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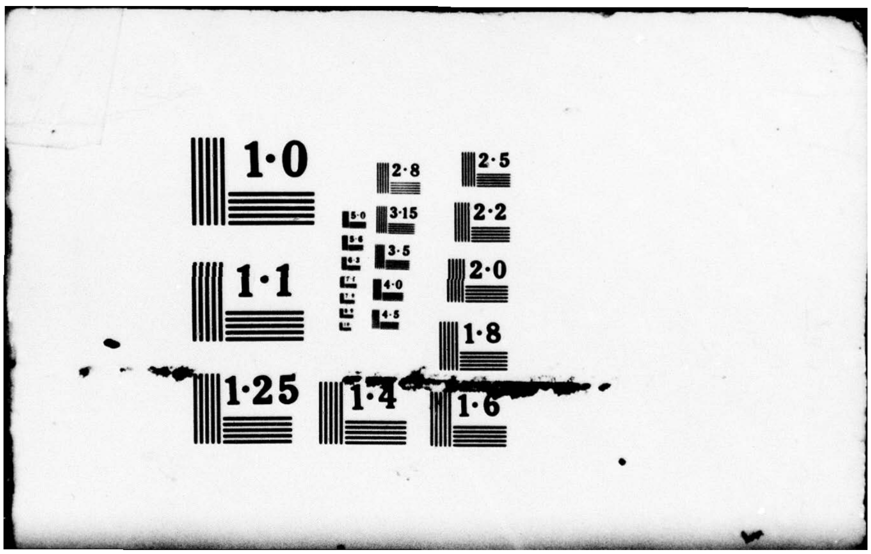
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FOREIGN TECHNOLOGY DIVISION

A WING WITH A SMALL ASPECT RATIO IN A RESTRICTED FLOW OF A NONVISCOUS FLUID

by

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α α ∝		Nu	Ν ν	
Beta	Β β		Xi	Ξ ξ	
Gamma	Γ γ		Omicron	Ο ο	
Delta	Δ δ		Pi	Π π	
Epsilon	Ε ε ϵ		Rho	Ρ ρ ϱ	
Zeta	Ζ ζ		Sigma	Σ σ ς	
Eta	Η η		Tau	Τ τ	
Theta	Θ θ ϑ		Upsilon	Υ υ	
Iota	Ι ι		Phi	Φ φ ϕ	
Kappa	Κ ϰ κ ϰ		Chi	Χ χ	
Lambda	Λ λ		Psi	Ψ ψ	
Mu	Μ μ		Omega	Ω ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	sin^{-1}
arc cos	cos^{-1}
arc tg	tan^{-1}
arc ctg	cot^{-1}
arc sec	sec^{-1}
arc cosec	csc^{-1}
arc sh	$sinh^{-1}$
arc ch	$cosh^{-1}$
arc th	$tanh^{-1}$
arc cth	$coth^{-1}$
arc sch	$sech^{-1}$
arc csch	$csch^{-1}$
rot	curl
lg	log

GRAPHICS DISCLAIMER

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A WING WITH A SMALL ASPECT RATIO IN A RESTRICTED FLOW OF A NON-VISCOUS FLUID

V. I. Kholyavko, Yu. F. Usik

The aerodynamic characteristics of a thin flat wing which is moving close to a solid or free surface (hydrofoil) are determined. Here, we use the theory of a thin body in accordance with which the three-dimensional flow around a thin body which is elongated in the direction of movement is replaced by a two-dimensional flow in transverse planes. In the case under consideration the problem is reduced to a study of the movement of a flat plate close to a surface of separation (Fig. 1).

Let a flat wing of small aspect ratio the maximum span of which coincides with the trailing edge move at a small angle of attack α with constant speed V_∞. During time dt a section of the wing $dx_1 = V_\infty dt$ passes through a fixed transversed plane ZOY. In an equivalent two-dimensional problem this movement of the wing corresponds to the vertical displacement of a flat plate which is moving with constant velocity $V_0 = V_\infty \alpha$ by the value $V_0 dt$ and to a change in the width of the plate from $l(x_1)$ to

$$l(x_1) + \frac{dl(x_1)}{dx_1} dx_1 = l(x_1) + \frac{dl(x_1)}{dx_1} V_\infty dt,$$

where $l(x_1)$ - the local wing semispan.

