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MATHEMATICAL MODELS FOR ACV MOTION IN RANDOM SEAS.(U)

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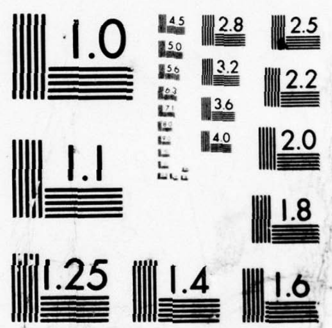

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MATHEMATICAL MODELS FOR ACV MOTION IN RANDOM SEAS

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Mathematical models are developed to predict the pitch, heave and roll motion of air cushion vehicles (ACV's) as they encounter arbitrarily specified wave fields while traveling at constant speed and course. The resulting dynamic equations are solved numerically using a digital computer.

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NOMENCLATURE

a	= wave amplitude
A_c	= cushion planform area
$(A_j)_i$	= area of jupe (cell) bottom opening
A_L	= leakage area
$(A_L)_i$	= area of orifice between loop and vertical cell
A_0	= equilibrium leakage area
b	= beam
C_n	= orifice coefficient
G	= cushion leakage fraction of gap height
h_i	= depth below origin of fully extended skirt at i^{th} jupe
I_x	= craft moment of inertia about the x-axix
I_y	= craft moment of inertia about the y-axis
K	= roll moment
k	= wavenumber
k_1, k_2	= increase in cell width on inside, outside per unit change in height
ℓ	= cushion length
M	= pitch moment
m	= mass of craft
m_c	= mass of air in cushion
P	= cushion gauge pressure
P_L	= loop pressure
Q	= volume rate of flow
$Q_{0,1,2}$	= fan performance parameters
ΔS_i	= length of skirt seal around cushion periphery
T	= effective width of skirt jupes

U_0 = craft speed

U_{sp} = unit step function

V_c = cushion volume

W = craft weight

x_c = x coordinate of cushion centroid

z_c = depth of top of cushion from origin

z_0 = heave coordinate

Z = force on craft in the z-direction

β_i = angle of outward normal at the i^{th} cell with respect to the x-axis

γ = angle of wave propagation direction with respect to the x-axis

γ_c = ratio of specific heats ($= c_p/c_v$)

η = wave height

θ = pitch angle

λ = wavelength

ρ = density

ϕ = roll angle

ω_e = encounter frequency

ω = wave frequency

<u>Subscripts</u>

a refers to atmosphere

i refers to i^{th} seal (jupe)

j refers to j^{th} subcushion

L refers to loop or leakage flow

0 refers to equilibrium conditions

I. Introduction

The purpose of this report is to present the results of GHR contract N00014-77-C-0063 in which mathematical models and computer simulations were developed to predict the motion of air cushion vehicles (ACV's) in random seas. The craft are considered to be traveling at constant speed through a specified random sea of arbitrary heading. The computer simulations predict the resulting pitch, heave and roll response of the craft. Numerical models are developed for two types of vehicles. In the first version, the craft has a single cushion, peripheral cell-stabilized support system; in the second, a divided cushion with a skirt consisting of finger seals is used. The computer programs are applied to two specific craft-the JEFF(A) and the JEFF(B)-as particular examples, respectively, of each type.

Previous work on the theoretical prediction of the seakeeping of air cushion vehicles which has been reported in the literature includes an analysis of the vertical plane dynamics of SES craft by Kaplan and Davis (1). They presented a simplified approach to the problem enabling some discussion of natural frequencies and damping ratios to be made before resorting to computer simulation. Lavis, Bartholomew and Jones (2) considered the response of a peripheral cell air cushion vehicle in random seas using a model which required empirical data for the roll and pitch stiffness and damping. In addition, they discussed atmospheric scaling and the consequent problem of lift system parameter distortion in scale modesl by using an analysis of the flow equations associated with heave motion. Linear models for hovercraft motion in regular waves were also developed by Reynolds (3) and Reynolds, West and Brooks (4) while a nonlinear analytical model for the heave motion of ACV's in waves has been proposed by Lebel and Swift (5). A nonlinear numerical model for the coupled pitch and heave motion of an air cushion vehicle in regular waves was developed by Doctors (6). The craft he considered was of the divided cushion type in which the main cushion is divided into subcushions by flexible longitudinal and transverse stability keels. More recently, Doctors has extended his analysis of divided cushion craft to include the effects of hydrodynamic influence and compressibility (7,8). Carrier, Lundblad and Swift (9) on the other hand, have presented the results of a non-linear analysis for the coupled pitch and heave motion for both a divided cushion craft and a vehicle of the single cushion, peripheral cell type. More recently, Carrier, Magnuson and Swift (10) have compared the predictions of computer simulations with seakeeping experiments made in a towing tank. The nonlinear models used in references (6-10) required a computer-evaluated numerical solution of the dynamic equations.

In related experimental investigations, Magnuson (11) and Fein, Magnuson and Moran (12) reported full scale seakeeping trial data for the performance of a 50-ton British Hovercraft. Magnuson and Mesalle (13) have also used the results of model experiments to develop linear equations of motion for the JEFF(B) in head waves, while Moran, Fein and Ricci (14) have investigated the seakeeping of high length-to-beam ratio SES craft.

In this report a non-linear model for pitch, heave and roll motions in waves is developed from first principles for a two types of ACV. The resulting dynamic equations are then solved on a digital computer using numerical techniques for two specific craft.

