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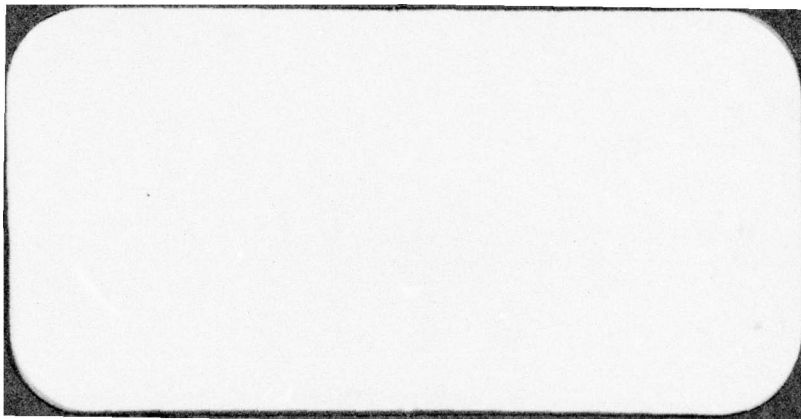
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COLLEGE OF ENGINEERING
CORNELL UNIVERSITY
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6 Report on the
~~FOURTH~~ INTERNATIONAL WORKSHOP ON GAME THEORY (4th):
Multiperson Game Theory and Its Applications
held at Cornell University, June 18-July 1, 1978.

by
10 W. F. Lucas

15 N00014-78-M-0048,
DAAG29-78-M-0105

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TABLE OF CONTENTS

	<u>Page</u>
Title Page	i
Table of Contents	ii
Report on Workshop Activities	1
Comments on Scientific Developments	4
List of Participants	8
Index to Abstracts	11
Abstracts of Talks	13

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FOURTH INTERNATIONAL WORKSHOP ON GAME THEORY:

Multiperson Game Theory and Its Applications

Report on Workshop Activities

The Fourth International Workshop was held at Cornell University in Ithaca, New York from Sunday, June 18 through Saturday, July 1, 1978. This Workshop followed in the tradition of the five conferences on game theory at Princeton University in 1953, 1955, 1957, 1961 and 1965, as well as the three previous international workshops in Jerusalem in 1965, in Berkeley in 1970 and near Bielefeld in 1974. The participants included a significant portion of distinguished experts in the field of game theory (excluding differential games and gaming) as well as a good number of excellent young scientists. A dozen countries in addition to the United States of America were represented at the Workshop. Significant advances in the theoretical aspects of the subject as well as in the development of new applications were announced. More details about these scientific achievements are presented in the next section of this report as well as in the abstracts which follow.

There were 55 participants in attendance at the Workshop (a few were only present part of the time), plus four doctoral student assistants from Cornell University. There were 21 participants from foreign countries: Israel (6), France (4), West Germany (4), and one each from Austria, Belgium, Cameroon, Canada, Colombia, Japan and India. Among those currently residing in the United States there were about 13 people who are nationals of other countries: Israel (6), and one or two each from Colombia, Egypt, Japan, India and Thailand. There were about 21 citizens of the United States who are currently employed in this Country. There were four faculty members from Cornell University. About 25 of those in attendance (including the four Cornell graduate students) could be classified as "young scientists" in the sense of age, years since (or until) their Ph.D., and having rank not exceeding an assistant professor or its equivalent; and they were a very talented group indeed. From the list of participants and their affiliations contained in this report, one can see that a good variety of specialties and disciplines were represented by the participants. From eight to twelve

(depending on one's definition of "academic") were from nonacademic institutions. Unfortunately, some scientists from Eastern Europe were unable to attend due to the lateness of the invitations.

There were 50 formal talks presented at the Workshop and abstracts for these are included in this report. Several informal presentations were also made, many long organized group discussions took place, working lists of unsolved problems were drawn up, and different meetings concerned with future directions, potential application, curriculum matters, publications, communications, and the overall development of game theory were held. A few of the highlights from these discussions are mentioned briefly in the next section. As was the case with the Princeton conferences and the other international workshops in game theory which were mentioned above, there will not be a detailed proceedings of this Workshop. As in the past, it is expected that most of the material presented at the Workshop will be published through the available journals in the appropriate disciplines involved.

This Workshop was under the direction of William F. Lucas of Cornell University and Robert M. Thrall of Rice University. They were assisted by an advisory committee consisting of Robert Aumann, Louis Billera, John Harsanyi, Michael Maschler, Reinhard Selten, Lloyd Shapley and Martin Shubik. Registration and a reception sponsored by Cornell's School of Operations Research and Industrial Engineering took place on Sunday, June 18. Full day sessions were held from Monday, June 19 through noon on Saturday, July 1, 1978, except for Sunday, June 25 and for an excursion on Wednesday, June 28. Most participants were housed in Cornell University dormitories and had their meals together on campus. The Cornell Conference Office assisted in making arrangements.

Financial support for the Fourth International Workshop was provided by the National Science Foundation through the Mathematical Sciences Section of the Division of Mathematical and Computer Sciences (Grant MCS77-17269), the Office of Naval Research through the Operations Research Program (Code 434) in the Mathematical and Information Sciences Division (Purchase Order No. N00014-78-M-0048, NR 047-180), and the U.S. Army Research Office through both the Mathematics Division, Research Triangle Park, N.C. (Purchase Order No. DAAG29-78-M-0105, RN 78-049) and the European Research Office, London,

England (foreign travel assistance). The organizers of the Workshop are most grateful for the support from these federal agencies and particularly to those individuals who helped to make this possible.

Note that for a few of the participants, the addresses given in the List of Participants do not agree with the affiliations denoted on their abstract. The former indicates their location in the fall of 1978, whereas the latter corresponds to their position at the time of the Workshop in June, 1978. Requests or questions regarding papers presented at the Workshop should be made directly to the authors.

Comments on Scientific Developments

Some brief remarks of a personal nature concerning some of the recent theoretical and applied developments in game theory will be made in this section. These general observations will not be limited to just the results presented at the Fourth International Workshop, since important developments have also been reported at other recent meetings as well as in various publications. Progress in the theoretical aspects of this field is currently being made at a rapid rate, and this includes several important new models as well as the significant maturing of some of the more traditional approaches. The great variety of new areas to which game theory is being applied is most impressive. Specialists from many other disciplines have, in the past few years, been seriously considering or actually employing game theoretical models. A few of these highlights concerning both theory and applications will be mentioned here.

One of the most impressive theoretical developments has been the maturing of value theory for n -person games, especially for the case of games with a continuum of players. This rich theory relates to developments in analysis and probability, and is ripe for sophisticated applications to economics and political science. Some movement in the latter directions is underway. Values for games with a continuum of players have already been recommended for use or actually implemented in a few real-world applications concerned with equity; e.g., for subsidizing or setting fair rates in systems with a large number of users. The necessary theory and numerical methods for making use of the value theories is also well advanced. The ultimate impact of these developments to mathematical modeling in the social sciences and operations research should prove to be quite profound. One or two big success stories in this direction might well cause these techniques to become broadly employed.

A few new models have been proposed for analyzing n -person cooperative games for finite n . Some of these make use of the extensive or normal form of a game, as well as the more typical approaches through the games in characteristic function form. Good progress is being made in analyzing these new models, and in providing new insights into some of the more classical

approaches. The newer game forms have been more successful in integrating fundamental aspects relating to dynamics and coalition formation than most traditional models in cooperative game theory.

Major results for the two-person repeated games and repeated games with incomplete information have also been achieved in recent years. An indication of this work can be obtained from papers by T. Bewley and E. Kohlberg, and J.-F. Mertens and S. Zamir in Mathematics of Operations Research.

New theories have also been proposed for bargaining situations, especially for the two-person case; and concepts from the social sciences have been introduced to better model interactions involving negotiation and power. Additional insights into the mathematical structure and nature of equilibrium points have been obtained, and several algorithms for finding equilibria are now available. Techniques for addressing multicriteria decisionmaking problems and the related questions in utility theory have received significant attention lately. Very interesting results on combinatorial games have been frequently forthcoming. Advances in other theoretical areas of game theory are also being announced, including stochastic games, random payoff games, approximations to optimal solution, and other questions involving probabilistic considerations. (The theory of differential games and results on gaming, simulation and experimental games are not being considered in this report.)

One of the exciting developments in the applications of game theory is its use in a number of situations concerned with equity and fairness. Problems concerned with equitable utility rates, fair allocation of costs and benefits, pricing and taxation, distributing joint overhead costs among different sorts of users, and suchlike are receiving significant attention. In particular, specific studies have been concerned with fair telephone rates, pricing spare parts, sharing library costs among using institutions in a just manner, subsidizing transportation systems, fair landing fees and other airport overhead charges, intermunicipality sharing of costs for joint water projects, taxation for environmental and pollution control, and so forth. Articles on multiperson game theory are beginning to appear in accounting journals. There appears to be a very bright future

for the use of game theory in such equity considerations. Many problems in the social sciences and traditional operations research are concerned with fair divisions, and the usual marginal pricing or optimization techniques employed are not always the most appropriate approaches. Various values and nucleoli are game theoretical solution concepts which can often be applied to such problems.

