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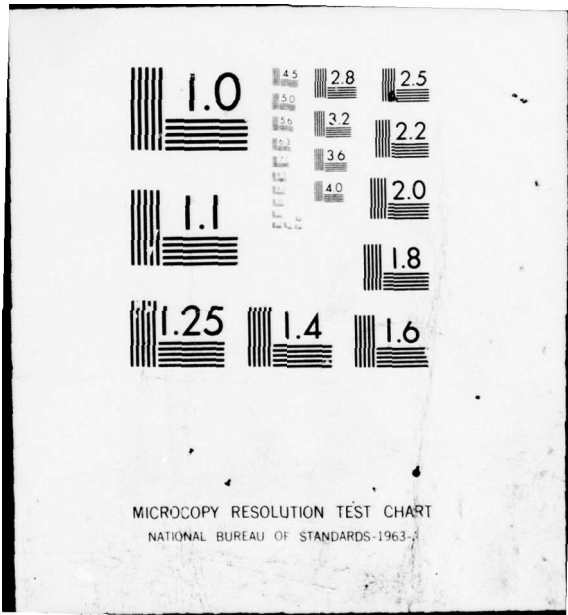
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ANALYSIS OF AXISYMMETRIC SHEET-METAL FORMING BY THE RIGID-PLASTIC, FINITE-ELEMENT METHOD

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MECHANICAL ENGINEERING
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA

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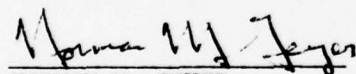
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report describes the development of a finite-element model for analyzing sheet-metal forming processes. Materials are assumed to be rigid-plastic with the view that the usefulness of an analysis method depends largely upon solution accuracy and computation efficiency. First, the variational formulation applicable to sheet-metal forming is described by considering solution uniqueness and the over slt			

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effect of geometry change involved in the forming processes. From this variational formulation, a finite-element process model based on the membrane theory is developed. Then, three basic sheet-metal forming processes, namely, the bulging of a sheet subject to hydrostatic pressure, the stretching of a sheet with a hemispherical head punch, and deep drawing of a sheet with a hemispherical head punch, are solved. The solutions arrived at by the rigid-plastic, finite-element method are compared with existing numerical solutions and the experimental data. The agreement is generally excellent and it is concluded that the rigid-plastic, finite-element method is efficient for analyzing sheet-metal forming problems with reasonable accuracy.

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SECTION I
INTRODUCTION

The metal forming processes basically involve large amounts of elastic deformation, and, due to the complexities of plasticity, the exact analysis of a process is infeasible in most of the cases. Thus, a number of approximate methods have been suggested, with varying degrees of approximation and idealization. Among these, techniques using the finite-element method take precedence because of their flexibility, ability to obtain a detailed solution, and the inherent proximity of their solutions to the exact one.

A prime objective of mathematical analysis of metalworking processes is to provide necessary information for proper design and control of these processes. Therefore, the method of analysis must be capable of determining the effects of various parameters on metal flow characteristics. Furthermore, the computation efficiency, as well as solution accuracy, is an important consideration for the method to be useful in analyzing metalworking problems.

With this viewpoint in mind, successful efforts have been carried out in analyzing various deformation processes, such as compression, heading, piercing, extrusion and drawing by the rigid-plastic, finite-element method (matrix method) [1]-[7].

The formulation of the matrix method, however, cannot be extended to the sheet-metal forming analysis due to the following reasons:

- (1) The classical variational formulation which is the basis of the matrix method does not necessarily determine a unique deformation mode. Physically, there is no inherent indeterminacy for work-hardening solids, but this indeterminacy is due rather to the fact

that the workhardening rate is not included in the mathematical formulation of the classical variational principle.

- (2) The kinematic assumption in the matrix method is not longer valid for the sheet-metal forming process. As long as bulk deformation or in-plane stretching are concerned, this kinematic assumption that the magnitude of the rate of rotation is negligible compared to the strain rate does not deviate much from the real situation and yields solutions consistent with reality. Geometric nonlinearity in sheet-metal forming, however, invalidates such a simplification.

The objective of the present investigation is, therefore, to develop and establish a finite-element method for sheet-metal forming processes.

In Section II various forms of variational formulations are reviewed in the light of uniqueness and geometry change which leads to a realization of the necessity of new formulations. In Section III a new formulation is obtained and the development of the finite-element model from it is described. With the particular example of sheet-metal forming processes in mind, the idealization of plane stress state and membrane theory is implemented. Furthermore, the development is confined to the case of axisymmetrical problems.

To establish the validity of the proposed method, three basic sheet-metal forming processes are analyzed and the solutions are compared with other available experimental data and numerical solutions. Hydrostatic bulging is treated in Section IV. Punch stretching with a hemispherical punch is discussed in Section V. To make the problem tractable, one moving contact boundary is considered first by neglecting die profile; then the analysis is extended to include two moving boundaries. In Section VI deep drawing with a hemispherical punch is solved.

SECTION II

BACKGROUND

1. Uniqueness

We consider the quasistatic deformation of a rigid-plastic solid.

On a portion S_V of the surface S of this body are prescribed given velocities, while the remainder S_T of the surface S is subjected to given surface tractions T_i . Assuming that these surface velocities and tractions are such that the entire body is in a state of plastic flow, we want to determine the stresses σ_{ij} and strain rates $\dot{\epsilon}_{ij}$ throughout the body.

The conventional formulation of variational principle for this problem is that among all kinematically admissible strain rate fields $\dot{\epsilon}_{ij}^*$, the actual one minimizes the expression (Hill [8]),

$$\pi_1 = \int \bar{\sigma} \dot{\epsilon}^* dv - \int_{S_T} T_i v_i^* dS, \quad (1)$$

where $\bar{\sigma}$ is the effective stress, $\dot{\epsilon}$ is the effective strain rate defined by

$$\bar{\sigma} = \sqrt{\frac{3}{2}} \sqrt{\sigma'_{ij} \sigma'_{ij}},$$

$$\dot{\epsilon} = \sqrt{\frac{2}{3}} \sqrt{\dot{\epsilon}_{ij} \dot{\epsilon}_{ij}},$$

respectively, where σ'_{ij} is the deviatoric component of σ_{ij} . Here a strain rate field $\dot{\epsilon}_{ij}^*$, defined throughout the body under consideration, is called kinematically admissible if it is derivable from a velocity field v_i^* which satisfies the condition of incompressibility $v_{i,i}^* = 0^+$ throughout the body

⁺The comma denotes the differentiation with respect to coordinates, e.g.,

$$v_{i,i} = \frac{\partial v_i}{\partial x_i}.$$

and the boundary conditions on S_v . The variational principle in this form has been successfully applied to the analysis of metal forming problems, such as extrusion [6]. As was found out later, and we will discuss this shortly, the success is related to the type of boundary conditions prescribed on the surface of the body undergoing deformation. In general, with the variational formulation of π_1 in Eq. (1), there is a question regarding uniqueness of deformation mode even though the stress field is uniquely determined [8], [9].

Consider an incipient flow in a rigid-plastic solid, workhardening or perfect plastic, governed by the following partial differential equations which are, of course, dual to the variational formulation π_1 . With respect to Cartesian reference frame x_i the following equations hold:

Equilibrium equations

$$\sigma_{ij,j} = 0 \text{ in the absence of body force} \quad (2a)$$

Strain rate-velocity relationship

$$\dot{\epsilon}_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \quad (2b)$$

Constitutive equation

$$\mu \sigma'_{ij} = \dot{\epsilon}_{ij}, \mu \text{ being an arbitrary constant} \quad (2c)$$

Yield criterion

$$\bar{\sigma} = \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}} = H(\bar{\epsilon}), \text{ where } \bar{\epsilon} \text{ is the effective strain defined} \quad (2d)$$

by $\bar{\epsilon} = \int d\bar{\epsilon}$ if $d\bar{\epsilon} = \sqrt{\frac{2}{3}} \sqrt{d\epsilon_{ij} d\epsilon_{ij}}$

Boundary conditions

$$n_j \sigma_{ij} = \hat{T}_i \text{ on } S_T, \quad (2e)$$

$$v_i = \hat{v}_i \text{ on } S_v,$$

where n_j is the unit normal vector to the surface of the body; \hat{T}_i and \hat{v}_i are prescribed values

Suppose that $(\sigma_{ij}^{(1)}, \dot{\epsilon}_{ij}^{(1)})$ is the solution to this boundary value problem. Construct a different set of stress fields and strain rate fields $(\sigma_{ij}^{(2)}, \dot{\epsilon}_{ij}^{(2)})$, where $\sigma_{ij}^{(2)} = \sigma_{ij}^{(1)}$, $\dot{\epsilon}_{ij}^{(2)} = C\dot{\epsilon}_{ij}^{(1)}$. C is any arbitrary factor and may vary from point to point throughout the body. Then, it is easily shown that this set $(\sigma_{ij}^{(2)}, \dot{\epsilon}_{ij}^{(2)})$ satisfies all the governing equations except for the boundary conditions on S_V . On S_V the velocity integrated from $\dot{\epsilon}_{ij}^{(2)}$ should coincide with the prescribed value \hat{v}_i . Since strain rate-velocity relation is linear, integrating $\dot{\epsilon}_{ij}^{(2)}$ would yield $C\hat{v}_i$ if $\dot{\epsilon}_{ij}^{(1)}$ is integrated to give \hat{v}_i , and therefore C must be unity on S_V . With this and the compatibility requirement the deformation mode may or may not be uniquely determined. One example of a well-established unique kinematic mode is in the plane-strain problem. In the plane-strain condition, unless one family of the characteristics is straight, the governing equation of the velocity field becomes the telegraphy equation which is hyperbolic and, therefore, the solution is uniquely determined if the boundary curve is not along a characteristic.

It can be readily shown that under certain boundary conditions the set $(\sigma_{ij}^{(1)}, C\dot{\epsilon}_{ij}^{(1)})$ also satisfies the boundary conditions on S_V and therefore the deformation mode is clearly not unique. The following is a partial list of such boundary conditions.

- (1) $S_V = 0$, i.e., all the boundaries are traction boundaries;
- (2) $\hat{v}_i = 0$ on S_V ;
- (3) On S_V only the ratio between the velocity components are prescribed, e.g., $\frac{\hat{v}_i}{\hat{v}_j} = \alpha$;
- (4) Mixed boundary condition; e.g., a normal component of \hat{v}_i and a tangential component of \hat{T}_i are prescribed over the surface, or vice versa. In this case, the additional condition of whether all the characteristics meet on a curve in the region should be checked [10].

Concrete examples are (1) the expansion of spherical shells [11] or cylindrical shells [12] under internal pressure, and (4) the indentation of a semi-infinite body by a flat punch under the plane-strain condition [13], torsion of a prismatic bar [10]. Among sheet-metal forming processes, hydrostatic bulging belongs to case (2) and punch stretching to case (4) or (3).

Note that the physical meaning of these boundary conditions is that the plastic flow is unconstrained and all or part of the body is free to deform. Mathematically, this nonuniqueness is due to the fact that the Levy-Mises theory, implied in the variational formulation π_1 and also appearing in the differential equations (2c), does not include the "viscosity effect" (in Prager's terminology [9]) and, therefore, this indeterminacy would be resolved if the workhardening effect is taken into account. In fact, for the workhardening solid there is no inherent indeterminacy in general; the apparent nonuniqueness is due simply to an inadequate formulation of the problem. In proper formulation, traction rate \dot{T}_i must be specified on S_T , and then from an infinite number of kinematically possible modes the actual mode can be singled out by the additional requirement that there must exist an equilibrium distribution of stress rate compatible with the implied rate of hardening everywhere in the body and with the given traction rate \dot{T}_i on S_T . Besides, the workhardening effect is explicitly brought into the constitutive equation in the form of

$$h\dot{\epsilon}_{ij} = \frac{\sigma'_{ij}}{\bar{\sigma}} \dot{\bar{\sigma}} \quad (3)$$

where $\dot{\bar{\sigma}}$ is the time rate of $\bar{\sigma}$, h the workhardening effect of the material being equal to $\frac{2}{3} \frac{d\bar{\sigma}}{d\bar{\epsilon}}$. It can be shown that the constitutive equation (3) can always be reduced to the constitutive equation (2c), but not necessarily

vice versa. Therefore, for a perfectly plastic solid, specifying the traction rate does not resolve the indeterminacy. Hill, then, showed that among all variational modes compatible with the boundary conditions for \hat{v}_i on S_V and the existing stress distribution σ_{ij} , the actual mode minimizes the following expression when geometry changes are neglected (Hill [8]):

$$\pi_2 = \frac{1}{2} \int h(\dot{\epsilon}_{ij}^*)^2 dv - \int_{S_T} \dot{T}_i v_i dS. \quad (4)$$

Note that the virtual mode $\dot{\epsilon}_{ij}^*$ in π_2 should be normal to the yield surface at the existing stress point in the stress space due to the compatibility requirement with existing stress distributions. For statically indeterminate problems, however, there is a coupling between stress field and strain rate field and we have to solve these two sets of variables simultaneously.

2. Geometry change

When the effect of geometry change cannot be neglected during deformation, it is necessary to reconsider the specification of the loading on S_T and the stresses since the changes in shape and area of surface elements are themselves unknown.

Let X_i be the position vector in a Cartesian reference frame at time t and after an infinitesimal time δt , x_i be the position. Let us call the configuration at time t undeformed configuration and the one at time $t + \delta t$ deformed configuration. When an actual force dP_i acts upon the area element da at time $t + \delta t$, there are various ways of reckoning this force.

First, the actual force dP_i is referred to the deformed configuration, or

$$dP_i = n_j \sigma_{ij} da, \quad (5a)$$

where n_j is the unit normal vector to the surface element of area da in a deformed configuration. The stress tensor σ_{ij} defined in this manner is called Cauchy stress tensor, or sometimes, true stress tensor.