These craft have skirt configurations which represent two different concepts in ACV flexible skirt design. Though in a generalized sense, both concepts make use of the idea of subdividing the cushion planform area, the divided cushion designs employ a small number of comparable sized subcushions, while the single cushion configuration utilizes a large main cushion with much smaller cells contained within the surrounding skirt.

More specifically, the divided cushion craft use transverse and longitudinal stability keels to compartmentalize the cushion planform into four sub-cushions.

The purpose of dividing the main cushion is to provide a restoring moment to perturbations in pitch and roll. The cushions are contained by a flexible skirt around the craft periphery consisting of finger seals which are open on the inside and are maintained in position by the cushion pressure. Each cushion is fed individually by fans; the air then leaks either to the atmosphere or beneath a stability keel to another compartment. Usually in craft of this type, the air supply for each cushion is fed through a peripheral bag which functions as an intermediate plenum. The effects of the plenum may be taken into account in the model by adjusting the pressure - flow relation for the cushion feed. The numerical model will be run at small wave amplitudes so that no wave impact on the peripheral bag occurs.

The single cushion craft, on the other hand, is essentially an extension of the peripheral jet concept by use of flexible trunks to maintain adequate clearance. In this configuration, a single cushion is contained by a flexible skirt made up of a series of cells. Air is fed through orifices to the cells or jupes from a loop plenum located above the jupes and extending around the craft periphery. The air supply inflates the cell and then escapes through the open bottom forming a vertical jet. The loop itself, as well as the main cushion, are each supplied from a separate fan system. The cellular skirt system provides the same static stability function as do the stability keels of the divided cushion craft. In addition, the downward jet flow beneath the skirt around the craft periphery forms a "curtain" effect minimizing leakage from the main cushion.

In this report, a mathematical model for each type of craft is developed separately in the next two sections. This is followed by a section describing the synthesis of the sea state excitation. In the last part, the computer programs obtained by solving the dynamic equations numerically are applied to the JEFF(A) and the JEFF(B). The computer program documentation, however, is contained in a separate report.

II. Peripheral Cell-Stabilized ACV

The position and orientation of the ACV is given by the heave coordinate z_0 (positive downward), pitch angle θ, and roll angle ϕ. An x, y, z coordinate system, fixed with respect to the craft is located so that its origin is at the center of mass (see Figure 1). The craft, traveling at constant speed U_0, encounters random seas in which the surface height, with respect to the mean elevation, is given by

$$\eta = \eta(x,y,t) \tag{1}$$

The equations of motion governing the craft response are:

$$\begin{aligned} m\ddot{z}_0 &= (Z)_{cushion} + (Z)_{seals} + W \\ I_y \ddot{\theta} &= (M)_{cushion} + (M)_{seals} \\ I_x \ddot{\phi} &= (K)_{cushion} + (K)_{seals} \end{aligned} \tag{2}$$

The cushion and seal forces and moments will be specified in terms of craft position and cushion and cell pressures.

The main cushion pressure must be such that the rate of change of the cushion mass equals the net rate of mass flow into the cushion. This is expressed by:

$$\frac{dm_c}{dt} = \rho_{in}Q_{in} - \rho_{out}Q_{out} \tag{3}$$

It will be assumed that the air density while entering and leaving the cushion is approximately the atmospheric air density.

$$\rho_{in} \simeq \rho_{out} \simeq \rho_a \tag{4}$$

The flow into the cushion is due to the fan air supply and is expressed in terms of the cushion pressure

