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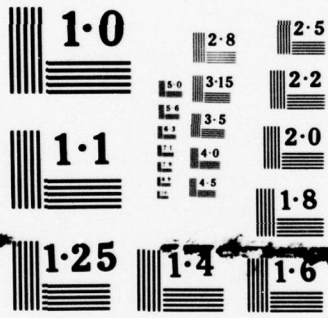
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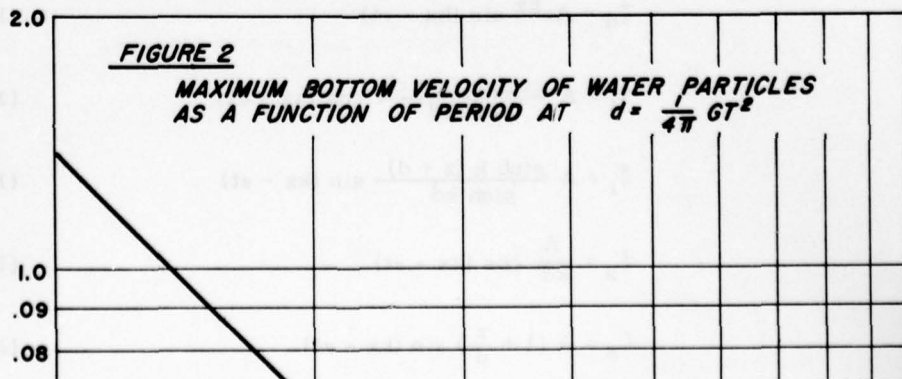
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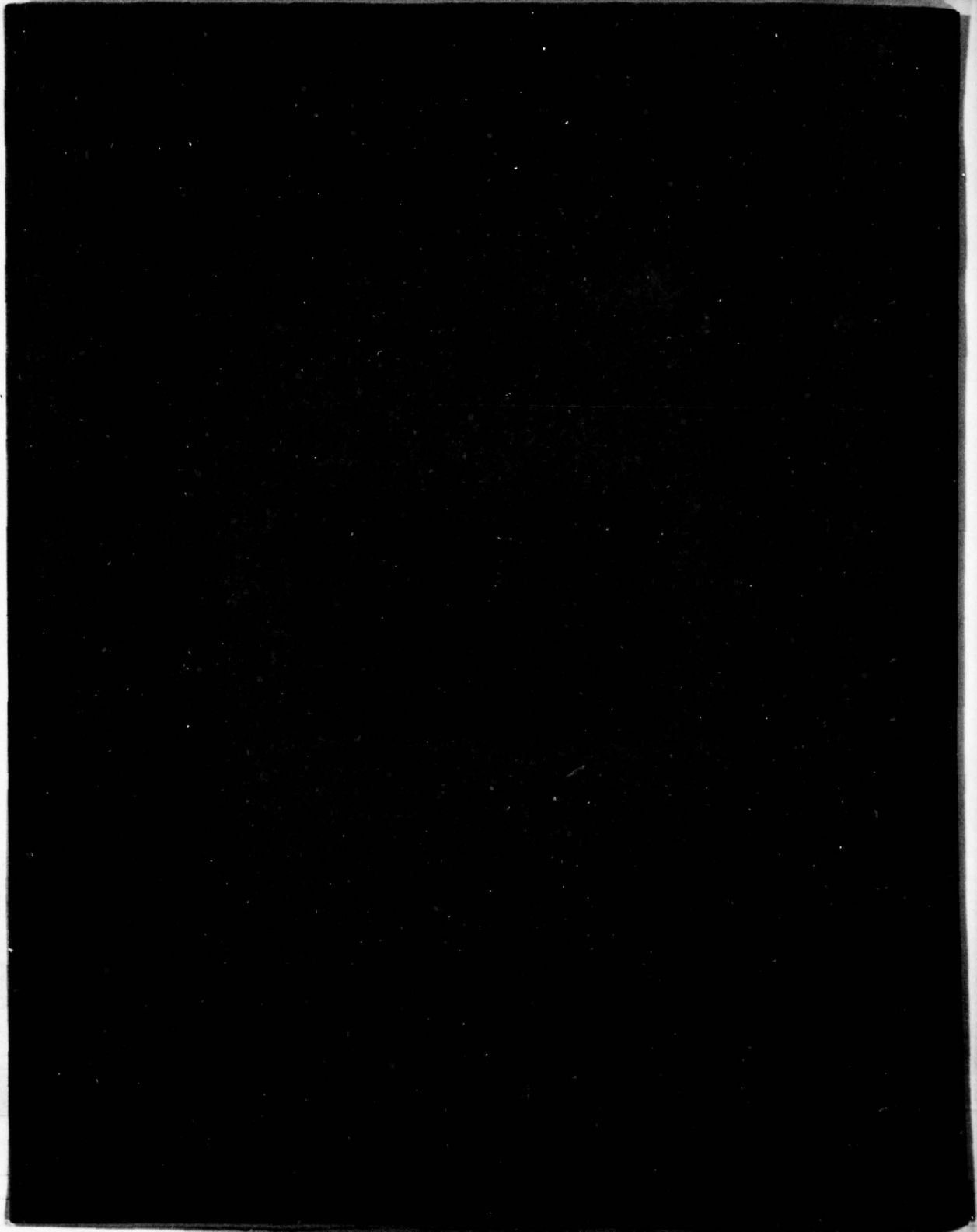
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⑥ Wave Generated Oscillatory Currents
Along the Bottom in the Eulittoral
and Sublittoral Zones,

(With graphs for determining maximum
horizontal velocity, maximum displacement,
and mean acceleration.)

⑩ Lee M. Hunt

⑪ May 1961

⑫ 37p.

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LIST OF SYMBOLS

A	=	wave amplitude = $\frac{H}{2}$
a	=	mean particle acceleration
C	=	shallow water phase or wave velocity
C _o	=	deep water phase or wave velocity
D	=	diameter of particle orbit
d	=	depth of water, measured from still water level to the bottom; taken as positive downward
e	=	base of Napierian logarithms = 2.718
g	=	acceleration of gravity - 32.2 ft/sec ²
H	=	shallow water wave height
H _o	=	deep water wave height
k	=	wave number = $\frac{2\pi}{L}$
L	=	shallow water wave length
L _o	=	deep water wave length
n	=	ratio of group velocity to phase velocity
T	=	wave period
t	=	time
U _d	=	deep water horizontal particle velocity
U _i	=	intermediate water horizontal particle velocity
U _s	=	shallow water horizontal particle velocity

- U_{\max} = maximum horizontal particle velocity
 W_d = deep water vertical particle velocity
 W_i = intermediate water vertical particle velocity
 W_s = shallow water vertical particle velocity
 x = horizontal coordinate (arbitrary origin), positive in direction of wave advance
 z = depth below still-water level; taken as negative downward
 ξ_d = deep water horizontal displacement of particle from its mean position
 ξ_i = intermediate water horizontal displacement of particle from its mean position
 ξ_s = shallow water horizontal displacement of particle from its mean position
 ξ_{\max} = maximum horizontal displacement of particle from its mean position
 ζ_d = deep water vertical displacement of particle from its mean position
 ζ_i = intermediate water vertical displacement of particle from its mean position
 ζ_s = shallow water vertical displacement of particle from its mean position
 ζ_{\max} = maximum vertical displacement of particle from its mean position
 π = 3.1416
 σ = angular frequency = $\frac{2\pi}{T}$

