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AIR FORCE MATERIALS LAB WRIGHT-PATTERSON AFB OH
SR-52 MAGNETIC CARD CALCULATOR SOLUTIONS TO COMPOSITE MATERIALS--ETC(U)
JAN 79 S W TSAI, H T HAHN, F HUBER

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**SR-52
MAGNETIC CARD CALCULATOR SOLUTIONS TO
COMPOSITE MATERIALS WORKBOOK**

*MECHANICS AND SURFACE INTERACTIONS BRANCH
NONMETALLIC MATERIALS DIVISION*

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JANUARY 1979

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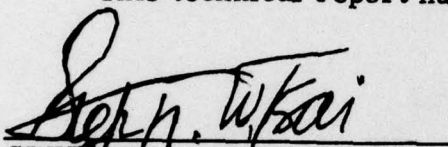
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This technical report has been reviewed and is approved for publication.


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Project Scientist

FOR THE COMMANDER


JEROME M. KELBLE, Chief
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This is a companion volume to AFML-TR-78-33, Composite Materials Workbook. Both workbook and its solutions are intended to present to the users of composite materials a set of tools to solve most commonly encounter- ed problems in design and testing. Attempts are made to simplify both the operational and conceptual aspects. Subjects are selected from the standpoint of practical application rather than elegance.		

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20. ABSTRACT (continued)

Programmable pocket calculators, preferably with magnetic card capability, are found to be most suitable to perform the calculations in the workbook. In this report, the description, operating instructions, program listing, and sample problems for these calculations have been compiled for Texas Instruments SR-52.

It is believed that composite materials are simple in concept and easy to describe quantitatively with the aid of card programmable calculators. The performance characteristics of composite materials can now be fully appreciated and utilized.

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FOREWORD

This report was prepared in the Mechanics and Surface Interactions Branch (AFML/MBM), Nonmetallic Materials Division, Air Force Materials Laboratory, Wright-Patterson AFB, Ohio. The work was performed under the joint support of Project No. 2419 "Nonmetallic Structural Materials," Task No. 241903 "Composite Materials and Mechanics Technology," and Project No. 2307 "Aerospace Sciences," Task No. 2307P1 "Life Analysis and Failure Mechanics in Engine and Airframe Structural Metals and Composites." The time period covered by the effort was 1 January 1976 to 1 March 1977. Stephen W. Tsai (AFML/MBM) was the laboratory project engineer.

Same numbers for equations and figures are used in this report as those in the AFML-TR-78-33, "Composite Materials Workbook," in order to facilitate cross referencing. Program card numbers are assigned the page number on which the instructions for the program card appear.

The technical assistance from Texas Instruments Incorporated during the preparation of this report is gratefully acknowledged. Standard production cards by Texas Instruments relevant to this report and the Workbook are listed in the Appendix.

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TITLE: SI/ENGINEERING CONVERSION				NO: 001	
A' Pa ~ kgf/m ²	B' J ~ ft lbf	C' J ~ cal	D' kgm ⁻³ ~ lb/yd ³	E' ms ⁻¹ ~ ft/sec	
A MPa ~ ksi	B MNm ⁻¹ ~ kip/in	C N ~ lbf	D kW ~ Hp	E MPam ^{1/2} ~ ksi ^{1/2} in	

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY	
SI → Eng Set angular mode switch to D (degree)	1	001 A&B		A	psi	
	2	Q _{ij} , σ _i	kPa	A	ksi	
			MPa	A	10 ⁶ psi	
			GPa	A	kgf/m ²	
			Pa	A'	kgf/mm ²	
			MPa	A'	kip/in	
			MNm ⁻¹	B	kgf/m	
			Nm ⁻¹	A'	lbf	
			N	C	kip	
			kN	C	kgf	
N			A'	ft lbf		
SI ← Eng Set angular mode switch to R (radian)	3	Q _{ij} , σ _i	D _{ij} , J	Nm	B'	kgf/m
				Nm	A'	kPa
				psi	A	MPa
				ksi	A	GPa
				10 ⁶ psi	A	Pa
				kgf/m ²	A'	MPa
				kgf/mm ²	A'	MNm ⁻¹
				kip/in	B	Nm ⁻¹
				kgf/m	A'	N
				lbf	C	kN
		B _{ij} , M _i	kip	C	N	
			kgf	A'	kN	
			ft lbf	B'	Nm	
			kgf m	A'	Nm	

PROGRAM TITLE: SI/ENGINEERING CONVERSION

NO: 001

PROGRAM LISTING:

000	46	045	51	090	03	135	95
001	11	046	89	091	07	136	81
002	42	047	01	092	06	137	46
003	00	048	93	093	95	138	89
004	00	049	03	094	81	139	59
005	51	050	04	095	46	140	32
006	89	051	01	096	18	141	90
007	93	052	95	097	42	142	87
008	01	053	81	098	00	143	43
009	04	054	46	099	00	144	00
010	05	055	15	100	51	145	00
011	95	056	42	101	89	146	65
012	81	057	00	102	93	147	56
013	46	058	00	103	02	148	46
014	12	059	51	104	03	149	87
015	42	060	89	105	08	150	43
016	00	061	93	106	09	151	00
017	00	062	09	107	95	152	00
018	51	063	00	108	81	153	55
019	89	064	09	109	46	154	56
020	05	065	08	110	19		
021	93	066	95	111	42		
022	07	067	81	112	00		
023	01	068	46	113	00		
024	95	069	16	114	51		
025	81	070	42	115	89		
026	46	071	00	116	01		
027	13	072	00	117	93		
028	42	073	51	118	06		
029	00	074	89	119	08		
030	00	075	93	120	06		
031	51	076	01	121	95		
032	89	077	00	122	81		
033	93	078	02	123	46		
034	02	079	95	124	10		
035	02	080	81	125	42		
036	04	081	46	126	00		
037	08	082	17	127	00		
038	95	083	42	128	51		
039	81	084	00	129	89		
040	46	085	00	130	03		
041	14	086	51	131	93		
042	42	087	89	132	02		
043	00	088	93	133	08		
044	00	089	07	134	01		
045	51	090	03	135	95		

TITLE: VOID CONTENT				NO: 003	
A' v_v	B' v_f'	C' v_f	D'	E'	
A ρ_f	B ρ_m	C m_f	D m_m	E ρ	

00	<u>REGISTER</u>					
01 ρ_f	02 ρ_m	03 m_f	04 m_m	05 ρ	06 v_v	
07 v_f'	08	09	10			
11	12	13	14	15	16	
17	18	19				
98	99	<u>INSTRUCTIONS</u>				

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Data	1	003 A&B	ρ_f	A	ρ_f
			ρ_m	B	ρ_m
			m_f	C	m_f
			m_m	D	m_m
			ρ	E	ρ
Compute	2			A'	v_v
				B'	v_f'
				C'	v_f

FORMULAS:

$m_{f,m}$: mass fraction of fiber, matrix

$\rho_{f,m}$: density of fiber, matrix

a. Void content

$$v_v = 1 - \rho (m_f/\rho_f + m_m/\rho_m)$$

b. Apparent fiber volume fraction ($v_v = 0$)

$$v'_f = \frac{1}{1 + \frac{m_m}{m_f} \frac{\rho_f}{\rho_m}}$$

c. True fiber volume fraction

$$v_f = (1 - v_v)v'_f$$

PROGRAM TITLE: VOID CONTENT

NO: 003

PROGRAM LISTING:

000	46	045	01	090	53
001	11	046	85	091	01
002	42	047	43	092	75
003	00	048	00	093	43
004	01	049	04	094	00
005	81	050	55	095	06
006	46	051	43	096	54
007	12	052	00	097	65
008	42	053	02	098	43
009	00	054	54	099	00
010	02	055	95	100	07
011	81	056	42	101	95
012	46	057	00	102	81
013	13	058	06		
014	42	059	81		
015	00	060	46		
016	03	061	17		
017	81	062	01		
018	46	063	55		
019	14	064	53		
020	42	065	01		
021	00	066	85		
022	04	067	43		
023	81	068	00		
024	46	069	04		
025	15	070	65		
026	42	071	43		
027	00	072	00		
028	05	073	01		
029	81	074	55		
030	46	075	43		
031	16	076	00		
032	01	077	03		
033	75	078	55		
034	43	079	43		
035	00	080	00		
036	05	081	02		
037	65	082	54		
038	53	083	95		
039	43	084	42		
040	00	085	00		
041	03	086	07		
042	55	087	81		
043	43	088	46		
044	00	089	18		

SAMPLE PROBLEM: 003 VOID CONTENT

NO: 003

1) Input: $\rho_f = 2600 \text{ Kg-m}^{-3}$
 $\rho_m = 1200 \text{ Kg-m}^{-3}$
 $m_f = .65$
 $m_m = .35$
 $\rho = 1800 \text{ Kg-m}^{-3}$

Output: $V_v = .025$
 $V_f = .462$
 $V_f = .450$

2) Input: $\rho_f = 1700 \text{ Kg-m}^{-3}$
 $\rho_m = 1200 \text{ Kg-m}^{-3}$
 $m_f = .70$
 $m_m = .30$
 $\rho = 1500 \text{ Kg-m}^{-3}$

Output: $V_v = .007$
 $V_f = .622$
 $V_f = .618$

TITLE: STRESS COMPONENTS				NO: 007	
A' σ_i	B'	C'	D'	E'	
A Initialize	B σ_1'	C σ_2'	D σ_6'	E δ	

00 $2(\theta - \delta)$	<u>REGISTER</u>				
01	02	03	04	05	06
07 σ_1', σ_1	08 σ_2', σ_2	09 σ_6', σ_6	10 $(\sigma_1 - \sigma_2)/2$		
11	12	13	14	15 I	16 R
17	18	19			
98	99 δ	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input σ_i	1	007 A&B	$\sigma_1', \sigma_2', \sigma_6'$	A' RUN RUN	$\sigma_1', \sigma_2', \sigma_6'$
Compute Invariants	2			A	δ^*
Input θ	3		θ (deg)	B C D E	σ_1' σ_2' σ_6' δ
Principal Stresses	4		δ (deg)	B C D	σ_I σ_{II} 0
Max Shear	5		$\delta + 45^\circ$	B C D	$\sigma_1' = I$ $\sigma_2' = I$ $\sigma_6' = R$
<p>*Note: Display will flash "0" when $\sigma_1 = \sigma_2$ (See Eq. 30). Stabilize display with CE and proceed, computations are not effected.</p>					

FORMULAS:

TABLE 3 STRESS TRANSFORMATION RELATIONS

	I	R
σ'_x	1	$\cos 2(\theta-\delta)$
σ'_y	1	$-\cos 2(\theta-\delta)$
σ'_{xy}	0	$-\sin 2(\theta-\delta)$

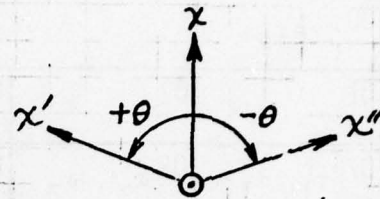


Figure 12 New coordinates $x'-y'$ in terms of old coordinates $x-y$, or given $x-y$ and θ , find $x'-y'$. Arrow of rotation is pointing up. If θ is negative, new coordinates are $x''-y''$.

$$I = \frac{\sigma_x + \sigma_y}{2} \quad (28)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \sigma_{xy}^2} \quad (29)$$

$$\tan 2\delta_1 = \frac{2\sigma_{xy}}{\sigma_x - \sigma_y} \quad (30)$$

$\delta = \delta_1 + 90$ if $\sigma_x < \sigma_y$; $\delta = \delta_1$ if $\sigma_x > \sigma_y$

Both I and R are invariants, as shown in the Mohr's Circle.

Special orientations:

- (1) When $\theta = \delta$, i.e., Principal Directions

$$\sigma'_x, \sigma'_y = \text{maximum or minimum}$$

$$\sigma'_{xy} = 0$$

- (2) When $\theta - \delta = \pm \pi/4$, i.e., max. shear orientation

$$\sigma'_x = \sigma'_y = I$$

$$\sigma'_{xy} = \text{max.} = R$$

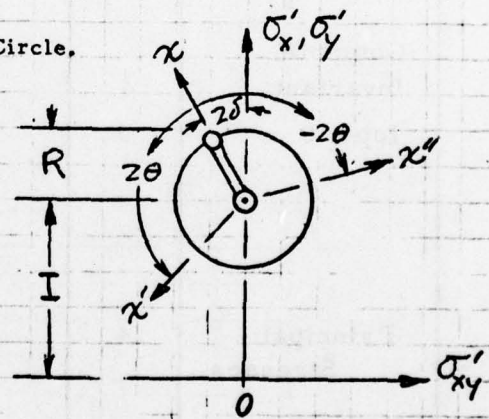


Figure 13 Mohr's Circle is defined by invariants I and R. Phase angle δ is defined by a specific combination of stress components σ_x, σ_y and σ_{xy} . As reference coordinates change by $+\theta$, the rotation in Mohr's Circle is 2θ .

PROGRAM TITLE: STRESS COMPONENTS

NO: 007

PROGRAM LISTING:

000	46	045	85	090	09	135	01
001	16	046	43	091	09	136	06
002	42	047	00	092	81	137	95
003	00	048	09	093	46	138	42
004	07	049	40	094	12	139	00
005	81	050	95	095	75	140	08
006	42	051	30	096	43	141	81
007	00	052	42	097	09	142	46
008	08	053	01	098	09	143	14
009	81	054	06	099	95	144	43
010	42	055	43	100	65	145	00
011	00	056	00	101	02	146	00
012	09	057	09	102	95	147	32
013	81	058	55	103	42	148	94
014	46	059	43	104	00	149	65
015	11	060	01	105	00	150	43
016	43	061	00	106	43	151	01
017	00	062	95	107	01	152	06
018	07	063	22	108	05	153	95
019	85	064	34	109	85	154	42
020	43	065	55	110	43	155	00
021	00	066	02	111	00	156	09
022	08	067	95	112	00	157	81
023	95	068	42	113	33	158	46
024	55	069	09	114	65	159	15
025	02	070	09	115	43	160	43
026	95	071	43	116	01	161	09
027	42	072	01	117	06	162	09
028	01	073	00	118	95	163	81
029	05	074	80	119	42		
030	43	075	87	120	00		
031	00	076	43	121	07		
032	07	077	09	122	81		
033	75	078	09	123	46		
034	43	079	85	124	13		
035	00	080	09	125	43		
036	08	081	00	126	01		
037	95	082	95	127	05		
038	55	083	42	128	75		
039	02	084	09	129	43		
040	95	085	09	130	00		
041	42	086	81	131	00		
042	01	087	46	132	33		
043	00	088	87	133	65		
044	40	089	43	134	43		

SAMPLE PROBLEM: 007 STRESS COMPONENTS

NO: 007

1. Given $\sigma_i = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (MPa)

$I = 1.5$ $R = -3.041$ $\delta = 49.731$

	$\delta-45$	$\delta-30$	$\delta-15$	δ	$\delta+15$	$\delta+30$	$\delta+45$	$\delta-\delta=0$
σ'_x	1.5	3.021	4.134	4.541	4.134	3.021	1.5	1
σ'_y	1.5	-.021	-1.134	-1.541	-1.134	-.021	1.5	2
σ'_{xy}	3.041	2.634	1.521	0	-1.521	-2.634	-3.041	3

2. Given $\sigma_i = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$ (MPa)

$I = -1.5$ $R = -3.041$ $\delta = -40.269$

	$\delta-45$	$\delta-30$	$\delta-15$	δ	$\delta+15$	$\delta+30$	$\delta+45$	$\delta-\delta=0$
σ'_x	-1.5	.021	1.134	1.541	1.134	.021	-1.5	-1
σ'_y	-1.5	-3.021	-4.134	-4.541	-4.134	-3.021	-1.5	-2
σ'_{xy}	3.041	2.634	1.521	0	-1.521	-2.634	-3.041	-3

TITLE: STRAIN COMPONENTS				NO: 011	
A' e_i	B'	C'	D'	E'	
A Initialize	B e_1'	C e_2'	D e_6'	E δ	

00 $2(\theta - \delta)$	<u>REGISTER</u>				
01	02	03	04	05 I	06 R
07	08	09	10 $(e_1 - e_2)/2$		
11	12	13	14	15	16
17 e_1', e_1	18 e_2', e_2	19 e_6', e_6			
98 δ	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input e_i	1	01: A&B	e_1 e_2 e_6	A' RUN RUN	e_1 e_2 e_6
Compute I's Invariants	2			A	δ^*
Input θ	3		θ (deg)	B	e_1'
Recall	3'			C	e_2'
				D	e_6'
				E	δ
Principal Strains	4		δ (deg)	B	e_I
				C	e_{II}
				D	0
Max Shear	5		$\delta + 45^\circ$	B	$e_1' = I$
				C	$e_2' = I$
				D	$e_6' = 2R$
*Note: Display will flash "0" when $e_x = e_y$ (See Eq. 44). Stabilize display with CE and proceed, computations are not effected.					

FORMULAS:

Only the shear component is the difference between the strain and stress transformation relations, i.e.,

TABLE 7 STRAIN TRANSFORMATION RELATIONS

	I	R
e'_x	1	$\cos 2(\theta - \delta)$
e'_y	1	$-\cos 2(\theta - \delta)$
e'_{xy}	0	$-2\sin 2(\theta - \delta)$

$$I = \frac{e_x + e_y}{2} \quad (42)$$

$$R = \sqrt{\left(\frac{e_x - e_y}{2}\right)^2 + \left(\frac{e_{xy}}{2}\right)^2} \quad (43)$$

$$\tan 2\delta = \frac{e_{xy}}{e_x - e_y} \quad (44)$$

$$\delta = \delta_1 \quad \text{if } e_x > e_y$$

$$\delta = \delta_1 + 90 \quad \text{if } e_x < e_y$$

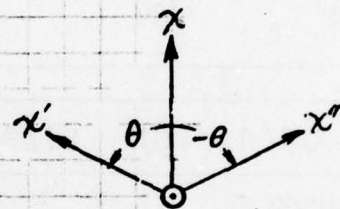


Figure 21 Positive and negative rotations for transformation.

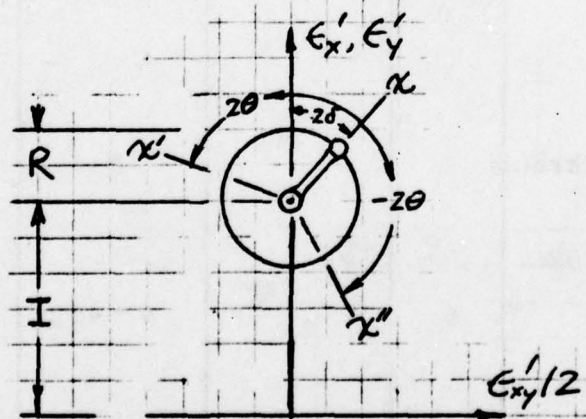


Figure 22 Mohr's Circle representation of strain transformation. Only the one-half factor for the shear strain is the difference between the strain representation and that for stress.

PROGRAM TITLE: STRAIN COMPONENTS

NO: 011

PROGRAM LISTING:

000	46	045	08	090	00	135	00
001	16	046	95	091	95	136	05
002	42	047	55	092	42	137	75
003	01	048	02	093	09	138	43
004	07	049	95	094	08	139	00
005	81	050	42	095	81	140	00
006	42	051	01	096	46	141	33
007	01	052	00	097	87	142	65
008	08	053	40	098	43	143	43
009	81	054	85	099	09	144	00
010	42	055	43	100	08	145	06
011	01	056	01	101	81	146	95
012	09	057	09	102	46	147	42
013	81	058	40	103	12	148	01
014	46	059	95	104	75	149	08
015	11	060	30	105	43	150	81
016	43	061	42	106	09	151	46
017	01	062	00	107	08	152	14
018	08	063	06	108	95	153	43
019	85	064	43	109	65	154	00
020	43	065	01	110	02	155	00
021	01	066	09	111	95	156	32
022	07	067	55	112	42	157	94
023	95	068	43	113	00	158	65
024	55	069	01	114	00	159	02
025	02	070	00	115	43	160	65
026	95	071	95	116	00	161	43
027	42	072	22	117	05	162	00
028	00	073	34	118	85	163	06
029	05	074	55	119	43	164	95
030	43	075	02	120	00	165	42
031	01	076	95	121	00	166	01
032	09	077	42	122	33	167	09
033	55	078	09	123	65	168	81
034	02	079	08	124	43	169	46
035	95	080	43	125	00	170	15
036	42	081	01	126	06	171	43
037	01	082	00	127	95	172	09
038	09	083	80	128	42	173	08
039	43	084	87	129	01	174	81
040	01	085	43	130	07		
041	07	086	09	131	81		
042	75	087	08	132	46		
043	43	088	85	133	13		
044	01	089	09	134	43		

SAMPLE PROBLEM: 011 STAIN COMPONENTS

NO: 011

1. Given $\epsilon_i = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (mm/m or $10^3 \mu\text{m/m}$)

$I = 1.5$ $R = -1.581$ $\delta = 54.217$

	$\delta-45$	$\delta-30$	$\delta-15$	δ	$\delta+15$	$\delta+30$	$\delta+45$	$\delta-\delta=0$
e'_x	1.5	2.291	2.869	3.081	2.869	2.291	1.5	1
e'_y	1.5	.709	.131	-.081	.131	.709	1.5	2
e'_{xy}	3.162	2.739	1.581	0	-1.581	-2.739	-3.162	3

2. Given $\epsilon_i = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$ (mm/m or $10^3 \mu\text{m/m}$)

$I = -1.5$ $R = -1.581$ $\delta = -35.783$

	$\delta-45$	$\delta-30$	$\delta-15$	δ	$\delta+15$	$\delta+30$	$\delta+45$	$\delta-\delta=0$
e'_x	-1.50	-.709	-.131	.081	.131	.709	-1.50	-1
e'_y	-1.50	-2.291	-2.869	-3.081	-2.869	-2.291	-1.50	-2
e'_{xy}	3.162	2.739	1.581	0	-1.581	-2.739	-3.162	-3

TITLE: STRAIN ROSETTES				NO: 015	
A'	B'	C'	D'	E'	
A θ_i	B e_i	C Initialize	D	E Solve 3x3	

00 $\cos^2 A$	<u>REGISTER</u>				
01 $\sin^2 A$	02 $\sin A \cos A$	03 e_A	04 z	05 $\cos^2 B$	06 $\sin^2 B$
07 $\sin B \cos B$	08 e_B	09 y	10 $\cos^2 C$		
11 $\sin^2 C$	12 $\sin C \cos C$	13 e_C	14 θ_A	15 θ_B	16 θ_C
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Compute Coeff	1	015 A&B		C	
Input Angles	2		θ_A (deg)	A	θ_A
			θ_B (deg)	RUN	θ_B
			θ_C (deg)	RUN	θ_C
Input Strains	3		e_A	B	e_A
			e_B	RUN	e_B
			e_C	RUN	e_C
Solve 3 x 3	4	MA1-05		E	e_x
				RUN	e_y
				RUN	e_{xy}

FORMULAS:

a. Three-Element Rosettes

Since there are 3 strain components at each point, 3-element rosettes are in general needed to solve for 3 unknowns. Assuming 3 elements are mounted at 3 different angles from some reference coordinates, each rosette must satisfy the first equation in Eq. 45, i.e.

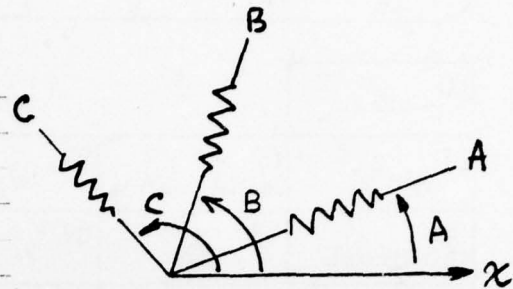


Figure 25 Strain rosette orientation with positive angles. If an angle is negative, it shall be so entered into Equation and

$$\epsilon_A = (\cos^2 A)\epsilon_x + (\sin^2 A)\epsilon_y + (\sin A \cos A)\epsilon_{xy}$$

$$\epsilon_B = (\cos^2 B)\epsilon_x + (\sin^2 B)\epsilon_y + (\sin B \cos B)\epsilon_{xy} \quad (45)$$

$$\epsilon_C = (\cos^2 C)\epsilon_x + (\sin^2 C)\epsilon_y + (\sin C \cos C)\epsilon_{xy}$$

In matrix form

$$\begin{bmatrix} \cos^2 A & \sin^2 A & \sin A \cos A \\ \cos^2 B & \sin^2 B & \sin B \cos B \\ \cos^2 C & \sin^2 C & \sin C \cos C \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_A \\ \epsilon_B \\ \epsilon_C \end{Bmatrix} \quad (46)$$

PROGRAM TITLE: STRAIN ROSETTES

NO: 015

PROGRAM LISTING:

000	46	045	01	090	06
001	11	046	43	091	33
002	42	047	01	092	40
003	01	048	04	093	42
004	04	049	33	094	01
005	81	050	65	095	00
006	42	051	43	096	43
007	01	052	01	097	01
008	05	053	04	098	06
009	81	054	32	099	32
010	42	055	95	100	40
011	01	056	42	101	42
012	06	057	00	102	01
013	81	058	02	103	01
014	46	059	43	104	43
015	12	060	01	105	01
016	42	061	05	106	06
017	00	062	33	107	33
018	03	063	40	108	65
019	81	064	42	109	43
020	42	065	00	110	01
021	00	066	05	111	06
022	08	067	43	112	32
023	81	068	01	113	95
024	42	069	05	114	42
025	01	070	32	115	01
026	03	071	40	116	02
027	81	072	42	117	81
028	46	073	00		
029	13	074	06		
030	43	075	43		
031	01	076	01		
032	04	077	05		
033	33	078	33		
034	40	079	65		
035	42	080	43		
036	00	081	01		
037	00	082	05		
038	43	083	32		
039	01	084	95		
040	04	085	42		
041	32	086	00		
042	40	087	07		
043	42	088	43		
044	00	089	01		

- 1) Given $A=0$, $B=60^\circ$, $C=-60^\circ$ and $\epsilon_A, \epsilon_B, \epsilon_C = 10, -3, -3$

Find Poisson's ratio along a-axis

$$\epsilon_x = 10$$

$$\epsilon_y = -7.333$$

$$\epsilon_{xy} = -4.618 \times 10^{-12} \approx 0$$

$$\nu = .733$$

TITLE: STRESS-STRAIN RELATION				NO: 019	
A'	S_{ij}	B'	C'	D'	E'
A	σ_i	B e_1	C e_2	D e_6	E

00	<u>REGISTER</u>				
01 S_{11}	02 S_{22}	03 S_{12}	04 S_{66}	05 S_{16}	06 S_{26}
07 σ_1	08 σ_2	09 σ_6	10		
11	12	13	14	15	16
17 e_1	18 e_2	19 e_6			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input S_{ij}	1	019 A&B	S_{11}	A'	S_{11}
			S_{22}	RUN	S_{22}
			S_{12}	RUN	S_{12}
			S_{66}	RUN	S_{66}
			S_{16}	RUN	S_{16}
			S_{26}	RUN	S_{26}
			Input σ_i	2	
σ_2	RUN	σ_2			
σ_6	RUN	σ_6			
Strains	3			B	e_1
				C	e_2
				D	e_6
Transf. Strains	4	011 A&B		A	δ^*
			θ (deg)	B	e_1'
				C	e_2'
				D	e_6'

FORMULAS:

$$\begin{aligned}
 \epsilon_1 &= S_{1j} \sigma_j = S_{11} \sigma_1 + S_{12} \sigma_2 + S_{16} \sigma_6 \\
 \epsilon_2 &= S_{2j} \sigma_j = S_{21} \sigma_1 + S_{22} \sigma_2 + S_{26} \sigma_6 \\
 \epsilon_6 &= S_{6j} \sigma_j = S_{61} \sigma_1 + S_{62} \sigma_2 + S_{66} \sigma_6
 \end{aligned}
 \tag{49}$$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{21} & S_{22} & S_{26} \\ S_{61} & S_{62} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}
 \tag{52}$$

S_{ij} is symmetric, i.e., $S_{ij} = S_{ji}$, or $S_{12} = S_{21}$, $S_{16} = S_{61}$, $S_{26} = S_{62}$

PROGRAM TITLE: STRESS-STRAIN RELATION

NO: 019

PROGRAM LISTING:

000	46	045	65	090	06
001	16	046	43	091	65
002	42	047	00	092	43
003	00	048	07	093	00
004	01	049	85	094	09
005	81	050	43	095	95
006	42	051	00	096	42
007	00	052	03	097	01
008	02	053	65	098	08
009	81	054	43	099	81
010	42	055	00	100	46
011	00	056	08	101	14
012	03	057	85	102	43
013	81	058	43	103	00
014	42	059	00	104	05
015	00	060	05	105	65
016	04	061	65	106	43
017	81	062	43	107	00
018	42	063	00	108	07
019	00	064	09	109	85
020	05	065	95	110	43
021	81	066	42	111	00
022	42	067	01	112	06
023	00	068	07	113	65
024	06	069	81	114	43
025	81	070	46	115	00
026	46	071	13	116	08
027	11	072	43	117	85
028	42	073	00	118	43
029	00	074	03	119	00
030	07	075	65	120	04
031	81	076	43	121	65
032	42	077	00	122	43
033	00	078	07	123	00
034	08	079	85	124	09
035	81	080	43	125	95
036	42	081	00	126	42
037	00	082	02	127	01
038	09	083	65	128	09
039	81	084	43	129	81
040	46	085	00		
041	12	086	08		
042	43	087	85		
043	00	088	43		
044	01	089	00		

1. Given:

$$\sigma_i = \begin{Bmatrix} 400 \\ .60 \\ 15 \end{Bmatrix} \text{ (MPa)} = \begin{Bmatrix} .40 \\ .06 \\ .015 \end{Bmatrix} \text{ (TPa)}$$

$$S_{ij} = \begin{bmatrix} 5.5249 & -1.5470 & 0 \\ -1.5470 & 97.0874 & 0 \\ 0 & 0 & 139.4700 \end{bmatrix} \text{ (TPa)}^{-1}$$

(for T-300/5208)

$$\epsilon_i = \begin{Bmatrix} 2.1171 \\ 5.2064 \\ 2.0921 \end{Bmatrix} \text{ (mm/m)}$$

2. Given:

$$\sigma_i = \begin{Bmatrix} .01 \\ .02 \\ .03 \end{Bmatrix} \text{ (TPa)}$$

$$S_{ij} \text{ (TPa)}^{-1} \text{ for T-300/5208}$$

$$\epsilon_i = \begin{Bmatrix} .0243 \\ 1.9263 \\ 4.1841 \end{Bmatrix} \text{ (mm/m)}$$

TITLE: STRAIN-STRESS RELATION				NO: 023
A' Q_{ij}	B'	C'	D'	E'
A e_i	B σ_1	C σ_2	D σ_6	E

00	<u>REGISTER</u>				
01	02	03	04	05	06
07 σ_1	08 σ_2	09 σ_6	10		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17 e_1	18 e_2	19 e_6			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Q_{ij}	1	023 A&B	Q_{11}	A'	Q_{11}
			Q_{22}	RUN	Q_{22}
			Q_{12}	RUN	Q_{12}
			Q_{66}	RUN	Q_{66}
			Q_{16}	RUN	Q_{16}
			Q_{26}	RUN	Q_{26}
Input e_i	2		e_1	A	e_1
			e_2	RUN	e_2
			e_6	RUN	e_6
Stresses	3			B	σ_1
				C	σ_2
				D	σ_6
Transf. Stresses	4	011 A&B		A	δ^*
			θ (deg)	B	σ_1'
				C	σ_2'
				D	σ_6'

FORMULAS:

$$\sigma_i = Q_{ij} e_j \quad (50)$$

$$\sigma_1 = Q_{11} e_1 = Q_{11} e_1 + Q_{12} e_2 + Q_{16} e_6$$

$$\sigma_2 = Q_{21} e_1 = Q_{21} e_1 + Q_{22} e_2 + Q_{26} e_6 \quad (51)$$

$$\sigma_6 = Q_{61} e_1 = Q_{61} e_1 + Q_{62} e_2 + Q_{66} e_6$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \\ e_6 \end{Bmatrix} \quad (53)$$

Q_{ij} is also symmetric, i.e., $Q_{ij} = Q_{ji}$, or $Q_{12} = Q_{21}$, $Q_{16} = Q_{61}$, $Q_{26} = Q_{62}$.

