ALEX(01)-TR-78-03

APPLICATION OF RINGDAL'S METHOD TO UNBIASED MEASUREMENT OF THE Ms-mb RELATIONSHIP

TECHNICAL REPORT NO. 15

VELA NETWORK EVALUATION AND AUTOMATIC PROCESSING RESEARCH

Prepared by
Alan C. Strauss

TEXAS INSTRUMENTS INCORPORATED
Equipment Group
Post Office Box 6015
Dallas, Texas 75222

Prepared for

AIR FORCE TECHNICAL APPLICATIONS CENTER
Alexandria, Virginia 22314

Sponsored by

ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Monitoring Research Office
ARPA Program Code No. 7F10
ARPA Order No. 2551

31 August 1978

Acknowledgment: This research was supported by the Advanced Research Projects Agency, Nuclear Monitoring Research Office, under Project VELA-UNIFORM, and accomplished under the technical direction of the Air Force Technical Applications Center under Contract Number F08606-77-C-0004.

ALEX(01)-TR-78-03

APPLICATION OF RINGDAL'S METHOD TO UNBIASED MEASUREMENT OF THE Ms-mb RELATIONSHIP

TECHNICAL REPORT NO. 15
'ELA NETWORK EVALUATION AND AUTOMATIC PROCESSING RESEARCH

Prepared by
Alan C. Strauss

TEXAS INSTRUMENTS INCORPORATED
Equipment Group
Post Office Box 6015
Dallas, Texas 75222

Prepared for

AIR FORCE TECHNICAL APPLICATIONS CENTER
Alexandria, Virginia 22314

Sponsored by

ADVANCED RESEARCH PROJECTS AGENCY
Nuclear Monitoring Research Office
ARPA Program Code No. 7F10
ARPA Order No. 2551

31 August 1978

Acknowledgment: This research was supported by the Advanced Research Projects Agency, Nuclear Monitoring Research Office, under Project VELA-UNIFORM, and accomplished under the technical direction of the Air Force Technical Applications Center under Contract Number F08606-77-C-0004.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
APPLICATION OF RINGDAL'S METHOD TO UNBIASED MEASUREMENT OF THE M_s-m_b RELATIONSHIP.
Technical Report [redacted] Network
E[redacted]essing Research.

5. TYPE OF REPORT & PERIOD COVERED
Technical rept. no. 15,

6. PERFORMING ORG. REPORT NUMBER
TI-ALEX(01)-TR-78-03

7. AUTHOR(s)
Alan C. Strauss

8. CONTRACT OR GRANT NUMBER(s)
F08606-77-C-0004,
ARPA Order-2551

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Texas Instruments Incorporated
Equipment Group
Dallas, Texas 75222

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
VELA T/8705/B/PMP

11. CONTROLLING OFFICE NAME AND ADDRESS
Advanced Research Projects Agency
Nuclear Monitoring Research Office
Arlington, Virginia 22209

12. REPORT DATE
31 Aug[redacted]78

13. NUMBER OF PAGES
48

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
Air Force Technical Applications Center
VELA Seismological Center
Alexandria, Virginia 22314

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE, DISTRIBUTION UNLIMITED

50p.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

ARPA Order No. 2551

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Seismology	Positive magnitude bias	Mean signal surface wave magnitude
Discrimination	Maximum likelihood	Unbiased surface wave magnitude
Network magnitude	M_s-m_b relationship	
Spectral magnitude	Seismic waveform magnitudes	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Ringdal's maximum likelihood method of removing magnitude bias was tested by removing the apparent bias of surface wave magnitude estimates. Bias removal was convincingly demonstrated by comparing maximum likelihood estimates of M_s obtained by a single sensor to those obtained by an array at the Alaskan Long Period Array (ALPA) site. Since the beamformed array has a lower detection threshold than the single-sensor reference site, → over

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

405 076

20. continued

it can serve as the standard by which to judge whether positive magnitude bias has been removed from the reference site surface wave magnitude estimates.

The results of this test indicate that below the 90 percent detection threshold of the reference site, positive magnitude bias becomes a significant factor in surface wave magnitude estimates. Comparison with beamformed array data indicates that application of Ringdal's method accurately removes this bias down to approximately the reference site 25 percent detection threshold. Below this point, the method over-corrects the magnitude estimates, resulting in abnormally low values. This test also indicates that correcting surface wave magnitude estimates for bias by Ringdal's method will improve the value of the M_s-m_b discriminant for network data by making it applicable to events with lower bodywave magnitudes. The results obtained in deriving an unbiased M_s-m_b relationship were consistent with various other published results and explain their diversity by showing that the slope of M_s-m_b depends on the average magnitude of the data base used to derive the relationship.

ABSTRACT

Ringdal's maximum likelihood method of removing magnitude bias was tested by removing the apparent bias of surface wave magnitude estimates. Bias removal was convincingly demonstrated by comparing maximum likelihood estimates of M_s obtained by a single sensor to those obtained by an array at the Alaskan Long Period Array (ALPA) site. Since the beamformed array has a lower detection threshold than the single-sensor reference site, it can serve as the standard by which to judge whether positive magnitude bias has been removed from the reference site surface wave magnitude estimates.

The results of this test indicate that below the 90 percent detection threshold of the reference site, positive magnitude bias becomes a significant factor in surface wave magnitude estimates. Comparison with beamformed array data indicates that application of Ringdal's method accurately removes this bias down to approximately the reference site 25 percent detection threshold. Below this point, the method over-corrects the magnitude estimates, resulting in abnormally low values. This test also indicates that correcting surface wave magnitude estimates for bias by Ringdal's method will improve the value of the $M_s - m_b$ discriminant for network data by making it applicable to events with lower bodywave magnitudes. The results obtained in deriving an unbiased $M_s - m_b$ relationship were consistent with various other published results and explain their diversity by showing that the slope of $M_s - m_b$ depends on the average magnitude of the data base used to derive the relationship.

ACKNOWLEDGMENTS

The author wishes to thank Dr. Robert L. Sax for his many helpful comments during the course of preparing this report.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

SECTION I
INTRODUCTION

A. THE MAGNITUDE BIAS PROBLEM

The conventional method of estimating seismic event magnitudes is to average the magnitude measurements made at those stations of a network which detect the event. The purpose of this averaging process is to 'smooth over' any abnormally low or high deviations of the magnitude estimates caused by source effects such as event radiation pattern, regional magnitude effects due to upper mantle structure under the station (e.g., Der, 1977), local receiver site geology and crustal structure effects (e.g., Ringdal et al., 1973; and Chang and von Seggern, 1978), and abnormal path attenuation effects (e.g., Der, 1977; and Sun, 1977). However, when estimating the magnitude of events near the network detection threshold, the averaging process itself is a biased procedure due to seismic noise, which limits signal detection to those few stations of the network which have the largest positive amplitude deviations.

It is important to note that the magnitude bias to be discussed in this report is not due to the interference of noise with weak signals. It is assumed that detected signals are measured at high enough signal-to-noise ratio to render negligible the influence of noise interference on magnitude measurement. The magnitude bias discussed here is grounded in two other quite different physical phenomena - the effect on signal detection of source-to-station statistical variation in signal amplitude for seismic events due to source, path, and local receiver effects, and the temporal statistical variation of the seismic noise levels at the stations. These factors influence the signal-to-noise ratio variation between observations which in turn determines that only the larger signals from an event are used to estimate the event magnitude. The net result is a positive bias in the event magnitude, which increases as the true event magnitude decreases.

A method for removing this positive magnitude bias was developed by Ringdal (1975). In brief, his method is to measure signal magnitudes at the detecting stations and to determine upper bounds on the signal magnitude at the non-detecting stations by measuring the magnitude of noise at the station as though it were a signal. A maximum likelihood estimate of the true network magnitude and standard deviation is then computed from the combined measurements of noise at non-detecting stations and measured signal magnitudes at detecting stations.

Any test of Ringdal's method must of necessity establish that bias in the measurement of the magnitude of small signals exists as a significantly observable effect and can be effectively removed down to some minimum level of detection probability. In light of this, the results of a paper by Evernden and Kohler (1976) should be mentioned. The authors of this paper investigated the question of bias in estimates of bodywave magnitude at small magnitudes. Their results indicate that

- Network bodywave magnitudes do have the potential of being biased.
- The probability of any significant bias occurring is so small that it can have only negligible effects on use of an M_s - m_b discriminant.

It therefore appears more profitable to use surface wave magnitudes rather than bodywave magnitudes in this study of positive magnitude bias.

B. THE TASK

The primary goal of this report is to test Ringdal's method of correcting surface wave magnitude bias. In the course of carrying out this test, the following points will be covered:

- Design an experiment to carry out this test
- Determine the differences between conventional network magnitude estimates and the estimates derived from Ringdal's method
- Describe qualitatively the conditions under which Ringdal's method can be expected to function properly
- Determine the effect of Ringdal's method on the $M_s - m_b$ discriminant.

Section II of this report discusses the methodology of Ringdal's method of magnitude estimation, the experiment designed to test this method, and the data base used to carry out the experiment. Section III presents an analysis of the results of performing the experiment in terms of the $M_s - m_b$ relationship and the $M_s - m_b$ discriminant. Section IV presents the conclusions of this report. Section V lists the references cited in this report.

SECTION II

METHODOLOGY

A. RINGDAL'S NETWORK MAGNITUDE ESTIMATION METHOD

Consider a large seismic event detected by each of the stations of a network. At each station, the magnitude of the event is measured. From Freedman (1967), one would expect these station magnitudes to be normally distributed. Therefore, with an unbiased selection of a large enough sample of magnitude measurements, the average of these station magnitudes, called the network magnitude estimate, would accurately represent the true magnitude of the event. However, as the true magnitude decreases toward the network detection threshold, those signals representing the lower side of the normal distribution are not detected due to relatively higher seismic noise. Averaging the magnitudes of the remaining portion of the distribution results in a positive bias in the network magnitude estimate relative to the true magnitude of the event.

Ringdal's method for removing positive bias in the measurement of the magnitude of a seismic event is to measure the magnitude of the propagated signals at all detecting stations and estimate an upper bound on the magnitude of signals at all non-detecting stations. These upper bounds are obtained by measuring the largest noise peak in the signal window and computing the corresponding 'noise magnitude'. The statistical method of maximum likelihood estimation is then applied to these signal and noise magnitudes to estimate the unbiased network magnitude.

