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HYDRODYNAMIC MASS OF BODIES IN A FLUID.(U)

OCT 64 K T PATTON

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U.S. NAVY UNDERWATER SOUND LABORATORY
FORT TRUMBULL, NEW LONDON, CONNECTICUT

HYDRODYNAMIC MASS OF BODIES IN A FLUID

By

Kirk T. Patton

USL Technical Memorandum No. 933-351-64

15 Oct[redacted]964

(9) Technical memo.

INTRODUCTION

This technical memorandum discusses hydrodynamic mass and presents a tabulation of several hydrodynamic mass factors and equations that pertain to bodies under translational motion in a fluid. This material is being submitted to the American Society of Mechanical Engineers for possible publication by that society.

(12) 16p.

NOTATION

A	area	L^2
a,b,c,d,l	dimensions of bodies	L
e	distance from boundary to bottom of body	L
F	force	F
i,j	indices used in tensor notation	
k	wave number	1/L
K	hydrodynamic mass factor	

NOTATION, Continued

m - mass FT^2/L

m_h - hydrodynamic mass FT^2/L

m_v - virtual mass FT^2/L

N - ratio of wing area to area of body section

ρ - density of fluid in which the body is immersed FT^2/L^4

ϕ - velocity potential

φ_i - normalized velocity potential

s - distance from free surface to center of body L

$\hat{n}$ - direction index

∇ - Laplacian operator $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)$

DISCUSSION

When the forces acting on a body that is moving at constant velocity in an ideal fluid of constant density are integrated over the surface of the body, the resultant force is found to be zero. This is commonly referred to as D'Alembert's Paradox because, in a real fluid, a body moving at constant velocity has a resisting force whereas the integration results imply that there is no resisting force.

For the case of a body moving with unsteady motion, the resultant force is found to be:

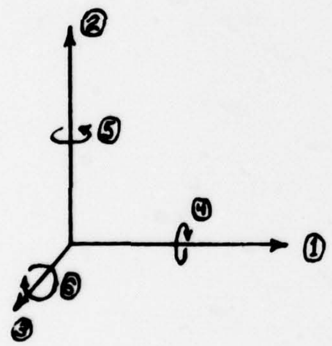
$$\vec{F} = \frac{d}{dt}\int_A \rho \phi \hat{n} \, dA \tag{1}$$

By introducing the normalized velocity potential φ , it is seen that the force F is identical with that induced by a mass of fluid added to that of the body and equal to:

$$m_{ij} = -\rho \int_A \frac{\partial \varphi_i}{\partial n} \varphi_j \, dA \qquad \begin{matrix} i = 1, 2, 3 \\ j = 1, 2, 3 \end{matrix} \qquad (2)$$

This mass is commonly referred to as added mass, increased inertia, or hydrodynamic mass. The sum of this additional mass and the body mass is usually called the virtual mass. In this memorandum, hydrodynamic mass is identified by the above expression; virtual mass is the sum of body mass and its hydrodynamic mass.

The hydrodynamic mass of the body is a second order tensor, as shown above. Thus, for a body having six degrees of freedom, as shown below, the hydrodynamic mass is represented by a 6 x 6 matrix. The subscripts indicate motion along or around 3 orthogonal axes. Subscripts 1 to 3 are for translation; subscripts 4 to 6 are for rotation.

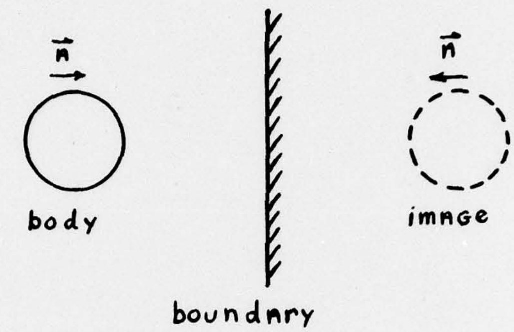


$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & m_{36} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} & m_{46} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} & m_{56} \\ m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{bmatrix}$$

Reference (b) proves that $m_{ij} = m_{ji}$. Thus, if all six modes of motion are considered, 21 terms are needed to completely describe the hydrodynamic mass of a body of arbitrary shape.

