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THEORY OF TWO-DIMENSIONAL ANTENNA ARRAYS WITH RANDOM ARRANGEMEN--ETC(U)

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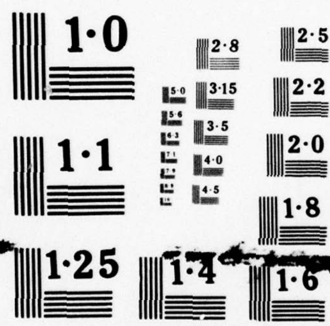


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THEORY OF TWO-DIMENSIONAL ANTENNA ARRAYS WITH RANDOM ARRANGEMENT OF EMITTERS (PART I)

by

L. G. Sodin

FOREIGN TECHNOLOGY DIVISION



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## U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

| Block | Italic | Transliteration | Block | Italic | Transliteration |
|---|---|---|---|---|---|
| А а | *А а* | A, a | Р р | *Р р* | R, r |
| Б б | *Б б* | B, b | С с | *С с* | S, s |
| В в | *В в* | V, v | Т т | *Т т* | T, t |
| Г г | *Г г* | G, g | У у | *У у* | U, u |
| Д д | *Д д* | D, d | Ф ф | *Ф ф* | F, f |
| Е е | *Е е* | Ye, ye; E, e* | Х х | *Х х* | Kh, kh |
| Ж ж | *Ж ж* | Zh, zh | Ц ц | *Ц ц* | Ts, ts |
| З з | *З з* | Z, z | Ч ч | *Ч ч* | Ch, ch |
| И и | *И и* | I, i | Ш ш | *Ш ш* | Sh, sh |
| Й й | *Й й* | Y, y | Щ щ | *Щ щ* | Shch, shch |
| К к | *К к* | K, k | Ъ ъ | *Ъ ъ* | " |
| Л л | *Л л* | L, l | Ы ы | *Ы ы* | Y, y |
| М м | *М м* | M, m | Ь ь | *Ь ь* | ' |
| Н н | *Н н* | N, n | Э э | *Э э* | E, e |
| О о | *О о* | O, o | Ю ю | *Ю ю* | Yu, yu |
| П п | *П п* | P, p | Я я | *Я я* | Ya, ya |

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

| Russian | English | Russian | English | Russian | English |
|---|---|---|---|---|---|
| sin | sin | sh | sinh | arc sh | $\sinh^{-1}$ |
| cos | cos | ch | cosh | arc ch | $\cosh^{-1}$ |
| tg | tan | th | tanh | arc th | $\tanh^{-1}$ |
| ctg | cot | cth | coth | arc cth | $\coth^{-1}$ |
| sec | sec | sch | sech | arc sch | $\mathrm{sech}^{-1}$ |
| cosec | csc | csch | csch | arc csch | $\mathrm{csch}^{-1}$ |

| Russian | English |
|---|---|
| rot | curl |
| lg | log |

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# THEORY OF TWO-DIMENSIONAL ANTENNA ARRAYS WITH RANDOM ARRANGEMENT OF EMITTERS (Part I).

L. G. Sodin.

Are investigated the strongly rarefied two-dimensional antenna-arrays with the random arrangement/position of emitters. Is shown the possibility of obtaining the low sidelobe levels and the high resolutions with a comparatively small number of cell/elements. Is carried out the comparison of these antennas with Mills's cross.

## Introduction.

In recent years rapidly increase the size/dimensions of the antennas, utilized in radiotelescopes. In a number of those operating, there are antennas with the size/dimensions of more than kilometer, and are design/projected antennas with size/dimensions into tens of kilometers. Logically, in this case, there cannot be the speeches about systems with continuous aperture. Actually can be used only the antenna arrays whose cell/elements are placed either along plane curve (circumference, cross, etc.), or it is irregular over certain area. First type antennas include widely known circular [7], cross-shaped [1, 2] and T-shaped [3, 4, 5, 6] systems, among the antennas, close to the second type, it is possible to call/name the connected system ESCAN [10] and project VLA [8, 9].

Antenna arrays with the cell/elements, arrange/located on plane curve, possess the essential deficiency/lack: along the directions, tangential to curve, into one point it is design/projected a large quantity of emitters. Due to this the radiation pattern (DN) has in some sections the large minor lobes to lower which by usual method - the adequate/approaching law of the excitation of emitters - is impossible. It is necessary to resort to extremely complex procedures, for example, to "time/temporary" synthesis of DN [11] or to the super-synthesis [is regulated the excited level of each pair of emitters, and them in a M-element antenna 0.5 M (M-1).]. In cross-shaped and T-shaped radiotelescopes is necessary to multiply arms of DN which is also inconvenient (is lost average value of DN, and the main DN of radiotelescope from power is determined DN from the field of its component antennas, which sharply increases the side-lobe level).

In connection with this more promising are "area" antennas with the irregular arrangement/position of emitters. To the study of such antennas is dedicated a series of works[1] [12, 13, 14, 15, 16, 19].

