

AFAPL-TR-78-6
Part II

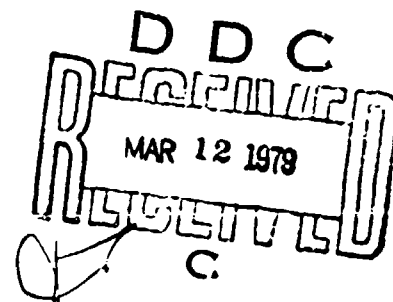
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**ROTOR-BEARING DYNAMICS
TECHNOLOGY DESIGN GUIDE
Part II Ball Bearings**

*SHAKER RESEARCH CORP.
BALLSTON LAKE, NEW YORK 12019*



FEBRUARY 1978

TECHNICAL REPORT AFAPL-TR-78-6, Part II
Interim Report for Period April 1976 - October 1977

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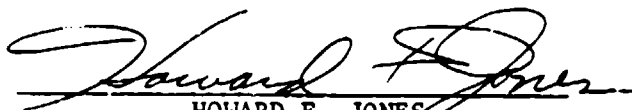
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This technical report has been reviewed and is approved for publication.



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The resulting program is reasonably small and easy to use. Lastly, the stiffness data included in the original Part IV have been updated and are included in appendices.

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The work reported herein was performed during the period 15 April 1976 to 15 November 1977, under the direction of John B. Schrand (AFAPL/SFL) and Dr. James F. Dill (AFAPL/SFL), Project Engineers. The report was released by the authors in December 1977.

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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
a	Semi-major axis of pressure ellipse	in.
b	Semi-minor axis of pressure ellipse	in.
B	Total curvature = $f_1 + f_2 - 1$	
B_{ij}	Damping component, change of force in i direction due to velocity in j direction; i = x, y, z; j = x, y, z.	$\frac{\text{lb-sec}}{\text{in}}$
$\underline{B}_N$	Damping matrix $\begin{bmatrix} (\underline{B}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{B}_N)_{\text{angular}} \end{bmatrix}$	
$(\underline{B})_{\text{lineal}}$	Damping matrix due to lateral velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}}$	$\frac{\text{lb-sec}}{\text{in}}$
$(\underline{B})_{\text{angular}}$	Damping matrix due to angular velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}}$	$\frac{\text{in-lb-sec}}{\text{radian}}$
C	Basic dynamic capacity	lb.

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
d	Ball diameter	in.
D	Diameter	in.
E	Pitch diameter	in.
E'	Operating pitch diameter	in.
E _B	Modulus of elasticity for ball	lbs/in ²
E _R	Modulus of elasticity for race	lbs/in ²
E(ε)	Complete elliptic integral of the second kind formed with the modulus sine where cosε = b/a	
i	r/d	
F _i	Applied load in i direction, i = x, y, z	lbs.
F _c	Ball centrifugal load	lbs.
<u>F</u>	Force matrix = $\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$	lbs.
i	Imaginary number unit vector = $\sqrt{-1}$	
J _x (ζ)	Radial load integral	
K	Ball-race stiffness	lb/in.
K _{ij}	Stiffness component, change of force in i direction due to displacement in j direction. i = x, y, z; j = x, y, z	lb/in.
K _N	Stiffness matrix $\begin{bmatrix} (K_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (K_N)_{\text{angular}} \end{bmatrix}$	

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
$(K)_{\text{lineal}}$	Stiffness matrix due to lateral displacements	lb/in.
	$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \text{ lineal}$	
$(K)_{\text{angular}}$	Stiffness matrix due to angular rotations	$\frac{\text{in.-lb}}{\text{rad}}$
	$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \text{ angular}$	
K_z	Axial deflection constant	
$K(\epsilon)$	Complete elliptic integral of first kind	
M	Applied moment	in.-lb
M_g	Gyroscopic moment	in.-lb
n	Number of balls	
P	Ball load	lb.
P_D	Diametral clearance	in.
Q	Spin torque	in.-lb
r	Raceway groove curvature radius	in.
R	Pitch radius	in.
$\underline{x}$	Column vector =	$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \dot{\theta}_x \\ \dot{\theta}_y \end{bmatrix}$

Symbol

Description

Units

$\underline{W}_N$

Column vector =

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \theta_x \\ \theta_y \end{bmatrix}$$

$\underline{Z}_N$

Impedance matrix = $\underline{K}_N + i \underline{v}_N \underline{B}_N$

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
α	Angle between ball rotational axis and center line of bearing	radians, ^o
β	Contact angle	radians, ^o
β'	Free contact angle	radians, ^o
γ	$\frac{d \cos \beta}{E}$	
Γ	Residues of Newton-Raphson equation	
δ	Displacement	in.
δ_x	Lateral displacement in x direction	in.
δ_y	Lateral displacement in y direction	in.
δ_z	Axial displacement	in.
Δ	Ball-race deflection or normal approach	in.
ζ	$= \frac{1}{2} \left[1 + \frac{P_D}{2 \delta_{\max}} \right]$	
ϵ_i	Hertzian contact parameter; Eqs. (73,88,91,92)	
η_B	$= \frac{4(1-\nu_B^2)}{E_B}$	in. ² /lb
η_R	$= \frac{4(1-\nu_R)^2}{E_R}$	in. ² /lbs
θ_x	Angular rotation about x axis	radians, ^o
θ_y	Angular rotation about y axis	radians, ^o
θ_z	Angular rotation about z axis	radians, ^o
μ	Coefficient of sliding friction	
ν	Frequency of vibration	radians/sec
ν_B	Poisson's ratio for ball	

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
ν_R	Poisson's ratio for race	radians/sec
ω_B	Rotational velocity of ball	radians/sec
ω_1	Rotational velocity of outer race relative to cage	radians/sec
ω_2	Rotational velocity of inner race relative to cage	radians/sec
ω_s	Ball spin rotational velocity	radians/sec
Ω_E	Absolute orbiting velocity of ball	radians/sec
Ω_1	Absolute rotational velocity of outer race	radians/sec
Ω_2	Absolute rotational velocity of inner race	radians/sec
ϕ	Angular location of ball	radians. ^o

SUBSCRIPTS

<u>Symbol</u>	<u>Description</u>
b	Refers to bearing
B	Refers to ball
c	Refers to centrifugal
i	Refers to outer (i = 1) or inner (i = 2) race
l	Refers to loaded extent of bearing
max	Refers to maximum condition
p	Refers to pedestal
q	Refers to ball circumferential position
R	Refers to race
x	Refers to x direction
y	Refers to y direction
z	Refers to z direction
1	Refers to outer race
2	Refers to inner race

SECTION I
INTRODUCTION AND SUMMARY

In recent years, the Rotor-Bearing Dynamics Design Technology Series, AFAPL-TR-65-45 (Parts I through X) has been considered by many engineers to be an important part of their basic analytical tool kit. However, since the issuance of the first volume, in May of 1965, the state-of-the-art has significantly advanced. Further, new techniques of data presentation have been developed, computer capabilities have increased and some minor typographical and technical errors were uncovered.

Part IV of AFAPL-TR-65-45 treated design data for typical deep-groove and angular contact bearings. The data was presented in graphical form and consisted of direct radial stiffness, load carrying capacity, and load levels. In addition design guidelines and limitations were discussed. The major deficiencies of this original volume were that centrifugal effects due to high speed were ignored, and axial and angular stiffness information were omitted.

Subsequent to the publication of Part IV, several extensive treatments of ball bearings including elastohydrodynamic, thermal, and cage effects have been published. The computer program of Mauriello, LaGasse, and Jones (3) considers both elastohydrodynamic and cage effects. The more recent computer based design guide prepared by Crecelius and Pirvics (4) treats elastohydrodynamic, thermal, and cage effects for a system of ball and roller bearings.

Thus very sophisticated analytical tools are available for the design and application of ball bearings. Neither of these tools, however, provide the user with the stiffness matrix required for solution of rotor dynamics problems. In addition both computer programs are very large and require an extensive computer facility for use.

The present volume is intended as an update of the original Part IV(1). Those aspects of the original Part IV(1) which treated general design

aspects of ball bearings, load capacity, speed limitations, etc. have been deleted since their coverage is superficial compared to the more sophisticated computer tools now available (3,4). Only those parts directly connected with preparation of input for the rotordynamic response programs (Volume I of the revised series) have been retained. The complete stiffness matrix is calculated including centrifugal effects. Considerations such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of ball bearing stiffness. The resulting program (Appendix A) is reasonably small and easy to use. Lastly, the stiffness data included in the original Part IV have been updated and are included in Appendices B, C and D.

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SECTION II

ANALYSIS

2.1 General Bearing Model and Coordinate System

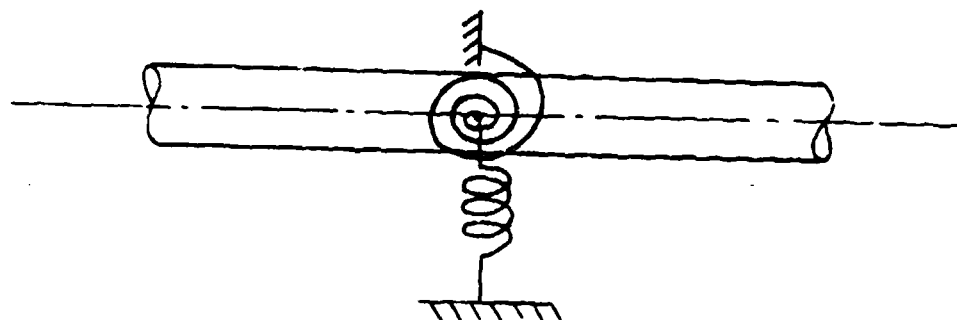
Accurate calculation of the lateral dynamic response of a high-speed rotor depends on realistic characterization of the support bearings. In the most general case, both linear and angular motions are restrained by the support bearings at the attachment location. In the analytical model, the reaction force and the reaction moment of each bearing are felt by the rotor through a single station of the rotor axis. As schematically illustrated in Figure 1a, a coil spring restraining the lateral displacement and a torsion spring which tends to oppose an inclination are attached to the same point of the rotor axis. A complete description of the characteristics of the support bearings, however, involves much more than the specification of the two spring constants. This is because:

- The lateral motion of the rotor axis is concerned with two displacement components and two inclination components.
- The restraining characteristics may include cross coupling among various displacement/inclination coordinates.
- The restraining force/moment may not be temporally in phase with the displacement/inclination.
- The restraining characteristics of the bearing may be dependent on either the rotor speed or the frequency of vibration, or both.
- Bearing pedestal compliance may not be negligible.

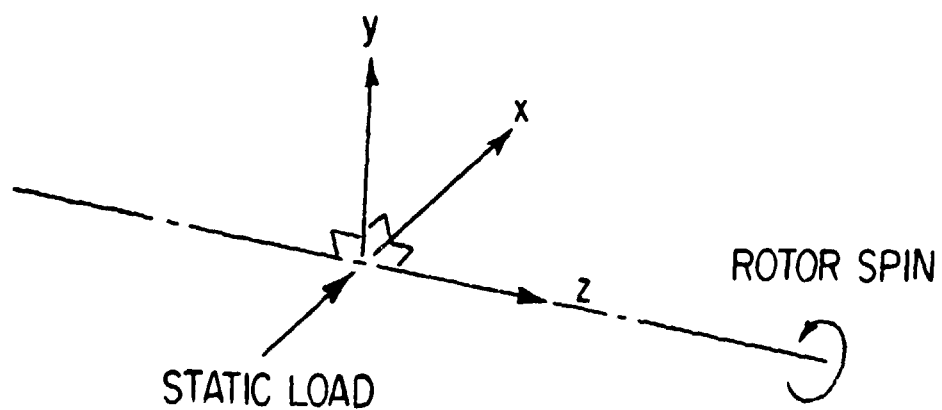
To accommodate the above considerations, the support bearing characteristics are described in Reference 2 by a four-degrees-of-freedom impedance matrix as defined in Equation (1):

$$\underline{R}_N = - \underline{Z}_N \cdot \underline{W}_N \quad (1)$$

where $\underline{W}_N$ is a column vector containing elements which are the two lateral displacements (δ_x, δ_y) and the two lateral inclinations (θ_x, θ_y) of the



(a) Bearing Stiffness Model



(b) Bearing Location
Coordinate System

Figure 1

rotor axis at the bearing station N.

Employing a right handed Cartesian representation in a lateral plane as depicted in Figure 1b, the z-axis is coincident with the spin vector of the rotor. The x-axis is oriented in the direction of the external static load, and the y-axis is perpendicular to both z and x axes forming the right handed triad (x, y, z). (δ_x, δ_y) are respectively lateral lineal displacement components of the rotor axis along the (x, y) directions. (θ_x, θ_y) are lateral inclination components respectively in the (z-x, z-y) planes. Note that θ_x is a rotation about the y-axis, while θ_y is a rotation about the negative x-axis.

$\underline{Z}_N$ is a complex (4 x 4 matrix), and in accordance with the common notation for stiffness and damping coefficients, may be expressed as

$$\underline{Z}_N = \underline{K}_N + i\nu\underline{B}_N \quad (2)$$

where $\underline{K}_N$ is the stiffness matrix and $\underline{B}_N$ is the damping matrix. ν is the frequency of vibration. Most commonly, lateral linear and angular displacements do not interact with each other so that the non-vanishing portions of $\underline{K}_N$ and $\underline{B}_N$ are separate 2 x 2 matrices. That is

$$\underline{K}_N = \begin{bmatrix} \begin{matrix} (\underline{K}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{K}_N)_{\text{angular}} \end{matrix} \end{bmatrix} \quad (3)$$

$$\underline{B}_N = \begin{bmatrix} \begin{matrix} (\underline{B}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{B}_N)_{\text{angular}} \end{matrix} \end{bmatrix} \quad (4)$$

Accordingly, a total characterization of a support bearing would include sixteen coefficients which make up the 4 (2 x 2) matrices:

$$(\underline{K})_{\text{lineal}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{lineal}} \quad (5)$$

$$(\underline{B})_{\text{lineal}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}} \quad (6)$$

$$(\underline{K})_{\text{angular}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{angular}} \quad (7)$$

$$(\underline{B})_{\text{angular}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}} \quad (8)$$

In the event that the pedestal compliance is significant, then the effective support impedance can be calculated from

$$\underline{Z}_N = (\underline{Z}_b^{-1} + \underline{Z}_p^{-1}) \quad (9)$$

where subscripts "p" and "b" refer to the pedestal and bearing respectively. Note that both pedestal inertia and damping may be included in $\underline{Z}_p$.

2.2 General Bearing Support Characteristics

The function of a bearing is to restrict the rotor axis to a nominal axis under realistic static and dynamic load environments. Deviation of any particular point of the rotor axis from the nominal line can be characterized by three lineal and two angular displacements. These may be designated as $(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)$ in accordance with a right-handed Cartesian reference system. The z-coordinate is coincident with the reference axis and is directed toward the spin vector. (θ_x, θ_y) are rotor axis inclinations respectively in the z-x and z-y planes. The x-coordinate is directed toward the predominant static load; e.g., earth gravity. Ideally, the bearing would resist the occurrence of any displacement so that the reaction force system imparted by the bearing to the rotor is generally expressed in matrix notation as

$$\underline{F} = - \underline{Z} \cdot \underline{x} \quad (10)$$

$\underline{F}$ is a column vector comprising the five reaction components (F_x, F_y, F_z, M_x, M_y), while $\underline{x}$ is the displacement vector ($\delta_x, \delta_y, \delta_z, \theta_x, \theta_y$). $\underline{Z}$ is a (5 x 5) matrix containing the elements Z_{ij} with both indices (i, j) ranging from 1 to 5. The values of Z_{ij} characterize how rotor displacements are being resisted by the bearing.

From the standpoint of dynamic perturbation, distinction is made between a static equilibrium component and a dynamic perturbation component for both the displacements and the reactions. Thus,

$$\underline{x} = \underline{x}_0 + \underline{x}'; \quad \underline{F} = \underline{F}_0 + \underline{F}' \quad (11)$$

($\underline{x}', \underline{F}'$) are respectively presumed to be infinitesimal in comparison with ($\underline{x}_0, \underline{F}_0$). Accordingly, Z_{ij} are regarded as dependent on $\underline{x}_0$ but not on $\underline{x}'$. To illustrate the idea of perturbation linearization, one may examine the one-dimensional load-displacement curve shown in Figure 2.

As illustrated, the load-displacement relationship is a 3/2 power law in accordance with the Hertzian point contact formula. It is not possible to describe the entire range by a linear approximation. However, if a small dynamic perturbation is taken around a static equilibrium point, $\delta'_x < \delta_{x_0}$, the small segment of the load-displacement curve can be approximated by a local tangent line. The corresponding force increment is

$$F'_x = \frac{\partial F_x}{\partial \delta x} \delta'_x \quad (12)$$

where δ'_x is the incremental displacement. $\partial F_x / \partial \delta x$ will depend on the amplitude of δ_{x_0} .

The question of history dependence is resolved by regarding $\underline{x}'$ as periodic motions at any frequency ν of interest, and Z_{ij} accordingly would have both real and imaginary parts and may also be dependent on both the rotor speed ω and the vibration frequency ν .

To avoid notational clumsiness, the primes will be dropped from ($\underline{F}', \underline{x}'$)

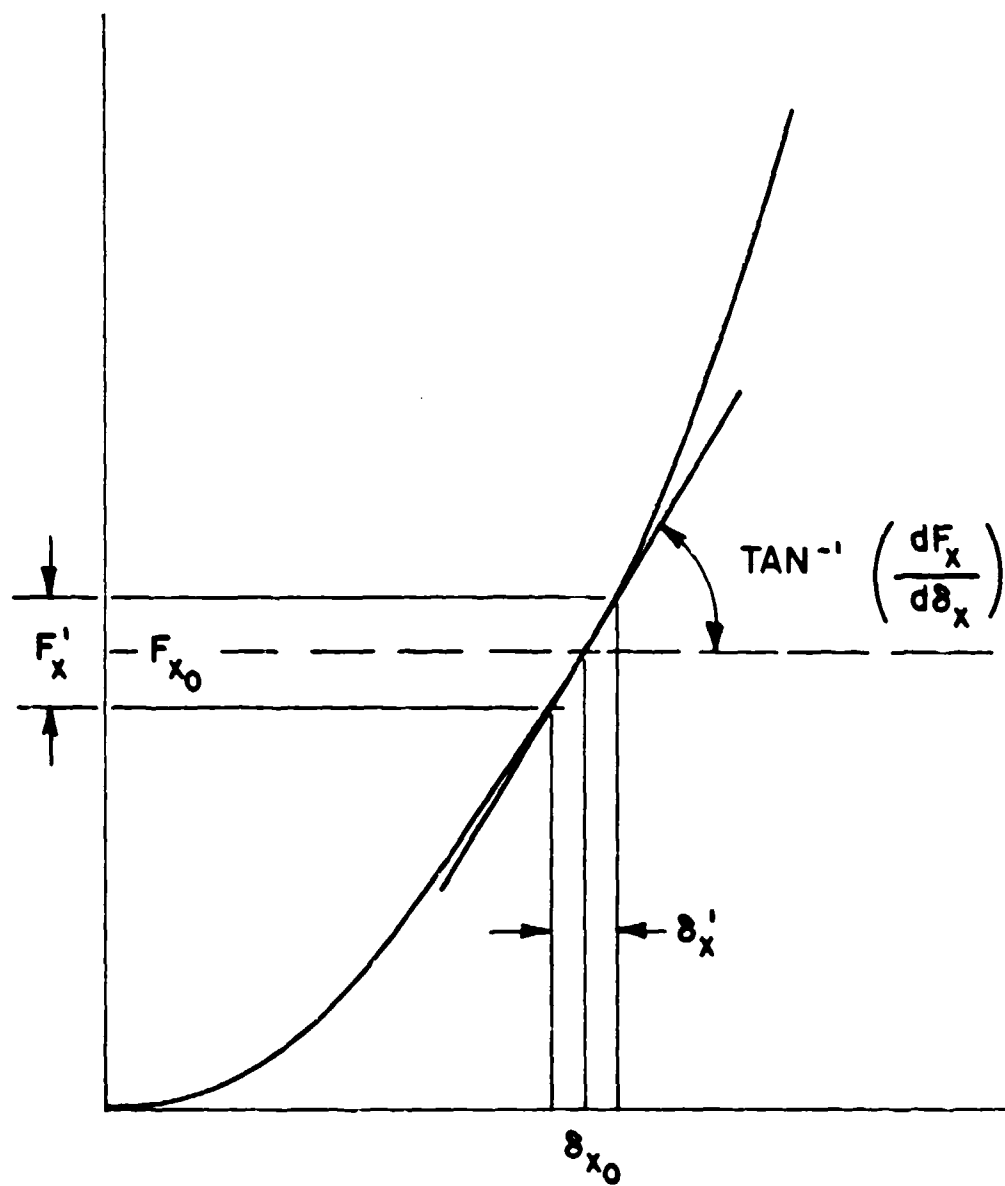


Figure 2 Linearization of Ball Bearing Stiffness

which are understood to be dynamic perturbation quantities unless the subscript "0" is used to designate the static equilibrium condition.

2.3 Ball Bearing Characterization

In many ways the ball bearing is much simpler to model from a rotor dynamics point of view than a fluid film bearing. In general, the following two simplifications can be made:

- The restraining characteristics do not include cross coupling among the various displacement/inclination coordinates.
- The restraining force/moment is normally temporally in phase with the displacement/inclination.

Figure 3 shows a ball bearing referred to in an orthogonal xyz coordinate system. The outer ring is fixed but the inner ring may move with respect to the coordinate system. Both rings are free to rotate about their axes.

Three lineal displacements, δ_x , δ_y , δ_z , and two angular displacements, θ_x , θ_y , are required to define the spatial position and attitude of the inner ring when it is displaced from its initial position. For purposes of derivation the initial situation is that existing when the bearing's end play is just taken up in the thrust direction. Figure 3 shows these displacements in the positive sense.

Figure 4 shows some important dimensions and establishes the convention of the ball-position index q . The contact angle β is the initial mounted contact angle and is shown in the positive sense.

2.3.1 Stiffness

The total characterization of a ball bearing's stiffness can be expressed by the matrix at the top of page 12.

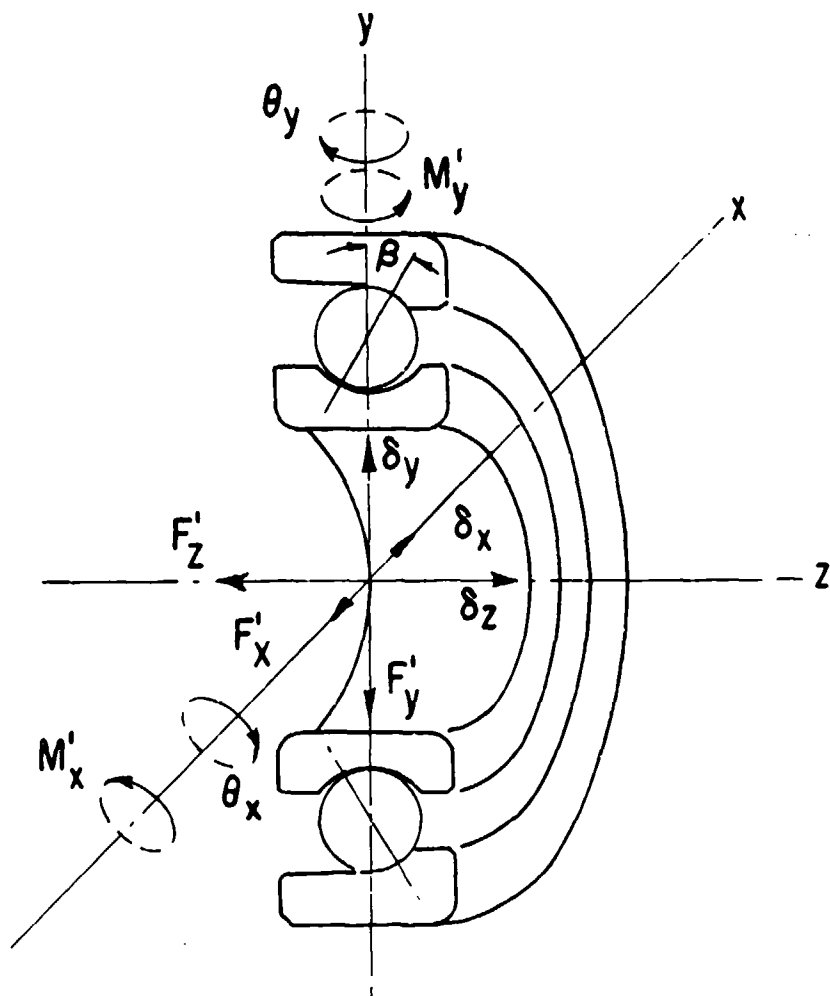


Figure 3 Ball Bearing Coordinate System

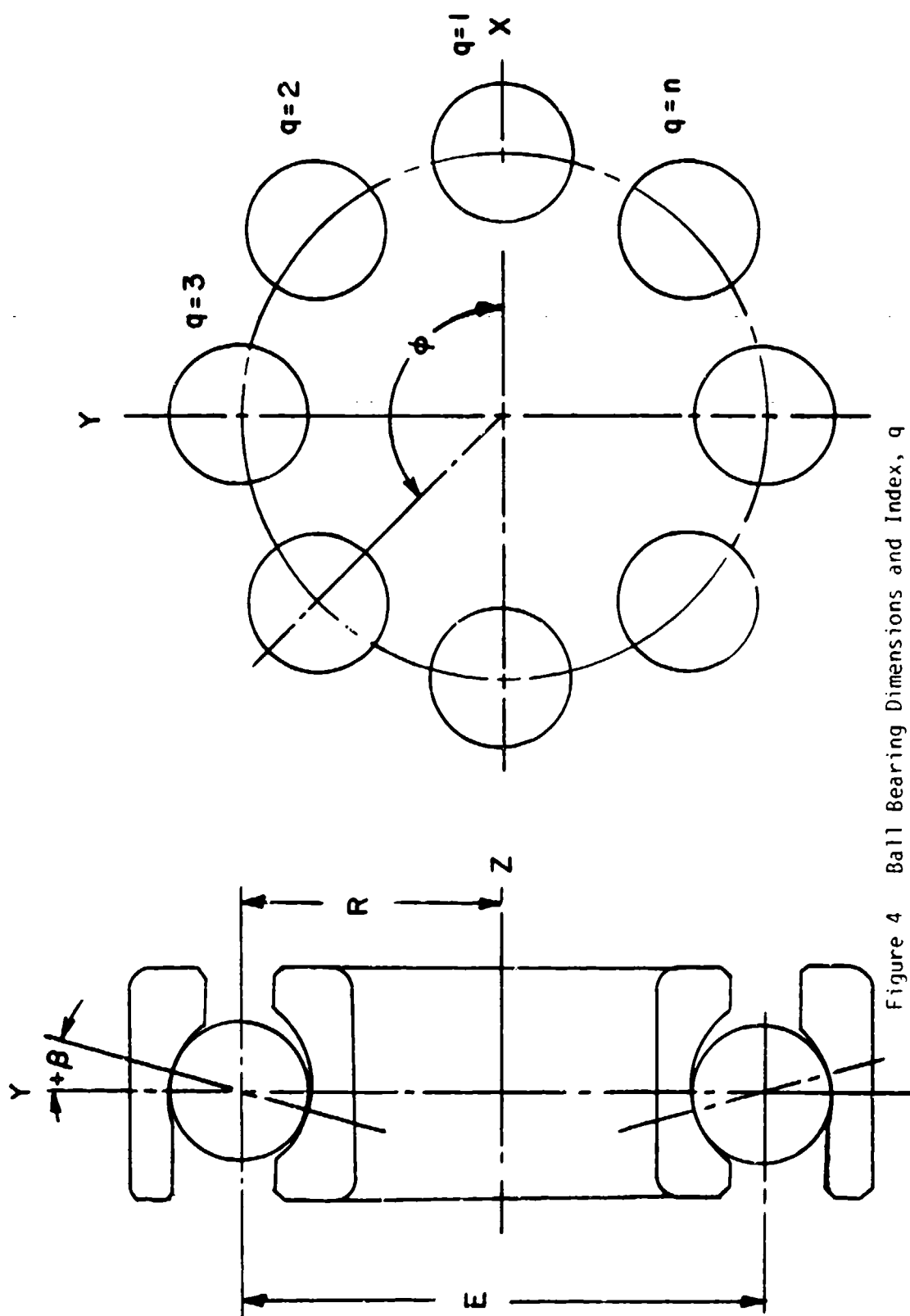


Figure 4 Ball Bearing Dimensions and Index, q

$$[K] = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} & \frac{\partial F_x}{\partial \theta_x} & \frac{\partial F_x}{\partial \theta_y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} & \frac{\partial F_y}{\partial \theta_x} & \frac{\partial F_y}{\partial \theta_y} \\ \frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_z}{\partial \theta_x} & \frac{\partial F_z}{\partial \theta_y} \\ \frac{\partial M_x}{\partial x} & \frac{\partial M_x}{\partial y} & \frac{\partial M_x}{\partial z} & \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_y}{\partial z} & \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{bmatrix} \quad (13)$$

The lineal and angular stiffness matrices (Equations 5 and 7) can be derived from Equation (13). For example:

$$(K)_{\text{lineal}} = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} \end{bmatrix} \quad (14)$$

$$(K)_{\text{angular}} = \begin{bmatrix} \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{bmatrix} \quad (15)$$

Note that although the axial components of stiffness are not utilized by the lateral rotor dynamics program (2), they have been retained in the general ball bearing stiffness matrix, Equation (13). The axial stiffness would be required, for example, if the reader was calculating the axial natural frequency of a ball bearing mounted shaft.

2.3.2 Damping

There is very little data on ball bearing damping. The data included in Reference 1, which is for nonrotating grease packed bearings, suggested a value in the order of 15-20 pound sec/in. This should be

used only as an approximate figure since this has not been confirmed.

2.4 Load-Deflection Relationships

For a given ball-raceway contact, the load deflection relationship is given by an equation of the form:

$$P = K \Delta^{3/2} \quad (16)$$

The total normal approach between two raceways under load separated by a rolling element is the sum of the approaches between the rolling element and each raceway. Hence,

$$\Delta = \Delta_1 + \Delta_2 \quad (17)$$

and

$$K = \left[\frac{1}{(1/K_1)^{2/3} + (1/K_2)^{2/3}} \right]^{3/2} \quad (18)$$

where K_1 and K_2 are a function of the ball-race geometry and material properties.

2.5 Ball Bearings Under Radial Load

For a rigidly supported bearing subjected to radial load, the radial deformation at any rolling element angular position is given by

$$\Delta_\phi = \delta_{\max} \cos\phi - (1/2)P_D \quad (19)$$

in which $\delta_{\max}$ is the maximum deflection, occurring at $\phi = 0$ and P_D is the diametral clearance. Figure 5 illustrates a radial bearing with clearance.

Equation (19) may be rearranged in terms of maximum deformation as follows:

$$\Delta = \Delta_{\max} \left\{ 1 - \frac{1}{2\zeta} (1 - \cos\phi) \right\} \quad (20)$$

in which

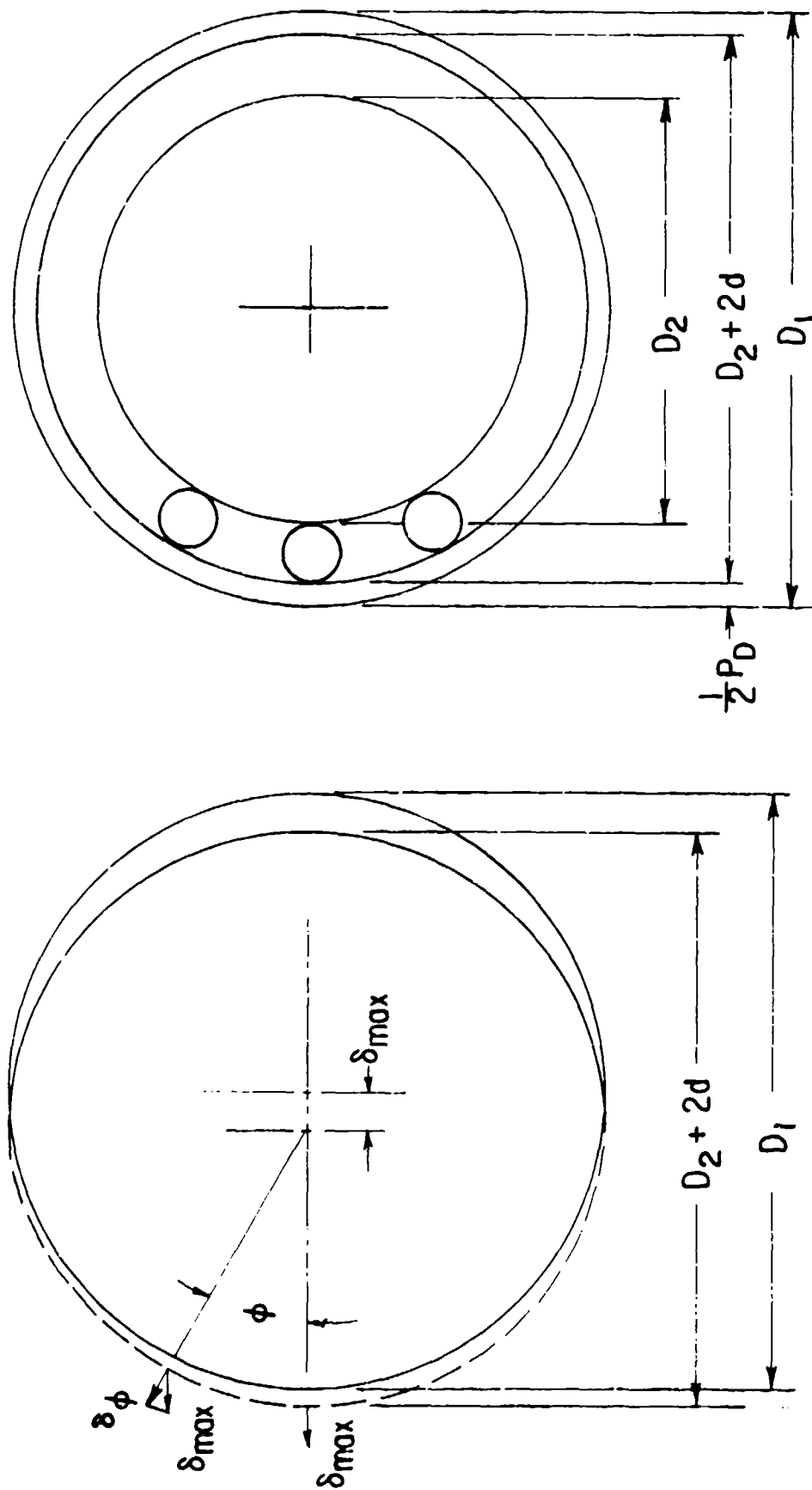


Figure 5 Radially Loaded Ball Bearing

$$\zeta = \frac{1}{2} \left[1 - \frac{P_D}{2\delta_{\max}} \right] \quad (21)$$

It is clear from Equation (21) that the angular extent of the load zone is determined by the diametral clearance such that

$$\phi_\ell = \cos^{-1} \left[\frac{P_D}{2\delta_{\max}} \right] \quad (22)$$

For zero clearance, $\phi = 90$

From Equation (16)

$$\left[\frac{P_\phi}{P_{\max}} \right] = \left[\frac{\Delta_\phi}{\Delta_{\max}} \right]^{3/2} \quad (23)$$

Therefore, from Equations (20) and (23)

$$P_\phi = P_{\max} \left[1 - \frac{1}{2\zeta} (1 - \cos\phi) \right]^{3/2} \quad (24)$$

For static equilibrium to exist, the applied radial load must equal the sum of the horizontal (parallel to the load direction) components of the rolling element loads:

$$F_x = \sum_{\phi=-\pi}^{\phi=+\pi} P_\phi \cos\phi \quad (25)$$

$$F_x = P_{\max} \sum_{\phi=-\pi}^{\phi=+\pi} \left[1 - \frac{1}{2\zeta} (1 - \cos\phi) \right]^{3/2} \cos\phi \quad (26)$$

The summation in above equations applies only to the angular extent of the load zone. Equation (26) can also be written in integral form:

$$F_x = nP_{\max} \times \frac{1}{2\pi} \int_{-\phi_\ell}^{+\phi_\ell} \left[1 - \frac{1}{2\zeta} (1 - \cos\phi) \right]^{3/2} \cos\phi d\phi \quad (27)$$

or

$$F_x = n P_{\max} J_x(\zeta) \quad (28)$$

in which

$$J_x(\zeta) = \frac{1}{2\pi} \int_{-\phi_\ell}^{+\phi_\ell} \left\{ 1 - \frac{1}{2\zeta} (1 - \cos\phi) \right\}^{3/2} \cos\phi d\phi \quad (29)$$

The radial integral of Equation (29) has been evaluated numerically by Harris (5) for various values of ζ .

