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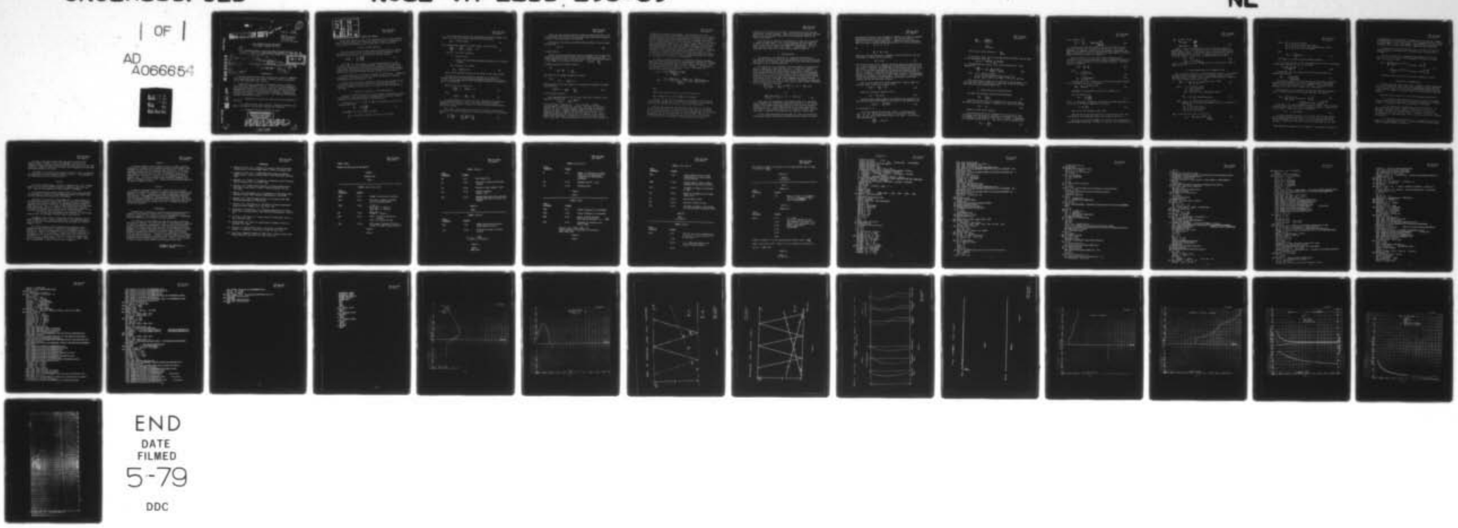

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A COMPUTER PROGRAM TO CALCULATE NORMAL MODE PROPAGATION OVER A FLAT HOMOGENEOUS OCEAN BOTTOM

by

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INTRODUCTION

This memorandum describes NUSL Program S1441 written in FORTRAN V. This program uses normal mode theory to predict acoustic propagation over a flat homogeneous ocean bottom.

Program S1441 is an extension of a program written for NUSL by A.D. Little Inc. (Reference 1 and 2). In the A.D. Little program the amplitude distribution of an acoustic signal as a function of depth is determined for a given mode by means of the numerical solution of the acoustic wave equation for given boundary conditions. Program S1441 calculates and produces Calcomp plots of the following for any mode, frequency and velocity profile:

a. Amplitude as a function of depth and the ray equivalent of any mode.

b. Group velocity, phase velocity, excitation pressure, and angle of incidence of sound waves striking the boundaries.

c. Propagation loss as a function of range.

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NORMAL MODE THEORY

Normal mode theory is based on the assumption that at large distances from a source standing waves formed by energy striking the boundaries in certain preferred directions form the main part of the sound field.

Solution of Wave Equation

There are three methods by which these preferred directions and the amplitude distribution of the standing waves may be determined. First one may find the solution in closed form of the wave equation.

$$\nabla^2 \Phi = \frac{1}{C^2} \frac{\partial^2 \Phi}{\partial t^2} \qquad (1)$$

Where Φ is a displacement or velocity potential and C is the velocity of sound in the medium considered. This is done in Reference 3 by integrating equation (1) subject to the boundary conditions in the complex plane and approximating the normal mode solution for the standing waves by the evaluation obtained from the residues in the integration.

A second method is the solution of equation (1) by direct numerical integration. This was done by A.D. Little Inc by means of a computer program described in References 1 and 2.

Third it has been shown in Reference 4 that one may consider the ray paths, which undergo total reflection at the boundaries and whose successive upgoing and downing rays are in phase, as the descriptions of the modes.

A. Numerical Integration of the Wave Equation

The basis of this program is the direct numerical integration of the wave equation. This method which is extremely well suited for analysis by means of a high speed computer is discussed below.

When equation (1) is solved for Φ the incremental pressure p is found by definition

$$p = -\rho \frac{\partial^2 \Phi}{\partial t^2} \qquad (2)$$

Where Φ is the displacement potential

ρ is the material density at the point considered

It is found that equation (1) is separable in terms of range r and depth z so that the pressure amplitude can be written as

$$p_a = F(r) \cdot u(z) \tag{3}$$

and a differential equation in terms of p_a may be obtained.

$$\frac{d^2u}{dz^2} + \left(\frac{\omega^2}{c^2} - k_r^2\right)u = 0 \tag{4}$$

where ω = the radial frequency

c = sound velocity

u = u(z) is the pressure amplitude distribution as a function of depth

k_r = horizontal wave number

$$k_r = \frac{\omega}{c}\sin\Theta \tag{5}$$

where Θ is measured relative to the normal to the ocean bottom.

The physical picture presented by equation (3) is that of a standing wave with a particular pressure amplitude distribution as a function of depth, $u(z)$, which travels unchanged in shape as it progresses in the r direction.

Equation (4) may be written as

$$\frac{d^2u}{dz^2} + f(z)u = 0 \tag{6}$$

where

$$f(z) = \frac{\omega^2}{c^2} - k_r^2 \tag{7}$$

In solving equation (4) there are three constraints which must be imposed upon any solution. These constraints are due to boundary conditions at the interface between the water and bottom material and at the waters surface.

First due to the necessity of continuity of pressure and particle velocity in the vertical direction at the bottom interface, it is necessary that

$$\frac{1}{u_1}\frac{du_1}{dz} = \frac{\rho_1}{\rho_b}\sqrt{k_r^2 - \frac{\omega^2}{c_b^2}} \tag{8a}$$

where the terms having subscript b refer to these quantities in the bottom material on one side of the interface while those with subscript 1 refer to the water side of the interface.

Second, since we assume a pressure release surface at the air-water boundary then

$$u = 0 \tag{8b}$$

at this surface.

Third, the modes by definition involve propagation with energy which strikes the bottom at angles larger than the critical angle so that "total reflection" occurs. Since the critical angle θ_c measured relative to the normal to the interface is given by $\sin\theta_c = \frac{c_1}{c_b}$

(naturally c_b must be creater than c_1)

then

$$\sin\theta > \frac{c_1}{c_b}$$

must hold for all energy striking the bottom.

Since

$$k_r = \frac{\omega}{c}\sin\theta$$

using euation (7) and the condition of continuity of pressure we obtain the result that

$$|\xi(z)| < \frac{\omega^2}{c_1^2} - \frac{\omega^2}{c_b^2} \tag{8c}$$

at the bottom interface.

Upon examining equation (6) it can be seen that a solution of the equation is of the form

$$u(z) = Be^{i\sqrt{\xi(z)}\,z}$$

for given values of c, ω and k_r. B is a constant. If $\xi(z)$ is positive $u(z)$ is in the form of a sinosoid. This kind of solution is obtained for the interference pattern between upgoing and downgoing waves in the water. If $\xi(z)$ is negative $u(z)$ is in the form of an exponential. $\xi(z)$ is negative for the following two cases. It is negative everywhere in the bottom and it is negative in the water at depths which correspond to shadow zones caused by vertexing of rays which form a mode. Both distributions are a result of the condition of continuity of pressure in the medium. Thus when there is "total reflection" at a level either by vertexing or reflection from a boundary and the pressure is finite at that level then at adjacent levels there may be a decay (whose rate is determined by the boundary conditions), but not a discontinuous step to zero pressure. An example of a distribution involving both exponentials is shown in Figure 1 which shows the amplitude distribution as a function of depth and Figure 2 which shows the ray equivalent of the mode. In Figure 2 it is seen that the ray equivalent of the normal mode vertexes at a depth of 48 feet. Therefore in Figure 1 the pressure amplitude distribution is in the form of an exponential between the surface of the ocean and a depth of 48 feet. It can also be seen in Figure 1 that the pressure amplitude distribution in the bottom is in the form of an exponential.

In order to integrate equation (6) numerically formulas relating u_{n+1} and its derivative u'_{n+1} with the quantities u_n and u'_n must be obtained (The subscripts signify the depth at which they are calculated). This is done by writing a Taylor series for u_{n+1} and u'_{n+1}. The details of this procedure is given in References 1 and 2. The results are summarized in equations (9) and (10)

$$u_{n+1} = \frac{(1 + \frac{h^2}{3} f_n) u_n + h u'_n}{1 + \frac{h^2}{6} f_{n+1}} \qquad (9)$$

$$u'_{n+1} = \frac{(1 - \frac{h^2}{3} f_{n+1}) u'_n - \frac{h}{2}(f_n + f_{n+1} - \frac{h^2}{6} f_n f_{n+1}) u_n}{1 + \frac{h^2}{6} f_{n+1}} \qquad (10)$$

where

h is the increment between level n and level $n+1$.

f_n and f_{n+1} are the values of $f(z)$ at n and $n+1$.

Equations (9) and (10) are recursion relationships such that given values for u_1 and u'_1 which are the values of u and u' just above the bottom we can calculate u and u' at all levels in the water.

Since we are interested in normalized values of u over the water column we can select u_1 to be any arbitrary value ($u_1 = 1$ is convenient). For an arbitrary value of the horizontal wave number k_r which is restricted by equation (8c) we can evaluate u'_1 by equation (8a). Then we can determine u for all levels by using equations (9) and (10) repeatedly. If the value of k_r corresponds to a mode, equation (8b) will be satisfied at the surface of the water. Each mode has at most one such solution for a given frequency. There is a low cut-off frequency for each mode so that at frequencies below the cut-off frequency equation (8b) cannot be satisfied.

After finding the amplitude distribution of a mode it is necessary to define the mode number. For a finite frequency the mode number is equal to the number of nodes in the amplitude distribution. Thus, the first mode has a node only at the surface. A representation of the amplitude distribution of the first mode is shown in Figure 1.

B. Ray Equivalent

Corresponding to the definition of a mode discussed above is a more physical approach in which the ray equivalent of the solution to the wave equation is considered. This approach has been discussed previously, References 5 and 6, and in Reference 7.

For simplicity let us consider a two layer medium of constant water depth H, density ρ_1, and sound velocity c_1, lying over a infinite bottom of density ρ_2 and sound velocity c_2 as shown in Figure 3. At large ranges from a point source we may consider sound to be propagated by plane waves. It is clear from Figure 3 that for certain waves whose direction is defined by an angle θ there will be constructive interference between it and a plane wave which undergoes one more bottom and surface reflections. In order for constructive interference to exist the phase difference between points A and B, Figure 3 must be $2(n-1)\pi$ degrees, or it must satisfy the equation:

$$\frac{2\pi}{\lambda_n}\left[\frac{H}{\cos\theta} + \frac{H}{\cos\theta}\cos 2\theta\right] - \epsilon - \pi = 2(n-1)\pi$$

or

$$\frac{2\pi}{\lambda_n}\left[2H\cos\theta\right] - \epsilon - \pi = 2(n-1)\pi \qquad (11)$$

where λ_n is the wavelength of the preferred mode, ϵ is the phase change undergone by a plane wave upon bottom reflection, n is the mode number, and the phase change upon reflection from the water surface is assumed to be $-\pi$. If the sound velocity in the water layer varies with depth the first term of equation (11) would be different from that given above. However, the discussion below applies to either case.

If for a given wavelength λ_n and angle θ there is constructive interference between plane waves suffering different numbers of bottom and surface reflections, then propagation consists of a series of upgoing and downgoing waves as shown in Figure 4. The left hand term of equation (11) is the phase change, 2Δ, undergone in the z direction when a ray makes a surface-bottom-surface cycle. For finite frequencies

$$\pi > \epsilon \geq 0$$

as

$$\frac{\pi}{2} > \theta \geq \theta_c$$

Therefore, the phase change, Δ, undergone in the z direction over the water depth is limited by the following

$$\Delta < n\pi \qquad (12)$$

The pressure is zero at the surface, the phase change upon reflection is $-\pi$, and the direction of propagation is reversed upon reflection from the surface. Therefore, the sound field in the vertical direction is the sum of two sine waves, representing the upgoing and downgoing waves in the z direction. These waves are shown for the first two modes in Figure 5. Because of equation (12) the number of nodes in this amplitude distribution is equal to the mode number. Thus there is a correspondence between the definitions of mode number in the solution of the wave equation and the ray equivalent solution.

