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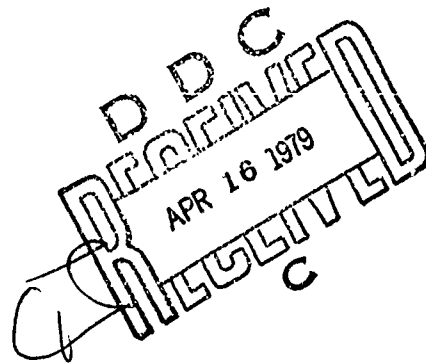
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SIMPLE AND EFFICIENT NUMERICAL TECHNIQUES FOR TREATING BODIES OF REVOLUTION

University of Mississippi

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TABLE OF CONTENTS

Section	Page
I. INTRODUCTION	1
II. NUMERICAL SOLUTION PROCEDURE FOR A BODY OF REVOLUTION	3
2.1 Formulation of the Integral Equations	3
2.2 Numerical Results for the Body of Revolution	34
III. CONCLUSION	53
APPENDIX A. SINGULARITY ANALYSES FOR SELF TERMS OF THE BODY OF REVOLUTION FORMULATION	55
A.1 Singularity Analysis of the ψ and ψ^p Integral Functions	56
A.2 Singularity Analysis of the Integral Function U_0	60
A.3 Singularity Analysis of the Integral Functions U_1 and U_1^p	63
A.4 Singularity Analysis of the Integral Functions U_2 and U_2^p	63
A.5 Singularity Analysis of the Integral Function U_3^p	65
A.6 Singularity Analysis of the Integral Function U_4^p	66
A.7 Singularity Analysis of the Integral Functions U_5 and U_5^p	67
A.8 Singularity Analysis of the Integral Function U_6	69
A.9 Elimination of Non-Integrable Singularities	69

	Page
APPENDIX B. DESCRIPTION AND LISTING OF THE CODE	73
B.1 Program Operation	74
B.2 Sample Case	77
B.3 Program Listing	87
REFERENCES	128

LIST OF FIGURES

Figure	Page
2.1. Geometry of the body of revolution.	4
2.2. Approximation of the generating arc by linear segments.	15
2.3. Cross section of a perfectly conducting cylinder with one open end.	18
2.4. Electric surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.	37
2.5. Magnetic surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.	38
2.6. Electric surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.	40
2.7. Magnetic surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.	41
2.8. Electric and magnetic surface current distributions on a "vacuum dielectric" cylinder illuminated by a axially incident plane wave.	43
2.9. Electric surface current distribution on a dielectric cylinder illuminated by an axially incident plane wave.	44
2.10. Magnetic surface current distribution on a dielectric cylinder illuminated by an axially incident plane wave.	45
2.11. Surface current distribution on a perfectly conducting cone-sphere structure, illuminated by a plane wave which is axially incident and propagates from the cone tip toward the spherical cap.	47
2.12. Surface current distribution on a perfectly conducting cone-sphere structure illuminated by a plane	50

wave which is axially incident and propagates from the spherical cap toward the cone tip.

- 2.13. Surface current distribution on a perfectly conducting open-ended cylinder illuminated by an axially incident plane wave. 52

Section I

INTRODUCTION

The determination of the skin currents induced on a missile in flight is an important first step in predicting the response of internal subsystems to an impressed electromagnetic field. The missile skin currents, however, may be affected by the presence of the rocket exhaust (plume) which consists of partially ionized gases or plasma and therefore has a finite conductivity. Because of the difficulty and expense involved in obtaining measured data, a reasonably accurate numerical solution procedure is desirable. Furthermore, numerical results can provide a partial validation of actual experiments. The more sophisticated numerical analyses generally involve a body of revolution model for both the missile and the plume. In this report we consider body of revolution models in general, for both perfect electric conductors and dielectric bodies, without reference to a particular structure.

The primary impetus for the development of the computer code described in this work has been the desire for simplicity of formulation and for increased accuracy and efficiency. Other numerical formulations for bodies of revolution are

already available [1, 2, 3] and, indeed, the formulation of [3] has been generalized to the missile/plume problem and results have been presented in a recent report (Wu, et. al. [3]). However, some stability problems have been encountered in these formulations for certain geometries, whereas the formulation presented in Section II has been extensively tested and has demonstrated no apparent stability problems. More importantly, however, this formulation and computer code serves as a basic building block from which an extremely complicated code is to be developed which treats a very sophisticated model of the missile/plume structure in which the inhomogeneous plume is modeled by layers of homogeneous material. This formulation will be described in a forthcoming report.

The numerical solution procedure for a body of revolution, which may be either a perfectly conducting body or a dielectric body, is presented in Section II. Numerical results for several cases are also presented and discussed. In Appendix B the implementation of the numerical formulation is discussed with direct reference to an available computer code which is also listed in the appendix.

Section II

NUMERICAL SOLUTION PROCEDURE FOR A BODY OF REVOLUTION

In this section integral equations are derived for equivalent surface currents induced on a dielectric body of revolution subject to plane wave illumination. Scattering by a perfectly conducting body of revolution can also be obtained as a special case. The numerical solution procedures are described and the method of moments is applied. Numerical results are obtained for several cases and are compared with other available data.

2.1 Formulation of the Integral Equations

Consider the body of revolution of Fig. 2.1, where the body is formed by rotating a planar curve, called the "generating arc," around the z -axis (also called the axis of the body of revolution). The regions exterior and interior to the body are denoted as regions 1 and 2, respectively. The t -coordinate, which is depicted in Fig. 2.1, follows the generating arc on the body surface S . The body, with constitutive parameters ($\mu_2, \epsilon_2, \sigma_2 = 0$), is considered

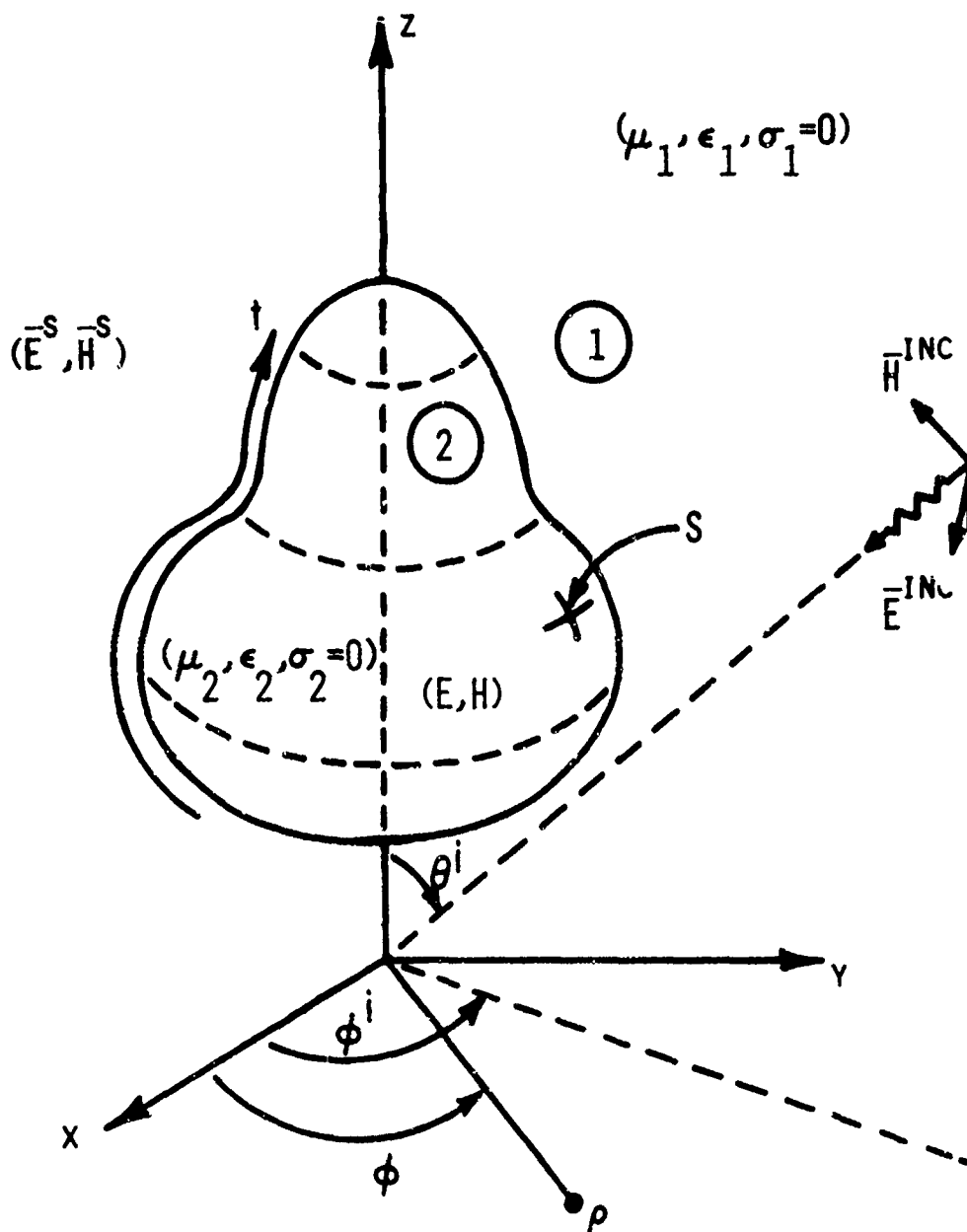


Figure 2.1. Geometry of the body of revolution.

to be immersed in an infinite homogeneous medium with parameters $(\mu_1, \epsilon_1, \sigma_1 = 0)$. The generalization to non-zero conductivity of either the body or exterior medium parameters is trivial, but has not been considered here.

Boundary conditions require that the total fields tangential to the surface of the body be continuous from Region 1 to Region 2. These conditions may be expressed as

$$\hat{n} \times (\bar{E} - \bar{E}^s) = \hat{n} \times \bar{E}^{inc} \quad (2.1a)$$

$$\hat{n} \times (\bar{H} - \bar{H}^s) = \hat{n} \times \bar{H}^{inc} \quad , \quad (2.1b)$$

where $(\bar{E}^{inc}, \bar{H}^{inc})$ constitute the incident fields, $(\bar{E}^s, \bar{H}^s)$ are the scattered fields in Region 1, and $(\bar{E}, \bar{H})$ are the total fields in region 2, and $\hat{n} = \hat{\phi} \times \hat{t}$ is the outward unit normal to the body surface. Via the equivalence principle, the body is replaced by two sets of electric currents $(\bar{J}_i, i = 1, 2)$ and magnetic currents $(\bar{M}_i, i = 1, 2)$ — one set just inside the surface, and the other just outside the surface. Each of the two sets of currents radiates in an infinite homogeneous medium having the constitutive parameters with the corresponding medium index, $i = 1, 2$. Thus the fields indicated in (2.1) may be expressed as

$$\bar{E}^s(\bar{r}) = -j\omega\bar{A}_1(\bar{r}) - \nabla\phi_1(\bar{r}) - \frac{1}{\epsilon_1} \nabla \times \bar{F}_1(\bar{r}) \quad (2.2a)$$

$$\vec{H}^s(\vec{r}) = -j\omega\vec{F}_1(\vec{r}) - \nabla\psi_1(\vec{r}) + \frac{1}{\mu_1} \nabla \times \vec{A}_1(\vec{r}) \quad (2.2b)$$

$$\vec{E}(\vec{r}) = -j\omega\vec{A}_2(\vec{r}) - \nabla\phi_2(\vec{r}) - \frac{1}{\epsilon_2} \nabla \times \vec{F}_2(\vec{r}) \quad (2.2c)$$

$$\vec{H}(\vec{r}) = -j\omega\vec{F}_2(\vec{r}) - \nabla\psi_2(\vec{r}) + \frac{1}{\mu_2} \nabla \times \vec{A}_2(\vec{r}) , \quad (2.2d)$$

where the potentials are defined by

$$\vec{A}_1(\vec{r}) = \frac{\mu_1}{4\pi} \iint_S \vec{J}_1(\vec{r}') G_1(\vec{r}, \vec{r}') dS' \quad (2.3a)$$

$$\vec{F}_1(\vec{r}) = \frac{\epsilon_1}{4\pi} \iint_S \vec{M}_1(\vec{r}') G_1(\vec{r}, \vec{r}') dS' \quad (2.3b)$$

$$\phi_1(\vec{r}) = \frac{1}{4\pi\epsilon_1} \iint_S \rho_1^e(\vec{r}') G_1(\vec{r}, \vec{r}') dS' \quad (2.3c)$$

$$\psi_1(\vec{r}) = \frac{1}{4\pi\mu_1} \iint_S \rho_1^m(\vec{r}') G_1(\vec{r}, \vec{r}') dS' , \quad (2.3d)$$

$$i = 1, 2 ,$$

and where

$$G_i(\bar{r}, \bar{r}') = \frac{e^{-jk_i R}}{R}, \quad (2.4a)$$

$$i = 1, 2,$$

$$R = |\bar{r} - \bar{r}'| = \left[\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi') + (z - z')^2 \right]^{1/2}. \quad (2.4b)$$

The continuity of the tangential fields at the boundary requires that

$$\bar{J}_1 = -\bar{J}_2 \equiv \bar{J} \quad (2.5a)$$

$$\bar{M}_1 = -\bar{M}_2 \equiv \bar{M} \quad (2.5b)$$

Note that we have introduced unsubscripted currents $\bar{J}$ and $\bar{M}$. One may then also define an unsubscripted charge by

$$\rho^e \equiv \rho_1^e = -\rho_2^e = \frac{1}{\omega} [\nabla'_s \cdot \bar{J}(\bar{r}')] \quad (2.6a)$$

$$\rho^m \equiv \rho_1^m = -\rho_2^m = \frac{1}{\omega} [\nabla'_s \cdot \bar{M}(\bar{r}')] \quad (2.6b)$$

Combining Eqs. (2.1) through (2.6) allows one to write two vector integro-differential equations which may be enforced on the surface of the body to obtain the unknown electric and magnetic currents:

$$\begin{aligned}
\hat{n} \times \bar{E}^{inc} = & \hat{n} \times \left\{ \frac{j\omega}{4\pi} \iint_S \bar{J} (\mu_1 G_1 + \mu_2 G_2) dS' \right. \\
& + \frac{1}{4\pi\omega} \nabla \iint_S (\nabla'_s \cdot \bar{J}) \left[\frac{G_1}{\epsilon_1} + \frac{G_2}{\epsilon_2} \right] dS' \\
& \left. + \frac{1}{4\pi} \nabla \times \iint_S \bar{M} (G_1 + G_2) dS' \right\} \quad (2.7a)
\end{aligned}$$

$$\begin{aligned}
\hat{n} \times \bar{H}^{inc} = & \hat{n} \times \left\{ \frac{j\omega}{4\pi} \iint_S \bar{M} (\epsilon_1 G_1 + \epsilon_2 G_2) dS' \right. \\
& + \frac{1}{4\pi\omega} \nabla \iint_S (\nabla'_s \cdot \bar{M}) \left[\frac{G_1}{\mu_1} + \frac{G_2}{\mu_2} \right] dS' \\
& \left. - \frac{1}{4\pi} \nabla \times \iint_S \bar{J} (G_1 + G_2) dS' \right\} , \quad (2.7b)
\end{aligned}$$

where the dependence of the appropriate quantities on source and/or field coordinates is understood. Since the term involving the curl operator appearing in (2.7) is not continuous at the boundary, the direct interchange of integration and differentiation is not allowed. It can be shown [4],

however, that on the surface S ,

$$\nabla \times \iint_S \bar{U}(G_1 + G_2) dS' = - \iint_S \bar{U} \times \nabla(G_1 + G_2) dS' , \quad (2.8)$$

$\bar{U} = \bar{J} \text{ or } \bar{M},$

where $\iint_S$ represents a Cauchy Principal Value integral.

Eqs. (2.7) can be written in a component operator form as

$$E_t^{inc} = \beta_{11}(J_t) + \beta_{12}(J_\phi) + \beta_{13}(M_t) + \beta_{14}(M_\phi) \quad (2.9a)$$

$$E_\phi^{inc} = \beta_{21}(J_t) + \beta_{22}(J_\phi) + \beta_{23}(M_t) + \beta_{24}(M_\phi) \quad (2.9b)$$

$$H_t^{inc} = \beta_{31}(J_t) + \beta_{32}(J_\phi) + \beta_{33}(M_t) + \beta_{34}(M_\phi) \quad (2.9c)$$

$$H_\phi^{inc} = \beta_{41}(J_t) + \beta_{42}(J_\phi) + \beta_{43}(M_t) + \beta_{44}(M_\phi) , \quad (2.9d)$$

where β_{ij} is the appropriate integro-differential operator.

It is desirable to express all of the quantities of (2.9)

in terms of the local coordinates (t, ϕ) on the body surface.

An orthogonal triad of unit vectors $(\hat{n}, \hat{\phi}, \hat{t})$ may also be

associated with each coordinate point (t, ϕ) , where $\hat{n}$, $\hat{\phi}$,

and $\hat{t}$ are defined as follows:

$$\hat{n} = \cos\gamma\cos\phi \hat{x} + \cos\gamma\sin\phi \hat{y} - \sin\gamma \hat{z} \quad (2.10a)$$

$$\hat{\phi} = -\sin\phi \hat{x} + \cos\phi \hat{y} \quad (2.10b)$$

$$\hat{t} = \sin\gamma\cos\phi \hat{x} + \sin\gamma\sin\phi \hat{y} + \cos\gamma \hat{z} , \quad (2.10c)$$

where γ is the angle between the tangent to the generating curve, $\hat{t}$, and the z-axis, defined to be positive if $\hat{t}$ points away from the z-axis and negative if $\hat{t}$ points toward the z-axis. We note that the surface divergence in this coordinate system becomes

$$\nabla'_s \cdot \bar{U} = \frac{1}{\rho'} \frac{\partial}{\partial t'} (\rho' U_t) + \frac{1}{\rho'} \frac{\partial}{\partial \phi'} (U_\phi) \quad , \quad (2.11)$$

$$\bar{U} = \bar{J} \text{ or } \bar{M}.$$

One may now expand (2.7) into components and compare to (2.9) to obtain expressions for the β_{ij} :

$$\begin{aligned} \beta_{11}(J_t) = & \frac{j\omega}{4\pi} \iint_S J_t [\sin\gamma \sin\gamma' \cos(\phi' - \phi) + \cos\gamma \cos\gamma'] \{\mu_1 G_1 + \mu_2 G_2\} dS' \\ & + \frac{1}{4\pi\omega} \frac{\partial}{\partial t} \iint_S \frac{1}{\rho'} \frac{\partial}{\partial t'} (\rho' J_t) \left\{ \frac{G_1}{\epsilon_1} + \frac{G_2}{\epsilon_2} \right\} dS' \quad (2.12a) \end{aligned}$$

$$\begin{aligned} \beta_{12}(J_\phi) = & -\frac{j\omega}{4\pi} \iint_S J_\phi \sin\gamma \sin(\phi' - \phi) \{\mu_1 G_1 + \mu_2 G_2\} dS' \\ & + \frac{1}{4\pi\omega} \frac{\partial}{\partial t} \iint_S \frac{1}{\rho'} \frac{\partial}{\partial \phi'} (J_\phi) \left\{ \frac{G_1}{\epsilon_1} + \frac{G_2}{\epsilon_2} \right\} dS' \quad (2.12b) \end{aligned}$$

$$\beta_{13}(M_t) = -\frac{1}{4\pi} \iint_S \frac{M_t}{R} [(\rho' \sin \gamma \cos \gamma' - \rho \cos \gamma \sin \gamma') \sin(\phi' - \phi) + (z - z') \sin \gamma \sin \gamma' \sin(\phi' - \phi)] \frac{d}{dR} \{G_1 + G_2\} dS' \quad (2.12c)$$

$$\beta_{14}(M_\phi) = -\frac{1}{4\pi} \iint_S \frac{M_\phi}{R} [\rho' \cos \gamma - \rho \cos \gamma \cos(\phi' - \phi) + (z - z') \sin \gamma \cos(\phi' - \phi)] \frac{d}{dR} \{G_1 + G_2\} dS' \quad (2.12d)$$

$$\beta_{21}(J_t) = \frac{j\omega}{4\pi} \iint_S J_t \sin \gamma' \sin(\phi' - \phi) \{\mu_1 G_1 + \mu_2 G_2\} dS' + \frac{j}{4\pi\omega\rho} \frac{\partial}{\partial \phi} \iint_S \frac{1}{\rho'} \frac{\partial}{\partial t'} (\rho' J_t) \left\{ \frac{G_1}{\epsilon_1} + \frac{G_2}{\epsilon_2} \right\} dS' \quad (2.12e)$$

$$\beta_{22}(J_\phi) = \frac{j\omega}{4\pi} \iint_S J_\phi \cos(\phi' - \phi) \{\mu_1 G_1 + \mu_2 G_2\} dS' + \frac{j}{4\pi\omega\rho} \frac{\partial}{\partial \phi} \iint_S \frac{1}{\rho'} \frac{\partial}{\partial \phi'} (J_\phi) \left\{ \frac{G_1}{\epsilon_1} + \frac{G_2}{\epsilon_2} \right\} dS' \quad (2.12f)$$

$$\beta_{23}(M_t) = -\frac{1}{4\pi} \iint_S \frac{M_t}{R} [\rho \cos \gamma' - \rho' \cos \gamma' \cos(\phi' - \phi) - (z - z') \sin \gamma' \cos(\phi' - \phi)] \frac{d}{dR} \{G_1 + G_2\} dS' \quad (2.12g)$$

$$\beta_{24}(M_\phi) = -\frac{1}{4\pi} \iint_S \frac{M_\phi}{R} (z - z') \sin(\phi' - \phi) \frac{d}{dR} \{G_1 + G_2\} dS' \quad , \quad (2.12h)$$

where R is defined by (2.4), and

$$\frac{dG_i}{dR} = -\frac{(1 + jk_i R)}{R^2} e^{-jk_i R} \quad , \quad (2.13)$$

$i = 1, 2 \quad .$

Eqs. (2.12) define the β_{ij} for (2.9a) and (2.9b). Recall that $\beta_{ij}(U)$ is an operator on the function U . One may also consider β_{ij} , $i = 1, 2$, $j = 1, 2$, to be dependent on the parameters of the media, i.e.

$$\beta_{ij}(U) = \beta_{ij}(U; \mu_1, \epsilon_1, \mu_2, \epsilon_2) \quad ,$$

$i = 1, 2,$
 $j = 1, 2.$

Similarly,

$$\beta_{ij}(U) = \beta_{ij}(U; \mu_1, \epsilon_1, \mu_2, \epsilon_2) \quad ,$$

$i = 3, 4,$
 $j = 3, 4.$

One then finds that

$$\beta_{33}(M_t; \mu_1, \epsilon_1, \mu_2, \epsilon_2) = \beta_{11}(M_t; \epsilon_1, \mu_1, \epsilon_2, \mu_2) \quad (2.14a)$$

$$\beta_{34}(M_\phi; \mu_1, \epsilon_1, \mu_2, \epsilon_2) = \beta_{12}(M_\phi; \epsilon_1, \mu_1, \epsilon_2, \mu_2) \quad (2.14b)$$

$$\beta_{43}(M_t; \mu_1, \epsilon_1, \mu_2, \epsilon_2) = \beta_{21}(M_t; \epsilon_1, \mu_1, \epsilon_2, \mu_2) \quad (2.14c)$$

$$\beta_{44}(M_\phi; \mu_1, \epsilon_1, \mu_2, \epsilon_2) = \beta_{22}(M_\phi; \epsilon_1, \mu_1, \epsilon_2, \mu_2) \quad (2.14d)$$

and

$$\beta_{31}(J_t) = -\beta_{13}(J_t) \quad (2.14e)$$

$$\beta_{32}(J_\phi) = -\beta_{14}(J_\phi) \quad (2.14f)$$

$$\beta_{41}(J_t) = -\beta_{23}(J_t) \quad (2.14g)$$

$$\beta_{42}(J_\phi) = -\beta_{24}(J_\phi) \quad (2.14h)$$

Thus Eqs. (2.14), which are actually statements of duality [5], serve to define the β_{ij} in (2.9c) and (2.9d) via Eqs. (2.12). Furthermore, if the body is a perfect conductor, (2.9) reduces to

$$E_t^{inc} = \beta_{11}(J_t; \mu_1, \epsilon_1, \mu_2=0, \epsilon_2=\infty) + \beta_{12}(J_\phi; \mu_1, \epsilon_1, \mu_2=0, \epsilon_2=\infty) \quad (2.15a)$$

$$E_\phi^{inc} = \beta_{21}(J_t; \mu_1, \epsilon_1, \mu_2=0, \epsilon_2=\infty) + \beta_{22}(J_\phi; \mu_1, \epsilon_1, \mu_2=0, \epsilon_2=\infty) \quad (2.15b)$$

For a numerical solution of (2.9) or (2.15) the generating arc is approximated as a sequence of linear segments as depicted in Fig.(2.2) where the approximation to the generating arc is shown in the plane $\phi = 0$. This segmented generating arc is rotated about the z -axis to obtain an approximation to the surface of the body of revolution. The points $t_0, t_1, \dots, t_{N+1}$ specify the end points of the linear segments approximating the generating arc and are written in terms of the coordinates ρ and z . The "half points" $t_{1/2}, t_{1 1/2}, \dots, t_{N+1/2}$ are defined as

$$t_{n-1/2} = \frac{(t_n + t_{n-1})}{2}, \quad (2.16)$$

$$1 \leq n \leq N+1.$$

The variations of the unknown electric and magnetic currents flowing on the surface are approximated by pulse functions in the t -direction and are expanded in Fourier series in the ϕ -direction. The expansion of the electric current is given by

$$\bar{J}(t', \phi') \approx \frac{\hat{t}}{2\pi\rho'} \sum_{m=-\infty}^{\infty} \sum_{n=1}^N I_t^{mn} P_1^n(t') e^{jm\phi'} + \hat{\phi} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{N+1} J_\phi^{mn} P_2^n(t') e^{jm\phi'}. \quad (2.17a)$$

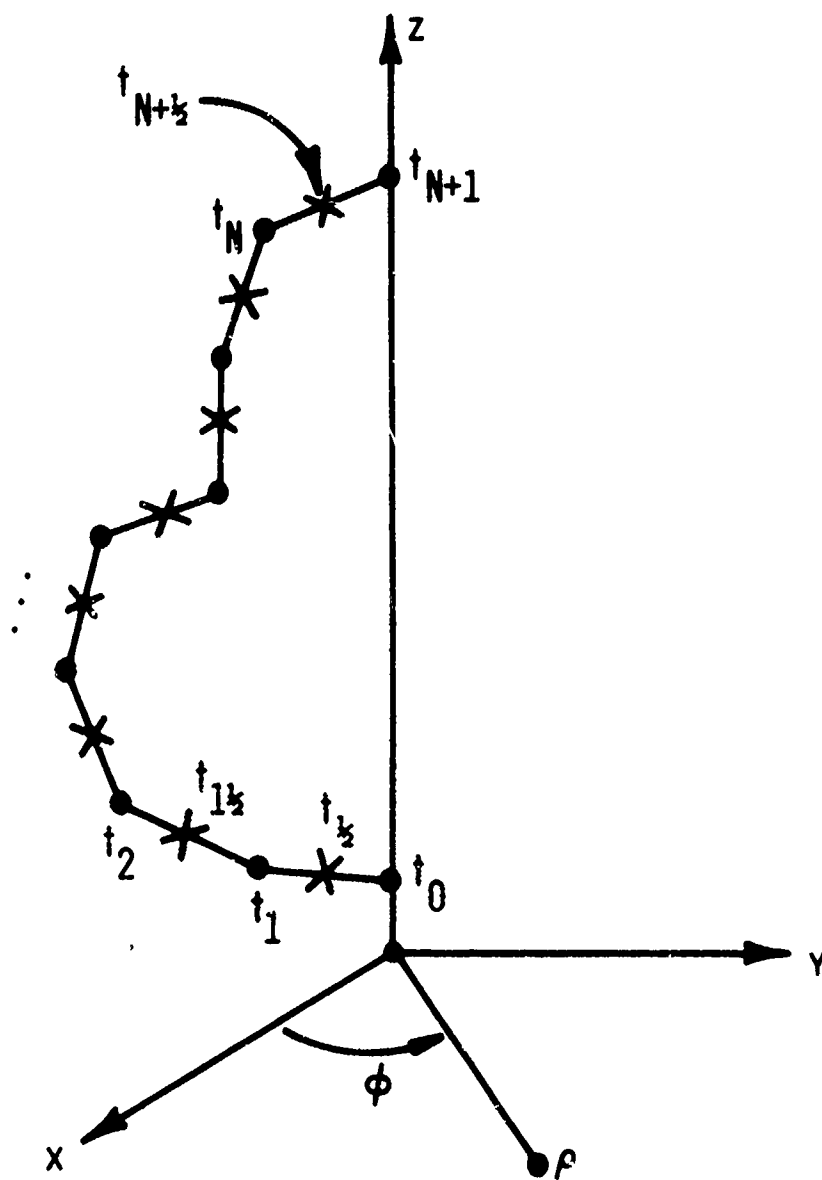


Figure 2.2. Approximation of the generating arc by linear segments.

The derivative of J_t with respect to t , which contributes to the charge, is approximated as

$$\frac{\partial}{\partial t'}[\rho' J_t(t', \phi')] \approx \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{N+1} \left\{ \frac{I_t^{mn} - I_t^{m,n-1}}{\Delta t_n} \right\} P_2^n(t') e^{jm\phi'} . \quad (2.17b)$$

In the preceding,

$$P_1^n(t') = \begin{cases} 1, & t_{n-\frac{1}{2}} \leq t' \leq t_{n+\frac{1}{2}} \\ 0, & \text{otherwise} \end{cases} , \quad (2.18a)$$

$$P_2^n(t') = \begin{cases} 1, & t_{n-1} \leq t' \leq t_n \\ 0, & \text{otherwise} \end{cases} , \quad (2.18b)$$

$$\begin{aligned} \Delta t_n &= |t_n - t_{n-1}| \\ &= [(\rho_n - \rho_{n-1})^2 + (z_n - z_{n-1})^2]^{\frac{1}{2}} , \end{aligned} \quad (2.19)$$

and I_t^{mn} is the coefficient of the "total" current flowing in the t -direction as defined by

$$I_t(t', \phi') = 2\pi \rho' J_t(t', \phi') . \quad (2.20)$$

This "total" current has no physical meaning (except for $m = 0$; for $m \neq 0$, the integrated current vanishes) but, as a mathematical concept, its use is advantageous in the

solution procedure. In (2.17b) it is assumed that

$$I_t^{m,0} \equiv I_t^{m,N+1} \equiv 0.$$

The magnetic current $\bar{M}(t',\phi')$ and its derivative, $\frac{\partial}{\partial t'}[\rho' M_t(t',\phi')]$, are defined similarly with $K_t^{mn} = 2\pi\rho' M_t^{mn}$ and M_ϕ^{mn} replacing I_t^{mn} and J_ϕ^{mn} , respectively.

The current expansion scheme (2.17) has a number of advantages. The first, and most obvious, is that the Fourier components of the current can be decoupled; thus one can solve for each unknown Fourier component of the current distribution independently. The remaining advantages arise from the use of the staggered pulse basis sets indicated by (2.18). To further illustrate some of these advantages we consider the case of a perfectly conducting cylinder (Fig. 2.3) which has one closed end and one open end. Note that the "total" current flowing in the t -direction (I_t) is zero for all Fourier components at $t=0$ and $t=b$ [6,7] and should therefore be well represented in the neighborhood of these two points by half pulses with a zero coefficient, as in the cases of the TE strip and the rectangular bent plate described in other works [8,9]. On the other hand, the current density in the ϕ -direction (J_ϕ) approaches zero ($m \neq \pm 1$) or a constant ($m = \pm 1$) as $t \rightarrow 0$ [6,7]

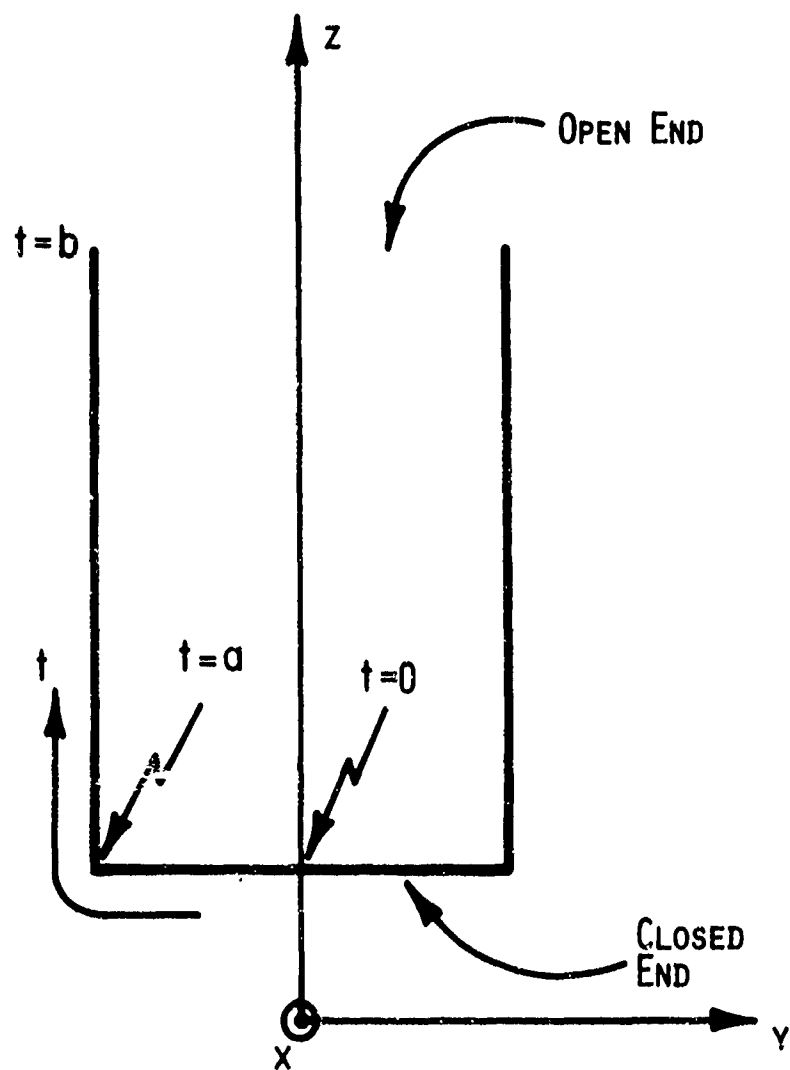


Figure 2.3. Cross section of a perfectly conducting cylinder with one open end.

and is always singular at $t = b$, as required by the so-called edge condition [10]. It has been demonstrated in the previous works [8,9] that the staggered subdomain scheme, such as (2.17) and (2.18), employing full pulses to represent J_ϕ adjacent to the points $t = 0$ and $t = b$, can accurately model the singular currents near edges and should clearly model a finite current as well. The full pulse, however, may not model a current which approaches zero well, but in view of the fact that such a situation occurs only at points where the surface intersects the axis of the body of revolution ($\rho = 0$) and that the current moment represented by the pulse is therefore relatively small, this deficiency should not significantly affect integral functionals of the current such as radar cross section, etc. Thus the staggered pulse-Fourier series basis set enjoys the same advantages described for the staggered subdomain scheme on the rectangular bent plate, namely, the current expansion ensures that t -directed components of current vanish at knife-like edges and are continuous at structure edges (sharp bends), and that the ϕ -directed current and the charge, both of which are singular at an edge, are allowed to vary independently on opposite sides of an edge. The basis set (2.17) therefore allows one to model both open and closed bodies, and bodies with sharp edges, with no special procedures required

at edges or points where the surface intersects the body axis. Of course, a *dielectric* body has no knife-like edges, but it may have structure edges at which, depending on the parameters of the medium, it may be necessary to represent currents which are singular [11]. The basis set (2.17), which is used for both the electric and magnetic currents, provides accurate modeling in the vicinity of such edges.