Game theory is becoming a frequently used tool in the increasingly more mathematical approaches being taken in political science. A two-week meeting of political scientists and game theorists, sponsored by the Mathematical Social Science Board, was held in Hyannis, Massachusetts in July of 1977. Several attempts to model international interactions via games are being made. These include international relations, monetary confrontations, trade, tariffs, custom unions, and import-export games. The stability of national and international coalitions is also of interest. Game theory has been very useful for measuring power in weighted voting systems and in other power considerations. The Banzhaf index has been accepted as a power measure in several court cases. Several new theories and indices have been suggested recently, and a good deal of study on such values has taken place. Models for lobbying and exploitation have also been put forward, as well as ones for allocating campaign resources. Proposals for alternates to some present voting schemes have been made. Interesting investigations into committee structures, coalitional dynamics, cabinet formation, and setting up and manipulating agenda are being pursued. Some interesting experimental work and a return to greater emphasis on the extensive form of a game is present in some of this work. Game theory appears to have a bright future in analyzing nontraditional forms of international conflict as well as more typical structures which arise in political science.

Some of the recent work in social choice theory has a strong game theoretical component. Other social science applications include papers on coping with deception, accidents and retribution, as well as new uses of prisoner dilemma type models. And a game model for reinsurance has been put forth.

Techniques from game theory have been used to study problems in energy. This includes energy policy, OPEC pricing curves over time, equilibrium

models for prices, cartel stability, and auctions and bidding strategies. Many publications continue to appear in the direction of applying game theory to economic theory. Equilibrium points, cores and value theory are frequently employed in mathematical economics, e.g. to various models for economic markets. Papers concerned with ownership, property and income distribution, public goods, fixed price economies, and several other topics have been presented at recent meetings and in various publications. Game theory has become an important methodology in modern economic theory.

Advances continue to be made in some of the traditional areas of applied game theory, such as in game models for allocations, duels, searches, divisions, attrition, inspection, and so forth. In addition, games are being employed more frequently in the direction of the life sciences where they have been used to study genetics, animal encounters, species behavior, and other interactions.

In brief, a good number of new theoretical discoveries, many interesting new applications, as well as major advances in traditional directions have appeared in game theory in the past few years. Many people from a large variety of different disciplines are directing their attention to this subject. There is a need for better communications to inform potential users of the new applications which are appearing as well as of still additional opportunities for employing game models. Several books in English on game theory and its applications have been published in the past few years. This includes books by the authors S. Brams, J. Conway, J. Harsanyi, J. Rosenmüller, M. Shubik, N.N. Vorob'ev, as well as others. Several other books should appear in the next year or two. Some of these are: a book by J. Case, an enlarged revision of G. Owen's text which is currently out of print, a TIMS study on games and energy, a monograph by W. Lucas on equity considerations, a book by E. Berlekamp, J. Conway and R. Guy on combinatorial games, plus proceedings from recent conferences such as the one on political science in Hyannis in July 1977 and the one on applied game theory in Vienna in June 1978. Some of these publications should appear in the New York University Press series entitled Studies in Game Theory and Mathematical Economics. Another series of major conferences in game theory has been the All Soviet conferences held in November 1968 in Erivan, June 1971 at Vilnius, September 1974 at Odessa, and May 1978; and proceedings from these conferences are eventually published (in Russian).

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Fourth International Workshop on Game Theory

Cornell University - June, 1978

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INDEX TO ABSTRACTS

Speakers	Page
1. Albers, Wulf	13
2. Aumann, Robert and Jacques Dreze	14
3. Bergstresser, K. and P.L. Yu	14
4. Billera, Louis J., David C. Heath and Joseph Raanan	15
5. Brock, Horace W.	15
6. Case, James	16
7. Chetty, V.K., D. Dasgupta and T.E.S. Raghavan	16
8. Hart, Sergiu	17
9. Ichiishi, Tatsuro and Shlomo Weber	17
10. Ichiishi, Tatsuro	18
11. Kalai, Ehud	18
12. Levine, Pierre	18
13. Levine, Pierre and Jean-Pierre Ponsard	19
14. Lucas, William F.	20
15. Mayberry, John P.	20
16. Megiddo, Nimrod	21
17. Mertens, Jean-Francoise and Shamuel Zamir	22
18. Mertens, Jean-Francoise	22
19. Moulin, Hervé	22
20. Moulin, Hervé and D. Gabay	23
21. Muto, Shigeo	23
22. Myerson, Roger B.	25
23. Myerson, Roger B.	25
24. Neyman, Abraham	26
25. Neyman, Abraham and Yair Tauman	27
26. Owen, Guillermo	29
27. Parthasarathy, T. and T.E.S. Raghavan	29
28. Peleg, Bezalel	31
29. Maschler, Michael and Micha Perles	31
30. Ponsard, Jean-Pierre and Sylvain Sorin	33
31. Postlewaite, Andrew	34
32. Raanan, Joseph	34
33. Rabie, Mohamed	35

34.	Reichardt, Robert	35
35.	Rosenmüller, Joachim	37
36.	Rosenthal, Robert W.	38
37.	Roth, Alvin E.	39
38.	Ruckle, William H.	40
39.	Ruckle, William H.	40
40.	Schmeidler, David	41
41.	Selten, Reinhard and Werner Güth	42
42.	Shapley, Lloyd S.	43
43.	Shenoy, Prakash P.	44
44.	Sussangkarn, Chal	45
45.	Thrall, Robert M., David Cardus and Charles Hammons	45
46.	Turbay-Bernal, Gabriel J.	48
47.	Vasilescu, Eugen N.	49
48.	Weber, Robert James	49
49.	Weber, Robert James	50
50.	Yu, Po Lung	51
51.	Zilcha, Itzhak	51

A SOLUTION CONCEPT BASED ON ASPIRATION PROFILES

Wulf Albers

Universität Augsburg

This concept can be applied to real and set valued characteristic function games, and to games on (Euclidian) spaces of alternatives, if the players' utility functions are strictly quasiconcave with a relative maximum (single peak).

The concept is based on the following three conditions concerning an aspiration profile $a \in \mathbb{R}^n$, i.e. an n -tupel of aspirations of the n players. (We call a coalition feasible with respect to a given aspiration profile a , if there is a payoff distribution that can be arranged by the coalition and which gives to any player of the coalition at least the utility of his aspiration level.)

a) Pareto-optimality: If any player i raises his aspiration level a_i and the levels of the other players do not decrease then there will be no feasible coalition for i .

b) Coalition forming ability: For any player there is at least one feasible coalition with respect to a .

c) No unilateral dependences: If a player i is so dependent upon a player j that any feasible coalition (with respect to a) which includes i also includes j , then the opposite dependence of j upon i shall also be given.

Note that this concept gives aspiration profiles. Usually for any aspiration profile there are different corresponding payoff vectors depending on the feasible coalition that is actually formed. (For quota games the aspiration profiles are the quotas and the corresponding payoff vectors are the quota solutions.) There are relations to a solution concept suggested by CROSS [1967]. Condition a) is equivalent to a condition of WILSON [1970]. This solution concept is discussed and demonstrated by means of some examples.

SHAPLEY VALUES OF FIXED PRICE ECONOMIES

Robert Aumann*
Hebrew University, Jerusalem

and

Jacques Dreze
CORE, Louvain

A fixed price economy with l goods is defined like an ordinary exchange economy, except that all traders are constrained to trade at exogenously given fixed prices p . These economies have become prominent in the last five years as models for disequilibrium phenomena such as unemployment. Value allocations in such economies are investigated, and are shown to be closely associated with certain coupon rationing schemes.

MULTICRITERIA n -PERSON GAMES

K. Bergstresser*
Washington State University

and

P. L. Yu
University of Kansas

Multicriteria problems occur naturally in situations involving decision making and (partial) conflict among n persons. In this paper it is demonstrated that existing solution concepts for single criterion n -person games in both normal form and characteristic function form induce domination structures in various spaces, including the payoff space, the imputation space and the coalition space. Using this discussion as a basis, several approaches for resolving multicriteria n -person games are proposed, including the multicriteria core and parametric methods.

* An asterisk denotes who presented the talk.

AN APPLICATION OF NON-ATOMIC VALUE THEORY

Louis J. Billera*
Cornell University

David C. Heath
Cornell University

and

Joseph Raanan
Bell Telephone Laboratories

The problem of determining rates is considered for a situation in which services are purchased in bulk, but they have to be paid for by a large number of small users. The desired rates must be "fair" and they must cover all costs. The problem is formulated as a non-atomic game and solved by using the value of the game. In addition to the general problem, a detailed actual case is presented together with computational methods and results. The specific case concerns internal telephone billing rates at Cornell University.