FOOTNOTE [1]. Sufficiently detailed bibliography can be found in [23]. ENDFOOTNOTE.

The most complete investigation is carried out in work [12]. However, some results of this work need refinement, a series of interesting questions there is not examined.

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In connection with this in the present work, will be made the attempt to investigate nonequidistant two-dimensional grating in more detail.

During the irregular arrangement/position of emitters in the plane of aperture, it is possible to distinguish two cases:

1) emitters are placed stochastic with the aid of random sampling;

2) emitters are placed with the aid of certain functional or theoretical-numerical algorithm analogously, for example, how are selected nodes in irregular quadrature formulas [17, 18].

Unfortunately, checking known theoretical-numerical algorithms did not yield positive results: in some sections the aperture of antenna decomposes on several rarely arranged groups, which leads to the appearance of intense diffraction lug/lobes in DN. In connection with this are further examined only the antennas with the random arrangement/position of cell/elements, in this case, the arrangement/position can be controlled by the assigned hit probabilities cell/element into one or the other point antenna. For example, it is possible not to allow/assume the incidence/impingement of a large number of emitters into one series, one diagonal, etc.

In work the parameters of antennas will be determined by average (on the final ensemble of uniform antennas) values, but it will be shown, that for sufficiently large antennas statistical average values are close to average in separate realization. The utilized procedure is analogous in essence used in [19] and is its development for two-dimensional antenna arrays with the random arrangement/position of emitters.

CALCULATION AND THE INVESTIGATION OF RADIATION PATTERN

Is examined rectangular grating with $N_1N_2$ by the nodes, numbered as follows:

$$k=-\frac{N_1-1}{3},\ -\frac{N_1-3}{2}\ \ldots 0 \ldots \frac{N_1-1}{2},$$

$$l=-\frac{N_2-1}{2},\ -\frac{N_2-3}{2}\ \ldots 0 \ldots \frac{N_2-1}{2}.$$

In node (k, l) can be located the emitter. In all in the lattice points is placed M of emitters, the current of each is equal to 1/M. To the presence of emitter in node (k, l) compare event $q_{kl}=1$, to absence - $q_{kl}=0$. The incidence/impingement of emitter into any node we consider equiprobable to: $P\{q_{kl}=1\}=\frac{M}{N_1N_2}$[1].

1.

FOOTNOTE [1]. Further instead of $N_1N_2$ let us write in certain cases $N^2$.

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The combined hit probability of emitters into nodes (k, l) and (r, s) let us take as equal to $\frac{M}{N^2}\frac{M-1}{N^2-1}$. By these we limit the ensemble of the random antennas only by such, in which is accurate M of cell/elements.

It would be possible to establish/install more rigorous conditions on probability. For example, to require, so that $P\{q_{kl}=1,\ q_{rs}=1\}\ll P\{q_{kl}=1\}$, i.e. so that one series could not hit too many cell/elements, etc. However, it is not difficult to show that with $N^2\gg 1$ this leads to a very small change in the average parameters of antenna.

In some works the emitters place so that their density would decrease to the edges of antenna [14-16]. In this case, average DN it will have the lowered/reduced sidelobe level. But in the strongly rarefied antennas the sidelobe level is determined average DN, but its fluctuations. The latter increase during the nonuniform arrangement/position of emitters in comparison with the case of uniform arrangement/position [12, 19].

In accordance with the selected probabilities

$$\overline{q_{kl}}=\frac{M^{1)}}{N^2},\quad \overline{q_{kl}q_{rs}}=\begin{cases}\frac{M}{N^2}, & k=r,\ l=s,\\ \frac{M}{N^2}\,\frac{M-1}{N^2-1} & \text{в остальных случаях.}^{(1)}\end{cases} \quad (1)$$

Key: (1). in the remaining cases.

FOOTNOTE 1. Feature indicates on top statistical averaging. ENDFOOTNOTE.

DN on field for the antenna in question is record/written as follows:

$$f(x,\ y)=\overline{f}(x,\ y)+\zeta(x,\ y)+i\,\eta(x,\ y)=\frac{1}{M}\sum_{k,\ l=-\frac{N-1}{2}}^{-N+1/2} q_{kl}e^{i(kx+ly)}. \quad (2)$$

Here $\overline{f}(x,\ y)$, $\zeta(x,\ y)$, $\eta(x,\ y)$ - middle field and its fluctuation (real and imaginary components),

$$x=\frac{2\pi d_1}{\lambda}u,\quad y=\frac{2\pi d_2}{\lambda}v,$$
$$u=\cos\varepsilon\sin A,\quad v=\cos\varepsilon\cos A,$$

$\varepsilon$, A - angle of elevation and azimuth, $d_1$ and $D_2$ - distance between the lattice points along principal axes, $\lambda$ - wavelength.