We next define the testing functions

$$T_1^{pq}(t, \phi) = P_1^q(t) e^{-jP\phi} \quad (2.21a)$$

$$T_2^{pq}(t, \phi) = P_2^q(t) e^{-jP\phi} \quad (2.21b)$$

Eqs. (2.9a) and (2.9c) are tested with (2.21a), while (2.9b) and (2.9d) are tested with (2.21b). The testing procedure results in two surface integrations being required for each β_{ij} . However, we note that the current has already been expanded as a Fourier series in the variable ϕ' . Likewise the various kernel terms appearing in (2.12) may be expanded in Fourier series of the form

$$\frac{e^{-jk_1 R_0}}{R_0} = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} G_{1m} e^{jm(\phi - \phi')} ,$$

$i = 1, 2 ,$

and

$$\frac{1}{R_0} \frac{d}{dR_0} \frac{e^{-jk_1 R_0}}{R_0} = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} G'_{1m} e^{jm(\phi - \phi')} ,$$

$i = 1, 2 ,$

with Fourier coefficients

$$G_{1m} = \int_{-\pi}^{\pi} \frac{e^{-jk_1 R_0}}{R_0} \cos(m\xi) d\xi \quad (2.22a)$$

$$G'_{1m} = \int_{-\pi}^{\pi} \frac{1}{R_0} \frac{d}{dR_0} \left\{ \frac{e^{-jk_1 R_0}}{R_0} \right\} \cos(m\xi) d\xi , \quad (2.22b)$$

where

$$R_0 = [\rho^2 + \rho'^2 - 2\rho\rho'\cos\xi + (z - z')^2]^{\frac{1}{2}} . \quad (2.22c)$$

Testing with the testing functions (2.21) permits all integrations in the variables ϕ and ϕ' to be carried out analytically, simultaneously decoupling the equations with respect to their dependence on the indices p and m . The double integral in the t -direction is reduced to a single integral via appropriate approximations (such as used in [8.9]). We also note that derivatives with respect to ϕ and ϕ' can be performed analytically during the testing procedure. By means of such approximations and procedures, one obtains

expressions for the elements $\beta_{ij_m}^{qn}$ of the generalized impedance matrix, where the subscript m refers to the Fourier coefficient index and the superscripts q and n refer to the indices of the field point and the current source, respectively. For convenience in writing these expressions, we first define two frequently used integral functions:

$$\psi_1(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} G_{1_m}(t_q, t') dt' \quad (2.23a)$$

$$\psi_1^0(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} G_{1_m}(t_q, t') \rho' dt' , \quad (2.23b)$$

where G_{1_m} is defined by (2.22). We also introduce the auxiliary "weight" functions arising from testing in the t -direction:

$$\chi_s(\Delta t_q, \gamma_q) = \frac{\Delta t_{q+1} \sin \gamma_{q+1} + \Delta t_q \sin \gamma_q}{2} \quad (2.24a)$$

$$\chi_c(\Delta t_q, \gamma_q) = \frac{\Delta t_{q+1} \cos \gamma_{q+1} + \Delta t_q \cos \gamma_q}{2} . \quad (2.24b)$$

The expressions for the elements of the generalized impedance matrix are given on the following pages:

$$\begin{aligned}
\beta_{11}^{qn} = & \frac{j\omega}{8\pi} \sin \gamma_n \chi_s (\Delta t_q, \gamma_q) [\mu_1 \psi_1(t_{n-k_2}, t_n; t_q, m+1) + \mu_2 \psi_2(t_{n-k_2}, t_n; t_q, m+1) \\
& + \mu_1 \psi_1(t_{n-k_2}, t_n; t_q, m-1) + \mu_2 \psi_2(t_{n-k_2}, t_n; t_q, m-1)] \\
& + \frac{j\omega}{8\pi} \sin \gamma_{n+1} \chi_s (\Delta t_q, \gamma_q) [\mu_1 \psi_1(t_n, t_{n+k_2}; t_q, m+1) + \mu_2 \psi_2(t_n, t_{n+k_2}; t_q, m+1) \\
& + \mu_1 \psi_1(t_n, t_{n+k_2}; t_q, m-1) + \mu_2 \psi_2(t_n, t_{n+k_2}; t_q, m-1)] \\
& + \frac{j\omega}{4\pi} \cos \gamma_n \chi_c (\Delta t_q, \gamma_q) [\mu_1 \psi_1(t_{n-k_2}, t_n; t_q, m) + \mu_2 \psi_2(t_{n-k_2}, t_n; t_q, m)] \\
& + \frac{j\omega}{4\pi} \cos \gamma_{n+1} \chi_c (\Delta t_q, \gamma_q) [\mu_1 \psi_1(t_n, t_{n+k_2}; t_q, m) + \mu_2 \psi_2(t_n, t_{n+k_2}; t_q, m)] \\
& + \frac{j}{4\pi\omega\Delta t_n} \left[\frac{1}{\varepsilon_1} \psi_1(t_{n-1}, t_n; t_{q+k_2}, m) + \frac{1}{\varepsilon_2} \psi_2(t_{n-1}, t_n; t_{q+k_2}, m) - \frac{1}{\varepsilon_1} \psi_1(t_{n-1}, t_n; t_{q-k_2}, m) \right. \\
& \quad \left. - \frac{1}{\varepsilon_2} \psi_2(t_{n-1}, t_n; t_{q-k_2}, m) \right] \\
& - \frac{j}{4\pi\omega\Delta t_{n+1}} \left[\frac{1}{\varepsilon_1} \psi_1(t_n, t_{n+1}; t_{q+k_2}, m) + \frac{1}{\varepsilon_2} \psi_2(t_n, t_{n+1}; t_{q+k_2}, m) - \frac{1}{\varepsilon_1} \psi_1(t_n, t_{n+1}; t_{q-k_2}, m) \right. \\
& \quad \left. - \frac{1}{\varepsilon_2} \psi_2(t_n, t_{n+1}; t_{q-k_2}, m) \right] ,
\end{aligned} \tag{2.25a}$$

$$\begin{aligned}
q &= 1, 2, \dots, N, \\
n &= 1, 2, \dots, N,
\end{aligned}$$

$$\beta_{12}^{qn} = -k\omega\chi_s(\Delta t_q, \gamma_q) [\mu_1 \psi_1^\rho(t_{n-1}, t_n; t_q, m+1) + \mu_2 \psi_2^\rho(t_{n-1}, t_n; t_q, m+1)]$$

$$- \mu_1 \psi_1^\rho(t_{n-1}, t_n; t_q, m-1) - \mu_2 \psi_2^\rho(t_{n-1}, t_n; t_q, m-1)]$$

$$- \frac{m}{2\omega} \left[\frac{1}{\varepsilon_1} \psi_1(t_{n-1}, t_n; t_{q+k_2}, m) + \frac{1}{\varepsilon_2} \psi_2(t_{n-1}, t_n; t_{q+k_2}, m) - \frac{1}{\varepsilon_1} \psi_1(t_{n-1}, t_n; t_{q-k_2}, m) \right]$$

$$- \frac{1}{\varepsilon_2} \psi_2(t_{n-1}, t_n; t_{q-k_2}, m)] ,$$

(2.25b)

$$\begin{aligned} q &= 1, 2, \dots, N, \\ n &= 1, 2, \dots, N+1, \end{aligned}$$

$$\beta_{21}^{qn} = \frac{\omega}{8\pi} \Delta t_q \sin \gamma_n [\mu_1 \psi_1(t_{n-k_2}, t_n; t_{q-k_2}, m+1) + \mu_2 \psi_2(t_{n-k_2}, t_n; t_{q-k_2}, m+1)]$$

$$- \mu_1 \psi_1(t_{n-k_2}, t_n; t_{q-k_2}, m-1) - \mu_2 \psi_2(t_{n-k_2}, t_n; t_{q-k_2}, m-1)]$$

$$+ \frac{\omega}{8\pi} \Delta t_q \sin \gamma_{n+1} [\mu_1 \psi_1(t_n, t_{n+k_2}; t_{q-k_2}, m+1) + \mu_2 \psi_2(t_n, t_{n+k_2}; t_{q-k_2}, m+1)]$$

$$- \mu_1 \psi_1(t_n, t_{n+k_2}; t_{q-k_2}, m-1) - \mu_2 \psi_2(t_n, t_{n+k_2}; t_{q-k_2}, m-1)]$$

$$- \frac{m \Delta t_q}{4\pi \omega \rho_{q-k_2}} \left[\frac{1}{\varepsilon_1} \psi_1(t_{n-1}, t_n; t_{q-k_2}, m) + \frac{1}{\varepsilon_2} \psi_2(t_{n-1}, t_n; t_{q-k_2}, m) \right]$$

$$+ \frac{m \Delta t_q}{4\pi \omega \rho_{q-k_2}} \left[\frac{1}{\varepsilon_1} \psi_1(t_n, t_{n+1}; t_{q-k_2}, m) + \frac{1}{\varepsilon_2} \psi_2(t_n, t_{n+1}; t_{q-k_2}, m) \right] ,$$

(2.25c)

$$\begin{aligned} q &= 1, 2, \dots, N+1, \\ n &= 1, 2, \dots, N, \end{aligned}$$

$$\begin{aligned}
\beta_{22}^{qn} = & k_2 j \omega \Delta t_q [\mu_1 \psi_1^\rho(t_{n-1}, t_n; t_{q-k_2}, m+1) + \mu_2 \psi_2^\rho(t_{n-1}, t_n; t_{q-k_2}, m+1) \\
& + \mu_1 \psi_1^\rho(t_{n-1}, t_n; t_{q-k_2}, m-1) + \mu_2 \psi_2^\rho(t_{n-1}, t_n; t_{q-k_2}, m-1)] \\
& - \frac{j m^2 \Delta t_q}{2 \omega \rho_{q-k_2}} \left[\frac{1}{\varepsilon_1} \psi_1(t_{n-1}, t_n; t_{q-k_2}, m) + \frac{1}{\varepsilon_2} \psi_2(t_{n-1}, t_n; t_{q-k_2}, m) \right], \quad (2.25d) \\
& \quad \begin{matrix} q = 1, 2, \dots, N+1, \\ n = 1, 2, \dots, N+1, \end{matrix}
\end{aligned}$$

$$\begin{aligned}
\beta_{13}^{qn} = & -j \frac{\cos \gamma_n}{4\pi} \chi_s(\Delta t_q, \gamma_q) U_3^\rho(t_{n-k_2}, t_n; t_q, m) - j \frac{\cos \gamma_{n+1}}{4\pi} \chi_s(\Delta t_q, \gamma_q) U_3^\rho(t_n, t_{n+k_2}; t_q, m) \\
& + j \rho_q \frac{\sin \gamma_n}{4\pi} \chi_c(\Delta t_q, \gamma_q) U_0(t_{n-k_2}, t_n; t_q, m) + j \rho_q \frac{\sin \gamma_{n+1}}{4\pi} \chi_c(\Delta t_q, \gamma_q) U_0(t_n, t_{n+k_2}; t_q, m) \\
& - j \frac{\sin \gamma_n}{4\pi} \chi_s(\Delta t_q, \gamma_q) U_1(t_{n-k_2}, t_n; t_q, m) - j \frac{\sin \gamma_{n+1}}{4\pi} \chi_s(\Delta t_q, \gamma_q) U_1(t_n, t_{n+k_2}; t_q, m), \quad (2.25e) \\
& \quad \begin{matrix} q = 1, 2, \dots, N, \\ n = 1, 2, \dots, N, \end{matrix}
\end{aligned}$$

$$\begin{aligned}
\beta_{14}^{qn} = & k_2 \chi_c(\Delta t_q, \gamma_q) U_4^\rho(t_{n-1}, t_n; t_q, m) - \rho_q \chi_c(\Delta t_q, \gamma_q) U_5^\rho(t_{n-1}, t_n; t_q, m) \\
& - k_2 \chi_s(\Delta t_q, \gamma_q) U_2^\rho(t_{n-1}, t_n; t_q, m), \quad (2.25z) \\
& \quad \begin{matrix} q = 1, 2, \dots, N, \\ n = 1, 2, \dots, N+1, \end{matrix}
\end{aligned}$$

$$\begin{aligned}
\beta_{23}^{qn} = & -\frac{\rho}{2\pi} \frac{q-k_2}{2\pi} \cos \gamma_n \Delta t U_q 5(t_{n-k_2}, t_n; t_{q-k_2}, m) - \frac{\rho}{2\pi} \frac{q-k_2}{2\pi} \cos \gamma_{n+1} \Delta t U_q 5(t_n, t_{n+k_2}; t_{q-k_2}, m) \\
& - \frac{1}{4\pi} \cos \gamma_n \Delta t U_q 6(t_{n-k_2}, t_n; t_{q-k_2}, m) - \frac{1}{4\pi} \cos \gamma_{n+1} \Delta t U_q 6(t_n, t_{n+k_2}; t_{q-k_2}, m) \\
& + \frac{1}{4\pi} \sin \gamma_n \Delta t U_q 2(t_{n-k_2}, t_n; t_{q-k_2}, m) + \frac{1}{4\pi} \sin \gamma_{n+1} \Delta t U_q 2(t_n, t_{n+k_2}; t_{q-k_2}, m), \quad (2.25g)
\end{aligned}$$

$q = 1, 2, \dots, N+1,$
 $n = 1, 2, \dots, N,$

$$\begin{aligned}
\beta_{24}^{qn} = & -\frac{k_2 j \Delta t}{2\pi} U_q 1^\rho(t_{n-1}, t_n; t_{q-k_2}, m) \cdot \quad (2.25h) \\
& q = 1, 2, \dots, N+1, \\
& n = 1, 2, \dots, N+1.
\end{aligned}$$

The U 's in (2.25) are integral functions defined by

$$U_0(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{\sin(m\xi) \sin \xi}{R_0} \frac{dG}{dR_0} d\xi dt' \quad (2.26a)$$

$$U_1(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{(z - z')}{R_0} \sin(m\xi) \sin \xi \frac{dG}{dR_0} d\xi dt' \quad (2.26b)$$

$$U_1^\rho(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{(z - z')}{R_0} \sin(m\xi) \sin \xi \frac{dG}{dR_0} \rho' d\xi dt' \quad (2.26c)$$

$$U_2(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{(z - z')}{R_0} \cos(m\xi) \cos \xi \frac{dG}{dR_0} d\xi dt' \quad (2.26d)$$

$$U_2^\rho(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{(z - z')}{R_0} \cos(m\xi) \cos \xi \frac{dG}{dR_0} \rho' d\xi dt' \quad (2.26e)$$

$$U_3^\rho(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{\rho'}{R_0} \sin(m\xi) \sin \xi \frac{dG}{dR_0} d\xi dt' \quad (2.26f)$$

$$U_4^\rho(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{(\rho - \rho')}{R_0} \cos(m\xi) \frac{dG}{dR_0} \rho' d\xi dt' \quad (2.26g)$$

$$U_5(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{\cos(m\xi)}{R_0} \sin^2(\xi/2) \frac{dG}{dR_0} d\xi dt' \quad (2.26h)$$

$$U_5^0(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{\cos(m\xi)}{R_0} \sin^2(\xi/2) \frac{dG}{dR_0} \rho' d\xi dt' \quad (2.26i)$$

$$U_6(t_1, t_2; t_q, m) = \int_{t_1}^{t_2} \int_{-\pi}^{\pi} \frac{(\rho - \rho')}{R_0} \cos(m\xi) \cos \xi \frac{dG}{dR_0} d\xi dt', \quad (2.26j)$$

where R_0 is defined in (2.22c) and

$$G = \frac{e^{-jk_1 R_0}}{R_0} + \frac{e^{-jk_2 R_0}}{R_0} \quad (2.27)$$

The integrals defined by the U 's are actually recombinations of expressions involving the Fourier coefficients (2.22b) and are expressed in the manner shown to facilitate the singularity analysis of self terms, which appears in Appendix A. We have now completely defined the elements of the impedance matrix corresponding to the operators appearing in (2.9a) and (2.9b); the matrix operators arising from (2.9c) and (2.9d) are obtained from the functional relationships (2.14). One should note that many of the integrals which must be computed in the calculations of the elements (2.25)

appear in several places and hence can be used several times to increase efficiency.

The incident plane wave fields are expressed as

$$\vec{E}^{inc} = (E_{\theta}^i \hat{\theta}^i + E_{\phi}^i \hat{\phi}^i) e^{-jk_1 \hat{n} \cdot \vec{r}} \quad (2.28a)$$

$$\vec{H}^{inc} = \frac{1}{\eta} (E_{\phi}^i \hat{\theta}^i - E_{\theta}^i \hat{\phi}^i) e^{-jk_1 \hat{n} \cdot \vec{r}}, \quad (2.28b)$$

where

$$\hat{\theta}^i = \cos\theta^i \cos\phi^i \hat{x} + \cos\theta^i \sin\phi^i \hat{y} - \sin\theta^i \hat{z} \quad (2.29a)$$

$$\hat{\phi}^i = -\sin\phi^i \hat{x} + \cos\phi^i \hat{y} \quad (2.29b)$$

$$\hat{n} = -\sin\theta^i \cos\phi^i \hat{x} - \sin\theta^i \sin\phi^i \hat{y} - \cos\theta^i \hat{z} \quad (2.29c)$$

$$\vec{r} = \rho \cos\phi \hat{x} + \rho \sin\phi \hat{y} + z \hat{z} \quad (2.29d)$$

and η is the free space impedance. Note that $\hat{n}$ is in the direction of propagation. The elements of the forcing function vector are thus determined by finding the components tangential to the surface and testing with (2.21). With reference to (2.9), these elements are given by the expressions on the following page, where $J_m(u)$ is the Bessel function of the first kind of order m . With regard to the expressions (2.30), we comment that, with no loss in generality, one may set $\phi^i = 0$.

$$\begin{aligned}
E_{t_m}^{inc^q} = & \{E_{\theta}^i \cos \theta^i \chi_s(\Delta t_q, \gamma_q) [j^{m-1} J_{m-1}(k_1 \rho_q \sin \theta^i) + j^{m+1} J_{m+1}(k_1 \rho_q \sin \theta^i)] \\
& - 2E_{\theta}^i \sin \theta^i \chi_c(\Delta t_q, \gamma_q) j^m J_m(k_1 \rho_q \sin \theta^i) \\
& - E_{\phi}^i \chi_s(\Delta t_q, \gamma_q) j^m [J_{m-1}(k_1 \rho_q \sin \theta^i) + J_{m+1}(k_1 \rho_q \sin \theta^i)] \} \pi e^{jk_1 z \cos \theta^i} e^{-jm\phi^i}, \\
& q = 1, 2, \dots, N, \quad (2.30a)
\end{aligned}$$

$$\begin{aligned}
E_{\phi_m}^{inc^q} = & \{E_{\theta}^i j^m \cos \theta^i [J_{m-1}(k_1 \rho_{q-\frac{1}{2}} \sin \theta^i) + J_{m+1}(k_1 \rho_{q-\frac{1}{2}} \sin \theta^i)] \\
& + E_{\phi}^i [j^{m-1} J_{m-1}(k_1 \rho_{q-\frac{1}{2}} \sin \theta^i) + j^{m+1} J_{m+1}(k_1 \rho_{q-\frac{1}{2}} \sin \theta^i)] \} \pi \Delta t_q e^{jk_1 z \cos \theta^i} e^{-jm\phi^i}, \\
& q = 1, 2, \dots, N+1, \quad (2.30b)
\end{aligned}$$

$$\begin{aligned}
H_{t_m}^{inc^q} = & \{E_{\phi}^i \cos \theta^i \chi_s(\Delta t_q, \gamma_q) [j^{m-1} J_{m-1}(k_1 \rho_q \sin \theta^i) + j^{m+1} J_{m+1}(k_1 \rho_q \sin \theta^i)] \\
& - 2E_{\phi}^i \sin \theta^i \chi_c(\Delta t_q, \gamma_q) j^m J_m(k_1 \rho_q \sin \theta^i) \\
& + E_{\theta}^i \chi_s(\Delta t_q, \gamma_q) j^m [J_{m-1}(k_1 \rho_q \sin \theta^i) + J_{m+1}(k_1 \rho_q \sin \theta^i)] \} \frac{\pi}{\eta} e^{jk_1 z \cos \theta^i} e^{-jm\phi^i}, \\
& q = 1, 2, \dots, N, \quad (2.30c)
\end{aligned}$$

$$\begin{aligned}
H_{\phi_m}^{inc^q} = & \{E_{\phi}^i j^m \cos \theta^i [J_{m-1}(k_1 \rho_{q-\frac{1}{2}} \sin \theta^i) + J_{m+1}(k_1 \rho_{q-\frac{1}{2}} \sin \theta^i)] \\
& - E_{\theta}^i [j^{m-1} J_{m-1}(k_1 \rho_{q-\frac{1}{2}} \sin \theta^i) + j^{m+1} J_{m+1}(k_1 \rho_{q-\frac{1}{2}} \sin \theta^i)] \} \frac{\pi \Delta t_q}{\eta} e^{jk_1 z \cos \theta^i} e^{-jm\phi^i}, \\
& q = 1, 2, \dots, N+1. \quad (2.30d)
\end{aligned}$$

A full solution for the currents on the body of revolution consists, in general, of an infinite number of Fourier components. In practice, however, the series is truncated such that $-M \leq m \leq M$, where M is the maximum number of positive Fourier components to be calculated. Furthermore, it can be shown that the current solution for $-m$ is related to the solution for $+m$, so that it is only necessary to compute the impedance matrix and drive vector for $m = 0, 1, 2, \dots, M$. This relationship is determined by comparing the signs of the impedance matrix and a decomposed drive vector for the positive and negative Fourier component indices. If one has available the impedance submatrices β_{ijm} for positive m , then the impedance matrix for the corresponding negative Fourier component, $-m$, is given by

$$\begin{bmatrix} \beta_{11m} & -\beta_{12m} & -\beta_{13m} & \beta_{14m} \\ -\beta_{21m} & \beta_{22m} & \beta_{23m} & -\beta_{24m} \\ -\beta_{31m} & \beta_{32m} & \beta_{33m} & -\beta_{34m} \\ \beta_{41m} & -\beta_{42m} & -\beta_{43m} & \beta_{44m} \end{bmatrix} \quad (2.31)$$

The submatrices constituting the drive vector can be resolved as follows:

$$\begin{bmatrix} E_{t_m}^{inc} \\ E_{\phi_m}^{inc} \\ H_{t_m}^{inc} \\ H_{\phi_m}^{inc} \end{bmatrix} = \begin{bmatrix} \alpha_{1_m}^{\theta} \\ \alpha_{2_m}^{\theta} \\ \alpha_{3_m}^{\theta} \\ \alpha_{4_m}^{\theta} \end{bmatrix} + \begin{bmatrix} \alpha_{1_m}^{\phi} \\ \alpha_{2_m}^{\phi} \\ \alpha_{3_m}^{\phi} \\ \alpha_{4_m}^{\phi} \end{bmatrix}, \quad (2.32)$$

where the superscripts θ and ϕ refer to the components of the incident field arising from E_{θ}^1 and E_{ϕ}^1 , respectively.

We then find that

$$\begin{bmatrix} \alpha_{1_{-m}}^{\theta} \\ \alpha_{2_{-m}}^{\theta} \\ \alpha_{3_{-m}}^{\theta} \\ \alpha_{4_{-m}}^{\theta} \end{bmatrix} = \begin{bmatrix} \alpha_{1_m}^{\theta} \\ -\alpha_{2_m}^{\theta} \\ -\alpha_{3_m}^{\theta} \\ \alpha_{4_m}^{\theta} \end{bmatrix} \quad (2.33a)$$

and that

$$\begin{bmatrix} \alpha_{1_{-m}}^{\phi} \\ \alpha_{2_{-m}}^{\phi} \\ \alpha_{3_{-m}}^{\phi} \\ \alpha_{4_{-m}}^{\phi} \end{bmatrix} = \begin{bmatrix} -\alpha_{1_m}^{\phi} \\ \alpha_{2_m}^{\phi} \\ \alpha_{3_m}^{\phi} \\ -\alpha_{4_m}^{\phi} \end{bmatrix}. \quad (2.33b)$$

Thus the decomposed negative Fourier component solution vectors are related to the corresponding positive component solution vectors by

$$\begin{bmatrix} I_{t-m}^{\theta} \\ J_{\phi-m}^{\theta} \\ K_{t-m}^{\theta} \\ M_{\phi-m}^{\theta} \end{bmatrix} = \begin{bmatrix} I_{t_m}^{\theta} \\ -J_{\phi_m}^{\theta} \\ -K_{t_m}^{\theta} \\ M_{\phi_m}^{\theta} \end{bmatrix} \quad (2.34a)$$

$$\begin{bmatrix} I_{t-m}^{\phi} \\ J_{\phi-m}^{\phi} \\ K_{t-m}^{\phi} \\ M_{\phi-m}^{\phi} \end{bmatrix} = \begin{bmatrix} -I_{t_m}^{\phi} \\ J_{\phi_m}^{\phi} \\ K_{t_m}^{\phi} \\ -M_{\phi_m}^{\phi} \end{bmatrix} \quad (2.34b)$$

which may be combined to obtain

$$\begin{bmatrix} I_{t_m} \\ J_{\phi_m} \\ K_{t_m} \\ M_{\phi_m} \end{bmatrix} = \begin{bmatrix} I_{t_m}^{\theta} \\ J_{\phi_m}^{\theta} \\ K_{t_m}^{\theta} \\ M_{\phi_m}^{\theta} \end{bmatrix} + \begin{bmatrix} I_{t_m}^{\phi} \\ J_{\phi_m}^{\phi} \\ K_{t_m}^{\phi} \\ M_{\phi_m}^{\phi} \end{bmatrix} \quad (2.35a)$$

$$\begin{bmatrix} I_{t-m} \\ J_{\phi-m} \\ K_{t-m} \\ M_{\phi-m} \end{bmatrix} = \begin{bmatrix} I_{t-m}^{\theta} \\ -J_{\phi-m}^{\theta} \\ -K_{t-m}^{\theta} \\ M_{\phi-m}^{\theta} \end{bmatrix} + \begin{bmatrix} -I_{t-m}^{\phi} \\ J_{\phi-m}^{\phi} \\ K_{t-m}^{\phi} \\ -M_{\phi-m}^{\phi} \end{bmatrix} \quad (2.35b)$$

Thus Eqs. (2.9) (or (2.14) for a perfect conductor) constitute a linear system of equations for the current coefficients in (2.17). The Fourier components of the current in the problem are decoupled, however, and one may solve for each Fourier component current distribution individually, using the impedance matrix elements given by (2.25) and the elements of the forcing function vector, (2.30). The forcing function vector is then decomposed into two drive vectors as indicated by (2.32) so that one may obtain both the positively and negatively indexed Fourier components of the current distributions simultaneously, through the relationships (2.35) .

2.2 Numerical Results for the Body of Revolution

The numerical procedures described in the previous section have been incorporated into a computer code, herein referred to as "DBR," and numerical results for the surface

currents on several structures have been obtained and compared with other available data. The code is capable of treating both dielectric and perfectly conducting bodies; in the latter case, the bodies may be either open or closed structures.

All of the structures considered in this section are assumed to be illuminated by a plane wave incident along the z-axis with an electric field polarized in the x-direction. Propagation may be in either the positive or negative z-direction. For a field incident along the axis of the body of revolution, only the Fourier components with $e^{\pm j\phi}$ variation ($m = \pm 1$) are excited. Using the relationships (2.35) the surface currents can then be written in the simple form

$$\begin{aligned}\bar{J}(t, \phi) &= J_t^\theta(t) [e^{j\phi} + e^{-j\phi}] + J_\phi^\theta(t) [e^{j\phi} - e^{-j\phi}] \\ &= J_t(t) \cos \phi + j J_\phi(t) \sin \phi\end{aligned}\quad (2.36a)$$

$$\begin{aligned}\bar{M}(t, \phi) &= M_t^\theta(t) [e^{j\phi} - e^{-j\phi}] + M_\phi^\theta(t) [e^{j\phi} + e^{-j\phi}] \\ &= j M_t(t) \sin \phi + M_\phi(t) \cos \phi\end{aligned}\quad , \quad (2.36b)$$

where

$$\begin{aligned}J_p(t) &= 2J_p^\theta(t) \\ M_p(t) &= 2M_p^\theta(t) , \\ p &= t \text{ or } \phi .\end{aligned}$$

The surface currents without superscripts in (2.36) therefore represent the variation in the t -direction of the actual surface current density on the body, consisting of a sum over the excited Fourier components and observed in either the $\phi = 0^\circ$ (J_t, M_ϕ) or the $\phi = 90^\circ$ (J_ϕ, M_t) plane. The figures in this section illustrate the t -variation of these actual current densities when viewed in the appropriate plane. We first consider results obtained for dielectric bodies.

In Figs. 2.4 and 2.5 the electric and magnetic surface current distributions, respectively, are illustrated for a dielectric sphere. Results calculated by means of DBR are compared with results obtained by Wu [12]. The radius of the sphere is $k_1 a = 1$ and its relative constitutive parameters are $\mu_r = 1$ and $\epsilon_r = 4$. The sphere is excited by a plane wave propagating in the positive z -direction. In general, the data obtained with the computer code DBR agree well with that of Wu. One should note, however, that the results calculated by means of DBR have a slight "glitch" near the points where the surface meets the body axis. It is speculated that the rapidly decreasing radius in this region may necessitate using a smoother current basis (e.g., linear) for the total current I_t in the vicinity of the points where the surface meets the body axis, where it is presently

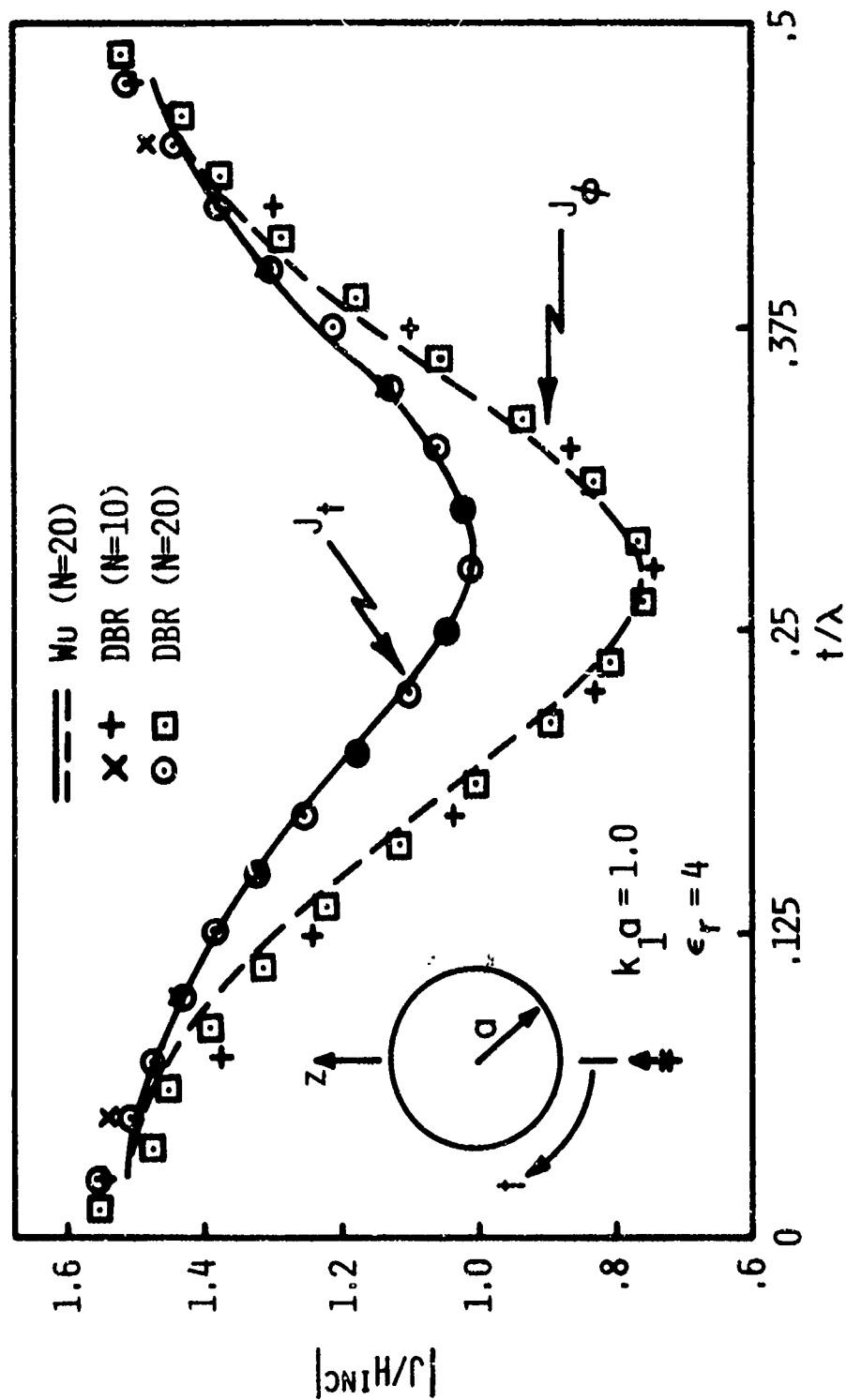


Figure 2.4. Electric surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.

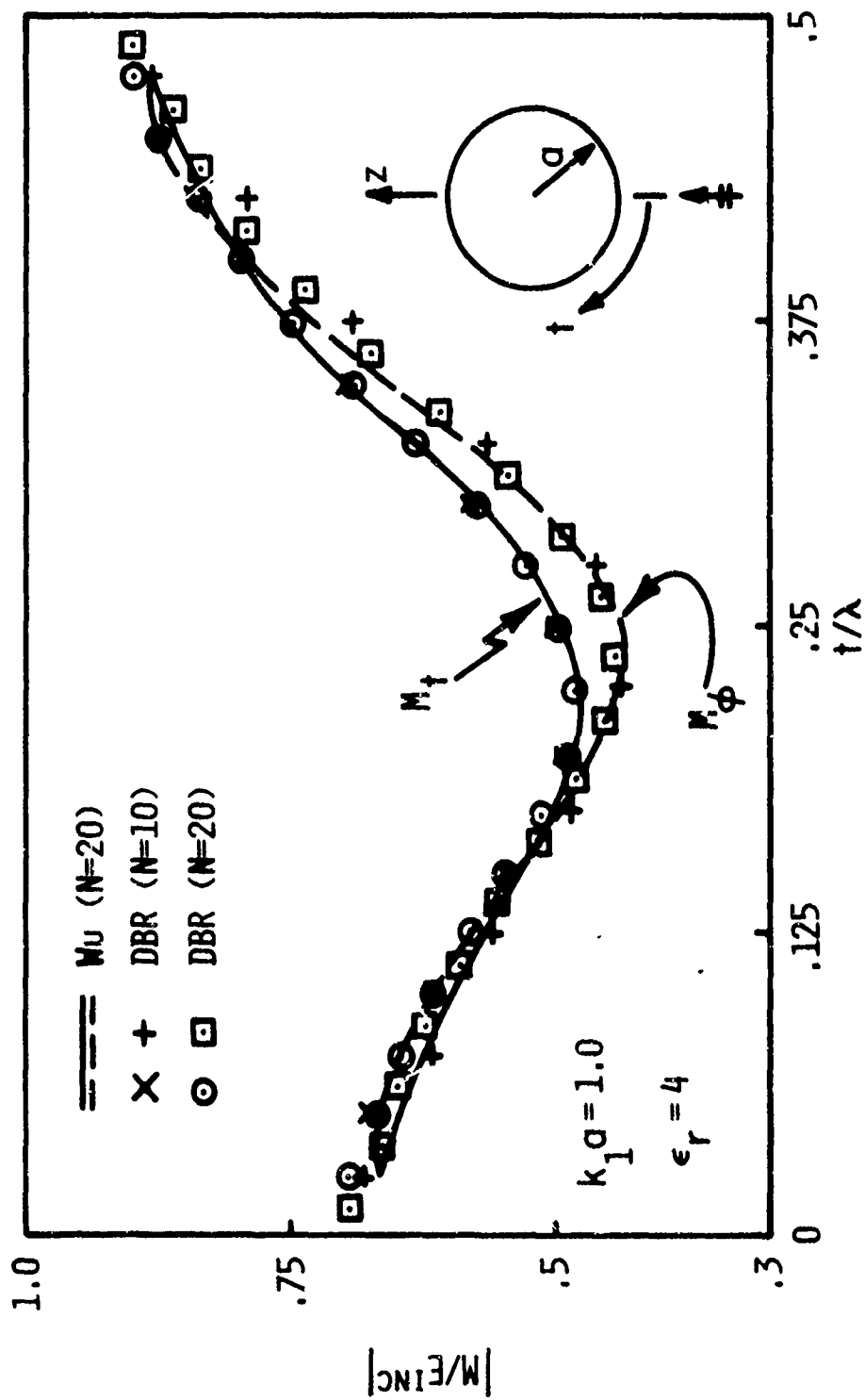


Figure 2.5. Magnetic surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.

modeled by a zero magnitude half pulse. This speculation has not yet been confirmed; however, numerous other attempts to correct the problem have been unsatisfactory. For open conducting bodies where the current density J_t also approaches zero near the ends of the body, no such "glitch" has been detected. The presence of the "glitch" should have a negligible effect on field calculations, however, except possibly very near the points where the surface meets the axis, since the total current moment for the pulses in this region is small.

The electric and magnetic surface current distributions for a dielectric sphere with radius $.2\lambda$ and $\epsilon_r = 80$ are shown in Figs. 2.6 and 2.7, respectively, and are again compared with results obtained by Wu [12]. One observes in Fig. 2.6 that the agreement of the results is excellent if one ignores the (apparently) non-physical oscillations exhibited by the ϕ -component of current in Wu's solution. Note that the results obtained with DBR for $N = 30$ are presented as continuous curves, rather than discrete points. We comment that the data points nearest the points where the surface meets the body axis have been ignored and the curves have been extrapolated in this region. The data points which were ignored, however, have been plotted in the figure as dots which do not lie on the continuous curve. All other

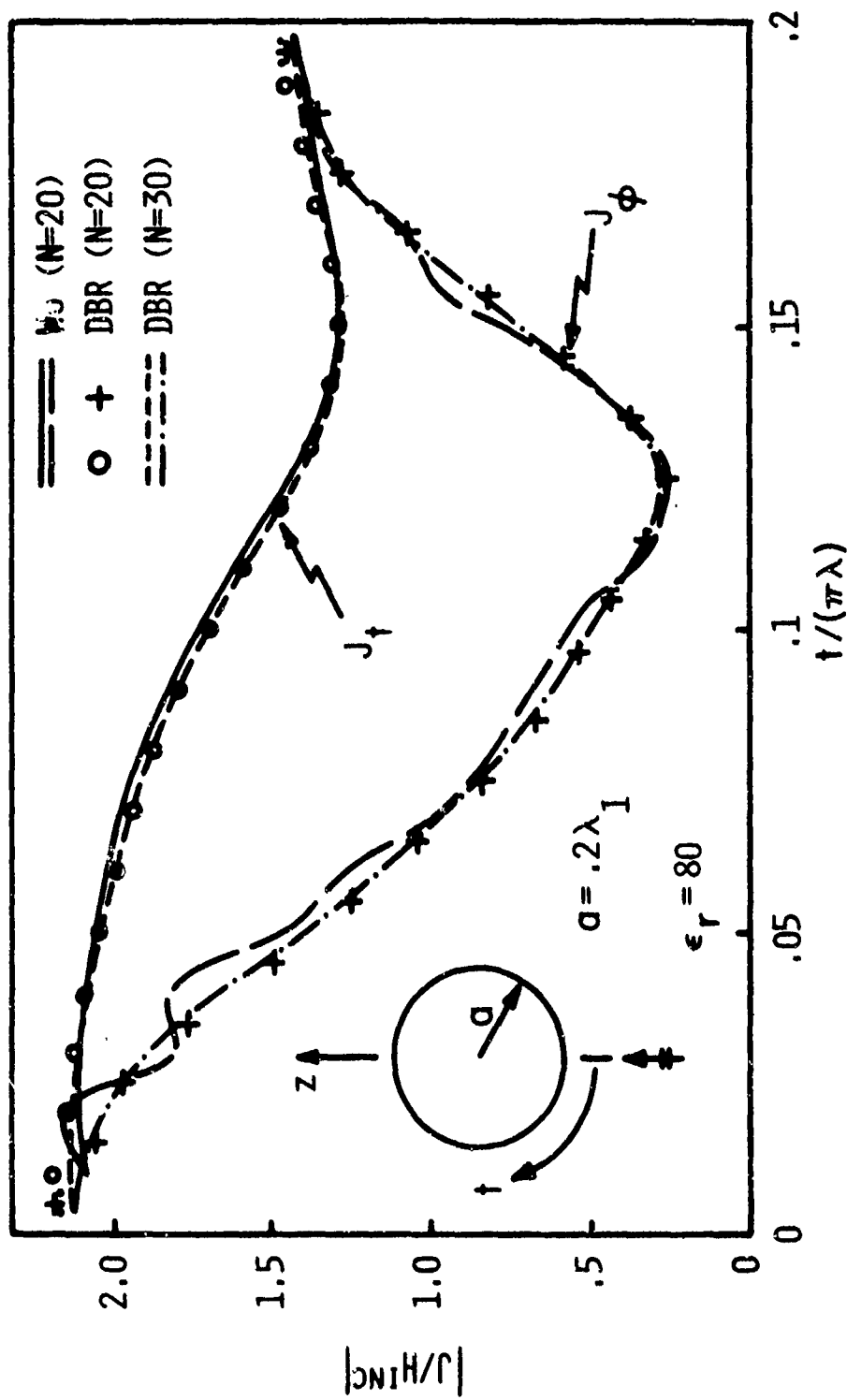


Figure 2.6. Electric surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.