The process of integrating equation (2) becomes quite complex as body shapes deviate from simple shapes, such as bodies of revolution or two-dimensional bodies. Also, if the body is oscillating, the computed hydrodynamic mass is valid only for low frequency oscillations because, in the analysis from which the hydrodynamic mass expression is derived, Laplace's equation, $\nabla^2 \phi = 0$, must be satisfied. A more general solution would satisfy the Helmholz equation, $(\nabla^2 + K^2)\phi = 0$, as well and would show that the hydrodynamic mass is a function of frequency.

If the body is near a boundary, the hydrodynamic mass is affected by the presence of the boundary. This can be accounted for analytically by the image method. (An image on the other side of the boundary is required to maintain the boundary condition such that the velocity normal to the boundary is equal to zero.)



BODIES UNDER TRANSLATIONAL MOTION

A full evaluation of the 6 x 6 hydrodynamic mass matrix is required to describe the motion of a body moving with six degrees of freedom. Many of the practical problems, however, are concerned with motion in one direction only.

Upon examination of the technical papers and literature on this subject matter, it was decided that it would be worthwhile to consolidate a certain number of hydrodynamic mass calculations and to present them in tabular form for easy reference. See Appendix I.

In this Appendix, the direction of translation relative to the body, the corresponding hydrodynamic mass and the information source are given for each body shown. The hydrodynamic mass is shown as a constant (the hydrodynamic mass factor) times the fluid density times a volume (characteristic of the body).

It should be kept in mind that the force due to the hydrodynamic mass is not the only force acting on an accelerating body because of the fluid; a resistance due to viscous forces will also be present for a body accelerating in a real fluid.

Attention is again invited to the fact that the mass that should be used to compute the natural frequency of an immersed body is the virtual mass, which is the summation ofhydrodynamic mass and body mass.

A first approximation to compute the force F that accelerates a body of mass m is:

$$\vec{F} = (m + \vec{m}_h)\vec{A}$$

where:
- a = acceleration
- m_h = hydrodynamic mass associated with the particular direction of motion
- m = body mass

CONCLUSIONS AND RECOMMENDATIONS

As mentioned in the previous section, all 21 significant terms of the 6 x 6 hydrodynamic mass matrix must be determined to analyze the motion of a body with 6 degrees of freedom. Work of this nature is being performed by the University of Rhode Island under a contract with Underwater Sound Laboratory.

Extensive experimentation is required to determine the hydrodynamic mass coefficients for various bodies.

The effect of frequency of oscillation on hydrodynamic mass should also be investigated.

Kirk T. Patton
KIRK T. PATTON
Mechanical Engineer

List of References

(a) H. Lamb, "Hydrodynamics," Cambridge University Press, 1932

(b) G. Birkhoff, "Hydrodynamics," Princeton University Press, 1963

(c) R. Bramig, "Experimental Determination of the Hydrodynamic Increase in Mass in Oscillating Bodies," DTMB Translation 118

(d) K. Wendel, "Hydrodynamic Masses & Hydrodynamic Moments of Inertia," DTMB Translation 260

(e) Kinsler-Frev, "Fundamentals of Acoustics," John Wiley & Sons, Inc., 1962

(f) K. Patton, "An Experimental Investigation of Hydrodynamic Mass and Mechanical Impedances," Thesis, University of Rhode Island, 1964

(g) M. Munk, "Fluid Mechanics, Part II," Aerodynamic Theory, Vol. I, edited by W. Durand, Dover Publications Inc., 1963

(h) L. Landweber, "Motion of Immersed and Floating Bodies," Handbook of Fluid Dynamics, edited by V. Streeter, McGraw-Hill Book Company, Inc., 1961

APPENDIX I

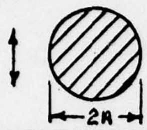
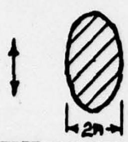
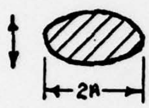
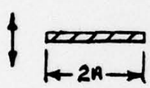
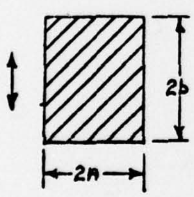
TABLE OF HYDRODYNAMIC MASS OF BODIES UNDER TRANSLATIONAL MOTION IN A FLUID