PROGRAM TITLE: STRAIN-STRESS RELATION

NO: 023

PROGRAM LISTING:

000	46	045	65	090	06
001	16	046	43	091	65
002	42	047	01	092	43
003	01	048	07	093	01
004	01	049	85	094	09
005	81	050	43	095	95
006	42	051	01	096	42
007	01	052	03	097	00
008	02	053	65	098	08
009	81	054	43	099	81
010	42	055	01	100	46
011	01	056	08	101	14
012	03	057	85	102	43
013	81	058	43	103	01
014	42	059	01	104	05
015	01	060	05	105	65
016	04	061	65	106	43
017	81	062	43	107	01
018	42	063	01	108	07
019	01	064	09	109	85
020	05	065	95	110	43
021	81	066	42	111	01
022	42	067	00	112	06
023	01	068	07	113	65
024	06	069	81	114	43
025	81	070	46	115	01
026	46	071	13	116	08
027	11	072	43	117	85
028	42	073	01	118	43
029	01	074	03	119	01
030	07	075	65	120	04
031	81	076	43	121	65
032	42	077	01	122	43
033	01	078	07	123	01
034	08	079	85	124	09
035	81	080	43	125	95
036	42	081	01	126	42
037	01	082	02	127	00
038	09	083	65	128	09
039	81	084	43	129	81
040	46	085	01		
041	12	086	08		
042	43	087	85		
043	01	088	43		
044	01	089	01		

SAMPLE PROBLEM: 023 STRAIN-STRESS RELATIONS

NO: 023

1. Given

$$e_i = \begin{Bmatrix} 2.1171 \\ 5.2064 \\ 2.0921 \end{Bmatrix} \text{ mm/m}$$

$$Q_{ij} = \begin{bmatrix} 181.811 & 2.897 & 0 \\ 2.897 & 10.346 & 0 \\ 0 & 0 & 7.170 \end{bmatrix} \text{ (GPa)}$$

(For T-300/5208)

$$\sigma_i = \begin{Bmatrix} 400 \\ 60 \\ 15 \end{Bmatrix} \text{ (MPa)}$$

2. Given

$$e_i = \begin{Bmatrix} .0243 \\ 1.9263 \\ 4.1841 \end{Bmatrix} \text{ mm/m}$$

A_{ij} (GPa) (for T-300/5208)

$$\sigma_i = \begin{Bmatrix} 10 \\ 20 \\ 30 \end{Bmatrix} \text{ MPa}$$

TITLE: MODULUS TO COMPLIANCE				NO: 027	
A' Q_{ij}	B'	C'	D'	E'	
A $ Q $	B	C	D	E	

00	<u>REGISTER</u>				
01 S_{11}	02 S_{22}	03 S_{12}	04 S_{66}	05 S_{16}	06 S_{26}
07	08	09	10 $ Q $		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Q_{ij}	1	027 A&B	Q_{11}	A'	Q_{11}
			Q_{22}	RUN	Q_{22}
			Q_{12}	RUN	Q_{12}
			Q_{66}	RUN	Q_{66}
			Q_{16}	RUN	Q_{16}
			Q_{26}	RUN	Q_{26}
Compute S_{ij}	2			A	S_{26}
				RCL 01	S_{11}
				RCL 02	S_{22}
				RCL 03	S_{12}
				RCL 04	S_{66}
				RCL 05	S_{16}
				RCL 06	S_{26}

FORMULAS:

In-Plane Modulus and Compliance

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix} \begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} \quad \text{or} \quad N_i = A_{ij} e_j^o \quad (203)$$

$$\text{Let Compliance} = a_{ij} = [A_{ij}]^{-1} \quad \text{or} \quad a_{ij} A_{jk} = \delta_{ik} \quad (204)$$

$$\begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} \quad \text{or} \quad e_i^o = a_{ij} N_j \quad (205)$$

where

$$\begin{aligned} a_{11} &= \frac{1}{\Delta} (A_{22}A_{66} - A_{26}^2), & a_{22} &= \frac{1}{\Delta} (A_{11}A_{66} - A_{16}^2) \\ a_{12} &= \frac{1}{\Delta} (A_{16}A_{26} - A_{12}A_{66}), & a_{66} &= \frac{1}{\Delta} (A_{11}A_{22} - A_{12}^2) \\ a_{16} &= \frac{1}{\Delta} (A_{12}A_{26} - A_{22}A_{16}), & a_{26} &= \frac{1}{\Delta} (A_{12}A_{16} - A_{11}A_{26}) \end{aligned} \quad (206)$$

$$\Delta = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix}$$

In-Plane Engineering Constants:

$$E_{11}^o = \frac{1}{ha_{11}}, \quad E_{22}^o = \frac{1}{ha_{22}}, \quad \nu_{12}^o = -\frac{a_{12}}{a_{11}}, \quad \nu_{21}^o = -\frac{a_{21}}{a_{22}}, \quad G_{12}^o = \frac{1}{ha_{66}} \quad (207)$$

PROGRAM TITLE: MODULUS TO COMPLIANCE

NO: 027

PROGRAM LISTING:

000	46	045	01	090	04	135	01	180	01
001	16	046	06	091	75	136	03	181	05
002	42	047	65	092	43	137	40	182	65
003	01	048	43	093	01	138	95	183	43
004	01	049	01	094	06	139	55	184	01
005	81	050	05	095	40	140	43	185	02
006	42	051	65	096	95	141	01	186	95
007	01	052	02	097	55	142	00	187	55
008	02	053	75	098	43	143	95	188	43
009	81	054	43	099	01	144	42	189	01
010	42	055	01	100	00	145	00	190	00
011	01	056	05	101	95	146	04	191	95
012	03	057	40	102	42	147	43	192	42
013	81	058	65	103	00	148	01	193	00
014	42	059	43	104	01	149	05	194	05
015	01	060	01	105	43	150	65	195	43
016	04	061	02	106	01	151	43	196	01
017	81	062	75	107	01	152	01	197	03
018	42	063	43	108	65	153	06	198	65
019	01	064	01	109	43	154	75	199	43
020	05	065	03	110	01	155	43	200	01
021	81	066	40	111	04	156	01	201	05
022	42	067	65	112	75	157	03	202	75
023	01	068	43	113	43	158	65	203	43
024	06	069	01	114	01	159	43	204	01
025	81	070	04	115	05	160	01	205	01
026	46	071	75	116	40	161	04	206	65
027	11	072	43	117	95	162	95	207	43
028	43	073	01	118	55	163	55	208	01
029	01	074	06	119	43	164	43	209	06
030	01	075	40	120	01	165	01	210	95
031	65	076	65	121	00	166	00	211	55
032	43	077	43	122	95	167	95	212	43
033	01	078	01	123	42	168	42	213	01
034	02	079	01	124	00	169	00	214	00
035	65	080	95	125	02	170	03	215	95
036	43	081	42	126	43	171	43	216	42
037	01	082	01	127	01	172	01	217	00
038	04	083	00	128	01	173	03	218	06
039	85	084	43	129	65	174	65	219	81
040	43	085	01	130	43	175	43		
041	01	086	02	131	01	176	01		
042	03	087	65	132	02	177	06		
043	65	088	43	133	75	178	75		
044	43	089	01	134	43	179	43		

SAMPLE PROBLEM: 027 MODULUS TO COMPLIANCE

NO: 027

$$Q_{ij} = \begin{bmatrix} 181.8111 & 2.8969 & 0 \\ 2.8969 & 10.3462 & 0 \\ 0 & 0 & 7.1700 \end{bmatrix} \text{ (MPa)}$$

$$S_{ij} = \begin{bmatrix} 5.5249 & -1.5469 & 0 \\ -1.5469 & 97.0870 & 0 \\ 0 & 0 & 139.4700 \end{bmatrix} \text{ (TPa)}^{-1}$$

TITLE: COMPLIANCE TO MODULUS				NO: 031	
A' s_{ij}	B'	C'	D'	E'	
A $ s $	B	C	D	E	

00	<u>REGISTER</u>				
01 s_{11}	02 s_{22}	03 s_{12}	04 s_{66}	05 s_{16}	06 s_{26}
07	08	09	10 $ s $		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input s_{ij}	1	031 A&B	s_{11}	A'	s_{11}
			s_{22}	RUN	s_{12}
			s_{12}	RUN	s_{12}
			s_{66}	RUN	s_{66}
			s_{16}	RUN	s_{16}
			s_{26}	RUN	s_{26}
Compute Q_{ij}	2		A	A	Q_{11}
			RCL 11	RCL 11	Q_{11}
			RCL 12	RCL 12	Q_{22}
			RCL 13	RCL 13	Q_{12}
			RCL 14	RCL 14	Q_{66}
			RCL 15	RCL 15	Q_{16}
			RCL 16	RCL 16	Q_{26}

PROGRAM TITLE: COMPLIANCE TO MODULUS

NO: 031

PROGRAM LISTING:

000	46	045	00	090	02	135	00	180	03
001	16	046	06	091	75	136	03	181	75
002	42	047	65	092	43	137	65	182	43
003	00	048	43	093	00	138	43	183	00
004	01	049	00	094	03	139	00	184	01
005	81	050	05	095	40	140	04	185	65
006	42	051	65	096	95	141	95	186	43
007	00	052	02	097	55	142	55	187	00
008	02	053	75	098	43	143	43	188	06
009	81	054	43	099	01	144	01	189	95
010	42	055	00	100	00	145	00	190	55
011	00	056	05	101	95	146	95	191	43
012	03	057	40	102	42	147	42	192	01
013	81	058	65	103	01	148	01	193	00
014	42	059	43	104	04	149	03	194	95
015	00	060	00	105	43	150	43	195	42
016	04	061	02	106	00	151	00	196	01
017	81	062	75	107	01	152	03	197	06
018	42	063	43	108	65	153	65	198	43
019	00	064	00	109	43	154	43	199	00
020	05	065	03	110	00	155	00	200	02
021	81	066	40	111	04	156	06	201	65
022	42	067	65	112	75	157	75	202	43
023	00	068	43	113	43	158	43	203	00
024	06	069	00	114	00	159	00	204	04
025	81	070	04	115	05	160	05	205	75
026	46	071	75	116	40	161	65	206	43
027	11	072	43	117	95	162	43	207	00
028	43	073	00	118	55	163	00	208	06
029	00	074	06	119	43	164	02	209	40
030	01	075	40	120	01	165	95	210	95
031	65	076	65	121	00	166	55	211	55
032	43	077	43	122	95	167	43	212	43
033	00	078	00	123	42	168	01	213	01
034	02	079	01	124	01	169	00	214	00
035	65	080	95	125	02	170	95	215	95
036	43	081	42	126	43	171	42	216	42
037	00	082	01	127	00	172	01	217	01
038	04	083	00	128	05	173	05	218	01
039	85	084	43	129	65	174	43	219	81
040	43	085	00	130	43	175	00		
041	00	086	01	131	00	176	05		
042	03	087	65	132	06	177	65		
043	65	088	43	133	75	178	43		
044	43	089	00	134	43	179	00		

TITLE: INVARIANTS OF ELASTICITY				NO: 033	
A'	B'	C'	D'	E'	
A Initialize	B	C	D	E	

00	$\frac{1}{m}$	<u>REGISTER</u>									
01	I_{1Q}	02	I_{2Q}	03	R_{1Q}	04	R_{2Q}	05	Q_{11}	06	Q_{22}
07		08		09	ν_{LT}	10	G_{LT}				
11	I_{1S}	12	I_{2S}	13	R_{1S}	14	R_{2S}	15	Q_{12}	16	S_{12}
17		18		19							
98	E_L	99	E_T	<u>INSTRUCTIONS</u>							

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Engineering Constants	1		E_L	STO 98	E_L
			E_T	STO 99	E_T
			ν_{LT}	STO 09	ν_{LT}
			G_{LT}	STO 10	G_{LT}
Compute invariants	2	033 A&B		A	R_{2S}
				RCL 11	I_{1S}
				RCL 12	I_{2S}
				RCL 13	R_{1S}
				RCL 14	R_{2S}
				RCL 01	I_{1Q}
				RCL 02	I_{2Q}
				RCL 03	R_{1Q}
	RCL 04	R_{2Q}			

FORMULAS:

TABLE 10 COMPLIANCE AND MODULUS IN TERMS OF ELASTIC SYMMETRIES

Symmetry (No Indep. Const)	Compliance Matrix S_{ij} (TPa) ⁻¹	Modulus Matrix Q_{ij} (MPa)
Anisotropic (6) or Generally Orthotropic (4)	$\begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & S_{16} \\ \frac{\nu_{21}}{E_{22}} & \frac{1}{E_{22}} & S_{26} \\ S_{61} & S_{62} & \frac{1}{G_{12}} \end{bmatrix}$	$\begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix}$
Specially Orthotropic (4)	$\begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{LT}}{E_L} & 0 \\ -\frac{\nu_{TL}}{E_T} & \frac{1}{E_T} & 0 \\ 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix}$	$\begin{bmatrix} mE_L & \nu_{TL}mE_L & 0 \\ \nu_{LT}mE_T & mE_T & 0 \\ 0 & 0 & G_{LT} \end{bmatrix}$

$$I_1 = \frac{1}{4} (Q_{11} + Q_{22} + 2Q_{12}) \quad (89)$$

$$I_2 = \frac{1}{8} (Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) \quad (90)$$

$$R_1 = \frac{1}{2} \sqrt{(-Q_{11} + Q_{22})^2} \quad (91)$$

$$R_2 = \frac{1}{8} \sqrt{(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})^2} \quad (92)$$

$$I_1 = \frac{1}{4} (S_{11} + S_{22} + 2S_{12}) \quad (81)$$

$$I_2 = \frac{1}{8} (S_{11} + S_{22} - 2S_{12} + S_{66}) \quad (82)$$

$$R_1 = \frac{1}{2} \sqrt{(-S_{11} + S_{22})^2} \quad (83)$$

$$R_2 = \frac{1}{8} \sqrt{(S_{11} + S_{22} - 2S_{12} - S_{66})^2} \quad (84)$$

$$m = \frac{1}{1 - \nu_{LT}\nu_{TL}}$$

$$= \frac{1}{1 - \nu_{LT}^2 \frac{E_T}{E_L}}$$

PROGRAM TITLE: INVARIANTS OF ELASTICITY

NO: 033

PROGRAM LISTING:

000 46	045 42	090 02	135 54	180 09
001 11	046 01	091 43	136 42	181 09
002 01	047 05	092 00	137 01	182 20
003 75	048 65	093 05	138 06	183 75
004 43	049 02	094 75	139 65	184 43
005 00	050 85	095 43	140 02	185 09
006 09	051 43	096 00	141 95	186 08
007 40	052 00	097 06	142 55	187 20
008 65	053 05	098 95	143 04	188 95
009 43	054 85	099 55	144 95	189 55
010 09	055 43	100 02	145 51	190 02
011 09	056 00	101 95	146 87	191 95
012 55	057 06	102 42	147 42	192 51
013 43	058 95	103 00	148 01	193 87
014 09	059 55	104 03	149 01	194 42
015 08	060 04	105 43	150 43	195 01
016 95	061 95	106 00	151 09	196 03
017 42	062 42	107 02	152 08	197 43
018 00	063 00	108 75	153 20	198 01
019 00	064 01	109 43	154 85	199 00
020 55	065 43	110 01	155 43	200 20
021 43	066 00	111 00	156 09	201 51
022 09	067 05	112 95	157 09	202 87
023 08	068 85	113 42	158 20	203 55
024 95	069 43	114 00	159 75	204 04
025 20	070 00	115 04	160 02	205 75
026 42	071 06	116 43	161 65	206 43
027 00	072 75	117 09	162 43	207 01
028 05	073 02	118 08	163 01	208 02
029 43	074 65	119 20	164 06	209 95
030 09	075 43	120 85	165 85	210 42
031 09	076 01	121 43	166 43	211 01
032 55	077 05	122 09	167 01	212 04
033 43	078 85	123 09	168 00	213 81
034 00	079 04	124 20	169 20	214 46
035 00	080 65	125 85	170 95	215 87
036 95	081 43	126 53	171 55	216 65
037 42	082 01	127 43	172 08	217 01
038 00	083 00	128 00	173 95	218 00
039 06	084 95	129 09	174 51	219 00
040 65	085 55	130 55	175 87	220 00
041 43	086 08	131 43	176 42	221 95
042 00	087 95	132 09	177 01	222 56
043 09	088 42	133 08	178 02	
044 95	089 00	134 94	179 43	

SAMPLE PROBLEM: 033 INVARIANTS OF ELASTICITY

NO: 033

Given:

Material	Type	E_L (GPa)	E_T (GPa)	ν_{LT}	G_{LT} (GPa)	Fiber Vol. ν_f	Specific Gravity
B/Ep	B(4) 5505	204	18.5	.23	5.79	.5	2.0
Gr/Ep	T-300 5208	181	10.3	.28	7.17	.70	1.60
Gl/Ep	Scotchply 1002	38.6	8.27	.26	4.14	.45	1.8

INVARIANTS FOR COMPLIANCE MATRIX

Material	I_{1S}	I_{2S}	R_{1S}	R_{2S}	$\delta_1 = \delta_2$
B/Ep	14.18	29.24	24.58	13.94	0
Gr/Ep	24.88	30.65	45.78	4.22	0
Gl/Ep	33.3	50.2	47.5	10.2	0

INVARIANTS FOR MODULUS MATRIX

Material	I_{1Q}	I_{2Q}	R_{1Q}	R_{2Q}	$\delta_1 = \delta_2$
B/Ep	58.03	29.77	93.20	23.98	0
Gr/Ep	49.49	26.88	85.73	19.71	0
Gl/Ep	13.0	7.47	15.4	3.33	0

TITLE: COMPLIANCE COMPONENTS

NO: 037

A'	B' S_{16}	C' S_{26}	D'	E'
A Invariants	B S_{11}	C S_{22}	D S_{12}	E S_{66}

00 θ	<u>REGISTER</u>				
01 S_{11}	02 S_{22}	03 S_{12}	04 S_{66}	05 S_{16}	06 S_{26}
07	08	09	10		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15	16
17 δ_{1S}	18 δ_{2S}	19 $2(\theta - \delta_1)$			
98 $4(\theta - \delta_2)$	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Invariants	1	037 A&B	I_{1S}	A	I_{1S}
			I_{2S}	RUN	I_{2S}
			R_{1S}	RUN	R_{1S}
			R_{2S}	RUN	R_{2S}
			δ_{1S}	RUN	δ_{1S}
			δ_{2S}	RUN	δ_{2S}
Input θ & Compute	2		θ (deg)	B	S_{11}
				C	S_{22}
				D	S_{12}
				E	S_{66}
				B'	S_{16}
	C'	S_{26}			

FORMULAS:

TABLE 17 TRANSFORMATION OF COMPLIANCE

	I_1	I_2	R_1	R_2
S'_{11}	1	1	$-\cos 2(\theta - \delta_1)$	$-\cos 4(\theta - \delta_2)$
S'_{22}	1	1	$\cos 2(\theta - \delta_1)$	$-\cos 4(\theta - \delta_2)$
S'_{12}	1	-1	0	$\cos 4(\theta - \delta_2)$
S'_{66}	0	4	0	$4 \cos 4(\theta - \delta_2)$
S'_{16}	0	0	$\sin 2(\theta - \delta_1)$	$2 \sin 4(\theta - \delta_2)$
S'_{26}	0	0	$\sin 2(\theta - \delta_1)$	$-2 \sin 4(\theta - \delta_2)$

$$I_1 = \frac{1}{4}(S_{11} + S_{22} + 2S_{12}) \quad (82)$$

$$I_2 = \frac{1}{8}(S_{11} + S_{22} - 2S_{12} + S_{66}) \quad (83)$$

$$R_1 = \frac{1}{2} \sqrt{(-S_{11} + S_{22})^2 + (S_{16} + S_{26})^2} \quad (84)$$

$$R_2 = \frac{1}{8} \sqrt{(S_{11} + S_{22} - 2S_{12} - S_{66})^2 + 4(S_{26} - S_{16})^2} \quad (85)$$

$$\tan 2\delta_1 = \frac{S_{16} + S_{26}}{S_{11} - S_{22}} \quad (86)$$

$$\tan 4\delta_2 = \frac{2(S_{16} - S_{26})}{S_{11} + S_{22} - 2S_{12} - S_{66}} \quad (87)$$

For orthotropic material

$$\delta_1 = \delta_2 \quad (88)$$

For anisotropic material

$$\delta_1 \neq \delta_2 \quad (89)$$

PROGRAM TITLE: COMPLIANCE COMPONENTS

NO: 037

PROGRAM LISTING:

000	46	045	01	090	06	135	85	180	14
001	11	046	03	091	43	136	43	181	43
002	42	047	85	092	09	137	01	182	00
003	01	048	53	093	08	138	09	183	03
004	01	049	53	094	33	139	33	184	81
005	81	050	43	095	65	140	65	185	46
006	42	051	00	096	43	141	43	186	15
007	01	052	00	097	01	142	01	187	43
008	02	053	75	098	04	143	03	188	00
009	81	054	43	099	85	144	75	189	04
010	42	055	01	100	43	145	43	190	81
011	01	056	08	101	01	146	09	191	46
012	03	057	54	102	01	147	08	192	17
013	81	058	65	103	75	148	33	193	43
014	42	059	04	104	43	149	65	194	00
015	01	060	54	105	01	150	43	195	05
016	04	061	42	106	02	151	01	196	81
017	81	062	09	107	95	152	04	197	46
018	42	063	08	108	42	153	95	198	18
019	01	064	32	109	00	154	42	199	43
020	07	065	65	110	03	155	00	200	00
021	81	066	02	111	75	156	02	201	06
022	42	067	65	112	43	157	75	202	81
023	01	068	43	113	01	158	43		
024	08	069	01	114	01	159	01		
025	81	070	04	115	85	160	09		
026	46	071	95	116	02	161	33		
027	12	072	42	117	65	162	65		
028	42	073	00	118	43	163	43		
029	00	074	05	119	01	164	01		
030	00	075	94	120	02	165	03		
031	75	076	85	121	95	166	65		
032	43	077	43	122	65	167	02		
033	01	078	01	123	04	168	95		
034	07	079	09	124	95	169	42		
035	95	080	32	125	42	170	00		
036	65	081	65	126	00	171	01		
037	02	082	43	127	04	172	81		
038	95	083	01	128	43	173	46		
039	42	084	03	129	01	174	13		
040	01	085	65	130	01	175	43		
041	09	086	02	131	85	176	00		
042	32	087	95	132	43	177	02		
043	65	088	42	133	01	178	81		
044	43	089	00	134	02	179	46		

Given:

Material	Type	I_{1S}	I_{2S}	R_{1S}	R_{2S}	$\delta_1 = \delta_2$
B/Ep	B(4) 5505	14.18	29.24	24.58	13.94	0
Gr/Ep	T-300 5208	24.88	30.65	45.78	4.22	0
Gl/Ep	Scotchply 1002	33.3	50.2	47.5	10.2	0

		T-300 / 5208					
		$S_{ij} \text{ (TPa)}^{-1}$			Engr. Const. (GPa)		
0°		5.52	-1.55	0	E_{11}'	=	181
					E_{22}'	=	10.3
		-1.55	97.09	0	ν_{12}'	=	.28
					ν_{21}'	=	.016
		0	0	139.47	G_{12}'	=	7.2
30°		34.75	-7.88	46.96	E_{11}'	=	28.78
					E_{22}'	=	12.42
		-7.88	80.53	32.34	ν_{12}'	=	.23
					ν_{21}'	=	.10
		46.96	32.34	114.15	G_{12}'	=	8.76
45°		59.75	-9.99	45.78	E_{11}'	=	16.74
					E_{22}'	=	16.74
		-9.99	59.75	45.78	ν_{12}'	=	.17
					ν_{21}'	=	.17
		45.78	45.78	105.71	G_{12}'	=	9.46

TITLE: MODULUS COMPONENTS				NO: 041	
A'	B' Q_{16}	C' Q_{26}	D'	E'	
A Invariants	B Q_{11}	C Q_{22}	D Q_{12}	E Q_{66}	

00 θ	<u>REGISTER</u>				
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05	06
07 δ_{1Q}	08 δ_{2Q}	09 $2(\theta - \delta_1)$	10		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17	18	19			
98	99 $4(\theta - \delta_2)$	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Invariants	1	041 A&B	I_{1Q}	A	I_{1Q}
			I_{2Q}	RUN	I_{2Q}
			R_{1Q}	RUN	R_{1Q}
			R_{2Q}	RUN	R_{2Q}
			δ_{1Q}	RUN	δ_{1Q}
			δ_{2Q}	RUN	δ_{2Q}
Input θ & Compute	2		θ (deg)	B	Q_{11}
				C	Q_{22}
				D	Q_{12}
				E	Q_{66}
				B'	Q_{16}
	C'	Q_{26}			

FORMULAS:

TABLE 20 TRANSFORMATION OF MODULUS

	I_1	I_2	R_1	R_2
Q'_{11}	1	1	$\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
Q'_{22}	1	1	$-\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
Q'_{12}	1	-1	0	$-\cos 4(\theta - \delta_2)$
Q'_{66}	0	1	0	$-\cos 4(\theta - \delta_2)$
Q'_{16}	0	0	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$-\sin 4(\theta - \delta_2)$
Q'_{26}	0	0	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$\sin 4(\theta - \delta_2)$

$$I_1 = \frac{1}{4} (Q_{11} + Q_{22} + 2Q_{12}) \quad (90)$$

$$I_2 = \frac{1}{8} (Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) \quad (91)$$

$$R_1 = \frac{1}{2} \sqrt{(-Q_{11} + Q_{22})^2 + 4(Q_{16} + Q_{26})^2} \quad (92)$$

$$R_2 = \frac{1}{8} \sqrt{(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})^2 + 16(Q_{16} - Q_{26})^2} \quad (93)$$

$$\tan 2\delta_1 = \pm \frac{2(Q_{16} + Q_{26})}{Q_{11} - Q_{22}} \quad (94)$$

$$\tan 4\delta_2 = \pm \frac{4(Q_{16} - Q_{26})}{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}} \quad (95)$$

$\delta_1 = \delta_2$ for orthotropic materials

$\delta_1 \neq \delta_2$ for anisotropic materials

PROGRAM TITLE: MODULUS COMPONENTS

NO: 041

PROGRAM LISTING:

000	46	045	94	090	06	135	65	180	15
001	11	046	65	091	43	136	43	181	43
002	42	047	43	092	09	137	00	182	01
003	00	048	00	093	09	138	03	183	04
004	01	049	03	094	33	139	85	184	81
005	81	050	75	095	94	140	43	185	46
006	42	051	53	096	65	141	00	186	17
007	00	052	53	097	43	142	01	187	43
008	02	053	43	098	00	143	85	188	01
009	81	054	00	099	04	144	43	189	05
010	42	055	00	100	85	145	00	190	81
011	00	056	75	101	43	146	02	191	46
012	03	057	43	102	00	147	95	192	18
013	81	058	00	103	02	148	42	193	43
014	42	059	08	104	95	149	01	194	01
015	00	060	54	105	42	150	02	195	06
016	04	061	65	106	01	151	85	196	81
017	81	062	04	107	04	152	43		
018	42	063	54	108	85	153	00		
019	00	064	42	109	43	154	09		
020	07	065	09	110	00	155	33		
021	81	066	09	111	01	156	65		
022	42	067	32	112	75	157	43		
023	00	068	65	113	02	158	00		
024	08	069	43	114	65	159	03		
025	81	070	00	115	43	160	65		
026	46	071	04	116	00	161	02		
027	12	072	95	117	02	162	95		
028	42	073	42	118	95	163	42		
029	00	074	01	119	42	164	01		
030	00	075	05	120	01	165	01		
031	75	076	85	121	03	166	81		
032	43	077	43	122	43	167	46		
033	00	078	09	123	09	168	13		
034	07	079	09	124	09	169	43		
035	95	080	32	125	33	170	01		
036	65	081	65	126	65	171	02		
037	02	082	43	127	43	172	81		
038	95	083	00	128	00	173	46		
039	42	084	04	129	04	174	14		
040	00	085	65	130	75	175	43		
041	09	086	02	131	43	176	01		
042	32	087	95	132	00	177	03		
043	55	088	42	133	09	178	81		
044	02	089	01	134	33	179	46		

Given:

Material	Type	I_{10}	I_{20}	R_{10}	R_{20}	$\delta_1 = \delta_2$
B/Ep	B(4) / 5505	58.03	29.77	93.20	23.98	0
Gr/Ep	T-300 5208	49.49	26.88	85.73	19.71	0
Gl/Ep	Scotchply 1002	13.0	7.47	15.4	3.33	0

	T-300 / 5208			B(4) / 5504		
	C_{ij}			C_{ij}		
0°	181.81	42.90	0	204.98	4.28	0
	42.90	10.35	0	4.28	18.59	0
	0	0	7.17	0	0	5.79
30°	109.38	32.46	-54.19	122.41	40.25	-61.13
	32.46	23.65	-20.05	40.25	29.21	-19.59
	-54.19	-20.05	36.74	-61.13	-19.59	41.76
45°	56.66	42.32	-42.89			
	42.32	56.66	-42.87			
	-42.87	-42.87	46.59			

TITLE: PLY DATA: T-300/5208 Gr/Ep				NO: 045	
A'	X, e_L	B'	X', e_L'	C'	Y, e_T
A	E_L	B	E_T	C	ν_{LT}
D'	Y', e_T'	E'	S, e_S	D	G_{LT}
					E'

00	S	<u>REGISTER</u>			
01	I_{1Q}	02	I_{2Q}	03	R_{1Q}
		04	R_{2Q}	05	E_L
		06	E_T		
07	0	08	0	09	ν_{LT}
		10	G_{LT}		
11	I_{1S}	12	I_{2S}	13	R_{1S}
		14	R_{2S}	15	X
		16	X'		
17	0	18	0	19	0
98	Y	99	Y'	<u>INSTRUCTIONS</u>	

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	045 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	e_L'
				C'	Y
				RUN	e_T
D'	Y'				
RUN	e_T'				
E'	S				
RUN	e_S				

PROGRAM TITLE: PLY DATA: T300/5208 Gr/Ep

NO: 045

PROGRAM LISTING:

000	46	045	02	090	03	135	08	180	43
001	11	046	08	091	04	136	42	181	00
002	01	047	05	092	93	137	00	182	05
003	08	048	93	093	02	138	07	183	95
004	01	049	07	094	02	139	43	184	81
005	42	050	03	095	00	140	00	185	46
006	00	051	02	096	42	141	05	186	18
007	05	052	42	097	01	142	81	187	43
008	01	053	00	098	04	143	46	188	09
009	00	054	03	099	01	144	12	189	08
010	93	055	01	100	05	145	43	190	81
011	03	056	09	101	00	146	00	191	55
012	42	057	93	102	00	147	06	192	43
013	00	058	07	103	42	148	81	193	00
014	06	059	01	104	01	149	46	194	06
015	93	060	00	105	05	150	13	195	95
016	02	061	42	106	01	151	43	196	81
017	08	062	00	107	05	152	00	197	46
018	42	063	04	108	00	153	09	198	19
019	00	064	02	109	00	154	81	199	43
020	09	065	04	110	42	155	46	200	09
021	07	066	93	111	01	156	14	201	09
022	93	067	09	112	06	157	43	202	81
023	01	068	08	113	04	158	01	203	55
024	07	069	00	114	00	159	00	204	43
025	42	070	42	115	42	160	81	205	00
026	01	071	01	116	09	161	46	206	06
027	00	072	01	117	08	162	16	207	95
028	04	073	03	118	02	163	43	208	81
029	09	074	00	119	04	164	01	209	46
030	93	075	93	120	06	165	05	210	10
031	04	076	06	121	42	166	81	211	43
032	08	077	04	122	09	167	55	212	00
033	08	078	07	123	09	168	43	213	00
034	42	079	42	124	06	169	00	214	81
035	00	080	01	125	08	170	05	215	55
036	01	081	02	126	42	171	95	216	43
037	02	082	04	127	00	172	81	217	01
038	06	083	05	128	00	173	46	218	00
039	93	084	93	129	00	174	17	219	95
040	08	085	07	130	42	175	43	220	81
041	08	086	08	131	00	176	01		
042	00	087	01	132	07	177	06		
043	42	088	42	133	42	178	81		
044	00	089	01	134	00	179	55		

TITLE: PLY DATA: B/5505 B/Ep				NO: 047	
A' X, e_L	B' X', e_L'	C' Y, e_T	D' Y', e_T'	E' S, e_S	
A E_L	B E_T	C ν_{LT}	D G_{LT}	E	

00 S	<u>REGISTER</u>				
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	047 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	e_L'
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA: B/5505 B/Ep

NO: 047

PROGRAM LISTING:

000	46	045	00	090	00	135	81	180	43
001	11	046	03	091	42	136	46	181	09
002	02	047	02	092	06	137	14	182	09
003	00	048	04	093	09	138	43	183	81
004	04	049	42	094	06	139	01	184	55
005	42	050	00	095	01	140	00	185	43
006	00	051	04	096	42	141	81	186	00
007	05	052	01	097	09	142	46	187	06
008	01	053	04	098	08	143	16	188	95
009	08	054	93	099	02	144	43	189	81
010	93	055	02	100	00	145	01	190	46
011	05	056	42	101	02	146	05	191	10
012	42	057	01	102	42	147	81	192	43
013	00	058	01	103	09	148	55	193	00
014	06	059	02	104	09	149	43	194	00
015	93	060	09	105	06	150	00	195	81
016	02	061	93	106	07	151	05	196	55
017	03	062	02	107	42	152	95	197	43
018	42	063	42	108	00	153	81	198	01
019	01	064	01	109	00	154	46	199	00
020	06	065	02	110	00	155	17	200	95
021	05	066	02	111	42	156	43	201	81
022	93	067	04	112	00	157	06		
023	07	068	93	113	07	158	09		
024	09	069	06	114	42	159	81		
025	42	070	42	115	00	160	55		
026	01	071	01	116	08	161	43		
027	00	072	03	117	42	162	00		
028	05	073	01	118	00	163	05		
029	08	074	03	119	09	164	95		
030	42	075	93	120	43	165	81		
031	00	076	09	121	00	166	46		
032	01	077	42	122	05	167	18		
033	02	078	01	123	81	168	43		
034	09	079	04	124	46	169	09		
035	93	080	01	125	12	170	08		
036	08	081	02	126	43	171	81		
037	42	082	06	127	00	172	55		
038	00	083	00	128	06	173	43		
039	02	084	42	129	81	174	00		
040	09	085	01	130	46	175	06		
041	03	086	05	131	13	176	95		
042	93	087	02	132	43	177	81		
043	02	088	05	133	01	178	46		
044	42	089	00	134	06	179	19		