From Equation (16),

$$P_{\max} = K \Delta_{\phi=0}^{3/2} = K \left\{ \delta_{\max} - \frac{1}{2} P_D \right\}^{3/2} \quad (30)$$

Therefore,

$$F_x = nK \left\{ \delta_{\max} - \frac{1}{2} P_D \right\}^{3/2} J_x(\zeta) \quad (31)$$

For a given bearing with a given clearance under a given load, Equation (31) may be solved by trial and error.

For ball bearings under pure radial load and zero clearance, $J_x = .2285$, and it can be shown from Reference 6 that

$$P_{\max} = \frac{4.37F}{n \cos\beta} \quad (32)$$

Accounting for nominal diametral clearance in the bearing, one may use the following approximation,

$$P_{\max} = \frac{5F}{n \cos\beta} \quad (33)$$

2.5.1 Approximation to Radial Stiffness

A complete definition of bearing stiffness requires calculation of the stiffness matrix defined by Equation (13). However, for many

applications, simplifications can be made which permit calculation of radial stiffness of a ball bearing under radial load alone.

Palmgren (6) gives a series of formulas to calculate bearing deflection for specific conditions of loading. For slow and moderate speed deep-groove and angular-contact ball bearings subjected to radial load which causes only radial deflection, that is $\delta_z = 0$,

$$\delta_x = 1.58 \times 10^{-5} \frac{P_{\max}^{2/3}}{d^{1/3} \cos \beta} \quad (34)$$

For self-aligning ball bearings,

$$\delta_x = 2.53 \times 10^{-5} \frac{P_{\max}^{2/3}}{d^{1/3} \cos \beta} \quad (35)$$

For this case the maximum rolling element load has previously been shown to be

$$P_{\max} = \frac{5F_x}{n \cos \beta} \quad (33)$$

Substituting for $P_{\max}$ in Equation (34) yields

$$\delta_x = \frac{4.62 \times 10^{-5} F_x^{2/3}}{n^{2/3} d^{1/3} \cos^{5/3} \beta} \quad (36)$$

Transposing, then taking the derivative with respect to x , the stiffness of the bearing is

$$K_{xx} = 4.77 \times 10^6 n d^{1/2} \cos^{5/2} \beta \delta_x^{1/2} \quad (37)$$

For self-aligning ball bearings

$$K_{xx} = 2.36 \times 10^6 n d^{1/2} \cos^{5/2} \beta \delta_x^{1/2} \quad (38)$$

From Equations (37) and (38) it is apparent that the deflection-stiffness relationship is nonlinear because it is dependent on the square

root of radial deflection. In this respect, a ball bearing is unlike a simple spring for which deflection is linear with respect to load.

By substituting for δ_x in Equations (37) and (38), bearing stiffness can be expressed in terms of load as:

Angular Contact Bearings

$$\frac{K_{xx}}{d^{1/3} n^{2/3} \cos^{5/3} \beta} = 3.247 \times 10^4 F_x^{1/3} \quad (39)$$

Self-Aligning Bearings

$$\frac{K_{xx}}{d^{1/3} n^{2/3} \cos^{5/3} \beta} = 2.028 \times 10^4 F_x^{1/3} \quad (40)$$

These expressions are plotted in Figure 6.

2.6 Bearings Under Thrust Load

Thrust ball bearings subjected to a centric thrust load have the load distributed equally among the rolling elements. Hence,

$$P = \frac{F_z}{n \sin \beta} \quad (41)$$

In Equation (41), β is the contact angle which occurs in the loaded bearing. For thrust ball bearings whose contact angles are nominally less than 90 degrees, the contact angle in the loaded bearing is greater than the initial contact angle β' which occurs in the unloaded bearing.

In the absence of centrifugal loading, the contact angles at inner and outer raceways are identical; however, they are greater than those in the unloaded condition. In the unloaded condition, contact angle is defined by

$$\cos \beta' = 1 - \frac{P_D}{2Bd} \quad (42)$$

in which P_D is the mounted diametral clearance. A thrust load, F_z , applied to the inner ring as shown by Figure 7 causes an axial deflection δ_z . This axial deflection is a component of a normal deflection along the line of contact such that from Figure 7

$$\Delta = Bd \left(\frac{\cos \beta'}{\cos \beta} - 1 \right) \quad (43)$$

Since $P = K\Delta^{3/2}$,

$$P = K(Bd)^{3/2} \left(\frac{\cos \beta'}{\cos \beta} - 1 \right)^{3/2} \quad (44)$$

Substitution of Equation (41) into Equation (44) yields

$$\frac{F_z}{nK(Bd)^{3/2}} = \sin \beta \left(\frac{\cos \beta'}{\cos \beta} - 1 \right)^{3/2} \quad (45)$$

Since K is a function of the final contact angle, β , Equation (45) must be solved by trial and error to yield an exact solution for β . Jones (7) has defined an axial deflection constant K_z as follows:

$$K_z = \frac{B}{g(+\gamma) + g(-\gamma)} \quad (46)$$

in which $\gamma = \frac{d \cos \beta}{E}$ and $g(+\gamma)$ refers to the inner raceway and $g(-\gamma)$ refers to the outer raceway. Jones further indicates that the sum of $g(+\gamma)$ and $g(-\gamma)$ remains virtually constant for all contact angles being dependent only on total curvature B . The axial deflection constant K_z is related to K as follows:

$$K = \frac{K_z d^{1/2}}{B^{3/2}} \quad (47)$$

Hence:

$$\frac{F_z}{nd^2 K_z} = \sin \beta \left(\frac{\cos \beta'}{\cos \beta} - 1 \right)^{3/2} \quad (48)$$

Taking K_z from Reference 7, Equation (48) may be solved numerically by the

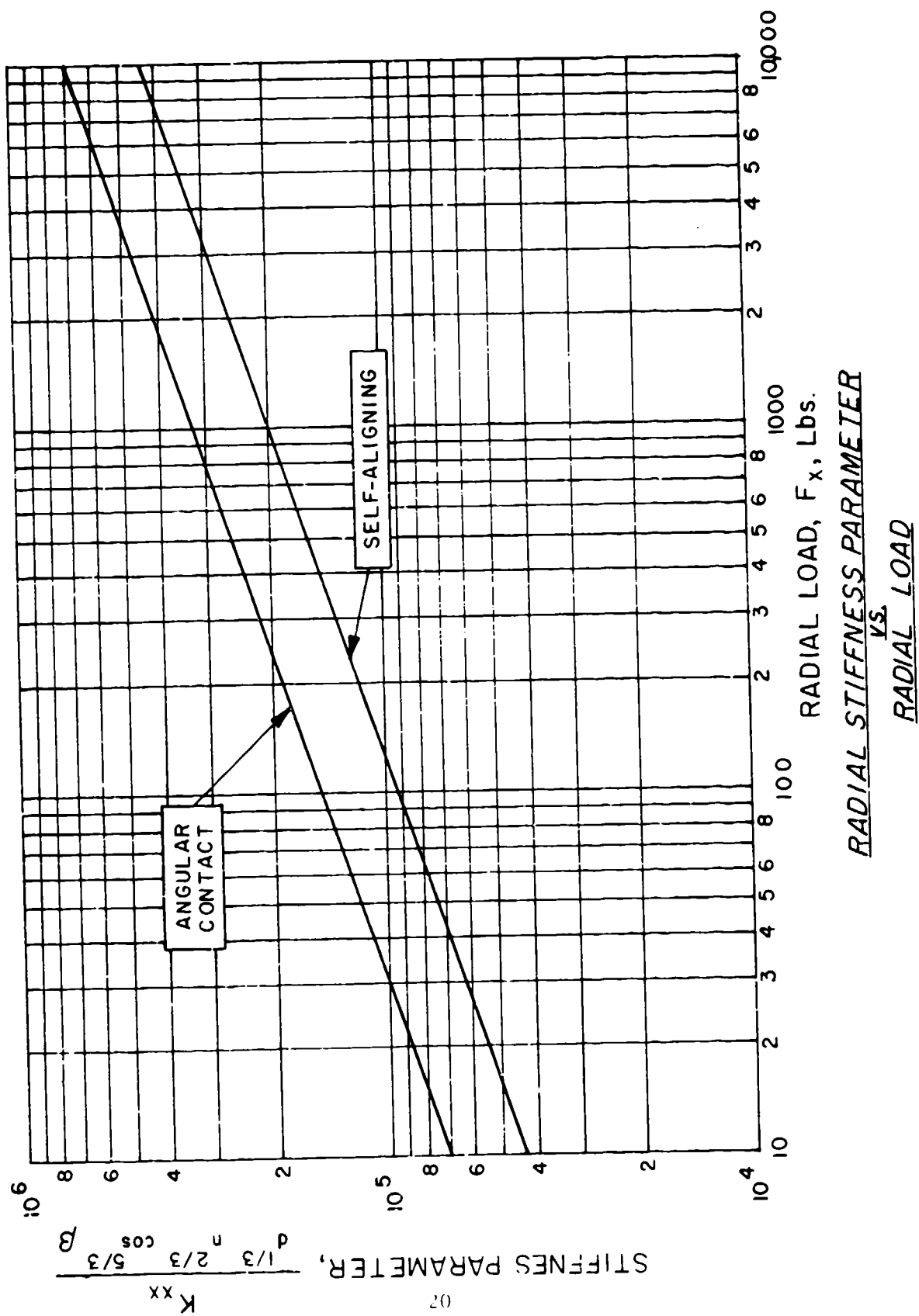


Figure 6 Radial Stiffness Parameter vs. Radial Load

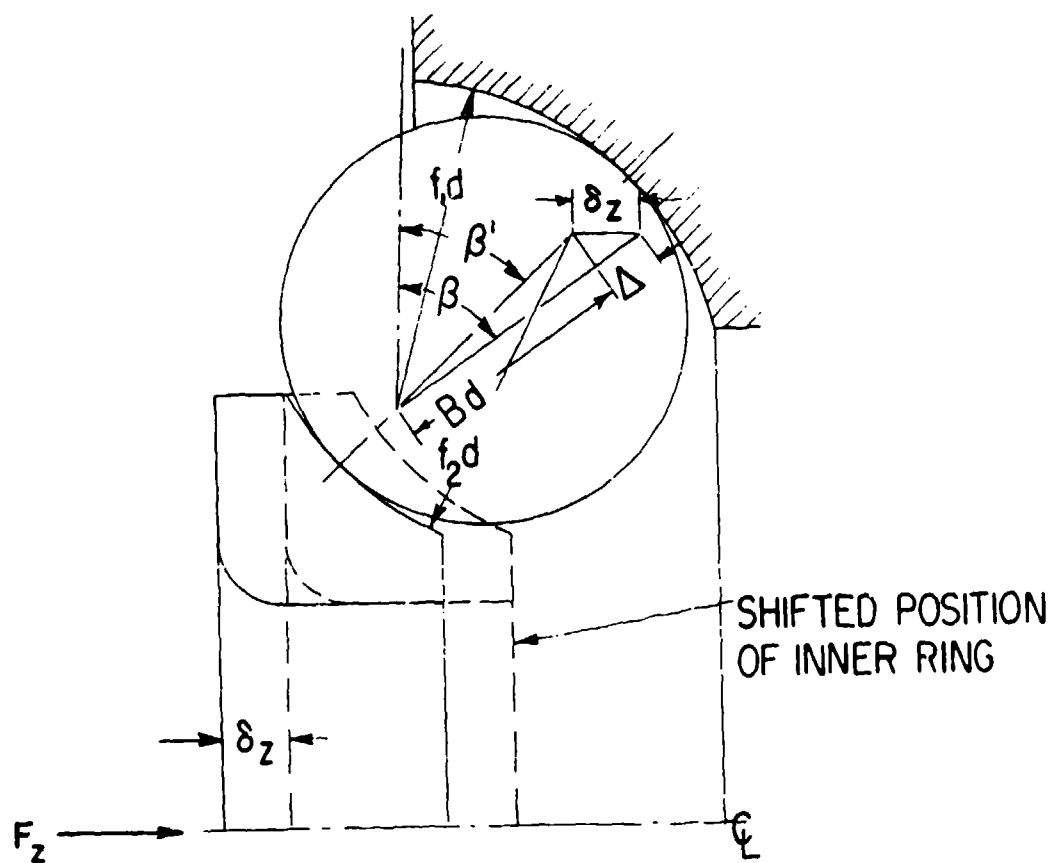


Figure 7 Angular Contact Ball Bearing Under Thrust Load

Newton-Raphson method.

The axial deflection δ_z corresponding to Δ may also be determined from Figure 7 as follows:

$$\delta_z = (Bd + \Delta) \sin\beta - Bd \sin\beta' \quad (49)$$

Substituting Δ from Equation (43) yields

$$\delta_z = \frac{Bd \sin(\beta - \beta')}{\cos\beta} \quad (50)$$

2.6.1 Approximation to Axial Stiffness

From examination of Figure 7, the following approximation can be made:

$$\Delta \approx \delta_z \sin\beta \quad (51)$$

Substituting for Δ in Equation (16) gives

$$\delta_z = \frac{1}{K^{2/3}} \frac{P^{2/3}}{\sin\beta} \quad (52)$$

Using the Jones (7) axial deflection parameter, Equation (52) becomes

$$\delta_z = \frac{B}{K_z^{2/3} d^{1/3}} \frac{P_{\max}^{2/3}}{\sin\beta} \quad (53)$$

A similar approximation was arrived at by Palmgren (6). Suggested values for axial deflection under pure axial load are:

Angular-Contact Bearings

$$\delta_z = 1.58 \times 10^{-5} \frac{P_{\max}^{2/3}}{d^{1/3} \sin\beta} \quad (54)$$

Self-Aligning Ball Bearings

$$\delta_z = 2.53 \times 10^{-5} \frac{P_{\max}^{2/3}}{d^{1/3} \sin\beta} \quad (55)$$

Thrust Ball Bearings

$$\epsilon_z = 1.9 \times 10^{-5} \frac{p_{\max}^{2/3}}{d^{1/3} \sin \beta} \quad (56)$$

The corresponding stiffnesses are:

Angular-Contact Bearings

$$\frac{K_{zz}}{d^{1/3} n^{2/3} \sin^{5/3} \beta} = 9.5 \times 10^4 F_z^{1/3} \quad (57)$$

Self-Aligning Ball Bearings

$$\frac{K_{zz}}{d^{1/3} n^{2/3} \sin^{5/3} \beta} = 5.9 \times 10^4 F_z^{1/3} \quad (58)$$

Thrust Ball Bearings

$$\frac{K_{zz}}{d^{1/3} n^{2/3} \sin^{5/3} \beta} = 7.9 \times 10^4 F_z^{1/3} \quad (59)$$

These relationships are plotted in Figure 8 for the three types of bearings.

2.7 Ball Bearings Under Combined Loading

Except for ball bearings under simple radial or thrust load, treated previously, there are very few solutions for bearing stiffness that can be evaluated by simple hand computation.

When a ball bearing operates at high speed, the body forces resulting from the ball's motion become significant and must be considered in any analysis. Figure 9 shows the forces and moments acting on the j^{th} ball in a high-speed ball bearing. The operating contact angle at the outer contact is less than that on the inner because of the body forces. Subscript 1 refers to an outer contact and subscript 2 to an inner.

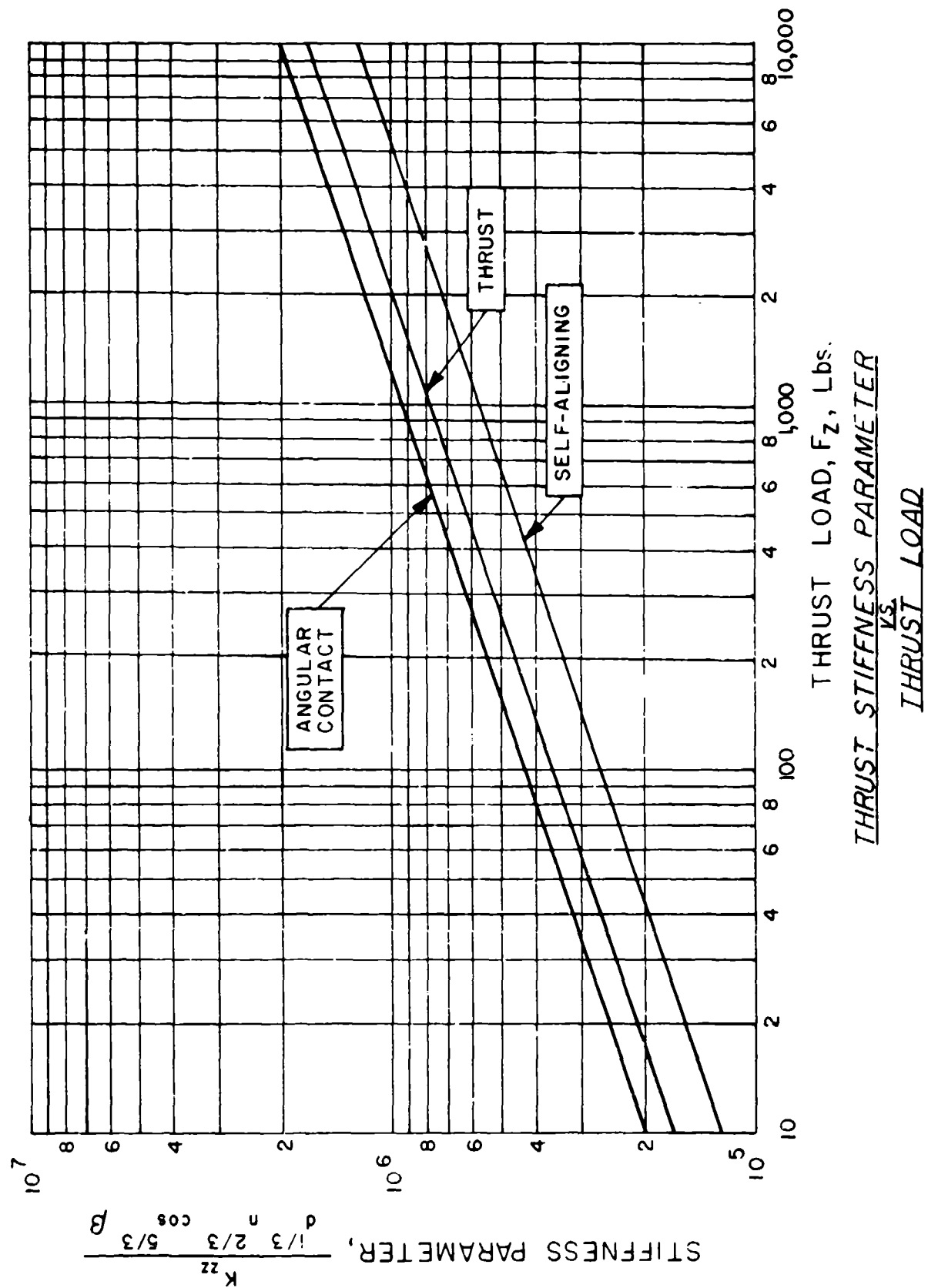


Figure 8 Thrust Stiffness Parameter vs. Thrust Load

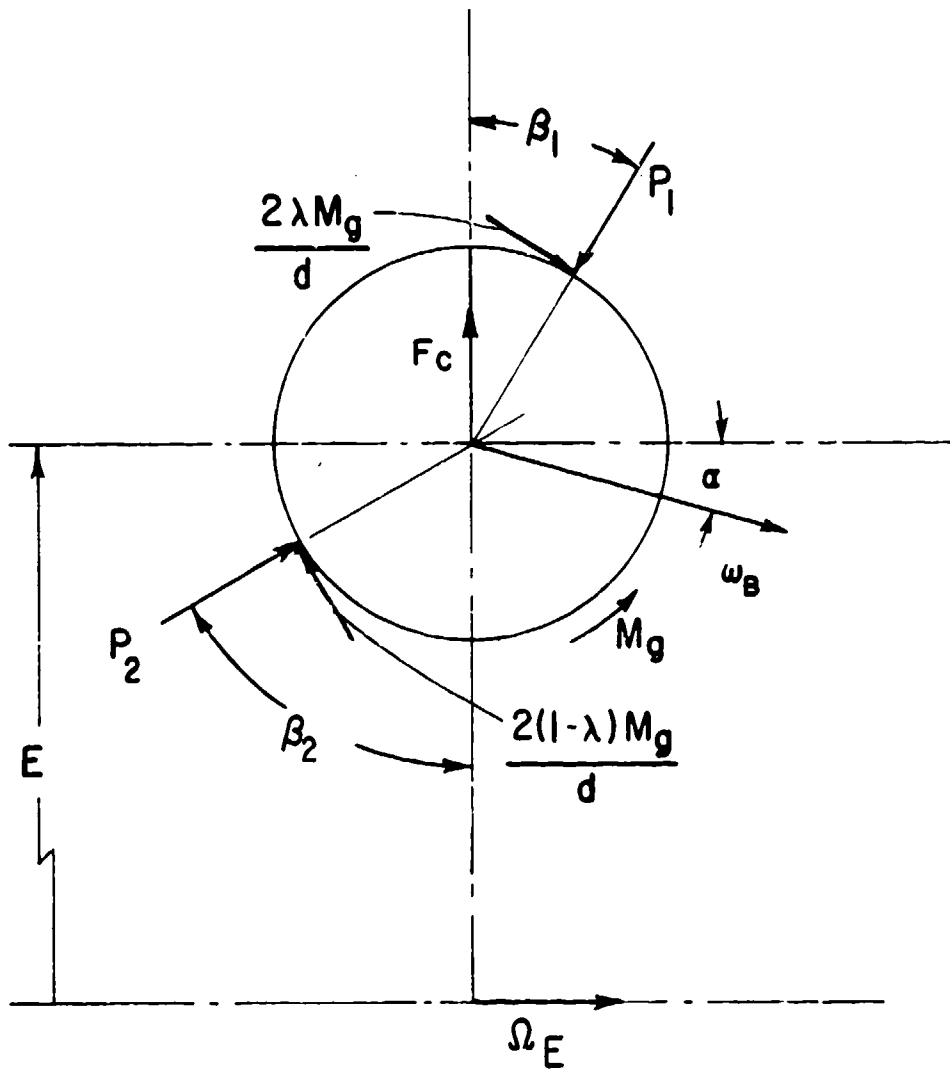


Figure 9 Ball Forces at High Speed Conditions

In Figure 10 the ball center is fixed in the plane of the paper. The ball rotates with the angular velocity ω_B directed at the angle α . The race rotates about the bearing axis with the angular velocity ω_1 , 1, 2 relative to the retainer. For the linear velocity of race and ball to be equal at a contact, the following relation must be satisfied:

$$\omega_B = \frac{C_1 E' (1 + C_1 \gamma_1) \omega_1}{d \cos(\beta_1 - \alpha)} \quad (60)$$

where $C_1 = 1$ and $C_2 = -1$

$$\gamma_1 = \frac{d \cos \beta_1}{E'} \quad (61)$$

For the 1th race to be stationary, the ball must orbit with the angular velocity Ω_E such that

$$\Omega_E = -\omega_1 \quad (62)$$

With stationary outer race and rotating inner, the actual angular velocity of the inner race is

$$\Omega_2 = \omega_2 + \Omega_E = \omega_2 - \omega_1 \quad (63)$$

For rotating outer and stationary inner, the actual angular velocity of the outer is

$$\Omega_1 = \omega_1 + \Omega_E = \omega_1 - \omega_2 \quad (64)$$

From Equation (60),

$$\frac{\omega_1}{\omega_2} = - \frac{(1 - \gamma_2) \cos(\beta_1 - \alpha)}{(1 + \gamma_1) \cos(\beta_2 - \alpha)} \quad (65)$$

considering that both races may rotate there results:

$$\omega_1 = \frac{(\Omega_1 - \Omega_2) (1 - \gamma_2) \cos(\beta_1 - \alpha)}{(1 + \gamma_1) \cos(\beta_2 - \alpha) + (1 - \gamma_2) \cos(\beta_1 - \alpha)} \quad (66)$$

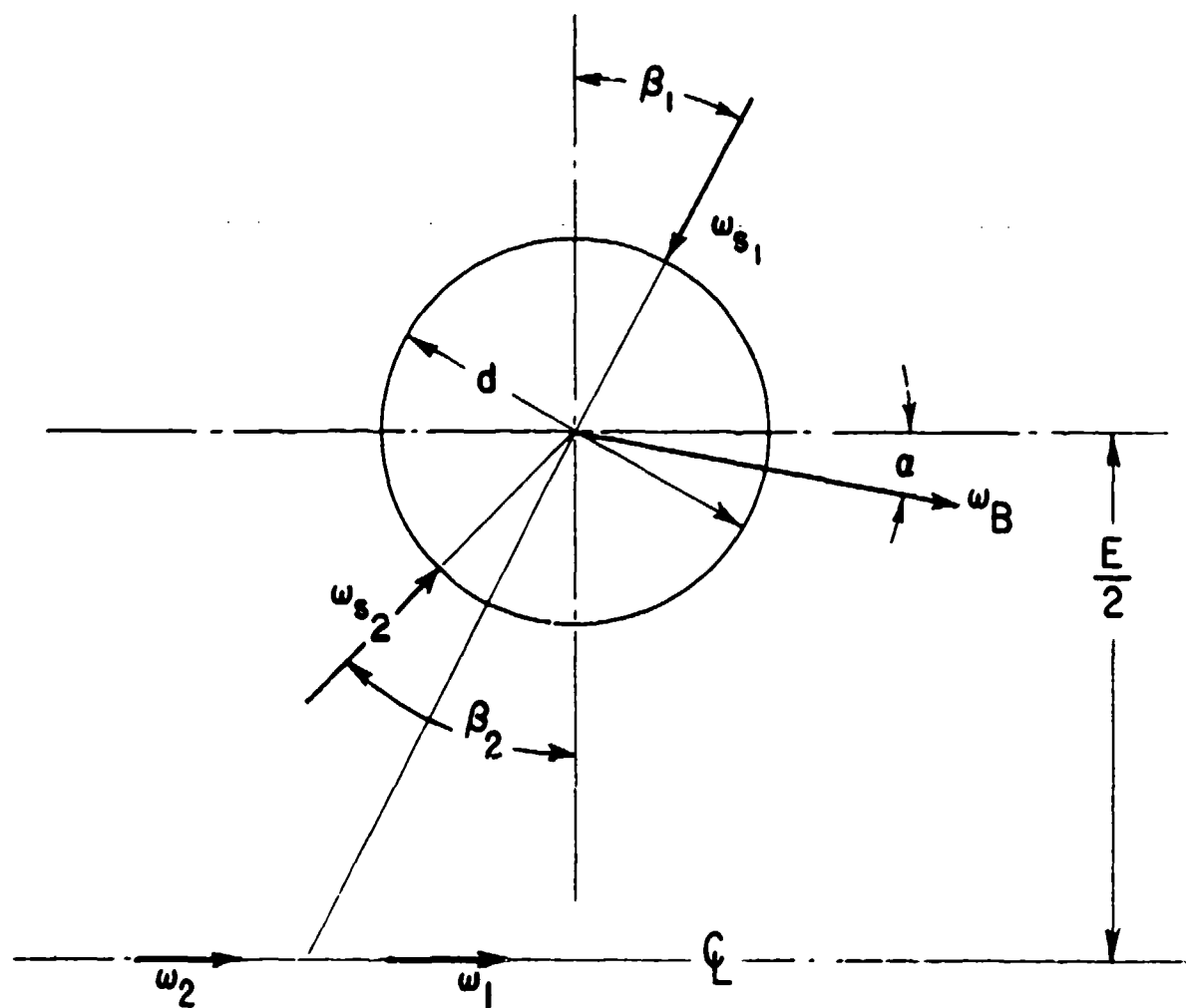


Figure 10 Ball Motion Vectors

$$\omega_2 = - \frac{(\Omega_1 - \Omega_2)(1 + \gamma_1)\cos(\beta_2 - \alpha)}{(1 + \gamma_1)\cos(\beta_2 - \alpha) + (1 - \gamma_2)\cos(\beta_1 - \alpha)} \quad (67)$$

$$\omega_B = \frac{E' \{ (\Omega_1 - \Omega_2)(1 + \gamma_1)(1 - \gamma_2) \}}{d \{ (1 + \gamma_1)\cos(\beta_2 - \alpha) + (1 - \gamma_2)\cos(\beta_1 - \alpha) \}} \quad (68)$$

$$\Omega_E = \frac{\Omega_1(1 + \gamma_1)\cos(\beta_2 - \alpha) + \Omega_2(1 - \gamma_2)\cos(\beta_1 - \alpha)}{(1 + \gamma_1)\cos(\beta_2 - \alpha) + (1 - \gamma_2)\cos(\beta_1 - \alpha)} \quad (69)$$

For an arbitrary choice of α there will be a spin of the ball relative to a race about the normal at the center of the contact area. This is illustrated in Figure 11.

From Figure 10,

$$\omega_{s_1} = C_1 \{ -\omega_1 \sin \beta_1 + \omega_B \sin(\beta_1 - \alpha) \} \quad (70)$$

The controlling race hypothesis assumes that all spin occurs at one contact while no spin occurs at the other. The contact at which no spin exists is called the controlling race. Lightly-loaded bearings may depart somewhat from this situation.

If ω_{s_1} is made zero and Equation (70) solved for α , there results for outer race control:

$$\alpha = \tan^{-1} \frac{\sin \beta_1 \cos \beta_1}{\cos^2 \beta_1 + \gamma_1} \quad (71)$$

and for inner race control:

$$\alpha = \tan^{-1} \frac{\sin \beta_2 \cos \beta_2}{\cos^2 \beta_2 - \gamma_2} \quad (72)$$

The existence of a particular type of control depends on the relative torques required to produce spin at the two contacts.

The torque required to produce spin is:

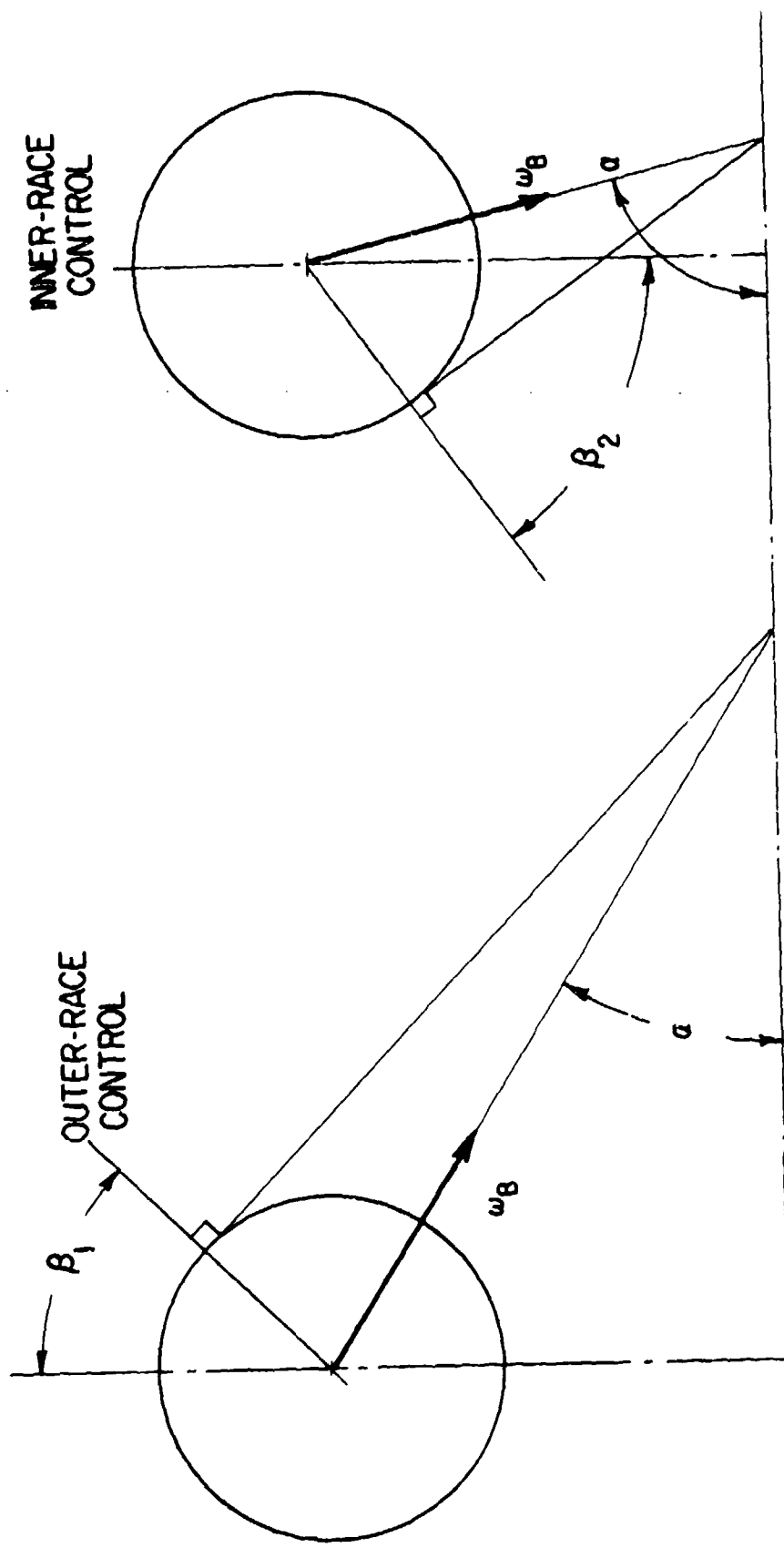


Figure 11 Illustration of Race Control Concept

$$Q = \frac{3\mu P a E(\epsilon)}{8} \quad (73)$$

- Q = spin torque, lb-in
 P = contact load, lbs.
 μ = coefficient of sliding friction
 a = semi-major axis of pressure ellipse, in.
 $E(\epsilon)$ = complete elliptic integral of the second kind
 formed with the modulus $\sin \epsilon$ where $\cos \epsilon = b/a$,
 b being the semi-minor axis of the ellipse

Figure 12 shows a ball acted on by the spin torque vectors Q_1 and Q_2 . Outer race control will exist if Q_1 projected on Q_2 is greater than Q_2 . Inner race control will exist if Q_2 projected on Q_1 is greater than Q_1 .