When the wave equation is solved numerically values of $\xi(z)$ are obtained. $\xi(z)$ is given in equation (7) by

$$\xi(z) = \frac{\omega^2}{c^2} - k_r^2 \qquad (13)$$

where k_r is given in equation (5) by

$$k_r = \frac{\omega}{c}\sin\theta \qquad (14)$$

Therefore given positive $\xi(z)$ one can determine from equations (13) and (14) the cosine of the angle of inclination of the equivalent ray

$$\cos\theta = \frac{c}{\omega}\sqrt{\xi(z)} \qquad (15)$$

as a function of z.

If the ray between two points z_1 and z_2 is continuous then $\Delta z = z_1 - z_2$ may be given in terms of the horizontal distance $\Delta R = R_1 - R_2$ and one particular value of the $\tan\theta$ over the path. This relationship is

$$\frac{\Delta R}{\Delta z} = \tan\theta$$

$$\frac{\Delta R}{\Delta z} = \frac{\sin\theta \cdot c}{\cos\theta \cdot c}$$

$$= \frac{c}{c_v \cos\theta}$$

where c_v the vertex velocity = $\frac{c}{\sin\theta}$

If the value of θ does not vary appreciably between z_1 and z_2 then we can approximate $\tan\theta$ by

$$\tan\theta = \frac{\tan\theta_1 + \tan\theta_2}{2}$$

where θ_1 and θ_2 are taken at $z = z_1$ and z_2 respectively. Thus (Reference 8)

$$\frac{\Delta R}{\Delta z} \approx \frac{1}{c_v} \frac{c_1 + c_2}{\cos\theta_1 + \cos\theta_2} \tag{16}$$

where c_v = vertex velocity
c_1, c_2 = are the sound velocity at z_1 and z_2
θ_1, θ_2 = angle relative to normal of ray at z_1 and z_2

Thus given values of $S(z)$ one can construct a ray equivalent. If vertexing takes place the depth z_v at which this occurs is the level whose value of sound velocity equals c_v.

C. Phase Velocity and Group Velocity

The phase velocity V_p is given by the relationship

$$V_p = \frac{c}{\sin\theta} \tag{17}$$

where θ is the direction of propagation of a plane wave where the sound velocity has a value C. Equation (17) can also be written

$$V_p = c_v \tag{18}$$

where c_v is the vertex velocity

The group velocity can be considered from two points of view. First group velocity V_g may be considered as a measure of the speed of propagation in the horizontal direction of a number of frequencies in a band $\Delta\omega$ centered about ω. V_g may be given by (Reference 9)

$$V_g = \frac{\Delta\omega}{\Delta k} \tag{19}$$

or by (Reference 10)

$$\frac{V_g}{c} = \frac{V_p}{c} + (kH)\frac{d\left(\frac{V_p}{c}\right)}{d(kH)} \qquad (20)$$

It can be seen that this approach to the calculation of group velocities involves the calculation of derivatives. This is not desirable in a computer program since this produces inaccuracies and makes it necessary to obtain an unnecessarily large number of values of group velocity as a function of frequency.

Tolstoy (Reference 11) has used a general theorem by Biot (Reference 12) to show the equivalence of V_g in equations 19 and 20 to the rate of energy transport in the horizontal direction. The group velocity is given by

$$V_g = \frac{1}{V_p}\,\frac{\nu}{\sigma} \qquad (21)$$

where

$$\nu = \int_{-\infty}^{+\infty} \rho\,\phi^2\,dz \qquad (22)$$

$$\sigma = \int_{-\infty}^{+\infty} \frac{\rho}{c^2}\,\phi^2\,dz \qquad (23)$$

where ρ and c are respectively the density and sound velocity at z, and ϕ is given in the equation

$$\Phi = \phi(z)\,e^{i(\xi x - \omega t)} \qquad (24)$$

where Φ is the displacement potential in equation (1).

Since by equation (2)

$$p = -\rho\frac{\partial^2 \Phi}{\partial t^2} \qquad (2)$$

and u is the value of pressure, normalized to maximum amplitude, as a function of z then by equations (2) and (24)

$$u \propto \rho\varphi \qquad (25)$$

where φ is the normalized value of ϕ.

Thus by (25) we can obtain φ once u is known since we are only interested in the normalized values of u and φ for given ω.

Given ρ_1 in the water and ρ_b in the bottom and u normalized to maximum pressure amplitude, in order to obtain φ, the normalization of

ϕ , we first obtain

$$\varphi_1, u_u(z) = \frac{u(z)}{\rho_1}$$
$$\varphi_b, u_u(z) = \frac{u(z)}{\rho_b} \qquad (25a)$$

where 1 and b signify water and bottom and u_u signifies unnormalized. Since the maximum value of u lies in the water, since ρ_b is greater than ρ_1 , and because $u(z)$ is normalized with respect to maximum amplitude we multiply both expressions in (25a) by ρ_1 to obtain φ normalized with respect to maximum amplitude so that

$$\varphi_1 = u(z)$$
$$\varphi_b = u(z)\,\rho_1/\rho_b \qquad (25b)$$

φ is the normalized value for ϕ and may be used in place of ϕ in equations (21-23).

D. Excitation Pressure and Propagation Loss

The sound field produced by a simple harmonic source in a two-layered half-space (shown in Figure 6) with a free surface at $z=0$ and the boundary between two fluids at $z=h$ is given by the solution of equation (1). This is given in Reference (9) by

$$\Phi = -i \frac{1}{\sqrt{r}} \frac{1}{\omega^2} \sum_m P_m e^{i(k_m r - \omega t - \pi/4)} \qquad (26)$$

where r = horizontal range
ω = radial frequency
m = mode number
k_m = horizontal wave number k_r for mode m.

$$P_m = p_m \frac{1}{\rho_s} \varphi_m(z)\, \varphi_m(z_0) \qquad (27)$$

where ρ_s is the water density at the source
$\varphi_m(z)$ is the normalized displacement potential, a function of depth
z_0 is the source depth
z is the receiver depth
m is the mode number

p_m is the excitation function given by

$$p_m = 2\pi(\rho_0 c_0 S)^{1/2} \frac{\rho_0}{\nu_m \sqrt{k_m}} \qquad (28)$$

where ρ_o is the water density
c_o is sound velocity at the source
S is the power output of an omnidirectional source[1]
k_m is the horizontal wave number

and

$$\nu_m = \int_{-\infty}^{+\infty} \rho \varphi_m^2 \, dz \tag{29}$$

where φ_m is the normalized displacement potential.

If the source produces a unit sound pressure level then the following relationship (Reference 9) must be satisfied

$$4\pi \left(\frac{S c_o \rho_o}{2\pi} \right)^{1/2} = 1 \tag{30}$$

Substituting (30) into equation (28) we obtain the exitation pressure P_m such that

$$P_m = \frac{\rho_o (2\pi)^{1/2}}{2\nu_m \sqrt{k_m}} \tag{31}$$

The exitation pressure is the sound pressure amplitude produced by a source which generates a unit source pressure level at unit range when both source and receiver are at antinodes. It is essentially a measure of the source level of mode m for a unit source.

From equation (26) we determine the pressure amplitude characteristics of the sound field and this amplitude, p_n, is given by

$$p_n = \frac{\rho_o}{\sqrt{r}} \left\{ \left[\sum_m P_m \cos\left(k_m r - \frac{\pi}{4}\right) \right]^2 + \left[\sum_m P_m \sin\left(k_m r - \frac{\pi}{4}\right) \right]^2 \right\}^{1/2} \tag{32}$$

Since p_n is the sound pressure amplitude at range r from a generator with unit source level, the value of propagation loss L_r at range r for a given depth of source and receiver is given by

$$L_r = -20 \log p_n \tag{33}$$

It can be seen from equations (27) and (32) that once φ_m is known one can easily determine the effect of the source and receiver depth on the sound field at a given range. If the source depth is such that $\varphi_m(z_o)$, where z_o is the source depth, is a node the mode m will

[1]The quantity is represented by the symbol of capital Π in Reference 9.

be suppressed in the sound field. Likewise, if z_0 is such that $\varphi_m(z_0)$ is an antinode the sound field of mode m will be greater than at any depth for which $\varphi(z)$ is less than $\varphi_m(z_0)$. The preceding also applies to a discussion of the effect of the receiver depth upon the sound field.

It can also be seen from equations (27), (28), and (29) that the pressure amplitude determined does not depend upon the normalization of $\phi(z)$.

For small attenuation of individual modes equation (32) may be rewritten to include losses at the boundaries as a function of range D_m for mode m so that

$$p_a = \frac{p_0}{\sqrt{r}} \left\{ \left[\sum_m P_m 10^{-D_m r/20} \cos(k_m r - \pi/4) \right]^2 + \left[\sum_m P_m 10^{-D_m r/20} \sin(k_m r - \pi/4) \right]^2 \right\}^{1/2} \tag{34}$$

where D_m is given in db of loss per unit increment in range.

DESCRIPTION OF PROGRAM S1441

Program S1441 uses normal mode theory to predict acoustic propagation over a flat homogeneous ocean bottom. The program may be used with any of three options, each of which provides different information about the sound field in a medium for a given frequency, velocity profile, and mode number.

The three options provide as output the following calcomp plots:

A. The first option produces two plots for each mode analyzed. One plot gives pressure normalized to the maximum amplitude as a function of depth. Another plot gives the ray equivalent of the mode.

B. The second option produces two plots for each mode. One plot gives three quantities: phase velocity, group velocity, and excitation pressure. These quantities are plotted as a function of frequency. Another plot gives the angle of incidence of energy at the two boundaries for the given modes. These angles are plotted as a function of frequency.

C. The third option produces a plot of propagation loss versus range for any combination of modes. These plots can be produced for any source or receiver depth or frequency.

For any of the three options used, two plots of the velocity profile, shown in Figures 7 and 8, are produced. The first plot (Figure 7) shows the sound velocity in both the water and the bottom. The second plot (Figure 8) shows in more detail via an expanded velocity scale the sound velocity in the water.

The format of the input data is shown in Tables I to IX. In Table II it is shown how to obtain either of options A, B, or C. The mechanics of the individual options are described below.

Option A

In option A the numerical solution to equation (4) or (6) is found subject to the boundary conditions given by equations (8a), (8b), and (8c). Then the ray equivalent of this solution is obtained.

For a given velocity profile inputed (Table IV) the sound velocity is calculated at N (Table V) levels equispaced between the surface and bottom by interpolation between the values given.

The value of the horizontal wave number k_r is varied subject to the restrictions given in equation (8c) and for each value of k_r, $u(z)$ is calculated over the water column of N equispaced levels by means of equations (9) and (10). The values of $u(z)$ is restricted such that $u(z)$ never exceeds U_m (Table V). Mode M must have M zero crossings. After $u(z)$ is calculated for a given k_r, k_r is incremented by Δk_r so as to obtain the smallest possible value of u at the surface.
If the conditions necessary for the existence of a given mode cannot be met, the statement "No Mode Found" is printed out.

Once $u(z)$ has been found for a given mode the ray equivalent can be found by equation (16). Sample calcomp plots of the amplitude distribution normalized to maximum amplitude and ray equivalent are found in Figures 1 and 2.

For a large negative velocity gradient high frequency sound is trapped near the oceans bottom. Under these circumstances, it is difficult to obtain a good approximation to the boundary condition at the surface. Then it is necessary to decrese the value by which k_r is incremented. This is done by using a large value (up to about 10) of IEX (Table II). A large value of IEX will increase the program time since it decreases the increment of k_r by a factor of 10^{-IEX}. Under most circumstances, a value of IEX of zero is adequate.

Option B

In option B group velocity, phase velocity, excitation pressure, and angle of incidence at the boundaries relative to the normal to the boundary are found over any frequency range (Table VI). For a given frequency, group velocity is determined by equations (21), (22), and (23); phase velocity by equation (18), and excitation pressure by equation (31). The angle of incidence at the boundaries is given by equation (15). If the ray vertexes before striking a boundary, the angle of incidence is given as 90°. Sample plots obtained from Option B are shown in Figures 9 and 10.

Option C

In Option C propagation loss for a source level at a one yard reference is obtained as a function of range for any given frequency and combination of modes. Propagation loss may be calculated by equations (32) and (33). The ranges over which loss is plotted and source and receiver depth are inputed by a card described in Table VII. Values of D_m are inputed as shown in Table VIII and the modes which make up the sound field are inputted as shown in Table IX. A sample calcomp plot is shown in Figure 11.

CONCLUSIONS

Program S1441 is designed to calculate and plot many quantities of interest in the study of a sound field in an ocean bounded by flat parallel boundaries. The problem of acoustic propagation is approached from the standpoint both of physical and ray acoustics.