TITLE: PLY DATA: SCOTCHPLY 1002				NO: 049	
A' X, e_L	B' X', $e_{L'}$	C' Y, e_T	D' Y', $e_{T'}$	E' S, e_S	
A E_L	B E_T	C ν_{LT}	D G_{LT}	E	

00 S	<u>REGISTER</u>				
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 ν_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	049 A&B		A	E_L
				B	E_T
				C	ν_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	$e_{L'}$
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	$e_{T'}$
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA: SCOTCHPLY 1002 G1/Ep NO: 049

PROGRAM LISTING:

000	46	045	42	090	05	135	06	180	19
001	11	046	00	091	02	136	81	181	43
002	03	047	03	092	42	137	46	182	09
003	08	048	03	093	06	138	14	183	09
004	93	049	93	094	09	139	43	184	81
005	06	050	03	095	02	140	01	185	55
006	42	051	03	096	00	141	00	186	43
007	00	052	42	097	42	142	81	187	00
008	05	053	00	098	09	143	46	188	06
009	08	054	04	099	08	144	16	189	95
010	93	055	03	100	01	145	43	190	81
011	02	056	03	101	03	146	01	191	46
012	07	057	93	102	08	147	05	192	10
013	42	058	03	103	42	148	81	193	43
014	00	059	42	104	09	149	55	194	00
015	06	060	01	105	09	150	43	195	00
016	93	061	01	106	02	151	00	196	81
017	02	062	05	107	09	152	05	197	55
018	06	063	00	108	42	153	95	198	43
019	42	064	93	109	00	154	81	199	01
020	01	065	02	110	00	155	46	200	00
021	06	066	42	111	00	156	17	201	95
022	04	067	01	112	42	157	43	202	81
023	93	068	02	113	00	158	06		
024	01	069	04	114	07	159	09		
025	04	070	07	115	42	160	81		
026	42	071	93	116	00	161	55		
027	01	072	05	117	08	162	43		
028	00	073	42	118	42	163	00		
029	01	074	01	119	00	164	05		
030	03	075	03	120	09	165	95		
031	42	076	01	121	43	166	81		
032	00	077	00	122	00	167	46		
033	01	078	93	123	05	168	18		
034	07	079	03	124	81	169	43		
035	93	080	42	125	46	170	09		
036	04	081	01	126	12	171	08		
037	07	082	04	127	43	172	81		
038	42	083	07	128	00	173	55		
039	00	084	05	129	06	174	43		
040	02	085	08	130	81	175	00		
041	01	086	42	131	46	176	06		
042	05	087	01	132	13	177	95		
043	93	088	05	133	43	178	81		
044	04	089	05	134	01	179	46		

TITLE: PLY DATA:				NO: 051	
A' X, e _L	B' X', e _L '	C' Y, e _T	D' Y', e _T '	E' S, e _S	
A E _L	B E _T	C v _{LT}	D G _{LT}	E	

00 S	<u>REGISTER</u>				
01 I _{1Q}	02 I _{2Q}	03 R _{1Q}	04 R _{2Q}	05 E _L	06 E _T
07 0	08 0	09 v _{LT}	10 G _{LT}		
11 I _{1S}	12 I _{2S}	13 R _{1S}	14 R _{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	051 A&B		A	E _L
				B	E _T
				C	v _{LT}
				D	G _{LT}
				A'	X
				RUN	e _L
				B'	X'
				RUN	e _L '
				C'	Y
				RUN	e _T
				D'	Y'
				RUN	e _T
				E'	S
				RUN	e _S

PROGRAM TITLE: PLY DATA

NO: 051

PROGRAM LISTING:

TITLE: PLY DATA:

NO: 053

A' X, e_L	B' X', $e_{L'}$	C' Y, e_T	D' Y', $e_{T'}$	E' S, e_S
A E_L	B E_T	C v_{LT}	D G_{LT}	E

00 S	<u>REGISTER</u>				
01 I_{1Q}	02 I_{2Q}	03 R_{1Q}	04 R_{2Q}	05 E_L	06 E_T
07 0	08 0	09 v_{LT}	10 G_{LT}		
11 I_{1S}	12 I_{2S}	13 R_{1S}	14 R_{2S}	15 X	16 X'
17 0	18 0	19 0			
98 Y	99 Y'	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	053: A&B		A	E_L
				B	E_T
				C	v_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	$e_{L'}$
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	$e_{T'}$
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA

NO: 053

PROGRAM LISTING:

TITLE: PLY DATA:				NO: 055	
A'	X, e_L	B'	X', $e_{L'}$	C'	Y, e_T
A	E_L	B	E_T	C	v_{LT}
				D	G_{LT}
				E'	S, e_S
				D	G_{LT}
				E	

00	S	<u>REGISTER</u>									
01	I_{1Q}	02	I_{2Q}	03	R_{1Q}	04	R_{2Q}	05	E_L	06	E_T
07	0	08	0	09	v_{LT}	10	G_{LT}				
11	I_{1S}	12	I_{2S}	13	R_{1S}	14	R_{2S}	15	X	16	X'
17	0	18	0	19	0						
98	Y	99	Y'	<u>INSTRUCTIONS</u>							

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	055 A&B		A	E_L
				B	E_T
				C	v_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	$e_{L'}$
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA

NO: 055

PROGRAM LISTING:

TITLE: PLY DATA:				NO: 057	
A'	X, e_L	B'	X', $e_{L'}$	C'	Y, e_T
A	E_L	B	E_T	C	v_{LT}
				D	G_{LT}
				E	

00	S	<u>REGISTER</u>			
01	I_{1Q}	02	I_{2Q}	03	R_{1Q}
		04	R_{2Q}	05	E_L
		06	E_T		
07	0	08	0	09	v_{LT}
		10	G_{LT}		
11	I_{1S}	12	I_{2S}	13	R_{1S}
		14	R_{2S}	15	X
		16	X'		
17	0	18	0	19	0
98	Y	99	Y'	<u>INSTRUCTIONS</u>	

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	057 A&B		A	E_L
				B	E_T
				C	v_{LT}
				D	G_{LT}
				A'	X
				RUN	e_L
				B'	X'
				RUN	$e_{L'}$
				C'	Y
				RUN	e_T
				D'	Y'
				RUN	e_T
				E'	S
				RUN	e_S

PROGRAM TITLE: PLY DATA

NO: 057

PROGRAM LISTING:

TITLE: STORAGE OF CONSTITUENT PROPERTIES

NO: 059

A' v_m	B' η_1	C' η_2	D' η_6 or η	E' v_f
A E_f, α_f	B G_f, α_m	C v_f	D E_m	E G_m

REGISTER						
00						
01 E_f	02 G_f	03 v_f	04 E_m	05 G_m	06 v_m	
07 η_1	08 η_2	09 η_6 or η	10 v_f			
11 v_m	12 α_f	13 α_m	14	15	16	
17	18	19				
98	99					

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	059 A&B		INV st fg.	
				0	
Input data	2		E_f	A	E_f
			G_f	B	G_f
			v_f	C	v_f
			E_m	D	E_m
			G_m	E	G_m
			v_m	A'	v_m
			η_1	B'	η_1
			η_2	C'	η_2
			η_6 or η	D'	η_6 or η
			v_f	E'	v_f
				st fg	
				0	
			α_f	A	
			α_m	B	

PROGRAM TITLE: STORAGE OF CONSTITUENT PROPERTIES NO: 059

PROGRAM LISTING:

000	46	045	46	090	46
001	11	046	14	091	17
002	60	047	60	092	42
003	00	048	00	093	00
004	00	049	00	094	07
005	01	050	05	095	81
006	01	051	06	096	46
007	42	052	42	097	18
008	00	053	00	098	42
009	01	054	04	099	00
010	81	055	81	100	08
011	42	056	42	101	81
012	01	057	01	102	46
013	02	058	05	103	19
014	81	059	81	104	42
015	46	060	46	105	00
016	12	061	15	106	09
017	60	062	60	107	81
018	00	063	00	108	46
019	00	064	00	109	10
020	02	065	07	110	42
021	06	066	01	111	01
022	42	067	42	112	00
023	00	068	00	113	01
024	02	069	05	114	75
025	81	070	81	115	43
026	42	071	42	116	01
027	01	072	01	117	00
028	03	073	06	118	95
029	81	074	81	119	42
030	46	075	46	120	01
031	13	076	16	121	01
032	60	077	60	122	81
033	00	078	00		
034	00	079	00		
035	04	080	08		
036	01	081	06		
037	42	082	42		
038	00	083	00		
039	03	084	06		
040	81	085	81		
041	42	086	42		
042	01	087	01		
043	04	088	07		
044	81	089	81		

FORMULAS:

$$E_1^* = \frac{1}{\eta_1 \nu_f + \nu_m} (\eta_1 \nu_f E_f + \nu_m E_m) \quad (128)$$

$$\nu_{12}^* = \frac{1}{\eta_1 \nu_f + \nu_m} (\eta_1 \nu_f \nu_f + \nu_m \nu_m) \quad (129)$$

$$\frac{1}{G_{cc}^*} = \left(\frac{\nu_f}{G_f} + \frac{\eta_c \nu_m}{G_m} \right) \frac{1}{\nu_f + \eta_c \nu_m} \quad (132)$$

$$\frac{1}{E^*} = \frac{\nu_f}{E_f} + \frac{\eta \nu_m}{E_m} \frac{1}{\nu_f + \eta \nu_m} \quad (146)$$

$$\nu^* = \frac{\nu_f \nu_f E_m + \eta \nu_m \nu_m E_f}{\nu_f E_m + \eta \nu_m E_f} \quad (147)$$

PROGRAM TITLE: COMPOSITE MODULI I

NO: 061

PROGRAM LISTING:

000 46	045 43	090 43	135 00	180 65
001 11	046 01	091 01	136 05	181 43
002 43	047 01	092 08	137 09	182 01
003 00	048 54	093 85	138 46	183 01
004 01	049 95	094 43	139 16	184 65
005 42	050 81	095 01	140 43	185 43
006 01	051 46	096 09	141 00	186 00
007 08	052 12	097 65	142 01	187 06
008 43	053 43	098 43	143 42	188 65
009 00	054 00	099 01	144 01	189 43
010 04	055 09	100 01	145 08	190 00
011 42	056 42	101 55	146 43	191 01
012 01	057 01	102 43	147 00	192 54
013 09	058 09	103 01	148 04	193 55
014 53	059 43	104 07	149 42	194 53
015 43	060 00	105 54	150 01	195 43
016 00	061 02	106 95	151 07	196 01
017 07	062 42	107 81	152 43	197 00
018 65	063 01	108 46	153 00	198 65
019 43	064 08	109 13	154 09	199 43
020 01	065 43	110 43	155 42	200 00
021 00	066 00	111 00	156 01	201 04
022 65	067 05	112 03	157 09	202 85
023 43	068 42	113 42	158 41	203 43
024 01	069 01	114 01	159 00	204 00
025 08	070 07	115 08	160 07	205 09
026 85	071 53	116 43	161 01	206 65
027 43	072 43	117 00	162 46	207 43
028 01	073 01	118 06	163 17	208 01
029 01	074 00	119 42	164 53	209 01
030 65	075 85	120 01	165 43	210 65
031 43	076 43	121 09	166 01	211 43
032 01	077 01	122 41	167 00	212 00
033 09	078 09	123 00	168 65	213 01
034 54	079 65	124 01	169 43	214 54
035 55	080 43	125 04	170 00	215 95
036 53	081 01	126 46	171 03	216 81
037 43	082 01	127 14	172 65	
038 00	083 54	128 43	173 43	
039 07	084 55	129 00	174 00	
040 65	085 53	130 08	175 04	
041 43	086 43	131 42	176 85	
042 01	087 01	132 01	177 43	
043 00	088 00	133 09	178 00	
044 85	089 55	134 41	179 09	

Input data with Program 059

$$\begin{aligned} E_f &= 10.5 \times 10^6 \\ G_f &= 4.38 \times 10^6 \\ \nu_f &= .2 \end{aligned}$$

$$\begin{aligned} E_m &= .5 \times 10^6 \\ G_m &= .185 \times 10^6 \\ \nu_m &= .35 \end{aligned}$$

$$\begin{aligned} n_1 &= 1 \\ n_2 &= .5 \\ n_6 \text{ or } n &= .4 \\ \nu_f &= .6 \end{aligned}$$

$$\begin{aligned} E_L &= 6.5 \times 10^6 \\ G_{LT} &= 758.6 \times 10^3 \\ \nu_{LT} &= .25 \\ G_{TT} &= 656.8 \times 10^6 \\ E &= 2.0152 \times 10^6 \\ \nu &= .3273 \end{aligned}$$

PROGRAM TITLE: COMPOSITE MODULI II

NO: 065

FORMULAS:

$$\frac{1}{E_a^*} = \left(\frac{\nu_f}{E_f} + \frac{\eta_a \nu_m}{E_m} \right) \frac{1}{\nu_f + \eta_a \nu_m} - \nu_f \nu_m \frac{(\eta_1 \eta_a E_f \nu_m - E_m \nu_f)(\nu_m / E_m - \nu_f / E_f)}{(\eta_1 \nu_f E_f + \nu_m E_m)(\nu_f + \eta_a \nu_m)} \quad (130)$$

$$\nu_{TT} = \frac{E_T}{2G_{TT}} - 1$$

PROGRAM TITLE: COMPOSITE MODULI II

NO: 065

PROGRAM LISTING:

000	46	045	65	090	53	135	55
001	11	046	53	091	53	136	43
002	53	047	43	092	43	137	01
003	43	048	00	093	00	138	09
004	01	049	07	094	07	139	54
005	00	050	65	095	65	140	20
006	55	051	43	096	43	141	75
007	43	052	00	097	01	142	01
008	00	053	08	098	00	143	95
009	01	054	65	099	65	144	81
010	85	055	43	100	43		
011	43	056	00	101	00		
012	00	057	01	102	01		
013	08	058	65	103	85		
014	65	059	43	104	43		
015	43	060	00	105	01		
016	01	061	06	106	01		
017	01	062	75	107	65		
018	55	063	43	108	43		
019	43	064	00	109	00		
020	00	065	04	110	04		
021	04	066	65	111	54		
022	54	067	43	112	65		
023	55	068	00	113	53		
024	53	069	03	114	43		
025	43	070	54	115	01		
026	01	071	65	116	00		
027	00	072	53	117	85		
028	85	073	43	118	43		
029	43	074	00	119	00		
030	00	075	06	120	08		
031	08	076	55	121	65		
032	65	077	43	122	43		
033	43	078	00	123	01		
034	01	079	04	124	01		
035	01	080	75	125	54		
036	54	081	43	126	54		
037	75	082	00	127	95		
038	43	083	03	128	20		
039	01	084	55	129	42		
040	00	085	43	130	01		
041	65	086	00	131	09		
042	43	087	01	132	81		
043	01	088	54	133	65		
044	01	089	55	134	02		

SAMPLE PROBLEM: 065 COMPOSITE MODULI

NO: 065

Input data with Program 059

$$E_f = 10.5 \times 10^6$$

$$G_f = 4.38 \times 10^6$$

$$\nu_f = .2$$

$$E_m = .5 \times 10^6$$

$$G_m = .185 \times 10^6$$

$$\nu_m = .35$$

$$\eta_1 = 1$$

$$\eta_2 = .5$$

$$\eta_6 \text{ or } \eta = .4$$

$$\nu_f = .6$$

$$G_{TT} = 656.8 \times 10^3$$

$$E_T = 1.9349 \times 10^6$$

$$\nu = .473$$

PROGRAM TITLE: THERMAL EXPANSION COEFFICIENTS NO: 069

FORMULAS:

$$a_L = a_1^* = (v_f E_f a_f + v_m E_m a_m) / (\eta_1 v_f E_f + v_m E_m) \quad (162)$$

$$a_T = a_2^* = v_f a_f + v_m a_m + (v_f a_f v_f + v_m a_m v_m) - (\eta_1 v_f v_f + v_m v_m) a_1^* \quad (163)$$

PROGRAM LISTING:

P

```
000 46 030 00 060 00 090 00
001 11 031 07 061 03 091 07
002 53 032 65 062 54 092 65
003 43 033 43 063 65 093 43
004 01 034 01 064 43 094 00
005 00 035 00 065 01 095 03
006 65 036 65 066 00 096 85
007 43 037 43 067 65 097 43
008 00 038 00 068 43 098 00
009 01 039 01 069 01 099 06
010 65 040 85 070 02 100 54
011 43 041 43 071 85 101 65
012 01 042 01 072 53 102 43
013 02 043 01 073 01 103 01
014 85 044 65 074 85 104 01
015 43 045 43 075 43 105 65
016 01 046 00 076 00 106 43
017 01 047 04 077 06 107 01
018 65 048 54 078 54 108 09
019 43 049 95 079 85 109 95
020 00 050 42 080 43 110 81
021 04 051 01 081 01
022 65 052 09 082 01
023 43 053 81 083 65
024 01 054 46 084 43
025 03 055 12 085 01
026 54 056 53 086 03
027 55 057 01 087 75
028 53 058 85 088 53
029 43 059 43 089 43
```

TITLE: LONGITUDINAL STRENGTH				NO: 071	
A' X (brittle)	B' X (ductile)	C' δ	D' X (Statist)	E'	
A X_f	B X_m	C δ_f	D X_{fo}	E α	

00	<u>REGISTER</u>				
01 E_f	02 G_f	03 ν_f	04 E_m	05 G_m	06 ν_m
07	08	09	10 ν_f		
11 ν_m	12	13	14 X_f	15 X_m	16 δ_f
17 X_{fo}	18 α	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Store data with Program 059	1				
Input data	2	071 A&B	X_f X_m δ_f X_{fo} α	A B C D E	X_f X_m δ_f X_{fo} α
Compute strength	3			A' B' C' RUN	X(Brittle) X(Ductile) δ X(Statistical)

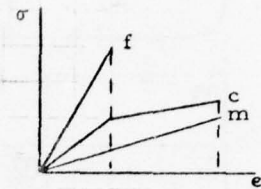
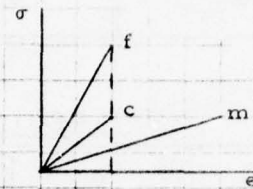
FORMULAS:

Brittle matrix

$$X = [v_f + (1-v_f)E_m/E_f]X_f \quad (164)$$

Ductile matrix (tough)

$$X = v_f X_f + v_m X_m \quad (165)$$



$$\bar{\delta} = \frac{\delta}{d_f} = \left[\frac{1}{2} \frac{E_f}{G_m} \frac{1 - \sqrt{v_f}}{\sqrt{v_f}} \right]^{1/2}$$

(188)

$$X = v_f (\text{strength of bundle of length } \delta)$$

$$= v_f X_B (L/\delta)^{-1/\alpha} \quad (182)$$

$$= v_f X_{fo} (\delta \alpha e)^{-1/\alpha} \quad (183)$$

PROGRAM TITLE: LONGITUDINAL STRENGTH

NO: 071

PROGRAM LISTING:

000	46	045	43	090	43
001	11	046	00	091	00
002	42	047	01	092	01
003	01	048	54	093	55
004	04	049	65	094	43
005	81	050	43	095	00
006	46	051	01	096	05
007	12	052	04	097	55
008	42	053	95	098	02
009	01	054	81	099	95
010	05	055	46	100	30
011	81	056	17	101	65
012	46	057	43	102	43
013	13	058	01	103	01
014	42	059	00	104	06
015	01	060	65	105	95
016	06	061	43	106	81
017	81	062	01	107	65
018	46	063	04	108	43
019	14	064	85	109	01
020	42	065	43	110	08
021	01	066	01	111	65
022	07	067	01	112	01
023	81	068	65	113	22
024	46	069	43	114	23
025	15	070	01	115	95
026	42	071	05	116	45
027	01	072	95	117	43
028	08	073	81	118	01
029	81	074	46	119	08
030	46	075	18	120	20
031	16	076	53	121	94
032	53	077	01	122	95
033	43	078	75	123	65
034	01	079	43	124	43
035	00	080	01	125	01
036	85	081	00	126	07
037	43	082	30	127	65
038	01	083	54	128	43
039	01	084	55	129	01
040	65	085	43	130	00
041	43	086	01	131	95
042	00	087	00	132	81
043	04	088	30		
044	55	089	65		

TITLE: IN-PLANE INVARIANTS FOR $[0_p / 90_q / 45_r / -45_s]$ NO: 075

A'	B' δ_{1A}	C' δ_{2A}	D'	E'
A Initialize	B I_{1A}	C I_{2A}	D R_{1A}	E R_{2A}

00	REGISTER				
01 $I_1 = I_{1A}$	02 $I_2 = I_{2A}$	03 R_1, R_{1A}	04 R_2, R_{2A}	05	06
07 δ_1	08 δ_2	09	10		
11	12	13	14	15	16
17 n_0	18 n_{90}	19 n			
98 n_{45}	99 n_{-45}	INSTRUCTIONS			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input ply data and initialize	1	045 A&B		A	E_L
Input n_θ and	2	075 A&B	N	A	N
Compute invariants of A_{ij}			n_0	RUN	n_0/N
			n_{90}	RUN	n_{90}/N
			n_{45}	RUN	n_{45}/N
			n_{-45}	RUN	n_{-45}/N
				B	I_{1A}
				C	I_{2A}
				D	R_{1A}
				E	R_{2A}
				B'	δ_{1A}
				C'	δ_{2A}

PROGRAM TITLE: IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/-45_s]$ NO: 075

FORMULAS:

TABLE 29 FORMULAS FOR IN-PLANE MODULUS FOR $[0_p/90_q/45_r/-45_s]$

	I_1	I_2	$\sqrt{V_1^2 + V_3^2} R_1$	$\sqrt{V_2^2 + V_4^2} R_2$
A'_{11}/h	1	1	$\cos 2(\theta - \delta_1)$	$\cos 4\theta$
A'_{22}/h	1	1	$-\cos 2(\theta - \delta_1)$	$\cos 4\theta$
A'_{12}/h	1	-1	0	$-\cos 4\theta$
A'_{66}/h	0	1	0	$-\cos 4\theta$
A'_{16}/h	0	0	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$-\sin 4\theta$
A'_{26}/h	0	0	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$\sin 4\theta$

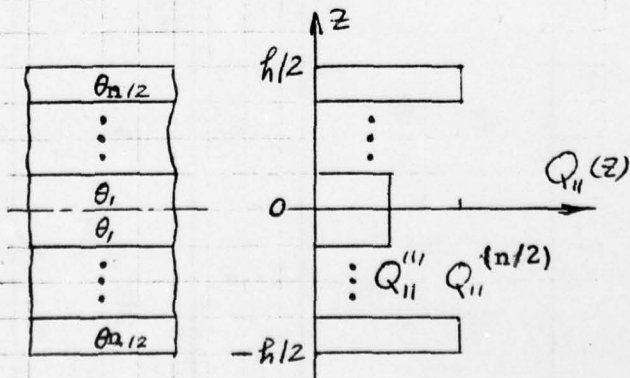
$$\sqrt{V_1^2 + V_3^2} = \frac{1}{N} \sqrt{(n_0 - n_{90})^2 + (n_{45} - n_{-45})^2} = R_{1A} / R_1 \quad (229)$$

$$\sqrt{V_2^2 + V_4^2} = \frac{1}{N} |(n_0 + n_{90} - n_{45} - n_{-45})| = R_{2A} / R_2 \quad (230)$$

$$\tan 2\delta_1 = \frac{n_{+45} - n_{-45}}{n_0 - n_{90}} \quad (231)$$

Laminate is orthotropic if $n_{45} = n_{-45}$.

Figure 61 Ply orientations and modulus of symmetric laminates.



PROGRAM TITLE: IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/-45_s]$ NO: 075

PROGRAM LISTING:

000	46	045	43	090	49	135	81	180	07
001	11	046	09	091	00	136	46	181	81
002	42	047	08	092	04	137	87	182	46
003	01	048	95	093	43	138	43	183	18
004	09	049	40	094	09	139	01	184	43
005	81	050	85	095	08	140	06	185	00
006	55	051	53	096	75	141	90	186	08
007	43	052	43	097	43	142	88	187	81
008	01	053	01	098	09	143	04		
009	09	054	07	099	09	144	05		
010	95	055	75	100	95	145	42		
011	42	056	43	101	42	146	00		
012	01	057	01	102	01	147	07		
013	07	058	08	103	06	148	41		
014	81	059	54	104	55	149	89		
015	55	060	40	105	53	150	46		
016	43	061	95	106	43	151	88		
017	01	062	30	107	01	152	00		
018	09	063	42	108	07	153	42		
019	95	064	00	109	75	154	00		
020	42	065	05	110	43	155	07		
021	01	066	49	111	01	156	41		
022	08	067	00	112	08	157	89		
023	81	068	03	113	54	158	46		
024	55	069	43	114	90	159	13		
025	43	070	01	115	87	160	43		
026	01	071	07	116	95	161	00		
027	09	072	85	117	94	162	02		
028	95	073	43	118	22	163	81		
029	42	074	01	119	34	164	46		
030	09	075	08	120	55	165	14		
031	08	076	75	121	02	166	43		
032	81	077	43	122	95	167	00		
033	55	078	09	123	42	168	03		
034	43	079	08	124	00	169	81		
035	01	080	75	125	07	170	46		
036	09	081	43	126	46	171	15		
037	95	082	09	127	89	172	43		
038	42	083	09	128	00	173	00		
039	09	084	95	129	42	174	04		
040	09	085	40	130	00	175	81		
041	81	086	30	131	08	176	46		
042	46	087	42	132	43	177	17		
043	12	088	00	133	00	178	43		
044	75	089	06	134	01	179	00		

SAMPLE PROBLEM: 075 IN-PLANE INVARIANTS FOR $[0_p/90_q/45_r/45_s]$

1. T-300/5208

$$I_1 = 49.49, I_2 = 26.88, R_1 = 85.732, R_2 = 19.717$$

$$\begin{aligned} \text{a) } [0/90] \quad I_{1A} &= 49.49 & R_{1A} &= 0 & \delta_{1A} &= 0 = \delta_2 \\ I_{2A} &= 26.88 & R_{2A} &= 19.72 \end{aligned}$$

$$\begin{aligned} \text{b) } [45/-45] \quad I_{1A} &= 49.49 & R_{1A} &= 0 & \delta_{1A} &= 0 = \delta_2 \\ I_{2A} &= 26.88 & R_{2A} &= 19.72 \end{aligned}$$

$$\begin{aligned} \text{c) } [0/90/45/-45] \quad I_{1A} &= 49.49 & R_{1A} &= 0 & \delta_{1A} &= 0 = \delta_2 \\ I_{2A} &= 26.88 & R_{2A} &= 0 \end{aligned}$$

$$\begin{aligned} \text{d) } [0/45] \quad I_{1A} &= 49.49 & R_{1A} &= 60.62 & \delta_{1A} &= -22.50 \\ I_{2A} &= 26.88 & R_{2A} &= 0 & \delta_{2A} &= 0 \end{aligned}$$

2. Same as 1 but for B(4)/5505

$$\begin{aligned} \text{a) } [0/90] \quad I_{1A} &= 58.0 & R_{1A} &= 0 & \delta_1 &= 0 = \delta_2 \\ I_{2A} &= 29.80 & R_{2A} &= 24.0 \end{aligned}$$

TITLE: IN-PLANE MODULUS AND COMPLIANCE				NO: 079	
A'	B' $\frac{A}{h}_{16}, ha_{16}$	C' $\frac{A}{h}_{26}, ha_{26}$	D'	E'	
A $\frac{1}{h} A_{ij} $	B $\frac{A}{h}_{11}, ha_{11}$	C $\frac{A}{h}_{22}, ha_{22}$	D $\frac{A}{h}_{12}, ha_{12}$	E $\frac{A}{h}_{66}, ha_{66}$	

00 θ	<u>REGISTER</u>				
01 I_{1A}, ha_{11}	02 I_{2A}, ha_{22}	03 R_{1A}, ha_{12}	04 R_{2A}, ha_{66}	05 ha_{16}	06 ha_{26}
07 δ_{1A}	08 δ_{2A}	09	10		
11 $\frac{A}{h}_{11}$	12 $\frac{A}{h}_{22}$	13 $\frac{A}{h}_{12}$	14 $\frac{A}{h}_{66}$	15 $\frac{A}{h}_{16}$	16 $\frac{A}{h}_{26}$
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 075					
Compute A_{ij}	3	041 A&B	θ (deg)	B	A_{11}/h
				C	A_{22}/h
				D	A_{12}/h
				E	A_{66}/h
				B'	A_{16}/h
				C'	A_{26}/h
Invert A_{ij}	4	027 A&B		A	ha_{26}
				RCL 01	ha_{11}
				RCL 02	ha_{22}
				RCL 03	ha_{12}
				RCL 04	ha_{66}
				RCL 05	ha_{16}
				RCL 06	ha_{26}

FORMULAS:

TABLE 29 FORMULAS FOR IN-PLANE MODULUS FOR $[0_p/90_q/45_r/-45_s]$

	I_1	I_2	R_{1A}	R_{2A}
A'_{11}/h	1	1	$\cos 2(\theta - \delta_1)$	$\cos 4\theta$
A'_{22}/h	1	1	$-\cos 2(\theta - \delta_1)$	$\cos 4\theta$
A'_{12}/h	1	-1	0	$-\cos 4\theta$
A'_{66}/h	0	1	0	$-\cos 4\theta$
A'_{16}/h	0	0	$-\frac{1}{2}\sin 2(\theta - \delta_1)$	$-\sin 4\theta$
A'_{26}/h	0	0	$-\frac{1}{2}\sin 2(\theta - \delta_1)$	$\sin 4\theta$

Let Compliance = $a_{ij} = [A_{ij}]^{-1}$ or $a_{ij} A_{jk} = \delta_{ik}$ (204)

where

$$\begin{aligned}
 a_{11} &= \frac{1}{\Delta} (A_{22}A_{66} - A_{26}^2), & a_{22} &= \frac{1}{\Delta} (A_{11}A_{66} - A_{16}^2) \\
 a_{12} &= \frac{1}{\Delta} (A_{16}A_{26} - A_{12}A_{66}), & a_{66} &= \frac{1}{\Delta} (A_{11}A_{22} - A_{12}^2) \\
 a_{16} &= \frac{1}{\Delta} (A_{12}A_{26} - A_{22}A_{16}), & a_{26} &= \frac{1}{\Delta} (A_{12}A_{16} - A_{11}A_{26})
 \end{aligned}
 \tag{206}$$

$$\Delta = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix}$$

In-Plane Engineering Constants:

$$E_{11}^o = \frac{1}{ha_{11}}, \quad E_{22}^o = \frac{1}{ha_{22}}, \quad \nu_{12}^o = -\frac{a_{12}}{a_{11}}, \quad \nu_{21}^o = -\frac{a_{12}}{a_{22}}, \quad G_{12}^o = \frac{1}{ha_{66}}$$

TITLE: IN-PLANE STRAIN

NO: 081

A'	B'	C'	D'	E'
A N_1	B e_1^o	C e_2^o	D e_6^o	E

REGISTER					
00					
01 a_{11}	02 a_{22}	03 a_{12}	04 a_{66}	05 a_{16}	06 a_{26}
07 N_1	08 N_2	09 N_6	10		
11	12	13	14	15	16
17 e_1^o	18 e_2^o	19 e_6^o			
98	99				

INSTRUCTIONS

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 079					
Input N_1	5	019 A&B	N_1	A	N_1
			N_2	RUN	N_2
			N_6	RUN	N_6
Compute e_1^o				B	e_1^o
				C	e_2^o
				D	e_6^o

FORMULAS:

$$\text{Let Compliance} = a_{ij} = [A_{ij}]^{-1} \text{ or } a_{ij} A_{jk} = \delta_{ik} \quad (204)$$

$$\begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} \quad \text{or } e_i^o = a_{ij} N_j \quad (205)$$

TITLE: PLY STRESS FOR $[0_p / 90_q / 45_r / -45_s]$

NO: 083

A'	B' Q_{16}	C' Q_{26}	D'	E'
A	B Q_{11}, σ_1	C Q_{22}, σ_2	D Q_{12}, σ_6	E Q_{66}

00 α_i	REGISTER				
01 I_1	02 I_2	03 R_1	04 R_2	05	06
07 σ_1	08 σ_2	09 σ_6	10		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17 ϵ_1^o	18 ϵ_2^o	19 ϵ_6^o			
98	99	INSTRUCTIONS			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 081					
Input ply data and initialize	6	045 A&B		A	E_L
Compute $Q_{ij}(\alpha)$	7	041 A&B	$\alpha_1(\text{deg})$	B	Q_{11}
Compute $\sigma_i(\alpha)$	8	023 A&B		B	σ_1
				C	σ_2
				D	σ_6
Compute $Q_{ij}(\beta)$	Rpt 7	041 A&B	$\alpha_i(\text{deg})$ ($i=2, 3, \dots, n$)	B	Q_{11}
Compute $\sigma_i(\beta)$	Rpt 8	023 A&B		B	σ_1
				C	σ_2
				D	σ_6
Repeat steps 7 and 8 for each successive ply.					