Figure 13 shows the relative positions of ball and race curvature centers before and after the application of the five displacements, δ_x , δ_y , δ_z , θ_x , and θ_y . The outer race curvature center is fixed. The inner race curvature center has moved to (A_1, A_2) and the ball center to (X_1, X_2) .

$$A_1 = B d \sin \beta + \delta_z + \delta_z'' + R[(\theta_x + \theta_x'') \sin \phi + (\theta_y + \theta_y'') \cos \phi] \quad (74)$$

$$A_2 = B d \cos \beta + (\delta_x + \delta_x'') \cos \phi + (\delta_y + \delta_y'') \sin \phi - \frac{P_D}{2} \quad (75)$$

$$B = f_1 + f_2 - 1 \quad (76)$$

$$R = E/2 + (f_2 - .5) d \cos \beta \quad (77)$$

- P_D = diametral clearance, in.
 δ_x'' = initial displacement along x, in.
 δ_y'' = initial displacement along y, in.
 δ_z'' = initial displacement along z, in.
 θ_x'' = initial misalignment about x, rad.
 θ_y'' = initial misalignment about y, rad.

For the forces and moments acting on the ball to be in equilibrium, the following must be satisfied:

$$P_1 \sin \beta_1 - P_2 \sin \beta_2 - \frac{2M}{d} \{ \lambda \cos \beta_1 - (1-\lambda) \cos \beta_2 \} = 0 \quad (78)$$

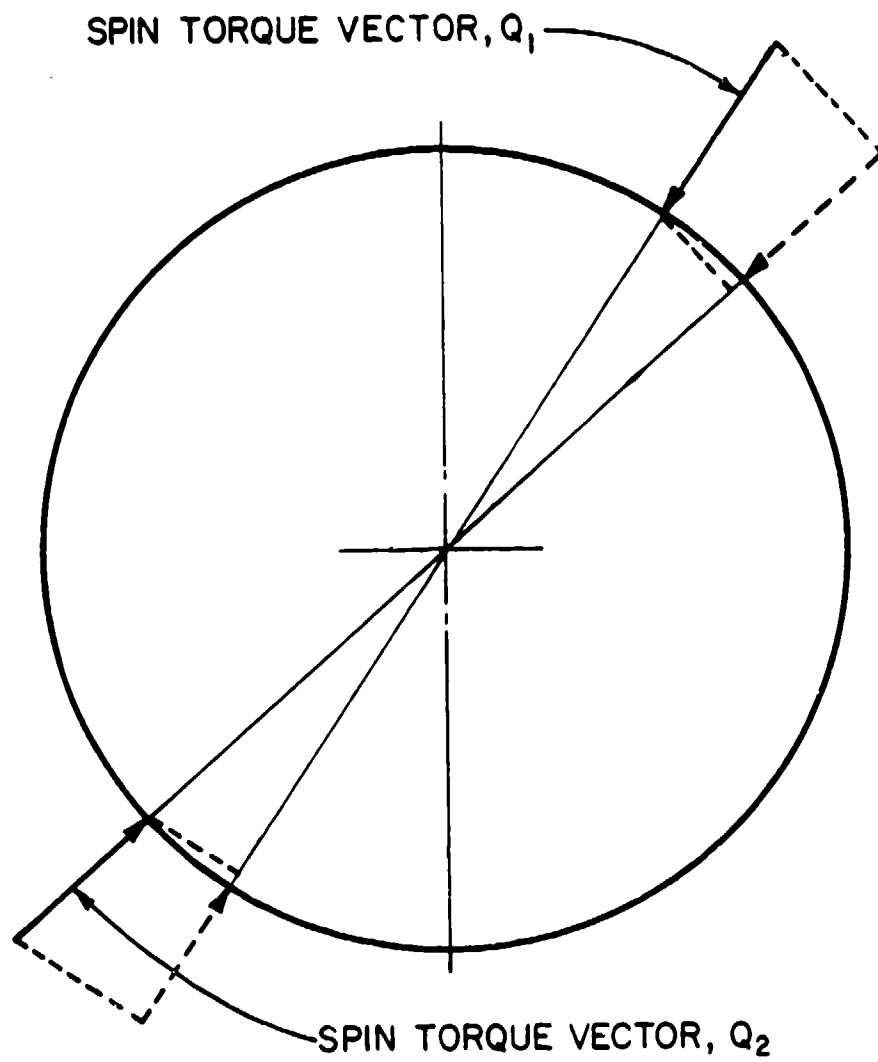


Figure 12 Spin Torque Vectors

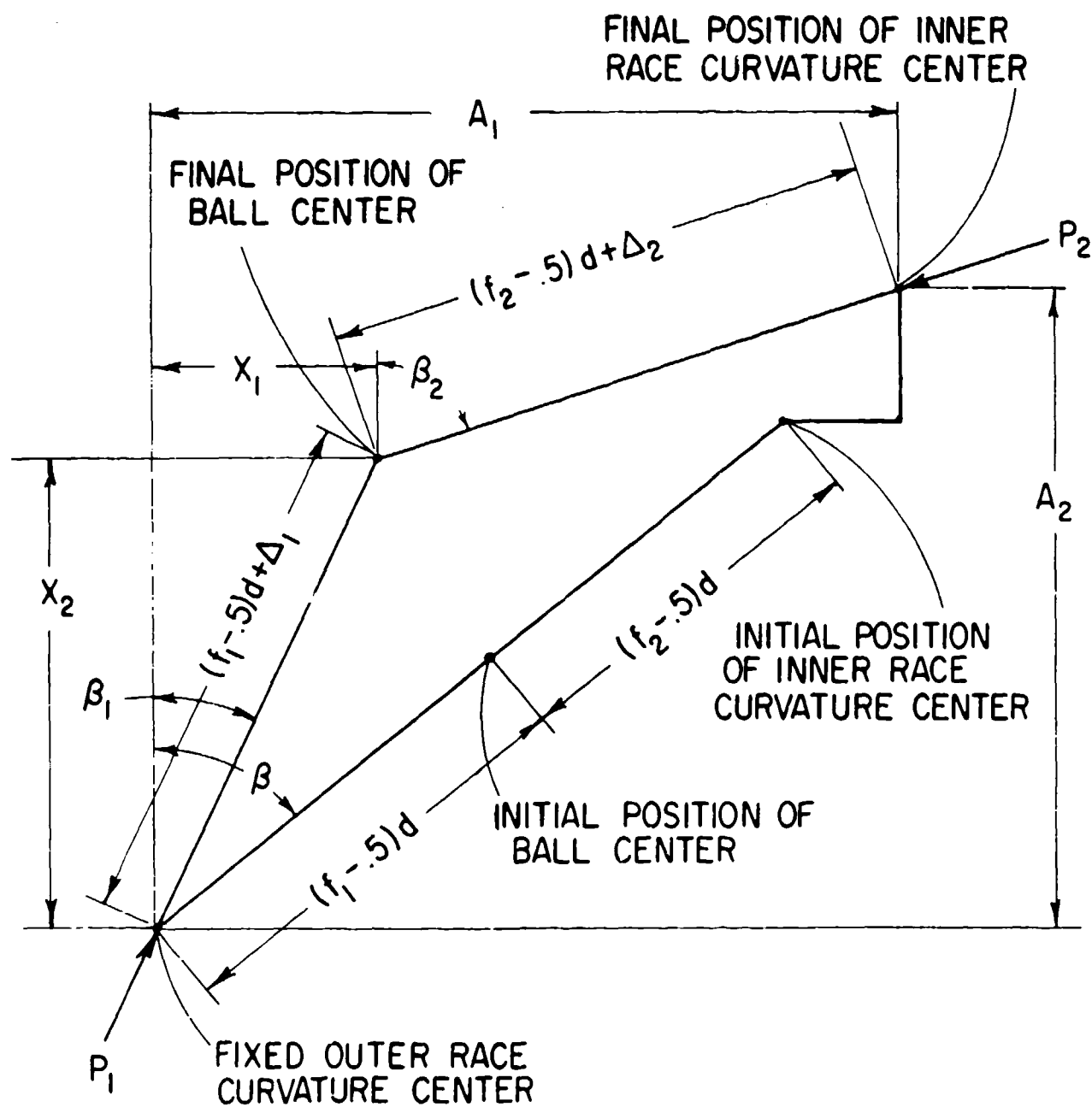


Figure 13 Race Curvature Center Deflection

$$P_1 \cos \beta_1 - P_2 \cos \beta_2 + \frac{2M}{d} \{ \lambda \sin \beta_1 - (1-\lambda) \sin \beta_2 \} - F_c = 0 \quad (79)$$

where λ is 1 for outer race control and 0 for inner race control and

$$F_c = M \Omega_E^2 E' / 2 \quad (80)$$

$$M = I_P \Omega_E \omega_B \sin \alpha \quad (81)$$

E' = Operating Pitch Diameter, in.

$$= E + 2X_2 - (f_1 - .5)d \cos \beta \quad (82)$$

From Figure 13

$$\beta_1 = \tan^{-1} \frac{X_1}{X_2} \quad (83)$$

$$\beta_2 = \tan^{-1} \frac{A_1 - X_1}{A_2 - X_2} \quad (84)$$

Δ_1 and Δ_2 are the elastic approaches of ball and raceways at the contacts

$$\Delta_1 = (X_1^2 + X_2^2)^{1/2} - (f_1 - .5)d > 0 \quad (85)$$

$$\Delta_2 = \{(A_1 - X_1)^2 + (A_2 - X_2)^2\}^{1/2} - (f_2 - .5)d > 0 \quad (86)$$

The contact loads are related to the elastic approaches through

$$P_i = K_i \Delta_i^{3/2} \quad (87)$$

where

$$K_i = \frac{11.94771}{(\eta_R + \eta_B) \cos \epsilon_1} \left[\frac{E(\epsilon_1)d}{\{K(\epsilon_1)\}^3 \{4 - 1/f_1 - \frac{2C_1 \gamma_1}{1+C_1 \gamma_1}\}} \right]^{1/2} \quad (88)$$

$$\eta_R = \frac{4(1-\nu_R^2)}{E_R} \quad (89)$$

$$\eta_B = \frac{4(1-\nu_B^2)}{E_B} \quad (90)$$

E_R and E_B are moduli of elasticity for race and ball.

ν_R and ν_B are Poisson's Ratios for race and ball.

$K(\epsilon)$ and $E(\epsilon)$ are the complete elliptic integrals of the first and second kind having the modulus $\sin \epsilon$.

$$\sin \epsilon_i = \sqrt{1 - \frac{b_i^2}{a_i^2}} \quad (91)$$

b_i and a_i are the semi-minor and semi-major axes of the pressure ellipse
 ϵ is related to the contact geometry through

$$\cot^2 \epsilon_i = \frac{(1 - \cos \tau_i) E(\epsilon_i)}{2(K(\epsilon_i) - E(\epsilon_i))} \quad (92)$$

$$\cos \tau_i = \frac{\frac{1}{f_i} - \frac{2C_i \gamma_i}{1 + C_i \gamma_i}}{4 - \frac{1}{f_i} - \frac{2C_i \gamma_i}{1 + C_i \gamma_i}} \quad (93)$$

Equations (78) and (79) are a set of nonlinear, simultaneous equations. When the relative ring displacements are held constant, the variables in Equations (78) and (79) are X_1 and X_2 . These are evaluated numerically.

The reactions of the bearing on the shaft are

$$F'_x = \sum_{i=1}^n \left[P_{2_i} \cos \beta_{2_i} + \frac{2(1-\lambda_i)M_{g_i}}{d} \sin \beta_{2_i} \right] \cos \phi_i \quad (94)$$

$$F'_y = \sum_{i=1}^n \left[P_{2_i} \cos \beta_{2_i} + \frac{2(1-\lambda_i)M_{g_i}}{d} \sin \beta_{2_i} \right] \sin \phi_i \quad (95)$$

$$F'_z = \sum_{i=1}^n \left[P_{2_i} \sin \beta_{2_i} - \frac{2(1-\lambda_i)M_{g_i}}{d} \cos \beta_{2_i} \right] \quad (96)$$

$$M'_x = \sum_{i=1}^n \left[RP_{2i} \sin \beta_{2i} - \frac{2(1-\lambda_i)M_{g_i}}{d} (R \cos \beta_{2i} - f_2 d) \right] \sin \phi_i \quad (97)$$

$$M'_y = \sum_{i=1}^n \left[RP_{2i} \sin \beta_{2i} - \frac{2(1-\lambda_i)M_{g_i}}{d} (R \cos \beta_{2i} - f_2 d) \right] \cos \phi_i \quad (98)$$

Then the equilibrium of external forces

$$\sum_{i=x,y,z} \{F'_i + F_i\} = 0 \quad (99)$$

F_x = external load along x, lbs.

F_y = external load along y, lbs.

F_z = external load along z, lbs.

Equation (99) is a set of three non-linear simultaneous equations in which the variables are δ_x , δ_y and δ_z . They are solved numerically in Appendix A using Newton-Raphson iteration.

If $\delta_{i,m-1}$ are current estimates of the three displacements, improved estimates are

$$\delta_{i,m} = \delta_{i,m-1} - [K_{ij}]^{-1} \{r_j\}_{m-1} \quad (100)$$

where $\{r_j\}_{m-1}$ are the residues of Equation (99) evaluated at current estimates of δ_i .

The elements K_{ij} of the coefficient matrix are the partial derivatives of the bearing's reactions with respect to the displacements δ_j and comprise the stiffness matrix, Equation (13).

The elements K_{ij} of the stiffness matrix, Equation (13), form the data input for the rotordynamic response computer program described in Reference 2.

SECTION III
APPLICATION OF COMPUTER PROGRAM

The analysis of Section 2.0 has been programmed in Fortran IV for a digital computer and is suitable for use on the CDC 6600. A program listing is presented in Appendix A.

3.1 Input Format

Instructions for preparation of the input cards are included as comment cards in the program listing in Appendix A. To illustrate a typical case consider the bearing in Table 1.

TABLE 1
SAMPLE CASE
DEEP GROOVE BALL BEARING

Number of Balls	=	9
Ball Diameter	=	.1875 inches
Pitch Diameter	=	.9252 inches
Contact Angle	=	0°
Outer-race Curvature	=	.530
Inner-race Curvature	=	.516
Poisson's Ratio	=	.25
Modulus of Elasticity	=	29×10^6 lb/in.
Ball Density	=	0.283 lb/in.^3
RPM of Inner Race	=	1000 rpm
RPM of Outer Race	=	0 rpm
Radial Load	=	1000 lbs.
Axial Load	=	0 lbs.

Figure 14 shows a printout of input for this case.

TEST CASE
TABLE I, PAGE 36 OF AFAPL-TR-77, PART II

DESIGN DATA FOR BEARING NO. 1

NO. OF BALLS	PITCH DIAMETER IN	INITIAL CONTACT ANGLE DESIGN DEG	RACE CURVATURE OUTER IN	INTERNAL CLEARANCE IN	MATERIAL TEMPERATURE LB/IN ²
9.2000-00	1.875-01	9.2520-01	5.3000-01	5.1600-01	0.0000
MODULUS OF ELASTICITY LB/IN ²	2.9000-07	2.9000-07	2.9000-07	2.9000-07	2.9000-07

INPUT DATA FOR RUN NO. 1 BEARING NO. 1

NO. OF BALLS	LOADS APPLIED TO SHAFT ALONG X LB	INITIAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER ALONG X IN	ABOUT Y RADIANS
9.2000	1.0000-03	0.0000	0.0000
DEFLECTION PERMITTED ALONG X ALONG Y	0.0000	0.0000	0.0000

Figure 14 Input Data for Sample Problem

3.2 Output Format

Figure 15 shows the output format for the sample case of Table 1. The output data includes the internal load distribution and various other stress and dynamic parameters.

The last section of data provides the complete stiffness matrix.

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OUTPUT DATA FOR RUN NO. 1 BEARING NO. 1														
BALL NUMBER	AXIAL DEG	OUTER LB	CONTACT LOAD LB	INNER LB	CONTACT ANGLE DEG	OUTER DEG	INNER DEG	CONTACT AREA IN	CONTACT AREA IN	CONTACT AREA IN	CONTACT AREA IN	CONTACT AREA IN	CONTACT AREA IN	CENTRICAL FORCE LB
4	1.2000-02	1.7117-02	1.7117-02	1.7117-02	0.0000	0.0000	0.0000	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	3.0412-03
5	1.6000-02	4.4102-02	4.4102-02	4.4102-02	0.0000	0.0000	0.0000	9.2317-02	9.2317-02	9.2317-02	9.2317-02	9.2317-02	9.2317-02	3.0412-03
6	2.0000-02	4.4102-02	4.4102-02	4.4102-02	0.0000	0.0000	0.0000	9.2317-02	9.2317-02	9.2317-02	9.2317-02	9.2317-02	9.2317-02	3.0412-03
7	2.0000-02	1.7117-02	1.7117-02	1.7117-02	0.0000	0.0000	0.0000	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	3.0412-03
BALL NUMBER	MEAN COMPRESSIVE STRESS PSI	OUTER PSI	INNER PSI	CONTACT DEFLECTION IN	OUTER IN	INNER IN	M/D VALUE	CONTACT AREA IN	CONTACT AREA IN	CONTACT AREA IN	CONTACT AREA IN	CONTACT AREA IN	CONTACT AREA IN	CENTRICAL FORCE LB
4	2.9188-05	2.9188-05	2.9188-05	9.2317-02	9.2317-02	9.2317-02	3.1277-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	3.0412-03
5	2.9188-05	2.9188-05	2.9188-05	9.2317-02	9.2317-02	9.2317-02	3.1277-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	3.0412-03
6	2.9188-05	2.9188-05	2.9188-05	9.2317-02	9.2317-02	9.2317-02	3.1277-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	3.0412-03
7	2.9188-05	2.9188-05	2.9188-05	9.2317-02	9.2317-02	9.2317-02	3.1277-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	6.7749-02	3.0412-03
REACTIONS OF BEARING ON SHAFT														
ALONG X	ALONG Y	ALONG Z	ABOUT X	ABOUT Y	ABOUT Z	ALONG X	ALONG Y	ALONG Z	ABOUT X	ABOUT Y	ABOUT Z	ALONG X	ABOUT X	ABOUT Y
-1.7000-03	5.3406-05	0.0000	0.0000	0.0000	0.0000	-1.0002-03	0.0000	0.0000	-1.0002-03	0.0000	0.0000	0.0000	0.0000	0.0000
PARTIAL DERIVATIVES OF REACTIONS WITH RESPECT TO DISPLACEMENTS														
DFX/DX	DFX/DY	DFX/DZ	DFX/DAX	DFX/DAY	DFX/DALX	DFX/DALY	DFX/DALZ	DFX/DALX	DFX/DALY	DFX/DALZ	DFX/DALX	DFX/DALY	DFX/DALZ	DFX/DALX
7.9779-05	-0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DFY/DX	DFY/DY	DFY/DZ	DFY/DAX	DFY/DAY	DFY/DALX	DFY/DALY	DFY/DALZ	DFY/DALX	DFY/DALY	DFY/DALZ	DFY/DALX	DFY/DALY	DFY/DALZ	DFY/DALX
-0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DFZ/DX	DFZ/DY	DFZ/DZ	DFZ/DAX	DFZ/DAY	DFZ/DALX	DFZ/DALY	DFZ/DALZ	DFZ/DALX	DFZ/DALY	DFZ/DALZ	DFZ/DALX	DFZ/DALY	DFZ/DALZ	DFZ/DALX
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DMX/DX	DMX/DY	DMX/DZ	DMX/DAX	DMX/DAY	DMX/DALX	DMX/DALY	DMX/DALZ	DMX/DALX	DMX/DALY	DMX/DALZ	DMX/DALX	DMX/DALY	DMX/DALZ	DMX/DALX
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DMY/DX	DMY/DY	DMY/DZ	DMY/DAX	DMY/DAY	DMY/DALX	DMY/DALY	DMY/DALZ	DMY/DALX	DMY/DALY	DMY/DALZ	DMY/DALX	DMY/DALY	DMY/DALZ	DMY/DALX
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

AFIN

Figure 15 Output Data for Sample Problem

SECTION IV

DESIGN DATA

Three separate sets of design charts were included in the original Part IV report. These were:

- a. Pure Radial Loaded Bearings (Deep Groove) Contact Angle
 $\beta' = 0^\circ$
- b. Pure Thrust Loaded Bearings (Deep Groove) Contact Angle
 $\beta' = 0^\circ$
- c. Angular Contact Bearings with Axial Preload and Applied
Radial Load $\beta' = 25^\circ, 15^\circ$

As a check of these design charts shows some differences between the original program prediction and the present program prediction, the design charts have been recalculated and are shown in Appendices B, C, and D.

Table 2 describes the dimensions and symbols used for the deep-grooved ball bearings. Table 3 contains information pertaining to the angular contact bearings. These bearings are identical to those studied previously in Reference 1.

4.1 Radial Stiffness Versus Radial Load

The first set of three charts (Appendix B) contains graphs of radial stiffness versus radial load. Load levels are indicated on the curves. The effects of bearing size and race curvatures are illustrated by these four charts. In general, a bearing with curvatures of $f_1 = .530$, $f_2 = .516$ is stiffer than the same bearing operating with curvatures of $f_1 = f_2 = .570$, for the same radial load. Radial stiffness is higher for a bearing with a larger bore diameter and/or a greater number of balls. Note, for pure radial load, the linear relationship between $\log K_{yy}$ and $\log F_y$. This was previously illustrated in Figure 6.

TABLE 2

DEEP GROOVE BEARINGS

Bearing Symbol	Bore (Inch)	Bore mm	O.D. (Inch)	Ball Diameter, Inch	Number of Balls	f_i	f_o
A1	.5906	15	1.2598	.1875	9	.516	.530
A2	.5906	15	1.2598	.1875	9	.570	.570
B1	.9843	25	1.8504	.250	10	.516	.530
B2	.9843	25	1.8504	.250	10	.570	.570
C1	1.378	35	2.4409	.3125	11	.516	.530
C2	1.378	35	2.4409	.3125	11	.570	.570
D1	2.1654	55	3.5433	.40625	13	.516	.530
D2	2.1654	55	3.5433	.40625	13	.570	.570
E1	2.9528	75	4.5276	.46875	15	.516	.530
E2	2.9528	75	4.5276	.46875	15	.570	.570
AA1	.5906	15	1.378	.2345	8	.516	.530
AA2	.5906	15	1.378	.2345	8	.570	.570
BB1	.9843	25	2.0472	.3125	9	.516	.530
BB2	.9843	25	2.0472	.3125	9	.570	.570
CC1	1.378	35	2.8346	.4375	9	.516	.530
CC2	1.378	35	2.8346	.4375	9	.570	.570
DD1	2.1654	55	3.937	.5625	10	.516	.530
DD2	2.1654	55	3.937	.5625	10	.570	.570
EE1	2.9528	75	5.1181	.6875	11	.516	.530
EE2	2.9528	75	5.1181	.6875	11	.570	.570

TABLE 3

ANGULAR CONTACT BEARINGS

$$\beta_o = 15^\circ, 25^\circ$$

Basic Static Load (lb)	Bearing Number	Bore (Inch)	Bore mm	O.D. (Inch)	d (in)	Number of Balls	f_i	f_o	Axial Preload (lb)		
									S.L.	M.	P.H.
630	PA 1	.5906	15	1.2598	.1875	11	.516	.530	20	50	100
	PA 2						.570	.570			
1400	PB 1	.9843	25	1.8504	.2500	13	.516	.530	50	100	200
	PB 2						.570	.570			
2600	PC 1	1.3780	35	2.4409	.3125	15	.516	.530	50	100	200
	PC 2						.570	.570			
5100	PD 1	2.1654	55	3.5433	.40625	18	.516	.530	100	200	300
	PD 2						.570	.570			
8600	PE 1	2.9528	75	4.5276	.46925	21	.516	.530	100	200	300
	PE 2						.570	.570			
760	PAA1	.5906	15	1.3780	.23425	10	.516	.530	20	50	100
	PAA2						.570	.570			
1640	PBB1	.9843	25	2.0472	.3125	12	.516	.530	50	100	200
	PBB2						.570	.570			
3750	PCC1	1.378	35	2.8346	.4375	12	.516	.530	50	100	200
	PCC2						.570	.570			
7300	PDD1	2.1654	55	3.9370	.5625	14	.516	.530	100	200	300
	PDD2						.570	.570			
12200	PEE1	2.9528	75	5.1181	.6875	16	.516	.530	100	200	300
	PEE2						.570	.570			

4.2 Axial Stiffness Versus Thrust Load

The second set of eight charts (Appendix C) contains graphs of axial stiffness and axial deflection versus axial thrust applied load. Load levels are tabulated in Table 4 for bearings undergoing a pure thrust load.

A similar observation as given above for radially loaded bearings, can be made for the thrust loaded bearing, i.e., a bearing operating with curvatures of $f_1 = .530$, $f_2 = .516$, is stiffer than the same bearing operating with curvatures of $f_1 = f_2 = .570$ for the same axial load. For all practical purposes, however, an average curve may be drawn for axial stiffness versus axial load for all bearing sizes. In particular, the bearing with the smaller bore and less balls is less stiff at light loads and more stiff at heavy loads as compared to the larger bore bearing. There is an approximate linear relationship between $\log K_{zz}$ and $\log F_z$ (also see Figure 8).

4.3 Radial Stiffness Versus Radial Load with Preload

The third set of (24) charts (Appendix D) contain graphs of radial stiffness versus radial load with preload. Load levels are indicated on the curves. The effects of bearing size, race curvatures, initial contact angle, and axial preload are illustrated by these 24 charts. For the same radial load and axial preload, a bearing operating with curvatures of $f_1 = .530$, $f_2 = .516$, is stiffer than the same bearing operating with curvatures of $f_1 = f_2 = .570$. The radial stiffness level is higher for a bearing with a larger bore diameter and/or a greater number of balls, and the smaller initial contact angle, ($\beta' = 15^\circ$). In general, the radial stiffness vs. radial load curve for an angular contact bearing is composed of three different behaving regions. One region shows the stiffness to be constant with varying radial load. (This is the light radial load region.) The middle, or moderate radial load region shows a minimum value for radial stiffness. The heavily radial loaded region shows a linear relationship between $\log K_{xx}$ and $\log F_x$. This third region is similar in behavior to that of the characteristics of a pure radial loaded deep grooved bearing. The basic cause for this curve having three separate regions is due to the axial preload. In region one, the axial preload has a great effect in holding the radial stiffness constant. In region two, where the applied radial

TABLE 4

AXIAL LOADED DEEP GROOVE BEARINGS

Table of approximate load values corresponding to $C/P = 5$ and $C/P = 10$ load levels.

Bearing Symbol	Load (lb)	
	$C/P = 5$	$C/P = 10$
A1	290	100
B1	600	250
C1	1100	450
D1	2250	950
E1	3650	1500
A2	70	30
B2	175	75
C2	300	100
D2	550	250
E2	950	350
AA1	380	155
BB1	800	300
CC1	1550	650
DD1	3000	1250
EE1	5050	2100
AA2	95	50
BB2	200	100
CC2	400	200
DD2	1500	350
EE2	2550	700

load becomes equal in magnitude to the axial preload the radial stiffness tends to decrease with increasing applied radial load to a minimum value. In the third region, the axial preload has little or no effect, and the angular contact bearing reflects the behavior of a pure radially loaded bearing i.e., a linear $\log K_{xx}$ versus $\log F_x$ relationship.

Thus another point one is led to observe is the role of axial preload magnitude on the three regions of a typical stiffness versus load curve. Three different preloads are represented in these charts and are tabulated in Table 3. These preloads are given the names selected light, moderate, and preferred heavy. The effect of increased preload is to increase the region one load range and decrease region three load range. Thus, the ultimate is a constant radial stiffness with varying radial load obtained with an infinite preload. The increased preload also has the effect of increasing the level of stiffness in regions one and two. However, it should be noted particularly that the level of stiffness in region three, for the same radial load, is the same for all preload values. This, as mentioned above, is because the axial preload effect is relieved entirely above a certain (radial load) (axial preload) ratio. (Approximately $F_x/F_z = 3$ for $\beta' = 25^\circ$ and $F_x/F_z = 4$ for $\beta' = 15^\circ$.)

In general, the light and extra light deep grooved ball bearings examined here will have a radial stiffness ranging from 10^5 to 2×10^6 for radial loads of from 10 to 2,000 lbs. The angular contact bearings will have radial stiffness values from 2×10^5 to 2×10^6 for radial loads of from 10 to 2,000 lbs. The deep grooved ball bearings will have an axial stiffness per bearing of from 2×10^4 to 4×10^6 for thrust loads of from 10 to 10^4 lbs. As in the case of the preloaded radial bearing, preloading will increase these values of axial stiffness.

APPENDIX A

COMPUTER PROGRAM FOR CALCULATING THE
STIFFNESS MATRIX OF A BALL BEARING

```

MAIN PROGRAM
CARD 1
1 1-10 ITEM NUMBER OF BALLS
1 1-10 BALL DIAMETER - IN
1 1-30 PITCH DIAMETER - IN
1 1-40 CONTACT ANGLE - DEGREES
1 1-50 OUTER-RACE CURVATURE FACTOR (AS .52)
1 1-60 INNER-RACE CURVATURE FACTOR (AS .52)
1 1-70 IF CONTACT ANGLE IS ZERO THIS ITEM IS THE TOTAL MOUNTED
DIA METRAL CLEARANCE FROM WHICH THE MOUNTED IS POSITIVE
ANGLE WILL BE CALCULATED. IF CONTACT ANGLE IS POSITIVE
IT IS THE INCREMENT IN INTERNAL CLEARANCE FROM WHICH THE
MOUNTED CONTACT ANGLE WILL BE CALCULATED. AN INCREASE IN
CLEARANCE IS POSITIVE. A REDUCTION IN CLEARANCE IS
NEGATIVE.
PUNCH 1 (NO DECIMAL POINT)
2 1-80 TITLE CARD - PUNCH ANYTHING.
3 1 LEAVE BLANK.
4 1-80 SECOND TITLE CARD - PUNCH ANYTHING. IF BLANK PROGRAM ASSUMES
1-10 POISSON'S RATIO FOR OUTER RING. IF BLANK PROGRAM ASSUMES
1-20 POISSON'S RATIO FOR INNER RING. IF BLANK PROGRAM ASSUMES
1-30 POISSON'S RATIO FOR BALLS. IF BLANK PROGRAM ASSUMES .25
1-40 MODULUS OF ELASTICITY FOR OUTER RING. IF BLANK PROGRAM
ASSUMES 29.E6
1-50 MODULUS OF ELASTICITY FOR INNER RING. IF BLANK PROGRAM
ASSUMES 29.E6
1-60 MODULUS OF ELASTICITY FOR BALLS. IF BLANK PROGRAM ASSUMES
29.E6
1-70 BALL DENSITY - LB/IN**3. IF BLANK PROGRAM ASSUMES .255
1-80 IF INNER-RACE CONTROL IS SUSPECTED PUNCH 2. HERE. IF
BLANK PROGRAM ASSUMES 1. CORRESPONDING TO OUTER-RACE
CONTROL. IF ASSUMPTION IS INCORRECT PROGRAM WILL CORRECT
IT.
5 1-10 RPM OF OUTER RING
1-20 RPM OF INNER RING
1-30 LOAD APPLIED TO INNER RING ALONG X - LB. MUST BE
1-40 LOAD APPLIED TO INNER RING ALONG Z - LB. MUST BE
NEGATIVE
1-50 INITIAL DISPLACEMENT OF INNER RING ALONG X - IN
1-60 INITIAL DISPLACEMENT OF INNER RING ALONG Y - IN
1-70 INITIAL DISPLACEMENT OF INNER RING ALONG Z - IN.
A POSITIVE VALUE LOADS THE BEARING AL THRUST
1-80 INITIAL DISPLACEMENT OF INNER ABOUT X - RADIANS
1-90 INITIAL DISPLACEMENT OF INNER ABOUT Y - RADIANS
1-10 INITIAL DISPLACEMENT OF INNER ABOUT Z - RADIANS
1-20 A 1. HERE PERMITS DEFLECTION ALONG X
1-30 A 1. HERE PERMITS DEFLECTION ALONG Y
1-40 A 1. HERE PERMITS DEFLECTION ALONG Z
6 1-10 IF INITIAL DISPLACEMENTS EXIST ALONG X, Y OR Z
DEFLECTIONS IN THOSE MODES DUE TO EXTERNAL LOADS ARE
NOT

```

UUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUUU

IF EXTERNAL LOADS ALONG X AND/OR Z EXIST DEFLECTIONS IN THOSE MODES ARE PERMITTED

DO NOT ENTER BOTH INITIAL DISPLACEMENT AND EXTERNAL LOAD FOR THE SAME HOLE

TO RUN ADDITIONAL LOADS WITH GAME BEARING REPEAT CARDS 5 AND 6 DIRECTLY AFTER LAST CARD A

TO RUN AN ADDITIONAL BEARING PLACE TWO BLANKS AFTER LAST CARD 6 AND DEAD CARDS 1 ET CERO.