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Average DN

$$\bar{f}(x, y) = \frac{\sin\frac{N_1 x}{2}}{N_1 \sin\frac{x}{2}} \frac{\sin\frac{N_2 y}{2}}{N_2 \sin\frac{y}{2}}, \quad (3)$$

of the fluctuation

$$\zeta + i\eta = \frac{1}{M} \sum_{k,\, l=-\frac{N-1}{2}}^{\frac{N-1}{2}} \left(q_{kl} - \frac{M}{N^2}\right) e^{i(kx+ly)}.$$

Finding the law of random number distribution $\zeta$ and $\eta$ - task is very complex. However, since with $N^2>>1$, $\zeta$ and $\eta$ - sum of a large number of almost independent term/component/addends, them it is possible to consider normally distributed according to central limit theorem. As it will be shown in the second part of the article, the calculations of concrete/specific/actual gratings confirm this assumption.

If $\zeta+i\eta$ - normal random vector, then for a complete descriptions of DN is sufficient to know:

$$\overline{\zeta(x, y)\zeta_1(x, y_1)},\ \overline{\eta\eta_1},\ \overline{\zeta\eta_1}\ \text{и}\ \overline{\zeta_1\eta}$$

Let us give the resultant expressions for these values:

$$\overline{\zeta\zeta_1} = \frac{N^2 - M}{2M(N^2-1)}\left\{\frac{\sin\frac{N_1}{2}(x-x_1)}{N_1\sin\frac{x-x_1}{2}}\,\frac{\sin\frac{N_2}{2}(y-y_1)}{N_2\sin\frac{y-y_1}{2}}+\right.$$

$$+\frac{\sin\frac{N_1}{2}(x+x_1)}{N_1\sin\frac{x+x_1}{2}}\,\frac{\sin\frac{N_2}{2}(y+y_1)}{N_2\sin\frac{y+y_1}{2}}-2\times$$

$$\left.\times\frac{\sin\frac{N_1x}{2}\sin\frac{N_1x_1}{2}\sin\frac{N_2y}{2}\sin\frac{N_2y_1}{2}}{N_1^2N_2^2\sin\frac{x_1}{2}\sin\frac{x}{2}\sin\frac{y}{2}\sin\frac{y_1}{2}}\right\}, \tag{4}$$

$$\overline{\eta\eta_1} = \frac{N^2 - M}{2M(N^2-1)}\left\{\frac{\sin\frac{N_1}{2}(x-x_1)}{N_1\sin\frac{x-x_1}{2}}\,\frac{\sin\frac{N_2}{2}(y-y_1)}{N_2\sin\frac{y-y_1}{2}}-\right.$$

$$\left.-\frac{\sin\frac{N_1}{2}(x+x_1)}{N_1\sin\frac{x+x_1}{2}}\,\frac{\sin\frac{N_2}{2}(y+y_1)}{N_2\sin\frac{y+y_1}{2}}\right\}, \tag{5}$$

$$\overline{\zeta\eta_1} = \overline{\zeta_1\eta} = 0. \tag{6}$$

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With $x=x_1$, $y=y_1$ from (4) and (5) we will obtain dispersions $\zeta$ and $\eta$:

$$\overline{\zeta^2} = \frac{N^2 - M}{2M(N^2-1)}\left(1+\frac{\sin N_1x\sin N_2y}{N^2\sin x\sin y}-2\,\frac{\sin^2\frac{N_1x}{2}\sin^2\frac{N_2y}{2}}{N^4\sin^2\frac{x}{2}\sin^2\frac{y}{2}}\right), \tag{7}$$

$$\overline{\eta^2} = \frac{N^2 - M}{2M(N^2-1)}\left(1-\frac{\sin N_1x\sin N_2y}{N^2\sin x\sin y}\right). \tag{8}$$

DN of the average "scattered" power

$$\overline{\Delta F}=\overline{\zeta^2}+\overline{\eta^2}=\frac{N^2-M}{M(N^2-1)}\left(1-\frac{\sin^2\frac{N_1x}{2}\sin^2\frac{N_2y}{2}}{N^4\sin^2\frac{x}{2}\sin^2\frac{y}{2}}\right). \qquad (9)$$

In the direction of major lobe of DN ($x=y=0$) $\overline{\zeta^2}=\overline{\eta^2}=0$ fluctuations are absent. Therefore the form of major lobe virtually coincides with the form of average radiation pattern.

In the directions of minor lobes $\left(x>\frac{\pi}{N_1},\ y>\frac{\pi}{N_2}\right)\overline{\zeta^2}=\overline{\eta^2}=\sigma^2\approx\frac{1}{2M}$ (with $N^2\gg M$), which coincides with the results of works [12, 14, 23].