This method takes as its basic assumption that, for a given event, world-wide magnitudes follow a Gaussian distribution with mean μ

and variance σ^2. The unknown mean of this distribution is called the true network magnitude of the event. It is also assumed that, at a given station, an event is detected if the station magnitude m exceeds a certain threshold value a . Thus, the probability of detecting a given event may be written as

$$P(\text{detect}/\mu,\sigma) = P(m \geq a/\mu,\sigma) = \Phi\left(\frac{\mu - a}{\sigma}\right) \tag{II-1}$$

where

$$\Phi = \int_{-\infty}^{x} \phi(t)\, dt \tag{II-2}$$

and

$$\phi(x) = \frac{1}{\sqrt{2\pi}} * e^{-x^2/2} \; . \tag{II-3}$$

Now assume that, for a given event, records from an n-station network are examined. Further, assume that the detection thresholds a_i, $i = 1, 2, \ldots, n$ are known, and that a magnitude m_i is computed at each of the detecting stations. Finally, assume that all station observations may be considered to be independent.

Given this set of assumptions, one computes as a function of the unknown event parameters (μ, σ) the likelihood function for a given set of signal and noise magnitudes using as starting points a preset value of σ and the mean signal magnitude for μ and then maximizes this likelihood function with respect to the unknown parameters. The likelihood function is:

$$L(m_1, \ldots, m_n/\mu,\sigma) = \prod_{\substack{\text{all} \\ \text{detections}}} \left[\frac{1}{\sigma} * \phi\left(\frac{m_i - \mu}{\sigma}\right)\right] * \prod_{\substack{\text{all non-} \\ \text{detections}}} \Phi\left(\frac{a_i - \mu}{\sigma}\right) \tag{II-4}$$

where ϕ and Φ are as defined in equations (II-2) and (II-3). Note that the first group of products of equation (II-4) represents the likelihood function when all non-detections are ignored, and is maximized by the conventional estimate of μ , the average of the magnitudes from the detecting stations.

The procedure is then to maximize equation (II-4) in terms of the unknowns μ and σ. This is carried out by an iterative procedure in the computer program. Fortunately, it appears the parameter σ can be restricted to within a fairly narrow range. For example, Vieth and Clawson (1972) found a value of $\sigma = 0.4$ to be representative of the WWSSN network. Bungum and Husebye (1974) showed that a value of $\sigma = 0.3$ appeared to be independent of event magnitude.

A formal development of this method may be found in Ringdal's report (1975). Ringdal demonstrated that his method effectively removed bias in estimates of network m_b by simulating normal magnitude errors and noise. Ringdal's method presumes that the magnitude deviation of signals are independent of the noise at the seismic stations where the signals are observed. There is some observational evidence that there is a strong dependence on regional magnitude deviations of signals and noise at the seismic stations as shown by Dietz and Sax (1977) which leads to nearly negligible m_b bias; but that random variations due to local geology and source factors are probably much larger as shown by Chang and von Seggern (1978). It is assumed here that no such complication exists in using Ringdal's method to remove bias in estimates of long-period M_s.

B. THE EXPERIMENTAL PLAN

In designing an experiment to test Ringdal's network magnitude bias correction method, one immediately encounters the problem of determining a standard by which to judge whether the bias has been removed. As originally formulated by Ringdal, this requires a perfect network (all stations detecting all events down to the network detection threshold) to provide the standard unbiased network magnitudes. Since no such network exists, the experiment was rearranged by interchanging the roles of stations and events. All observations of long-period signals were made at the Alaskan Long Period

Array (ALPA). The assumption that averaging magnitudes of long-period signals is the major source of bias effects in the M_s-m_b relationship rather than bodywave magnitudes was prompted by the observations of Section I, which indicate that magnitude bias may be more evident in surface wave magnitudes than in bodywave magnitudes. The approach will therefore be to use Ringdal's method on ALPA data to estimate the average surface wave magnitude of many events having the same bodywave magnitude. Since the beamforming process reduces the 50 percent detection threshold of ALPA by approximately one-half m_b unit, for that range of m_b the beamformed array data will detect essentially all events while the single-sensor reference site of the ALPA array will detect less than half of the events. Thus, for this range of m_b values, the array can serve as the standard against which to judge whether Ringdal's method has removed the magnitude bias. (Since the beamforming process attenuates the signal slightly, one can expect the reference site unbiased magnitudes to be slightly greater than the beamformed array magnitudes.)

This approach of course requires that M_s values recorded at a given station for events of the same m_b be normally distributed as are the m_b values of a given event recorded by a network. To examine this, event M_s minus mean signal M_s differences were computed for all events above the 90 percent detection threshold. These values were normalized by the transformation

$$y_i = \frac{x_i - \bar{x}}{\sigma} \qquad \text{(II-5)}$$

where

x_i = event M_s for a given m_b
$\bar{x}$ = mean signal M_s at a given m_b
σ = standard deviation associated with $\bar{x}$.

The y_i values were grouped into 0.2 unit bins, producing the histogram in Figure II-1 where the data were normalized so that the highest bar had a value of one. A normal distribution (zero mean, unit standard deviation) is

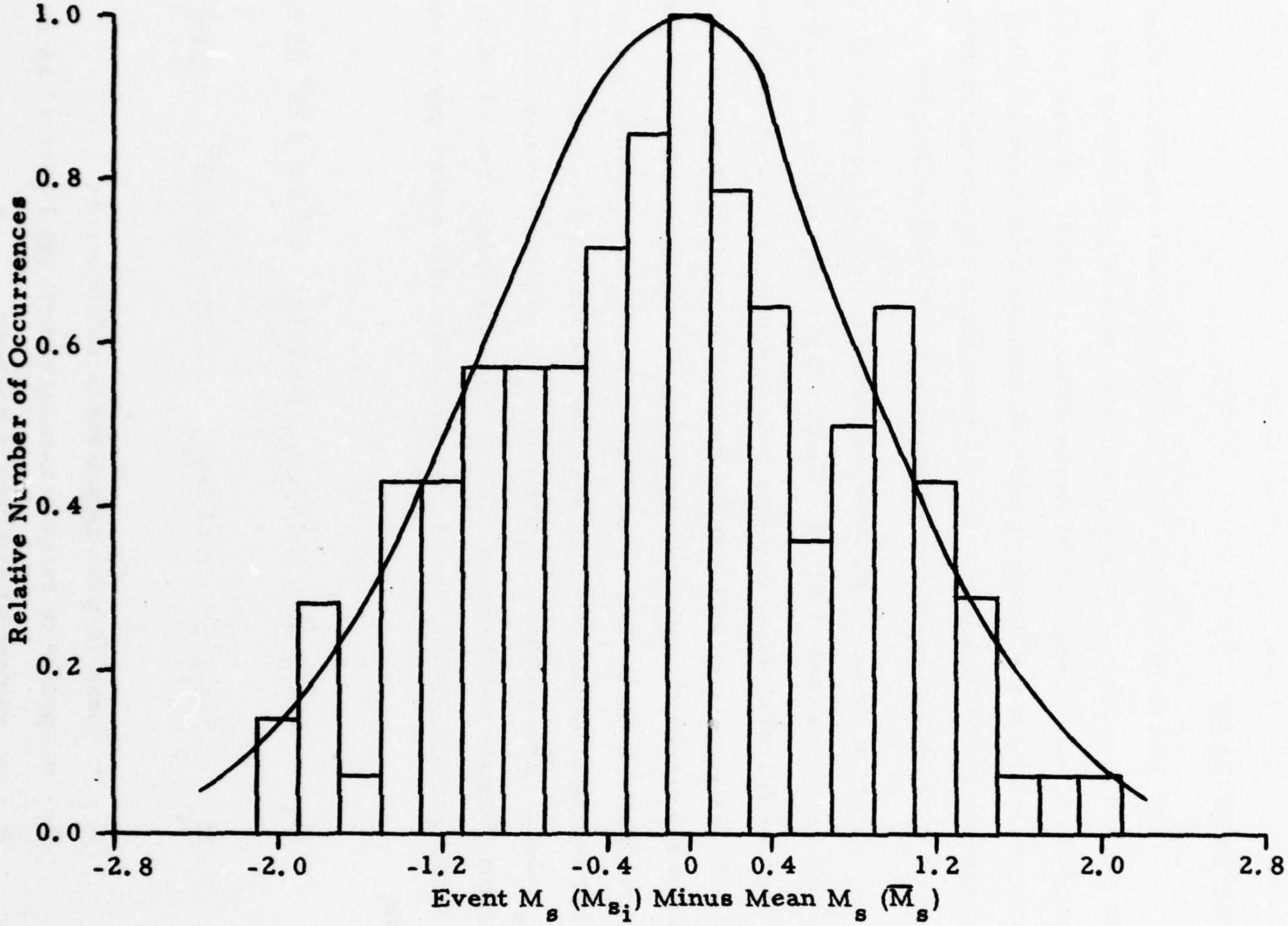


FIGURE II-1

NORMALIZED DISTRIBUTION OF (M_{s_i} - $\overline{M}_s$) VALUES also shown in this figure. This result indicates only small deviations between the observed histogram and the presumed normal distribution.

C. THE DATA BASE

The source of the data used to test Ringdal's magnitude bias correction method is a suite of events processed and analyzed during the evaluation of the Alaskan Long Period Array (Strauss, 1973). A total of 233 events with epicenters in the Kurile Islands-Kamchatka area were selected from the available ALPA data base. Events from these regions which were not selected were rejected due to the presence of interfering waveforms.

Plots of these events were examined to determine which events had detectable surface waves at ALPA. Surface wave magnitudes were computed for each event, whether detected or not, by a program which searches the signal gate for zero crossings and the interval between each pair of zero crossings for the maximum absolute amplitude. Using these amplitudes and the period computed from the zero crossings, a series of M_s values was computed. This series of values was then searched for the maximum 20 second period M_s values. This method of computing M_s is described in detail in an earlier report (Strauss and Weltman, 1977). The only change made is that, following Nuttli and Kim (1975), the M_s values used in this report were computed from

$$M_s = \log_{10} \frac{A}{T} + 1.07 \log_{10} \Delta + 1.16 \qquad 10^\circ \leq \Delta \leq 25^\circ \qquad \text{(II-5)}$$

$$M_s = \log_{10} \frac{A}{T} + 1.66 \log_{10} \Delta + 0.30 \qquad \Delta > 25^\circ \qquad \text{(II-6)}$$

where

A = peak-to-peak amplitude in millimicrons

T = period in seconds measured between 17 and 23 seconds

Δ = epicentral distance in degrees.

In order to 'bin' the measured signal and noise surface wave magnitudes, one must have an m_b value for each event. Since the parameters of the events in the data base were originally derived from the PDE, NORSAR, and LASA event bulletins, and since the m_b values of these bulletins often disagree, one with the other, it was necessary to find a means of assigning an m_b value to each event such that event-to-event m_b variance would be minimized.