[A publication will appear in Operations Research, fall, 1978.]

INTERPERSONAL COMPARISONS OF UTILITY IN
GAME THEORY AND ETHICS: A UNIFIED TREATMENT

Horace W. Brock
Stanford Research Institute

It is possible to distinguish two different types of interpersonal comparisons of utility. First, there are the "classical" types of "absolute" comparisons which arise in moral theory, e.g., in Harsanyi's rule utilitarianism. Second, there are the "intrinsic" comparisons which arise in Nash's bargaining theory, and in the non-transferable Shapley value. The relationship between these two types of comparisons will be set forth. As a corollary, it may be possible to better understand exactly what is being "contributed" by player 1 according to Shapley's generalized value.

AN IMPORT-EXPORT GAME

James Case
Federal Trade Commission

Two- and three-player versions of a game played regularly by the importers and exporters of such commodities as coffee, sugar, and grain are considered. Various solution concepts are examined and a practical solution proposed.

POWER AND INCOME DISTRIBUTION IN PRODUCTION ECONOMIES

V. K. Chetty*

Indian Statistical Institute, New Delhi

D. Dasgupta

Indian Statistical Institute, New Delhi

and

T. E. S. Raghavan

University of Chicago, Chicago Circle

The problem concerns pricing factors of production when the technology is characterized by concave or convex production functions. Three elementary axioms for collective choice are shown to be equivalent to the maximum decision rule, and the nucleolus for a class of n -person cooperative games. This rule is then used to determine the prices of factors of production. This solution concept is found to reflect the 'power structure' of the society. Also, for a variety of production models, the rewards for all factors except one, which turns out to be powerful, is according to the marginal worth. Thus it leads to results which are closer to traditional economic thinking on distribution of income.

It is also shown that in economies with steeply diminishing marginal output, the workers benefit more by absenting themselves at a positive rate. This leads to inefficiency. The results are reported in the following two papers:

- (1) "Power and Distribution of Income" by V. K. Chetty, D. Dasgupta and T. E. S. Raghavan, Indian Stat. Inst. Discussion Paper No. 139, 1976.

- (2) "Absenteeism, Efficiency and Income Distribution",
by V. K. Chetty and T. E. S. Raghavan, Indian Stat.
Inst. Discussion Paper No. 160, 1976.

MEASURE-BASED VALUES OF MARKET GAMES

Sergiu Hart
Stanford University

The idea of "marginal contribution" (or, "worth") is best captured in the game theoretic concept of value. Its relation to the usual economic equilibrium can be stated as the following Value Principle. In a perfectly competitive economy, every value allocation is competitive, and the two sets of allocations are identical if the economy is sufficiently differentiable. However, when modelling perfect competitiveness by a non-atomic space of agents, the (asymptotic) value may fail to exist in the general (non-differentiable) case. The purpose of this paper is to extend the existence of value to the class of market games, by adding a suitable requirement: "consistency" with the given "population measure". Furthermore, an explicit formula is obtained for the competitive price vector corresponding to this measure-based value; it can be interpreted as the expected equilibrium prices for the sub-economy formed by a random coalition (or, sample) of the original one.

SOME THEOREMS ON THE CORE OF A NON-SIDEPAYMENT GAME WITH A MEASURE SPACE OF PLAYERS

Tatsuro Ichiishi*
Carnegie-Mellon University

and

Shlomo Weber
Hebrew University

The present paper studies games without sidepayments in which an arbitrary positive σ -finite measure space of players is given. Several necessary and sufficient conditions for non-emptiness of the core, and also a sufficient condition, are obtained for a wide class of games.

MANAGEMENT VERSUS OWNERSHIP

Tatsuro Ichiishi
Carnegie-Mellon University

A model of a modern capitalistic economy is formulated, which emphasizes the separation between management and ownership of the firm. The firm is managed by its laborers with objective of maximization of laborers' utilities. Ownership (or equivalently, share holding) is determined by portfolio selection (which in turn is ultimately motivated by utility maximization). The formation of firms (hence the issue of shares) is endogenously determined. A concept of competitive equilibrium is proposed and its existence proved, which synthesizes the Nash equilibrium (non-cooperative concept) and the core (cooperative concept) within this economic context.

SOCIAL CHOICE ON PUBLIC, PRIVATE AND INTERMEDIATE GOODS

Ehud Kalai
Northwestern University

The existence of an Arrow type social welfare function on a restricted domain is investigated. The existing model is enlarged to include cases of private goods. It is shown that there exists an n -person social welfare function if and only if there exists a 2-person one for $n \geq 2$.

GAMES WITH STRATEGIC PREJUDGMENTS

Pierre Levine
Université de Paris VI

We introduce a model where the agents make interdependent decisions by means of two kinds of data. The first ones, called strategic prejudgments, are common knowledge and give information on the behavior of the agents. The second kind of data, the choice functions, are private information of the agents. They use them to make their decision in view of the information they have

collected with the help of the strategic prejudgments. In the first part, we define a concept of solution for this model called a game with strategic prejudgments and we give existence conditions for the solution.

In the second part, we use this model to analyse games in normal form with complete information. We discuss the relation between our solution and the concept of Nash equilibrium.

POWER AND NEGOTIATION

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and

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This paper is concerned with the formalization of the notion of power in the context of two person negotiations. The notion of power is defined as follows: the amount of power of A over B is related to the level of achievement of B's objectives in the interaction A-B. As for the origin of power, it is taken as related to the strategies available to both parties in their interaction (such as rewards, punishments, etc.).

The development of these ideas relies mainly on the concepts of game theory but specific factors drawn from the psychological and psychosociological literature, such as the notion of representation, are explicitly introduced in the model.

Besides the formalization, the paper includes a detailed discussion of two examples enhancing the role of representation in power analysis. An axiomatic treatment of the model is briefly reported in the appendix.

ON THE EXISTENCE OF STABLE SETS

William F. Lucas
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Several recent discoveries indicate some progress in verifying the conjecture that there exists n -person cooperative games in characteristic function form which have an empty core and no von Neumann-Morgenstern solution. A major step was to demonstrate that the union of all stable sets for a game need not be a connected set (nor empty).

On the other hand, recent results on the existence of stable sets for special classes of games, such as some symmetric games, provide some hope for obtaining general existence theorems for some such special cases.

CONWAY'S "NUMBERS AND GAMES"

John P. Mayberry
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In recent years, game theory has become less concerned with actual games: Conway's "On Numbers and Games" (Academic Press, 1976) almost denies connection with the "theory of games".

This talk attempts to whet the listener's appetite for the banquet of results in Conway's book. We present his definition of "games" (which include all two-player zero-sum games of perfect information - not only finite), and his definition of "numbers" (which are a subset of games).

Von Neumann's theory of sets, the transfinite ordinals, Dedekind cuts, impartial games and the Sprague-Grundy theory, and Knuth's surreal numbers - all are subsumed within Conway's theory, which also permits the analysis of many games, new and old. Parts of the book are devoted to the mathematical theory of impartial games, of which Nim is the prototype. Misère form of impartial games (i.e., when the last player to move loses) produces complications which must astonish anyone aware of the relative simplicity of Misère Nim.

We describe also the rules, and give examples, for

two of the games whose values are all numbers: we present a Hackenbush position which Black wins because he is ahead by exactly $5/64$ of a move, and a number $(0.29289\dots (2 + \sqrt{2})^{-1})$ which gives, in the game of Contorted Fractions, an advantage of exactly $1/5$ of a move. We share Conway's amazement that these statements have precise meanings.

COMPUTATIONAL COMPLEXITY IN SOME GAME THEORY PROBLEMS

Nimrod Megiddo
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The notion of worst-case asymptotic time-complexity is examined in some game theory problems. It is shown that finding the Shapley value or the nucleolus of a weighted-majority game is NP-hard. However, these solutions have low complexity in cost allocation problems over trees.

The complexity of approximating the Shapley value for a weighted-majority game is shown to depend on the measure of approximation. Percentage error requirements lead to exponential number of samplings in the direct Monte-Carlo method for the value.

An example of a combinatorial game, namely, a game of pursuit in a tree, is given and shown to be solvable in linear time.

It is suggested that complexity should be incorporated in the models. For instance, how should player 1 act in a two-person zero-sum game where player 2 faces computational complications and player 1 does not?

A SURVEY OF REPEATED GAMES I AND II

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and

Shamuel Zamir*
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A detailed survey of developments in the theory of repeated games in normal form including the zero-sum and general-sum cases, with emphasis on recently obtained major results, is presented. Some of these, as well as related references, appear in publications by the authors in Mathematics of Operations Research. [Important results on repeated games with incomplete information by Truman Bewley and Elon Kolhberg also appear in two papers in this journal.]

VALUES AND DERIVATIVES

Jean-Francoise Mertens
CORE, Louvain

A formula in terms of derivatives gives a value on a rather large space of games.

