

AD-A066 165

FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB OHIO F/G 20/4
SINGULAR INTEGRODIFFERENTIAL EQUATION OF THE PROBLEM OF OUTFLOW--ETC(U)
JUL 78 E DUYSEYEV
FTD-ID(RS)T-1119-78

UNCLASSIFIED

NL

| OF |

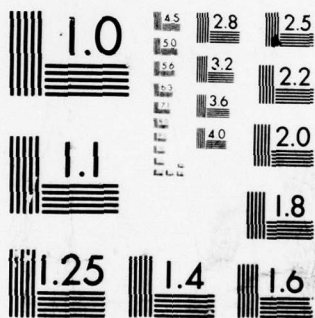
AD
A066 / 65



END
DATE
FILMED

'5--79'

DDC



MICROCOPY RESOLUTION TEST CHART
 NATIONAL BUREAU OF STANDARDS-1963-A

1

AD-A066165

FOREIGN TECHNOLOGY DIVISION



SINGULAR INTEGRODIFFERENTIAL EQUATION OF THE PROBLEM
OF OUTFLOW OF LIQUID FROM UNDER A SHIELD

By

E. Duysheyev



DDC
REFORMED
19 MAR 1979
E

Approved for public release;
distribution unlimited.

78 12 22 233

EDITED TRANSLATION

FTD-ID(RS)T-1119-78

17 July 1978

MICROFICHE NR: *AD-78C-000983*

CSB77011688

SINGULAR INTEGRODIFFERENTIAL EQUATION OF THE
PROBLEM OF OUTFLOW OF LIQUID FROM UNDER A SHIELD

By: E. Duysheyev

English pages: 10

Source: Ploskoye i Prostranstvennoye
Teheniye Zhidkosti i Gaza, Izd-Vo
"Ilim," Frunze, 1972, pp. 86-93

Country of Origin: USSR
Translated by: Robert Allen Potts
Requester: FTD/TQTA
Approved for public release;
distribution unlimited.

ACCESSION for	
NTIS	White Section <input checked="" type="checkbox"/>
DDC	Buff Section <input type="checkbox"/>
UNANNOUNCED <input type="checkbox"/>	
JUSTIFICATION _____	
BY	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	Avail and/or SPECIAL
A	

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:
TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З э	<i>З э</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh ⁻¹
cos	cos	ch	cosh	arc ch	cosh ⁻¹
tg	tan	th	tanh	arc th	tanh ⁻¹
ctg	cot	cth	coth	arc cth	coth ⁻¹
sec	sec	sch	sech	arc sch	sech ⁻¹
cosec	csc	csch	csch	arc csch	csch ⁻¹

Russian	English
rot	curl
lg	log

1119

SINGULAR INTEGRODIFFERENTIAL EQUATION OF THE PROBLEM OF OUTFLOW OF LIQUID FROM UNDER A SHIELD

E. Duysheyev.

Various problems of the outflow of liquid from under a shield, taking into account force of gravity and force of surface tension, were studied by many authors. A detailed review of works, dedicated to such problems, is available in [1, 2]. Let us examine the steady motion of incompressible liquid (taking into account the simultaneous action of forces of gravity and surface tension) between horizontal walls PA and POE. In this case the upper wall PA on the right terminates with shield AB, slanted at angle α ($0 < \alpha < \pi$), and the lower - coincides with the axis of abscissa. The axis of ordinates passes through point O and the bottom edge B of shield AB. Liquid, with density ρ , flows out from under the shield with free surface BE.

Let the magnitude of speed v_0 and the thickness of stream h_0 at point E be assigned. It is required to find speed v on BE. In such a setting this problem is reduced in [3] to a system of two equations, the study of which was produced by the method of successive approximations for certain values of parameters. However, the question about the solvability of the problem for other small values of parameters in [3] remained open.

In this work the system of equations of the problem is studied for small Froude number $\varepsilon = \frac{v_0^2}{g h_0}$ characterizing the forms of the flow lines.