The mean squares $\zeta$ and $\eta$, of the undertaken on the period one realization DN,

$$\langle\zeta^2\rangle=\frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\zeta^2(x,y)\,dxdy,\ \langle\eta^2\rangle=$$
$$=\frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\eta^2(x,y)\,dxdy.$$

Integrating real and imaginary parts (2) on period, we will obtain

$$\langle\zeta^2\rangle=\langle\eta^2\rangle=\frac{N^2-M}{2MN^2}.$$

It is evident that and with $N^2\gg M$ the average in realization coincide with statistical average in the region of minor lobes. With $x>\pi/N_1$ and $y>\pi/N_2$ for the strongly rarefied antennas from (3), (7) and (8) it is evident that $\overline{\zeta^2}\approx\overline{\eta^2}\gg f^2(x,y)$.

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Consequently, in the field of minor lobes of DN is completely determined by random component.

Correlation functions $\zeta$ and $\eta$ in this region take the form

$$\left.\begin{matrix}\overline{\zeta\zeta_1}\\ \overline{\eta\eta_1}\end{matrix}\right\} = \sigma^2\left[\frac{\sin\frac{N_1}{2}(x-x_1)\sin\frac{N_2}{2}(y-y_1)}{N^2\sin\frac{x-x_1}{2}\sin\frac{y-y_1}{2}} \pm \right.$$
$$\left. \pm \frac{\sin\frac{N_1}{2}(x+x_1)\sin\frac{N_2}{2}(y+y_1)}{N^2\sin\frac{x+x_1}{2}\sin\frac{y+y_1}{2}}\right]. \quad (10)$$

If we examine DN in one half-planes (for example, x>0, by second term in (10) it is possible to disregard. Since $\overline{\zeta\zeta_1}$ and $\overline{\eta\eta_1}$ find to be dependent on differences in the arguments, $\zeta$ and $\eta$ the uniform stray fields. The fluctuations of field in the second half-plane are completely correlated with fluctuations in the first half-plane:

$$\overline{\zeta(x,\ y)\zeta(-x-y)} \approx 1,\quad \overline{\eta(x,\ y)\eta(-x,-y)} \approx 1$$

and they it is possible not to examine.

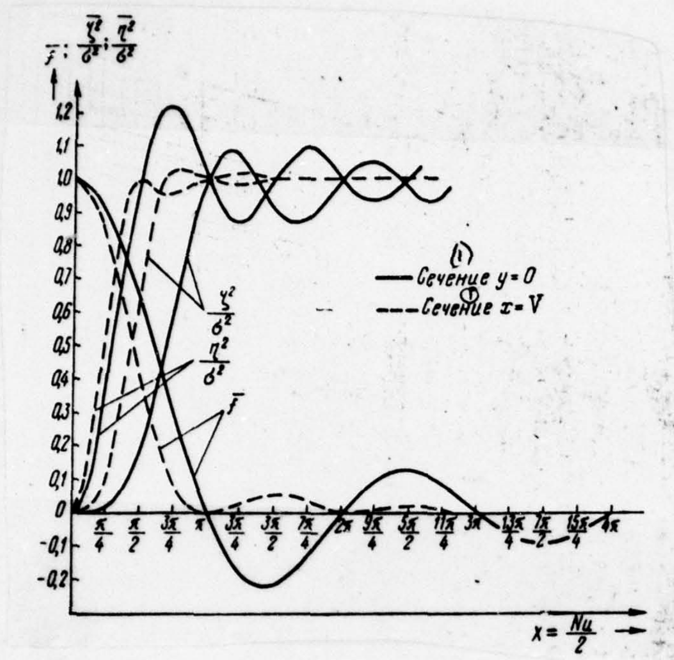


Fig. 1. Key: (1). Section.

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Figure 1 gives dependences $\bar{f}$, $\bar{\zeta}^2/\sigma^2$ and $\bar{\eta}^2/\sigma^2$ on x for two typical sections y=0 and y=x. From the figure one can see that out of the region of major lobe $\bar{\zeta}^2$ and $\bar{\eta}^2$ it is in effect constant.

All this shows that in the region of minor lobes of DN $Z(x, y) = \sqrt{\zeta^2+\eta^2}$ forms uniform Rayleigh field with one-dimensional density of distribution [1]

$$p(Z) = \frac{Z}{\sigma^2} \exp \frac{-Z^2}{2\sigma^2}.$$

FOOTNOTE [1]. More precise than DN forms field with the distribution of Rice at average value (3). For real tasks the neglect of middle field admissibly leads to essential simplification in the formulas. Precise calculation although leads to bulky formulas, it is not complex. ENDFOOTNOTE.

Average value $\bar{Z}=\sqrt{\frac{\pi}{2}}\sigma\approx\frac{\sqrt{\pi}}{2\sqrt{M}}$ constantly in the region of minor lobes and does not depend on the size/dimensions of antenna $N_1$ and $N_2$. By the force of this $\bar{Z}$, badly/poorly are described the properties of real antenna. So, with $N_1$, $N_2\to\infty$ DN acquires the narrow large diffraction lug/lobes, which do not virtually affect average value. It is much better to describe properties of DN with the aid of the overshoots of stray field Z(x, y) above certain fixed level.

