

AD-A067 554

IBM THOMAS J WATSON RESEARCH CENTER YORKTOWN HEIGHTS N Y F/G 12/1
TOPICS IN MATHEMATICAL PROGRAMMING. RESEARCH IN NONDIFFERENTIAB--ETC(U)
NOV 78 E L JOHNSON, P S WOLFE F49620-77-C-0014

UNCLASSIFIED

AFOSR-TR-79-0420

NL

1 OF 1
ADA
067554



END
DATE
FILMED

6-79
DDC

AFOSR-TR. 79-0420

LEVEL

12

TOPICS IN MATHEMATICAL PROGRAMMING

Research in nondifferentiable and discrete optimization

ADA067554

E.L. Johnson and P. Wolfe
Mathematical Sciences Department
IBM Thomas J. Watson Research Center
P. O. Box 218
Yorktown Heights, NY 10598

November 30, 1978

DDC
RECEIVED
APR 17 1979
C

DDC FILE COPY

Final Technical Report on Contract F49620-77-C-0014,
1 October 1976 - 30 September 1978

Approved for public release, distribution unlimited

Prepared for
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
AFSC, United States Air Force
Bolling AFB, DC 20332

79 04 13 004

Approved for public release;
distribution unlimited.

20. Abstract continued.

("relaxation") methods for solving large systems of linear inequalities. (4) The subadditive characterization of facets of integer programming polyhedra has been extended to a very general class of pure integer problems. (5) Work has continued on use of subadditive functions to give a satisfactory duality theory for integer programming, to provide pricing information, and eventually to solve problems. (6) The theories of blocking pairs of polyhedra and anti-blocking pairs of polyhedra have been extended, and we have characterized pairs of polyhedra which are, respectively, the blocker and anti-blocker of some unspecified third polyhedron.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR-79-0420	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) TOPICS IN MATHEMATICAL PROGRAMMING RESEARCH IN NONDIFFERENTIABLE AND DISCRETE OPTIMIZATION	5. TYPE OF REPORT & PERIOD COVERED Final rept.	
7. AUTHOR(s) E.L. Johnson and P.S. Wolfe	6. PERFORMING ORG. REPORT NUMBER 1 Oct 76 - 30 Sep 78	
	8. CONTRACT OR GRANT NUMBER(s) F49620-77-C-0014	
9. PERFORMING ORGANIZATION NAME AND ADDRESS International Business Machines P.O. Box 218 Yorktown Heights, NY 10598	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304 A6	
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM Bolling AFB, Washington, DC 20332	12. REPORT DATE Nov 1978	
	13. NUMBER OF PAGES 9	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 1211p.	13. SECURITY CLASS. (of this report) UNCLASSIFIED	
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE		
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Nondifferentiable optimization Integer programming Discrete optimization Subadditivity Duality		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Activity and research in Nondifferentiable Optimization (NDO) and Discrete Optimization are described. (1) Conjugate descent meth- ods have been extended to constrained NDO problems. (2) A robust and efficient algorithm for the minimization of an arbitrary convex univariate function has been devised. (3) Substantial improvements have been made to subgradient optimization		

TOPICS IN MATHEMATICAL PROGRAMMING

Research in nondifferentiable and discrete optimization

**E.L. Johnson and P. Wolfe
Mathematical Sciences Department
IBM Thomas J. Watson Research Center
P. O. Box 218
Yorktown Heights, NY 10598**

November 30, 1978

ACCESSION for	
NTIS	Write Section ☒
BDC	Buff Section ☐
UNANNOUNCED	☐
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
SPECIAL	
A	

**Final Technical Report on Contract F49620-77-C-0014,
1 October 1976 - 30 September 1978**

Approved for public release, distribution unlimited

**Prepared for
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
AFSC, United States Air Force
Bolling AFB, DC 20332**

**AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DDC
This technical report has been reviewed and is
approved for public release IAW AFR 190-12 (7b).
Distribution is unlimited.
A. D. BLOSE
Technical Information Office**

79 04 13 004

PREFACE

The "topics in mathematical programming" that constituted our research under this contract are those of Nondifferentiable Optimization (NDO) and Integer Programming, the specialties of the two principal investigators. They are dealt with below under separate headings.

