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ON THE EFFECT OF BOTTOM SLOPE ON RANGE ACCURACY IN THE BOTTOM B--ETC(U)
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ON THE EFFECT OF BOTTOM SLOPE ON RANGE ACCURACY IN THE BOTTOM BOUNCE MODE

JULY 1966

SUBMITTED TO:

U.S. NAVY ELECTRONICS LABORATORY
SAN DIEGO, CALIFORNIA

CONTRACT NO.:

NObsr - 93023

REPORT NO.:

TRG-023-TM-66-25

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6 ON THE EFFECT OF BOTTOM SLOPE ON RANGE ACCURACY
IN THE BOTTOM BOUNCE MODE

12 29 p

10 by
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Report No. 023-TM-66-25

Contract No. NObsr-93023

15

14 TRG-023-TM-66-25

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ABSTRACT

An analysis was performed to determine range accuracy for echo ranging, bottom-bounce sonar systems. The model for analysis assumes isovelocity conditions and a sloping bottom. The range to the target is determined from a knowledge of the echo travel time and measurement of the required environmental parameter (e.g., water depth, bottom slope). The results indicate that the range error due to bottom slope is independent of range for small bottom slopes. The range errors due to inaccuracies in measuring the bottom depth and slant range are also examined and the rms error in range is determined for a representative environment.




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SECTION I
INTRODUCTION

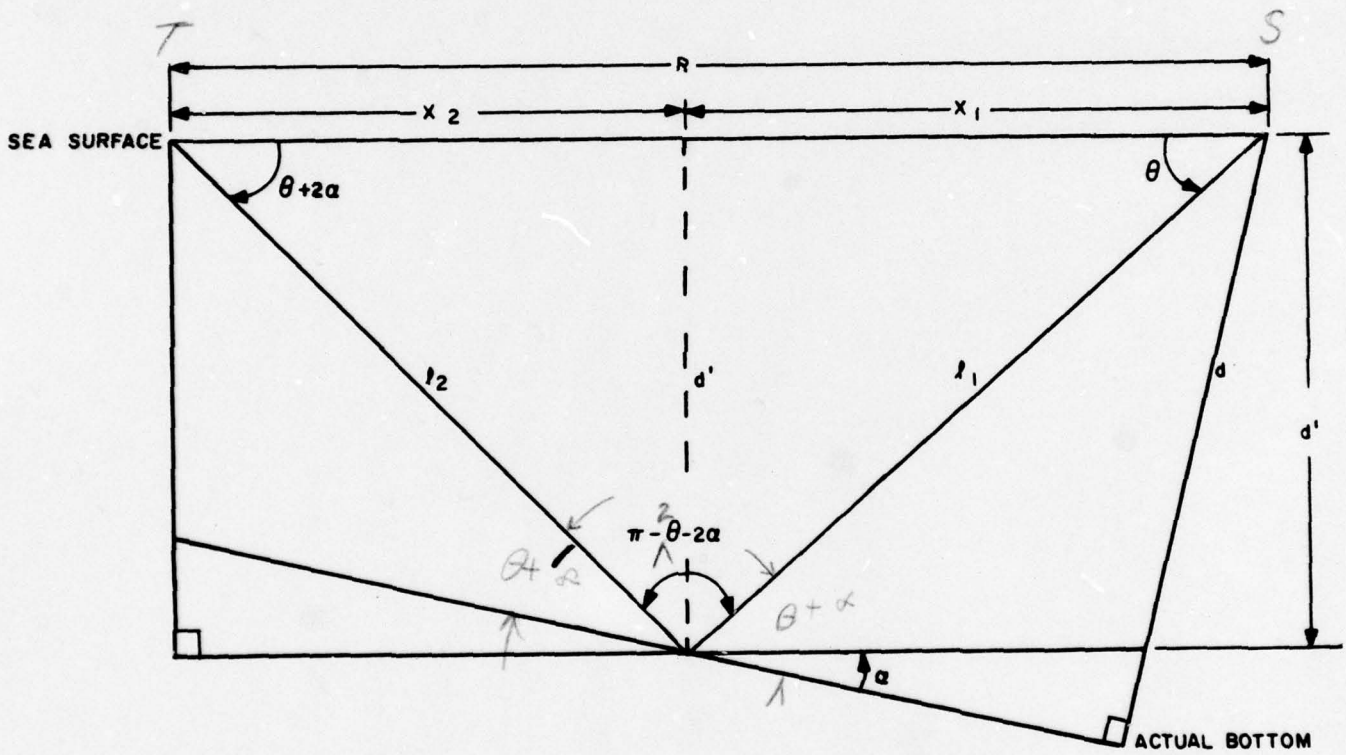
For effective utilization of a sonar system, the user must estimate the target range from a knowledge of the environmental parameters and measurement of the echo travel time. In addition, an estimate of the range accuracy is required if the sonar detection is to be used as a basis for a fire control solution⁽¹⁾.

