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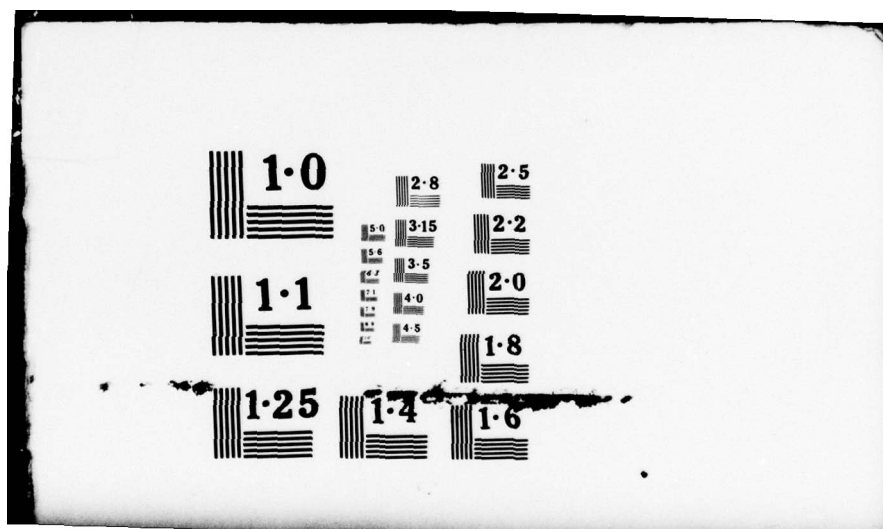
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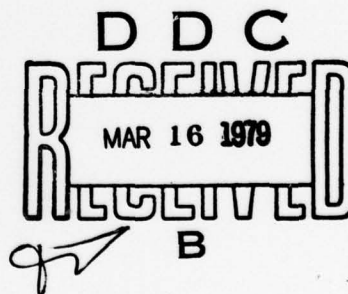
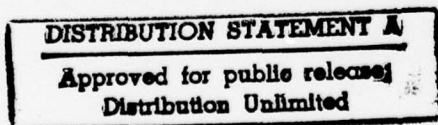


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79-0005

**Transonic Shock-Turbulent Boundary Layer
Interaction with Suction and Blowing**

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State University, Blacksburg, Va.*



**17th AEROSPACE SCIENCES
MEETING**

New Orleans, La./January 15-17, 1979

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 79-0005	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Transonic Shock-Turbulent Boundary Layer Interaction with Suction and Blowing.	5. TYPE OF REPORT & PERIOD COVERED TECHNICAL rept.	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) G. R. Inger	8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0456 NR 061-197	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Virginia Polytechnic Institute and State University Blacksburg, VA Dept. of Aerospace and Ocean Engineering	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61153N RR 023-01-01	
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Code 438 Arlington, VA 22217	12. REPORT DATE 15-17 January 1979	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 16p.	13. NUMBER OF PAGES 14	
	15. SECURITY CLASS. (of this report) UNCLASSIFIED	
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE NA	
16. DISTRIBUTION STATEMENT (of this Report) APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED		16 RR02301 12 RR0230101
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Shock-boundary layer interaction, boundary layers with mass transfer, shock turbulent boundary layer interaction		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An approximate non-asymptotic theory of weak normal shock-unseparated turbulent boundary layer interactions is given which includes the effect of surface mass transfer. The results of a parametric study of Reynolds number effects on various interaction properties without mass transfer, including skin friction are first presented along with detailed comparisons with experiment including supercritical airfoil data. The extension and application of the theory to include moderate amounts of either suction or blowing through the surface is then discussed, especially as regards the influence of mass transfer on the		

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20. ABSTRACT (continued)

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TRANSONIC SHOCK-TURBULENT BOUNDARY LAYER INTERACTION WITH SUCTION AND BLOWING

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Abstract

An approximate non-asymptotic theory of weak normal shock-unseparated turbulent boundary layer interactions is given which includes the effect of surface mass transfer. The results of a parametric study of Reynolds number effects on various interaction properties without mass transfer, including skin friction, are first presented along with detailed comparisons with experiment including supercritical airfoil data. The extension and application of the theory to include moderate amounts of either suction or blowing through the surface is then discussed, especially as regards the influence of mass transfer on the skin friction behavior. As a consequence of its influence on the boundary layer profile shape away from the wall, small suction ($-\dot{m}_w/\rho_e u_e \leq 5 \times 10^{-4}$) appreciably reduces the upstream influence and thickening effects of the interaction but hastens the onset of separation in the shock foot region.

Nomenclature

B	Mass transfer parameter, $\dot{m}_w/\rho_e u_e$
C_f	Skin friction coefficient, $2\tau_w/\rho_e u_e^2$
L	Distance to undisturbed shock location
$\dot{m}_w$	Mass flux rate (per sec.) across surface ($= \rho_w v_{w0}$)
M	Mach number
ΔP	Pressure rise across incident normal shock
p'	Interaction pressure perturbation ($p - p_1$)
Re_L, Re_δ	Reynolds numbers $\rho_e u_e L/\mu_e, \rho_e u_e \delta/\mu_e$
T	Absolute static temperature
u, v	Flow velocity components along x, y respectively
x, y	Streamwise and normal coordinates, respectively
γ	Ratio of specific heats
δ	Undisturbed boundary layer thickness
δ^*	Displacement thickness
η	y/δ
Δn	Interactive displacement growth, $\Delta \delta^*/\delta_0^*$
μ	Coefficient of viscosity
ν	Kinematic viscosity coefficient, μ/ρ
ρ	Density
τ	Shear stress
θ^*	Momentum thickness

Subscripts

1,2,3	Interaction regions, Fig. 2
e	Edge of boundary layer
0	Undisturbed solid wall boundary layer conditions

Subscripts (Continued)

ref	Eckert reference temperature value
w	Wall surface conditions

1. Introduction

The study of transonic shock-turbulent boundary layer interactions is important in the aerodynamic design of high speed aircraft wings, turbine and cascade blades in turbomachinery, and airbreathing engine inlets and diffusers. Consequently, the control and suppression of interaction effects in these applications by suction or blowing is of interest since boundary layer control (BLC) by surface mass transfer has achieved practical status. Experimental studies have established the value of using distributed suction normal to the surface for shock-boundary layer interaction separation suppression on wings^{1,2}, porous wind tunnel walls³ and within supersonic inlets⁴. There is also interest in normal blowing effects in connection with transonic shock-boundary layer interactions on wings⁵ and on mass transfer - cooled hot turbine blades⁶. Moreover, the study of interactions in the presence of distributed normal mass transfer is of fundamental interest in its own right⁷ and provides an idealized model for optimum performance estimates of BLC systems on wings and flaps and rational criteria for separation prevention by suction⁸.