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INTRODUCTION

ABSTRACT ↓
Wave generated oscillatory currents along the ocean floor provide one of the more important forces participating in the erosion, transportation, and deposition of marine sediments. These currents are especially active in the eulittoral zone which ranges from the high tide level to a depth of 150 feet. Significant currents in the sublittoral zone, which continues from the lower border of the eulittoral zone to a depth of 600 feet or the edge of the continental shelf, are generated only by exceptionally long period waves. The effect of these currents has long been of importance to marine geologists and coastal engineers, and their importance can only be enhanced by the increasing amount of instruments and installations being placed on the ocean bottom in these zones. ← *ABSTRACT*

Due to turbulence as well as the oscillatory nature of wave generated bottom currents, attempts to measure their magnitudes have met with considerable difficulty. Since observations made in controlled laboratory tests and in some field tests show reasonable agreement between actual and theoretical particle motion^(1,2,3,4) these currents are usually calculated from theory. This report discusses briefly the theory upon which these calculations are based and then, in order to facilitate the use of the theory, provides graphs of maximum particle velocity, maximum particle displacement, and mean particle acceleration at the bottom as a function of depth, wave period, and wave height. The curves are drawn such that if the latter three variables are known the former three can be read directly from the graphs. If maximum particle velocity and displacement over the entire water column is required the reader is referred to GRAPHS FOR OBTAINING ORBITAL DISPLACEMENTS AND VELOCITIES, Scripps Institution of Oceanography, Wave Project No. 71, Nov. 12, 1947.

In using the graphs it should be kept in mind that the theory does not take into consideration the spectral nature of some naturally occurring waves in shallow water. The values presented then, represent a somewhat simplified picture of current characteristics under these conditions. It might be pointed out, however, that due to the filtering out of waves with periods smaller than the one considered, bottom currents calculated from theory are more accurate than those higher in the water column.

CHARACTERISTICS OF ORDINARY GRAVITY WAVES TRANSITING FROM "DEEP" TO "SHALLOW" WATER

Ordinary Gravity Waves

Waves having periods ranging from 1 to 30 seconds are known as ordinary gravity waves as distinguished from ultra-gravity waves with a period range of 0.1 to 1 second, and infra-gravity waves whose period range is 30 seconds to 5 minutes. Ordinary gravity waves include the wind generated waves known as seas while under the direct influence of the wind, and swell after they have begun to decay in a region of lighter wind or calm. Seas generally have a shorter period than swell with the division being at around 9 seconds. There is, however, some overlap. This whole band of ordinary gravity waves contains a large fraction of the total wave energy and is, therefore, important to a variety of problems.

Ordinary gravity waves are generated by the pressure and tangential stresses applied to the water surface by wind action. Dimensional characteristics, therefore, depend upon wind velocity, duration, and fetch as well as the distance the waves have traveled from their area of generation. This last point is due to the decay undergone by waves once the generating force is removed. Decay may be a slow process as shown by the summer swell which breaks on the California coast after traveling more than 4000 miles from their generation area in the "Roaring Forties" and "Furious Fifties" of the South Pacific.

Transition from "Deep" to "Shallow" Water Waves

Ordinary gravity waves may be divided into deep (short) and shallow (long) water waves through a consideration of the relationship between phase velocity, wave length, and water depth. The phase velocity can be evaluated with sufficient accuracy from the classical equation for gravity waves of small steepness^(1,2,5):

$$C^2 = \frac{gL}{2\pi} \tanh kd. \quad (1)$$

If the relative depth (d/L) is greater than 0.5, then the hyperbolic tangent can be replaced by unity to an accuracy of one per cent. Equation (1) is thereby reduced to

$$C^2 = \frac{gL}{2\pi}. \quad (2)$$

Waves under conditions where d/L is greater than 0.5 and where the phase velocity is independent of depth are called deep water waves, the wave length of which is given by

$$L_0 = \frac{g}{2\pi} T^2. \quad (3)$$

If, on the other hand, d/L is less than 0.05, the hyperbolic tangent can be replaced by the argument $2\pi d/L$ to the same accuracy and equation (1) becomes

$$C^2 = gd. \quad (4)$$

Waves under these conditions are independent of wave length, but dependent upon depth, and are called shallow water waves.

Waves whose relative depth lies between 0.05 and 0.5 are termed intermediate and equation (1) must be used.

Throughout the remainder of this paper, unless otherwise specified, shallow water will be used to indicate any relative depth less than the depth at the deep to shallow water wave transition point as given by

$$d = \frac{1}{4\pi} gT^2 \quad (5)$$

or $1/2 L_0$.

Orbital Motion of Water Particles

Associated with the motion of waves is a motion of the water particles themselves. In the deep water case this motion is in the form of circles in

the vertical plane with the water particles moving in the direction of wave propagation under the wave crest and in the opposite direction under the trough. The time required for one complete orbit is equal to the period of the wave. The orbital diameter is given by

$$D = He^{kz}, \quad (6)$$

hence D decreases exponentially with increasing depth. The following rule is often useful: The orbital diameter, (equal to the wave height at the surface), is reduced by one-half with each depth increase equal to one-ninth of the wave length.

As an example a deep water wave having dimensions $T = 10$ seconds, $L_0 = 512$ feet, and $H = 22$ feet will have a surface orbit of 22 feet in diameter. At a depth equal to one-half the wave length (256 feet) the orbital diameter is 0.95 feet, and at a depth equal to the wave length (512 feet) the orbital diameter is 0.042 feet. Since the orbital diameter at $1/2 L_0$ is only $1/23$ that of the surface orbit it can be seen that the bottom can have no significant effect on the character of the waves as long as d exceeds $1/2 L_0$.

As any wave crosses its particular transition point from deep to shallow water, an increasing potential orbital diameter is brought in contact with the bottom. However, at the sea bed, the component of motion normal to the bottom must vanish, and the response of the water particles over the entire water column to this transition is a tendency toward elliptical rather than circular orbits. Moreover, the eccentricity of the elliptical orbits increases with decreasing depth.

Particle Velocity, Displacement, and Acceleration

The speed with which water particles move around their orbits — essentially uniform while they are circular — is no longer so after they are transformed into ellipses, but is greatest near the crest and trough of the wave. This discrepancy between the speeds along different parts of the elliptical orbit increases with decreasing depth, since it is proportional to the length of the major axis of the ellipse. Since the transformation of the orbit from circle to ellipse consists of an expansion of the horizontal axis, with the vertical axis changing only as much as the height of the waves, the speed with which the water particle advances in the crest and recedes in the trough grows greater as the depth decreases.

Figures 1 and 2 graphically illustrate the relative depth of transition as a function of period, the depth range over which the maximum bottom particle velocity at the transition point is exceeded, and the relationship between maximum particle velocity at the transition point and wave period.