The first step in assigning these m_b values was to form a 'combined m_b value' for each event. If only one of the event bulletins reported a given event, the combined m_b was simply the m_b derived from that bulletin. If two event bulletins reported an event, the average of the two reported magnitudes served as the combined m_b. If all three of the lists reported the event, the two magnitudes with the closest values were averaged to form the combined m_b. Since the LASA event bulletin reported the greatest number of events, the 'combined m_b' values are closer to LASA m_b values than the PDE or NORSAR m_b values.

At this point, there were four sets of m_b values assigned to the events of the data base: PDE, NORSAR, LASA, and combined. Only the combined m_b values had a magnitude for every event of the data base. In order to decide which set of m_b values should be used in 'binning' the surface wave magnitudes, maximum likelihood detection capability curves were computed for the surface wave detection statistics using each of the four types of m_b. The idea behind this approach is that the more precisely an m_b estimation procedure represents the events, the less variance will appear in the maximum likelihood fits to the detection statistics. The four fits to the Rayleigh wave reference site detection statistics are shown in Figure II-2 to illustrate the approach. The upper portion of each sub-figure shows the detection statistics for each data set. The lower portion of each figure shows the maximum likelihood fit (solid line) to the detection statistics. The dashed

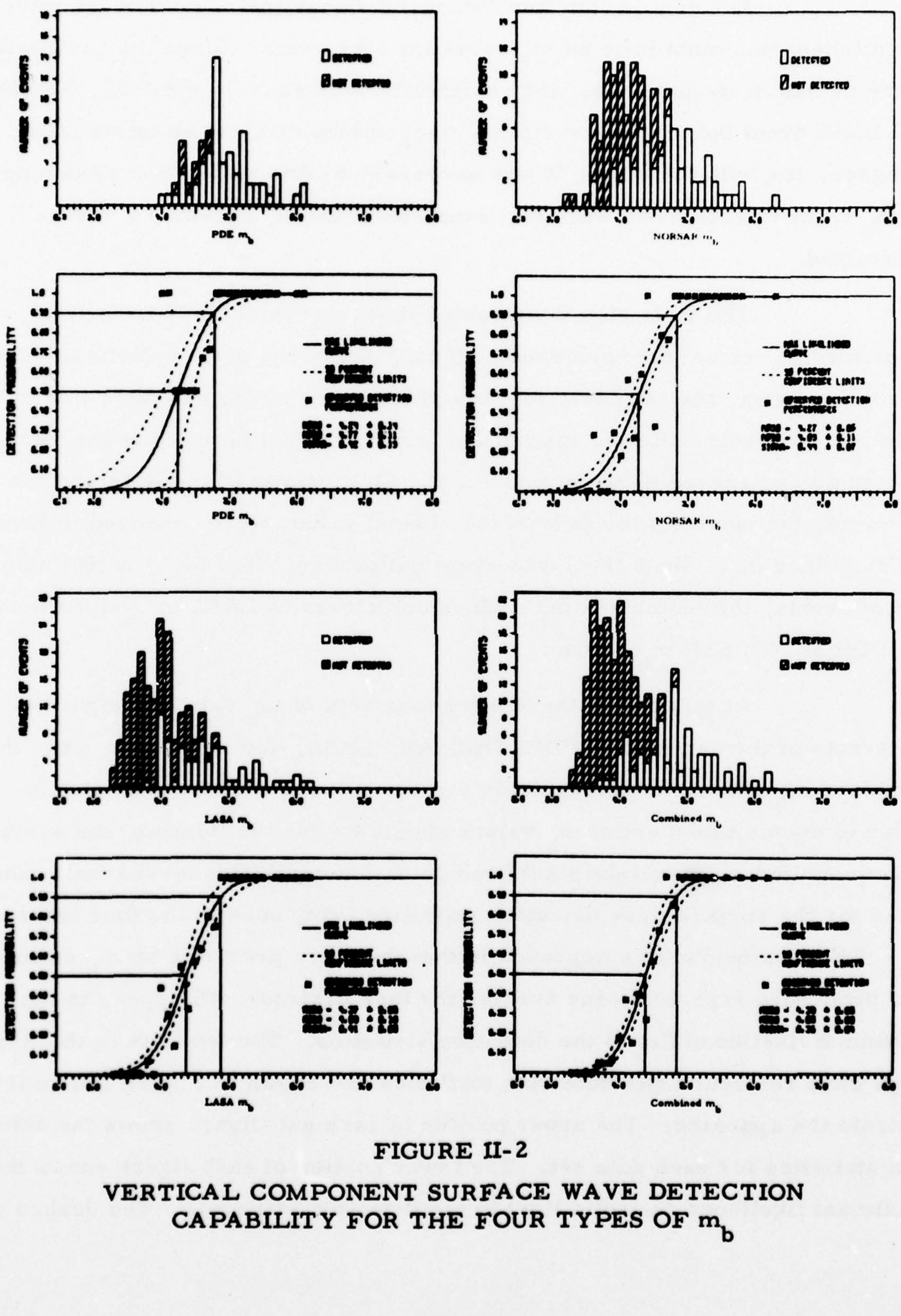


FIGURE II-2

VERTICAL COMPONENT SURFACE WAVE DETECTION CAPABILITY FOR THE FOUR TYPES OF m_b

lines represent the 95 percent confidence limits on this fit. This method of presenting detection statistics is described by Ringdal (1974). From this figure one can see that the PDE and NORSAR bulletins did not assign m_b values to enough events of the data base. Of the four sets of data, the combined m_b values appear to be the best in terms of internal consistency. To illustrate this matter further, the fitting errors at the 50 percent and 90 percent detection thresholds are listed in Table II-1. The smallest errors occur when the combined m_b values are used, indicating that the combined m_b values best represent the relative size of the events.

Having measured the signal and noise surface wave magnitudes for the reference site and beamformed array, and having assigned each event a bodywave magnitude, it is now possible to test Ringdal's method for removing magnitude bias.

TABLE II-1

ERROR IN FIT TO DETECTION STATISTICS AT 50 PERCENT AND 90 PERCENT DETECTION THRESHOLDS

m_b Type	m_{b50}	m_{b90}	Number of Events
PDE	$\pm$ 0.14	$\pm$ 0.11	75
NORSAR	$\pm$ 0.06	$\pm$ 0.11	134
LASA	$\pm$ 0.05	$\pm$ 0.10	222
Combined	$\pm$ 0.05	$\pm$ 0.08	233

All detection statistics measured on LR-V reference site.

SECTION III

DATA ANALYSIS

A. REMOVAL OF SURFACE WAVE MAGNITUDE BIAS

The first step in the analysis of the data was to run Ringdal's maximum likelihood magnitude estimation program using the long-period surface wave magnitude measurements made on detected signals and the 'noise magnitude' measurements made on non-detected signals. The program operates by binning these magnitude measurements on the basis of the bodywave magnitude of each event. Thus, for each m_b value, there is a set of M_s values. Each set of M_s values is processed by an algorithm designed to estimate the maximum likelihood magnitude and associated standard deviation. The algorithm uses as its starting point the mean of the detected surface waves and standard deviation set equal to 0.35. The algorithm then proceeds to determine the true magnitude and associated standard deviation by an iterative procedure which increments and decrements the magnitude and standard deviation until the likelihood function (equation (II-4)) reaches a maximum value. The resulting estimate of the unbiased surface wave magnitude of a set of events observed at one station is operationally equivalent to estimating the surface wave magnitude of a single event from station magnitude measurements made on a network of seismic stations.

The output of this program is plotted in Figure III-1 for vertical Rayleigh and in Figure III-2 for transverse Love surface wave measurements. The open circles in these figures represent the reference site maximum likelihood estimate of the surface wave magnitude. The solid dots represent the corresponding reference site mean signal surface wave magnitudes. The crosses represent the beamformed array mean signal surface wave magnitudes