MANIPULATION OF VOTING SCHEMES

Hervé Moulin
Université de Paris, IX

A voting scheme is a game form that describes any strategic procedure to select one among a fixed set of alternatives. The well-known Gibbard-Satterthwaite theorem asserts that such a procedure is necessarily dictatorial as soon as at least three different alternatives can be effectively selected. Next the concept of a dominance-solvable voting scheme is presented as a weakening of the strategy-proofness requirement: it relies on successive elimination of dominated strategies and generalizes the well-known concept of "sophisticated voting".

Dominance-solvable voting schemes turn out to contain many usual voting procedures such as voting by veto, king-maker, and voting by binary choice.

The procedure of voting by elimination is proved to be a dominance solvable voting scheme always selecting an efficient alternative.

UNIQUENESS AND STABILITY OF NASH EQUILIBRIA

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and

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Using recent results on iterative solutions of non linear equations in several variables, we first obtain sufficient conditions for the uniqueness and global stability of the Nash equilibrium of a large class of normal form games. If u_i is player i 's payoff function and x_i his strategic variable, we assume that x_i is restricted on R_+ and that u_i is smooth. Our sufficient condition amounts to saying that the matrix

$$\begin{bmatrix} \frac{\partial^2 u_i}{\partial x_i \partial x_j} \end{bmatrix}_{i,j=1,\dots,n}$$

is a strictly dominant diagonal matrix and/or a M-matrix.

This result is then applied to the general quantity setting oligopoly model where it turns out that our condition generalizes the classical condition of McManus-Quandt, and others.

Next we consider the local stability of a Nash equilibrium that we define in terms of iterative eliminations of dominated strategies. This leads us to almost necessary and sufficient conditions for stability.

ON SYMMETRIC STABLE SETS

Shigeo Muto
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Stable sets and subsolutions are described mainly for symmetric n -person games $(n;k)$ in which k -person coalitions are strongly vital, i.e., $v(s) \leq \frac{s}{k} v(k)$ for $k \leq s \leq n-1$ and $v(s) = 0$ for all $s < k$, $k > 1$.

In the first part, two types (i.e., systematic and semi-symmetric) of stable sets are defined and their

existence are investigated. Furthermore symmetric stable sets are determined for some classes of $(n;k)$.

In the latter part, the production game defined by S. Hart, which is a kind of $(n;k)$, is taken into consideration and his open questions are studied.

Finally some results on subsolutions are presented.

Our main results are summarized as follows:

1. Existence of systematic stable sets and determination of symmetric stable sets for $(n;k)$ games with $v(k) \leq 2/(n-k+2)$.
2. Existence of semi-symmetric stable sets for
 - (i) $(n;2)$ games,
 - (ii) $(n;k)$ games ($n = qk+r$, $q \geq 2$ and $0 \leq r \leq k-1$) with one-point core, and
 - (iii) $(n;k)$ games ($n = 2k-1$) with one-point core.
3. (i) Determination of finite symmetric stable sets for $(n;k)$ games ($k \leq (n+1)/2$) with $v(k) \geq k/(n-k+1)$.
(ii) Uniqueness of such stable sets.
4. (i) Determination of symmetric stable sets for $(n;2)$, $(n;3)$, $(n;4)$ and $(n;5)$ games.
(ii) Their uniqueness for $(n;2)$ and $(n;3)$.
5. Uniqueness of Lucas' symmetric stable sets for $(n;n-1)$ games.
6. For Hart's production games ($n = qk+r$, $q \geq 2$ and $0 \leq r \leq k-1$), the following are obtained:
 - (i) Determination of symmetric stable sets for
 - a) $r = 0$ and $[(k+1)/2] \leq q \leq k-1$,
 - b) $r \geq 1$ and $[(k-r)/2] \leq q \leq k-(r+2)$,
 - c) $r \geq 1$ and $[(k-1)/(k+1)] \leq q \leq k-(r+2)$,
 - d) $r = 0$, $k = 2\ell+1$ ($\ell \geq 2$) and $q = \ell$, and
 - e) $r = 0$, $k = 6$ and $q = 2$.
 - (ii) Uniqueness of Hart's symmetric stable sets.
 - (iii) Existence of semi-symmetric stable sets.
7. For subsolutions, the following are obtained.
 - (i) Determination of finite symmetric subsolutions for $(n;2)$ games.
 - (ii) Determination of symmetric subsolutions for $(n;n-1)$ games which are smaller than the

symmetric stable sets defined by Lucas.

- (iii) Coincidence of the core with the essential standard for general (not necessarily $(n;k)$) symmetric games.

THREAT EQUILIBRIA AND FAIR SETTLEMENTS IN COOPERATIVE GAMES

Roger B. Myerson
Northwestern University

The role of threats is studied in cooperative normal form games. A threats-game is constructed, in which every set of players selects a joint threat strategy and then a settlement function determines the final outcome. Threat equilibria and cooperative solutions are defined for any settlement function. Two axioms are introduced which determine a unique settlement function for games with transferable utility. This settlement function is closely related to the Shapley value, and has attractive Pareto-optimality and individual-rationality properties. A simple oligopoly problem is studied to illustrate these ideas.

LINEARITY, CONCAVITY, AND SCALE INVARIANCE IN SOCIAL CHOICE FUNCTIONS

Roger B. Myerson
Northwestern University

Three theorems are derived about social choice functions, which are defined on comprehensive convex subsets of utility-allocation space. Theorem 1 asserts that a linearity condition, together with Pareto-optimality, implies that a social choice function must be utilitarian. Theorem 2 asserts that a concavity condition, together with Pareto-optimality and independence of irrelevant alternatives, implies that a social choice function must be either utilitarian or colinear, where colinearity is a property closely related to the maximin criterion. Theorem 3 asserts that only dictatorships can satisfy scale invariance and independence of irrelevant alternatives.

VALUES OF LARGE GAMES

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There exist an asymptotic value on $bv'NA$ which solves the open problem raised by Aumann and Shapley in their book, Values of Non-Atomic Games. The main tool in proving the result is a new renewal theorem for sampling without replacement. This renewal theorem is actually "equivalent" to the existence of an asymptotic value on the simplest single jump function.

In order to state the renewal theorem for sampling without replacement we need several notations. Let π be a finite set, λ a probability measure on π , $0 < x < 1$ and $a \in \pi$. Let $P(a,x)$ be the probability that in a random order of π , a is the first element (in the order) for which the λ -accumulated sum exceeds x , i.e., if for every order R of π and a in π , we denote $P_a^R = \{b: b \in \pi, bRa\}$ the set of elements of π preceding a in the order R , then

$$P(a,x) = |\{R: \lambda(P_a^R) \leq x \leq \lambda(P_a^R) + \lambda(a)\}| / (|\pi|!)$$

(where $|\pi|$ denotes the number of elements in π). The distance between the probability measure $P(\cdot, x)$ and $\lambda(\cdot)$ on π is defined by $\|P(\cdot, x) - \lambda(\cdot)\| =$

$\sum_{a \in \pi} |P(a,x) - \lambda(a)|$. The renewal theorem for sampling without replacement asserts that $\|P(\cdot, x) - \lambda(\cdot)\|$ tends to 0 as $\max_{a \in \pi} \lambda(a)$ tends to 0. In fact we prove a

stronger result: For every $\epsilon > 0$ there exists $\delta > 0$ and $K > 0$ such that if $\rho = \max_{a \in \pi} \lambda(a) < \delta$, and $K \cdot \rho < x < 1 - K \cdot \rho$ then $\|P(\cdot, x) - \lambda(\cdot)\| < \epsilon$.

The existence of an asymptotic value on $bv'NA$ enables us to prove the existence of the asymptotic value on many other spaces of interest. There exist an asymptotic value on the algebra A generated by the continuous games in $bv'NA$, and even on the space $A^*va'NA$, the space generated by games of the form $u \cdot v$, u in A and v in $bv'NA$.

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THE PARTITION VALUE

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and

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In their book, Values of Non-Atomic Games, R. J. Aumann and L. S. Shapley extended the concept of value to certain classes of Non Atomic Games. A natural interest arises in values which are limits, in an appropriate sense, of Shapley values of finite games that approximate non-atomic games. Indeed, the asymptotic value stems from such an approach. Roughly speaking, the asymptotic value is defined on each game v for which all the sequences of Shapley values, corresponding to sequences of finite games that approximate v , have the same limit.

The asymptotic value has desirable properties like continuity diagonality and uniqueness; unfortunately, however, the asymptotic value does not exist for many economically important games.

In this paper we suggest a different approach, which yields a new kind of value: the partition value. Like the asymptotic value the partition value is defined by means of a limiting process of finite games. But in contrast to the asymptotic value, the sort of limit used for the partition value is so weak that any other conceivable constructive approach to the value concept,

by means of limits of finite games, will necessarily yield a value which is also a partition value.