Second, the actual force dP_i is referred to the undeformed configuration, or

$$dP_i = N_j S_{ij} dA, \quad (5b)$$

where N_j is the unit normal vector to the surface element of area dA in an undeformed configuration. The stress tensor S_{ij} defined in this manner is called the first kind of Kirchhoff stress tensor, or sometimes, nominal stress tensor. This tensor has the disadvantage of not being symmetric and therefore awkward to use in a constitutive equation with a symmetric strain tensor. Nonetheless, sometimes this stress tensor is used with nonsymmetric velocity gradients [14].

Third, to obtain a stress tensor, which is symmetric and referred to the undeformed configuration, we proceed as follows. Instead of the actual force dP_i , consider a force $d\tilde{P}_i$ related to the force dP_0 in the same way that a material vector dX_i is related by the deformation to the corresponding vector dx_i . That is,

$$d\tilde{P}_i = \frac{\partial X_i}{\partial x_j} dP_j. \quad (5c)$$

Refer this pseudo-force $d\tilde{P}_i$ to the undeformed configuration to define the second kind of Kirchhoff stress tensor τ_{ij} :

$$d\tilde{P}_i = N_j \tau_{ij} dA. \quad (5d)$$

Using the expression relating the area change of the same element during deformation [15],

$$n_i da = \frac{\rho_0}{\rho} N_j \frac{\partial X_j}{\partial x_i} dA, \quad (6)$$

where ρ_0 and ρ are densities of the volume element before and after the deformation, the relationship between different stress measures is obtained. From Eqs. (6), (5a), and (5b),

$$\begin{aligned} dP_i &= n_j \sigma_{ij} da = \sigma_{ij} \frac{\rho_0}{\rho} N_k \frac{\partial X_k}{\partial x_j} dA \\ &= N_j S_{ij} dA \end{aligned} \quad (7a)$$

or

$$S_{ij} = \frac{\rho_0}{\rho} \frac{\partial X_j}{\partial x_k} \sigma_{ik} \quad (7b)$$

and from Eqs. (6), (5a), (5c), and (5d),

$$\tau_{ij} = \frac{\rho_0}{\rho} \frac{\partial X_i}{\partial x_k} \sigma_{kr} \frac{\partial X_j}{\partial x_r}. \quad (7c)$$

All these different stress tensors become exactly the same when we bring the deformed configurations to the undeformed configurations and make them identical in the limit. Stress rates, however, are not the same. Let δu_i be the increment of displacement of the element; then

$$\delta S_{ij} = \delta \sigma_{ij} - \sigma_{kj} \frac{\partial}{\partial x_k} \delta u_i \quad \text{to the first order,}$$

neglecting plastic volume change. Or, in terms of rates,

$$\dot{S}_{ij} = \dot{\sigma}_{ij} - \sigma_{kj} v_{i,k}. \quad (8)$$

Let us compare the magnitude of the second term with that of the first term in the right-hand side of Eq. (8). Since

$$\begin{aligned}\frac{\partial v_i}{\partial x_k} &= \frac{1}{2} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right) + \frac{1}{2} \left(\frac{\partial v_i}{\partial x_k} - \frac{\partial v_k}{\partial x_i} \right) \\ &= \frac{1}{2} \dot{\epsilon}_{ik} + \frac{1}{2} \dot{\omega}_{ik},\end{aligned}\tag{9a}$$

where $\dot{\epsilon}_{ik}$ is the rate of deformation and $\dot{\omega}_{ik}$ the rate of rotation. Then

$$\sigma_{kj} \frac{\partial v_i}{\partial x_k} = \frac{1}{2} \sigma_{kj} \dot{\epsilon}_{ik} + \frac{1}{2} \sigma_{kj} \dot{\omega}_{ik}.\tag{9b}$$

Now, if workhardening characteristics are given by the relation

$$\bar{\sigma} = H(\bar{\epsilon}),\tag{10a}$$

then

$$H' = \frac{d\bar{\sigma}}{d\bar{\epsilon}}$$

or

$$d\bar{\epsilon} = \frac{d\bar{\sigma}}{H'}.\tag{10b}$$

For an order of magnitude calculation we can write approximately

$$\dot{\epsilon}_{ik} = \frac{\dot{\bar{\sigma}}_{ik}}{H'}\tag{10c}$$

or

$$\sigma_{kj} \dot{\epsilon}_{ik} = \frac{\sigma_{kj} \dot{\bar{\sigma}}_{ik}}{H'}.\tag{10d}$$

Then, from Eqs. (9b) and (10d), Eq. (8) becomes

$$\dot{S}_{ij} = \dot{\bar{\sigma}}_{ij} \left(1 - \frac{\sigma_{ij}}{H'} \right) + \frac{1}{2} \sigma_{kj} \dot{\omega}_{ik}.\tag{11}$$

From Eq. (11) we conclude that if the order of the rate of rotation $\dot{\omega}_{ij}$ is the same as or less than the order of strain rate $\dot{\epsilon}_{ij}$, and if the work-hardening rate H' is greater than the stress level, then $\dot{S}_{ij} = \dot{\sigma}_{ij}$. Otherwise, geometry change should not be neglected.

It could be shown [16] that when geometry change is taken into account, the condition for continuing equilibrium requires that

$$\frac{\partial S_{ij}}{\partial x_i} = 0 \quad \text{in the absence of body force.}$$

Using this condition, Hill subsequently derived the following variational formulation [17]:

$$\pi_3 = \frac{1}{2} \int h(\dot{\epsilon}_{ij}^*)^2 dv - \frac{1}{2} \int \sigma_{kj} v_{i,k}^* v_{j,i}^* dv - \int_{S_T} T_i v_i^* dS. \quad (12)$$

Formulation π_3 follows essentially the same line of formulation π_2 except that now geometry change is considered. In the formulation π_3 , as well as in π_2 , virtual mode must be compatible with the existing stress distribution and the boundary condition on S_V . As has been discussed earlier for statically indeterminate problems this is not an appropriate formulation.

Summarizing the development so far, the kinematic mode in sheet-metal forming of a rigid-plastic solid is not uniquely determined by considering the first-order expansion of the potential alone. Consideration up to second-order expansion of the potential, or equivalent consideration of workhardening rate in a physical sense, needs stress rate terms explicitly in the variational formulation. When geometry change cannot be neglected, these stress rate- are related to stress distribution, which is not known for statically indeterminate problems. The approach of viewing the deformation

as determining the incipient flow by assuming the deformed configuration coincident with the undeformed configuration clearly does not lead to a workable variational formulation for sheet-metal forming of a rigid-plastic solid. In this respect, it is intended to develop an appropriate variational formulation in the next section.

SECTION III
FINITE-ELEMENT FORMULATION

1. Variational formulation

Let x_i be the position vector in a Cartesian frame of reference at time t , the moment under consideration. Let σ_{ij} be the true stress at time t and $\sigma_{ij} + d\sigma_{ij}$ the true stress in the same material element after an infinitesimal time dt , both tensors being associated with the same Cartesian axes. Let ds_{ij} be the increment in nominal stress in the same element in time dt , based on the dimensions at time t . Let du_i be the increment of displacement of the element, then

$$ds_{ij} = d\sigma_{ij} - \sigma_j \frac{\partial(du_i)}{\partial x_k} . \quad (13)$$

Requiring continuing equilibrium of stresses, the virtual work principle gives

$$\int_V \left(\sigma_{ij} + d\sigma_{ij} - \sigma_{kj} \frac{\partial(du_i)}{\partial x_k} \right) \delta \left(\frac{\partial(du_j)}{\partial x_i} \right) dV = \int_{S_F} (E_j + dF_j) \delta(du_j) ds, \quad (14)$$

where $T_j = \ell_i \sigma_{ij}$ and $dT_j = \ell_i ds_{ij}$, ℓ_i being the unit normal to the surface at time t . The variational formulation is obtained from Eq. (14) as follows:

$$\delta\phi = \delta \left\{ \int_V \sigma_{ij} d\epsilon_{ij} dV + \int_V \frac{1}{2} h d\epsilon_{ij}^2 dV - \int_V \frac{1}{2} \sigma_{kj} \frac{\partial(du_i)}{\partial x_k} \frac{\partial(du_j)}{\partial x_i} dV - \int_{S_F} (T_j + dT_j) du_j ds \right\} = 0,$$

where

$$d\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial(du_i)}{\partial x_j} + \frac{\partial(du_j)}{\partial x_i} \right) \quad (15)$$

and $h = \frac{2}{3} H'$, with H' the slope of the stress and strain curve. The first three terms of the functional ϕ represent the energy dissipated during the time dt up to the second order. If it is assumed that the principal axes of true strain-rate keep the same directions in the element and the principal components of strain-rate maintain the constant ratios during the time dt , the dissipated energy can be expressed directly [18] as

$$\sum (\sigma_p dE_p + \frac{1}{2} h dE_p^2)$$

per unit volume, where dE_p is the logarithmic strain components. The final form of the functional becomes

$$\phi = \int_V \bar{\sigma} d\bar{E} dV + \frac{1}{2} \int H' (d\bar{E})^2 dV - \int_{S_F} (T_j + dT_j) du_j dS, \quad (16)$$

where $d\bar{E}$ is defined by

$$d\bar{E} = \sqrt{\frac{2}{3} \sum (dE_p)^2}$$

2. Theory of the finite-element method

An important step in finite-element modeling is obtaining approximate state equations in a region. The weighted residual method derives the state equations directly from the governing differential equations. Let us write the governing differential equation as

$$Lu - f = 0, \quad (17)$$

where L is the differential operator, f is the known function, and u is the solution. With the trial solution u^* , Eq. (17) is not satisfied, but there remains an error or residual R such that

$$R = Lu^* - f. \quad (18)$$

This residual is multiplied by weight function w and integrated over the domain and the state equations are derived from the condition that this integral vanishes with a given choice of weight function w :

$$\int wR \, dv = 0. \quad (19)$$

One well-known method among weighted residual methods is Galerkin's approach.

A more frequently used approach is the derivation from a variational principle which is a dual expression of the governing differential equation. Assume that a functional Φ , which is equivalent to the differential equation, has been established. Let a continuum be divided into a finite collection M of subdomains called elements interconnected at a finite number of nodes N . If it is true that the total functional is equal to the sum of the contributions of each element $\phi^{(m)}$, then we may write as follows:

$$\Phi = \sum_{m=1}^M \phi^{(m)}(u). \quad (20)$$

In each element let us approximate the solution with a linear combination of trial functions v_i such that

$$u \approx \sum \alpha_i v_i \quad (21)$$

holds, where α_i are unknown coefficients to be determined later. By substituting Eq. (21) into Eq. (20), we have

$$\begin{aligned}
\Phi &= \sum_{m=1}^M \varphi^{(m)}(\alpha_i, v_i) \\
&= \sum_{m=1}^M \varphi^{(m)}(\alpha_i) \quad \text{since } v_i \text{'s are known} \\
&= \Phi(\alpha_i).
\end{aligned} \tag{22}$$

The original Φ of u is now discretized with a function φ of parameters α_i , and the initial variational problem reduces to determining the α_i that minimizes φ . The minimization of φ with respect to α_i may be written as

$$\delta\varphi = \frac{\delta\varphi}{\delta\alpha_i} \delta\alpha_i = 0, \tag{23}$$

where δ denotes the first variation. Since α_i 's are independent, expression (23) is equivalent to a set of simultaneous equations,

$$\frac{\partial\varphi}{\partial\alpha_i} = 0. \tag{24}$$

This is, in fact, the classical Ritz technique. It is the choice of trial functions that makes the finite-element method different from the Ritz method and renders it successful; they are piecewise polynomials. Besides, the coefficients α_i , called nodal values in the finite-element literature, do have a definite physical meaning, such as displacement or velocity.

The trial function v_i must satisfy certain requirements to enable convergence as the subdivision into ever smaller elements is attempted. First, as the element size decreases, the functions in the integral must tend to be single-valued and well behaved in physical problems. This is called the "completeness" requirement and is satisfied if the trial function is of class c^p when p is the highest order in the integrand of the

functional. Second, the validity of the summation implied in Eq. (20) must be preserved. This is called the "compatibility" requirement and is satisfied if v_i is of class C^{p-1} [19], [20]. When admissible trial functions are used, the functional converges monotonically with an increasing number of elements (or decreasing size) at a rate proportional to h^2 where h is a characteristic element dimension.

3. Modeling of axisymmetric problems

The general outline of the finite-element modeling stated above will be expanded in detail for the case of axisymmetric thin shells subject to axisymmetric loading. This particular problem is of interest since some basic sheet-metal forming processes belong to this category. When the ratio of thickness to the radius of curvature is sufficiently small, bending moment and shearing forces may be neglected without serious error and the *membrane theory* may be justified [21]. Moreover, the state of stress can be treated as an approximate plane so long as $\frac{dt}{ds}$ is small compared with unity, where t is the local thickness and s is the distance in any direction parallel to the surface. We now may rewrite Φ with the substitution of $t \, dA = dv$ to Eq. (16):

$$\Phi = \int \bar{\sigma}(d\bar{E})t \, dA + \frac{1}{2} \int H'(d\bar{E})^2(t \, dA) - \int (T + dT)du_j \, dA \quad (25)$$

for the unit included angle of the element, where A is the area of the element and t is the sheet thickness.

From the symmetry of the problem it is easily shown that the circumferential direction and the meridian direction are the principal directions and if the friction between the shell and the external agent is negligible, the thickness direction will be the third principal direction. Within the

order of approximation taken in the formulation, the logarithmic strain increment may be used as the strain increment measure. Then the definitions of strain increments are

$$dE = \begin{Bmatrix} dE_1 \\ dE_2 \end{Bmatrix} = \begin{Bmatrix} dE_r \\ dE_\theta \end{Bmatrix} = \begin{Bmatrix} \ln \frac{s}{s_0} \\ \ln \frac{r}{r_0} \end{Bmatrix}, \quad (26)$$

if, during an incremental deformation, an element of undeformed length s_0 is stretched to the length s and the point currently at the radial distance r_0 moves to the deformed radial location r . Subscripts r , θ refer to the meridian and the circumferential direction, respectively.