Horizontal and vertical particle velocity for deep, intermediate, and shallow water are given by

$$U_d = A\sigma^{kz} \cos(kx - \sigma t) \quad (7)$$

$$W_d = A\sigma^{kz} \sin(kx - \sigma t) \quad (8)$$

$$U_i = A\sigma \frac{\cosh k(z+d)}{\sinh kd} \cos(kx - \sigma t) \quad (9)$$

$$W_i = A\sigma \frac{\sinh k(z+d)}{\sinh kd} \sin(kx - \sigma t) \quad (10)$$

$$U_s = \frac{A\sigma}{kd} \cos(kx - \sigma t) \quad (11)$$

$$W_s = A\sigma \left(1 + \frac{z}{d}\right) \sin(kx - \sigma t) \quad (12)$$

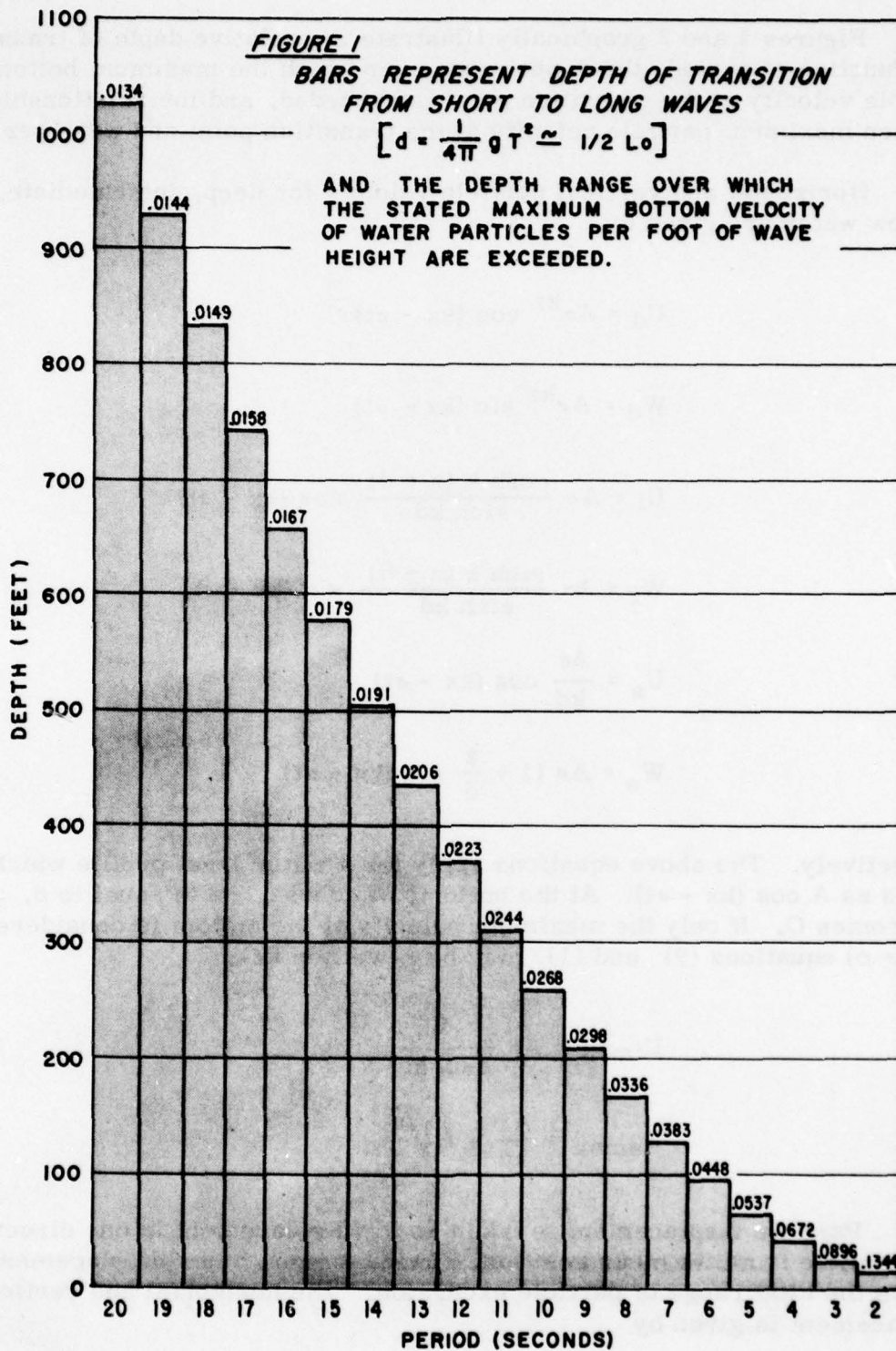
respectively. The above equations apply for a water level profile which varies as $A \cos(kx - \sigma t)$. At the bottom, of course, $-z$ is equal to d , and W becomes 0. If only the maximum velocity at the bottom is considered ($x, t = 0$) equations (9) and (11) may be rewritten as

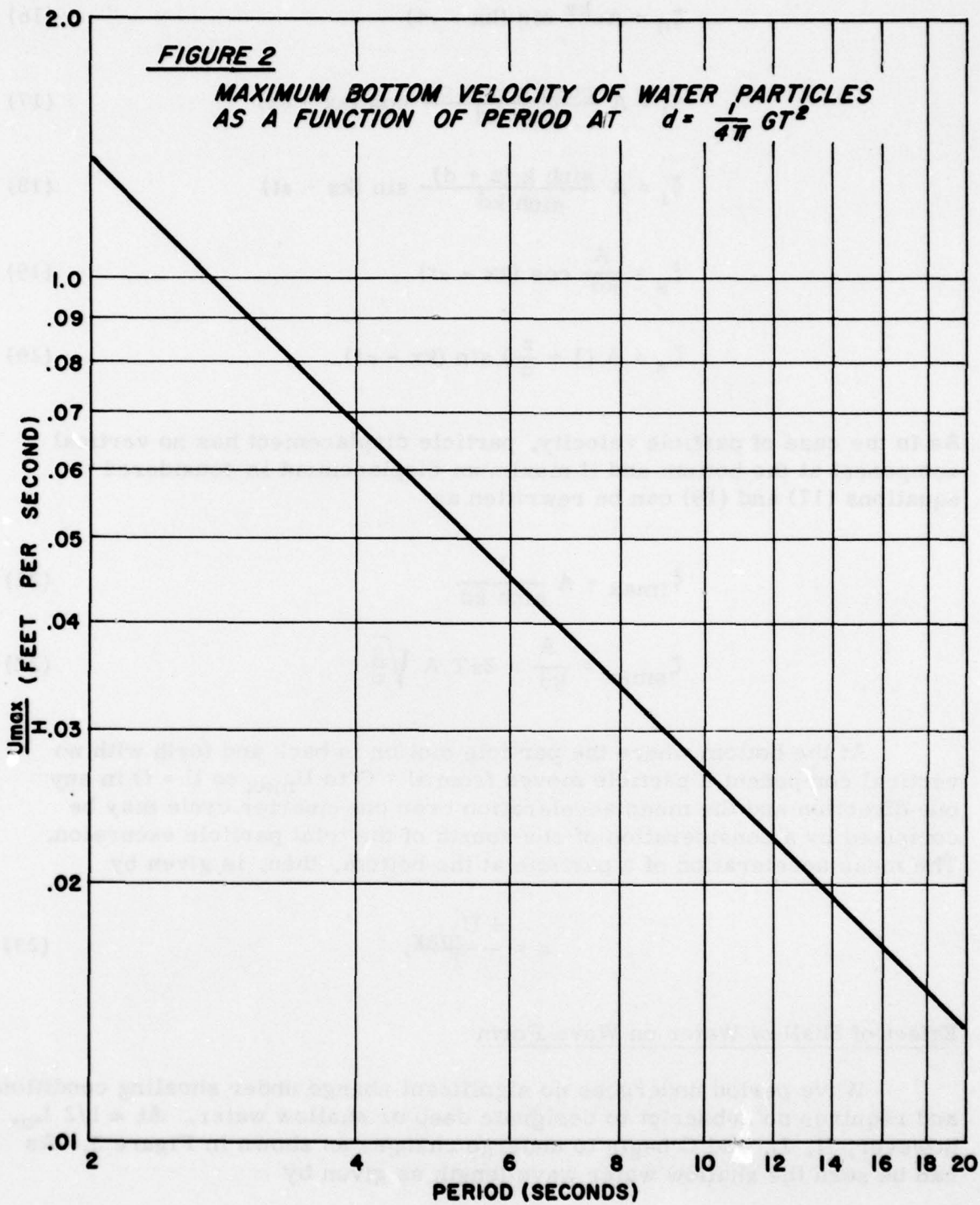
$$U_{i\max} = A\sigma \frac{1}{\sinh kd} \quad (13)$$