Although some basic theory for the transonic shock-turbulent boundary layer interaction problem has accumulated for the zero mass transfer case⁹⁻¹¹, only rather crude overall integral methods are presently available to treat the effects of suction or blowing⁴; a more detailed theory including mass transfer is thus desirable to provide a proper analytical framework and understanding in the aforementioned applications. Likewise, there has evidently been no systematic study of attendant Reynolds number effects on unseparated interactions at realistic values pertaining to practical applications, although these effects may be significant. The purpose of this paper is to present the results of a combined study of these two features for the case of non-separating flow with normal wall (unvectored) mass transfer.

Our approach is based on extending a previously-developed approximate non-asymptotic theory of weak normal shock-turbulent boundary layer interactions¹¹ on the premise that, notwithstanding the existence of powerful numerical methods, there will be a continuing need for such analytical methods that delineate the essential physical features and parametric trends of the problem at realistic Reynolds numbers. In Section 2 the basic formulation and features of the theoretical model including surface mass transfer

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effects are given. Section 3 then discusses typical numerical results: first for zero mass transfer where heretofore - unpublished Reynolds number effects including those on upstream influence and skin friction are presented plus several comparisons with experimental data; results showing the mass transfer effects on important interaction properties are then given. Section 4 concludes with a discussion of the limitations of the theory and recommendations for further studies.

2. Theoretical Formulation

2.1 Basic Features of the Interaction Flow Model

It is well-known experimentally that when separation occurs, the disturbance flow pattern associated with normal shock-boundary layer interaction is a very complicated one involving a bifurcated shock pattern¹², whereas the unseparated case pertaining to turbulent boundary layers up to $M_1 \leq 1.3$ has instead a much simpler type of interaction pattern which is more amenable to analytical treatment (see Fig. 1). With some judicious simplifications, it is possible to construct a fundamentally-based approximate analytical theory of the problem in this latter case. For the sake of orientation and completeness, a brief summary of this theory will now be given (full details can be found in Ref. 11).

The flow consists of a known incoming isobaric turbulent boundary layer profile $M_0(y)$ subjected to small transonic perturbations due to an impinging weak normal shock. In the practical Reynolds number range of interest here [$Re_L \sim 0(10^6)$] we purposely employ a non-asymptotic disturbance flow model in the turbulent boundary layer patterned after the Lighthill-Stratford-Honda double-deck approach¹³⁻¹⁵ that has proven highly successful in treating a variety of other problems involving turbulent boundary layer response to strong rapid adverse pressure gradients^{16,17}. This was done because of the large body of turbulent boundary layer-shock interaction data that strongly supports such a model in this Reynolds number range¹⁸⁻²⁸ (including the transonic regime²⁹⁻³¹) and because of the findings of a separate general theoretical study³² showing that asymptotic theory results for very high Reynolds numbers^{9,10}, although rigorous in this limit, do not extrapolate down to the present Reynolds number range. The resulting flow model consists of an inviscid disturbance flow surrounding a non-linear shock discontinuity and underlain by a thin viscous disturbance sublayer as schematically illustrated in Fig. 2. The introduction of some further simplifications (including the assumption of small linearized disturbances ahead of and behind the nonlinear shock jump* plus neglect of the detailed shock structure within the boundary layer³⁵, which give accurate results for the overall

*As far as the overall interaction solution for $10^5 \leq Re_L \leq 10^8$ is concerned, these nonlinear shock jump conditions plus the various non-uniform viscous flow effects within the boundary layer reduce the lower Mach No. limit otherwise pertaining to the linearized supersonic theory in purely inviscid potential uniform flow - see the AIAA paper version of Ref. 11 for more detailed discussion.

properties of engineering interest provided M_1 is not too close the unity ($M_1 \leq 1.05$), then yields an approximate analytical solution consisting of linearized potential supersonic disturbance flow in region 1 plus the subsonic disturbance flow in quadrant 3 caused by the interaction-generated interface displacement $\eta_3(x)$ and the postshock perturbations along $x = 0^+$ due to the impingement of region 1 Mach wave disturbances on the shock; beneath this is a rotational inviscid disturbance flow region 2 inside the boundary layer underlain by a thin shear stress-disturbance sublayer which carries the major upstream influence, displacement thickness effect and skin friction perturbation of the interaction. In the Reynolds number range of interest here, we can approximate this thin sublayer to lie within the linear part of the basic velocity profile where $U_0 = (u_w/u_w)y$ (i.e., within the laminar sublayer of the turbulent profile**); consequently, a known solution¹³ for this sublayer can be extended to the present problem to obtain the displacement effect (effective wall position seen by the overlying inviscid disturbance flow) and the corresponding disturbance skin friction caused by the pressure disturbance field.

The matching of these regional solutions yields linear integral equations that can be readily solved by operational methods¹¹ for the disturbance pressure along both the boundary layer edge and wall. The remaining interactive flow properties can then be determined in terms of p_w' , including the interactive growth in displacement thickness and the corresponding skin friction perturbation. It is noted that this latter perturbation solution has been recently extended to include the region downstream as well as upstream of the shock and to include non-linear inertia effects in an adverse pressure gradient using the general non-dimensional wall shear-pressure solution ahead of separation given by triple deck theory³⁶ (converted to turbulent flow by expressing all results in terms of C_{f_0} instead of Re_L); the results yield the approximate analytical expression

$$C_f(x) \approx C_{f_0} - \sqrt{\beta C_{f_0}} \left[1 + \left(\frac{C_f(x) - C_{f_0}}{1.234 C_{f_0}} \right)^{1/2} \right] C_{p_w}' F(x/\delta_0) \quad (1)$$

where the non-dimensional function F is essentially unity ahead of the shock $x \leq 0$ and then decreases behind it, decaying slowly like $F \sim (x/\delta_0)^{-1/3}$ far downstream. Eq. 1 indicates that, depending on Reynolds number (C_{f_0}), a sufficiently strong interactive pressure rise can cause incipient separation [$C_f(x) \rightarrow 0$] near the shock.

This solution contains the essential global features of the mixed transonic character of the non-separating normal shock-turbulent boundary layer interaction problem including the significant lateral pressure gradient effects, upstream

**As shown below, this gives good results for $Re_L \sim 10^5 - 10^8$; however, it begins to significantly break down for $Re_L > 10^8$ where the effect of the logarithmic portion of the profile on the sublayer solution becomes significant³².

influence and interactive skin friction for an arbitrary input turbulent boundary layer profile. Moreover, many detailed comparisons with all available experimental data (Ref. 31 and below) plus recent streamlining of the computation program^{37,38}, have shown the method to give a very good account of all the important engineering features of the interaction for $Re_L \leq 10^6$ at low computational cost. Hence the theory provides a sound basis for interpreting experimental data on unseparated flows and for further extensions, in particular to allow mass transfer through the surface.