To bring the model closer to reality in the present investigation, normal anisotropy is included and the corresponding stress-strain increment relation is obtained, using Hill's criterion [13], as

$$\frac{dE_r}{(1+R)\sigma_r - R\sigma_\theta} = \frac{dE_\theta}{(1+R)\sigma_\theta - R\sigma_r} = \frac{d\bar{E}}{(1+R)\bar{\sigma}}, \quad (27)$$

where R is the planar isotropy parameter which is the ratio of width strain to the thickness strain in uniaxial tension. The effective stress and the effective strain are defined[†] as

$$\bar{\sigma} = \sqrt{\sigma_\theta^2 - \frac{2R}{1+R}\sigma_r\sigma_\theta + \sigma_r^2}, \quad (28a)$$

$$d\bar{E} = \frac{1+R}{\sqrt{1+2R}} \sqrt{dE_r^2 + \frac{2R}{1+R}dE_\theta dE_r + dE_\theta^2}. \quad (28b)$$

[†]Note that $H' = \frac{d\bar{\sigma}}{d\bar{E}}$ must be consistent with these definitions.

The effective strain, $d\bar{E}$, may be written in matrix form as

$$d\bar{E} = \sqrt{\frac{2}{3}} [d\bar{E}_D^T D dE]^{1/2}, \quad (29a)$$

where

$$D = \frac{3(1+R)}{2(1+2R)} \begin{vmatrix} 1+R & R \\ R & 1+R \end{vmatrix}. \quad (29b)$$

The sheet geometry is approximated by a series of conical frustra, as shown in Fig. 1. Linear trial functions, or shape functions, as they are often called in the finite-element literature, are enough since the integrand in the functional is of class C^1 . The unknown coefficients, or nodal values, are taken to be the incremental displacement at nodes. Then we may write

$$\begin{aligned} \underline{u}^{(m)} &= \langle dv_1, dw_1, dv_2, dw_2 \rangle^T \\ &= \langle du_1, du_2, du_3, du_4 \rangle^T \end{aligned} \quad (30)$$

for a representative element m , where dv_i, dw_i are the radial and the axial components of incremental displacement of the i -th node. Then the incremental displacement field inside the element may be written as

$$\begin{aligned} \underline{u} = \begin{Bmatrix} dv \\ dw \end{Bmatrix} &= \begin{vmatrix} \frac{1+t'}{2} & 0 & \frac{1-t'}{2} & 0 \\ 0 & \frac{1+t'}{2} & 0 & \frac{1-t'}{2} \end{vmatrix} \begin{Bmatrix} dv_1 \\ dw_1 \\ dv_2 \\ dw_2 \end{Bmatrix} \\ &= \underline{Nu}^{(m)}, \end{aligned} \quad (31)$$

where t' is the local coordinate varying from the value of -1 at node 2

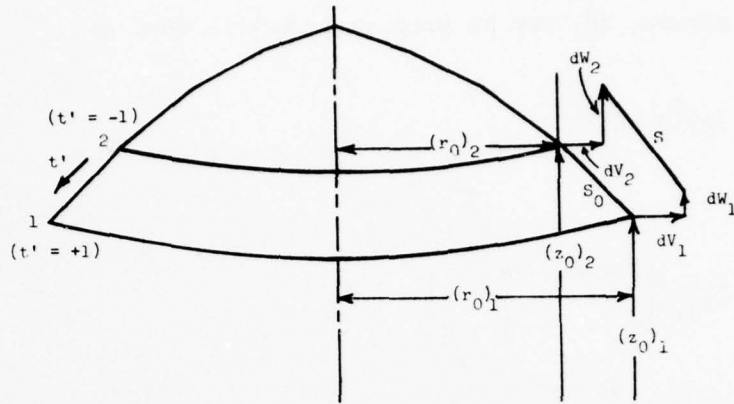


Figure 1. Approximation of the Sheet Geometry into a Series of Conical Frustra

to +1 at node 1. (See Fig. 1.) Due to this incremental displacement field, an element of length s_0 ,

$$s_0 = \sqrt{\{(r_0)_1 - (r_0)_2\}^2 + \{(z_0)_2 - (z_0)_1\}^2},$$

is stretched to a new length s ,

$$\begin{aligned} s &= \sqrt{\{(r_0)_1 - (r_0)_2 + dv_1 - dv_2\}^2 + \{(z_0)_2 - (z_0)_1 + dw_2 - dw_1\}^2} \\ &= \sqrt{(r_1 - r_2)^2 + (z_2 - z_1)^2}, \end{aligned} \quad (32)$$

where $(r_0)_i$, $(z_0)_i$ are the radial and the vertical positions of the i -th node at the undeformed configuration and $(r)_i$, $(z)_i$ at the deformed configuration. Since the element is straight, any point of t' in the local coordinate is shown to have a global radial position r_0 determined by

$$r_0 = \left(\frac{1+t'}{2}\right)(r_0)_1 + \left(\frac{1-t'}{2}\right)(r_0)_2. \quad (33)$$

The new position r of the same particle is given by

$$r = r_0 + \frac{(1+t')}{2} dv_1 + \frac{(1-t')}{2} dv_2. \quad (34)$$

We are now at the position of calculating the strain increment field. Recall the equation (25) and substitute Eqs. (32), (33), and (34) into it to obtain

$$d\tilde{E} = \left\{ \begin{array}{l} \frac{1}{2} \ln \frac{\{(r_0)_1 - (r_0)_2 + dv_1 - dv_2\}^2 + \{(z_0)_1 - (z_0)_2 + dw_2 - dw_1\}^2}{s_0^2} \\ \ln \frac{r_0 + \frac{(1+t')}{2} dv_1 + \frac{(1-t')}{2} dv_2}{r_0} \end{array} \right\} \quad (35)$$

We may write $\phi^{(m)}$, a contribution from the m-th element to the total functional Φ , in terms of nodal values, for unit angle included:

$$\begin{aligned} \phi^{(m)} &= \int \left\{ \bar{\sigma} d\tilde{E} + \frac{1}{2} H' (d\tilde{E})^2 \right\} t \, dA - \int (T_i + dT_i) v_i \, dA \\ &= \int \bar{\sigma} \left(\sqrt{\frac{2}{3}} t \right) [d\tilde{E}^T \underline{D} \, d\tilde{E}]^{1/2} \, dA + \frac{1}{2} \int H' \left(\frac{2}{3} t \right) [d\tilde{E}^T \underline{D} \, d\tilde{E}] \, dA - \int \underline{T}^T \underline{N} u^{(m)} \, dA, \end{aligned} \quad (36)$$

where

$$\underline{T} = \begin{Bmatrix} T_1 + dT_1 \\ T_2 + dT_2 \\ T_3 + dT_3 \\ T_4 + dT_4 \end{Bmatrix}.$$

Minimization gives a set of simultaneous equations:

$$\begin{aligned} \frac{\partial \phi^{(m)}}{\partial \underline{u}^{(m)}} &= \int \left(\sqrt{\frac{2}{3}} t \right) \bar{\sigma} [d\tilde{E}^T \underline{D} \, d\tilde{E}]^{-1/2} \frac{\partial (d\tilde{E})^T}{\partial \underline{u}^{(m)}} \underline{D} \, d\tilde{E} \, dA + \int \left(\frac{2}{3} t \right) H' \frac{\partial (d\tilde{E})^T}{\partial \underline{u}^{(m)}} \underline{D} \, d\tilde{E} \, dA \\ &\quad - \int \underline{N}^T \underline{T} \, dA. \end{aligned} \quad (37)$$

From Eq. (35),

$$\frac{\partial (dE)}{\partial \underline{u}^{(m)}} = Q = \begin{vmatrix} \frac{\partial (dE_1)}{\partial u_1} & \frac{\partial (dE_2)}{\partial u_1} \\ \frac{\partial (dE_1)}{\partial u_2} & \frac{\partial (dE_2)}{\partial u_2} \\ \frac{\partial (dE_1)}{\partial u_3} & \frac{\partial (dE_2)}{\partial u_3} \\ \frac{\partial (dE_1)}{\partial u_4} & \frac{\partial (dE_2)}{\partial u_4} \end{vmatrix} = \begin{vmatrix} \frac{r_1 - r_2}{s^2} & \frac{1 + t'}{2r} \\ \frac{-(z_2 - z_1)}{s^2} & 0 \\ \frac{-(r_1 - r_2)}{s^2} & \frac{1 - t'}{2r} \\ \frac{(z_2 - z_1)}{s^2} & 0 \end{vmatrix} \quad (38)$$

Therefore, Eq. (37) becomes

$$\frac{\partial \phi^{(m)}}{\partial \underline{u}^{(m)}} = \int \left(\frac{2}{3} t\right) \bar{\sigma} \left[\frac{2}{3} d\underline{E}^T \underline{D} d\underline{E}\right]^{-1/2} \underline{QD} d\underline{E} dA + \int \left(\frac{2}{3} t\right) H' \underline{QD} d\underline{E} dA - \int N^T \underline{T} dA = 0. \quad (39)$$

These equations, being valid for an m-th element, are now to be combined under the condition of compatibility that the first-order derivative of nodal value may be discontinuous across element boundaries but the nodal value itself must be continuous,

$$\frac{\partial \phi}{\partial \underline{u}} = \sum \frac{\partial \phi^{(m)}}{\partial \underline{u}^{(m)}} = 0. \quad (40)$$

4. Linearization

Eqs. (39) and (40) are nonlinear equations and it is very difficult to solve them without linearizing. One way is to take an initial guess of the solution to the equation as u^* and rewrite Eq. (39) in terms of the differences between this initial guess and the correct solution Δu , where $u_{\text{correct}} = u^* + \Delta u$, and expand it. Where the initial guess is sufficiently close to the correct solution, we may neglect higher-order terms of Δu and

thereby linearize successfully. This can be done mathematically in a systematic way and is called the Newton-Raphson method [22]. Say we have a nonlinear equation $\psi(u) = 0$, then we may expand into a series with respect to the correct solution u_0 such that

$$\begin{aligned}\psi(u) &= \psi(u_0) + \left(\frac{d\psi}{du}\right)_{u=u_0} (u - u_0) + \frac{1}{2}\left(\frac{d^2\psi}{du^2}\right)_{u=u_0} (u - u_0)^2 + \dots \\ &= \psi^* + \left(\frac{d\psi}{du}\right)^* \Delta u + \frac{1}{2}\left(\frac{d^2\psi}{du^2}\right) (\Delta u)^2 + \dots = 0.\end{aligned}$$

If u and u_0 are sufficiently close, we may neglect the higher-order terms and write

$$\psi = \psi^* + \left(\frac{d\psi}{du}\right)^* \Delta u = 0. \quad (41)$$

In our formulations the equations to be minimized are $\frac{\partial \Phi^{(m)}}{\partial u^{(m)}} = 0$, and, therefore, the expressions corresponding to Eq. (41) are

$$\left| \frac{\partial^2 \Phi^{(m)}}{\partial u_i^{(m)} \partial u_j^{(m)}} \right|^* (\Delta u) = \left| -\frac{\partial \Phi^{(m)}}{\partial u_i} \right|^*. \quad (42)$$

It may be shown that

$$\frac{\partial^2 \Phi^{(m)}}{\partial u_i^{(m)} \partial u_j^{(m)}} = p^{(m)} = \frac{2}{3} \int \frac{1}{d\bar{E}} \left\{ \left[\bar{\sigma} + H' d\bar{E} \right] \left(K - \frac{2}{3} \frac{bb^T}{d\bar{E}} \right) + \frac{2}{3} \frac{H' bb^T}{d\bar{E}^2} \right\} t \, dA \quad (45a)$$

where

$$\underline{b} = QD \, d\bar{E}$$

$$\underline{K} = QDQ^T$$

and that

$$\frac{\partial \phi^{(m)}}{\partial u_i^{(m)}} = \underline{H}^{(m)} - \underline{F}^{(m)}, \quad (43b)$$

where

$$H^{(m)} = \frac{2}{3} \int \frac{1}{d\bar{E}} (\bar{\sigma} + H' d\bar{E}) b t \, dA,$$

$$F^{(m)} = \int \underline{N}^T \underline{T} \, dA.$$

By assembling the equations obtained for an element, we finally have

$$P^* \Delta u = F - H^* \quad (44)$$

We evaluate the integrals with the Gaussian quadrature formulation.

We have yet to introduce the boundary conditions for solving a physical problem. For an incremental displacement prescribed boundary, the corresponding perturbations should vanish and, for a traction prescribed boundary, the prescribed traction value will enter into the \underline{F} vector. The solution procedure is as follows:

- (1) Assume an initial guess u_1 , and compute P , H , F corresponding to this guess.
- (2) Solve Eq. (3.31) and obtain Δu .
- (3) Obtain a new initial guess $u_2 = u_1 + \Delta u$.

Repeat this process until convergence is achieved. Convergence is checked by the fractional norm. A norm is defined by a square root value, i.e.,

$$\|u\| = \sqrt{u_1^2 + u_2^2 + \dots}$$

and

$$\|\Delta u\| = \sqrt{(\Delta u_1)^2 + (\Delta u_2)^2 + \dots}$$

The fractional norm is the ratio $\frac{\|\Delta u\|}{\|u\|}$ and when, for subsequent iterations, this value reaches the magnitude smaller than a predetermined value, say, 10^{-6} , the iteration stops and the solution is thus obtained.

SECTION IV

HYDROSTATIC BULGING

1. Introduction

The ductility of sheet metal under biaxial stress is often examined by means of the so-called bulge test. A uniform plane sheet is placed over a die with an aperture and is firmly clamped around the perimeter. An increasing hydrostatic pressure is applied to one side of the sheet, causing it to bulge through the aperture. From the measured profile and thickness of the plastically deformed sheet near the pole, it is possible to calculate the local state of stress in terms of the applied pressure. If, in addition, the state of strain is measured by means of a grid, the stress-strain characteristics of the metal under biaxial tension are obtained. The advantage of this test over any other simple one is that a greater range of pre-instability strain can be obtained.