PLACE THREE PLANS AFTER LAST CARD 6 IN STOP

```

COMMO: 1(3,3),AMAJ(2,60),B(2),BFA,RD,RET,BELM,RET,UT(2,60),UMI
1N(2,60),RELV(60),C(2),CRV(2),CUSE,COR(3),CH(2),CSEP(2),CHAL(2),C
ONENS,DEL(3),DEL(3),CTV(5,5),CUX(2,2),DEL(2),LDELX(2,2),DANAG(2),
DCCX(2,2),DPA(2,2),DALY(2),UCX(2),DGYRO(2),DPSIX(2,2),DUELA(2,2),
DCCA(2,2),DPA(2,2),DGY(2),UPA(2,2),DPSIA(2),DAA(2),E,EL(2),ER(3),
DEREV(60),PSAVE(2),F(2),FRFE(3),G2(2),DPSIA(2),H(2),HH(2),H1(3),H2(2),
DEHON(2),H7,ILAD,ILUI,ICTL(60),IC,JPASS,N,NOL,ON,NZ,PU,PR(2),PU,
720M,DALF(60),P(2,60),X(2),PSI(2),K,RPV(2),ROT(2),SIN3,STR5(2),SU
6X),XALSS,IN,XF(3),SUAH(2),TSAVE(2),TRO(2),VI(2),V8(2),WYIH(2),
COMMO: DUV(2,5),DEX(2),YR(2),YAL(60),TOL(5)
DIMENSION AA(3,3),CORO(3),ERP(3),WOPD(2)
DATA WORD(1),WORD(2)/4H OPT,4H INN/
TOL(1)=1,
TOL(2)=TOL(1)
TOL(3)=TOL(2)
TOL(4)=TOL(3)
TOL(5)=TOL(4)
C(1)=1,
C(2)=1,
DAD(1,1)=1,
DAD(1,4)=6,
DAD(1,5)=8,
DAD(2,1)=0,
H(2)=0,
HH(1)=0,
H1(1)=1,
H1(2)=0,
H2(1)=0,
H2(2)=1,
DAD(2,4)=0,
DAD(2,5)=0,
CEA(1)=0,
DEX(2)=2,
DPA(1,1)=0,
DPA(1,2)=0,

```

```

10 DGA(1,1)=0.
20 DGA(1,2)=0.
   DDELA(1,1)=0.
   DDELA(1,2)=0.
   IPR=0
   READ(5,20)XN,0,E,BETA,F(1),F(2),PD
   FORMAT(8E10,0)
   IF(XN.EQ.0.)STOP
   NEXM
   ISTOP=C
   READ(5,30)
   FORMAT(1H,79H
1
2
30
35
40
50
60
70
75
80
90
100
110
120
130
140
   WRITE(6,30)
   READ(5,20)PR(1),PR(2),PR,PR(1),PR(2),YF,DFNS,XIC
   IF(XIC.EQ.0.)XIC=1.
   DO 35 J=1,N
   ICTL(J)=XIC
   ILOADED
   IPR=IPR+1
   RR=(F(1)+F(2)-1.)*D
   DO 40 K=1,2
   CRV(K)=(F(K)-.5)*D
   RET=BETA/57.295780
   TEMPE=CUS(RET)
   A1=2.*LD*(1.-TEMP)
   PD=0.
   PDME=A1+PD
   IF(PDM-2.*PD)70,50,50
   WRITE(6,60)
   FORMAT(47H THIS BEARING HAS TOO MUCH INTERNAL LOOSENESS)
   ISTOP=1
   GO TO 75
   IF(PDM)75,75,80
   RET=0.
   SIN=0.
   COS=1.
   PD=PD
   GO TO 90
   RET=ACOS((2.*BD-PDM)/(2.*BD))
   SIN=SIN(RET)
   COS=COS(RET)
   RET=RET*57.29578
   DO 140 K=1,2
   IF(YP(K))110,110,120
   YP(K)=29.56
   IF(PR(K))130,130,140
   PR(K)=.25
   CONTINUE
   RE=.5*E+CRV(2)*COSB
   IF(IPR)150,150,160

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[illegible]

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310 310 X=1.2
IF (FREQ(K).GT.D.) KKK=KK+KK**2
CONTINUE
DEL(1)=0.
DEL(2)=0.
DEL(3)=0.
IF (AF(1,1).F.O.) DFL(1)=.095*D/(1.+SINR)
TEMP=.054/COSR
IF (AF(2,1).F.O.) DFL(2)=-SIGN(TEMP,XF(2))
DO 330 IF K=1,20
DEL(1)=0
DO 320 X=.95
XF(1(K))=0.
DO 320 L=1,5
DTV(L,K)=0.
DO 350 J=1,4
XDEJ
PHI=6.283185*(XJ-1.)/XW
PIE(J)=PI
SHESTG(PH)
CPHESOS(PH)
JPASSEN
CALL BALL(SPH,CPH,PDX)
IF (ICOUNT) GO TO 360,330
K2=340
330 IF (0.350) K7,ITER,J,(DFL(K),K=1,3)
FORMAT(13) MAIN PROGRAM,374,1P3F12.4)
GO TO 375
360 CONTINUE
IF (KKK-1) 350,370,380
COR1=(AF1(1)+VF(1))/DTV(1,1)
DEL(1)=DEL(1)-COR1
IF (ABS(CO(1))-TOL(3)) 550,530,530
IF (KKK.GT.4) GO TO 390
FC2=XF1(2)+XF(2)
COR2=FE2/TV(2,2)
DEL(2)=DEL(2)-COR2
IF (ABS(CO(2))-TOL(4)) 550,530,530
IF (KKK.GT.5) GO TO 410
F2=XF1(1)+XF(1)
F2=XF1(2)+XF(2)
DET=DTV(1,1)*DTV(2,2)-DTV(2,1)*DTV(1,2)
COR1=(LRI+DTV(2,2))-ERC*DTV(1,2))/DET
COR2=(LTV(1,1)*ER2-DTV(2,1)*ER1)/DET
DEL(1)=DEL(1)-COR1
DEL(2)=DEL(2)-COR2
IF (ABS(CO(1))-TOL(3)) 400,530,530
IF (ABS(CO(2))-TOL(4)) 550,530,530
IF (KKK.GT.9) GO TO 420
COR3=XF1(1)/DTV(3,3)
DEL(3)=DEL(3)-COR3
IF (ABS(CO(3))-TOL(5)) 550,530,530

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[illegible]

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IF (P(2,J).LE.0.)GO TO 590
PHI=PHI(J)*57.29578
WRITE(6,580)J,PHI,P(1,J),P(2,J),RT(1,J),RT(2,J),AMAJ(1,J),AMAJ(2,J)
1)BMIN(1,J),BMIN(2,J),XCX(J)
FORMAT(17,5X,1P10E12.4)
CONTINUE
WRITE(6,600)
FORMAT(132H0 BALL MFAN COMPRESSIVE STRESS' CONTACT DEFLECT
1)ION H/D VALUE TYPE OF ROTATIONAL ORBITAL
2)GYMOSCOPIC/130H NUMBER OUTER CONTROL INNER VELOCITY
3)INNER MOMENT/17X,3HPSI,0X,3HPSI,10X,2HIN,10X,2HIN,45X,3HHRPM,9X,3
4)ITY SHRPM,8X,5HLS*IN)
5)NO 030 J,1,IN
IF (P(2,J).LE.0.)GO TO 630
WRDE=WORD(1)
IF (1CTL(J).EQ.2)WRDE=WORD(2)
NO 010 K=1,2
STRS(K)=1.273240*P(K,J)/(AMAJ(K,J)*8MIN(K,J))
TFMP=ASIN(.5*AMAJ(K,J)/(F(K)*D))AT(K,J)/57.29578
HQQ(K)=F(K)*(1-COS(TFMP))
WRITE(6,620)J,STRS(K),STRS(2),XDEL(1,J),XDEL(2,J),HUU(1),HUU(2),
1)WDD,REV(J),EPEV(J),XGYRO(J)
FORMAT(17,5X,1P6E12.4,3X,4,2HEP,3X,1P5E12.4)
CONTINUE
WRITE(6,640)
FORMAT(1H0,15X,29HREACTIONS OF BEARING ON SHAFT,21X,49HOTAL DISPL
1)ACEMENT OF INNER WITH RESPECT TO OUTER/119H ALONG X ALONG Y AL
2)ONG 2 ABOUT Y ABOUT Y/119H L2 L3 L4 L5 L6 L7 L8 L9 L10 L11 L12 L13 L14 L15 L16 L17 L18 L19 L20 L21 L22 L23 L24 L25 L26 L27 L28 L29 L30 L31 L32 L33 L34 L35 L36 L37 L38 L39 L40 L41 L42 L43 L44 L45 L46 L47 L48 L49 L50 L51 L52 L53 L54 L55 L56 L57 L58 L59 L60 L61 L62 L63 L64 L65 L66 L67 L68 L69 L70 L71 L72 L73 L74 L75 L76 L77 L78 L79 L80 L81 L82 L83 L84 L85 L86 L87 L88 L89 L90 L91 L92 L93 L94 L95 L96 L97 L98 L99 L100 L101 L102 L103 L104 L105 L106 L107 L108 L109 L110 L111 L112 L113 L114 L115 L116 L117 L118 L119 L120 L121 L122 L123 L124 L125 L126 L127 L128 L129 L130 L131 L132 L133 L134 L135 L136 L137 L138 L139 L140 L141 L142 L143 L144 L145 L146 L147 L148 L149 L150 L151 L152 L153 L154 L155 L156 L157 L158 L159 L160 L161 L162 L163 L164 L165 L166 L167 L168 L169 L170 L171 L172 L173 L174 L175 L176 L177 L178 L179 L180 L181 L182 L183 L184 L185 L186 L187 L188 L189 L190 L191 L192 L193 L194 L195 L196 L197 L198 L199 L200 L201 L202 L203 L204 L205 L206 L207 L208 L209 L210 L211 L212 L213 L214 L215 L216 L217 L218 L219 L220 L221 L222 L223 L224 L225 L226 L227 L228 L229 L230 L231 L232 L233 L234 L235 L236 L237 L238 L239 L240 L241 L242 L243 L244 L245 L246 L247 L248 L249 L250 L251 L252 L253 L254 L255 L256 L257 L258 L259 L260 L261 L262 L263 L264 L265 L266 L267 L268 L269 L270 L271 L272 L273 L274 L275 L276 L277 L278 L279 L280 L281 L282 L283 L284 L285 L286 L287 L288 L289 L290 L291 L292 L293 L294 L295 L296 L297 L298 L299 L300 L301 L302 L303 L304 L305 L306 L307 L308 L309 L310 L311 L312 L313 L314 L315 L316 L317 L318 L319 L320 L321 L322 L323 L324 L325 L326 L327 L328 L329 L330 L331 L332 L333 L334 L335 L336 L337 L338 L339 L340 L341 L342 L343 L344 L345 L346 L347 L348 L349 L350 L351 L352 L353 L354 L355 L356 L357 L358 L359 L360 L361 L362 L363 L364 L365 L366 L367 L368 L369 L370 L371 L372 L373 L374 L375 L376 L377 L378 L379 L380 L381 L382 L383 L384 L385 L386 L387 L388 L389 L390 L391 L392 L393 L394 L395 L396 L397 L398 L399 L400 L401 L402 L403 L404 L405 L406 L407 L408 L409 L410 L411 L412 L413 L414 L415 L416 L417 L418 L419 L420 L421 L422 L423 L424 L425 L426 L427 L428 L429 L430 L431 L432 L433 L434 L435 L436 L437 L438 L439 L440 L441 L442 L443 L444 L445 L446 L447 L448 L449 L450 L451 L452 L453 L454 L455 L456 L457 L458 L459 L460 L461 L462 L463 L464 L465 L466 L467 L468 L469 L470 L471 L472 L473 L474 L475 L476 L477 L478 L479 L480 L481 L482 L483 L484 L485 L486 L487 L488 L489 L490 L491 L492 L493 L494 L495 L496 L497 L498 L499 L500 L501 L502 L503 L504 L505 L506 L507 L508 L509 L510 L511 L512 L513 L514 L515 L516 L517 L518 L519 L520 L521 L522 L523 L524 L525 L526 L527 L528 L529 L530 L531 L532 L533 L534 L535 L536 L537 L538 L539 L540 L541 L542 L543 L544 L545 L546 L547 L548 L549 L550 L551 L552 L553 L554 L555 L556 L557 L558 L559 L560 L561 L562 L563 L564 L565 L566 L567 L568 L569 L570 L571 L572 L573 L574 L575 L576 L577 L578 L579 L580 L581 L582 L583 L584 L585 L586 L587 L588 L589 L590 L591 L592 L593 L594 L595 L596 L597 L598 L599 L600 L601 L602 L603 L604 L605 L606 L607 L608 L609 L610 L611 L612 L613 L614 L615 L616 L617 L618 L619 L620 L621 L622 L623 L624 L625 L626 L627 L628 L629 L630 L631 L632 L633 L634 L635 L636 L637 L638 L639 L640 L641 L642 L643 L644 L645 L646 L647 L648 L649 L650 L651 L652 L653 L654 L655 L656 L657 L658 L659 L660 L661 L662 L663 L664 L665 L666 L667 L668 L669 L670 L671 L672 L673 L674 L675 L676 L677 L678 L679 L680 L681 L682 L683 L684 L685 L686 L687 L688 L689 L690 L691 L692 L693 L694 L695 L696 L697 L698 L699 L700 L701 L702 L703 L704 L705 L706 L707 L708 L709 L710 L711 L712 L713 L714 L715 L716 L717 L718 L719 L720 L721 L722 L723 L724 L725 L726 L727 L728 L729 L730 L731 L732 L733 L734 L735 L736 L737 L738 L739 L740 L741 L742 L743 L744 L745 L746 L747 L748 L749 L750 L751 L752 L753 L754 L755 L756 L757 L758 L759 L760 L761 L762 L763 L764 L765 L766 L767 L768 L769 L770 L771 L772 L773 L774 L775 L776 L777 L778 L779 L780 L781 L782 L783 L784 L785 L786 L787 L788 L789 L790 L791 L792 L793 L794 L795 L796 L797 L798 L799 L800 L801 L802 L803 L804 L805 L806 L807 L808 L809 L810 L811 L812 L813 L814 L815 L816 L817 L818 L819 L820 L821 L822 L823 L824 L825 L826 L827 L828 L829 L830 L831 L832 L833 L834 L835 L836 L837 L838 L839 L840 L841 L842 L843 L844 L845 L846 L847 L848 L849 L850 L851 L852 L853 L854 L855 L856 L857 L858 L859 L860 L861 L862 L863 L864 L865 L866 L867 L868 L869 L870 L871 L872 L873 L874 L875 L876 L877 L878 L879 L880 L881 L882 L883 L884 L885 L886 L887 L888 L889 L890 L891 L892 L893 L894 L895 L896 L897 L898 L899 L900 L901 L902 L903 L904 L905 L906 L907 L908 L909 L910 L911 L912 L913 L914 L915 L916 L917 L918 L919 L920 L921 L922 L923 L924 L925 L926 L927 L928 L929 L930 L931 L932 L933 L934 L935 L936 L937 L938 L939 L940 L941 L942 L943 L944 L945 L946 L947 L948 L949 L950 L951 L952 L953 L954 L955 L956 L957 L958 L959 L960 L961 L962 L963 L964 L965 L966 L967 L968 L969 L970 L971 L972 L973 L974 L975 L976 L977 L978 L979 L980 L981 L982 L983 L984 L985 L986 L987 L988 L989 L990 L991 L992 L993 L994 L995 L996 L997 L998 L999 L1000 L1001 L1002 L1003 L1004 L1005 L1006 L1007 L1008 L1009 L1010 L1011 L1012 L1013 L1014 L1015 L1016 L1017 L1018 L1019 L1020 L1021 L1022 L1023 L1024 L1025 L1026 L1027 L1028 L1029 L1030 L1031 L1032 L1033 L1034 L1035 L1036 L1037 L1038 L1039 L1040 L1041 L1042 L1043 L1044 L1045 L1046 L1047 L1048 L1049 L1050 L1051 L1052 L1053 L1054 L1055 L1056 L1057 L1058 L1059 L1060 L1061 L1062 L1063 L1064 L1065 L1066 L1067 L1068 L1069 L1070 L1071 L1072 L1073 L1074 L1075 L1076 L1077 L1078 L1079 L1080 L1081 L1082 L1083 L1084 L1085 L1086 L1087 L1088 L1089 L1090 L1091 L1092 L1093 L1094 L1095 L1096 L1097 L1098 L1099 L1100 L1101 L1102 L1103 L1104 L1105 L1106 L1107 L1108 L1109 L1110 L1111 L1112 L1113 L1114 L1115 L1116 L1117 L1118 L1119 L1120 L1121 L1122 L1123 L1124 L1125 L1126 L1127 L1128 L1129 L1130 L1131 L1132 L1133 L1134 L1135 L1136 L1137 L1138 L1139 L1140 L1141 L1142 L1143 L1144 L1145 L1146 L1147 L1148 L1149 L1150 L1151 L1152 L1153 L1154 L1155 L1156 L1157 L1158 L1159 L1160 L1161 L1162 L1163 L1164 L1165 L1166 L1167 L1168 L1169 L1170 L1171 L1172 L1173 L1174 L1175 L1176 L1177 L1178 L1179 L1180 L1181 L1182 L1183 L1184 L1185 L1186 L1187 L1188 L1189 L1190 L1191 L1192 L1193 L1194 L1195 L1196 L1197 L1198 L1199 L1200 L1201 L1202 L1203 L1204 L1205 L1206 L1207 L1208 L1209 L1210 L1211 L1212 L1213 L1214 L1215 L1216 L1217 L1218 L1219 L1220 L1221 L1222 L1223 L1224 L1225 L1226 L1227 L1228 L1229 L1230 L1231 L1232 L1233 L1234 L1235 L1236 L1237 L1238 L1239 L1240 L1241 L1242 L1243 L1244 L1245 L1246 L1247 L1248 L1249 L1250 L1251 L1252 L1253 L1254 L1255 L1256 L1257 L1258 L1259 L1260 L1261 L1262 L1263 L1264 L1265 L1266 L1267 L1268 L1269 L1270 L1271 L1272 L1273 L1274 L1275 L1276 L1277 L1278 L1279 L1280 L1281 L1282 L1283 L1284 L1285 L1286 L1287 L1288 L1289 L1290 L1291 L1292 L1293 L1294 L1295 L1296 L1297 L1298 L1299 L1300 L1301 L1302 L1303 L1304 L1305 L1306 L1307 L1308 L1309 L1310 L1311 L1312 L1313 L1314 L1315 L1316 L1317 L1318 L1319 L1320 L1321 L1322 L1323 L1324 L1325 L1326 L1327 L1328 L1329 L1330 L1331 L1332 L1333 L1334 L1335 L1336 L1337 L1338 L1339 L1340 L1341 L1342 L1343 L1344 L1345 L1346 L1347 L1348 L1349 L1350 L1351 L1352 L1353 L1354 L1355 L1356 L1357 L1358 L1359 L1360 L1361 L1362 L1363 L1364 L1365 L1366 L1367 L1368 L1369 L1370 L1371 L1372 L1373 L1374 L1375 L1376 L1377 L1378 L1379 L1380 L1381 L1382 L1383 L1384 L1385 L1386 L1387 L1388 L1389 L1390 L1391 L1392 L1393 L1394 L1395 L1396 L1397 L1398 L1399 L1400 L1401 L1402 L1403 L1404 L1405 L1406 L1407 L1408 L1409 L1410 L1411 L1412 L1413 L1414 L1415 L1416 L1417 L1418 L1419 L1420 L1421 L1422 L1423 L1424 L1425 L1426 L1427 L1428 L1429 L1430 L1431 L1432 L1433 L1434 L1435 L1436 L1437 L1438 L1439 L1440 L1441 L1442 L1443 L1444 L1445 L1446 L1447 L1448 L1449 L1450 L1451 L1452 L1453 L1454 L1455 L1456 L1457 L1458 L1459 L1460 L1461 L1462 L1463 L1464 L1465 L1466 L1467 L1468 L1469 L1470 L1471 L1472 L1473 L1474 L1475 L1476 L1477 L1478 L1479 L1480 L1481 L1482 L1483 L1484 L1485 L1486 L1487 L1488 L1489 L1490 L1491 L1492 L1493 L1494 L1495 L1496 L1497 L1498 L1499 L1500 L1501 L1502 L1503 L1504 L1505 L1506 L1507 L1508 L1509 L1510 L1511 L1512 L1513 L1514 L1515 L1516 L1517 L1518 L1519 L1520 L1521 L1522 L1523 L1524 L1525 L1526 L1527 L1528 L1529 L1530 L1531 L1532 L1533 L1534 L1535 L1536 L1537 L1538 L1539 L1540 L1541 L1542 L1543 L1544 L1545 L1546 L1547 L1548 L1549 L1550 L1551 L1552 L1553 L1554 L1555 L1556 L1557 L1558 L1559 L1560 L1561 L1562 L1563 L1564 L1565 L1566 L1567 L1568 L1569 L1570 L1571 L1572 L1573 L1574 L1575 L1576 L1577 L1578 L1579 L1580 L1581 L1582 L1583 L1584 L1585 L1586 L1587 L1588 L1589 L1590 L1591 L1592 L1593 L1594 L1595 L1596 L1597 L1598 L1599 L1600 L1601 L1602 L1603 L1604 L1605 L1606 L1607 L1608 L1609 L1610 L1611 L1612 L1613 L1614 L1615 L1616 L1617 L1618 L1619 L1620 L1621 L1622 L1623 L1624 L1625 L1626 L1627 L1628 L1629 L1630 L1631 L1632 L1633 L1634 L1635 L1636 L1637 L1638 L1639 L1640 L1641 L1642 L1643 L1644 L1645 L1646 L1647 L1648 L1649 L1650 L1651 L1652 L1653 L1654 L1655 L1656 L1657 L1658 L1659 L1660 L1661 L1662 L1663 L1664 L1665 L1666 L1667 L1668 L1669 L1670 L1671 L1672 L1673 L1674 L1675 L1676 L1677 L1678 L1679 L1680 L1681 L1682 L1683 L1684 L1685 L1686 L1687 L1688 L1689 L1690 L1691 L1692 L1693 L1694 L1695 L1696 L1697 L1698 L1699 L1700 L1701 L1702 L1703 L1704 L1705 L1706 L1707 L1708 L1709 L1710 L1711 L1712 L1713 L1714 L1715 L1716 L1717 L1718 L1719 L1720 L1721 L1722 L1723 L1724 L1725 L1726 L1727 L1728 L1729 L1730 L1731 L1732 L1733 L1734 L1735 L1736 L1737 L1738 L1739 L1740 L1741 L1742 L1743 L1744 L1745 L1746 L1747 L1748 L1749 L1750 L1751 L1752 L1753 L1754 L1755 L1756 L1757 L1758 L1759 L1760 L1761 L1762 L1763 L1764 L1765 L1766 L1767 L1768 L1769 L1770 L1771 L1772 L1773 L1774 L1775 L1776 L1777 L1778 L1779 L1780 L1781 L1782 L1783 L1784 L1785 L1786 L1787 L1788 L1789 L1790 L1791 L1792 L1793 L1794 L1795 L1796 L1797 L1798 L1799 L1800 L1801 L1802 L1803 L1804 L1805 L1806 L1807 L1808 L1809 L1810 L1811 L1812 L1813 L1814 L1815 L1816 L1817 L1818 L1819 L1820 L1821 L1822 L1823 L1824 L1825 L1826 L1827 L1828 L1829 L1830 L1831 L1832 L1833 L1834 L1835 L1836 L1837 L1838 L1839 L1840 L1841 L1842 L1843 L1844 L1845 L1846 L1847 L1848 L1849 L1850 L1851 L1852 L1853 L1854 L1855 L1856 L1857 L1858 L1859 L1860 L1861 L1862 L1863 L1864 L1865 L1866 L1867 L1868 L1869 L1870 L1871 L1872 L1873 L1874 L1875 L1876 L1877 L1878 L1879 L1880 L1881 L1882 L1883 L1884 L1885 L1886 L1887 L1888 L1889 L1890 L1891 L1892 L1893 L1894 L1895 L1896 L1897 L1898 L1899 L1900 L1901 L1902 L1903 L1904 L1905 L1906 L1907 L1908 L1909 L1910 L1911 L1912 L1913 L1914 L1915 L1916 L1917 L1918 L1919 L1920 L1921 L1922 L1923 L1924 L1925 L1926 L1927 L1928 L1929 L1930 L1931 L1932 L1933 L1934 L1935 L1936 L1937 L1938 L1939 L1940 L1941 L1942 L1943 L1944 L1945 L1946 L1947 L1948 L1949 L1950 L1951 L1952 L1953 L1954 L1955 L1956 L1957 L1958 L1959 L1960 L1961 L1962 L1963 L1964 L1965 L1966 L1967 L1968 L1969 L1970 L1971 L1972 L1973 L1974 L1975 L1976 L1977 L1978 L1979 L1980 L1981 L1982 L1983 L1984 L1985 L1986 L1987 L1988 L1989 L1990 L1991 L1992 L1993 L1994 L1995 L1996 L1997 L1998 L1999 L2000 L2001 L2002 L2003 L2004 L2005 L2006 L2007 L2008 L2009 L2010 L2011 L2012 L2013 L2014 L2015 L2016 L2017 L2018 L2019 L2020 L2021 L2022 L2023 L2024 L2025 L2026 L2027 L2028 L2029 L2030 L2031 L2032 L2033 L2034 L2035 L2036 L2037 L2038 L2039 L2040 L2041 L2042 L2043 L2044 L2045 L2046 L2047 L2048 L2049 L2050 L2051 L2052 L2053 L2054 L2055 L2056 L2057 L2058 L2059 L2060 L2061 L2062 L2063 L2064 L2065 L2066 L2067 L2068 L2069 L2070 L2071 L2072 L2073 L2074 L2075 L2076 L2077 L2078 L2079 L2080 L2081 L2082 L2083 L2084 L2085 L2086 L2087 L2088 L2089 L2090 L2091 L2092 L2093 L2094 L2095 L2096 L2097 L2098 L2099 L2100 L2101 L2102 L2103 L2104 L2105 L2106 L2107 L2108 L2109 L2110 L2111 L2112 L2113 L2114 L2115 L2116 L2117 L2118 L2119 L2120 L2121 L2122 L2123 L2124 L2125 L2126 L2127 L2128 L2129 L2130 L2131 L2132 L2133 L2134 L2135 L2136 L2137 L2138 L2139 L2140 L2141 L2142 L2143 L2144 L2145 L2146 L2147 L2148 L2149 L2150 L2151 L2152 L2153 L2154 L2155 L2156 L2157 L2158 L2159 L2160 L2161 L2162 L2163 L2164 L2165 L2166 L2167 L2168 L2169 L2170 L2171 L2172 L2173 L2174 L2175 L2176 L2177 L2178 L2179 L2180 L2181 L2182 L2183 L2184 L2185 L2186 L2187 L2188 L2189 L2190 L2191 L2192 L2193 L2194 L2195 L2196 L2197 L2198 L2199 L2200 L2201 L2202 L2203 L2204 L2205 L2206 L2207 L2208 L2209 L2210 L2211 L2212 L2213 L2214 L2215 L2216 L2217 L2218 L2219 L2220 L2221 L2222 L2223 L2224 L2225 L2226 L2227 L2228 L2229 L2230 L2231 L2232 L2233 L2234 L2235 L2236 L2237 L2238 L2239 L2240 L2241 L2242 L2243 L2244 L2245 L2246 L2247 L2248 L2249 L2250 L2251 L2252 L2253 L2254 L2255 L2256 L2257 L2258 L2259 L2260 L2261 L2262 L2263 L2264 L2265 L2266 L2267 L2268 L2269 L2270 L2271 L2272 L2273 L2274 L2275 L2276 L2277 L2278 L2279 L2280 L2281 L2282 L2283 L2284 L2285 L2286 L2287 L2288 L2289 L2290 L2291 L2292 L2293 L2294 L2295 L2296 L2297 L2298 L2299 L2300 L2301 L2302 L2303 L2304 L2305 L2306 L2307 L2308 L2309 L2310 L2311 L2312 L2313 L2314 L2315 L2316 L2317 L2318 L2319 L2320 L2321 L2322 L2323 L2324 L2325 L2326 L2327 L2328 L2329 L2330 L2331 L2332 L2333 L2334 L2335 L2336 L2337 L2338 L2339 L2340 L2341 L2342 L2343 L2344 L2345 L2346 L2347 L2348 L2349 L2350 L2351 L2352 L2353 L2354 L2355 L2356 L2357 L2358 L2359 L2360 L2361 L2362 L2363 L2364 L2365 L2366 L2367 L2368 L2369 L2370 L2371 L2372 L2373 L2374 L2375 L2376 L2377 L2378 L2379 L2380 L2381 L2382 L2383 L2384 L2385 L2386 L2387 L2388 L2389 L2390 L2391 L2392 L2393 L2394 L2395 L2396 L2397 L2398 L2399 L2400 L2401 L2402 L2403 L2404 L2405 L2406 L2407 L2408 L2409 L2410 L2411 L2412 L2413 L2414 L2415 L2416 L2417 L2418 L2419 L2420 L2421 L2422 L2423 L2424 L2425 L2426 L2427 L2428 L2429 L2430 L2431 L2432 L2433 L2434 L2435 L2436 L2437 L2438 L2439 L2440 L2441 L2442 L2443 L2444 L2445 L2446 L2447 L2448 L2449 L2450 L2451 L2452 L2453 L2454 L2455 L2456 L2457 L2458 L2459 L2460 L2461 L2462 L2463 L2464 L2465 L2466 L2467 L2468 L2469 L2470 L2471 L2472 L2473 L2474 L2475 L2476 L2477 L2478 L2479 L2480 L2481 L2482 L2483 L2484 L2485 L2486 L2487 L2488 L2489 L2490 L2491 L2492 L2493 L2494 L2495 L2496 L2497 L2498 L2499 L2500 L2501 L2502 L2503 L2504 L2505 L2506 L2507 L2508 L2509 L2510 L2511 L2512 L2513 L2514 L2515 L2516 L2517 L2518 L2519 L2520 L2521 L2522 L2523 L2524 L2525 L2526 L2527 L2528 L2529 L2530 L2531 L2532 L2533 L2534 L2535 L2536 L2537 L2538 L2539 L2540 L2541 L2542 L2543 L2544 L2545 L2546 L2547 L2548 L2549 L2550 L2551 L2552 L2553 L2554 L2555 L2556 L2557 L2558 L2559 L2560 L2561 L2562 L2563 L2564 L2565 L256
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700	FORMAT(5900)	DFZ/DY	DFZ/DZ	DFZ/DALX	DFZ/D
	WRIIE(0,620)				
	WRIIE(0,260)	DTV(1,2),DTV(1,3),DTV(1,4),DTV(1,5)			
710	FORMAT(5900)	DMX/DX	DMX/DZ	DMX/DALX	DMX/D
	WRIIE(0,720)				
720	FORMAT(5900)	LB*IN/IN	LB*IN/IN	LB*IN/KAD	LB*IN/
	WRIIE(0,260)	DTV(4,2),DTV(4,3),DTV(4,4),DTV(4,5)			
730	FORMAT(5900)	DMY/DX	DMY/DZ	DMY/DALX	DMY/D
	WRIIE(0,720)				
	WRIIE(0,260)	DTV(5,2),DTV(5,3),DTV(5,4),DTV(5,5)			
	GO TO 275				
	END				
	SUBROUTINE BALL(SPH,CPH,PDX)				
	COMMON A(3,3),AMAU(2,60),B(2),BFIA,RQ,BFT,REIM,BBEL,BT(2,60),BMI				
	INC(2,60),REVE(60),C(2),CRV(2),COSR,COR(3),CB(2),CSEP(2),CBAL(2),U,				
	REFNS,DFL1(5),DFL(3),DTV(5,5),DPX(2,2),DEL(2),DFLX(2,2),DXNGG(2),				
	3DCX(2,2),DMX(2,2),DALX(2),UCX(2),DGYRQ(2),DPSIX(2,2),DDEL(2,2),				
	4DPA(2,2),DGA(2,2),DPA(2,2),DPA(2,2),DPA(2,2),DPA(2,2),DPA(2,2),				
	SEPEV(60),ESAVE(2),F(2),ERFE(3),G(2),GQC(2),H(2),H4(2),H1(2),H2(2),				
	6HOD(2),IRP,ILCAD,IQUIT,ICTL(60),IC,JPASS,N,NOLCAN,NZ,PU,PR(2),P3,				
	7PCM,PHEF(60),P(2,60),PX(2),PSI(2),R,RPY(2),ROT(2),SIGN,STRS(2),SB,				
	R2),SNEF(2),SQ(2),S8AL(2),TSAVE(2),TRO(2),V1(2),V2(2),WUMH(2),				
	9XN,AMASS,XIN,XE(3),XF1(5),XCX(60),XDEL(2,60),XGYRO(60),X(2),XK(2)				
	COMMOU,DAU(2,5),DEX(2),YP(2),XAL(60),TOL(5)				
	JEUPASS				
	DAD(1,4)=ER*SPH				
	DAD(1,5)=ER*CPH				
	DAD(2,6)=CPH				
	DAD(2,3)=SPH				
	ICNTEO				
	IC=ICTL(J)				
	A1=BD*SIGN+DFL(1)+DFL11(1)+R*(DFL11(4)*SPH+DFL11(5)*CPH)				
	A2=BD*COSR+(DFL(2)+DFL11(2))*CPH+(DFL(3)+DFL11(3))*SPH-PUX*.5				
	A1A=A1*2+A2*2				
	SQTE=SQRT(A1A2)				
	TEMP=A1				
	CVRG2=U				
	IF(ABS(A1)-1.E-6)30,30,10				
	IF(A1)20,20,50				
	IF=-1.				
	GO TO 30				
	IF(A2-BD-1.E-8)70,40,40				
	X(1)=0				
	X(2)=.55*(A2-BD)+CRV(1)				
	GO TO 190				

BALL 22

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50  TG=1.
60  IF((A2-CRV(1))**2+A1**2-CRV(2)**2)70.70.80
70  NOLoad=NOLoad+1
    P(2,J)=0.
    RETURN
80  TR=ABS(A1)/A2
    A1=ABS(A1)
90  IF(SQT-BD)130.90.150
    AY=A1*CRV(1)/SQT
100  AY=A2*CRV(1)/SQT
    XX=(AX*TR+AY-A2)/TR
110  IF(XX)140.110.110
    X(1)=(AX+.0*(AX-XX))*TG
120  X(2)=A2-.6*(A2-AY)
    A1=TFWF
130  CLM=(CRV(1)**2+A1**2-CRV(2)**2)/(2.*CRV(1)*SQT)
    ALAM=ATAN(SQT(1.-CLM**2)/CLM)
    RI=ATAN(TR)-ALAM
    AY=CRV(1)*SIN(RI)
    AY=SQT(COV(1)**2-AX**2)
140  GO TO 100
    X(1)=6*AY*TG
    X(2)=AY+.1*AX*TR
150  GO TO 120
    DC 480 L=1.2
    G=0*SQT(1./(1.+TR**2))*C(L)/E
    T1=1./F(L)-2.*G/(1.+G)
    T2=4.-2./F(L)+T1
    CALL ELIPIN(T1,T2,EE,FK,CSE2,ROUT)
160  IF(ROUT)180.180.180
    WRITE(6,170)J,L
170  FORMAT(11P,FALL - 170.214)
    RETURN
    YK(L)=1.84771/EL(L)*SQT(EE*U)/(EK**3*CSE2*T2)
180  AP1=(SQT-20)/(1.+(X(2)/XK(1))**.66666667)
    APO=SQT-BD-AP1N
    AY=(CRV(1)+APO)*TR/SQT(1.+TR**2)
    AY=SQT(COV(1)+APO)**2-AY**2
190  GO TO 100
    DC 360 IT=1.25
    CVR02=CVR02+.2
    IF(CVR02-1.)210.210.200
200  CVR02=1.
    B(1)=ATAN(X(1)/X(2))
    B(2)=ATAN((A1-X(1))/(A2-X(2)))
210  DC 200 K=1.2
    SE(N)=SIN(B(K))
    CS(N)=COS(B(K))
    CX(1,1)=CS(1)**2/A(C)
    CX(1,2)=-A(1)*DBX(1,1)/X(2)
    CX(2,1)=-CS(2)**2/(A2-X(2))
    CX(2,2)=DBX(2,1)/X(2)