In this paper an analytic expression is developed for the determination of target range in an isovelocity medium with a sloping bottom, given the depth of the water, the slope of the bottom and the travel time for the echo. Expressions are subsequently developed for the range errors resulting from the uncertainties in the measurement of these parameters and representative RMS range errors are calculated for various ranges and pulse lengths.

The work by T.N. Reynolds⁽²⁾ is extended by considering errors in sound velocity, bottom depth determination, and transmission time measurement. In this paper a two-dimensional model is considered, and the expression obtained herein reduces to Reynolds' result for small bottom slope. The rate of change of the range with depth and slant range (travel time) are also evaluated.

Figure 1 illustrates the geometry for the analysis. In the figure:

- $L \equiv$ slant range to target and equals $cT/2$, where c is the velocity of sound and T is the travel time,
- $d \equiv$ perpendicular distance from sonar to the bottom of the ocean,
- $d' \equiv$ vertical depth to the bottom at point of reflection,



Redraw.

FIGURE 1. GEOMETRY FOR THE ANALYSIS

- depression*
- $\theta \equiv$ bearing angle measured from the sea surface,
 - $\alpha \equiv$ angle of bottom slope, positive as shown,
 - $R \equiv$ horizontal range from sonar to target,
 - $l_1 \equiv$ distance from sonar to point of specular reflection on bottom,
 - $l_2 \equiv$ distance from specular reflection on bottom to target,
 - $x_1 \equiv$ horizontal distance from sonar to specular reflection point, and
 - $x_2 \equiv$ horizontal distance from specular reflection point to target.

The difference between the depth of the sonar and the depth of the target is small compared to the total depth of the ocean. Consequently, the error introduced by assuming that both are at the surface, for deep bottoms, is negligible. In the error analysis later on, their finite depths will be considered in greater detail.

SECTION II
DEVELOPMENT OF THE RANGE FORMULA

The slant range L is given by

$$L = l_1 + l_2 \quad (1)$$

and the horizontal target range is seen to be

$$R = x_1 + x_2 \quad (2)$$

By the pythagorean Theorem

$$(l_1)^2 = x_1^2 + d'^2 \quad (3)$$

$$(l_2)^2 = x_2^2 + d'^2 \quad (4)$$

Referring to Figure 1, the following expressions are easily written:

$$\sin \theta = \frac{d'}{l_1} \quad (5)$$

$$\cos \theta = \frac{x_1}{l_1} \quad (6)$$

$$\sin (\theta + \alpha) = \frac{d'}{l_1} \quad (7)$$

and,

$$\sin (\theta + \alpha) = \frac{d'}{l_2} \quad (8)$$

Subtracting (4) from (3) gives

$$(l_1)^2 - (l_2)^2 = (x_1)^2 - (x_2)^2 \quad (9)$$

Now substituting (5) and (8) into (1) yields

$$L = \frac{d'}{\sin \theta} + \frac{d'}{\sin(\theta + 2\alpha)} \quad (10)$$

Equations (5) and (7) may be solved for d'

$$d' = \frac{d \sin \theta}{\sin(\theta + \alpha)} \quad (11)$$

Substituting (11) into (10) gives

$$L = \frac{d \sin \theta}{\sin(\theta + \alpha)} \left[\frac{1}{\sin \theta} + \frac{1}{\sin(\theta + 2\alpha)} \right] \quad (12)$$

utilization of the identity: $\sin A + \sin B = 2 \left[\sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right]$

results in

$$\sin(\theta + 2\alpha) = \frac{2d}{L} \cos \alpha \quad (13)$$

By the Law of Sines, an expression for R in terms of L , θ , and α may be obtained from Figure 1 as follows:

$$\frac{l_2}{\sin \theta} = \frac{R}{\sin(2\theta + 2\alpha)} \quad (14)$$

$$\frac{l_1}{\sin(\theta + 2\alpha)} = \frac{R}{\sin(2\theta + 2\alpha)} \quad (15)$$

or,

$$l_2 = \frac{R \sin \theta}{\sin(2\theta + 2\alpha)} \quad (16)$$

$$l_1 = \frac{R \sin(\theta + 2\alpha)}{\sin(2\theta + 2\alpha)} \quad (17)$$

$\sin L = l_1 + l_2$ one obtains

$$L = \frac{R}{\sin(2\theta + 2\alpha)} \left[\sin \theta + \sin(\theta + 2\alpha) \right] \quad (18)$$

but, $\sin \theta + \sin(\theta + 2\alpha) = 2 \sin(\theta + \alpha) \cos \alpha$ by the expansion for the $\sin(A+B)$, and $\sin(2\theta + 2\alpha) = 2 \sin(\theta + \alpha) \cos(\theta + \alpha)$;

combining

$$\begin{aligned} \text{a. } L &= \frac{R \cos \alpha}{\cos(\theta+\alpha)} \\ \text{b. } \frac{R}{L} &= \frac{\cos(\theta+\alpha)}{\cos \alpha} \end{aligned} \quad (19)$$