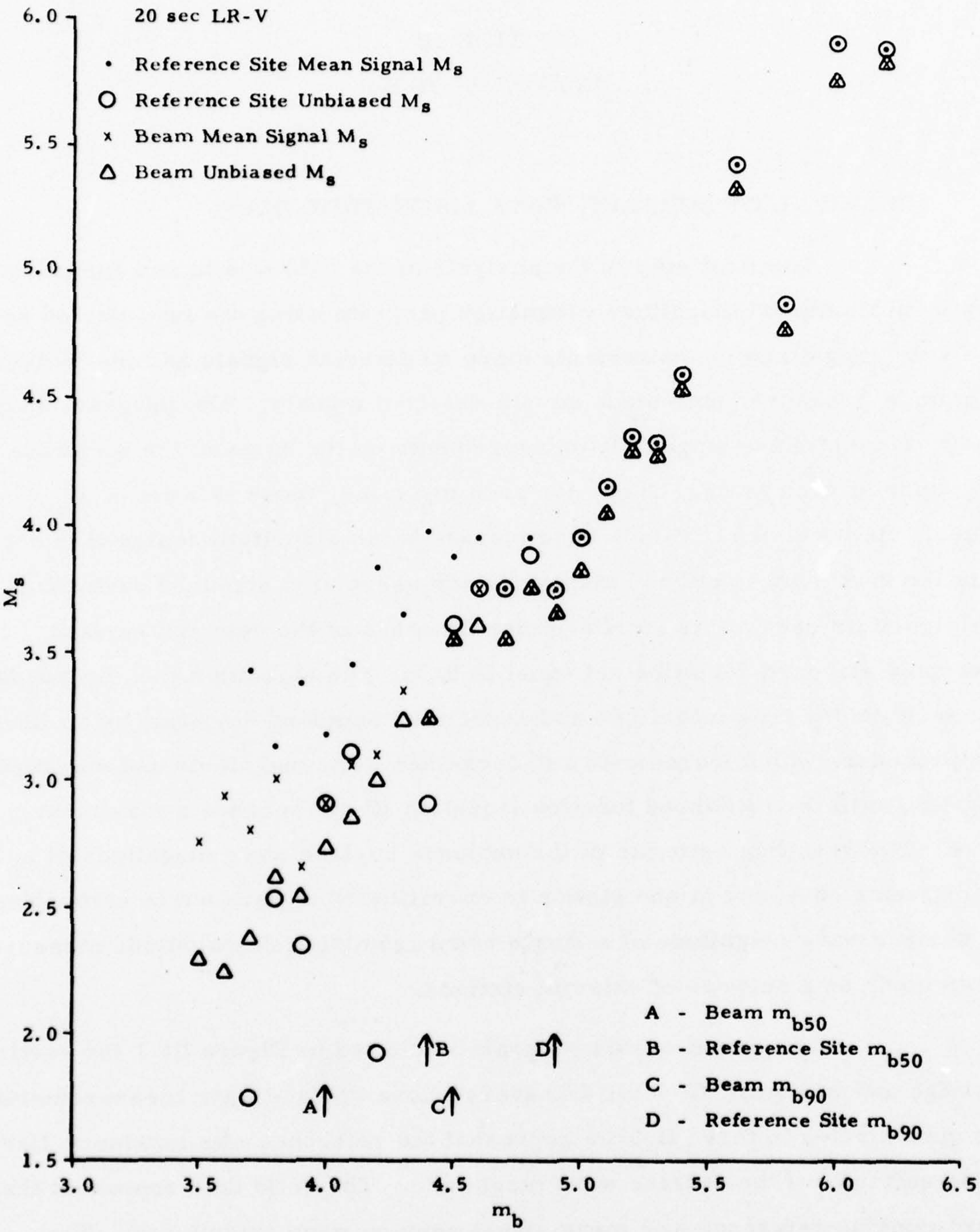


FIGURE III-1

VERTICAL COMPONENT RAYLEIGH WAVE SURFACE WAVE MAGNITUDES MEASURED AT 20 SECONDS PERIOD

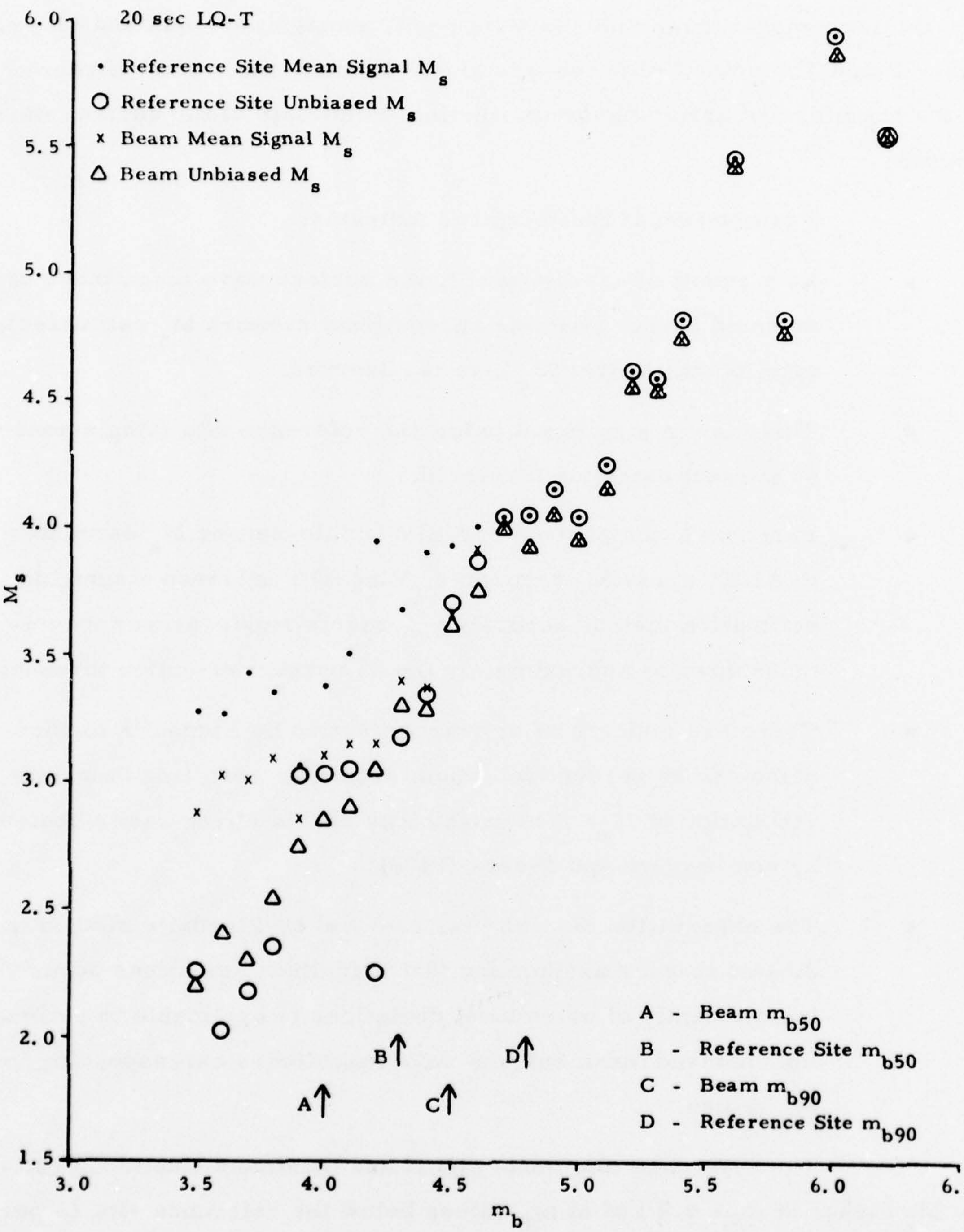


FIGURE III-2

TRANSVERSE COMPONENT LOVE WAVE SURFACE WAVE MAGNITUDES MEASURED AT 20 SECONDS PERIOD

which are to serve as the standard against which to judge whether the reference site mean signal magnitudes contain positive magnitude bias and, if so, whether Ringdal's method removes this bias. Finally, the triangles represent the beamformed array maximum likelihood estimate of the surface wave magnitude.

Examination of these figures indicates:

- As a result of averaging only the surface wave magnitudes of detected events (forming conventional network M_s estimates), significant positive M_s bias is observed.
- This bias is significant below the reference site (single-sensor) 90 percent detection threshold.
- Based on a comparison of ALPA single-sensor M_s estimates to ALPA array M_s estimates, Ringdal's unbiased magnitude estimation method accurately corrects single-sensor magnitudes down to approximately the 25 percent detection threshold.
- There are indications of overcorrection by Ringdal's method below the 25 percent detection threshold, resulting in underestimation of M_s. The possibility of this effect was indicated by von Seggern and Rivers (1978).
- The observation of valid bias removal by Ringdal's method indicates that his assumption that magnitude deviations occur independently of noise level deviations is applicable to estimating unbiased mean surface wave magnitudes corresponding to m_b values.

The reference site mean signal and maximum likelihood estimate M_s values at $m_b = 4.2$ and at m_b values below the reference site 25 percent detection threshold show large departures from the trend of the surrounding data. For these points, there were only one or two detections and ten to twenty non-detections in the event population. This indicates that Ringdal's method may not work well when the detection population is sparse. Therefore, these data points will be dropped from the remainder of the analysis.

For m_b values where there were no surface wave non-detections, the beamformed array mean signal M_s values trend on the average approximately 0.08 M_s units below the corresponding reference site M_s values. This is to be expected, since the beamforming process attenuates the signal amplitudes slightly. In the evaluation of ALPA (Strauss, 1973) this signal attenuation was estimated to be approximately 1 dB or 0.05 magnitude units.

From Figures III-1 and III-2, it appears that Ringdal's method does remove positive magnitude bias from surface wave magnitude data. However, each M_s estimate shown in the two figures carries with it a standard deviation. (Examples of these are listed in Table III-1 for the vertical Rayleigh data.) If these standard deviations are very large, one could not state with any degree of certainty that the mean signal magnitudes were not equal to the maximum likelihood estimate magnitudes. To resolve this matter, a hypothesis test procedure was carried out.

Following the procedure detailed by Johnson and Leone (1964), the hypothesis that the reference site mean signal M_s equals the reference site maximum likelihood M_s was tested using the Student's t statistic. The test statistic t was computed from

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \qquad \text{(III-1)}$$

with ν degrees of freedom, where

TABLE III-1

SURFACE WAVE MAGNITUDES WITH ASSOCIATED STANDARD DEVIATIONS

m_b	Reference Site				Beam	
	Mean Signal		Maximum Likelihood		Mean Signal	
	M_s	S	M_s	S	M_s	S
4.0	3.18	0.20	2.90	0.27	2.93	0.20
4.1	3.46	0.29	3.09	0.42	3.04	0.42
4.3	3.65	0.22	3.23	0.47	3.34	0.39
4.4	3.98	0.17	2.89	0.87	3.23	0.43
4.5	3.87	0.53	3.61	0.52	3.54	0.61
4.6	3.95	0.29	3.76	0.42	3.77	0.43
4.7	3.74	0.11	3.74	0.11	3.55	0.19
4.8	3.94	0.30	3.88	0.31	3.76	0.38
4.9	3.73	0.04	3.73	0.04	3.66	0.02
5.0	3.95	0.39	3.95	0.39	3.82	0.41

S = sample standard deviation

$$\nu = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1+1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2+1}} - 2. \qquad \text{(III-2)}$$

In these equations

$\bar{x}_i$ = M_s estimate

S_i = sample standard deviation of $\bar{x}_i$

n_i = number of points used to compute S_i and $\bar{x}_i$.

The results of this test showed that the hypothesis failed, i.e., the reference site mean signal M_s and the reference site maximum likelihood estimate M_s were not equal, with 90 percent confidence, for m_b = 4.0, 4.1, 4.3, and 4.4. (Recall that the m_b = 4.2 data were dropped from the analysis.) Based on this test, at m_b values greater than 4.4, the reference site mean signal M_s and unbiased M_s could not be said to be unequal.

Since the beamformed array mean signal M_s values are the standard of comparison, it is important to perform this test with these values and the reference site maximum likelihood estimate M_s values. In this case, the computed values of the t statistic were such that at no m_b value could these two estimates of M_s be said to be unequal.