```

```

230 X(2,2)=-DEX(2,1)*(A1-X(1))/(A2-X(2))
231 E=EX+2*(2)-CRV(1)*COSR
232 DEL(1)=SQRT(X(1)*2+X(2)*2)-CRV(1)
233 DEL(2)=SQRT((A1-X(1))*2+(A2-X(2))*2)-CRV(2)
234 DO 300 K=1,2
235 IF (DEL(K)-1.E-6)235,235,230
236 CONTINUE
237 DO 300 K=1,2
238 CALL ADENOM(VALU+1
239 P(2,J)=0.
240 P(2,J)=0.
241 DO 300 K=1,2
242 CC(K)=1+CC(K)/E2
243 CC(K)=1+CC(K)*E2(K)
244 T1=1./F(K)-2.*C(K)*G2(K)/G(K)
245 T2=4.-2./T(K)+T1
246 CALL FLIP(T1,T2,EE,K,CSE2,INIT)
247 IF (INIT)270,270,250
248 WRITE(6,250)J,K
249 FORMAT(11)MALL - 260,214)
250 RETUR
251 FRAVE(N)=0
252 TCAVE(N)=T2
253 CSEP(K)=SQRT(CSE2)
254 SNEP(K)=SQRT(1.-CSE2)
255 X(N)=11.04771/EL(K)*SQRT(D*EE/(EK**3*CSF2*T2))
256 SA(N)=EK(K)*SQRT(DEL(N))
257 DELX(N)=1/EC(K)*SQ(K)
258 DELY(N)=2/EC(K)*CR(K)
259 CTAVE1/T1
260 STAVE=SQRT(1.-CTAU**2)
261 DTDEP=*(T2-T1)*C(K)/(T2*GG(K))*2*STAU)
262 DEDE=(EE-LK*CSE2)/(SNEP(K)*CSEP(K))
263 DEDE=(EE-LK)/SNEP(K)*CSEP(K)
264 DTAVE=(EE*SNEP(K))*2*STAU*.5/(CSEP(K)*(-2.*EE*(EK-EE)/SNEP(K)+(E
265 I*DKDEP-5.0238555*SQRT(D/(T2*EE*FK**5)))/(EL(K)*CSF2)*((LK*DEDEP-3.
266 I*EE*DKDEP)*CSF2(K)+2.*EK*EE*SNEP(K))
267 DKDT2=-X(K)*.5/T2
268 DKKG(N)=DKDEP*DEDTAU*DTG-DXKDT2*2.*C(K)/GG(K)**2
269 DO 300 L=1,2
270 DO 300 M=1,2
271 DGA(L,M)=0./E2**2*(E2*SB(L)*DBX(L,M)+CB(L)*DEX(M))
272 DBX(L,M)=1.5*SQ(L)*DELY(L,M)+DEL(L)*SQRT(DEL(L))*DXKDEP(L,M)*DGX(L,M)
273 )
274 TALE=SQ(1C)/(CC(1C)+C(1C)*D/E2)
275 ALXEATAN(TAL)
276 SALESTH(ALX)
277 CAL=CCOS(ALX)
278 DO 300 K=1,2
279 TEMPER(K)=ALX
280 SPAL(K)=STN(TEMP)

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7.

3.

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360 GO TO 380
370 CONTINUE
380 WRITE(0,370)J
390 FORMAT(11HUBALL - 370.14)
400 IQUIT=1
410 RETURN
420 ICNT=ICNT+1
430 DO 390 K=1,2
440 WOTH(K)=(.2387324 *ESAVE(K)*CXEP(K)*EL(K)*PX(K)*D/TSAVE(K))*3333
450 1728(K)=.375*WOTH(K)*PX(K)*ESAVE(K)/CXEP(K)
460 R2B1=COS(7(2)-R(1))
470 IF(ICNT-3)420,400,400
480 IF(LOOP)410,410,510
490 LOOP=1
500 IC=1
510 GO TO 5
520 IF(IC-1)430,430,460
530 IF(TRA(2)*B2B1-TRA(1))450,450,440
540 IC1=2
550 GO TO 490
560 IC1=1
570 GO TO 490
580 IF(TRA(1)*B2B1-TRA(2))480,480,470
590 IC1=1
600 GO TO 490
610 IC1=2
620 IF(IC1-IC)500,510,500
630 IC=IC1
640 GO TO 5
650 DELA(2,1)=SB(2)
660 DELA(2,2)=CB(2)
670 DPA(2,1)=CU(2)*2/(A2-X(2))
680 DPA(2,2)=-(A1-X(1))*(CH(2)/(A2-X(2)))*2
690 DGA(2,1)=U*TRA(2,1)*SB(2)/E2
700 DGA(2,2)=U*TRA(2,2)*SB(2)/E2
710 DO 530 K=1,2
720 DALA(1)=Y2*TRA(1,K)-Y3*(-2.*Y3*TRA(1,K)+C(IC)*TRA(1,K))*Y4
730 V1(K)=UG(1)*SRA(2)*DGA(2,K)-LALA)
740 V2(K)=UG(1)*SRA(1)*DGA(1,K)-DALA)
750 DGBA(1)=5*(U)*(-GG(1)*DGA(2,K)+GG(2)*DGA(1,K))-GG(1)*GG(2)*(-V1(K)+
760 1CAL(2)*DGA(1,K)-V2(K)-CBAL(1)*DGA(2,K))
770 DGBA(1)=DN*(PPM(1)*(-V1(K)+CBAL(2)*DGA(2,K)-V2(K)-CBAL(1)*DGA(2,K))/DN
780 2*2
790 DCA=XMASS*E2*REVE*U*U*U*U
800 DCA(K)=XTN*(PEVA*U*U*U*U+CLVE*U*U*U*U)*CAL+PEVA*KEVF*CAL+DALA)
810 DO 520 LE1,2
820 DPA(L,K)=1.5*SO(L)*DELA(1,K)+SEL(L)*SOPT(DE(L))*DPA(L,K)+JGA(L,K)
830 DPA(L,K)=Y1*TRA(1,K)-Y2*TRA(2,K)+TRA(1,K)+SB(1)-TRA(2,K)*SO(2)-Y7*
840 1 DPA(K)

```



```

20 FORMAT(12HUEFLIPIN - 10)
30 CTE=CSE2/SNES
XIG=ALOG(1./CSE2)
C=1.+CSE2*(.44479204+CSE2*.085099193+CSE2*.040905094))+CSE2*(.02
10697940+CSE2*(.081502240+CSE2*.013820000))*XLG
C=1.+CSE2*(.097032891+CSE2*(.054544409+.032024666*CSE2))
1+((.5+CSE2*(.124750742+CSE2*(.060118519+.010944912*CSE2)))*XLG
EUK=EK/EE
COK=(1.-CTAU-2.*(EUK-1.)*CTE2)*SNES*SNF/((8.-4.*SNES)*EUK-2.*CSE2*
1EUK**2-6.)
CSE=SE-CKN
IF (ABS(CKN)-5.E-7)60,40,40
40 CONTINUE
50 COKIF (6,0)
60 FORMAT(12HUEFLIPIN - 40)
70 RETURN
END
SUBROUTINE SIMULT(A,N,B,X,KX)
DIMENSION A(3,7),ROV(5),R(3),X(3),KOL(3)
KOL(1)=1
10697940=N
KOL(2)=KOL(1)+1
10697940=N
COUNT=1,N
LARGESTEN=COUNT+1
10697940=N
KOL(1)=1
JCOL=1
IF (N-100) 1, 5035,5014,5006
5007 IF (N-100) 1, 5035,5014,5006
5008 A(1)=ABS(A(1,1))
5009 IF (N-100) 1, 5035,5014,5006
5010 IF (N-100) 1, 5035,5014,5006
5011 IF (N-100) 1, 5035,5014,5006
5012 IF (N-100) 1, 5035,5014,5006
5013 IF (N-100) 1, 5035,5014,5006
5014 IF (N-100) 1, 5035,5014,5006
5015 IF (N-100) 1, 5035,5014,5006
5016 IF (N-100) 1, 5035,5014,5006
5017 IF (N-100) 1, 5035,5014,5006
5018 IF (N-100) 1, 5035,5014,5006
5019 IF (N-100) 1, 5035,5014,5006
5020 IF (N-100) 1, 5035,5014,5006
5021 IF (N-100) 1, 5035,5014,5006
5022 IF (N-100) 1, 5035,5014,5006
5023 IF (N-100) 1, 5035,5014,5006
5024 IF (N-100) 1, 5035,5014,5006
5025 IF (N-100) 1, 5035,5014,5006
5026 IF (N-100) 1, 5035,5014,5006
5027 IF (N-100) 1, 5035,5014,5006
5028 IF (N-100) 1, 5035,5014,5006
5029 IF (N-100) 1, 5035,5014,5006
5030 IF (N-100) 1, 5035,5014,5006
5031 IF (N-100) 1, 5035,5014,5006
5032 IF (N-100) 1, 5035,5014,5006
5033 IF (N-100) 1, 5035,5014,5006
5034 IF (N-100) 1, 5035,5014,5006
5035 IF (N-100) 1, 5035,5014,5006
5036 IF (N-100) 1, 5035,5014,5006
5037 IF (N-100) 1, 5035,5014,5006
5038 IF (N-100) 1, 5035,5014,5006
5039 IF (N-100) 1, 5035,5014,5006
5040 IF (N-100) 1, 5035,5014,5006
5041 IF (N-100) 1, 5035,5014,5006
5042 IF (N-100) 1, 5035,5014,5006
5043 IF (N-100) 1, 5035,5014,5006
5044 IF (N-100) 1, 5035,5014,5006
5045 IF (N-100) 1, 5035,5014,5006
5046 IF (N-100) 1, 5035,5014,5006
5047 IF (N-100) 1, 5035,5014,5006

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5023 DO 5024 JCOL=1,N
5024 KOL(JCOL)=KOL(JCOL+1)
5025 A(N,JCOL)=ROW(JCOL)
5026 KOL(N)=IERASE
5027 DO 5034 KOUNT=1,N
5028 IF (KOL(KOUNT)-KOUNT) 5035,5034,5028
5029 IF (KOL(KOUNT)-KOUNT) 5035,5034,5028
5030 IF (KOL(IROW)-KOUNT) 5035,5034,5028
5031 DO 5031 JCOL=1,N
5032 ROW(I)=A(JCOL,IROW)
5033 A(JCOL,IROW)=A(JCOL,KOUNT)
5034 IERASE=KOL(KOUNT)
5035 KOL(KOUNT)=KOL(IROW)
5036 KOL(IROW)=IERASE
5037 GO TO 5034
5038 CONTINUE
5039 GO TO 5035
5040 CONTINUE
5041 IF (KX.NE.3) GO TO 508
5042 KX=0
5043 DO 5042 IROW=1,N
5044 X(IROW)=0
5045 DO 5042 JKOL=1,N
5046 X(IROW)=X(IROW)+A(IROW,JKOL) * B(JKOL)
5047 KX=0
5048 RETURN
5049 KX=1
5050 RETURN
5051 END