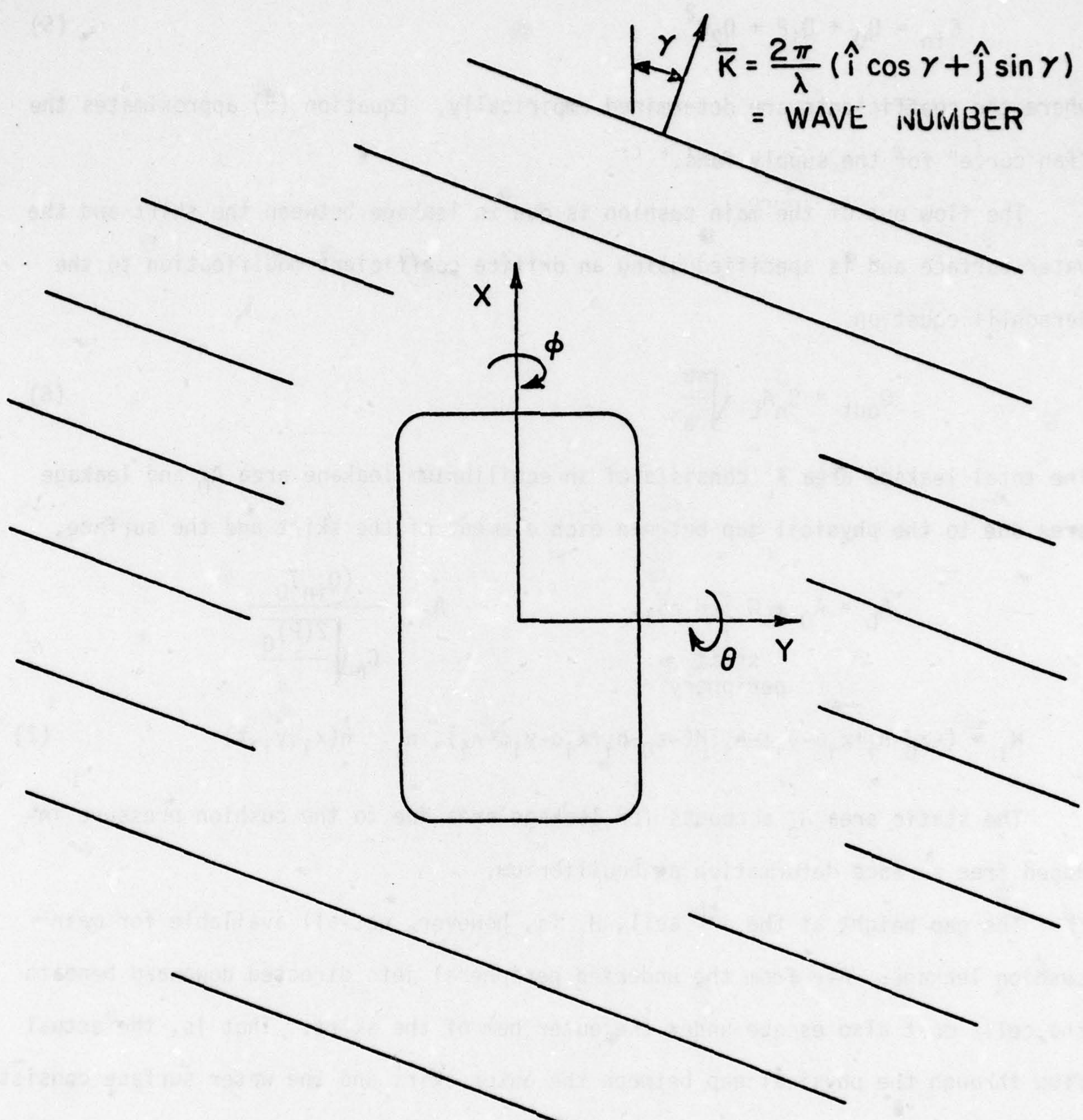

Figure 1 Craft Coordinate System

$$Q_{in} = Q_0 + Q_1 P + Q_2 P^2 \qquad (5)$$

where the coefficients are determined empirically, Equation (5) approximates the "fan curve" for the supply fans.

The flow out of the main cushion is due to leakage between the skirt and the water surface and is specified using an orifice coefficient modification to the Bernoulli equation

$$Q_{out} = C_n A_L \sqrt{\frac{2P}{\rho_a}} \qquad (6)$$

The total leakage area A_L consists of an equilibrium leakage area A_0 and leakage area due to the physical gap between each element of the skirt and the surface,

$$A_L = A_0 + G \sum_{\substack{i \\ \text{craft} \\ \text{periphery}}} H_i \Delta S_i , \qquad A_0 = \frac{(Q_{in})_0}{C_n \sqrt{\frac{2(P)_0}{\rho_a}}}$$

$$H_i = (-z_0 - h_i + x_i\theta - y_i\phi - \eta_i) H(-z_0 - h_i + x_i\theta - y_i\phi - \eta_i), \quad \eta_i = \eta(x_i, y_i, t) \qquad (7)$$

The static area A_0 accounts for leakage area due to the cushion pressure induced free surface deformation at equilibrium.

The gap height at the i^{th} cell, H_i is, however, not all available for main cushion leakage. Air from the underfed peripheral jets directed downward beneath the cells must also escape under the outer hem of the skirt. That is, the actual flow through the physical gap between the outer skirt and the water surface consists of flow from the main cushion and deflected jet flow from the cellular skirt. The effective gap height for the main cushion is then a fraction G of the physical gap height. The proportion of the gap alloted to main cushion flow will be approximated by the fraction known to exist at equilibrium. This would be equal to the ratio of equilibrium cushion supply to the total fan feed at equilibrium.

The rate of change of the mass of air in the main cushion is expanded by writing

$$\frac{dm_c}{dt} = \rho \frac{dV_c}{dt} + V_c \frac{d\rho}{dt} \qquad (8)$$

The cushion volume depends on the craft position relative to the waves:

$$V_c = A_c(-z_0+x_c\theta-z_c) - \iint_{\substack{\text{Cushion}\\ \text{Planform}}} \eta(x,y,t)dA$$

The first term in the above expression is due to changes in average height of the cushion, while the second term represents the wave "pumping" effect.

Compressibility effects of the cushion air mass are taken into account by the last term in Equation (8). The cushion density is related to cushion pressure by assuming that the thermodynamic processes are reversible and adiabatic. The rate of density change may then be expressed by

$$\frac{d\rho}{dt} = \frac{1}{\gamma} \rho^{1-\gamma} \left(\frac{\rho_a^{\gamma}}{P_a}\right) \frac{dP}{dt} \qquad (10)$$

Thus, Equations (3) - (10), which represent conservation of mass for the main cushion air flow, relate the cushion pressure to craft position, surface wave motion and the rates of motion of the vehicle.

The cushion forces on the craft are the resultants of the cushion air pressure acting over the area of the cushion supported externally. The z-direction force is

$$(Z)_{cushion} = -PA_C, \qquad (11)$$

while the pitch and roll moments are

$$(M)_{cushion} = \sum_i (-\cos\beta_i)P[\eta_i-x_i\theta+y_i\phi]\Delta S_i(-z_0-\eta_i)-x_c(Z)_{cushion} \qquad (12)$$

and

$$(K)_{cushion} = \sum_i (\sin\beta_i)P[\eta_i-x_i\theta+y_i\phi]\Delta S_i(-z_0-\eta_i) \qquad (13)$$

respectively.