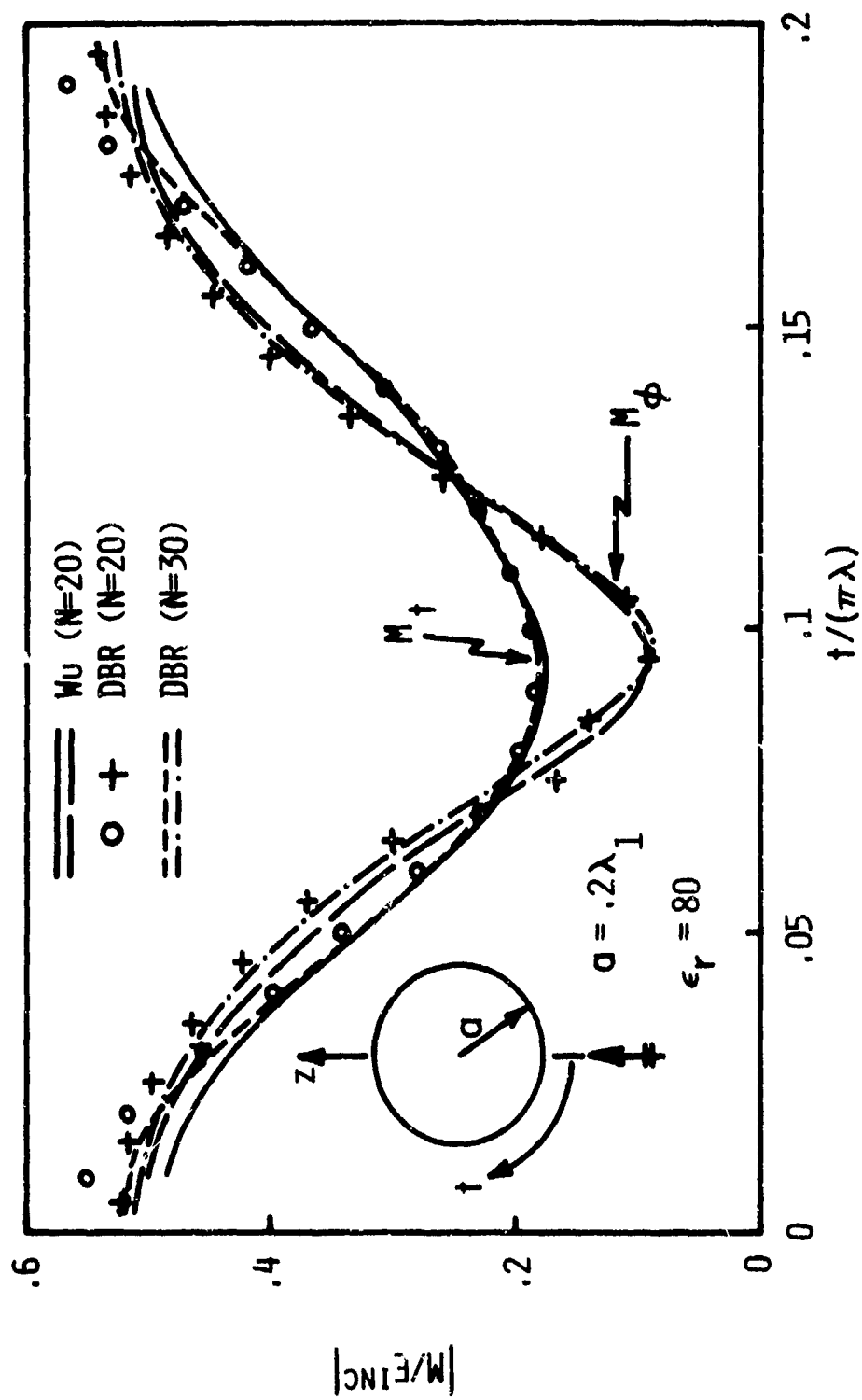


Figure 2.7. Magnetic surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.

data points lie on the curves. The results for the magnetic current (Fig. 2.7) are also in good agreement. For the ϕ -component of magnetic current, however, the data obtained with DBR for $N = 20$ exhibit a slight downward bulge near $t/(\pi\lambda) \approx .075$, whereas Wu's results show no such behavior. The bulge disappears when more unknowns are used, as illustrated by the curve for $N = 30$.

In Fig. 2.8 numerical data are compared with the exact solution for the current densities on a "vacuum dielectric" ($\epsilon_r = 1$) cylinder. The exact solution is, of course, given by

$$\begin{aligned}\bar{J} &= \hat{n} \times \bar{H}^{inc} \\ \bar{M} &= \bar{E}^{inc} \times \hat{n} \quad .\end{aligned}$$

The numerical data agree reasonably well with the exact solution when one considers the fact that the ϕ -components of current must exhibit a step-function jump in magnitude at the edges of the cylinder. The ability of a computer code to model such a case accurately may be important if one wishes to consider objects of very low dielectric contrast. Figs. 2.9 and 2.10 show calculated electric and magnetic current distributions, respectively, for the same size cylinder when the dielectric constant is increased to $\epsilon_r = 4$.

We next consider numerical results for two perfectly

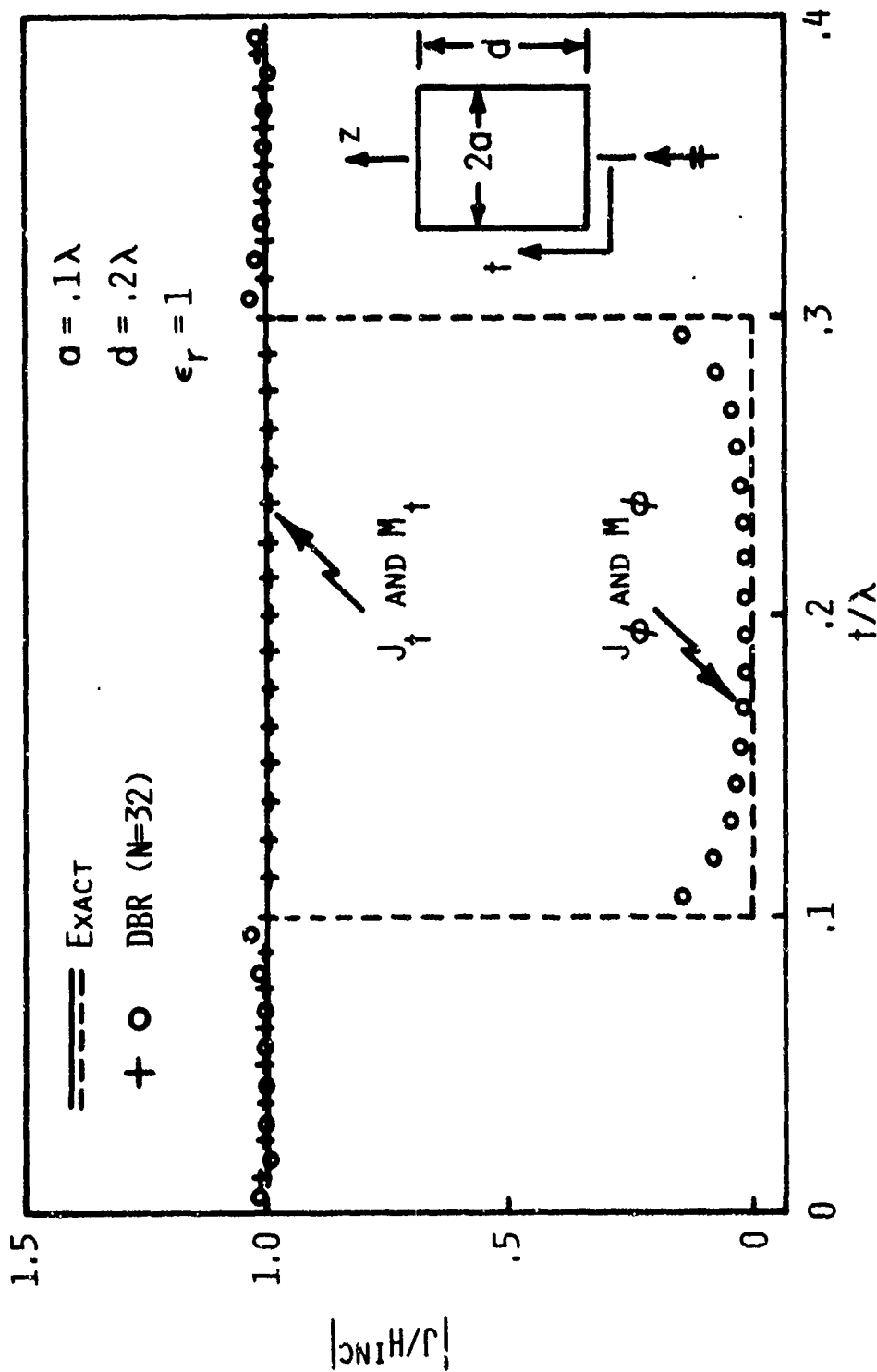


Figure 2.8. Electric and magnetic surface current distributions on a "vacuum dielectric" cylinder illuminated by an axially incident plane wave.

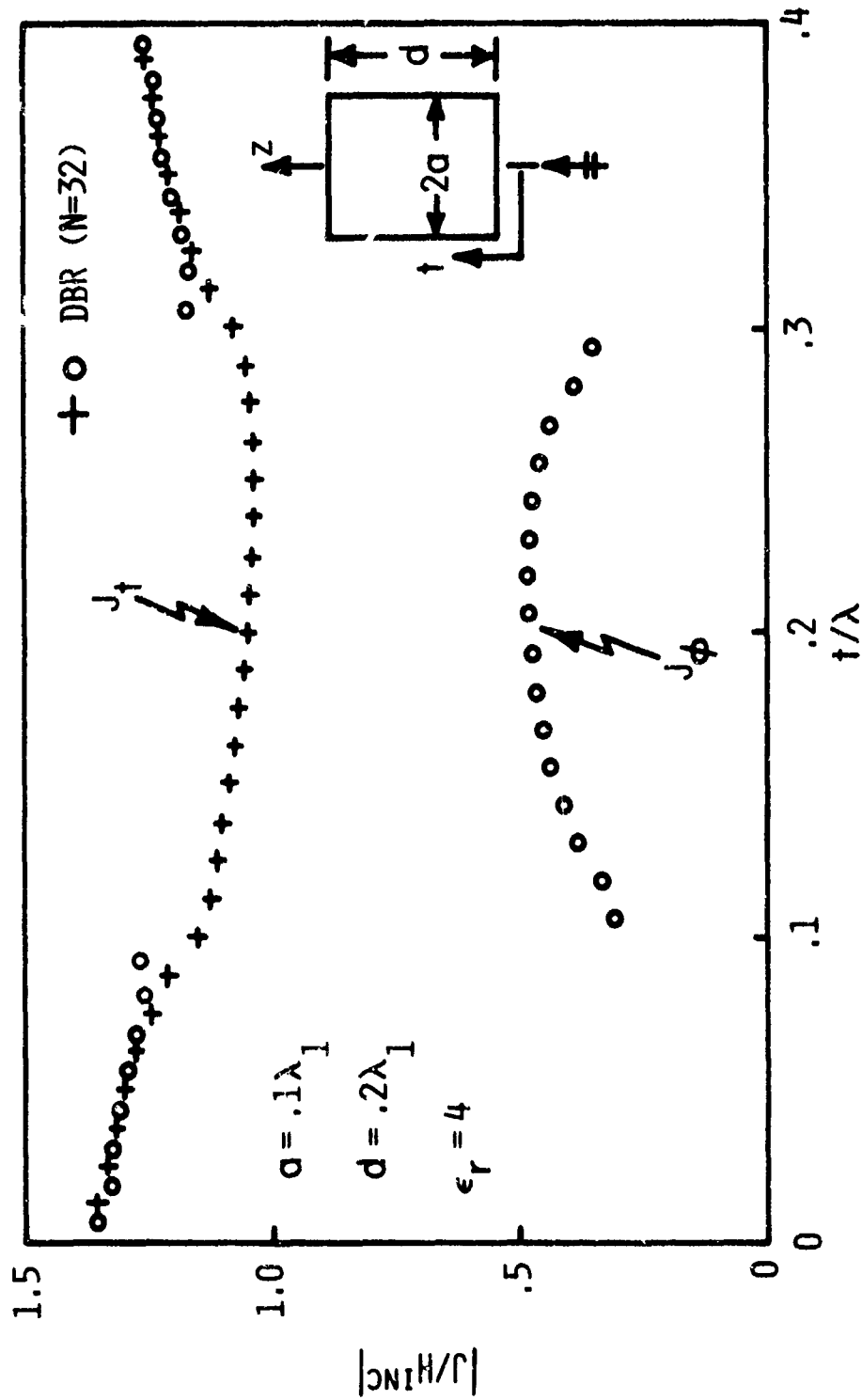


Figure 2.9. Electric surface current distribution on a dielectric cylinder illuminated by an axially incident plane wave.

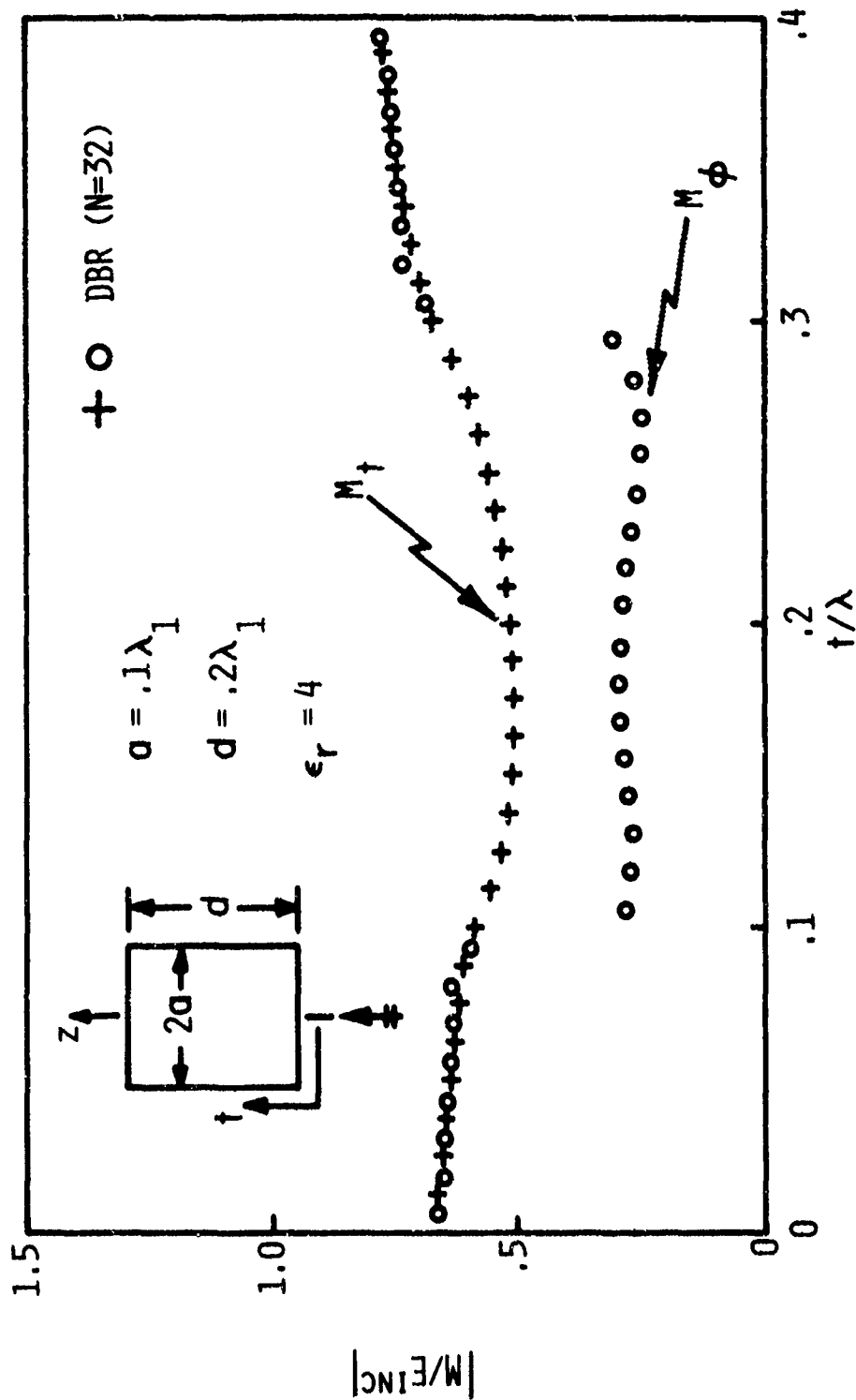


Figure 2.10. Magnetic surface current distribution on a dielectric cylinder illuminated by an axially incident plane wave.

conducting bodies. Fig. 2.11 illustrates the current distribution on a cone-sphere structure illuminated by an axially incident plane wave propagating from the cone tip toward the spherical cap. Numerical results obtained with DBR are compared with results of Mautz and Harrington [1] and Poggio and Miller [13]. The results of Mautz and Harrington are obtained via an EFIE formulation, while Poggio and Miller employed the MFIE. Results obtained with DBR are in excellent agreement with those obtained with the MFIE and do not exhibit the (apparently) non-physical oscillations in the current distributions obtained by Mautz and Harrington.

We also note at this point that there may exist a theoretical relationship between the t - and ϕ -components of current at the points where the surface meets the body axis. Such a relationship is of interest when treating body of revolution structures. Consider the region near the tip of the cone-sphere structure depicted in Fig. 2.11. The charge density near the tip for the m^{th} Fourier component is given by

$$\begin{aligned} \rho &= \frac{1}{\omega r} \left[\frac{\partial}{\partial t} (r J_t) + \frac{\partial}{\partial \phi} (J_\phi) \right] \\ &= \frac{1}{\omega r} \left[J_t \frac{\partial}{\partial t} (r) + r \frac{\partial}{\partial t} (J_t) + \frac{\partial}{\partial \phi} (J_\phi) \right], \end{aligned}$$

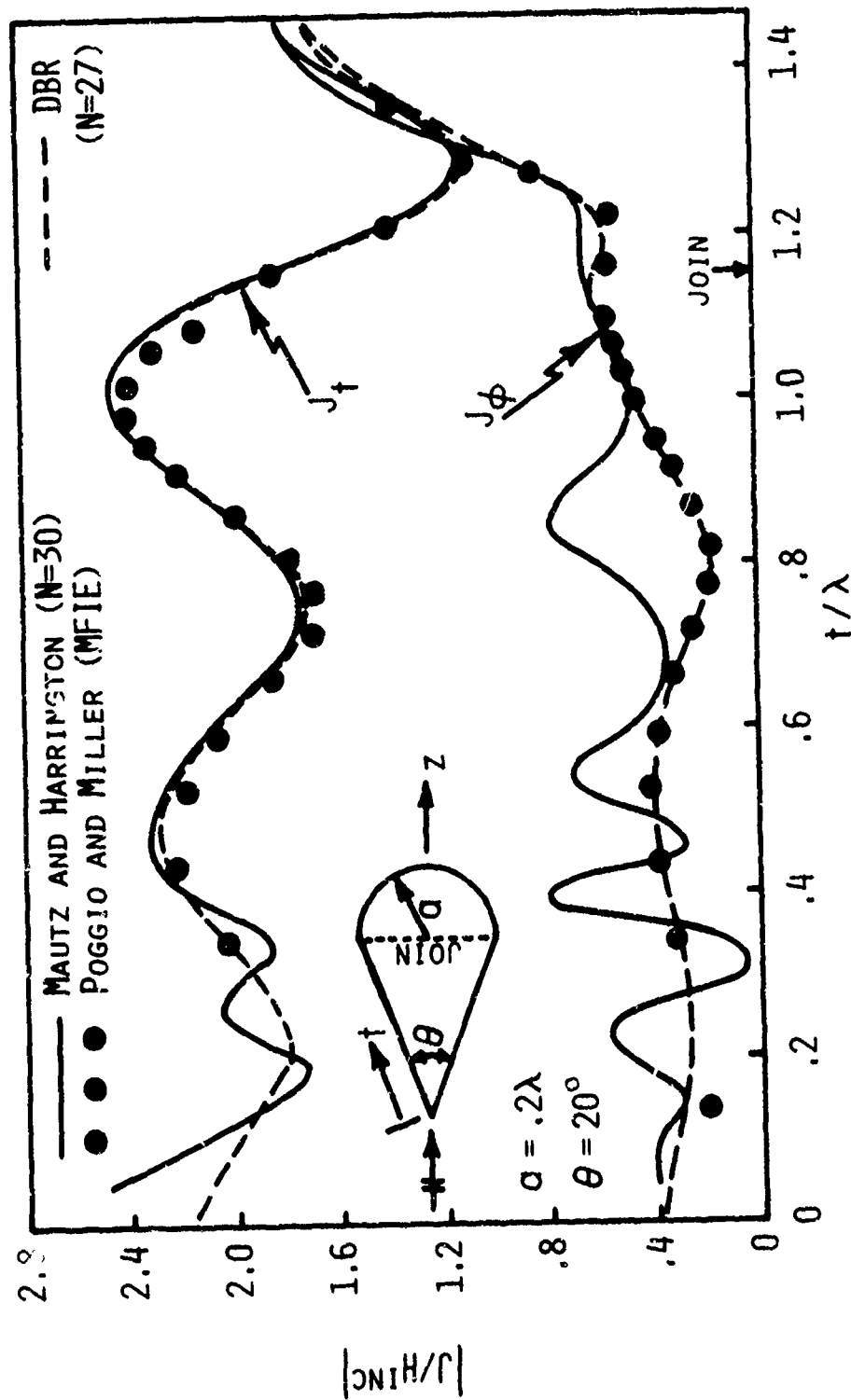


Figure 2.11. Surface current distribution on a perfectly conducting cone-sphere structure illuminated by a plane wave which is axially incident and propagates from the cone tip toward the spherical cap.

or

$$\rho = \frac{j}{\omega r} \left[J_t \sin(\theta/2) + r \frac{\partial}{\partial t} (J_t) + jmJ_\phi \right] , \quad (2.37)$$

where r is the cylindrical coordinate variable representing the distance from the z -axis and where, on the conical surface, $r = t \sin(\theta/2)$. The "total" charge is therefore

$$q = j \frac{2\pi}{\omega} \left[J_t \sin(\theta/2) + r \frac{\partial}{\partial t} (J_t) + jmJ_\phi \right] . \quad (2.38)$$

From symmetry considerations, however, it can be shown that the total charge at the points where the surface meets the body axis is zero for $m \neq 0$. At this point we rely on a visual inspection of the numerical results to infer that for $m = \pm 1$

$$\lim_{t \rightarrow 0} r \frac{\partial}{\partial t} (J_t) = 0 . \quad (2.39)$$

A rigorous theoretical investigation of the fields in the neighborhood of the cone tip is necessary to determine the validity of (2.39). Such an investigation, however, is seriously hampered by the lack of tabulated zeros of the associated Legendre function, and has therefore not been pursued in this work. With (2.38) we then have for $m = \pm 1$

$$J_t \sin(\theta/2) = \mp j J_\phi , \quad (2.40)$$

which relates the value of the two current components at the point where the surface meets the body axis. The special case for $\theta = \pi$ and $m = \pm 1$ was developed previously by Mautz and Schuman [7]. Eq. (2.40) may be of some interest for "conically-tipped" surfaces when $m = \pm 1$. Referring to Fig. 2.11 again, one should note that results computed by means of DBR (which enforces no special conditions at the body axis) do indeed satisfy (within 5%) the condition (2.40) both at the cone tip and at the point on the spherical cap at the axis.

When the cone-sphere structure is illuminated by an axially incident plane wave propagating from the spherical cap to the cone tip, the current distributions shown in Fig. 2.12 are obtained. Results computed with DBR are again compared with those of Mautz and Harrington [1] and Poggio and Miller [13]. For the ϕ -component of current, agreement with the MFIE approach is again excellent, except very near the cone tip. However, for the t -component of current, the MFIE approach yields results which appear to oscillate slightly. Results obtained by means of DBR show essentially no oscillations. Also, the computed results again satisfy the condition (2.40) both at the cone tip and at the point on the spherical cap at the axis.

Fig. 2.13 illustrates the current distribution on a

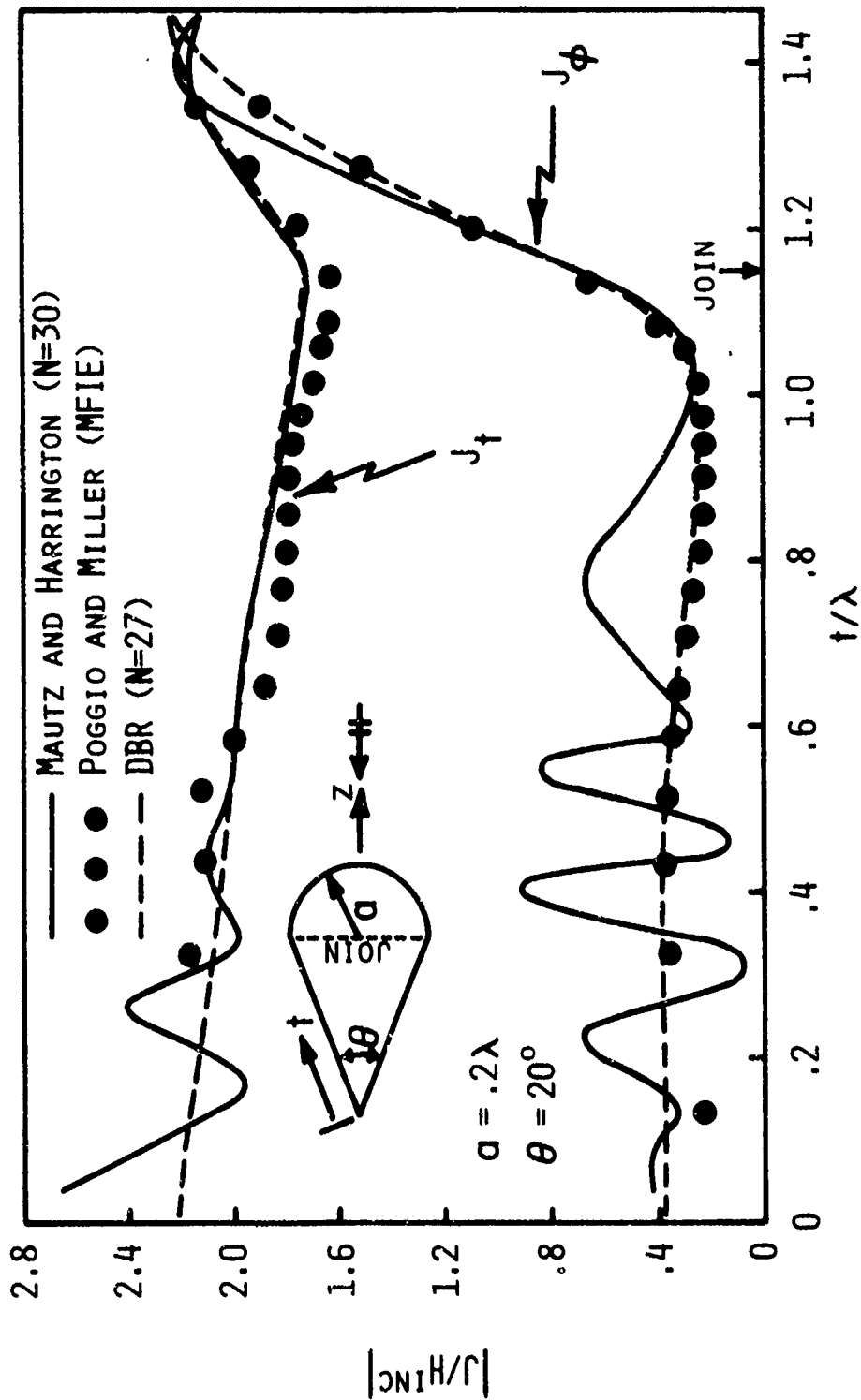


Figure 2.12. Surface current distribution on a perfectly conducting cone-sphere structure illuminated by a plane wave which is axially incident and propagates from the spherical cap toward the cone tip.

perfectly conducting open-ended cylinder subject to an axially incident plane wave. The results computed with DBR are compared with those of Davis, which were obtained using a set of hybrid integral equations [14]. Davis apparently encountered stability problems in the solution of the usual EFIE, and developed the hybrid equations as a means to avoid these problems. A comparison of the data in Fig. 2.13 should confirm that the formulation presented in this work apparently does not suffer from stability problems of the kind encountered by Davis. This problem also presents the opportunity to check the *a posteriori* correction for pulses which represent singular currents near edges [15]. The two square symbols near the edges of the plot in Fig. 2.13 represent computed currents which have been corrected *a posteriori* by dividing the computed current magnitude by $\sqrt{2}$. These corrected values are in excellent agreement with the results obtained by Davis, who used spline functions containing the edge condition. On the other hand, if one wishes to calculate the fields using the computed currents, it is probably better not to correct the value of the edge current, since the computed (uncorrected) value should accurately represent the current *moment* over the pulse region.

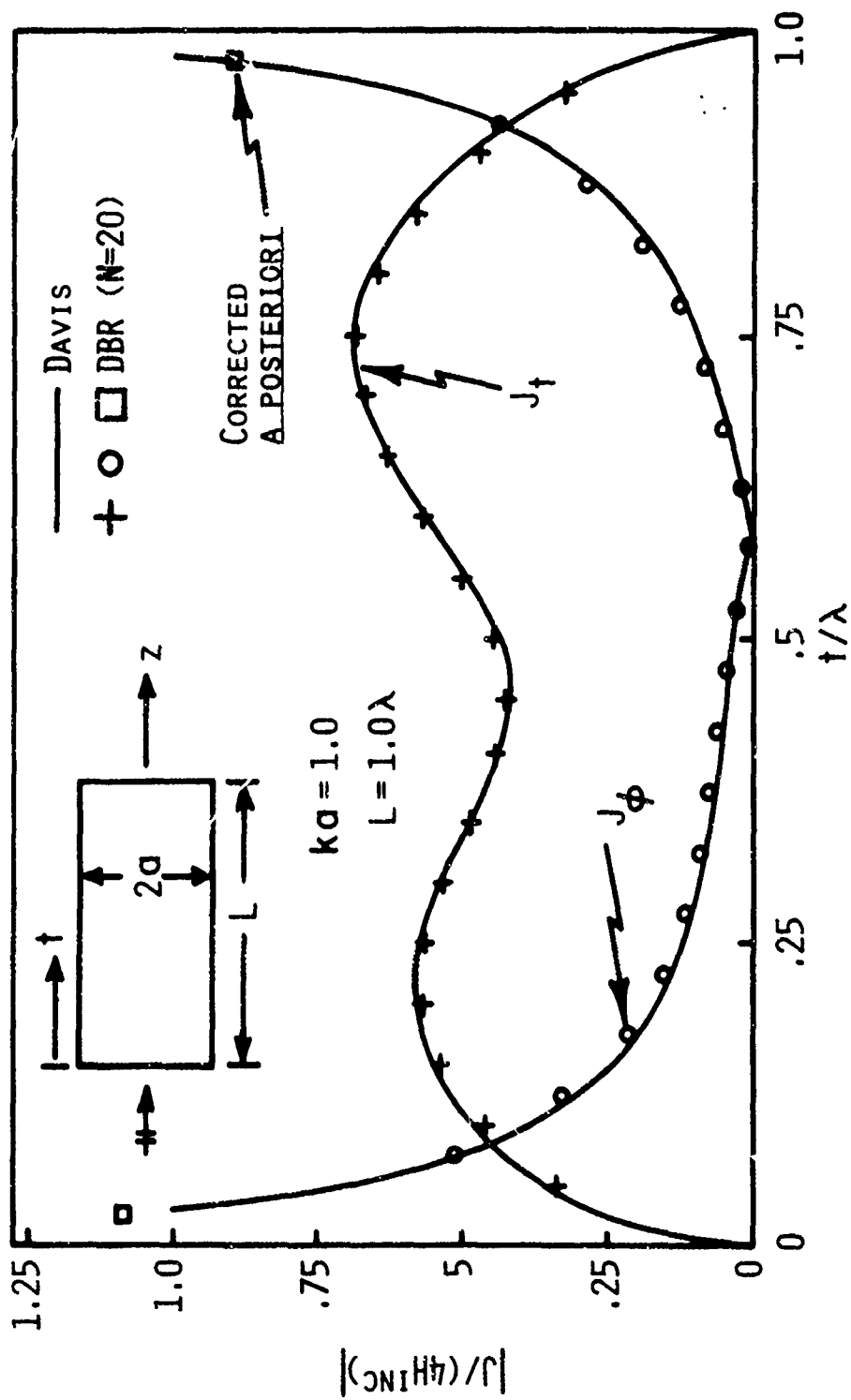


Figure 2.13. Surface current distribution on a perfectly conducting open-ended cylinder illuminated by an axially incident plane wave.

Section III

CONCLUSION

In this report we have presented a simple and efficient numerical formulation for determining the currents induced on both perfectly conducting and dielectric bodies of revolution. The formulation has been implemented in a computer code (DBR) which is described and listed in Appendix B. Numerical results have been presented for the current induced on several types of bodies of revolution and the excellent agreement of the results with other available data has been demonstrated. It has also been noted that other researchers have encountered stability problems with some body of revolution configurations. There has been no evidence of solution instability for any structural geometry when the procedures described in this work are employed. A slight "glitch" in the solution for the current has been detected at points where the surface meets the axis of the body of revolution. However, this glitch is relatively small and should have no significant effect on integral functionals of the current such as fields, radar cross-section, etc. Furthermore, it has been demonstrated that the techniques described in this work provide accurate treatment of sharp bends and/or knife-like

edges in the structure.

The numerical formulation presented in this report serves as a basis for a very sophisticated numerical model of the missile/plume structure in which the inhomogeneous plume is modeled by layers of homogeneous material. This sophisticated numerical model now appears to have been successfully implemented and will be the subject of a forthcoming report.

APPENDIX A

SINGULARITY ANALYSES FOR SELF TERMS OF THE BODY OF REVOLUTION FORMULATION

When calculating elements of the impedance matrix for the body of revolution via (2.25), one should be aware that the integrands of the integral functions ψ and U may possess a singularity when the field point is within the source region. In this appendix each integrand is investigated to determine if a singularity is present and, if so, a numerical treatment of the integral function is presented. In all of the investigations we employ the following coordinate parameterization valid for $t_{j-1} \leq t \leq t_j$:

$$z = z_{j-1} + \ell \cos \gamma_j \quad (\text{A.1a})$$

$$\rho = \rho_{j-1} + \ell \sin \gamma_j \quad , \quad (\text{A.1b})$$

$$0 \leq \ell \leq \Delta t_j ,$$

where $\ell = t - t_{j-1}$. For all self terms we then have that

$$(z - z') = (\ell - \ell') \cos \gamma_j \quad (\text{A.2a})$$

$$(\rho - \rho') = (\ell - \ell') \sin \gamma_j \quad . \quad (\text{A.2b})$$

A.1 Singularity Analysis of the ψ and ψ^0 Integral Functions

For a self term, the ψ integral function of (2.23a) may be written

$$\psi_i = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \frac{e^{-jk_i R_0}}{R_0} \cos(m\xi) d\xi d\ell' , \quad (A.3a)$$

$i = 1, 2 ,$

where

$$R_0 = [(\rho - \rho')^2 + 2\rho\rho'(1 - \cos\xi) + (z - z')^2]^{\frac{1}{2}} . \quad (A.3b)$$

As $t \rightarrow t'$ and $\xi \rightarrow 0$, $R_0 \rightarrow 0$ and the integrand of (A.3a) is clearly singular. Thus we express ψ as

$$\psi_i = I_{1_i} + I_{2_i} , \quad (A.4)$$

where

$$I_{1_i} = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left[\frac{e^{-jk_i R_0}}{R_0} \cos(m\xi) - \frac{1}{R_0} \right] d\xi d\ell' \quad (A.5a)$$

$$I_{2_i} = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \frac{1}{R_0} d\xi d\ell' , \quad (A.5b)$$

and where the integrand of I_{1_i} is no longer singular.