RESEARCH IN NONDIFFERENTIABLE OPTIMIZATION

1. The IIASA Meeting

A meeting, titled "Task Force on Nondifferentiable Optimization", was held at the International Institute for Applied Systems Analysis (IIASA) in Laxenburg, Austria, on March 28 - April 8, 1977. It had its origins in the identification of the subject of Nondifferentiable Optimization (NDO) through the publication of the volume Nondifferentiable Optimization¹ and was organized by C. Lemarechal and R. Mifflin of IIASA, with the counsel of Balinski and Wolfe. In the format of a small workshop, it was attended by R. Fletcher (U.K.), J. Gauvin (Canada), J.-L. Goffin (Canada), Lemarechal, R. Marsten (USA), Mifflin, B.T. Polyak (USSR), B.N. Pschenichnyi (USSR), and Wolfe. Each day was devoted to the work of one of the attendees, who presented it in the form of a lecture and then discussed it in detail with the group. We all found the exchange of great value. Of particular interest to our own work was Marsten's detailed report on his "Boxstep" method, Polyak's summary of the work of N.Z. Shor (the earliest user of subgradient optimization in real optimization problems), and Lemarechal's report on computational experience with subgradient optimization, Wolfe's and his own versions of "conjugate descent", and Shor's recent "dilatation" method. Lemarechal reported that on the several problems tried the methods ranked in efficiency in the order just listed. (His observation is supported by our own experiments.) and indicate what progress we made in them.

2. Identification of conjugate descent methods

Lemarechal and Wolfe, two closely related methods which can be viewed as extensions of the method of conjugate gradients (used for the minimization of smooth functions) to nondifferentiable functions. Both procedures make essential use of the accumulation of a "bundle" of previously calculated subgradients which is

¹ M.L. Balinski and P. Wolfe. Nondifferentiable Optimization. Mathematical Programming Study 3. North-Holland Publishing Company, Amsterdam, 1976.

used to determine a direction of descent. The methods differ in the ordering of certain "resetting" steps that both algorithms perform. We wrote a set of computer routines in which such variants can readily be tried, and found almost no difference in performance between the two methods when they were used in the same context. Henceforth we can speak with some justification of the conjugate descent method as a general procedure (involving conjugation via a "nearest point calculation", and a line search) of which the particular schemes mentioned -- and others -- are variant implementations.

3. Constraints in conjugate descent

We devised a means for using conjugate descent for constrained optimization problems. For the problem

$$\text{Min } f(x) : g(x) \leq 0$$

(we take a single constraint for simplicity), we minimize

$$F(x) = f(x) + K \text{ Max } 0, g(x)$$

for sufficiently large K . If the point x is on the boundary, and b, c are respective subgradients of f, g there, then $b + Kc$ is a subgradient of F . We wish, of course, to make use of such a subgradient when $-g$ itself is not a feasible direction. The new procedure is based on the observation that if both b and $b + Kc$ are members of the bundle of subgradients used to define the new descent direction at x then for sufficiently large K that direction will be the projection of $-b$ onto the tangent plane to the constraint; in our view, the most desirable outcome. The idea works for any number of constraints. We have tested it on some small-scale problems with linear constraints, and it seems to be quite effective. The report "Constraints in Conjugate Descent" describing the procedure is in preparation.

4. Univariate optimization

Finding the minimum of a convex function of a single variable (to a specified degree of approximation) is an essential subtask of an efficient conjugate descent procedure. Our procedure for the piecewise-linear convex function has been further polished, exercised on a variety of problems, and seems to be foolproof. (It was the subject of IBM Invention Disclosure Y08-77-0033, January, 1977.) It has been extended to general convex functions using some of the ideas of our previous algorithm for smooth functions, as reported in "Minimization of nonsmooth univariate functions" (see Publications.) The procedure is extraordinarily robust and, we think, efficient, although there is no question that the piecewise-linear minimizer should be used when a problem is known to have that character.