It is possible at high frequencies when the velocity profile has more than one "channel" to obtain solutions of the differential equation (6) which are physically unrealizable. This can only be detected by observing the values of $\mathfrak{K}(z)$ which are shown in the printout from this program. If two depths which have positive values of $\mathfrak{K}(z)$ which indicate propagation of rays are separated by negative values of $\mathfrak{K}(z)$ which indicate a "shadow zone" one has a physically unrealizable situation. Since the velocity profiles taken over the BIFI range show in general a monotonically decreasing or increasing sound velocity with depth this limitable does not severely handicap analysis. However, caution should be taken in analysis when complex velocity gradients are used.

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REFERENCES

1. Arthur D. Little, Inc., "Design of An Array to Excite Individual Normal Modes in the BIFI Range" Report C-70673, 31 January 1969.

2. Arthur D. Little, Inc., "A Memorandum Describing the Computer Program Used to Calculate Normal Modes in the BIFI Range" February 1969.

3. Pekeris, C.L., "Theory of Propagation of Explosive Sound in Shallow Water" GSA Memoir #27, 15 October 1948.

4. Officer, C.B., "Normal Mode Propagation in Three Layered Liquid Half-Space by Ray Theory" Journal of Applied Physics, Vol 23, 997, 1952.

5. Kanabis, W.G. and Bernier, H.F., "Propagation in BIFI Range Using Explosive Sources" NUSL Technical Memorandum No. 2211-94-68.

6. Kanabis, W.G., "BIFI Propagation Tests of 30 January 1968" NUSL Technical Memorandum No. 2211-185-68.

7. Officer, C.B., "Introduction to the Theory of Sound Transmission" McGraw-Hill Book Company, Inc., New York, 1958.

8. Weinberg, H. and Einstein, L.T., "Eigenray Analysis for an Ocean Model with a Sloping Bottom" NUSL Technical Memorandum No. 2070-25-67.

9. Tolstoy, I. and Clay, C.B., "Ocean Acoustics" McGraw-Hill Book Co., Inc., New York, 1966.

10. Brekhovskikh, L.M. "Waves in Layered Media" Academic Press Inc., New York, N.Y. 1960.

11. Tolstoy, I., "Shallow Water Test of the Theory of Layered Wave Guides" J. Acoustical Society Am., Volume 30, 340, 1958.

12. Biot, M.A., "General Theorems on Equivalence of Group Velocity and Energy Transport" Phys. Rev. Vol. 105, 1129, 1957.

FORMAT (20A4)

HEADING ON EACH PAGE OF THE OUTPUT

TABLE I

Heading Card

CARD 1

FORMAT (I10, F10.0, 4I10)

INPUT PARAMETER	COLUMNS	
NUMV	1-10	Number of velocities in profile
VEL 1	11-20	Velocity at origin of calcomp plot of velocity profile
IVØP	21-30	If IRP = 0 When IVOP = 0 Option A When IVOP = 1 Option B
IRP	31-40	When IVOP = 1 If IRP = 1 Option C
IVRP	41-50	IVRP = 1 Velocity profile not plotted IVRP = 0 Profile plotted
IEX	51-60	IEX changes increment of Kr by a factor of 10^{-IEX}. Values from 0-10.

TABLE II

CARD II

FORMAT (5F10.3)

INPUT PARAMETER	COLUMNS	
ZM	1-10	Water depth (ft)
CB	11-20	Velocity of sound in the bottom (ft/sec)
RO	21-30	Density of water (grams 1 cm^3)
RB	31-40	Density of Bottom (grams 1 cm^3)
FSC	41-50	Maximum Depth Plotted in velocity profiles and plots in option A is 200*FSC

TABLE III

CARD 3

FORMAT (2F10.3)

INPUT PARAMETER	COLUMNS	
Z(I)	1-10	Height above bottom at which sound velocity is C(I)
C(I)	11-20	Velocity of sound in (ft/sec) at Z(I)

I = 1, ... NUMV
in order of increasing Z

TABLE IV

GROUP 4
NUMV Cards

FORMAT (I10, 2F10.3)

INPUT PARAMETER	COLUMNS	
N	1-10	Number of intervals into which depth is to be sub-divided in integration of differential equations
UM	11-20	Maximum value of $u(z)$
FQ	21-30	Frequency (Hz)

TABLE V

CARD 5

FORMAT (4I10)

INPUT PARAMETER	COLUMNS	
ISFQ	1-10	Highest frequency to be analyzed
IEFQ	11-20	Lowest frequency to be analyzed
NMOD	21-30	Number of modes analyzed Mode numbers analyzed = 1, ... NMOD
INCF	31-40	Decrement in frequency from ISFQ to IEFO

Options A and C ISFQ = IEFQ = FQ
Option B Range of frequencies is selected by ISFQ, IEFQ, INCF

TABLE VI

CARD 6

FORMAT (4I10, 3F10.3)

INPUT PARAMETER	COLUMNS	
IRST	1-10	Lowest range in feet at which propagation loss versus range will be plotted
IREN	11-20	Largest range in feet at which propagation loss will be plotted
IRIC	21-30	Increment in range to be plotted in feet
NPS	31-40	Number of propagation loss versus range plots
ZS	41-50	Source depth in feet
ZRC	51-60	Receiver in depth in feet
FMI	61-70	Increment of range on the calcomp plot per division in nautical miles

TABLE VII

CARD 7
For Option C

FORMAT (4F10.5)

INPUT PARAMETER	COLUMNS	
DD(J)	1-10	DD(J) is the loss at boundaries as a function of range in db per foot for the mode J.
	11-20	
	21-30	J = 1, NMOD where NMOD is the number of modes analyzed
	31-40	

The number of cards in this group is the integer greater than or equal to the quantity $\frac{NMOD}{4}$

TABLE VIII

GROUP 8
For Option C

Card a.

FORMAT I10

INPUT PARAMETER	COLUMNS	
NMS	1-10	Number of modes to be summed in the plot of propagation less versus range(NMS≤NMOD)

Card(s) b.

FORMAT 6I10

INPUT PARAMETER	COLUMNS	
MDS(J)	1-10	J = 1, NMS The value of MDS(J) are the mode numbers of the modes to be summed in the propagation loss versus range plots. MDS(J)≤NMOD
	11-20	
	21-30	
	31-40	
	41-50	
	51-60	

Number of card(s) b. is the integer greater than or equal to $\frac{NMS}{6}$

Cards a and b form a set. There is a set of cards for each plot in Option C. (NPS sets)