Definition: Let Q be a symmetric subspace of BV . A value ϕ on Q is a partition value if for each v in Q and for each coalition S there exists an admissible sequence of partitions $(\pi_n)_{n=1}^{\infty}$ such that

$$\pi_n \succ \{S, I-S\}$$

$$\lim_{n \rightarrow \infty} (\psi_{v, \pi_n})(\bar{S}) = (\psi v)(S)$$

where ψ is the Shapley value for finite games.

To state the results we use the following notations: A is the algebra spanned by games of the form $f \circ \mu$, where f is a continuous function in bv and μ is a probability measure on NA .

$A^*bv'NA$ is the linear symmetric subspace spanned by $A \cdot bv'NA$.

The spaces $bv'NA^*bv'NA$ and $A^*bv'NA^*bv'NA$ are defined the same as $A^*bv'NA$.

The results are:

- 1) Every partition value ϕ is continuous in the variation norm, and $\|\phi\| \leq 1$.
- 2) Every partition value is a diagonal value.
- 3) If ϕ is a partition value on a symmetric subspace A of BV , then ϕ has a unique extension to a partition value ϕ on the space generated by $Q \cup ASYMP$.

It turns out that the partition value is a useful tool in proving the existence of values.

Main Theorem: There exists a partition value on each of the following spaces: $ASYMP$, $bv'NA$, A , $A^*bv'NA$, $bv'NA^*bv'NA$, $A^*bv'NA^*bv'NA$. Moreover there exists a partition value on the space W generated by all the above mentioned spaces. (The existence of values on the spaces A , $A^*bv'NA$, $bv'NA$, $A^*bv'NA^*bv'NA$ and W was not previously known.)

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VALUES OF GAMES WITH A PRIORI UNIONS

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A modification of the Shapley value is suggested, to take into account the possibility that, due to affinities, agreements, etc., certain sets of players (unions) will be negotiating as blocs. Both heuristic and axiomatic formulations give rise to the same modified value. Extensions are discussed, both to the case where the a priori unions are given only in probability, and to the case where there may be some hierarchy of structure within the given unions. Some examples are treated in detail.

ON STOCHASTIC GAMES WHERE JUST ONE PLAYER CONTROLS THE TRANSITIONS

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and

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A stochastic game is roughly described as follows. Everyday players I and II play one of the matrix games A_1, A_2, \dots, A_S . If they play the game A_B today the rules will specify which matrix game will be played tomorrow and this rule will depend only on the matrix played today and the choices of the row and column chosen by player I and player II respectively. Player I accumulates the immediate payoffs discounted at a discount rate β . Here $0 \leq \beta \leq 1$. The aim of player I is to maximize the total discounted reward over an infinite period. Player II wants to minimize the same. A second payoff is the \liminf of the average payoff per day. The following is the main theorem.

Theorem: If the rules of movement from one payoff matrix to another depend only on the current state and the choice of say player II then the value exists in stationary strategies (i.e. mixed strategies at each matrix that are used all the time no matter when and how the matrix is reached). A pair of good stationary strategies can be computed by linear programming for the discounted case and hence the data defining the stochastic game and the entries defining a solution to the stochastic game (such as value for each initial state, the coordinates of stationary good strategies) all lie in the same ordered subfield of the real field containing the entries of the data. There exists a good stationary strategy for player II which remains good for all β sufficiently near 1. For the cesaro average payoff the value exists and the data and entries of a solution lie in the same ordered field. These assertions hold also for two-person non-zero sum stochastic games on discounted and cesaro average payoffs. If both players control the law of motion the theorem is not true in general. The following is a counter example. In payoff matrix A , if row 1 and column 1 or row 2 and column 2 are chosen by the players the next payoff matrix is still A . Otherwise it is A_2 . Here A_2 is absorbing with just the payoff 0. The following are the payoffs.

$$A_1 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \quad A_2 = [0] .$$

Then the value of the game if we start at A_1 is $(-4 + 2\sqrt{13})/3$. Trivially it is 0 if we start at A_2 .

REPRESENTATIONS OF SIMPLE GAMES BY SOCIAL CHOICE FUNCTIONS

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We investigate possible constructions of choice procedures (social choice functions) for committees (simple games). The notion of a capacity of a committee is derived from our construction. We determine the capacity of strong, symmetric and weak simple games. We also provide an upper bound on the capacity of a simple game without veto players.

A SUPERADDITIVE SOLUTION TO NASH BARGAINING GAMES

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Let K be the class of nonempty, compact, convex and comprehensive domains in R_+^2 , which are regarded as Nash bargaining games with $(0,0)$ being a common conflict payoff. Let K_0 be a subclass consisting of those domains whose weak Pareto optimum coincides with the strong Pareto optimum. A superadditive solution is a function $\bar{u}: K \rightarrow R_+^2$ satisfying the following axioms:

- A.1 Strong Pareto optimality.
- A.2 Invariance under change of utility scale.
- A.3 Symmetry.
- A.4 Superadditivity, i.e., for all T, S in K ,

$$\bar{u}^T + \bar{u}^S \leq \bar{u}^{T+S} .$$
- A.5 Continuity over K_0 .

Theorem. There exists a superadditive solution over K and it is unique.

If S is in K , \bar{u}^S can be defined by the following

equation:

$$(1) \int_p^{\bar{u}^S} \sqrt{-du_1 du_2} = \int_{\bar{u}^S}^q \sqrt{-du_1 du_2}, \bar{u}^S \in \text{Pareto optimum of } S,$$

where the integrals are taken along the weak Pareto frontier, ∂S , $p = \partial S \cap u_2$ axis, $q = \partial S \cap u_1$ axis.

For S in K_0 , one can define a status quo line $u(t)$ which connects the origin to the agreement point \bar{u}^S as follows: Denote by t^S the value of the integrals in (1). For t in $[0, t^S]$ define $u_1(t)$ and $u_2(t)$ by the relations

$$(2) \int_{u_2(t)}^{p^S} \sqrt{\frac{-du_1}{du_2}} du_2 = t = \int_{u_1(t)}^{q^S} \sqrt{\frac{-du_2}{du_1}} du_1$$

(an obvious extension holds for domains in K/K_0). The slope of $u(t)$ turns out to be the geometric mean of the slopes of ∂S at points $A(t)$ and $B(t)$, where $A_1(t) = u_1(t)$ and $B_2(t) = u_2(t)$. This procedure gives rise to a theory for variable treat games; however, unlike the Nash's case, the resulting game need not behave as a constant sum game.

The axioms are inconsistent if we allow S to contain payments which yield a player less than his own conflict payoff.

A number of open problems arise - of which the most important one concerns possible generalization to higher dimensions. Possibilities of extensions are discussed such as dropping the comprehensibility requirement and giving up the continuity axiom.

ZERO-SUM GAMES WITH INCOMPLETE INFORMATION

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and

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The class of games under consideration is the following:

- (i) Let G be a finite two-person game tree with its rules (sequence of moves and information sets), and
- (ii) Let M_H be the zero-sum payoff associated with a play H of G , M_H is a discrete random variable: $\text{Prob}(M_H = m_{r,s}^H) = p_r q_s$, both players know the probability distributions $p \in P$ and $q \in Q$, moreover player A knows r and player B knows s from the very beginning of G .

General properties on value and optimal strategies as functions of p and q can be derived from the linear programs associated with games in this class. In particular the value is a convex concave function which is piecewise bilinear (there exists two finite polyhedra partitions of P and Q , $\{P_e\}$, $\{Q_k\}$ such that the value is bilinear on each $P_e \times Q_k$).

Properties on the value apply directly to any finitely repeated game, the value of which was already known to be concave-convex. However, the piecewise bilinearity does not carry over to the case of infinitely repeated games.

If G is a game tree with perfect information, it was known that the value, $v(p,q)$, can be obtained directly through a recursive method operating on partial histories h of the game. If there is lack of information on one side only, we show that the optimal behavioral strategies of the informed and of the non informed players can be computed explicitly using $v(p,q)$ and the respective $v^h(p,q)$. They may be described respectively by a sequence of conditional probabilities in P and a sequence of security level hyperplanes ($w = (w_r)$ $r \in R$).

GAME FORMS AND NASH EQUILIBRIA

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University of Illinois

The problem considered involves an economic planner who wants to design an economic allocation process. He must describe the messages the agents can send and the outcome which will arise for each set of joint messages. The concept of Nash equilibrium is used to capture the notion of possible manipulation by the agents. The problem is then to devise a "game form" as described above such that the Nash equilibria will be "desirable" (e.g., Pareto optimal) regardless of the agents' preferences.

NONATOMIC LINEAR PRODUCTION GAMES

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Linear Production Games for finitely many players were introduced by Owen (Math. Prog., 1975). There, he investigated the cores of such games and the relationship between the core and the set of competitive prices. The theory is extended here to the non-atomic case. We show how the results can also be extended and strengthened. In particular, we show that in the non-atomic case the core contains only those measures generated by shadow prices.

Linear Production Games are generated with linear programs and their duals. In case the solution to the dual program is unique, we show that, in the non-atomic case, it generates the only member of the core, which is also the value of the game.