Let us introduce

$$\varepsilon_0 = \frac{g h_0}{v_0^2}; \quad \lambda = \frac{\sigma}{2 h_0 v_0^2}; \quad V = \frac{v}{v_0}; \quad \frac{y}{h_0} = y, \quad (1)$$

$$u_1(t) = \varepsilon_0 (y(t) - 1), \quad u_2(t) = \lambda \theta(t). \quad (2)$$

Then from system (1.14), (1.6) of work [3] we easily obtain the system of nonlinear singular integrodifferential equations with small parameter $\varepsilon < 1$:

$$\varepsilon \frac{du_s}{dt} = \mathcal{D}(t, u_1, \delta, \Omega), \quad u_s(t) = \lambda [\Omega(t) - f_s(t)], \quad (3)$$

$$|t| \leq 1,$$

$$u_s(-1) = 0, \quad S = 1, 2. \quad (4)$$

Here for brevity of writing the following designations are used:

$$P(\cdot) = -\frac{\sin[\Omega(t) - f_0(t)]}{\pi(t+\epsilon) \cdot V(u, \delta)}, \quad \Omega(t) = \frac{\sqrt{t-t^2}}{\pi} \int_{-1}^1 \frac{\ln V(u, (x), \delta)}{(x-\epsilon)\sqrt{1-x^2}} dx,$$

$$V(\cdot) = \sqrt{1 - 2u_1(t) + \delta^2(t) - \delta(t)}, \quad (5)$$

$$\delta(t) = (1+t)u_1'(t), \quad u_1(\cdot) > 0;$$

$$f_0(t) = \frac{2\alpha}{\pi} \cdot \arctg \sqrt{\frac{(\alpha-1)(1+t)}{\alpha+1(1-t)}}, \quad \alpha > 1.$$

The solution of problem (3), (4) we will seek in the form:

$$\begin{aligned} u_s(t, \epsilon) &= W_s(t) + \epsilon \eta_s(t, \epsilon) + \Pi_s(z, \epsilon), \\ \Omega(\cdot) &= \omega(t) + \epsilon T(t, \epsilon) + q(t, \epsilon), \end{aligned} \quad (6)$$

where $T(t, \epsilon)$, $q(t, \epsilon)$ are expressed through sought functions $\eta_s(t, \epsilon)$ and $\Pi_s(t, \epsilon)$, and $W_s(t)$ - solution of the confluent system:

$$\omega(t) = \frac{\sqrt{t-t^2}}{\pi} \int_{-1}^1 \frac{\ln V(W_s(x), \delta)}{(x-\epsilon)\sqrt{1-x^2}} dx = f_0(t), \quad W_s(t) = 0. \quad (7)$$

The physical sense of equations (7) involves the fact that with $\epsilon=0$ the free surface BE of the stream flowing out from under the shield AB is horizontal. Taking into account (4), from (7) we find:

$$W_s(t) = \frac{t}{2} \left[1 - \exp\left(-\frac{\pi}{2} \int_{-1}^1 \left(\frac{1}{\alpha-1} - \frac{1}{\alpha+1}\right) f_0(x) dx \right) \right], \quad W_s(t) = 0(\delta)$$

Substituting now (6) in (3), we will have:

$$\begin{aligned} \epsilon^2 \eta_s'(t, \epsilon) + \epsilon W_s'(t) &= P(t, W_s + \epsilon \eta_s, \epsilon \delta_s, \omega + \epsilon T) - P(t, W_s, 0, \omega), \\ \epsilon \eta_s(t, \epsilon) &= \epsilon T(t, \epsilon) = \frac{\lambda \sqrt{t-t^2}}{\pi} \int_{-1}^1 \frac{\ln V(W_s + \epsilon \eta_s, \epsilon \delta_s) - \ln V(W_s(x), 0)}{(x-\epsilon)\sqrt{1-x^2}} dx, \end{aligned} \quad (8)$$

$$\begin{aligned} \varepsilon \Pi'_i(t, \varepsilon) &= \mathcal{P}(t, \theta_1 + \Pi_i, \theta_2 + \tau_i, \theta_3 + q) - \mathcal{P}(t, \theta_1, \theta_2, \theta_3), \\ \Pi_i(t, \varepsilon) &= \lambda q(\tau, \varepsilon) = \frac{\lambda \sqrt{1-\varepsilon^2}}{\varepsilon} \int_{-1}^1 \frac{\ln V(\theta_1 + \Pi_i, \theta_2 + \tau_i) - \ln V(\theta_1, \theta_2)}{(x-1)\sqrt{1-x^2}} dx, \end{aligned} \quad (10)$$

$$\begin{aligned} \theta_1 &= W_i(t) + \varepsilon \eta_i(t, \varepsilon), \quad \theta_2 = \varepsilon \alpha_i(t, \varepsilon) = \varepsilon(1+\varepsilon) \eta'_i(t, \varepsilon), \\ \theta_3 &= \omega(t) + \varepsilon T(t, \varepsilon), \quad \tau_i(t, \varepsilon) = (1-\varepsilon) \Pi'_i(t, \varepsilon). \end{aligned} \quad (11)$$