TABLE IX

GROUP 9
For Option C

APPENDIX
S1441


```
      DOUBLE PRECISION     F,U,UP
      DOUBLE PRECISION               K2,    W2,P2,H,ZP,   CZ,G,H2,DEN
      DOUBLE PRECISION S,K20,DK2,GAM,BA
      DOUBLE PRECISION E,V
      DOUBLE PRECISION  UMAX
      DOUBLE PRECISION  COS2 , CCC  , CV
      DIMENSION HED(20),Z(100),C(100),F(500),UP(500) ,U(500) ,
     1  ZMP(1000), XMP(1000), COS2(500), CCC(500)
     1  , ZZP(500) , ZPP(500),  VV(500),  DATA(1024) , UU(500)
     1  ,PM(5000)          ,DDT(100)
     1  , ZR(500) , CR(500) ,RP(100) ,ZBT(5) , ZBF(5)
     1,DD(100),PPT(1000),PCT(1000),PST(1000),MDS(100),USR(100),URR(100),
     1  PMM(100),FMR(100)  ,FDS(100)
     1 ,GVEL(4000), PVEL(4000), FQP(4000), T1(4000), T2(4000)
      COMMON MS,JM,U
      COMMON JMS
      CALL PLOTS  ( DATA(1), 1024,  6  )
    4 JPM = 25
      READ(3,1)HED
    1 FORMAT (20A4)
      READ (3,112)        NUMV, VEL1  , IVOP  ,IRP  , IVPL  , IEX
  112 FORMAT ( I10,F10.0,4I10)
      NUM  =  0
      READ(3,2) ZM,CB,RO,RB ,FSC
    2 FORMAT (5F10.3)
      ZMB = (200.0*FSC  -ZM)/(20.0*FSC)
      ZBT(1) = ZM
      ZBT(2) = ZM
      ZBT(3) = 200.0 *FSC
      ZBT(4) = -20.0*FSC
      ZBF(1) =  0.0
      ZBF(2) = 10.0
      ZBF(3) =  0.0
      ZBF(4) =  1.0
      DO 3 I=1,100
      Z(I) = 0.
    3 C(I) = 0.
      I=1
    5 READ (3,30) Z(I),C(I)
   30 FORMAT(2F10.3)
      IF(DABS(Z(I)-ZM)-.01)6,6,7
    7 I = I+1
      GO TO 5
    6 I = 1
      IF (IVPL.EQ.1)  GO TO 8
      DO 130  K  =  1, NUMV
      CR(NUMV -K +1)  =  C(K)
  130 ZR(NUMV -K +1)  =  Z(K)
      ZR(NUMV +1) = ZM -0.01
      ZR(NUMV +2) = ZM +50.0
      CR(NUMV  +1)  =  CB
      CR(NUMV  +2)  =  CB
      DO 185   NINY  = 1, NUMV
  185 ZR(NINY)  =  ZM - ZR(NINY)
      ZR(NUMV +3) = 200.0*FSC
      ZR(NUMV +4) = -20.0*FSC
      CR(NUMV  +3)  =  VEL1
      CR(NUMV  +4)  =  50.0
```

```
      CALL PLOT (0.0,0.0,-3)
      CALL LINE (CR,ZR,NUMV +2, 1,0,0  )
      CALL LINE (ZBF,ZBT,2,1,0,0)
      CALL SYMBOL (10.25,ZMB,0.14,6HBOTTOM,0.0,6 )
      CALL AXIS (0.0,0.0,14HSOUND VELOCITY,14,10.,0.0,CR(NUMV  +3) ,
     1CR(NUMV+4),10.0  )
      CALL AXIS (0.0,0.0,11HWATER DEPTH,11,10.0,90.0,ZR(NUMV +3)   ,
     1ZR(NUMV+4),10.0  )
      CALL PLOT (15.0,0.0,-3)
      ZR(NUMV +1) = 200.0*FSC
      ZR(NUMV +2) = -20.0*FSC
      CR(NUMV +1) =VEL1
      CR(NUMV +2) =10.0
      CALL LINE (CR,ZR,NUMV,1,0,0)
      CALL LINE (ZBF,ZBT,2,1,0,0)
      CALL SYMBOL (10.25,ZMB,0.14,6HBOTTOM,0.0,6 )
      CALL AXIS (0.0,0.0,14HSOUND VELOCITY,14,10.,0.0,CR(NUMV  +1) ,
     1CR(NUMV+2),10.0  )
      CALL AXIS (0.0,0.0,11HWATER DEPTH,11,10.0,90.0,ZR(NUMV +1)   ,
     1ZR(NUMV+2),10.0  )
      CALL PLOT  ( 15.0,0.0, -3)
    8 JP = 3
  188 WRITE(4,9)HED
    9 FORMAT (1H1,9X,20A4)
      WRITE(4,10)
   10 FORMAT(1H0,15X,4HZ,FT,5X,8HC,FT/SEC)
   11 WRITE(4,12)ZR(I),CR(I)
   12 FORMAT(1H0,9X,F10.1,F12.1)
      IF(DABS(Z(I)-ZM)-.01)14,14,13
   13 I = I+1
      JP = JP+1
      IF(JP-JPM)11,8,8
   14 READ(3,15)N,  UM ,FQ
   15 FORMAT(I10,2F10.3)
      NNN  =  N
  151 READ (3,150) ISFQ, IEFQ, NMOD, INCF
  150 FORMAT (4I10)
      IF (IRP.NE.1)  GO TO 120
      READ (3,121) IRST, IREN, IRIC, NPS, ZS, ZRC  ,FMI
  121 FORMAT (4I10, 3F10.3)
      ZS = ZM - ZS
      ZRC = ZM - ZRC
      READ (3.122)  (DD(J), J= 1,NMOD)
  122 FORMAT (4F10.5)
  120 DO 154  M=  1,NMOD
      DO 155  L= ISFQ, IEFQ,-INCF
      IF (IVOP.NE.1)   GO TO 162
      FQ  =  L
  162 P2 = 6.283185306
      W2 = (P2*FQ)*(P2*FQ)
      N  =  NNN
      FMOD = M
      IREL = 0
      IF(N)29,4,29
   29 K20 = ((W2*(CB-C(1))*(CB+C(1)))/(C(1)*C(1)*CB*CB))
      DK2= .099*K20
      K2 = -K20
   31 K2 = K2 + DK2
```

```
      IF(K2-K20)16,32,32
   32 WRITE(4,33)
   33 FORMAT(1H0,13HNO MODE FOUND)
      IEFQ  =  L  + INCF
      IF (IVOP.EQ.1)  GO TO 160
      GO TO 111
   16 FLB  =  FLOAT(N)
      H = ZM/ DBLE(FLB)
      ZP = 0.
      J = 1
      I = 1
   17 IF (ZP- Z(I+1)) 18,18,19
   19 I = I+1
      GO TO 17
   18 CZ = (C(I)*(Z(I+1)-ZP)+C(I+1)*(ZP-Z(I)))/(Z(I+1)-Z(I))
      CCC(J)  =  CZ
      G =-(W2*(C(1)-CZ)*(C(1)+CZ))/(CZ*CZ*C(1)*C(1))
      F(J) = K2 - G
      IF(ZP-ZM) 20,21,21
   20 J=J+1
      FLA  =  FLOAT(J -1)
      ZP  =  ZM*DBLE(FLA)/DBLE(FLB)
      GO TO 17
   21 U(1) = 1.0
      UP(1) =(RO/RB)*DSQRT ((W2*(CB-C(1))*(CB+C(1)))/(C(1)*(C(1)*CB*CB))
     1-K2)
      J = 1
   22 JP = 3
   25 FLA  =  FLOAT(J -1)
      FLB  =  FLOAT(N)
      ZP  =  ZM*DBLE(FLA)/DBLE(FLB)
      IF(J-N-1)27,34,27
   27 IF(DABS(U(J))-UM)28,34,34
   28 J = J+1
      JP= JP+1
      H2= H*H
      DEN = 1.0 + (H2*F(J))/6.0
      U(J)= ((1.0-(H2/3.0)*F(J-1))*U(J-1)+H*UP(J-1))/DEN
      UP(J)=((1.0-(H2*F(J))/3.0)*UP(J-1)-(F(J-1)+F(J)-(H2*F(J)*F(J-1))/
     16.0)*H*U(J-1)* 0.5)/DEN
      IF (JP-JPM)25,22,25
   34 JM = J
      CALL COUNT
  195 IF (MS-M)31,35,35
   35 K2 = K2 - DK2
      DK2= DK2 / 10.0
      IF(DK2- .000001*K20 *10**(-IEX))36,36,31
   36 J = 2
      S = H*U(1)*U(1)/2.0
      S2= H*U(1)*U(1)/(2.0*CCC(1)*CCC(1))
   37 IF(J-JMS)38,39,38
   38 S = H*U(J)*U(J)+S
      S2= H*U(J)*U(J)/(CCC(J)*CCC(J))   +S2
      J = J+1
      GO TO 37
   39 S =(H*U(J)*U(J)/2.0) + S
      S2= H*U(J)*U(J)/(2.0*CCC(J)*CCC(J))   +S2
      IF (IVOP.EQ.1)  GO TO 164
```

```
      J = 1
   42 JP = 3
      WRITE(4,23)HED,N,K2,UM,FQ,M
   23 FORMAT(1H1,5X,20A4,/,2X,3HN =,I5,2X,4HK2 =,F15.8,2X,4HUM =,F6.1,
     12X,6HFREQ = ,F6.1,2X,3HM =,I5)
      WRITE(4,47) ZM,CB,RO,RB
   47 FORMAT(3X,6HZMAX = ,F10.3,4HCB = ,F10.3,4HRO = ,F15.8,4HRB = ,
     1F15.8)
      WRITE(4,24)
   24 FORMAT(1H0,17X,4HZ,FT,10X,1HU,10X,5HDU/DZ,11X,4HF(Z))
   43 ZP = ZM*( FLOAT(J-1)/ FLOAT(N))
      ZQ = ZM - ZP
      WRITE(4,26)ZQ,U(J),UP(J),F(J)
      IF(J-N-1)45,44,45
   45 IF(DABS(U(J))-UM)46,44,44
   46 J = J+1
      JP = JP+1
      IF(JP-JPM)43,42,43
   44 WRITE(4,40) S
   40 FORMAT (1H0,10X, 3HS = ,F15.5)
  164 UMAX = 0.0
      NM  =  N  +1
      DO  104  K=  1, NM
      IF  ( DABS (U(K)) - UMAX)  104,104,103
  103 UMAX  =  DABS(U(K) )
  104 CONTINUE
      DO  105  K=  1, NM
  105 UU(K) =  U(K)/UMAX
      DO 106  K= 1, NM
  106 ZZP(K)  =  ZM*(FLOAT(K -1)/FLOAT(N)  )
      GAM = DSQRT(K20 - K2)
      BA = (RO*C(1))/(2.0*RB*CB*GAM*S )
      WRITE(4,41) BA,GAM
   41 FORMAT(1H0,10X,5HB/A = ,F15.5,5X,7HGAMMA = ,F15.5 )
      SS=  RO*S/(UMAX*UMAX)+RO* RO*U(1)*U(1)/(UMAX*UMAX*2.0*GAM*RB)
      SSS= RO*S2/(UMAX*UMAX)           +       RO*RO*U(1)*U(1)/(UMAX*UMAX
     1*2.0*GAM*RB*CB*CB)
      IF(IVOP.EQ.1)  GO TO 153
      E = DEXP(-GAM*H)
      ZP = 0.0
      V = 1.0
      NN   =  1
      ZPP(1)  =  0.0
      VV(1)  = UU(1)
      WRITE(4,23)HED,N,K2,UM,FQ,M
      WRITE(4,47) ZM,CB,RO,RB
      WRITE(4,48)
   48 FORMAT(1H0,17X,4HZ,FT,10X,1HU )
   49 WRITE(4,26)ZP,V
   26 FORMAT(1H0,F22.2,F13.3,F13.3,F16.5)
   50 V = V*E
      NN  = NN  +1
      VV(NN)  =  VV(NN -1)*E
      ZPP(NN)  =  ZP -      H
      ZP = ZP-H
      IF  (ZPP(NN)  + 50.0)        110, 110,  49
  110 CALL PLOT  (  5.0,0.0, -3)
      DO 184   NINX  = 1, NN
```

```
184 ZPP(NINX) =  ZM - ZPP(NINX)
    NINV  =  N + 1
    DO 183   NINW  = 1, NINV
183 ZZP(NINW) =  ZM - ZZP(NINW)
    VV(NN +1)  =  -1.0
    VV(NN +2)  =  0.5
    ZPP(NN +1) = 200.0*FSC
    ZPP(NN +2) = -20.0*FSC
    ZZP(N +2)  = 200.0*FSC
    ZZP(N +3)  = -20.0*FSC
    UU(N+2)  =  -1.0
    UU(N+3)  =  0.5
    CALL  AXIS (0.0,0.0, 9HAMPLITUDE,  9,4.0,0.0,UU(N+2),UU(N+3),10.0)
    CALL AXIS (0.0,0.0,16HWATER DEPTH (FT),16,10.0,90.0,ZZP(N+2),
   1  ZZP(N +3),10.0)
    CALL LINE  (UU, ZZP,  N+1, 1,0,0)
    CALL LINE (VV, ZPP,  NN,  1,0,0  )
    CALL LINE (ZBF,ZBT,2,1,0,0)
    CALL SYMBOL (10.25,ZMB,0.14,6HBOTTOM,0.0,6 )
    CALL SYMBOL(1.3,9.50,0.14,9HAMPLITUDE,0.0,9)
    CALL SYMBOL(1.7,9.25,0.10,6HVERSUS,0.0,6)
    CALL SYMBOL(1.6,9.00,0.14,5HDEPTH,0.0,5)
    CALL SYMBOL(0.8,8.75,0.14,19HFREQUENCY         HZ,0.0,19)
    CALL NUMBER(2.2,8.75,0.14,FQ,0.0,-1)
    CALL SYMBOL(1.4,8.50,0.14,4HMODE,0.0,4)
    CALL NUMBER(2.1,8.50,0.14,FMOD,0.0,-1)
    NUM  =  NUM  +1
153 XMP(1)  =  0.0
    ZMP(1)  =  0.0
    NM  =  N  +1
    DO 170 K=  1,NM
    IF(F(1).GT.0.0)   GO TO 179
    IF(F(K).GT.0.0)    GO TO 171
170 IRTB = K
171 COS2(IRTB +1) = F(IRTB +1)*CCC(IRTB +1)*CCC(IRTB +1)/W2
    CV  =  CCC(IRTB +1)/DSQRT(1.0 - COS2(IRTB +1))
    ZMP(2) = H*(CV - CCC(IRTB +1))/(CCC(IRTB)-CCC(IRTB +1))
    XMP(2) =  (ZMP(2) - ZMP(1))*(CCC(IRTB +1) + CV)/(CV*(DSQRT(COS2
   1 (IRTB +1))))
    ZMP(1) = FLOAT(IRTB)*H - ZMP(2)
    ZMP(2) = ZMP(1) + ZMP(2)
    NM = NM -IRTB +1
    DO 172  K = 3,NM
    TH1 = 90.0
    KO = IRTB -1 +K
    COS2(KO-1) = F(KO -1)*CCC(KO -1)*CCC(KO -1)/W2
    COS2(KO)=F(KO)*CCC(KO)*CCC(KO)/W2
    TH2 = (ACOS(SQRT(COS2(NM +IRTB -1))))*57.2958
    IF(CCC(KO).GE.CV)   GO TO 142
    XMP(K) = XMP(K  -1) +H*(CCC(KO -1) + CCC(KO))/(CV*(DSQRT(COS2(KO-1
   1)) + DSQRT(COS2(KO))))
172 ZMP(K)  =  (KO -1)*H
    N =  NM -1
    GO TO 173
179 COS2(1)  =  ABS(F(1))*CCC(1)*CCC(1)/W2
    CV  =  CCC(1)/DSQRT(1.0 - COS2(1) )
    DO 140  K=  2,NM
    COS2(K -1) = ABS(F(K -1))*CCC(K -1)*CCC(K -1)/W2
```

```
      COS2(K)  =  ABS(F(K))*CCC(K)*CCC(K)/W2
      TH1 =(ACOS(SQRT(COS2(1))))*57.2958
      TH2 =(ACOS(SQRT(COS2(NM))))*57.2958
      IF( CCC(K).GE.CV)  GO TO 142
      XMP(K) = XMP(K -1) + H*(CCC(K-1) + CCC(K))/(CV*(DSQRT(COS2(K-1))+
     1 DSQRT(COS2(K))))
  140 ZMP(K)  =  (K-1)*H
  173 DO 141  K=  1,N
      XMP(NM +K) = 2.0*XMP(NM)  -  XMP(NM - K)
  141 ZMP(NM +K) = ZMP(NM -K)
      GO TO 143
  142 ZMP(K) = ZMP(K -1)  +  H*(CV - CCC(K-1))/(CCC(K) - CCC(K -1))
      TH2  =  90.0
      XMP(K) = XMP(K -1) + (ZMP(K) - ZMP(K-1))*(CCC(K-1) + CV)/(CV*(
     1  DSQRT(COS2(K -1))))
      N = K -1
      NM  =  K
      DO 144  K=  1,N
      XMP(N+1+K) = 2.0*XMP(N+1) - XMP(N+1-K)
  144 ZMP(NM +K) = ZMP(NM -K)
  143 XMP(2*NM)  =  0.0
  182 XMP(2*NM +1)  =  1000.0
      NIZZ = 2*NM -1
      DO 186 NINZ = 1, NIZZ
  186 ZMP(NINZ) = ZM - ZMP(NINZ)
      ZMP(2*NM)  =  200.0*FSC
      ZMP(2*NM +1)  = -20.0*FSC
      WRITE (4,145) TH1,TH2
      IF (IVOP.EQ.1)  GO TO 157
  145 FORMAT (2F10.3)
      CALL PLOT (15.0,0.0,-3)
      CALL LINE (XMP,ZMP,NM +N,1,0,0)
      CALL LINE (ZBF,ZBT,2,1,0,0)
      CALL SYMBOL (10.25,ZMB,0.14,6HBOTTOM,0.0,6 )
      CALL AXIS (0.0,0.0,10HRANGE (FT),10,12.0,0.0,XMP(2*NM)    ,
     1XMP(2*NM +1),10.0)
      CALL AXIS (0.0,0.0,16H      DEPTH (FT),16,10.0,90.0,ZMP(2*NM),
     1 ZMP(2*NM +1),10.0 )
      CALL SYMBOL(5.0,9.50,0.14,14HRAY EQUIVALENT,0.0,14)
      CALL SYMBOL(5.0,9.25,0.14,19HFREQUENCY          HZ,0.0,19)
      CALL NUMBER(6.4,9.25,0.14,FQ,0.0,-1)
      CALL SYMBOL(5.5,9.00,0.14,4HMODE,0.0,4)
      CALL NUMBER(6.2,9.00,0.14,FMOD,0.0,-1)
      CALL PLOT (15.0,0.0,-3)
      IF (IVOP.NE.1)   GO TO 154
  157 T1(L) = TH1
      T2(L) = TH2
      FMH = SQRT(P2*FQ /(30.48*CV) )
      PM(L) = SQRT(P2)*      RO/(FMH*2.0*(SS) )
      GVEL(L) = SS/(CV*SSS)
      FQP(L) = L
      WRITE ( 4,166)  PM(L) , FMH, SS  , GVEL(L)
  166 FORMAT(4F10.3)
      IF (IRP.EQ.1) PM(M) = PM(L)
      FMR(M) = P2*FQ/CV
      IZS = ZS*FLB/ZM   +1.0
      IZR =ZRC*FLB/ZM   +1.0
      USR(M) = U(IZS)/UMAX
```

```
      URR(M) = U(IZR)/UMAX
      PMM(M) = PM(M)*USR(M)*URR(M)/RO
  155 PVEL(L) = CV
      IF (IRP.EQ.1) GO TO 154
  160 IDF    = (ISFQ - IEFQ)/INCF  +1
      LIFQ = IDF -1
      DO 161 LLL= 1, IDF
      ILL  =  (LLL  -1)*INCF
      FQP(LLL)  =  FQP(IEFQ+ILL)
      GVEL(LLL)  =  GVEL(IEFQ+ILL)
      T1(LLL)  =  T1(IEFQ+ILL)
      T2(LLL)  =  T2(IEFQ+ILL)
      PM(LLL) = PM(IEFQ + ILL)
  161 PVEL(LLL)  =  PVEL(IEFQ+ILL)
      WRITE(4,163)  (PVEL(J), GVEL(J), FQP(J), PM(J), J= 1,IDF)
  163 FORMAT (4F10.3)
      FQP(LIFQ +2)  =  0.0
      FQP(LIFQ +3) =  ISFQ/10
      PVEL(LIFQ +2)  =  3500.0
      PVEL(LIFQ +3)  =  250.0
      GVEL(LIFQ +2)  =  3500.0
      GVEL(LIFQ +3)  =  250.0
      PM(LIFQ  +2)  =  0.0
      PM(LIFQ  +3)  =  0.1
      CALL PLOT (15.0,0.0,-3)
      CALL LINE (FQP,PVEL,LIFQ +1,1,10,14)
      CALL LINE (FQP,GVEL,LIFQ +1,1,10,28)
      CALL LINE (FQP,PM,LIFQ +1,1,10,4)
      CALL AXIS (0.0,0.0,14HFREQUENCY (HZ),14,10.0,0.0,FQP(LIFQ +2),
     1 FQP(LIFQ +3),10.0)
      CALL AXIS (0.0,0.0,27HSOUND VELOCITY (FT PER SEC),27,10.0,90.0,
     1  PVEL (LIFQ +2),PVEL(LIFQ +3),10.0)
      CALL AXIS(10.0,0.0,18HEXITATION PRESSURE,-18,10.,90.0,PM(LIFQ +2),
     1 PM(LIFQ +3),10.0)
      CALL SYMBOL(2.4,9.50,0.14,38HSOUND VELOCITY AND EXCITATION PRESSUR
     1E,0.0,38)
      CALL SYMBOL(4.7,9.25,0.10,6HVERSUS,0.0,6)
      CALL SYMBOL(4.4,9.00,0.14,9HFREQUENCY,0.0,9)
      CALL SYMBOL(4.5,8.75,0.14,4HMODE,0.0,4)
      CALL NUMBER(5.2,8.75,0.14,FMOD,0.0,-1)
      CALL SYMBOL(4.7,8.50,0.10,14HPHASE VELOCITY,0.0,14)
      CALL SYMBOL(4.2,8.50,0.10,14,0.0,-1)
      CALL SYMBOL(4.7,8.25,0.10,14HGROUP VELOCITY,0.0,14)
      CALL SYMBOL(4.2,8.25,0.10,28,0.0,-1)
      CALL SYMBOL(4.7,8.00,0.10,19HEXCITATION PRESSURE,0.0,19)
      CALL SYMBOL(4.2,8.00,0.10,4,0.0,-1)
       T1(LIFQ  +2)  = 90.0
       T1(LIFQ +3)  = -10.0
       T2(LIFQ +2)  = 90.0
       T2(LIFQ  +3)  = -10.0
      CALL PLOT (15.0,0.0,-3)
      CALL LINE (FQP, T1,LIFQ +1,1,10,4)
      CALL LINE (FQP, T2,LIFQ +1,1,10,5)
      CALL AXIS (0.0,0.0,14HFREQUENCY (HZ),-14,10.,0.0,FQP(LIFQ +2),
     1 FQP(LIFQ +3),10.0)
      CALL AXIS (0.0,0.0,28HANGLE OF INCIDENCE (DEGREES),28, 9.0,90.0,
     1  T1(LIFQ +2), T1(LIFQ +3),10.0)
      CALL SYMBOL(3.7,9.50,0.14,18HANGLE OF INCIDENCE,0.0,18)
```

```
      CALL SYMBOL(4.7,9.25,0.10,6HVERSUS,0.0,6)
      CALL SYMBOL(4.4,9.00,0.14,9HFREQUENCY,0.0,9)
      CALL SYMBOL(4.5,8.75,0.14,4HMODE,0.0,4)
      CALL NUMBER(5.2,8.75,0.14,FMOD,0.0,-1)
      CALL SYMBOL(4.5,8.50,0.10,25HBOTTOM ANGLE OF INCIDENCE,0.0,25)
      CALL SYMBOL(4.0,8.50,0.10,4,0.0,-1)
      CALL SYMBOL(4.5,8.25,0.10,26HSURFACE ANGLE OF INCIDENCE,0.0,26)
      CALL SYMBOL(4.0,8.25,0.10,5,0.0,-1)
  154 CALL PLOT (15.0,0.0,-3)
      IF (IRP.NE.1)  GO TO 111
  123 DO 124 IP=  1,NPS
      READ (3,125)  NMS
  125 FORMAT (I10)
      READ (3,126)  (MOS(J),  J=1,NMS)
  126 FORMAT (6I10)
      DO 131 IR = IRST, IREN, IRIC
      IREO = (IR -IRST)/IRIC  +1
      PCT(IREO)  =  0.0
  131 PST(IREO)  =  0.0
      DO 127  IQ= 1,NMS
      M=  MOS(IQ)
      DO 129 IR = IRST, IREN, IRIC
      RR  =  IR
      IREO = (IR -IRST)/IRIC  +1
      DDT(M) = 10.0**(-DD(M)*RR/20.0)
      PST(IREO)  =  PST(IREO)+PMM(M)*DDT(M)        *SIN(FMR(M)*RR-P2/8.0)
      PCT(IREO)=    PCT(IREO)+PMM(M)*DDT(M)        *COS(FMR(M)*RR-P2/8.0)
  129 CONTINUE
  127 CONTINUE
      DO 132 IR = IRST, IREN, IRIC
      RR  =  IR
      IREO = (IR -IRST)/IRIC  +1
      PPT(IREO) = SQRT(PCT(IREO)*PCT(IREO) + PST(IREO)*PST(IREO))*RO/
     1SQRT(RR/3.0)
      PPT(IREO        ) =-20.0*ALOG10(PPT(IREO))
  132 RP(IREO        )  = FLOAT(IR)/6000.0
      WRITE (4,128) (PPT(J),  J= 1,IREO    )
  128 FORMAT (10F10.5)
      RP(IREO +1)  =  0.0
      RP(IREO +2)  =  FMI
      PPT(IREO +1)  =  30.0
      PPT(IREO +2)  =  10.0
      ZS = ZM - ZS
      ZRC = ZM - ZRC
      CALL LINE (RP,PPT,IREO,1,10,4)
      CALL AXIS(0.0,0.0,13HRANGE (MILES),13,20.0,0.0,RP(IREO +1),
     1 RP(IREO +2),10.0)
      CALL AXIS(0.0,0.0,21HPROPAGATION LOSS (DB),21,10.0,90.0,
     1 PPT(IREO +1),PPT(IREO +2),10.0)
      CALL SYMBOL(8.9,9.50,0.14,16HPROPAGATION LOSS,0.0,16)
      CALL SYMBOL(9.7,9.25,0.10,6HVERSUS,0.0,6)
      CALL SYMBOL(9.7,9.00,0.14,5HRANGE,0.0,5)
      CALL SYMBOL(8.9,8.75,0.14,18HFREQUENCY         HZ,0.0,18)
      CALL NUMBER(10.3,8.75,.14,FQ,0.0,-1)
      CALL SYMBOL(8.6,8.50,0.14,20HSOURCE DEPTH        FT,0.0,20)
      CALL NUMBER(10.5,8.50,0.14,ZS,0.0,-1)
      CALL SYMBOL(8.6,8.25,0.14,22HRECEIVER DEPTH        FT,0.0,22)
      CALL NUMBER(10.6,8.25,0.14,ZRC,0.0,-1)
```

```
      CALL SYMBOL (8.6,8.0,0.14,5HMODES,0.0,5)
      DO 187  J=  1,NMS
      FUS(J)  =  MDS(J)
  187 CALL NUMBER (9.1  +0.5*J,8.0,0.14,FUS(J),0.0,-1)
      CALL PLOT (15.0,0.0,-3)
  124 CONTINUE
      CALL PLOT (15.0,0.0,-3)
  111 CALL PLOT (15.0,0.0,-3)
      END
```

```
      SUBROUTINE COUNT
      DIMENSION U(500)
      DOUBLE PRECISION U
      COMMON MS,JM,U
      COMMON JMS
      MS=0
      J=1
      IS=1
      JMS=1
    5 IF (U(J)) 1,2,3
    3 IS1=IS
      IS=1
    7 IF (IS-IS1) 4,2,4
    4 MS=MS+1
      JMS=J
    2 J=J+1
      IF (J-JM-1) 5,6,5
    1 IS1=IS
      IS=0
      GO TO 7
    6 RETURN
      END
```