During the development, a number of auxiliary results are obtained which are of interest by themselves. These are results about the properties of any measure in the core of a vector-measure game that is positively homogeneous of degree 1. These results may help resolve some open problems about the relationships between cores and values of general non-atomic games.

The talk is concluded with a few examples that show the different relationships between cores and values of Linear Production Games in the finite and the non-atomic case.

"NO GAP" FOR THE 4-PERSON VETO-POWER GAME

Mohamed Rabie
Cornell University

Consider the class of simple games $G(N,v)$ where for any $S \subset N$

$$\begin{aligned} v(S) &= 1 && \text{if } n \in S, |S| \geq n-1, \text{ and} \\ v(S) &= 0 && \text{otherwise.} \end{aligned}$$

It has not been known whether or not this game has a solution which leaves a "gap" in the x_n direction, i.e., whether there is any solution K and value a ($0 \leq a \leq 1$) such that

$$K \cap \{x \in A \mid x_n = a\} = \emptyset.$$

In case of $n = 3$ this game does not have a "gap", since any solution for this game consists of a continuous curve from the point core, $(0,0,1)$, to a point on the edge $x_3 = 0$.

It is shown here that for $n = 4$ this game also does not have a "gap". To prove this result, first assume there is a solution K with a "gap". From this step one can find a "cycle"; i.e., $(x,y,z \in K$ with $x_4 = y_4 = z_4$, $x_1 > y_1$, $x_2 > y_2$ or $y_2 > z_2$, $y_3 > z_3$ and $z_3 > x_3$, $z_1 > x_1$). Second if there is a cycle in K one can find another cycle "above" it, and from this supposition one can arrive eventually at a contradiction.

OVERLAPPING COALITIONS - THE THREE-PERSON GAME CASE

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In social life, situations abound in which a person is simultaneously a member of different social units. This situation is modelled as follows:

Given a three-person cooperative game without side-payments in normal form with the special feature that

only players $\{1,2\}$ and $\{2,3\}$ can communicate, whereas the communication between players 1 and 3 is impossible. Player 2 is called the pivot-player. We assume for the first step, that player 1 knows only the second player's and his own payoff but not those of the third player, believing to be involved together with player 2 in a game against Nature. A similar assumption is made about player 3's state of mind.

Then coalitions $\{1,2\}$ and $\{2,3\}$ form, playing a joint mixed centroid maximin strategy if by this both players will get quickly more than the amount of their individual security level. Socially imposed rules regulate who determines the random device and who connects its signals to the different pure-strategy components of the chosen joint mixed strategy. If the pivot-player obtains with a positive probability this role in both coalitions $\{1,2\}$ and $\{2,3\}$, he is in the position to choose the same random device for both coalitions and to relate its signals to the pure strategy vectors in such a way as to increase his expected value.

A sufficient, but not necessary condition, is given for the case, that this can be done without inconsistent strategy choices for the pivot-player stemming from his two different alliances using Aumann's notion of c-acceptability for cooperative games without side payments.

In many cases, another type of optimization of the pivot-player's expected value is "break-optimal signal-linkage" in which he connects the first and the third player's pure strategies (according to the given probability distribution) but in such a way as to maximize his own expected value by breaking the pure-strategy obligations to him by the signal-linkage. An algorithm for achieving this task is given. The vector of expected values for the three players then can be computed.

Let us now change part of our initial conditions. Thus, the players 1 and 2 may now know the entire payoff structure, as well as the communication network

of the game. They now may be suspicious that the pivot-player may exploit his central position. One or both of these players may then counteract, by anticipating the way in which the pivot-player establishes a signal-linkage as well as breaks his obligations. It can be shown that such a counteraction is advantageous for a player only if the correlation between his and the pivot-player's payoff is highly negative. It may well be to a player's benefit to stick with the given strategy allocations, still knowing that the pivot-player may break some of his obligations. This is the case, as is conjectured, if a measure - a correlation coefficient with weighted entries - increases from its original value, if a break-optimal signal-linkage for player 2 is established.

The establishment of a signal-linkage can be thought of as modelling the situation, encountered very often in social life, in which a person interacting with two contrasting social units puts a semantic interpretation on the events coming up from the outside world in such a way as to reconcile differing views and enabling the two units to react jointly on them.

VALUES OF NON-SIDEPAYMENT GAMES, LOCATION CONFLICTS AND PUBLIC GOODS

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A location conflict is represented by a planning area (a convex closed subset of R^n) and a system of n utility functions for players 1, ..., n such that each utility function is, say, concave, differentiable, and has a single maximizing point ("bliss point" of player i). Applications may be seen in a pure geometrical set-up or either an economy with public goods.

Game theoretically a location conflict leads to a pure bargaining game without sidepayments; however, the threat point is replaced by a bliss point and hence Nash's bargaining solution or other concepts do not

apply immediately.

Several concepts of values for non-sidepayment games with bliss point character are being discussed. There is also a natural "extension procedure" (a la Harsanyi), which allows to discuss values also in the case that "pure bargaining" is not given and thus medium sized coalitions do have some influence.

Given an economy with public goods location conflicts arise in a natural way (c.f. Zeckhauser - Weinstein). If a value concept is selected, then as a consequence, it is possible to define the notion of a corresponding equilibrium. This equilibrium may or may not depend on a prescribed subset of prices for public goods (for instance "equal taxation" seems to be feasible). If the full set of Lindahl-prices is admitted, then the value attached equilibrium boils down to the Lindahl-equilibrium.

SEQUENCES OF GAMES WITH VARYING OPPONENTS

R. W. Rosenthal

Bell Telephone Laboratories

This paper considers a problem faced by players who are involved in a sequence of games: not necessarily the same games, not necessarily with the same opponents, and not necessarily under conditions of complete information. The players are assumed to act in response to stationary Markovian hypotheses which they form about the actions of their opponents. Conditions are explored which require that these hypotheses be correct on average and that the players' actions be optimal in response to their hypotheses.

THE λ -TRANSFER VALUE: SOME DIFFICULTIES

Alvin E. Roth
University of Illinois, Urbana

The λ -transfer value for a game G without transferable utility is obtained by considering a related game g in which utility is transferable among the players at an exchange rate given by a vector λ . The game g has a larger set of feasible outcomes than the game G , but if the Shapley value of g is feasible in G then it is the λ -transfer value for G .

Although the Shapley value for games with transferable utility is often interpreted as an expected value of the game, it seems difficult to carry over such an interpretation to the λ -transfer value. For example, let $p \in [0, \frac{1}{2})$ and consider the 3-player game $G(p)$ without transferable utility, whose characteristic function is

$$\begin{aligned} V_p(i) &= \{(u_1, u_2, u_3) \mid u_1 \leq 0\} \quad \text{for } i = 1, 2, 3 \\ V_p(12) &= \{(u_1, u_2, u_3) \mid (u_1, u_2) \leq (\frac{1}{2}, \frac{1}{2})\} \\ V_p(13) &= \{(u_1, u_2, u_3) \mid (u_1, u_3) \leq (p, 1-p)\} \\ V_p(23) &= \{(u_1, u_2, u_3) \mid (u_2, u_3) \leq (p, 1-p)\}, \text{ and} \\ V_p(123) &= \{u \mid u \leq y \text{ for some } y \text{ in the convex hull} \\ &\quad \text{of } \{(\frac{1}{2}, \frac{1}{2}, 0), (p, 0, 1-p), (0, p, 1-p)\}\}. \end{aligned}$$

Then $G(p)$ is the game in which players 1 and 2 together may obtain the outcome $x = (\frac{1}{2}, \frac{1}{2}, 0)$, players 1 and 3 may obtain $y = (p, 0, 1-p)$, players 2 and 3 may obtain $z = (0, p, 1-p)$, and the three players together can secure any convex combination of x , y , and z . This game exhibits a property which never occurs in games with transferable utility: players 1 and 2 can simultaneously obtain their maximum feasible utility payoffs, at the outcome $x = (\frac{1}{2}, \frac{1}{2}, 0)$. Since p is less than $\frac{1}{2}$, players 1 and 2 both strictly prefer x to any outcome which gives positive weight to y or z . In the game $G(0)$ this is particularly clear: since $p = 0$, cooperation with player 3 offers no gains to either player 1 or player 2. It therefore seems difficult

to defend any outcome besides x as resulting from a play of the game by utility-maximizing players.

Nevertheless, the components of x , y , or z sum to 1, so when utility is transferable (for $\lambda = (1,1,1)$) the corresponding game g is the symmetric 3-player majority game whose Shapley value is $u = (1/3, 1/3, 1/3)$. Since u is in the convex hull of x, y , and z , it is a feasible outcome in the game $G(p)$, and consequently u is the λ -transfer value for $G(p)$.