Let us examine certain section of DN: $x=t\cos\phi$, $y=t\sin\phi$. In this section minor lobes form the Rayleigh random process Z(t) with the correlation function of its orthogonal components

$$\overline{\zeta(t)\zeta(t+\tau)}=\sigma^2\frac{\sin N_1\frac{\tau\cos\varphi}{2}\sin N_2\frac{\tau\sin\varphi}{2}}{N^2\sin\frac{\tau\cos\varphi}{2}\sin\frac{\tau\sin\varphi}{2}}.$$

The overshoots of this process form random flow with average value [21] in the unit interval

$$\bar{\nu}_1(Z)=\left(-\frac{R''(0)}{2\pi}\right)^{1/2}\frac{Z}{\sigma^2}\exp\frac{-Z^2}{2\sigma^2}$$

and with the asymptotically Poisson distribution of a number of overshoots with Z>>σ

$$p(\nu)=\frac{\bar{\nu}^{\nu}}{\nu!}e^{-\bar{\nu}}. \qquad (11)$$

Here R" (0) - the second derivative of the coefficient of correlation ζ(t) of τ=0.

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Since Z(t)=Z(-t) further overshoots will be computed on the half minimum period of DN: 0≤t≤π. Average number of overshoots in this interval:

$$\bar{\nu}(Z)=\left[\frac{\pi}{24}(N_1^2\cos^2\varphi+N_2^2\sin^2\varphi)\right]^{1/2}\frac{Z}{\sigma}\exp\frac{-Z^2}{2\sigma^2}.$$

Set/assuming further $\sigma=(2M)^{-1/2}$ and $N_1^2\cos^2\varphi+N_2^2\sin^2\varphi\approx N^2$, if $N_1\approx N_2$, then

$$\bar{\nu}(Z)=\left(\frac{\pi}{12}\right)^{1/2}NZM^{1/2}\exp(-MZ^2) \qquad (12)$$

virtually for any section of DN.

In accordance with (11) the dispersion of a number of overshoots is equal to $\bar{V}_{\upsilon}$ consequently, the difference between realizations by the ne-equidistant curve of antenna is noticeable for such levels of DN where $\bar{\nu}$ is small. Exponential dependence $\bar{\nu}(Z)$ (12) leads to the fact that the transition from small ones to large ones $\nu$ requires very insignificant change Z. Hence it follows that the difference between the DN of the separate realizations of antenna is small. In other words, in the ensemble of nonequidistant antennas in question the overwhelming majority is close to average, very poor and very good antennas are encountered extremely rarely. During the design of concrete/specific/actual antenna, poor versions, i.e., versions with the mound arrangement/position of cell/elements, easily are screenned. As far as versions are concerned best, they temporarily exceed average (according to a number of overshoots of DN), antenna. In confirmation this, let us make a rough estimate of the side-lobe level in the best version. Since for any version $\bar{z} \frac{1}{2}\sqrt{\frac{\pi}{M}}$, best must have Chebyshev type DN: in the region of the minor lobes $z(t)\approx A|\cos at|$.

For this DN the side-lobe level is connected with average level by simple correlation $z_{\text{макс}}\approx\frac{\pi}{2}z=\frac{\pi^{3/2}}{4M^{1/2}}$. It is obvious that to carry out the appropriate arrangement/position of emitters is impossible (at least because $z_{\text{макс}}$ it does not depend on N), and by the force of this

$$z_{\text{макс}}>\frac{\pi^{3/2}}{4M^{1/2}}\approx\frac{1,39}{M^{1/2}}. \tag{13}$$

For a real antenna let us accept the following determination of side-lobe level $z_0$. In each section of DN above level $z_0$, must be on the average one overshoot. In this case, of (12) for $z_0$ we obtain the transcendental equation

$$M^{1/2} z_0 e^{-Mz_0^2} = \frac{1,95}{N},$$

solution of which in interval $100 \leqslant N^2 \leqslant 2 \cdot 10^7$ takes the form:

$$z_0 = \frac{1,5 \div 3}{M^{1/2}}. \qquad (14)$$

Comparison (13) and (14) shows the unessential differences between the best antenna and the average.

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Therefore, if we take any realization of nonequidistant antenna with the random arrangement/position of cell/elements and artificial to remove some anomalies in the location of emitters, will be obtained DN very close to calculated. In works [14, 15] they are investigated DN of the antennas, position of emitters in which were optimized on computers by the method of dynamic programming. However, those given in these works with the DN of antennas, as a rule, are worse than average. In following work [16] examined statistical arrangement/position of cell/elements on antenna are obtained the results, which well coincide with calculated ones in the given above formulas, but the main thing, best, than during the use of the roughly optimized arrangement/position of cell/elements.