FORMULAS:

a. Calculation of In-Plane Strains

$$\begin{aligned} e_1^o &= a_{11}N_1 + a_{12}N_2 + a_{16}N_6 \\ e_2^o &= a_{21}N_1 + a_{22}N_2 + a_{26}N_6 \\ e_6^o &= a_{61}N_1 + a_{62}N_2 + a_{66}N_6 \end{aligned} \tag{233}$$

or

$$\begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} \quad \text{or } e_i^o = a_{ij}N_j \tag{234}$$

b. Calculation of Stresses of the t-th ply.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}^{(t)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix}^{(t)} \begin{Bmatrix} e_1^o \\ e_2^o \\ e_6^o \end{Bmatrix} \tag{235}$$

For the a oriented ply, replace index "t" by "a" in the equation above

c. TABLE 31 SAMPLE CALCULATIONS OF PLY STRESSES
(For T-300/5208)

Laminate	Imposed N_i	Resulting e_i^o	$\sigma_i^{(0)}$	$\sigma_i^{(90)}$	$\sigma_i^{(45)}$	$\sigma_i^{(-45)}$
[0, 90]	1	.0104	1.893	.1088		
	0	-.0003	.0269	.0269		
	0	0	0	.0282		
[0/90]	1	.0101	1.8662	-.2279		
	1	.0101	.1338	1.4495		
	1	.1395	1.0	.9432		
[45/-45]	1	.0104			.5769	.5591
	0	-.003			.4231	.4224
	0	0			-.4331	.4268
[0/90/45/-45]	1	.0144	2.5971	.1412	.6399	.6060
	0	-.0042	-.0024	-.7294	.3679	.3584
	0	0	0	-.0393	-.4144	.4236

TITLE: INVARIANTS OF FLEXURAL RIGIDITY

NO: 085

A'	B'	C'	D'	E'
A Initialize	B	C	D	E

00 $2\alpha_t$	<u>REGISTER</u>				
01 I_1, I_{1D}	02 I_2, I_{2D}	03 R_1, R_{1D}	04 R_2, R_{2D}	05 t	06 $3t^2 - 3t + 1$
07 δ_{1D}	08 δ_{2D}	09	10		
11 F_{c2D}	12 F_{s2D}	13 F_{c4D}	14 F_{s4D}	15 $2z_o/h$	16
17	18	19 $4\alpha_t$			
98	99 $n/2$	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input ply data	1	045 A&B		A	E_L
Input data	2		$n/2$	STO 99	$n/2$
			$2z_o/h$	STO 15	$2z_o/h$
Initialize	3	085 A&B		A	$F_t(1)$
Compute in-variants of D_{ij}			α_1 (deg)	B	$F_t(2)$
			α_2 (deg)	RUN	$F_t(3)$
			.	.	.
			.	.	.
			.	.	.
			$\alpha_{n/2}$	RUN	$\frac{R_{1D}}{R_1}$
Recall				RCL 01	I_{1D}
				RCL 02	I_{2D}
				RCL 03	R_{1D}
				RCL 04	R_{2D}
				RCL 07	δ_1
				RCL 08	δ_2

FORMULAS:

TABLE 33 FORMULAS FOR FLEXURAL RIGIDITY WITH ISOTROPIC SUBSTRUCTURE

	$\left[1 - \left(\frac{2z_o}{h}\right)^3\right] I_1$	$\left[1 - \left(\frac{2z_o}{h}\right)^3\right] I_2$	$\sqrt{V_1^2 + V_3^2} R$	$\sqrt{V_2^2 + V_4^2} R_2$
$\frac{12}{h^3} D'_{11} - \left(\frac{2z_o}{h}\right)^3 Q'_{11}$	1	1	$\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{22} - \left(\frac{2z_o}{h}\right)^3 Q'_{11}$	1	1	$-\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{12} - \left(\frac{2z_o}{h}\right)^3 Q'_{12}$	1	-1	0	$-\cos 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{66} - \frac{1}{2} \left(\frac{2z_o}{h}\right)^3 (Q'_{11} - Q'_{12})$	0	1	0	$-\cos 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{16}$	0	0	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$-\sin 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{26}$	0	0	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$\sin 4(\theta - \delta_2)$

$$\sqrt{V_1^2 + V_3^2} = \left(\frac{2}{N}\right)^3 \sqrt{(\sum F_t \cos 2\alpha_t)^2 + (\sum F_t \sin 2\alpha_t)^2} = R_{1D} / R_1 \quad (259)$$

$$\sqrt{V_2^2 + V_4^2} = \left(\frac{2}{N}\right)^3 \sqrt{(\sum F_t \cos 4\alpha_t)^2 + (\sum F_t \sin 4\alpha_t)^2} = R_{2D} / R_2 \quad (260)$$

$$\tan 2\delta_2 = \frac{2(D_{16} + D_{26})}{D_{11} - D_{22}} = -\frac{V_3}{V_1} \quad (261)$$

$$\tan 4\delta_2 = \frac{4(D_{16} - D_{26})}{D_{11} + D_{22} - 2D_{12} - 4D_{66}} = -\frac{V_4}{V_2} \quad (262)$$

$$F_t = 3\left(t + \frac{z_o}{h_o}\right)^2 - 3\left(t + \frac{z_o}{h_o}\right) + 1 \quad (263)$$

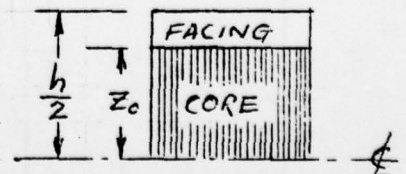


Figure 70 Sandwich Plate

PROGRAM TITLE: INVARIANTS OF FLEXURAL RIGIDITY NO: 085

PROGRAM LISTING:

000	46	045	65	090	00	135	05	180	40
001	11	046	03	091	06	136	85	181	85
002	01	047	75	092	95	137	01	182	43
003	75	048	43	093	44	138	95	183	01
004	43	049	00	094	01	139	41	184	04
005	01	050	05	095	03	140	88	185	40
006	05	051	65	096	43	141	46	186	95
007	45	052	03	097	00	142	87	187	30
008	03	053	85	098	00	143	43	188	55
009	95	054	01	099	32	144	01	189	43
010	49	055	95	100	65	145	02	190	09
011	00	056	42	101	43	146	55	191	09
012	01	057	00	102	00	147	43	192	45
013	49	058	06	103	06	148	01	193	03
014	00	059	81	104	95	149	01	194	95
015	02	060	46	105	44	150	94	195	49
016	00	061	12	106	01	151	95	196	00
017	42	062	65	107	02	152	22	197	04
018	01	063	04	108	43	153	34	198	43
019	01	064	94	109	01	154	55	199	01
020	42	065	95	110	09	155	02	200	01
021	01	066	42	111	32	156	95	201	40
022	02	067	01	112	65	157	42	202	85
023	42	068	09	113	43	158	00	203	43
024	01	069	55	114	00	159	07	204	01
025	03	070	02	115	06	160	43	205	02
026	42	071	95	116	95	161	01	206	40
027	01	072	42	117	44	162	04	207	95
028	04	073	00	118	01	163	55	208	30
029	01	074	00	119	04	164	43	209	55
030	85	075	33	120	43	165	01	210	43
031	43	076	65	121	09	166	03	211	09
032	01	077	43	122	09	167	94	212	09
033	05	078	00	123	75	168	95	213	45
034	65	079	06	124	43	169	22	214	03
035	43	080	95	125	00	170	34	215	95
036	09	081	44	126	05	171	55	216	49
037	09	082	01	127	95	172	04	217	00
038	95	083	01	128	90	173	95	218	03
039	46	084	43	129	87	174	42	219	81
040	88	085	01	130	22	175	00		
041	42	086	09	131	80	176	08		
042	00	087	33	132	87	177	43		
043	05	088	65	133	43	178	01		
044	40	089	43	134	00	179	03		

TITLE: FLEXURAL RIGIDITY AND COMPLIANCE				NO: 088	
A'	B' D_{16}, d_{16}	C' D_{26}, d_{26}	D'	E'	
A $ D_{ij} $	B D_{11}, d_{11}	C D_{22}, d_{22}	D D_{12}, d_{12}	E D_{66}, d_{66}	

00	REGISTER					
01 I_1, d_{11}	02 I_2, d_{22}	03 R_1, d_{12}	04 R_2, d_{66}	05 d_{16}	06 d_{26}	
07 δ_1	08 δ_2	09	10			
11 D_{11}	12 D_{22}	13 D_{12}	14 D_{66}	15 D_{16}	16 D_{26}	
17	18	19				
98	99	INSTRUCTIONS				

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 085					
Compute D_{ij}	4	041, A&B	θ (deg)	B C D E B' C'	D_{11} D_{22} D_{12} D_{66} D_{16} D_{26}
Invert D_{ij}	5	027 A&B		A RCL 01 RCL 02 RCL 03 RCL 04 RCL 05 RCL 06	d_{26} d_{11} d_{22} d_{12} d_{66} d_{16} d_{26}

PROGRAM TITLE: FLEXURAL RIGIDITY AND COMPLIANCE NO: 088

FORMULAS:

TABLE 33 FORMULAS FOR FLEXURAL RIGIDITY WITH ISOTROPIC SUBSTRUCTURE

	$\left[1 - \left(\frac{2z_o}{h}\right)^3\right]_{I_1}$	$\left[1 - \left(\frac{2z_o}{h}\right)^3\right]_{I_2}$	R_{1D}	R_{2D}
$\frac{12}{h^3} D'_{11} - \left(\frac{2z_o}{h}\right)^3 Q_{11}^o$	1	1	$\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{22} - \left(\frac{2z_o}{h}\right)^3 Q_{11}^o$	1	1	$-\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{12} - \left(\frac{2z_o}{h}\right)^3 Q_{12}^o$	1	-1	0	$-\cos 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{66} - \frac{1}{2} \left(\frac{2z_o}{h}\right)^3 (Q_{11}^o - Q_{12}^o)$	0	1	0	$-\cos 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{16}$	0	0	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$-\sin 4(\theta - \delta_2)$
$\frac{12}{h^3} D'_{26}$	0	0	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$\sin 4(\theta - \delta_2)$

AD-A067 987

AIR FORCE MATERIALS LAB WRIGHT-PATTERSON AFB OH
SR-52 MAGNETIC CARD CALCULATOR SOLUTIONS TO COMPOSITE MATERIALS--ETC(U)
JAN 79 S W TSAI, H T HAHN, F HUBER
AFML-TR-77-50

F/6 9/2

UNCLASSIFIED

NL

2 of 2
AD
A067987

END
DATE
FILMED
6-79
DDC

TITLE: CURVATURE DUE TO BENDING

NO: 090

A'	B'	C'	D'	E'
A M_i	B k_1	C k_2	D k_6	E

00	<u>REGISTER</u>				
01 d_{11}	02 d_{22}	03 d_{12}	04 d_{66}	05 d_{16}	06 d_{26}
07 M_1	08 M_2	09 M_6	10		
11	12	13	14	15	16
17 k_1	18 k_2	19 k_6			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from VI-1b					
Input M_i	6	019 A&B	M_1 M_2 M_6	A RUN RUN	M_1 M_2 M_6
Compute k_i				B C D	k_1 k_2 k_6

FORMULAS:

TABLE 50 FLEXURE-CURVATURE RELATIONS

	k_1	k_2	k_6
M_1	D_{11}	D_{12}	D_{16}
M_2	D_{21}	D_{22}	D_{26}
M_6	D_{61}	D_{62}	D_{66}

	M_1	M_2	M_6
k_1	d_{11}	d_{12}	d_{16}
k_2	d_{21}	d_{22}	d_{26}
k_6	d_{61}	d_{62}	d_{66}

Since $e_1 = zk_1$, $e_2 = zk_2$, $e_6 = zk_6$

Finally, from stress-strain relation of the t-th layer.

$$\left. \begin{aligned} \sigma_1 &= Q_{11}e_1 + Q_{12}e_2 + Q_{16}e_6 \\ \sigma_2 &= Q_{21}e_1 + Q_{22}e_2 + Q_{26}e_6 \\ \sigma_6 &= Q_{61}e_1 + Q_{62}e_2 + Q_{66}e_6 \end{aligned} \right\} \quad (281)$$

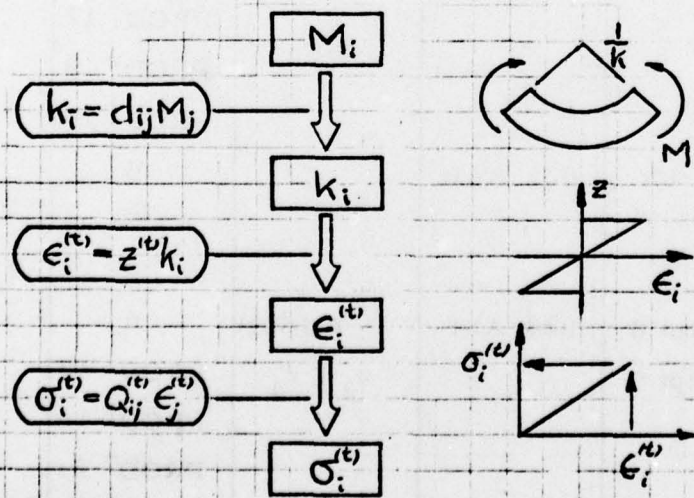


Figure 73 Flow diagram for ply stress calculation.

TITLE: PLY STRESS DUE TO BENDING				NO: 092	
A'	B' Q_{16}	C' Q_{26}	D'	E'	
A	B Q_{11}, σ_1	C Q_{22}, σ_2	D Q_{12}, σ_6	E Q_{66}	

00	<u>REGISTER</u>				
01 I_1	02 I_2	03 R_1	04 R_2	05	06
07 σ_1	08 σ_2	09 σ_6	10		
11 Q_{11}	12 Q_{22}	13 Q_{12}	14 Q_{66}	15 Q_{16}	16 Q_{26}
17 ϵ_1	18 ϵ_2	19 ϵ_6			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 00					
Input ply data & initialize	7	045 A&B		A	E_L
Compute $Q_{ij}(\alpha_1)$	8	041 A&B	$\alpha_1(\text{deg})$	B	Q_{11}
Compute $\epsilon_i(z_1)$	9		z_1	PROD* 17 PROD* 18 PROD* 19	
Compute $\sigma_i(z_1)$	10	023 A&B		B C D	σ_1 σ_2 σ_6
Compute $Q_{ij}(\alpha_2)$	Rpt 8	023: A&B	$\alpha_2(\text{deg})$	B	Q_{11}
Compute $\epsilon_i(z_2)$	Rpt 9		z_2 / z_1	PROD* 17 PROD* 18 PROD* 19	
Compute $\sigma_i(z_2)$	Rpt 10	023 A&B		B, C, D	σ_i

TITLE: INVARIANTS OF COUPLING MODULUS				NO: 093	
A' σ_i	B'	C'	D'	E'	
A Initialize	B a_t, R_{1B}	C δ_{1B}	D R_{2B}	E δ_{2B}	

00 $-2a_t$	<u>REGISTER</u>				
01	02	03 R_1, R_{1B}	04 R_2, R_{2B}	05 t	06 $2t - 1$
07 δ_{1B}	08 δ_{2B}	09	10		
11 F_{c2B}	12 F_{s2B}	13 F_{c4B}	14 F_{s4B}	15	16
17	18	19 $-4a_t$			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input ply data	1	045 A&B		A	E_L
Input data	2	093 A&B	n	A	n
Compute invariants	3		a_1 (deg)	B	a_1
			a_2 (deg)	RUN	a_2
			.	.	.
			.	.	.
Recall	4		a_N	RUN	R_{1B}
				C	δ_{1B}
				D	R_{2B}
				E	δ_{2B}

FORMULAS:

TABLE 56 FORMULA FOR TRANSFORMED COUPLING MODULUS

	$\sqrt{v_1^2 + v_3^2} R_1$	$\sqrt{v_2^2 + v_4^2} R_2$
$\frac{h^2}{2} B'_{11}$	$\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{22}$	$-\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{12}$	0	$-\cos 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{66}$	0	$-\cos 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{16}$	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$-\sin 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{26}$	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$\sin 4(\theta - \delta_2)$

$$v_1 = \frac{1}{N^2} \sum F_t \cos 2\alpha, \quad v_2 = \frac{1}{N^2} \sum F_t \cos 4\alpha \quad (322)$$

$$v_3 = -\frac{1}{N^2} \sum F_t \sin 2\alpha, \quad v_4 = -\frac{1}{N^2} \sum F_t \sin 4\alpha \quad (323)$$

$$\tan 2\delta_1 = -\frac{v_3}{v_1} \quad (320)$$

$$\tan 4\delta_2 = -\frac{v_4}{v_2} \quad (321)$$

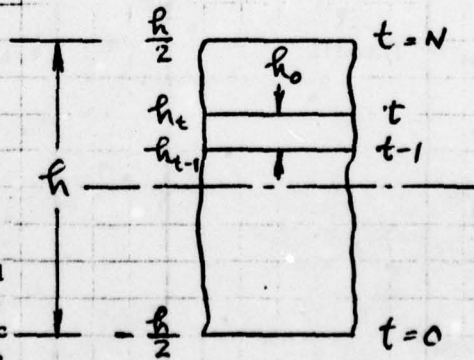


Figure 78 Integration of unsymmetric laminates must be performed for the entire thickness from $-h/2$ to $h/2$. (For symmetric laminates, integration can be limited to from 0 to $h/2$).

PROGRAM TITLE: INVARIANTS OF COUPLING MODULUS NO: 093

PROGRAM LISTING:

000	46	045	02	090	65	135	43	180	85
001	11	046	95	091	43	136	01	181	43
002	42	047	42	092	00	137	04	182	01
003	09	048	01	093	06	138	55	183	02
004	09	049	09	094	95	139	43	184	40
005	00	050	43	095	44	140	01	185	95
006	42	051	00	096	01	141	03	186	30
007	01	052	00	097	04	142	94	187	55
008	01	053	33	098	43	143	95	188	53
009	42	054	65	099	09	144	22	189	43
010	01	055	43	100	09	145	34	190	09
011	02	056	00	101	75	146	55	191	09
012	42	057	06	102	43	147	04	192	65
013	01	058	95	103	00	148	95	193	02
014	03	059	44	104	05	149	42	194	54
015	42	060	01	105	95	150	00	195	40
016	01	061	01	106	90	151	08	196	95
017	04	062	43	107	87	152	43	197	49
018	42	063	01	108	43	153	01	198	00
019	00	064	09	109	00	154	03	199	03
020	05	065	33	110	05	155	40	200	43
021	46	066	65	111	85	156	85	201	00
022	88	067	43	112	01	157	43	202	03
023	42	068	00	113	95	158	01	203	81
024	00	069	06	114	41	159	04	204	46
025	05	070	95	115	88	160	40	205	13
026	65	071	44	116	46	161	95	206	43
027	02	072	01	117	87	162	30	207	00
028	75	073	03	118	43	163	55	208	07
029	01	074	43	119	01	164	53	209	81
030	95	075	00	120	02	165	43	210	46
031	42	076	00	121	55	166	09	211	14
032	00	077	32	122	43	167	09	212	43
033	06	078	65	123	01	168	65	213	00
034	81	079	43	124	01	169	02	214	04
035	46	080	00	125	94	170	54	215	81
036	12	081	06	126	95	171	40	216	46
037	65	082	95	127	22	172	95	217	15
038	02	083	44	128	34	173	49	218	43
039	94	084	01	129	55	174	00	219	00
040	95	085	02	130	02	175	04	220	08
041	42	086	43	131	95	176	43	221	81
042	00	087	01	132	42	177	01		
043	00	088	09	133	00	178	01		
044	65	089	32	134	07	179	40		

NO: 093

TITLE: COUPLING MODULUS				NO: 097	
A'	B' B_{16}	C' B_{26}	D'	E'	
A	B B_{11}	C B_{22}	D B_{12}	E B_{66}	

00	<u>REGISTER</u>				
01	02	03 R_{1B}	04 R_{2B}	05	06
07 δ_{1B}	08 δ_{2B}	09	10		
11	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Continue from 093					
Compute B_{ij}	3	041 A&B	θ (deg)	B	B_{11}
Recall	3'			C	B_{22}
				D	B_{12}
				E	B_{66}
				B'	B_{16}
				C'	B_{26}

FORMULAS:

TABLE 56 FORMULA FOR TRANSFORMED COUPLING MODULUS

	$\sqrt{V_1^2 + V_3^2} R_1$	$\sqrt{V_2^2 + V_4^2} R_2$
$\frac{h^2}{2} B'_{11}$	$\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{22}$	$-\cos 2(\theta - \delta_1)$	$\cos 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{12}$	0	$-\cos 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{66}$	0	$-\cos 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{16}$	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$-\sin 4(\theta - \delta_2)$
$\frac{h^2}{2} B'_{26}$	$-\frac{1}{2} \sin 2(\theta - \delta_1)$	$\sin 4(\theta - \delta_2)$

$$V_1 = \frac{1}{N^2} \sum F_t \cos 2\alpha, \quad V_2 = \frac{1}{N^2} \sum F_t \cos 4\alpha \quad (322)$$

$$V_3 = \frac{1}{N^2} \sum F_t \sin 2\alpha, \quad V_4 = \frac{1}{N^2} \sum F_t \sin 4\alpha \quad (323)$$

$$\tan 2\delta_1 = -\frac{V_3}{V_1} \quad (320)$$

$$\tan 4\delta_2 = -\frac{V_4}{V_2} \quad (321)$$

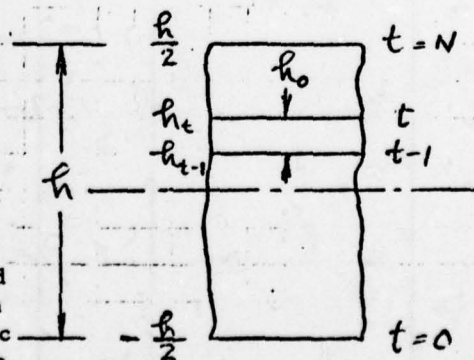


Figure 78 Integration of unsymmetric laminates must be performed for the entire thickness from $-h/2$ to $h/2$. (For symmetric laminates, integration can be limited to from 0 to $h/2$).

TITLE: MOISTURE CONTENT AND SWELLING STRAIN

NO: 099

A' e_T^H	B'	C'	D'	E'
A s	B v_v	C c	D α_T^H	E c_o

00	<u>REGISTER</u>				
01 s	02 v_v	03 C	04	05	06
07	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input data	1	099 A	s	A	s
Compute c	2		v_v c v_m s_m	B C RUN RUN	v_v c_m c_v c
Compute α_T^H	3		v_m	D	α_T^H
Compute c_o	4			E	c_o
Compute e_T^H	5		p p'	A' RUN RUN	e_T^H

PROGRAM TITLE: MOISTURE CONTENT AND SWELLING STRAIN

NO: 099

FORMULAS:

$$c = \frac{M' - M}{M} = \frac{c_m m_m + c_f m_f + M_{vw}}{M}$$

$$= \frac{(c_m v_m \rho_m + c_f v_f \rho_f + v_v \rho_w)}{\rho}$$

$$= \frac{(c_m v_m s_m + c_f v_f s_f + v_v)}{s} \quad (374)$$

$$\alpha_T^H = \frac{1 + v_m}{3} s$$

$$c_o = v_v / s$$

$$e_T^H = \frac{1}{2} \left[(1 + c) \frac{\rho}{\rho'} - 1 \right] \quad \text{cf. (402)}$$

PROGRAM LISTING:

```

000 46 030 00 060 01
001 11 031 03 061 85
002 42 032 81 062 43
003 00 033 46 063 00
004 01 034 14 064 03
005 81 035 85 065 54
006 46 036 01 066 65
007 12 037 95 067 81
008 42 038 55 068 55
009 00 039 03 069 81
010 02 040 65 070 75
011 81 041 43 071 01
012 46 042 00 072 95
013 13 043 01 073 55
014 81 044 95 074 02
015 65 045 81 075 95
016 81 046 46 076 81
017 65 047 15
018 81 048 43
019 85 049 00
020 43 050 02
021 00 051 55
022 02 052 43
023 95 053 00
024 55 054 01
025 43 055 95
026 00 056 81
027 01 057 46
028 95 058 16
029 42 059 53
    
```

TITLE: RESIDUAL STRESSES (MICRO)				NO: 101
A' Input Data	B'	C'	D'	E'
A e_f	B e_m	C $\sigma_{mL}^R, \sigma_{fL}^R$	D	E

00	<u>REGISTER</u>				
01 E_f	02	03	04 E_m	05	06
07 η_1	08	09	10 v_f		
11 v_m	12 e_f	13 e_m	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	101 A	E_f E_m η_1 v_f e_f	A' RUN RUN RUN RUN	
Compute stress	2		e_m	RUN A RUN	σ_{mL}^R σ_{fL}^R

PROGRAM TITLE: RESIDUAL STRESSES (MICRO)

NO: 101

FORMULAS:

Matrix

$$\sigma_{mL}^R = \frac{\nu_f E_f E_m (\epsilon_f - \eta_1 \epsilon_m)}{\eta_1 \nu_f E_f + \nu_m E_m} \quad (387)$$

$$\sigma_{mT}^R = 0 \quad (388)$$

Fiber

$$\sigma_{fL}^R = -\frac{\nu_m}{\nu_f} \sigma_{mL}^R = \frac{\nu_m E_m E_f (\eta_1 \epsilon_m - \epsilon_f)}{\eta_1 \nu_f E_f + \nu_m E_m} \quad (389)$$

$$\sigma_{fT}^R = 0 \quad (390)$$

PROGRAM LISTING:

000	46	030	01	060	55	090	01
001	16	031	03	061	53	091	00
002	42	032	81	062	43	092	94
003	00	033	46	063	00	093	95
004	01	034	11	064	07	094	81
005	81	035	43	065	65		
006	42	036	01	066	43		
007	00	037	00	067	01		
008	04	038	65	068	00		
009	81	039	43	069	65		
010	42	040	00	070	43		
011	00	041	01	071	00		
012	07	042	65	072	01		
013	81	043	43	073	85		
014	42	044	00	074	43		
015	01	045	04	075	01		
016	00	046	65	076	01		
017	94	047	53	077	65		
018	85	048	43	078	43		
019	01	049	01	079	00		
020	95	050	02	080	04		
021	42	051	75	081	54		
022	01	052	43	082	95		
023	01	053	00	083	81		
024	81	054	07	084	65		
025	42	055	65	085	43		
026	01	056	43	086	01		
027	02	057	01	087	01		
028	81	058	03	088	55		
029	42	059	54	089	43		

TITLE: STRESS ANALYSIS PARAMETERS

NO: 103

A' Eng'g Const.	B'	C'	D'	E'
A Parallel	B Perpendicular	C k, n, et al	D	E

00	<u>REGISTER</u>				
01 E_L	02 E_T	03 G_{LT}	04 ν_{LT}	05 S_{11}	06 S_{22}
07 S_{66}	08 S_{12}	09 k	10 n		
11 $\sqrt{n^2 - 4k}$	12 μ_1	13 μ_2	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	103 A&B	E_L E_T G_{LT} ν_{LT}	A' RUN RUN RUN	S_{12}
Compute parameters Crack parallel to fibers	2			A RUN RUN RUN	k n μ_1 μ_2
Crack perpendicular to fibers				B RUN RUN RUN	k n μ_1 μ_2

PROGRAM TITLE: STRESS ANALYSIS PARAMETERS

NO: 103

FORMULAS:

$$\mu_1 = \frac{1}{2} \left[n + (n^2 - 4k)^{1/2} \right], \quad \mu_2 = \frac{1}{2} \left[n - (n^2 - 4k)^{1/2} \right] \quad (511)$$

$$n = \left[2 \left(\sqrt{\frac{S_{22}}{S_{11}}} + \frac{S_{12}}{S_{11}} \right) + \frac{S_{66}}{S_{11}} \right]^{1/2}, \quad k = \left(\frac{S_{22}}{S_{11}} \right)^{1/2} \quad (512)$$

PROGRAM TITLE: STRESS ANALYSIS PARAMETERS

NO: 103

PROGRAM LISTING:

000	46	045	95	090	81	135	01
001	16	046	94	091	40	136	20
002	42	047	42	092	75	137	42
003	00	048	00	093	04	138	00
004	01	049	08	094	65	139	06
005	81	050	81	095	43	140	43
006	42	051	46	096	00	141	00
007	00	052	11	097	09	142	02
008	02	053	43	098	95	143	20
009	81	054	00	099	30	144	42
010	42	055	06	100	42	145	00
011	00	056	55	101	01	146	05
012	03	057	43	102	01	147	43
013	81	058	00	103	85	148	00
014	42	059	05	104	43	149	03
015	00	060	95	105	01	150	20
016	04	061	30	106	00	151	42
017	43	062	42	107	95	152	00
018	00	063	00	108	95	153	07
019	01	064	09	109	55	154	41
020	20	065	81	110	02	155	11
021	42	066	85	111	95	156	81
022	00	067	43	112	42		
023	05	068	00	113	01		
024	43	069	08	114	02		
025	00	070	55	115	81		
026	02	071	43	116	43		
027	20	072	00	117	01		
028	42	073	05	118	00		
029	00	074	95	119	75		
030	06	075	65	120	43		
031	43	076	02	121	01		
032	00	077	85	122	01		
033	03	078	43	123	95		
034	20	079	00	124	55		
035	42	080	07	125	02		
036	00	081	55	126	95		
037	07	082	43	127	42		
038	43	083	00	128	01		
039	00	084	05	129	03		
040	04	085	95	130	81		
041	55	086	30	131	46		
042	43	087	42	132	12		
043	00	088	01	133	43		
044	01	089	00	134	00		

SAMPLE PROBLEM: 103 STRESS ANALYSIS PARAMETERS NO: 103

$$E_L = 181 \times 10^9$$

$$E_T = 10.3 \times 10^9$$

$$G_{LT} = 7.17 \times 10^9$$

$$\nu_{LT} = .28$$

Parallel to fibers

$$k = 4.192$$

$$n = 5.750$$

$$\mu_1 = 4.894$$

$$\mu_2 = 856.6 \times 10^{-3}$$

Perpendicular to fibers

$$k = 238.5 \times 10^{-3}$$

$$n = 1.372$$

$$\mu_1 = 1.167$$

$$\mu_2 = 204.3 \times 10^{-3}$$

TITLE: CRACK TIP STRESSES

NO: 107

A'	B'	C'	D'	E'
A σ_x	B σ_y	C τ_{xy}	D	E

00	<u>REGISTER</u>				
01 E_L	02 E_T	03 G_{LT}	04 ν_{LT}	05 S_{11}	06 S_{12}
07 S_{66}	08 S_{12}	09 k	10 n		
11 $\sqrt{n^2 - 4k}$	12 μ_1	13 μ_2	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Compute parameters with 103	1				
Compute crack tip stresses	2	107 A&B		A B C	$\sigma_x \sqrt{2\pi r}/K_I$ $\sigma_y \sqrt{2\pi r}/K_I$ $\tau_{xy} \sqrt{2\pi r}/K_I$

FORMULAS:

$$\theta = 90^\circ$$

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \frac{k}{\sqrt{2} \bar{\mu}} (\sqrt{\mu_1} - \sqrt{\mu_2}), \quad \sigma_y = \frac{K_I}{\sqrt{2\pi r}} \frac{1}{\sqrt{2} \bar{\mu}} \left(\frac{\mu_1}{\sqrt{\mu_2}} - \frac{\mu_2}{\sqrt{\mu_1}} \right)$$

(518)

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \frac{k}{\sqrt{2} \bar{\mu}} \left(\frac{1}{\sqrt{\mu_1}} - \frac{1}{\sqrt{\mu_2}} \right), \quad \bar{\mu} = \mu_1 - \mu_2$$

PROGRAM LISTING:

000	46	030	75	060	54
001	11	031	43	061	95
002	43	032	01	062	46
003	00	033	03	063	87
004	09	034	55	064	55
005	65	035	43	065	02
006	53	036	01	066	30
007	43	037	02	067	55
008	01	038	30	068	53
009	02	039	51	069	43
010	30	040	87	070	01
011	75	041	81	071	02
012	43	042	46	072	75
013	01	043	13	073	43
014	03	044	43	074	01
015	30	045	00	075	03
016	54	046	09	076	54
017	51	047	65	077	95
018	87	048	53	078	56
019	81	049	43		
020	46	050	01		
021	12	051	02		
022	43	052	30		
023	01	053	20		
024	02	054	75		
025	55	055	43		
026	43	056	01		
027	01	057	03		
028	03	058	30		
029	30	059	20		

TITLE: Y FOR CENTER-CRACKED PLATE AND COD

NO: 109

A' Constants	B' $E_L \cdot \gamma$	C' τ_y	D'	E'
A a	B w	C Y	D COD	E

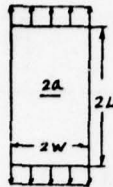
00	<u>REGISTER</u>				
01 a	02 w	03 a/w	04 1.5254	05 -0.2881	06 0.1282
07 E_L	08 $\gamma = 2n$	09 τ_y	10 Y		
11 σ	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter constants	1	109 A&B	0.1282	A'	.1282
			-0.2881	RUN	-0.2881
			1.5254	RUN	1.5254
Enter data	2		a	A	a
			w	B	w
Compute Y	3			C	Y
Enter data	4		E_L	B'	E_L
			γ	RUN	γ
Compute COD	5		τ_y	C'	τ_y
			σ	D	COD

PROGRAM TITLE: Y FOR CENTER-CRACKED PLATE AND
COD

NO: 109

FORMULAS:



Isotropic plates

$$Y = 1 + 0.1282 (a/w) - 0.2881 (a/w)^2 + 1.5254 (a/w)^3 \quad (516)$$

Total COD

COD = elastic COD + plastic CTOD

$$= \frac{Y a \sigma}{E_L} \left(\gamma + \frac{\pi}{2} \frac{Y}{r} \sigma \right) \quad (526)$$

PROGRAM TITLE: Y FOR CENTER-CRACKED PLATE AND
COD

NO: 109

PROGRAM LISTING:

000	46	045	06	090	09
001	11	046	85	091	54
002	42	047	01	092	95
003	00	048	95	093	81
004	01	049	42	094	46
005	81	050	01	095	16
006	46	051	00	096	42
007	12	052	81	097	00
008	42	053	46	098	06
009	00	054	14	099	81
010	02	055	42	100	42
011	81	056	01	101	00
012	46	057	01	102	05
013	13	058	65	103	81
014	55	059	43	104	42
015	43	060	01	105	00
016	00	061	00	106	04
017	01	062	65	107	81
018	95	063	43	108	46
019	20	064	00	109	17
020	42	065	01	110	42
021	00	066	55	111	00
022	03	067	43	112	07
023	45	068	00	113	81
024	03	069	07	114	42
025	65	070	65	115	00
026	43	071	53	116	08
027	00	072	43	117	81
028	04	073	00	118	46
029	85	074	08	119	18
030	43	075	85	120	42
031	00	076	59	121	00
032	03	077	55	122	09
033	40	078	02	123	81
034	65	079	65		
035	43	080	43		
036	00	081	01		
037	05	082	00		
038	85	083	65		
039	43	084	43		
040	00	085	01		
041	03	086	01		
042	65	087	55		
043	43	088	43		
044	00	089	00		

TITLE: ECDZ AND NOTCHED STRENGTH

NO: .113

A'	B'	C'	D'	E'
A	B	C	D	E
Y, c _o	Constants	X	YX _n / X	

00	<u>REGISTER</u>				
01 a	02 w	03 a/w	04 1.5254	05 -0.2881	06 0.1282
07 X	08	09	$10 \left(\frac{x}{YX_n} \right)^2 - 1$		
11 used	12 used	13 c _o	14 a	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter constants	1	113 A&B	0.1282	A	0.1282
			-0.2881	RUN	-0.2881
			1.5254	RUN	1.5254
Enter unnotched strength	2		X	B	X
Enter data to determine c _o Repeat for all data	3			C	
			a	RUN	a
			w	RUN	Y
			X _n	RUN	c _o
				st flg	
Compute YX _n / X	4			6	
				RUN	c _o (best fit)
				D	
			c _o	RUN	c _o
a _o	RUN	a _o			
θ(deg)	RUN	YX _n / X			



PROGRAM TITLE: ECDZ AND NOTCHED STRENGTH

NO: 113

FORMULAS:

$$c_o = \frac{a}{\left(\frac{X}{YX_n}\right)^2 - 1} \quad \text{cf. Eq. (540)}$$

$$c_o \text{ (best fit)} = \frac{\sum a \left[\left(\frac{X}{YX_n}\right)^2 - 1 \right]}{\sum \left[\left(\frac{X}{YX_n}\right)^2 - 1 \right]^2}$$

$$\frac{YX_n}{X} = \left(\frac{c_o}{a \sin \theta + c_o} \right)^{1/2} \quad (546)$$

PROGRAM TITLE: ECDZ AND NOTCHED STRENGTH

NO: 113

PROGRAM LISTING:

000	46	045	00	090	01	135	01
001	13	046	03	091	00	136	02
002	22	047	65	092	95	137	81
003	50	048	43	093	81	138	50
004	00	049	00	094	60	139	00
005	42	050	06	095	00	140	81
006	01	051	85	096	01	141	46
007	01	052	01	097	00	142	14
008	42	053	95	098	06	143	81
009	01	054	65	099	41	144	42
010	02	055	81	100	00	145	01
011	81	056	95	101	01	146	03
012	42	057	20	102	02	147	81
013	00	058	65	103	43	148	42
014	01	059	43	104	01	149	01
015	81	060	00	105	02	150	04
016	42	061	07	106	55	151	81
017	00	062	95	107	43	152	32
018	02	063	40	108	01	153	65
019	55	064	75	109	01	154	43
020	43	065	01	110	95	155	01
021	00	066	95	111	81	156	04
022	01	067	42	112	46	157	85
023	95	068	01	113	11	158	43
024	20	069	00	114	42	159	01
025	42	070	40	115	00	160	03
026	00	071	44	116	06	161	95
027	03	072	01	117	81	162	20
028	45	073	01	118	42	163	65
029	03	074	43	119	00	164	43
030	65	075	00	120	05	165	01
031	43	076	01	121	81	166	03
032	00	077	65	122	42	167	95
033	04	078	43	123	00	168	30
034	85	079	01	124	04	169	81
035	43	080	00	125	81	170	00
036	00	081	95	126	46		
037	03	082	44	127	12		
038	40	083	01	128	42		
039	65	084	02	129	00		
040	43	085	43	130	07		
041	00	086	00	131	81		
042	05	087	01	132	01		
043	85	088	55	133	01		
044	43	089	43	134	42		

TITLE: OFF-AXIS FATIGUE STRENGTH				NO: 117	
A'	B'	C'	D'	E'	
A Constants	B $a_{\theta}, b_{\theta}, \text{etal}$	C $S_{\theta}(S)$	D $S_{\theta}(T)$	E	

00	<u>REGISTER</u>				
01 a_S	02 b_S	03 a_T	04 b_T	05 θ	06 1/mn
07 $a_{\theta}(S)$	08 $b_{\theta}(S)$	09 m^2	10 $a_{\theta}(T)$		
11 $b_{\theta}(T)$	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	117 A&B	a_S	A	a_S
			b_S	RUN	b_S
			a_T	RUN	a_T
			b_T	RUN	b_T
Compute a_{θ}, b_{θ}	2		θ (deg)	B	$a_{\theta}(S)$
				RUN	$b_{\theta}(S)$
				RUN	$a_{\theta}(T)$
				RUN	$b_{\theta}(T)$
Compute S_{θ} at N	3		N	C	$S_{\theta}(S)$
			N	D	$S_{\theta}(T)$

PROGRAM TITLE: OFF-AXIS FATIGUE STRENGTH

NO: 117

FORMULAS:

$$a_{\theta} = \frac{a_s}{mn}, \quad b_{\theta} = \frac{b_s}{mn}$$

$$a_{\theta} = \frac{a_s}{n^2}, \quad b_{\theta} = \frac{b_s}{n^2}$$

PROGRAM LISTING:

```
000 46 030 20 060 42 090 65
001 11 031 42 061 00 091 43
002 42 032 00 062 09 092 00
003 00 033 06 063 43 093 07
004 01 034 65 064 00 094 95
005 81 035 43 065 03 095 94
006 42 036 00 066 55 096 85
007 00 037 01 067 43 097 43
008 02 038 95 068 00 098 00
009 81 039 42 069 09 099 08
010 42 040 00 070 95 100 95
011 00 041 07 071 42 101 81
012 03 042 81 072 01 102 46
013 81 043 43 073 00 103 14
014 42 044 00 074 81 104 28
015 00 045 06 075 43 105 65
016 04 046 65 076 00 106 43
017 81 047 43 077 04 107 01
018 46 048 00 078 55 108 00
019 12 049 02 079 43 109 95
020 42 050 95 080 00 110 94
021 00 051 42 081 09 111 85
022 05 052 00 082 95 112 43
023 33 053 08 083 42 113 01
024 65 054 81 084 01 114 01
025 43 055 43 085 01 115 95
026 00 056 00 086 81 116 81
027 05 057 05 087 46
028 32 058 32 088 13
029 95 059 40 089 28
```

TITLE: FATIGUE STRENGTH OF ANGLE-PLY LAMINATE NO: 119

A'	B'	C'	D'	E'
A Data	B u_1, u_2	C k_S, k_T	D	E

00	<u>REGISTER</u>				
01 E_L	02 E_T	03 G_{LT}	04 ν_{LT}	05 a_S	06 b_S
07 a_T	08 b_T	09 $2\theta'$	10 u_1		
11 u_2	12 k_S	13 k_T	14	15 used	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	119 A&B	E_L	A	E_L
			E_T	RUN	E_T
			G_{LT}	RUN	G_{LT}
			ν_{LT}	RUN	ν_{LT}
			a_S	RUN	a_S
			b_S	RUN	b_S
			a_T	RUN	a_T
Compute parameters	2		b_T	RUN	b_T
				B	u_1
				RUN	u_2
			$\theta(\text{deg})$	C	k_S
			RUN	k_T	

FORMULAS:

$$k_T = \frac{1}{2} \left[1 - \sec 2\theta + \frac{(u_1 + \sec 2\theta) \tan^2 2\theta}{u_2 + \tan^2 2\theta} \right] \quad (555)$$

$$k_S = -\frac{1}{2} \frac{(u_1 + \sec 2\theta) \tan 2\theta}{u_2 + \tan^2 2\theta} \quad (556)$$

$$u_1 = \frac{1 - E_L/E_T}{1 + 2\nu_{LT} + E_L/E_T} \quad (557)$$

$$u_2 = \frac{E_L/G_{LT}}{1 + 2\nu_{LT} + E_L/E_T} \quad (558)$$

PROGRAM TITLE: FATIGUE STRENGTH OF ANGLE-PLY LAMINATE NO: 119

PROGRAM LISTING:

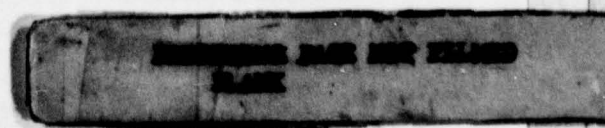
000	46	045	95	090	13	135	95	180	09
001	11	046	55	091	65	136	94	181	32
002	42	047	53	092	02	137	55	182	40
003	00	048	01	093	95	138	02	183	55
004	01	049	85	094	42	139	95	184	43
005	81	050	02	095	00	140	42	185	01
006	42	051	65	096	09	141	01	186	06
007	00	052	43	097	53	142	03	187	75
008	02	053	00	098	43	143	81	188	01
009	81	054	04	099	01	144	01	189	54
010	42	055	85	100	00	145	85	190	65
011	00	056	43	101	65	146	43	191	43
012	03	057	00	102	43	147	01	192	00
013	81	058	01	103	00	148	00	193	09
014	42	059	55	104	09	149	65	194	33
015	00	060	43	105	33	150	43	195	20
016	04	061	00	106	65	151	00	196	95
017	81	062	02	107	43	152	09	197	55
018	42	063	54	108	00	153	32	198	02
019	00	064	42	109	09	154	40	199	95
020	05	065	01	110	32	155	55	200	42
021	81	066	05	111	85	156	53	201	01
022	42	067	95	112	43	157	43	202	04
023	00	068	42	113	00	158	01	203	81
024	06	069	01	114	09	159	01		
025	81	070	00	115	32	160	65		
026	42	071	81	116	54	161	43		
027	00	072	43	117	55	162	00		
028	07	073	00	118	53	163	09		
029	81	074	01	119	43	164	33		
030	42	075	55	120	01	165	40		
031	00	076	43	121	01	166	85		
032	08	077	00	122	65	167	43		
033	81	078	03	123	43	168	00		
034	46	079	95	124	00	169	09		
035	12	080	55	125	09	170	32		
036	01	081	43	126	33	171	40		
037	75	082	01	127	40	172	54		
038	43	083	05	128	85	173	42		
039	00	084	95	129	43	174	01		
040	01	085	42	130	00	175	06		
041	55	086	01	131	09	176	85		
042	43	087	01	132	32	177	53		
043	00	088	81	133	40	178	43		
044	02	089	46	134	54	179	00		

TITLE: STRENGTH PREDICTION (FAILURE POTENTIAL) NO: 123

A'	B'	C'	D'	E'
A Data	B x_o , etc.	C	D	E

00	<u>REGISTER</u>				
01 \hat{t}	02 L	03 C_1	04 β	05 α	06 C_2
07 C_3	08 x_o	09 $\Gamma(\frac{1}{\alpha(\beta+1)}+1)$	10 $\Gamma(\frac{2}{\alpha(\beta+1)}+1)$		
11 γ	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	123 A&B	\hat{t}	A	\hat{t}
			β	RUN	β
			C_1	RUN	C_1
			α	RUN	α
			C_3	RUN	C_3
			C_2	RUN	C_2
			L	RUN	L
Compute	2		$\gamma(0.5772)$	RUN	γ
				B	x_o
				RUN	$1+1/\alpha(\beta+1)$
			$F(1+1/\alpha(\beta+1))$	RUN	$1+2/\alpha(\beta+1)$
			$\Gamma(1+2/\alpha(\beta+1))$	RUN	\bar{x} (power)
	RUN	c. v. (power)			
			C	\bar{x} (exp.)	



PROGRAM TITLE: STRENGTH PREDICTION (FAILURE
POTENTIAL)

NO: 123

FORMULAS:

$$\bar{x}(\text{power}) = x_0 \Gamma \left[1 + \frac{1}{\alpha(\beta+1)} \right]$$

$$x_0 = Lt_0 = \left[\hat{t} L C_1 \beta (\beta+1) \right]^{1/(\beta+1)} \quad (580)$$

$$C.V. = \left[\frac{\Gamma[1 + 2/\alpha(\beta+1)]}{\Gamma^2[1 + 1/\alpha(\beta+1)]} - 1 \right]^{1/2}$$

$$\bar{x} = \frac{C_3}{\alpha} \left[\alpha \ln(\hat{t} L C_2 / C_3) - \gamma \right] \quad \gamma: \text{Euler constant } (=0.5772) \quad (585)$$

PROGRAM TITLE: STRENGTH PREDICTION (FAILURE
POTENTIAL)

NO: 123

PROGRAM LISTING:

000	46	045	43	090	43	135	09	180	54
001	11	046	00	091	00	136	40	181	95
002	42	047	04	092	04	137	75	182	81
003	00	048	85	093	85	138	01	183	00
004	01	049	01	094	01	139	95	184	00
005	81	050	54	095	54	140	30	185	00
006	42	051	95	096	95	141	95		
007	00	052	45	097	20	142	81		
008	04	053	53	098	42	143	46		
009	81	054	43	099	01	144	13		
010	42	055	00	100	02	145	43		
011	00	056	04	101	85	146	00		
012	03	057	85	102	01	147	07		
013	81	058	01	103	95	148	55		
014	42	059	54	104	81	149	43		
015	00	060	20	105	42	150	00		
016	05	061	95	106	00	151	05		
017	81	062	65	107	09	152	65		
018	42	063	43	108	43	153	53		
019	00	064	00	109	01	154	43		
020	07	065	03	110	02	155	00		
021	81	066	45	111	65	156	05		
022	42	067	53	112	02	157	65		
023	00	068	43	113	85	158	53		
024	06	069	00	114	01	159	43		
025	81	070	04	115	95	160	00		
026	42	071	55	116	81	161	01		
027	00	072	53	117	42	162	65		
028	02	073	43	118	01	163	43		
029	81	074	00	119	00	164	00		
030	42	075	04	120	43	165	02		
031	01	076	85	121	00	166	65		
032	01	077	01	122	08	167	43		
033	81	078	54	123	65	168	00		
034	46	079	54	124	43	169	06		
035	12	080	95	125	00	170	55		
036	43	081	42	126	09	171	43		
037	00	082	00	127	95	172	00		
038	01	083	08	128	81	173	07		
039	65	084	81	129	43	174	54		
040	43	085	43	130	01	175	23		
041	00	086	00	131	00	176	75		
042	02	087	05	132	55	177	43		
043	65	088	65	133	43	178	01		
044	53	089	53	134	00	179	01		

TITLE: RESIDUAL STRENGTH (a)

NO: 127

A' Parameters	B'	C'	D'	E'
A N_o	B R_l	C R_r	D	E

00	<u>REGISTER</u>				
01 α_s	02 x_o	03 α_r	04 α_l	05 β	06 C_1
07 n	08 S_{max}	09 \hat{x}	10 N_o		
11	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter parameters	1	127 A&B	α_s	A'	α_s
			x_o	RUN	x_o
			α_r	RUN	α_r
			α_l	RUN	α_l
			β	RUN	β
			C_1	RUN	C_1
			\hat{x}	RUN	\hat{x}
Compute N_o	2		S_{max}	A	N_o
Compute R_l	3		n	B	R_l
Compute R_r	4		n	C	R_r

FORMULAS:

$$\begin{aligned}
 R_r(\bar{x}_r) &= \exp \left[- \frac{\bar{x}_r^{\alpha_s} + (\alpha_s / \alpha_l) (S_{\max} / C_1)^{\beta \alpha_l} n^{\alpha_l}}{\bar{x}_o^{\alpha_s}} + \left(\frac{S_{\max}}{\bar{x}_o} \right)^{\alpha_s} \right] \\
 &= \exp \left[- \left(\frac{\bar{x}_r}{\bar{x}_o} \right)^{\alpha_s} - \left(\frac{n}{N_o} \right)^{\alpha_l} + \left(\frac{S_{\max}}{\bar{x}_o} \right)^{\alpha_s} \right] \quad (622)
 \end{aligned}$$

$$R_l(N) = \exp \left[- \left(\frac{N}{N_o} \right)^{\alpha_l} \right] \quad (623)$$

$$N_o (S_{\max} / C_1)^{\beta / \alpha_l} = (\bar{x}_o^{\alpha_s} \alpha_l / \alpha_s)^{1 / \alpha_l} \quad (624)$$

PROGRAM TITLE: RESIDUAL STRENGTH (a)

NO: 127

PROGRAM LISTING:

000	46	045	43	090	45	135	43
001	16	046	00	091	43	136	00
002	42	047	01	092	00	137	02
003	00	048	95	093	04	138	54
004	01	049	65	094	95	139	45
005	81	050	43	095	94	140	43
006	42	051	00	096	22	141	00
007	00	052	04	097	23	142	01
008	02	053	55	098	81	143	95
009	81	054	43	099	46	144	22
010	42	055	00	100	13	145	23
011	00	056	01	101	42	146	81
012	03	057	95	102	00		
013	81	058	55	103	07		
014	42	059	53	104	55		
015	00	060	43	105	43		
016	04	061	00	106	00		
017	81	062	08	107	02		
018	42	063	55	108	95		
019	00	064	43	109	45		
020	05	065	00	110	43		
021	81	066	06	111	00		
022	42	067	54	112	01		
023	00	068	45	113	95		
024	06	069	43	114	94		
025	81	070	00	115	75		
026	42	071	05	116	53		
027	00	072	95	117	43		
028	09	073	45	118	00		
029	81	074	43	119	07		
030	46	075	00	120	55		
031	11	076	04	121	43		
032	42	077	20	122	01		
033	00	078	95	123	00		
034	08	079	42	124	54		
035	53	080	01	125	45		
036	43	081	00	126	43		
037	00	082	81	127	00		
038	02	083	46	128	04		
039	55	084	12	129	85		
040	43	085	55	130	53		
041	00	086	43	131	43		
042	09	087	01	132	00		
043	54	088	00	133	08		
044	45	089	95	134	55		

TITLE: RESIDUAL STRENGTH (b)

NO: 131

A' Parameters	B'	C'	D'	E'
A N_o	B R_l	C R_r	D	E

00	<u>REGISTER</u>				
01 α_s	02 x_o	03 α_r	04 α_l	05 β	06 C_1
07 n	08 S_{max}	09 \hat{x}	10 N_o		
11	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter parameters	1	131 A&B	α_s x_o α_r α_l β C_1 \hat{x}	A' RUN RUN RUN RUN RUN RUN	
Compute N_o	2		S_{max}	A	N_o
Compute R_l	3		n	B	R_l
Compute R_r	4		n	C	R_r

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FORMULAS:

$$\begin{aligned}
 R_r(\bar{x}_r) &= \exp \left[- \frac{\left(\frac{\alpha_r}{\bar{x}_r} + \alpha_r (S_{\max}/C_1)^\beta \right) \alpha_s / \alpha_r}{\bar{x}_o^{\alpha_s}} + \left(\frac{S_{\max}}{\bar{x}_o} \right)^{\alpha_s} \right] \\
 &= \exp \left\{ - \left[\left(\frac{\bar{x}_r}{\bar{x}_o} \right)^{\alpha_r} + \left(\frac{n}{N_o} \right)^{\alpha_s / \alpha_r} + \left(\frac{S_{\max}}{\bar{x}_o} \right)^{\alpha_s} \right] \right\} \quad (627)
 \end{aligned}$$

$$R_f(N) = \exp \left[- \left(\frac{N}{N_o} \right)^{\alpha_s / \alpha_r} \right] \quad (628)$$

$$N_o (S_{\max}/C_1)^\beta = \frac{\alpha_r}{\bar{x}_o^{\alpha_s / \alpha_r}} \quad (629)$$

PROGRAM TITLE: RESIDUAL STRENGTH (b)

NO: 131

PROGRAM LISTING:

000	46	045	20	090	23	135	02
001	16	046	65	091	81	136	54
002	42	047	53	092	46	137	45
003	00	048	43	093	13	138	43
004	01	049	00	094	42	139	00
005	81	050	02	095	00	140	01
006	42	051	55	096	07	141	95
007	00	052	43	097	55	142	22
008	02	053	00	098	43	143	23
009	81	054	09	099	00	144	81
010	42	055	54	100	02		
011	00	056	45	101	95		
012	03	057	43	102	45		
013	81	058	00	103	43		
014	42	059	03	104	00		
015	00	060	95	105	03		
016	04	061	55	106	85		
017	81	062	43	107	43		
018	42	063	00	108	00		
019	00	064	03	109	07		
020	05	065	95	110	55		
021	81	066	42	111	43		
022	42	067	01	112	01		
023	00	068	00	113	00		
024	06	069	81	114	95		
025	81	070	46	115	45		
026	42	071	12	116	53		
027	00	072	55	117	43		
028	09	073	43	118	00		
029	81	074	01	119	01		
030	46	075	00	120	55		
031	11	076	95	121	43		
032	42	077	45	122	00		
033	00	078	53	123	03		
034	08	079	43	124	54		
035	55	080	00	125	95		
036	43	081	01	126	94		
037	00	082	55	127	85		
038	06	083	43	128	53		
039	95	084	00	129	43		
040	45	085	03	130	00		
041	43	086	54	131	08		
042	00	087	95	132	55		
043	05	088	94	133	43		
044	95	089	22	134	00		

TITLE: CUMULATIVE DAMAGE				NO: 135	
A' Data	B'	C'	D'	E'	
A R_{ln_1}	B R_{ln_1}	C	D	E	

00	<u>REGISTER</u>				
01 N_{o1}	02 N_{o2}	03 n_1	04 α_l	05 α_s	06 α_r
07	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	135 A	N_{o1}	A'	N_{o1}
			N_{o2}	RUN	N_{o2}
			n_1	RUN	n_1
			α_l	RUN	α_l
			α_s	RUN	α_s
			α_r	RUN	α_r
Compute R_{ln_1} model (a)	2		N_2	A	R_{ln_1}
			N_2	B	R_{ln_1}
model (b)					

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FORMULAS:

$$(a) \alpha_r = \alpha_s$$

$$R_{ln1}(N_2) = \exp \left[- \left(\frac{n_1}{N_{o1}} \right)^{\alpha_l} - \left(\frac{N_2}{N_{o2}} \right)^{\alpha_l} \right] \quad (634)$$

$$(b) \alpha_l = 1$$

$$R_{ln1}(N_2) = \exp \left[- \left(\frac{n_1}{N_{o1}} + \frac{N_2}{N_{o2}} \right)^{\alpha_s / \alpha_r} \right] \quad (636)$$

PROGRAM TITLE: CUMULATIVE DAMAGE

NO: 135

PROGRAM LISTING:

000	46	045	00
001	16	046	01
002	42	047	54
003	00	048	45
004	01	049	43
005	81	050	00
006	42	051	04
007	00	052	95
008	02	053	94
009	81	054	22
010	42	055	23
011	00	056	95
012	03	057	81
013	81	058	46
014	42	059	12
015	00	060	55
016	04	061	43
017	81	062	00
018	42	063	02
019	00	064	85
020	05	065	43
021	81	066	00
022	42	067	03
023	00	068	55
024	06	069	43
025	81	070	00
026	46	071	01
027	11	072	95
028	55	073	45
029	43	074	53
030	00	075	43
031	02	076	00
032	95	077	05
033	45	078	55
034	43	079	43
035	00	080	00
036	04	081	06
037	95	082	54
038	85	083	95
039	53	084	94
040	43	085	22
041	00	086	23
042	03	087	95
043	55	088	81
044	43		

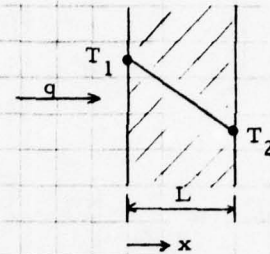
FORMULAS:

Heat Transfer (temperature effects)

$$q = -KVT \quad (638)$$

One dimensional steady

$$q_x = +K_x \frac{T_1 - T_2}{L} \quad (639)$$

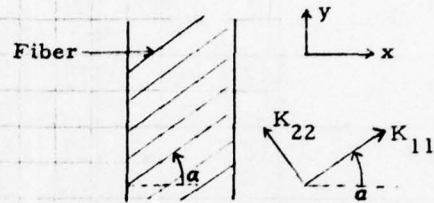


Composite Materials

$$K_x = K_{11} \cos^2 \alpha + K_{22} \sin^2 \alpha \quad (640)$$

Approximations ($v_f < 0.785$)

$$K_{11} = (1 - v_f)K_m + v_f K_f \quad (641)$$



$$K_{22} = K_m \left\{ (1 - 2\sqrt{v_f/\pi}) + \frac{1}{B_K} \left[\pi - \frac{4}{\sqrt{1 - B_K^2 v_f/\pi}} \tan^{-1} \frac{\sqrt{1 - (B_K^2 v_f/\pi)}}{1 + B_K (v_f/\pi)^{1/2}} \right] \right\} \quad (642)$$

$$B_K \equiv 2 \left(\frac{K_m}{K_f} - 1 \right) \quad (643)$$

PROGRAM TITLE: COMPOSITE CONDUCTIVITIES

NO: 139

PROGRAM LISTING:

000	46	045	02	090	20	135	81
001	11	046	75	091	22	136	46
002	42	047	01	092	34	137	13
003	00	048	95	093	65	138	42
004	01	049	65	094	04	139	00
005	81	050	02	095	55	140	00
006	42	051	95	096	53	141	33
007	00	052	42	097	01	142	40
008	02	053	00	098	75	143	65
009	81	054	04	099	43	144	43
010	42	055	40	100	00	145	00
011	00	056	65	101	05	146	07
012	03	057	43	102	54	147	85
013	43	058	00	103	30	148	43
014	00	059	03	104	95	149	00
015	01	060	55	105	75	150	06
016	65	061	59	106	59	151	65
017	53	062	95	107	95	152	43
018	01	063	42	108	94	153	00
019	75	064	00	109	55	154	00
020	43	065	05	110	43	155	32
021	00	066	43	111	00	156	40
022	03	067	00	112	04	157	95
023	54	068	03	113	85	158	42
024	85	069	55	114	01	159	00
025	43	070	59	115	75	160	08
026	00	071	54	116	02	161	81
027	03	072	30	117	65		
028	65	073	65	118	53		
029	43	074	43	119	43		
030	00	075	00	120	00		
031	02	076	04	121	03		
032	95	077	85	122	55		
033	42	078	01	123	59		
034	00	079	54	124	54		
035	07	080	55	125	30		
036	81	081	53	126	95		
037	46	082	01	127	65		
038	12	083	75	128	43		
039	43	084	43	129	00		
040	00	085	00	130	01		
041	01	086	05	131	95		
042	55	087	54	132	42		
043	43	088	30	133	00		
044	00	089	95	134	06		

TITLE: MOISTURE GAIN AND DISTRIBUTION				NO: 143	
A'	B'	C' t	D'	E'	
A D_x, S, h, x^*	B	C t^*	D C^*	E G	

00	<u>REGISTER</u>				
01 D_x	02 D_x/S^2	03 h	04 $x^*=x/h$	05 $t^*=D_x t/S^2$	06 (2j+1)
07 SUM G	08 SUM C^*	09 exp[]	10 G		
11 C^*	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Input Data	1	143 A&B	D_x	A	D_x
			S	RUN	D_x/S^2
			h	RUN	h
			x	RUN	$x^*=x/h$
			t	RUN	$t^*=D_x t/S^2$
Compute C^* and G	2		j=0	B	
			1	B	
			2	B	
			3	B	
			.	.	
			.	.	
			.	.	
			.	.	
			C	C	G
			D	D	C^* t



FORMULAS:

Moisture Effects

$$\Gamma = -DVC \quad (644)$$

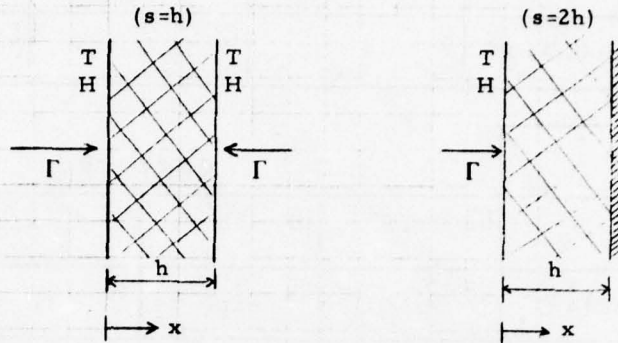
Steady Bound. Cond.