Now expanding the left side of equation (13),

$$\sin \left\{ [(\theta+\alpha)] + \alpha \right\} = \sin(\theta+\alpha) \cos \alpha + \cos(\theta+\alpha) \sin \alpha \quad (20)$$

Divide both sides of equation (13) by $\cos \alpha$ and substitute equation (19b), then

$$\sin(\theta+\alpha) + \frac{R}{L} \sin \alpha = \frac{2d}{L} \quad (21)$$

Solving for $\sin(\theta+\alpha)$ yields

$$\sin(\theta+\alpha) = \sqrt{1 - \cos^2(\theta+\alpha)} = \frac{2d}{L} - \frac{R}{L} \sin \alpha \quad (22)$$

$$\text{But, from (19b) } \frac{R^2}{L^2} \cos^2 \alpha = \cos^2(\theta+\alpha)$$

Hence, substituting into (22), squaring both sides and collecting terms, yields an expression in R^2

$$R^2 - 4dR \sin \alpha + (4d^2 - L^2) = 0 \quad (23)$$

Using the quadratic formula to solve for R and choosing the positive sign of the square root on physical grounds, results in

$$R = 2d \sin \alpha + \sqrt{4d^2 \sin^2 \alpha + (L^2 - 4d^2)} \quad (24)$$

From Figure 2 below

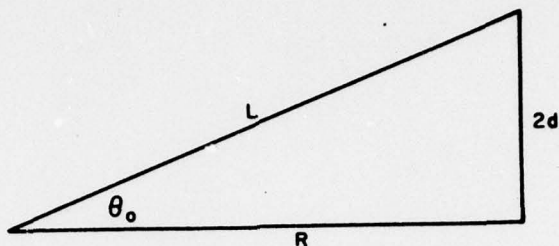


FIGURE 2. GEOMETRY FOR CALCULATION OF RANGE

where:

$R \equiv$ range

$L \equiv$ slant range

$2d \equiv$ which is twice the perpendicular distance to the bottom measured from the sonar,

one obtains

$$\begin{aligned} \text{a) } \sin \theta_0 &= \frac{2d}{L} \\ \text{and} \quad \text{b) } \cos^2 \theta_0 &= \frac{L^2 - 4d^2}{L^2} \end{aligned} \quad (25)$$

Hence, (24) may be written

$$R = 2d \sin \alpha + L \cos \theta_0 \sqrt{1 + \tan^2 \theta_0 \sin^2 \alpha} \quad (26)$$

Now, since $\sin^2 \alpha$ is small, the binomial expansion is applicable. Therefore,

$$\sqrt{1 + \tan^2 \theta_0 \sin^2 \alpha} \approx 1 + \frac{\sin^2 \alpha}{2} \tan^2 \theta_0$$

But

$$\frac{\sin^2 \alpha}{2} \tan^2 \theta_0 \ll -1$$

In this approximation, (26) becomes

$$R = 2d \sin \alpha + L \cos \theta_0 \quad (27)$$

or,

$$R = L \cos \theta_0 + 2d\alpha$$

since α is small, $\sin \alpha \approx \alpha$.

Expression (27) is composed of two terms: $R_0 = L \cos \theta_0$, and $\Delta R = 2d \sin \alpha$. The R_0 term is the range for zero bottom slope and the ΔR term arises due to the non-zero bottom slope. The interesting observation is that this term, interpreted as an error (or correction factor) is independent of the range!

Figure 3 is a plot of the range correction term, $\Delta R = R - R_0$ vs bottom slope for a water depth of 5 kyd and a 30 kyd slant range to the target. Note that the graph is for positive α (depth decreasing from sonar to target); the range correction term becomes negative for negative α .