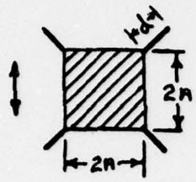
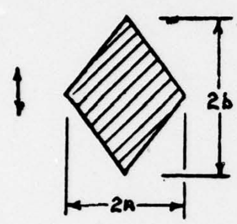
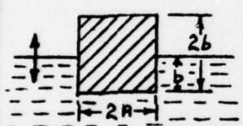
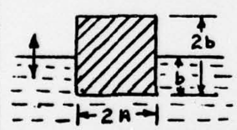
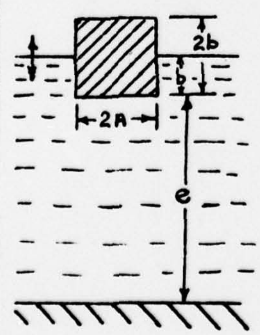
Note: * Numbers listed under "source" refer to the publications shown in the "List of References".

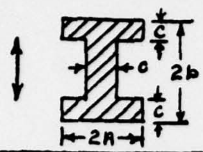
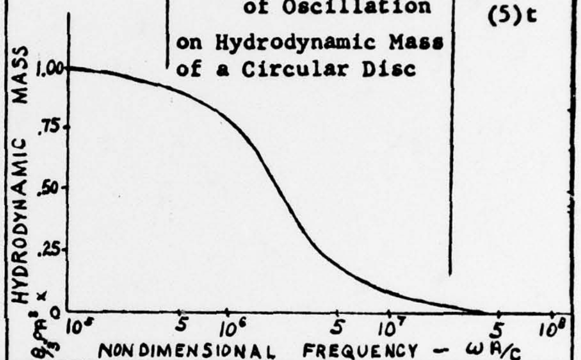
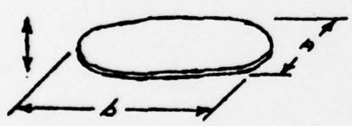
Letters t and e listed under "source" indicate that the data were obtained from theory and experimentation.

APPENDIX I

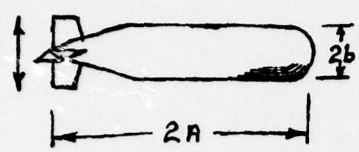
TWO DIMENSIONAL BODIES

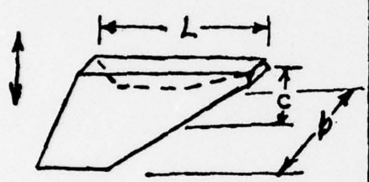
Section Through Body	Translational Direction	Hydrodynamic Mass per Unit Length	Source
	Vertical	$m_h = 1 \pi \rho a^2$	(4)t
	Vertical	$m_h = 1 \pi \rho a^2$	(4)t
	Vertical	$m_h = 1 \pi \rho a^2$	(4)t
	Vertical	$m_h = 1 \pi \rho a^2$	(4)t, (6)e
 a/b= ∞	Vertical	$m_h = 1 \pi \rho a^2$	(4)t
a/b=10		$m_h = 1.14 \pi \rho a^2$	(4)t
a/b= 5		$m_h = 1.21 \pi \rho a^2$	(4)t
a/b= 2		$m_h = 1.36 \pi \rho a^2$	(4)t
a/b= 1		$m_h = 1.51 \pi \rho a^2$	(4)t
a/b=1/2		$m_h = 1.70 \pi \rho a^2$	(4)t
a/b=1/5		$m_h = 1.98 \pi \rho a^2$	(4)t
a/b=1/10		$m_h = 2.23 \pi \rho a^2$	(4)t