One Dimensional

Temp Constant

Uniform Initial Distributions

Moisture Distribution:



$$C^* = \frac{C - C_i}{C_m - C_i} = 1 - \frac{4}{\pi} \sum_{j=0}^{\infty} \frac{1}{(2j+1)} \sin \frac{(2j+1)\pi x}{h} \exp \left\{ - \left[(2j+1)^2 \pi^2 \right] \frac{D_x t}{s^2} \right\} \quad (645)$$

Moisture Content (weight gain):

$$M = \frac{W - W_D}{W_D} = G(M_m - M_i) + M_i \quad (646)$$

$$G = 1 - \frac{8}{\pi^2} \sum_{j=0}^{\infty} \frac{\exp \left[- (2j+1)^2 \pi^2 \left(\frac{D_x t}{s^2} \right) \right]}{(2j+1)^2} \quad (647)$$

Time to reach 99.9% saturation:

$$t_m = \frac{0.67 s^2}{D_x} \quad (648)$$

PROGRAM TITLE: MOISTURE GAIN AND DISTRIBUTION

NO: 143

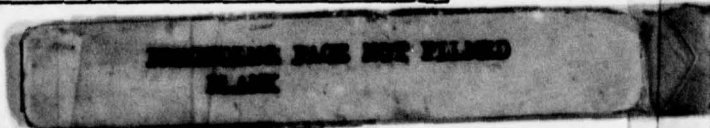
PROGRAM LISTING:

045	07	000	46	090	59	135	54
046	42	001	11	091	40	136	10
047	00	002	42	092	75	137	30
048	08	003	00	093	01	138	14
049	81	004	01	094	95	139	00
050	46	005	81	095	94	140	10
051	12	006	40	096	42	141	68
052	65	007	55	097	01	142	34
053	02	008	43	098	00	143	41
054	85	009	00	099	43	144	14
055	01	010	01	100	00	145	10
056	95	011	95	101	06	146	10
057	42	012	20	102	65	147	68
058	00	013	42	103	43	148	44
059	06	014	00	104	00	149	31
060	40	015	02	105	04	150	04
061	65	016	81	106	65	151	10
062	59	017	42	107	59	152	50
063	40	018	00	108	95	153	35
064	65	019	03	109	22	154	04
065	43	020	81	110	38	155	20
066	00	021	55	111	32	156	50
067	05	022	43	112	36	157	19
068	95	023	00	113	04	158	08
069	94	024	03	114	92	159	30
070	22	025	95	115	38		
071	23	026	42	116	34		
072	42	027	00	117	52		
073	00	028	04	118	00		
074	09	029	00	119	14		
075	55	030	42	120	49		
076	43	031	00	121	04		
077	00	032	08	122	80		
078	06	033	81	123	30		
079	40	034	65	124	04		
080	95	035	43	125	80		
081	44	036	00	126	50		
082	00	037	02	127	56		
083	07	038	95	128	50		
084	43	039	42	129	95		
085	00	040	00	130	55		
086	07	041	05	131	17		
087	65	042	00	132	50		
088	08	043	42	133	49		
089	55	044	00	134	29		

TITLE: BEST FIT FOR WEIBULL DISTRIBUTION				NO: 147	
A'	B'	C'	D'	E'	
A	x	B	j	C	$r^2(C.D.)$
		D	α	E	
				x_0	

00	<u>REGISTER</u>				
01	02	03	04	05	06
07	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Initialize	1	147 A&B	N	CM _s STO 1 4	N
Enter data Repeat for all data	2		x j	A B	lnx j
Compute	3			C RUN E	α x_0 $r^2(C.D.)$



PROGRAM TITLE: BEST FIT FOR WEIBULL DISTRIBUTION NO: 147

FORMULAS:

$$F = 1 - \exp \left[- (x/x_0)^\alpha \right] \quad (702)$$

$$F = \frac{j - 0.3}{N + 0.4}$$

PROGRAM TITLE: BEST FIT FOR WEIBULL DISTRIBUTION NO: 147

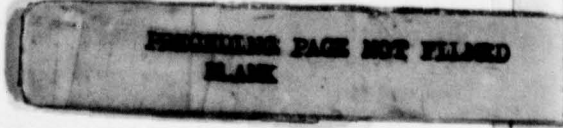
PROGRAM LISTING:

000	46	045	02	090	43	135	00	180	41
001	88	046	44	091	00	136	04	181	87
002	44	047	00	092	04	137	95	182	46
003	00	048	04	093	75	138	55	183	13
004	00	049	40	094	43	139	43	184	14
005	43	050	44	095	00	140	00	185	65
006	00	051	00	096	05	141	00	186	43
007	00	052	07	097	95	142	95	187	01
008	75	053	43	098	94	143	94	188	03
009	01	054	00	099	42	144	55	189	55
010	95	055	02	100	01	145	43	190	53
011	30	056	65	101	03	146	01	191	43
012	42	057	43	102	55	147	01	192	00
013	00	058	00	103	53	148	95	193	04
014	08	059	01	104	43	149	22	194	40
015	25	060	95	105	00	150	23	195	55
016	43	061	44	106	06	151	56	196	43
017	00	062	00	107	75	152	46	197	00
018	00	063	05	108	43	153	16	198	00
019	56	064	43	109	00	154	51	199	94
020	46	065	00	110	03	155	77	200	85
021	12	066	01	111	65	156	94	201	43
022	75	067	44	112	43	157	85	202	00
023	93	068	00	113	01	158	43	203	07
024	03	069	03	114	00	159	01	204	95
025	95	070	40	115	95	160	02	205	56
026	55	071	44	116	42	161	95	206	46
027	53	072	00	117	01	162	55	207	11
028	43	073	06	118	01	163	46	208	23
029	01	074	01	119	56	164	87	209	42
030	04	075	41	120	46	165	43	210	00
031	85	076	88	121	77	166	01	211	01
032	93	077	46	122	42	167	01	212	56
033	04	078	14	123	01	168	95		
034	54	079	43	124	02	169	56		
035	95	080	00	125	46	170	46		
036	94	081	03	126	15	171	17		
037	85	082	55	127	14	172	51		
038	01	083	43	128	65	173	77		
039	95	084	00	129	43	174	85		
040	23	085	00	130	00	175	53		
041	94	086	65	131	03	176	43		
042	23	087	42	132	94	177	01		
043	42	088	01	133	85	178	02		
044	00	089	00	134	43	179	65		

TITLE: A AND B ALLOWABLES (WEIBULL)				NO: 151
A' Parameters	B'	C'	D'	E'
A x_A	B x_B	C $i + 1$	D	E

00	<u>REGISTER</u>				
01 α	02 n	03 $X_{2n;\gamma}^2$	04 j	05 0.99	06 0.90
07 $\sum x_i \alpha$	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter parameters	1	151 A	α	A'	α
			n	RUN	n
			$X_{2n;\gamma}^2$	RUN	$X_{2n;\gamma}^2$
Enter data Repeat for all data	2		x_i	C	$i + 1$
				RUN	
Compute	3			A	x_A
				B	x_B



FORMULAS:

For "A" allowable x_A

$$\hat{R}_\gamma = 0.99, \quad \gamma = 0.95$$

For "B" allowable x_B

$$\hat{R}_\gamma = 0.90, \quad \gamma = 0.95$$

$$x_{A,B} = \left[-2ntn\hat{R}_\gamma / X_{2n;\gamma}^2 \right]^{1/\alpha} \hat{x}_o \quad (728)$$

$$\hat{x}_o = \left[\frac{1}{n} \sum_{i=1}^n x_i^\alpha \right]^{1/\alpha} \quad (729)$$

PROGRAM TITLE: A AND B ALLOWABLES (WEIBULL)

NO: 151

PROGRAM LISTING:

000	46	045	46	090	65	135	03	180	43
001	16	046	11	091	43	136	94	181	01
002	81	047	43	092	00	137	85	182	02
003	42	048	00	093	02	138	43	183	65
004	00	049	07	094	65	139	00	184	41
005	01	050	55	095	93	140	04	185	87
006	81	051	43	096	09	141	95	186	46
007	42	052	00	097	00	142	55	187	13
008	00	053	04	098	23	143	43	188	14
009	02	054	65	099	95	144	00	189	65
010	81	055	02	100	55	145	00	190	43
011	42	056	65	101	43	146	95	191	01
012	00	057	43	102	00	147	94	192	03
013	03	058	00	103	03	148	55	193	55
014	81	059	02	104	95	149	43	194	53
015	46	060	65	105	94	150	01	195	43
016	13	061	93	106	45	151	01	196	00
017	00	062	09	107	43	152	95	197	04
018	42	063	09	108	00	153	22	198	40
019	00	064	23	109	01	154	23	199	55
020	04	065	95	110	20	155	56	200	43
021	42	066	55	111	95	156	46	201	00
022	00	067	43	112	81	157	16	202	00
023	07	068	00	113	00	158	51	203	94
024	01	069	03	114	03	159	77	204	85
025	44	070	95	115	65	160	94	205	43
026	00	071	94	116	43	161	85	206	00
027	04	072	45	117	01	162	43	207	07
028	43	073	43	118	00	163	01	208	95
029	00	074	00	119	95	164	02	209	56
030	04	075	01	120	42	165	95	210	46
031	81	076	20	121	01	166	55	211	11
032	45	077	95	122	01	167	46	212	23
033	43	078	81	123	56	168	87	213	42
034	00	079	46	124	46	169	43	214	00
035	01	080	12	125	77	170	01	215	01
036	95	081	43	126	42	171	01	216	56
037	44	082	00	127	01	172	95		
038	00	083	07	128	02	173	56		
039	07	084	55	129	46	174	46		
040	41	085	43	130	15	175	17		
041	00	086	00	131	14	176	51		
042	02	087	04	132	65	177	77		
043	04	088	65	133	43	178	85		
044	81	089	02	134	00	179	53		

TITLE: SIZE EFFECT (a)

NO: 155

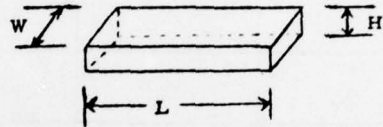
A' Dimensions	B' Dimensions	C'	D'	E'
A v_t, A_t, l_t	B v_f, A_f, l_f	C Volume, 3-pt	D Volume, 4-pt	E

00	<u>REGISTER</u>				
01 w_t	02 L_t	03 H_t	04 w_f	05 L_f	06 H_f
07 v_t	08 A_t	09 l_t	10 v_f		
11 A_f	12 B_f	13 l_f	14 α	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter dimensions	1	155 A&B	w_t L_t H_t w_f L_f H_f	A' RUN RUN RUN B' B' B'	w_t L_t H_t w_f L_f H_f
Compute	2			A RUN RUN B RUN RUN	v_t A_t l_t v_f A_f l_f
Compute (volume model)	3		α α	C RUN D RUN	X_{of}/X_{ot} (3-pt) X_{of}/X_{ot} (4-pt)

FORMULAS:

3-pt. flexure (center point loading)	4-pt. flexure (quarter point loading)
Volume $\frac{(X_o)_f}{(X_o)_t} = \left[2(a+1)^2 \frac{V_t}{V_f} \right]^{\frac{1}{a}}$	$\frac{(X_o)_f}{(X_o)_t} = \left[\frac{4(a+1)^2}{a+2} \frac{V_t}{V_f} \right]^{\frac{1}{a}}$
$V_t = WLH,$ $A_t = 2L(W+H),$ $l_t = 4L,$	$V_f = WLH$ $A_f = WL, B_f = LH$ $l_f = 2L$



PROGRAM TITLE: SIZE EFFECT (a)

NO: 155

PROGRAM LISTING:

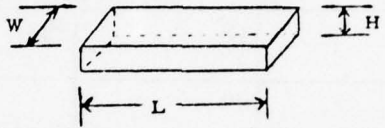
000	46	045	00	090	06	135	04	180	00
001	16	046	03	091	95	136	85	181	07
002	81	047	54	092	42	137	01	182	55
003	42	048	95	093	01	138	95	183	43
004	00	049	42	094	00	139	40	184	01
005	01	050	00	095	81	140	65	185	00
006	81	051	08	096	43	141	02	186	95
007	42	052	81	097	00	142	65	187	45
008	00	053	04	098	04	143	43	188	43
009	02	054	65	099	65	144	00	189	01
010	81	055	43	100	43	145	07	190	04
011	42	056	00	101	00	146	55	191	20
012	00	057	02	102	05	147	43	192	95
013	03	058	95	103	95	148	01	193	81
014	81	059	42	104	42	149	00	194	53
015	46	060	00	105	01	150	95	195	43
016	11	061	09	106	01	151	45	196	00
017	43	062	81	107	81	152	43	197	04
018	00	063	46	108	43	153	01	198	40
019	01	064	17	109	00	154	04	199	55
020	65	065	81	110	05	155	20	200	43
021	43	066	42	111	65	156	95	201	00
022	00	067	00	112	43	157	81	202	00
023	02	068	04	113	00	158	46	203	94
024	65	069	81	114	06	159	14	204	85
025	43	070	42	115	95	160	81	205	43
026	00	071	00	116	42	161	42	206	00
027	03	072	05	117	01	162	01	207	07
028	95	073	81	118	02	163	04	208	95
029	42	074	42	119	81	164	85	209	56
030	00	075	00	120	43	165	01	210	46
031	07	076	06	121	00	166	95	211	11
032	81	077	81	122	05	167	40	212	23
033	43	078	46	123	65	168	65	213	42
034	00	079	12	124	02	169	04	214	00
035	02	080	43	125	95	170	55	215	01
036	65	081	00	126	42	171	53	216	56
037	02	082	04	127	01	172	43		
038	65	083	65	128	03	173	01		
039	53	084	43	129	81	174	04		
040	43	085	00	130	46	175	85		
041	00	086	05	131	13	176	02		
042	01	087	65	132	81	177	54		
043	85	088	43	133	42	178	65		
044	43	089	00	134	01	179	43		

TITLE: SIZE EFFECT (b)				NO: 159	
A'	B'	C'	D'	E'	
A X_{of}/X_{ot}	B Surface, 3-pt	C Surface, 4-pt	D Edge, 3-pt	E Edge, 4-pt	

00		<u>REGISTER</u>									
01	w_t	02	L_t	03	H_t	04	w_f	05	L_f	06	H_f
07	v_t	08	A_t	09	l_t	10	v_f				
11	A_f	12	B_f	13	l_f	14	α	15	16		
17	18		19								
98	99		<u>INSTRUCTIONS</u>								

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Compute dimensions with 155	1				
Compute Surface, 3-pt	2	159 A&B		A	
			α	RUN	X_{of}/X_{ot}
Surface, 4-pt			α	RUN	X_{of}/X_{ot}
Edge, 3-pt			α	RUN	X_{of}/X_{ot}
Edge, 4-pt			α	RUN	X_{of}/X_{ot}

FORMULAS:

3-pt. flexure (center point loading)	4-pt. flexure (quarter point loading)
Surface $\frac{(X_o)_f}{(X_o)_t} = \left[\frac{(a+1) A_t}{A_f + B_f/(a+1)} \right]^{\frac{1}{a}}$	$\frac{(X_o)_f}{(X_o)_t} = \left[\frac{(a+1)^2 A_t}{B_f/2 + (a+1)(A_f + B_f)/2 + (a+1)^2 A_f/2} \right]^{\frac{1}{a}}$
Edge $\frac{(X_o)_f}{(X_o)_t} = \left[(a+1) \frac{l_t}{l_f} \right]^{\frac{1}{a}}$	$\frac{(X_o)_f}{(X_o)_t} = \left[\frac{a+1}{a+2} \frac{l_t}{l_f/2} \right]^{\frac{1}{a}}$
<div style="display: flex; align-items: center;"> <div style="flex: 1;"> <p> $V_t = WLH,$ $V_f = WLH$ $A_t = 2L(W+H),$ $A_f = WL, B_f = LH$ $l_t = 4L,$ $l_f = 2L$ </p> </div> <div style="flex: 1; text-align: center;">  </div> </div>	

PROGRAM TITLE: SIZE EFFECT (b)

NO: 159

PROGRAM LISTING:

000	46	045	85	090	43	135	85	180	00
001	11	046	01	091	01	136	01	181	07
002	81	047	95	092	01	137	95	182	55
003	42	048	40	093	55	138	55	183	43
004	01	049	65	094	02	139	53	184	01
005	04	050	43	095	54	140	43	185	00
006	85	051	00	096	95	141	01	186	95
007	01	052	08	097	45	142	04	187	45
008	95	053	55	098	43	143	85	188	43
009	65	054	53	099	01	144	02	189	01
010	43	055	43	100	04	145	54	190	04
011	00	056	01	101	20	146	65	191	20
012	08	057	02	102	95	147	43	192	95
013	55	058	55	103	81	148	00	193	81
014	53	059	02	104	46	149	09	194	53
015	43	060	85	105	13	150	55	195	43
016	01	061	53	106	81	151	43	196	00
017	01	062	43	107	42	152	01	197	04
018	85	063	01	108	01	153	03	198	40
019	43	064	04	109	04	154	65	199	55
020	01	065	85	110	85	155	02	200	43
021	02	066	01	111	01	156	95	201	00
022	55	067	54	112	95	157	45	202	00
023	53	068	65	113	65	158	43	203	94
024	43	069	53	114	43	159	01	204	85
025	01	070	43	115	00	160	04	205	43
026	04	071	01	116	09	161	20	206	00
027	85	072	01	117	55	162	95	207	07
028	01	073	85	118	43	163	81	208	95
029	54	074	43	119	01	164	85	209	56
030	54	075	01	120	03	165	01	210	46
031	95	076	02	121	95	166	95	211	11
032	45	077	54	122	45	167	40	212	23
033	43	078	55	123	43	168	65	213	42
034	01	079	02	124	01	169	04	214	00
035	04	080	85	125	04	170	55	215	01
036	20	081	53	126	20	171	53	216	56
037	95	082	43	127	95	172	43		
038	81	083	01	128	81	173	01		
039	46	084	04	129	46	174	04		
040	12	085	85	130	14	175	85		
041	81	086	01	131	81	176	02		
042	42	087	54	132	42	177	54		
043	01	088	40	133	01	178	65		
044	04	089	65	134	04	179	43		

TITLE: DATA AVERAGING (a)				NO: 163	
A'	B'	C'	D'	E'	
A σ_2	B σ_6	C A_{ij}, B_i	D	E	

00	<u>REGISTER</u>				
01 σ_2	02 σ_2^2	03 σ_6	04	05	06
07	08	09	10 $\Sigma \sigma_2^2$		
11 $\Sigma \sigma_2^3$	12 $\Sigma \sigma_2 \sigma_6^2$	13 $\Sigma \sigma_2^4$	14 $\Sigma \sigma_2^2 \sigma_6^2$	15 $\Sigma \sigma_6^4$	16 $\Sigma \sigma_2$
17 $\Sigma \sigma_2^2$	18 $\Sigma \sigma_6^2$	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	163 A&B	σ_2	A	$\Sigma \sigma_2^2$
Repeat for all data			σ_6	B	$\Sigma \sigma_6^2$
Display results	2			C	A_{11}
				RUN	A_{12}
				RUN	A_{13}
				RUN	A_{22}
				RUN	A_{23}
				RUN	A_{33}
				RUN	B_1
				RUN	B_2
				RUN	B_3



PROGRAM TITLE: DATA AVERAGING (a)

NO: 163

FORMULAS:

$$[A] = [\sigma]^T [\sigma]$$

cf. Eq. (769)

$$\{B\} = [\sigma]^T \{1\}$$

PROGRAM LISTING:

000 46	030 42	060 01	090 01
001 11	031 00	061 05	091 81
002 42	032 03	062 43	092 43
003 00	033 40	063 00	093 01
004 01	034 65	064 01	094 02
005 40	035 43	065 44	095 81
006 42	036 00	066 01	096 43
007 00	037 01	067 06	097 01
008 02	038 95	068 43	098 03
009 44	039 44	069 00	099 81
010 01	040 01	070 02	100 43
011 00	041 02	071 44	101 01
012 65	042 43	072 01	102 04
013 43	043 00	073 07	103 81
014 00	044 02	074 43	104 43
015 01	045 65	075 00	105 01
016 95	046 43	076 03	106 05
017 44	047 00	077 40	107 81
018 01	048 03	078 44	108 43
019 01	049 40	079 01	109 01
020 43	050 95	080 08	110 06
021 00	051 44	081 81	111 81
022 02	052 01	082 46	112 43
023 40	053 04	083 13	113 01
024 44	054 43	084 43	114 07
025 01	055 00	085 01	115 81
026 03	056 03	086 00	116 43
027 81	057 40	087 81	117 01
028 46	058 40	088 43	118 08
029 12	059 44	089 01	119 81

TITLE: DATA AVERAGING (b)				NO: 165	
A'	B'	C'	D'	E'	
A F_{ij}	B f_m	C	D	E	

00	<u>REGISTER</u>				
01 F_2	02 F_{22}	03 F_{66}	04 σ_2	05	06
07	08	09	10		
11	12	13	14	15	16
17	18	19			
98	99	<u>INSTRUCTIONS</u>			

OPERATION	STEP	READ	ENTER	PRESS	DISPLAY
Enter data	1	165 A		A	
			F_2	RUN	F_2
			F_{22}	RUN	F_{22}
			F_{66}	RUN	F_{66}
Compute	2		σ_2	B	$F_2 \sigma_2 + F_{22} \sigma_2^2$
			σ_6	RUN	f_n

PROGRAM TITLE: DATA AVERAGING (b)

NO: 165

FORMULAS:

$$f_m(\sigma_2, \sigma_6) = F_2\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2. \quad (775)$$

PROGRAM LISTING:

000 46	020 65	040 95	060 01	080 08
001 11	021 43	041 81	061 05	081 81
002 81	022 00	042 43	062 43	
003 42	023 01	043 00	063 00	
004 00	024 85	044 02	064 01	
005 01	025 43	045 65	065 44	
006 81	026 00	046 43	066 01	
007 42	027 02	047 00	067 06	
008 00	028 65	048 03	068 43	
009 02	029 43	049 40	069 00	
010 81	030 00	050 95	070 02	
011 42	031 04	051 44	071 44	
012 00	032 40	052 01	072 01	
013 03	033 85	053 04	073 07	
014 81	034 81	054 43	074 43	
015 46	035 40	055 00	075 00	
016 12	036 65	056 03	076 03	
017 42	037 43	057 40	077 40	
018 00	038 00	058 40	078 44	
019 04	039 03	059 44	079 01	

APPENDIX A

TEXAS INSTRUMENTS PRERECORDED CARDS

CARD NO.	TITLE	PAGE
MA1-05	SIMULTANEOUS EQUATIONS IN 2 OR 3 UNKNOWN	168
MA1-14-1, 2	MATRIX INVERSION AND DETERMINANT (3 x 3)	170
MA1-16	MATRIX ARITHMETIC (2) (MULTIPLICATION)	173
ST1-01	BASIC STATISTICS FOR 1 OR 2 VARIABLES	175
ST1-03-1, 2	MEANS AND MOMENTS	178
ST1-08	LINEAR REGRESSION	181
ST1-09	POWER CURVE FIT	183
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SIMULTANEOUS EQUATIONS IN 2 OR 3 UNKNOWNNS

Given the coefficients of two simultaneous equations with two unknowns each, the solutions are effected as follows.

$$\begin{aligned} a_0x + a_1y &= a_2 \\ b_0x + b_1y &= b_2 \end{aligned}$$

$$x = \frac{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}{\begin{vmatrix} a_0 & a_1 \\ b_0 & b_1 \end{vmatrix}} = \frac{a_1b_2 - a_2b_1}{a_0b_1 - a_1b_0}$$

$$y = \frac{\begin{vmatrix} a_0 & a_2 \\ b_0 & b_2 \end{vmatrix}}{\begin{vmatrix} a_0 & a_1 \\ b_0 & b_1 \end{vmatrix}} = \frac{a_0b_2 - a_2b_0}{a_0b_1 - a_1b_0} \text{ where } a_0b_1 - a_1b_0 \neq 0.$$

For three unknowns:

$$\begin{aligned} a_0x + a_1y + a_2z &= a_3 \\ b_0x + b_1y + b_2z &= b_3 \\ c_0x + c_1y + c_2z &= c_3 \end{aligned}$$

The program solves for x in the first equation and substitutes the result into the second and third equations. Now only two, two-unknown equations exist and are solved as above.

NOTE: $a_0, b_0,$ and $c_0 \neq 0$.

Example: Solve for x and y in the following two equations:

$$5.2x + 3y = 2.56$$

$$10.9x - 4.3y = -34.87$$

Enter	Press	Display	Comments
5.2	2nd CHS	0.	Clear data memories
3	2nd I	5.2	a_0
2.56	RUN	3.	a_1
10.9	RUN	2.56	a_2
4.3	2nd I	10.9	b_0
34.87	RUN	-4.3	b_1
	RUN	-34.87	b_2
	RUN	-1.7	x
	RUN	3.8	y

Example: Solve for x, y, and z.

$$.25x - 3.1y - 3z = 675$$

$$1.25x + .18y + .73z = .901$$

$$-.8x + 1.1y - 1.6z = -2.074$$

$$x = .18, y = -3., z = 1.0$$

TEXAS INSTRUMENTS C 1976

50-1VM SIMULTANEOUS EQUATIONS

ΦA SIMULTANEOUS EQUATIONS

a_0, a_1, a_2	b_0, b_1, b_2	c_0, c_1, c_2, c_3	x, y, z
b_0, b_1, b_2, b_3	b_0, b_1, b_2, b_3	c_0, c_1, c_2, c_3	x, y, z

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Clear Data Memories		2nd CHS	
	Perform Steps 3-4 for			
	2 unknowns OR			
	Perform Steps 5-6 for			
	3 unknowns			
3*	Enter coefficients for	a_0	2nd I	a_0
	2 unknowns	a_1	RUN	a_1
		a_2	RUN	a_2
		b_1	2nd I	b_1
		b_2	RUN	b_2
		b_3	RUN	b_3
4	Calculate x and y		D	x
			RUN	y
5*	Enter coefficients for	a_0	A	a_0
	3 unknowns	a_1	RUN	a_1
		a_2	RUN	a_2
		a_3	RUN	a_3
		b_0	B	b_0
		b_1	RUN	b_1
		b_2	RUN	b_2
		b_3	RUN	b_3
		c_0	C	c_0
		c_1	RUN	c_1
		c_2	RUN	c_2
		c_3	RUN	c_3
6	Calculate x, y, and z		E	x
			RUN	y
			RUN	z

*NOTE: To correct input error return to last user defined key and reenter associated values

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	'LBL	028 06	6	056 03	3	084 65	X
001 11	A	029 56	'rtn	057 56	'rtn	085 43	RCL
002 42	STO	030 42	STO	058 46	'LBL	086 01	1
003 00	0	031 00	0	059 14	D	087 01	1
004 00	0	032 07	7	060 10	'E'	088 95	=
005 56	'rtn	033 56	'rtn	061 46	'LBL	089 55	+
006 42	STO	034 42	STO	062 85	+	090 53	(
007 00	0	035 00	0	063 43	RCL	091 43	RCL
008 01	1	036 08	8	064 00	0	092 00	0
009 56	'rtn	037 56	'rtn	065 09	9	093 06	6
010 42	STO	038 46	'LBL	066 56	'rtn	094 65	X
011 00	0	039 13	C	067 43	RCL	095 43	RCL
012 02	2	040 42	STO	068 00	0	096 01	1
013 56	'rtn	041 01	1	069 04	4	097 02	2
014 42	STO	042 00	0	070 56	'rtn	098 75	--
015 00	0	043 56	'rtn	071 46	'LBL	099 43	RCL
016 03	3	044 46	'LBL	072 10	'E'	100 00	0
017 56	'rtn	045 17	'B'	073 43	RCL	101 07	7
018 46	'LBL	046 42	STO	074 00	0	102 65	X
019 12	B	047 01	1	075 06	6	103 43	RCL
020 42	STO	048 01	1	076 65	X	104 01	1
021 00	0	049 56	'rtn	077 43	RCL	105 01	1
022 05	5	050 42	STO	078 01	1	106 95	=
023 56	'rtn	051 01	1	079 03	3	107 42	STO
024 46	'LBL	052 02	2	080 75	--	108 00	0
025 16	'A	053 56	'rtn	081 43	RCL	109 04	4
026 42	STO	054 42	STO	082 00	0	110 65	X
027 00	0	055 01	1	083 08	8	111 43	RCL

*Denotes 2nd function key

REGISTERS

00	a ₀	05 - b ₀	10 - C ₀	15
01	a ₁	06 b ₁ , a ₀	11 c ₁ , b ₀	16
02	a ₂	07 b ₂ , a ₁	12 c ₂ , b ₁	17
03	a ₃	08 b ₃ , a ₂	13 c ₃ , b ₂	18
04	Z, Y	09 Y, X	14	19

FLAGS

0	1	2	3	4
---	---	---	---	---

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 00	0	140 02	2	168 42	STO	196 95	=
113 07	7	141 49	'PROD	169 00	0	197 56	'rtn
114 94	+/-	142 00	0	170 09	9	198 41	GTO
115 85	+	143 03	3	171 02	2	199 85	+
116 43	RCL	144 43	RCL	172 44	SUM	200 46	'LBL
117 00	0	145 00	0	173 01	1	201 19	'D'
118 08	8	146 05	5	174 04	4	202 43	RCL
119 95	=	147 42	STO	175 19	'D'	203 00	0
120 55	+	148 00	0	176 19	'D'	204 04	4
121 43	RCL	149 04	4	177 19	'D'	205 65	X
122 00	0	150 01	1	178 10	'E'	206 36	'IND
123 06	6	151 42	STO	179 65	X	207 43	RCL
124 95	=	152 00	0	180 43	RCL	208 00	0
125 42	STO	153 09	9	181 00	0	209 09	9
126 00	0	154 06	6	182 01	1	210 95	=
127 09	9	155 42	STO	183 94	+/-	211 94	+/-
128 56	'rtn	156 01	1	184 75	-	212 36	'IND
129 46	'LBL	157 04	4	185 43	RCL	213 44	SUM
130 15	E	158 19	'D'	186 00	0	214 01	1
131 43	RCL	159 19	'D'	187 04	4	215 04	4
132 00	0	160 19	'D'	188 65	X	216 01	1
133 00	0	161 43	RCL	189 43	RCL	217 44	SUM
134 20	'1/x	162 01	1	190 00	0	218 00	0
135 49	'PROD	163 00	0	191 02	2	219 09	9
136 00	0	164 42	STO	192 85	+	220 44	SUM
137 01	1	165 00	0	193 43	RCL	221 01	1
138 49	'PROD	166 04	4	194 00	0	222 04	4
139 00	0	167 01	1	195 03	3	223 56	'rtn

*Denotes 2nd function key

MATRIX INVERSION AND DETERMINANT (3 x 3)

This program will evaluate and take the inverse of a 3 x 3 matrix.

$$\text{Matrix } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

The determinant of matrix A is

$$|A| = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

The inverse of matrix A is $B = A^{-1}$, where $A \times A^{-1} = 1$.

The elements of inverted matrix B are

$$\begin{aligned} b_{11} &= (a_{22}a_{33} - a_{32}a_{23}) / |A| \\ b_{21} &= (a_{32}a_{13} - a_{12}a_{33}) / |A| \\ b_{31} &= (a_{12}a_{23} - a_{32}a_{13}) / |A| \\ b_{12} &= (a_{31}a_{23} - a_{21}a_{33}) / |A| \\ b_{22} &= (a_{11}a_{33} - a_{31}a_{13}) / |A| \\ b_{32} &= (a_{21}a_{13} - a_{11}a_{23}) / |A| \\ b_{13} &= (a_{21}a_{32} - a_{31}a_{22}) / |A| \\ b_{23} &= (a_{31}a_{12} - a_{11}a_{32}) / |A| \\ b_{33} &= (a_{11}a_{22} - a_{21}a_{12}) / |A| \end{aligned}$$

- NOTES: 1. If $|A| = 0$, the display will flash 0 when C is pressed.
2. The determinant must be calculated before its inverse.

Example: Let $A = \begin{pmatrix} 2 & 4 & 9 \\ 1 & 3 & 5 \\ 7 & 8 & 6 \end{pmatrix}$. Evaluate $|A|$ and find A^{-1} .

Enter	Press	Display	Comments
2	<input type="checkbox"/> A	2	a_{11}
4	<input type="checkbox"/> RUN	4	a_{12}
9	<input type="checkbox"/> RUN	9	a_{13}
1	<input type="checkbox"/> RUN	1	a_{21}
3	<input type="checkbox"/> RUN	3	a_{22}
5	<input type="checkbox"/> RUN	5	a_{23}
7	<input type="checkbox"/> RUN	7	a_{31}
8	<input type="checkbox"/> RUN	8	a_{32}
6	<input type="checkbox"/> RUN	6	a_{33}
	<input type="checkbox"/> B	-45	$ A $
	<input type="checkbox"/> C	.4888888889	b_{11}
	<input type="checkbox"/> RUN	-1.0666666667	b_{21}
	<input type="checkbox"/> RUN	.1555555556	b_{31}
	<input type="checkbox"/> RUN	-.6444444444	b_{12}
	<input type="checkbox"/> RUN	1.1333333333	b_{22}
	<input type="checkbox"/> RUN	.0222222222	b_{32}
	<input type="checkbox"/> RUN	.2888888889	b_{13}
	<input type="checkbox"/> RUN	-.2666666667	b_{23}
	<input type="checkbox"/> RUN	-.0444444444	b_{33}

Enter card 2

TEXAS INSTRUMENTS		©1976
A 3X3 MATRIX INV, DET (1)		B
1/A		MA1-14-1
811-33		

TEXAS INSTRUMENTS		©1976
(2) DE ANI XIRIYV EXC B		B
A 3X3 MATRIX INV, DET (2)		MA1-14-2
1/A		

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Card 1 (Side A)			
2	Enter Matrix A	B ₁₁	A	B ₁₁
		B ₁₂	RUN	B ₁₂
		B ₁₃	RUN	B ₁₃
		B ₂₁	RUN	B ₂₁
		B ₂₂	RUN	B ₂₂
		B ₂₃	RUN	B ₂₃
		B ₃₁	RUN	B ₃₁
		B ₃₂	RUN	B ₃₂
		B ₃₃	RUN	B ₃₃
3	Calculate determinant		B	A
4	Enter Card 2 (A and B)			
5	Calculate inverse matrix		C	B ₁₁
			RUN	B ₂₁
			RUN	B ₃₁
			RUN	B ₁₂
			RUN	B ₂₂
			RUN	B ₃₂
			RUN	B ₁₃
			RUN	B ₂₃
			RUN	B ₃₃

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	'LBL	028 12	B	056 03	3
001 11	A	029 43	RCL	057 65	X
002 48	'EXC	030 00	0	058 43	RCL
003 00	0	031 09	9	059 00	0
004 00	0	032 65	X	060 04	4
005 01	1	033 43	RCL	061 65	X
006 48	'EXC	034 00	0	062 43	RCL
007 00	0	035 01	1	063 00	0
008 00	0	036 65	X	064 08	8
009 46	'LBL	037 43	RCL	065 75	-
010 68	'8'	038 00	0	066 43	RCL
011 36	'IND	039 05	5	067 00	0
012 42	STO	040 85	+	068 07	7
013 00	0	041 53	(069 65	X
014 00	0	042 43	RCL	070 43	RCL
015 56	'rtn	043 00	0	071 00	0
016 48	'EXC	044 02	2	072 05	5
017 00	0	045 65	X	073 65	X
018 00	0	046 43	RCL	074 43	RCL
019 85	+	047 00	0	075 00	0
020 01	1	048 06	6	076 03	3
021 95	=	049 65	X	077 75	-
022 48	'EXC	050 43	RCL	078 43	RCL
023 00	0	051 00	0	079 00	0
024 00	0	052 07	7	080 01	1
025 41	GTO	053 85	+	081 65	X
026 68	'8'	054 43	RCL	082 43	RCL
027 46	'LBL	055 00	0	083 00	0
				084 08	8
				085 65	X
				086 43	RCL
				087 00	0
				088 06	6
				089 75	-
				090 43	RCL
				091 00	0
				092 04	4
				093 65	X
				094 43	RCL
				095 00	0
				096 02	2
				097 65	X
				098 43	RCL
				099 00	0
				100 09	9
				101 95	=
				102 42	STO
				103 01	1
				104 09	9
				105 56	'rtn
				106 00	
				107 00	
				108 00	
				109 00	
				110 00	
				111 00	

*Denotes 2nd function key

REGISTERS

REGISTER	VALUE
00	05 a ₃₂
01 a ₁₁	06 a ₃₃
02 a ₁₂	07 a ₃₁
03 a ₁₃	08 a ₃₂
04 a ₂₁	09 a ₃₃
05	10
06	11
07	12
08	13
09	14
10	15
11	16
12	17
13	18
14	19
15	Det.