A DIVISION GAME

William H. Ruckle
Clemson University

Fifty indivisible units are to be divided among N players according to the following scheme:

- (a) The players are ranked $N, N-1, \dots, 3, 2, 1$.
- (b) Player N proposes a division among the N participants.
- (c) If a majority of the N players accept the proposal it carries. (Majority = $50\% + 1$)
- (d) If N 's proposal does not carry, N must drop out and the game continues with players $N-1, N-2, \dots, 3, 2, 1$ according to the same rules.

Under the assumption that each player wishes to maximize his share, a unique solution to this game exists for $N = 1, 2, 3, 4, 5, 6$. For $N = 7$ there are two solutions, and thereafter the possibility of several solutions occurs at each stage. As an experiment groups of four to six students were involved in enactments of this game to determine how fast they could learn to obtain a maximum share.

SEARCH AND AMBUSH PROBLEMS AS GEOMETRIC GAMES

William H. Ruckle
Clemson University

This is a survey of results and problems treated in the following papers:

1. Ambushing Random Walks I: Finite Models, *Opr. Res.* 24(1976) p. 314.

2. Ambushing Random Walks II: Continuous Models, Clemson University Tech. Report 235, to appear in Oprs. Res.
3. Ambushing Random Walks III: More Continuous Models, Clemson University Tech. Report 284.
4. Some Examples of Geometric Games, Clemson University Tech. Report 273.

Some of the games described are: (1) A hunter trying to shoot a bird while the bird flies over a field. (2) The game of the intersection of subintervals of an interval in which the payoff to one player is the length of the intersection. (3) A finite ambush game. (4) A transit game with a gap. (5) Pursuit on a complete lattice. Two open problems described are: (1) Pursuit on a cyclic lattice. (2) The game in which BLUE chooses a convex subset B having a given shape within a convex set A and RED chooses a line ℓ through A . The payoff to BLUE is 1 if $A \cap \ell = \emptyset$ and 0 otherwise.

WALRASIAN ANALYSIS VIA STRATEGIC OUTCOME FUNCTIONS

David Schmeidler
Tel Aviv University

The main purpose of this note is to construct a game in strategic form whose Nash equilibria are strong and coincide with the Walras equilibria of the underlying Arrow-Debreu pure exchange economy. The game's definition is independent of the economy and applies to all economies with neoclassical agents' characteristics. The informational decentralization aspects of such constructions and other modelling problems are discussed.

APPLICATIONS OF A NON-COOPERATIVE SOLUTION
CONCEPT FOR EXTENSIVE GAMES, I AND II

Reinhard Selten*
University of Bielefeld

and

Werner Güth*
University of Köln

A non-cooperative solution concept developed by John C. Harsanyi and Reinhard Selten permits the selection of a unique equilibrium point out of many equilibrium points. The concept is applied to perturbed agent normal forms of extensive games, in order to achieve perfectness by going to the limit. The concept applies Harsanyi's tracing procedure to a prior distribution, whose inductive definition considers the structural properties of the game.

Two economic game models have been selected for the application of the solution concept. The first model describes an oligopoly situation, where three firms make simultaneous entry decisions. Those firms, which enter the market, then play a Cournot-oligopoly game with linear demand and proportional costs. The three firms have different costs of entry. There are three pure strategy equilibrium points, each of them with two firms in the market and one outside. The solution concept selects the one, where the two firms with the lower entry costs enter the market; a result, which is in agreement with intuitive expectation.

The second model describes a bargaining situation, where one firm A has developed a new product and may either offer production rights or selling rights only to two firms B_1 and B_2 ; the same contract must be offered to both B-players. B_2 has higher opportunity costs than B_1 . This situation is compared with another one, where B_1 and B_2 are merged with one player B. Does unity give strength?

The comparison of a decentralized and a centralized bargaining game permits the investigation of this question. It turns out that centralization is

advantageous for some combination of parameter values and disadvantageous for others. The disadvantages of centralization are by no means counterintuitive. They are connected to relatively high opportunity costs differences, which give B_2 a high incentive to insist on the more profitable contract. This reduces player B_1 's risk of behaving in the same way.

The applications show that the solution concept is a promising tool for the analysis of economic game models.

POLITICAL GAMES - A NONSYMMETRIC POWER INDEX

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The Rand Corporation

The symmetric Shapley-Shubik power index assumes that the voters are aligned by the various issues that may confront them, with each possible alignment being equally probable. "Power" is then defined to be the probability of pivoting under the given voting rule (simple game) (N, w) . In the present model, the voters $i \in N$ are differentiated by their "ideological characteristics", represented as points $x^i \in R^m$, and their "political powers" are their probabilities of pivoting when they are ordered by the inner products (ξ, x^i) , where $\xi \in R^m$ is chosen according to the uniform distribution on the unit sphere $\|\xi\| = 1$. One may interpret the vector ξ as the direction or political tendency associated with the issues that are to be decided by the voters.

In the non-atomic version of the model, there is a measure space (E, C, μ) of voters and an ideological mapping $\alpha: E \rightarrow R^m$. For any $\xi \in R^m$, $\|\xi\| = 1$, the corresponding voter-alignment is given by $A_\xi(i) = (\xi, \alpha(i))$, $i \in E$, and there is an associated pivot level $h_0 = h_0(\xi)$, defined as $\sup \{h: A_\xi^{-1}([h, \infty))$ is a winning coalition\}. In order to allocate the "pivoting credit" among the voters i for whom

$A_\xi(i) = h_0$, we define for each $S \in C$ and $\delta \in R$ an " S, δ -shifted" alignment

$$A_\xi(S, \delta; i) = \begin{cases} A_\xi(i) + \delta & \text{if } i \in S \\ A_\xi(i) & \text{if } i \notin S. \end{cases}$$

The derivative $dh_0(S, \delta)/dS$, evaluated at $\delta = 0$, provides the requisite allocation of "pivoting credit". Under suitable conditions of nonatomicity, measurability, and continuity, this derivative will exist for almost all $\xi \in R^m$ and we may integrate over $\|\xi\| = 1$ to obtain the desired political power index, which will be a measure δ on (E, C) . For the power density or "power per capita", we may divide this by the population measure μ .

ON COMMITTEE DECISION MAKING:
A GAME-THEORETICAL APPROACH

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In this paper, we study the committee decision making process using game theory. The committee is modeled as a game. A new solution concept called the one-core is introduced and studied. The concept of a bargaining set, first introduced by Aumann and Maschler in the context of games with side payments, is defined with appropriate modifications for committee games. These solution concepts are compared with two other well-known solution concepts - the core and the Condorcet solution.

A POSSIBILITY THEOREM FOR TOTALLY ACCEPTABLE GAME FORMS

Chal Sussangkarn
University of California, Berkeley

A game form is simply a function that maps players' strategies into outcomes. A result of Gibbard and Satterwaite says that if the number of outcomes is at least three, there is no non-dictatorial game form for which each player has a dominant strategy for any preference profile. Recently, attention has turned towards acceptable game forms, i.e., those which possess a Nash-equilibrium for every profile and every Nash-equilibrium is weakly Pareto-optimal. If the number of players is one or two, then it is known that every acceptable game form is dictatorial. However, non-dictatorial acceptable game forms exist for three or more players. A major problem is that in general a game form will possess many Nash-equilibria and there is no guarantee that some Nash-equilibrium will result when the game is played. The reason is because players may focus on different Nash-equilibria and choose their strategies accordingly. Only if all Nash-equilibria are interchangeable can one be sure that a Nash-equilibrium will result. Thus, the question arises whether there are any acceptable game forms in which all Nash-equilibria are interchangeable. Unfortunately, it is shown that all such game forms must be dictatorial.

DECISION MAKING WITH MULTIPLE OBJECTIVES

AND BENEFIT/COST ANALYSIS

Robert M. Thrall
Rice University

David Cardus
Texas Institute of Rehabilitation and Research

and

Charles Hammons
Rice University

It is clearly in the interest of game theory and of its disciples to have it more used by decision makers (or managers). Many major issues involve the resolution of

conflicting interests and the successful manager must carefully account for and balance them.

Game theorists as well as other model builders often complain that they cannot get decision makers to use their models, and decision makers complain that the models given to them neglect some of the important factors. One possible explanation is that, whereas management decisions are conditioned by factors which do not have "hard data" measures, in academia (where most model builders reside) "soft data" is held in such low repute that it tends to be neglected. The methodology of soft data generation has become significant albeit no means as complete or well established as that for hard data. This methodology includes (a) hypothetical lotteries, (b) Delphi type procedures, (c) factor analysis, and (d) cluster analysis.

Much has been written about social choice functions and we must live with the consequences of a number of impossibility theorems of which the Arrow Paradox is perhaps the best known. There is a wide range of thought as to the possibility, feasibility, or even desirability of setting up a utility function to measure benefits of some proposed course of action.

An important barrier to measuring in addition to individual reluctance to make certain comparative value judgements, another barrier to measuring benefits is the presence of deep lying differences of viewpoints as to the relative importance of various benefit factors. For example, a researcher has a natural inclination to place a high value on a scientific achievement, a controller has a similar inclination to regard direct monetary benefits and costs as primary, and an arthritic is likely to focus on relief of pain and restoration of mobility.