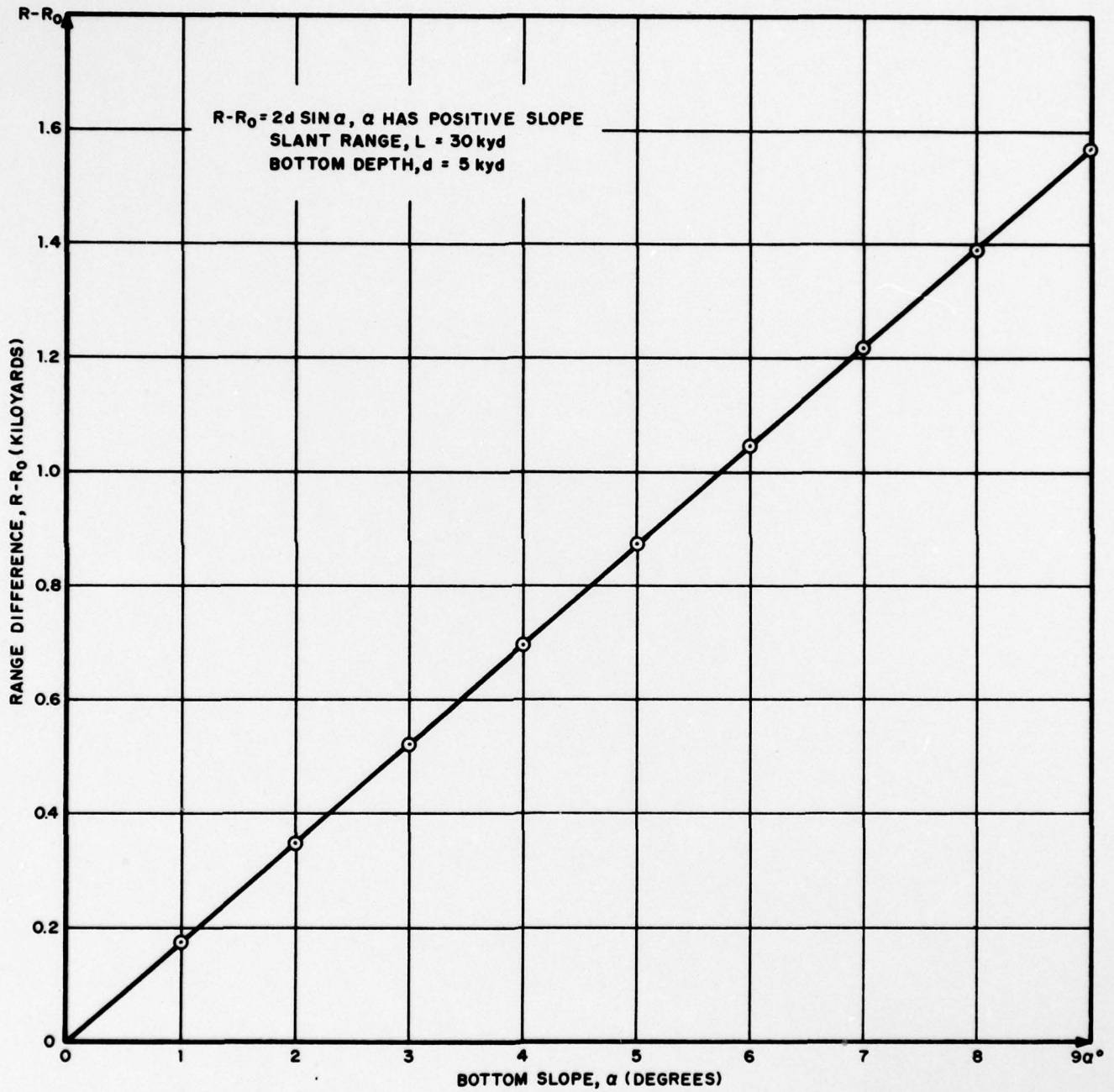


FIGURE 3. PLOT OF RANGE DIFFERENCE, $R - R_0$ VS. BOTTOM SLOPE α

SECTION III

DETERMINATION OF RANGE DERIVATIVES

From equations (13) and (19) the range changes with respect to α , L and d may be found.

Since $R = R(L, d, \alpha)$, using the chain rule,

$$\frac{dR}{d\alpha} = \frac{\partial R}{\partial \alpha} + \frac{\partial R}{\partial \alpha} \frac{\partial \theta}{\partial \alpha}$$

$$\frac{dR}{d\alpha} = -L \frac{\sin(\theta+2\alpha)}{\cos^2 \alpha} + \frac{2d}{L} \left[\frac{\sin \alpha}{\cos(\theta+2\alpha)} + 2 \right] \left[\frac{L \sin(\theta+\alpha)}{\cos \alpha} \right] \quad (28)$$

$$\frac{dR}{dL} = \frac{\partial R}{\partial L} + \frac{\partial R}{\partial \theta} \frac{\partial \theta}{\partial L}$$

$$\frac{dR}{dL} = \frac{\cos(\theta+\alpha)}{\cos \alpha} + \frac{2d}{L} \frac{\sin(\theta+\alpha)}{\cos(\theta+2\alpha)} \quad (29)$$

and

$$\frac{dR}{dd} = \frac{\partial R}{\partial d} + \frac{\partial R}{\partial \theta} \frac{\partial \theta}{\partial d}$$

$$\frac{dR}{dd} = -2 \frac{\sin(\theta+\alpha)}{\cos(\theta+2\alpha)} \quad (30)$$

(Where it is understood that the "total" derivatives are really partial derivatives with the appropriate parameters held constant).

For $\alpha = 0$ equations (28) to (30) reduce to:

$$\frac{dR}{d\alpha} = L \sin \theta_0 = 2d \quad (\text{The rate of change of } R \text{ with respect to } \alpha \text{ is independent of } R \text{ for fixed travel time!}) \quad (31)$$

$$\frac{dR}{dL} = \sec \theta \quad (32)$$

$$\frac{dR}{dd} = -2 \tan \theta \quad (33)$$

The result of equation (31) is contrary to intuition but may be more readily seen by inspecting equation (27).