The seal forces will be specified by considering each jupe or cell in a manner roughly analogous to the treatment of the main cushion. The open cells themselves may be thought of as a series of mini-cushions around the bow and sides of the craft. For the i^{th} cushion, conservation of mass is required for the air flow through the jupe. This would be a statement that the volume rate of flow into the cell from the loop plenum must equal the flow out the cell bottom opening plus the rate of cell volume increase. In equation form, this becomes

$$C_n(A_L)_i \sqrt{\frac{2|P_L-(P_j)_i|}{\rho_a}}\ \mathrm{sgn}\,[P_L-(P_j)_i] = C_n[(A_0)_i+(1-G)\Delta S_i(-z_0+x_i\theta-y_i\phi-$$

$$-h_i-n_i)]U_{sp}[(A_0)_i+(1-G)\Delta S_i(-z_0+x_i\theta-y_i\phi-h_i-n_i)] \sqrt{\frac{2(P_j)_i}{\rho_a}} + T\ \Delta S_i$$

$$(-\dot{z}_0+x_i\dot{\theta}-y_i\dot{\phi}-\dot{n}_i)U_{sp}(z_0-x_i\theta+y_i\phi+h_i+n_i) \qquad (14)$$

where $(A_0)_i$ is the equilibrium leakage area under the i^{th} jupe. Since the equilibrium jupe pressure is the same as for the main cushion, $(A_0)_i$ can be expressed in terms of the cell's equilibrium flow rate by

$$(A_0)_i = \frac{(Q_i)_0}{C_n\sqrt{\frac{2P_0}{\rho_a}}} \qquad (15)$$

For the closed jupes across the stern, the second term in Equation (14) is omitted since there is no escape flow out the bottom.

Though the loop plenum is fed separately from the main cushion, the gap exit flow areas for the cushion and the loop-skirt system are proportional. The flow out from the two systems is also nearly proportional since the pressure of the cell is usually very close to the cushion pressure. Because the total flow through the loop-skirt system is nearly proportional to flow through the cushion, the loop pressure, $P_L \propto P$. The constant of proportionality will be taken as the ratio of the respective pressures at equilibrium. With this approximation and conservation of mass, the pressure within each cell may be related to cushion pressure, craft motion and wave motion.

The additional vertical force (above that already included in the main cushion forcing term) on the i^{th} seal, Z_i, is expressed in terms of the cell pressure by

$$Z_i = -[(P_j)_i - P](A_j)_i + \{-[(P_j)_i - P](k_1)_i[z_0 - x_i\theta + y_i\phi + h_i + \eta_i] -$$

$$(P_j)_i(k_2)_i[z_0 - x_i\theta + y_i\phi + h_i + \eta_i]\}\Delta S_i U_{sp}(z_0 - x_i\theta + y_i\phi + h_i + \eta_i) \quad (16)$$

The resultant seal forces and moments are then:

$$(Z)_{seal} = \sum_i Z_i, \quad (M)_{seal} = \sum_i (-x_i)Z_i, \quad (K)_{seal} = \sum_i (y_i)Z_i \quad (17)$$

III. Divided Cushion ACV

The motion of a divided cushion ACV in the arbitrary wave field (specified formally by Equation (1)) is governed by the same equations of motion, Equations (2) as the single cushion craft. The cushion and seal forcing terms as well as sub-cushion conservation of mass expressions will, of course, be different.

Let subscript j represent either I, II, III or IV corresponding to the sub-cushions shown in Figure 2. Then the conservation of mass expression for the j^{th} cushion is given by

$$\begin{aligned} \frac{d}{dt}(m_c)_j &= \rho_a[(Q_{\dot{m}})_j - (Q_{out})_j] \\ &= \frac{d}{dt}(\rho V_c)_j \\ &= \rho_j \frac{d}{dt}(V_c)_j + (V_c)_j \frac{d}{dt}\rho_j \end{aligned} \quad (18)$$

In the craft considered, each subcushion is supplied air separately by fans. The air supply for the j^{th} cushion is expanded to second order in terms of the corresponding cushion pressure:

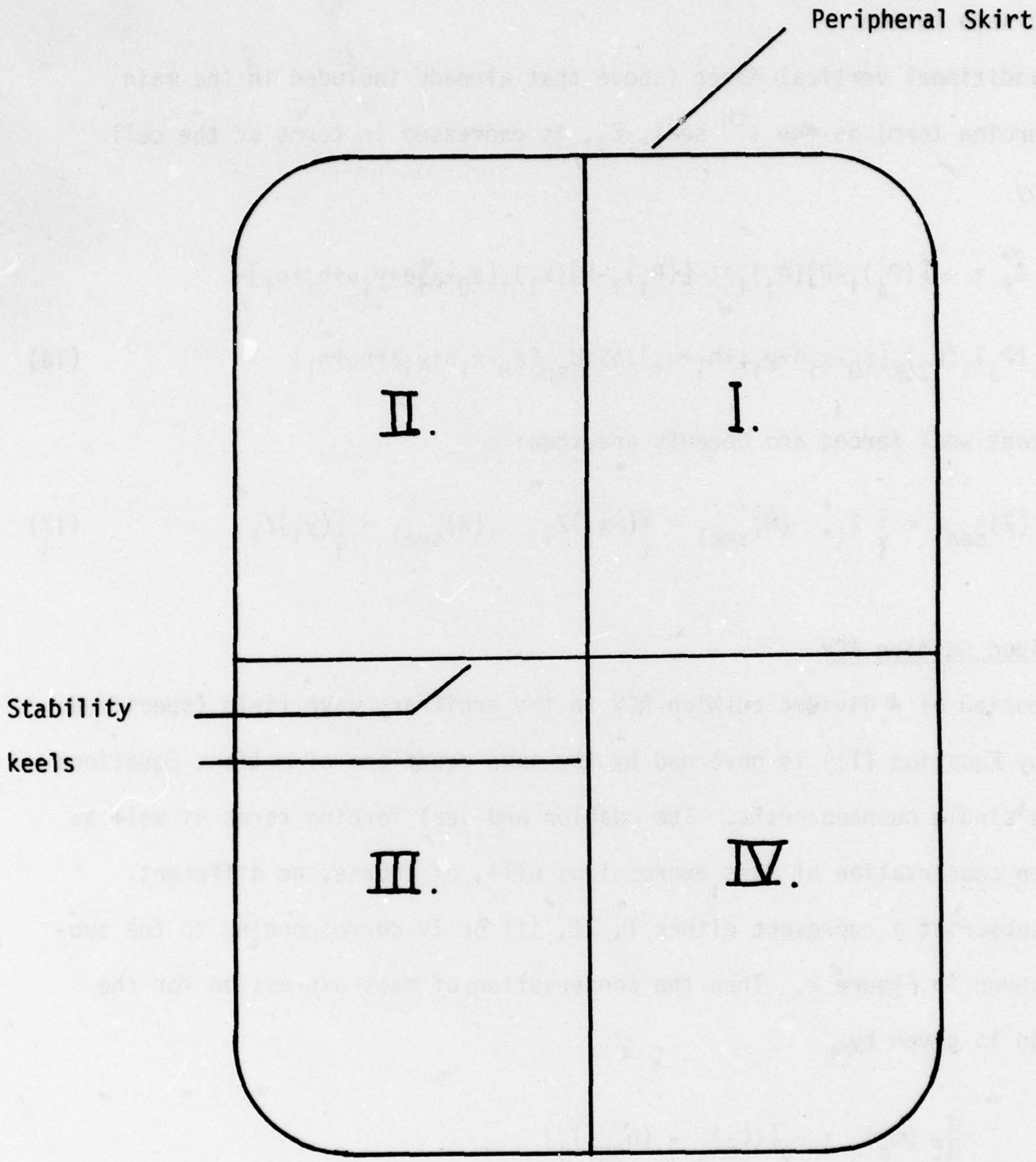

Figure 2 Nomenclature for divided cushion support system

$$(Q_{\dot{m}})_j = C_{oj} + C_{1j}\, P_j + C_{2j}\, P_j^{\,2} \tag{19}$$

where the empirical constants $C_{1,2,3j}$ include the effects of ducting from fan to cushion.

The flow out of the j^{th} cushion due to leakage between skirt and water surface is expressed by

$$(Q_{out})_j = C_n\, (A_L)_j \sqrt{\frac{2P_j}{\rho_a}} + C_n \sum_{m=1}^{4} A_{mj} \sqrt{\frac{2|P_j - P_m|}{\rho_a}}\ \mathrm{sgn}(P_j - P_m) \tag{20}$$

The leakage area $(A_L)_j$ represents gap openings between the j^{th} cushion and the atmosphere,

$$(A_L)_j = (A_o)_j + \Big(\sum_{\substack{i \\ \text{External} \\ \text{Periphery}}} H_i\, \Delta S_i \Big)_j \tag{21}$$

where the summation is around that portion of the skirt separating cushion j from the atmosphere. The area A_{mj} is the leakage area between cushion m and cushion j and is expressed by

$$A_{mj} = \Big(\sum H_i\, \Delta S_i\Big)_{mj} \tag{22}$$

in which the summation is across the skirt separating cushion m from cushion j.

The effective static leakage area for the j^{th} cushion, $(A_o)_j$, is given by

$$(A_o)_j = \frac{Q_{\dot{in}}\left(\frac{W}{A_j} R_j\right)}{C_n \sqrt{\frac{2(W/A_j\, R_j)}{\rho_a}}} \tag{23}$$

and accounts for free surface compliance effects on the leakage area while the craft is in equilibrium. The pressure term $\frac{W}{A_j} R_j$ is the static gauge pressure of cushion j, $(P_o)_j$, and

$$R_j = \begin{cases} \dfrac{(-X_c)_{III}}{2[(-X_c)_{III} + (X_c)_I]} & \text{when } j = I, II \\ \\ \dfrac{(X_c)_I}{2[(-X_c)_{III} + (X_c)_I]} & \text{when } j = III, IV \end{cases} \tag{24}$$

To simplify the analysis, the cross flow terms are linearized using an equivalent linearization procedure with the result that

$$C_n A_{mj} \sqrt{\frac{2|P_j - P_m|}{\rho_a}} \; \text{sgn} \, (P_j - P_m) = \frac{\pi C_n A_{mj}}{V_{mj} \rho_a} (P_j - P_m) \tag{25}$$

in which V_{mj} is a characteristic crossflow velocity and is taken to be $\sqrt{\frac{2(P_o)_j}{\rho_a}}$.