2.2 The influence of Mass Transfer

The influence of surface mass transfer on the interaction solution is two-fold: (1) it alters the incoming undisturbed flow (on which the disturbance solution depends) by changing δ_0 , τ_w and the profile shape away from the wall; (2) it further introduces new mass transfer-induced terms in the disturbance equations. Now the secondary effects (2) may in fact be neglected with good approximation under the assumed conditions of small-to-moderate normal mass transfer rates ($\dot{m}_w/\rho_e u_e \leq 10^{-3}$) typical of practical applications, according to the following considerations. Under the continued assumption that the viscous disturbance sublayer lies within the laminar sublayer region of a turbulent boundary layer, the mass transfer effect in the leading approximation does not introduce any curvature into the $u_0(y)$ profile but only alters its slope (τ_w); consequently in the viscous disturbance sublayer perturbation equations those terms proportional to $d^2 u_0/dy^2$ and v_w (which in turn is proportional to $d^2 u_0/dy^2$ via the mean flow momentum equation near the wall) can be neglected, leaving the form of these equations unchanged. Likewise, under the continued assumption that turbulent fluctuations are uncorrelated with the interactive disturbances, the explicit new terms in the overlying rotational-inviscid perturbation equations that are proportional to v_w can be neglected also, since detailed studies of the hydrodynamic stability equations³⁹ (which are very similar to those of the present problem) have shown that these terms have an altogether negligible effect throughout a high Reynolds number parallel shear flow boundary layer unless the surface mass transfer is quite large (approaching blow-off). Thus to a consistent degree of first approximation, the form of both the viscous and inviscid disturbance equations is unchanged by moderate blowing or suction provided their effect on the mean flow-based coefficients in these equations is included. It is reemphasized that the primary mass transfer effects on the viscous sublayer field and its thickness are thus fully accounted for.

The present interactive perturbation solution in principle may be used with any mean turbulent boundary layer profile input and hence could be coupled with either an experimental measurement or any desired state-of-the-art numerical prediction code. Here, to bring out clearly and efficiently where the various mass transfer effects enter, we have chosen an accurate analytical profile model⁴⁰ that has proven especially well-suited to such non-uniform flow perturbation problems⁴¹. We assume for simplicity that the blowing or suction is on the average uniform and normal to the wall, that its streamwise extent is large com-

pared to the short interaction range, and that it extends far enough upstream to have established a well-defined local equilibrium profile in the incoming boundary layer. Then the mass transfer effect on turbulent skin friction in terms of the basic parameter $B = \dot{m}_w/\rho_e u_e$ can be described by the relation⁴²

$$\frac{C_f}{C_{f0}} = 1 - 23 \left(\frac{T_{ref}}{T_e} \frac{1}{2C_{f0}} \right)^{1/2} B \quad (2)$$

where $T_{ref}/T_e = 1 + 0.038M_e^2 + 0.50(T_w/T_e - 1)$ for $\gamma = 1.4$, and C_{f0} is the zero-blowing value (we used a reference temperature-based Schulz-Grünow relation⁷). According to Eq. (2) suction, for example, increases the skin friction.

The corresponding mass transfer effect on δ_0 can be estimated as follows. Since it is well-known that the momentum thickness to boundary layer thickness ratio and the Crocco energy equation solution are both insensitive to moderate amounts of mass transfer⁴³, we can use the following approximate zero blowing relationship based on a power-law ($U_0 \sim y^{1/N}$) profile:

$$\frac{\delta_0}{\delta^*} = \frac{(N+1)(N+2)}{N} \left\{ 1 + \left[\frac{N(\gamma-1)M_e^2}{2(N+1)(N+2)} + \frac{T_w - T_e}{(N+1)T_e} \right] \right\} \quad (3A)$$

where δ^* is found from the incompressible form of the momentum equation including mass transfer:

$$\frac{d\delta^*}{dx} = \frac{C_f}{2} + B \quad (3B)$$

and x here is the running length from some upstream reference point. Thus under the aforementioned assumption that B is a constant over some region $x_1 \leq x \leq x_2$ (and zero outside) with x_1 , x_2 far in front of and behind, respectively, the interaction zone $x \sim L$, and using Eq. (2) plus the approximate power law formula $C_{f0} \sim x^{-2(N+3)}$ to facilitate analytical integration, Eq. 3B yields

$$\frac{\delta^*}{L} \approx \frac{(N+3)}{2(N+1)} \cdot C_{f0} + B \left[1 - \frac{23(N+3)}{2(N+2)} \sqrt{\left(\frac{T_{ref}}{T_e} \right) C_{f0}} \right] \quad (3C)$$

which in conjunction with Eq. 3A yields δ_0 .

In addition to τ_w , mass transfer also influences the profile shape away from the wall as given by the following turbulent boundary layer shear stress profile relation recommended by Conrad and Donaldson⁴⁰ (similar expressions also have been proposed by others⁴³):

$$\frac{\tau}{\tau_w} = 1 - 3\eta^2 + 3\eta^3 + (1 - \eta^2)(u/u_e)(2B/C_{f0}) \quad (4)$$

where $\eta \equiv y/\delta$. Further, Eq. 4 is to be used with the basic turbulent shear stress definition that

$$\frac{d(u/u_e)}{dn} = \frac{C_f}{2} Re_\delta \left(\frac{T}{T_e} \right) \left(\frac{\tau/\tau_w}{v_{eff}/v_e} \right) \quad (5)$$

Regarding the turbulent kinematic eddy viscosity distribution v_{eff} , the available experimental evidence⁴⁰⁻⁴⁵ implies that its functional form is significantly affected only by relatively large surface mass transfer rates provided the mass transfer effect on the value of τ_w is taken into account; to a consistent degree of approximation, then, the two-layer piecewise-continuous viscosity formulation developed for use in interaction problems without mass transfer by Inger and Williams⁴¹ may be applied also to the weak-to-moderate blowing or suction cases studied here. Thus we have for $u-T$:

$$\left. \frac{v_{eff}}{v_e} \right|_{INNER} = (T_w/T_e)^{1+\omega}, \quad \text{for } \eta \leq \eta_S^* \quad (6A)$$

$$\left. \frac{v_{eff}}{v_e} \right|_{OUTER} = (T_w/T_e)^{1+\omega} + .051 Re_\delta \left[\frac{T_{ref} C_f}{2 T_e} \right]^{1/2} \quad (6B)$$

$$\text{for } \eta > \eta_S^* + .16$$

and

$$\frac{v_{eff}}{v_e} = \left. \frac{v_{eff}}{v_e} \right|_{INNER} + \left[\frac{\eta - \eta_S^*}{.16} \right] \left[\left. \frac{v_{eff}}{v_e} \right|_{OUTER} - \left. \frac{v_{eff}}{v_e} \right|_{INNER} \right] \quad (6C)$$

$$\text{for } \eta_S^* < \eta < \eta_S^* + .16$$

where F is the Klebanoff intermittency factor and η_S^* is found by requiring that the no slip condition $U_0(0) = 0$ be satisfied upon inward integration of Eq. 5 from the outer initial condition $U_0(1) = U_0$. Thus the substitution of Eqs. (4) and (6) plus the Crocco integral temperature profile $T(u)$ into Eq. 5 and subsequent integration yields accurate yet fundamentally-based incoming turbulent boundary layer velocity and Mach number profiles including compressibility, heat transfer and moderate amounts of wall suction or injection. The results satisfy the proper boundary conditions including vanishing gradients at the boundary layer edge, conform to the Law of the Wall near the surface, are continuous across the entire boundary layer with a velocity defect-type behavior in the outer part, and are in good agreement with experiment over a wide range of transonic-to-moderately supersonic Mach numbers⁴¹ including the effects of surface mass transfer⁴³.