SECTION IV
CALCULATION OF RMS ERROR IN RANGE

Consider the error due to errors in α , L, and d.

$$\Delta R \doteq \frac{dR}{d\alpha} \Delta\alpha + \frac{dR}{dL} \Delta L + \frac{dR}{dd} \Delta d \quad (34)$$

Since the differentials are independent, the expressions for the RMS error in range is

$$\sigma_R = \sqrt{(\Delta R_\alpha)^2 + (\Delta R_L)^2 + (\Delta R_d)^2} \quad (35)$$

To obtain some representative estimates of range errors which could occur, calculations were performed for the environment values listed in Table 1.

TABLE I
Table of Assumed Environmental Values

Quantity	Value	Uncertainty
Speed of Sound(fps)	5000	2.5(.05%)(3,4)
Water Depth (yd)	5000	25 ⁽⁵⁾
Target depth (yd)	-	150
Bottom Slope (deg)	0	1.5 ⁽²⁾

SECTION V
SLANT RANGE ERRORS

There are two sources of error attributable to the velocity profile. The first is due to limitations in the precision of measurement apparatus, and the second is due to horizontal velocity gradients (i.e., the velocity profile varies with range and bearing). The exact magnitude of the range errors associated with these parameters uncertainties is not known. In the isovelocity model, it is felt that this source of error may be adequately represented as an uncertainty in the mean sound velocity.

Since

$$L = \frac{cT}{2} \quad (38)$$

$$dL = \frac{c}{2} dt + \frac{T}{2} dc \quad (39)$$

and the error in L, ΔL is given by

$$\Delta L \doteq \frac{c}{2} \Delta T + \frac{cT}{2} \frac{\Delta c}{c} \quad (40)$$

or, equivalently,

$$\Delta L = L \frac{dc}{c} + \frac{cT}{2} \quad (41)$$

where the uncertainty in travel time is simply the pulse length, τ , for CW signals (or, for linear FM signals, the resolution is given by the reciprocal of the bandwidth). The uncertainty in average velocity is estimated to be .05% ($\frac{\Delta c}{c} = .0005$).

For $R = 15$ Kyds. and $\tau = 10$ msec, one obtains (see Figure 2):

$$L = \sqrt{R^2 + 4D^2} = \sqrt{(15)^2 + 4(5)^2} = 18.02, \quad (42)$$

and

$$\theta_0 = \sin^{-1} \frac{10}{18.02} = 33^\circ 40'$$

From equation (32), we get

$$\begin{aligned} \Delta R_L &= \Delta L \sec \theta \\ &= \left[L \frac{dc}{c} + \frac{cT}{2} \right] \sec \theta \\ &= \left\{ \left[18.02 \times 10^3 \right]^2 \left[5 \times 10^{-4} \right]^2 + \left[\frac{1633}{2} \times 10^{-2} \right]^2 \right\}^{1/2} \sec 33^\circ 40' \\ &= 14.6 \text{ yds.} \end{aligned} \tag{44}$$

SECTION VI
BOTTOM DEPTH ERRORS

To estimate the range error associated with depth uncertainties, one must consider:

- (1) the depth to the bottom of the ocean,
- (2) the depth of the sonar, and
- (3) the depth of the target.

Note that in this study the water depth is measured perpendicular to the bottom (see Figure 1). Figure 4 is similar to Figure 1 with the addition of finite sonar and target depths.

The virtual depth of the target is now seen to be:

$2d - (d_t + d_s) \sec \alpha$.
For convenience, define

$$D^* = d - 1/2 (d_t + d_s) \sec \alpha \quad (45)$$

To find the errors in the depths, take the derivative of D^*

$$dD^* = dd - 1/2 (dd_t + dd_s) \sec \alpha + 1/2 (d_t + d_s) \tan \alpha \sec \alpha \quad (46)$$

Since for most cases of interest the sonar depth error is negligible, the dd_s term may be dropped. For a zero mean bottom slope, equation (46) reduces to