$$U_{s\max} = \frac{A\sigma}{kd} = A\sqrt{\frac{g}{d}}. \quad (14)$$

Particle displacement is taken as the displacement in one direction of a particle from its mean position. Twice the maximum displacement then is the total range of particle excursion. The horizontal and vertical displacement is given by

$$\xi_d = A e^{kz} \cos(kx - \sigma t) \quad (15)$$





$$\zeta_d = A e^{kz} \sin(kx - \sigma t) \quad (16)$$

$$\xi_i = A \frac{\cosh k(z+d)}{\sinh kd} \cos(kx - \sigma t) \quad (17)$$

$$\zeta_i = A \frac{\sinh k(z+d)}{\sinh kd} \sin(kx - \sigma t) \quad (18)$$

$$\xi_s = \frac{A}{kd} \cos(kx - \sigma t) \quad (19)$$

$$\zeta_s = A \left(1 + \frac{z}{d}\right) \sin(kx - \sigma t). \quad (20)$$

As in the case of particle velocity, particle displacement has no vertical component at the bottom and if maximum displacement is considered equations (17) and (19) can be rewritten as

$$\xi_{imax} = A \frac{1}{\sinh kd} \quad (21)$$

$$\zeta_{smax} = \frac{A}{kd} = 2\pi T A \sqrt{\frac{g}{d}}. \quad (22)$$

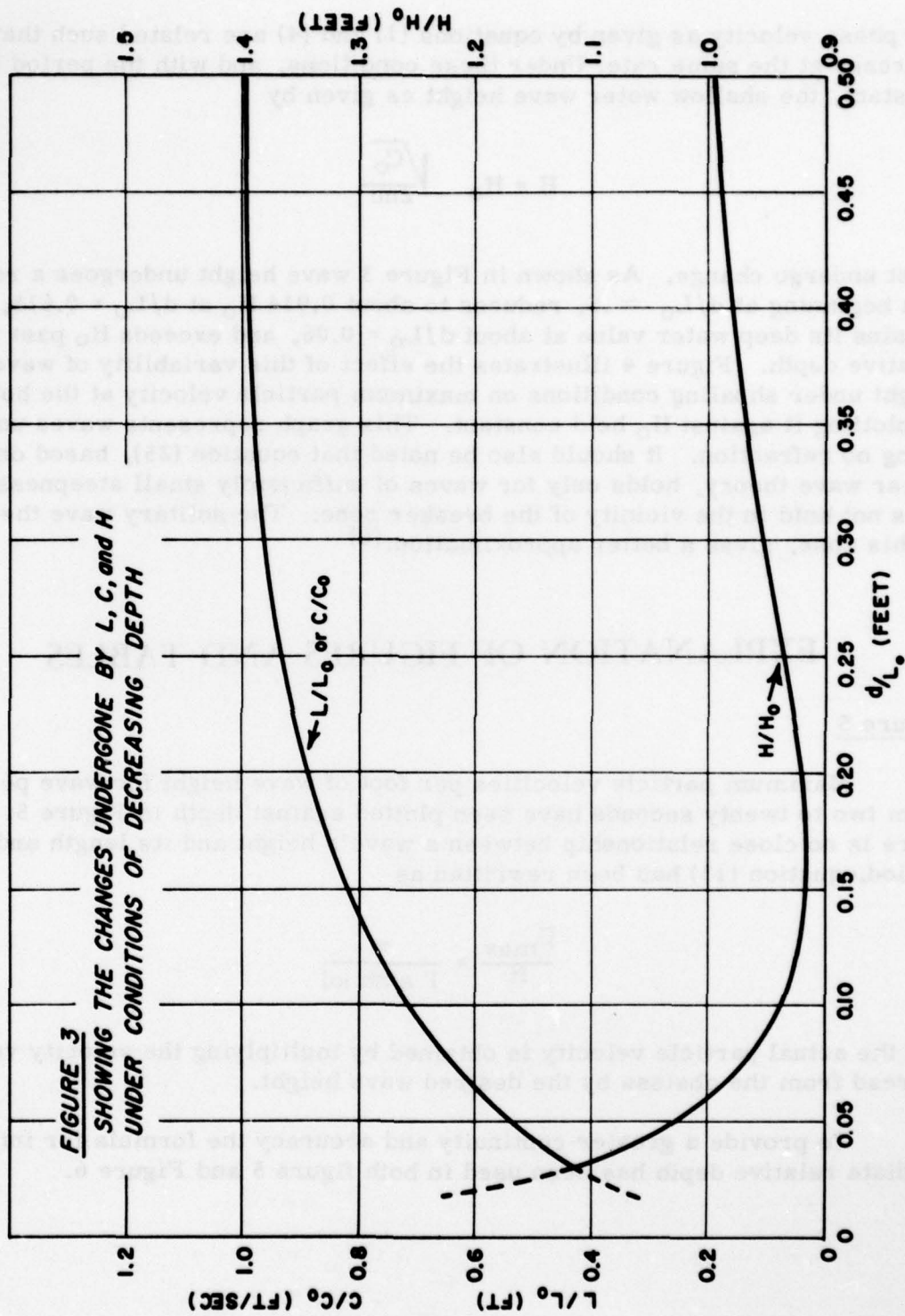
At the bottom where the particle motion is back and forth with no vertical component a particle moves from $U = 0$ to U_{max} to $U = 0$ in any one direction and the mean acceleration over one-quarter cycle may be computed by a consideration of one-fourth of the total particle excursion. The mean acceleration of a particle at the bottom, then, is given by

$$a = \frac{4 U_{max}}{T}. \quad (23)$$

Effect of Shallow Water on Wave Form

Wave period undergoes no significant change under shoaling conditions and requires no subscript to designate deep or shallow water. At $\approx 1/2 L_0$, however, H , L , and C begin to undergo changes as shown in Figure 3. As can be seen the shallow water wave length as given by

$$L = L_0 \tanh kd \quad (24)$$



and phase velocity as given by equations (1) and (4) are related such that they decrease at the same rate. Under these conditions, and with the period being constant, the shallow water wave height as given by

$$H = H_0 \sqrt{\frac{C_0}{2nc}} \quad (25)$$

must undergo change. As shown in Figure 3 wave height undergoes a reduction beginning at $d/L_0 \approx .5$, reduces to about $0.914 H_0$ at $d/L_0 = 0.615$, regains its deep water value at about $d/L_0 = 0.06$, and exceeds H_0 past this relative depth. Figure 4 illustrates the effect of this variability of wave height under shoaling conditions on maximum particle velocity at the bottom by plotting it against H_0 held constant. This graph represents waves undergoing no refraction. It should also be noted that equation (25), based on linear wave theory, holds only for waves of sufficiently small steepness and does not hold in the vicinity of the breaker zone. The solitary wave theory, in this zone, gives a better approximation. (6)