The overshoots of one-dimensional section of DN cannot be considered the as comprehensive characteristic. More detailed description give the overshoots of the two-dimensional stray field z(x, y). Strict theory of overshoots of two-dimensional field at present is not yet constructed. But, if level z, above which are examined the overshoots, it is sufficiently high, so so that in the section of overshoots the horizontal plane z would be only simply connected regions, it is possible to use the method described in [22]. In accordance with [22] over the single area of plane (x, y) the stray field has on the average $n_1$ of the overshoots above level z, where

$$n_1(z) = -\int_0^{\infty} z_y dy \int_{-\infty}^{\infty} P_4(z,\ 0,\ z_y,\ z_{xx})\, z_{xx} dz_{xx};$$

here $z_y = \frac{\partial z}{\partial y}$, $z_x = \frac{\partial z}{\partial x}$, $z_{xx} = \frac{\partial^2 z}{\partial x^2}$, $P_4(z,\ z_x,\ z_y,\ z_{xx})$

- the four-dimensional density of field z and it derivatives. $P_4$ let us find through eight-dimensional density $\zeta$, $\eta$ and them derivatives:

$$P_8(\zeta,\ \eta,\ \zeta_x,\ \eta_x,\ \zeta_y,\ \eta_y,\ \zeta_{xx},\ \eta_{xx}) = P_2(\eta,\ \eta_{xx})\, P_1(\zeta_x)\, P_1(\zeta_y)\, P_1(\eta_x)\, P_1(\eta_y).$$

$P_2$ - two-dimensional, $P_1$ - one-dimensional normal density, moreover:

$$\overline{\zeta^2} = \overline{\eta^2} = \sigma^2;\quad \overline{\zeta_x^2} = \overline{\eta_x^2} = \overline{\zeta_y^2} = \overline{\eta_y^2} = \sigma_1^2 = \sigma^2 \frac{\partial^2 \rho(x,\ y)}{\partial x^2}\bigg|_{x=y=0} = \frac{\sigma^2 N_1 N_2}{12},$$

$$\overline{\zeta_{xx}^2} = \overline{\eta_{xx}^2} = \sigma_1^2;\quad \overline{\zeta\zeta_{xx}} = \overline{\eta\eta_{xx}} = -\sigma_1^2;\quad \sigma^2 = \frac{N^2 - M}{2M(N^2 - 1)}.$$

During transition to polar coordinates $\zeta = z\cos\varphi$, $\eta = z\sin\varphi$ (jacobian of transformation $z^4$) and during integration we will obtain

$$\overline{n}_1(z) = \frac{N_1 N_2 M}{12\pi} z^2 e^{-Mz^2}\left(1 - \frac{0{,}25}{Mz^2}\right). \quad (15)$$

Actually (15) it is used with $Mz^2 \gg 1$, in connection with which second term in brackets we further disregard. By the force of parity $z(x, y)$ we examine overshoots in one half-planes on half-period $0<x<\pi$, $-\pi<y<\pi$.

$$\bar{n}(z) = \frac{\pi}{6} N^2 M z^2 e^{-Mz^2}. \qquad (16)$$

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The total area of this region of DN is equal to $2\pi^2$ and total number of overshoots

Figure 2 gives to the dependence of a number of overshoots on a number of cell/elements of antenna with $N^2=10^3$, $10^4$, $10^5$ and $z^2=0.001-0.1$. Curve/graphs these are constructed taking into account a precise value of $\sigma^2$ (8). For the illustration of the possible evacuation/rarefaction of antenna Fig. 3 gives dependences $M/N^2$ (in percentages) on $N^2$ for $z^2=0.01$ and by 0.0316 and $\bar{n}=1$ and 10.

According to an average number of overshoots (16) it is possible to find quasi-maximum sidelobe level $z_M$ - the level for which probability $P_0$ of the absence of overshoot is sufficiently close to unit.

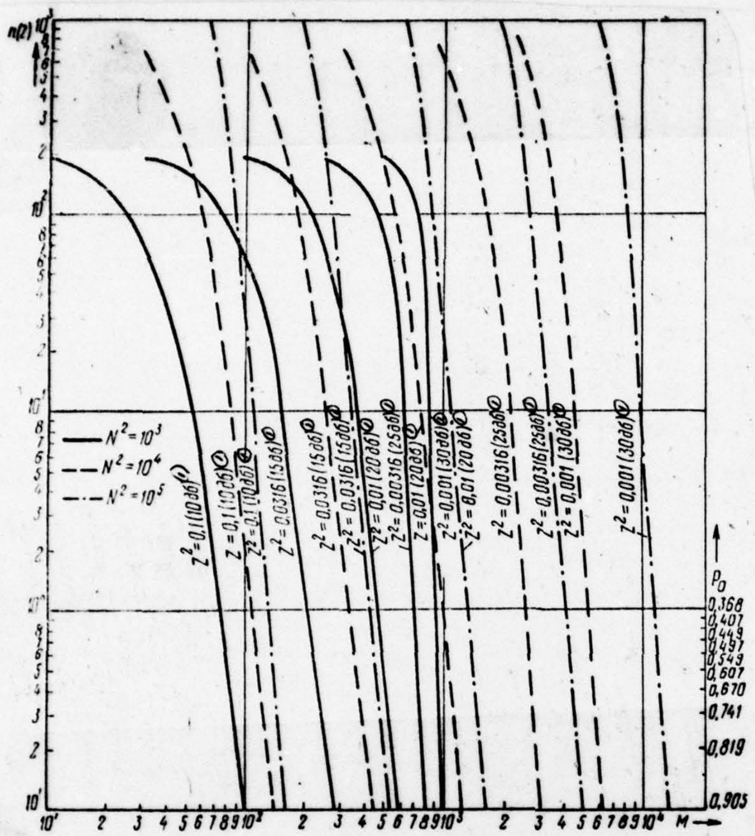


Fig. 2.