Hydrostatic bulging is not only important as a material property test, but also as a forming operation. Thus, a number of theoretical investigations, dealing with axisymmetric hydrostatic bulging (Fig. 2) has appeared in the literature.

The classical analysis of bulging is the one by Hill [23]. His solutions are, however, special ones. Instead of analyzing deformation with a given stress-strain characteristic, Hill first adopted special kinematic assumptions and from them deduced the necessary stress-strain characteristics which satisfy all the governing equations under the prescribed kinematic mode. The kinematic assumptions are first, that any material element describes a circular path which is, moreover, orthogonal to the



Figure 2 Schematic view of hydrostatic bulging.

momentary profile, and second, that circumferential strain is numerically equal to the tangential strain. The required stress-strain characteristic is found to be an exponential type. Hill's other solution on a linear workhardening solid uses the method of successive approximation by adopting a yield criterion which is neither von Mises nor Tresca, for the purpose of mathematical simplicity.

Analyses of work by Woo [24], Yamada [25], and Wang [26] are based upon the realistic choices of stress-strain characteristics and the yield criterion. In applying the deformation theory of rigid plasticity, Wang experiences a mathematical difficulty and attributes this to the fact that the differential equations associated with the deformation theory possesses a singularity which has the effect of restricting the range of calculation within a certain value of the polar strain. Besides, the agreement of deformation theory predictions with the experiment is rather poorer than the incremental theory prediction [27].

In applying the incremental theory of rigid plasticity, researchers experience a difficulty in satisfying the boundary condition at the fixed edge, i.e., $\dot{\epsilon}_{\theta} = 0$. To avoid this difficulty, Woo uses the deformation theory, while Yamada reasons that introducing an elastic strain component into the formulation will resolve this "mathematical difficulty" (in Yamada's terms) and turns to the elasto-plastic constitutive law. Another theoretical work of interest comes from Wang, using the parametric representation of the stresses.

The only published solution on hydrostatic bulging using the finite-element method is the one by Iseki et al. [28], with the incremental theory of elasto-plasticity.

2. Computational procedures

In adopting the finite-element model to hydrostatic bulging, it is necessary to reconsider the external work increment term, since the pressure is uniform over the entire surface of a closed shell. In this case the increment of external work may be written as [29], [30],

$$\Delta w = p\bar{\nabla V}, \quad (45)$$

where $\bar{\nabla V}$ is the increase of the volume enclosed by the deformed sheet and p is the pressure acting on the deformed configuration.

As an initial condition, Hill's special solution is utilized. In other words, the initial profile of the bulge is assumed to be a part of a sphere whose radius is given by $r = \frac{1}{2} \left(\frac{a^2}{h} + h \right)$, where a is the radius of the original blank and h is the polar height at the moment. With this geometry, a pressure p is prescribed. This pressure should be greater, at least, than the pressure which makes the sheet having initial geometry everywhere plastic. The initial guess on the incremental displacement is also obtained from Hill's special solution by assuming normal trajectory of the element particle to the bulge profile. The program for computing the initial guess is given in Appendix A.

When a converged solution is obtained for the given pressure, a new bulge profile is determined from the initial bulge profile and incremental displacement grid. Then the pressure is assigned a higher value and the converged solution for the previous step is used as the initial guess for the incremental displacement field and the computation continues in this way. The program for the analysis of hydraulic bulge is given in Appendix B.

3. Results and discussion

To examine the validity of the present FEM for hydrostatic bulging, the solution is compared with those achieved by the elasto-plastic FEM and the experiment.

The following conditions were employed for the comparison with the elasto-plastic FEM:

$$\begin{aligned}\text{Workhardening characteristics: } \bar{\sigma} &= 105(.0019 + \bar{\epsilon})^{0.2} \times 10^9 \text{ kg/m}^2 \\ &= 1.036(.0019 + \bar{\epsilon})^{0.2} \times 10^9 \text{ N/m}^2\end{aligned}$$

$$\text{Thickness: } 3.0 \times 10^{-4} \text{ m } (= 0.3 \text{ mm})$$

$$\text{Radius of the sheet: } 2.4 \times 10^{-2} \text{ m } (= 24 \text{ mm})$$

$$\text{Anisotropy parameter: } 1.0$$

An identical problem was also solved by Yamada [25], using the finite-difference method with the elastic-plastic theory. Fig. 3 shows the relationship between hydrostatic pressure and the polar thickness strain. The solid line represents the elasto-plastic FEM (and also the finite-difference method) and the points indicate the solution given by the rigid-plastic FEM. The deviation of the first point by the rigid-plastic FEM is thought to reflect the approximation involved in the initial condition that the sheet is everywhere plastic and that the initial geometry is a part of a sphere. The solution can be improved numerically by taking a smaller value of h in generating the initial condition. Nevertheless, the solutions after this first step are in extremely good agreement with the elasto-plastic FEM and any disturbance in the initial conditions does not matter after an initial deformation of a small magnitude. The pressure increment is raised by twice after some deformation and it is to be noted that the solutions

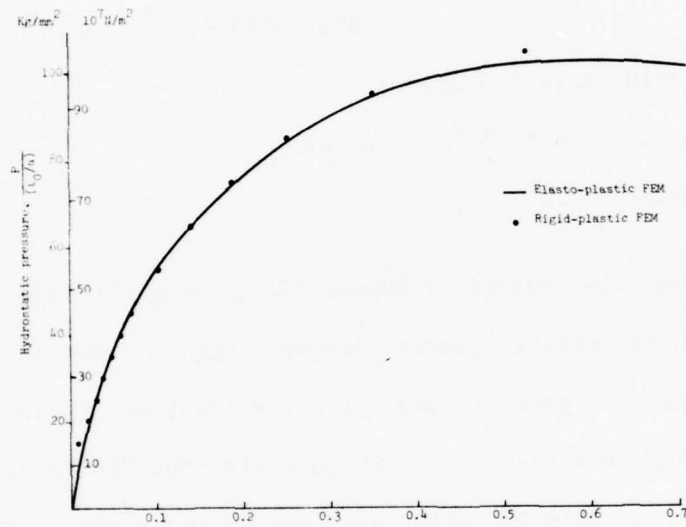


Figure 3. Hydrostatic pressure vs. polar thickness strain.

with the larger pressure increment size are still accurate. This means that the method is computationally economical with a reasonable accuracy. After the last point in the diagram the solution diverges and it is thought that the pressure maximum has been reached. The convergence is excellent; in every step, five to seven iterations seem to be sufficient. Fig. 4(a), (b) show the comparisons of strain distributions. The circumferential strain distributions are in good agreement. The tangential strain distribution by the rigid-plastic FEM deviates somewhat at the edge from that by the elastoplastic FEM. The tangential strain is more sensitive to the method employed than the circumferential strain, but this deviation of tangential strain is not serious because the solution closely follows that by the finite-difference method and we may conclude that the strain distribution is accurately predicted. Fig. 5 shows the distributions of stresses when the polar thickness strain is (-0.4). Fig. 6 shows the bulge profile at some stages of deformation. A number of material elements are traced during deformation and are shown on each bulge profile.

Next, the solution is compared with Mellor's [31] experiment on half-hard aluminum.

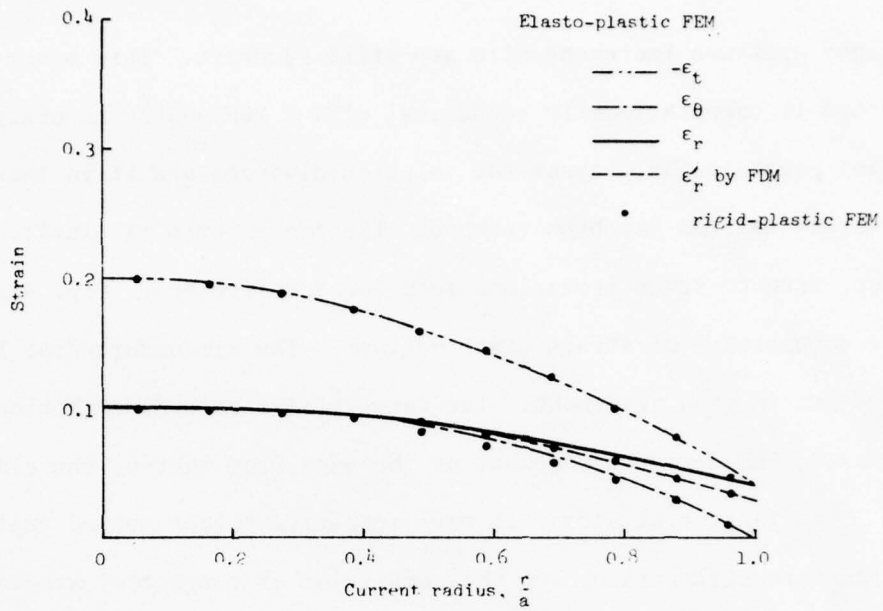
$$\begin{aligned} \text{Workhardening characteristics: } \sigma &= 15,460(1 + 0.76\epsilon) \text{ psi} \\ &= 1.066(1 + 0.76\epsilon) \times 10^8 \text{ N/m}^2 \end{aligned}$$

$$\text{Radius of the sheet: } 5.0 \text{ inches} = 1.27 \text{ m}$$

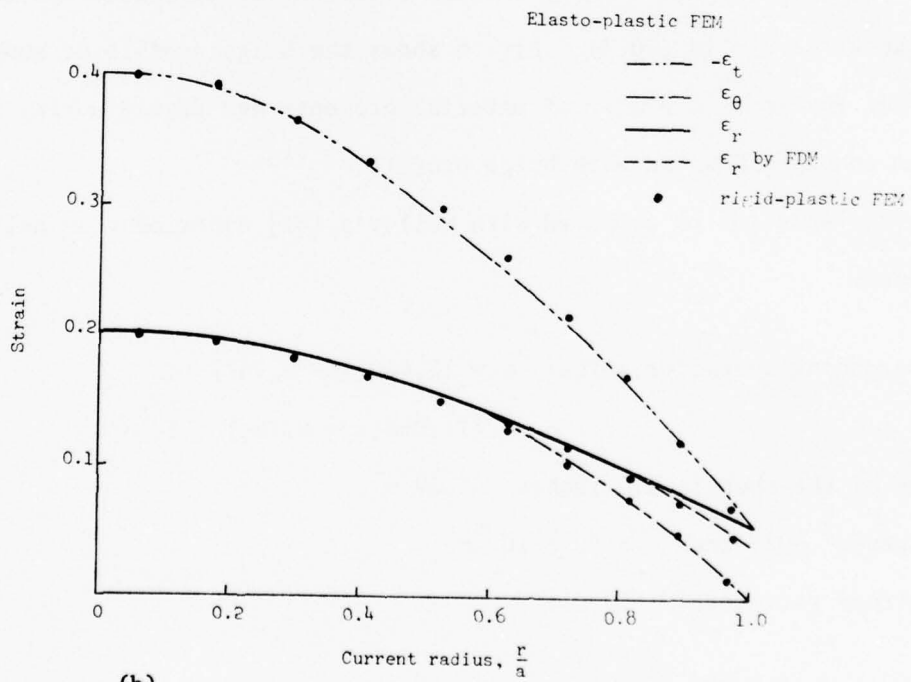
$$\text{Thickness: } .035 \text{ inch} = 8.89 \times 10^{-4} \text{ m}$$

$$\text{Anisotropy parameter: } 1.0$$

One thing to be mentioned is that in the actual experiment, the die has a round profile of radius $\frac{3}{8}$ in., but in the analysis this profile has been



(a)



(b)

Figure 4. (a) and (b) Distribution of Strains.

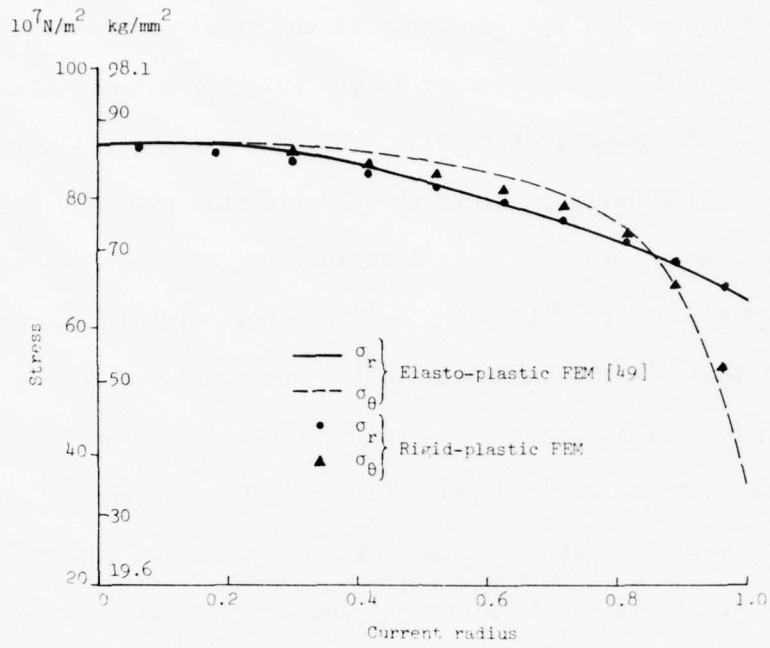


Figure 5. Distribution of Stresses

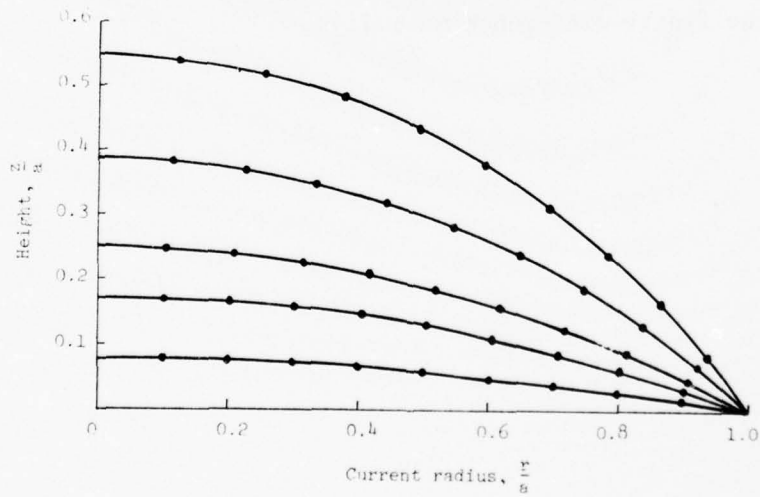


Figure 6. Bulge Profile

neglected. Fig. 7 shows that the agreement of the relation between pressure and polar height is good. The agreement in the bulge profile is also excellent, as in Fig. 8. As shown in Fig. 9(a), (b), the theoretical circumferential strain still closely predicts the experimental one, but there is some discrepancy in thickness strain distribution. As has been mentioned, the actual die has a round profile which has been neglected in the analysis, and it is thought that the thickness strain is more sensitive to the profile than is the circumferential strain. Initially, there is virtually no discrepancy, but increases at later stages. This may be explained by the fact that initially the sheet is not in contact with the profile, but as deformation continues, more of the sheet is brought into contact with the profile and makes the actual situation different from the one used in the analysis.