5. Benchmark problems

In collaboration with Lemarechal and Mifflin (while they were at IIASA) and Goffin at the University of Montreal we have defined a set of four (at the moment) test problems that we agree are representative and important and will serve as a basis for comparing results. They will be maintained in machine-readable form and are available to anyone.

6. The solution of large systems of linear inequalities

The 1957 work of Agmon and of Motzkin and Schoenberg on "relaxation" methods for the solution of linear inequality systems was the first application of what we now call "subgradient optimization" to mathematical programming. It did not have much impact at the time because the simplex method proved more effective for the small problems of that era. Our first work specifically aimed at NDO problems reestablished the importance of that approach for large-scale optimization generally; subsequently, a number of important applications emerged which can be modeled as requiring the approximate solution of systems of many thousands of (very sparse) linear inequalities in many thousands of variables: electron-beam photolithography and x-ray tomography are two such applications we have worked on. Such problems seriously tax, or even exceed, the capabilities of any current implementation of the simplex method, which further can benefit but little from the fact that in many cases only approximate solutions are needed. Our work has been directed both at seeking to understand what problem features are important for successful use of the method and at improving on standard implementations. We have experimented with (1) randomly generated problems of a certain type, (2) the constraint sets defined by some standard small and medium-size linear programming problems, and (3) some small models of the E-beam photolithography problem. Quite unexpectedly, for random problems with a given number of variables, the method converged more rapidly for problems with many inequalities than for few. We now think we can explain that: the degree to which the set defined by the inequalities "approximates" a sphere increases with the number of inequalities, and the method can be shown to solve such "spherical" problems with nearly perfect efficiency. We have devised a simple estimation procedure which can eliminate the need for calculating many of the inequalities in some large systems (this is the subject of IBM Invention Disclosure Y08-77-0202, April, 1977; a research report on this is in preparation). We have also proved the feasibility of a certain method of efficiently handling inequalities in small "blocks", making use of our very quick algorithm for finding the nearest point in a given polyhedron to a given external point. In our test

M. Held, P. Wolfe, and H.P. Crowder: "Validation of Subgradient Optimization", Mathematical Programming 6 (1974), 62-88)
P. Wolfe. Finding the Nearest Point in a Polytope. Mathematical Programming 11 (1976) 128-149.

problems these techniques have shown an order-of-magnitude improvement in computing time as compared with our implementation of "standard" methods.

7. Bibliography

We have continued the maintenance of a Bibliography of papers related to NDO which now runs to some 260 items.

Summary of the two-year effort in NDO

We began our work concentrating on the exploitation of conjugate descent methods, which at the time seem to hold the greatest promise for efficient solution of general NDO problems, and we feel that their success has been demonstrated for problems of middling size when calculation of the function being minimized is expensive.

Unfortunately, the amount of work per step required by such methods increases at least linearly with the number of variables in the problem, and they do not seem feasible for the very interesting problems mentioned above involving thousands of variables. Subgradient optimization shows little dependence on number of variables; its speed of convergence depends much more on specific features of the problem we are just beginning to understand. We do know, though, that the calculations required for it can be performed with great economy in the typical case that the data of the problem are very sparse. Our work so far has shown that large improvements in the procedure can be made using rather simple devices, and we think that there is a great deal more along that line to exploit.

RESEARCH IN INTEGER PROGRAMMING

1. Subadditive Functions for Describing Facets of Additive Systems

The thrust of the work here has been to obtain a central theory offering some description of polyhedra for pure integer programming problems. The subadditive function characterization of facets has been extended to a very general combinatorial optimization problem involving an additive system. The paper, "On the Generality of the Subadditive Characterization of Facets", describes this work. The theory encompasses, in the same framework, antiblocking (or packing) type problems, as well as blocking, so that there is no necessity, as in Araoz, for a separate development employing superadditive functions.

The increased generality in treating semigroups not only takes us away from the group relaxation in allowing us to directly represent integer programming problems, but allows us to represent problems of a noncommutative nature without having to write down a linear programming relaxation, which may be awkward.