```

APPENDIX B
BEARING STIFFNESS DESIGN CHARTS
PURE RADIAL LOAD

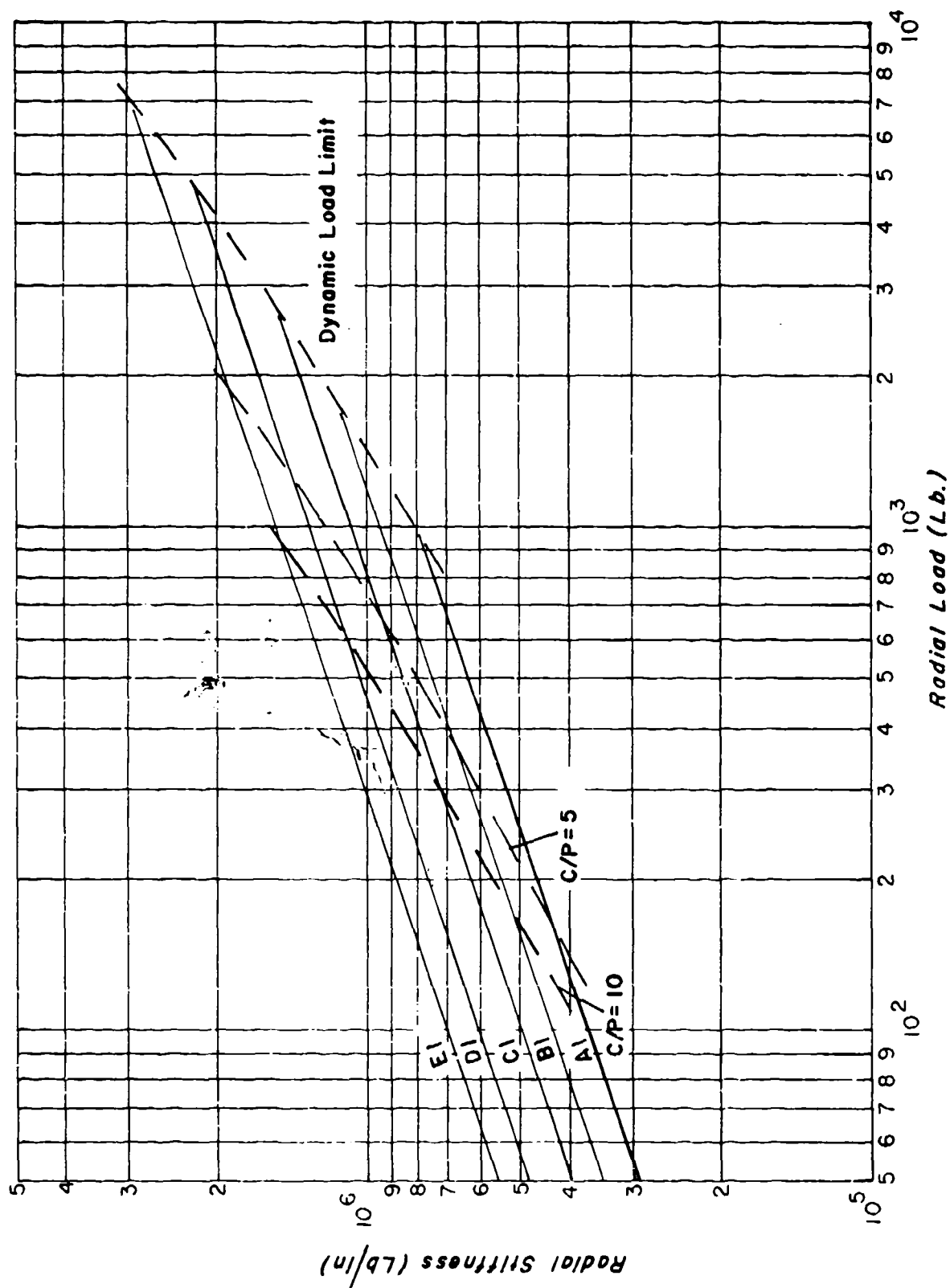


Fig. B-1 Radial Stiffness for Deep Groove Ball Bearing, Pure Radial Load

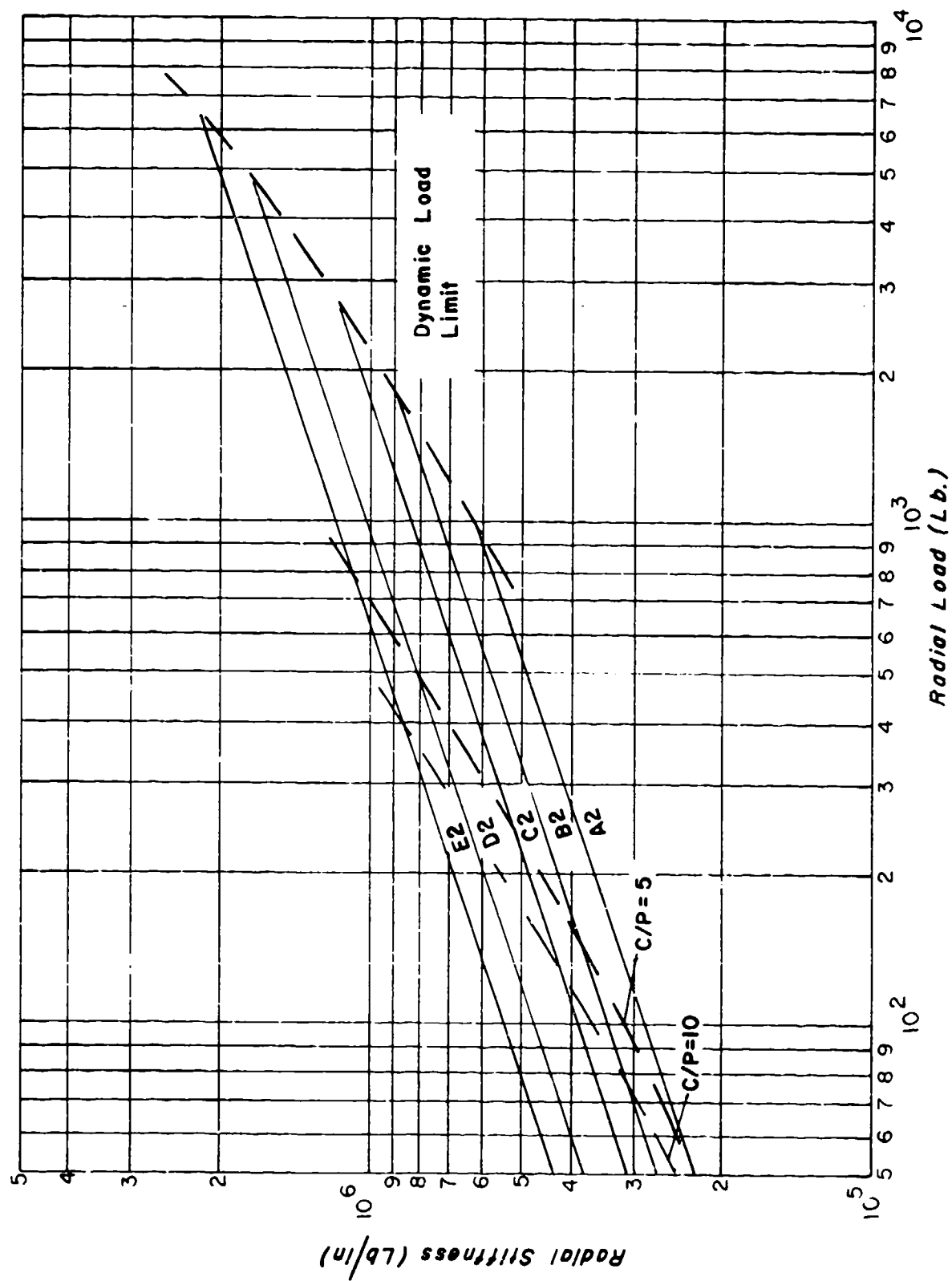


Fig. B-2 Radial Stiffness for Deep Groove Ball Bearings, Pure Radial Load

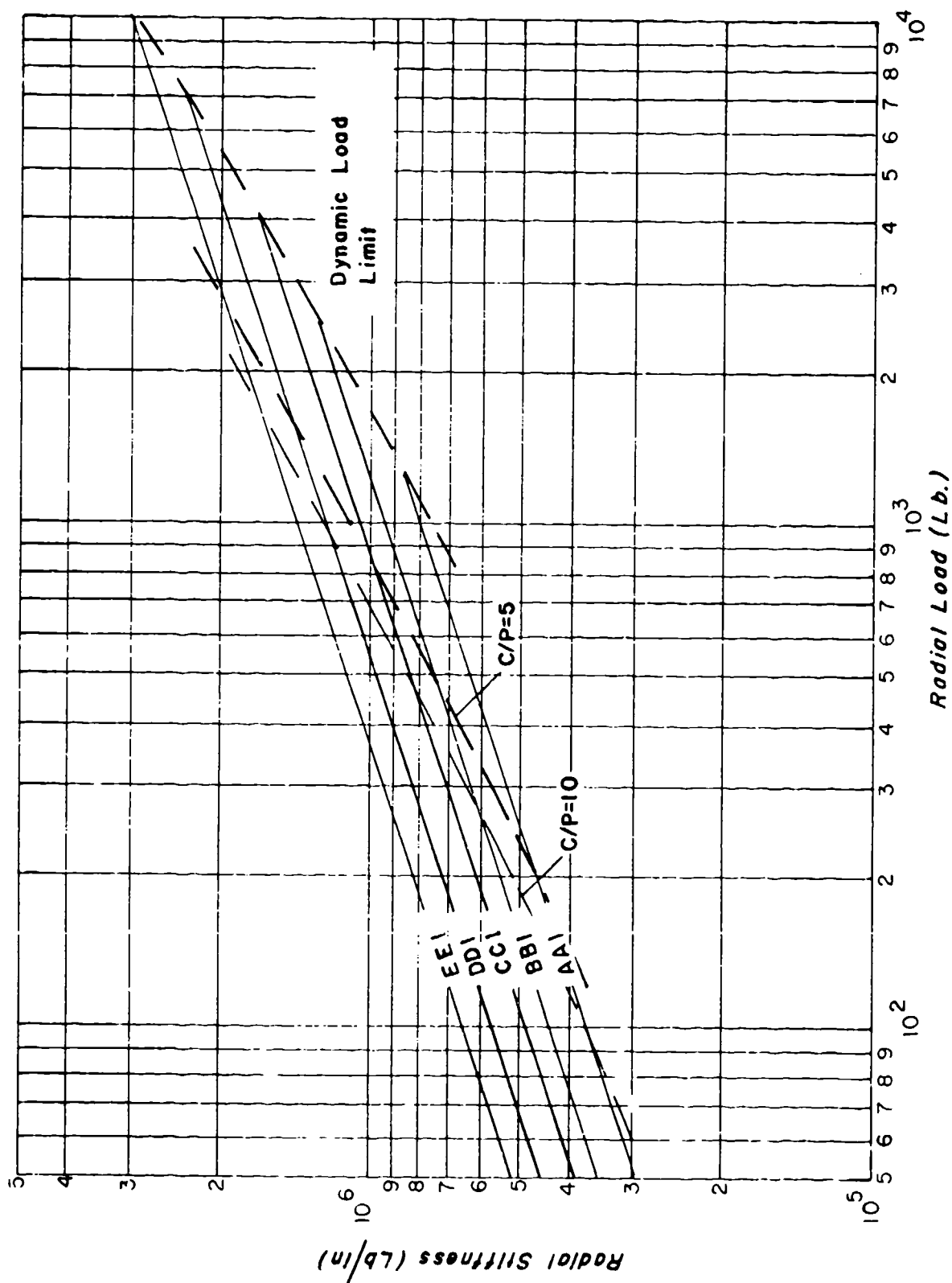


Fig. B-3 Radial Stiffness for Deep Groove Ball Bearings, Pure Radial Load

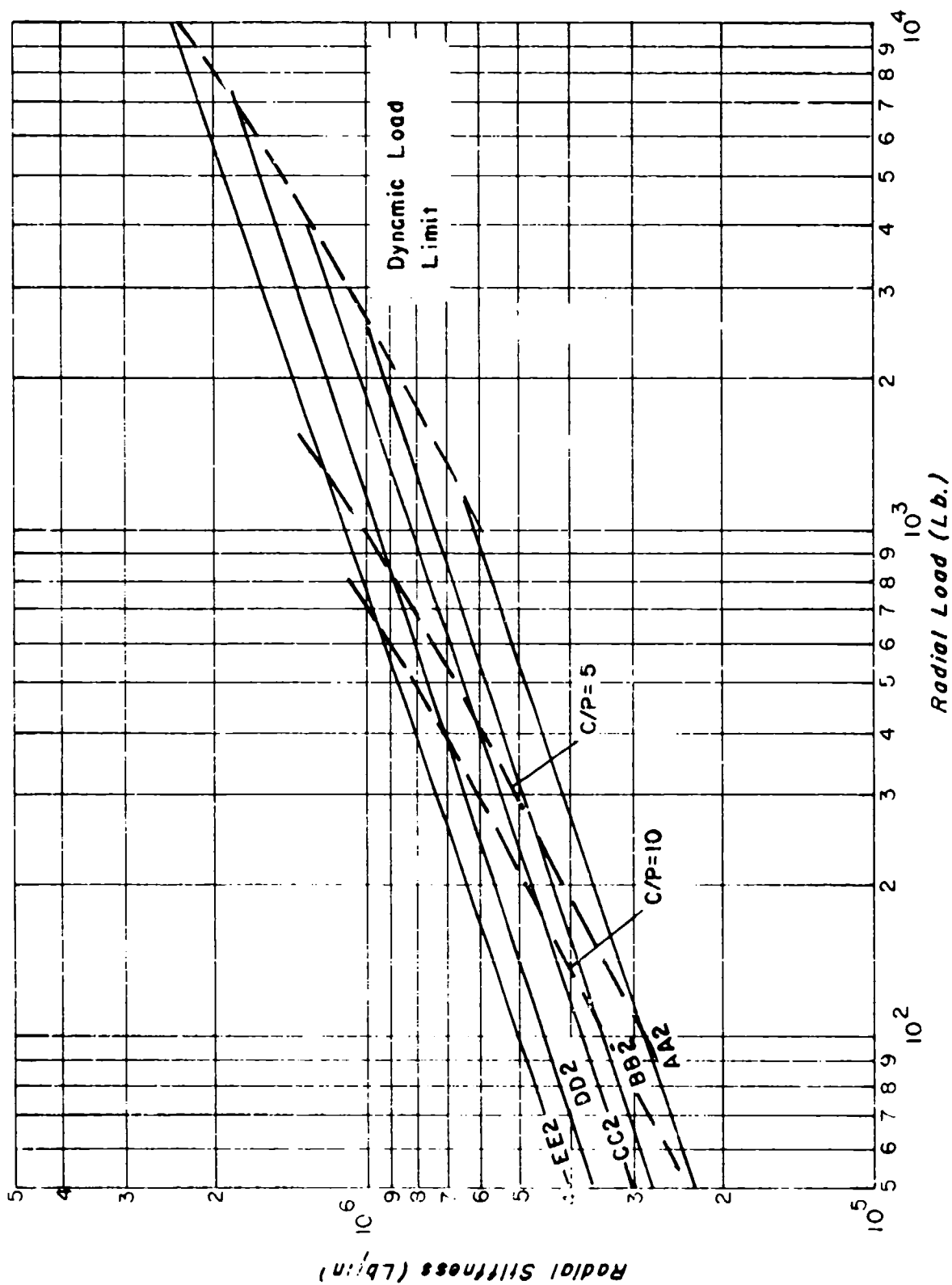


Fig. B-4 Radial Stiffness for Deep Groove Ball Bearings, Pure Radial Load

APPENDIX C
BEARING STIFFNESS DESIGN CHARTS
PURE THRUST LOAD

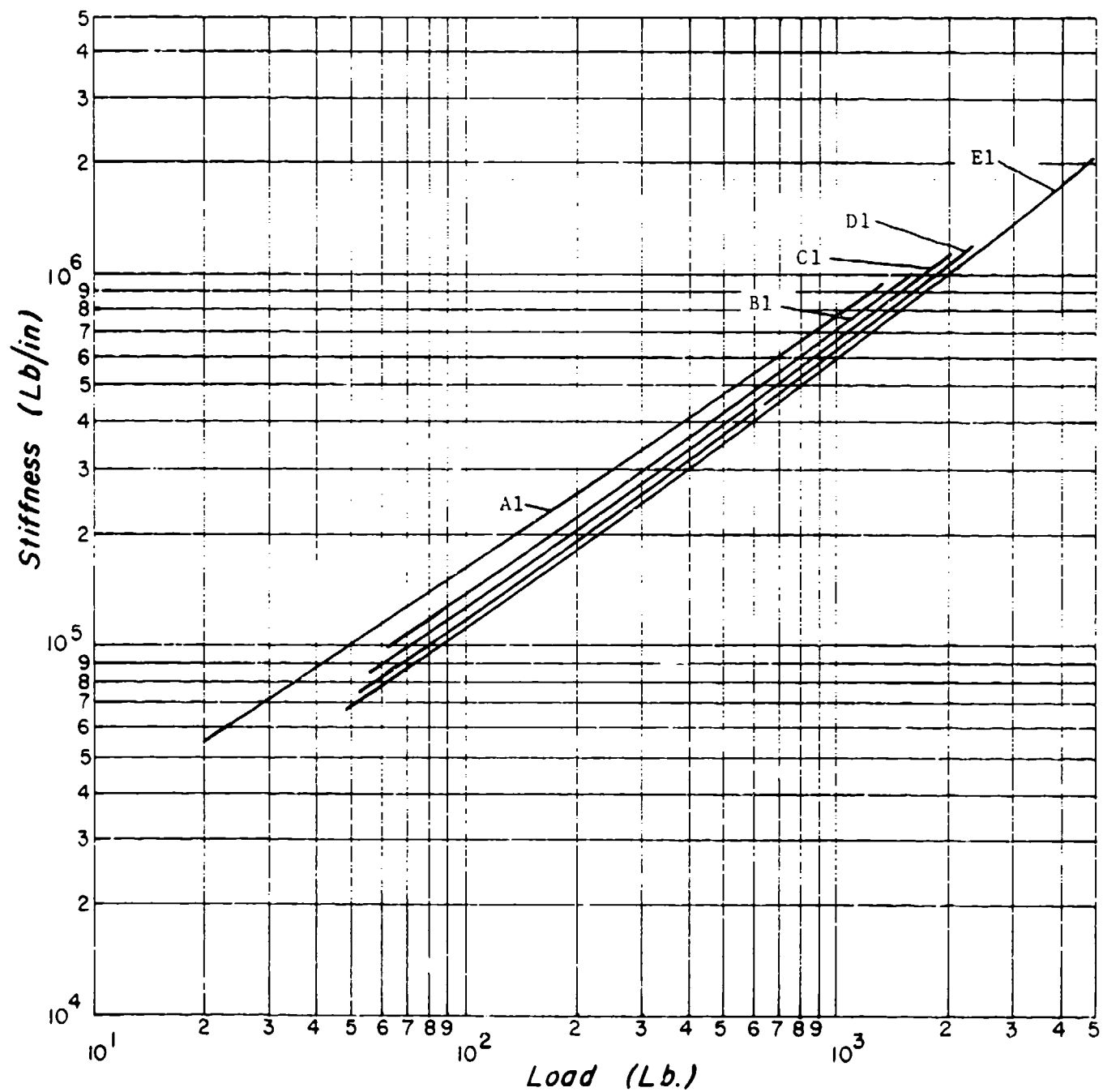


Fig. C-1 Axial Stiffness versus Axial Load,
No Radial Load

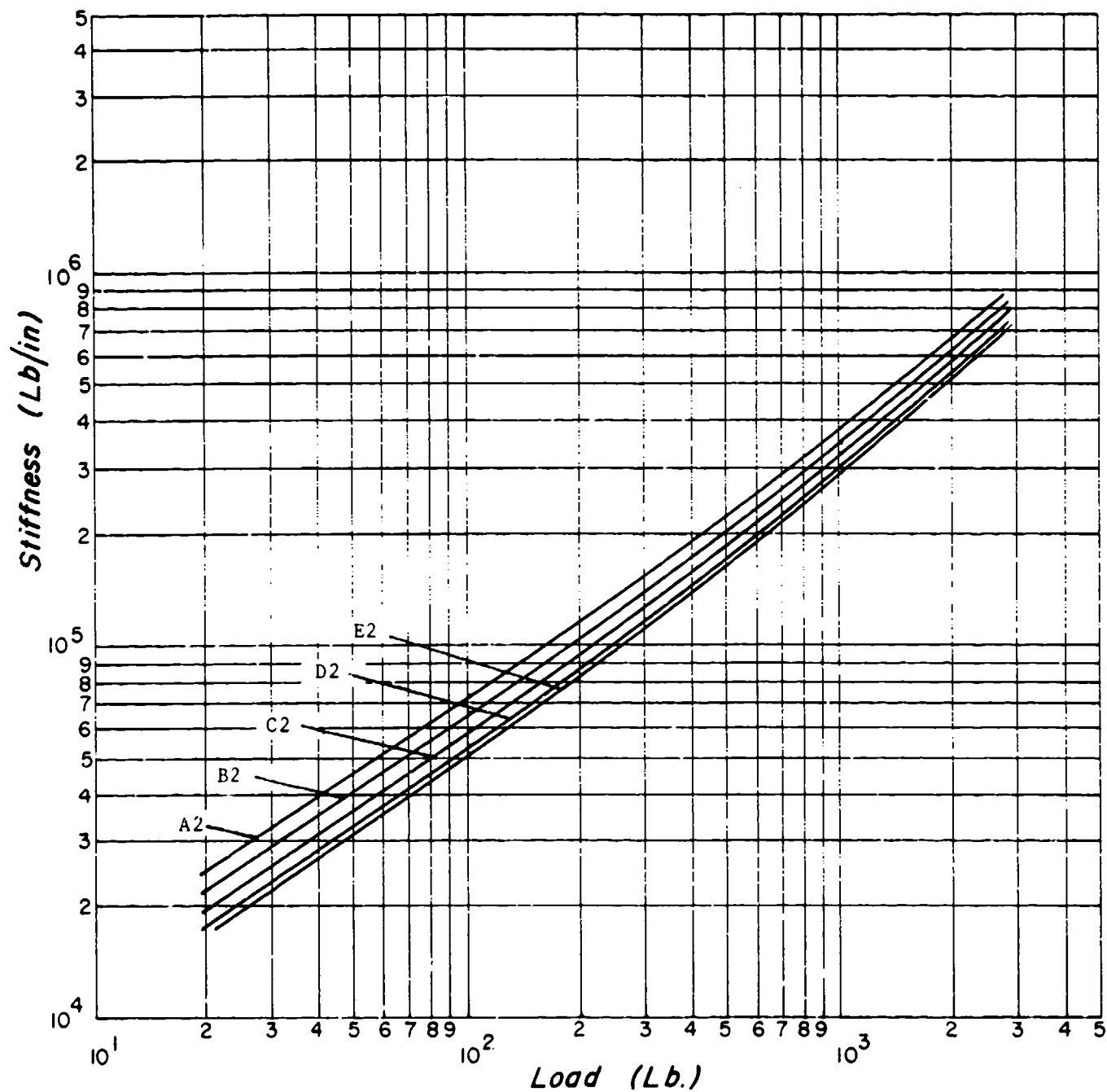


Fig. C-2 Axial Stiffness versus Axial Load,
No Radial Load

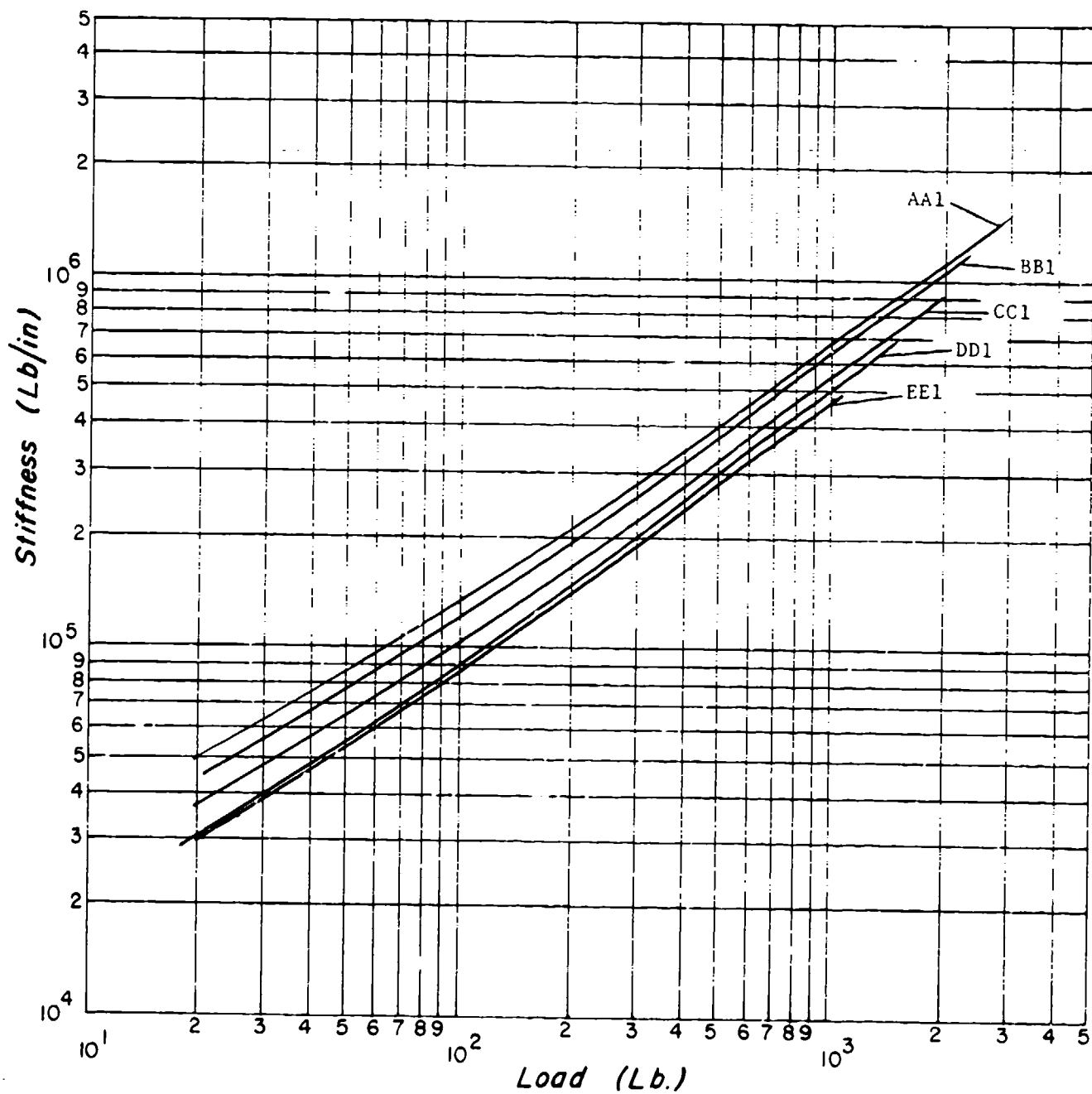


Fig. C-3 Axial Stiffness versus Axial Load,
No Radial Load

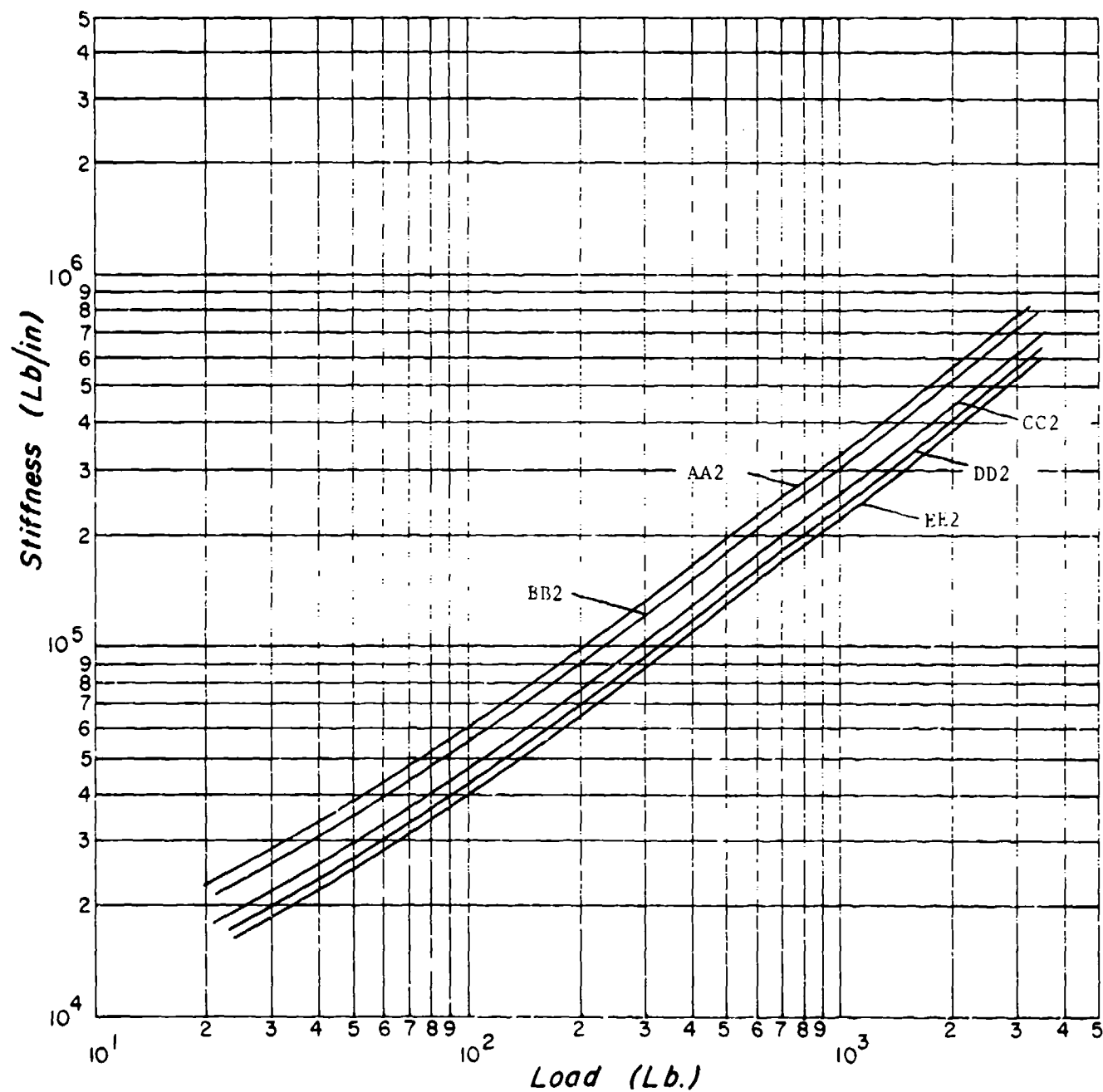


Fig. C-4 Axial Stiffness versus Axial Load,
No Radial Load

APPENDIX D

BEARING STIFFNESS DESIGN CHARTS

ANGULAR CONTACT BEARINGS WITH PRELOAD

$$\beta = 25^{\circ}$$

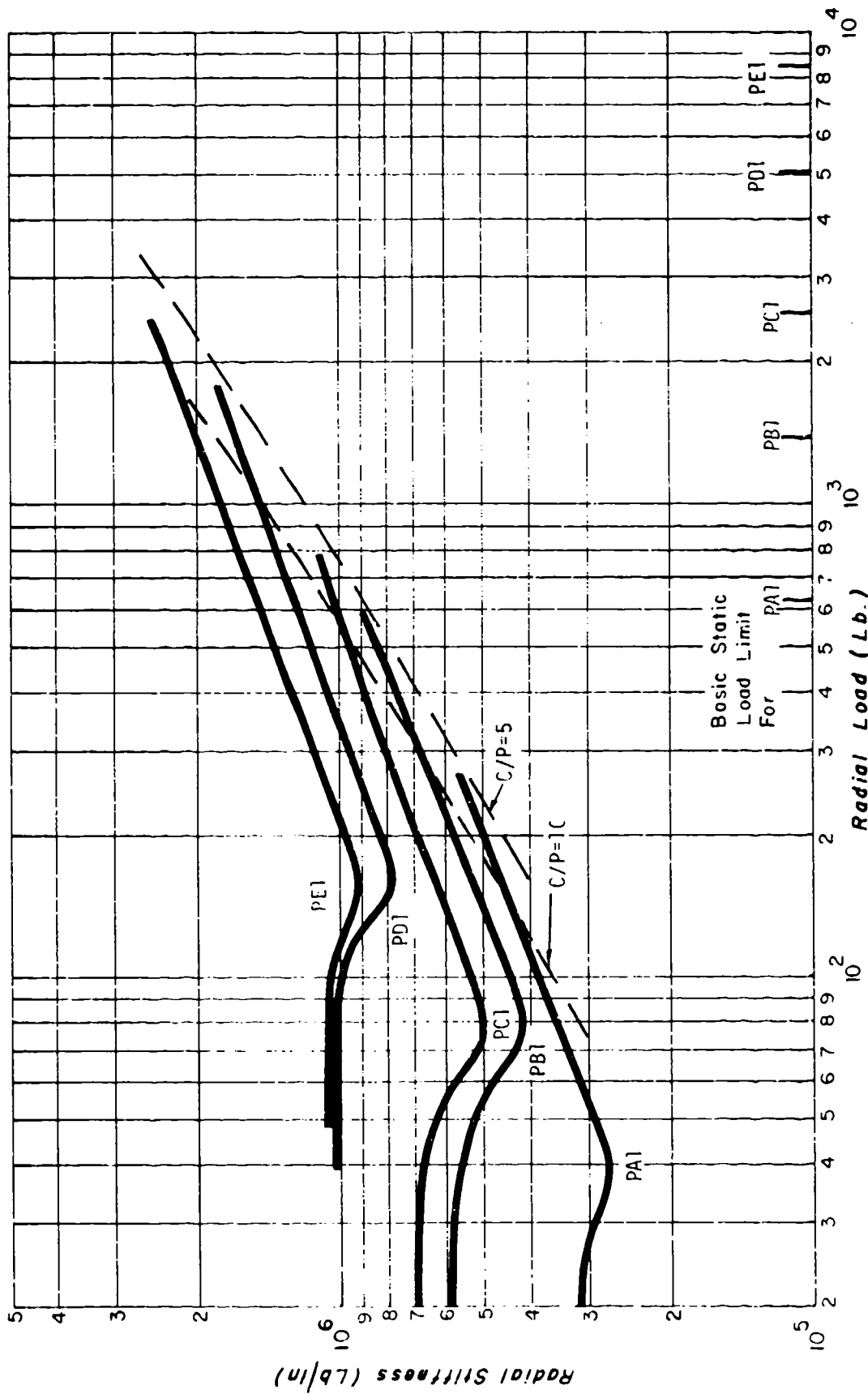


Fig. D-1 Radial Stiffness for Angular Contact Bearing,
Preload -- Selected Light,
B = 250

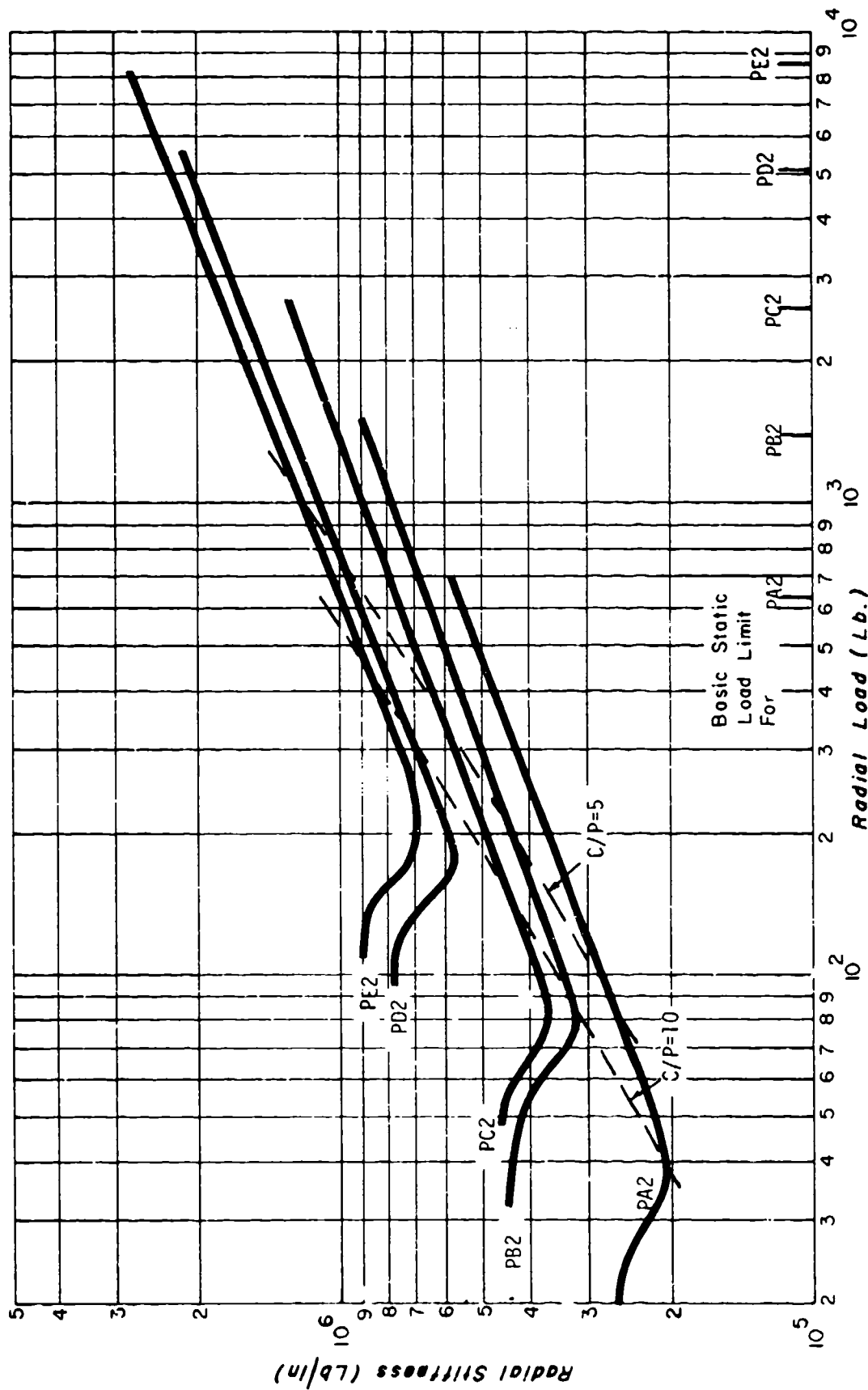


Fig. D-2 Radial Stiffness for Angular Contact Bearing,
Preload - Selected Light,
 $\beta = 25^\circ$

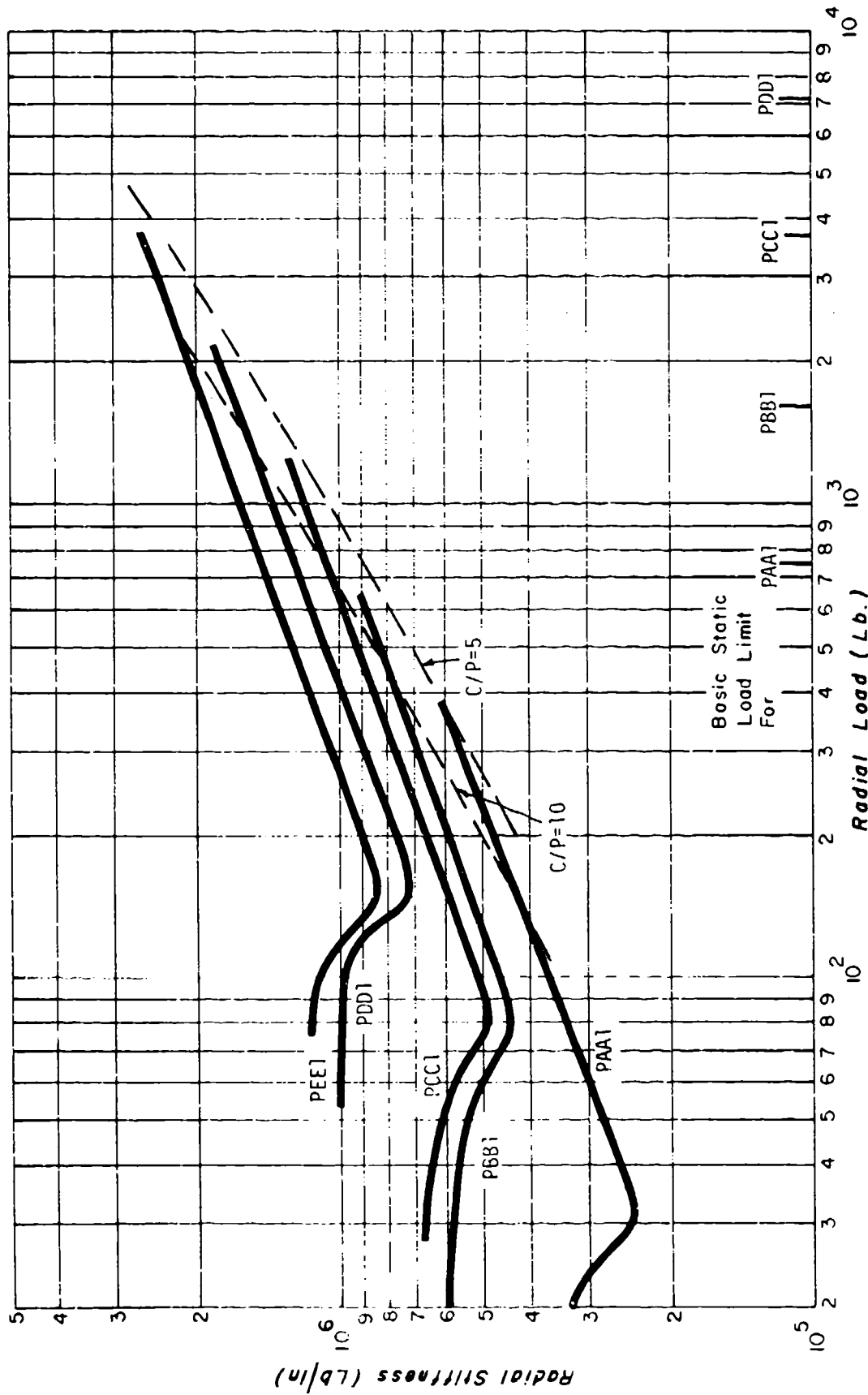


Fig. D-3 Radial Stiffness for Angular Contact Bearing,
Preload -- Selected Light,
 $\phi = 250$

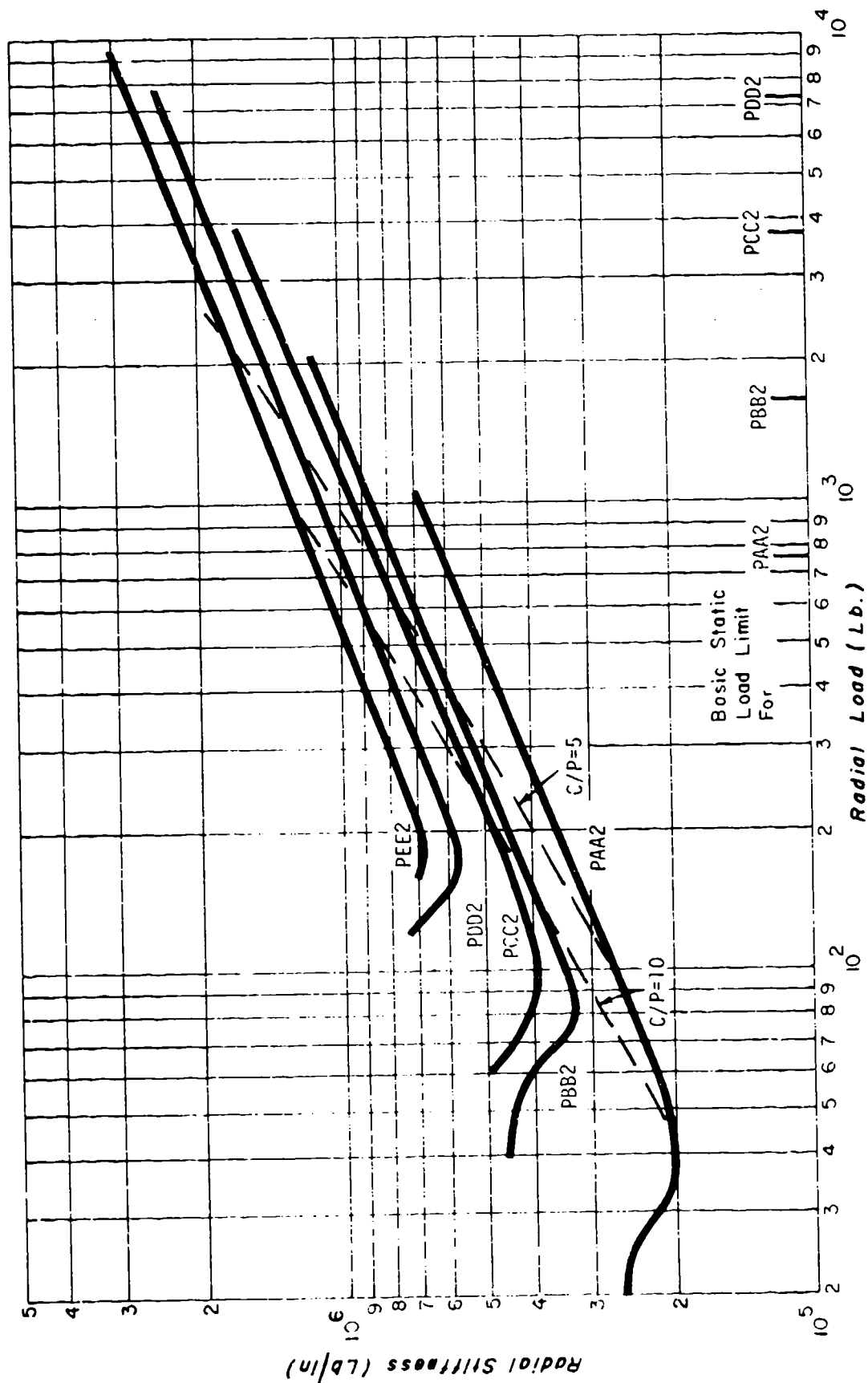


Fig. D-4 Radial Stiffness for Angular Contact Bearing,
Preload -- Selected Light,
P = 250

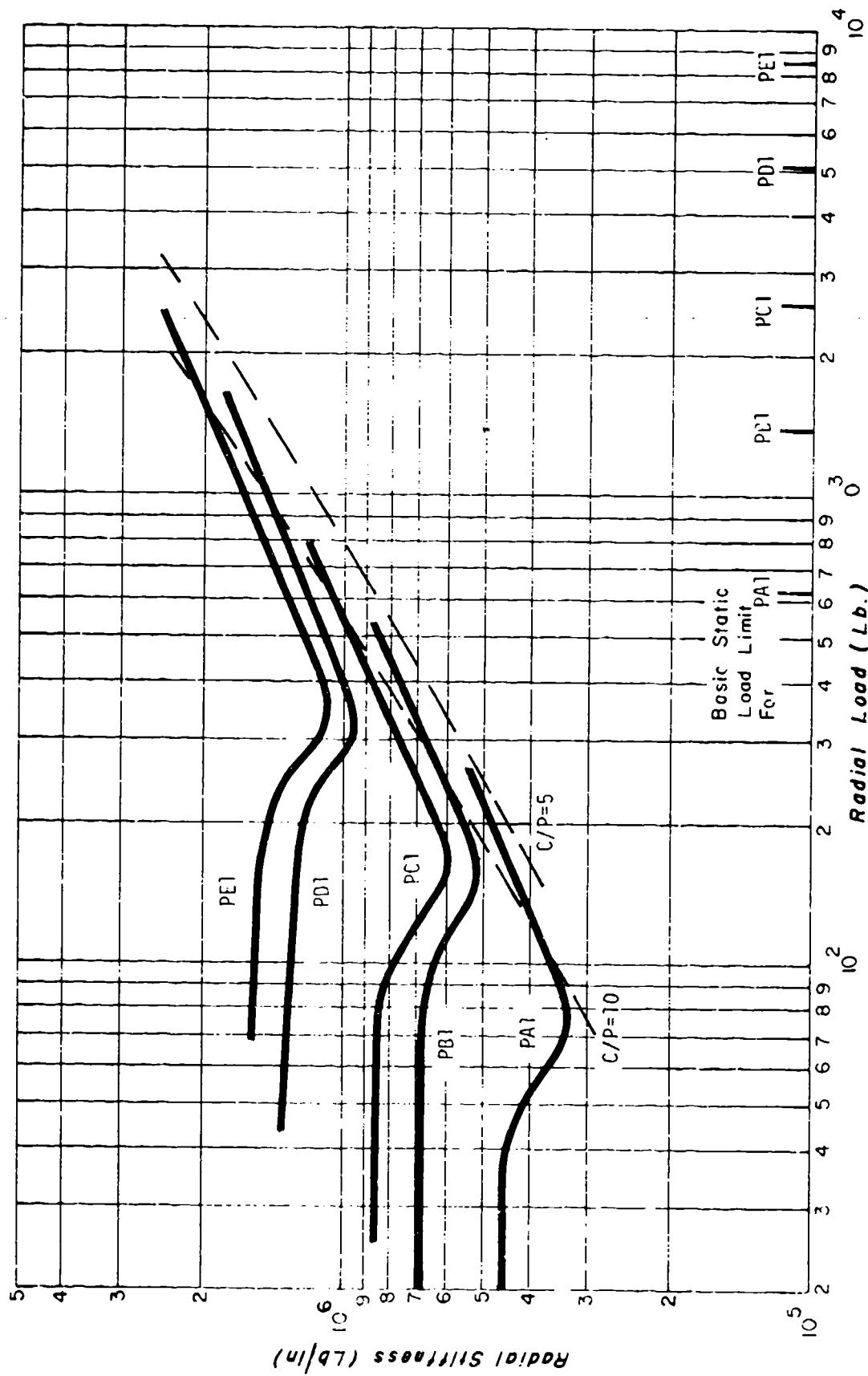


Fig. D-5 Radial Stiffness for Angular Contact Bearing,
Preload -- Moderate,
8 - 250

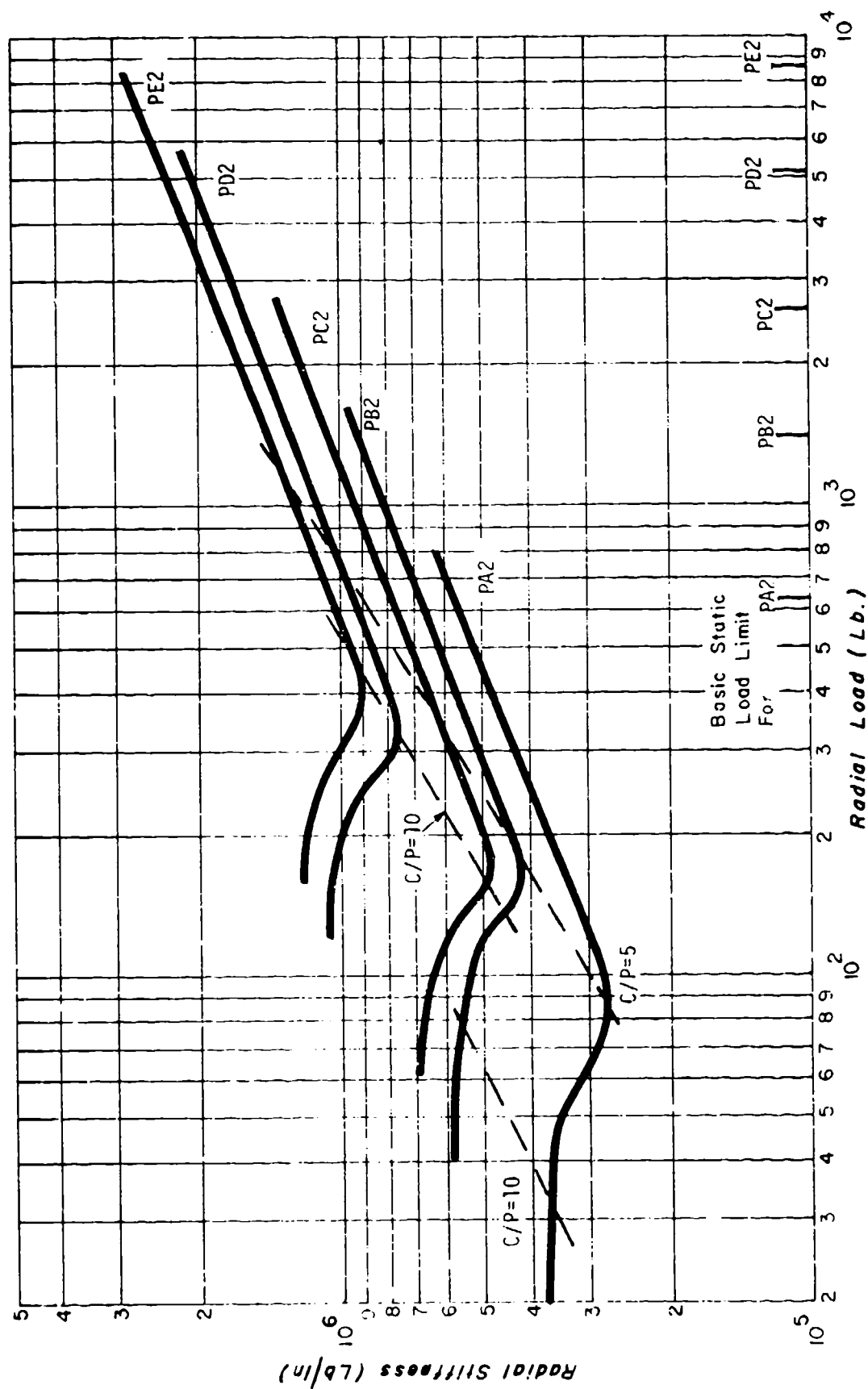


Fig. D-6 Radial Stiffness for Angular Contact Bearing,
Preload -- Moderate,
 $\alpha = 25^\circ$