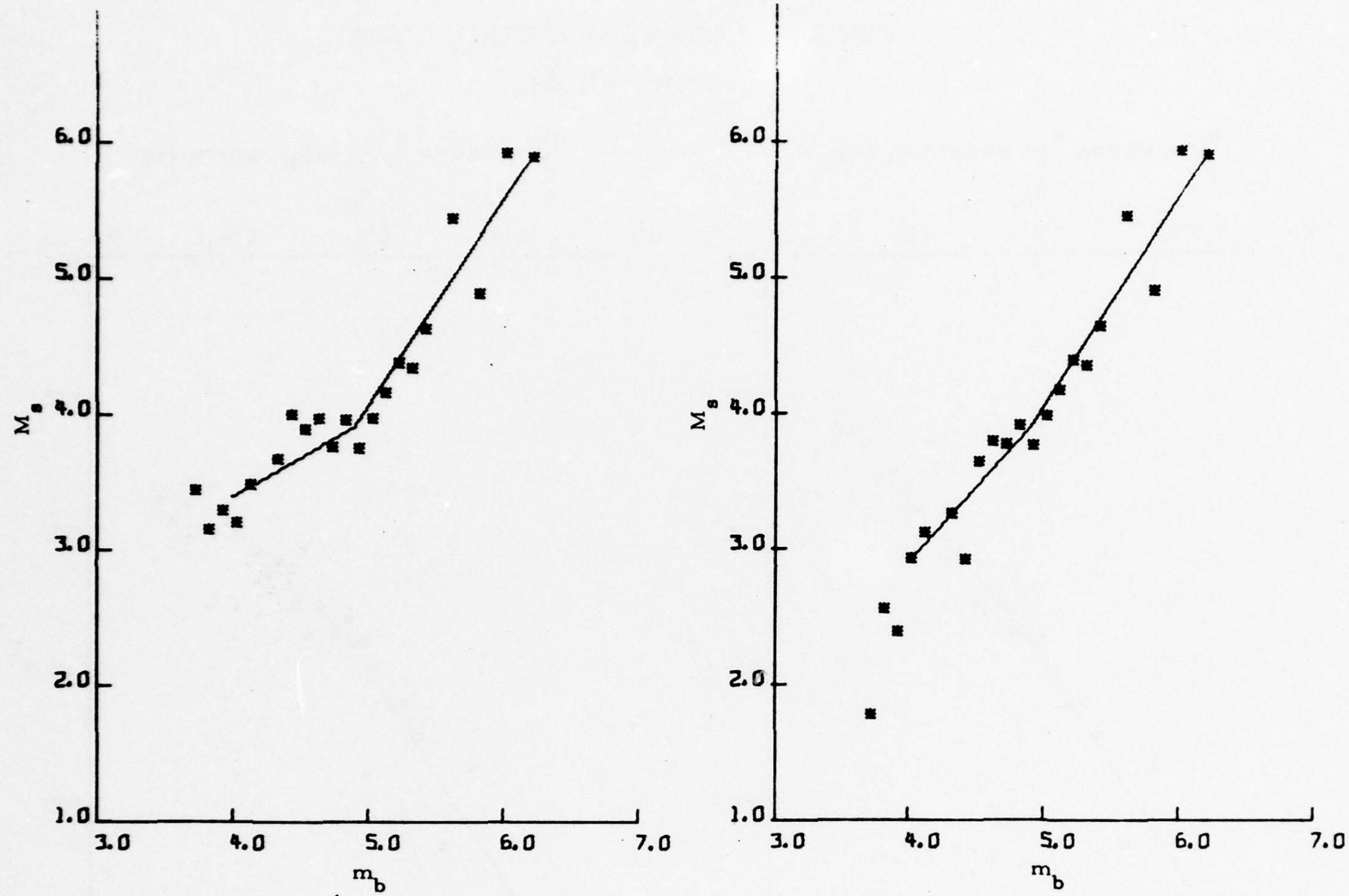
The above analysis indicates that the conventional M_s estimates (average of the individual signal M_s values) do contain a positive bias for M_s measured on events with m_b below the 90 percent detection threshold and that Ringdal's method does remove this bias, at least down to the beamformed array 50 percent detection threshold (equivalent to the reference site 25 percent detection threshold). This implies an extension of the M_s - m_b curve by about 0.8 m_b units for this data.

The statistical tests indicate that the observed bias corrections were significant at a 90 percent confidence level. No significant difference could be observed between the bias corrected single sensor reference site magnitudes and the presumed unbiased array magnitude measurements, also at a confidence level of 90 percent.

It is of interest to examine the least-squares line fits to the various data sets. In each of the following figures, one line was fit through the $M_s - m_b$ points from $m_b = 4.0$ to $m_b = 4.9$ and a second line through the $M_s - m_b$ points from $m_b = 4.9$ to $m_b = 6.2$. These fits were required to tie at the intersecting point. The selection of the $m_b = 4.9$ intersection point was guided by the observation that positive magnitude bias was significant below $m_b = 4.9$ for reference site data. Thus, breaking the $M_s - m_b$ data into two sets at this point should most clearly demonstrate the effect of this bias on the $M_s - m_b$ relationship. Fits to the beamformed array $M_s - m_b$ data were made over the same ranges in m_b for ease of comparison with the reference site $M_s - m_b$ fits.

In the fitting procedure, each mean or unbiased $M_s - m_b$ point was weighted by the number of individual M_s values used to compute the mean or unbiased M_s value. The standard deviation associated with each mean or unbiased M_s value was likewise weighted to produce the standard deviation of M_s of the $M_s - m_b$ fit. The standard deviations so computed are larger than one would expect from the $M_s - m_b$ points shown in the following figures, since the $M_s - m_b$ points shown are the mean values representing the M_s population at each m_b value.

Figure III-3a shows the reference site mean signal $M_s - m_b$ data, while Figure III-3b shows the corresponding unbiased $M_s - m_b$ data. Figure III-4a shows the beamformed array mean signal $M_s - m_b$ data, while Figure III-4b shows the corresponding unbiased $M_s - m_b$ data. Although not used in the fitting procedure, the points below $m_b = 4.0$ are shown for the

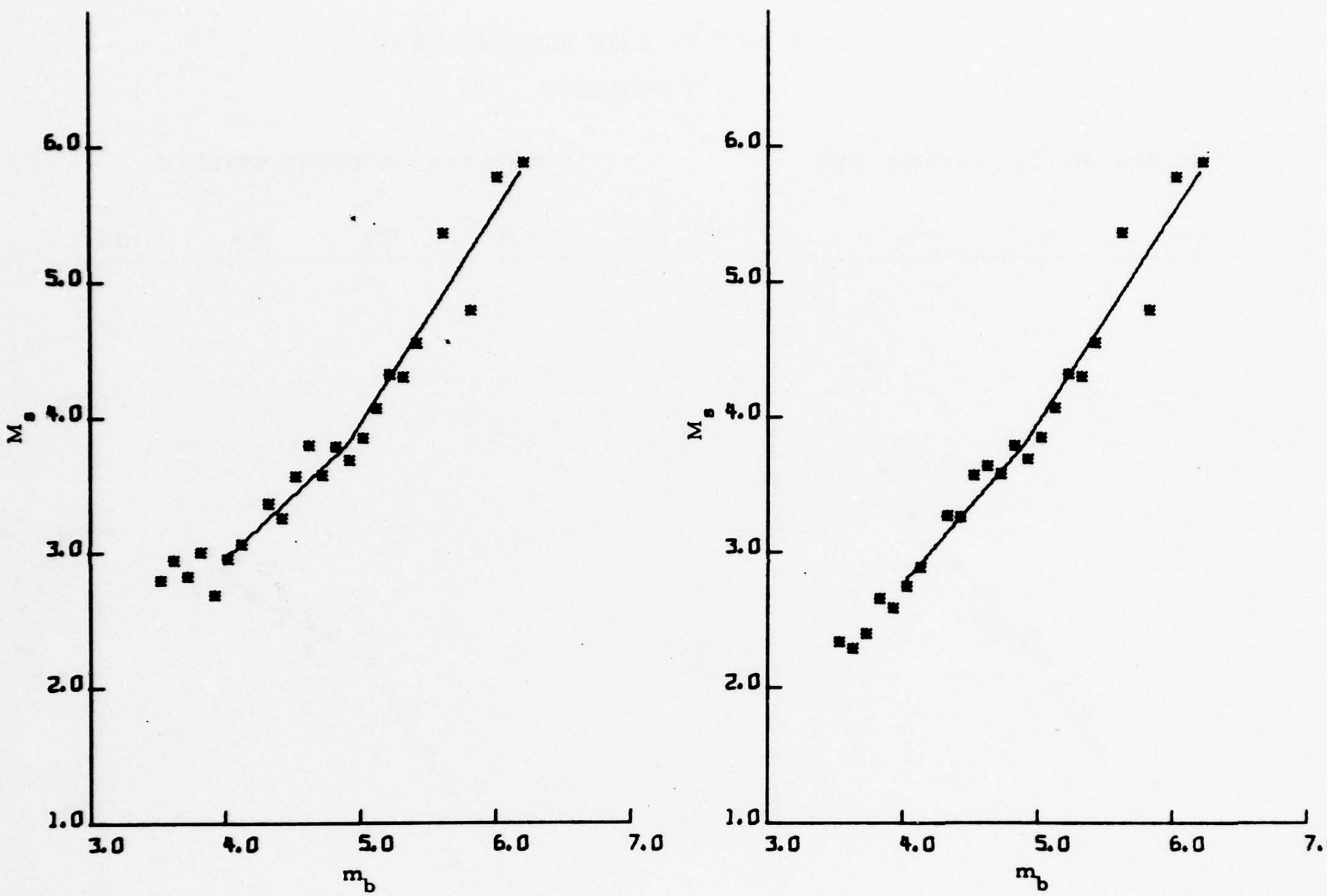


(a) Mean Signal M_s versus m_b

(b) Unbiased M_s versus m_b

FIGURE III-3

REFERENCE SITE M_s - m_b DATA



(a) Mean Signal M_s versus m_b

(b) Unbiased M_s versus m_b

FIGURE III-4

BEAMFORMED ARRAY M_s - m_b DATA

sake of completeness. These figures all show the vertical component 20-second Rayleigh wave $M_s - m_b$ data. The corresponding figures for transverse component Love wave $M_s - m_b$ data are not shown since they are essentially identical to the Rayleigh wave data. The equations for the $M_s - m_b$ fits shown in Figures III-3 and III-4 are listed in Table III-2 along with their associated standard deviations. Note that these standard deviations are all essentially the same, with the exception of the standard deviation (0.27) associated with equation (III-3). Since this equation is based on the smallest amount of data, it is felt that this difference is not significant. For comparison, note that Lambert et al. (1974) found a value of 0.37 for the standard deviation of fits to HGLP $M_s - m_b$ data, while Strauss and Weltman (1977) found a value of 0.35 for the standard deviation of fits to SRO $M_s - m_b$ data.

Referring to Figure III-3a, note that the data show a pronounced change of slope at $m_b = 4.9$, indicating magnitude bias below this point for the reference site mean signal $M_s - m_b$ data. The corresponding unbiased $M_s - m_b$ data of Figure III-3b shows a much smaller change in slope. The beamformed array mean signal $M_s - m_b$ data of Figure III-4a show a small change in slope, indicating the potential presence of a small amount of magnitude bias. Again, the corresponding maximum likelihood beam estimate $M_s - m_b$ data of Figure III-4b show a smaller change in slope. In fact, comparison of Figures III-3b and III-4b (which shows essentially parallel $M_s - m_b$ curves), indicates that Ringdal's method corrects the reference site M_s data for positive magnitude bias for m_b values between the 90 percent and the 25 percent detection thresholds. This results in an extension of the linear $M_s - m_b$ relationship from approximately the reference site 90 percent detection threshold to the reference site 25 percent detection threshold. For this case, it is shown that an unbiased reference curve could be represented down to 4.0 using Ringdal's method versus 4.8 by averaging the M_s of detected long-period signals.