The flow in plane ZOY which is caused by the vertical displacement of the plate and the change in its width leads to a change in the connected mass of the plate by a value which corresponds to the increase in the wing lift on length dx_1, i.e.,

$$\frac{dY}{dx_1} = \frac{d(mV_0)}{dt} = V_0 \frac{dm(x_1)}{dt} = V_0 \frac{dm(x_1)}{dx_1}\frac{dx_1}{dt} = V_\infty^2 \alpha \frac{dm(x_1)}{dx_1}.$$

Integrating this equation along the length of the cord $0 \leq x_1 \leq b$, we obtain the wing lift

$$Y = \int_0^b \frac{dY}{dx_1} dx_1 = V_\infty^2 \alpha m(b), \tag{1}$$

where m(b) - the attached mass of the plate which is determined in the wing cross section along the maximum span.

The value of the transverse aerodynamic moment relative to the axis OZ_1 which passes through the apex of the wing is calculated from the formula

$$M_{z_1} = -\int_0^b x_1 \frac{dY}{dx_1} dx_1 = -V_\infty^2 \alpha \int_0^b x_1 dm(x_1) = -V_\infty^2 \alpha \times$$
$$\times \left[m(b)b - \int_0^b m(x_1) dx_1\right]. \tag{2}$$

The pressure drag (induced drag) of the wing with consideration of the suction effect on the leading edges is determined from the relationship

$$X_i = \frac{1}{2} Y \alpha. \tag{3}$$

Let us introduce into consideration instead of the forces and moment (1)-(3) the corresponding coefficients

$$c_y = \frac{Y}{q_\infty S};$$
$$c_{x_i} = \frac{X_i}{q_\infty S}; \quad m_{z_1} = \frac{M_{z_1}}{q_\infty Sb}.$$

where S - the area of the wing in a plane;

$$S = 2\int_0^b l(x_1)\,dx_1;$$

q_∞ - the dynamic head;

$$q_\infty = \frac{\rho V_\infty^2}{2}.$$

Then the aerodynamic characteristics of a wing with small aspect ratio can be presented in the following form

$$c_y = 2\frac{m(b)}{\rho S}\alpha; \quad c_{x_i} = \frac{1}{2}c_y\alpha = \frac{m(b)}{\rho S}\alpha^2;$$

$$m_{z_1} = -2\frac{m(b)}{\rho S}\alpha\left[1 - \frac{1}{bm(b)}\int_0^b m(x_1)\,dx_1\right]. \qquad (4)$$

If the coefficients of lift and lateral moment are known, the position of the wing's center of pressure in relation to its apex in fractions of the root cord can be calculated from the formula

$$x_д = -\frac{m_{z_1}}{c_y} = 1 - \frac{1}{bm(b)}\int_0^b m(x_1)\,dx_1. \qquad (5)$$

Formulas (4) and (5) were obtained from the general relationships of the theory of a thin body and are suitable in all cases where the assumptions of this theory are valid. Let us note two special features which follow from formulas (4) and (5).

1. In accordance with (4) the lift and moment coefficients are a linear function of the angle of attack. The results of experimental studies confirm the linear nature of these relationships for wings of small aspect ratios to $\alpha \leq 6°$.

2. The lift coefficient does not depend on the shape of the wing in plane $z_1 = l(x_1)$ and is determined only by the wing cross section on the trailing edge. Experimental data and calculations in accordance with precise theories also confirm this special feature for wings with an aspect ratio $\lambda \leq 1.5$.

Thus, despite limitedness by the case $\lambda \to 0$, the theory of a thin body leads to qualitatively correct results for wings of finite aspect ratio. It can be assumed that under certain conditions this theory will also provide satisfactory quantitative results.

From (3) and (4) it follows that the task of determining the aerodynamic characteristics of a wing of small aspect ratio in a restricted flow is reduced to finding the connected mass of a plate close to the surface of separation. To calculate the connected mass, we use the method of special features and we distribute the vortical layer with continuous intensity $\gamma(z)$ along the length of the plate within limits $-l \le z \le l$. Obviously, in this case $\gamma(z) = -\gamma(-z)$. We consider the effect of the surface of separation by the mirror reflection method (Fig. 1).