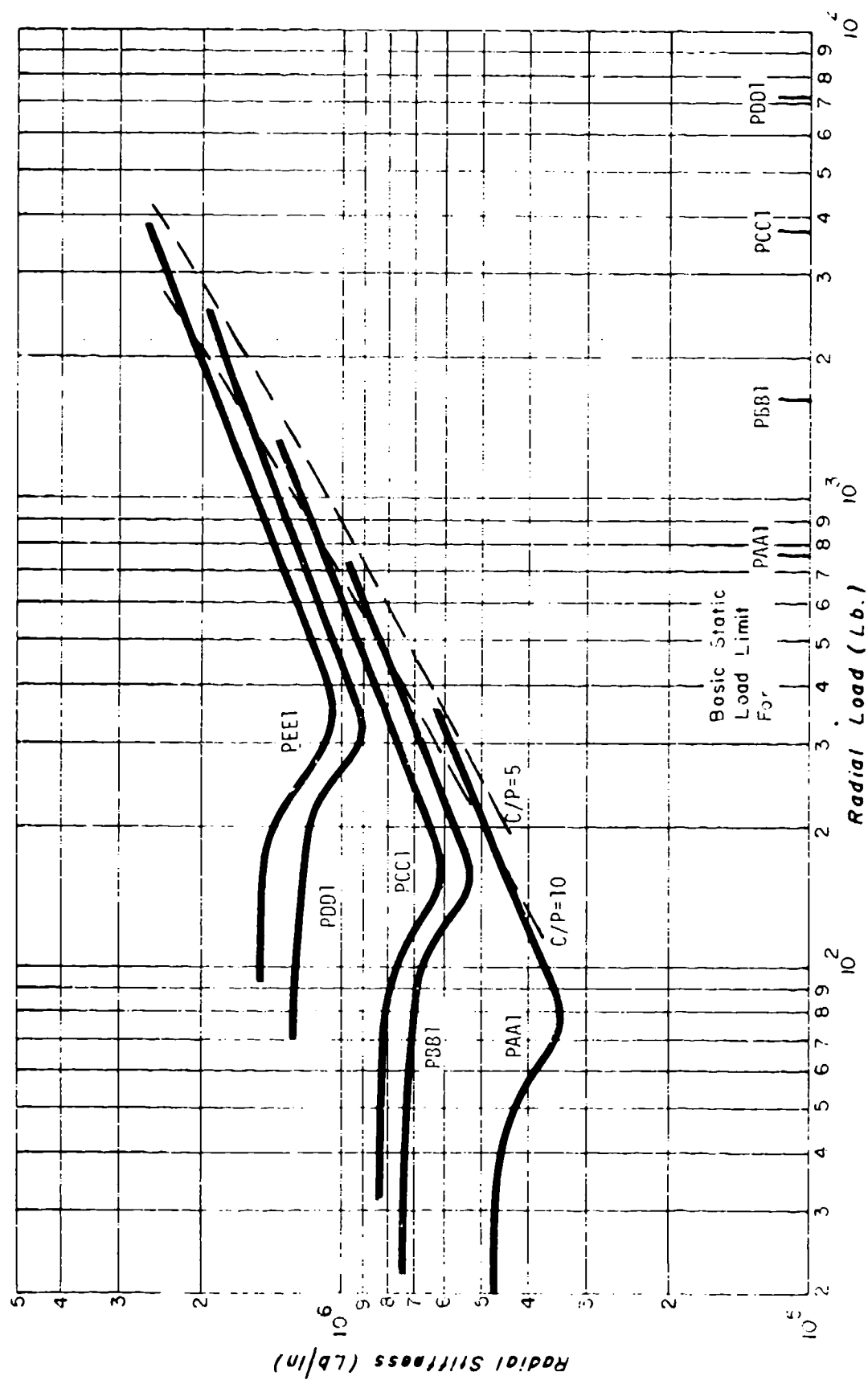


Fig. D-7 Radial Stiffness for Angular Contact Bearing,
Preload -- Moderate,
 $\mu = 0.50$

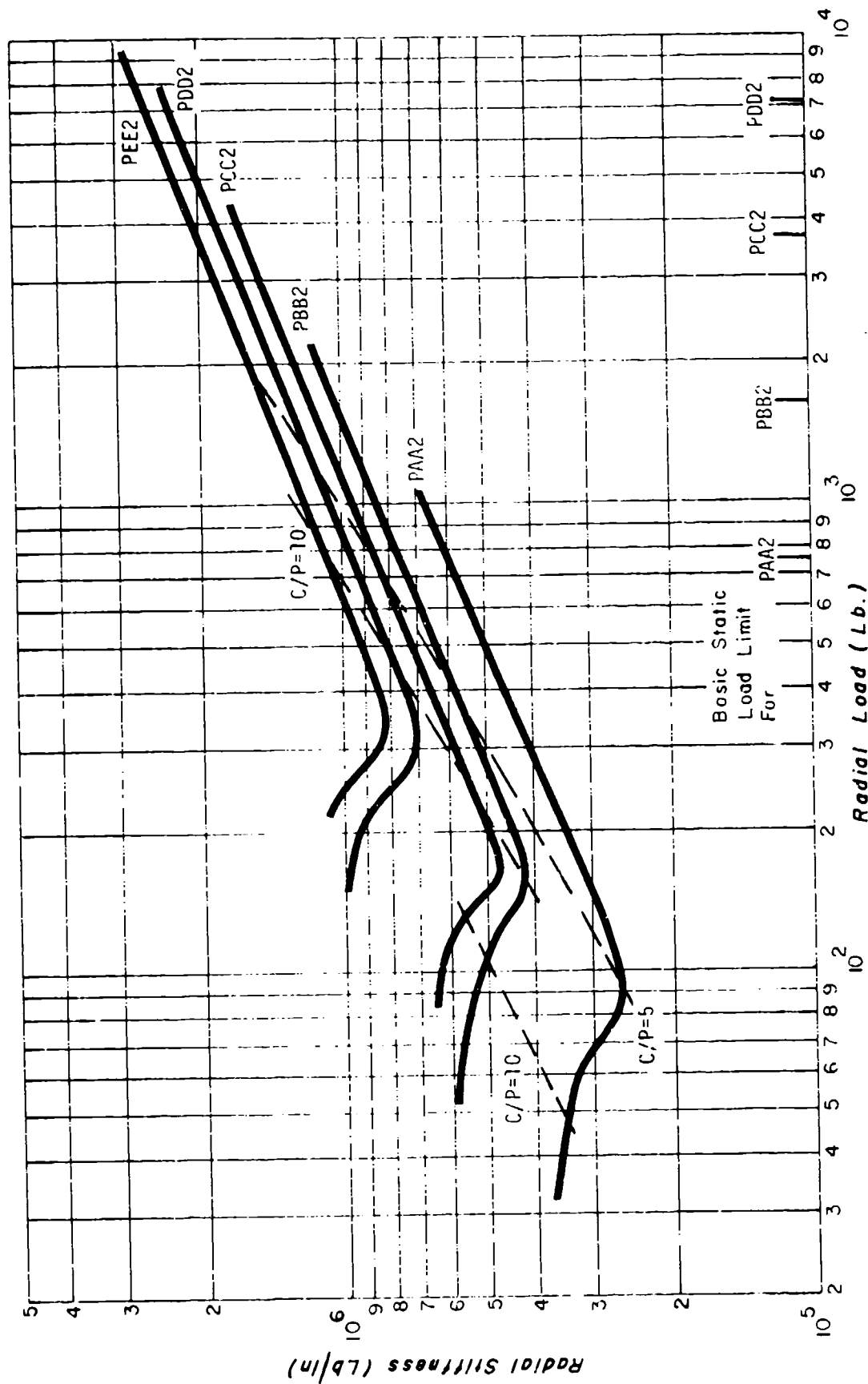


FIG. D-8 Radial Stiffness for Angular Contact Bearing,
Preload -- Moderate,
 $\beta = 25^\circ$

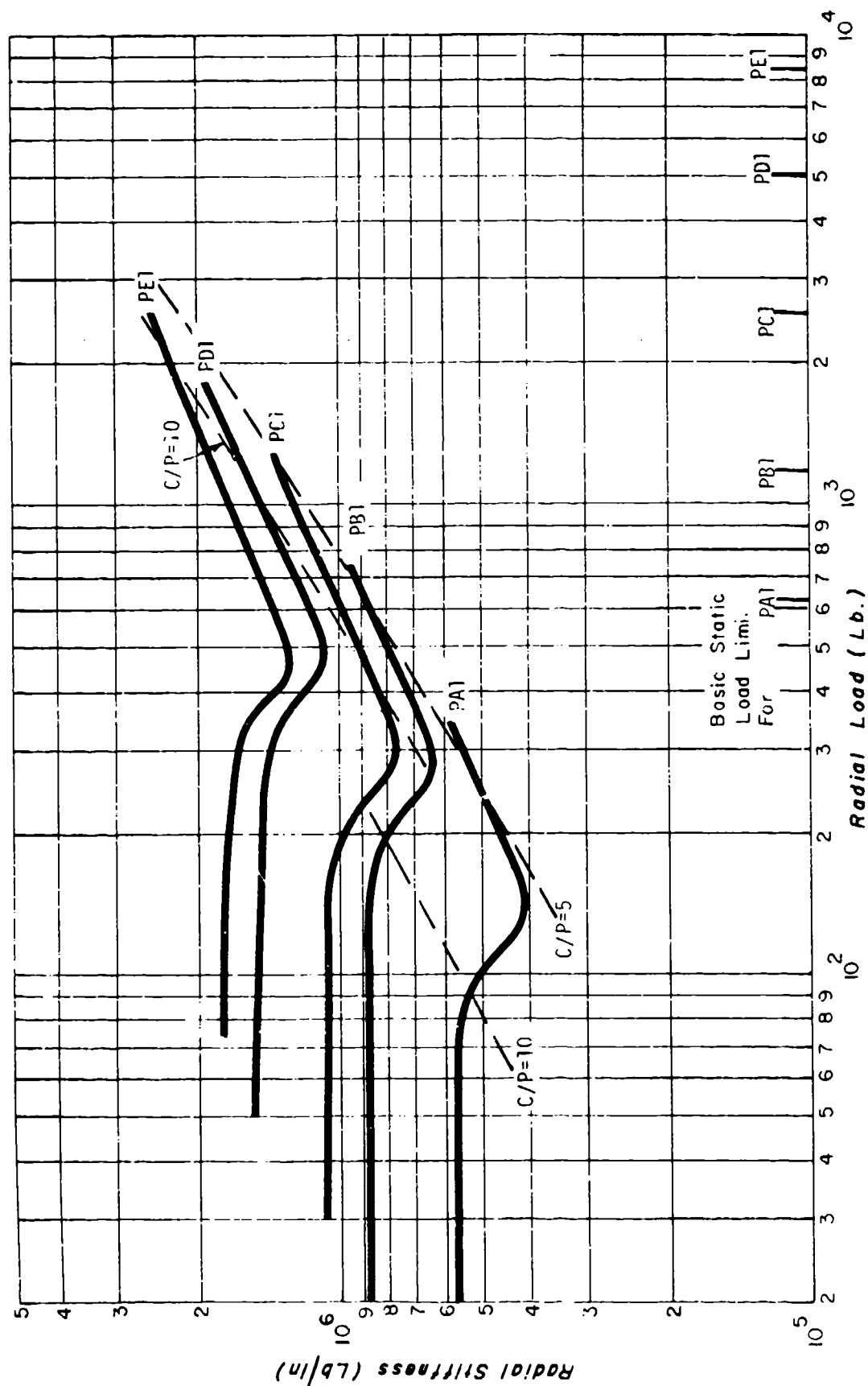


Fig. D-9 Radial Stiffness for Angular Contact Bearing,
Preload -- Preferred Heavy,
 $\beta = 25^\circ$

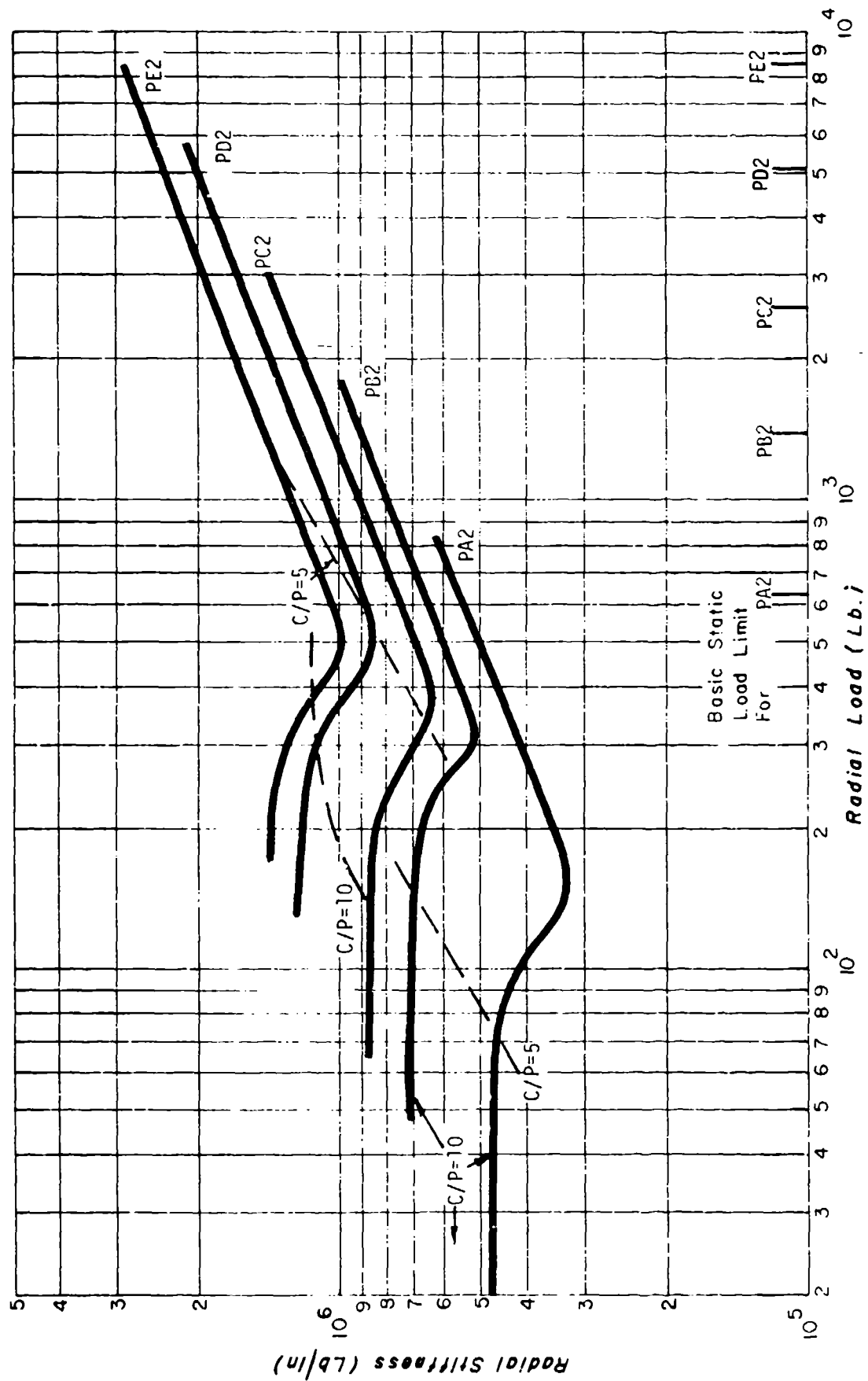


Fig. D-10 Radial Stiffness for Angular Contact Bearing,
Preload -- Preferred Heavy,
 $\psi = 250$

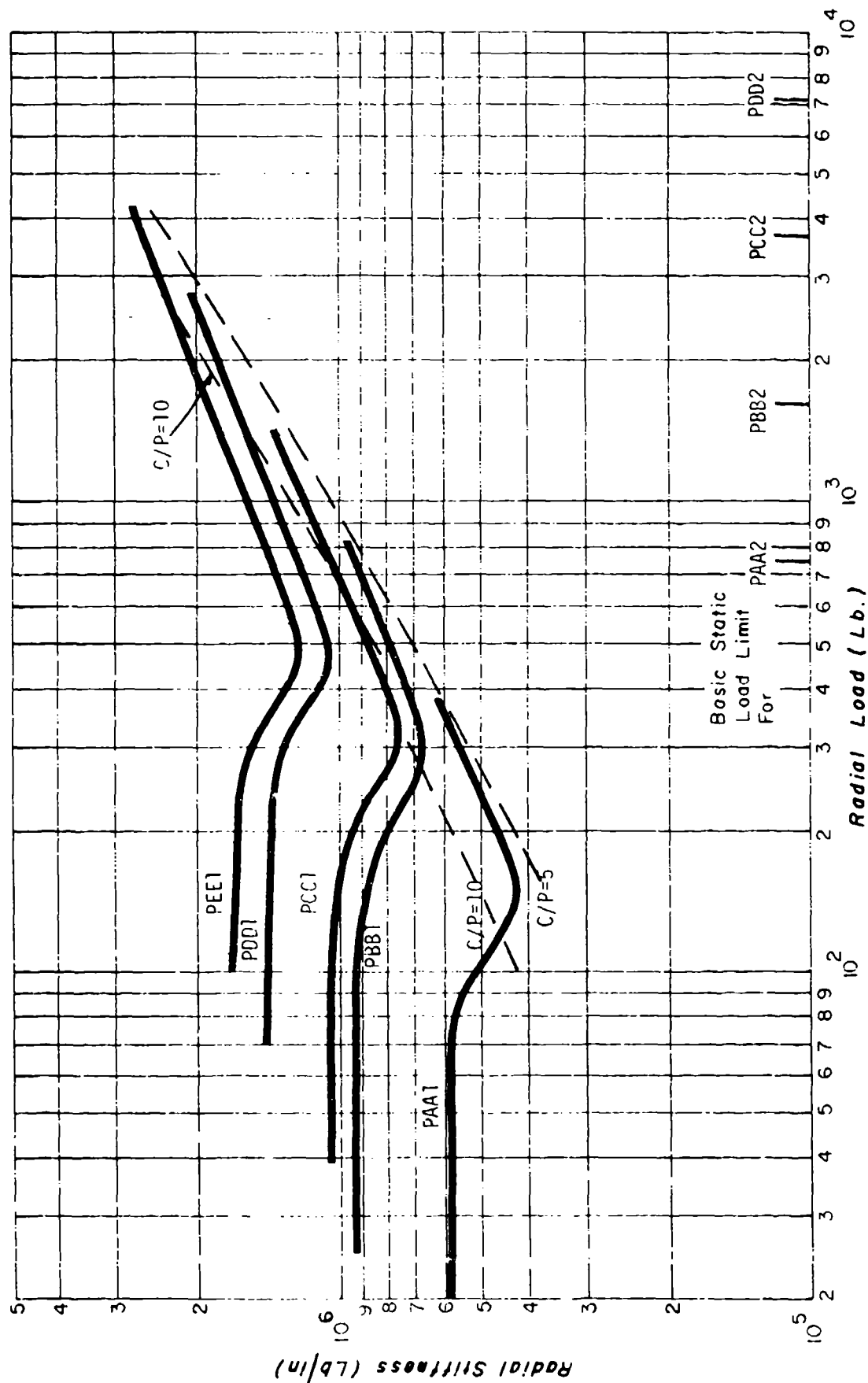


Fig. D-11 Radial Stiffness for Angular Contact Bearing,
Preload -- Pretorrei Heavy,
 $\mu = 250$

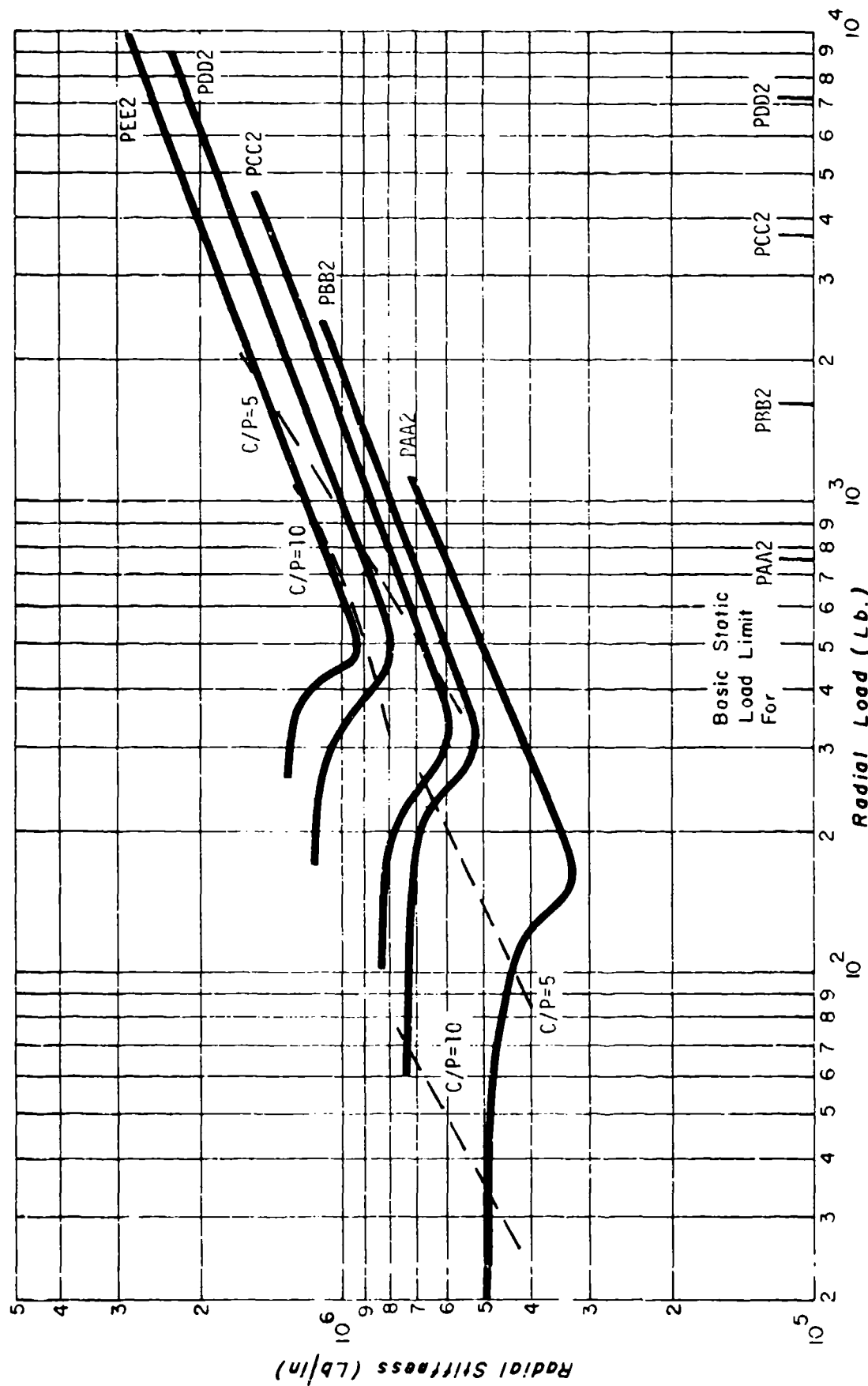


Fig. D-12 Radial Stiffness for Angular Contact Bearing,
Preload -- Preferred Heavy,
 $\beta = 25^\circ$

APPENDIX E

BEARING STIFFNESS DESIGN CHARTS

ANGULAR CONTACT BEARING WITH PRE-LOAD

$\beta = 15^{\circ}$

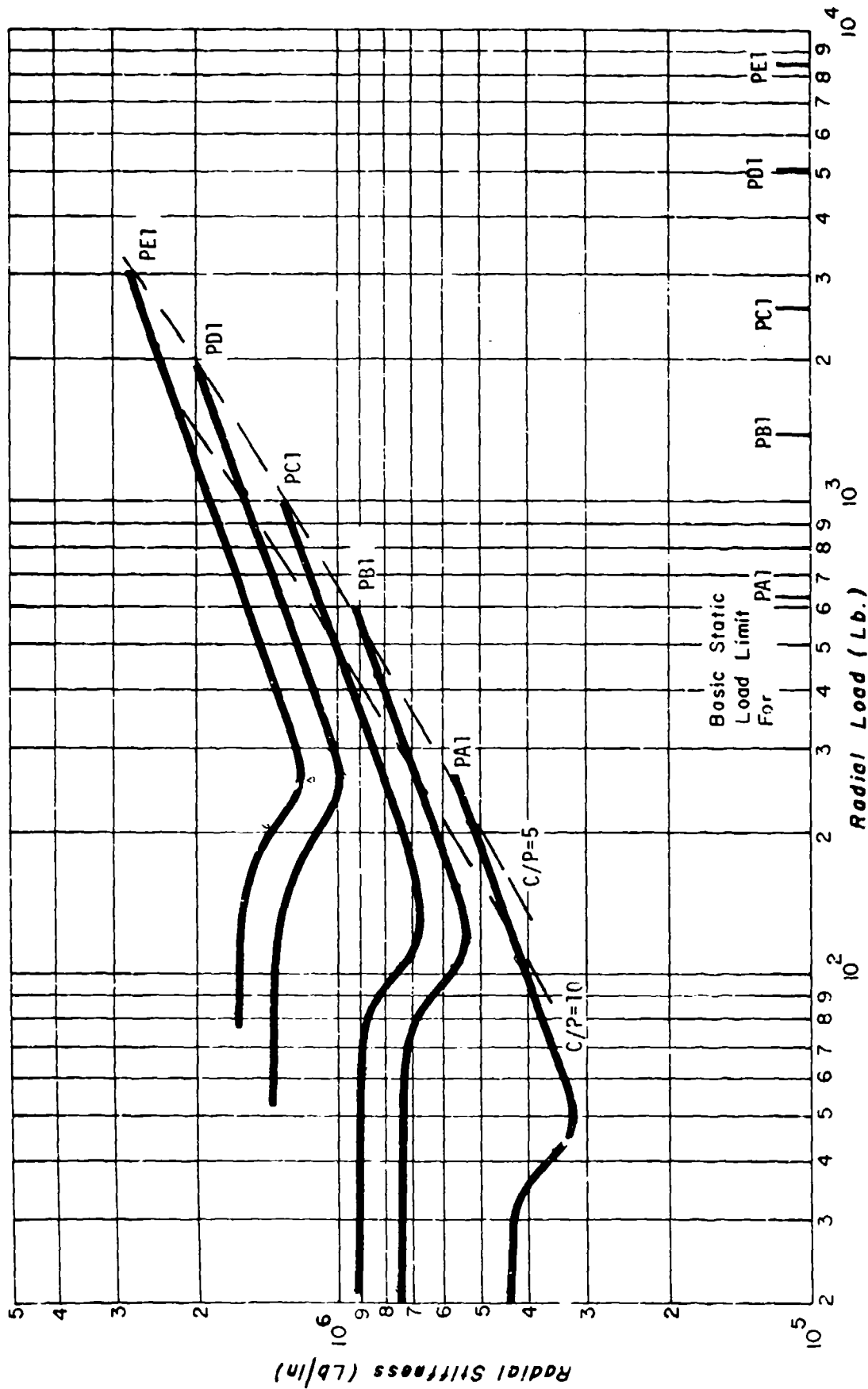


Fig. E-1 Radial Stiffness for Angular Contact Bearing,
Preload -- Selected Light,
 $\delta = 150$

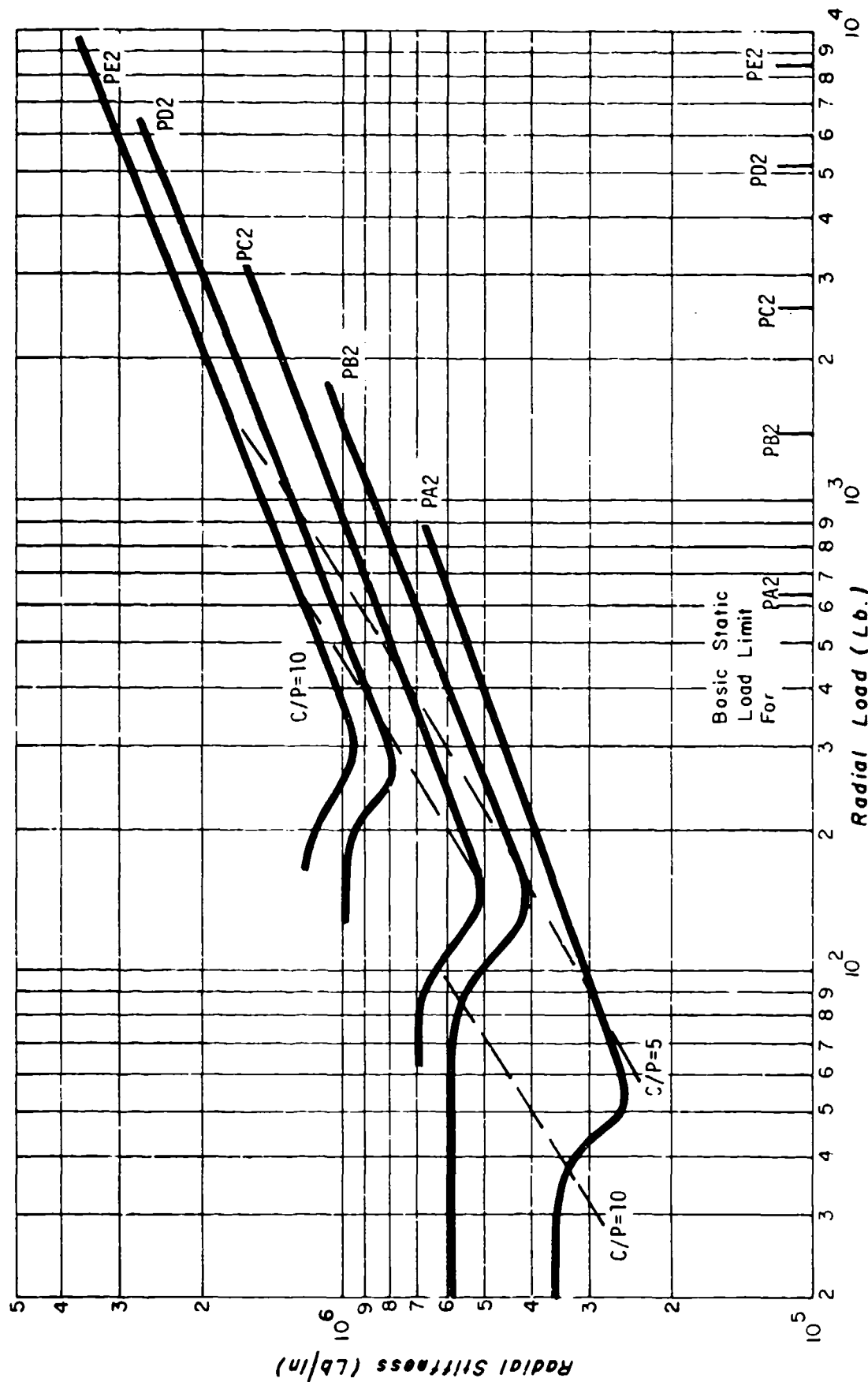


Fig. E-2 Radial Stiffness for Angular Contact Bearing,
Preload -- Selected light,
 $\delta = 15^\circ$

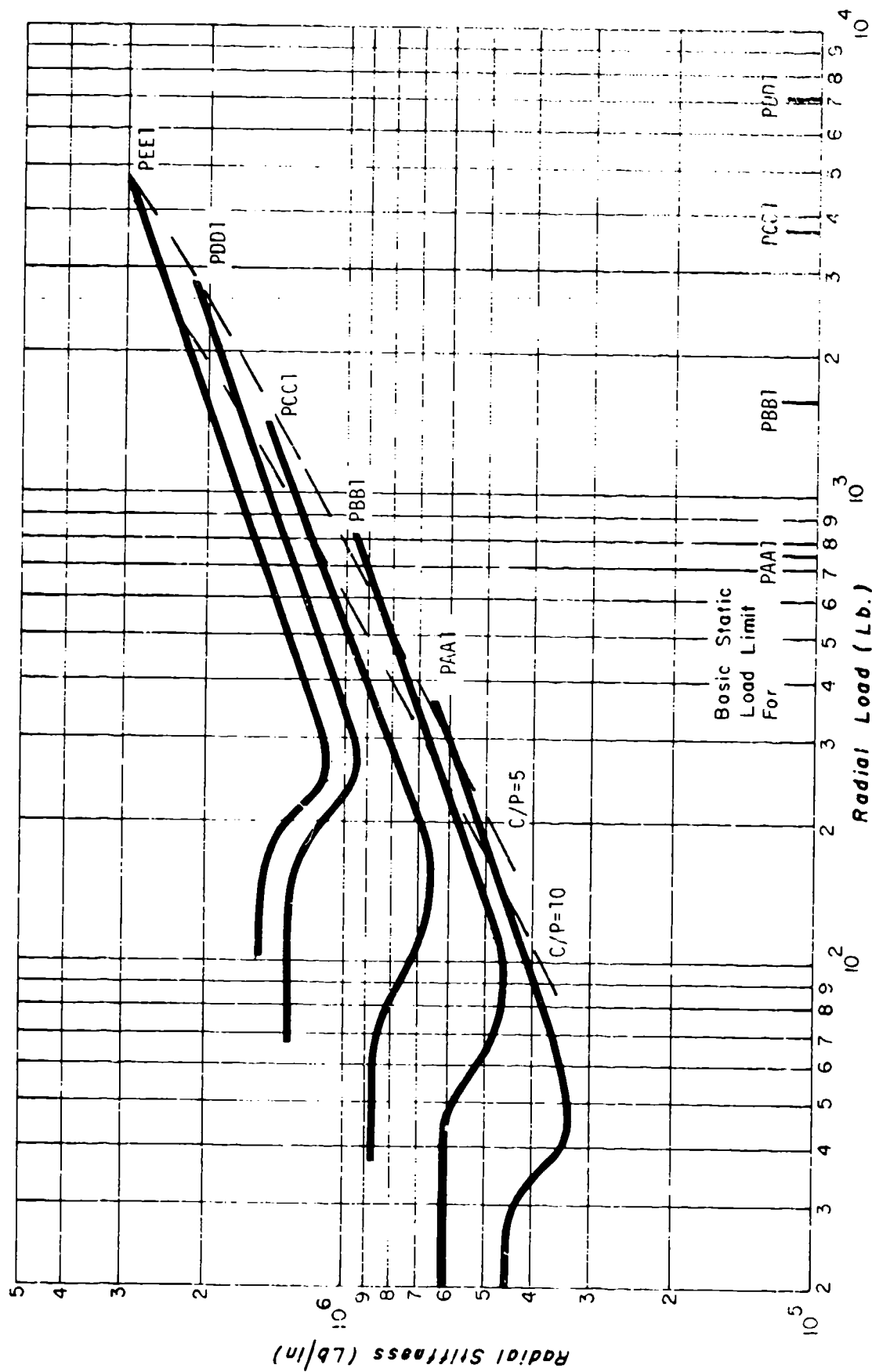


Fig. E-3 Radial Stiffness for Angular Contact Bearing,
Preload -- Selected Light,
 $\beta = 15^\circ$