$$dD^* = dd - 1/2 dd_t \quad (47)$$

and since the errors are independent,

$$\Delta D^* = \sqrt{(dd)^2 + (-1/2 dd_t)^2} \quad (48)$$

Now from equations (33) and (48) the error in range due to

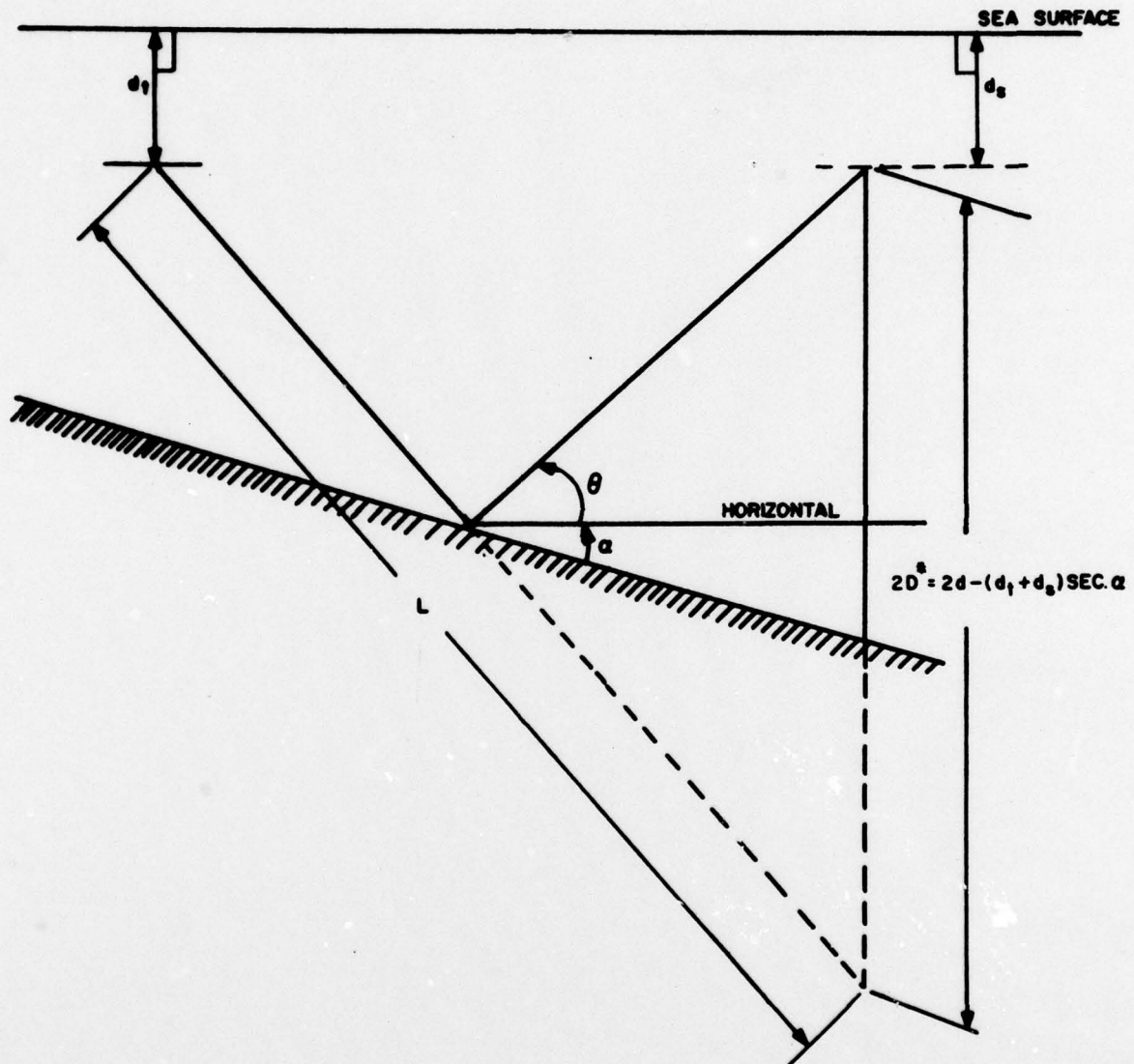


FIGURE 4. GEOMETRY FOR CALCULATION OF BOTTOM DEPTH ERRORS

depth errors is found to be:

$$\Delta R_D^* = - 2\Delta D^* \tan \theta \quad (49)$$

Substituting the values from the table

$$\begin{aligned} \Delta R_D &= -2\sqrt{(25)^2 + (1/2 \ 150)^2} \tan 33^\circ 40' \\ &= 105 \text{ yds.} \end{aligned}$$

SECTION VII
ERROR IN RANGE DUE TO BOTTOM SLOPE

A mean bottom slope of $1-1/2^\circ$ is assumed based on Reynolds' (2) measurements. This is probably pessimistic, since he apparently attributed the entire range error which he measured to bottom slope and neglected other sources of error as discussed in this paper.

From equation (31), the error in range due to bottom slope may be written as:

$$\Delta R_\alpha = 2d\Delta_\alpha \quad (50)$$

Hence, substituting the values from Table 1, one obtains

$$\Delta R_\alpha = \frac{2\pi}{180} (5000) (1.5) = 261 \text{ yds}$$

SECTION VIII
RMS ERROR IN RANGE

Combining the errors found in (44), (49) and (50) and substituting into (35) we get

$$\begin{aligned}\sigma_R &= (261)^2 + (105)^2 + (14.6)^2 \\ &= 281 \text{ yds.}\end{aligned}\tag{51}$$

Table 2 gives the components namely, ΔR_α , ΔR_D and ΔR_L that make up the RMS error in range at three different ranges and for six pulse lengths. Figure 5 is a plot of range error (σ_R) vs pulse length (τ) for constant target range, and Figure 6 is a plot of range error (σ_R) vs target range (R) for fixed pulse lengths.