In general, the theoretical prediction by the rigid-plastic FEM is in good agreement with both the experiments and the analyses by the elasto-plastic FEM and the finite-difference method.

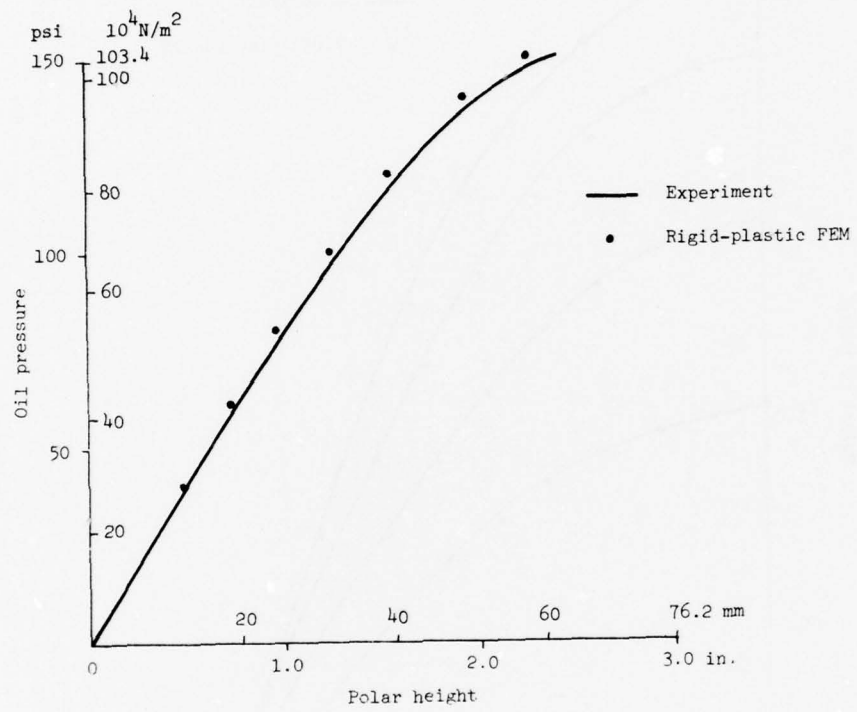


Figure 7. Polar Height vs. Pressure

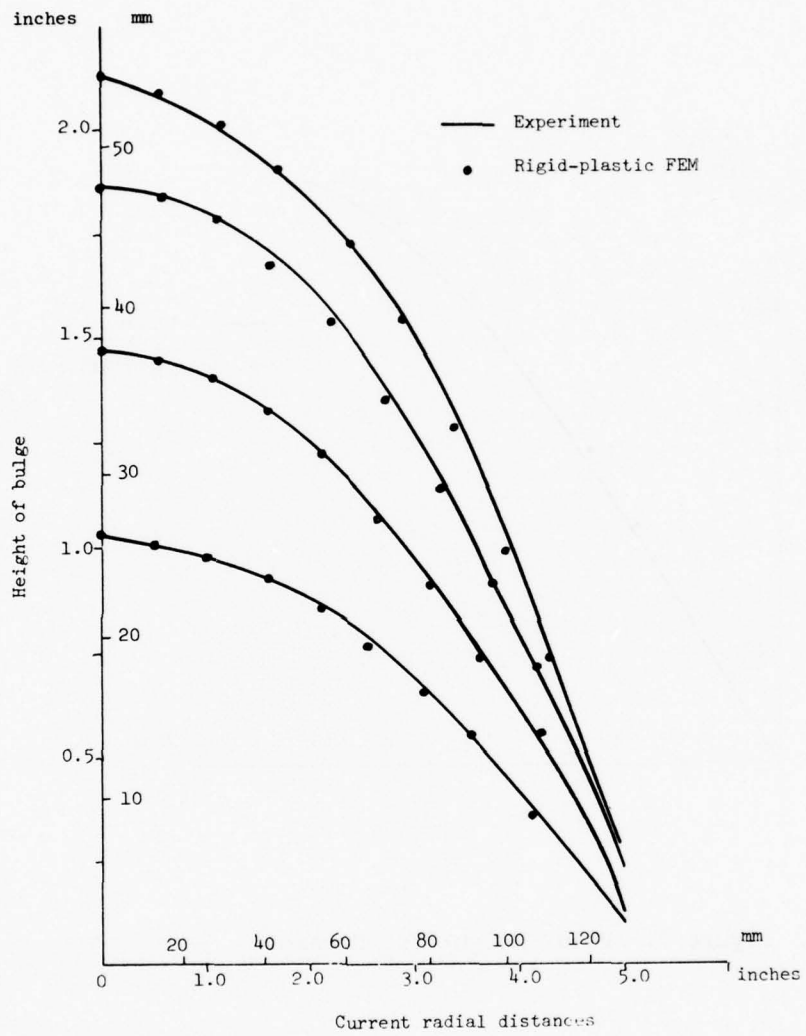


Figure 8. Bulge Profile

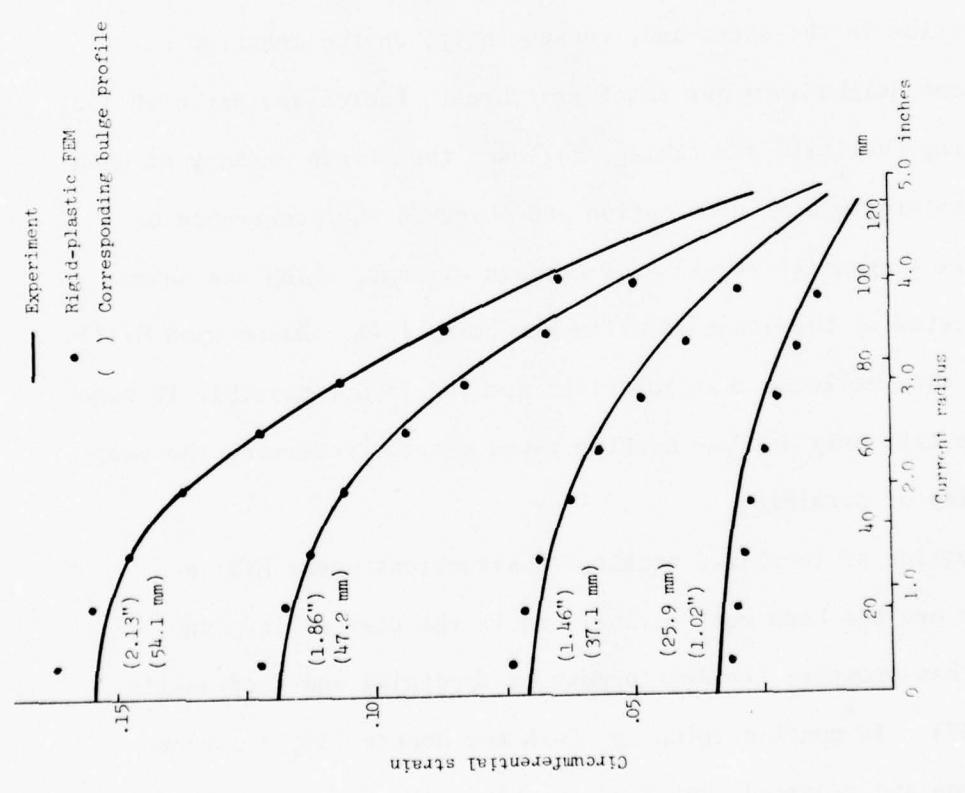
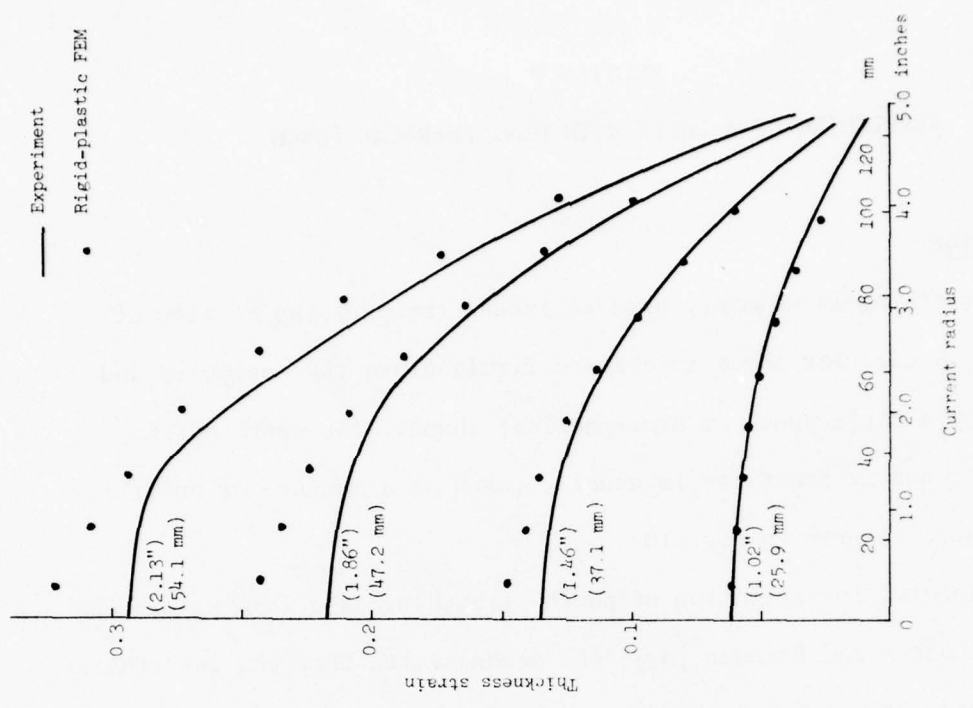


Figure 9. (a) Circumferential Strain Distribution; (b) Thickness Strain Distribution

SECTION V

STRETCHING OF A SHEET WITH HEMISPHERICAL PUNCH

1. Introduction

Punch stretching is commonly used to assess the pressing quality of sheet metals. A circular sheet is clamped firmly along the periphery and is stretched by a rigid punch of hemispherical shape. The depth of the deformed sheet when it fractures is usually taken as a measure of ductility. See the schematic diagram in Fig. 10.

An experimental investigation of punch stretching as a forming problem dates back to Loxely and Freeman [32], who demonstrated that the interfacial friction between the punch and the sheet has a significant effect on the strain distribution in the sheet and, consequently, on the location of fracture and dome height when the sheet fractures. Keeler and Backofen [33], in characterizing the limit stretching, followed the strain history of each element with the progress of deformation and observed the occurrence of discontinuity in tangential strain at a certain element, which was subsequently interpreted as the onset of diffuse necking [34]. Based upon Hill's analysis [35], they believed that localized necking is not possible in punch stretching, but that only diffuse necking takes place, increasing the overall nonuniformity of straining.

The observation of localized necking in situations where Hill's analysis denies one has been well established in the case of in-plane stretching and has prompted the development of Marciniak and Kuczynski's theory [36], [37]. In punch stretching, Gosh and Hecker [38] observed localized necking and reported that local necking sets in even though

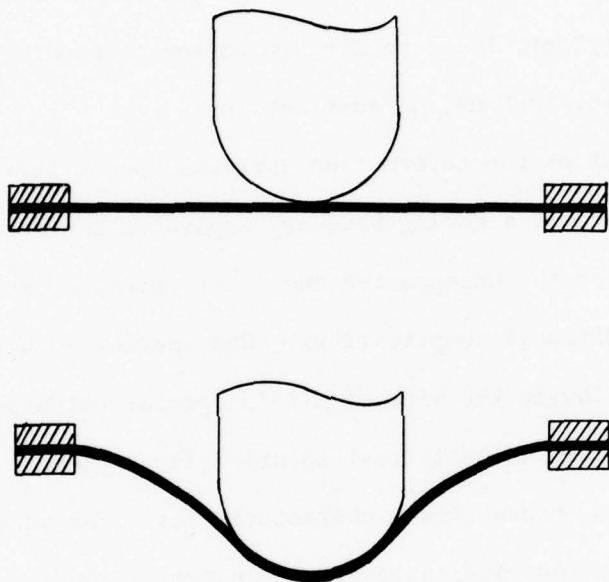


Figure 10. Schematic View of the Stretching of a Sheet with a Hemispherical Head Punch

the plane-strain condition, which is thought to be responsible for local necking in in-plane stretching, is not achieved. This is attributed to the fact that in punch stretching an increment in tangential strain is geometrically tied to an increment in circumferential strain and, therefore, the approach to plane-strain condition becomes slower. Another experimental investigation of punch stretching is the one by Alexander and Kaftanoglu [39]. They observed that the deformation is limited by the "strain propagation instability" or, local necking in common terminology, and not by "maximum load instability" or, diffuse necking.