FLAGS

FLAG	VALUE
0	1
1	2
2	3
3	4

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 10	'E'	140 00	0	168 06	6	196 43	RCL
113 42	STO	141 05	5	169 10	'E'	197 01	1
114 01	1	142 75	-	170 42	STO	198 09	9
115 02	2	143 43	RCL	171 01	1	199 95	=
116 43	RCL	144 00	0	172 00	0	200 70	'iferr
117 00	0	145 04	4	173 56	'rtn	201 67	'7'
118 04	4	146 65	x	174 01	1	202 56	'rtn
119 65	x	147 43	RCL	175 01	1	203 46	'LBL
120 43	RCL	148 00	0	176 42	STO	204 67	'7'
121 00	0	149 02	2	177 00	0	205 00	0
122 03	3	150 10	'E'	178 00	0	206 20	'1/x
123 75	-	151 42	STO	179 46	'LBL	207 00	0
124 43	RCL	152 01	1	180 89	'3'	208 56	'rtn
125 00	0	153 08	8	181 36	'IND	209 00	
126 01	1	154 43	RCL	182 43	RCL	210 00	
127 65	x	155 00	0	183 00	0	211 00	
128 43	RCL	156 05	5	184 00	0	212 00	
129 00	0	157 65	x	185 56	'rtn	213 00	
130 06	6	158 43	RCL	186 01	1	214 00	
131 10	'E'	159 00	0	187 44	SUM	215 00	
132 42	STO	160 09	9	188 00	0	216 00	
133 01	1	161 75	-	189 00	0	217 00	
134 05	5	162 43	RCL	190 41	GTO	218 00	
135 43	RCL	163 00	0	191 89	'3'	219 00	
136 00	0	164 08	8	192 46	'LBL	220 00	
137 01	1	165 65	x	193 10	'E'	221 00	
138 65	x	166 43	RCL	194 95	=	222 00	
139 43	RCL	167 00	0	195 55	+	223 00	

*Denotes 2nd function key

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	'LBL	028 75	-	056 42	STO	084 02	2
001 13	C	029 43	RCL	057 01	1	085 75	-
002 43	RCL	030 00	0	058 01	1	086 43	RCL
003 00	0	031 07	7	059 43	RCL	087 00	0
004 07	7	032 65	x	060 00	0	088 01	1
005 65	x	033 43	RCL	061 01	1	089 65	x
006 43	RCL	034 00	0	062 65	x	090 43	RCL
007 00	0	035 05	5	063 43	RCL	091 00	0
008 06	6	036 10	'E'	064 00	0	092 08	8
009 75	-	037 43	STO	065 09	9	093 10	'E'
010 43	RCL	038 01	1	066 75	-	094 42	STO
011 00	0	039 06	6	067 43	RCL	095 01	1
012 04	4	040 43	RCL	068 00	0	096 07	7
013 65	x	041 00	0	069 07	7	097 43	RCL
014 43	RCL	042 08	8	070 65	x	098 00	0
015 00	0	043 65	x	071 43	RCL	099 02	2
016 09	9	044 43	RCL	072 00	0	100 65	x
017 10	'E'	045 00	0	073 03	3	101 43	RCL
018 42	STO	046 03	3	074 10	'E'	102 00	0
019 01	1	047 75	-	075 42	STO	103 06	6
020 03	3	048 43	RCL	076 01	1	104 75	-
021 43	RCL	049 00	0	077 04	4	105 43	RCL
022 00	0	050 02	2	078 43	RCL	106 00	0
023 04	4	051 65	x	079 00	0	107 05	5
024 65	x	052 43	RCL	080 07	7	108 65	x
025 43	RCL	053 00	0	081 65	x	109 43	RCL
026 00	0	054 09	9	082 43	RCL	110 00	0
027 08	8	055 10	'E'	083 00	0	111 03	3

*Denotes 2nd function key

REGISTERS	
03	05 a ₂₂
04	06 a ₂₃
05	07 a ₃₁
06	08 a ₃₂
07	09 a ₃₃
08	10 b ₁₁
09	11 b ₁₂
10	12 b ₁₃
11	13 b ₂₁
12	14 b ₂₂
13	15 b ₂₃
14	16 b ₃₁
15	17 b ₃₂
16	18 b ₃₃
17	19 Det.

FLAGS			
0	1	2	3
			4

MATRIX ARITHMETIC (2)

The product AB in that order of the $m \times p$ matrix $A = [a_{ij}]$ and the $p \times n$ matrix $B = [b_{ij}]$ is the $m \times n$ matrix $C = [c_{ij}]$ where $0 \leq m \leq 3$, $0 \leq n \leq 3$, $0 \leq p \leq 3$.

$$AB = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = C$$

NOTE: if $m < 3$, $n < 3$, $p < 3$, it is not necessary to enter zeros for the remainder of the matrix elements. Matrix 'A' is not destroyed, thus you may do constant multiplication by matrix 'A'.

Reference: *Computer Methods for Science and Engineering*, Robert L. LaFara, Hayden Book Company, 1973.

Example: Evaluate $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Enter	Press	Display	Comments
	E	0.	Initialize
2	A	1.	a_{11}
3	RUN	1.	a_{12}
1	B	1.	a_{21}
2	RUN	1.	a_{22}
	D	1.	Multiply
1	A	0.	$a_{31} \times b_{11}$
2	RUN	0.	$a_{31} \times b_{21}$
3	B	0.	$a_{32} \times b_{12}$
4	RUN	0.	$a_{32} \times b_{22}$
	2nd LS	11.	C_{11}
	RUN	16.	C_{12}
	2nd RS	7.	C_{21}
	RUN	10.	C_{22}

Example: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 32 \\ 32 \end{bmatrix}$

Example: $\begin{bmatrix} 1 & 4 & -2 \\ 6 & 3 & 0 \\ 6 & -1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 \\ -12 & 6 & 22 \\ 4 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 30 & 30 \\ -7 & 16 & 2 \end{bmatrix}$

TEXAS INSTRUMENTS		© 1976	
91-1VM		MA1-16	
MATRIX ARITHMETIC (2)		MATRIX ARITHMETIC (2)	
← A	← MATRIX ARITHMETIC (2)	← MULT	← INIT
← ci	← ci	← ci	← ci
← a1(b1)	← a2(b2)	← a3(b3)	← INIT

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Initialize		E	
3	Enter matrix	a_{11} (b_{11})	A	$a_{31} \times b_{11}$
4		a_{12} (b_{12})	RUN	$a_{31} \times b_{12}$
5		a_{13} (b_{13})	RUN	$a_{31} \times b_{13}$
6		a_{21} (b_{21})	B	$a_{32} \times b_{21}$
7		a_{22} (b_{22})	RUN	$a_{32} \times b_{22}$
8		a_{23} (b_{23})	RUN	$a_{32} \times b_{23}$
9		a_{31} (b_{31})	C	$a_{33} \times b_{31}$
10		a_{32} (b_{32})	RUN	$a_{33} \times b_{32}$
11		a_{33} (b_{33})	RUN	$a_{33} \times b_{33}$
	(If Matrix B has been entered go to 13)			
12	Setup for multiply (Repeat 3-11 for Matrix B)		D	
13	Recall Matrix C		2nd LS	C_{11}
			RUN	C_{12}
			RUN	C_{13}
			2nd RS	C_{21}
			RUN	C_{22}
			RUN	C_{23}
			2nd CS	C_{31}
			RUN	C_{32}
			RUN	C_{33}
14	To use Matrix A again, go to Step 12			

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 25	CLR	028 52	EE	056 56	*rtn	084 00	0
001 56	*rtn	029 19	*D	057 46	*LBL	085 00	0
002 46	*LBL	030 44	SUM	058 42	STO	086 07	7
003 15	E	031 00	0	059 19	*D	087 51	SBR
004 47	*CMs	032 01	1	060 44	SUM	088 24	CE
005 86	*rset	033 10	*E	061 00	0	089 56	*rtn
006 46	*LBL	034 44	SUM	062 03	3	090 42	STO
007 19	*D	035 00	0	063 10	*E	091 00	0
008 42	STO	036 04	4	064 44	SUM	092 00	0
009 01	1	037 10	*E	065 00	0	093 07	7
010 09	9	038 44	SUM	066 06	6	094 51	SBR
011 46	*LBL	039 00	0	067 10	*E	095 42	STO
012 10	*E	040 07	7	068 44	SUM	096 56	*rtn
013 03	3	041 56	*rtn	069 00	0	097 46	*LBL
014 44	SUM	042 46	*LBL	070 09	9	098 88	*2
015 01	1	043 24	CE	071 56	*rtn	099 08	8
016 09	9	044 19	*D	072 46	*LBL	100 51	SBR
017 43	RCL	045 44	SUM	073 14	D	101 52	EE
018 00	0	046 00	0	074 50	*stflg	102 56	*rtn
019 00	0	047 02	2	075 00	0	103 42	STO
020 65	x	048 10	*E	076 56	*rtn	104 00	0
021 36	*IND	049 44	SUM	077 46	*LBL	105 00	0
022 43	RCL	050 00	0	078 87	*1	106 08	8
023 01	1	051 05	5	079 07	7	107 51	SBR
024 09	9	052 10	*E	080 51	SBR	108 24	CE
025 95	=	053 44	SUM	081 52	EE	109 56	*rtn
026 56	*rtn	054 00	0	082 56	*rtn	110 42	STO
027 46	*LBL	055 08	8	083 42	STO	111 00	0

*Denotes 2nd function key

REGISTERS

00 Used	02 C22	10 a11	15 a22
01 C11	06 C23	11 a12	16 a31
02 C12	07 C31	12 a13	17 a32
03 C13	08 C32	13 a21	18 a33
04 C21	09 C33	14 a22	19 Used

FLAGS

0 Used	1	2	3	4
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PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 00	0	140 00	0	168 00	0	196 00	0
113 08	8	141 00	0	169 00	0	197 46	*LBL
114 51	SBR	142 60	*if flg	170 60	*if flg	198 65	x
115 42	STO	143 00	0	171 00	0	199 01	1
116 56	*rtn	144 87	*1	172 88	*2	200 44	SUM
117 46	*LBL	145 01	1	173 01	1	201 00	0
118 89	*3	146 00	0	174 03	3	202 00	0
119 09	9	147 46	*LBL	175 41	GTO	203 25	CLR
120 51	SBR	148 32	sin	176 32	sin	204 36	*IND
121 52	EE	149 48	*EXC	177 46	*LBL	205 43	RCL
122 56	*rtn	150 00	0	178 13	C	206 00	0
123 42	STO	151 00	0	179 42	STO	207 00	0
124 00	0	152 46	*LBL	180 00	0	208 56	*rtn
125 00	0	153 33	cos	181 00	0	209 41	GTO
126 09	9	154 36	*IND	182 60	*if flg	210 65	x
127 51	SBR	155 42	STO	183 00	0	211 46	*LBL
128 24	CE	156 00	0	184 89	*3	212 17	*B
129 56	*rtn	157 00	0	185 01	1	213 03	3
130 42	STO	158 01	1	186 06	6	214 41	GTO
131 00	0	159 44	SUM	187 41	GTO	215 85	+
132 00	0	160 00	0	188 32	sin	216 46	*LBL
133 09	9	161 00	0	189 46	*LBL	217 18	*C
134 51	SBR	162 56	*rtn	190 16	*A	218 06	6
135 42	STO	163 41	GTO	191 00	0	219 41	GTO
136 56	*rtn	164 33	cos	192 46	*LBL	220 85	+
137 46	*LBL	165 46	*LBL	193 85	+	221 00	0
138 11	A	166 12	B	194 42	STO	222 00	0
139 42	STO	167 42	STO	195 00	0	223 00	0

*Denotes 2nd function key

BASIC STATISTICS FOR ONE OR TWO VARIABLES

This program calculates means, standard deviations, and standard errors of the mean for one or two variables, and covariance and correlation coefficient for two variables.

The input is a set of data points:

$$\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n), \dots, (x_m, y_m)\}$$

in which the y_i 's may be omitted if only one variable is being used.

The statistics computed are:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \text{mean of } x$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i = \text{mean of } y$$

$$S_x = \sqrt{\frac{\sum_{i=1}^n x_i^2 - n\bar{x}^2}{n-1}} = \text{std. dev. of } x \text{ by } n-1 \text{ method}$$

$$S'_x = \sqrt{\frac{\sum_{i=1}^n x_i^2 - n\bar{x}^2}{n}} = \text{std. dev. of } x \text{ by } n \text{ method}$$

$$S_y = \sqrt{\frac{\sum_{i=1}^n y_i^2 - n\bar{y}^2}{n-1}} = \text{std. dev. of } y \text{ by } n-1 \text{ method}$$

$$S'_y = \sqrt{\frac{\sum_{i=1}^n y_i^2 - n\bar{y}^2}{n}} = \text{std. dev. of } y \text{ by } n \text{ method}$$

$$S_{\bar{x}} = \frac{S_x}{\sqrt{n}} = \text{std. error of the mean of } x \text{ by } n-1 \text{ method}$$

$$S'_{\bar{x}} = \frac{S'_x}{\sqrt{n}} = \text{std. error of the mean of } x \text{ by } n \text{ method}$$

$$S_{\bar{y}} = \frac{S_y}{\sqrt{n}} = \text{std. error of the mean of } y \text{ by } n-1 \text{ method}$$

$$S'_{\bar{y}} = \frac{S'_y}{\sqrt{n}} = \text{std. error of the mean of } y \text{ by } n \text{ method}$$

$$S_{xy} = \frac{1}{n-1} \left[\sum_{i=1}^n x_i y_i - \frac{1}{n} \sum_{i=1}^n x_i \sum_{i=1}^n y_i \right] = \text{covariance of } x \text{ and } y \text{ by } n-1 \text{ method}$$

$$S'_{xy} = \frac{1}{n} \left[\sum_{i=1}^n x_i y_i - \frac{1}{n} \sum_{i=1}^n x_i \sum_{i=1}^n y_i \right] = \text{covariance of } x \text{ and } y \text{ by } n \text{ method}$$

$$r_{xy} = \frac{S_{xy}}{S_x S_y} = \frac{S'_{xy}}{S'_x S'_y} = \text{correlation coefficient of } x \text{ and } y$$

NOTES: 1. The number of data points must be > 1 .

2. This program may be used with the Linear Regression program (ST1-08) to determine additional statistics. Data points entered on either program may be used for the other. Do not initialize if data from the other program is being used.

Example: Calculate \bar{x} , \bar{y} , S_x , S_y , S'_x , S'_y , $S_{\bar{x}}$, $S_{\bar{y}}$, $S'_{\bar{x}}$, $S'_{\bar{y}}$, S_{xy} , S'_{xy} , r_{xy} for the data in the following table, then change the x and y values for $i = 5$ from (23, 17) to (5, 19) and find the new r_{xy} and S_{xy} .

i	x_i	y_i
1	31	6
2	20.5	19
3	14	14
4	9	12.5
5	23	17
6	37	22
7	38	19.5
8	18	21
9	29	16

TEXAS INSTRUMENTS

10-115 STATS FOR 1 OR 2 VARIABLES

STATS FOR 1 OR 2 VARIABLES

ST1-01

delete	f, n	S _y , S _y '	S _y , S _y '	r _{xy}
x, y	k, n	S _x , S _x '	S _x , S _x '	S _{xy} , S _{xy} '

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Initialize		2nd [MS]	
3	Enter Data	x ₁	A	1
	If only one variable is used	y ₁	RUN	1
	omit the y steps	x ₂	A	2
		y ₂	RUN	2
4	To delete (x, y)	x ₁	2nd [L]	
		y ₁	RUN	
5	Compute outputs		B	\bar{x}
			RUN	n
	All outputs except those obtained by pressing run, may be obtained in any order. Outputs obtained by pressing run must be obtained immediately following the corresponding keypush shown		2nd [R]	\bar{y}
			RUN	n
			C	S _x
			RUN	S _x '
			2nd [C]	S _y
			RUN	S _y '
			D	S _x
			RUN	S _x '
			2nd [D]	S _y
			RUN	S _y '
			E	S _{xy}
			RUN	S _{xy} '
			2nd [E]	r _{xy}

Enter	Press	Display	Comment
31	2nd [MS]		Initialize
6	A	1.	x ₁
20.5	RUN	1.	y ₁
19	A	2.	x ₂
14	RUN	2.	y ₂
14	A	3.	x ₃
etc.	RUN	3.	y ₃
	B	24.38888889	compute \bar{x}
	2nd [R]	16.33333333	compute \bar{y}
	C	10.07403152	compute S _x
	RUN	9.497888004	compute S _y
	2nd [C]	4.981214711	compute S _x
	RUN	4.696334268	compute S _y
	D	3.358010507	compute S _x
	RUN	3.165962668	compute S _y
	2nd [D]	1.660404904	compute S _x
	RUN	1.565444756	compute S _y
	E	10.85416667	compute S _{xy}
	RUN	9.648148148	compute S _{xy}
	2nd [E]	2163006968	compute r _{xy}
	2nd [L]		delete 23 (23, 17)
23	RUN	8.	delete 17 (23, 17)
17	A	8.	insert 5 (5, 19)
5	RUN	9.	insert 19 (5, 19)
19	A	0825567005	compute new r _{xy}
	2nd [E]	5.006944445	compute new S _x
	E	4.450617284	compute new S _y
	RUN		

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	'LBL	028 00	0	056 56	'rtn	084 44	SUM
001 11	A	029 56	'rtn	057 46	'LBL	085 00	0
002 42	STO	030 42	STO	058 16	'A'	086 04	4
003 00	0	031 00	0	059 95	=	087 40	'x ²
004 01	1	032 02	2	060 94	+/-	088 94	+/-
005 44	SUM	033 44	SUM	061 42	STO	089 41	GTO
006 00	0	034 00	0	062 00	0	090 32	sin
007 03	3	035 04	4	063 01	1	091 46	'LBL
008 40	'x ²	036 40	'x ²	064 44	SUM	092 12	B
009 44	SUM	037 46	'LBL	065 00	0	093 43	RCL
010 00	0	038 32	sin	066 03	3	094 00	0
011 06	6	039 44	SUM	067 40	'x ²	095 03	3
012 46	'LBL	040 00	0	068 94	+/-	096 46	'LBL
013 34	tan	041 07	7	069 44	SUM	097 33	cos
014 43	RCL	042 43	RCL	070 00	0	098 65	X
015 00	0	043 00	0	071 06	6	099 43	RCL
016 00	0	044 01	1	072 02	2	100 00	0
017 30	'\x	045 65	X	073 94	+/-	101 00	0
018 42	STO	046 43	RCL	074 44	SUM	102 20	'1/x
019 00	0	047 00	0	075 00	0	103 54)
020 08	8	048 02	2	076 00	0	104 56	'rtn
021 01	1	049 95	=	077 51	SBR	105 43	RCL
022 44	SUM	050 44	SUM	078 34	tan	106 00	0
023 00	0	051 00	0	079 56	'rtn	107 00	0
024 00	0	052 05	5	080 42	STO	108 56	'rtn
025 25	CLR	053 43	RCL	081 00	0	109 46	'LBL
026 43	RCL	054 00	0	082 02	2	110 17	'B'
027 00	0	055 00	0	083 94	+/-	111 43	RCL

*Denotes 2nd function key

REGISTERS

00 n	05 Σxy	10	15
01 x	06 Σx ²	11	16
02 y	07 Σy ²	12	17
03 Σx	08 √n-1	13	18
04 Σy	09	14	19

FLAGS

0	1	2	3	4
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PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 00	0	140 43	RCL	168 13	C	196 00	0
113 04	4	141 65	X	169 41	GTO	197 08	8
114 41	GTO	142 43	RCL	170 42	STO	198 40	'x ²
115 33	cos	143 00	0	171 46	'LBL	199 95	=
116 46	'LBL	144 08	8	172 19	'D'	200 56	'rtn
117 13	C	145 46	'LBL	173 18	'C'	201 65	X
118 43	RCL	146 42	STO	174 41	GTO	202 43	RCL
119 00	0	147 55	±	175 42	STO	203 00	0
120 06	6	148 43	RCL	176 46	'LBL	204 08	8
121 75	-	149 00	0	177 15	E	205 40	'x ²
122 53	(150 00	0	178 43	RCL	206 55	±
123 12	B	151 30	'\x	179 00	0	207 43	RCL
124 46	'LBL	152 95	=	180 05	5	208 00	0
125 24	CE	153 56	'rtn	181 75	-	209 00	0
126 40	'x ²	154 41	GTO	182 43	RCL	210 95	=
127 65	X	155 43	RCL	183 00	0	211 56	'rtn
128 43	RCL	156 46	'LBL	184 03	3	212 46	'LBL
129 00	0	157 18	'C'	185 65	X	213 10	'E'
130 00	0	158 43	RCL	186 43	RCL	214 15	E
131 54)	159 00	0	187 00	0	215 55	±
132 30	'\x	160 07	7	188 04	4	216 53	(
133 55	-	161 75	-	189 55	±	217 53	(
134 43	RCL	162 53	(190 43	RCL	218 13	C
135 00	0	163 17	'B'	191 00	0	219 55	±
136 08	8	164 41	GTO	192 00	0	220 53	(
137 95	=	165 24	CE	193 95	=	221 53	(
138 56	'rtn	166 46	'LBL	194 55	-	222 18	'C'
139 46	'LBL	167 14	D	195 43	RCL	223 56	'rtn

*Denotes 2nd function key

MEANS AND MOMENTS

For a given set of input data, $\{x_1, x_2, \dots, x_n\}$, with associated frequencies, $\{f_1, f_2, \dots, f_n\}$ (for grouped data), the following means, moments, and skewness and kurtosis of distribution are calculated, if $f_i = 1$ for all i , the calculations are for ungrouped data, otherwise for grouped data.

Means

$$\text{Arithmetic} = A = \frac{1}{N} \sum_{i=1}^n f_i x_i$$

$$\text{Harmonic} = H = N / \sum_{i=1}^n \frac{f_i}{x_i}$$

$$\text{Geometric} = G = \left(\prod_{i=1}^n x_i^{f_i} \right)^{1/n}$$

$$\text{Generalized} = M(t) = \left(\frac{1}{N} \sum_{i=1}^n f_i (x_i)^t \right)^{1/t}$$

Moments

$$m_1 = A = \frac{1}{N} \sum_{i=1}^n f_i x_i$$

$$m_2 = \frac{1}{N} \sum_{i=1}^n f_i (x_i - A)^2$$

$$\text{Kurtosis of distribution} = \frac{m_4}{(m_2)^2}$$

$$\text{Skewness of distribution} = \frac{m_3}{(m_2)^{3/2}}$$

$$N = \sum_{i=1}^n f_i \quad n = \text{Number of different groups}$$

- NOTES: 1. The initialization routine assumes a frequency of 1 and $t = 1$.
 2. The value of t may be changed before any x_i 's are entered, if changed after any x_i , $M(t)$ will be meaningless.
 3. For grouped data if $f_i = f_{i-1}$, it need not be entered.
 4. A negative frequency will cause the next x entered to be deleted f_i times.
 5. A negative x will cause the geometric and generalized means to be invalid.

Error Indications:

A frequency or t of 0 will cause the display to flash 9.999999999 99 and the old value to be retained.

Example: The ungrouped ($f_i = 1$) data $\{x_i = 1.1, 2.3, 4.7, 3.5, 1.8, 2.9\}$ yields the following: $A = 2.716666667$, $G = 2.451843924$, $H = 2.187595899$, $m(1) = 2.716666667$, $m_2 = 1.368055556$, $m_3 = .5367592591$, $m_4 = 3.903019215$; kurtosis of distribution = 2.085418497, skewness of distribution = .3354470243

References: *Handbook of Mathematical Tables and Formulas*, R. S. Burington, McGraw-Hill, Fourth Edition, 1965, p. 164. *International Dictionary of Applied Mathematics*, W. F. Frieberger, D. Van Nostrand, 1960

TEXAS INSTRUMENTS			
1-EO-TLS	(1) S I N E M O W	MEANS & MOMENTS	ST1-03-1
A	G	H	M(t)
x → Σf	t	1	del x
			INIT

TEXAS INSTRUMENTS			
2-EO-TLS	(2) S I N E M O W	MEANS & MOMENTS(2)	ST1-03-2
A	G	H	M(t)
m2	m3	m4	kurt
			skew

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program ST1-03-1			
2	Initialize		E	1
3	Enter t (for gen. mean)	t ≠ 0	C	t
4	Enter data			
	a. for grouped data,			
	ent. f_i if $f_i \neq f_{i-1}$,	t ≠ 0	B	f_i
	b. enter x	x	A	Σf
	repeat a and b for each i			
5	Delete data			
	a. for grouped data,			
	enter f_i if $f_i \neq f_{i-1}$,	t ≠ 0	B	f_i
	b. enter x to be deleted	x	D	corrected Σf
6	Calculate			
	a. arithmetic mean		2nd F1	A
	b. geometric mean		2nd F2	G
	c. harmonic mean		2nd F3	H
	d. generalized mean		2nd F4	M(t)
.7	Enter Program ST1-03-2			
8	Calculate			
	a. second moment		A	m_2
	b. third moment		B	m_3
	c. fourth moment		C	m_4
	d. kurtosis		D	kurtosis
	e. skewness		E	skewness
	f. means, see step 6			

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	'LBL	028 45	y'	056 43	RCL
001 11	A	029 43	RCL	057 00	0
002 42	STO	030 00	0	058 05	5
003 00	0	031 02	2	058 02	2
004 04	4	032 95	=	059 65	X
005 65	X	033 49	'PROD	060 44	SUM
006 43	RCL	034 00	0	061 00	0
007 00	0	035 06	6	062 09	9
008 02	2	036 43	RCL	063 43	RCL
009 95	=	037 00	0	064 00	0
010 44	SUM	038 04	4	065 04	4
011 00	0	039 45	y'	066 65	X
012 05	5	040 43	RCL	067 44	SUM
013 43	RCL	041 00	0	068 01	1
014 00	0	042 03	3	069 00	0
015 04	4	043 65	X	070 43	RCL
016 20	'1/x	044 43	RCL	071 00	0
017 65	X	045 00	0	072 04	4
018 43	RCL	046 02	2	073 95	=
019 00	0	047 95	=	074 44	SUM
020 02	2	048 44	SUM	075 01	1
021 95	=	049 00	0	076 01	1
022 44	SUM	050 08	8	077 43	RCL
023 00	0	051 43	RCL	078 00	0
024 07	7	052 00	0	079 02	2
025 43	RCL	053 04	4	080 44	SUM
026 00	0	054 40	'x ²	081 00	0
027 04	4	055 65	X	082 01	1
				083 43	RCL
				084 00	0
				085 55	5
				086 55	5
				087 43	RCL
				088 00	0
				089 01	1
				090 95	=
				091 42	STO
				092 01	1
				093 02	2
				094 25	CLR
				095 43	RCL
				096 00	0
				097 01	1
				098 56	'rtn
				099 46	'LBL
				100 12	B
				101 90	'ifzro
				102 87	'1'
				103 42	STO
				104 00	0
				105 02	2
				106 56	'rtn
				107 46	'LBL
				108 15	E
				109 25	CLR
				110 47	'CMs
				111 01	1

Denotes 2nd function key

REGISTERS

00	05 ∑f	10 ∑fx ²	15
01 n	06 ∑fx	11 ∑fx ⁴	16
02 f	07 ∑1/x	12 ∑fx/n	17
03 f	08 ∑fx'	13	18
04 last x	09 ∑fx ²	14	19

FLAGS

0	1	2	3	4
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PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 42	STO	140 11	A	168 46	'LBL
113 00	0	141 01	1	169 18	'C'
114 02	2	142 94	+/-	170 43	RCL
115 42	STO	143 22	INV	171 00	0
116 00	0	144 49	'PROD	172 01	1
117 06	6	145 00	0	173 55	=
118 46	'LBL	146 02	2	174 43	RCL
119 13	C	147 43	RCL	175 00	0
120 90	'ifzro	148 00	0	176 07	7
121 87	'1'	149 01	1	177 95	=
122 42	STO	150 56	'rtn	178 56	'rtn
123 00	0	151 46	'LBL	179 46	'LBL
124 03	3	152 16	'A'	180 19	'D'
125 56	'rtn	153 43	RCL	181 43	RCL
126 46	'LBL	154 01	1	182 00	0
127 14	D	155 02	2	183 08	8
128 42	STO	156 56	'rtn	184 55	=
129 00	0	157 46	'LBL	185 43	RCL
130 04	4	158 17	'B'	186 00	0
131 01	1	159 43	RCL	187 01	1
132 94	+/-	160 00	0	188 95	=
133 22	INV	161 06	6	189 35	√y
134 49	'PROD	162 35	√y	190 43	RCL
135 00	0	163 43	RCL	191 00	0
136 02	2	164 00	0	192 03	3
137 43	RCL	165 01	1	193 95	=
138 00	0	166 95	=	194 56	'rtn
139 04	4	167 56	'rtn	195 46	'LBL
				196 47	'1'
				197 20	'1/x
				198 56	'rtn
				199 00	
				200 00	
				201 00	
				202 00	
				203 00	
				204 00	
				205 00	
				206 00	
				207 00	
				208 00	
				209 00	
				210 00	
				211 00	
				212 00	
				213 00	
				214 00	
				215 00	
				216 00	
				217 00	
				218 00	
				219 00	
				220 00	
				221 00	
				222 00	
				223 00	

Denotes 2nd function key

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	'LBL	028 03	3	056 55	-	084 09	9
001 11	A	029 65	X	057 43	RCL	085 55	-
002 43	RCL	030 43	RCL	058 00	0	086 43	RCL
003 01	1	031 01	1	059 01	1	087 00	0
004 02	2	032 02	2	060 75	-	088 01	1
005 40	'x ²	033 65	X	061 04	4	089 75	-
006 94	+/-	034 43	RCL	062 65	X	090 03	3
007 85	+	035 00	0	063 43	RCL	091 65	X
008 53	(036 09	9	064 01	1	092 43	RCL
009 43	RCL	037 55	-	065 02	2	093 01	1
010 00	0	038 43	RCL	066 65	X	094 02	2
011 09	9	039 00	0	067 43	RCL	095 40	'x ²
012 55	-	040 01	1	068 01	1	096 40	'x ²
013 43	RCL	041 85	+	069 00	0	097 95	=
014 00	0	042 43	RCL	070 55	-	098 56	'rtn
015 01	1	043 01	1	071 43	RCL	099 46	'LBL
016 95	=	044 02	2	072 00	0	100 14	D
017 56	'rtn	045 65	X	073 01	1	101 11	A
018 46	'LBL	046 40	'x ²	074 85	+	102 40	'x ²
019 12	B	047 65	X	075 06	6	103 42	STO
020 43	RCL	048 02	2	076 65	X	104 01	1
021 01	1	049 95	=	077 43	RCL	105 03	3
022 00	0	050 56	'rtn	078 01	1	106 13	C
023 55	+	051 46	'LBL	079 02	2	107 46	'LBL
024 43	RCL	052 13	C	080 40	'x ²	108 87	'1'
025 00	0	053 43	RCL	081 65	X	109 55	-
026 01	1	054 01	1	082 43	RCL	110 43	RCL
027 75	-	055 01	1	083 00	0	111 01	1

*Denotes 2nd function key

REGISTERS

00	05	∑/x	03	∑/x ³	15	
01	n	06	x'	11	∑/x ⁴	16
02	f	05	∑/x	02	∑/x/n	17
03	t	08	∑/x'	13	Used	18
04	lastx	09	∑/x ²	14		19

FLAGS

0	1	2	3	4
---	---	---	---	---

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 03	3	140 35	√y	168 43	RCL	196 00	
113 95	=	141 43	RCL	169 00	0	197 00	
114 56	'rtn	142 00	0	170 03	3	198 00	
115 46	'LBL	143 01	1	171 95	=	199 00	
116 15	E	144 95	=	172 56	'rtn	200 00	
117 11	A	145 56	'rtn	173 00		201 00	
118 65	X	146 46	'LBL	174 00		202 00	
119 40	'x ²	147 18	'C'	175 00		203 00	
120 95	=	148 43	RCL	176 00		204 00	
121 30	√x	149 00	0	177 00		205 00	
122 95	=	150 01	1	178 00		206 00	
123 42	STO	151 55	-	179 00		207 00	
124 01	1	152 43	RCL	180 00		208 00	
125 03	3	153 00	0	181 00		209 00	
126 12	B	154 07	7	182 00		210 00	
127 41	GTO	155 95	=	183 00		211 00	
128 87	'1'	156 56	'rtn	184 00		212 00	
129 46	'LBL	157 46	'LBL	185 00		213 00	
130 16	'A'	158 19	'D'	186 00		214 00	
131 43	RCL	159 43	RCL	187 00		215 00	
132 01	1	160 00	0	188 00		216 00	
133 02	2	161 08	8	189 00		217 00	
134 56	'rtn	162 55	-	190 00		218 00	
135 46	'LBL	163 43	RCL	191 00		219 00	
136 17	'B'	164 00	0	192 00		220 00	
137 43	RCL	165 01	1	193 00		221 00	
138 00	0	166 95	=	194 00		222 00	
139 06	6	167 35	√y	195 00		223 00	

*Denotes 2nd function key

LINEAR REGRESSION

The linear least-squares fit of input data points (x,y) is calculated using the following:

$$\text{Slope} = m = \frac{\frac{\sum x \cdot y}{n} - \frac{\sum xy}{n}}{\left(\frac{\sum x^2}{n}\right) - \frac{(\sum x)^2}{n}}$$

$$\text{Intercept} = b = \frac{\sum y - m \sum x}{n}$$

The y' for entered x and x' for entered y are calculated as follows:

$$y' = mx + b$$

$$x' = (y - b)/m$$

The coefficient of determination is calculated as:

$$r^2 = m \left(\frac{\sum x \cdot y}{n} - \frac{\sum xy}{n} \right) / \left(\sum y^2 - \frac{(\sum y)^2}{n} \right)$$

- NOTES:** 1. The calculator must be initialized by pressing **2nd** **CS** before each new set of data is entered.
 2. The number of data points entered must be greater than 1 before any of the calculations are attempted.
 3. This program may be used with the Basic Statistics program (ST1-01) to determine additional statistics. Data points entered on either program may be used for the other. Do not initialize if data from the other program is being used.