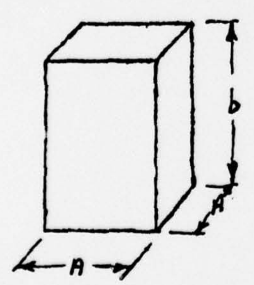
Section Through Body	Translational Direction	Hydrodynamic Mass per Unit Length	Source
 d/a=.05	Vertical	$m_h = 1.61\ \pi\rho\ a^2$	(4)t
d/a=.10		$m_h = 1.72\ \pi\rho\ a^2$	(4)t
d/a=.25		$m_h = 2.19\ \pi\rho\ a^2$	(4)t
 a/b= 2	Vertical	$m_h = .85\ \pi\rho\ a^2$	(4)t
a/b= 1		$m_h = .76\ \pi\rho\ a^2$	(4)t
a/b=1/2		$m_h = .67\ \pi\rho\ a^2$	(4)t
a/b=1/5		$m_h = .61\ \pi\rho\ a^2$	(4)t
 a/b= 1	Vertical (normal to free surface)	$m_h = .75\ \pi\rho\ a^2$	(4)t
 a/b =1	Horizontal (parallel to free surface)	$m_h = .25\ \pi\rho\ a^2$	(4)t
 a/b= 1; e/b=∞	Vertical (normal to free surface)	$m_h = .75\ \pi\rho\ a^2$	(4)t
e/b=2.6		$m_h = .83\ \pi\rho\ a^2$	(4)t
e/b=1.8		$m_h = .89\ \pi\rho\ a^2$	(4)t
e/b=1.5		m_h 1.00 $\pi\rho\ a^2$	(4)t
e/b=.5		m_h 1.35 $\pi\rho\ a^2$	(4)t
e/b=.25		m_h 2.00 $\pi\rho\ a^2$	(4)t

Body Shape	Translational Direction	Hydrodynamic Mass	Source
	Vertical	$m_h = 2.11\,\pi\rho a^2$	(6)e
THREE DIMENSIONAL BODIES			
1. Flat Plates **Circular Disc** 	Vertical	$m_h = \frac{8}{3}\rho a^3$ Effect of Frequency of Oscillation on Hydrodynamic Mass of a Circular Disc 	(1)t, (5)t (5)t
Elliptical Disc 	As Shown	$m_h = Kba^2\,\frac{\pi}{6}\rho$	(7)t

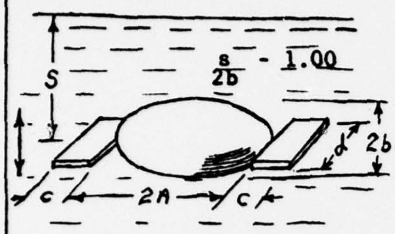
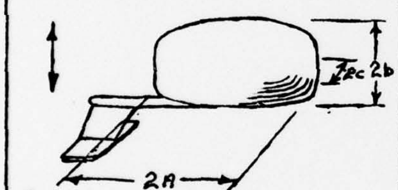
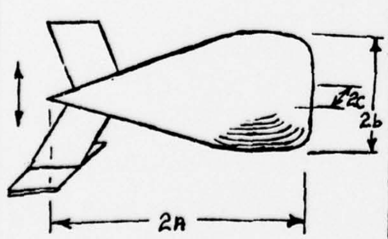
b/a	K
∞	1.00
14.3	.991
12.75	.987
10.43	.985
9.57	.983
8.19	.978
7.00	.972
6.00	.964
5.02	.952
4.00	.933
3.00	.900
2.00	.826
1.50	.748
1.00	.637

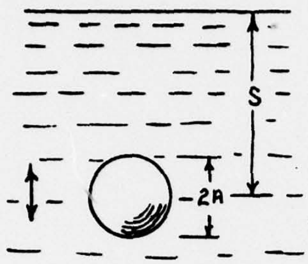
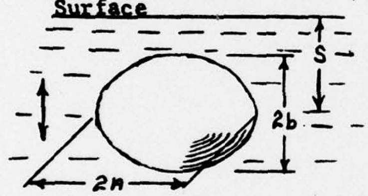
<table>
<tr><th>Body Shape</th><th>Translational Direction</th><th>Hydrodynamic Mass</th><th>Source</th></tr>
<tr><td>"Torpedo" Type Body


a/b=5.0
Area of Horizontal "Tail" = 10% of Area of Body Maximum Horizontal Section.</td><td>Vertical</td><td>$m_h = .818 \pi \rho b^2 (2a)$</td><td>(6)e</td></tr>
<tr><td>V-Fin Type Body