The presence of these differences in viewpoint gives the situation some of the features of game theory and raise the possibility that game theory may have a useful role to play. Another consequence of these differences is that it seems unlikely that there can be any generally

accepted method for assigning a single number to measure benefits.

But what if, instead of demanding a model which yields a single number, we ask rather for a multidimensional utility? With careful selection of the individual component dimensions, it may be possible to get reasonable agreement on a measurement vector. For example, our researcher, controller, and arthritic might agree on a monetary measurement for purely economic factors, on a measure of merit for scientific achievement, and also on a measure of pain relief. The agreement on these individual dimensions in no way impinges on the right or the likelihood of disagreement on the relative values of a unit increase in each of the three dimensions. The number of basic dimensions to be used should be governed by the following principles: (1) inability to obtain reasonable agreement on values in a proposed basic dimension may indicate a need to subdivide it into two or more parts and (2) the smallest number of dimensions which will permit reasonable componentwise agreement may serve to encourage the use of soft data and thus may help establish bridges between model builders and decision makers.

In principle few will deny that a decision to embark on some course of action should be made only if the total expected benefits exceed the total expected costs. Thus some form of benefit/cost model is very attractive provided an acceptable utility measure is available. However, there are quite a few variants of benefit/cost models. The benefit/cost difference is appropriate for unconstrained decisions whereas the benefit/cost ratio is preferred in the presence of cost or other constraints. There is a considerable confusion in terminology as well as genuine differences in features of benefit/cost models.

Working in the context of a multiobjective, multicriteria, benefit/cost ratio model, one of the authors (R.M. Thrall, "Benefit/Cost Estimation, Alternatives, Requirements, Advantages and Disadvantages," pp. 25-35,

in Computer Applications in Health Care Delivery, 1976, Symposia Specialists, Miami) proposed a five stage process: "(1) identification of objectives, (2) measurement of level of achievement of each objective, (3) scaling of each objective in a multidimensional value space, (4) determination of relative weights for the several value dimensions by the relevant classes of evaluators, and (5) synthesis of the outputs of the earlier stages by the responsible decision maker."

Each of the first four stages involves substantial work by the model builder; the fifth stage belongs primarily to the responsible decision maker. Despite this apparent separation of function, the model builder should strive throughout for substantial interaction with the decision maker and his staff. Perhaps game theory can be a useful contributor to this interaction and to the decision maker in his attempt to reconcile or compromise between conflicting evaluations of various criteria.

This paper is to appear in a forthcoming book Decision Information (Academic Press).

A THEORY OF SOLUTIONS FOR N-PERSON COOPERATIVE GAMES

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University of New Orleans

In an attempt to answer fundamental questions that cannot be answered satisfactorily within the present state of the theory of games, a new technique for game theoretical analysis is introduced under the name of "Utility Transfer Analysis". This technique which is based on the so called theorems of the alternatives (like the Farkas Lemma), and also on an extension of the concept of balanced sets of L. S. Shapley and O. N. Bondareva, allows us to characterize sets of coalitions and payoff vectors in a state of relative equilibrium.

A theory of solutions based on the Utility Transfer Analysis technique is developed, and all the computational and theoretical power of linear programming is made available when the solutions of the game are equivalently

defined by means of associated linear programming problems.

The solutions defined, always exist, are invariant under strategic equivalence, and are computationally tractable.

STRICT DETERMINACY IMPLIES ALMOST COMPLETE INFORMATION IN FINITE GAMES

Eugen N. Vasilescu
Baruch College, CUNY

It is established that a finite game is strictly determined for a player i iff in its complete inflation player i has complete information about the other players. This settles a conjecture of Birch. In the process of proving this result a class of completely mixed games is introduced.

PROBABILISTIC VALUES FOR GAMES

Robert James Weber
Yale University

The Shapley value is characterized by axioms concerning linearity, symmetry, efficiency, and the treatment of dummy players. We consider values, for games on a fixed finite set of players, which satisfy various subsets of these axioms. In particular, the linearity and dummy axioms, in conjunction with a monotonicity axiom, characterize the family of "probabilistic" values; the addition of the efficiency axiom yields the family of "random-order" values. We also discuss the effect of restricting our consideration solely to super-additive games, or to simple games. Our approach yields a new, and remarkably simple, derivation of the Shapley value formula.

If we broaden the context of our investigation to finite-carrier games in an infinite universe of players, or to non-atomic games, we encounter the family of "semivalues". Results concerning these values have been recently obtained by various subsets of {Pradeep Dubey, Abraham Neyman, Robert Weber}.

A two-person noncooperative resource-allocation game can be constructed "on top of" a characteristic-function game. In joint work with Martin Shubik, we have found that certain elemental semivalues arise as the equilibrium allocations in the noncooperative game. In particular, the Banzhaf value can be obtained in this manner.

COVERS AND EXTENSIONS OF GAMES

Robert James Weber
Yale University

A cover of a characteristic-function game v with player set N is another game w on N which satisfies $w(S) \geq v(S)$ for all $S \subset N$. A number of methods have been proposed for constructing covers; these yield the superadditive cover, the balanced cover, the exact cover, and a variety of constant-sum covers. A motivating principle is that, if a game and its cover assign the same "worth" to the grand coalition N , then the games behave similarly with respect to certain solution concepts.

An extension of a game v on N is a function $f: [0,1]^n \rightarrow R$ which satisfies $f(1^S) = v(S)$ for all $S \subset N$. Owen's multilinear extension is well-known, and is a useful tool for work concerning values of games. The balanced extension of v is defined by

$$f(x_1, \dots, x_n) = \max [\gamma_S v(S),$$

where the maximization is over all $[\gamma_S: S \subset N]$ such that $\gamma_S \geq 0$ for all $S \subset N$, $\sum \gamma_S = 1$, and $\sum_{S \ni i} \gamma_S = x_i$ for all $i \in N$.

The balanced extension is useful in work dealing with the core of a game, and with other domination-related concepts. It provides a direct geometric generalization of a number of Shapley's results concerning the "line-graph" of a symmetric game. (Aubin's "fuzzy 'extension'" of a game--which appears in Schmeidler's "Cores of Exact Games"--is actually the balanced extension of the balanced cover.)

NAIVE SECOND ORDER GAME

Po Lung Yu
University of Kansas

The paradigm of decision dynamics is used to describe the decision dynamics involving more than one decision maker. The framework supplied in this note is different from traditional game theory or differential games. Traditional simplicity assumptions are replaced by a more complicated, but more realistic, setting. Although much mathematically beautiful results in the traditional game theory or differential games have disappeared in the second order games, the more realistic setting of the latter does make it easier for the decision makers to find a "good" decision. Concepts of time-optimality and time-stability, and their necessary and/or sufficient conditions are described. Unconventional concepts of strategies and uncertainty involved in gaming phenomena are discussed. A highlight of the article is a systematic discussion on reframing tactics of gaming situations, which do not exist in the context of traditional game theory or differential games. Various research topics are discussed at the end of the paper.

PARETO OPTIMALITY OF COMPETITIVE EQUILIBRIA IN AN
INFINITE HORIZON MONETARY ECONOMIES: SURVEY OF
SOME RECENT DEVELOPMENTS

Itzhak Zilcha
Cornell University

The model we consider is a general consumption-loan type (see Samuelson [1]). There are infinitely many generations. The number of periods traders live is finite, the number of consumers in each generation is arbitrary but finite, the number of commodities is any finite constant. There is no assumption of stationarity, namely endowments and utilities may change over time. We assume that all transactions are costless, while we do not allow trade between consumers at certain date unless both are alive in that period.

As Samuelson noticed, since there is no central market as in the usual finite economy, some type of financial intermediation like money is necessary to establish Pareto optimality of a competitive equilibrium. We assume that there is fiat money in the economy "contrived" by the first generation. Money carries no intrinsic utility. An equilibrium is monetary if the price of money at this equilibrium is positive, otherwise it is barter. Consider the range of competitive equilibria with money. The conclusions of Samuelson's stationary case (i.e., when all traders have the same preferences and endowments) are:

- a) There is a monetary equilibrium if and only if there is no barter equilibrium which is Pareto optimal.
- b) If there is a monetary equilibrium, then there is also a monetary equilibrium which is Pareto optimal.

It was shown by D. Cass, M. Okuno and Itzhak Zilcha [2] that by relaxing the assumption that all traders are essentially identical (except their date of birth) the above theorems do not hold any more. There are examples where both monetary and barter equilibria exists but none is Pareto optimal. On the other hand it is possible that there are monetary and barter equilibria which are both Pareto optimal.

Another work by Okuno-Zilcha [3] shows that competitive equilibria exist under fairly general assumptions and investigates the Pareto optimality of such equilibria. A characterization of competitive equilibria which are Pareto optimal is given.

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