EXPLANATION OF FIGURES AND TABLES

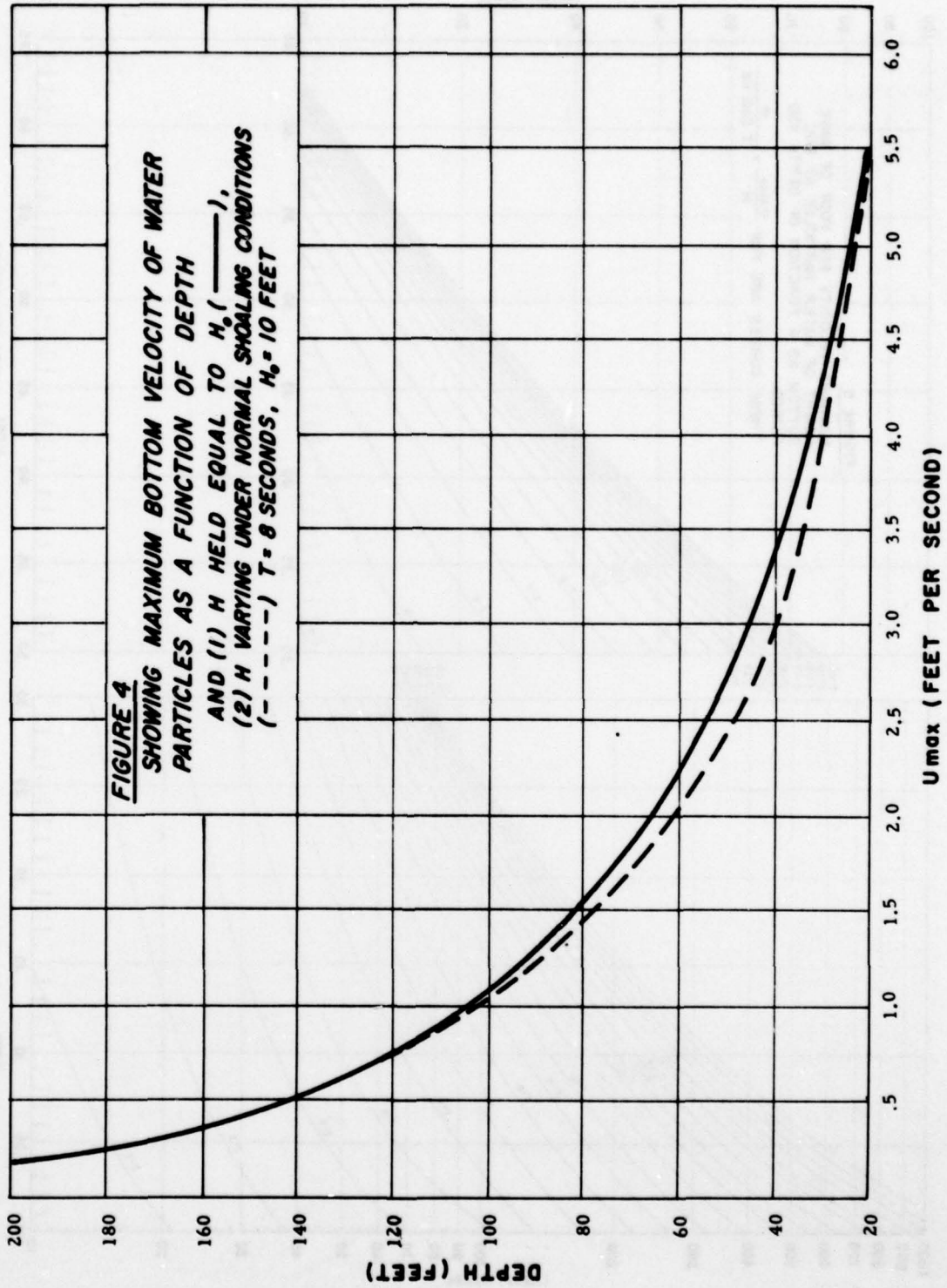
Figure 5

Maximum particle velocities per foot of wave height for wave periods from two to twenty seconds have been plotted against depth in Figure 5. Since there is no close relationship between a wave's height and its length and period, equation (13) has been rewritten as

$$\frac{U_{\max}}{H} = \frac{\pi}{T \sinh kd} \quad (26)$$

and the actual particle velocity is obtained by multiplying the velocity value as read from the abscissa by the desired wave height.

To provide a greater continuity and accuracy the formula for intermediate relative depth has been used in both figure 5 and Figure 6.



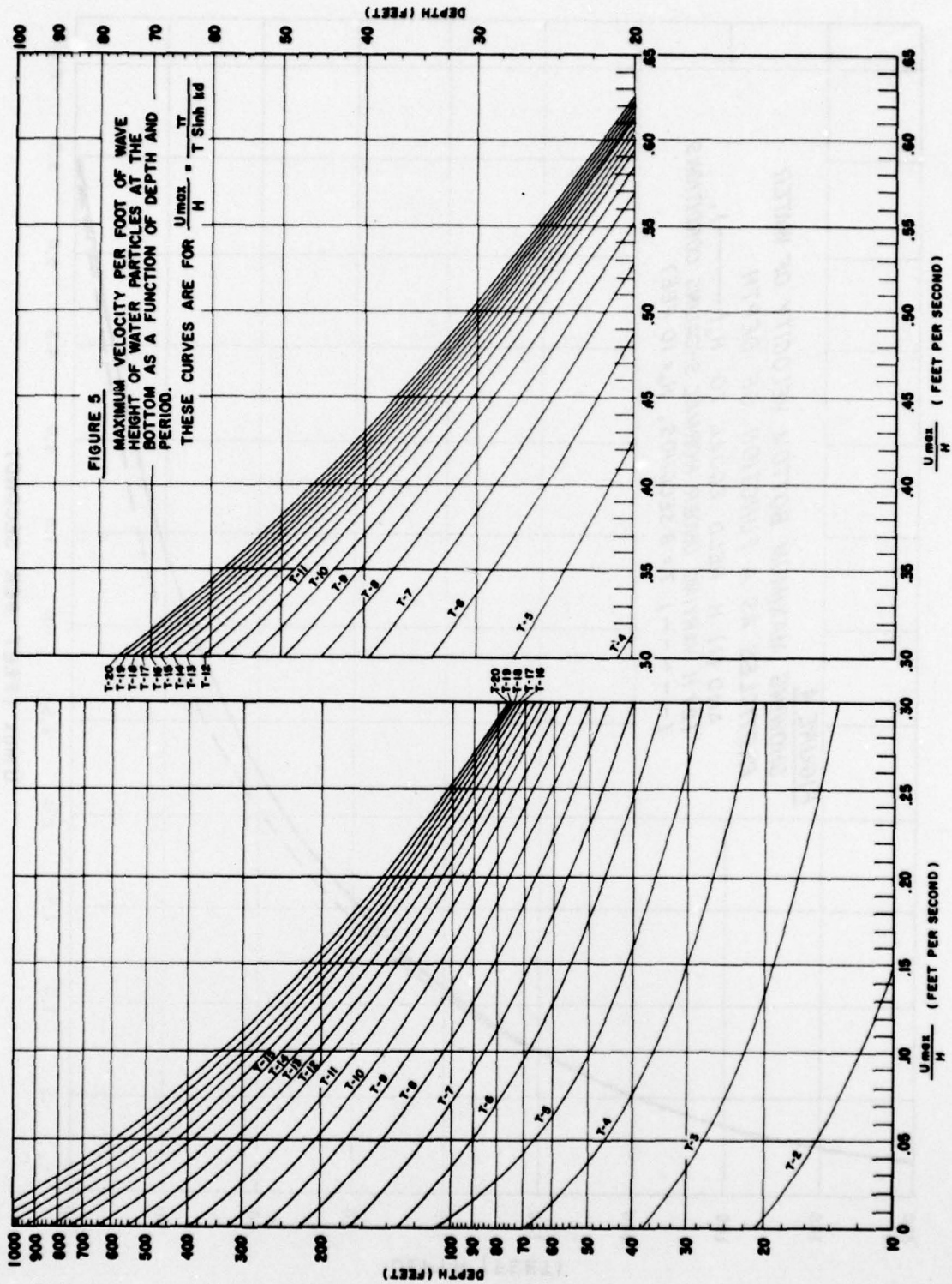


Figure 6

Maximum particle displacement per foot of wave height for periods from two to twenty seconds have been plotted against depth in Figure 6. As in the case of particle velocity equation (21) has been rewritten as