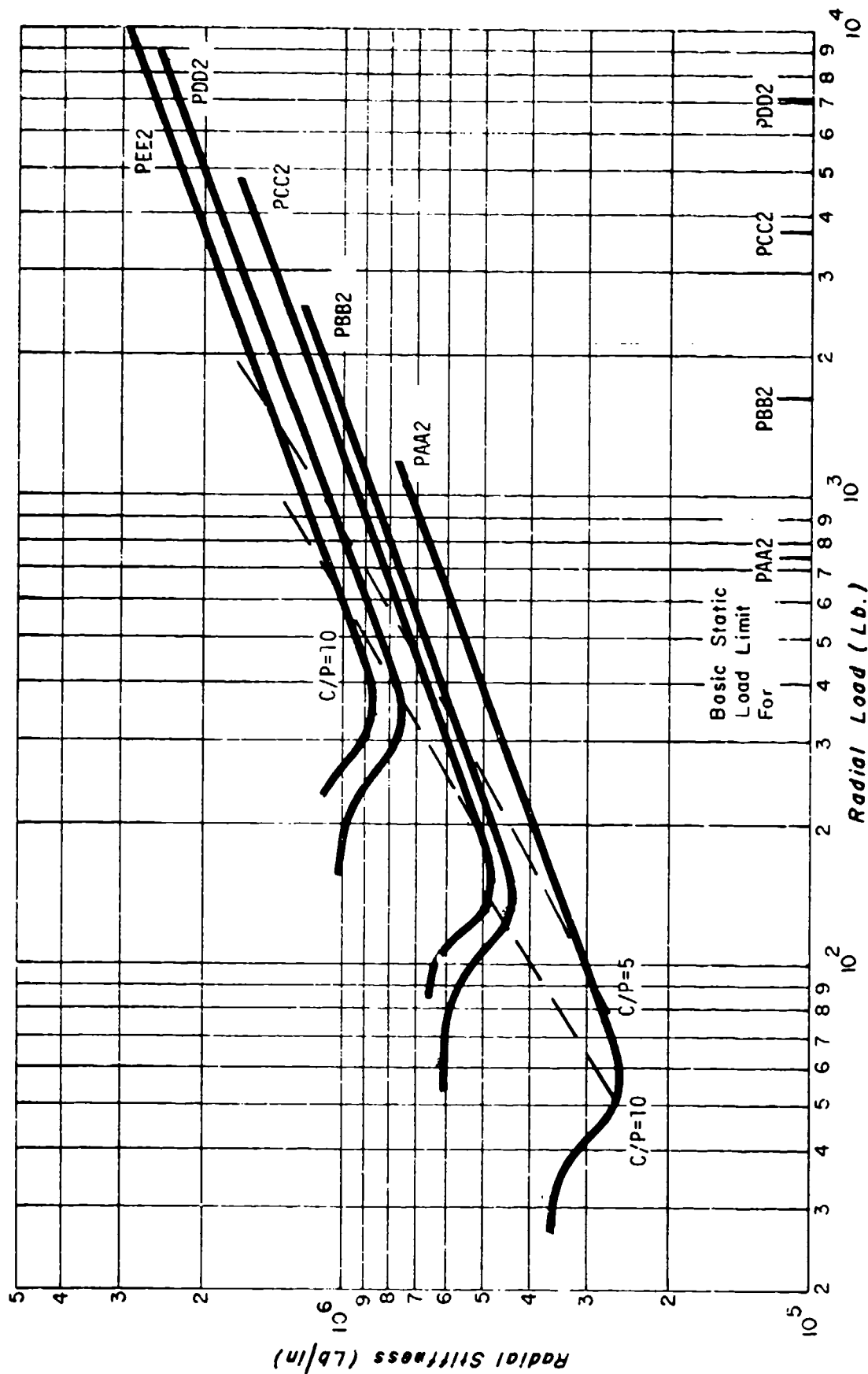


Fig. E-4 Radial Stiffness for Angular Contact Bearing,
Preload -- Selected Light,
 $\beta = 150$

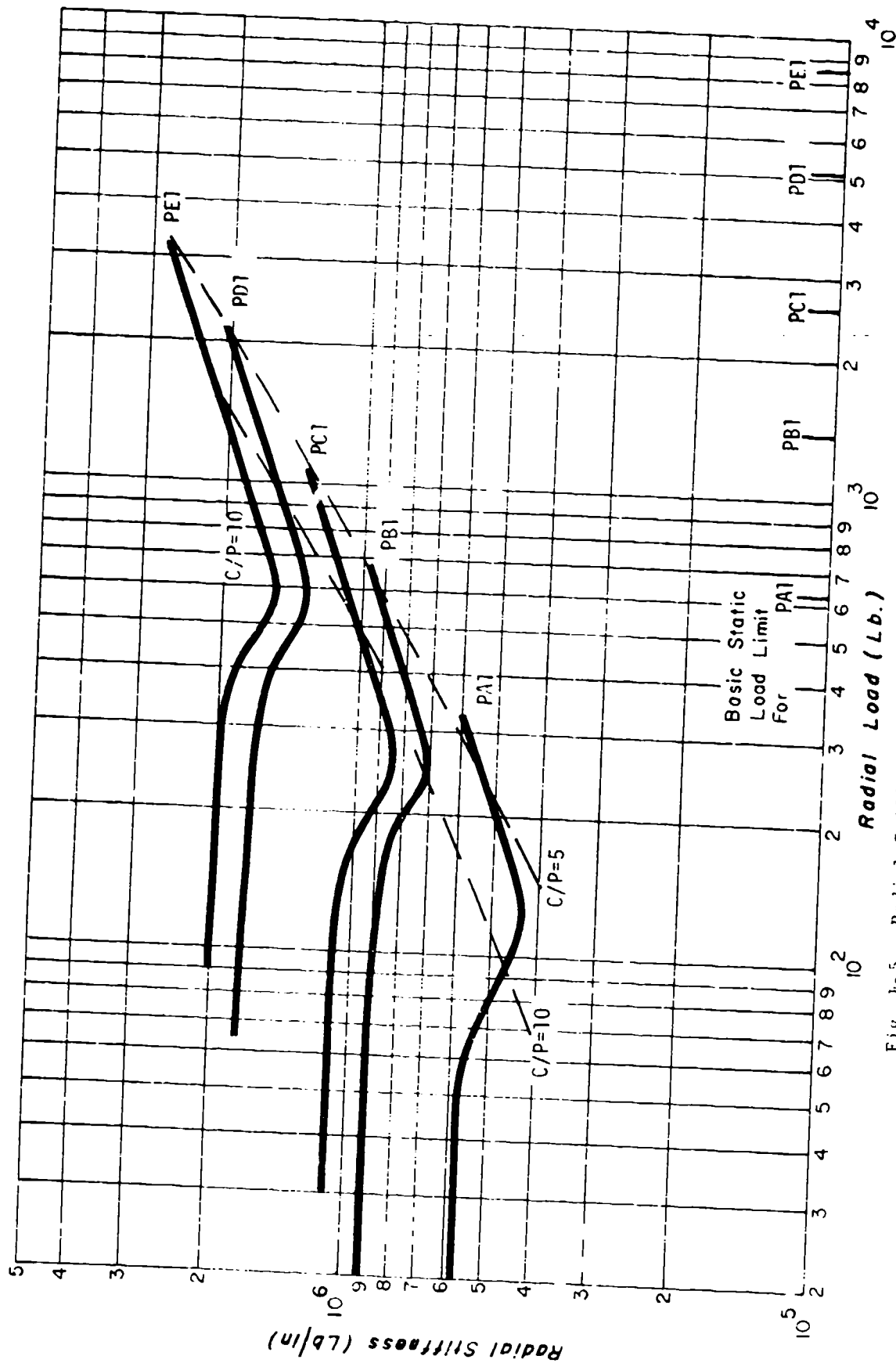


Fig. L-5 Radial Stiffness for Angular Contact Bearing,
Preload -- Moderate,
 $\beta = 150$

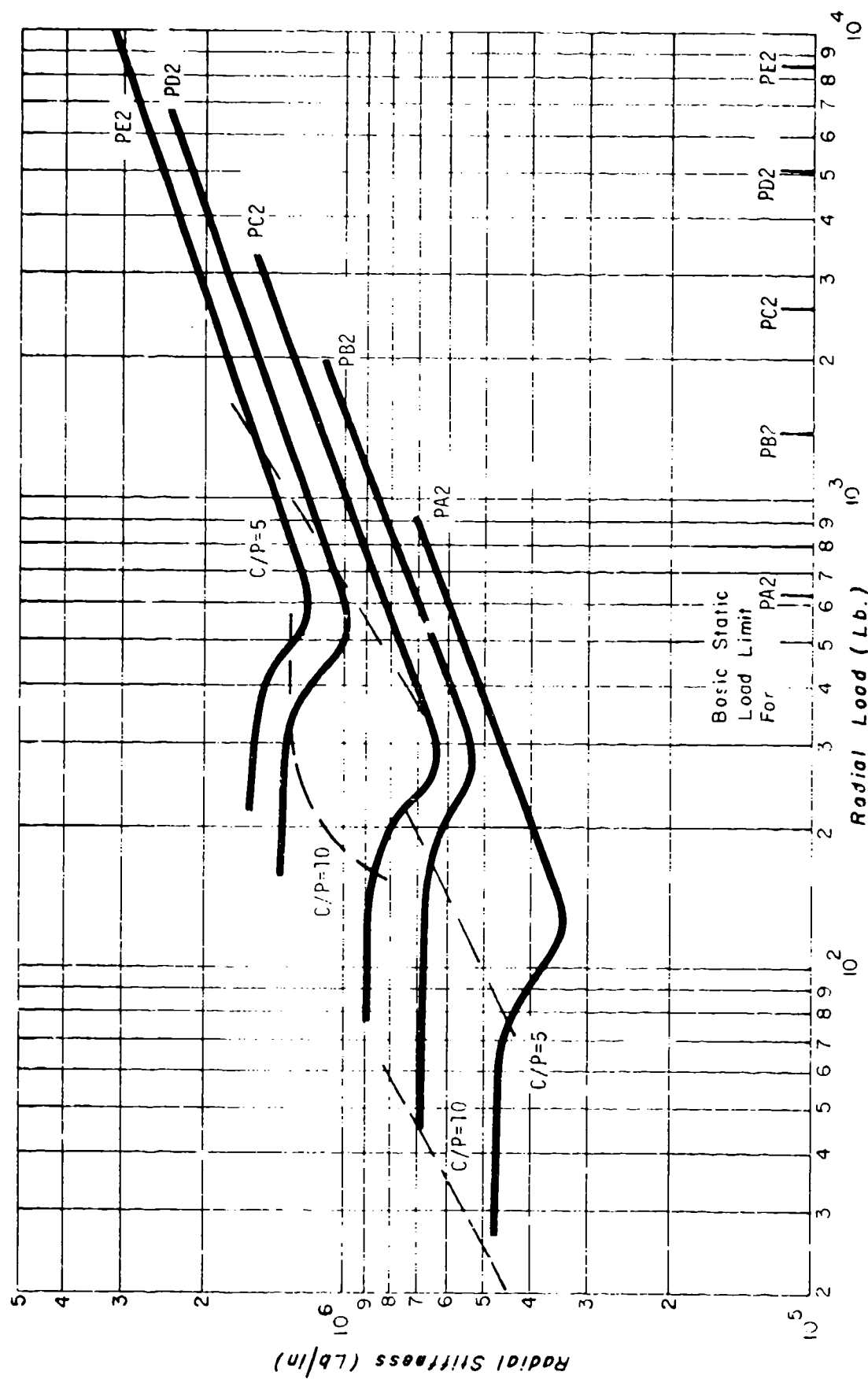


Fig. E-6 Radial Stiffness for Angular Contact Bearing,
Preload -- Moderate.
R-154

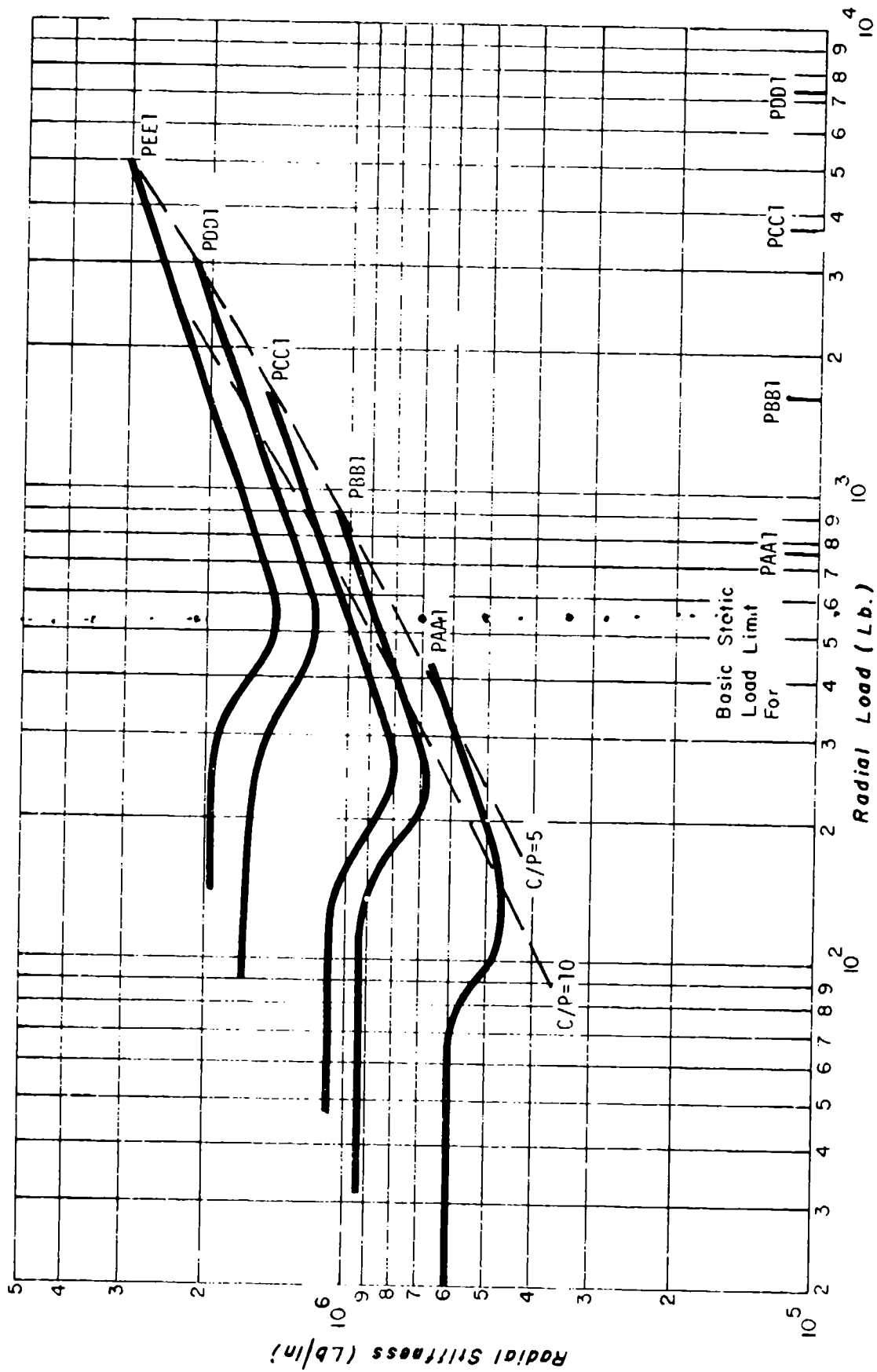


Fig. E-7 Radial Stiffness for Angular Contact Bearings,
Preload -- Moderate,
 $\theta = 15^\circ$

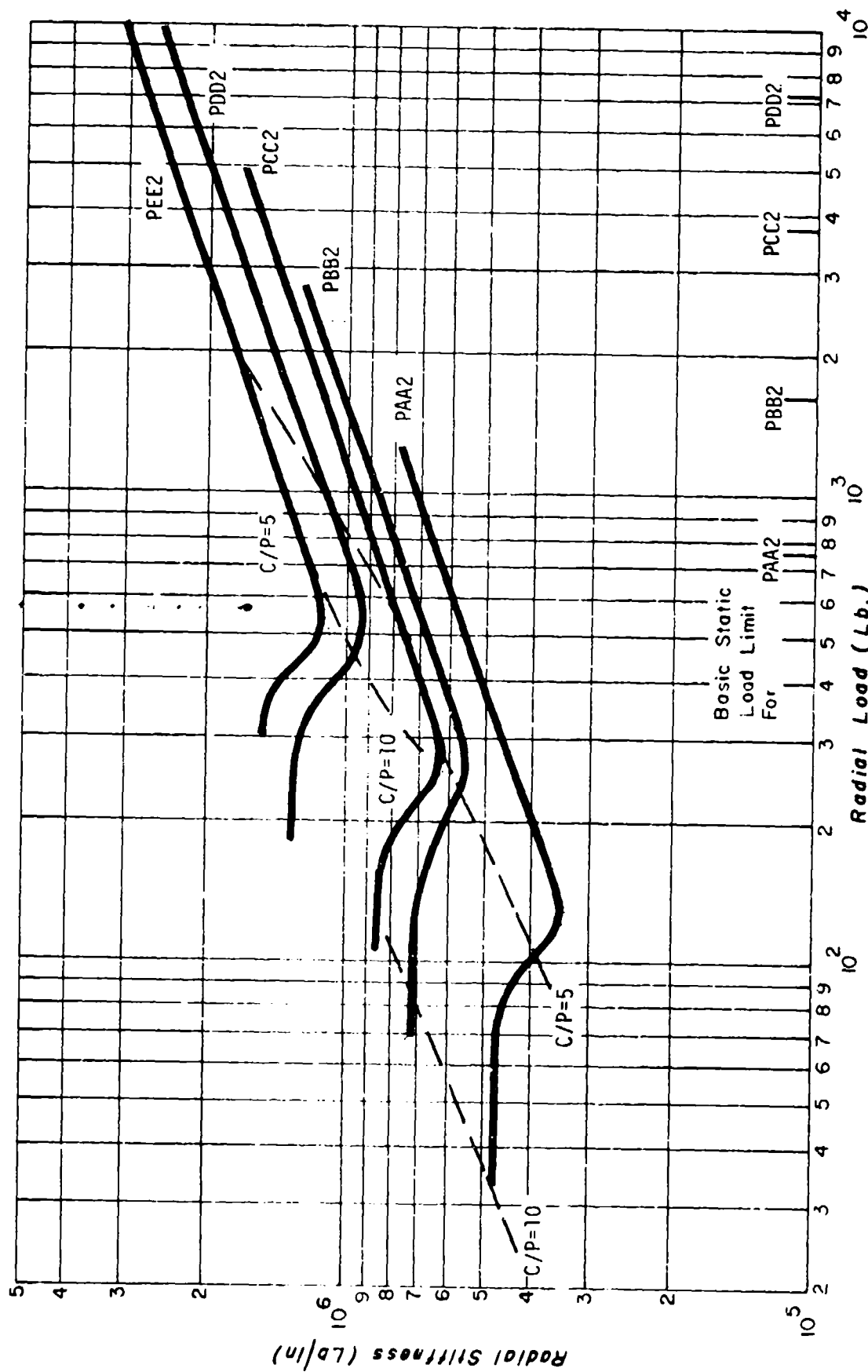


Fig. E-8 Radial Stiffness for Angular Contact Bearing,
Preload -- Moderate,
 $\beta = 150$

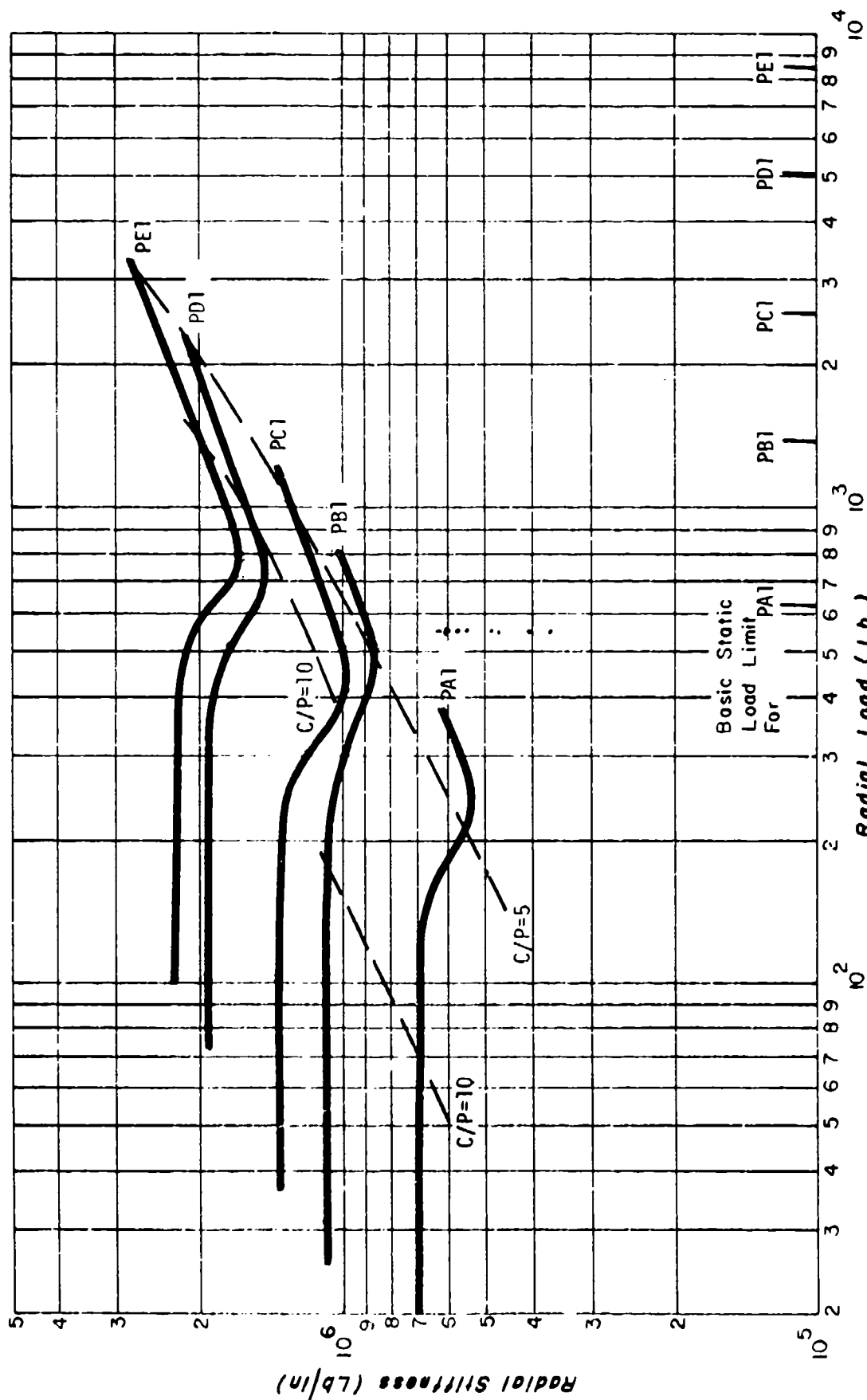


Fig. E-9 Radial Stiffness for Angular Contact Bearing,
Preload -- Preferred Heavy,
B - 15°

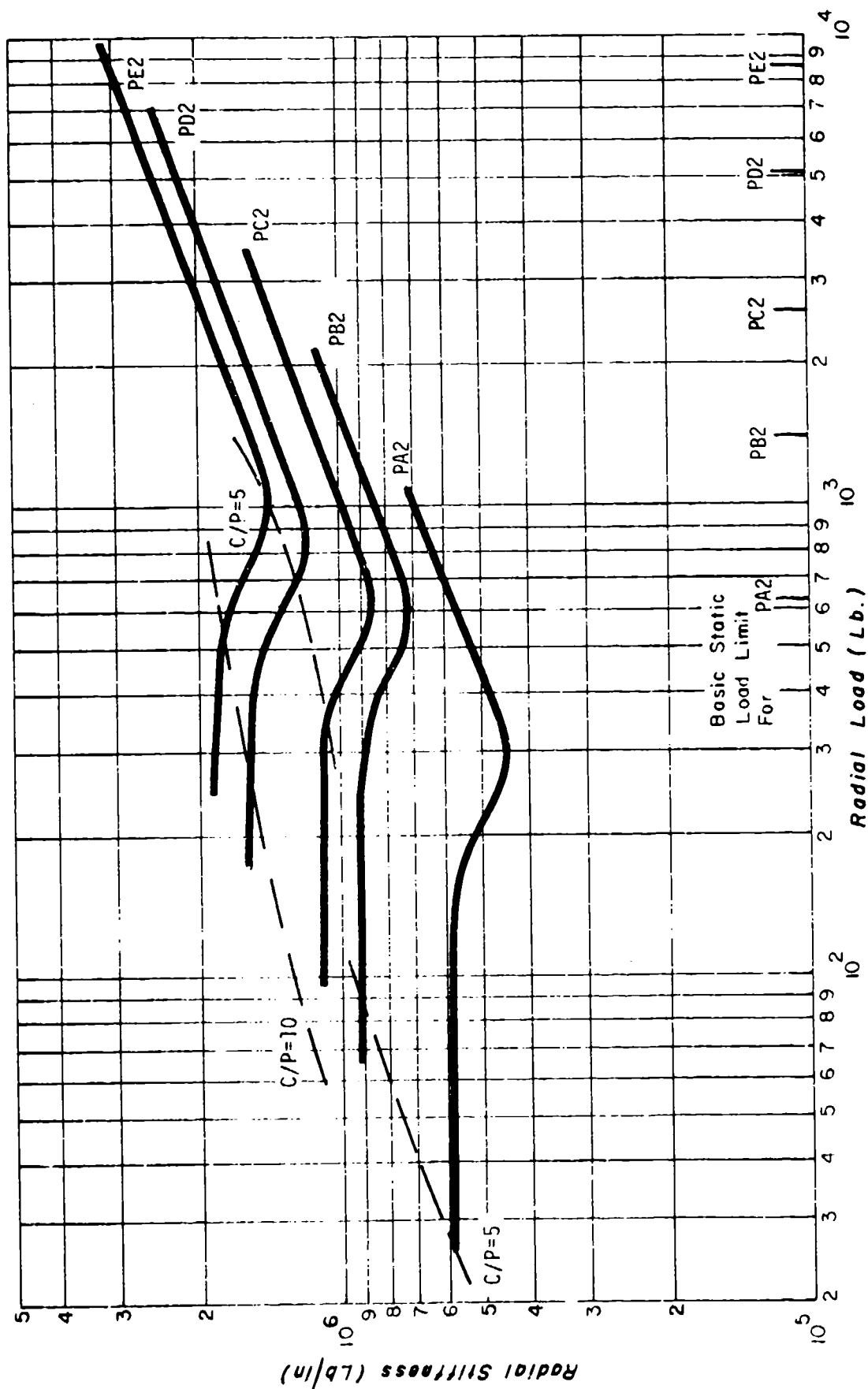


Fig. E-10 Radial Stiffness for Angular Contact Bearings,
Preload -- Preferred Heavy,
B x 150

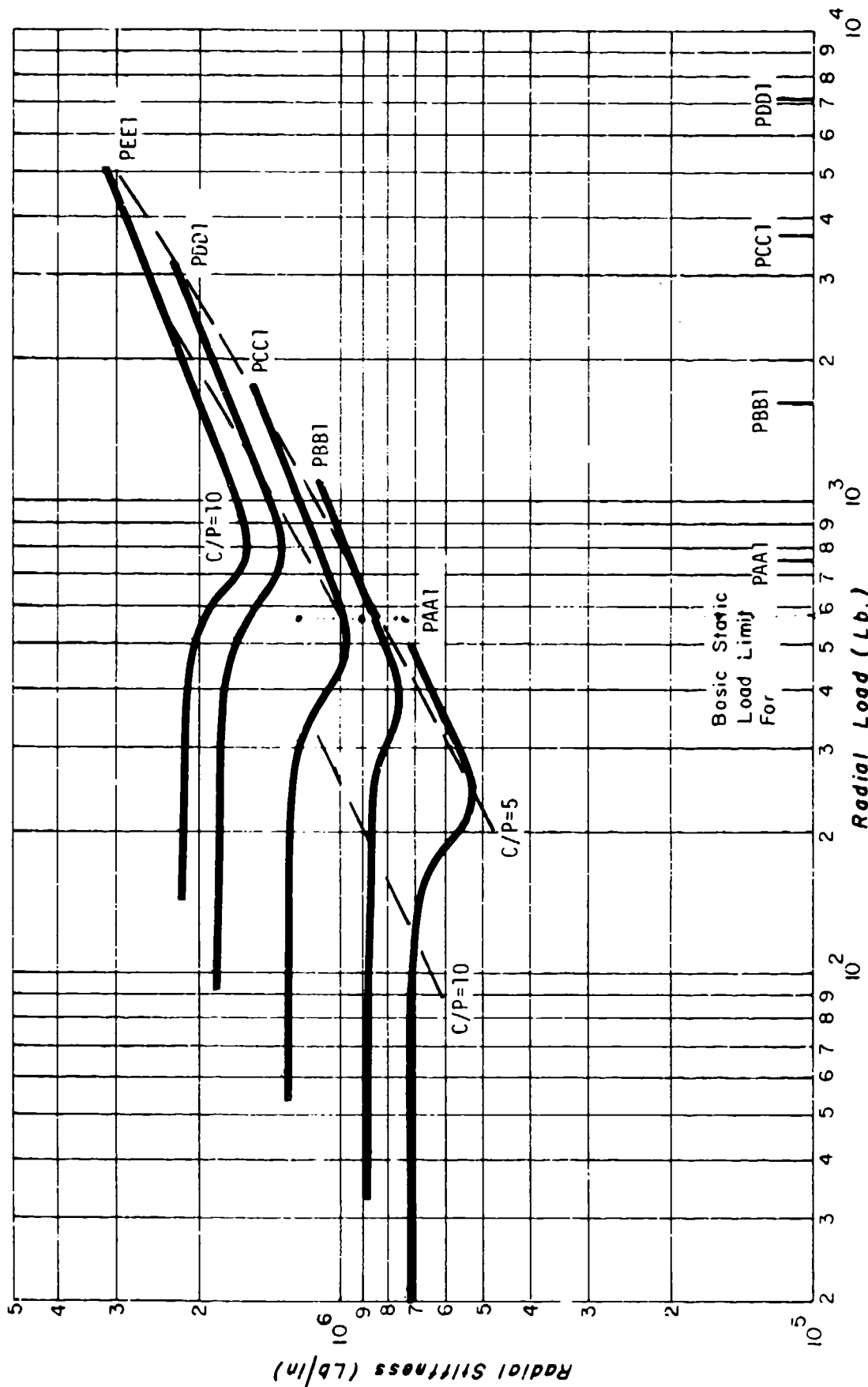


Fig. E-11 Radial Stiffness for Angular Contact Bearing,
Preload -- Preferred Heavy,
 $\beta = 15^\circ$

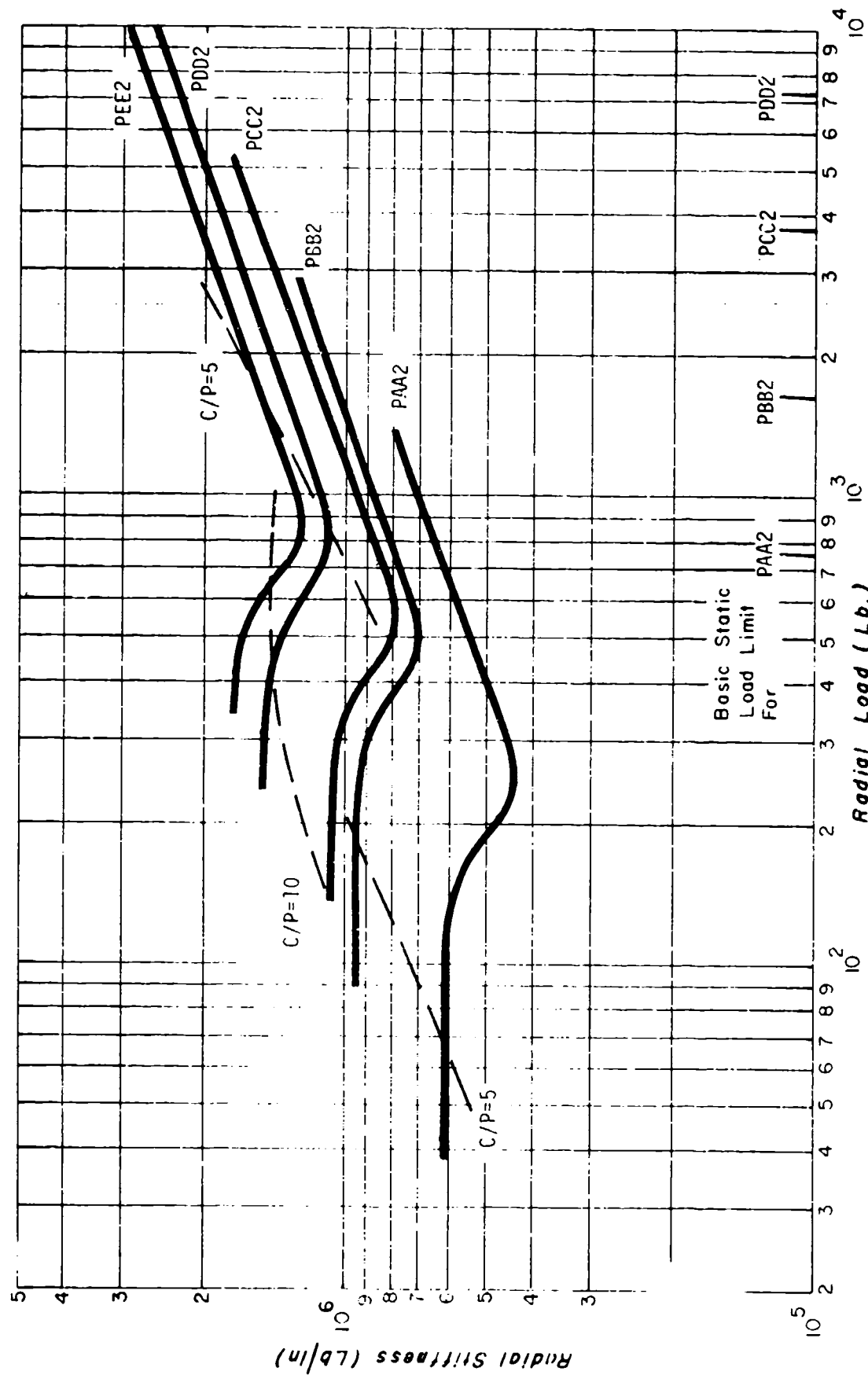


Fig. E-12 Radial Stiffness for Angular Contact Bearing,
Preload -- Preferred Heavy,
 $\beta = 15^\circ$

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