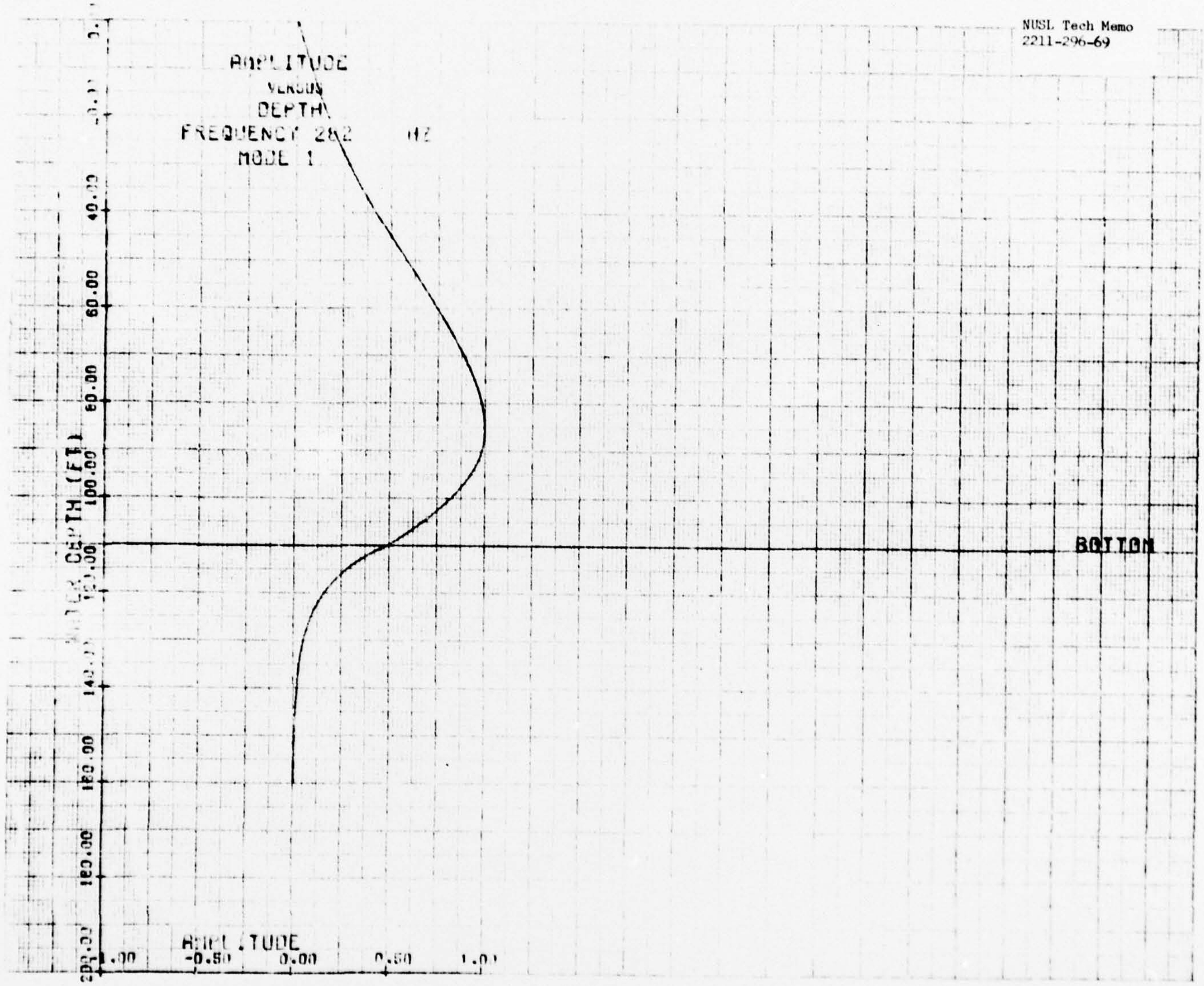
AMPLITUDE
VERSUS
DEPTH
FREQUENCY 282 HZ
MODE 1
BOTTOM
AMPLITUDE
-0.50
0.00
0.50
40.00
60.00
80.00
100.00
140.00
160.00
180.00