The reversible, adiabatic assumption applied to the j^{th} cushion yields

$$\frac{d}{dt} \rho_j = \frac{1}{\gamma_c} \left[\frac{(m_c)_j}{(V_c)_j}\right]^{1-\gamma_c} \left(\frac{\rho_a}{P_a}\right)^{\gamma_c} \frac{d}{dt} P_j, \tag{26}$$

while the volume of cushion j, $(V_c)_j$, is given by

$$(V_c)_j = A_j[-z_o + (x_c)_j \theta - (y_c)_j \phi - z_c] - \iint\limits_{\substack{\text{Planform} \\ \text{Cushion } j}} \eta(x,y,t)dA \tag{27}$$

The cushion forces and moments are the resultants of the cushion pressure acting over externally supported areas:

$$(Z)_{cushion} = \sum_{j=1}^{IV} (-P_j)A_j$$

$$(M)_{cushion} \sum_{j=1}^{IV} \Big[\sum_{\substack{i \\ \text{periphery} \\ \text{cushion } j}} (-\cos \beta_i) P_j (h_i - x_i \theta - y_i \phi) \cdot \Delta S_i (-z_o - \eta_i) \Big]_j + P_j A_j (x_c)_j$$

$$(K)_{cushion} = \sum_{j=I}^{IV} \left\{ \left[\sum_{\substack{i \\ \text{periphery} \\ \text{cushion } j}} (\sin \beta_i)\, P_j\, (\eta_i - x_i\theta - y_i\phi)\Delta S_i \right. \right.$$

$$\left. \left. (-z_o - \eta_i) \right]_j - P_j\, A_j\, (y_c)_j \right\} \qquad (28)$$

The seal forces are calculated assuming that the seal fabric has no inertia so that the hydrodynamic pressure on the seal equals the air pressure contained by the skirt; thus:

$$(Z)_{seals} = \sum_{j=I}^{IV} - P_j \left[\sum_{\substack{i \\ \text{external} \\ \text{periphery}}} k_i (z_o - x_i\theta + y_i\phi + h_i + \eta_i) U_{sp}(z_o - x_i\theta + y_i\phi + \eta_i)\Delta S_i \right]_j$$

$$M_{seals} = \sum_{j=I}^{IV} P_j \left[\sum_{\substack{i \\ \text{external} \\ \text{periphery}}} x_i k_i (z_o - x_i\theta + y_i\phi + h_i + \eta_i) U_{sp}(z_o - x_i\theta + y_i\phi + h_i + \eta_i)\Delta S_i \right]_j$$

$$K_{seals} = \sum_{j=1}^{IV} P_j \left[\sum_{\substack{i \\ \text{external} \\ \text{periphery}}} (-y_j) k_j (z_o - x_j\theta + y_j\phi + h_i + \eta_i) U_{sp}(z_o - x_i\theta + y_i + h_i + \eta_i)\Delta S_i \right]_j . \qquad (29)$$

IV. Wave Surface

The wave surface height with respect to the mean elevation is specified in the form of a finite Fourier sine series:

$$\eta(x,y,t) = \sum_{m=1}^{N} a_m \sin\left\{ \frac{2\pi}{\lambda_m} \left[x \cos \gamma_m + y \sin \gamma_m\right] - (\omega_e)_m\, t + \phi_m \right\} \qquad (30)$$

By using this general Fourier representation, any arbitrary sea surface may be generated as a superposition of small-amplitude, deep water waves. For the m[th] harmonic constituent, the encounter frequency, wavenumber and phase velocity are related by the following expressions:

$$(\omega_e)_m = \omega_m - \frac{\omega_m^2 U_o}{g} \cos \gamma_m$$

$$k_m = \frac{2\pi}{\lambda_m} = \frac{\omega_m}{c_m} = g/c_m^2 \qquad (31)$$

Each wavelet is defined when its amplitude a_m, heading γ_m, frequency ω_m, and phase angle ϕ_m are given. This method offers considerable flexibility in the specification of the sea surface. Single frequency components may be used as well as super positions of regular waves.

For a fully developed, random sea the Pierson-Moskowitz spectrum can be applied to obtain the amplitudes. In this approach, the sea is assumed to consist of long-crested waves propagating at a single heading γ. The random sea is synthesized by superimposing N harmonic wave components, as in Equation (30), in which the component phases, ϕ_m, are chosen as random numbers with equal probability between 0 and 2π. The range of frequencies for which there is appreciable wave energy is divided into N bands of width $\Delta\omega$ and center frequency ω_m. Then the m^{th} wave component will have wave frequency ω_m and amplitude a_m given by

$$a_m = \sqrt{2\, S\, (\omega_m) \Delta\omega} \qquad (32)$$

where

$$S(\omega_m) = \frac{\alpha_\omega g^2}{\omega_m^5} \exp \left\{ -\beta_w \left(\frac{g}{V\omega_m}\right)^4 \right\} \qquad (33)$$

is the Pierson-Moskowitz wave energy spectrum. The parameter $\alpha=8.10\times10^{-3}$ and $\beta = .74$, while V is the speed of the wind generating the sea.

V. Numerical Model Application

The dynamic equations obtained from the theroetical model are solved numerically using a digital computer. After experimenting with several algorithms for solving differential equations, it was found that a polynomial extrapolation method (taken for Gear (15)) was the best means for extrapolating time dependent variables from one time step to the next. The details of the computational procedure, including complete listings of the programs, are contained in the "Computer Program Manual for ACV Simulations."

In the simulations, the craft trajectories are calculated as the vehicle runs at constant speed and course through a specified wave field. The time response of dynamic variables is then analyzed to determine averages and root mean squares. Also the first harmonic magnitude and phase is calculated at each encounter frequency used in specifying the wave field. These results are normalized by dividing the magnitude of the response variable's first harmonic by the corresponding quantity in the wave field excitation.

The computer programs were applied to two specific craft - a single cushion craft, the JEFF(A), and a divided cushion craft, the JEFF(B). The craft particulars for each example, as used in the programs, are given in Tables 1 and 2 respectively.

Before the programs were run using random seas, they were tested using a single frequency component wave excitation. The procedure was to run the craft at constant speed through regular waves until a steady state response was achieved. This was repeated for various speeds, waveheights, wave lengths and wave propagation directions. The results were then presented in the form of frequency response plots. Figures (3) and (4) show a frequency response for, respectively, the JEFF(A) and the JEFF(B) obtained while the craft were encountering head seas. The JEFF(A), in particular, was extensively studied in this manner since towing tank experimental data was available for comparison. The results of this study were presented in reference (10).