Let functions $\eta_i(\cdot), T(\cdot), \Pi_i(\cdot), q(\cdot)$ be represented with the aid of

$$\eta_i(t, \varepsilon) = \sum_{n=1}^{\infty} \eta_{in}(t, \varepsilon) \varepsilon^{n-1}, \quad T(t, \varepsilon) = \sum_{n=1}^{\infty} T_n(t) \varepsilon^{n-1}, \quad (12)$$

$$\begin{aligned} \Pi_i(t, \varepsilon) &= \sum_{n=0}^{\infty} \Pi_{in}(t, \varepsilon) \lambda_i^n, \quad q(t, \varepsilon) = \sum_{n=0}^{\infty} q_n(t, \varepsilon) \lambda_i^n, \\ \lambda_i &= \frac{1}{\varepsilon}, \end{aligned} \quad (13)$$

where $\eta_{in}(t)$ and $\Pi_{in}(t, \varepsilon)$ satisfy Holder boundary condition, and

$$\eta_{in}(-1) = \alpha_n \lambda_i^{n-1}, \quad \Pi_{in}(-1, \varepsilon) = \alpha_{m_i} \varepsilon^{n-1}, \quad \alpha_r = \text{const.} \quad (14)$$

The problem now involves finding functions $\eta_{in}(t)$ and $\Pi_{in}(t, \varepsilon)$.

Let us assume that analytical, with respect to u, δ and Ω , functions $\mathcal{P}(\cdot), \ln V(\cdot)$ together with their derivatives of $(i+k+j)$ and $(i+k)$ orders

$$\begin{aligned} P_{i,j}(t) &= \frac{P_{i,j}(t, W_i, Q_i)}{i! j!}, \quad \beta_{i,j}(t) = \frac{B_{i,j}(W_i, Q_i)}{i! j!}, \\ B &= \ln V. \end{aligned}$$

satisfy Holder boundary condition, where from structure $\mathcal{P}(\cdot)$ and (7)

follows $R_{00} = P_{0,0} = R_{2,0} = 0$, $\kappa, \tau \geq 1$. \mathcal{P} Using the analyticity of functions $\mathcal{P}(\cdot)$, $\ln V(\cdot)$ and placing (12) in (9), and then equating the coefficients with identical powers of ϵ , we obtain a system of linear singular integral equations:

$$T_n(t) = \Psi_{0n}(t) \equiv (1+t) [f_{0n}(t) - \gamma'_{2,n-1}(t)] V(W_2, 0), \quad n \geq 1, \quad (15)$$

$$\gamma_{2n}(t) = \lambda T_n(t) \equiv \frac{\lambda \sqrt{1-t^2}}{\pi} \int_{-1}^1 \frac{\beta_{20}(x) \gamma_{2n}(x) + \beta_{02}(x) \alpha_{2n}(x) + f_{0n}(x)}{(x+t)\sqrt{1-x^2}} dx.$$

Here for convenience of reasonings there are used designations:

$$\alpha_{2n}(t) = (1+t) \gamma'_{2n}(t), \quad \gamma'_{20}(t) = W_2'(t), \quad f_{02} = f_{20} = 0,$$

$$f_{0n}(t) = \sum_{i=0}^{n-1} C_{n-i}^i P_{00,i}(t) \prod_{j=i}^{i+1} T_n(t) + \sum_{\substack{i=0 \\ i \neq n}}^n P_{00,i}(t) \cdot$$

$$\cdot \left[\left(\sum_{\alpha=1}^{n-i} \gamma_{2\alpha}(t) \right)^i \left(\sum_{\alpha=1}^{n-i} T_\alpha(t) \right)^{\kappa} \right] + P_{0i,n}(t) \left[\left(\sum_{\alpha=1}^{n-i} \alpha_{2\alpha}(t) \right)^i \cdot \right.$$

$$\left. \cdot \left(\sum_{\alpha=1}^{n-i} T_\alpha(t) \right)^{\kappa} \right] + \sum_{\substack{i=0 \\ i \neq n}}^n P_{i,n}(t) \left[\left(\sum_{\alpha=1}^{n-i} \gamma_{2\alpha}(t) \right)^i \left(\sum_{\alpha=1}^{n-i} \alpha_{2\alpha}(t) \right)^{\kappa} \left(\sum_{\alpha=1}^{n-i} T_\alpha(t) \right)^{\tau} \right],$$

