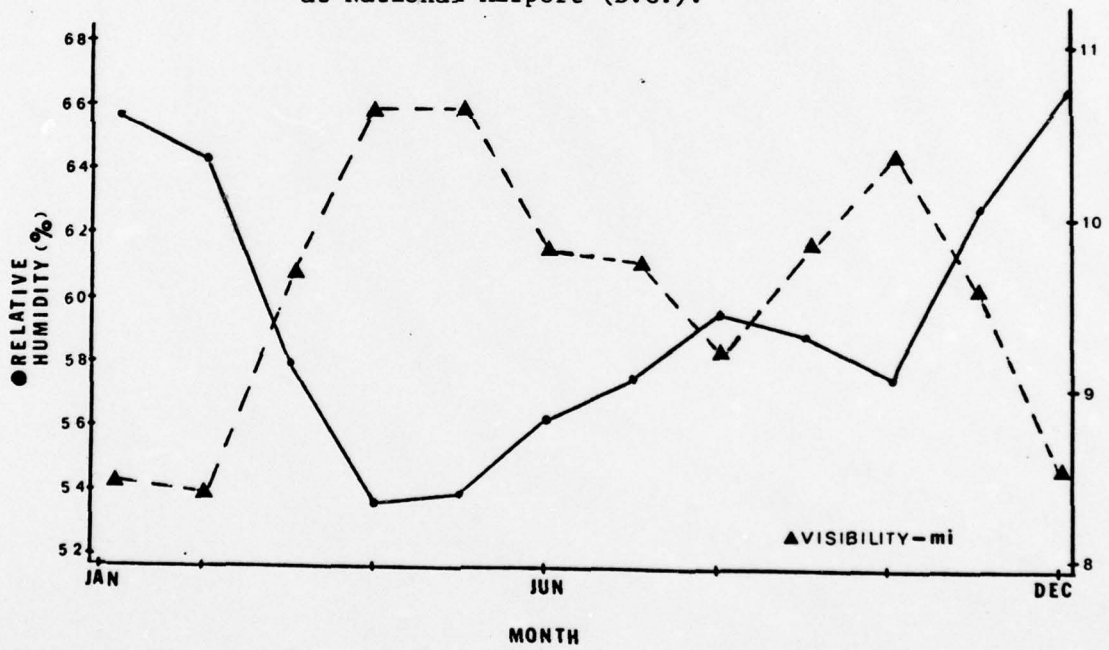


Figure 9b. Bimodal annual oscillation of relative humidity at Cleveland, Oh.

(ii) Cross-Spectrum Analysis

Looking further into the correlation between monthly mean visibility and relative humidity, the cross correlation function was computed for each station using the entire 216 point time series. The cross correlation coefficients calculated in this manner were quite low. Most were negative, and less than $|0.2|$, with a few positive values (Table 9). The low cross correlation coefficients are probably due to using the entire time series for the cross correlation. The relative humidity varies less from year to year than the visibility and shows no trend toward higher or lower values, as indicated by the monthly mean values. Still, the similarity of all the stations indicates confidence in a correlation between visibility and relative humidity.

Table 9. Cross correlation coefficients for visibility versus relative humidity.

Station	r	Station	r
Washington, D. C.	-0.038	Cleveland	+0.048
Raleigh-Durham	+0.057	Ft. Wayne	-0.167
Atlanta	-0.053	Milwaukee	-0.046
Birmingham	+0.042	Peoria	-0.104
Houston	-0.009	St. Louis	-0.001
Austin	-0.040	Phoenix	-0.248
Youngstown	+0.053	Tucson	-0.254

Cross correlation functions and cross correlation coefficients were also calculated for the visibility between stations using the entire time series. Cross correlations were performed for stations within geographical groups and between groups, as shown earlier in Figure 1. The results presented in Table 10 show that the within-group correlations were much higher, with correlation coefficient values of about 0.7. The between group correlations were about 0.2. This result indicates that topographic and climatic similarities have great bearing on visibility. The exceptions to this are the correlation between Tucson and Phoenix, with a correlation coefficient of 0.2, and between Raleigh-Durham and National Airport in Washington, D. C., with a correlation coefficient of 0.3.

Table 10. Cross correlation coefficients for visibility among stations within groups and between groups.

Stations	r	Stations	r
NAT - RDU	0.3	HOU - AUS	0.6
RDU - ATL	0.8		
ATL-- BHM	0.6	PNX - TUS	0.2
YNG - CLE	0.8	TUS - AUS	0.4
CLE - FWY	0.7	TUS - RDU	0.4
FWY - MKE	0.7	TUS - FWY	0.4
MKE - PIA	0.7	AUS - RDU	0.2
PIA - STL	0.8	AUS - FWY	0.2
		RDU - FWY	0.2

Cross spectra and coherence squared, i.e., the correlation coefficient squared at each frequency, were also determined for a few stations which had the highest correlation between visibility and relative humidity. Overall, the cross-spectra were somewhat less pronounced in showing the six and twelve month periodicities. However, these two predominant harmonics in the time series are shown by the cross spectra analysis (Figure 10). The spectrum of relative humidity shows little energy at any frequency, but there is a suggestion of peaks at six and twelve month periods (Figure 11). The results of the cross spectrum analysis show little correlation, as indicated by the low correlation coefficients. This is also indicated by the coherence squared for National Airport (Figure 12). The strongest correlations are at twelve and five to six months. The high value indicated at the low frequency end of the spectrum of the relative humidity (Figure 11) seems to indicate that there may be a long term trend in the relative humidity. However, this possibility was not considered in this study.