3. Numerical Results and Discussion

3.1 Zero Mass Transfer

To provide a basis for appreciating and scaling the influence of mass transfer on the interaction it is desirable to examine first some theoretical results for the impermeable wall case. Some general results for this case emphasizing the influence of Mach number have already been

given in Ref. 11 and thus need not be repeated; we concentrate here on more recent and heretofore-unpublished results for the effect of Reynolds number and comparisons with experimental data.

The predicted influence of Reynolds number on the interaction pressure field for a typical Mach number case is shown in Fig. 3. It is seen that there is a moderate Reynolds number effect even without unseparation: the extent of the interaction upstream and downstream of the incident shock decreases with increasing Reynolds number, tending toward a solution typical of the response to a simple step pressure rise at very high Reynolds number, in agreement with both experimental observations^{4,12,18,23-30} and Navier-Stokes numerical simulation⁴⁶ of turbulent boundary layer-shock wave interactions. Moreover, at the boundary layer edge the strengths of the local shock jump and post-shock expansion increase and decrease, respectively, with increasing Reynolds number; at sufficiently high Re_δ the post-shock expansion region predicted by present theory becomes very small and weak and hence probably difficult to detect experimentally.

The corresponding upstream influence (defined as the distance x_{up} ahead of the shock where the local interaction-induced pressure rise is only 5% of the overall total) at various shock strengths as a function of Reynolds number is shown in Fig. 4, plotted in ratio to δ_0 (which also of course experiences Mach and Reynolds effects). These values are of order unity ($x_{up} \sim \delta_0$), as we should indeed expect for the short-range type of interactions characteristic of turbulent boundary layers⁴⁷, and decrease markedly with both the shock strength and Reynolds number; at moderate Reynolds numbers, x_{up}/δ_0 decreases monotonically with Re_δ approximately as a power law. It is emphasized that in addition to being in full qualitative agreement with many experimental observations, both the magnitude and parametric trends of these theoretical results are completely concordant with several detailed correlation studies^{19-21,27} of upstream influence data on interacting turbulent boundary layers that directly verify the present non-asymptotic triple deck flow model.

The scale effect on the corresponding interactive displacement thickness growth (Fig. 5) is also of practical interest since this thickening often has a significant back-effect on the inviscid flow and shock position on airfoils^{31,48,50} or in channel flows^{31,37,49}. It is seen that the predicted displacement growth decreases significantly with increasing Reynolds number as would be expected (again, this trend agrees with experiment).

The typical distribution of interactive skin friction along the interaction is shown in Fig. 6 and illustrates how C_f typically decreases toward the shock owing to the adverse pressure gradient disturbance induced by the shock-boundary layer interaction (increasing shock Mach number enhances this owing to the stronger local interaction pressure gradient involved¹¹). When the interaction is strong enough, the present theory predicts vanishing skin friction and a very short separation bubble slightly behind the shock foot, which is directly confirmed by several detailed studies of the skin friction behavior across tran-

sonic turbulent boundary layer interactions^{12,29,30,51}. Reynolds number has the expected influence on the skin friction and incipient separation behavior: the relative effect of the interaction at a given shock strength decreases with Re_L , incipient separation occurring more readily at lower Reynolds number as observed experimentally.

To further validate and illustrate the theory, it is desirable to make some direct comparisons with experiment under the assumed transonic flow conditions; this is difficult, however, because most of the existing transonic shock-boundary layer interaction data on airfoils involve high local Mach numbers ($M_l > 1.3-1.4$) with a distinct lambda-shock interaction pattern and pronounced boundary layer separation which cannot be compared meaningfully with the present theory. Nevertheless, two suitable non-separated cases were found in some published NAE wind tunnel tests of supercritical airfoil sections⁵²; the measured pressure distributions and corresponding theoretical predictions (based on the local pre-shock Mach number and Reynolds number conditions at the experimentally-observed shock location) are shown in Fig. 7. The theory is seen to predict the upstream influence well, whereas it overestimates the pressure recovery downstream. This is typical of such airfoil tests and has been shown^{31,48,50} to be caused by the fact that, in contrast to the normal incident shock theoretically assumed, the actual shock occurring in airfoil experiments is usually oblique (albeit still with subsonic post-shock flow) owing to the interactive displacement thickness back-effect on the surrounding inviscid flow; this lowers the actual overall shock pressure rise in transonic flow 20 - 30% below the normal shock value at the same incoming flow Mach number. As illustrated by the good comparison with some recent DFVLR-Göttingen interaction data^{50,51} on a supercritical wing section shown in Fig. 8, when this obliquity effect is incorporated the present theory gives a satisfactory account of the interaction downstream as well as upstream of the shock.

Ackeret, Feldman and Rott's famed experimental study¹² of shock-boundary layer interaction on a plate and wall in the choked transonic flow of a slightly-curved wind tunnel nozzle provides some further examples of unseparated turbulent flow that can be rendered suitable for comparison with the present theory; we have chosen those for which both wall and inviscid pressure distributions, as well as displacement thickness, are given. It should be emphasized, however, that direct comparisons with their data involve numerous uncertainties: (a) the inviscid flow edge is only approximately defined, (b) the shock location and shape are uncertain to within .25 to .50 $\times \delta_0$, (c) error in reading the curves, (d) a significant background inviscid pressure gradient beclouds interpretation of the outer interaction zone and the incoming turbulent layer profile, (e) the upstream boundary layer history is only partially understood, especially following forced transition cases, and (f) a significant channel blockage effect occurs from the interactive boundary layer thickening, which reduces the effective theoretical shock strength and hence the downstream interaction pressure level^{31,37,49}. A typical non-separating inter-

active pressure field measured by AF & R¹² is illustrated in Fig. 9a; experimental pressure distributions along both the surface and the approximate boundary layer edge are compared in Fig. 9b with our theoretical prediction (corrected for the estimated interactive blockage effect using the Inger-Panaras method^{31,37}), while the corresponding displacement thicknesses are shown in Fig. 9c. In view of the aforementioned uncertainties from the data and limitations of the theory, the overall agreement is seen to be quite good. In particular, the following definitive features of the interaction theory are well-corroborated: (1) the magnitude, sign and streamwise extent of the lateral pressure gradient effect both ahead and behind the shock; (2) the existence of a long slow interactive pressure rise (algebraic rather than exponential¹¹) downstream of the shock; (3) the overall streamwise scale and upstream influence distance; (4) the magnitude and shape of the interactive displacement thickness growth; (5) the local inviscid pressure jump across the shock at the boundary layer edge; (6) a non-singular inviscid subsonic expansion region behind the shock due to the viscous-inviscid interaction.⁵⁰