Furthermore, we have that

$$\begin{aligned} R_0 &= [(\rho - \rho')^2 + (z - z')^2 + 4\rho\rho'\sin^2(\xi/2)]^{\frac{1}{2}} \\ &= R_1[1 + \beta_1^2\sin^2(\xi/2)]^{\frac{1}{2}}, \end{aligned}$$

where

$$\begin{aligned} R_1 &= [(\rho - \rho')^2 + (z - z')^2]^{\frac{1}{2}} \\ \beta_1 &= \frac{2[\rho\rho']^{\frac{1}{2}}}{R_1}. \end{aligned}$$

Thus, with a simple change of variables, I_{2_i} may be written as [16]

$$\begin{aligned} I_{2_i} &= 4 \int_{\ell_1}^{\ell_2} \int_0^{\frac{\pi}{2}} \frac{1}{R_1[1 + \beta_1^2\sin^2\zeta]^{\frac{1}{2}}} d\zeta d\ell' \\ &= 4 \int_{\ell_1}^{\ell_2} \frac{1}{R_1[1 + \beta_1^2]^{\frac{1}{2}}} K\left(\frac{1}{[1 + \beta_1^2]^{\frac{1}{2}}}\right) d\ell' \\ &= 4 \int_{\ell_1}^{\ell_2} \frac{1}{R_2} K(\beta_2) d\ell', \end{aligned} \tag{A.6}$$

where $K(\beta_2)$ is the complete elliptic integral of the first kind defined by

$$K(u) = \int_0^{\frac{\pi}{2}} \frac{1}{[1 - u^2 \sin^2 \phi]^{\frac{1}{2}}} d\phi \quad , \quad (A.7a)$$

and

$$R_2 = [(\rho + \rho')^2 + (z - z')^2]^{\frac{1}{2}} \quad (A.7b)$$

$$\beta_2^* = \frac{2[\rho\rho']^{\frac{1}{2}}}{R_2} \quad . \quad (A.7c)$$

The integrand of (A.6) is still singular, however, and near the singularity at $t = t'$ varies as

$$\frac{1}{R_2} K(\beta_2) \xrightarrow[t \rightarrow t']{} \frac{1}{2\rho} [\ln 4 + \ln(R_2) - \ln(R_1)] \quad . \quad (A.8)$$

Only the last term is singular, so we add and subtract the singular term from (A.6) to obtain

$$I_{2i} = I_{2i}^a + I_{2i}^b \quad , \quad (A.9)$$

where

$$I_{2i}^a = 4 \int_{\ell_1}^{\ell_2} \left[\frac{1}{R_2} K(\beta_2) + \frac{1}{2\rho} \ln(R_1) \right] d\ell' \quad (A.10a)$$

$$I_{2_i}^b = -\frac{2}{\rho} \int_{\ell_1}^{\ell_2} \ln(R_1) d\ell' \quad . \quad (A.10b)$$

The integral $I_{2_i}^a$ no longer has a singular integrand and the integral $I_{2_i}^b$ can be evaluated analytically by using the parameterization (A.1) as follows:

$$\begin{aligned} I_{2_i}^b &= -\frac{2}{\rho} \int_{\ell_1}^{\ell_2} \ln(R_1) d\ell' \\ &= -\frac{2}{\rho} \int_{\ell_1}^{\ell_2} \ln|\ell - \ell'| d\ell' \\ &= \frac{2}{\rho} [(\ell_2 - \ell_1) - (\ell_2 - \ell) \ln(\ell_2 - \ell) \\ &\quad - (\ell - \ell_1) \ln(\ell - \ell_1)] \quad . \quad (A.11) \end{aligned}$$

The integrals I_{1_i} and $I_{2_i}^a$ can thus be integrated numerically, while $I_{2_i}^b$ is given by (A.11), and we have

$$\psi_1 = I_{1_i} + I_{2_i}^a + I_{2_i}^b \quad . \quad (A.12)$$

We comment here that for *non-self* terms it is more

convenient to calculate ψ_i as

$$\psi_i = I_{1_i} + I_{2_i} \quad , \quad (A.13)$$

where I_{2_i} is defined by (A.6), than to compute ψ_i directly from (A.3). Following the procedures described above it is trivial to show that the self term of ψ_i^ρ is calculated via

$$\psi_i^\rho = I_{1_i}^\rho + \rho I_{2_i}^a + \rho I_{2_i}^b \quad , \quad (A.14)$$

where

$$I_{1_i}^\rho = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ \frac{e^{-jk_1 R_0}}{R_0} \rho' \cos(m\xi) - \frac{\rho}{R_0} \right\} d\xi d\ell' \quad . \quad (A.15)$$

A.2 Singularity Analysis of the Integral Function U_0

The definition of the integral function U_0 is given by (2.26a) as

$$U_0 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} - \frac{\sin(m\xi) \sin\xi}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] d\xi d\ell' \quad , \quad (A.16)$$

where R_0 is defined by (A.3b). As stated in Section II, all of the integral functions U may be Cauchy Principal Value integrals. Thus one may evaluate the integrals by direct numerical integration, if the quadrature points are chosen in accordance with the definition of the Cauchy Principal Value Integral. On the other hand, evaluation of the integrals in this manner may result in the subtraction of very large numerical values of the integrands and hence lead to possible loss of precision in the calculations. To avoid such problems we analyze the U integral functions in the manner presented in Section A.1. Near the singular point the integrand of (A.16) can be shown to vary as

$$\frac{-2m\xi^2}{[(\ell - \ell')^2 + \rho^2\xi^2]^{(3/2)}} .$$

Therefore we write

$$U_0 = I_3 + I_4 , \quad (A.17)$$

where

$$\begin{aligned}
I_3 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ -\frac{\sin(m\xi)\sin\xi}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} \right. \right. \\
\left. \left. + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] - \frac{-2m\xi^2}{[(\ell - \ell')^2 + \rho^2 \xi^2]^{(3/2)}} \right\} d\xi d\ell' \quad (A.18a)
\end{aligned}$$

$$I_4 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \frac{-2m\xi^2}{[(\ell - \ell')^2 + \rho^2 \xi^2]^{(3/2)}} d\xi d\ell' \quad (A.18b)$$

The integrand of I_3 is non-singular and can be evaluated numerically. The integral I_4 can be integrated analytically and is given by

$$\begin{aligned}
I_4 = -\frac{4m}{\rho^3} \left\{ (\ell - \ell_1) \ln \left[\rho\pi + \sqrt{(\ell - \ell_1)^2 + \rho^2 \pi^2} \right] \right. \\
+ (\ell_2 - \ell) \ln \left[\rho\pi + \sqrt{(\ell_2 - \ell)^2 + \rho^2 \pi^2} \right] \\
\left. - (\ell - \ell_1) \ln(\ell - \ell_1) - (\ell_2 - \ell) \ln(\ell_2 - \ell) \right\} \quad (A.19)
\end{aligned}$$

Eq. (A.17) then provides the desired result.

A.3 Singularity Analysis of the Integral Functions U_1

and U_1^0

The integrand of the integral function U_1 , as defined by (2.26b), is identical with the integrand of the integral function U_0 except for the product factor $(z - z')$. It should therefore be clear from the analysis of U_0 that as $R_0 \rightarrow 0$ the integrands of U_1 and U_1^0 are proportional to

$$\frac{-2m\xi^2(\ell - \ell')\cos\gamma'}{[(\ell - \ell')^2 + \rho^2\xi^2]^{(3/2)}},$$

which is non-singular and hence can be integrated numerically.

A.4 Singularity Analysis of the Integral Functions U_2

and U_2^0

By comparison with the integrand of U_1 , one can immediately see that as $R_0 \rightarrow 0$ the integrand of U_2 approaches

$$\frac{-2(\ell - \ell')\cos\gamma'}{[(\ell - \ell')^2 + \rho^2\xi^2]^{(3/2)}}.$$

This function is singular and we therefore attempt to compute U_2 as

$$U_2 = I_5 + I_6 \quad , \quad (A.20)$$

where

$$I_5 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ -\frac{(z - z') \cos(m\xi) \cos\xi}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] + \frac{2(\ell - \ell') \cos\gamma'}{[(\ell - \ell')^2 + \rho^2 \xi^2]^{(3/2)}} \right\} d\xi d\ell' \quad (A.21a)$$

$$I_6 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} -\frac{2(\ell - \ell') \cos\gamma'}{[(\ell - \ell')^2 + \rho^2 \xi^2]^{(3/2)}} d\xi d\ell' \quad . \quad (A.21b)$$

The integrand of I_5 is non-singular and may be integrated numerically. Analytical evaluation of I_6 yields

$$I_6 = \frac{4 \cos\gamma'}{\rho} \left\{ \ell n \left[\frac{\rho\pi + \sqrt{(\ell - \ell_1)^2 + \rho^2 \pi^2}}{\rho\pi + \sqrt{(\ell - \ell_2)^2 + \rho^2 \pi^2}} \right] + \ell n \left| \frac{\ell - \ell_2}{\ell - \ell_1} \right| \right\} \quad , \quad (A.22)$$

which is infinite if $\ell = \ell_1$ or $\ell = \ell_2$. These values may occur in the evaluation of (2.25g), so U_2 has a non-integrable singularity. The consequences of this result are discussed in Section A.9. We similarly express

$$U_2^0 = I_7 + \rho I_6, \quad (A.23)$$

where

$$I_7 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ -\frac{(z - z') \rho' \cos(m\xi) \cos \xi}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} \right. \right. \\ \left. \left. + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] + \frac{2\rho(\ell - \ell') \cos \gamma'}{[(\ell - \ell')^2 + \rho^2 \xi^2]^{(3/2)}} \right\} d\xi d\ell' \quad (A.24)$$

Thus U_2^0 also has a non-integrable singularity and will be treated in Section A.9.

A.5 Singularity Analysis of the Integral Function U_3^0

By direct comparison with U_0 one can immediately write

$$U_3^0 = I_8 + \rho I_4, \quad (A.25)$$

where

$$I_8 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ -\frac{\rho' \sin(m\xi) \sin \xi}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} \right. \right. \\ \left. \left. + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] + \frac{2m\rho \xi^2}{[(\ell - \ell')^2 + \rho^2 \xi^2]^{(3/2)}} \right\} d\xi d\ell' \quad (A.26)$$

and where I_4 is given by (A.19). The integral I_8 has a non-singular integrand and can be integrated numerically.

A.6 Singularity Analysis of the Integral Function U_4^0

By analogy with U_2^0 one can compute

$$U_4^0 = I_9 + I_{10} \quad , \quad (A.27)$$

where

$$I_9 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ -\frac{\rho'(\rho - \rho') \cos(m\xi)}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] + \frac{2\rho(\ell - \ell') \sin \gamma'}{[(\ell - \ell')^2 + \rho^2 \xi^2]^{(3/2)}} \right\} d\xi d\ell' \quad (A.28)$$

$$I_{10} = 4 \sin \gamma' \left\{ \ln \left[\frac{\rho\pi + \sqrt{(\ell - \ell_1)^2 + \rho^2 \pi^2}}{\rho\pi + \sqrt{(\ell - \ell_2)^2 + \rho^2 \pi^2}} \right] + \ln \left| \frac{\ell - \ell_2}{\ell - \ell_1} \right| \right\} \quad (A.29)$$

The integral I_9 can be computed numerically, but I_{10} is infinite if $\ell = \ell_1$ or $\ell = \ell_2$, and is therefore considered in Section A.9.

A.7 Singularity Analysis of the Integral Function U_5

and U_5^0

The integral function U_5 is defined by (2.26h) as

$$U_5 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} -\frac{\cos(m\xi) \sin^2(\xi/2)}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] d\xi d\ell' \quad (A.30)$$

As $R_0 \rightarrow 0$ the integrand of U_5 approaches

$$\frac{-\xi^2}{2 \left[(\ell - \ell')^2 + \rho^2 \xi^2 \right]^{(3/2)}} \quad .$$

The integrand is therefore singular and we compute U_5 as

$$U_5 = I_{11} + I_{12} \quad , \quad (A.31)$$

where

$$I_{11} = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ -\frac{\cos(m\xi) \sin^2(\xi/2)}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] + \frac{\xi^2}{2 \left[(\ell - \ell')^2 + \rho^2 \xi^2 \right]^{(3/2)}} \right\} d\xi d\ell' \quad (A.32a)$$

$$\begin{aligned}
I_{12} &= \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} - \frac{\xi^2}{2 \left[(\ell - \ell')^2 + \rho^2 \xi^2 \right]^{(3/2)}} d\xi d\ell' \\
&= -\frac{1}{\rho^3} \left\{ (\ell - \ell_1) \ln \left[\rho\pi + \sqrt{(\ell - \ell_1)^2 + \rho^2 \pi^2} \right] \right. \\
&\quad + (\ell_2 - \ell) \ln \left[\rho\pi + \sqrt{(\ell_2 - \ell)^2 + \rho^2 \pi^2} \right] \\
&\quad \left. - (\ell - \ell_1) \ln(\ell - \ell_1) - (\ell_2 - \ell) \ln(\ell_2 - \ell) \right\}. \quad (A.32b)
\end{aligned}$$

The integral I_{11} is integrated numerically since the integrand is now non-singular. Similarly,

$$U_5^0 = I_{13} + \rho I_{12}, \quad (A.33)$$

where

$$\begin{aligned}
I_{13} &= \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ - \frac{\rho' \cos(m\xi) \sin^2(\xi/2)}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} \right. \right. \\
&\quad \left. \left. + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] + \frac{\rho \xi^2}{2 \left[(\ell - \ell')^2 + \rho^2 \xi^2 \right]^{(3/2)}} \right\} d\xi d\ell'. \quad (A.34)
\end{aligned}$$

A.8 Singularity Analysis of the Integral Function U_6

Analysis of the integral function U_6 is directly analogous to that of U_4^0 . We have

$$U_6 = I_{14} + \frac{1}{\rho} I_{10} \quad , \quad (A.35)$$

where

$$I_{14} = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ - \frac{(\rho - \rho') \cos(m\xi) \cos\xi}{R_0^3} \left[(1 + jk_1 R_0) e^{-jk_1 R_0} \right. \right. \\ \left. \left. + (1 + jk_2 R_0) e^{-jk_2 R_0} \right] + \frac{2(\ell - \ell') \sin\gamma'}{[(\ell - \ell')^2 + \rho^2 \xi^2]^{(3/2)}} \right\} d\xi d\ell' \quad , \quad (A.36)$$

and where I_{10} is given by (A.29). The integral I_{14} can be evaluated numerically, whereas the analytical result for I_{10} may be infinite and is therefore treated in the next section.

A.9 Elimination of Non-Integrable Singularities

The non-integrable singularities which have appeared in the analyses of the integral functions U_2 , U_2^c , U_4^0 , and U_6 are, of course, also non-physical. We therefore proceed

to determine how these apparent singularities may be removed. Consider now the quantity K which is the sum of the third and fifth terms of (2.25g). Then for a self term we have

$$\begin{aligned}\frac{4\pi K}{\Delta t_n} &= -U_6 \cos \gamma_n + U_2 \sin \gamma_n \\ &= -I_{14} \cos \gamma_n - I_{10} \frac{\cos \gamma_n}{\rho} + I_5 \sin \gamma_n + I_6 \sin \gamma_n.\end{aligned}\quad (\text{A.37})$$

Then note that

$$\begin{aligned}-I_{10} \frac{\cos \gamma_n}{\rho} + I_6 \sin \gamma_n &= \left\{ \frac{-4 \cos \gamma_n \sin \gamma_n}{\rho} + \frac{4 \sin \gamma_n \cos \gamma_n}{\rho} \right\} A, \\ &\equiv 0\end{aligned}\quad (\text{A.38a})$$

where

$$A = \ln \left[\frac{\rho \pi + \sqrt{(\ell - \ell_1)^2 + \rho^2 \pi^2}}{\rho \pi + \sqrt{(\ell - \ell_2)^2 + \rho^2 \pi^2}} \right] + \ln \left| \frac{\ell - \ell_2}{\ell - \ell_1} \right|. \quad (\text{A.38b})$$

Thus, the result for K does converge as expected. The numerical procedure for the evaluation of the integral functions U_2 and U_6 is therefore to compute

$$U_2 = I_5 \quad (\text{A.39a})$$

$$U_6 = I_{14}, \quad (\text{A.38b})$$

thus removing the canceling non-integrable terms from the evaluation of the remaining terms in the integral.

If we next consider the quantity K which is the sum of the first and third terms of (2.25f), then for a self term we have, for example,

$$\begin{aligned} 2K &= \chi_c(\Delta t_n, \gamma_n) U_4^0 - \chi_s(\Delta t_n, \gamma_n) U_2^0 \\ &= \chi_c(\Delta t_n, \gamma_n) (I_9 + I_{10}) - \chi_s(\Delta t_n, \gamma_n) (I_7 + \rho I_6) , \end{aligned} \quad (A.40)$$

and

$$\begin{aligned} \chi_c(\Delta t_n, \gamma_n) I_{10} - \chi_s(\Delta t_n, \gamma_n) \rho I_6 &= \{ 2[\Delta t_{n+1} \cos \gamma_{n+1} + \Delta t_n \cos \gamma_n] \sin \gamma_m \\ &\quad - 2[\Delta t_{n+1} \sin \gamma_{n+1} + \Delta t_n \sin \gamma_n] \cos \gamma_m \} A , \end{aligned} \quad (A.41)$$

where A is defined by (A.38b) and where, for a self term, either $m = n$ or $m = n+1$. By way of example we choose $m = n$, which gives

$$\chi_c I_{10} - \chi_s \rho I_6 = 2A \Delta t_{n+1} \{ \cos \gamma_{n+1} \sin \gamma_n - \sin \gamma_{n+1} \cos \gamma_n \} . \quad (A.42)$$

The trigonometric functions in (A.42) do not cancel unless $\gamma_n = \gamma_{n+1}$ (which implies that there is no bend in the surface

at $t = t_n$) and the result may therefore be infinite. The difficulty lies in the mathematical representation of the problem — the field point is placed between two current subdomains each of which has an independent associated current coefficient and this point may be at a bend in the surface where $\gamma_n \neq \gamma_{n+1}$. It is relatively easy to demonstrate that if the actual current on the structure appeared in the integrals, the integrals would be convergent if evaluated in the Cauchy Principal Value sense. To eliminate this problem, we define in this situation

$$U_2^0 = I_7 \quad (\text{A.43a})$$

$$U_4^0 = I_9 \quad , \quad (\text{A.43b})$$

thus removing the singular terms from the evaluation of the integrals. While this procedure is perhaps not entirely rigorous, it has been found to work satisfactorily.

Appendix B

DESCRIPTION AND LISTING OF THE CODE

The computer code DBR is designed to calculate the induced currents on a general body of revolution, which may be either a perfectly conducting or a dielectric body. Perfectly conducting bodies may be either "closed" or "open," i.e. the body surface may or may not intersect the axis of the body of revolution, respectively. Dielectric bodies must be closed. The code is not capable of treating bodies in which the generating arc forms a closed loop (such as for toroidal bodies) nor is it capable of treating bodies having a finite conductivity. The modifications necessary to include such cases, however, are fairly straightforward. The excitation which induces current on the body is assumed to be a plane wave for perfect conductors or dielectrics and/or a single delta-gap voltage source for perfect conductors. Any delta-gap voltage source is assumed to be ϕ -independent.

The output of the code for each excitation consists of a description of the input parameters and pertinent calculated parameters, listings of each Fourier component of current, and listings of sums over the computed Fourier current components observed in specified planes of constant ϕ .

The code itself contains sufficient information in comment cards for user operation. The general purpose auxiliary routines needed by the code are also listed in this appendix for the reader's convenience.

B.1 Program Operation

The program MAIN reads all input data except for input data which describes the generating arc. The input data is adequately described by comment cards in the program. The subroutine GENCUR is called to read the data describing the generating arc in terms of the ρ - and z -coordinates of the points t_n (see Fig. 2.2). GENCUR also calculates and stores vectors corresponding to the quantities $t_{n+\frac{1}{2}}$, Δt_n , and γ_n . The number of points used should be sufficient to adequately approximate the generating arc as well as to represent the variation of the current on the body. The body surface should not intersect the axis of the body of revolution except at the end points of the generating arc. If the surface does intersect the axis at either or both ends of the generating arc, then the points t_0 and/or t_{N+1} should be placed on the axis. The general form of the input data for a single case is given on the next page in terms of the program variables:

NFLDS	MODEB	MODEE	NGQ	IDB
LAMBDA	EPSR	MUR		
THETA(1)	PHI(1)	ETHETA(1)	EPHI(1)	ANTFD(1)
:				
:				
THETA(NFLDS)	PHI(NFLDS)	ETHETA(NFLDS)	EPHI(NFLDS)	ANTFD(NFLDS)
RHO(1) Z(1)				
RHO(2) Z(2)				
:				
:				
RHO(NPTS) Z(NPTS)				
9999.0	9999.0			

Note that the number of excitations entered must correspond to the input parameter NFLDS. On the other hand, the points read which describe the generating arc are terminated by the presence of the numbers 9999.0 and the quantity NPTS is calculated internally. Multiple cases may be run by placing a new data set immediately following the first data set.

Once all data is read for a particular case MAIN computes the dimensions required to run the case. If the absolute dimensions provided in MAIN are inadequate, execution is aborted for the case and data is read for a new case, if present; otherwise, SLTN is called to compute and print the results. The impedance matrix and drive vector are computed for each Fourier component by calls to the subroutines ZMATRX and CVFILL, respectively. The elements computed by these routines are twice those indicated by Eq. (2.25) and (2.30). The impedance matrix is inverted by the routine CSMINV and the current coefficients indicated in (2.17) are computed as

$I = Z^{-1}V$ for each Fourier component by the multiplication routine ICRMUL. The coefficients of the exponential series (2.17) are transformed to coefficients of a trigonometric series by the subroutine SUMOUT. The current coefficients are then printed and the process is repeated for the next Fourier component.

As mentioned previously, the elements of the impedance matrix calculated by the program are twice that indicated by (2.25). Except for this, the elements are calculated using the expressions (2.25) and the duality relations (2.14), where appropriate. The integral functions ψ and U appearing in (2.25) are calculated by numerical integration. Gaussian quadrature integration (subroutines CGQ1, CGQ1T) is used in the t -direction and the order of the approximation is controlled by the input parameter NGQ. Second order Gaussian quadrature (NGQ = 2) has generally been found sufficient. Gaussian quadrature integration, however, represents a fixed order approximation and it is not entirely satisfactory for performing integrations in the ϕ -direction since the integrands of the ψ and U functions may vary rapidly as a function of ϕ . We have therefore chosen to perform integrations in the ϕ -direction using adaptive trapezoidal rule integration (subroutine TRPADP). The adaptive numerical integration is terminated when the relative error between the two most recent

results is less than 1%. The adaptive integration procedure assures that the integrations in the ϕ -direction will be performed accurately regardless of the body size or Fourier component considered. For self-terms, in which the integrands of the ψ and U functions may be singular, the procedures indicated in Appendix A are followed. Any analytically evaluated portion of an integral function U is included by the function subroutine CANALY. Analytically evaluated portions of the ψ functions are included automatically in the subroutines ELLPTC and ELLPTR. For non-self terms the ψ functions are evaluated as indicated by Eq. (A.13) in order to increase the convergence rate of the adaptive integration in the ϕ -direction by smoothing the integrand.

B.2 Sample Case

A sample case is provided in this section to provide a convenient check for user implementation only. The sample run is not intended to adequately model any structure, but simply to exercise most sections of the code. The input data for the sample case follows:

1	0	1	2	1	
1.0	4.0		1.0		
90.	0.		-1.0	0.0	0.0
0.0	0.0				
.25	0.0				
.5	.25				
.5	.5				
.3	.65				
.15	.65				
0.0	.6				
9999.	9999.				

The resulting output of the program is presented on the following pages.

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*
* CASE NUMBER: 1 *
*
* EXCITATION NUMBER: 1 *
*

POINT	RHO	Z	DELTA-T	GAMMA
1	0.000000E+00	0.000000E+00		
	1.250000E-01	0.000000E+00	2.500000E-01	1.570796E+00
2	2.500000E-01	0.000000E+00		
	3.750000E-01	1.250000E-01	3.535534E-01	7.853982E-01
3	5.000000E-01	2.500000E-01		
	5.000000E-01	3.750000E-01	2.500000E-01	0.000000E+00
4	5.000000E-01	5.000000E-01		
	4.000000E-01	5.750000E-01	2.500000E-01	-9.272952E-01
5	3.000000E-01	6.500000E-01		
	2.250000E-01	6.500000E-01	1.500000E-01	-1.570796E+00
6	1.500000E-01	6.500000E-01		
	7.500000E-02	6.250000E-01	1.581139E-01	-1.892547E+00
7	0.000000E+00	6.000000E-01		

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*
* INPUT DATA *
*

MAXIMUM NUMBER OF MODES TO BE USED = 2

ORDER OF GAUSSIAN QUADRATURE = 2

FREE SPACE WAVELENGTH = 1.0000000E+00

A DIELECTRIC BODY HAS BEEN ASSUMED:

RELATIVE DIELECTRIC CONSTANT = 4.0000000E+00

RELATIVE PERMEABILITY = 1.0000000E+00

INCIDENT FIELD DATA:

THETA = 9.0000000E+01

PHI = 0.0000000E+00

E-THETA = -1.0000000E+00

E-PHI = 0.0000000E+00

*
* COMPUTED DATA *
*

PARAMETERS OF FREE SPACE:

DIELECTRIC CONSTANT = 8.8541850E-12

PERMEABILITY = 1.2566370E-06

WAVENUMBER = 6.2831850E+00

SPEED OF LIGHT = 2.9979250E+08

PARAMETERS OF THE DIELECTRIC BODY:

DIELECTRIC CONSTANT = 3.5416740E-11

PERMEABILITY = 1.2566370E-06

WAVENUMBER = 1.2566370E+01

SPEED OF LIGHT = 1.4989630E+08

WAVELENGTH = 5.0000000E-01

OMEGA = 1.8836520E+09

FREQUENCY = 2.9979250E+08

"T" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

RHO	Z	REAL TOTAL	IMAGINARY TOTAL	MAGNITUDE TOTAL	PHASE	REAL DENSITY	IMAGINARY DENSITY	MAGNITUDE DENSITY
***	***	***	*****	*****	****	****	*****	*****
MODE NUMBER	0							
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
2.50000E-01	0.00000E+00	1.51233E-03	1.39029E-03	2.05428E-03	4.25926E+01	9.62777E-04	8.85089E-04	1.30779E-03
5.00000E-01	2.50000E-01	-4.55590E-03	4.88770E-03	6.68181E-03	1.32980E+02	-1.45021E-03	1.55580E-03	2.12689E-03
5.00000E-01	5.00000E-01	-3.69480E-03	3.48112E-03	5.08120E-03	1.36648E+02	-1.17609E-03	1.11030E-03	1.61740E-03
3.00000E-01	6.50000E-01	1.68047E-03	3.71937E-03	4.08138E-03	6.56858E+01	8.91517E-04	1.97319E-03	2.16524E-03
1.50000E-01	6.50000E-01	5.57119E-04	5.32033E-04	7.70333E-04	4.36838E+01	5.91122E-04	5.64569E-04	8.17412E-04
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00

INV. COND. NO.: 3.3222760E-07 DETERMINANT: (3.1273050E-05 -6.4957890E-05) TIMES (10.)**(-200.0)

"PHI" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

RHO	Z	REAL TOTAL	IMAGINARY TOTAL	MAGNITUDE TOTAL	PHASE	REAL DENSITY	IMAGINARY DENSITY	MAGNITUDE DENSITY
***	***	***	*****	*****	****	****	*****	*****
MODE NUMBER	0							
1.25000E-01	0.00000E+00	-7.38063E-13	3.555961E-13	8.19418E-13	1.54252E+02	-9.39731E-13	4.53223E-13	1.04332E-12
3.75000E-01	1.25000E-01	9.07931E-13	2.06482E-12	2.25552E-12	6.62642E+01	3.85338E-13	8.76336E-13	9.57314E-13
5.00000E-01	3.75000E-01	6.34647E-12	4.97980E-13	6.36597E-12	4.48656E+00	2.02014E-12	1.58512E-13	2.02635E-12
4.00000E-01	5.75000E-01	-9.43897E-12	3.42090E-12	1.00322E-11	1.60063E+02	-3.75246E-12	1.36113E-12	3.99170E-12
2.25000E-01	6.50000E-01	-5.21204E-12	-5.92272E-12	7.88948E-12	-1.31348E+02	-3.68676E-12	-4.18946E-12	5.58066E-12
7.50000E-02	6.25000E-01	-8.71204E-13	-2.67353E-12	2.81190E-12	-1.08049E+02	-1.84875E-12	-5.67342E-12	5.96704E-12

INV. COND. NO.: 3.3222760E-07 DETERMINANT: (3.1273050E-05 -6.4957890E-05) TIMES (10.)**(-200.0)

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"I" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

RHO	Z	MODE NUMBER	REAL TOTAL	IMAGINARY TOTAL	MAGNITUDE TOTAL	PHASE	REAL DENSITY	IMAGINARY DENSITY	MAGNITUDE DENSITY
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	0.00000E+00	0.00000E+00	0.00000E+00
2.50000E-01	0.00000E+00	-2.37470E-09	-2.37470E-09	-3.39079E-09	4.13964E-09	-1.25005E+02	-1.51178E-09	-2.15864E-09	2.63538E-09
5.00000E-01	2.50000E-01	8.85322E-10	8.85322E-10	-2.63334E-09	2.77818E-09	-7.14175E+01	2.81307E-10	-8.38220E-10	8.84323E-10
5.00000E-01	5.00000E-01	-4.24101E-09	-4.24101E-09	2.56733E-09	4.95755E-09	1.48811E+02	-1.34995E-09	8.17205E-10	1.57804E-09
3.00000E-01	6.50000E-01	-6.20803E-12	-6.20803E-12	-8.37803E-10	8.37106E-10	-9.04248E+01	-3.29240E-12	-4.44086E-10	4.44086E-10
1.50000E-01	6.50000E-01	7.50407E-10	7.50407E-10	-1.08616E-09	1.32017E-09	-5.53601E+01	7.96206E-10	-1.15245E-09	1.40074E-09
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	0.00000E+00	0.00000E+00	0.00000E+00

INV. COND. NO.: 3.3222760E-07 DETERMINANT: (3.1273050E-05 -6.4957890E-05) TIMES (10.)**(-200.0)

"PHI" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

RHO	Z	MODE NUMBER	REAL TOTAL	IMAGINARY TOTAL	MAGNITUDE TOTAL	PHASE	REAL DENSITY	IMAGINARY DENSITY	MAGNITUDE DENSITY
1.25000E-01	0.00000E+00	1.27316E-01	1.27316E-01	1.80166E-01	2.20610E-01	5.47527E+01	1.62103E-01	2.29394E-01	2.80890E-01
3.75000E-01	1.25000E-01	3.45628E-01	3.45628E-01	6.6916E-02	3.52860E-01	1.09694E+01	1.46689E-01	2.84321E-02	1.49419E-01
5.00000E-01	3.75000E-01	-1.40593E-01	-1.40593E-01	-6.24160E-01	6.39798E-01	-1.02694E+02	-4.47521E-02	-1.98676E-01	2.03654E-01
4.00000E-01	5.75000E-01	6.48447E-02	6.48447E-02	-1.02929E-01	1.21652E-01	-5.77893E+01	2.58009E-02	-4.09542E-02	4.84039E-02
2.25000E-01	6.50000E-01	7.42488E-02	7.42488E-02	1.83489E-01	1.97942E-01	6.79693E+01	5.25203E-02	1.29792E-01	1.40015E-01
7.50000E-02	6.25000E-01	3.42608E-02	3.42608E-02	1.05092E-01	1.10542E-01	7.19339E+01	7.27461E-02	2.23013E-01	2.34578E-01

INV. COND. NO.: 3.3222760E-07 DETERMINANT: (3.1273050E-05 -6.4957890E-05) TIMES (10.)**(-200.0)

"Psi" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 1

COS(PHI) COEFFICIENTS

RHO	Z	REAL TOTAL ****	IMAGINARY TOTAL *****	MAGNITUDE TOTAL *****	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *****	MAGNITUDE DENSITY *****
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???
2.50000E-01	0.00000E+00	1.52079E-03	1.01818E-03	1.83017E-03	3.38026E+01	9.68168E-04	6.48195E-04	1.16512E-03
5.00000E-01	2.50000E-01	2.82837E-04	-1.28673E-04	1.31745E-03	-7.76030E+01	9.00297E-05	-4.09579E-04	4.19357E-04
5.00000E-01	5.00000E-01	-1.26725E-03	-9.66056E-04	1.59348E-03	-1.42681E+02	-4.03378E-04	-3.07505E-04	5.07221E-04
3.00000E-01	6.50000E-01	1.65503E-03	1.51015E-03	2.24046E-03	4.23793E+01	8.78019E-04	8.01160E-04	1.18860E-03
1.50000E-01	6.50000E-01	7.63971E-04	-6.54471E-07	7.63972E-04	-4.90836E-02	8.10599E-04	-6.94416E-07	8.10599E-04
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???

INV. COND. NO.: 2.6376860E-07 DETERMINANT: (4.0919200E-06 -5.6265210E-06) TIMES (10.) **(-200.0)

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 3.9475058E-01

83

"Phi" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 1

SIN(PHI) COEFFICIENTS

RHO	Z	REAL TOTAL ****	IMAGINARY TOTAL *****	MAGNITUDE TOTAL *****	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *****	MAGNITUDE DENSITY *****
1.25000E-01	0.00000E+00	-1.49471E-03	-5.80045E-04	1.60331E-03	-1.58790E+02	-1.90313E-03	-7.38537E-04	2.04140E-03
3.75000E-01	1.25000E-01	-6.28147E-04	1.80451E-04	6.53553E-04	1.63972E+02	-2.66594E-04	7.65858E-05	2.77376E-04
5.00000E-01	3.75000E-01	-4.06397E-04	-8.65970E-04	9.56589E-04	-1.15141E+02	-1.29360E-04	-2.75647E-04	3.04492E-04
4.00000E-01	5.75000E-01	9.08867E-04	-6.23358E-04	1.10222E-03	-3.44546E+01	3.61627E-04	-2.48117E-04	4.38561E-04
2.25000E-01	6.50000E-01	1.50175E-03	-8.15038E-06	1.50177E-03	-3.10956E-01	1.06227E-03	-5.76521E-06	1.06228E-03
7.50000E-02	6.25000E-01	-7.09715E-05	-4.15060E-04	4.21085E-04	-9.97032E+01	-1.50606E-04	-8.80786E-04	8.93569E-04

INV. COND. NO.: 2.6376860E-07 DETERMINANT: (4.0919200E-06 -5.6265210E-06) TIMES (10.) **(-200.0)

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 1.0000000E+00

"T" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 1

SIN(PHI) COEFFICIENTS

RHO	Z	REAL TOTAL ****	IMAGINARY TOTAL *****	MAGNITUDE TOTAL *****	PHASE ****	REAL DENSITY ****	IMAGINARY DENSITY *****	MAGNITUDE DENSITY *****
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???
2.50000E-01	0.00000E+00	-2.98247E-01	-1.74145E-01	3.45366E-01	-1.49720E+02	-1.89870E-01	-1.10864E-01	2.19867E-01
5.00000E-01	2.50000E-01	-4.02491E-01	4.57507E-01	6.08693E-01	1.31269E+02	-1.27799E-01	1.45629E-01	1.93753E-01
5.00000E-01	5.00000E-01	4.94885E-01	-7.92825E-01	9.34603E-01	-5.80274E+01	1.57527E-01	-2.52364E-01	2.97493E-01
3.00000E-01	6.50000E-01	2.46446E-01	4.18585E-01	4.85746E-01	5.95122E+01	1.30744E-01	2.22066E-01	2.57696E-01
1.50000E-01	6.50000E-01	-6.07100E-01	5.20082E-01	7.99410E-01	1.39414E+02	-6.44154E-01	5.51824E-01	8.48200E-01
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???

INV. COND. NO.: 2.6376860E-07. DETERMINANT: (4.0919200E-06 -5.6265210E-06) TIMES (10.) **(-200.0)

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 1.000000E+00

4

"PHI" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 1

COS(PHI) COEFFICIENTS

RHO	Z	REAL TOTAL ****	IMAGINARY TOTAL *****	MAGNITUDE TOTAL *****	PHASE ****	REAL DENSITY ****	IMAGINARY DENSITY *****	MAGNITUDE DENSITY *****
1.25000E-01	0.00000E+00	4.54932E-01	-2.53718E-01	5.20099E-01	-2.91486E+01	5.79237E-01	-3.23044E-01	6.63229E-01
3.75000E-01	1.25000E-01	-1.28526E+00	1.59347E+00	2.04721E+00	1.28889E+02	-5.45483E-01	6.76290E-01	8.68862E-01
5.00000E-01	3.75000E-01	-1.90686E+00	2.34260E+00	3.02050E+00	1.29145E+02	-6.06974E-01	7.45673E-01	9.61481E-01
4.00000E-01	5.75000E-01	-1.20882E+00	1.42639E+00	1.86972E+00	1.30280E+02	-4.80974E-01	5.67543E-01	7.43936E-01
2.25000E-01	6.50000E-01	3.13761E-02	3.14860E-01	3.16420E-01	8.43092E+01	2.21941E-02	2.22718E-01	2.23821E-01
7.50000E-02	6.25000E-01	2.69081E-01	-1.95520E-01	3.32431E-01	-3.59593E+01	5.71008E-01	-4.14242E-01	7.05441E-01

INV. COND. NO.: 2.6376860E-07. DETERMINANT: (4.0919200E-06 -5.6265210E-06) TIMES (10.) **(-200.0)

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 9.8722930E-01

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"T" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

SUM OF MODES 0 THROUGH 1									
OBSERVED AT PHI = 0.0 DEGREES									
RHO	Z	***	REAL TOTAL ***	IMAGINARY TOTAL *****	MAGNITUDE TOTAL *****	PHASE ****	REAL DENSITY ***	IMAGINARY DENSITY *****	MAGNITUDE DENSITY *****
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???
2.50000E-01	0.00000E+00	0.00000E+00	3.03312E-03	2.40848E-03	3.87306E-03	3.84516E+01	1.93094E-03	1.53328E-03	2.46567E-03
5.00000E-01	2.50000E-01	5.00000E-01	-4.27315E-03	3.62297E-03	5.58809E-03	1.39879E+02	-1.36018E-03	1.14622E-03	1.77874E-03
3.00000E-01	5.00000E-01	3.00000E-01	-4.96205E-03	2.52207E-03	5.56622E-03	1.53057E+02	-1.57947E-03	8.02799E-04	1.77178E-03
1.50000E-01	6.50000E-01	1.50000E-01	3.33550E-03	5.22952E-03	6.20269E-03	5.74694E+01	1.76954E-03	2.77435E-03	3.29063E-03
0.00000E-01	0.00000E-01	0.00000E-01	1.32109E-03	5.31439E-03	1.42398E-03	2.19136E+01	1.40172E-03	5.63874E-04	1.51089E-03
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???
ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 3.9475050E-01									

"PHI" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

SUM OF MODES 0 THROUGH 1									
OBSERVED AT PHI = 90.0 DEGREES									
RHO	Z	***	REAL TOTAL ***	IMAGINARY TOTAL *****	MAGNITUDE TOTAL *****	PHASE ****	REAL DENSITY ***	IMAGINARY DENSITY *****	MAGNITUDE DENSITY *****
1.25000E-01	0.00000E+00	0.00000E+00	-1.49471E-03	5.80045E-04	1.60331E-03	-1.58790E+02	-1.94312E-03	-7.38537E-04	2.04140E-03
5.00000E-01	1.25000E-01	5.00000E-01	-6.28147E-04	1.80451E-04	6.53553E-04	1.63972E+02	-2.66594E-04	7.65858E-05	2.77376E-04
4.00000E-01	5.00000E-01	4.00000E-01	-4.06397E-04	8.65970E-04	9.56589E-04	-1.15141E+02	-1.29360E-04	-2.75647E-04	3.04492E-04
2.25000E-01	6.50000E-01	2.25000E-01	9.08867E-04	-6.23586E-04	1.10222E-03	-3.44546E+01	3.61627E-04	-2.48117E-04	4.38561E-04
7.50000E-02	0.00000E-01	0.00000E-01	1.50175E-03	-8.15038E-06	1.50177E-03	-3.10957E-01	1.06227E-03	-5.76522E-06	1.06228E-03
0.00000E+00	0.00000E+00	0.00000E+00	-7.09715E-05	-4.15060E-04	4.21085E-04	-9.97032E+01	-1.50606E-04	-8.80786E-04	8.93569E-04
ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 1.0000000E+00									

-T- COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

SUM OF MODES 0 THROUGH 1

OBSERVED AT PHI = 90.0 DEGREES

RHO	Z	REAL TOTAL ***	IMAGINARY TOTAL *****	MAGNITUDE TOTAL *****	PHASE ****	REAL DENSITY ***	IMAGINARY DENSITY *****	MAGNITUDE DENSITY *****
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???
2.50000E-01	0.00000E+00	-2.98247E-01	-1.74145E-01	3.45366E-01	-1.49720E+02	-1.89870E-01	-1.10864E-01	2.19867E-01
5.00000E-01	2.50000E-01	-4.01491E-01	4.57507E-01	6.08693E-01	1.31269E+02	-1.27799E-01	1.45629E-01	1.93753E-01
5.00000E-01	5.00000E-01	4.94885E-01	-7.92825E-01	9.34603E-01	-5.80274E+01	1.57527E-01	-2.52364E-01	2.97493E-01
3.00000E-01	6.50000E-01	2.46446E-01	4.18585E-01	4.85746E-01	5.95122E+01	1.30744E-01	2.22066E-01	2.57696E-01
1.50000E-01	6.50000E-01	-6.07100E-01	5.20082E-01	7.99410E-01	1.39414E+02	-6.44154E-01	5.51824E-01	8.48200E-01
0.00000E+00	6.00000E-01	0.00000E+00	9.00000E+00	9.00000E+00	???	???	???	???