In clarifying the maxima and minima, the cross-spectra analysis is not as good as was expected due to the fact that only seventeen degrees of freedom were available with a bandwidth of 0.02 cycles per month. However, it reinforces the previous result that the six and twelve month components are the most important in the time series and in the correlation between visibility and relative humidity.

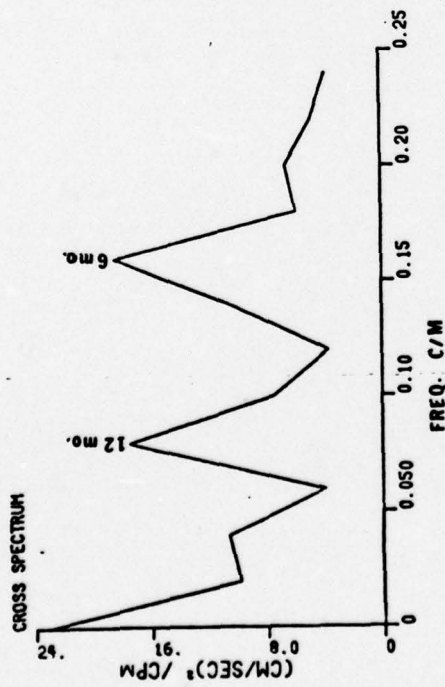


Figure 10. Cross-spectrum of visibility vs relative humidity. (NAT)

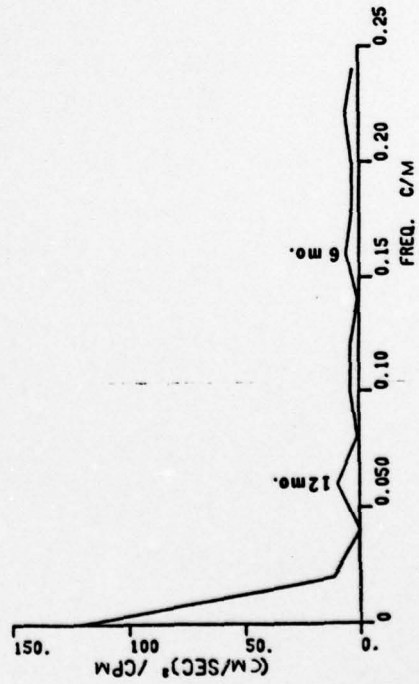


Figure 11. Spectrum of relative humidity. (NAT)

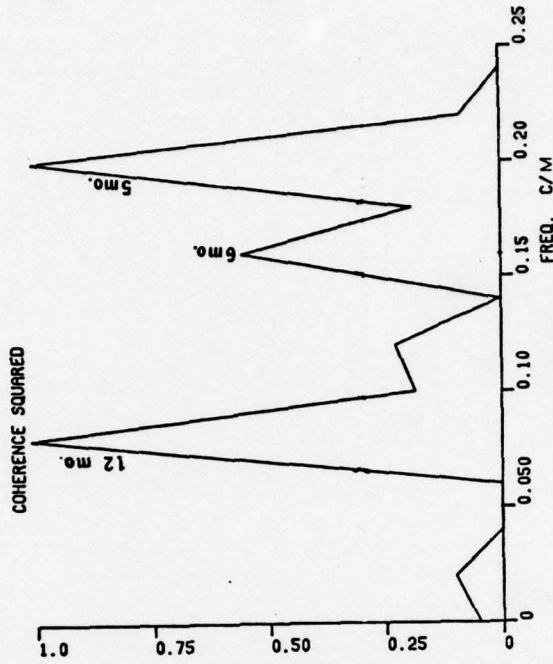


Figure 12. Coherence squared for visibility vs relative humidity. (NAT)

SUMMARY

Some of the difficulties inherent to an investigation of visibility were brought out in this study. Problems were lessened in this study by using a large data base of 18 years. One particular problem encountered by this study was the use of the "+" in reporting visibility. Unfortunately the "+" symbol was not in the archived data and the results biased by changes in the usage of the "+" symbol cannot be estimated. If the "+" symbol did impose an influence on the outcome of the trend analysis, the trend deduced should have been discontinuous. This was shown in the Austin, Texas visibility data. In general, the "+" symbol has not seriously affected this study.

The first major finding of this study was the determination that a downward trend in the visibility existed at all 14 stations. This was shown by the linear regression method and more dramatically by the method developed by Holzworth and Maga (1960). This result may be due to new urban and/or industrial developments around the airports or to an increase in the amount of aircraft traffic. The deterioration of the visibility may also be due to an increase in the sulfate percentage of the total particulate matter from 1964 to 1970.

Another finding of this study was a bimodal annual oscillation in the mean monthly visibility cycle. Spectral analysis indicated that the two major components of the total variance occur with periods of approximately six and twelve months. The positive correlation of the visibility time series was high among stations within similar geographical areas, indicating that geographical region is one major factor in determining the periodicity of the visibility.

Further examination of this result determined that the monthly mean relative humidity also exhibits a bimodal annual oscillation, but was 180 degrees out of phase with the visibility. There seems little doubt that seasonal variability in relative humidity is a major, if not the dominant, cause of the bimodal oscillation in the annual visibility cycle. Other factors such as wind speed, wind direction, the number of daily observations used in this study, and station location may also play an important role in the periodic variability of visibility.

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