The distribution $\gamma(z)$ should satisfy the following condition on the plate: a particle of the fluid which is adjacent to the plate at any point S should possess a constant vertical velocity V_0 (Fig. 1). Using this boundary condition, we obtain the integral equation for the determination of the unknown function $\gamma(z)$:

$$\int_{-l}^{l} \frac{\gamma(z)\,dz}{z - z_s} \pm \int_{-l}^{l} \frac{\gamma(z)(z - z_s)\,dz}{(z - z_s)^2 + 4h^2} = 2\pi V_0. \qquad (6)$$

Here, the "plus" sign pertains to the movement of a wing beneath a free surface (hydrofoil), and the "minus" sign - to the movement of the wing close to a solid surface (ground). The values of l and h are determined in the lateral plane of flow ZOY depending on coordinate x_1 which enters into equation (6) as a parameter. To calculate the lift and induced drag coefficients of the wing the values of l and h should be determined in the cross section of the maximum span.

We introduce the new variable ϑ from the relationship $z = l \cos \vartheta$ (with $-l \le z \le l$ we have $\pi < \vartheta < 0$) and we conduct the replacement $\gamma(z) = \gamma(l \cos \vartheta) = \gamma(\vartheta)$, designating $\bar{h} = h/b$. Then equation (6) is written as follows:

$$\int_0^\pi \frac{\gamma(\vartheta)\sin\vartheta\, d\vartheta}{\cos\vartheta - \cos\vartheta_0} \pm \int_0^\pi \frac{\gamma(\vartheta)(\cos\vartheta - \cos\vartheta_0)\sin\vartheta}{(\cos\vartheta - \cos\vartheta_0)^2 + 4\bar{h}^2} = 2\pi V_0. \qquad (7)$$

Let us note a partial solution of equations (6) and (7) which corresponds to the motion of a wing in a limitless fluid ($h = \infty$). From (6) and (7) with $h = \infty$ we obtain

$$\int_{-l}^{l} \frac{\gamma(z)\, dz}{z - z_0} = \int_0^\pi \frac{\gamma(\vartheta)\sin\vartheta}{\cos\vartheta - \cos\vartheta_0}\, d\vartheta = 2\pi V_0,$$

whence

$$\gamma = 2V_0 \frac{z}{\sqrt{l^2 - z^2}} = 2V_0 \operatorname{ctg}\vartheta. \qquad (8)$$

For the solution of equation (7) in the general form with $h \neq \infty$ we present the unknown function $\gamma(\vartheta)$ as a series

$$\gamma(\vartheta) = 2V_0 \left[A_0 \operatorname{ctg}\vartheta + \sum_{n=1}^{\infty} A_{2n} \sin 2n\vartheta\right]. \qquad (9)$$

in which the first term with $A_0 = 1$ is determined by the solution of (8) and the second term and $A_0 \neq 1$ characterize the additional load which arises on the plate from the influence of the flow boundaries. Obviously, with $h \to \infty$ coefficient $A_0 \to 1$, and $A_{2n} \to 0$ ($n = 1, 2 \ldots$).

In recording series (9) we used the condition

$$\gamma(\vartheta) = -\gamma(\pi - \vartheta).$$

If the solution to (9) is known, then the associated mass is calculated in the following manner. The values of the function of the velocity potential on the surface of the plate with $V_0 = 1$ is determined with an accuracy to a constant which subsequently is not significant:

$$\varphi_{B,H} = \pm\frac{1}{2}\int\gamma(z)\,dz = \mp\frac{l}{2}\int\gamma(\vartheta)\sin\vartheta\,d\vartheta,$$

where the upper sign pertains to the upper surface (ϕ_B), and the lower - to the lower surface (ϕ_H). The general formula for the determination of the associated mass has the form [1]

$$m = -\rho\int_c \varphi\frac{\partial\varphi}{\partial n}\,dS.$$

Since, in this case, on the lower surface of the plate $\frac{\partial\varphi}{\partial n} = +1$, and on the upper $\frac{\partial\varphi}{\partial n} = -1$, and, besides, dS = dz,

$$m = -4\rho\int_0^l \varphi_H(z)\,dz = -4\rho l\int_0^{\frac{\pi}{2}} \varphi_H(\vartheta)\sin\vartheta\,d\vartheta = \\ = 2\rho l^2\int_0^{\frac{\pi}{2}}\sin\vartheta\,d\vartheta\int\gamma(\vartheta)\sin\vartheta\,d\vartheta.$$