$\frac{l}{b} = 1.0$ $\frac{l}{c} = 2.0$</td><td>Vertical</td><td>$m_h = .3975 \rho L^3$</td><td>(6)e</td></tr>
<tr><td>Parallelepipeds

</td><td>Vertical</td><td>$m_h = K \rho a^2 b$

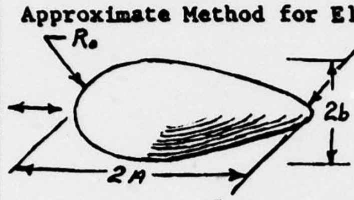
b/a — K
1 — 2.32
2 — .86
3 — .62
4 — .47
5 — .37
6 — .29
7 — .22
10 — .10</td><td>(6)e</td></tr>
</table>

Body Shape	Translational Direction	Hydrodynamic Mass	Source
Ellipsoid with Attached Rectangular Flat Plates Near a Free Surface. 	Vertical	$m_h = K\cdot\frac{4}{3}\pi\rho ab^2$ $a/b = 2.00;\ c = b$ $c.d = N\pi ab$ N K 0 .9130 .20 1.0354 .30 1.3010 .40 1.4610 .50 1.5706	(6)e
Streamlined Body  $\frac{a}{b}$ - 2.38 $\frac{a}{c}$ - 2.11	Vertical	$m_h = 1.124\rho\left[\frac{4}{3}\pi ad^2\right]$ $d = \frac{c+b}{2}$	(6)e
Area of Horizontal "Tail" =25%	of Area of	Body Maximum Horizont	al Section.
Streamlined Body 	Vertical	$m_h = .672\rho\left[\frac{4}{3}\pi\ ad^2\right]$ $d = \frac{c+b}{2}$	(6)e
Area of Horizontal "Tail" =20%	of Area of	Body Maximum Horizontal	Section.

Body Shape	Translational Direction	Hydrodynamic Mass	Source
Sphere Near a Free Surface  	Vertical	$m_h = K\frac{2}{3}\pi\rho a^3$ s/2a — K 0 — .50 .5 — .88 1.0 — 1.08 1.5 — 1.16 2.0 — 1.18 2.5 — 1.18 3.0 — 1.16 3.5 — 1.12 4.0 — 1.04 4.5 — 1.00	(6)e
Ellipsoid Near a Free <u>Surface</u>  	Vertical	$m_h = K\cdot\frac{4}{3}\pi\rho ab^2$ a/b = 2.00 s/2b — K 1.00 — .913 2.00 — .905	(6)e
<u>2. Bodies of Arbitrary Shape</u> Ellipsoid with Attached Rectangular Flat Plates  	Vertical	$m_h = K\cdot\frac{4}{3}\pi\rho ab^2$ a/b = 2.00; c=b c.d = N π ab N — K 0 — .7024 .20 — .8150 .30 — 1.0240 .40 — 1.1500 .50 — 1.2370	(6)e

Body Shape	Translational Direction	Hydrodynamic Mass	Source
Ellipsoids (continued)		a/b K axial K lateral 8.01 .029 .945 9.02 .024 .954 9.97 .021 .960 ∞ 0 1.000	

Approximate Method for Elongated Bodies of Revolution.



$$m_h = K_1 \rho V = Ke \left[1 + 17.0(Cp - 2/3)^2 + 2.49(M - 1/2)^2 + .283 \left[(r_o - 1/2)^2 + (r_1 - 1/2)^2\right]\right]$$

where; K_1 - Hydrodynamic Mass coefficient for axil motion

K_e - Hydrodynamic Mass Coefficient for axil motion of an ellipsoid of the same ratio of a/b

V - Volume of body

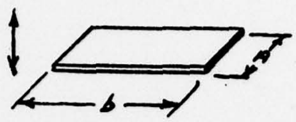
C_p - Prismatic coefficient $= \dfrac{4V}{b^2(2a)}$

M - Nondimensional abscissa $X_{m/l}$ corresponding to maximum ordinate

r_o, r_1 - Dimensionless radii of curvature at nose and tail

$$r_o = \frac{Ro\ (2a)}{b^2} \qquad r_1 = \frac{R1\ (2a)}{b^2}$$

Body Shape	Translational Direction	Hydrodynamic Mass	Source
	Lateral Motion	Munk has shown that the hydrodynamic mass of an elongated body of revolution can be reasonably approximated by the product of the density of the fluid, the volume of the body and the k - factor for an ellipsoid of the same a/b ratio.	