3.2 Mass Transfer Effects

Preliminary study⁵³ indicated that the dominant influence of mass transfer comes from the effect on the profile shape away from the wall; including only the τ_w - effect gives a significant error in both magnitude and sign of the profile changes. The typical consequences of this on the interaction solution itself are illustrated in Fig. 10, which shows how the various contributions to the suction/blowing effect on the Mach no. profile influence the wall pressure distribution (analogous results were obtained for the interactive displacement thickness and skin friction). Whereas the contribution of the mass transfer effect on δ_0 is negligible compared to that on τ_w , the influence on overall profile shape is large and overwhelmingly opposite to the τ_w - effect. This conclusion was found to apply over a wide range of conditions and is concordant with the fact that transonic interactions are known to be very sensitive to the upstream turbulent boundary layer profile form factor³⁷.

Referring hereafter to the complete mass transfer model we observe that suction, because of its predominant effect in decreasing the Mach number gradient and hence enhancing the profile "fullness" away from the wall, reduces the streamwise extent and thickening of the interaction, making it appear more inviscid-like in character with a steeper adverse pressure gradient. Thus suction is qualitatively equivalent to an increase in Reynolds number. Blowing has the expected opposite effects of spreading out the interaction pressure field and increasing the displacement thickness.

Results of a systematic study of suction/blowing effects on the interaction pressure field for a typical case are presented in Fig. 11. In Fig. 11A it is seen that moderate amounts of suction significantly reduce the upstream influence distance and overall streamwise extent of the interaction and steepen the adverse wall pressure gradient, whereas blowing has equally the opposite

effect. Concordant with these trends, suction also strengthens the local shock jump at the boundary layer edge while reducing (perhaps even eliminating at high enough B) the degree and extent of the post-shock expansion region (Fig. 11B). The non-dimensional upstream influence distance versus the mass transfer parameter B is plotted in Fig. 12.

The predicted influence of suction and blowing on the interactive displacement thickness distribution is shown in Fig. 13, while the total downstream thickness is plotted vs. B in Fig. 14. It is seen (as expected) that this thickening is significantly influenced by mass transfer; for example, the moderate suction value $B = -0.0003$ reduces $\Delta\delta^*(\infty)/\delta_0^*$ nearly three-fold. It is noted that the influence of Reynolds number on the interaction in the presence of surface mass transfer effects described below was also studied⁵³; suffice it here to state that over the range of values $|B| \leq .0005$, its relative effect was found to be quite similar to that shown above in Section 3.1.

The interactive local skin friction is of particular interest since it identifies possible flow separation and provides re-initialization data for continuing boundary layer calculations downstream of the interaction zone. We note in this regard that although the present theory is no longer valid for separated flow [$C_f(x) < 0$ over some portion of the wall] it is still useful to indicate trends toward this situation, i.e., where and when incipient separation ($C_f \rightarrow 0$ at some x) first occurs. As indicated in Fig. 15, the influence of mass transfer involves two opposing effects: far upstream or downstream the skin friction-increasing effect of suction dominates (which tends to delay separation)[†], whereas in the neighborhood of the shock foot ($|x/\delta_0| \leq 1$) the suction-induced steepening of the local adverse pressure gradient becomes of controlling importance and C_f is actually reduced. Thus in contrast to what occurs in non-interacting boundary layers, slight suction here actually has a locally-adverse effect in hastening interactive incipient separation under the shock; rather, it is small amounts of blowing which can delay (and in the present example eliminate altogether) such separation. When the blowing is sufficiently strong, Fig. 15 indicates that the upstream skin friction reduction will eventually exceed that in the shock foot region, whereupon $C_{f,min}$ and hence the incipient separation point begin to move forward in front of the shock. Presumably, at even higher blowing rates separation ultimately will occur well upstream of the shock; however, the present theory must be modified to study this question since several of the underlying simplifying assumptions become doubtful at these rates ($B \geq 10^{-3}$).

The aforementioned favorable local effect of small blowing is further emphasized by the skin friction result shown in Fig. 12 for $M_1 = 1.30$ (this Mach number being the often-quoted nominal incipient separation value for turbulent boundary

layer normal shock interactions^{1,4}). The present theory indeed shows a small separation region approximately one boundary layer thickness in length under the shock foot, in rough agreement with experimental findings. Moreover, it is seen that this can be eliminated by small blowing rates exceeding $B \geq 5 \times 10^{-3}$ to produce a fully attached unseparated flow throughout the interaction zone, albeit one which emerges downstream with an extremely low C_f level that is completely out of local equilibrium.

It is noted that while there presently exists no systematic quantitative experimental data on transonic shock-boundary layer interactions with surface mass transfer, some observations in a channel flow test have been reported⁵⁴ that qualitatively support the foregoing conclusions. Some recent experimental results on a supersonic compressive interaction flow with wall suction⁵⁵ also observe our theoretically-predicted reduction of interactive thickening and upstream influence. However, to the authors' knowledge, there is no data regarding the influence of mass transfer in the shock foot region of transonic shock-unseparated turbulent boundary layer interactions.

4. Concluding Remarks

This non-asymptotic study of weak normal shock-turbulent layer interactions for two-dimensional non-separating flows including mass transfer has shown that even small amounts of suction ($-\dot{m}_w/\rho_e u_e \leq 5 \times 10^{-4}$) appreciably reduce both the streamwise scale and thickening effect of the interaction but hasten the onset of separation slightly behind the shock foot. Equal amounts of weak blowing on the other hand can completely eliminate interaction-induced separation. These results were found to be mainly a consequence of the mass transfer effect on the incoming boundary layer Mach number profile shape which in turn significantly affects the interaction pressure distribution and hence the local skin friction. The present theory provides a useful analytical framework for the evaluation and parametric study of the interaction mass transfer effects in a variety of practical applications. Moreover, it provides the basis for further improvement: extension of the analytical model to larger mass transfer rates by including their explicit effect on the viscous disturbance sublayer and mean turbulent boundary layer eddy viscosity equations. With such added features, it should be possible to examine the basic question of how mass transfer influences incipient separation over a very wide range of suction or blowing rates as well as Mach and Reynolds numbers without the need for present-day empiricisms.