Substituting the expression $\gamma(\vartheta)$ from (9) into this formula, we obtain

$$m = \pi\rho l^2\left(A_0 + \frac{A_2}{2}\right). \qquad (10)$$

Thus, to determine the associated mass of a plate it is necessary to know the first two coefficients A_0 and A_2 of the expansion (9). The remaining coefficients A_{2n} (n = 2, 3, 4...) characterize the load distribution over the span of a plate and do not have a direct influence on the total aerodynamic characteristics of the wing.

With h = ∞, for the solution of (8) it follows that $A_0 = 1$ and $A_{2n} = 0$; therefore, it accordance with (10)

$$m_\infty = \pi\rho l^2. \qquad (11)$$

The aerodynamic characteristics of the wing with consideration of (11) are determined from formulas (4) and (5)

$$C_{y_\infty} = \frac{\pi\lambda}{2}\alpha; \quad C_{x_i} = \frac{1}{2}C_{y_\infty}\alpha = \frac{1}{\pi\lambda}C^2_{y_\infty};$$
$$x_d = 1 - \frac{1}{bl^2(b)}\int_0^b l^2(x_1)\,dx_1. \tag{12}$$

Here $\lambda = \frac{4l^2(b)}{S}$ - the aspect ratio of the wing, $z_1 = l(x_1)$ - the equation of the form of a wing in a plane.

Formulas (12) are known relationships of a wing of small aspect ratio in a limitless fluid [2].

Let us move on to the calculation of the expansion coefficients A_{2n}(n = 0, 1, 2...). Equation (7) with consideration of (9) is transformed as follows:

$$A_0 - \sum_{n=1}^{\infty} A_{2n}\cos 2n\vartheta_s \pm I(\bar{h}, \vartheta_s) = 1, \tag{13}$$

where we introduce the designations

$$I(\bar{h}, \vartheta_s) = A_0\left(\frac{1}{2}J_0 - \cos\vartheta_s J_1 + \frac{1}{2}J_2\right) -$$
$$-\frac{1}{2}\cos\vartheta_s\sum_{n=1}^{\infty}A_{2n}(J_{2n-1} + J_{2n+1}) + \frac{1}{4}\sum_{n=1}^{\infty}A_{2n}(J_{2n-2} - J_{2n+2});$$
$$J_m = \frac{1}{\pi}\int_0^{\pi}\frac{\cos m\vartheta\,d\vartheta}{(\cos\vartheta - \cos\vartheta_s)^2 + 4\bar{h}^2}.$$

After calculation of the integrals J_m and further transformations of formula (13) we finally obtain the equations for the expansion coefficients A_{2n}(n = 0, 1, 2, ...).

1. For the motion of a wing close to a solid surface

$$A_0\zeta_0(\bar{h}, \vartheta_s) + 1 = \sum_{n=1}^{\infty} A_{2n}\left[\zeta_{2n}(\bar{h}, \vartheta_s) - \cos 2n\vartheta_s\right]. \tag{14}$$

2. For the motion of a wing beneath a free surface (hydrofoil)

$$A_0\left[2 + \zeta_0(\bar{h}, \vartheta_s)\right] - 1 = \sum_{n=1}^{\infty} A_{2n}\left[\zeta_{2n}(\bar{h}, \vartheta_s) - \cos 2n\vartheta_s\right]. \tag{15}$$

The following designations are introduced into (14) and (15):

$$\zeta_0(\bar{h}, \vartheta_s) = -\frac{u}{V}|\cos\vartheta_s| - 2\bar{h}\frac{u}{V};$$

$$\zeta_{2n}(\bar{h}, \vartheta_s) = \left(\frac{u + \cos\vartheta_s}{t}\right)^{2n} T_{2n}(t);$$

$$u = \frac{1}{2}\sqrt{V - (\sin^2\vartheta_s + 4\bar{h}^2)};$$

$$\bar{u} = \frac{1}{2}\sqrt{V + \sin^2\vartheta_s + 4\bar{h}^2};$$

$$V = \sqrt{(\sin^2\vartheta_s + 4\bar{h}^2)^2 + 16\bar{h}^2\cos^2\vartheta_s};$$

$$t = \frac{u}{\sqrt{u^2 + 4\bar{h}^2}}; \quad T_{2n}(t) = \cos(2n \arccos t),$$

where $T_{2n}(t)$ - the Chebyshev polynomial type I.