TABLE III-2

RAYLEIGH WAVE $M_s - m_b$ RELATIONSHIPS

Data Type	Magnitude Estimate Type	$4.0 \le m_b \le 4.9$		Equation Number	$4.9 \le m_b \le 6.2$		Equation Number
		Relationship	S. D.		Relationship	S. D.	
Reference Site	Mean Signal M_s	$M_s = 0.57\ m_b + 1.10$	0.27	(III-3)	$M_s = 1.51\ m_b - 3.47$	0.40	(III-4)
	Unbiased M_s	$M_s = 1.09\ m_b - 1.47$	0.38	(III-5)	$M_s = 1.53\ m_b - 3.61$	0.40	(III-6)
Beamformed Array	Mean Signal M_s	$M_s = 0.93\ m_b - 0.77$	0.39	(III-7)	$M_s = 1.56\ m_b - 3.84$	0.40	(III-8)
	Unbiased M_s	$M_s = 1.14\ m_b - 1.79$	0.41	(III-9)	$M_s = 1.55\ m_b - 3.79$	0.40	(III-10)

One further point remains to be made concerning these $M_s - m_b$ fits. In the past it has been common practice to compute a linear fit to the $M_s - m_b$ data throughout the range in m_b of the data used. This results in $M_s - m_b$ relationships which are dependent on this range in m_b. For example, consider the following $M_s - m_b$ relationships:

$$M_s = 1.18\, m_b - 1.66 \qquad \text{(Lambert et al., 1974)} \qquad \text{(III-11)}$$

$$M_s = 1.36\, m_b - 1.44 \qquad \text{(Basham, 1969)} \qquad \text{(III-12)}$$

$$M_s = 1.59\, m_b - 3.97 \qquad \text{(Richter, 1958).} \qquad \text{(III-13)}$$

Equation (III-11) was derived from ALPA, NORSAR, and HGLP surface wave data with a center of mass at $m_b = 4.37$. This equation is almost identical to equation (III-9) of Table III-2, which is defined over the range $4.0 \leq m_b \leq 4.9$. Equation (III-12), with a center of mass at $m_b = 4.85$ and a slope midway between those of equations (III-9) and (III-10) of Table III-2, may be considered to be essentially an average fit to the data used to compute equations (III-9) and (III-10). Equation (III-13), which was computed from large m_b events ($m_b > 6.5$), is almost identical to equation (III-10), which was computed for the range $4.9 \leq m_b \leq 6.2$. All of this indicates that the $M_s - m_b$ data is best fitted by two lines. While the evidence is not conclusive, it appears that the point of intersection of these fits should be at about $m_b = 5.0$.

Although the results of the unbiased estimates of $M_s - m_b$ relationship are consistent with other authors' results, some idea of the physical significance and generality of the results can be obtained by comparison with bodywave source spectrum studies. In a study of P wave spectral measurements by Sax (1975), the low frequency spectral displacement amplitude, defined as the spectrum below the corner frequency, scales with m_b as follows.

$$\text{Log}\ \Omega'_o = 1.14\, m_b - 1.21 \qquad \text{(III-14)}$$

where $\text{Log}\ \Omega_o'$ is the spectral magnitude corrected for distance as

$$\text{Log}\ \Omega_o' = \text{Log}\ \Omega_o + k \log \Delta + C \qquad \text{(III-15)}$$

where

regional $k = 3,\ C = -1.59$

teleseismic $k = 1,\ C = 1.89.$

$\text{Log}\ \Omega_o$ is the measured log (displacement amplitude), and Δ is the epicentral distance of propagation. Aki (1967) treats M_s measurement as equivalent to the spectral magnitude of surface waves to within an additive constant. Comparing equation (III-14) with equation (III-9), it is observed that the low frequency spectral magnitude of bodywaves also scales similarly to M_s to within a constant, 0.58, or approximately a factor of 4. This can be shown to be reasonable by observing the scaling relationship for computing magnitude from spectrum, given by Thatcher and Hanks (1973) for local magnitude by defining local magnitude as the square root of the propagated energy.

$$M_L = \text{Log}\ \Omega_o' + \frac{3}{2} \log f_c . \qquad \text{(III-16)}$$

Sax (1975) extended the spectral scaling law to the magnitude of P waves, m_{bs}, and teleseismic distances as

$$m_{bs} = \text{Log}\ \Omega_o' + \frac{3}{2} \log f_c \qquad \text{(III-17)}$$

by means of the attenuation function correction given by equation (III-15), which was determined by fitting data. Sax (1975) observed that the average logarithmic corner frequency of P waves decreases with m_b as

$$\text{Log}\ f_c = -0.074\ m_b + 0.74 \qquad \text{(III-18)}$$

which leads to

$$\text{Log}\ \Omega_o' = 1.11\ m_{bs} - 1.11 \quad . \qquad \text{(III-19)}$$

The predicted scaling of the low frequency spectral magnitude of P waves is reasonably close to the observed scaling law given by equation (III-14). Thus, the small increment in the slope of the M_s - m_b relationship probably reflects the decrease in the corner frequencies of larger magnitude events.

The results for scaling bodywave spectral amplitudes did differ from the results in the M_s - m_b relationship in that no break in the slope was observed. This might be explained by insufficient observations of events above $m_b = 5$ to determine the break in slope of bodywave spectral amplitude versus m_b, should it exist.

The break in slope of the M_s - m_b relationship may be due to the effect of the short-period system response curve on the measurement of m_b. The corrections for system magnification would be expected to be more accurate for smaller events with corner frequencies near the peak of the system response. Errors in magnification corrections could be introduced progressively in larger magnitude events due to bias in visual measurement of period of a signal. The bias would be toward measuring shorter periods than would be the case if the ground displacement wavelets were observed directly. This could result in compression of the m_b scale above $m_b = 5$.

B. THE EFFECT OF MAGNITUDE BIAS CORRECTION ON THE M_s - m_b DISCRIMINANT

Figure III-5 shows a typical M_s - m_b scatter plot. The stars represent the M_s - m_b values for the Kurile Islands - Kamchatka events of the data base as measured on the beamformed array traces. The open circles represent M_s - m_b values for a suite of Eurasian presumed nuclear explosions

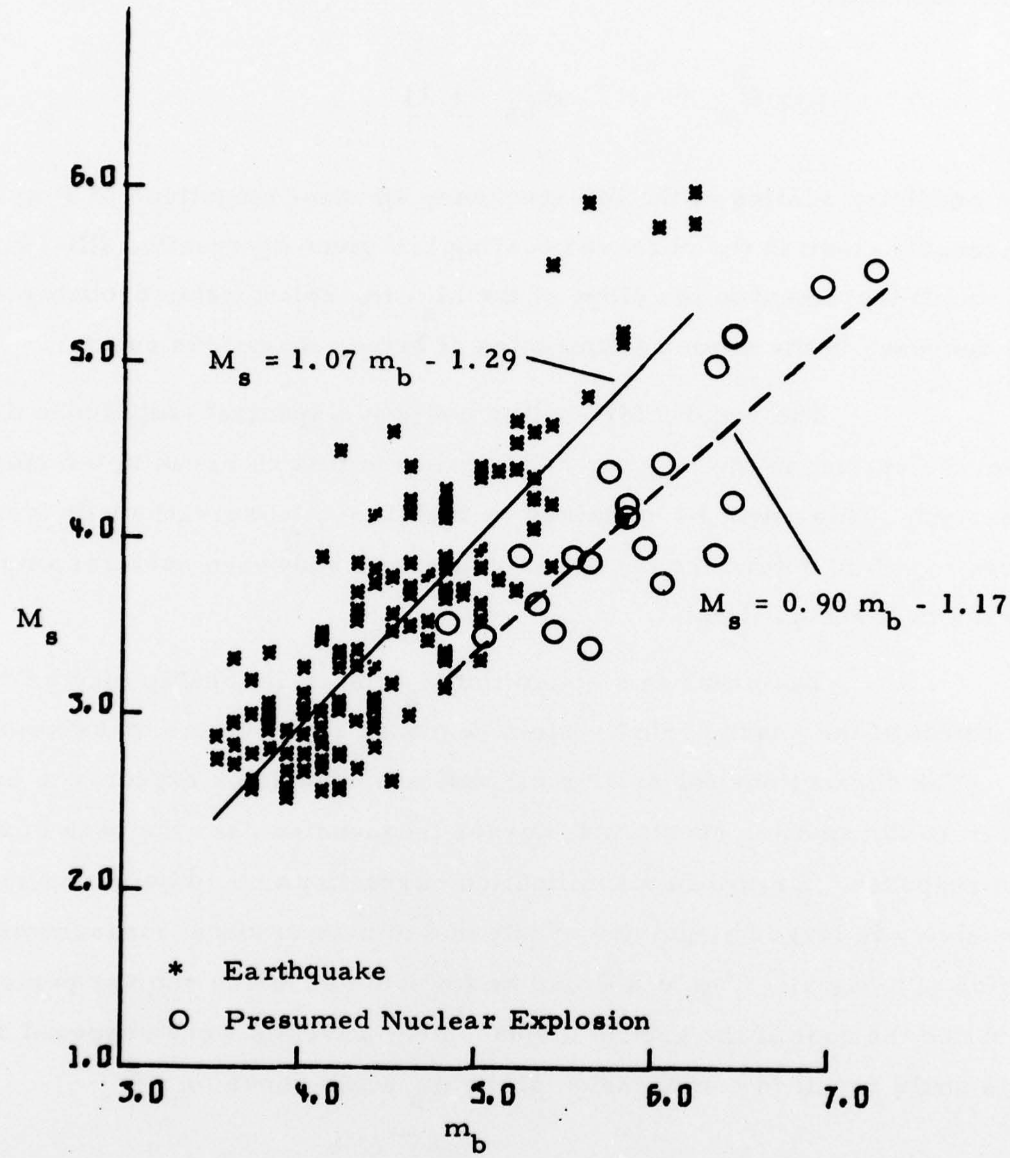


FIGURE III-5

20 SECOND LR-V CONVENTIONAL M_s - m_b PLOT FOR BEAM DATA

as recorded at ALPA and measured on the beamformed array traces. The solid line is the least-squares fit to the earthquake M_s - m_b data. The dashed line is the least-squares fit to the presumed nuclear explosion data. The equations of these fits are shown on the figure.