From the viewpoint of the deformation analysis, punch stretching is a complicated problem because a moving boundary separates the region in contact with the punch head from the unsupported one. The friction over the punch head gives rise to additional complications. One special solution is by Chakrabarty [40]. Following the line of Hill's special solution on hydrostatic bulging he obtained an analytical solution for a special material having exponential type stress-strain characteristics. For more general materials the only solutions available are the numerical ones. Numerical solutions of importance are those by Woo [41] and by Wang [42], [43].

Woo's and Wang's solutions were obtained by the finite-difference method. The only solution by the finite-element method on punch stretching is one by Wifi [44]. His elasto-plastic, finite-element model does not neglect the bending moment nor the effect of shear stress and uses two-dimensional triangular elements to take the thickness variation into account. Friction, which is of primary significance compared with the secondary effect of bending and thickness, is assumed to be perfect, meaning that once the element touches the punch head, it does not slide over the punch but sticks to it.

2. Computational procedures

In applying the finite-element method to punch stretching, a thought should be given to the implementation of boundary conditions. The boundary conditions in punch stretching are stated not only by prescribing tractions and incremental displacements but sometimes by their ratios. In this report, the problem is similar to the ball indentation problem (Lee et al. [45]).

The radial and vertical positions of the material elements in the contact region are not independent but they are related to each other through a mathematical expression for the geometrical requirement that they must be actually on the surface of the punch head. The expression is

$$(r_0 + v)^2 + (c + z_0 + w)^2 = r_p^2, \quad (46)$$

where r_0 , z_0 are radial and vertical positions of the element at the present undeformed configuration; v , w are the increments of horizontal and vertical displacements, and c is a parameter related to the punch height h by the expression

$$c = r_p - h.$$

See Fig. 11. Recall that the finite-element formulation in Chapter IV has already been linearized and what it really solves for are the perturbation terms. Therefore, we also linearize the boundary condition (46) to obtain

$$2(r_0 + v^*)\Delta v + 2(c + z_0 + w^*)\Delta w = r_p^2 - (r_0 + v^*)^2 - (c + z_0 + w^*)^2, \quad (47)$$

where starred (*) quantities are initial guesses, and Δv , Δw are perturbations. By rearranging (47), we have

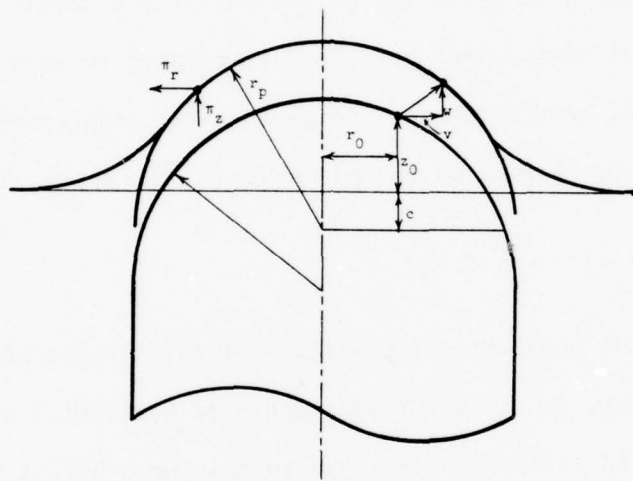


Figure 11. Geometrical Requirement for the Node on the Contact Region

$$\Delta v = \frac{1}{\alpha} \Delta w + \beta, \quad (48a)$$

where

$$\alpha = \frac{(r_0 + v^*)}{(c + z_0 + w^*)} = \frac{1}{\tan \theta} \quad (48b)$$

and

$$\beta = \frac{r_p^2 - (c + z_0 + w^*)^2 - (r_0 + v^*)^2}{2(r_0 + v^*)}. \quad (48c)$$

When the finite-element model is implemented, all the tractions are transformed into generalized nodal forces. Therefore, it is convenient to write the boundary condition in terms of the generalized nodal forces $\pi_{(r)}$ and $\pi_{(z)}$, the horizontal and vertical components, respectively. See Fig. 11.

Now

$$\begin{aligned} \pi_{(r)} &= N \cos \theta - S \sin \theta \\ \pi_{(z)} &= N \sin \theta + S \cos \theta \end{aligned} \quad (49)$$

where N and S are generalized forces normal and tangential to the punch head. We eliminate N through the relation

$$\cos^2 \theta + \sin^2 \theta = 1$$

and obtain

$$\pi_z \cos \theta = \pi_r \sin \theta + k, \quad (50)$$

where k is the frictional force at nodes. However, from geometry we know that the following holds:

$$\cos \theta = \frac{r_0 + v^*}{r_p}, \quad (51a)$$

$$\sin \theta = \frac{z_0 + w^* + c}{r_p}. \quad (51b)$$

So, (50) may be written as

$$\pi(z) + \frac{\pi(r)}{\alpha} = \frac{kr_p}{(r_0 + v^*)}. \quad (52)$$

If the die has a round profile of the radius r_D , then the requirement for a material element to lie geometrically on the profile is similar to the requirement to be satisfied on the punch head. Therefore, we have (similar to Eq. (46)),

$$(a - r_0 - v)^2 + (r_D - z_0 - w)^2 = r_D^2, \quad (53)$$

where a is the radius of the sheet. Linearization of Eq. (53) gives

$$2(a - r_0 - v^*)\Delta v + 2(r_D - z_0 - w^*)\Delta w = -r_D^2 + (a - r_0 - v^*)^2 + (r_D - z_0 - w^*)^2 \quad (54)$$

or, rewriting,

$$\Delta v = \frac{\Delta w}{\gamma} + \Omega, \quad (55a)$$

where

$$\gamma = -\frac{(a - r_0 - v^*)}{(r_D - z_0 - w^*)} \quad (55b)$$

and

$$\Omega = \frac{(a - r_0 - v^*)^2 + (r_D - z_0 - w^*)^2 - r_D^2}{2(a - r_0 - v^*)}. \quad (55c)$$

The tractional boundary condition over the die profile can be written similarly as

$$\pi(z) + \frac{\pi(r)}{\gamma} = -\frac{k\gamma_D}{(a - r_0 - v^*)} \quad (56)$$

For the portion of the sheet which is not in contact with the punch head nor with the die profile, the displacement increment in the radial direction and the displacement increment in the axial direction are not bound to each other, as is the case for the contact region, but remains as independent variables. Traction is, however, given the value of zero.

With the advancement of the punch head, the portion of the sheet in contact with the punch or die profile increases and, consequently, the boundary separating this "contact region" from the "unsupported region" changes. The presence of this moving boundary is always a source of complications in the numerical analysis of punch stretching because it requires a basically trial-and-error approach. The treatment of the moving boundary used in the present analysis for punch stretching without round die corners is explained in detail as follows:

First, assume the position of the boundary in future configurations. In the FEM this means assuming which nodes will be in contact with the punch head in the future configuration. Then, obtain a converged solution based upon this assumption and check to see if it is true. Since the position of the boundary is already known in the current configuration, in practice we assume and check how much this boundary advances.

(1) Check whether the boundary is assumed to advance too fast.

Compute the normal component of the generalized nodal force

to the punch head for the nodes in contact with the punch head. If every generalized normal force is directed outward from the punch head, then all the nodes which are assumed to be in contact with the punch head actually do so. On the other hand, the generalized normal force in the direction toward the punch head for any node means that external force other than the one exerted by the punch is necessary for this particular node to conform with the punch geometry. Since there is physically no source of applied force other than the punch, the assumption that this particular node is in contact with the punch head is not correct and the position of the boundary should be re-assumed to exclude this node from the contact region.

- (2) Check whether the boundary is assumed to advance too slowly.

Compute the distance between the nodes in the unsupported region and the center of the hemisphere of the punch head. If this distance is shorter than the radius of the punch head for any node, it means that this particular node is inside the punch head. Since this is physically impossible, the assumption that this particular node is not in contact with the punch head is not correct and the position of the boundary should be re-assumed to include this particular node.

Although this basically trial-and-error approach seems to be very time consuming, in actual computation we can predict the movement of the boundary fairly accurately based upon the distance between the free nodes and the punch surface. Furthermore, since we already know the position of the boundary in the current configuration, it is enough to check the

boundary assumption for only a few nodes neighboring the previous position of the boundary, not for whole nodes.

The procedure described above for the contact region on a punch head is also applicable for the contact region at the die corner. Handling two moving boundaries simultaneously really does not invoke any new theoretical difficulties but only takes more computation time and may be impractical for inefficient numerical methods.

In order to implement Coulomb friction between sheet and punch or die, we first prescribe a tangential friction force S and obtain a converged solution and then compute generalized nodal forces. From Eqs. (49) we then are able to compute the normal component N and the friction coefficient $\mu = \frac{S}{N}$ corresponding to the initially prescribed value of S . If the computed friction coefficient is not what is intended, then we modify the S value and repeat the process. It should be noted here that the correction of frictional force S needs the necessary modification only in the F matrix (Eq. (44)), while the stiffness matrix P , which is the most time-consuming part, remains the same.

The deformation step is controlled by the punch head increment, which is designed in the present codes to yield the maximum increment of effective strain roughly equal to a preset value. In the present work the optimum size is shown to be a 0.04 increment of effective strain. The solution generally converged after 10 \sim 15 iterations for a single step within the fractional norm of 10^{-6} . The actual program is shown in Appendix C.

3. Results and discussion

The present rigid-plastic FEM is compared with the finite-difference methods by Wang [43] and Woo [41], and also with the experiment by Kaftanoglu and Alexander [39].

(1) Comparison with the finite-difference solution by Wang

The parameters used in Wang's example are as follows:

Material: copper

$$\begin{aligned}\text{Stress-strain characteristics: } \sigma &= 30.5\epsilon^{0.326} \text{ ton/in.}^2 \\ &= 4.6361\epsilon^{0.326} \times 10^8 \text{ N/m}^2\end{aligned}$$

Anisotropy: $R = 1.0$

Friction: $\mu = 0.04$

Thickness: $t = 0.035 \text{ in.} = 8.89 \times 10^{-2} \text{ m}$

Punch radius: $r_p = 1.0 \text{ in.} = 2.54 \times 10^{-2} \text{ m}$

Radius of sheet: $a = 1.15 \text{ in.} = 2.921 \times 10^{-2} \text{ m}$

Initial radius is sometimes denoted by r_0 .

The two methods are in excellent agreement in predicting the punch head for a given punch travel. See Fig. 12; the solid line represents Wang's solution and the points represent the rigid-plastic FEM. Fig. 13 shows the thickness strain distribution. Again, a good agreement between the two solutions is apparent.

The second example has the following parameters:

$$\text{Stress-strain characteristics: } \sigma = k\epsilon^{0.2}$$

Anisotropy: $R = 1.0$

Friction: $\mu = 0.2$

Punch radius: $r_p = 1.0$

Radius of sheet: $r_0 = 1.0$

In Wang's work all the results are reported in the dimensionless number. Figs. 14 and 15 show the circumferential strain distribution and thickness strain distribution, respectively. The solid line represents Wang and points