If in relationships (14) or (15) we record a number of values of ϑ_s, we obtain systems of algebraic equations for the determination of expansion coefficients A_{2n}($n = 0, 1 \ldots$).

Here, the *number* of recorded points ϑ_{si} ($i = 1, 2 \ldots$) determines the number of equations in systems (14) and (15) and, consequently, the number of unknown coefficients A_{2n} ($n = 0, 1, 2, \ldots, i - 1$). The remaining coefficients ($n = i, i + 1, \ldots$) should be taken as equal to zero.

In accordance with (14) and (15), calculations of the coefficients were conducted with the fixing of 1 point (approximation I), three (approximation II), and five (approximation III).

Some of the calculations for the motion of a wing close to a solid surface are presented in Fig. 2. From the known coefficients A_0 and A_2 and formula (10), we can find the associated mass of the plate, and from formulas (4) and (5) - the aerodynamic characteristics of the wing.

Figure 3 presents the values of the derivative of the lift coefficient in accordance with angle of attack C_y^α referred to the value $C_{y\infty}^\alpha$ in a limitless flow (12). Here, the dots designate the mean values of $\bar{C}_y^\alpha$ which were obtained in accordance with the theory of a vortical lifting surface for rectangular wings with $\lambda = 0.4-1.6$ [4].

An analysis of the calculations which were conducted shows that satisfactory results right up to $\bar{h} \geq 0.1$ can be obtained already in approximation III for a wing close to a solid surface and, in II - for a hydrofoil.

If we restrict ourselves to values of $\bar{h} \geq 0.4$ (wing close to the ground) and $\bar{h} \geq 0.25$ (hydrofoil), the aerodynamic characteristics of a wing with a small aspect angle can be determined from approximation I. In this case simple analytical relations occur which follow from (14) and (15) with $\vartheta_s = \pi/2$ and $A_{2n} = 0$ (n = 1, 2, 3 ...).

1. For a wing close to a solid surface

$$A_0 = \frac{\sqrt{1+4\bar{h}^2}}{2\bar{h}}, \quad m = \pi\rho l^2 \frac{\sqrt{1+4\bar{h}^2}}{2\bar{h}}.$$

Since

$$\bar{C}_y^\alpha = \frac{m}{m_\infty},$$

Then

$$\bar{C}_y^\alpha = \frac{\sqrt{1+4\bar{h}^2}}{2\bar{h}}.$$

2. For a hydrofoil

$$A_0 = \frac{1}{2 - \frac{2\bar{h}}{\sqrt{1+4\bar{h}^2}}}; \quad m = \pi\rho l^2 \frac{1}{2 - \frac{2\bar{h}}{\sqrt{1+4\bar{h}^2}}};$$

$$\bar{C}_y^\alpha = \frac{1}{2 - \frac{2\bar{h}}{\sqrt{1+4\bar{h}^2}}}. \qquad (17)$$

The calculations from formulas (16) and (17) are shown in Fig. 2 by the solid lines.

Let us note that if in the expression $\frac{\sqrt{1+(2\bar{h})^2}}{2\bar{h}}$ we replace 2 by a larger number (for example 3), formulas (16) and (17) provide a satisfactory agreement with calculations in accordance with approximation III right up to $\bar{h} \geq 0.10$.

Thus, we can recommend the following approximate formulas for the calculation of the associated mass of a plate in a restricted flow with $\bar{h} \geq 0.10$.