The computer programs were then tested using random sea input. The data for two runs, one for each craft, are presented in Tables 3 and 4. It was found that the computer programs operated successfully when the wave input was changed from regular waves to the superpositions of regular waves used to synthesize

Table 1. Physical Parameters for JEFF(A)

W	= 320,000 lbs.
I_y	= 4.75×10^6 ft-lb-sec^2
I_x	= 1.21×10^6 ft-lb-sec^2
ℓ	= 82.2 ft.
b	= 42 ft.
h	= 10 ft.
x_c	= 0 ft.
z_c	= 5 ft.
β_i	= +90°, -90° along starboard, port side of craft; 0 at bow; 180° at stern
k_1	= 1 along bow and sides; 4 at stern
k_2	= .6 along bow and sides; 4 at stern
ΔS_i	= 4.33 ft.
T	= 8.0 ft.
Q_0	= 4200 ft^3/sec
Q_1	= -11 ft^5/lb-sec
Q_2	= 0
P_a	= 2120 lb/ft^2
ρ_a	= 2.2×10^{-3} slug/ft^3
C_p	= 1.3
C_n	= .7
G	= 1/4
$(A_L)_i$	= 2.5 ft^2
$(A_j)_i$	= 20 ft^2 for bow and sides; 0 at stern

Table 2. Physical Parameters for JEFF(B)

W	= 325,000 lbs.
m	= 10,093 slugs
I_y	= 4.37×10^6 ft-lb-sec^2
I_x	= 1.1×10^6 ft-lb-sec^2
ℓ	= 76.4 ft.
ℓ_j	= 38.2 ft for j = I - IV
C_n	= .7
b	= 48 ft.
b_j	= 24 ft., j = I - IV
h_i	= 9.5 ft on peripheral skirt, 8.5 on stability keels
$(x_c)_j$	=> $(X_c)_{I,II}$ = 19.1 ft.; $(X_c)_{III,IV}$ = -19.1 ft.
$(y_c)_j$	=> $(Y_c)_{I,IV}$ = 12 ft.; $(Y_c)_{II,III}$ = -12 ft.
k_i	= 1
$(z_c)_j$	= 4.5 ft., j = I-IV
$(Q_{in})_j$	= 2010 - 3.5 P_j
C_{oj}	= 2010 ft^3/sec, C_{1j} = -3.5 ft^3/sec $\left(\frac{1}{\text{lb/ft}^2}\right)$, $C_{2j} = 0$
P_a	= 14.7 psi = 2.12×10^3 lb/ft^2
ρ_a	= 2.2×10^{-3} slug/ft^3