Example:

x	20.4	19.7	21.8	20.1	20.7
y	9.2	8.9	11.4	9.4	10.3

$r^2 = .9256678121$

$m = 1.22906793$

$b = -15.40505529$

For $y = 12$, $x = 22.29742931$

For $x = 22.3$, $y = 12.00315956$

TEXAS INSTRUMENTS			
80-115	NOISSEGEER RVEM1		
LINEAR REGRESSION			
Y → X'	X → Y	delete	ST1-08
X	Y	slope	intcpt
		r ²	

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Initialize		2nd CS	
3	Enter data	X	A	X
	Repeat 3 for each x y	Y	B	n
4	Delete data			
	a. Enter x (not necessary if deleting last pair entered)	X	A	X
	b. Enter y	Y	B	n-1
5	Calculate			
	a. r ²		C	r ²
	b. slope		D	m
	c. intercept		E	b
	d. x	Y	2nd A	X'
	e. y	X	2nd B	Y'

POWER CURVE FIT

Given a set of data points:

$$(x_i, y_i), i = 1, 2, \dots, n$$

Where $x_i > 0$ and $y_i > 0$, this program will fit a power curve:

$$y = ax^b \quad (a > 0)$$

The problem may be interpreted as a linear regression problem by using the equation in the form:

$$\ln y = b \ln x + \ln a$$

The computed statistics are:

1. Regression coefficients

$$a = \exp \left[\frac{\sum \ln y_i}{n} - b \frac{\sum \ln x_i}{n} \right]$$

$$b = \frac{\frac{\sum (\ln x_i)(\ln y_i)}{n} - \left(\frac{\sum \ln x_i}{n} \right) \left(\frac{\sum \ln y_i}{n} \right)}{\frac{\sum (\ln x_i)^2}{n} - \left(\frac{\sum \ln x_i}{n} \right)^2}$$

NOTE: n is a positive integer $\neq 1$.

2. Correlation coefficient

$$r^2 = \frac{\left[\frac{\sum (\ln x_i)(\ln y_i)}{n} - \left(\frac{\sum \ln x_i}{n} \right) \left(\frac{\sum \ln y_i}{n} \right) \right]^2}{\left[\frac{\sum (\ln x_i)^2}{n} - \left(\frac{\sum \ln x_i}{n} \right)^2 \right] \left[\frac{\sum (\ln y_i)^2}{n} - \left(\frac{\sum \ln y_i}{n} \right)^2 \right]}$$

3. Estimated value x' for given y

$$x' = \sqrt[n]{y/a}$$

4. Estimated value y' for given x

$$y' = ax'^b$$

Example: x_i | 3 6 8 11 13.5 15 18.5 20
 y_i | .8 1.12 1.3 1.53 1.95 2.4 3.0 3.7

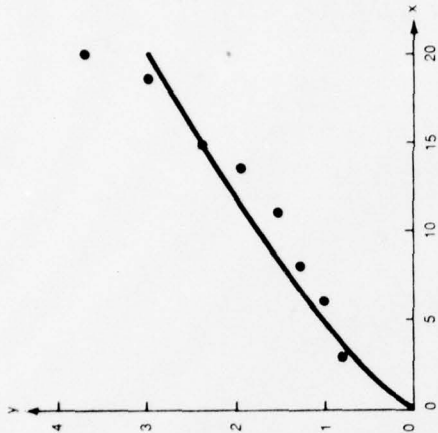
$$a = .2901902009$$

$$b = .7759169351$$

$$r^2 = 0.921727263$$

$$\text{for } x = 9, y' = 1.596230726$$

$$\text{for } y = 2.8, x' = 18.56904361$$



TEXAS INSTRUMENTS	
60-115 POWER CURVE FIT	
ST1-09	
delete	INIT
$y \rightarrow x'$	$x \rightarrow y'$
x_i	y_i
b	a

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Initialize		A	0
3	Enter data	x_i	B	x_i
	(repeat for all i)	y_i	C	y_i
4	Delete data			
	a. Enter x	x	B	x
	b. Enter y	y	2nd A	$n-1$
5	Calculate regression coefficients		D	a
			2nd F	b
6	Calculate correlation coefficient		E	r^2
7	Compute estimated value for			
	x given y	y	2nd F	x
	y' given x	x	2nd C	y'

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 20	'1/x	140 08 8		168 46	'LBL	196 01 1	
113 42	STO	141 94 +/-		169 17	'B'	197 00 0	
114 01 1		142 85 X		170 55 =		198 65 X	
115 00 0		143 43 RCL		171 43	RCL	199 53 (
116 65 X		144 00 0		172 00 0		200 43 RCL	
117 53 (145 01 1		173 09 9		201 00 0	
118 43 RCL		146 85 +		174 95 =		202 05 5	
119 00 0		147 43 RCL		175 35 \y		203 75 -	
120 04 4		148 00 0		176 43 RCL		204 43 RCL	
121 75 -		149 03 3		177 00 0		205 00 0	
122 43 RCL		150 95 =		178 08 8		206 03 3	
123 00 0		151 55 +		179 95 =		207 40 'x'	
124 01 1		152 43 RCL		180 56 'rtn		208 55 -	
125 65 X		153 00 0		181 46 'LBL		209 43 RCL	
126 43 RCL		154 00 0		182 18 'C'		210 00 0	
127 00 0		155 95 =		183 45 y'		211 00 0	
128 03 3		156 22 INV		184 43 RCL		212 54)	
129 55 +		157 23 lnX		185 00 0		213 20 '1/x	
130 43 RCL		158 42 STO		186 08 8		214 65 X	
131 00 0		159 00 0		187 65 X		215 43 RCL	
132 00 0		160 09 9		188 43 RCL		216 01 1	
133 54)		161 56 'rtn		189 00 0		217 01 1	
134 42 STO		162 46 'LBL		190 09 9		218 40 'x'	
135 01 1		163 19 'D'		191 95 =		219 95 =	
136 01 1		164 43 RCL		192 56 'rtn		220 42 STO	
137 95 =		165 00 0		193 46 'LBL		221 01 1	
138 42 STO		166 08 8		194 15 E		222 02 2	
139 00 0		167 56 'rtn		195 43 RCL		223 56 'rtn	

*Denotes 2nd function key

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	'LBL	028 00 0		056 46	'LBL	084 00 0	
001 11 A		029 06 6		057 16	'A'	085 02 2	
002 47 'C'MS		030 44 SUM		058 23	lnX	086 43 RCL	
003 25 'CLR		031 00 0		059 42	STO	087 00 0	
004 56 'rtn		032 01 1		060 00 0		088 07 7	
005 46 'LBL		033 49 'PROD		061 07 7		089 22 INV	
006 12 B		034 00 0		062 22	INV	090 44 SUM	
007 23 lnX		035 07 7		063 44	SUM	091 00 0	
008 42 STO		036 40 'x'		064 00 0		092 04 4	
009 00 0		037 44 SUM		065 03 3		093 01 1	
010 06 6		038 00 0		066 40 'x'		094 94 +/-	
011 22 INV		039 02 2		067 22	INV	095 41 GTO	
012 23 lnX		040 43 RCL		068 44	SUM	096 87 '1'	
013 56 'rtn		041 00 0		069 00 0		097 46 'LBL	
014 46 'LBL		042 07 7		070 05 5		098 14 D	
015 13 C		043 44 SUM		071 43	RCL	099 43 RCL	
016 23 lnX		044 00 0		072 00 0		100 00 0	
017 42 STO		045 04 4		073 06 6		101 02 2	
018 00 0		046 01 1		074 22	INV	102 75 -	
019 07 7		047 46 'LBL		075 44	SUM	103 43 RCL	
020 44 SUM		048 87 '1'		076 00 0		104 00 0	
021 00 0		049 44 SUM		077 01 1		105 01 1	
022 03 3		050 00 0		078 49	'PROD	106 40 'x'	
023 40 'x'		051 00 0		079 00 0		107 55 +	
024 44 SUM		052 43 RCL		080 07 7		108 43 RCL	
025 00 0		053 00 0		081 40	'x'	109 00 0	
026 05 5		054 00 0		082 22	INV	110 00 0	
027 43 RCL		055 56 'rtn		083 44	SUM	111 95 =	

*Denotes 2nd function key

REGISTERS

00	Used	17
01	Used	5
02	r ²	17
03	b	18
04	a	19

FLAGS

1	1
---	---

EXPONENTIAL CURVE FIT

Given a set of data points:

$$\{(x_i, y_i), i = 1, 2, \dots, n\} \quad y_i > 0$$

This program finds the least-squares fit for an exponential function of the form:

$$y = ae^{bx} \quad (a > 0)$$

The problem is computed according to the linear equation

$$\ln y = \ln a + bx$$

Statistical outputs:

1. Coefficients a, b

$$a = \exp \left[\frac{\sum \ln y_i}{n} - b \frac{\sum x_i}{n} \right]$$

$$b = \frac{\sum x_i \ln y_i - \frac{\sum x_i}{n} \frac{\sum \ln y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

Note: n is positive integer $\neq 1$.

2. Correlation coefficient

$$r^2 = \frac{\left[\sum x_i \ln y_i - \frac{\sum x_i}{n} \frac{\sum \ln y_i}{n} \right]^2}{\left[\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right] \left[\sum (\ln y_i)^2 - \frac{(\sum \ln y_i)^2}{n} \right]}$$

3. Estimated value for x' given y

$$x' = \frac{1}{b} \ln \left(\frac{y}{a} \right)$$

4. Estimated value for y' given x

$$y' = ae^{bx}$$

Example:	x_i	1.42	1.94	2.63	3.11	3.7	4.25
	y_i	.63	.9	1.42	1.9	2.43	2.89

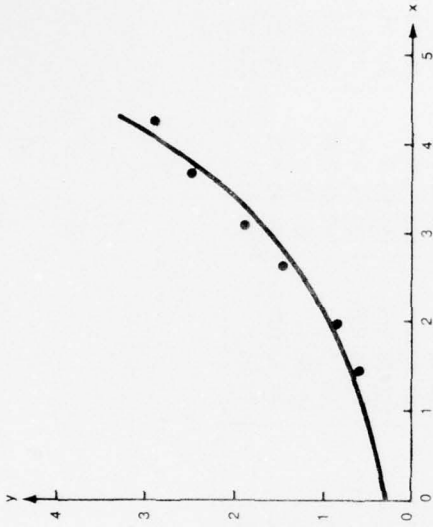
$$a = .3131935086$$

$$b = .5477927735$$

$$r^2 = .9810626986$$

$$\text{For } x = 1.6, \quad y' = .7524162355$$

$$\text{For } y = 2.2, \quad x' = 3.558629277$$



TEXAS INSTRUMENTS			
JL-1115 EXPONENTIAL CURVE FIT			
ST11-10			
delete	$y \rightarrow x'$	$x \rightarrow y'$	
INIT	x_i	y_i	r^2

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			0.
2	Initialize		A	x_i
3	Enter data (repeat for all i)	x_i y_i	B C	n
4	Delete data			
	a. Enter x	x	B	x
	b. Enter y	y	2nd A	$n-1$
5	Calculate coefficients		D	a
			2nd B	b
6	Calculate correlation coefficient		E	r^2
7	Compute estimated value for			
	x' given y	y	2nd F	x'
	y' given x	x	2nd G	y'

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	'LBL	028 00 0	056 56 'rtn	084 44 SUM			
001 10	'E'	029 05 5	057 46 'LBL	085 00 0			
002 43	RCL	030 43 RCL	058 16 'A'	086 02 2			
003 00 0		031 00 0	059 23 Inx	087 43 RCL			
004 00 0		032 06 6	060 42 STO	088 00 0			
005 56	'rtn	033 44 SUM	061 00 0	089 07 7			
006 46	'LBL	034 00 0	062 07 7	090 22 INV			
007 11	A	035 01 1	063 22 INV	091 44 SUM			
008 47	'CMs	036 49 'PROD	064 44 SUM	092 00 0			
009 25	CLR	037 00 0	065 00 0	093 04 4			
010 56	'rtn	038 07 7	066 03 3	094 01 1			
011 46	'LBL	039 40 'x ²	067 40 'x ²	095 94 +/-			
012 12	B	040 44 SUM	068 22 INV	096 41 GTO			
013 42	STO	041 00 0	069 44 SUM	097 87 '1'			
014 00 0		042 02 2	070 00 0	098 46 'LBL			
015 06 6		043 43 RCL	071 05 5	099 14 D			
016 56	'rtn	044 00 0	072 43 RCL	100 43 RCL			
017 46	'LBL	045 07 7	073 00 0	101 00 0			
018 13	C	046 44 SUM	074 06 6	102 02 2			
019 23	Inx	047 00 0	075 22 INV	103 75 -			
020 42	STO	048 04 4	076 44 SUM	104 43 RCL			
021 00 0		049 01 1	077 00 0	105 00 0			
022 07 7		050 46 'LBL	078 01 1	106 01 1			
023 44	SUM	051 87 '1'	079 49 'PROD	107 40 'x ²			
024 00 0		052 44 SUM	080 00 0	108 55 ÷			
025 03 3		053 00 0	081 07 7	109 10 'E'			
026 40	'x ²	054 00 0	082 40 'x ²	110 95 =			
027 44	SUM	055 10 'E'	083 22 INV	111 20 '1/x			

*Denotes 2nd function key

REGISTERS

03 n	05 ∑ (Iny) ²	10 Used	15
01 ∑ x	06 x	11 Used	16
02 ∑ x ²	07 x Iny	12 r ²	17
03 ∑ Iny	08 b	1	18
04 ∑ x Iny	09 a	14	19

FLAGS

0	1	2	3	4
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PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 42	STO	140 43 RCL	168 09 9	196 00 0			
113 01 1		141 00 0	169 95 =	197 65 X			
114 00 0		142 01 1	170 23 Inx	198 53 (
115 65 X		143 85 +	171 55 ÷	199 43 RCL			
116 53 (144 43 RCL	172 43 RCL	200 00 0			
117 43	RCL	145 00 0	173 00 0	201 05 5			
118 00 0		146 03 3	174 08 8	202 75 -			
119 04 4		147 95 =	175 95 =	203 43 RCL			
120 75 -		148 55 ÷	176 56 'rtn	204 00 0			
121 43	RCL	149 10 'E'	177 46 'LBL	205 03 3			
122 00 0		150 95 =	178 18 'C'	206 40 'x ²			
123 01 1		151 22 INV	179 65 X	207 55 +			
124 65 X		152 23 Inx	180 43 RCL	208 10 'E'			
125 43	RCL	153 42 STO	181 00 0	209 54)			
126 00 0		154 00 0	182 08 8	210 20 '1/x			
127 03 3		155 09 9	183 95 =	211 65 X			
128 55 ÷		156 56 'rtn	184 22 INV	212 43 RCL			
129 10 'E'		157 46 'LBL	185 23 Inx	213 01 1			
130 54)		158 19 'D'	186 65 X	214 01 1			
131 42	STO	159 43 RCL	187 43 RCL	215 40 'x ²			
132 01 1		160 00 0	188 00 0	216 95 =			
133 01 1		161 08 8	189 09 9	217 42 STO			
134 95 =		162 56 'rtn	190 95 =	218 01 1			
135 42	STO	163 46 'LBL	191 56 'rtn	219 02 2			
136 00 0		164 17 'B'	192 46 'LBL	220 56 'rtn			
137 08 8		165 55 ÷	193 15 E	221 00			
138 94 +/-		166 43 RCL	194 43 RCL	222 00			
139 65 X		167 00 0	195 01 1	223 00			

*Denotes 2nd function key

LOGARITHMIC CURVE FIT

Given a set of data points:

$$\{(x_i, y_i), i = 1, 2, \dots, n\}$$

Where: $x_i > 0$, this program fits a logarithmic curve:

$$y = a - b \ln x$$

Statistical outputs:

1. Regression coefficients

$$a = \frac{\sum y_i - b \sum \ln x_i}{n}$$

$$b = \frac{\sum y_i \ln x_i - \frac{\sum \ln x_i \sum y_i}{n}}{\sum (\ln x_i)^2 - \frac{(\sum \ln x_i)^2}{n}}$$

2. Correlation coefficient

$$r^2 = \frac{[\sum y_i \ln x_i - \frac{\sum \ln x_i \sum y_i}{n}]^2}{[\sum (\ln x_i)^2 - \frac{(\sum \ln x_i)^2}{n}] [\sum y_i^2 - \frac{(\sum y_i)^2}{n}]}$$

3. Estimated value for x given y

$$x' = \exp \left[\frac{y - a}{b} \right]$$

4. Estimated value for y given x

$$y' = a - b \ln x$$

NOTE: n is positive integer $\neq 1$.

Example:

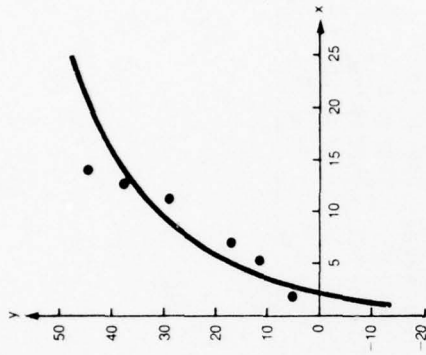
x_i	2	5	7	11	13	14
y_i	5.3	10.8	17.5	28.2	36.9	44.4

$$a = -13.49953513$$

$$b = 18.91052316$$

$$r^2 = .8418448418$$

For $x = 21$, $y' = 44.07397695$
 For $y = 40$, $x = 16.93001448$



TEXAS INSTRUMENTS			
LOGARITHMIC CURVE FIT			
ST1-11	delete	INIT	ST1-11
	$y \rightarrow x'$	$x \rightarrow y$	
	x_i	y_i	r^2
		a	
		b	

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Initialize		A	0
3	Enter data	x_i	B	x_i
	(repeat for all i)	y_i	C	n
4	Delete data			
	a. Enter x	x	B	x
	b. Enter y	y	2nd A	n-1
5	Calculate coefficients		D	a
			2nd B	b
6	Calculate correlation coefficient		E	r^2
7	Compute estimated value for			
	x' given y	y	2nd A	x'
	y' given x	x	2nd B	y

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 00 0		140 43 RCL		188 65 X		196 75 -	
113 65 X		141 00 0		189 53 (197 43 RCL	
114 53 (142 01 1		170 43 RCL		198 00 0	
115 43 RCL		143 85 +		171 00 0		199 09 9	
116 00 0		144 43 RCL		172 05 5		200 95 =	
117 04 4		145 00 0		173 75 -		201 55 ÷	
118 75 -		146 03 3		174 43 RCL		202 43 RCL	
119 43 RCL		147 95 =		175 00 0		203 00 0	
120 00 0		148 55 ÷		176 03 3		204 08 8	
121 01 1		149 43 RCL		177 40 *x ²		205 95 =	
122 65 X		150 00 0		178 55 +		206 22 INV	
123 43 RCL		151 00 0		179 43 RCL		207 23 Inx	
124 00 0		152 95 =		180 00 0		208 56 *rtn	
125 03 3		153 42 STO		181 00 0		209 46 *LBL	
126 55 +		154 00 0		182 54)		210 18 *C'	
127 43 RCL		155 09 9		183 20 *1/x		211 23 Inx	
128 00 0		156 56 *rtn		184 65 X		212 65 X	
129 00 0		157 46 *LBL		185 43 RCL		213 43 RCL	
130 54)		158 19 *D'		186 01 1		214 00 0	
131 42 STO		159 43 RCL		187 01 1		215 08 8	
132 01 1		160 00 0		188 40 *x ²		216 85 +	
133 01 1		161 08 8		189 95 =		217 43 RCL	
134 95 =		162 56 *rtn		190 42 STO		218 00 0	
135 42 STO		163 46 *LBL		191 01 1		219 09 9	
136 00 0		164 15 E		192 02 2		220 95 =	
137 08 8		165 43 RCL		193 56 *rtn		221 56 *rtn	
138 94 +/-		166 01 1		194 46 *LBL		222 00	
139 65 X		167 00 0		195 17 *B'		223 00	

*Denotes 2nd function key

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46 *LBL		028 00 0		056 00 0		084 00 0	
001 11 A		029 01 1		057 07 7		085 07 7	
002 47 *CMs		030 49 *PROD		058 22 INV		086 22 INV	
003 25 CLR		031 00 0		059 44 SUM		087 44 SUM	
004 56 *rtn		032 07 7		060 00 0		088 00 0	
005 46 *LBL		033 40 *x ²		061 03 3		089 04 4	
006 12 B		034 44 SUM		062 40 *x ²		090 01 1	
007 42 STO		035 00 0		063 22 INV		091 94 +/-	
008 00 0		036 02 2		064 44 SUM		092 41 GTO	
009 06 6		037 43 RCL		065 00 0		093 87 *1'	
010 56 *rtn		038 00 0		066 05 5		094 46 *LBL	
011 46 *LBL		039 07 7		067 43 RCL		095 14 D	
012 13 C		040 44 SUM		068 00 0		096 43 RCL	
013 42 STO		041 00 0		069 06 6		097 00 0	
014 00 0		042 04 4		070 23 Inx		098 02 2	
015 07 7		043 01 1		071 22 INV		099 75 -	
016 44 SUM		044 46 *LBL		072 44 SUM		100 43 RCL	
017 00 0		045 87 *1'		073 00 0		101 00 0	
018 03 3		046 44 SUM		074 01 1		102 01 1	
019 40 *x ²		047 00 0		075 49 *PROD		103 40 *x ²	
020 44 SUM		048 00 0		076 00 0		104 55 ÷	
021 00 0		049 43 RCL		077 07 7		105 43 RCL	
022 05 5		050 00 0		078 40 *x ²		106 00 0	
023 43 RCL		051 00 0		079 22 INV		107 00 0	
024 00 0		052 56 *rtn		080 44 SUM		108 95 =	
025 06 6		053 46 *LBL		081 00 0		109 20 *1/x	
026 23 Inx		054 16 *A'		082 02 2		110 42 STO	
027 44 SUM		055 42 STO		083 43 RCL		111 01 1	

*Denotes 2nd function key

REGISTERS

00 n	0 ⁵ ∑y ²	11 Used	15
01 ∑Inx	0 ⁶ x	11 Used	16
02 ∑(Inx) ²	0 ⁷ y	1 ² r ²	1 ⁷
03 ∑y	0 ⁸ b	1 ³	1 ⁸
04 ∑y Inx	0 ⁹ a	1 ⁴	1 ⁹

FLAGS

0	1	2	3	4
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NORMAL DISTRIBUTION

In order to calculate the standard normal distribution, the following probability function is used:

$$Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Then: $Q(x) = Z(x)(b_1t + b_2t^2 + b_3t^3 + b_4t^4 + b_5t^5) + \epsilon(x)$

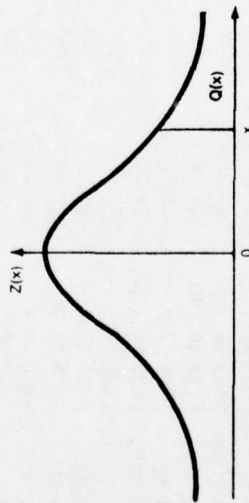
Where: $t = \frac{1}{1 + px}$

- $|\epsilon(x)| < 7.5 \times 10^{-8}$
- $p = .2316419$
- $b_1 = .319381530$
- $b_2 = -.356563782$
- $b_3 = 1.781477937$
- $b_4 = -1.821255978$
- $b_5 = 1.330274429$

- NOTES: 1. $Z(-x) = Z(x)$, $P(x) = 1 - Q(x)$
 2. $|x| < 21.25$
 3. $Z(x)$ must be calculated before $Q(x)$

Reference: *Handbook of Mathematical Functions*, National Bureau of Standards, 1964

Example: For $x = 2.02$, $Z(x) = .0518635767$, $Q(x) = .0216916245$



TEXAS INSTRUMENTS			
51-1115	NORMAL DISTRIBUTION	ST1-15	ST1-15
←A= NORMAL DISTRIBUTION			
x → Z(x)	Q(x)		

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
.2	Enter data	x	A	Z(x)
3	Calculate Q(x)		B	Q(x)

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 30	*√x	056 65	X	084 43	RCL
001 11	A	029 95	=	057 93	*	085 00	0
002 22	INV	030 42	STO	058 03	3	086 02	2
003 50	*st flg	031 00	0	059 01	1	087 45	y'
004 01	1	032 01	1	060 09	9	088 03	3
005 80	*if pos	033 56	*rtn	061 03	3	089 65	X
006 87	*1'	034 46	*LBL	062 08	8	090 01	1
007 94	+/-	035 12	B	063 01	1	091 93	*
008 50	*st flg	036 25	CLR	064 05	5	092 07	7
009 01	1	037 93	*	065 03	3	093 08	8
010 46	*LBL	038 02	2	066 75	-	094 01	1
011 87	*1'	039 03	3	067 43	RCL	095 04	4
012 42	STO	040 01	1	068 00	0	096 07	7
013 00	0	041 06	6	069 02	2	097 07	7
014 00	0	042 04	4	070 45	y'	098 09	9
015 40	*x ²	043 01	1	071 02	2	099 03	3
016 55	÷	044 09	9	072 65	X	100 07	7
017 02	2	045 65	X	073 93	*	101 75	-
018 95	=	046 43	RCL	074 03	3	102 43	RCL
019 94	+/-	047 00	0	075 05	5	103 00	0
020 22	INV	048 00	0	076 06	6	104 02	2
021 23	lnx	049 85	+	077 05	5	105 45	y'
022 55	÷	050 01	1	078 06	6	106 04	4
023 53	(051 95	=	079 03	3	107 65	X
024 59	*π	052 20	*1/x	080 07	7	108 01	1
025 65	X	053 42	STO	081 08	8	109 93	*
026 02	2	054 00	0	082 02	2	110 08	8
027 54)	055 02	2	083 85	+	111 02	2

*Denotes 2nd function key

REGISTERS

00 Used	05	10	15
01 Used	06	11	16
02 Used	07	12	17
03	08	13	18
04	09	14	19

FLAGS

0	1 Used	3	4
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PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 01	1	140 00	0	168 00		196 00	
113 02	2	141 01	1	169 00		197 00	
114 05	5	142 95	=	170 00		198 00	
115 05	5	143 60	*if flg	171 00		199 00	
116 09	9	144 01	1	172 00		200 00	
117 07	7	145 69	'9'	173 00		201 00	
118 08	8	146 56	*rtn	174 00		202 00	
119 85	+	147 46	*LBL	175 00		203 00	
120 43	RCL	148 69	'9'	176 00		204 00	
121 00	0	149 94	+/-	177 00		205 00	
122 02	2	150 85	+	178 00		206 00	
123 45	y'	151 01	1	179 00		207 00	
124 05	5	152 95	=	180 00		208 00	
125 65	X	153 56	*rtn	181 00		209 00	
126 01	1	154 00		182 00		210 00	
127 93	*	155 00		183 00		211 00	
128 03	3	156 00		184 00		212 00	
129 03	3	157 00		185 00		213 00	
130 00	0	158 00		186 00		214 00	
131 02	2	159 00		187 00		215 00	
132 07	7	160 00		188 00		216 00	
132 04	4	161 00		189 00		217 00	
134 04	4	162 00		190 00		218 00	
135 02	2	163 00		191 00		219 00	
136 09	9	164 00		192 00		220 00	
137 95	=	165 00		193 00		221 00	
138 65	X	166 00		194 00		222 00	
139 43	RCL	167 00		195 00		223 00	

*Denotes 2nd function key

CHI-SQUARE DISTRIBUTION

The chi-square density function is calculated by

$$p, f(x) = \frac{e^{-\frac{x}{2}} x^{\frac{\nu-1}{2}}}{2^{\frac{\nu}{2}} \Gamma(\frac{\nu}{2})}$$

where ν = degrees of freedom, $0 < \nu \leq 141$, $x > 0$ and $\ln x \leq \frac{200 \ln 10}{(\nu - 2)}$

The following series expansion calculates the cumulative distribution:

$$P(x, \nu) = \left(\frac{x}{2}\right)^{\frac{\nu}{2}} \frac{e^{-\frac{x}{2}}}{\Gamma(\frac{\nu}{2})} \left[1 + \sum_{r=1}^{\infty} \frac{x^r}{(r+2)(r+4) \dots (r+2r)} \right]$$

Where: $\Gamma(\nu+1) = \nu! \Gamma(\nu) = \nu! = \nu(\nu-1)!$

and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

NOTES: 1. If the degrees of freedom is entered as a non-integer, it is rounded up to an integer for the calculation.

2. If ν or x is negative or zero, a flashing 0 is returned.

3. A ν must be entered before an x , and both must be entered before P is calculated.

4. A new x may be entered without changing ν .

5. An x must be entered each time P is to be calculated.

Example: For $\nu = 10$, $\Gamma\left(\frac{10}{2}\right) = 24$ and if $x = 8.5$, $f(x) = .0969533757$, and

$P = .4198816863$

Reference: *An Introduction to Probability and Stochastic Processes*; James L. Melisa, Andrew P. Sage, Prentice-Hall Electrical Eng. Series, 1973

TEXAS INSTRUMENTS			
91-TLS	NONINTEGRAL CHI-SQUARE DISTRIBUTION		
↔A	↔B	ST1-16	
↔	↔	↔	↔
$\nu \rightarrow \Gamma(\nu/2)$	$x \rightarrow f(x)$	P	

USER INSTRUCTIONS

STEP	PROCEDURE	ENTER	PRESS	DISPLAY
1	Enter Program (A and B)			
2	Enter degrees of freedom	ν	A	$\Gamma(\frac{\nu}{2})$
3	Enter x	x	B	$f(x)$
4	Calculate P		C	P

PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
000 46	*LBL	028 75	-	056 04 4	
001 43	RCL	029 48	*EXC	057 49	*PROD
002 02	2	030 00 0		058 00 0	
003 44	SUM	031 06 6		059 03 3	
004 00 0		032 95 =		060 46	*LBL
005 08 8		033 22 INV		061 42	STO
006 43	RCL	034 90	*ifzro	062 43	RCL
007 00 0		035 43 RCL		063 00 0	
008 02 2		036 43 RCL		064 04 4	
009 55 +		037 00 0		065 75 -	
010 43	RCL	038 06 6		066 93 *	
011 00 0		039 49	*PROD	067 05 5	
012 08 8		040 00 0		068 95 =	
013 65 X		041 05 5		069 22	INV
014 43	RCL	042 43 RCL		070 90	*ifzro
015 00 0		043 00 0		071 44	SUM
016 04 4		044 05 5		072 59	*π
017 85 +		045 56	*rtn	073 30	*√x
018 42	STO	046 46	*LBL	074 49	*PROD
019 00 0		047 44	SUM	075 00 0	
020 04 4		048 43	RCL	076 03 3	
021 43	RCL	049 00 0		077 43	RCL
022 00 0		050 04 4		078 00 0	
023 06 6		051 75 -		079 03 3	
024 95 =		052 01 1		080 56	*rtn
025 52	EE	053 95 =		081 46	*LBL
026 22	INV	054 42	STO	082 11	A
027 52	EE	055 00 0		083 22	INV
				084 52	EE
				085 57	*fix
				086 00 0	
				087 52	EE
				088 22	INV
				089 52	EE
				090 57	*fix
				091 09 9	
				092 22	INV
				093 80	*if pos
				094 10	*E
				095 90	*ifzro
				096 10	*E
				097 42	STO
				098 00 0	
				099 07 7	
				100 55 +	
				101 01 1	
				102 42	STO
				103 00 0	
				104 03 3	
				105 02 2	
				106 95 =	
				107 42	STO
				108 00 0	
				109 01 1	
				110 42	STO
				111 00 0	

*Denotes 2nd function key

REGISTERS

01	Used
02	Used
03	Used
04	Used
05	Used
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07	Used
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98	Used
99	Used
100	Used

FLAGS

0	1	2	3	4
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PROGRAM LISTING

DISPLAY	KEY	DISPLAY	KEY	DISPLAY	KEY
112 04 4		140 56	*rtn	168 94	+/-
113 75 -		141 46	*LBL	169 54)	
114 53 (142 12	B	170 22	INV
115 57 *fix		143 22	INV	171 23	Invx
116 00 0		144 52	EE	172 55 -	
117 75 -		145 22	INV	173 43	RCL
118 93 *		146 80	*if pos	174 00 0	
119 05 5		147 10	*E	175 03 3	
120 54)		148 90	*ifzro	176 55 +	
121 52	EE	149 10	*E	177 02 2	
122 22	INV	150 42	STO	178 45	Y
123 52	EE	151 00 0		179 43	RCL
124 57	*fix	152 02 2		180 00 0	
125 09 9		153 45	Y	181 01 1	
126 95 =		154 53 (182 95 =	
127 22	INV	155 43	RCL	183 42	STO
128 90	*ifzro	156 00 0		184 00 0	
129 42	STO	157 01 1		185 04 4	
130 43	RCL	158 75 -		186 42	STO
131 00 0		159 01 1		187 00 0	
132 01 1		160 95 =		188 05 5	
133 75 -		161 65	X	189 56	*rtn
134 01 1		162 53 (190 46	*LBL
135 95 =		163 43	RCL	191 13	C
136 29	*xi	164 00 0		192 43	RCL
137 42	STO	165 02 2		193 00 0	
138 00 0		166 55 +		194 02 2	
139 03 3		167 02 2		195 55 +	

*Denotes 2nd function key