1. Close to a solid surface

$$m = \pi\rho \frac{\sqrt{1+9\bar{h}^2}}{3\bar{h}} l^2. \qquad (18)$$

2. Under a free surface

$$m = \frac{\pi\rho l^2}{2 - \frac{3\bar{h}}{\sqrt{1+9\bar{h}^2}}}. \qquad (19)$$

Formulas (18) and (19) are convenient with a calculation of moment characteristics and the position of the wings' center of pressure. For example, for a wing close to a solid surface from (5) and (18) we find

$$x_0 = 1 - \frac{\bar{h}}{b l^2(b)\sqrt{1+9\bar{h}^2}} \int_0^1 l^2(x_1) \sqrt{\frac{l^2(x_1) + 9h^2(x_1)}{h(x_1)}}\, dx_1.$$

Before the integral $\bar{h} = h/l(b)$, h - the position of the trailing edge of the wing in relation to the interfaces, $l(b)$ - the wing semispan.

In the integrand the values h and l depend on the coordinates x_1. With small angles of attack

$$h(x_1) = h + (b - x_1)\alpha = l(b)\left[\bar{h} + \frac{b}{l(b)}(1 - x_1)\alpha\right].$$

In the assumptions of the theory of a thin body $\frac{b}{l(b)} \sim \frac{1}{\lambda}$, under certain conditions the second term in the right side can also have the order of the first term; therefore, the position of the center of pressure on a flat wing close to a surface of separation depends on the angles of attack.

Let us limit ourselves to the case a → 0; then $h(x_1) = l(b)\bar{h} = \text{const}$ and the position of the center of pressure is determined from the formula

$$x_0 = 1 - \frac{1}{\sqrt{1+9\bar{h}^2}} \int_0^1 \bar{l}^2(x_1)\sqrt{\bar{l}^2(x_1)+9\bar{h}^2}\,dx_1. \tag{20}$$

where $\bar{l}(x_1) = \frac{l(x_1)}{l(b)}$ - the relative local wing semispan; $\bar{x}_1 = \frac{x_1}{b}$.

Let us examine a family of wings in which the form in a plane is given by the equation $\bar{l}(x_1) = x_1^{-m}$.

The exponent m varies within limits of $0 < m < \infty$. With m = 1 we obtain a delta wing and with m → 0 - rectangular.

In accordance with formula (20) we will have

$$x_0 = 1 - \frac{1}{\sqrt{1+9\bar{h}^2}} \int_0^1 \bar{x}_1^{2m}\sqrt{\bar{x}_1^{2m}+9\bar{h}^2}\,d\bar{x}_1. \tag{21}$$

If we accomplish such conversions for a hydrofoil, then from (5) and (19) with α → 0 it follows that

$$x_0 = 1 - \left(2 - \frac{3\bar{h}}{\sqrt{1+9\bar{h}^2}}\right)\int_0^1 \frac{\bar{l}^2(\bar{x}_1)\sqrt{\bar{l}^2(\bar{x}_1)+9\bar{h}^2}}{2\sqrt{\bar{l}^2(\bar{x}_1)+9\bar{h}^2}-3\bar{h}}\,dx_1. \tag{22}$$

For a family of wings $\bar{l}(\bar{x}_1) = x_1^m$

$$x_0 = 1 - \left(2 - \frac{3\bar{h}}{\sqrt{1+9\bar{h}^2}}\right)\int_0^1 \frac{\bar{x}_1^{2m}\sqrt{\bar{x}_1^{2m}+9\bar{h}^2}}{2\sqrt{\bar{x}_1^{2m}+9\bar{h}^2}-3\bar{h}}\,dx_1. \tag{23}$$

With $\bar{h} = \infty$ (the flow of a limitless fluid) from (21) and (23)

$$x_{0\infty} = \frac{2m}{2m+1}. \qquad (24)$$

This result can also be determined from formula (12).

In the general case the integrals in formulas (21) and (23) are expressed by hypergeometric functions [3]. Figure 4 presents the calculations of $\bar{x}_0 = \frac{x_0}{x_{0\infty}}$ for wings with m = 1/2, 1, 2 (solid lines - for a wing close to a solid surface, dashed lines - for a hydrofoil).

An analysis of the results which have been obtained shows that with the approach of a wing to a solid surface (ground) the center of pressure is shifted toward the trailing edge, in which regard this displacement is greater the smaller the exponent m. In particular, the greatest displacement should be observed for a rectangular wing (m → 0).