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 1.0000000E+00

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"PHI" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

SUM OF MODES 0 THROUGH 1

OBSERVED AT PHI = 0.0 DEGREES

RHO	Z	REAL TOTAL ***	IMAGINARY TOTAL *****	MAGNITUDE TOTAL *****	PHASE ****	REAL DENSITY ***	IMAGINARY DENSITY *****	MAGNITUDE DENSITY *****
1.25000E-01	0.00000E+00	5.82248E-01	-7.35522E-02	5.86875E-01	-7.19973E+00	7.41341E-01	-9.36496E-02	7.47232E-01
3.75000E-01	1.25000E-01	-9.39637E-01	1.66046E+00	1.90789E+00	1.19505E+02	-3.98794E-01	7.04722E-01	8.09735E-01
5.00000E-01	3.75000E-01	-2.04746E+00	1.71844E+00	2.67304E+00	1.39993E+02	-6.51726E-01	5.46996E-01	8.50854E-01
4.00000E-01	5.75000E-01	-1.14397E+00	1.32346E+00	1.74935E+00	1.30839E+02	-4.55173E-01	5.26589E-01	6.96045E-01
2.25000E-01	6.50000E-01	1.05625E-01	4.98349E-01	5.09420E-01	7.80333E+01	7.47143E-02	3.52510E-01	3.60034E-01
7.50000E-02	5.25000E-01	3.03362E-01	-9.01147E-02	3.16464E-01	-1.65442E+01	6.43755E-01	-1.91229E-01	6.71557E-01

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 9.8722930E-01

B.3 Program Listing

C	PROGRAM DBR	BR-	10
C		BR-	20
C	VERSION 4.01	BR-	30
C		BR-	40
C	JULY 28, 1978	BR-	50
C	*****	BR-	60
C		BR-	70
C	PROGRAM DBR CALCULATES THE CURRENTS (TOTAL AND DENSITY) FOR A	BR-	80
C	BODY OF REVOLUTION, EITHER DIELECTRIC OR PERFECTLY CONDUCTING.	BR-	90
C		BR-	100
C		BR-	110
C	PROGRAM "MAIN." READS ALL THE INPUT DATA EXCEPT FOR DATA	BR-	120
C	CONCERNING THE GENERATING CURVE OF THE BODY OF REVOLUTION.	BR-	130
C	SUBROUTINE "GENCUR" IS CALLED TO READ THIS DATA. "MAIN." THEN	BR-	140
C	COMPUTES THE DIMENSIONS NECESSARY TO RUN THE PROGRAM FOR THE CASE	BR-	150
C	IN QUESTION AND ABORTS EXECUTION OF THE CASE IF THE ABSOLUTE	BR-	160
C	DIMENSIONS PROVIDED IN "MAIN." ARE INSUFFICIENT. IF THE	BR-	170
C	DIMENSIONS ARE ADEQUATE "MAIN." CALLS SUBROUTINE "SLTN" TO	BR-	180
C	SOLVE THE PROBLEM.	BR-	190
C		BR-	200
C	AUXILIARY ROUTINES NEEDED:	BR-	210
C		BR-	220
C	CGQ1 - INTEGRATION OF A COMPLEX FUNCTION BY GAUSSIAN QUAD.	BR-	230
C	TRPADP - INTEGRATION OF A COMPLEX FUNCTION BY ADAPTIVE	BR-	240
C	TRAPEZOIDAL RULE	BR-	250
C	CSMINV - INVERSION OF A COMPLEX MATRIX	BR-	260
C	ICRMUL - COMPLEX MATRIX MULTIPLICATION	BR-	270
C	BESSEL - COMPUTATION OF BESSEL FUNCTION OF ARBITRARY ORDER	BR-	280
C	ELIC1K - COMPUTES COMPLETE ELLIPTIC INTEGRAL OF FIRST KIND	BR-	290
C		BR-	300
C	ASSIGNMENTS:	BR-	310
C		BR-	320
C	READ FROM DSK: #2	BR-	330
C	WRITE PRINTED OUTPUT ON LPT: #5 THROUGH #15	BR-	340
C	(AS NECESSARY: FOR MULTIPLE EXCITATIONS)	BR-	350
C		BR-	360
C		BR-	370
C	*****	BR-	380
	IMPLICIT COMPLEX (C)	BR-	390
	REAL LAMBDA, MU, MUR	BR-	400
C	"LOCATE 1"	BR-	410
	DIMENSION C(10296), R(306)	BR-	420
	COMMON/MBE/MODEB, MODEE	BR-	430
	COMMON/PNT/NPTS, NUNKT, NUNKPH, NUNK2	BR-	440
	COMMON/INT/NMODEM, IDB, NGQ	BR-	450
	COMMON/WVL/LAMBDA, EPSR, MUR	BR-	460
	COMMON/CAS/ICASE	BR-	470
C	"LOCATE 2"	BR-	480
	NDMSNC=10296	BR-	490
	NDMSNR=306	BR-	500
C		BR-	510

C-----NOTE: WHEN THE DIMENSION OF "C" OR "R" IS CHANGED, THE VALUE OF BR- 520
 "NDMSNC" OR "NDMSNR", RESPECTIVELY, MUST BE CHANGED TO THE BR- 530
 APPROPRIATE VALUE FOR PROGRAM OPERATION. BR- 540
 BR- 550
 INPUT DATA DESCRIPTION: BR- 560
 (ALL DATA IS READ IN "FREE" FORMAT) BR- 570
 BR- 580
 NFLDS = NUMBER OF DIFFERENT EXCITATIONS TO BE SOLVED FOR BR- 590
 THIS STRUCTURE. BR- 600
 BR- 610
 MODEB = STARTING FOURIER COMPONENT NUMBER BR- 620
 BR- 630
 MODEE = ENDING FOURIER COMPONENT NUMBER BR- 640
 BR- 650
 IDB = A CONTROL VARIABLE: BR- 660
 = 1, IF THE STRUCTURE IS A DIELECTRIC BODY BR- 670
 = 0, IF THE STRUCTURE IS A PERFECT CONDUCTOR BR- 680
 BR- 690
 ANTFD = NUMBER OF SUBDOMAIN WHICH IS FED BY A DELTA GAP SOURCE. BR- 700
 IF = 0, THERE IS NO SOURCE. THE SECOND COORDINATE BR- 710
 POINT READ IN IS CONSIDERED TO BE THE FIRST SUBDOMAIN, BR- 720
 AND SO ON. BR- 730
 BR- 740
 EPSR = RELATIVE DIELECTRIC CONSTANT OF THE BODY BR- 750
 (IF DIELECTRIC) BR- 760
 BR- 770
 MUR = RELATIVE PERMEABILITY OF THE BODY (IF DIELECTRIC) BR- 780
 BR- 790
 LAMBDA = WAVELENGTH (IN METERS) IN FREE SPACE BR- 800
 BR- 810
 NGQ = ORDER OF GAUSSIAN QUADRATURE INTEGRATION TO BE USED BR- 820
 BR- 830
 THETA = SPHERICAL COORDINATE ANGLE OF INCIDENCE (IN DEGREES) BR- 840
 BR- 850
 PHI = SPHERICAL COORDINATE ANGLE OF INCIDENCE (IN DEGREES) BR- 860
 BR- 870
 ETHETA = SIGNED MAGNITUDE OF THE E-THETA INCIDENT FIELD BR- 880
 BR- 890
 EPHI = SIGNED MAGNITUDE OF THE E-PHI INCIDENT FIELD BR- 900
 BR- 910
 BR- 920
 DO 40 ICASE = 1, 50 BR- 930
 READ(2,10000,END=50)NFLDS,MODEB,MODEE,NGQ,IDB BR- 940
 READ(2,10001) LAMBDA, EPSR, MUR BR- 950
 NMODEM=MODEE-MODEB+1 BR- 960
 I2=NFLDS BR- 970
 I3=NFLDS+I2 BR- 980
 I4=NFLDS+I3 BR- 990
 I5=NFLDS+I4 BR- 1000
 I6=NFLDS+I5 BR- 1010
 DO 10 I = 1, NFLDS BR- 1020
 READ(2,10001) THETA, PHI, ETHETA, EPHI, ANTFD BR- 1030
 R(I)=THETA BR- 1040
 R(I2+I)=PHI BR- 1050
 R(I3+I)=ETHETA BR- 1060
 R(I4+I)=EPHI BR- 1070
 R(I5+I)=ANTFD BR- 1080
 10 CONTINUE BR- 1090
 CALL GENCUR(R(I6+1),I6,NDMSNR,NPTS,NREQR) BR- 1100
 NREQR=NREQR+I6 BR- 1110
 NUNKT=NPTS-2 BR- 1120