Acknowledgment

This work was supported by the Office of Naval Research under contract N00014-75-C-0456.

[†] Thus in the large scale, suction always has the beneficial influence of promoting a more rapid equilibration of the turbulent skin friction downstream.

[‡] It should be noted that the often-important blockage corrections in such tests due to interactive thickening of the wall boundary layers can be significantly reduced by suction (see Fig. 13).

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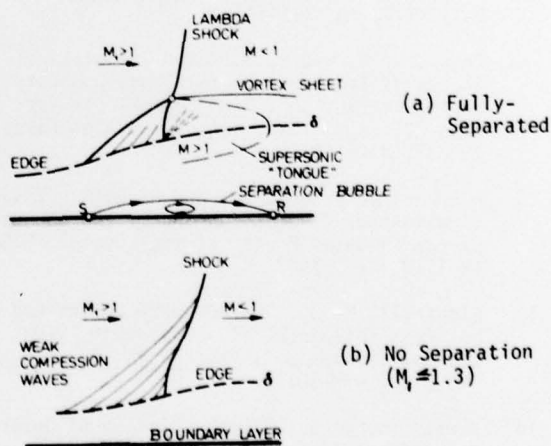


Fig. 1 Effect of Separation on Interaction Flow Pattern

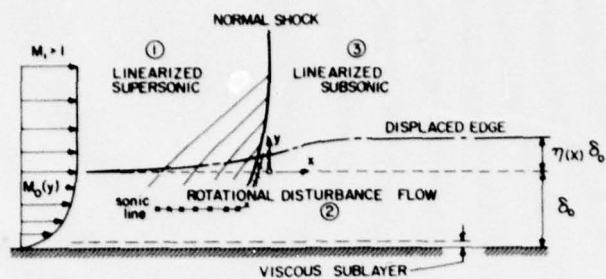


Fig. 2 Theoretical Model of Non-Separating Interaction (Schematic)

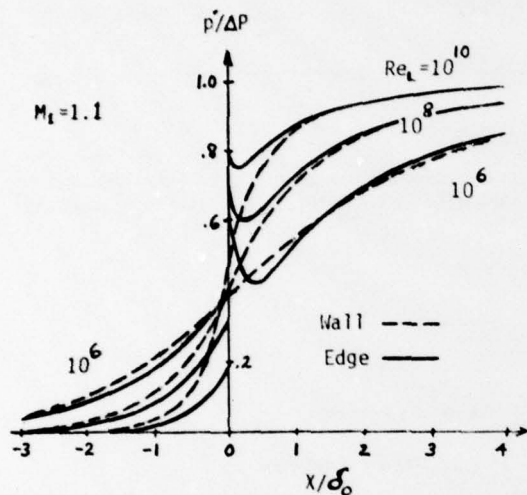


Fig. 3 Reynolds Number Influence on Interaction Pressure Field

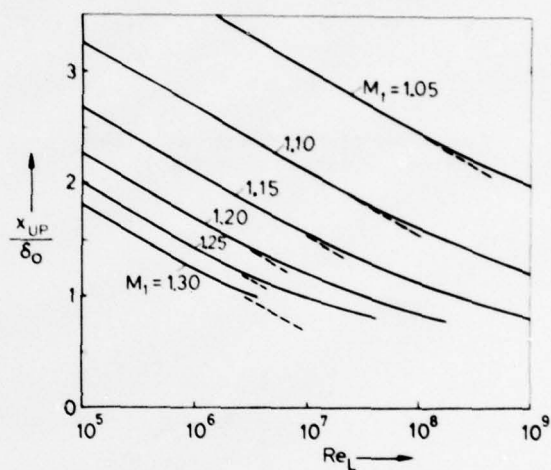


Fig. 4 Upstream Influence vs. Mach and Reynolds Number

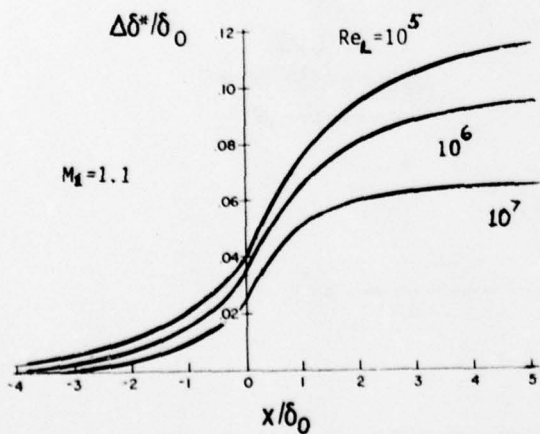


Fig. 5 Scale Effect on Interactive Displacement Thickness Growth

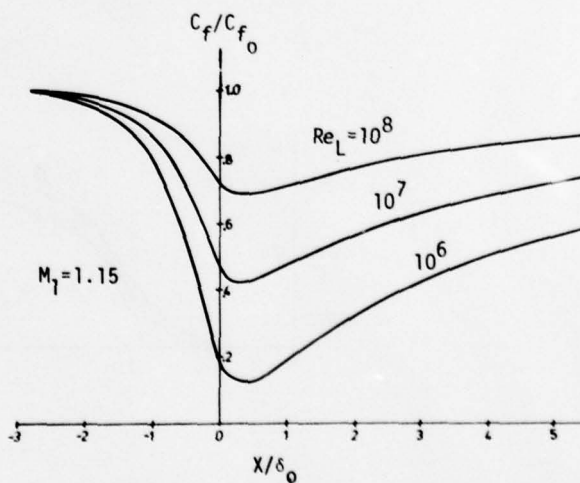
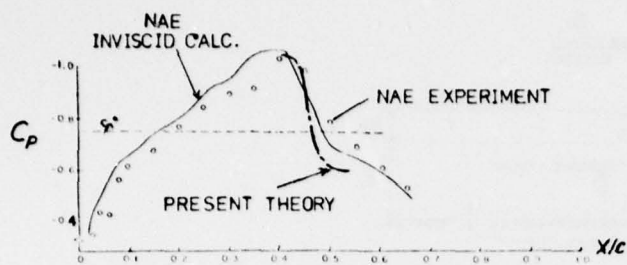


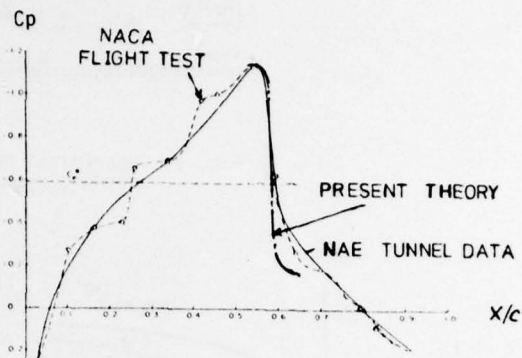
Fig. 6 Reynolds Number Influence on Interactive Skin Friction Distribution