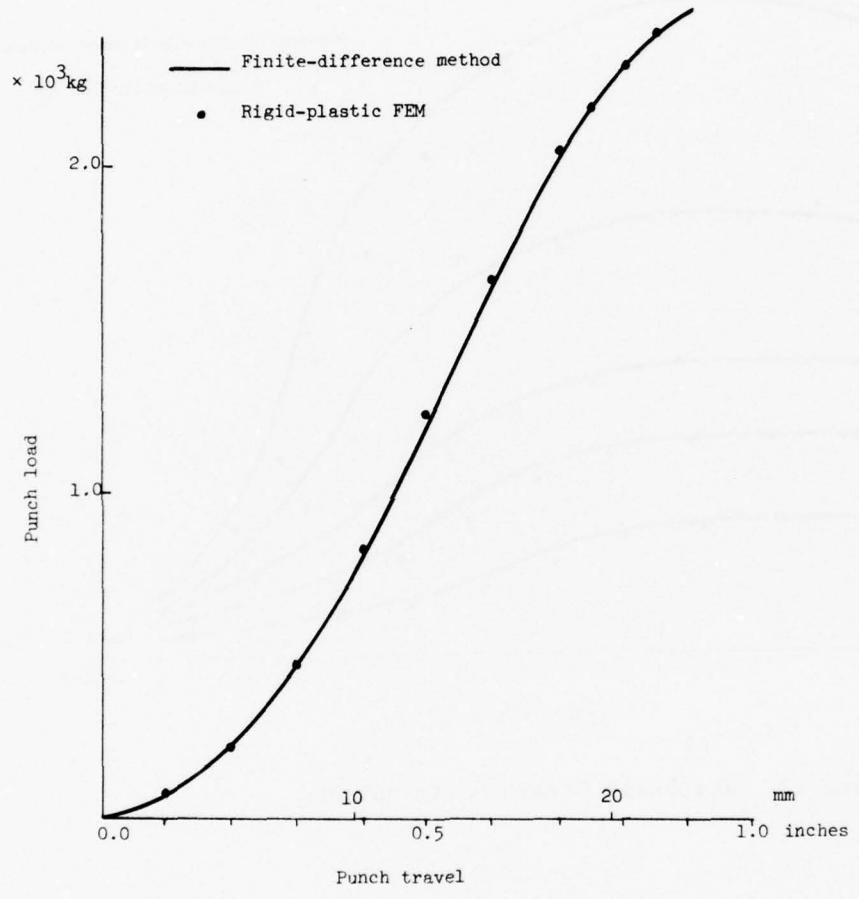


Figure 12. Punch Head vs. Punch Travel

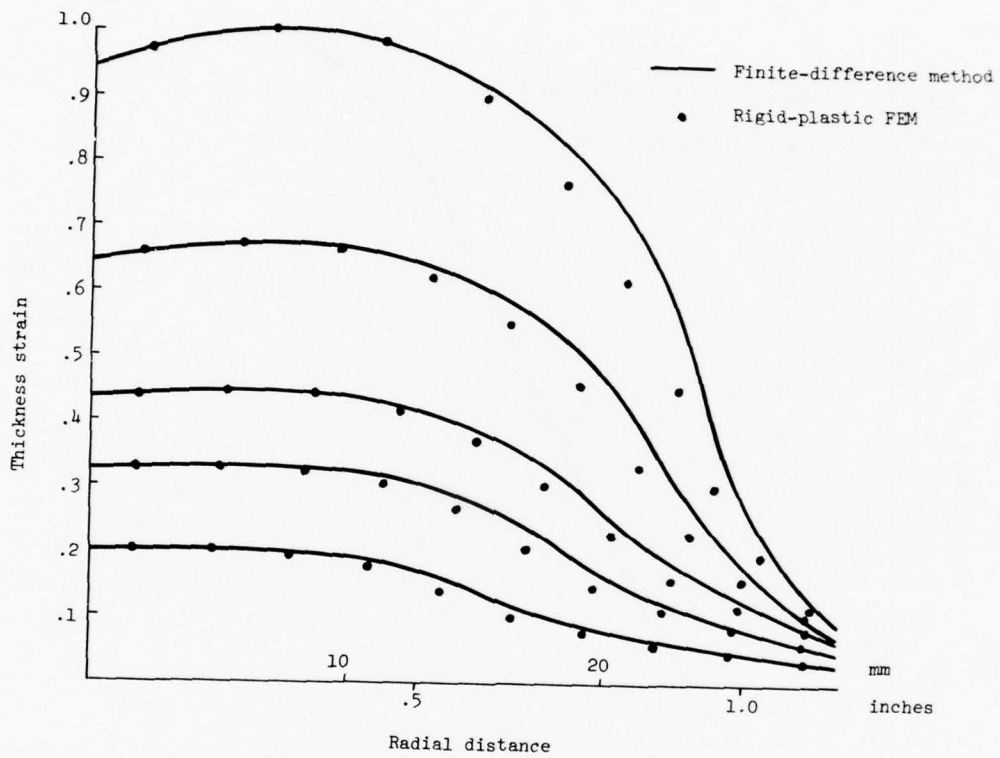


Figure 13. Thickness Strain Distribution

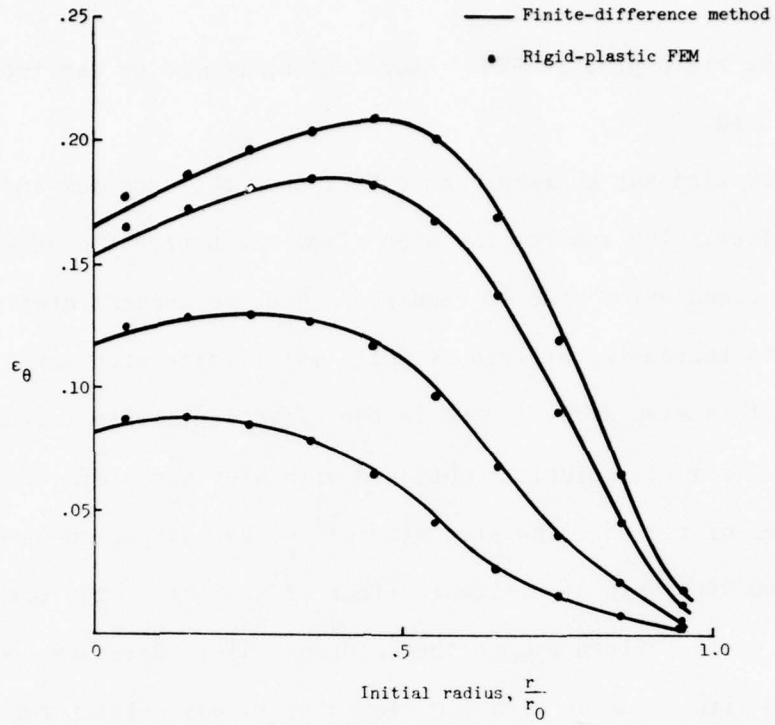


Figure 14. Circumferential Strain Distribution

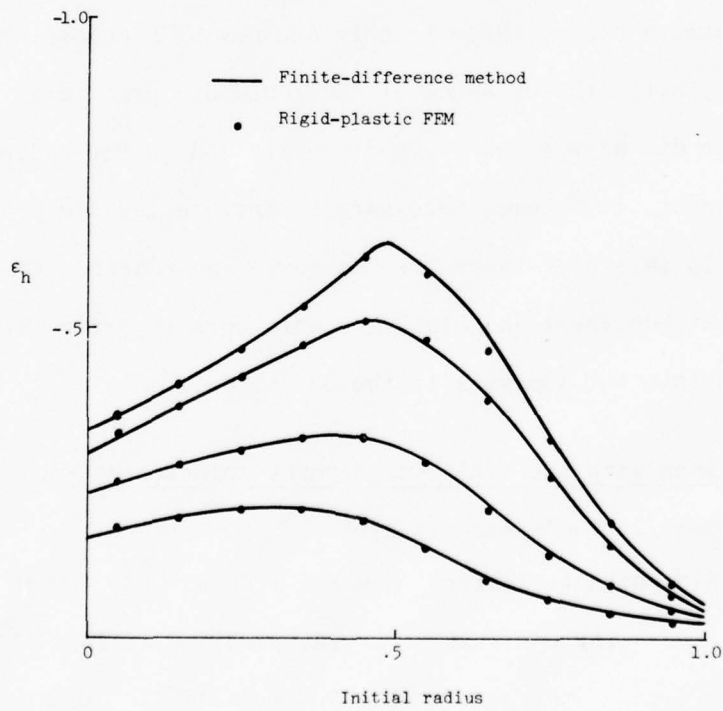


Figure 15. Thickness Strain Distribution

represent the rigid-plastic FEM. Excellent agreement of the two solutions is demonstrated.

The step size has an important effect upon the accuracy and efficiency of the solution. The smaller the step size, the better the accuracy, although more computation time is required. Fig. 16 demonstrates that there is a limit to increasing efficiency while maintaining accuracy. For example, solutions with a step size of 0.08 in the effective strain increment deviates considerably from the solutions obtained with step sizes of .02 or .04. In the remainder of the work the step size of .04 is most often used.

Compared with this significant effect of step size, the mesh size does not exert a great influence upon the solution, as is demonstrated in Fig. 17. The solution with a coarse mesh (10 elements) is essentially the same as the one with a finer mesh (40 elements), even though the latter will be helpful in pinpointing the exact location of peak strain.

In the examples above, there is only one moving boundary, that between punch and sheet, since the presence of the round die profile is neglected. In practice, the die always has a round profile and as the radius of the profile gets larger, it becomes necessary to include the die profile in the analysis. In this case there are two moving boundaries, the second being the one between sheet and die. The only work reported which includes the die profile into the analysis is the one by Woo.

(2) Comparison with the finite-difference solution by Woo

The parameters in Woo's example are:

$$\begin{aligned} \text{Stress-strain characteristics: } \sigma &= 5.4 + 27.8\epsilon^{0.504} \text{ ton/in.}^2 \\ \text{for } \epsilon < 0.36: &= (0.08208 + 0.422569\epsilon^{0.504}) \times 10^9 \text{ N/m}^2 \\ &= 5.4 + 24.4\epsilon^{0.375} \text{ ton/in.}^2 \\ \text{for } \epsilon > 0.36: &= (0.08208 + 0.37089\epsilon^{0.375}) \times 10^9 \text{ N/m}^2 \end{aligned}$$

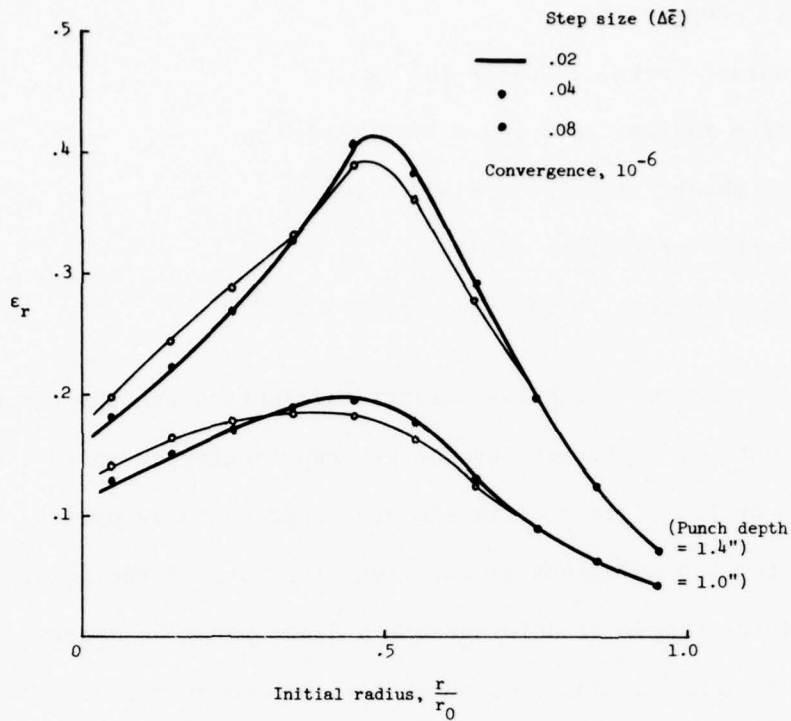


Figure 16. Effect of Step Size

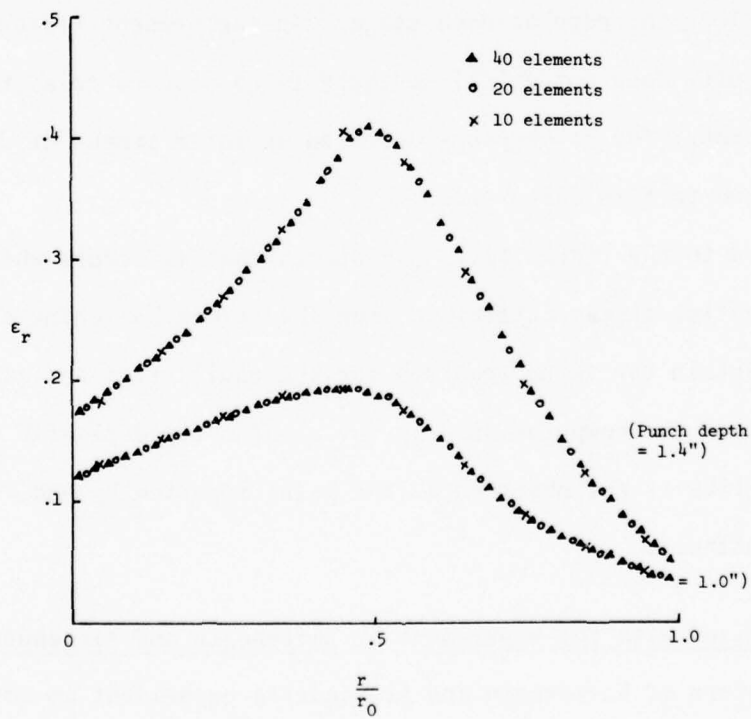


Figure 17. Effect of Mesh Size

Material: copper

Punch radius: 1 in. = 2.54×10^{-2} m

Die profile radius: 0.3 in. = 7.62×10^{-3} m

Radius of sheet: 1.3 in. = 3.302×10^{-2} m

Coefficient of friction: 0.04

Thickness of sheet: 0.035 in. = 8.89×10^{-4} m

Figs. 18 and 19 are the thickness strain distribution and the circumferential strain distribution. Solutions by Woo are represented by solid lines and the solutions by the rigid-plastic FEM are represented by points. Agreement between the two solutions is excellent for most of the deformation. However, at later stages of deformation, a discrepancy is observed around the edges. Re-examining Woo's computational procedure reveals that in order to avoid the difficulty of satisfying boundary conditions exactly along the fixed edge ($\epsilon_{\theta} = 0$), he allowed a small increment of circumferential strain along the edge at each stage. In the present rigid-plastic FEM such difficulty does not exist, so there is no need to relax the boundary condition. The discrepancy observed at later stages of deformation may be attributed to this difference.

With regard to the instability, Woo stated that it occurs when the resultant tangential stress determined from the strain hardening characteristics cannot obtain the value required for the equilibrium and at that instant he stopped the computation. In the present rigid-plastic analysis such an instability is not observed at the point reported by Woo, and the computation continues.