From these data one can see that at some value of m_b the fitted lines will intersect. This lack of parallelism indicates that below some m_b the M_s - m_b approach will fail to discriminate. For this data, it appears that the discriminant is of little value below $m_b = 5.5$.

Now consider the individual M_s values for each m_b to represent the M_s measurements made at stations of a network for one event. The mean signal M_s - m_b points are plotted in Figure III-6, where it is assumed that there is one event per m_b value. The stars in Figure III-6 and III-7 represent the mean M_s value at each m_b. The open circles represent those presumed explosion m_b values for which there was only one observation. The solid circles represent those presumed explosions for which there were several signal and noise magnitude measurements. These are indicated separately, since Ringdal's method could only be applied to these presumed explosions. Figure III-7 shows the corresponding M_s - m_b data after the M_s values have been corrected by Ringdal's method. The fits to the earthquake population are the two-line fits tying at $m_b = 4.9$ as described earlier. In a similar fashion, two lines were fitted to the presumed explosion M_s - m_b data. The point of intersection for these fits was set at $m_b = 5.9$, since this was the lowest m_b at which all presumed explosions were detected. The equations of these fits are shown on the figures.

Comparing Figures III-6 and III-7, one can see that the three presumed explosion points to which Ringdal's method could be applied are in line with the trend of the larger m_b ($m_b \geq 5.9$) presumed explosion M_s - m_b points. This indicates that, given sufficient observations of the presumed explosions, removal of positive magnitude bias may be expected to extend

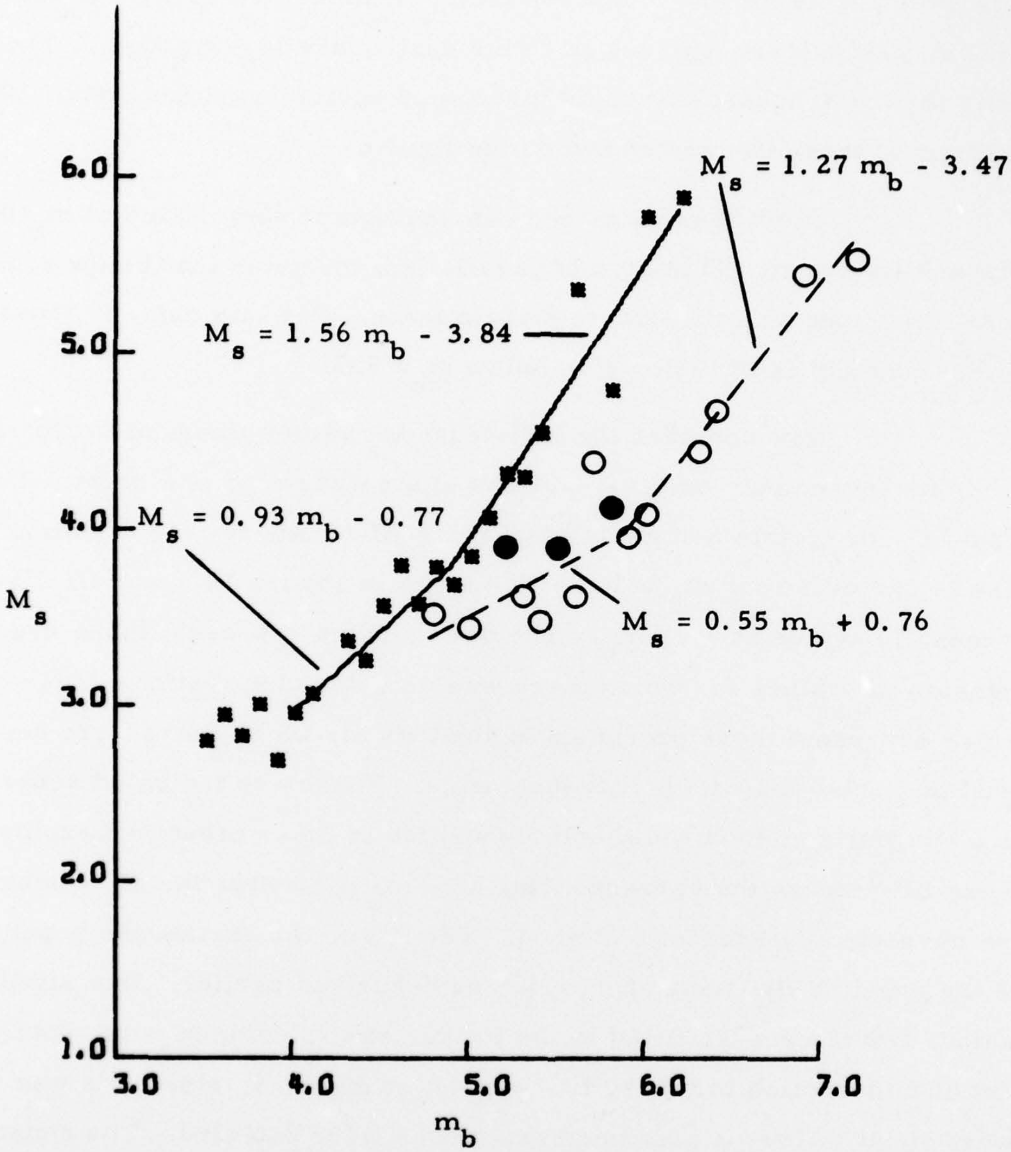


FIGURE III-6

THE M_s - m_b DISCRIMINANT BEAM MEAN SIGNAL M_s

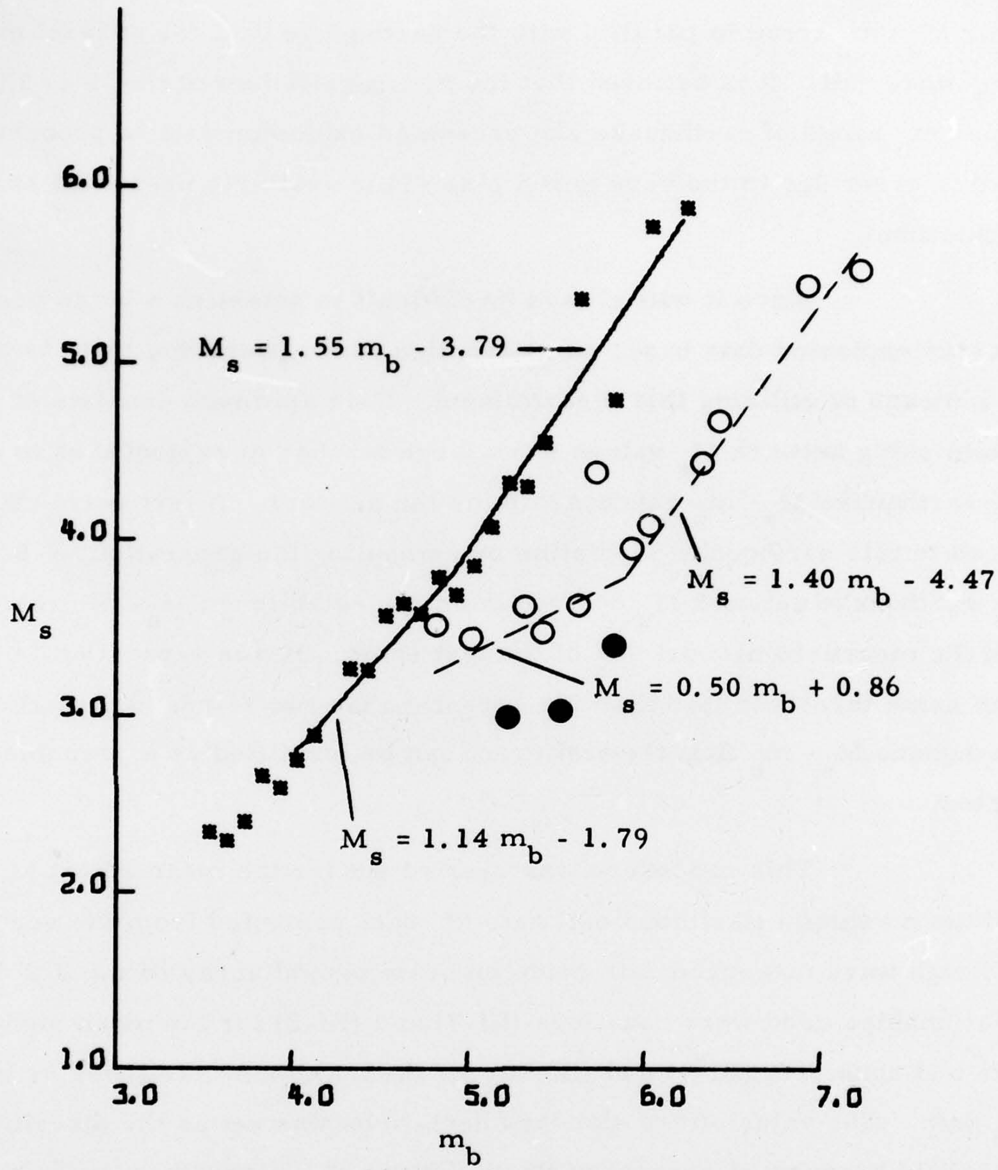


FIGURE III-7

THE M_s - m_b DISCRIMINANT BEAM UNBIASED SIGNAL M_s

their M_s - m_b trend in parallel with the earthquake data for at least one-half magnitude unit. It is believed that the non-parallelism of the lines fit to the upper m_b range of earthquake and presumed explosion data is probably estimation error due to the very small size of the available presumed explosion population.

Since it will always be difficult to establish a large presumed nuclear explosion data base, an alternative to the preceding plots is offered as a means of utilizing this discriminant. This approach consists of first establishing network M_s values for a large number of earthquakes to establish the earthquake M_s - m_b relationship for the network. A test event can be compared to this earthquake population by computing the separation S between the earthquake network M_s derived from the established M_s - m_b relationship and the measured network M_s of the test event. If this separation is greater than some threshold (for example, greater than two standard deviations of the earthquake M_s - m_b fit), the test event can be identified as a presumed nuclear explosion.

This procedure was carried out for the mean signal M_s data and the maximum likelihood estimate M_s data computed from the vertical Rayleigh wave measurements made on beamformed array data. The M_s - m_b relationships used were equations (III-7) and (III-8) for the mean signal M_s data and equations (III-9) and (III-10) for the maximum likelihood estimate M_s data. The value of two standard deviations was set as the discrimination threshold because 95 percent of the members of a normally distributed population lie within two standard deviations of the mean. Thus, the probability that a test event is a member of the earthquake population when it is separated from the earthquake mean by a positive deviation of more than two standard deviations is less than 0.025. In this example, a value of 0.4 was used as the standard deviation.