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NUNKPH=NPTS-1
IF(IDB .EQ. 0) GO TO 20
NUNKT=NUNKT*2
NUNKPH=NUNKPH*2
20 NUNK=NUNKT+NUNKPH
NUNK2=NUNK*NUNK
NREQC=NUNK*NFLDS*6+NUNK2
IF( NREQC .GT. NDMSNC .OR. NREQR .GT. NDMSNR) GO TO 30
NPTSM1=NPTS-1
MR2=NFLDS+1
MR3=NFLDS+MR2
MR4=NFLDS+MR3
MR5=NFLDS+MR4
MR6=NFLDS+MR5
MR7=NPTSM1+MR6
MR8=NPTSM1+MR7
MR9=NPTSM1+MR8
MR10=NPTSM1+MR9
MR11=NPTS+MR10
C-----MR12=NPTS+MR11
MC2=NUNK2+1
MC3=NUNK*NFLDS+MC2
MC4=NUNK*NFLDS+MC3
MC5=NUNK*NFLDS+MC4
MC6=NUNK*NFLDS+MC5
C-----MC7=NUNK*NFLDS+MC6
CALL ZERO(C,NREQC)
CALL SLTN(
$          C(1),C(MC2),C(MC3),C(MC4),C(MC5),C(MC6),
$          R(1),R(MR2),R(MR3),R(MR4),R(MR5),R(MR6),R(MR7),R(MR8),
$          R(MR9),R(MR10),R(MR11),NUNK,NFLDS
GO TO 40
30 CONTINUE
WRITE(5,10002) ICASE, NDMSNC, NDMSNR, NREQC, NREQR
40 CONTINUE
50 CONTINUE
10000 FORMAT(9I)
10001 FORMAT(9E)
10002 FORMAT(' CASE NUMBER'13,' EXECUTION ABORTED: DIMENSIONS INSUFFICI
#ENT'/' DIMENSION GIVEN IN PROGRAM:'//5X,'C('16,')'5X,'R('16,')'//
#' DIMENSIONS REQUIRED FOR THIS CASE:'//5X,'C('16,')'5X,'R('16,')'')
STOP
END
SUBROUTINE GENCUR(S,NFLDS4,NDMSNS,NPTS,NREQR)
C*****
C
C SUBROUTINE "GENCUR" READS THE GENERATING CURVE DATA FOR THE BODY
C OF REVOLUTION AND USES THIS INFORMATION TO COMPUTE THE DIMENSIONS
C NECESSARY OF THE REAL VECTOR "S". IT ALSO USES THE GENERATING
C CURVE DATA TO COMPUTE INTERMEDIATE COORDINATE LOCATIONS, VALUES
C OF STEP SIZE, AND ANGLES CORRESPONDING TO THE DIRECTION OF EACH
C ELEMENTAL SURFACE.
C
C*****
C
C DIMENSION S(1)
C NDMSN=NDMSNS-NFLDS4
C
C INPUT DATA DESCRIPTION:
C
C RHO = CYLINDRICAL COORDINATE RHO (IN METERS)

```

BR- 1130
BR- 1140
BR- 1150
BR- 1160
BR- 1170
BR- 1180
BR- 1190
BR- 1200
BR- 1210
BR- 1220
BR- 1230
BR- 1240
BR- 1250
BR- 1260
BR- 1270
BR- 1280
BR- 1290
BR- 1300
BR- 1310
BR- 1320
BR- 1330
BR- 1340
BR- 1350
BR- 1360
BR- 1370
BR- 1380
BR- 1390
BR- 1400
BR- 1410
BR- 1420
BR- 1430
BR- 1440
BR- 1450
BR- 1460
BR- 1470
BR- 1480
BR- 1490
BR- 1500
BR- 1510
BR- 1520
BR- 1530
BR- 1540
BR- 1550
BR- 1560
BR- 1570
BR- 1580
BR- 1590
BR- 1600
BR- 1610
BR- 1620
BR- 1630
BR- 1640
BR- 1650
BR- 1660
BR- 1670
BR- 1680
BR- 1690
BR- 1700
BR- 1710
BR- 1720
BR- 1730

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C	Z = CYLINDRICAL COORDINATE Z (IN METERS)	BR- 1740
C		BR- 1750
C	NOTE: THE LAST RECORD OF THE GENERATING CURVE DATA MUST BE	BR- 1760
C	9999. 9999.	BR- 1770
C	(THE SPACING IS ARBITRARY).	BR- 1780
C		BR- 1790
C		BR- 1800
	I=1	BR- 1810
	DO 10 K = 1, NDMSN	BR- 1820
10	S(K)=0.0	BR- 1830
20	READ(2,10000) RHO, Z	BR- 1840
	NPTS=I-1	BR- 1850
	IF(RHO .EQ. 9999. .AND. Z .EQ. 9999.) GO TO 30	BR- 1860
	K=2*I	BR- 1870
	IF(K .GT. NDMSN) GO TO 20	BR- 1880
	S(K-1)=RHO	BR- 1890
	S(K)=Z	BR- 1900
	I=I+1	BR- 1910
	GO TO 20	BR- 1920
C		BR- 1930
C	-----COMPUTE THE DIMENSION OF "S" REQUIRED FOR THIS CASE	BR- 1940
C		BR- 1950
	30 NREQR=6*NPTS-4	BR- 1960
	IF(NREQR .GT. NDMSN) RETURN	BR- 1970
	NSTRT=4*NPTS-4	BR- 1980
C		BR- 1990
C	-----REORDER THE INPUT DATA	BR- 2000
C		BR- 2010
	DO 40 I = 1, NPTS	BR- 2020
	K=2*I	BR- 2030
	NI=NSTRT+I	BR- 2040
	S(NI)=S(K-1)	BR- 2050
	S(NPTS+NI)=S(K)	BR- 2060
40	CONTINUE	BR- 2070
	NPTSM1=NPTS-1	BR- 2080
	DO 50 I = 1, NPTSM1	BR- 2090
	NI=NSTRT+I	BR- 2100
	RI=S(NI)	BR- 2110
	RIPI=S(NI+1)	BR- 2120
	NI=NPTS+NI	BR- 2130
	ZI=S(NI)	BR- 2140
	ZIPI=S(NI+1)	BR- 2150
C		BR- 2160
C	-----STORE RHO AND Z HALFWAY POINTS	BR- 2170
C		BR- 2180
	S(I)=(RI+RIPI)/2.0	BR- 2190
	S(I+NPTSM1)=(ZI+ZIPI)/2.0	BR- 2200
	DIPI=SQRT((RIPI-RI)**2+(ZIPI-ZI)**2)	BR- 2210
	ARGN=(RIPI-RI)/DIPI	BR- 2220
	ARGD=(ZIPI-ZI)/DIPI	BR- 2230
C		BR- 2240
C	-----STORE DELTA-T VECTOR	BR- 2250
C		BR- 2260
	S(I+2*NPTSM1)=DIPI	BR- 2270
C		BR- 2280
C	-----STORE GAMMA VECTOR	BR- 2290
C		BR- 2300
	S(I+3*NPTSM1)=ATAN2(ARGN,ARGD)	BR- 2310
	50 CONTINUE	BR- 2320
	10000 FORMAT(9E)	BR- 2330
	RETURN	BR- 2340

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END
SUBROUTINE SLTN(
$      CZ,CIT,CIP,CVT,CVP,CITOLD,
$      THETA,PHI,ETHETA,EPI,ANTFD,RHOPH,ZPH,DELTAT,
$      GAMMA,RHO,Z,NUNK,NFLDS )
C*****BR- 2350
C      SUBROUTINE "SLTN" COMPUTES AND OUTPUTS THE RESULTS. BR- 2360
C      PRINTED RESULTS ARE OUTPUT TO DEVICES NUMBER 5 THROUGH 15. BR- 2370
C      (FOR MULTIPLE EXCITATIONS, EACH CASE IS OUTPUT TO A NEW BR- 2380
C      DEVICE #, BEGINNING WITH DEVICE #5) BR- 2390
C*****BR- 2400
C      IMPLICIT COMPLEX (C) BR- 2410
C      REAL LAMBDA,MUR,MU1,MU2 BR- 2420
C      DIMENSION POL(2),FTC(2) BR- 2430
C      DIMENSION CZ(NUNK,NUNK),CIT(NUNK,NFLDS),CIP(NUNK,NFLDS) BR- 2440
C      DIMENSION CVT(NUNK,NFLDS),CVP(NUNK,NFLDS) BR- 2450
C      DIMENSION CITOLD(NUNK,NFLDS,2) BR- 2460
C      DIMENSION THETA(1),PHI(1),ETHETA(1),EPI(1),ANTFD(1) BR- 2470
C      DIMENSION RHO(1),Z(1),RHOPH(1),ZPH(1),DELTAT(1),GAMMA(1) BR- 2480
C      COMMON/MBE/MODEB,MODEE BR- 2490
C      COMMON/SCAFAC/ISCALE BR- 2500
C      COMMON/PNT/NPTS,NUNKT,NUNKPH,NUNK2 BR- 2510
C      COMMON/INT/NMODEM,IDB,NGQ BR- 2520
C      COMMON/WVL/LAMBDA,EPSR,MUR BR- 2530
C      COMMON/CAS/ICASE BR- 2540
C      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA BR- 2550
C      COMMON/PRM/EPS1,EPS2,MU1,MU2 BR- 2560
C      DATA POL/'THETA','PHI'/ BR- 2570
C      DATA FTC/'SIN','COS', BR- 2580
C      NPTSM1=NPTS-1 BR- 2590
C      ZER=0.0 BR- 2600
C      PI=3.1415926536 BR- 2610
C      AK1=2.0*PI/LAMBDA BR- 2620
C      SL1=2.997925E8 BR- 2630
C      OMEGA=AK1*SL1 BR- 2640
C      FREQ=OMEGA/2.0/PI BR- 2650
C      MU1=PI*4.0E-7 BR- 2660
C      EPS1=1.0/(MU1*SL1*SL1) BR- 2670
C      IF(IDB.EQ.0) GO TO 10 BR- 2680
C      EPS2=EPS1*EPSR BR- 2690
C      MU2=MU1*MUR BR- 2700
C      SL2=1.0/SQRT(MU2*EPS2) BR- 2710
C      AK2=OMEGA/SL2 BR- 2720
C      WVL2=2.0*PI/AK2 BR- 2730
C      10 CONTINUE BR- 2740
C      C-----PARAMETER PRINT-OUT OPTION BR- 2750
C      GO TO 50 BR- 2760
C      DO 40 JJ=1,NFLDS BR- 2770
C      JW=JJ+4 BR- 2780
C      WRITE(JW,10036) ICASE, JJ BR- 2790
C      WRITE(JW,10000) BR- 2800
C      DO 30 J=1,NPTS BR- 2810
C      WRITE(JW,10001) J, RHO(J), Z(J) BR- 2820
C      IF(J.EQ.NPTS) GO TO 20 BR- 2830
C      WRITE(JW,10002) RHOPH(J), ZPH(J), DELTAT(J), GAMMA(J) BR- 2840
C      20 CONTINUE BR- 2850

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IF(NFLDS .GT. 11) WRITE(JW,10033)	BR- 2960
30 CONTINUE	BR- 2970
40 CONTINUE	BR- 2980
50 CONTINUE	BR- 2990
DO 66 J=1,NFLDS	BR- 3000
JW=J+4	BR- 3010
WRITE(JW,10003)	BR- 3020
WRITE(JW,10004) NMODEM, NGQ	BR- 3030
WRITE(JW,10005) LAMBDA	BR- 3040
IF(IDB .EQ. 0) WRITE(JW,10006)	BR- 3050
IF(IDB .EQ. 1) WRITE(JW,10007) EPSR, MUR	BR- 3060
WRITE(JW,10008) THETA(J), PHI(J), ETHETA(J), EPHI(J)	BR- 3070
IANTFD=IFIX(ANTFD(J))	BR- 3080
IF(IANTFD .NE. 0) WRITE(JW,10025) RHO(IANTFD+1), Z(IANTFD+1)	BR- 3090
WRITE(JW,10017)	BR- 3100
WRITE(JW,10022)	BR- 3110
WRITE(JW,10018) EPS1, MU1, AK1, SL1	BR- 3120
IF(IDB .EQ. 1) WRITE(JW,10019)	BR- 3130
IF(IDB .EQ. 1) WRITE(JW,10018) EPS2, MU2, AK2, SL2	BR- 3140
IF(IDB .EQ. 1) WRITE(JW,10023) WVL2	BR- 3150
WRITE(JW,10020) OMEGA, FREQ	BR- 3160
60 CONTINUE	BR- 3170
M0ONLY=1	BR- 3180
M1ONLY=1	BR- 3190
DO 70 JJ=1,NFLDS	BR- 3200
IF(EPHI(JJ) .NE. 0.0 .OR. ETHETA(JJ) .NE. 0.0) M0ONLY=0	BR- 3210
70 IF(THETA(JJ) .NE. 0.0 .AND. THETA(JJ) .NE. 180.0) M1ONLY=0	BR- 3220
IF(M1ONLY .EQ. 1) NMODEM=1	BR- 3230
IF(M0ONLY .EQ. 1) NMODEM=1	BR- 3240
DO 340 JM=1,NMODEM	BR- 3250
MODE=JM-1+MODEB	BR- 3260
IF(M1ONLY .EQ. 1) MODE=1	BR- 3270
IF(M0ONLY .EQ. 1) MODE=0	BR- 3280
IF(JM .NE. 1) CALL ZERO(CZ,NUNK*NUNK)	BR- 3290
CALL ZMATRX(CZ,RHO,Z,RHOPH,ZPH,DELTAT,GAMMA,NUNK,MODE)	BR- 3300
C	BR- 3310
C??	BR- 3320
C MATRIX PRINTOUT OPTION	BR- 3330
C??	BR- 3340
C	BR- 3350
C WRITE(5,11200) (((II,JJ,CZ(II,JJ)),JJ=1,NUNK),II=1,NUNK)	BR- 3360
C	BR- 3370
C CALL CVFILL(CVT,CVP,RHO,Z,RHOPH,ZPH,DELTAT,GAMMA,THETA,PHI,	BR- 3380
ETHETA,EPHI,ANTFD,NUNK,NFLDS,MODE)	BR- 3390
C	BR- 3400
C??	BR- 3410
C DRIVE MATRIX PRINTOUT OPTION	BR- 3420
C??	BR- 3430
C	BR- 3440
C WRITE(5,11100) ((CVT(I,1),CVP(I,1)),I=1,NUNK)	BR- 3450
C	BR- 3460
C ISCALE=0	BR- 3470
C CALL CSMINV(CZ,NUNK,NUNK,CDETRM,ACOND,IERR)	BR- 3480
C	BR- 3490
C??	BR- 3500
C INVERSE MATRIX PRINTOUT OPTION	BR- 3510
C??	BR- 3520
C	BR- 3530
C WRITE(5,11200) (((II,JJ,CZ(II,JJ)),JJ=1,NUNK),II=1,NUNK)	BR- 3540
C	BR- 3550
C POWER=-FLOAT(ISCALE)*10.	BR- 3560

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ITHETA=1	BR- 3570
CALL ICRMUL(CZ,CVT,CIT,NUNK,NUNK,NUNK,NFLDS)	BR- 3580
CALL SUMOUT(CIT,NUNK,NFLDS,MODE,ITHETA)	BR- 3590
ITHETA=0	BR- 3600
CALL ICRMUL(CZ,CVT,CIP,NUNK,NUNK,NUNK,NFLDS)	BR- 3610
CALL SUMOUT(CIP,NUNK,NFLDS,MODE,ITHETA)	BR- 3620
80 CONTINUE	BR- 3630
CZERO=CMPLX(0.0,0.0)	BR- 3640
NMODE1=NMODEM-1	BR- 3650
C	BR- 3660
C-----CURRENT NORMALIZATION OPTION	BR- 3670
C	BR- 3680
C CALL NORMAL(CIT,ETHETA,EPHI,NUNK,NFLDS)	BR- 3690
C CALL NORMAL(CIP,ETHETA,EPHI,NUNK,NFLDS)	BR- 3700
C	BR- 3710
90 CONTINUE	BR- 3720
NTE=NUNK	BR- 3730
IF(IDB.EQ. 1) NTE=NTE/2	BR- 3740
NPHE=NTE+1	BR- 3750
DO 330 J=1,NFLDS	BR- 3760
JW=J+4	BR- 3770
M0PR=0	BR- 3780
M1PR=0	BR- 3790
IF(EPHI(J).EQ. 0. .AND. ETHETA(J).EQ. 0.) M0PR=1	BR- 3800
IF(THETA(J).EQ. 0. .OR. THETA(J).EQ. 180.) M1PR=1	BR- 3810
IF(M0PR.EQ. 1) M1PR=0	BR- 3820
IF(M0PR.EQ. 1) GO TO 110	BR- 3830
IF(M1PR.EQ. 1) GO TO 110	BR- 3840
DO 100 JSUM=1,NUNK	BR- 3850
CVT(JSUM,J)=CITOLD(JSUM,J,1)	BR- 3860
CVP(JSUM,J)=CITOLD(JSUM,J,2)	BR- 3870
OBI=1.0	BR- 3880
OB2=SIN(FLOAT(MODE)*3.1415926/2.0)	BR- 3890
IF(MODE.EQ. 0) OB2=1.0	BR- 3900
OBSERV=OB1	BR- 3910
IF(JSUM.GT. NTE) OBSERV=OB2	BR- 3920
IF(JSUM.GT. 2*NTE+NPHE) OBSERV=OB1	BR- 3930
CITOLD(JSUM,J,1)=CITOLD(JSUM,J,1)+CIT(JSUM,J)*CMPLX(OBSERV,0.)	BR- 3940
OBSERV=OB2	BR- 3950
IF(JSUM.GT. NTE) OBSERV=OB1	BR- 3960
IF(JSUM.GT. 2*NTE+NPHE) OBSERV=OB2	BR- 3970
CITOLD(JSUM,J,2)=CITOLD(JSUM,J,2)+CIP(JSUM,J)*CMPLX(OBSERV,0.)	BR- 3980
100 CONTINUE	BR- 3990
IF(MODE.EQ. 0) GO TO 110	BR- 4000
LOC2=NTE+1	BR- 4010
CALL RMSERR(CITOLD(1,J,1),CVT(1,J),NTE,ETTPC)	BR- 4020
CALL RMSERR(CITOLD(LOC2,J,1),CVT(LOC2,J),NPHE,EPTPC)	BR- 4030
CALL RMSERR(CITOLD(1,J,2),CVP(1,J),NTE,ETPPC)	BR- 4040
CALL RMSERR(CITOLD(LOC2,J,2),CVP(LOC2,J),NPHE,EPPPC)	BR- 4050
IF(IDB.EQ. 0) GO TO 110	BR- 4060
LOC1=LOC2+1	BR- 4070
LOC2=LOC1+N	BR- 4080
CALL RMSERR(CITOLD(LOC1,J,1),CVT(LOC1,J),NTE,ETTDI)	BR- 4090
CALL RMSERR(CITOLD(LOC2,J,1),CVT(LOC2,J),NPHE,EPTDI)	BR- 4100
CALL RMSERR(CITOLD(LOC1,J,2),CVP(LOC1,J),NTE,ETPDI)	BR- 4110
CALL RMSERR(CITOLD(LOC2,J,2),CVP(LOC2,J),NPHE,EPPDI)	BR- 4120
110 CONTINUE	BR- 4130
IFINAL=0	BR- 4140
120 CONTINUE	BR- 4150
IF(M0PR.EQ. 1 .AND. MODE.NE. 0) GO TO 320	BR- 4160
IF(M1PR.EQ. 1 .AND. MODE.NE. 1) GO TO 320	BR- 4170

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DO 310 ITP=1,2	BR- 4180
IF(M0PR .EQ. 1) GO TO 130	BR- 4190
IF(ITP .EQ. 1 .AND. ETHETA(J) .EQ. 0.) GO TO 310	BR- 4200
130 IF(ITP .EQ. 2 .AND. EPHI(J) .EQ. 0.) GO TO 310	BR- 4210
PSENS=POL(ITP)	BR- 4220
IF(ITP .EQ. 1) GO TO 150	BR- 4230
DO 140 ITR=1,NUNK	BR- 4240
CIT(ITR,J)=CIP(ITR,J)	BR- 4250
140 CONTINUE	BR- 4260
150 CONTINUE	BR- 4270
PHI1=0.	BR- 4280
PHI2=90.	BR- 4290
ET1=ETTPC	BR- 4300
ET2=ETTDI	BR- 4310
EPI=EPTPC	BR- 4320
EP2=EPTDI	BR- 4330
IF(ITF .EQ. 1) GO TO 160	BR- 4340
ET1=ETPPC	BR- 4350
EPI=EPPPC	BR- 4360
ET2=ETPDI	BR- 4370
EP2=EPPDI	BR- 4380
160 CONTINUE	BR- 4390
IF(IFINAL .NE. 1) GO TO 180	BR- 4400
DO 170 ITR=1,NUNK	BR- 4410
170 CIT(ITR,J)=CITOLD(ITR,J,ITP)	BR- 4420
180 CONTINUE	BR- 4430
WRITE(JW,10009)	BR- 4440
WRITE(JW,10030) PSENS	BR- 4450
IF(IFINAL .NE. 1) WRITE(JW,10029) MODE	BR- 4460
IF(IFINAL .EQ. 1) WRITE(JW,10032) MODEE	BR- 4470
IF(MODE .EQ. 0 .OR. IFINAL .EQ. 1) GO TO 190	BR- 4480
IFTC=2	BR- 4490
IF(ITP .EQ. 2) IFTC=1	BR- 4500
WRITE(JW,10031) FTC(IFTC)	BR- 4510
190 CONTINUE	BR- 4520
IF(IFINAL .NE. 1) GO TO 200	BR- 4530
PHIOBS=PHI1	BR- 4540
IF(ITP .EQ. 2) PHIOBS=PHI2	BR- 4550
WRITE(JW,10035) PHIOBS	BR- 4560
200 CONTINUE	BR- 4570
WRITE(JW,10010)	BR- 4580
DEN=2.0*PI*RHO(1)	BR- 4590
IF(DEN.NE.0..OR.MODE.NE.1)WRITE(JW,10011) RHO(1),Z(1),CZERO,CZERO,	BR- 4600
\$CZERO	BR- 4610
IF(DEN.EQ.0..AND.MODE.EQ.1)WRITE(JW,10012) RHO(1),Z(1),CZERO,ZER	BR- 4620
DO 210 I=1,(NPTS-2)	BR- 4630
CALL MAGPHS(CIT(I,J),CMP)	BR- 4640
DEN=2.0*PI*RHO(I+1)	BR- 4650
CITD=CIT(I,J)/CMPLX(DEN,0.0)	BR- 4660
A=CABS(CITD)	BR- 4670
210 WRITE(JW,10013) RHO(I+1), Z(I+1), CIT(I,J), CMP, CITD, A	BR- 4680
DEN=2.0*PI*RHO(NPTS)	BR- 4690
IF(DEN.NE.0..OR.MODE.NE.1)WRITE(JW,10011) RHO(NPTS),Z(NPTS),	BR- 4700
\$CZERO,CZERO,CZERO	BR- 4710
IF(DEN.EQ.0..AND.MODE.EQ.1)WRITE(JW,10012) RHO(NPTS),Z(NPTS),CZERO,	BR- 4720
\$ZER	BR- 4730
IF(IFINAL .NE. 1) WRITE(JW,10021) ACOND,CDETRM,POWER	BR- 4740
IF(MODE .NE. 0 .AND. M1PR .NE. 1) WRITE(JW,10034) ET1	BR- 4750
WRITE(JW,10014)	BR- 4760
WRITE(JW,10030) PSENS	BR- 4770
IF(IFINAL .NE. 1) WRITE(JW,10029) MODE	BR- 4780

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IF(IFINAL .EQ. 1) WRITE(JW,10032) MODEE	BR- 4790
IF(MODE .EQ. 0 .OR. IFINAL .EQ. 1) GO TO 220	BR- 4800
IFTC=1	BR- 4810
IF(ITP .EQ. 2) IFTC=2	BR- 4820
WRITE(JW,10031) FTC(IFTC)	BR- 4830
220 CONTINUE	BR- 4840
IF(IFINAL .NE. 1) GO TO 230	BR- 4850
PHIOBS=PHI2	BR- 4860
IF(ITP .EQ. 2) PHIOBS=PHI1	BR- 4870
WRITE(JW,10035) PHIOBS	BR- 4880
230 CONTINUE	BR- 4890
WRITE(JW,10010)	BR- 4900
IS=NPTS-1	BR- 4910
IE=NPTS*2-3	BR- 4920
DO 240 I=IS,IE	BR- 4930
IR=I-IS+1	BR- 4940
CALL MAGPHS(CIT(I,J),CMP)	BR- 4950
AMD=REAL(CMP)	BR- 4960
PHASE=AIMAG(CMP)	BR- 4970
DEN=2.0*PI*RHOPH(IR)	BR- 4980
CITT=CIT(I,J)*CMPLX(DEN,0.0)	BR- 4990
A=CABS(CITT)	BR- 5000
240 WRITE(JW,10013) RHOPH(IR), ZPH(IR), CITT, A, PHASE, CIT(I,J), AMD	BR- 5010
IF(IFINAL .NE. 1) WRITE(JW,10021) ACOND,CDETRM,POWER	BR- 5020
IF(MODE .NE. 0 .AND. M1PR .NE. 1) WRITE(JW,10034) EPI	BR- 5030
IF(IDB .EQ. 0) GO TO 310	BR- 5040
WRITE(JW,10015)	BR- 5050
WRITE(JW,10030) PSENS	BR- 5060
IF(IFINAL .NE. 1) WRITE(JW,10029) MODE	BR- 5070
IF(IFINAL .EQ. 1) WRITE(JW,10032) MODEE	BR- 5080
IF(MODE .EQ. 0 .OR. IFINAL .EQ. 1) GO TO 250	BR- 5090
IFTC=1	BR- 5100
IF(ITP .EQ. 2) IFTC=2	BR- 5110
WRITE(JW,10031) FTC(IFTC)	BR- 5120
250 CONTINUE	BR- 5130
IF(IFINAL .NE. 1) GO TO 260	BR- 5140
PHIOBS=PHI2	BR- 5150
IF(ITP .EQ. 2) PHIOBS=PHI1	BR- 5160
WRITE(JW,10035) PHIOBS	BR- 5170
260 CONTINUE	BR- 5180
WRITE(JW,10010)	BR- 5190
DEN=2.0*PI*RHO(1)	BR- 5200
IF(DEN.NE.0..OR.MODE.NE.1)WRITE(JW,10011) RHO(1),Z(1),CZERO,CZERO,	BR- 5210
\$CZERO	BR- 5220
IF(DEN.EQ.0..AND.MODE.EQ.1)WRITE(JW,10012) RHO(1),Z(1),CZERO,ZER	BR- 5230
IS=IS+NPTS-1	BR- 5240
IE=IE+NPTS-2	BR- 5250
DO 270 I=IS,IE	BR- 5260
IR=I-IS+2	BR- 5270
CALL MAGPHS(CIT(I,J),CMP)	BR- 5280
DEN=2.0*PI*RHO(IR)	BR- 5290
CITD=CIT(I,J)/CMPLX(DEN,0.0)	BR- 5300
A=CABS(CITD)	BR- 5310
270 WRITE(JW,10013) RHO(IR), Z(IR), CIT(I,J), CMP, CITD, A	BR- 5320
DEN =2.0*PI*RHO(NPTS)	BR- 5330
IF(DEN.NE.0..OR.MODE.NE.1)WRITE(JW,10011) RHO(NPTS),Z(NPTS),	BR- 5340
\$CZERO,CZERO,CZERO	BR- 5350
IF(DEN.EQ.0..AND.MODE.EQ.1)WRITE(JW,10012) RHO(NPTS),Z(NPTS),CZERO,	BR- 5360
\$ZER	BR- 5370
IF(IFINAL .NE. 1) WRITE(JW,10021) ACOND,CDETRM,POWER	BR- 5380
IF(MODE .NE. 0 .AND. M1PR .NE. 1) WRITE(JW,10034) ET2	BR- 5390

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WRITE(JW,10016)
WRITE(JW,10030) PSENS
IF(IFINAL.NE. 1) WRITE(JW,10029) MODE
IF(IFINAL.EQ. 1) WRITE(JW,10032) MOLEE
IF(MODE.EQ. 0 .OR. IFINAL.EQ. 1) GO TO 280
IFTC=2
IF(ITP.EQ. 2) IFTC=1
WRITE(JW,10031) FTC(IFTC)
280 CONTINUE
IF(IFINAL.NE. 1) GO TO 290
PHIOBS=PHI1
IF(ITP.EQ. 2) PHIOBS=PHI2
WRITE(JW,10035) PHIOBS
290 CONTINUE
WRITE(JW,10010)
IS=IS+NPTS-2
DO 300 I=IS,NUNK
IR=I-IS+1
CALL MAGPHS(CIT(I,J),CMP)
AMD=REAL(CMP)
PHASE=AIMAG(CMP)
DEN=2.0*PI*RHOPH(IR)
CITT=CIT(I,J)*CMPLX(DEN,0.0)
A=CABS(CITT)
300 WRITE(JW,10013) RHOPH(IR), ZPH(IR), CITT, A, PHASE, CIT(I,J), AMD
IF(IFINAL.NE. 1) WRITE(JW,10021) ACOND,CDETRM,POWER
IF(MODE.NE. 0 .AND. MIPR.NE. 1) WRITE(JW,10034) EP2
310 CONTINUE
320 CONTINUE
IF(JM.EQ. NMODEM .AND. NMODEM.NE. 1 .AND. MODEB.EQ. 0)
SIFINAL = IFINAL + 1
IF(IFINAL.EQ. 1) GO TO 120
330 CONTINUE
340 CONTINUE
10000 FORMAT('///POINT'6X,'RHO'13X,'Z'11X,'DELTA-T'9X,'GAMMA'//)
10001 FORMAT(I4,1P2E15.6)
10002 FORMAT(4X,1P4E15.6)
10003 FORMAT('1'14(''')/' *'12X,'*'/' * INPUT DATA '*'/' *'12X,'*'/'
$' '14(''')//)
10004 FORMAT(' MAXIMUM NUMBER OF MODES TO BE USED = 'I3/' ORDER OF GAUSS-
SSIAN QUADRATURE = 'I3/)
10005 FORMAT(' FREE SPACE WAVELENGTH = '1PE/)
10006 FORMAT(' A PERFECT CONDUCTOR HAS BEEN ASSUMED.//)
10007 FORMAT(' A DIELECTRIC BODY HAS BEEN ASSUMED: '//6X,'RELATIVE DIELEC-
STRIC CONSTANT = '1PE//6X,'RELATIVE PERMEABILITY = '1PE/)
10008 FORMAT(' INCIDENT FIELD DATA: '//6X,'THETA = '1PE//6X,'PHI = '1PE//6X,
'E-THETA = '1PE//6X,'E-PHI = '1PE/)
10009 FORMAT('1'43X,'T" COMPONENT OF ELECTRIC CURRENT')
10010 FORMAT(7X,'RHO'11X,'Z'10X,'REAL'7X,'IMAGINARY'4X,'MAGNITUDE'6X,
$'PHASE'8X,'REAL'7X,'IMAGINARY'4X,'MAGNITUDE'/32X,'TOTAL'8X,
$'TOTAL'8X,'TOTAL'20X,'DENSITY'6X,'DENSITY'6X,'DENSITY'6X,
$' ***'10X,'***'9X,'***'7X,9('''),4X,9('''),6X,'***'8X,
$'***'7X,9('''),4X,9(''')//)
10011 FORMAT(' '1P5E13.5,' ??? '1P3E13.5)
10012 FORMAT(' '1P5E13.5,1X,4(' ??? ')
10013 FORMAT(' '1P9E13.5)
10014 FORMAT('1'42X,'PHI" COMPONENT OF ELECTRIC CURRENT')
10015 FORMAT('1'43X,'T" COMPONENT OF MAGNETIC CURRENT')
10016 FORMAT('1'42X,'PHI" COMPONENT OF MAGNETIC CURRENT')
10017 FORMAT('0'17(''')/' *'15X,'*'/' * COMPUTED DATA '*'/' *'15X,'*'/'1X,
$17(''')//)

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10018 FORMAT(5X,'DIELECTRIC CONSTANT = '1PE//5X,'PERMEABILITY = '1PE//5XBR- 6010
      $,'WAVENUMBER = '1PE//5X,'SPEED OF LIGHT = '1PE/') BR- 6020
10019 FORMAT(' PARAMETERS OF THE DIELECTRIC BODY: '/') BR- 6030
10020 FORMAT(' OMEGA = '1PE// ' FREQUENCY = '1PE/') BR- 6040
10021 FORMAT('/0INV. COND. NO.: '1PE,3X,'DETERMINANT: (' BR- 6050
      $1P2E,') TIMES (10.)*('F6.1,')') BR- 6060
10022 FORMAT(' PARAMETERS OF FREE SPACE: '/') BR- 6070
10023 FORMAT(5X,'WAVELENGTH = '1PE/) BR- 6080
10024 FORMAT(' THE MAXIMUM NUMBER OF MODES WAS USED: '/' THE MAXIMUM (OVER- 6090
      SER NO. OF INC. FIELDS) "RMS" ERROR IN THE CURRENT VECTORS WAS: ' BR- 6100
      $1PE/) BR- 6110
10025 FORMAT(' A DELTA-GAP SOURCE IS ASSUMED AT RHO = '1PE,' AND Z = ' BR- 6120
      $1PE/) BR- 6130
10026 FORMAT(46X,'OUTPUT IN "MODE 1" FORMAT')/ BR- 6140
10027 FORMAT(8E) BR- 6150
10028 FORMAT(8I) BR- 6160
10029 FORMAT('0'52X,'MODE NUMBER 'I2/) BR- 6170
10030 FORMAT(39X,'FOR 'A5,' DIRECTED PORTION OF INCIDENT FIELD') BR- 6180
10031 FORMAT(49X,A3,'(PHI) COEFFICIENTS')/ BR- 6190
10032 FORMAT('0'47X,'SUM OF MODES 0 THROUGH 'I2/) BR- 6200
10033 FORMAT('0WARNING: TOO MANY INCIDENT FIELDS HAVE BEEN USED. ONLY TBR- 6210
      SHE FIRST 11 HAVE BEEN PRINTED.') BR- 6220
10034 FORMAT('0ROOT-MEAN-SQUARE ERROR AT THIS STAGE: '1PE) BR- 6230
10035 FORMAT(45X,'OBSERVED AT PHI = 'F5.1,' DEGREES')/ BR- 6240
10036 FORMAT('1'50X,27('')/51X,'*25X,'*'/51X,'* CASE NUMBER: ' BR- 6250
      $ I3,8X,'*'/51X,'*25X,'*'/51X,'* EXCITATION NUMBER: 'I3,2X, BR- 6260
      $ '*'/51X,'*25X,'*'/51X,27('')/ BR- 6270
11100 FORMAT(1P2E,10X,1P2E) BR- 6280
11200 FORMAT(3(' CZ('I2,', 'I2,')='1P2E,1X)) BR- 6290
      RETURN BR- 6300
      END BR- 6310
      SUBROUTINE ZMATRX(CZ,RHO,Z,RHOPH,ZPH,DELTAT,GAMMA,NUNK,M) BR- 6320
C***** BR- 6330
C BR- 6340
C      SUBROUTINE "ZMATRX" FILLS THE IMPEDANCE MATRIX FOR EACH MODE. BR- 6350
C BR- 6360
C***** BR- 6370
      IMPLICIT COMPLEX (C) BR- 6380
      COMPLEX G1R,G2R,G4R,G5R BR- 6390
      COMPLEX G0,G1,G2,G3,G5,G6 BR- 6400
      COMPLEX G1M,G2M BR- 6410
      REAL MU1,MU2,COSG BR- 6420
      DIMENSION CZ(NUNK,NUNK) BR- 6430
      DIMENSION RHO(1),Z(1),RHOPH(1),ZPH(1),DELTAT(1),GAMMA(1) BR- 6440
      COMMON/PNT/NPTS,NUNKT,NUNKPH,NUNK2 BR- 6450
      COMMON/INT/NMODEM,IDB,NGQ BR- 6460
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA BR- 6470
      COMMON/PRM/EP1,EP2,MU1,MU2 BR- 6480
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM BR- 6490
      COMMON/TPI/TWOPI,MAXP,ER,API BR- 6500
      COMMON/SLF/ISELF BR- 6510
      EXTERNAL G1,G2,G3,G5,G6,G0,G1R,G2R,G4R,G5R BR- 6520
      EXTERNAL G1M,G2M BR- 6530
      CRX(DUMMY)=CMPLX(DUMMY,0.0) BR- 6540
      CIX(DUMMY)=CMPLX(0.0,DUMMY) BR- 6550
      NPTS1=NPTS-1 BR- 6560
      ER=1.E-2 BR- 6570
      NTE=NUNKT BR- 6580
      MAXP=10000 BR- 6590
      NPHE=NUNKPH BR- 6600
      API=PI BR- 6610

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TWOPI=2.0*PI	BR- 6620
IF(IDB.EQ. 0) GO TO 10	BR- 6630
NTE=NTE/2	BR- 6640
NPHE=NPHE/2	BR- 6650
NTM=NTE	BR- 6660
NPHM=NPHE	BR- 6670
10 MODE=M	BR- 6680
MODEP1=MODE+1	BR- 6690
MODEM1=MODE-1	BR- 6700
NGQ2=NGQ*2	BR- 6710
KTS=NTE+NPHE	BR- 6720
KPHS=KTS+NTM	BR- 6730
CH=CMPLX(0.5,0.0)	BR- 6740
CZERO=CMPLX(0.0,0.0)	BR- 6750
CJO=CMPLX(0.0,OMEGA)	BR- 6760
CMU1=CMPLX(MU1,0.0)	BR- 6770
CMU2=CMPLX(MU2,0.0)	BR- 6780
CEPS1=CMPLX(EPS1,0.0)	BR- 6790
CEPS2=CMPLX(EPS2,0.0)	BR- 6800
CO=CMPLX(OMEGA,0.0)	BR- 6810
CMP=CMPLX(FLOAT(MODEP1),0.0)	BR- 6820
CM=CMPLX(FLOAT(MODE),0.0)	BR- 6830
CMM=CMPLX(FLOAT(MODEM1),0.0)	BR- 6840
CM2=CM*CM	BR- 6850
C	BR- 6860
C-----EVALUATE ALL EXPRESSIONS IN WHICH THE FIELD POINT IS LOCATED AT	BR- 6870
C A POSSIBLE BEND (NON-HALF POINTS)	BR- 6880
C	BR- 6890
TL=0.0	BR- 6900
TM=0.5	BR- 6910
TU=1.0	BR- 6920
DO 100 IF1=1,NTE	BR- 6930
IF3=IF1+KTS	BR- 6940
IF1P1=IF1+1	BR- 6950
RK=RHO(IF1P1)	BR- 6960
ZK=Z(IF1P1)	BR- 6970
DTK=DELTAT(IF1)	BR- 6980
DTKP1=DELTAT(IF1P1)	BR- 6990
SGK=SIN(GAMMA(IF1))	BR- 7000
SGKP1=SIN(GAMMA(IF1P1))	BR- 7010
ACGK=COS(GAMMA(IF1))	BR- 7020
ACGKP1=COS(GAMMA(IF1P1))	BR- 7030
BRCKTS=(DTKP1*SGKP1+DTK*SGK)/2.0	BR- 7040
BRCKTC=(DTKP1*ACGKP1+DTK*ACGK)/2.0	BR- 7050
DO 90 IS1=1,NPHE	BR- 7060
IS2=IS1+NTE	BR- 7070
IS3=IS1+KTS	BR- 7080
IS4=IS1+KPHS	BR- 7090
IS1M1=IS1-1	BR- 7100
IS3M1=IS3-1	BR- 7110
R1M1=RHO(IS1)	BR- 7120
Z1M1=Z(IS1)	BR- 7130
DTI=DELTAT(IS1)	BR- 7140
SGI=SIN(GAMMA(IS1))	BR- 7150
ACGI=COS(GAMMA(IS1))	BR- 7160
RF=RK	BR- 7170
ZF=ZK	BR- 7180
RSL=R1M1	BR- 7190
ZSL=Z1M1	BR- 7200
DEL=DTI	BR- 7210
SING=SGI	BR- 7220

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COSG=ACGI	BR- 7230
C1B11=CIX(DTI*OMEGA*SGI*BRCKTS/2.0)	BR- 7240
C3B11=CIX(DTI*OMEGA*ACGI*BRCKTC)	BR- 7250
C1B12=CRX(-DTI*OMEGA*PI*BRCKTS)	BR- 7260
IF(IDB .EQ. 0) GO TO 20	BR- 7270
C1B13=CIX(-DTI*ACGI*BRCKTS)	BR- 7280
C3B13=CIX(DTI*SGI*RK*BRCKTC)	BR- 7290
C5B13=CIX(-DTI*SGI*BRCKTS)	BR- 7300
C1B14=CRX(DTI*TWOPI*BRCKTC)	BR- 7310
C2B14=CRX(-4.0*PI*DTI*RK*BRCKTC)	BR- 7320
C3B14=CRX(-DTI*TWOPI*BRCKTS)	BR- 7330
20 TF=TL	BR- 7340
ISELF=0	BR- 7350
IF(IS1 .EQ. IF1P1) ISELF=1	BR- 7360
CALL ELLPTC(TL,TM,NGQ,CANEL)	BR- 7370
CALL ELLPTR(TL,TM,NGQ,CANELR)	BR- 7380
GM=FLOAT(MODEP1)	BR- 7390
CALL CGQ1T(G1M,TL,TM,NGQ,CANS,CANSR)	BR- 7400
CALLP=CANS+CANEL	BR- 7410
CALLPR=CANSR+CANELR	BR- 7420
GM=FLOAT(MODEM1)	BR- 7430
CALL CGQ1T(G1M,TL,TM,NGQ,CANS,CANSR)	BR- 7440
CALLM=CANS+CANEL	BR- 7450
CALLMR=CANSR+CANELR	BR- 7460
IF(IS1 .EQ. 1) GO TO 30	BR- 7470
GM=FLOAT(MODE)	BR- 7480
CALL CGQ1(G1M,TL,TM,NGQ,CANS)	BR- 7490
CALL=CANS+CANEL	BR- 7500
CZ(IF1,IS1M1)=CZ(IF1,IS1M1)+C1B11*CMU1*(CALLP+CALLM)	BR- 7510
CZ(IF1,IS1M1)=CZ(IF1,IS1M1)+C3B11*CMU1*CALL	BR- 7520
IF(IDB .EQ. 0) GO TO 50	BR- 7530
CZ(IF3,IS3M1)=CZ(IF3,IS3M1)+C1B11*CEPS1*(CALLP+CALLM)	BR- 7540
CZ(IF3,IS3M1)=CZ(IF3,IS3M1)+C3B11*CEPS1*CALL	BR- 7550
30 IF(IDB .EQ. 0) GO TO 50	BR- 7560
GM=FLOAT(MODEP1)	BR- 7570
CALL CGQ1T(G2M,TL,TM,NGQ,CANS,CANSR)	BR- 7580
CA2LP=CANS+CANEL	BR- 7590
CA2LPR=CANSR+CANELR	BR- 7600
GM=FLOAT(MODEM1)	BR- 7610
CALL CGQ1T(G2M,TL,TM,NGQ,CANS,CANSR)	BR- 7620
CA2LM=CANS+CANEL	BR- 7630
CA2LMR=CANSR+CANELR	BR- 7640
GM=FLOAT(MODE)	BR- 7650
IF(IS1 .EQ. 1) GO TO 40	BR- 7660
CALL CGQ1(G2M,TL,TM,NGQ,CANS)	BR- 7670
CA2L=CANS+CANEL	BR- 7680
CZ(IF1,IS1M1)=CZ(IF1,IS1M1)+C1B11*CMU2*(CA2LP+CA2LM)	BR- 7690
CZ(IF1,IS1M1)=CZ(IF1,IS1M1)+C3B11*CMU2*CA2L	BR- 7700
CZ(IF3,IS3M1)=CZ(IF3,IS3M1)+C1B11*CEPS2*(CA2LP+CA2LM)	BR- 7710
CZ(IF3,IS3M1)=CZ(IF3,IS3M1)+C3B11*CEPS2*CA2L	BR- 7720
CALL CGQ1(G0,TL,TM,NGQ,CANS0L)	BR- 7730
IF(ISELF .EQ. 1) CANS0L=CANS0L+CANALY(TL,TM,0,0)	BR- 7740
CALL CGQ1(G1,TL,TM,NGQ,CANS1L)	BR- 7750
CALL CGQ1(G3,TL,TM,NGQ,CANS3L)	BR- 7760
IF(ISELF .EQ. 1) CANS3L=CANS3L+CANALY(TL,TM,3,0)	BR- 7770
CB13=C1B13*CANS3L+C3B13*CANS0L+C5B13*CANS1L	BR- 7780
CZ(IF1,IS3M1)=CZ(IF1,IS3M1) + CB13	BR- 7790
40 CALL CGQ1(G4R,TL,TM,NGQ,CANS4L)	BR- 7800
CALL CGQ1(G5R,TL,TM,NGQ,CANS5L)	BR- 7810
IF(ISELF .EQ. 1) CANS5L=CANS5L+CANALY(TL,TM,5,1)	BR- 7820
CALL CGQ1(G2R,TL,TM,NGQ,CANS2L)	BR- 7830

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50 TF=TM	BR- 7840
ISELF=0	BR- 7850
IF(IS1.EQ. IF1) ISELF=1	BR- 7860
CALL ELLPTC(TM,TU,NGQ,CANEU)	BR- 7870
CALL ELLPTR(TM,TU,NGQ,CANEUR)	BR- 7880
GM=FLOAT(MODEP1)	BR- 7890
CALL CGQIT(G1M,TM,TU,NGQ,CANS,CANSR)	BR- 7900
CALUP=CANS+CANEU	BR- 7910
CALUPR=CANSR+CANEUR	BR- 7920
GM=FLOAT(MODEM1)	BR- 7930
CALL CGQIT(G1M,TM,TU,NGQ,CANS,CANSR)	BR- 7940
CALUM=CANS+CANEU	BR- 7950
CALUMR=CANSR+CANEUR	BR- 7960
IF(IS1.EQ. NPHE) GO TO 60	BR- 7970
GM=FLOAT(MODE)	BR- 7980
CALL CGQI(G1M,TM,TU,NGQ,CANS)	BR- 7990
CALU=CANS+CANEU	BR- 8000
CZ(IF1,IS1)=CZ(IF1,IS1)+C1B11*CMU1*(CALUP+CALUM)+C3B11*CMU1*CALU	BR- 8010
IF(IDB.EQ. 0) GO TO 80	BR- 8020
CZ(IF3,IS3)=CZ(IF3,IS3)+C1B11*CEPS1*(CALUP+CALUM)+C3B11*CEPS1*CALU	BR- 8030
60 IF(IDB.EQ. 0) GO TO 80	BR- 8040
GM=FLOAT(MODEP1)	BR- 8050
CALL CGQIT(G2M,TM,TU,NGQ,CANS,CANSR)	BR- 8060
CA2UP=CANS+CANEU	BR- 8070
CA2UPR=CANSR+CANEUR	BR- 8080
GM=FLOAT(MODEM1)	BR- 8090
CALL CGQIT(G2M,TM,TU,NGQ,CANS,CANSR)	BR- 8100
CA2UM=CANS+CANEU	BR- 8110
CA2UMR=CANSR+CANEUR	BR- 8120
GM=FLOAT(MODE)	BR- 8130
IF(IS1.EQ. NPHE) GO TO 70	BR- 8140
CALL CGQI(G2M,TM,TU,NGQ,CANS)	BR- 8150
CA2U=CANS+CANEU	BR- 8160
CZ(IF1,IS1)=CZ(IF1,IS1)+C1B11*CMU2*(CA2UP+CA2UM)+C3B11*CMU2*CA2U	BR- 8170
CZ(IF3,IS3)=CZ(IF3,IS3)+C1B11*CEPS2*(CA2UP+CA2UM)+C3B11*CEPS2*CA2U	BR- 8180
CALL CGQI(G0,TM,TU,NGQ,CANS0U)	BR- 8190
IF(ISELF.EQ. 1) CANS0U=CANS0U+CANALY(TM,TU,0,0)	BR- 8200
CALL CGQI(G1,TM,TU,NGQ,CANS1U)	BR- 8210
CALL CGQI(G3,TM,TU,NGQ,CANS3U)	BR- 8220
IF(ISELF.EQ. 1) CANS3U=CANS3U+CANALY(TM,TU,3,0)	BR- 8230
CBI3=C1B13*CANS3U+C3B13*CANS0U+C5B13*CANS1U	BR- 8240
CZ(IF1,IS3)=CZ(IF1,IS3) + CBI3	BR- 8250
70 CALL CGQI(G4R,TM,TU,NGQ,CANS4U)	BR- 8260
CALL CGQI(G5R,TM,TU,NGQ,CANS5U)	BR- 8270
IF(ISELF.EQ. 1) CANS5U=CANS5U+CANALY(TM,TU,5,1)	BR- 8280
CALL CGQI(G2R,TM,TU,NGQ,CANS2U)	BR- 8290
CBI4=C1B14*(CANS4L+CANS4U)+C2B14*(CANS5L+CANS5U)	BR- 8300
CBI4=CBI4+C3B14*(CANS2L+CANS2U)	BR- 8310
CZ(IF1,IS4)=CZ(IF1,IS4) + CBI4	BR- 8320
CA2MR=CA2LMR+CA2UMR	BR- 8330
CA2PR=CA2LPR+CA2UPR	BR- 8340
CZ(IF1,IS2)=CZ(IF1,IS2)+C1B12*CMU2*(CA2PR-CA2MR)	BR- 8350
CZ(IF3,IS4)=CZ(IF3,IS4)+C1B12*CEPS2*(CA2PR-CA2MR)	BR- 8360
80 CALMR=CALLMR+CALUMR	BR- 8370
CALPR=CALLPR+CALUPR	BR- 8380
CZ(IF1,IS2)=CZ(IF1,IS2)+C1B12*CMU1*(CALPR-CALMR)	BR- 8390
IF(IDB.EQ. 1) CZ(IF3,IS4)=CZ(IF3,IS4)+C1B12*CEPS1*(CALPR-CALMR)	BR- 8400
90 CONTINUE	BR- 8410
100 CONTINUE	BR- 8420
C	BR- 8430
C-----COMPUTE ALL EXPRESSIONS IN WHICH THE FIELD POINT IS LOCATED AT	BR- 8440

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C	A HALF POINT ON THE GENERATING CURVE.	BR- 8450
C		BR- 8460
	DO 270 IF1=1,NPHE	BR- 8470
	IF2=IF1+NTE	BR- 8480
	IF3=IF1+KTS	BR- 8490
	IF4=IF1+KPHS	BR- 8500
	IF1M1=IF1-1	BR- 8510
	IF3M1=IF3-1	BR- 8520
	RK=RHOPH(IF1)	BR- 8530
	ZK=ZPH(IF1)	BR- 8540
	DTK=DELTAT(IF1)	BR- 8550
	SGK=SIN(GAMMA(IF1))	BR- 8560
	ACGK=COS(GAMMA(IF1))	BR- 8570
	DO 260 IS1=1,NPHE	BR- 8580
	IS2=IS1+NTE	BR- 8590