A. Comparison of Predicted Local Interaction Pressures with NAE Experiments for a Supercritical NACA 64A410 Airfoil: $M_\infty = .70$, $Re_{c\infty} = 8 \times 10^6$

Fig. 7

Comparison of Theory with NAE Supercritical Airfoil Data (Ref. 52)



B. Comparison of Predicted and Experimental Pressures for the NACA 64A410 Airfoil: $M_\infty = .751$, $Re_{c\infty} = 35 \times 10^6$

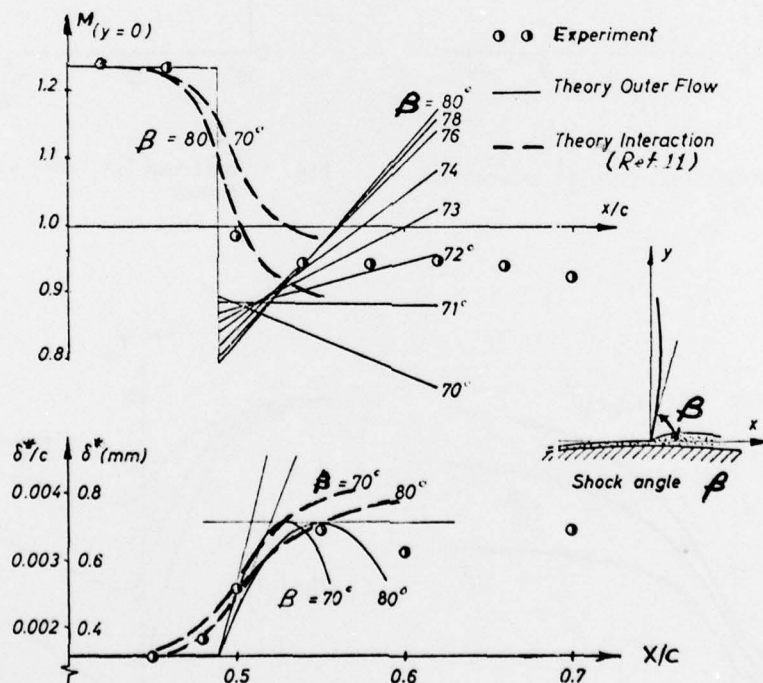
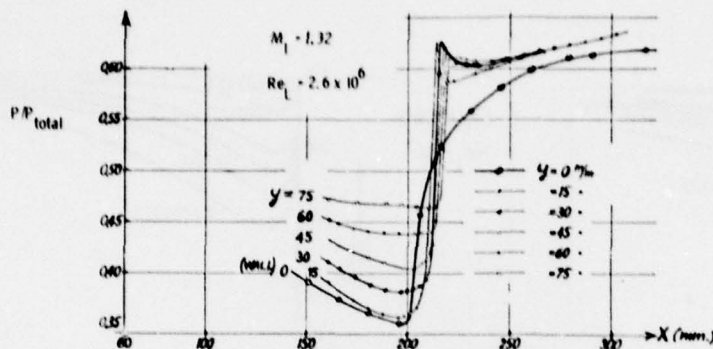
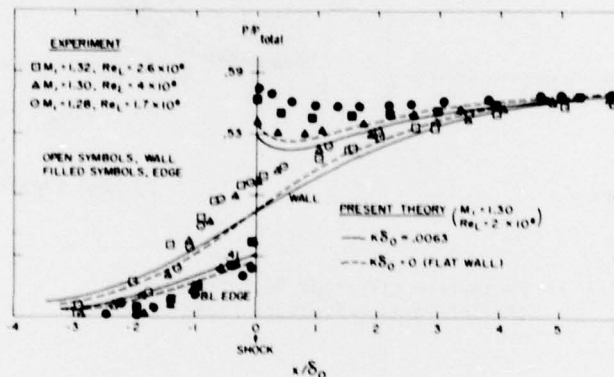


Fig. 8

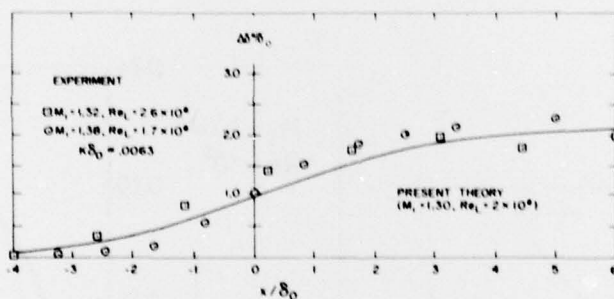
Comparison of Present Theory with DFVLR-Göttingen Flow Measurements on Supercritical Wing Section (Ref. 51)



(a) Typical Pressure Data

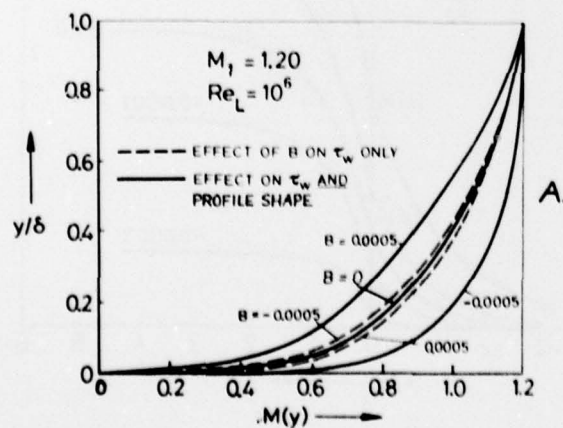


(b) Wall and Boundary Layer Edge Pressure Comparisons

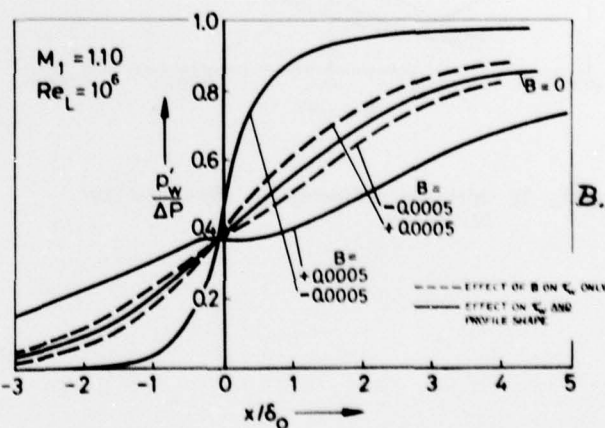


(c) Interactive Displacement Thicknesses

Fig.9 Comparison with Wind Tunnel Data of Ackeret, Feldman and Rott¹²

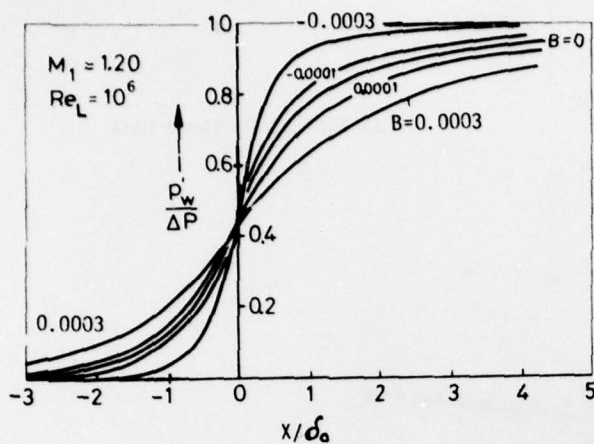


A.