$$\frac{\xi_{\max}}{H} = \frac{1}{2 \sinh kd} \quad (27)$$

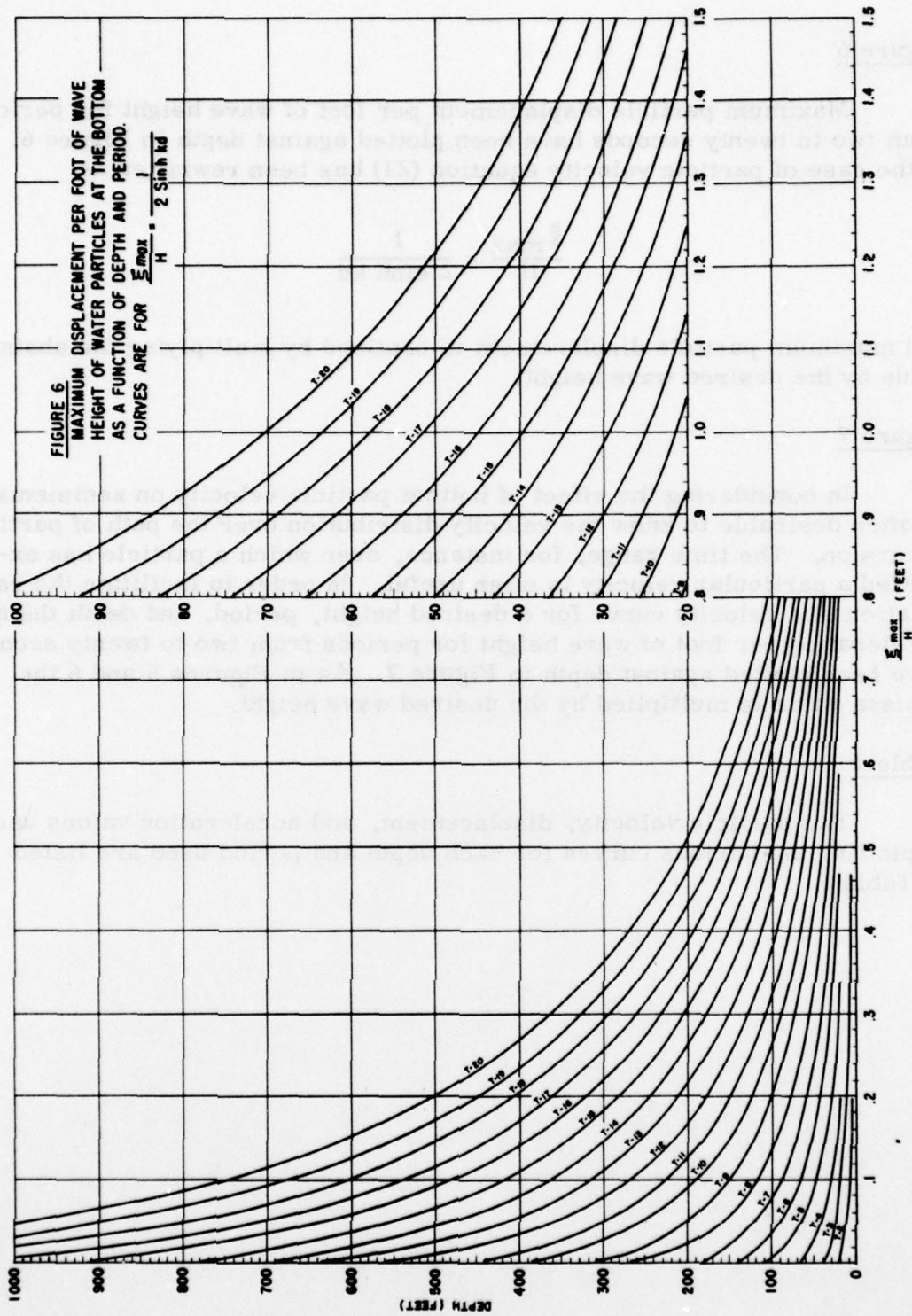
and maximum particle displacement is obtained by multiplying the abscissa value by the desired wave height.

Figure 7

In considering the effect of bottom particle velocity on sediments it is often desirable to know the velocity distribution over the path of particle excursion. The time range, for instance, over which a particle has exceeded a particular velocity is often useful. In order to facilitate the calculation of a velocity curve for a desired height, period, and depth the mean acceleration per foot of wave height for periods from two to twenty seconds have been plotted against depth in Figure 7. As in Figures 5 and 6 the abscissa value is multiplied by the desired wave height.

Table I

The particle velocity, displacement, and acceleration values used in plotting the various curves for each depth and period used are listed in Table I.