Key: (1). dB.

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In accordance with (11)

$$P_0 = e^{-\bar{n}} = \exp\left(-\frac{\pi N^2 M z^2}{6} e^{-Mz^2}\right).$$

In Fig. 2 in right lower angle is plotted scale $P_0$. Usually is sufficient to take $P_0=0.5$. In this case, the half of the selected randomly realizations of antenna will not have minor lobes above $z_M$. After assigning $P_0, z_M$ and by a number of emitters M, it is possible to determine the permissible size/dimensions of antenna. For illustration is given by table 1, designed for $P_0=0.5$ and $z_M^2=0,04$.

In [12], was made an attempt at calculation $P_0$ on the assumption that the overshoots appear in a finite number of directions, for which the fluctuations of DN are not depended. Logically, obtained in this case formula is not precise [1].

FOOTNOTE [1]. Besides in addition to this, in [12] is permitted the error: not registration/accounting symmetry of DN. Error this is corrected in [23], and here is given the corrected formula. ENDFOOTNOTE.

For one-dimensional section of DN formula (12) takes the form

$$P_L=(1-e^{-Mz^2})^{L/\lambda}.$$

Table 1.

| $M$ | 127 | 223 | 370 | 500 |
|---|---|---|---|---|
| $N^2$ | 350 | 2100 | $2.07 \cdot 10^5$ | $2.3 \cdot 10^7$ |

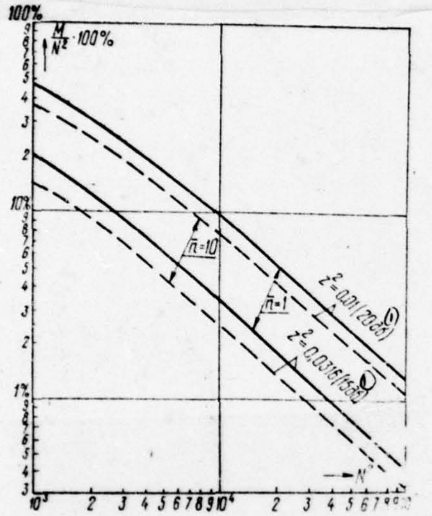


Fig. 3.

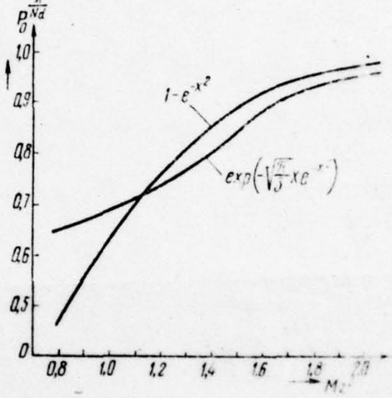


Fig. 4.

Fig. 3.

Key: (1). dB.

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Here L - length of antenna which entered into formula in connection with the fact that DN is examined in the region of "visibility" $|u|<1$.

$<1, |x|<\frac{2\pi d}{\lambda})$. Correct formula

$$P_0 = \left[\exp\left(-\sqrt{\frac{\pi}{3}}\, zM^{1/2} e^{-Mz^2}\right)\right]^{L/\lambda}.$$

In the region of DN where $P \to 1$,

$$P_L \approx 1 - \frac{L}{\lambda} e^{-Mz^2},\quad P_0 \approx 1 - \sqrt{\frac{\pi}{3}}\sqrt{M}z\frac{L}{\lambda}e^{-Mz^2}.$$

Figure 4 gives curve/graphs $(P_0)^{\lambda/L}$ and $(P_L)^{\lambda/L}$, from which it is evident that formula (12) gives the high probability of the absence of overshoot.

Advantages and disadvantages in the proposed antennas.

1. Let us compare antenna with random arrangement/position of cell/elements with Mills's cross [1].

FOOTNOTE [1]. Among antennas with the regular arrangement/position of emitters, are optimum circular and T-shaped [24]. However, is more resistant to fluctuations Mills's cross. ENDFOOTNOTE.

Let us suppose both of systems have the identical resolution, equivalent to grating from NxN cell/elements. In this case Mills's cross contains on 2N emitters in each arm, in all 4N emitters[2].

FOOTNOTE [2]. They sometimes erroneously assume that Mills's cross, from k+k cell/elements is equivalent on resolution to antenna from kxk cell/elements. ENDFOOTNOTE.