From these figures one may observe that for the 10ms and 100 ms pulse lengths there is almost no difference in range accuracy because the dominant source of error is due to the bottom slope term. For the 500 ms pulse length, the uncertainty due to the pulse length dominates the other terms and causes a significant loss in range accuracy.

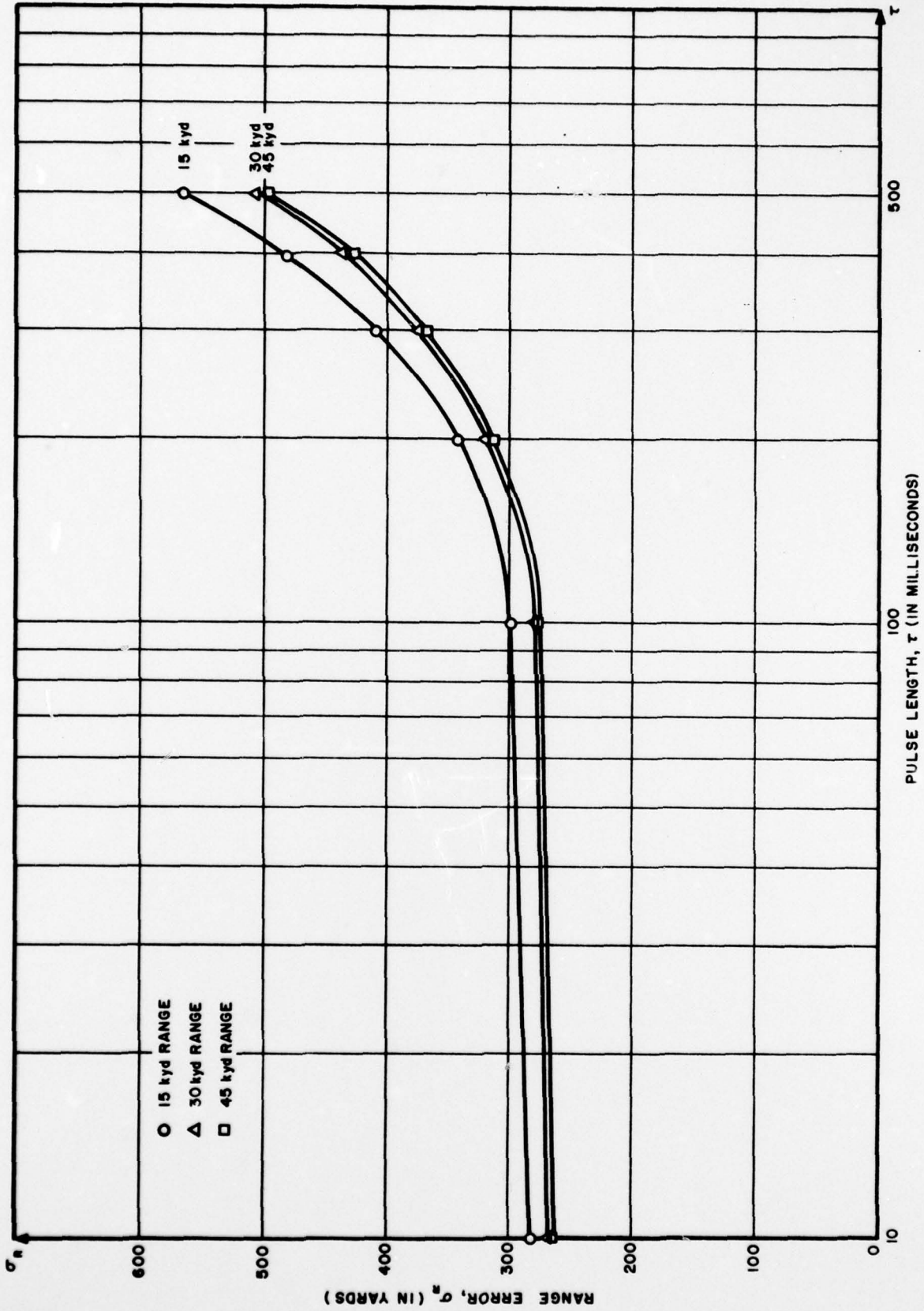


FIGURE 5. RANGE ERROR (σ_R) VS. PULSE LENGTH (τ)