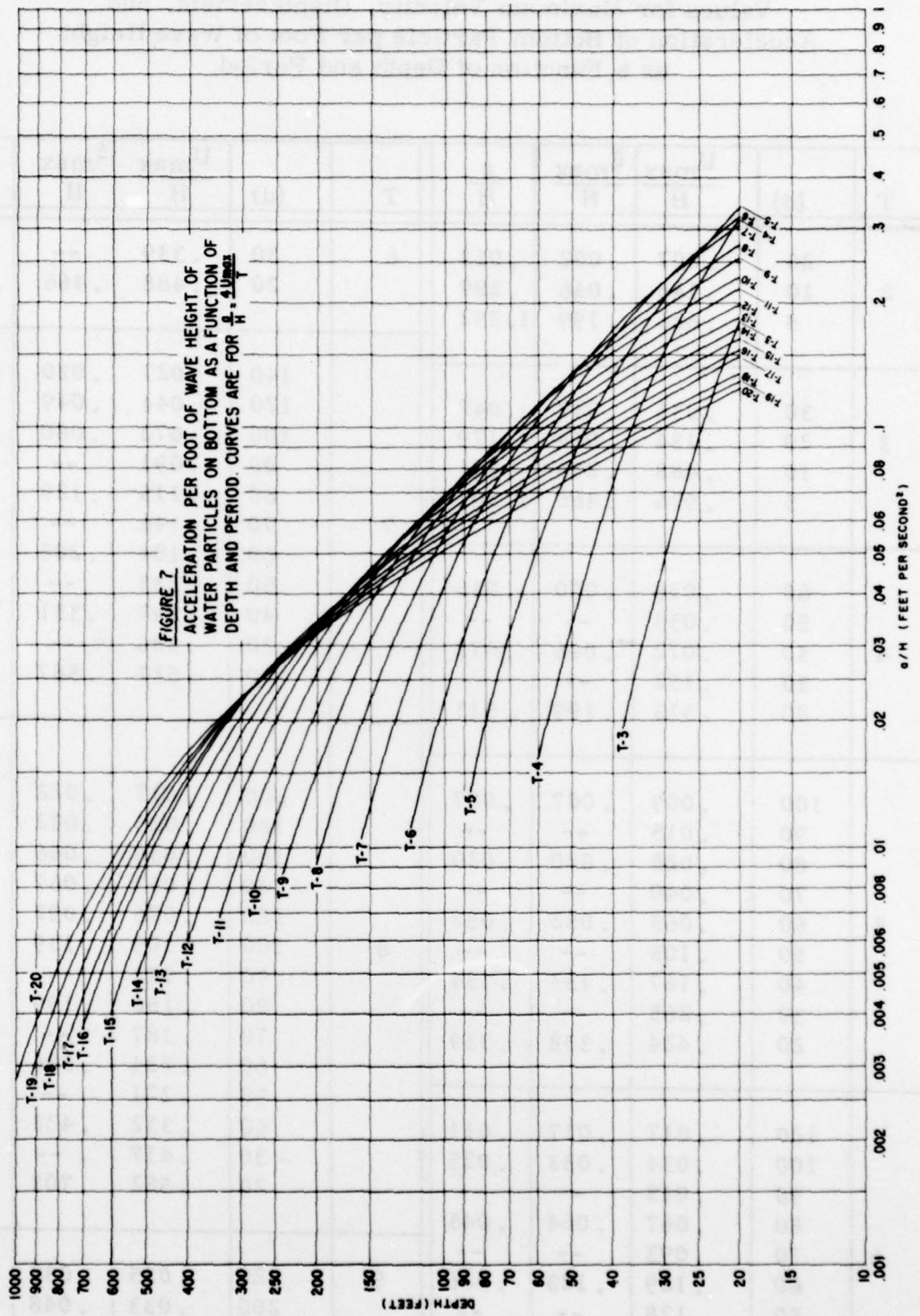


FIGURE 7
ACCELERATION PER FOOT OF WAVE HEIGHT OF
WATER PARTICLES ON BOTTOM AS A FUNCTION OF
DEPTH AND PERIOD. CURVES ARE FOR $\frac{H}{L} = \frac{1}{100}$

TABLE I

Values for Maximum Velocity, Displacement, and
Acceleration of Bottom Particle per Foot of Wave Height
as a Function of Depth and Period

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
2	20	.007	.002	.013	6	30	.339	--	--
	10	.145	.046	.289		20	.488	.466	.325
	5	.626	.199	1.252	7	140	.027	.030	.015
3	30	.035	.017	.047		120	.044	.049	.025
	20	.134	.064	.179		100	.072	.080	.041
	10	.488	.233	.651		90	.091	--	--
	5	.976	.466	1.301		80	.115	.129	.066
4	60	.016	.010	.016		70	.146	--	--
	50	.034	--	--		60	.184	.205	.105
	40	.072	.046	.072		50	.233	--	--
	30	.152	--	--		40	.297	.331	.170
	20	.313	.199	.313		30	.386	--	--
5	100	.009	.007	.007	20	.527	.587	.301	
	90	.015	--	--	8	200	.017	.022	.009
	80	.025	.020	.020		180	.025	.032	.013
	70	.040	--	--		160	.036	.046	.018
	60	.065	.052	.052		140	.053	.067	.027
	50	.105	--	--		120	.076	.097	.038
	40	.167	.133	.134		100	.109	.139	.055
	30	.265	--	--		90	.131	--	--
20	.424	.338	.339	80		.156	.199	.078	
6	120	.017	.017	.011		70	.187	--	--
	100	.034	.033	.023		60	.224	.286	.112
	90	.048	--	--	50	.271	--	--	
	80	.067	.064	.045	40	.332	.423	.166	
	70	.093	--	--	30	.417	--	--	
	60	.129	.123	.086	20	.552	.703	.276	
	50	.178	--	--	9	220	.025	.035	.011
	40	.244	.233	.163		200	.033	.048	.015
				180		.045	.064	.020	

TABLE I (con't)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
9	160	.060	.086	.027	11	220	.059	.104	.021
	140	.080	.115	.036		200	.072	.125	.026
	120	.107	.153	.048		180	.087	.152	.032
	100	.141	.203	.063		160	.105	.183	.038
	90	.163	--	--		140	.127	.222	.046
	80	.188	.270	.084		120	.153	.268	.056
	70	.218	--	--		100	.186	.327	.068
	60	.254	.364	.113		90	.207	--	--
	50	.298	--	--		80	.230	.403	.084
	40	.356	.511	.158		70	.258	--	--
	30	.438	--	--		60	.291	.509	.106
20	.569	.816	.253	50	.332	--	--		
10	280	.020	.032	.008	40	.387	.678	.141	
	260	.026	.041	.010	30	.465	--	--	
	240	.033	.052	.013	20	.592	1.036	.215	
	220	.041	.066	.016	12	400	.017	.033	.006
	200	.053	.084	.021		380	.020	.039	.007
	180	.066	.106	.026		360	.024	.046	.008
	160	.084	.133	.034		340	.028	.054	.009
	140	.105	.168	.042		320	.034	.064	.011
	120	.132	.211	.053		300	.040	.076	.013
	100	.167	.266	.067		280	.047	.089	.016
	90	.188	--	--		260	.055	.105	.018
	80	.212	.338	.085		240	.065	.123	.022
	70	.240	--	--		220	.076	.145	.025
	60	.275	.438	.110		200	.089	.170	.030
50	.318	--	--	180		.104	.200	.035	
40	.374	.596	.150	160		.122	.233	.041	
30	.454	--	--	140		.144	.275	.048	
20	.582	.927	.233	120	.170	.324	.057		
11	320	.022	.039	.008	100	.202	.386	.067	
	300	.027	.047	.010	90	.221	--	--	
	280	.033	.058	.012	80	.244	.466	.081	
	260	.040	.070	.015	70	.271	--	--	
	240	.049	.085	.018	60	.303	.579	.101	
				50	.343	--	--		
				40	.397	.759	.132		

TABLE I (con't)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
12	30	.474	--	--	14	360	.045	.101	.013
	20	.598	1.143	.199		340	.051	.114	.015
13	460	.017	.035	.005		320	.058	.129	.017
	440	.020	.039	.006		300	.065	.145	.019
	420	.023	.047	.007		280	.073	.162	.021
	400	.026	.054	.008		260	.082	.183	.023
	380	.030	.062	.009		240	.092	.205	.026
	360	.035	.072	.011		220	.103	.231	.029
	340	.040	.082	.012		200	.116	.260	.033
	320	.046	.095	.014		180	.131	.293	.037
	300	.053	.109	.016		160	.148	.331	.042
	280	.060	.125	.018		140	.169	.376	.048
	260	.069	.143	.021		120	.193	.431	.055
	240	.079	.164	.024		100	.224	.500	.064
	220	.091	.188	.028	90	.242	--	--	
	200	.104	.215	.032	80	.263	.587	.075	
	180	.119	.246	.037	70	.289	--	--	
	160	.137	.283	.042	60	.320	.713	.091	
	140	.158	.326	.049	50	.359	--	--	
	120	.183	.378	.056	40	.411	.916	.117	
100	.214	.443	.066	30	.486	--	--		
90	.233	--	--	20	.607	1.355	.173		
80	.255	.527	.078	15	600	.016	.038	.004	
70	.281	--	--		580	.018	.042	.005	
60	.312	.647	.096		560	.020	.047	.005	
50	.352	--	--		540	.022	.052	.006	
40	.405	.838	.125		520	.024	.058	.006	
30	.480	--	--		500	.027	.064	.007	
20	.603	1.249	.186		480	.030	.071	.008	
14	520	.017	.038		.005	460	.033	.079	.009
	500	.019	.043		.005	440	.037	.088	.010
	480	.022	.049		.006	420	.041	.098	.011
	460	.025	.055		.007	400	.045	.109	.012
	440	.028	.063		.008	380	.050	.120	.013
	420	.032	.071		.009	360	.056	.133	.015
	400	.036	.080		.010	340	.062	.148	.017
	380	.040	.090	.011	320	.068	.164	.018	
				300	.076	.181	.020		
				280	.084	.200	.022		