Figure 1

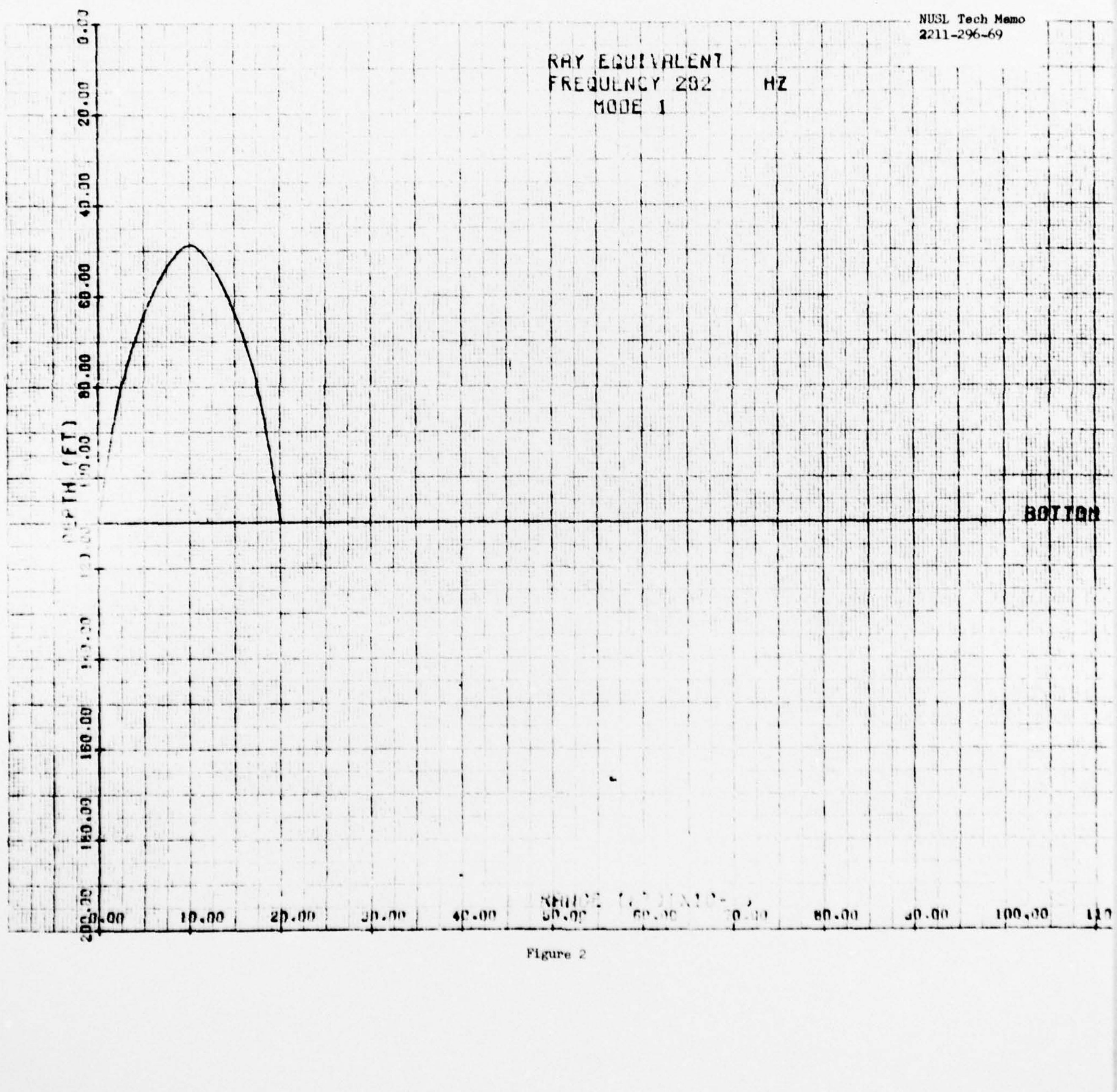
RAY EQUIVALENT
FREQUENCY 282 HZ
MODE 1
BOTTOM
DEPTH (FT)
0.00
20.00
40.00
60.00
80.00
160.00
200.00
10.00
20.00
40.00
80.00
100.00

Figure 2

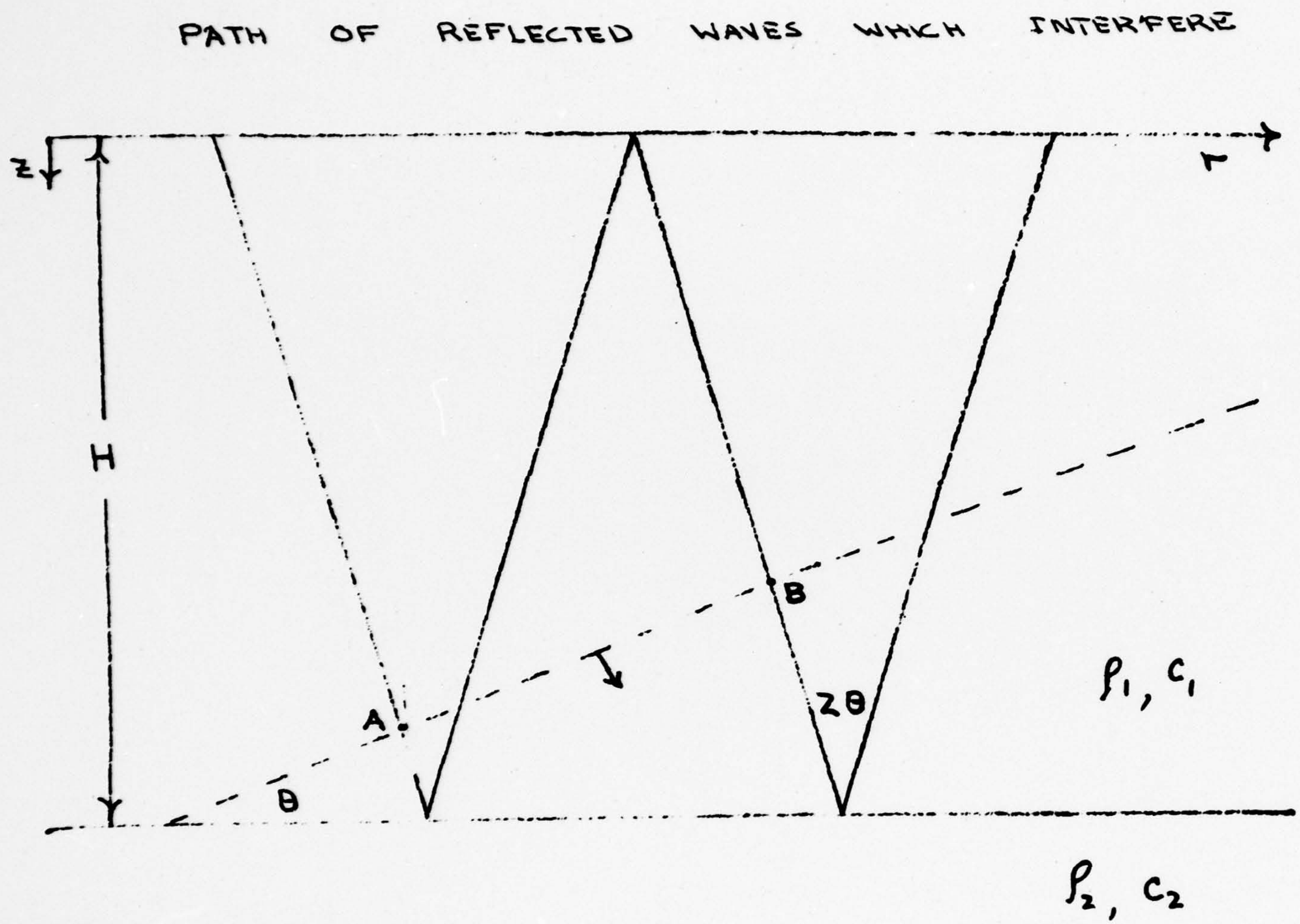
PATH OF REFLECTED WAVES WHICH INTERFERE
z
r
H
A
B
θ
2θ
ρ1, c1
ρ2, c2