$$f_{2n}(t) = \sum_{i=1}^{n-1} C_{n-i}^i \left[\beta_{i+1,0}(t) \prod_{j=i}^{i+1} \gamma_{2n_j}(t) + \beta_{0,i+1}(t) \prod_{j=i}^{i+1} \alpha_{2n_j}(t) \right] +$$

$$+ \sum_{\substack{i=0 \\ i \neq n}}^n \beta_{i,n}(t) \left[\left(\sum_{\alpha=1}^{n-i} \gamma_{2\alpha}(t) \right)^i \left(\sum_{\alpha=1}^{n-i} \alpha_{2\alpha}(t) \right)^{\kappa} \right], \quad \sum_{j=1}^i \kappa_j = n,$$

$$i, \kappa, j \geq 1,$$

where sums \sum' are expanded by rules indicated in [4]. From the structure of functions $T_n(t)$, $\mathcal{P}(\cdot)$, $\ln V(\cdot)$ it follows that $f_{0n}(t)$ and $f_{2n}(t)$ are limited and satisfy Holder boundary condition. Since $\Psi_{01}(t)$ - known function, then from system (15) it is possible to easily find $\gamma_{21}(t)$. Let $\gamma_{2\tau}(t)$, $\tau = \overline{1, n-1}$ be found. Then, by applying the theories of linear singular integral equations, for any $n \geq 1$ from expression (15) we find

$$\begin{aligned} \eta_{0n}(t) &= \lambda \psi_{0n}(t), \quad \eta_{1n}(t) = -\beta_{10}^{-1}(t) [\beta_{10}(t-1) \eta_{1n}(t-1) + \\ &+ f_{1n}(t-1) + \beta_{00}(t)(1+t) \lambda \psi_{0n}'(t) + f_{1n}(t) + \\ &+ \frac{1}{\pi} \int_{-1}^1 \left(\frac{1}{x-t} - \frac{1}{x+1} \right) \psi_{0n}(x) dx] \end{aligned} \quad (16)$$

Similarly, from the analyticity of functions \mathcal{P} , $\ln V$ and (10), (13) we will have the following recurrence equations for

$$\begin{aligned} \Pi_{1n}(t, \varepsilon) : \\ \varepsilon \Pi_{10}'(t, \varepsilon) &= \sum_{i=0}^n \mathcal{A}_{1i0}(t, \varepsilon) \Pi_{1i0}'(t, \varepsilon) q_0^i(t, \varepsilon), \quad \Pi_{120}(t, \varepsilon) = 0, \\ q_0(t, \varepsilon) &= \frac{\sqrt{1-t^2}}{\pi} \int_{-1}^1 \frac{\sum_{i=0}^n \beta_{1i0}(x, \varepsilon) \Pi_{1i0}'(x, \varepsilon)}{(x-t)\sqrt{1-x^2}} dx, \end{aligned} \quad (17)$$

$$\begin{aligned} \varepsilon \Pi_{1n}'(t, \varepsilon) - \alpha_{100}(t, \varepsilon) \Pi_{1n}(t, \varepsilon) - \frac{\alpha_{000}(t, \varepsilon) \sqrt{1-t^2}}{\pi} \\ \times \int_{-1}^1 \frac{\beta_{100}(x, \varepsilon) \Pi_{1n}(x, \varepsilon)}{(x-t)\sqrt{1-x^2}} dx = F_n'(t, \varepsilon), \quad (18) \\ \Pi_{12n}(t, \varepsilon) = \mathcal{F} q_{n-1}(t, \varepsilon), \quad n \geq 1, \end{aligned}$$

$$\begin{aligned} q_n(\cdot) &= \frac{\sqrt{1-t^2}}{\pi} \int_{-1}^1 \frac{\beta_{10}(x, \varepsilon) \Pi_{1n}(x, \varepsilon)}{(x-t)\sqrt{1-x^2}} dx + \psi_n(t, \varepsilon), \\ \psi_n(\cdot) &= \frac{\sqrt{1-t^2}}{\pi} \int_{-1}^1 \frac{\beta_{00}(x, \varepsilon) \eta_{1n}(x, \varepsilon) + \alpha_n(x, \varepsilon)}{(x-t)\sqrt{1-x^2}} dx, \end{aligned}$$