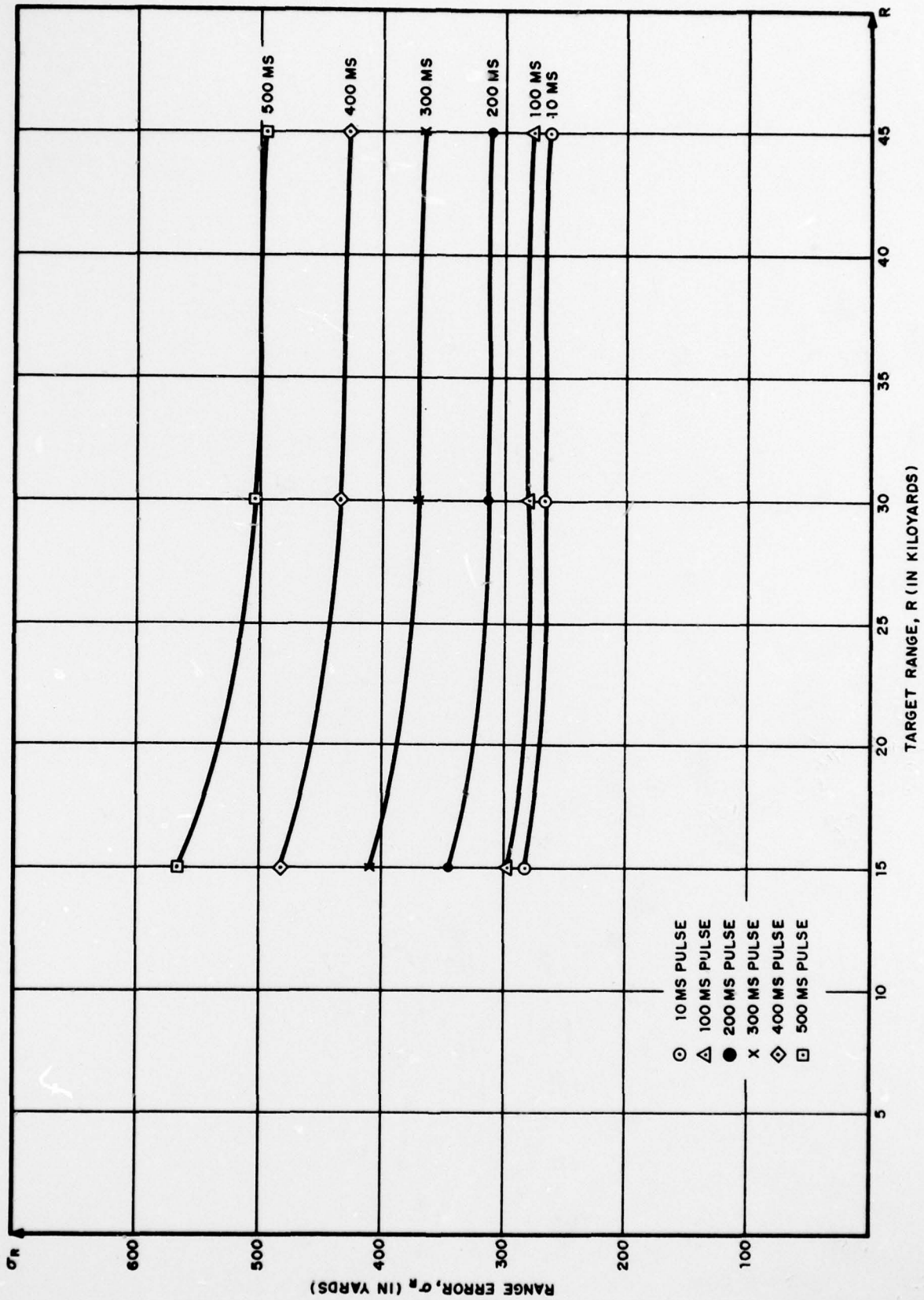


FIGURE 6. PLOT OF RANGE ERROR (σ_R) VS. TARGET RANGE (R)

TABLE 2

Pulse Length	R	d	L	θ	ΔR_{α}	ΔR_d	ΔR_L	σ_R
10 ms	15	5	18.02	33°40'	261	105	14.6	281
100 ms	15	5	18.02	33°40'	261	105	98.6	298
200 ms	15	5	18.02	33°40'	261	105	196.4	343
300 ms	15	5	18.02	33°40'	261	105	295	408
400 ms	15	5	18.02	33°40'	261	105	392	482
500 ms	15	5	18.02	33°40'	261	105	490	565
10 ms	30	5	31.62	18°15'	261	52	18.7	267
100 ms	30	5	31.62	18°15'	261	52	87.4	280
200 ms	30	5	31.62	18°15'	261	52	173.1	317
300 ms	30	5	31.62	18°15'	261	52	259	371
400 ms	30	5	31.62	18°15'	261	52	344	435
500 ms	30	5	31.62	18°15'	261	52	430	505
10 ms	45	5	46.1	12°32'	261	35	25.5	264
100 ms	45	5	46.1	12°32'	261	35	87.0	277
200 ms	45	5	46.1	12°32'	261	35	168.9	313
300 ms	45	5	46.1	12°32'	261	35	251	364
400 ms	45	5	46.1	12°32'	261	35	334	425
500 ms	45	5	46.1	12°32'	261	35	419	495

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