<table>
<tr><th>Body Shape</th><th>Translational Direction</th><th>Hydrodynamic Mass</th><th>Source</th></tr>
<tr><td>Rectangular Plates

</td><td>Vertical</td><td>$m_h = K \pi \rho \frac{a^2}{4} b$
<table>
<tr><th>b/a</th><th>K</th></tr>
<tr><td>1.0</td><td>.478</td></tr>
<tr><td>1.5</td><td>.680</td></tr>
<tr><td>2.0</td><td>.840</td></tr>
<tr><td>2.5</td><td>.953</td></tr>
<tr><td>3.0</td><td>1.00</td></tr>
<tr><td>3.5</td><td>1.00</td></tr>
<tr><td>4.0</td><td>1.00</td></tr>
<tr><td>∞</td><td>1.00</td></tr>
</table></td><td>(6)e</td></tr>
<tr><td>Triangular Plates
</td><td>Vertical</td><td>$m_h = \frac{\rho}{3} a^3 \frac{(\tan\theta)^{3/2}}{(\pi)}$</td><td>(6)e</td></tr>
<tr><td><u>3. Bodies of Revolution</u>
Spheres

</td><td>Vertical</td><td>$m_h = \frac{2}{3} \pi \rho a^3$</td><td>(1)t, (2)t</td></tr>
<tr><td>Ellipsoids

</td><td>Vertical</td><td>$m_h = K \cdot \frac{4}{3} \pi \rho ab^2$
<table>
<tr><th>a/b</th><th>K for Axial Motion</th><th>K for Lateral Motion</th></tr>
<tr><td>1.00</td><td>.500</td><td>.500</td></tr>
<tr><td>1.50</td><td>.305</td><td>.621</td></tr>
<tr><td>2.00</td><td>.209</td><td>.702</td></tr>
<tr><td>2.51</td><td>.156</td><td>.763</td></tr>
<tr><td>2.99</td><td>.122</td><td>.803</td></tr>
<tr><td>3.99</td><td>.082</td><td>.860</td></tr>
<tr><td>4.99</td><td>.059</td><td>.895</td></tr>
<tr><td>6.01</td><td>.045</td><td>.918</td></tr>
<tr><td>6.97</td><td>.036</td><td>.933</td></tr>
</table></td><td>(1)t</td></tr>
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Distribution List

External

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Commercial Engrg Corp., Houston, Texas (NObsr 85537)
Telephonics Corp., Huntington, LI,NY (NObsr 77626)
McKiernan-Terry Corp., Div. of Litton Ind. Clifton, N.J. (NObsr 81240)
Defense Res. Lab., Univ of Texas, (Mr. Hillery) (NObsr 72627)
DTMB, (Code 530)
DTMB (Code 547)
DTMB (Code 548)
DTMB (Code 549)
NUOS, Newport, R.I.
Tech. Dir., Woods Hole Oceano Inst., Woods Hole, Mass.
NASL (Code 9360)
NEL, San Diego, Calif (Mr. R. Miller Code 3140)

Internal

Code 100	Code 930	Code 935.6	Code 930S(3)
Code 101	Code 930A	Code 935.7	Code 920
Code 900	Code 933	Code 230	H. Phelps
Code 900A	Code 933.2	Code 232	R. Welsh
Code 900B	Code 933.3	Code 233	G.V.Williams
Code 900C	Code 933.4	F. Contrata	A. Markowitz
G. Christensen	K. T. Patton (3)	R. Galligan	Code 902
	Code 904	Code 904.2S(5)	