The effect of the boundary of a free surface for a hydrofoil is opposite to the effect of the ground.

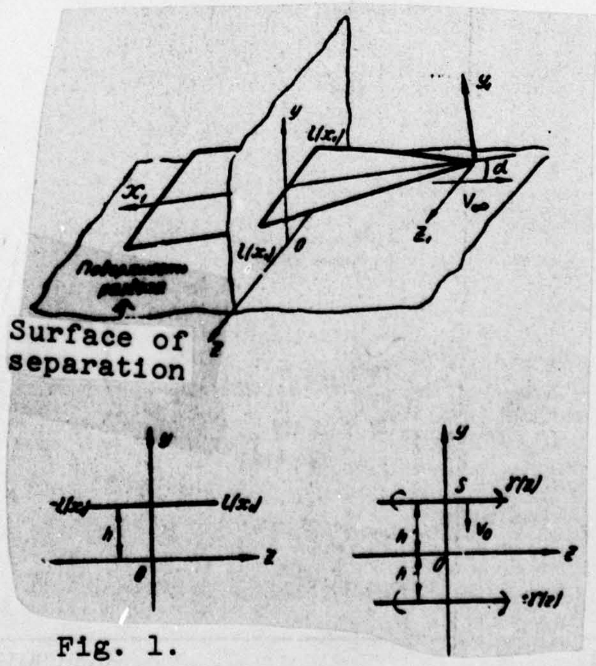

Fig. 1.

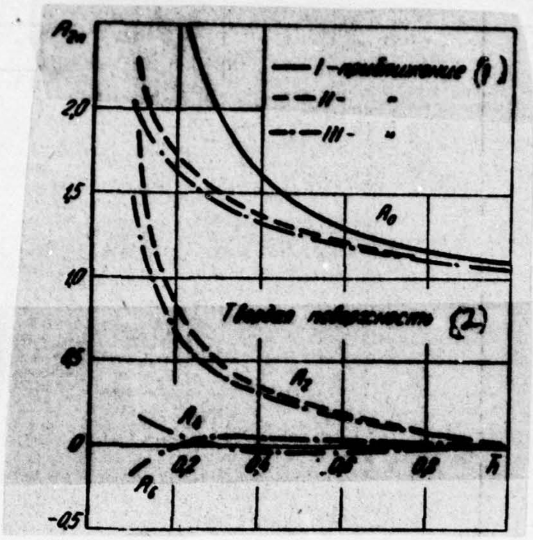

Fig. 2.

Key: (1) Approximation; (2) Solid surface.

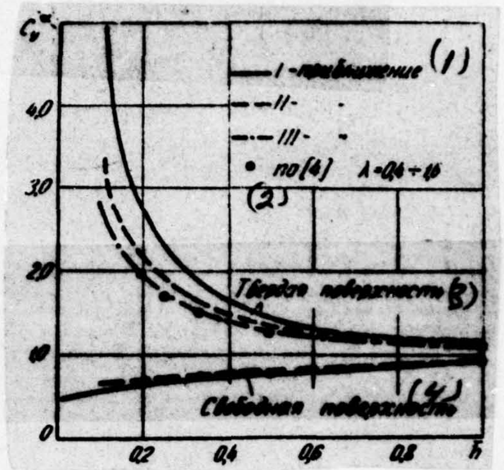

Fig. 3.

Key: (1) Approximation; (2) According to; (3) Solid surface; (4) Free surface.

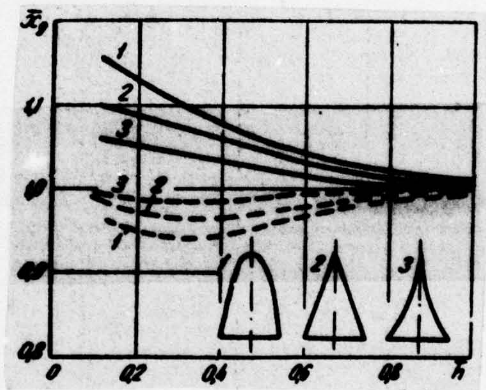

Fig. 4.

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