In order to provide more insight into the effect on the M_s - m_b discriminant of removing magnitude bias, the separation S is presented in Table III-3 for the mean signal M_s (S_{sig}) and the maximum likelihood estimate M_s (S_{mle}). This table shows that for a discrimination threshold of two standard deviations:

- In neither case are any of the earthquakes mis-identified.
- Eleven of the fifteen presumed nuclear explosions are so identified when the mean signal M_s is used.
- Twelve of the fifteen presumed nuclear explosions are so identified when the unbiased M_s is used.
- Those presumed explosions which were not identified as such are the smaller m_b events for which there was only one observation available. The one event which was identified as an earthquake using mean signal M_s and as an explosion using unbiased M_s was one of the three for which there were sufficient observations to apply Ringdal's method. (The other two of the three were identified as explosions using both types of M_s.)

If a more conservative threshold of three standard deviations is used, only seven of the fifteen presumed nuclear explosions are identified when the mean signal M_s is used, while ten are identified as explosions when the unbiased M_s is used.

These results indicate that Ringdal's method of removing positive magnitude bias from network m_b can improve the value of the M_s - m_b discriminant. Determination of the degree by which it will improve the discriminant must await the collection of a presumed explosion data base containing multiple observations of each event, i.e., by a network of seismic stations.

TABLE III-3

SEPARATION S FROM EARTHQUAKE MEAN M_s

Earthquakes			Presumed Nuclear Explosions		
m_b	S_{sig}	S_{mle}	m_b	S_{sig}	S_{mle}
4.0	0.02	0.05	4.8	0.19	0.18
4.1	0.00	0.02	5.0	0.54	0.54
4.2	0.05	-0.01	5.2	0.39	1.30
4.3	-0.11	-0.13	5.3	0.83	0.83
4.4	0.09	0.00	5.4	1.13	1.13
4.5	-0.12	-0.20	5.5	0.87	1.74
4.6	-0.26	-0.16	5.6	1.30	1.29
4.7	0.05	0.02	5.7	0.69	0.69
4.8	-0.07	-0.08	5.8	1.11	1.82
4.9	0.13	0.14	5.9	1.43	1.42
5.0	0.14	0.14	6.0	1.46	1.45
5.1	0.08	0.08	6.3	1.57	1.56
5.2	-0.02	-0.02	6.4	1.49	1.48
5.3	0.16	0.16	6.9	1.51	1.50
5.4	0.06	0.06	7.2	1.89	1.88
5.6	-0.43	-0.44			
5.8	0.45	0.44			
6.0	-0.22	-0.23			
6.2	-0.02	-0.03			

S_{sig} denotes separation between mean signal M_s-m_b values and the line fit to the earthquake mean signal M_s-m_b data

S_{mle} denotes separation between unbiased M_s-m_b values and the line fit to the earthquake unbiased M_s-m_b data

One standard deviation is taken as 0.4 magnitude units.

SECTION IV

CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The following conclusions have been reached in the course of this test of Ringdal's method of removing positive magnitude bias:

- Surface wave magnitudes are strongly affected by positive magnitude bias for events with bodywave magnitudes below the 90 percent detection threshold.
- Below this detection threshold, this bias is the controlling factor in the trend of the $M_s - m_b$ relationship.
- Ringdal's method of removing this bias successfully removed the bias from the surface wave magnitudes for events with bodywave magnitudes down to approximately the 25 percent detection threshold.
- For good results, the method requires at least three detections with corresponding signal magnitudes.
- The earthquake $M_s - m_b$ relationships derived from unbiased ALPA-recorded surface wave magnitude data are:

Single Site
$$\begin{cases} M_s = 1.09\, m_b - 1.47 & 4.0 \le m_b \le 4.9 \\ M_s = 1.53\, m_b - 3.61 & 4.9 \le m_b \le 6.2 \end{cases}$$

Array
$$\begin{cases} M_s = 1.14\, m_b - 1.79 & 4.0 \le m_b \le 4.9 \\ M_s = 1.55\, m_b - 3.79 & 4.9 \le m_b \le 6.2\, . \end{cases}$$

- Removal of magnitude bias from those presumed explosions for which there were multiple observations brought the M_s of these events into line with the trend of the M_s data for those presumed explosions with m_b at or above the array 100 percent detection threshold of surface waves.

B. RECOMMENDATIONS

In light of the preceding discussion of positive magnitude bias removal and M_s - m_b relationships, and considering the difficulty of assembling a presumed nuclear explosion data base large enough to establish a reliable explosion M_s - m_b relationship, it would appear that the optimum manner in which to use the M_s - m_b discriminant would be the determination of the separation between each of the M_s - m_b points and the earthquake M_s - m_b relationship as described in Section III. This approach permits one to assign a single number as the discrimination value rather than a pair of numbers (M_s and m_b). This discrimination number can then be compared to some pre-set threshold (such as two or three standard deviations from the mean) in order to identify the event.

It is recommended that a test of this procedure using unbiased M_s data be carried out using the SRO-ASRO stations as a network. From the signal and noise magnitudes measured at the stations for a suite of earthquakes, the unbiased M_s - m_b relationship can be established. Then a suite of test events can be used to determine the discrimination capability of this method in terms of the percentage of correct event identifications and the m_b range over which it is valid.

The results of this study also indicate that an in-depth investigation of M_s - m_b relationships using unbiased mean M_s data would be fruitful. In particular, more large ($m_b > 5.0$) earthquakes are needed to better establish the M_s - m_b least-squares trend line for large m_b. The key points to be established are whether M_s - m_b data is best fitted with two lines, as illustrated in this report and, if so, where the point of intersection lies. The same points need to be answered for explosion M_s - m_b data. The difficulty of assembling a sufficiently large explosion data base at any one station could be circumvented by removing the station instrument response and by comparing similar measurements of M_s at all stations at all times. A study of this nature can be used to test the hypothesis that the earthquake and explosion M_s - m_b relationships are parallel.

SECTION V
REFERENCES

Aki, K., 1967; Scaling Law of Seismic Spectrum, Journal of Geophysical Research, vol. 72, No. 4, 1217-1231.

Basham, P. W., 1969; Canadian Magnitudes of Earthquakes and Nuclear Explosions in Southwestern North America, Geophysical Journal, vol. 17, 1-13.

Bungum, H., and E. S. Husebye, 1974; Analysis of the Operational Capabilities for Detection and Location of Seismic Events at NORSAR, Bulletin of the Seismological Society of America, vol. 64, 637-656.

Chang, A., and D. von Seggern, 1978; A Study of Amplitude Variation and m_b Bias at the LASA Subarrays, Technical Report SDAC-TR-77-11, Teledyne Geotech, Alexandria, VA.

Der, Z. A., 1977; On the Existence, Magnitude and Causes of Broad Regional Variations in Bodywave Amplitudes, Report Number SDAC-TR-76-08, Teledyne Geotech, Alexandria, VA.

Evernden, J. F., and W. M. Kohler, 1976; Bias in Estimates of m_b at Small Magnitudes, Bulletin of the Seismological Society of America, vol. 66, 1887-1904.

Freedman, H. W., 1967; Estimating Earthquake Magnitude, Bulletin of the Seismological Society of America, vol. 57, 747-760.

Johnson, N. L., and F. C. Leone, 1964; Statistics and Experimental Design, Volume I, John Wiley and Sons, Incorporated, New York, NY 224-226.

Lambert, D. G., A. I. Tolstoy, and E. S. Becker, 1974; Seismic Detection and Discrimination Capabilities of the Very Long Period Experiment Final Report; Technical Report No. 7, ALEX(01)-TR-74-07, AFTAC Contract Number F08606-74-C-0033, Texas Instruments Incorporated, Dallas, TX.

Nuttli, O. W., and S. G. Kim, 1975; Surface-Wave Magnitudes of Eurasian Earthquakes and Explosions, Bulletin of the Seismological Society of America, vol. 65, 693-709.

Richter, C. F., 1958; Elementary Seismology, W. H. Freeman and Company, 348-349.

Ringdal, F., and R. L. Whitelaw, 1973; Continued Evaluation of the Norwegian Short-Period Array, Special Report No. 9, AFTAC Contract Number F33657-72-C-0725, Texas Instruments Incorporated, Dallas, TX.

Ringdal, F., 1974; Estimation of Seismic Detection Thresholds, Texas Instruments Report No. ALEX(01)-TR-74-02, AFTAC Contract Number F08606-74-C-0033, Texas Instruments Incorporated, Dallas, TX.

Ringdal, F., 1975; Maximum Likelihood Estimation of Seismic Event Magnitude from Network Data, Texas Instruments Report No. ALEX(01)-TR-75-01, AFTAC Contract Number F08606-75-C-0029, Texas Instruments Incorporated, Dallas, TX.

Sax, R. L., 1975; Application of P-Wave Spectral Measurements to Short-Period Discrimination, Texas Instruments Report No. ALEX(01)-TR-75-08, AFTAC Contract Number F08606-75-C-0029, Texas Instruments Incorporated, Dallas, TX.

Strauss, A. C., 1973; Final Evaluation of the Detection and Discrimination Capability of the Alaskan Long Period Array, Special Report No. 8, AFTAC Contract Number F33657-72-C-0725, Texas Instruments Incorporated, Dallas, TX.

Strauss, A. C., and L. C. Weltman, 1977; Continuation of the Seismic Research Observatories Evaluation, Texas Instruments Report No. ALEX(01)-TR-77-02, AFTAC Contract Number F08606-77-C-0004, Texas Instruments Incorporated, Dallas, TX.

Sun, D., 1977; Determination of Seismic Source Parameters from Long-Period Surface Wave Data, Technical Report No. 11, Texas Instruments Report No. ALEX(01)-TR-77-11, AFTAC Contract Number F08606-77-C-0004, Texas Instruments Incorporated, Dallas, TX.

Thatcher, W., and T. C. Hanks, 1973; Source Parameters of Southern California Earthquakes, Journal of Geophysical Research, vol. 78, 8547-8576.

Veith, K. F., and G. E. Clawson, 1972; Magnitude from Short-Period P-Wave Data, Bulletin of the Seismological Society of America, vol. 62, 435-452.

von Seggern, D., and D. W. Rivers, 1978; Comments on the Use of Truncated Distribution Theory for Improved Magnitude Estimation, (preprint) to be published in Bulletin of the Seismological Society of America, Oct.