$$\begin{aligned}
 Q_n(\cdot) &= \sum_{i=1}^{n-1} C_{n-i}^i [b_{i+0}(t, \varepsilon)] \prod_{j=1}^{i+1} \Pi_{i,j}(t, \varepsilon) + b_{n,i+1}(t, \varepsilon) \cdot \\
 &\times \prod_{j=1}^{i+1} \tau_{i,j}(t, \varepsilon) + \sum_{\substack{2 \leq i+k \leq n \\ k \geq 1}} b_{i+k}(t, \varepsilon) \left[\left(\sum_{d=1}^{i+k} \Pi_{i,d}(t, \varepsilon) \right)^i \left(\sum_{d=1}^{i+k} \tau_{i,d}(t, \varepsilon) \right)^k \right], \\
 &\sum_{j=1}^{i+1} k_j = n, \\
 F_n(\cdot) &= a_{n00}(t, \varepsilon) \tau_{n0}(t, \varepsilon) + a_{n01}(t, \varepsilon) \cdot \psi_n(t, \varepsilon) + \\
 &+ \sum_{i=1}^{n-1} C_{n-i}^i [a_{i+0,00}(t, \varepsilon)] \prod_{j=1}^{i+1} \Pi_{i,j}(t, \varepsilon) + a_{n0,00}(t, \varepsilon) \prod_{j=1}^{n-1} \tau_{n,j}(t, \varepsilon) + \\
 &+ \sum_{\substack{2 \leq i+k \leq n \\ k \geq 1}} \left\{ a_{i+k}(t, \varepsilon) \left[\sum_{d=1}^{i+k} \Pi_{i,d}(t, \varepsilon) \right]^i \left(\sum_{d=1}^{i+k} \tau_{i,d}(t, \varepsilon) \right)^k + a_{i+k}(t, \varepsilon) \cdot \right. \\
 &\times \left. \left[\left(\sum_{d=1}^{i+k} \tau_{i,d}(t, \varepsilon) \right)^i \left(\sum_{d=1}^{i+k} q_{i,d}(t, \varepsilon) \right)^k \right] + \sum_{\substack{2 \leq i+k \leq n \\ k \geq 1}} a_{i+k}(t, \varepsilon) \left[\sum_{d=1}^{i+k} \Pi_{i,d}(t, \varepsilon) \right]^i \cdot \right. \\
 &\times \left. \left. \left(\sum_{d=1}^{i+k} \tau_{i,d}(t, \varepsilon) \right)^k \left(\sum_{d=1}^{i+k} q_{i,d}(t, \varepsilon) \right)^k \right], \right. \\
 a_{i+0,0}(t, \varepsilon) &= \sum_{\substack{i+k \leq n \\ k \geq 1}} A_{i+k,j}(t, \varepsilon) C_i^k \Pi_{i,0}^{i-k} C_j^k q_0^{i-k}, \quad i \geq 2, j \geq 2, \\
 b_{i+k}(t, \varepsilon) &= \sum_{\substack{i+k \leq n \\ k \geq 1}} d_{i+k}(t, \varepsilon) C_i^k \Pi_{i,0}^{i-k}(t, \varepsilon), \\
 \tau_{i,k} &= (1+\varepsilon) \cdot \Pi_{i,k}(t, \varepsilon),
 \end{aligned}$$

where $A_{i+k,j}(\cdot)$, $d_{i+k}(\cdot)$ - derivatives of $(i+k, j)$ and $(i+k)$ orders of function \mathcal{P} and $\ln V$ with respect to dependent variable.

Since, by virtue of the structure of formulas (5), functions $A_{i+k,j}(\cdot)$ are integrated and $d_{i+k}(\cdot)$ are continuous according to Holder, and the right side of equation (17) satisfies Lifshits condition, then equation (17) is solvable.

Let us assume that $A_{100}(t, \varepsilon) < 0$, $a_{100}(t, \varepsilon) < 0$ and the solution of equation (17) is found [4].

For assigned $n = 1$ from (18) it is not difficult to determine $\Pi_{1n}(t, \varepsilon)$. Let functions $\Pi_{0n}(t, \varepsilon)$, $\tau = t, \bar{n} - t$ already be found. Then $\Pi_{2n}(\cdot)$, and consequently $F_n(t, \varepsilon)$, - is known function. Considering that the first equation of system (18) can easily lead to an equation of Fredholm type, the solution of which is presented in the form:

$$\Pi_{1n}(t, \varepsilon) = Q_{1n}(t, \varepsilon) + \int_{-1}^t R(t, x, \varepsilon) Q_{1n}(x, \varepsilon) dx, \quad n \geq 1, \quad (19)$$