Figure 3

DOWNGOING AND UPGOING WAVES

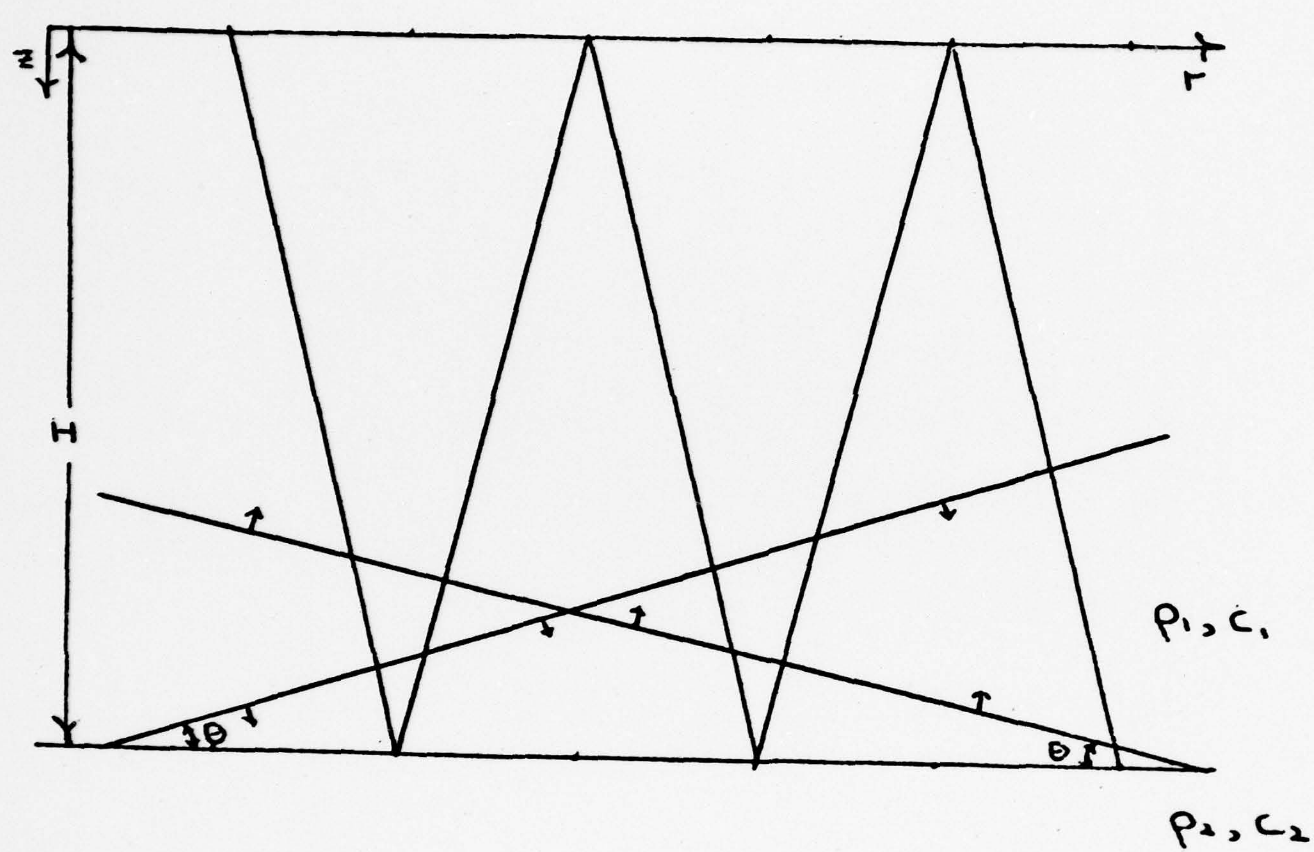

Figure 4

PRESSURE AMPLITUDE DISTRIBUTIONS OF UPGOING AND DOWNGOING RAYS AND RESULTANT

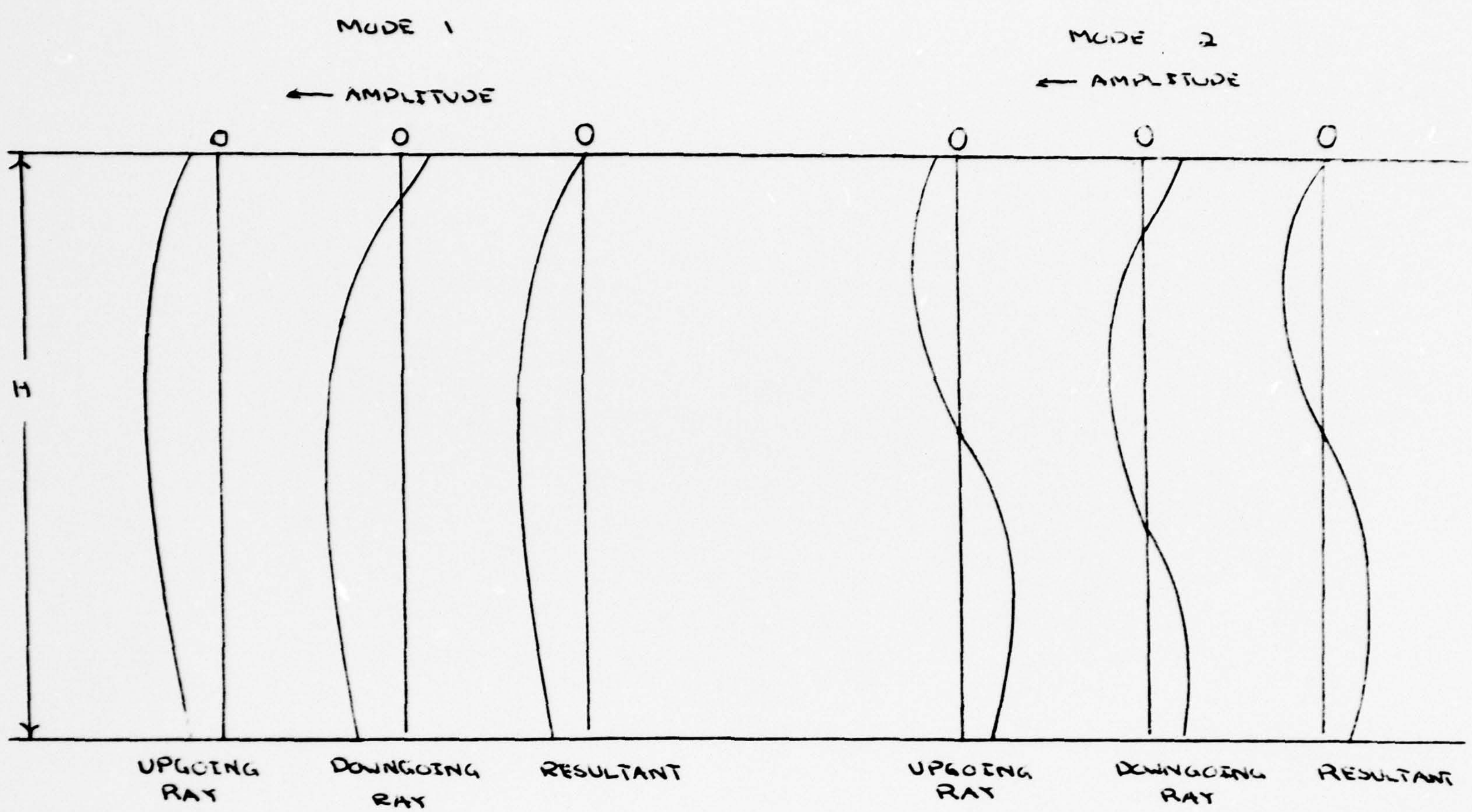

Figure 5

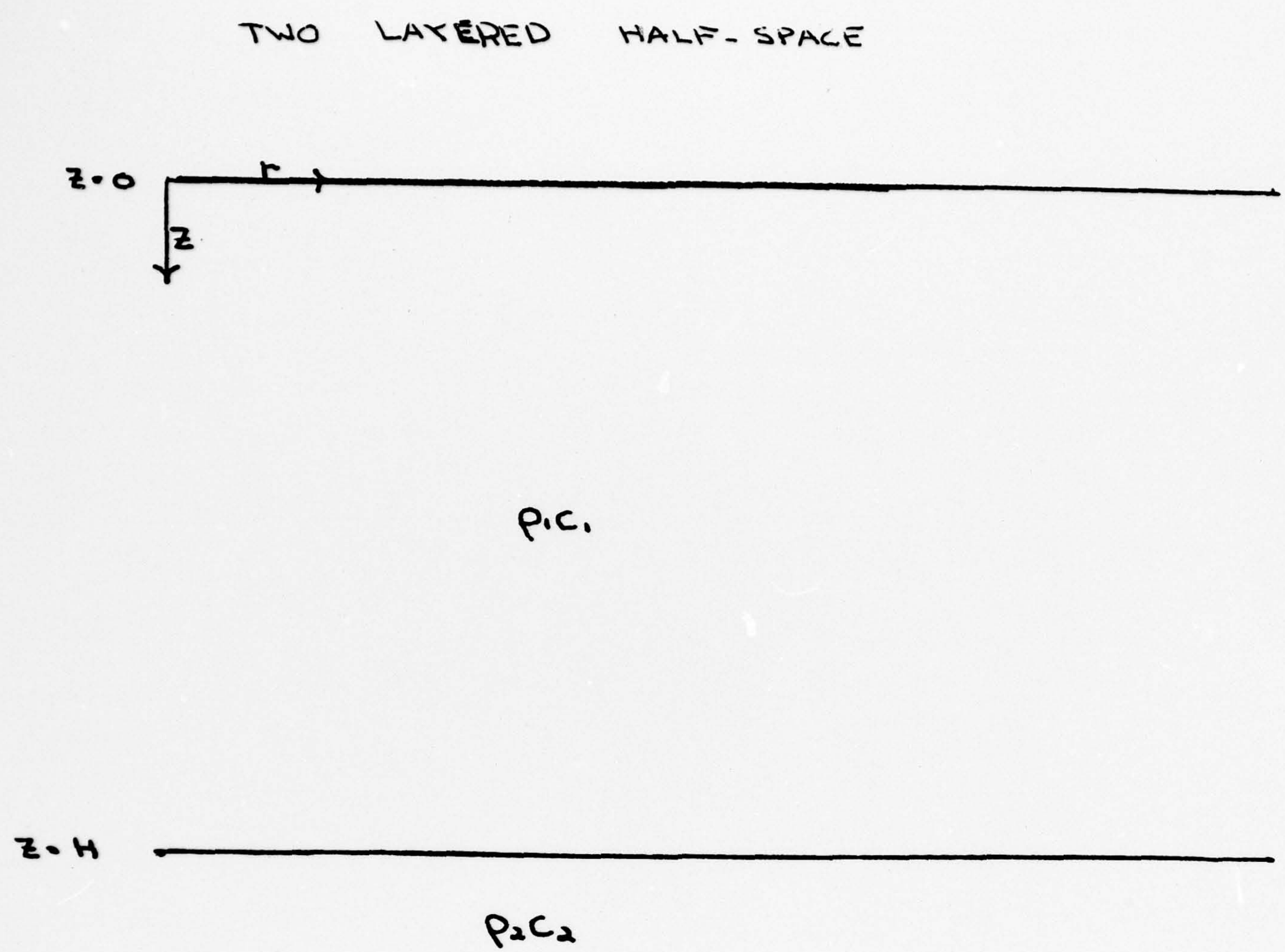

Figure 6

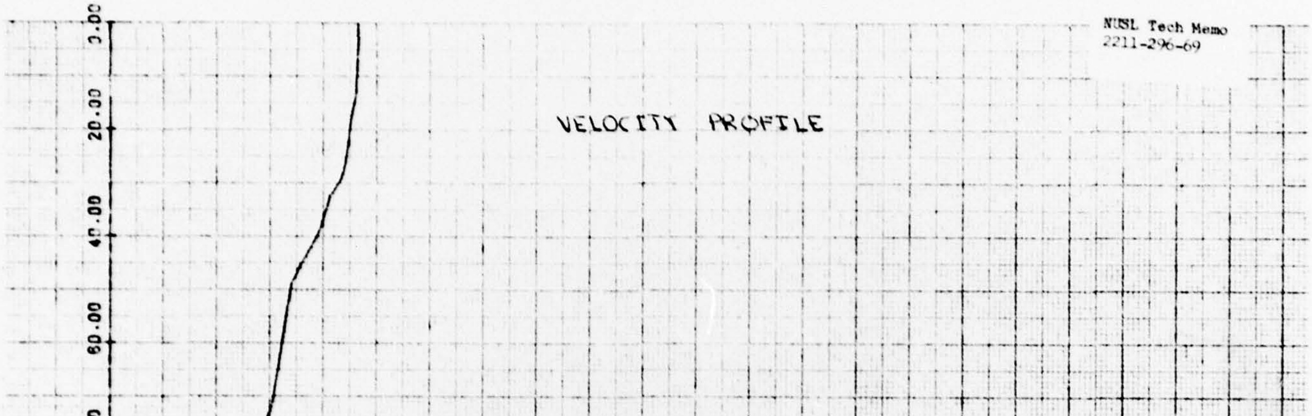
VELOCITY PROFILE
WATER DEPTH
0.00
20.00
40.00
60.00
80.00
100.00
140.00
160.00
180.00
SOUND VELOCITY
4850.00
4900.00
4950.00
5000.00
5050.00
BOTTOM