	IS3=IS1+KTS	BR- 8600
	IS4=IS1+KPHS	BR- 8610
	IS1M1=IS1-1	BR- 8620
	IS3M1=IS3-1	BR- 8630
	RIM1=RHOPH(IS1)	BR- 8640
	ZIM1=Z(IS1)	BR- 8650
	DTI=DELTAT(IS1)	BR- 8660
	SGI=SIN(GAMMA(IS1))	BR- 8670
	ACGI=COS(GAMMA(IS1))	BR- 8680
	RF=RK	BR- 8690
	ZF=ZK	BR- 8700
	RSL=RIM1	BR- 8710
	ZSL=ZIM1	BR- 8720
	DEL=DTI	BR- 8730
	SING=SG	BR- 8740
	COSG=ACGI	BR- 8750
	C5B11=CIX(1.0/OMEGA)	BR- 8760
	C2B12=CRX(-DTI*TWOPI/OMEGA)	BR- 8770
	C1B22=CIX(DTK*DTI*PI*OMEGA)	BR- 8780
	C2B22=CIX(-DTK*DTI*TWOPI/(OMEGA*RK))	BR- 8790
	C1B21=CRX(DTK*DTI*OMEGA*SGI/2.0)	BR- 8800
	C3B21=CRX(-DTK/(OMEGA*RK))	BR- 8810
	IF(IDB.EQ.0) GO TO 110	BR- 8820
	C1B23=CRX(-2.0*DTK*DTI*RK*ACGI)	BR- 8830
	C3B23=CRX(-DTK*DTI*ACGI)	BR- 8840
	C5B23=CRX(DTK*DTI*SGI)	BR- 8850
	C1B24=CIX(-DTK*TWOPI*DTI)	BR- 8860
110	TF=TM	BR- 8870
	ISELF=0	BR- 8880
	IF(IS1.EQ. IF1) ISELF=1	BR- 8890
	CALL ELLPTC(TL,TM,NGQ,CANEL)	BR- 8900
	CALL ELLPTP(TL,TM,NGQ,CANELR)	BR- 8910
	GM=FLOAT(MODEP1)	BR- 8920
	CALL CGQ1T(G1M,TL,TM,NGQ,CANS,CANSR)	BR- 8930
	CALLP=CANS+CANEL	BR- 8940
	CALLPR=CANSR+CANELR	BR- 8950
	GM=FLOAT(MODEM1)	BR- 8960
	CALL CGQ1T(G1M,TL,TM,NGQ,CANS,CANSR)	BR- 8970
	CALLM=CANS+CANEL	BR- 8980
	CALLMR=CANSR+CANELR	BR- 8990
	GM=FLOAT(MODE)	BR- 9000
	CALL CGQ1(G1M,TL,TM,NGQ,CANS)	BR- 9010
	CALL=CANS+CANEL	BR- 9020
	IF(IS1.EQ. 1) GO TO 120	BR- 9030
	CZ(IF2,IS1M1)=CZ(IF2,IS1M1)+C1B21*CMU1*(CALLP-CALLM)	BR- 9040
	IF(IDB.EQ.0) GO TO 140	BR- 9050

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	CZ(IF4,IS3M1)=CZ(IF4,IS3M1)+C1B21*CEPS1*(CALP-CALM)	BR- 9060
120	IF(IDB.EQ.0) GO TO 140	BR- 9070
	GM=FLOAT(MODEP1)	BR- 9080
	CALL CGQ1T(G2M,TL,TM,NGQ,CANS,CANSR)	BR- 9090
	CA2LP=CANS+CANEL	BR- 9100
	CA2LPR=CANSR+CANELR	BR- 9110
	GM=FLOAT(MODEM1)	BR- 9120
	CALL CGQ1T(G2M,TL,TM,NGQ,CANS,CANSR)	BR- 9130
	CA2LM=CANS+CANEL	BR- 9140
	CA2LMR=CANSR+CANELR	BR- 9150
	GM=FLOAT(MODE)	BR- 9160
	CALL CGQ1(G2M,TL,TM,NGQ,CANS)	BR- 9170
	CA2L=CANS+CANEL	BR- 9180
	IF(IS1.EQ.1) GO TO 130	BR- 9190
	CZ(IF2,IS1M1)=CZ(IF2,IS1M1)+C1B21*CMU2*(CA2LP-CA2LM)	BR- 9200
	CZ(IF4,IS3M1)=CZ(IF4,IS3M1)+C1B21*CEPS2*(CA2LP-CA2LM)	BR- 9210
	CALL CGQ1(G5,TL,TM,NGQ,CANS5L)	BR- 9220
	IF(ISELF.EQ.1) CANS5L=CANS5L+CANALY(TL,TM,5,0)	BR- 9230
	CALL CGQ1(G6,TL,TM,NGQ,CANS6L)	BR- 9240
	CALL CGQ1(G2,TL,TM,NGQ,CANS2L)	BR- 9250
	CB23=C1B23*CANS5L+C3B23*CANS6L+C5B23*CANS2L	BR- 9260
	CZ(IF2,IS3M1)=CZ(IF2,IS3M1)+CB23	BR- 9270
130	CALL CGQ1(G1R,TL,TM,NGQ,CANS1L)	BR- 9280
140	CONTINUE	BR- 9290
	CALL ELLPTC(TM,TU,NGQ,CANEU)	BR- 9300
	CALL ELLPTR(TM,TU,NGQ,CANEUR)	BR- 9310
	GM=FLOAT(MODEP1)	BR- 9320
	CALL CGQ1T(G1M,TM,TU,NGQ,CANS,CANSR)	BR- 9330
	CALUP=CANS+CANEU	BR- 9340
	CALUPR=CANSR+CANEUR	BR- 9350
	GM=FLOAT(MODEM1)	BR- 9360
	CALL CGQ1T(G1M,TM,TU,NGQ,CANS,CANSR)	BR- 9370
	CALUM=CANS+CANEU	BR- 9380
	CALUMR=CANSR+CANEUR	BR- 9390
	GM=FLOAT(MODE)	BR- 9400
	CALL CGQ1(G1M,TM,TU,NGQ,CANS)	BR- 9410
	CALU=CANS+CANEU	BR- 9420
	CAL=CAL+CALU	BR- 9430
	CALPR=CALPR+CALUPR	BR- 9440
	CALMR=CALMR+CALUMR	BR- 9450
	IF(IS1.EQ.NPHE) GO TO 150	BR- 9460
	CZ(IF2,IS1)=CZ(IF2,IS1)+C1B21*CMU1*(CALUP-CALUM)	BR- 9470
	IF(IDB.EQ.0) GO TO 230	BR- 9480
	CZ(IF4,IS3)=CZ(IF4,IS3)+C1B21*CEPS1*(CALUP-CALUM)	BR- 9490
150	IF(IDB.EQ.0) GO TO 230	BR- 9500
	GM=FLOAT(MODEP1)	BR- 9510
	CALL CGQ1T(G2M,TM,TU,NGQ,CANS,CANSR)	BR- 9520
	CA2UP=CANS+CANEU	BR- 9530
	CA2UPR=CANSR+CANEUR	BR- 9540
	GM=FLOAT(MODEM1)	BR- 9550
	CALL CGQ1T(G2M,TM,TU,NGQ,CANS,CANSR)	BR- 9560
	CA2UM=CANS+CANEU	BR- 9570
	CA2UMR=CANSR+CANEUR	BR- 9580
	GM=FLOAT(MODE)	BR- 9590
	CALL CGQ1(G2M,TM,TU,NGQ,CANS)	BR- 9600
	CA2U=CANS+CANEU	BR- 9610
	IF(IS1.EQ.NPHE) GO TO 160	BR- 9620
	CZ(IF2,IS1)=CZ(IF2,IS1)+C1B21*CMU2*(CA2UP-CA2UM)	BR- 9630
	CZ(IF4,IS3)=CZ(IF4,IS3)+C1B21*CEPS2*(CA2UP-CA2UM)	BR- 9640
	CALL CGQ1(G5,TM,TU,NGQ,CANS5U)	BR- 9650
	IF(ISELF.EQ.1) CANS5U=CANS5U+CANALY(TM,TU,5,0)	BR- 9660

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CALL CGQ1(G6,TM,TU,NGQ,CANS6U)	BR- 9670
CALL CGQ1(G2,TM,TU,NGQ,CANS2U)	BR- 9680
CB23=C1B23*CANS5U+C3B23*CANS6U+C5B23*CANS2U	BR- 9690
CZ(IF2,IS3)=CZ(IF2,IS3) + CB23	BR- 9700
160 CALL CGQ1(G1R,TM,TU,NGQ,CANS1U)	BR- 9710
CZ(IF2,IS4)=CZ(IF2,IS4)+C1B24*(CANS1L+CANS1U)	BR- 9720
CA2=CA2L+CA2U	BR- 9730
CA2PR=CA2LPR+CA2UPR	BR- 9740
CA2MR=CA2LMR+CA2UMR	BR- 9750
CA2EPS=CA2/CEPS2	BR- 9760
CAMU=CA1/CMU1+CA2/CMU2	BR- 9770
CZ(IF2,IS2)=CZ(IF2,IS2)+C1B22*CMU2*(CA2PR+CA2MR)	BR- 9780
CZ(IF2,IS2)=CZ(IF2,IS2)+C2B22*CA2EPS*CM2	BR- 9790
CB44=C1B22*(CEPS1*(CA1PR+CA1MR)+CEPS2*(CA2PR+CA2MR))	BR- 9800
CB44=CB44+CM2*C2B22*CAMU	BR- 9810
CZ(IF4,IS4)=CZ(IF4,IS4) + CB44	BR- 9820
CB12=C2B12*CM*CA2EPS	BR- 9830
CB34=C2B12*CM*CAMU	BR- 9840
CB21=C3B21*CM*CA2EPS	BR- 9850
CB43=C3B21*CM*CAMU	BR- 9860
CB11=C5B11*CA2EPS	BR- 9870
CB33=C5B11*CAMU	BR- 9880
IF(IF1.EQ. 1) GO TO 170	BR- 9890
CZ(IF1M1,IS2)=CZ(IF1M1,IS2) + CB12	BR- 9900
CZ(IF3M1,IS4)=CZ(IF3M1,IS4) + CB34	BR- 9910
170 IF(IF1.EQ. NPHE) GO TO 180	BR- 9920
CZ(IF1,IS2)=CZ(IF1,IS2) - CB12	BR- 9930
CZ(IF3,IS4)=CZ(IF3,IS4) - CB34	BR- 9940
180 IF(IS1.EQ. 1) GO TO 200	BR- 9950
CZ(IF2,IS1M1)=CZ(IF2,IS1M1) - CB21	BR- 9960
CZ(IF4,IS3M1)=CZ(IF4,IS3M1) - CB43	BR- 9970
IF(IF1.EQ. 1) GO TO 190	BR- 9980
CZ(IF1M1,IS1M1)=CZ(IF1M1,IS1M1) - CB11	BR- 9990
CZ(IF3M1,IS3M1)=CZ(IF3M1,IS3M1) - CB33	BR-10000
190 IF(IF1.EQ. NPHE) GO TO 200	BR-10010
CZ(IF1,IS1M1)=CZ(IF1,IS1M1) + CB11	BR-10020
CZ(IF3,IS3M1)=CZ(IF3,IS3M1) + CB33	BR-10030
200 IF(IS1.EQ. NPHE) GO TO 220	BR-10040
CZ(IF2,IS1)=CZ(IF2,IS1) + CB21	BR-10050
CZ(IF4,IS3)=CZ(IF4,IS3) + CB43	BR-10060
IF(IF1.EQ. 1) GO TO 210	BR-10070
CZ(IF1M1,IS1)=CZ(IF1M1,IS1) + CB11	BR-10080
CZ(IF3M1,IS3)=CZ(IF3M1,IS3) + CB33	BR-10090
210 IF(IF1.EQ. NPHE) GO TO 220	BR-10100
CZ(IF1,IS1)=CZ(IF1,IS1) - CB11	BR-10110
CZ(IF3,IS3)=CZ(IF3,IS3) - CB33	BR-10120
220 CONTINUE	BR-10130
230 CONTINUE	BR-10140
CALEPS=CA1/CEPS1	BR-10150
CB21=C3B21*CM*CALEPS	BR-10160
CB12=C2B12*CM*CALEPS	BR-10170
CB11=C5B11*CALEPS	BR-10180
CZ(IF2,IS2)=CZ(IF2,IS2)+C1B22*CMU1*(CA1PR+CA1MR)	BR-10190
CZ(IF2,IS2)=CZ(IF2,IS2)+C2B22*CALEPS*CM2	BR-10200
IF(IF1.NE. 1) CZ(IF1M1,IS2)=CZ(IF1M1,IS2) + CB12	BR-10210
IF(IF1.NE. NPHE) CZ(IF1,IS2)=CZ(IF1,IS2) - CB12	BR-10220
IF(IS1.EQ. 1) GO TO 240	BR-10230
CZ(IF2,IS1M1)=CZ(IF2,IS1M1) - CB21	BR-10240
IF(IF1.NE. 1) CZ(IF1M1,IS1M1)=CZ(IF1M1,IS1M1) - CB11	BR-10250
IF(IF1.NE. NPHE) CZ(IF1,IS1M1)=CZ(IF1,IS1M1) + CB11	BR-10260
240 IF(IS1.EQ. NPHE) GO TO 250	BR-10270

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CZ(IF2,IS1)=CZ(IF2,IS1) + CB21
IF(IF1.NE. 1) CZ(IF1M1,IS1)=CZ(IF1M1,IS1) + CB11
IF(IF1.NE. NPHE) CZ(IF1,IS1)=CZ(IF1,IS1) - CB11
250 CONTINUE
260 CONTINUE
270 CONTINUE
C
C-----NOW WE HAVE:
C
C      (BETA-31) = -(BETA-13)
C      (BETA-32) = -(BETA-14)
C      (BETA-41) = -(BETA-23)
C      (BETA-42) = -(BETA-24)
C
IF(IDB.EQ. 0) GO TO 290
NE=NTE+NPHE
NEL=NE+1
DO 280 IF=1,NE
IFP=IF+NE
DO 280 IS=NEL,NUNK
ISP=IS-NE
CZ(IFP,ISP) = -CZ(IF,IS)
280 CONTINUE
290 CONTINUE
RETURN
END
SUBROUTINE CVFILL(CVTHC,CVPHC,RHO,Z,RHOPH,ZPH,DELTAT,GAMMA,
#      THETA,PHI,ETHETA,EPI,ANTFD,NUNK,NFLDS,MODE)
C*****
C      SUBROUTINE CVFILL COMPUTES THE FORCING FUNCTION VECTOR (OR
C      MATRIX FOR MULTIPLE EXCITATIONS).
C*****
C      IMPLICIT COMPLEX (C)
C      DOUBLE PRECISION BARG1,BJ(50),BY(2)
C      REAL COSTH,CGK,CGKP1,MU
C      DIMENSION CVTHC(NUNK,NFLDS),CVPHC(NUNK,NFLDS)
C      DIMENSION RHO(1),Z(1),RHOPH(1),ZPH(1),GAMMA(1),DELTAT(1)
C      DIMENSION THETA(1),PHI(1),ETHETA(1),EPI(1),ANTFD(1)
C      COMMON/PNT/NPTS,NUNKT,NUNKPH,NUNK2
C      COMMON/INT/NMODEM,IDB,NGQ
C      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
C      CALL ZERO(CVTHC,NUNK*NFLDS)
C      CALL ZERO(CVPHC,NUNK*NFLDS)
C      MODEP1=MODE+1
C      MODEM1=MODE-1
C      NTE=NUNKT
C      NPHE=NUNKPH
C      NTM=0
C      NPHM=0
C      IF(IDB.EQ. 0) GO TO 10
C      NTE=NTE/2
C      NPHE=NPHE/2
C      NTM=NTE
C      NPHM=NPHE
10 CONTINUE
C      TPI=2.0*PI
C      FPI=4.0*PI
C      JMP1=MODEP1+1
C      JM=MODEP1

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JMML=MODE	BR-10890
MU=FPI*1.E-7	BR-10900
ETA=OMEGA*MU/AK1	BR-10910
CJ=CMPLX(0.0,1.0)	BR-10920
C	BR-10930
C----- EVALUATE FIELDS AT NON-HALF POINTS	BR-10940
C	BR-10950
DO 110 IF1=1,NTE	BR-10960
IF3=IF1+NTE+NPHE	BR-10970
IF1P1=IF1+1	BR-10980
RF=RHO(IF1P1)	BR-10990
ZF=Z(IF1P1)	BR-11000
DTK=DELTAT(IF1)	BR-11010
DTKP1=DELTAT(IF1P1)	BR-11020
SGK=SIN(GAMMA(IF1))	BR-11030
SGKP1=SIN(GAMMA(IF1P1))	BR-11040
CGK=COS(GAMMA(IF1))	BR-11050
CGKP1=COS(GAMMA(IF1P1))	BR-11060
BRCKTS=(DTKP1*SGKP1+DTK*SGK)/2.0	BR-11070
BRCKTC=(DTKP1*CGKP1+DTK*CGK)/2.0	BR-11080
DO 100 I=1,NFLDS	BR-11090
COSTH=COS(THETA(I)*PI/180.0)	BR-11100
SINTH=SIN(THETA(I)*PI/180.0)	BR-11110
BARG=AK1*RF*SINTH	BR-11120
BARG1=DBLE(BARG)	BR-11130
IF(BARG) 20, 20, 40	BR-11140
20 DO 30 K=2,JMP1	BR-11150
30 BJ(K)=0.0	BR-11160
BJ(1)=1.0	BR-11170
GO TO 50	BR-11180
40 CALL BESSEL(BARG1,MODEP1,BJ,BY,1)	BR-11190
50 BET1=TPI*COSTH*BRCKTS*ETHETA(I)	BR-11200
BET2=-FPI*SINTH*BRCKTC*ETHETA(I)	BR-11210
BET3=TPI*BRCKTS*EPI(I)	BR-11220
BHT1=TPI*BRCKTS*COSTH*EPI(I)/ETA	BR-11230
BHT2=-FPI*BRCKTC*SINTH*EPI(I)/ETA	BR-11240
BHT3=-TPI*BRCKTS*ETHETA(I)/ETA	BR-11250
CJMP1=CMPLX(SNGL(BJ(JMP1)),0.0)	BR-11260
CJM=CMPLX(SNGL(BJ(JM)),0.0)	BR-11270
IF(JMML) 60, 60, 70	BR-11280
60 CJMM1=-CJMP1	BR-11290
GO TO 80	BR-11300
70 CJMM1=CMPLX(SNGL(BJ(JMML)),0.0)	BR-11310
80 CONTINUE	BR-11320
CJPM2=CJ** (MODEM1-1)	BR-11330
CJPM1=CJ*CJPM2	BR-11340
CJPM=CJ*CJPM1	BR-11350
CJPM1=CJ*CJPM	BR-11360
EARG=AK1*ZF*COSTH-PHI(I)*FLOAT(MODE)*PI/180.0	BR-11370
CARG=CMPLX(Z.0,EARG)	BR-11380
CE=CEXP(CARG)	BR-11390
CV=(CJMM1*CJPM1+CJPM1*CJMP1)*CMPLX(BET1,0.0)	BR-11400
CV=CV+CJPM*CJM*CMPLX(BET2,0.0)	BR-11410
CVTHC(1F1,I)=CV*CE	BR-11420
CV=(CJPM2*CJMM1-CJPM*CJMP1)*CMPLX(BET3,0.0)	BR-11430
CVPHC(1F1,I)=CV*CE	BR-11440
IF(IDB) 90, 100, 90	BR-11450
90 CV=(CJPM2*CJMM1-CJPM*CJMP1)*CMPLX(BHT3,0.0)	BR-11460
CVTHC(1F3,I)=CV*CE	BR-11470
CV=(CJPM1*CJMM1+CJPM1*CJMP1)*CMPLX(BHT1,0.0)	BR-11480
CV=CV+CJPM*CJM*CMPLX(BHT2,0.0)	BR-11490

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CVPHC(IF3,I)=CV*CE	BR-11500
100 CONTINUE	BR-11510
110 CONTINUE	BR-11520
C	BR-11530
C----- EVALUATE FIELDS AT HALF POINTS	BR-11540
C	BR-11550
DO 200 IF1=1,NPHE	BR-11560
IF2=IF1+NTE	BR-11570
IF4=IF2+NPHE+NTM	BR-11580
RF=RHOPH(IF1)	BR-11590
ZF=ZPH(IF1)	BR-11600
DTK=DELTAT(IF1)	BR-11610
DO 190 I=1,NFLDS	BR-11620
COSTH=COS(THETA(I)*PI/180.0)	BR-11630
SINTH=SIN(THETA(I)*PI/180.0)	BR-11640
BARG=AK1*RF*SINTH	BR-11650
BARG1=DBLE(BARG)	BR-11660
IF(BARG) 120, 120, 140	BR-11670
120 DO 130 K=2,JMP1	BR-11680
130 BJ(K)=0.0	BR-11690
BJ(1)=1.0	BR-11700
GO TO 150	BR-11710
140 CALL BESSEL(BARG1,MODEP1,BJ,BY,1)	BR-11720
150 BEP1=-DTK*TPI*COSTH*ETHETA(I)	BR-11730
BEP2=DTK*TPI*EPHI(I)	BR-11740
BHP1=-DTK*TPI*COSTH*EPHI(I)/ETA	BR-11750
BHP2=-DTK*TPI*ETHETA(I)/ETA	BR-11760
CJMP1=CMPLX(SNGL(BJ(JMP1)),0.0)	BR-11770
IF(JMM1) 160, 160, 170	BR-11780
160 CJMM1=-CJMP1	BR-11790
GO TO 180	BR-11800
170 CJMM1=CMPLX(SNGL(BJ(JMM1)),0.0)	BR-11810
180 CONTINUE	BR-11820
CJPM2=CJ** (MODEM1-1)	BR-11830
CJPM1=CJ*CJPM2	BR-11840
CJPM=CJ*CJPM1	BR-11850
CJPM1=CJPM*CJ	BR-11860
EARG=AK1*ZF*COSTH-PHI(I)*FLOAT(MODE)*PI/180.	BR-11870
CARG=CMPLX(0.0,EARG)	BR-11880
CE=CEXP(CARG)	BR-11890
CV=(CJPM2*CJMM1-CJPM*CJMP1)*CMPLX(BEP1,0.0)	BR-11900
CVTHC(IF2,I)=CV*CE	BR-11910
CV=(CJPM1*CJMM1+CJPM1*CJMP1)*CMPLX(BEP2,0.0)	BR-11920
CVPHC(IF2,I)=CV*CE	BR-11930
IF(IDB.EQ.0) GO TO 190	BR-11940
CV=(CJPM2*CJMM1-CJPM*CJMP1)*CMPLX(BHP1,0.0)	BR-11950
CVPHC(IF4,I)=CV*CE	BR-11960
CV=(CJPM1*CJMM1+CJPM1*CJMP1)*CMPLX(BHP2,0.0)	BR-11970
CVTHC(IF4,I)=CV*CE	BR-11980
190 CONTINUE	BR-11990
200 CONTINUE	BR-12000
C	BR-12010
C----- INCLUDE VOLTAGE SOURCE IF PRESENT	BR-12020
C	BR-12030
IF(MODE.NE.0) GO TO 220	BR-12040
DO 210 I=1,NFLDS	BR-12050
IANTFD=IFIX(ANTFD(I))	BR-12060
IF(IANTFD.LE.0) GO TO 210	BR-12070
CVTHC(IANTFD,I)=CVTHC(IANTFD,I)+FPI	BR-12080
210 CONTINUE	BR-12090
220 CONTINUE	BR-12100

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RETURN	BR-12110
END	BR-12120
SUBROUTINE SUMOUT(CI,NUNK,NFLDS,MODE,ITHETA)	BR-12130
C*****	BR-12140
C	BR-12150
C SUBROUTINE "SUMOUT" TRANSFORMS THE EXPONENTIAL COMPONENT CURRENT	BR-12160
C COEFFICIENTS TO SINE AND COSINE COMPONENT CURRENT COEFFICIENTS.	BR-12170
C	BR-12180
C*****	BR-12190
IMPLICIT COMPLEX (C)	BR-12200
DIMENSION CI(NUNK,NFLDS)	BR-12210
COMMON/INT/NMODEM,ILB,NGQ	BR-12220
COMMON/PNT/NPTS,NUNKT,NUNKPH,NUNK2	BR-12230
IF(MODE.EQ.0) RETURN	BR-12240
NTE=NUNKT	BR-12250
NPHE=NUNKPH	BR-12260
NTM=0	BR-12270
NPHM=0	BR-12280
IF(IDB.EQ.0) GO TO 10	BR-12290
NTE=NTE/2	BR-12300
NPHE=NPHE/2	BR-12310
NTM=NTE	BR-12320
NPHM=NPHE	BR-12330
10 CONTINUE	BR-12340
CF1=CMPLX(2.0,9.0)	BR-12350
CF2=CMPLX(0.0,2.0)	BR-12360
IF(ITHETA.EQ.1) GO TO 20	BR-12370
CF=CF1	BR-12380
CF1=CF2	BR-12390
CF2=CF	BR-12400
20 CONTINUE	BR-12410
DO 60 J=1,NFLDS	BR-12420
DO 50 I=1,NUNK	BR-12430
IF(I.GT.NTE) GO TO 40	BR-12440
30 CI(I,J)=CI(I,J)*CF1	BR-12450
GO TO 50	BR-12460
40 IF(I.GT.NTE+NPHE+NTM) GO TO 30	BR-12470
CI(I,J)=CI(I,J)*CF2	BR-12480
50 CONTINUE	BR-12490
60 CONTINUE	BR-12500
RETURN	BR-12510
END	BR-12520
SUBROUTINE RMSERR(CIT,CITOLD,NUNK,RMSMXE)	BR-12530
C*****	BR-12540
C	BR-12550
C SUBROUTINE "RMSERR" COMPUTES THE ROOT-MEAN-SQUARE RELATIVE	BR-12560
C ERROR BETWEEN THE PRESENT CURRENT SOLUTION AND THE PREVIOUS	BR-12570
C SOLUTION.	BR-12580
C	BR-12590
C*****	BR-12600
IMPLICIT COMPLEX (C)	BR-12610
DIMENSION CIT(NUNK), CITOLD(NUNK)	BR-12620
SQERR=0.0	BR-12630
DO 10 I=1,NUNK	BR-12640
DEN=CABS(CIT(I))	BR-12650
ANUM=CABS(CIT(I)-CITOLD(I))	BR-12660
IF(DEN.LT.1.E-9) DEN=CABS(CITOLD(I))	BR-12670
IF(DEN.LT.1.E-9) DEN=1.0	BR-12680
RELERR=ANUM/DEN	BR-12690
SQERR=SQERR+RELERR**2	BR-12700
10 CONTINUE	BR-12710

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SQMERR=SQERR/FLOAT(NUNK)	BR-12720
RMSMXE=SQRT(SQMERR)	BR-12730
RETURN	BR-12740
END	BR-12750
SUBROUTINE ZERO(C,N)	BR-12760
C	BR-12770
C----- INITIALIZATION ROUTINE	BR-12780
C	BR-12790
COMPLEX C(N)	BR-12800
C0=CMPLX(0.0,0.0)	BR-12810
DO 10 J=1,N	BR-12820
10 C(J)=C0	BR-12830
RETURN	BR-12840
END	BR-12850
SUBROUTINE MAGPHS(CRI,CMF)	BR-12860
C	BR-12870
C----- ROUTINE TO CALCULATE MAGNITUDE AND PHASE OF A COMPLEX NUMBER	BR-12880
C RESULT IS STORED IN A COMPLEX NUMBER AS WELL	BR-12890
C	BR-12900
IMPLICIT COMPLEX (C)	BR-12910
REAL I,M	BR-12920
COMMON/FRQ/PI,A,B,D,E,F	BR-12930
R=REAL(CRI)	BR-12940
I=AIMAG(CRI)	BR-12950
M=CABS(CRI)	BR-12960
P=ATAN2(I,R)*180.0/PI	BR-12970
CMF=CMPLX(M,P)	BR-12980
RETURN	BR-12990
END	BR-13000
SUBROUTINE NORMAL(CI,ETHETA,EPI,NUNK,NFLDS)	BR-13010
C*****	BR-13020
C	BR-13030
C SUBROUTINE NORMAL NORMALIZES THE ELECTRIC CURRENT BY THE	BR-13040
C INCIDENT H-FIELD AND THE MAGNETIC CURRENT BY THE INCIDENT	BR-13050
C E-FIELD.	BR-13060
C	BR-13070
C*****	BR-13080
IMPLICIT COMPLEX (C)	BR-13090
DIMENSION CI(NUNK,NFLDS)	BR-13100
DIMENSION ETHETA(1),EPI(1)	BR-13110
COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-13120
COMMON/INT/NMODEM,IDB,NGQ	BR-13130
NUNKE=NUNK	BR-13140
IF(IDB.EQ.1) NUNKE=NUNK/2	BR-13150
NUNKEL=NUNKE+1	BR-13160
DO 30 K=1,NFLDS	BR-13170
ENORM=SQRT(ETHETA(K)**2+EPI(K)**2)	BR-13180
HNORM=ENORM*SL1*8.85418533E-12	BR-13190
CEN=CMPLX(ENORM,0.0)	BR-13200
CHN=CMPLX(HNORM,0.0)	BR-13210
DO 10 I=1,NUNKE	BR-13220
10 CI(I,K)=CI(I,K)/CHN	BR-13230
IF(IDB.EQ.0) GO TO 30	BR-13240
DO 20 I=NUNKEL,NUNK	BR-13250
20 CI(I,K)=CI(I,K)/CEN	BR-13260
30 CONTINUE	BR-13270
RETURN	BR-13280
END	BR-13290
COMPLEX FUNCTION GIM(TP)	BR-13300
IMPLICIT COMPLEX (C)	BR-13310
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-13320

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COMMON/ARC/T	BR-13338
EXTERNAL CG1M	BR-13348
T=TP	BR-13358
CALL TRPADP(CG1M,0.0,PI,ER,MAXP,IER,CANS)	BR-13368
G1M=CANS/CMPLX(TWOPI,0.0)	BR-13378
RETURN	BR-13388
END	BR-13398
COMPLEX FUNCTION CG1M(PHIP)	BR-13408
IMPLICIT COMPLEX (C)	BR-13418
REAL COSG	BR-13428
COMMON/ARC/TP	BR-13438
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM	BR-13448
COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-13458
RS=RSL+TP*DEL*SING	BR-13468
ZS=ZSL+TP*DEL*COSG	BR-13478
R=SQRT(RF*RF+RS*RS+(ZF-ZS)**2-2.0*RF*RS*COS(PHIP))	BR-13488
CARG=CMPLX(0.0,-AK1*R)	BR-13498
CR=CMPLX(R,0.0)	BR-13508
ACMP=COS(GM*PHIP)	BR-13518
CG1M=(CMPLX(ACMP,0.0)*CEXP(CARG)-CMPLX(1.0,0.0))/CR	BR-13528
RETURN	BR-13538
END	BR-13548
COMPLEX FUNCTION G2M(TP)	BR-13558
IMPLICIT COMPLEX (C)	B-13568
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-13578
COMMON/ARC/T	BR-13588
EXTERNAL CG2M	BR-13598
T=TP	BR-13608
CALL TRPADP(CG2M,0.0,PI,ER,MAXP,IER,CANS)	BR-13618
G2M=CANS/CMPLX(TWOPI,0.0)	BR-13628
RETURN	BR-13638
END	BR-13648
COMPLEX FUNCTION CG2M(PHIP)	BR-13658
IMPLICIT COMPLEX (C)	BR-13668
REAL COSG	BR-13678
COMMON/ARC/TP	BR-13688
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM	BR-13698
COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-13708
RS=RSL+TP*DEL*SING	BR-13718
ZS=ZSL+TP*DEL*COSG	BR-13728
R=SQRT(RF*RF+RS*RS+(ZF-ZS)**2-2.0*RF*RS*COS(PHIP))	BR-13738
CARG=CMPLX(0.0,-AK2*R)	BR-13748
CR=CMPLX(R,0.0)	BR-13758
ACMP=COS(GM*PHIP)	BR-13768
CG2M=(CMPLX(ACMP,0.0)*CEXP(CARG)-CMPLX(1.0,0.0))/CR	BR-13778
RETURN	BR-13788
END	BR-13798
SUBROUTINE ELLPTC(TL,TU,NGQ,CANS)	BR-13808
IMPLICIT COMPLEX (C)	BR-13818
REAL COSG	BR-13828
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-13838
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM	BR-13848
COMMON/SLF/ISELF	BR-13858
EXTERNAL CGME	BR-13868
CALL CGQ1(CGME,TL,TU,NGQ,CANS1)	BR-13878
CANS=CANS1*CMPLX(2.0/PI,0.0)	BR-13888
IF(ISELF.EQ.0) RETURN	BR-13898
TLLN=0.0	BR-13908
TULN=0.0	BR-13918
IF(TF.GT.TL) TLLN=(TF-TL)*ALOG(TF-TL)	BR-13928
IF(TF.LT.TU) TULN=(TU-TF)*ALOG(TU-TF)	BR-13938

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ANS=((TU-TL)*(1.0-ALOG(DEL))-TLLN-TULN)/RF/PI	BR-13940
CANS=CANS+CMPLX(ANS,0.0)	BR-13950
RETURN	BR-13960
END	BR-13970
COMPLEX FUNCTION CGME(TP)	BR-13980
IMPLICIT COMPLEX (C)	BR-13990
REAL COSG	BR-14000
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM	BR-14010
COMMON/SLF/ISELF	BR-14020
RS=RSL+TP*DEL*SING	BR-14030
ZS=ZSL+TP*DEL*COSG	BR-14040
ZFMZS2=(ZF-ZS)**2	BR-14050
RFMRS2=(RF-RS)**2	BR-14060
RN=RFMRS2+ZFMZS2	BR-14070
RD=(RF+RS)**2+ZFMZS2	BR-14080
R2=SQRT(RD)	BR-14090
R3=SQRT(RN)	BR-14100
AM1=RN/RD	BR-14110
ANS=ELIC1K(AM1)	BR-14120
ANS=ANS/R2	BR-14130
IF(ISELF.EQ.0) GO TO 10	BR-14140
ANS=ANS+ALOG(R3)/2.0/RF	BR-14150
10 CGME=CMPLX(ANS,0.0)	BR-14160
RETURN	BR-14170
END	BR-14180
SUBROUTINE ELLPTR(TL,TU,NGQ,CANS)	BR-14190
IMPLICIT COMPLEX (C)	BR-14200
REAL COSG	BR-14210
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-14220
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM	BR-14230
COMMON/SLF/ISELF	BR-14240
EXTERNAL CGMER	BR-14250
CALL CGQ1(CGMR,TL,TU,NGQ,CANS1)	BR-14260
CANS=CANS1*CMPLX(2.0/PI,0.0)	BR-14270
IF(ISELF.EQ.0) RETURN	BR-14280
TLLN=0.0	BR-14290
TULN=0.0	BR-14300
IF(TF.GT.TL) TLLN=(TF-TL)*ALOG(TF-TL)	BR-14310
IF(TF.LT.TU) TULN=(TU-TF)*ALOG(TU-TF)	BR-14320
ANS=((TU-TL)*(1.0-ALOG(DEL))-TLLN-TULN)/PI	BR-14330
CANS=CANS+CMPLX(ANS,0.0)	BR-14340
RETURN	BR-14350
END	BR-14360
COMPLEX FUNCTION CGMER(TP)	BR-14370
IMPLICIT COMPLEX (C)	BR-14380
REAL COSG	BR-14390
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM	BR-14400
COMMON/SLF/ISELF	BR-14410
RS=RSL+TP*DEL*SING	BR-14420
ZS=ZSL+TP*DEL*COSG	BR-14430
ZFMZS2=(ZF-ZS)**2	BR-14440
RFMRS2=(RF-RS)**2	BR-14450
RN=RFMRS2+ZFMZS2	BR-14460
RD=(RF+RS)**2+ZFMZS2	BR-14470
R2=SQRT(RD)	BR-14480
R3=SQRT(RN)	BR-14490
AM1=RN/RD	BR-14500
ANS=ELIC1K(AM1)	BR-14510
ANS=ANS*RS/R2	BR-14520
IF(ISELF.EQ.0) GO TO 10	BR-14530
ANS=ANS+ALOG(R3)/2.0	BR-14540

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10 CGMER=CMPLX(ANS,0.0)	BR-14550
RETURN	BR-14560
END	BR-14570
COMPLEX FUNCTION CGPOR(DUMMY)	BR-14580
C	BR-14590
C----- FUNCTION: (1/(R0))*(D(G1+G2)/D(R0))	BR-14600
C	BR-14610
IMPLICIT COMPLEX (C)	BR-14620
REAL COSGS	BR-14630
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM	BR-14640
COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-14650
COMMON/ARC/TP	BR-14660
COMMON/ANG/PHIP	BR-14670
RS=RSL+TP*DEL*SINGS	BR-14680
ZS=ZSL+TP*DEL*COSGS	BR-14690
R=SQRT(RF*RF+RS*RS+(ZF-ZS)**2-2.0*RF*RS*COS(PHIP))	BR-14700
RK1=AK1*R	BR-14710
RK2=AK2*R	BR-14720
CARG1=CMPLX(0.0,-RK1)	BR-14730
CARG2=CMPLX(0.0,-RK2)	BR-14740
C1PJK1=CMPLX(1.0,RK1)	BR-14750
C1PJK2=CMPLX(1.0,RK2)	BR-14760
CB=C1PJK1*CEXP(CARG1)+C1PJK2*CEXP(CARG2)	BR-14770
CGPOR=-CB/CMPLX(R**3,0.0)	BR-14780
RETURN	BR-14790
END	BR-14800
COMPLEX FUNCTION G0(TP)	BR-14810
IMPLICIT COMPLEX (C)	BR-14820
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-14830
COMMON/ARC/T	BR-14840
EXTERNAL CG0	BR-14850
T=TP	BR-14860
CALL TRPADP(CG0,0.0,PI,ER,MAXP,IER,CANS)	BR-14870
G0=CANS/CMPLX(TWOPI,0.0)	BR-14880
RETURN	BR-14890
END	BR-14900
COMPLEX FUNCTION G1(TP)	BR-14910
IMPLICIT COMPLEX (C)	BR-14920
COMMON/ISWTCH/ISWR	BR-14930
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-14940
COMMON/ARC/T	BR-14950
EXTERNAL CG1	BR-14960
ISWR=0	BR-14970
T=TP	BR-14980
CALL TRPADP(CG1,0.0,PI,ER,MAXP,IER,CANS)	BR-14990
G1=CANS/CMPLX(TWOPI,0.0)	BR-15000
RETURN	BR-15010
END	BR-15020
COMPLEX FUNCTION G1R(TP)	BR-15030
IMPLICIT COMPLEX (C)	BR-15040
COMMON/ISWTCH/ISWR	BR-15050
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-15060
COMMON/ARC/T	BR-15070
EXTERNAL CG1	BR-15080
ISWR=1	BR-15090
T=TP	BR-15100
CALL TRPADP(CG1,0.0,PI,ER,MAXP,IER,CANS)	BR-15110
G1R=CANS/CMPLX(TWOPI,0.0)	BR-15120
RETURN	BR-15130
END	BR-15140
COMPLEX FUNCTION G2(TP)	BR-15150

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IMPLICIT COMPLEX (C)	BR-15160
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-15170
COMMON/ARC/T	BR-15180
COMMON/ISWTCH/ISWR	BR-15190
EXTERNAL CG2	BR-15200
T=TP	BR-15210
ISWR=0	BR-15220
CALL TRPADP(CG2,0.0,PI,ER,MAXP,IER,CANS)	BR-15230
G2=CANS/CMPLX(TWOPI,0.0)	BR-15240
RETURN	BR-15250
END	BR-15260
COMPLEX FUNCTION C2R(TP)	BR-15270
IMPLICIT COMPLEX (C)	BR-15280
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-15290
COMMON/ARC/T	BR-15300
COMMON/ISWTCH/ISWR	BR-15310
EXTERNAL CG2	BR-15320
T=TP	BR-15330
ISWR=1	BR-15340
CALL TRPADP(CG2,0.0,PI,ER,MAXP,IER,CANS)	BR-15350
G2R=CANS/CMPLX(TWOPI,0.0)	BR-15360
RETURN	BR-15370
END	BR-15380
COMPLEX FUNCTION G3(TP)	BR-15390
IMPLICIT COMPLEX (C)	BR-15400
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-15410
COMMON/ARC/T	BR-15420
EXTERNAL CG3	BR-15430
T=TP	BR-15440
CALL TRPADP(CG3,0.0,PI,ER,MAXP,IER,CANS)	BR-15450
G3=CANS/CMPLX(TWOPI,0.0)	BR-15460
RETURN	BR-15470
END	BR-15480
COMPLEX FUNCTION G4R(TP)	BR-15490
IMPLICIT COMPLEX (C)	BR-15500
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-15510
COMMON/ARC/T	BR-15520
COMMON/ISWTCH/ISWR	BR-15530
EXTERNAL CG4	BR-15540
T=TP	BR-15550
ISWR=1	BR-15560
CALL TRPADP(CG4,0.0,PI,ER,MAXP,IER,CANS)	BR-15570
G4R=CANS/CMPLX(TWOPI,0.0)	BR-15580
RETURN	BR-15590
END	BR-15600
COMPLEX FUNCTION G5(TP)	BR-15610
IMPLICIT COMPLEX (C)	BR-15620
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-15630
COMMON/ARC/T	BR-15640
COMMON/ISWTCH/ISWR	BR-15650
EXTERNAL CG5	BR-15660
T=TP	BR-15670
ISWR=0	BR-15680
CALL TRPADP(CG5,0.0,PI,ER,MAXP,IER,CANS)	BR-15690
G5=CANS/CMPLX(TWOPI,0.0)	BR-15700
RETURN	BR-15710
END	BR-15720
COMPLEX FUNCTION G5R(TP)	BR-15730
IMPLICIT COMPLEX (C)	BR-15740
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-15750
COMMON/ARC/T	BR-15760

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COMMON/ISWICH/ISWR	BR-15770
EXTERNAL CG5	BR-15780
T=TP	BR-15790
ISWR=1	BR-15800
CALL TRPADP(CG5,0.0,PI,ER,MAXP,IER,CANS)	BR-15810
G5R=CANS/CMPLX(TWOPI,0.0)	BR-15820
RETURN	BR-15830
END	BR-15840
COMPLEX FUNCTION G6(TP)	BR-15850
IMPLICIT COMPLEX (C)	BR-15860
COMMON/TPI/TWOPI,MAXP,ER,PI	BR-15870
COMMON/ARC/T	BR-15880
EXTERNAL CG6	BR-15890
T=TP	BR-15900
CALL TRPADP(CG6,0.0,PI,ER,MAXP,IER,CANS)	BR-15910
G6=CANS/CMPLX(TWOPI,0.0)	BR-15920
RETURN	BR-15930
END	BR-15940
COMPLEX FUNCTION CG0(PHIP)	BR-15950
IMPLICIT COMPLEX (C)	BR-15960
REAL COSGS	BR-15970
COMMON/ANG/PHI	BR-15980
COMMON/ARC/TP	BR-15990
COMMON/SLF/ISELF	BR-16000
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM	BR-16010
COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-16020
PHI=PHIP	BR-16030
SMPSP=SIN(GM*PHIP)*SIN(PHIP)	BR-16040
CINTG=CMPLX(SMPSP,0.0)*CGPOR(XXXXXX)	BR-16050
IF(ISELF.EQ.0) GO TO 10	BR-16060
ANUM=2.0*GM*PHIP*PHIP	BR-16070
DEN=((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5	BR-16080
CINTG=CINTG+CMPLX(ANUM/DEN,0.0)	BR-16090
10 CG0=CINTG	BR-16100
RETURN	BR-16110
END	BR-16120
COMPLEX FUNCTION CG1(PHIP)	BR-16130
IMPLICIT COMPLEX (C)	BR-16140
REAL COSGS	BR-16150
COMMON/ISWICH/ISWR	BR-16160
COMMON/ANG/PHI	BR-16170
COMMON/ARC/TP	BR-16180
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM	BR-16190
COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-16200
PHI=PHIP	BR-16210
ZS=ZSL+TP*DEL*COSGS	BR-16220
SMPSP=SIN(GM*PHIP)*SIN(PHIP)	BR-16230
IF(ISWR.EQ.1) SMPSP=SMPSP*(RSL+TP*DEL*SINGS)	BR-16240
CG1=CMPLX((ZF-ZS)*SMPSP,0.0)*CGPOR(XXXXXX)	BR-16250
RETURN	BR-16260
END	BR-16270
COMPLEX FUNCTION CG2(PHIP)	BR-16280
IMPLICIT COMPLEX (C)	BR-16290
REAL COSGS,CMPCP	BR-16300
COMMON/ANG/PHI	BR-16310
COMMON/ISWICH/ISWR	BR-16320
COMMON/ARC/TP	BR-16330
COMMON/SLF/ISELF	BR-16340
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM	BR-16350
COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-16360
PHI=PHIP	BR-16370

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ZS=ZSL+TP*DEL*COSGS	BR-16380
CMPCP=COS(GM*PHIP)*COS(PHIP)	BR-16390
IF(ISWR.EQ. 1) CMPCP=CMPCP*(RSL+TP*DEL*SINGS)	BR-16400
CINTG=CMPLX((ZF-ZS)*CMPCP,0.0)*CGPOR(XXXXXX)	BR-16410
IF(ISELF.EQ. 0) GO TO 10	BR-16420
ANUM=2.0*(TF-TP)*DEL*COSGS	BR-16430
IF(ISWR.EQ. 1) ANUM=ANUM*RF	BR-16440
DEN=((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5	BR-16450
CINTG=CINTG+CMPLX(ANUM/DEN,0.0)	BR-16460
10 CG2=CINTG	BR-16470
RETURN	BR-16480
END	BR-16490
COMPLEX FUNCTION CG3(PHIP)	BR-16500
IMPLICIT COMPLEX (C)	BR-16510
REAL COSGS	BR-16520
COMMON/ANG/PHI	BR-16530
COMMON/ARC/TP	BR-16540
COMMON/SLF/ISELF	BR-16550
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM	BR-16560
COMMON/FRO/PI,AK1,AK2,SL1,SL2,OMEGA	BR-16570
PHI=PHIP	BR-16580
RS=RSL+TP*DEL*SINGS	BR-16590
SMPSP=SIN(GM*PHIP)*SIN(PHIP)	BR-16600
CINTG=CMPLX(RS*SMPSP,0.0)*CGPOR(XXXXXX)	BR-16610
IF(ISELF.EQ. 0) GO TO 10	BR-16620
ANUM=2.0*RF*GM*PHIP*PHIP	BR-16630
DEN=((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5	BR-16640
CINTG=CINTG+CMPLX(ANUM/DEN,0.0)	BR-16650
10 CG3=CINTG	BR-16660
RETURN	BR-16670
END	BR-16680
COMPLEX FUNCTION CG4(PHIP)	BR-16690
IMPLICIT COMPLEX (C)	BR-16700
REAL COSGS,CMP	BR-16710
COMMON/ANG/PHI	BR-16720
COMMON/ISWTCH/ISWR	BR-16730
COMMON/ARC/TP	BR-16740
COMMON/SLF/ISELF	BR-16750
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM	BR-16760
COMMON/FRO/PI,AK1,AK2,SL1,SL2,OMEGA	BR-16770
PHI=PHIP	BR-16780
RS=RSL+TP*DEL*SINGS	BR-16790
CMP=COS(GM*PHIP)	BR-16800
IF(ISWR.EQ. 1) CMP=CMP*RS	BR-16810
CINTG=CMPLX((RF-RS)*CMP,0.0)*CGPOR(XXXXXX)	BR-16820
IF(ISELF.EQ. 0) GO TO 10	BR-16830
ANUM=2.0*(TF-TP)*DEL*SINGS	BR-16840
IF(ISWR.EQ. 1) ANUM=ANUM*RF	BR-16850
DEN=((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5	BR-16860
CINTG=CINTG+CMPLX(ANUM/DEN,0.0)	BR-16870
10 CG4=CINTG	BR-16880
RETURN	BR-16890
END	BR-16900
COMPLEX FUNCTION CG5(PHIP)	BR-16910
IMPLICIT COMPLEX (C)	BR-16920
REAL COSGS	BR-16930
COMMON/ISWTCH/ISWR	BR-16940
COMMON/ANG/PHI	BR-16950
COMMON/ARC/TP	BR-16960