In the main sections of the DN of cross according to power, is equivalent the DN on the field of regular antenna from 2Nx2N cell/elements. Out of main sections they are multiplied by the DN of two arms of cross. Therefore the greatest side-lobe level is observed in main sections. Disregarding minor lobes by the regular DN of cross, let us assume at first that the sidelobe level of each arm is connected with the amplitude-phase errors for the currents of its cell/elements. Then dispersion of DN in the region of minor lobes [20] is equal to[3]:

$$\sigma_+^2 = \frac{\sigma_\alpha^2 + \sigma_\varphi^2}{2(2N)},$$

while DN itself will be the Rayleigh stationary random process, which has above level R on the average

$$\bar{n}_+ = \sqrt{\frac{\pi}{6}} N \frac{R}{\sigma_+} \exp \frac{-R^2}{2\sigma_+^2}$$

of overshoots.

FOOTNOTE [3]. Errors we consider not correlated. ENDFOOTNOTE.

For a nonequidistant antenna level of DN on field let us designate as before through z. Obviously, during the comparison of the cross of Mills and antenna with the irregular arrangement/position of emitters one should take $R=z^2$ and

$$\bar{n}_+ = \frac{4\sqrt{\pi}}{\sqrt{6}} \frac{N^{3/2} z^3}{\sqrt{\sigma_\alpha^2 + \sigma_\varphi^2}} \exp \frac{-2Nz^4}{\sigma_\alpha^2 + \sigma_\varphi^2 \; ^{4)}}.$$

FOOTNOTE 4. Takinj into account that the overshoots are concentrated in two main sections of DN, into this formula is adiei fa:tor by 2. ENDFOOTNOTE.

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Typical valuas $\sigma_\alpha^2$ and $\sigma_\varphi^2$ [4] compose 0.015 and

$$\bar{n}_+ \approx 16{,}7N^{3/2} z^2 \exp(-67Nz^4). \quad (16a)$$

The "random" antenna of the same resolution and with the same number of emitters will have

$$\bar{n}_c \approx 2{,}1N^3z^2 \exp(-4Nz^2) \quad (17)$$

the overshoots abɔve level z.

Comparison (16a) and (17) shows that with small ɔnes $zn_+ > n_c$. Since $z^2$=0.01 $n_+$ and $n_c$ depending on N are given in Table 2.

Table 2 shows that than the more resolution, the mɔrə noticeable the advantage of antenna with the irregular arrangement/position of emitters. In nonequidistant antenna will be in actuality advantages, also, with small N. In this case minor lobes the average DN of

Table 2.

| $N$ | 100 | 400 | 1000 |
|---|---|---|---|
| $n_+$ | 84 | 93 | 0,7 |
| $n_c$ | 380 | 0,15 | $10^{-13}$ |

Mills's cross will have noticeable value in comparison with the overshoots of the random component (but above them they disregarded). So with N=100 the average DN of cross it has 240 overshoots above level $z^2=0.01$.

2. In nonequidistant antenna average distance between adjacent emitters into $\frac{N}{M^{1/2}}$ of times more than in antenna with regular arrangement/position of emitters, which composes (3-10) once in typical versions. Due to this the communication/connection between emitters and, as a result, dependence of the impedance of emitter on the position of ray/beam with electrical phasing it is attenuate/weakened. This simplifies the task of the agreement of emitters with the system of phasing.

3. Main disadvantage in antenna array with random location of cell/elements is complexity of system of feeder communications and phasing. The necessary quantity of antenna cables and phase inverters proves to be greater than for regular antennas of the type of the cross of Mills or T-obraznoy (with an identical number of emitters). In more detail this question will be examined in the second part of the work.

4. Noticeably differs nonequidistant grating from cross of Mills and T-obraznoy antenna in value of front-to-rear factor and in its

dependence on position of ray/beam. Front-to-rear factor of nonequidistant antenna is close to $MD_0$, where $D_0$ - front-to-rear factor of elementary source. Therefore during a change in the position of the ray/beam of a change in front-to-rear factor is repeated the DN of emitter. In turnstile antennas, on the contrary, front-to-rear factor weakly depends on the direction of phasing, but its value in the direction of normal to grating is less than $MD_0$ [4]. Is in more detail the calculation of front-to-rear factor and comparison with T-shaped antenna will be made in II part.

table 2.

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Conclusions.

The possibility of the essential decrease of a number of antenna cell/elements with high size/dimensions and on low side-lobe level makes the described above antenna arrays promising ones. Apparently, the antennas very high size/dimensions will be constructed precisely according to this principle. Independence of DN from the concrete/specific/actual version of the arrangement/position of emitters is the important factor, which simplifies the design of large antennas.

Let us note, however, that with a number of emitters (N<50-100) the optimization of their location can give noticeable results. Figure 2 shows that for N<50 the statistical arrangement/position of emitters is possible only with the small degree of evacuation/rarefaction, or on high side-lobe level.

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