Figure 7

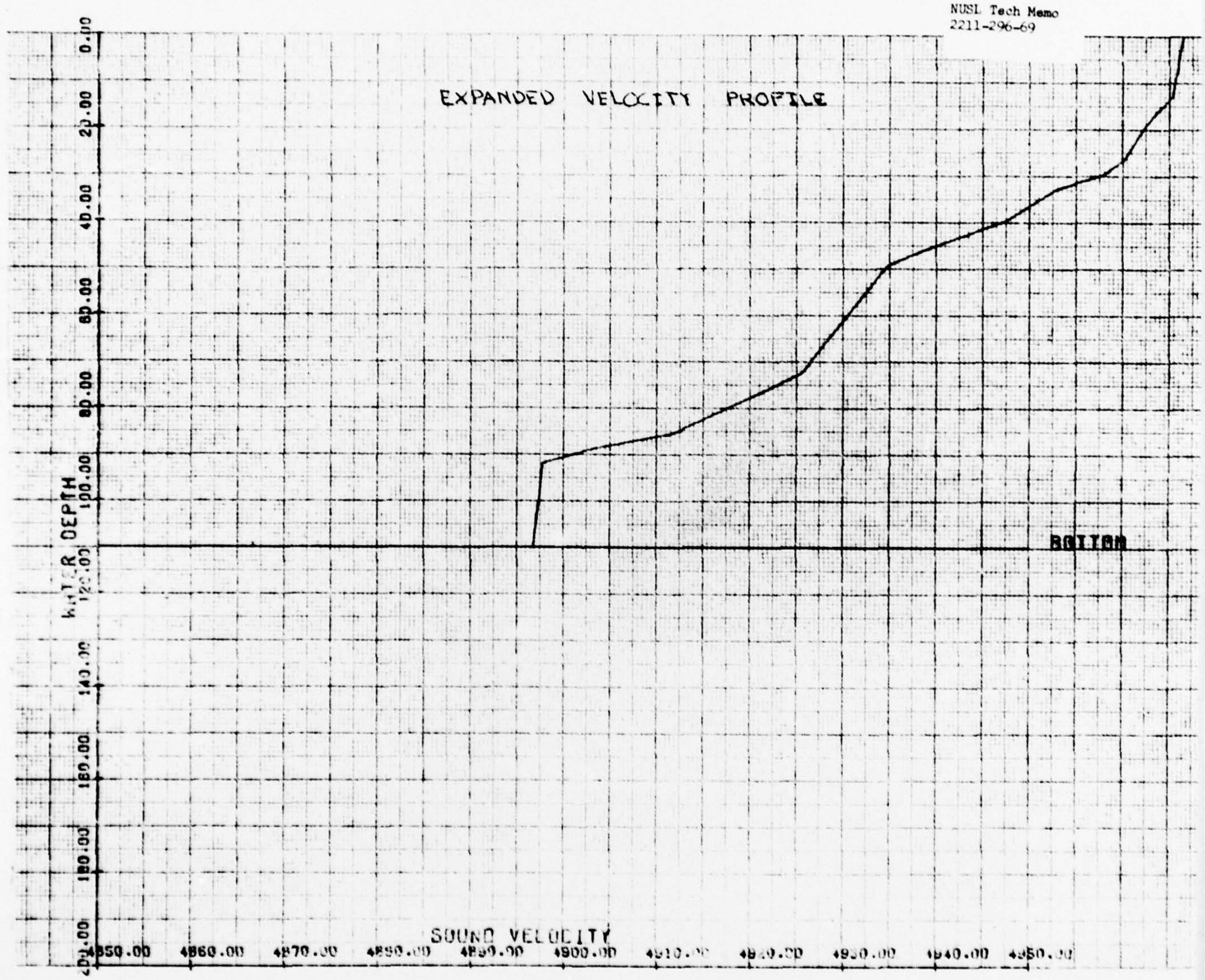
EXPANDED VELOCITY PROFILE
WATER DEPTH
SOUND VELOCITY
BOTTOM

Figure 8

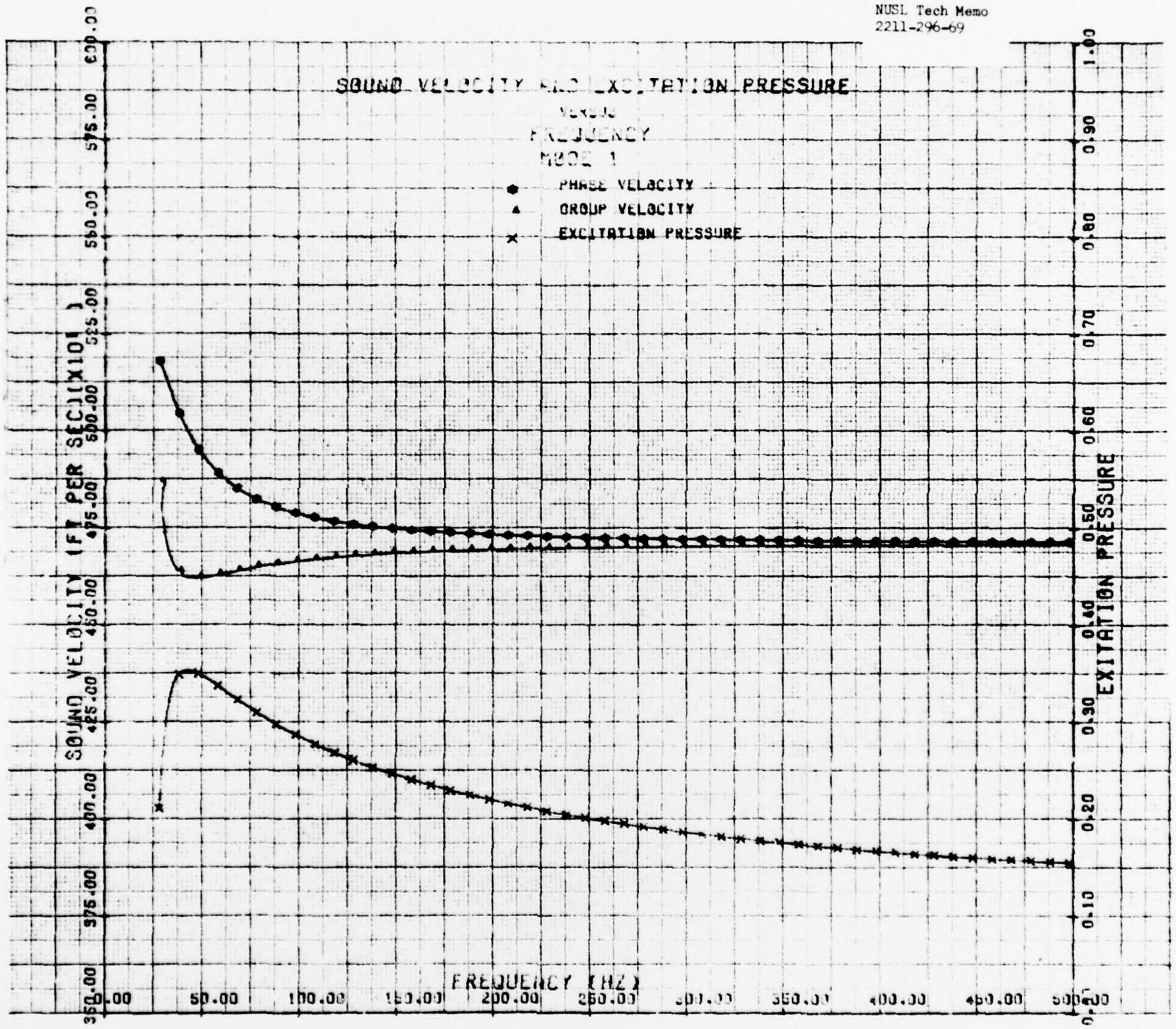
SOUND VELOCITY AND EXCITATION PRESSURE
VERSUS
FREQUENCY
MODE 1
PHASE VELOCITY
GROUP VELOCITY
EXCITATION PRESSURE
SOUND VELOCITY (FT PER SEC)(X10¹)
EXITATION PRESSURE
FREQUENCY (HZ)

Figure 9

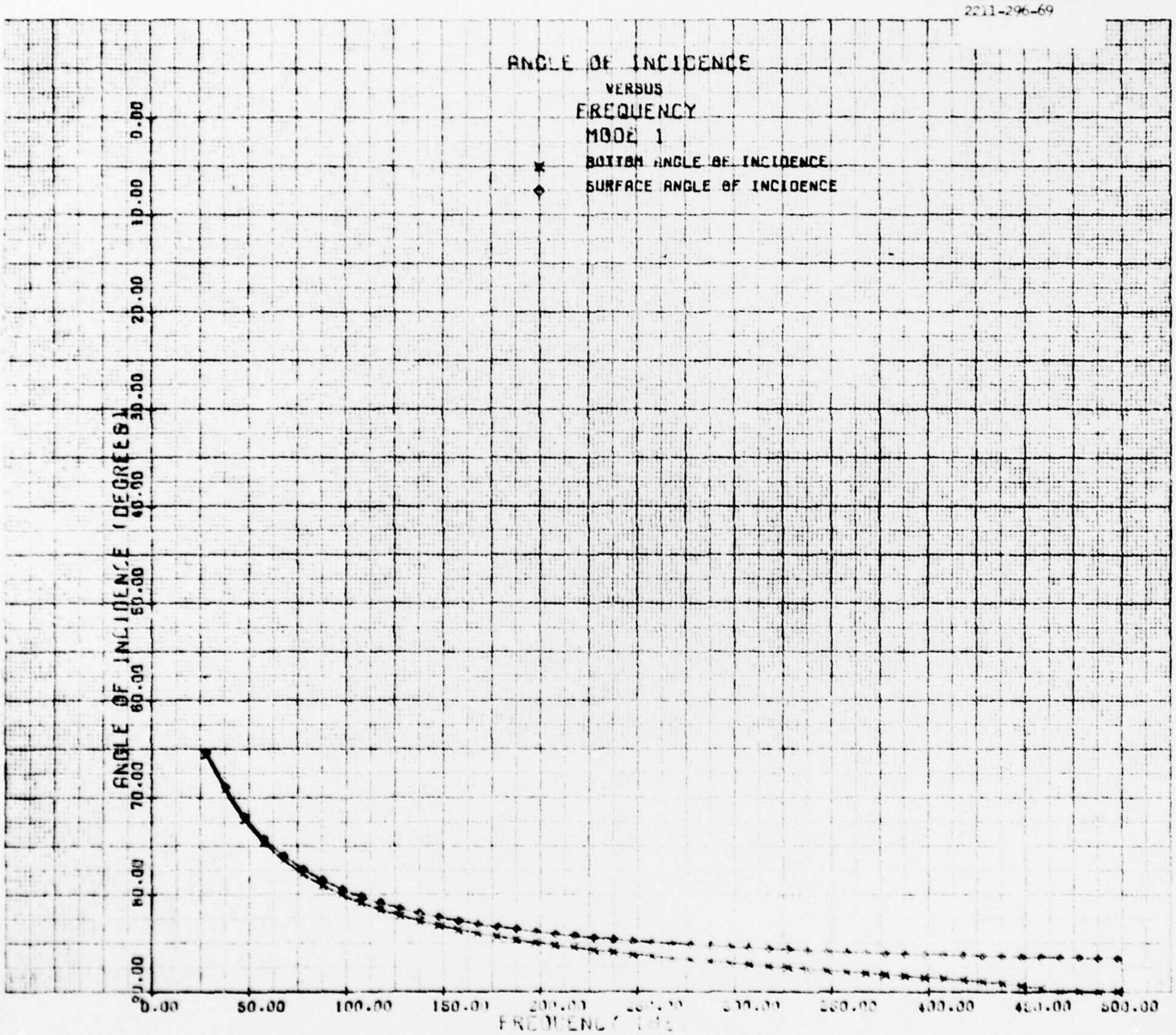
ANGLE OF INCIDENCE
VERSUS
FREQUENCY
MODE 1
BOTTOM ANGLE OF INCIDENCE
SURFACE ANGLE OF INCIDENCE
ANGLE OF INCIDENCE (DEGREES)
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
50.00
100.00
150.00
500.00

Figure 10

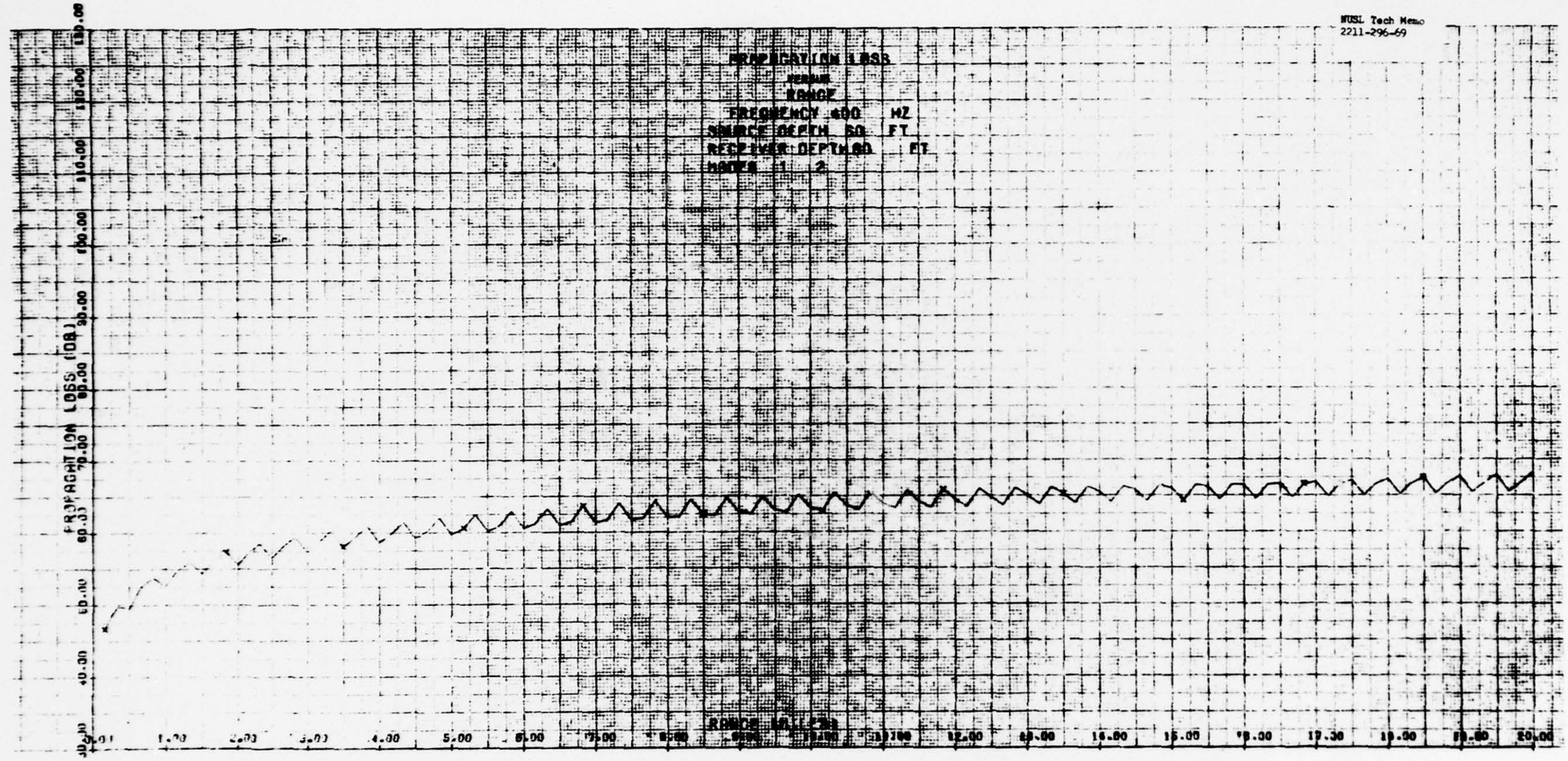
PROPAGATION LOSS
VERSUS
RANGE
FREQUENCY 400 HZ
SOURCE DEPTH 60 FT
RECEIVER DEPTH 60 FT
MODES 1 2
PROPAGATION LOSS (DB)
RANGE (MILES)

Figure 11