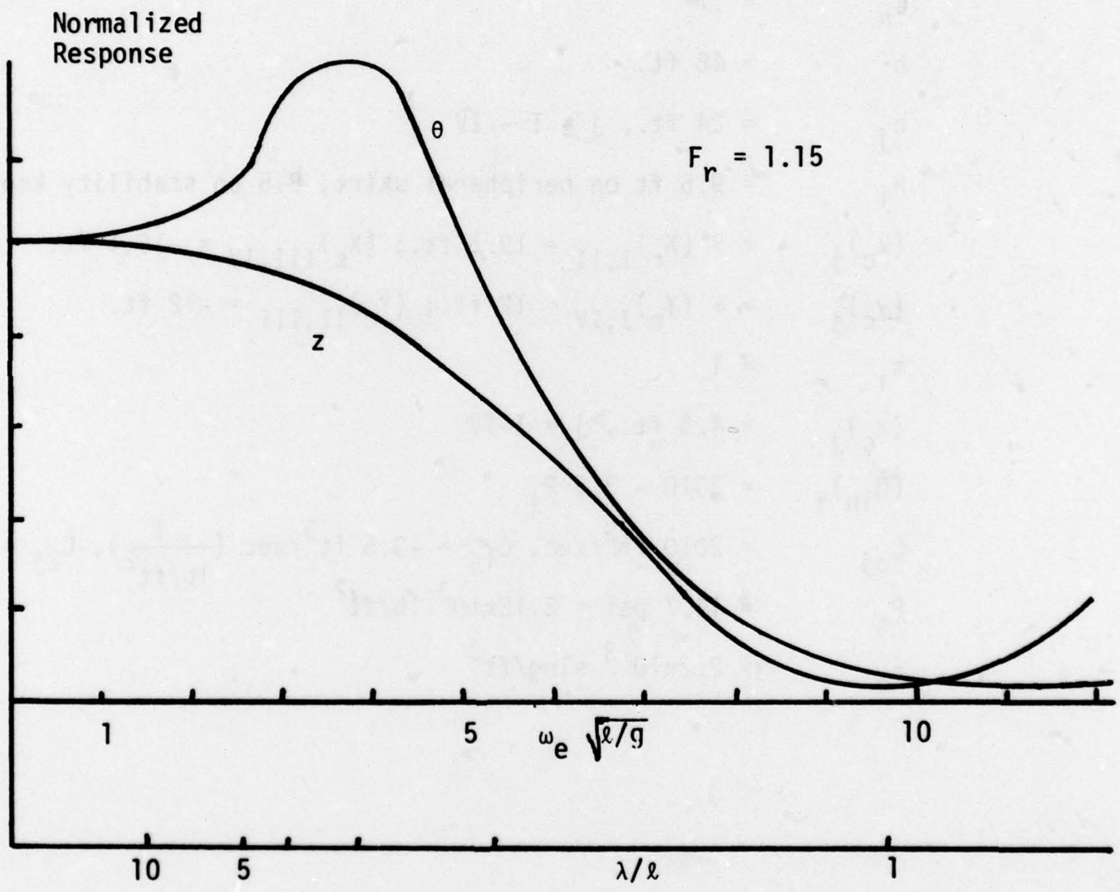

Figure 3 JEFF(A) head seas frequency response

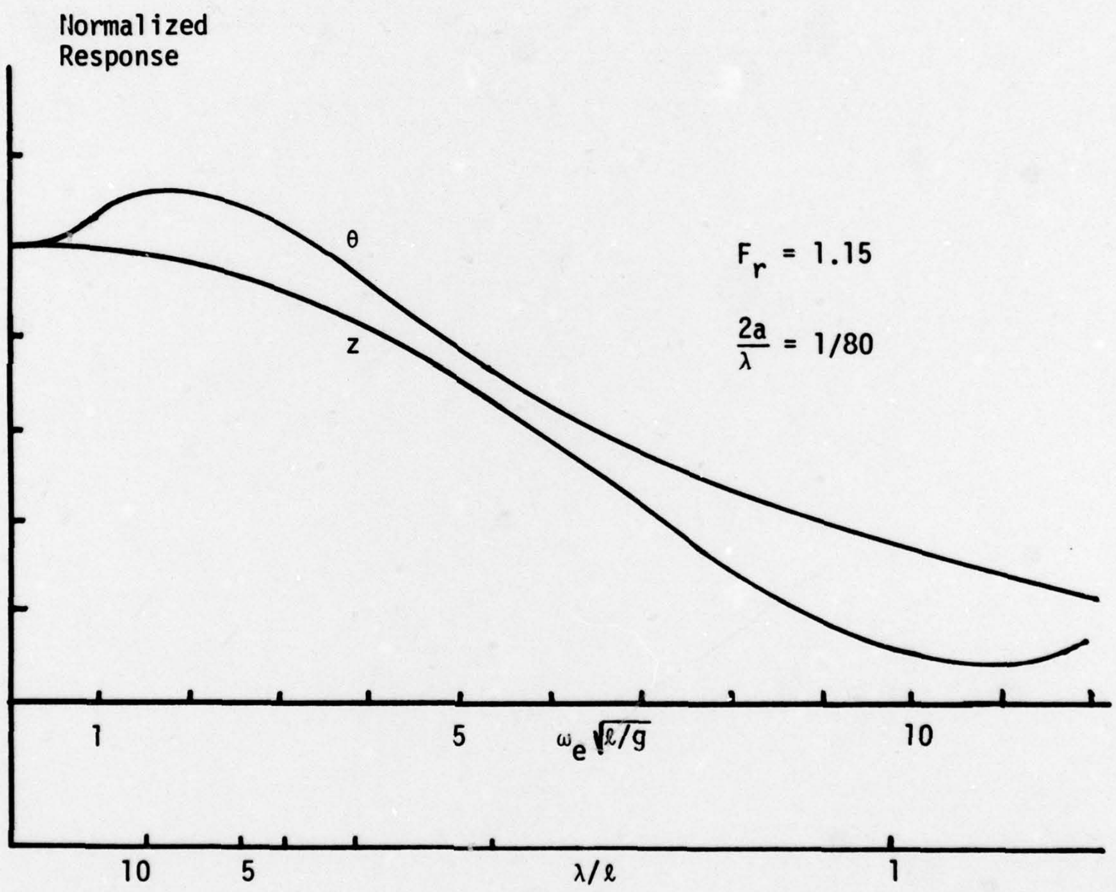

Figure 4 JEFF(B) head seas frequency response

random seas. Due to limitations on the amount of uninterrupted computer time available however, it was not possible to make random sea runs which are long enough to yield completely reliable statistics for the ACV's random sea motion. Hence, the short runs described in Tables 3 and 4 should be interpreted as a demonstration of the programs' capabilities only.

Table 3. JEFF(A) Response to a Random Sea

Record Length = 11.1 seconds

Encounter frequency	Wave Input (Head Sea) Amplitude	Phase	Craft Response Heave (Normalized 1st Harmonic)	Pitch
1.09	0.027	5.83	0.79	0.53
1.40	0.245	4.58	0.73	0.94
1.75	0.566	5.72	0.95	1.13
2.13	0.761	5.32	0.79	0.79
2.55	0.807	4.03	0.88	0.95
3.00	0.766	0.52	0.73	0.90
3.49	0.690	4.13	0.60	0.57
4.02	0.607	5.38	0.41	0.38
4.58	0.528	4.02	0.30	0.13
5.18	0.458	2.46	0.22	0.11

Table 4. JEFF(B) Response to a Random Sea

F_r = 1.15 Record Length = 13.5 seconds

Encounter frequency	Wave Input Amplitude	Wave Input Phase	Craft Response Heave (Normalized 1st Harmonic)	Craft Response Pitch (Normalized 1st Harmonic)
1.09	0.027	5.83	1.37	1.38
1.40	0.245	4.58	1.03	1.07
1.75	0.566	5.72	0.94	1.01
2.13	0.761	5.32	0.90	0.90
2.55	0.807	4.03	0.86	0.87
3.00	0.766	0.52	0.76	0.77
3.49	0.690	4.13	0.65	0.71
4.02	0.607	5.38	0.65	0.72
4.58	0.528	4.02	0.47	0.60
5.18	0.458	2.46	0.17	0.48

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