where $R(\cdot)$ - resolvent of kernel

$$H(\cdot) = \frac{f(t, \varepsilon) b_{10}(x, \varepsilon)}{\sqrt{\varepsilon} \sqrt{1-x^2}} \int_{-1}^t f^{-1}(\tau, \varepsilon) \frac{\sqrt{1-\tau^2} \cdot a_{100}(\tau, \varepsilon)}{x-\tau} d\tau,$$

$$f(t, \varepsilon) = \exp\left(\frac{t}{\varepsilon} \int_{-1}^t a_{100}(s, \varepsilon) ds\right), \quad a_{100}(\cdot) < 0,$$

$$Q_{1n}(\cdot) = f(t, \varepsilon) \left[\Pi_{1n}(-1, \varepsilon) + \varepsilon^{-1} \int_{-1}^t f^{-1}(x, \varepsilon) F_n(x, \varepsilon) dx \right].$$

Estimates for functions $\eta_{1n}(t)$, $\Pi_{1n}(t, \varepsilon)$ and convergence of series (12) and (13) are established similarly [4].

Thus, by knowing functions (8), (12), (13), (16), (13) and (19), we can determine the solution of initial problem by formula (6), where with $\varepsilon \rightarrow 0$ $u_n(t, \varepsilon)$ strives toward the solution of confluent system. On the basis of (5), (8), (12), (13), (15) and (18), (19) the magnitude of velocity $V(\cdot) = V_0(t, d, \varepsilon, \lambda)$, and consequently, $\Omega(t) = \Omega_0(t, d, \varepsilon, \lambda)$

is known function of t , depending also on α - angle of slope of shield AB, small Froude number $\varepsilon < 1$ and parameter λ , considering the coefficient of surface tension. Omitting detail, let us present the formula of the coefficient of compression of the stream in the following form:

$$K(\alpha, \varepsilon, \lambda) = \frac{\mathcal{F}}{\mathcal{F} + 2} \frac{1}{1 - \mathcal{J}(\alpha, \varepsilon, \lambda)}, \quad \mathcal{J}(\cdot) = \frac{2}{\mathcal{F} + 2} [1 - R(\alpha, \varepsilon, \lambda)]^2 \quad (20)$$

$$R(\cdot) = -\frac{1}{2} \int_{-1}^1 \frac{\sin[f_0(t) - \Omega_0(t, \alpha, \varepsilon, \lambda)]}{(1+t)V_0(t, \alpha, \varepsilon, \lambda)} dt, \quad \mathcal{J}(\cdot) < 1.$$

REFERENCES

1. Гуревич М.И. Теория струй. Механика в СССР за 50 лет, т.2, 1970.
2. Биркгов Г., Сарантонелло Э. Струи, следы и каверны. 1964.
3. Дуйшеев Э. Сборник исследований по интегро-дифференциальным уравнениям в Киргизии, вып.3, 1965.
4. Дуйшеев Э. Труды Киргиз. Гос. университета, серия матем. наук, вып.7, 1970.

1. Gurevich, M. I. Stream theory. Mechanics in the USSR for 50 years, Vol. 2, 1970.
2. Birkjov G., Sarantonello E. Streams, traces and pockets. 1964.
3. Dushheyev E. Collection of investigations on integrodifferential equations in Kirgizia, iss. 3, 1965.
4. Dushheyev E. Works of Kirgiz. State University, series of math. science, iss. 7, 1970.

end 1119

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

<u>ORGANIZATION</u>	<u>MICROFICHE</u>	<u>ORGANIZATION</u>	<u>MICROFICHE</u>
A205 DMATC	1	E053 AF/INAKA	1
A210 DMAAC	2	E017 AF/RDXTR-W	1
B344 DIA/RDS-3C	9	E403 AFSC/INA	1
C043 USAMIIA	1	E404 AEDC	1
C509 BALLISTIC RES LABS	1	E408 AFWL	1
C510 AIR MOBILITY R&D	1	E410 ADTC	1
LAB/FIO		E413 ESD	2
C513 PICATINNY ARSENAL	1	FTD	
C535 AVIATION SYS COMD	1	CCN	1
C591 FSTC	5	ASD/FTD/NICD	3
C619 MIA REDSTONE	1	NIA/PHS	1
D008 NISC	1	NICD	2
H300 USAICE (USAREUR)	1		
P005 ERDA	1		
P005 CIA/CRS/ADB/SD	1		
NAVORDSTA (50L)	1		
NASA/KSI	1		
AFIT/LD	1		