COMMON/SLF/ISELF	BR-16970
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM	BR-16980

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COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-16990
PHI=PHIP	BR-17000
S2PCMP=COS(GM*PHIP)*(SIN(PHIP/2.0)**2)	BR-17010
IF(ISWR.EQ.1) S2PCMP=S2PCMP*(RSL+TP*DEL*SINGS)	BR-17020
CINTG=CMPLX(S2PCMP,0.0)*CGPOR(XXXXXX)	BR-17030
IF(ISELF.EQ.0) GO TO 10	BR-17040
ANUM=PHIP*PHIP/2.0	BR-17050
IF(ISWR.EQ.1) ANUM=ANUM*RF	BR-17060
DEN=((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5	BR-17070
CINTG=CINTG+CMPLX(ANUM/DEN,0.0)	BR-17080
10 CG5=CINTG	BR-17090
RETURN	BR-17100
END	BR-17110
COMPLEX FUNCTION CG6(PHIP)	BR-17120
IMPLICIT COMPLEX (C)	BR-17130
REAL COSGS,CMPCP	BR-17140
COMMON/ANG/PHI	BR-17150
COMMON/ARC/TP	BR-17160
COMMON/SLF/ISELF	BR-17170
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM	BR-17180
COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-17190
PHI=PHIP	BR-17200
RS=RSL+TP*DEL*SINGS	BR-17210
CMPCP=COS(GM*PHIP)*COS(PHIP)	BR-17220
CINTG=CMPLX((RF-RS)*CMPCP,0.0)*CGPOR(XXXXXX)	BR-17230
IF(ISELF.EQ.0) GO TO 10	BR-17240
ANUM=2.0*(TF-TP)*DEL*SINGS	BR-17250
DEN=((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5	BR-17260
CINTG=CINTG+CMPLX(ANUM/DEN,0.0)	BR-17270
10 CG6=CINTG	BR-17280
RETURN	BR-17290
END	BR-17300
COMPLEX FUNCTION CANALY(TL,TU,I,ISWR)	BR-17310
C----- ROUTINE TO ADD RESULT OF ANALYTICAL INTEGRATION OF THE	BR-17320
C SINGULAR PORTION OF THE INTEGRANDS	BR-17330
C	BR-17340
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,ACOSGS,GM	BR-17350
COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA	BR-17360
ID=I+1	BR-17370
TUMTDT=(TU-TF)*DEL	BR-17380
TLMTDT=(TL-TF)*DEL	BR-17390
RPI=RF*PI	BR-17400
IF(TUMTDT) 10, 10, 20	BR-17410
10 TU1=0.0	BR-17420
TU2=0.0	BR-17430
GO TO 30	BR-17440
20 RAD=SQRT(TUMTDT**2+RPI**2)	BR-17450
TU1=ALOG(RPI+RAD)	BR-17460
TU2=ALOG(TUMTDT)	BR-17470
30 IF(TLMTDT) 50, 40, 40	BR-17480
40 TL1=0.0	BR-17490
TL2=0.0	BR-17500
GO TO 60	BR-17510
50 RAD=SQRT(TLMTDT**2+RPI**2)	BR-17520
TL1=ALOG(RAD+RPI)	BR-17530
TL2=ALOG(-TLMTDT)	BR-17540
60 TERM=(TU-TF)*(TU1-TU2)+(TF-TL)*(TL1-TL2)	BR-17550
TERM=-TERM/(PI*RF)	BR-17560
IF(ISWR.EQ.0) TERM=TERM/RF	BR-17570
GO TO (70, 100, 100, 80, 100, 90) ID	BR-17580
	BR-17590

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70	TERM=TERM*2.0*GM/RF	BR-17600
	GO TO 110	BR-17610
80	TERM=TERM*2.0*GM	BR-17620
	GO TO 110	BR-17630
90	TERM=TERM/(2.0*RF)	BR-17640
	GO TO 110	BR-17650
100	TERM=0.0	BR-17660
110	CANALY=CMPLX(TERM,0.0)	BR-17670
	RETURN	BR-17680
	END	BR-17690

	SUBROUTINE CSMINV(A,NDIM,N,DETERM,COND,IERR)	MI- 10
C*****		MI- 20
C		MI- 30
C	CSMINV IS A SUBROUTINE WHICH WILL ACCEPT A SINGLE PRECISION	MI- 40
C	COMPLEX MATRIX AND RETURN THE INVERSE OF THE MATRIX IN ITS	MI- 50
C	PLACE. THE SUBROUTINE WILL ALSO COMPUTE THE NORMALIZED	MI- 60
C	DETERMINANT OF THE MATRIX, AND THE INVERSE CONDITION NUMBER	MI- 70
C	OF THE MATRIX.	MI- 80
C		MI- 90
C	N - THE ORDER OF THE MATRIX TO BE INVERTED	MI- 100
C	A - COMPLEX DOUBLE PRECISION INPUT MATRIX (DESTROYED)	MI- 110
C	THE INVERSE OF A IS RETURNED IN ITS PLACE.	MI- 120
C	NDIM - THE SIZE TO WHICH A IS DIMENSIONED IN THE CALLING	MI- 130
C	PROGRAM	MI- 140
C	DETERM - THE NORMALIZED DETERMINANT OF A WHICH IS RETURNED	MI- 150
C	COND - THE INVERSE OF MITTRA'S CONDITION NUMBER OF	MI- 160
C	THE MATRIX.	MI- 170
C	IERR - ERROR INDICATOR WHOSE VALUE IS ZERO UNLESS TOO	MI- 180
C	LARGE A MATRIX IS PASSED (.GT. 250 X 250),	MI- 190
C	IN WHICH CASE IERR=1	MI- 200
C		MI- 210
C	PREPARED BY MICHAEL G. HARRISON E.E. DEPT	MI- 220
C	JUNE 23, 1972	MI- 230
C*****		MI- 240
C	COMPLEX A(NDIM,NDIM),PIVOT(250),AMAX,T,SWAP,DETERM,U	MI- 250
C	INTEGER*4 IPIVOT(250),INDEX(250,2)	MI- 260
C	REAL TEMP,ALPHA(250)	MI- 270
C		MI- 280
C	INITIALIZATION	MI- 290
C		MI- 300
	IERR=0	MI- 310
	IF(NDIM.LE.250) GO TO 5	MI- 320
	IERR=1	MI- 330
	WRITE(5,4) NDIM	MI- 340
4	FORMAT('0CSMINV ERROR. ATTEMPT TO INVERT A MATRIX 'I4,	MI- 350
	1' ON A SIDE, '/' WHEN 250 X 250 IS THE MAXIMUM ALLOWED.')	MI- 360
	RETURN	MI- 370
5	CONTINUE	MI- 380
	DETERM = CMPLX(1.0,0.0)	MI- 390
	SUMAXA=0.	MI- 400
	DO 20 J=1,N	MI- 410
	ALPHA(J)=0.0	MI- 420
	SUMROW=0.	MI- 430
	DO 10 I=1,N	MI- 440
	ALPHA(J)=ALPHA(J)+A(J,I)*CONJG(A(J,I))	MI- 450
10	SUMROW=SUMROW+CABS(A(J,I))	MI- 460
	ALPHA(J)=SQRT(ALPHA(J))	MI- 470

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	IF(SUMROW.GT.SUMAXA) SUMAXA=SUMROW	MI-	480
20	IPIVOT(J)=0	MI-	490
	DO 600 I=1,N	MI-	500
C		MI-	510
C	SEARCH FOR PIVOT ELEMENT	MI-	520
C		MI-	530
	AMAX=CMPLX(0.0,0.0)	MI-	540
	DO 105 J=1,N	MI-	550
	IF (IPIVOT(J)-1) 60, 105, 60	MI-	560
60	DO 100 K=1,N	MI-	570
	IF (IPIVOT(K)-1) 80, 100, 740	MI-	580
80	TEMP=AMAX* CONJG(AMAX)-A(J,K) * CONJG(A(J,K))	MI-	590
	IF(TEMP) 85,85,100	MI-	600
85	IROW=J	MI-	610
	ICOLUM=K	MI-	620
	AMAX=A(J,K)	MI-	630
100	CONTINUE	MI-	640
105	CONTINUE	MI-	650
	IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1	MI-	660
C		MI-	670
C	INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL	MI-	680
C		MI-	690
	IF (IROW-ICOLUM) 140, 260, 140	MI-	700
140	DETERM=-DETERM	MI-	710
	DO 200 L=1,N	MI-	720
	SWAP=A(IROW,L)	MI-	730
	A(IROW,L)=A(ICOLUM,L)	MI-	740
200	A(ICOLUM,L)=SWAP	MI-	750
	SWAP=ALPHA(IROW)	MI-	760
	ALPHA(IROW)=ALPHA(ICOLUM)	MI-	770
	ALPHA(ICOLUM)=SWAP	MI-	780
260	INDEX(I,1)=IROW	MI-	790
	INDEX(I,2)=ICOLUM	MI-	800
	PIVOT(I)=A(ICOLUM,ICOLUM)	MI-	810
	U = PIVOT(I)	MI-	820
	ALPHA1=ALPHA(ICOLUM)	MI-	830
C		MI-	840
C	----- THE FOLLOWING SUBROUTINE CALL IS FOR UNDERFLOW PROTECTION DURING	MI-	850
C	CALCULATION OF THE NORMALIZED DETERMINANT	MI-	860
C		MI-	870
	CALL DTRNT(DETERM,U,ALPHA1)	MI-	880
	TEMP=PIVOT(I) * CONJG(PIVOT(I))	MI-	890
	IF(TEMP) 330,720,330	MI-	900
C		MI-	910
C	DIVIDE PIVOT ROW BY PIVOT ELEMENT	MI-	920
C		MI-	930
	330 A(ICOLUM,ICOLUM) = CMPLX(1.0,0.0)	MI-	940
	DO 350 L=1,N	MI-	950
	U = PIVOT(I)	MI-	960
	350 A(ICOLUM,L) = A(ICOLUM,L)/U	MI-	970
C		MI-	980
C	REDUCE NON-PIVOT ROWS	MI-	990
C		MI-	1000
	380 DO 550 L1=1,N	MI-	1010
	IF(L1-ICOLUM) 400, 550, 400	MI-	1020
400	T=A(L1,ICOLUM)	MI-	1030
	A(L1,ICOLUM)= CMPLX(0.0,0.0)	MI-	1040
	DO 450 L=1,N	MI-	1050
	U = A(ICOLUM,L)	MI-	1060
450	A(L1,L) = A(L1,L)-U*T	MI-	1070
550	CONTINUE	MI-	1080

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600 CONTINUE	MI- 1090
C	MI- 1100
C INTERCHANGE COLUMNS	MI- 1110
C	MI- 1120
620 DO 710 I=1,N	MI- 1130
L=N+1-I	MI- 1140
IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630	MI- 1150
630 JROW=INDEX(L,1)	MI- 1160
JCOLUM=INDEX(L,2)	MI- 1170
DO 705 K=1,N	MI- 1180
SWAP=A(K,JROW)	MI- 1190
A(K,JROW)=A(K,JCOLUM)	MI- 1200
A(K,JCOLUM)=SWAP	MI- 1210
705 CONTINUE	MI- 1220
710 CONTINUE	MI- 1230
SUMAXI=0.	MI- 1240
DO 910 I=1,N	MI- 1250
SUMROW=0.	MI- 1260
DO 900 J=1,N	MI- 1270
900 SUMROW=SUMROW + CABS(A(I,J))	MI- 1280
IF(SUMROW.GT.SUMAXI) SUMAXI=SUMROW	MI- 1290
910 CONTINUE	MI- 1300
COND = 1./(SUMAXA*SUMAXI)	MI- 1310
RETURN	MI- 1320
720 WRITE(5,730)	MI- 1330
730 FORMAT('0',10('*****')/'0MATRIX IS SINGULAR'/'0',10('*****'))	MI- 1340
740 RETURN	MI- 1350
END	MI- 1360
SUBROUTINE DTRMNT(DETERM,U,A)	MI- 1370
C*****	MI- 1380
C	MI- 1390
C THE SOLE PURPOSE OF THIS ROUTINE IS TO SCALE THE NORMALIZED	MI- 1400
C DETERMINANT SO THAT MACHINE UNDERFLOWS WILL NOT OCCUR.	MI- 1410
C THIS IS NECESSARY BECAUSE THE DYNAMIC RANGE OF THE DEC-1077	MI- 1420
C MACHINE IS ONLY ABOUT 10**-38 TO 10**+39.	MI- 1430
C	MI- 1440
C THE PARAMETER ISCALE MUST BE RETURNED VIA COMMON TO ANY	MI- 1450
C PROGRAM NEEDING THE VALUE OF THE NORMALIZED DETERMINANT.	MI- 1460
C	MI- 1470
C THE VALUE OF THE NORMALIZED DETERMINANT IS THE VALUE RETURNED	MI- 1480
C BY CSMINV TIMES TEN RAISED TO THE POWER (-ISCALE*TEN).	MI- 1490
C	MI- 1500
C	MI- 1510
C IF CSMINV IS CALLED MORE THAN ONCE BY A PROGRAM, "ISCALE" MUST BE	MI- 1520
C INITIALIZED TO ZERO FOR EACH CALL AFTER THE FIRST.	MI- 1530
C	MI- 1540
C*****	MI- 1550
COMPLEX DETERM,U	MI- 1560
COMMON/SCAFAC/ISCALE	MI- 1570
DATA ISCALE/0/	MI- 1580
IF(CABS(DETERM) .GT. 1.E-10) GO TO 100	MI- 1590
DETERM=DETERM*1.E10	MI- 1600
ISCALE=ISCALE+1	MI- 1610
100 DETERM=DETERM*U/CMPLX(A,0.0)	MI- 1620
RETURN	MI- 1630
END	MI- 1640

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SUBROUTINE CGQ1 (CF,XL,XU,N,CVAL)
C*****
C
C   PERFORMS INTEGRATION OF A FUNCTION OF A SINGLE VARIABLE
C   BY GAUSSIAN QUADRATURE.
C
C   N = ORDER OF GAUSSIAN QUADRATURE APPROXIMATION
C       (2,4,8,10,12,16,32)
C   CF = EXTERNALLY SUPPLIED FUNCTION....MUST BE FUNCTION OF ONE
C       VARIABLE FOR CGQ1.
C   XL = LOWER BOUND OF VARIABLE
C   XU = UPPER BOUND OF VARIABLE
C   CVAL = RESULTING VALUE OF THE INTEGRATION
C
C   CF MUST BE LISTED IN AN EXTERNAL STATEMENT
C
C   PREPARED BY MICHAEL G. HARRISON E.E. DEPT    JUNE 22, 1972
C*****
C   IMPLICIT COMPLEX (C)
C   DIMENSION Q1(52),Q2(24),Q3(32),NQ(8),NS(8),QG(108)
C   EQUIVALENCE (Q1(1),QG(1)),(Q2(1),QG(53)),(Q3(1),QG(77))
C   DATA Q1
C   $      / .283675134594812882E0,0.5E0,.43056815579702629E0,
C   $ .17392742256872693E0,.16999052179242813E0,.32607257743127307E0,
C   $ 0.48014452824876812E0,.50614268145188130E-1,.39833323870681337E0,
C   $ .11119051722668724E0,.26276620495816449E0,.15685332293894364E0,
C   $ .9171732124782490E-1,.18134189168918099E0,.4869532642585858E0,
C   $ .3333567215434407E-1,.43253168334449225E0,.747256745752903E-1,
C   $ .3397047841496122E0,.10954318125799102E0,.2166976970646236E0,
C   $ .13463335915499818E0,.74437169490815605E-1,.14776211235737644E0,
C   $ 0.49078031712335963E0,.23587668193255914E-1,.45205862818523743E0,
C   $ .53469662997659215E-1,.38495133709715234E0,.8003916427167311E-1,
C   $ .29365897714330872E0,.10158371336153296E0,.18391574949909010E0,
C   $ .11674626826917740E0,.62616704255734458E-1,.12457352290670139E0,
C   $ .49470046749582497E0,.13576229705877047E-1,.47228751153661629E0,
C   $ .31126761969323946E-1,.43281560119391587E0,.47579255841246392E-1,
C   $ .37770220417750152E0,.62314485627766936E-1,.30893812220132187E0,
C   $ .7479799440828837E-1,.22900838802861369E0,.8457825969750127E-1,
C   $ .14080177538962946E0,.9130170752246179E-1,.47506254918818720E-1,
C   $ .9472530522753425E-1 /
C   DATA Q2
C   $      / 0.49759360999851068E+0 , 0.61706148999935998E-2 ,
C   *      0.48736427798565475E+0 , 0.14265694314466832E-1 ,
C   *      0.46913727600136638E+0 , 0.22138719408709903E-1 ,
C   *      0.44320776350220052E+0 , 0.29649292457718890E-1 ,
C   *      0.41000099298695146E+0 , 0.36673240705540153E-1 ,
C   *      0.37006209578927718E+0 , 0.43095080765976638E-1 ,
C   *      0.32404682596848778E+0 , 0.48909326052056944E-1 ,
C   *      0.27271073569441977E+0 , 0.53722135057982817E-1 ,
C   *      0.21689675381302257E+0 , 0.57752834026862801E-1 ,
C   *      0.15752133974808169E+0 , 0.60835236463901695E-1 ,
C   *      0.95559433736808150E-1 , 0.62918728173414142E-1 ,
C   *      0.32078446431302813E-1 , 0.63969097673376078E-1 /
C   DATA Q3
C   $      / 0.49863193092474078E+0 , 0.35093050047350403E-2 ,
C   *      0.49280575577263417E+0 , 0.81371973654528350E-2 ,
C   *      0.48238112779375322E+0 , 0.12696032654531030E-1 ,
C   *      0.46745303796886904E+0 , 0.17136931456510717E-1 ,
C   *      0.44816057788302606E+0 , 0.21417949011113340E-1 ,

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*      0.42468380686628499E+0 ,      0.25499029631188088E-1 ,Q - 620
*      0.39724189798397120E+0 ,      0.29342046739267774E-1 ,Q - 630
*      0.36609105937014484E+0 ,      0.32911111388180923E-1 ,Q - 640
*      0.33152213346510760E+0 ,      0.36172897054424253E-1 ,Q - 650
*      0.29385787862038116E+0 ,      0.39096947893535153E-1 ,Q - 660
*      0.25344995446611470E+0 ,      0.41655962113473378E-1 ,Q - 670
*      0.21067563806531767E+0 ,      0.43826046502201906E-1 ,Q - 680
*      0.16593430114106382E+0 ,      0.45586939347881942E-1 ,Q - 690
*      0.11964368112606854E+0 ,      0.46922199540402283E-1 ,Q - 700
*      0.72235980791398250E-1 ,      0.47819360039637430E-1 ,Q - 710
*      0.24153832843869158E-1 ,      0.48270044257363900E-1 /Q - 720
$ NQ/2,4,8,10,12,16,24,32/,      Q - 730
$ NS/1,3,7,15,25,37,53,77/      Q - 740
DO 300 L=1,8                      Q - 750
IF(N.EQ.NQ(L)) GO TO 301          Q - 760
300 CONTINUE                      Q - 770
WRITE(5,905) N                    Q - 780
905 FORMAT('0 CALLING PARAMETER =',I5,' INTEGRATION NOT POSSIBLE'//)Q - 790
RETURN                            Q - 800
301 CONTINUE                      Q - 810
NP=NS(L)                         Q - 820
NE=NP+N-1                        Q - 830
AX=0.5*(XU+XL)                   Q - 840
BX=XU-XL                         Q - 850
CVAL=(0.,0.)                     Q - 860
DO 350 J=NP,NE,2                 Q - 870
DX=QG(J)*BX                      Q - 880
CVAL=CVAL+QG(J+1)*(CF(AX+DX)+CF(AX-DX)) Q - 890
350 CONTINUE                     Q - 900
CVAL=CVAL*BX                     Q - 910
RETURN                           Q - 920
END                               Q - 930

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SUBROUTINE CGQ1T(CF,XL,XU,N,CVAL,CVALR)      QR- 10
C*****                                     QR- 20
C      SUBROUTINE CGQ1T IS A VERSION OF THE GAUSSIAN QUADRATURE      QR- 30
C      INTEGRATION ROUTINE "CGQ1" WHICH HAS BEEN MODIFIED TO        QR- 40
C      SIMULTANEOUSLY INTEGRATE THE FUNCTIONS (CF) AND ((RHC-PRIME)*CF). QR- 50
C      THE ARGUMENTS ARE THE SAME EXCEPT FOR THE INCLUSION OF THE QR- 60
C      ADDITIONAL RESULT "CVALR."                                     QR- 70
C      QR- 80
C      QR- 90
C      QR- 100
C*****                                     QR- 110
IMPLICIT COMPLEX (C)           QR- 120
DIMENSION Q1(52),Q2(24),Q3(32),NQ(8),NS(8),QG(100) QR- 130
REAL COSGS                     QR- 140
COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM QR- 150
EQUIVALENCE (Q1(1),QG(1)),(Q2(1),QG(53)),(Q3(1),QG(77)) QR- 160
DATA Q1                        QR- 170
$      /.288675134594812882E0,0.5E0,.43056815579702629E0, QR- 180
$ .17392742256872693E0,.16999052179242813E0,.32607257743127307E0, QR- 190
$ 0.48014492824876812E0,.50614268145188130E-1,.39033323870681337E0,QR- 200
$ .11119051722668724E0,.26276620495816449E0,.15685332293994364E0, QR- 210
$ .9171732124782490E-1,.18134189168918099E0,.48695326425858586E0, QR- 220
$ .3333567215434407E-1,.43253168334449225E0,.747256745752903E-1, QR- 230
$ .3397047841496122E0,.10954318125799102E0,.2166976970646236E0, QR- 240
$ .13463335915499818E0,.74437169490815605E-1,.1477611235737644E0, QR- 250

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$0.49078031712335963E0,.23587668193255914E-1,.45205862818523743E0, QR- 260
$.53469662997659215E-1,.38495133709715234E0,.8003916427167311E-1, QR- 270
$.29365897714330872E0,.10158371336153296E0,.18391574949909010E0, QR- 280
$.11674626826917740E0,.62616704255734458E-1,.12457352290670139E0, QR- 290
$.49470046749582497E0,.13576229705877047E-1,.47228751153661629E0, QR- 300
$.31126761969323946E-1,.43281560119391587E0,.47579255041246392E-1, QR- 310
$.37770220417750152E0,.62314485627766936E-1,.30893812220132187E0, QR- 320
$.7479799440828837E-1,.22900838882861369E0,.8457825969750127E-1, QR- 330
$.14080177538962946E0,.9130170752246179E-1,.47506254918818720E-1, QR- 340
$.9472530522753425E-1 / QR- 350
DATA Q2 QR- 360
$ / 0.49759360999851068E+0 , 0.6170614899935998E-2 , QR- 370
* 0.40736427798565475E+0 , 0.14265694314466832E-1 , QR- 380
* 0.46913727600136638E+0 , 0.22138719408709903E-1 , QR- 390
* 0.44320776350220052E+0 , 0.29649292457718890E-1 , QR- 400
* 0.41000099298695146E+0 , 0.36673240705540153E-1 , QR- 410
* 0.37006209578927718E+0 , 0.43095000/659/6638E-1 , QR- 420
* 0.32404682596848778E+0 , 0.48809326052056944E-1 , QR- 430
* 0.27271073569441977E+0 , 0.53722135057982817E-1 , QR- 440
* 0.21689675381302257E+0 , 0.57752634026862801E-1 , QR- 450
* 0.15752133984808169E+0 , 0.60835236463901696E-1 , QR- 460
* 0.95539433/368008150E-1 , 0.62918728173414148E-1 , QR- 470
* 0.32028446431302813E-1 , 0.63969097673376078E-1 /QR- 480
DATA Q3 QR- 490
$ /0.49863193092474078E+0 , 0.3509305004/350483E-2 , QR- 500
* 0.49280575577263417E+0 , 0.81371973654528350E-2 , QR- 510
* 0.48238112779375322E+0 , 0.12696032654631030E-1 , QR- 520
* 0.46745303790886984E+0 , 0.17136931455510717E-1 , QR- 530
* 0.44816057788302606E+0 , 0.2141794901.113340E-1 , QR- 540
* 0.42468380686628499E+0 , 0.25499029631180088E-1 , QR- 550
* 0.39724189798397120E+0 , 0.29342046739267774E-1 , QR- 560
* 0.36609105937014484E+0 , 0.32911111388180923E-1 , QR- 570
* 0.33152213346510760E+0 , 0.36172897054424253E-1 , QR- 580
* 0.29385/8/862038116E+0 , 0.39096947893535153E-1 , QR- 590
* 0.253449954466114/0E+0 , 0.416559621134/3378E-1 , QR- 600
* 0.21067563806531767E+0 , 0.43826046502201906E-1 , QR- 610
* 0.16593430114106382E+0 , 0.45506939347881942E-1 , QR- 620
* 0.11964368112606854E+0 , 0.46922199540402283E-1 , QR- 630
* 0.72235980791398250E-1 , 0.47819360039637430E-1 , QR- 640
* 0.24153832843869158E-1 , 0.48270044257363900E-1 /QR- 650
$,NQ/2,4,8,10,12,16,24,32/, QR- 660
$ NS/1,3,7,15,25,37,53,77/ QR- 670
DO 300 L=1,8 QR- 680
IF(N.EQ.NQ(L)) GO TO 301 QR- 690
300 CONTINUE QR- 700
WRITE(5,905) N QR- 710
905 FORMAT('0 CALLING PARAMETER =',I5,' INTEGRATION NOT POSSIBLE'//) QR- 720
RETURN QR- 730
301 CONTINUE QR- 740
NP=NS(L) QR- 750
NE=NP+N-1 QR- 760
AX=0.5*(XU+XL) QR- 770
BX=XU-XL QR- 780
CVAL=(0.,0.) QR- 790
CVALR=CMPLX(0.,0.) QR- 800
DO 350 J=NP,NE,2 QR- 810
DX=QG(J)*BX QR- 820
AXP=AX+DX QR- 830
AXM=AX-DX QR- 840
RSP=RSL+AXP*DEL*SINGS QR- 850
RSM=RSL+AXM*DEL*SINGS QR- 860

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CFP=CF(AXP)	QR-	870
CFM=CF(AXM)	QR-	880
CVAL=CVAL+QG(J+1)*(CFP+CFM)	QR-	890
CVALR=CVALR+QG(J+1)*(CFP*CMPLX(RSP,0.0)+CFM*CMPLX(RSM,0.0))	QR-	900
350 CONTINUE	QR-	910
CVAL=CVAL*BK	QR-	920
CVALR=CVALR*BK	QR-	930
RETURN	QR-	940
END	QR-	950

SUBROUTINE ICRMUL(CM,CV,CVS,NMROWS,NMCLS,NVROWS,NVCLS)	MM-	10
C*****	MM-	20
C	MM-	30
C PERFORMS [CVS] = [CM] X [CV]	MM-	40
C	MM-	50
C CM = INPUT MATRIX (NMROWS X NMCLS)	MM-	60
C CV = INPUT MATRIX (NVROWS X NVCLS)	MM-	70
C CVS = SOLUTION MATRIX (NMROWS X NVCLS)	MM-	80
C NMROWS = # OF ROWS IN [CM]	MM-	90
C NMCLS = # OF COLUMNS IN [CM]	MM-	100
C NVROWS = # OF ROWS IN [CV]	MM-	110
C NVCLS = # OF COLUMNS IN [CV]	MM-	120
C	MM-	130
C*****	MM-	140
C IMPLICIT COMPLEX (C)	MM-	150
C DIMENSION CV(NVROWS,NVCLS),CM(NMROWS,NMCLS),CVS(NMROWS,NVCLS)	MM-	160
C IF(NMCLS.NE.NVROWS) GO TO 40	MM-	170
C C0=CMPLX(0.0,0.0)	MM-	180
C DO 30 J=1,NVCLS	MM-	190
C DO 20 I=1,NMROWS	MM-	200
C CSUM=C0	MM-	210
C DO 10 K=1,NMCLS	MM-	220
C 10 CSUM=CSUM+CM(I,K)*CV(K,J)	MM-	230
C 20 CVS(I,J)=CSUM	MM-	240
C 30 CONTINUE	MM-	250
C RETURN	MM-	260
C 40 WRITE(5,10000) NMCLS,NVROWS	MM-	270
C 10000 FORMAT(' MULTIPLICATION NOT POSSIBLE.')	MM-	280
C \$' NMCLS = 'I,10X,' NVROWS = 'I)	MM-	290
C STOP	MM-	300
C END	MM-	310

SUBROUTINE BESSEL (X,N,BJ,BY,NX)	JY-	10
IMPLICIT REAL*8 (A-H,O-Z)	JY-	20
DIMENSION BJ(1),BY(1)	JY-	30
REAL*4 SNGL	JY-	40
D = 1. D=07	JY-	50
C	JY-	60
C CHECK FOR ERRORS IN N AND X	JY-	70
C	JY-	80
C IF (N) 710,720,720	JY-	90
710 WRITE (5,900)	JY-	100
900 FORMAT (30H N IS NEGATIVE)	JY-	110
GO TO 999	JY-	120
720 IF (X) 730,730,740	JY-	130

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730	WRITE (5,910)	JY-	140
910	FORMAT (30H X IS NEGATIVE OR ZERO)	JY-	150
	GO TO 999	JY-	160
740	IF (NX .EQ. 1) GO TO 30	JY-	170
	IF (NX .EQ. 2) GO TO 600	JY-	180
	IF (NX .EQ. 3) GO TO 30	JY-	190
	WRITE (5,920)	JY-	200
920	FORMAT (30H ERROR IN PARAMETER NX)	JY-	210
999	WRITE (5,930) X,N,NX	JY-	220
930	FORMAT (30H PARAMETER VALUES X,N,NX ,G15.5,215)	JY-	230
	STOP	JY-	240
C		JY-	250
C	CALCULATION OF J BESSEL FUNCTION	JY-	260
C		JY-	270
30	L=N	JY-	280
31	IF(X-15.)32,32,34	JY-	290
32	NTEST=20.+10.*X-X** 2. / 3.	JY-	300
	GO TO 36	JY-	310
34	NTEST=90.+X/2.	JY-	320
36	IF(L-NTEST)40,38,38	JY-	330
38	WRITE (5,940)	JY-	340
940	FORMAT (30H RANGE OF X VIOLATED)	JY-	350
	GO TO 999	JY-	360
40	N1 = L + 1	JY-	370
	BJ(N1)=0.0	JY-	380
	BPREV=.0	JY-	390
C		JY-	400
C	COMPUTE STARTING VALUE OF M	JY-	410
C		JY-	420
	IF(X-5.)50,60,60	JY-	430
50	MA=X+6.	JY-	440
	GO TO 70	JY-	450
60	MA=1.4*X+60./X	JY-	460
70	MB = L + 1FIX (SNGL (X))/4 +2	JY-	470
	MZERO=MAX0(MA,MB)	JY-	480
C		JY-	490
C	SET UPE\9L\\\TOO6MM	JY-	500
C		JY-	510
	MMA=NTEST	JY-	520
100	DO 190 M=MZERO,MMA,3	JY-	530
C		JY-	540
C	SET F(M),F(M-1)	JY-	550
C		JY-	560
	FMI=1.0E-28	JY-	570
	FM=.0	JY-	580
	ALPHA=.0	JY-	590
	IF(M-(M/2)*2)120,110,120	JY-	600
110	JT=-1	JY-	610
	GO TO 130	JY-	620
120	JT=1	JY-	630
130	M2=M-2	JY-	640
	DO 160 K=1,M2	JY-	650
	MK=M-K	JY-	660
	BMK = 2.*DFLOAT(MK)*FMI/X-FM	JY-	670
	FM=FMI	JY-	680
	FMI=BMK	JY-	690
	IF(MK-L-1)150,140,150	JY-	700
140	BJ(L+1)=BMK	JY-	710
150	JT=-JT	JY-	720
	S=1+JT	JY-	730
160	ALPHA=ALPHA+BMK*S	JY-	740

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BMK=2.*FMI/X-FM	JY-	751
IF(L)180,170,180	JY-	760
170 BJ(L+1)=BMK	JY-	770
180 ALPHA=ALPHA+BMK	JY-	780
BJ(L+1)=BJ(L+1)/ALPHA	JY-	790
ERR=BJ(L+1)-BPREV	JY-	800
IF(DABS(ERR)-DABS(D*BJ(L+1)))200,200,185	JY-	810
185 BPREV=BJ(L+1)	JY-	820
190 ERR=ERR/BJ(L+1)*100.	JY-	830
WRITE (5,950) ERR	JY-	840
950 FORMAT(59H REQUIRED ACCURACY IN BJN NOT OBTAINED, PER CENT DIFF.	JY-	850
11S ,G20.6)	JY-	860
200 IF(L-N)210,220,220	JY-	870
220 L=L-1	JY-	880
IF(L.LT. 0)GO TO 240	JY-	890
GO TO 31	JY-	900
210 IF(L.EQ. 0)GO TO 240	JY-	910
DO 230 I=2,N	JY-	920
I=N-I	JY-	930
230 BJ(L+1)=2.*DFLOAT(L+1)*BJ(L+2)/X-BJ(L+3)	JY-	940
240 IF (NX.EQ. 3) GO TO 600	JY-	950
RETURN	JY-	960
C	JY-	970
C CALCULATION OF Y BESSEL FUNCTION	JY-	980
C	JY-	990
600 PI = 3.141592653	JY-	1000
C	JY-	1010
C BRANCH IF X LESS THAN OR EQUAL 4	JY-	1020
C	JY-	1030
IF (X-4.) 640,640,630	JY-	1040
C	JY-	1050
C COMPUTE Y0 AND Y1 FOR X GREATER THAN 4	JY-	1060
C	JY-	1070
C	JY-	1080
630 T=4./X	JY-	1090
P0=.3989422793	JY-	1100
Q0=-.0124669441	JY-	1110
P1=.3989422819	JY-	1120
Q1=.0374228364	JY-	1130
A=T*T	JY-	1140
B=A	JY-	1150
P0=P0-.0017530620*A	JY-	1160
Q0=Q0+.0004564324*A	JY-	1170
P1=P1+.029218256*A	JY-	1180
Q1=Q1-.00063904*A	JY-	1190
A=A*A	JY-	1200
P0=P0+.00017343*A	JY-	1210
Q0=Q0-.0000869791*A	JY-	1220
P1=P1-.000223203*A	JY-	1230
Q1=Q1+.0001064741*A	JY-	1240
A=A*B	JY-	1250
P0=P0-.0000487613*A	JY-	1260
Q0=Q0+.0000342468*A	JY-	1270
P1=P1+.0000580759*A	JY-	1280
Q1=Q1-.0000398708*A	JY-	1290
A=A*B	JY-	1300
P0=P0+.0000173565*A	JY-	1310
Q0=Q0-.0000142078*A	JY-	1320
P1=P1-.000020092*A	JY-	1330
Q1=Q1+.00001622*A	JY-	1340
A=A*B	JY-	1350
P0=P0-.0000037043*A		

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Q0=Q0+.0000032312*A	JY- 1360
P1=P1+.0000042414*A	JY- 1370
Q1=Q1-.0000036594*A	JY- 1380
A = DSQRT(2.*PI)	JY- 1390
B=4.*A	JY- 1400
P0=A*P0	JY- 1410
Q0=B*Q0/X	JY- 1420
P1=A*P1	JY- 1430
Q1=B*Q1/X	JY- 1440
A=X-PI/4.	JY- 1450
B = DSQRT (2./(PI*X))	JY- 1460
Y0 = B*(P0*DSIN(A)+Q0*DCOS(A))	JY- 1470
Y1 = B*(-P1*DCOS(A)+Q1*DSIN(A))	JY- 1480
GO TO 690	JY- 1490
C	JY- 1500
C	JY- 1510
C	JY- 1520
640	JY- 1530
XX=X/2.	JY- 1540
X2=XX*XX	JY- 1550
T=DLOG(XX)+.5772156649	JY- 1560
SUM=0.	JY- 1570
TERM=T	JY- 1580
Y0=T	JY- 1590
DO 670 L = 1,15	JY- 1600
IF(L-1) 650,660,650	JY- 1610
650	JY- 1620
SUM=SUM+1./DFLOAT(L-1)	JY- 1630
660	JY- 1640
FL=L	JY- 1650
TS=T-SUM	JY- 1660
TERM=(TERM*(-X2)/FL**2)*(1.-1./(FL*TS))	JY- 1670
670	JY- 1680
Y0=Y0+TERM	JY- 1690
TERM = XX*(T-.5)	JY- 1700
SUM=0.	JY- 1710
Y1=TERM	JY- 1720
DO 680 L = 2,16	JY- 1730
SUM=SUM+1./DFLOAT(L-1)	JY- 1740
FL=L	JY- 1750
FL1=FL-1.	JY- 1760
TS=T-SUM	JY- 1770
TERM=(TERM*(-X2)/(FL1*FL))*(TS-.5/FL)/(TS+.5/FL1)	JY- 1780
680	JY- 1790
Y1=Y1+TERM	JY- 1800
PI2=2./PI	JY- 1810
Y0=PI2*Y0	JY- 1820
Y1=-PI2/X+PI2*Y1	JY- 1830
C	JY- 1840
C	JY- 1850
C	JY- 1860
690	JY- 1870
IF(N-1) 500,500,530	JY- 1880
C	JY- 1890
C	JY- 1900
C	JY- 1910
500	JY- 1920
IF(N) 510,520,510	JY- 1930
510	JY- 1940
BY(2)=Y1	JY- 1950
520	JY- 1960
BY(1)=Y0	
RETURN	
C	
C	
C	
530	
BY(1)=Y0	
BY(2)=Y1	
DO 545 K=2,N	
T=DFLOAT(2*(K-1))/X	

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C	BY(K+1)=T*BY(K)-BY(K-1)	JY- 1970
C	BESSEL Y FUNCTION HAS EXCEEDED 10**35	JY- 1980
C		JY- 1990
C	IF (DABS(BY(K+1))-1.0D35) 545,545,541	JY- 2000
541	WRITE (5,980)	JY- 2010
980	FORMAT (30H Y BESSEL FCN EXCEEDED 10**35)	JY- 2020
	GO TO 999	JY- 2030
545	CONTINUE	JY- 2040
	RETURN	JY- 2050
	END	JY- 2060

	FUNCTION ELICIK(AM1)	CE- 10
C	*****	CE- 20
C		CE- 30
C	COMPLETE ELLIPTIC INTEGRAL OF THE FIRST KIND K(M),	CE- 40
C	WHERE AM1=1-M	CE- 50
C		CE- 60
C	REFERENCE:	CE- 70
C		CE- 80
C	ABRAMOWITZ AND STEGUN, EQ. 17.3.34	CE- 90
C		CE- 100
C	MAGNITUDE(ERROR) .LE. 2.0E-8	CE- 110
C		CE- 120
C	*****	CE- 130
	DATA A0,A1,A2,A3,A4,B0,B1,B2,B3,B4/	CE- 140
\$	1.38629436112, .09666344259, .03590092383, .03742563713,	CE- 150
\$	.01451196212, .5, .12498593597, .06880248576,	CE- 160
\$	.03328355346, .00441787012/	CE- 170
	A=A0+A1*AM1	CE- 180
	B=B0+B1*AM1	CE- 190
	IF(AM1 .LT. 1.E-18) GO TO 10	CE- 200
	AM12=AM1*AM1	CE- 210
	A=A+A2*AM12	CE- 220
	B=B+B2*AM12	CE- 230
	IF(AM1 .LT. 1.E-12) GO TO 10	CE- 240
	AM13=AM12*AM1	CE- 250
	A=A+A3*AM13	CE- 260
	B=B+B3*AM13	CE- 270
	IF(AM1 .LT. 1.E-9) GO TO 10	CE- 280
	AM14=AM13*AM1	CE- 290
	A=A+A4*AM14	CE- 300
	B=B+B4*AM14	CE- 310
10	CONTINUE	CE- 320
	ELICIK=A-B*ALOG(AM1)	CE- 330
	RETURN	CE- 340
	END	CE- 350

	SUBROUTINE TRAPDF(CF,XLOW,XHIGH,ER,MAXP,IER,CANS)	AT- 10
C	*****	AT- 20
C		AT- 30
C	INTEGRATION OF A FUNCTION BY ADAPTIVE TRAPEZOIDAL RULE	AT- 40
C		AT- 50
C		AT- 60
C	CF = EXTERNALLY SUPPLIED COMPLEX FUNCTION TO BE INTEGRATED	AT- 70
C		AT- 80
C	XLOW = LOWER LIMIT OF INTEGRATION	AT- 90

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C		AT-	100
C	XHIGH = ((UPPER LIMIT OF INTEGRATION MINUS LOWER LIMIT) DIVIDED	AT-	110
C	BY TWO) + LOWER LIMIT	AT-	120
C		AT-	130
C	ER = RELATIVE ERROR CRITERION FOR CONVERGENCE CHECK	AT-	140
C		AT-	150
C	MAXP = MAXIMUM NUMBER OF POINTS TO BE USED	AT-	160
C		AT-	170
C	IER = ERROR CODE:	AT-	180
C	= 1, IF INTEGRATION DID NOT CONVERGE WITHIN (MAXP)	AT-	190
C	POINTS	AT-	200
C	= 0, OTHERWISE	AT-	210
C		AT-	220
C	CANS = RESULT OF THE INTEGRATION	AT-	230
C		AT-	240
C		AT-	250
C	CF MUST BE LISTED IN AN EXTERNAL STATEMENT IN THE	AT-	260
C	CALLING PROGRAM.	AT-	270
C		AT-	280
C	THE RESULT (CANS) IS INTEGRAL(CF) BETWEEN THE LIMITS (XLOW) AND	AT-	290
C	(XLOW+(XHIGH-XLOW)*2). SYMMETRY OF THE FUNCTION (CF) IS ASSUMED	AT-	300
C	ABOUT THE POINT (XHIGH).	AT-	310
C		AT-	320
C	*****	AT-	330
	IMPLICIT COMPLEX(C)	AT-	340
	COMMON/NPOINT/NP	AT-	350
	IER=0	AT-	360
	DELO=XHIGH-XLOW	AT-	370
	NPOLD=2	AT-	380
	C2=CMPLX(2.0,0.0)	AT-	390
	IF(MAXP.EQ.0) MAXP=10000	AT-	400
	IF(ER.EQ.0) ER=1.E-4	AT-	410
	CDELO=CMPLX(DELO,0.0)	AT-	420
	COLD=(CF(XLOW)+CF(XHIGH))/C2	AT-	430
	C=COLD	AT-	440
10	NP=NPOLD+NPOLD-1	AT-	450
	DEL=DELO/2.0	AT-	460
	CDEL=CMPLX(DEL,0.0)	AT-	470
	K=NP-NPOLD	AT-	480
	K=K+K	AT-	490
	DO 20 J=2,K,2	AT-	500
	X=XLOW+DEL*FLOAT(J-1)	AT-	510
20	C=C+CF(X)	AT-	520
	CO=COLD*CDELO	AT-	530
	CN=C*CDEL	AT-	540
	CND=CN	AT-	550
	ABSND=CABS(CND)	AT-	560
	IF(ABSND.LT.1.E-20) ABSND=1.E-20	AT-	570
	DIFF=CABS(CN-CO)	AT-	580
	ERR=DIFF/ABSND	AT-	590
	IF(ERR.LT.ER) GO TO 40	AT-	600
	IF(NP.GE.MAXP) GO TO 30	AT-	610
	DELO=DEL	AT-	620
	CDELO=CDEL	AT-	630
	NPOLD=NP	AT-	640
	COLD=C	AT-	650
	GO TO 10	AT-	660
30	IER=1	AT-	670
40	CANS=CN*C2	AT-	680
	RETURN	AT-	690
	END	AT-	700

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