This work also raises a whole set of questions as to what results from Gomory's original group paper carry over to this more general framework. The mapping notion there, for deriving facets of a problem from its subgroups, can be generalized to give results relating facets for different additive systems. For example, for a non-Abelian finite group, simply requiring commutativity as a relation gives a mapping T on an Abelian group such that $T(g+h) = T(g)+T(h)$ for all g, h in the non-Abelian group. Thus, facets for the Abelian group give valid inequalities and, in many cases, facets, for the non-Abelian problem.

2. Algorithmic implications

The theory developed provides a framework into which computational methods can be placed. For one special case, the knapsack problem, we have indicated how traditional methods fit into this framework. However, the "lifting methods", using subadditive functions, open up many new algorithmic directions. In addition, a reasonable satisfactory duality theory is finally provided for integer programs. This duality theory is based on linear programming duality and provides optimality criteria for an integer primal solution and a dual subadditive function.

As far as solving problems, the effort has been redirected toward finding suitable subadditive functions to apply directly to the original problem. Two classes of functions have been identified as being potentially useful for solving the knapsack problem. The paper "Subadditive Methods for the Knapsack Problem" discusses these functions and relates our methods to previous methods.

3. The NSF-CBMS Regional Conference

A series of ten lectures was presented at an NSF-CBMS regional conference at the State University of New York in Buffalo in June 1978. The lecture notes from that conference, "Integer Programming: Facets, Subadditivity, and Duality for Group and Semigroup Problems" will appear in the SIAM series. The lectures detailed the extension of the subadditive function approach to semigroup problems and related them to the blocking pair theory of Fulkerson.

4. Mixed Integer Programming

The extension of the subadditive approach to the mixed problem should be possible once the gains made on the pure problem have been consolidated and put to algorithmic use.

Summary of the two-year effort in integer programming

In combinatorial polyhedra, the main results are in the paper "Support functions, blocking pairs, and antiblocking pairs". There, a unified framework is provided allowing many more combinatorial optimization problems to be viewed as either blocking pairs or anti-blocking pairs. A main result is necessary and sufficient conditions for two polyhedra to be, respectively, the blocker and antiblocker of some given polyhedron.

In mixed integer programming, the main results are in the paper "Duality and pricing in multiple choice right-hand side problems". There, subadditive functions are shown to provide an adequate dual problem but not complete pricing information. Computational efforts for the mixed problem have been delayed by new results on the pure problem as outlined below.

The subadditive function approach for pure problems has been pushed forward in two directions. First, it has been seen to be applicable to a wide variety of problems, even non-Abelian and nonassociative addition systems. Secondly, some new functions have been developed in order to directly attack pure integer problems without going through the linear programming relaxation. This work has been detailed for the knapsack problem.

PUBLICATIONS

E.L. Johnson. Support functions, blocking pairs, and anti-blocking pairs. Mathematical Programming Studies 8 (1978), M.L. Balinski and A.J. Hoffman (eds.), 167-196.

E.L. Johnson. On the group problem and a subadditive approach to integer programming. IBM Research Center Report RC-6815, October, 1977. To appear in Annals of Discrete Mathematics 3, Discrete Optimization, P.L. Hammer, E.L. Johnson, and B. Korte (eds.).

E.L. Johnson. Integer programming: Facets, Subadditivity, and Duality for Group and Semi-group Problems. IBMRC-7450, December, 1978. To appear in SIAM series on Regional Conferences.

E.L. Johnson. On the Generality of the Subadditive Characterization of Facets. IBMRC-7521, February, 1979. Submitted to Mathematics of Operations Research.

D. Granot, F. Granot, and E.L. Johnson. Duality and Pricing in Multiple Right-hand Side Linear Programming Problems.

P. Wolfe. Minimization of nonsmooth univariate functions. Proceedings of the IIASA Workshop on Nondifferentiable Optimization, March-April 1977, Laxenburg, Austria, C. Lemarechal and R. Mifflin (eds.).

(The two following items are invention disclosures)

H.P. Crowder, E.L. Johnson, and P. Wolfe. Minimization of piecewise linear functions. IBM Technical Disclosure Bulletin 20 (1977), 842-845.

H.P. Crowder, E.L. Johnson, and P. Wolfe. Estimation of inequalities in optimization. IBM Technical Disclosure Bulletin 20 (1977), 2085-88.