TABLE I (con't)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
15	260	.093	.222	.025	16	280	.094	.238	.024
	240	.103	.246	.027		260	.102	.261	.026
	220	.114	.273	.030		240	.112	.286	.028
	200	.127	.303	.034		220	.123	.314	.031
	180	.141	.338	.038		200	.136	.346	.034
	160	.158	.378	.042		180	.150	.381	.038
	140	.178	.424	.047		160	.166	.423	.042
	120	.202	.482	.054		140	.185	.472	.046
	100	.231	.553	.062		120	.208	.531	.052
	90	.249	--	--		100	.238	.606	.060
	80	.270	.646	.072		90	.255	--	--
	70	.295	--	--		80	.276	.703	.069
	60	.326	.779	.087		70	.301	--	--
	50	.364	--	--		60	.331	.843	.083
	40	.415	.992	.111		50	.369	--	--
30	.489	--	--	40	.419	1.070	.105		
20	.612	1.462	.163	30	.493	--	--		
				20	.615	1.566	.154		
16	680	.015	.038	.004	17	760	.015	.039	.0035
	660	.016	.042	.004		740	.016	.043	.0037
	640	.019	.049	.005		720	.017	.047	.0040
	620	.020	.051	.005		700	.019	.051	.0044
	600	.022	.056	.006		680	.020	.055	.0047
	580	.024	.061	.006		660	.022	.060	.0051
	560	.026	.067	.007		640	.024	.065	.0056
	540	.029	.074	.007		620	.026	.070	.0061
	520	.032	.081	.008		600	.028	.076	.0065
	500	.035	.088	.009		580	.031	.083	.0072
	480	.038	.097	.010		560	.033	.090	.0077
	460	.042	.106	.011		540	.036	.098	.0084
	440	.046	.116	.012		520	.039	.106	.0091
	420	.050	.127	.013		500	.042	.115	.0098
	400	.055	.139	.014		480	.046	.124	.0108
	380	.060	.152	.015		460	.050	.135	.0117
360	.065	.167	.016	440	.054	.146	.0127		
340	.072	.182	.018	420	.058	.158	.0136		
320	.078	.199	.020	400	.063	.171	.0148		
300	.086	.218	.022						

TABLE I (con'd)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
17	380	.068	.185	.0160	18	520	.046	.133	.0102
	360	.074	.200	.0174		500	.050	.142	.0111
	340	.080	.217	.0188		480	.053	.153	.0117
	320	.087	.235	.0204		460	.057	.164	.0126
	300	.094	.254	.022		440	.061	.176	.0135
	280	.102	.276	.024		420	.066	.189	.0146
	260	.110	.299	.026		400	.071	.203	.0157
	240	.120	.325	.028		380	.076	.218	.0168
	220	.131	.354	.031		360	.081	.233	.0180
	200	.143	.387	.034		340	.088	.251	.0195
	180	.157	.424	.037		320	.094	.270	.0208
	160	.173	.477	.041		300	.101	.290	.0224
	140	.191	.518	.045		280	.109	.312	.0242
	120	.214	.580	.051		260	.117	.337	.0260
	100	.243	.658	.057		240	.127	.364	.0282
	90	.260	--	--		220	.137	.394	.0304
	80	.281	.760	.066		200	.149	.427	.0031
	70	.305	--	--		180	.163	.466	.0362
	60	.335	---	--		160	.178	.511	.0395
	50	.373	--	--		140	.197	.564	.0437
40	.423	1.145	.100	120	.219	.628	.0486		
30	.496	--	--	100	.247	.709	.0548		
20	.627	1.697	.148	90	.265	--	--		
18	840	.014	.041	.0031	80	.285	.816	.0633	
	820	.015	.044	.0033	70	.309	--	--	
	800	.017	.048	.0037	60	.338	1.031	.0751	
	780	.018	.051	.0040	50	.376	--	--	
	760	.019	.055	.0042	40	.425	1.220	.0944	
	740	.021	.060	.0046	30	.499	--	--	
	720	.022	.064	.0048	20	.619	1.775	1.375	
	700	.024	.069	.0053	19	940	.013	.041	.0027
	680	.026	.074	.0057		920	.014	.043	.0029
	660	.028	.080	.0062		900	.015	.046	.0031
	640	.030	.086	.0066		880	.016	.050	.0033
	620	.032	.093	.0071		860	.018	.053	.0037
	600	.035	.100	.0077		840	.019	.057	.0040
	580	.037	.107	.0082		820	.020	.061	.0042
	560	.040	.115	.0088		800	.021	.065	.0044
	540	.043	.123	.0095	780	.023	.069	.0048	

TABLE I (con'd)

T	(d)	$\frac{U_{\max}}{H}$	$\frac{\xi_{\max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{\max}}{H}$	$\frac{\xi_{\max}}{H}$	$\frac{a}{H}$
19	760	.024	.074	.0050	20	1040	.013	.041	.0026
	740	.026	.079	.0054		1020	.014	.043	.0028
	720	.028	.084	.0058		1000	.014	.046	.0028
	700	.030	.090	.0063		980	.015	.049	.0030
	680	.032	.096	.0067		960	.016	.052	.0032
	660	.034	.102	.0071		940	.017	.055	.0034
	640	.036	.109	.0075		920	.018	.059	.0036
	620	.038	.116	.0080		900	.020	.062	.0040
	600	.041	.124	.0086		880	.021	.066	.0042
	580	.044	.132	.0092		860	.022	.070	.0044
	560	.047	.141	.0098		840	.023	.074	.0046
	540	.050	.150	.0105		820	.025	.079	.0050
	520	.053	.160	.0111		800	.026	.084	.0052
	500	.056	.171	.0117		780	.028	.089	.0056
	480	.060	.182	.0126		760	.030	.094	.0060
	460	.064	.194	.0134		740	.031	.100	.0062
	440	.068	.206	.0143		720	.033	.106	.0066
	420	.073	.220	.0153		700	.035	.112	.0070
	400	.077	.234	.0162		680	.037	.119	.0074
	380	.083	.250	.0174		660	.040	.126	.0080
	360	.088	.267	.0185		640	.042	.133	.0084
	340	.094	.285	.0197		620	.044	.141	.0088
	320	.100	.304	.0210		600	.047	.150	.0094
	300	.107	.325	.0225		580	.050	.158	.0100
	280	.115	.348	.0242		560	.052	.166	.0104
	260	.123	.373	.0258		540	.056	.178	.0112
	240	.133	.401	.0280		520	.059	.188	.0118
	220	.143	.432	.0301		500	.063	.199	.0126
	200	.154	.477	.0324		480	.066	.211	.0132
	180	.168	.507	.0353		460	.070	.223	.0140
	160	.183	.554	.0385		440	.074	.237	.0148
	140	.201	.608	.0423		420	.079	.251	.0158
120	.223	.675	.0469	400	.083	.266	.0166		
100	.251	.760	.0528	380	.089	.282	.0178		
90	.268	--	--	360	.094	.299	.0188		
80	.288	.872	.0606	340	.100	.318	.0200		
70	.312	--	--	320	.106	.338	.0212		
60	.341	1.033	.0717	300	.113	.359	.0226		
50	.378	--	--	280	.120	.383	.024		
40	.428	1.296	.0901	260	.128	.409	.026		
30	.500	--	--	240	.137	.438	.027		
20	.620	1.876	1.305	220	.148	.470	.030		

TABLE I (con'd)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
20	200	.159	.506	.032	20	80	.291	.927	.058
	180	.172	.548	.034		70	.315	--	--
	160	.187	.596	.037		60	.344	1.096	.069
	140	.205	.653	.041		50	.380	--	--
	120	.227	.723	.045		40	.430	1.370	.086
	100	.254	.810	.051		30	.503	--	--
	90	.271	--	--		20	.623	1.984	.125

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