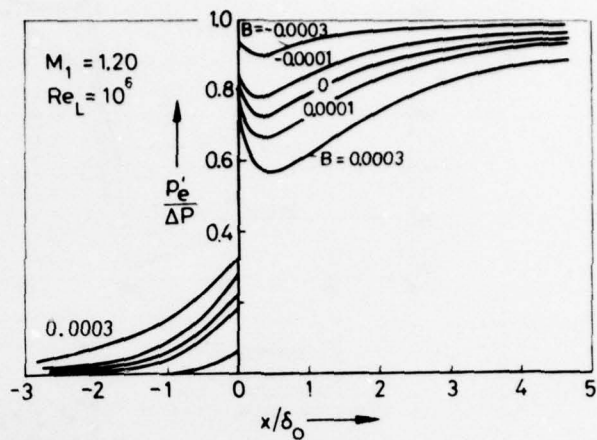


B.

Fig. 10 Influence of the Mass Transfer Effect-Model on the Interaction Solution



(a) Wall Pressure



(b) Edge Pressure

Fig. 11 Parametric Effect of Mass Transfer on the Interaction Pressure Field

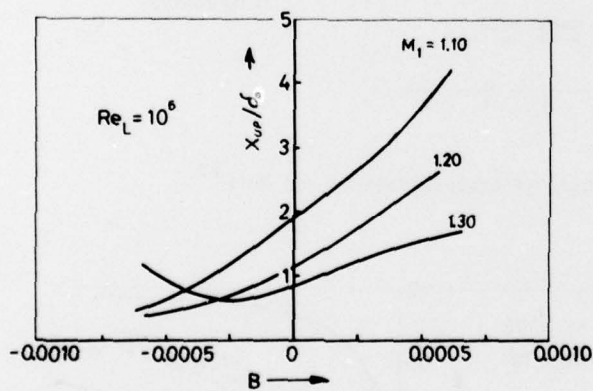


Fig. 12 Upstream Influence vs. Mass Transfer Parameter

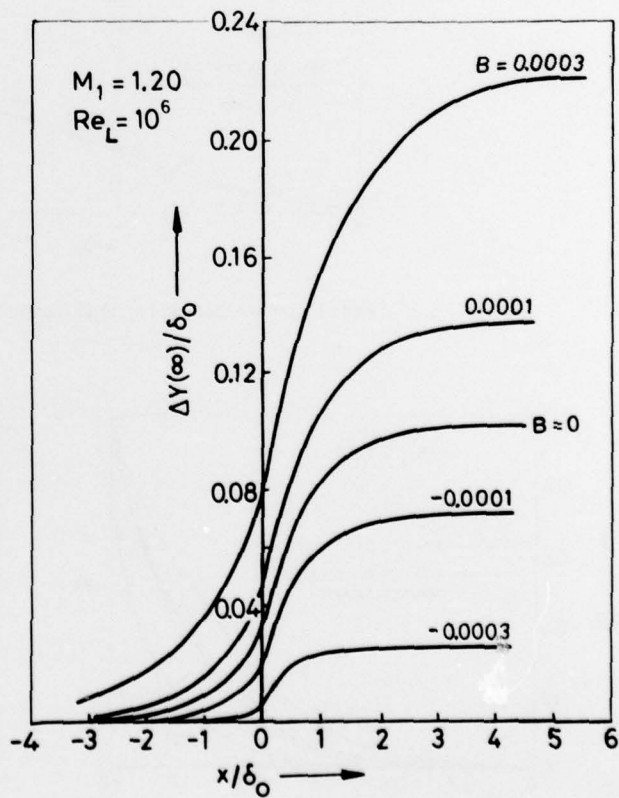


Fig. 13 Interactive Displacement Thickness Distributions with Suction or Blowing

Fig. 14

Influence of Mass Transfer on Downstream
Interactive Thickening of the Boundary
Layer

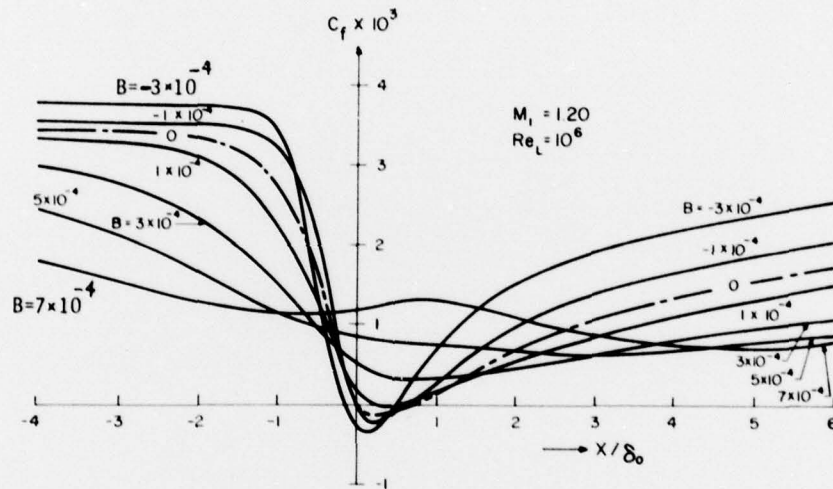
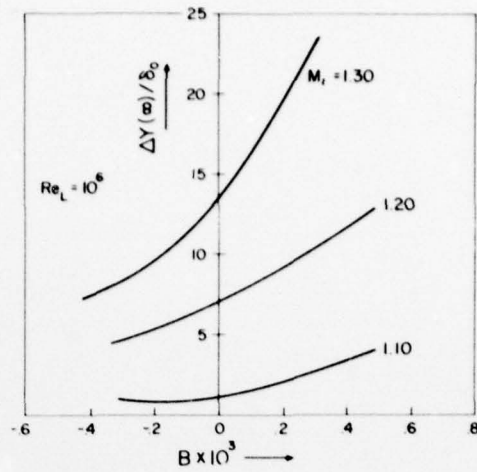


Fig. 15. Suction and Blowing Effects on Interactive Skin Friction, $M_1 = 1.20$

Fig. 16

Blowing Effect on Skin Friction and
Incipient Separation, $M_1 = 1.30$