(3) Comparison with the experiment by Kaftanoglu and Alexander

The parameters of Kaftanoglu and Alexander's experiment on soft copper are:

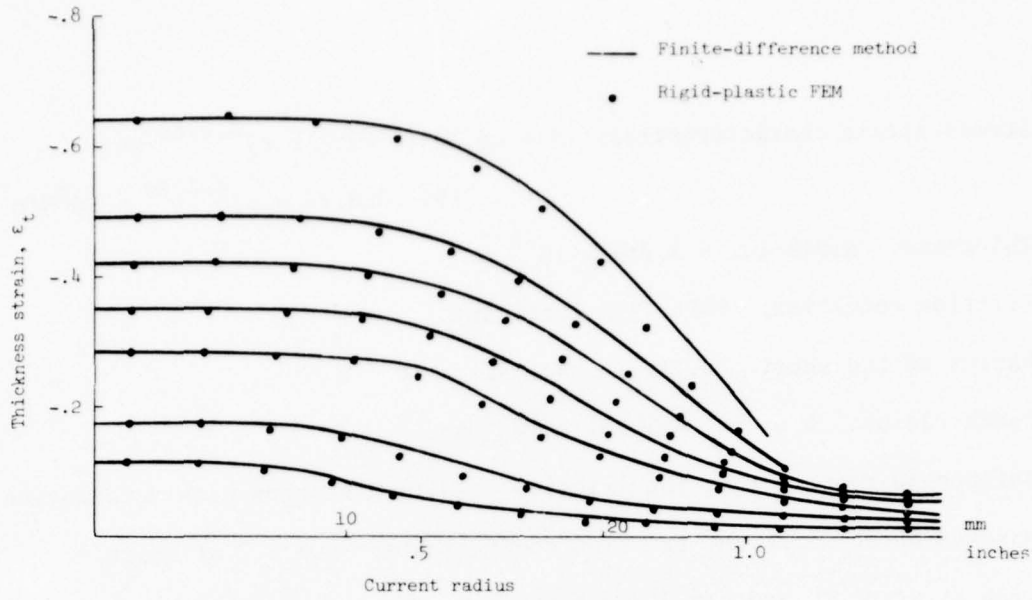


Figure 18. Distribution of Thickness Strain When Die Profile Is Considered

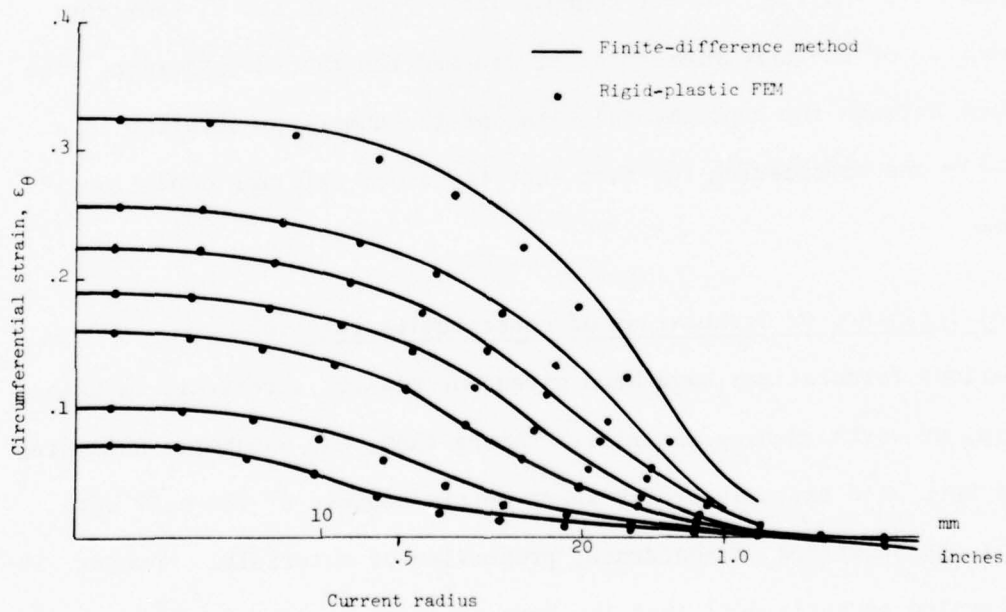


Figure 19. Distribution of Circumferential Strain When Die Profile Is Considered

Stress-strain characteristics: $\sigma = 68,394(0.0122 + \epsilon)^{0.3789}$ psi
 $= 4.7156 (0.0122 + \epsilon)^{0.3789} \times 10^8 \text{ N/m}^2$

Thickness: 0.048 in. = 1.219×10^{-3} m

Friction condition: PTFE film lubricant

Radius of the sheet: 0.717 in. = 1.821×10^{-3} m

Punch radius: 0.65 in. = 1.651×10^{-3} m

Kaftanoglu reports that the friction condition changes with deformation and measures three different friction coefficients: $\mu = 0.2$ at stage 1, $\mu = 0.135$ at stage 2, and $\mu = 0.07$ at stage 3. To include the changing friction coefficient into the analysis, we need more information on the friction history, which is difficult to obtain experimentally. Therefore, as a representative value, we use the mean of three values of the friction coefficient, $\mu = 0.135$, for our computation. Figs. 20 and 21 show the distribution of the circumferential strain and the thickness strain. The agreement between the experimental data and the numerical solution is a reasonable one considering the fact that the exact friction condition is not known.

(4) Influence of formulation of constitutive relation

Various formulations have been given for plastic stress-strain relationships of workhardening materials. Among them, the parabolic hardening law has been used extensively for sheet metals because of the ease with which it characterizes workhardening properties of materials. However, it was suggested recently [46] that the Voce equation [47] is a better representation of materials behavior when solving plasticity problems involving workhardening rate. The forming limit curves were compared using the

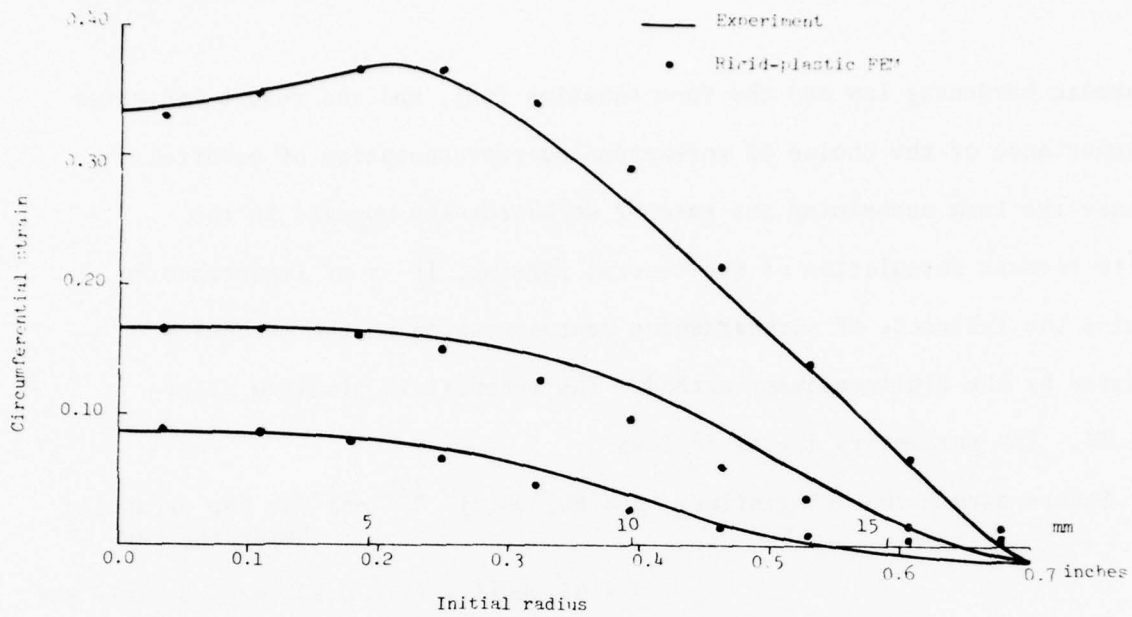


Figure 20. Comparison of the Numerical Solution with the Experimental Data for Circumferential Strain Distribution

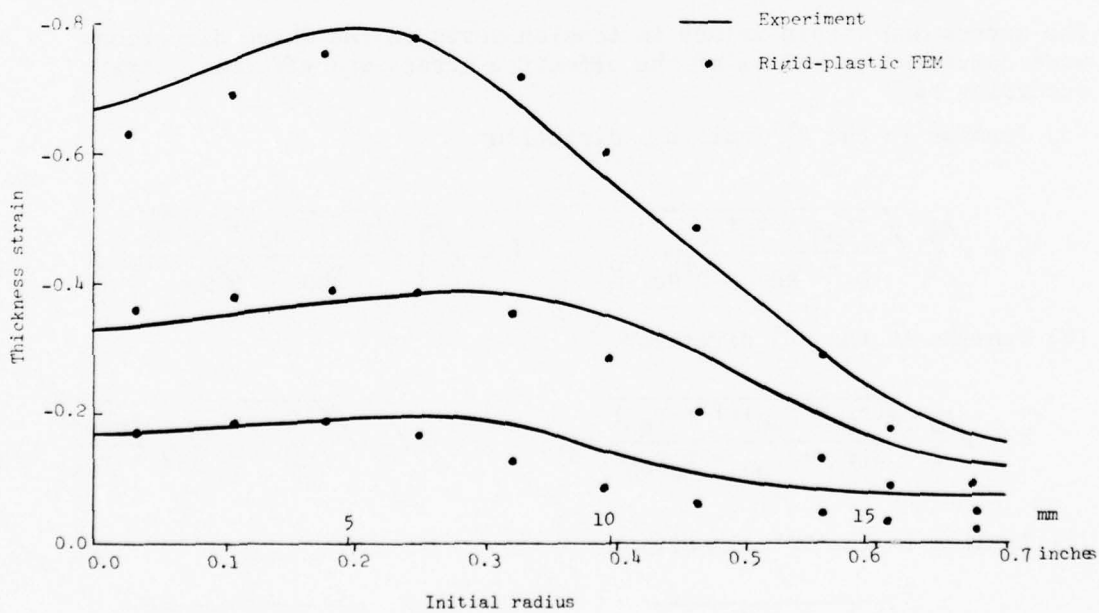


Figure 21. Comparison with the Experiment for Thickness Strain Distribution

parabolic hardening law and the Voce equation [48], and the result indicates an importance of the choice of workhardening representation of materials. Because the term containing the rate of workhardening appears in the finite-element formulation of sheet-metal forming, it is of importance to examine the influence of workhardening representation on the mechanics computed by the finite-element method. The material is aluminum alloy 2036-T4. The parameters are as follows:

$$\begin{aligned} \text{Stress-strain characteristics: } \sigma &= 86,000(\epsilon)^{0.222} \text{ psi for the parabolic} \\ &\hspace{15em} \text{hardening law} \\ \sigma &= 65,000\{1 - (1 - 0.508)\exp(-8.51\epsilon)\} \text{ psi} \\ &\hspace{15em} \text{for the Voce equation} \end{aligned}$$

Fig. 22 shows the two stress-strain curves together with tension test data from the specimens cut in the three directions (0° , 45° , 90°).[†]

[†](1) The stress and strain values in tension tests in the three directions were converted to values of the effective stress and effective strain according to

(a) Tension in the 0° (rolling) direction:

$$\bar{\sigma} = \sqrt{\frac{3}{2}} \sqrt{\frac{r_{90} + r_0 r_{90}}{r_0 + r_{90} + r_0 r_{90}}} \sigma_{0'}, \quad \bar{\epsilon} = \sqrt{\frac{2}{3}} \sqrt{\frac{r_0 + r_{90} + r_0 r_{90}}{r_{90} + r_0 r_{90}}} \epsilon_{0'}$$

(b) Tension in the 45° direction:

$$\bar{\sigma} = \sqrt{\frac{3}{2}} \sqrt{\frac{(r_0 + r_{90})(1 + r_{45})}{2(r_0 + r_{90} + r_0 r_{90})}} \sigma_{45'}, \quad \bar{\epsilon} = \sqrt{\frac{2}{3}} \sqrt{\frac{2(r_0 + r_{90} + r_0 r_{90})}{(r_0 + r_{90})(1 + r_{45})}} \epsilon_{45'}$$

(c) Tension in the 90° direction:

$$\bar{\sigma} = \sqrt{\frac{3}{2}} \sqrt{\frac{r_0 + r_0 r_{90}}{r_0 + r_{90} + r_0 r_{90}}} \sigma_{90'}, \quad \bar{\epsilon} = \sqrt{\frac{2}{3}} \sqrt{\frac{r_0 + r_{90} + r_0 r_{90}}{r_0 + r_0 r_{90}}} \epsilon_{90'}$$

(Footnote continued on next page)

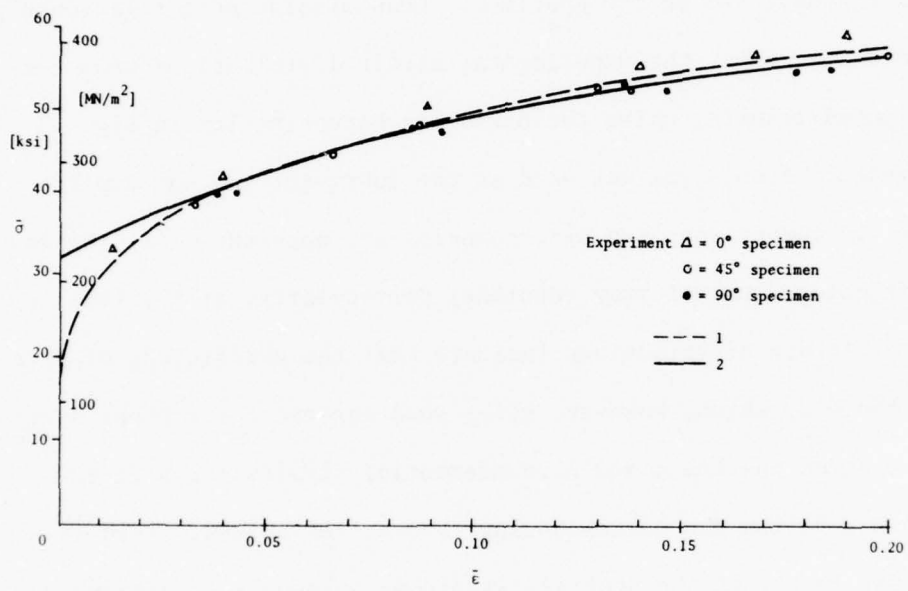


Figure 22. Stress-strain Curve for Al. 2036-T4

r-value: $r_0 = 0.66$, $r_{45} = 0.69$, $r_{90} = 0.70$, and $r_a = 0.685$

Radius of die opening: (0.80 in.)

Blank thickness:

Radius of punch head: 0.75 in. and 0.45 in.

Coefficient of friction: 0 and 0.2

Punch stretching was performed on a horizontal hydraulic press. Tests were interrupted for strain measurements (thickness and circumferential strains) from the grids photprinted on the specimen. Load-displacement relationships were also recorded. First, the experimental strain distributions were compared with computed results, using the parabolic hardening law in Fig. 23. In the experiment Johnson's wax was used as the lubricant and was applied at each stage. In comparison, two discrepancies are apparent: (i) the coefficient of friction does not stay constant; particularly, at the last stage, the experimental strain distributions indicate that the coefficient of friction is less than 0.2, which, however, gives good agreement for other stages, and (ii) the measured thickness and circumferential strains for a given punch depth do not follow the corresponding theoretical curves. This is attributed to the fact that the accurate strain measurements is extremely difficult for critical comparison between theory and experiment. The load values summarized in Table 1 show an excellent agreement between the two.

where r_0 , r_{45} , r_{90} are the r-values obtained from the tension of specimens cut in the 0° , 45° , and 90° directions, respectively.

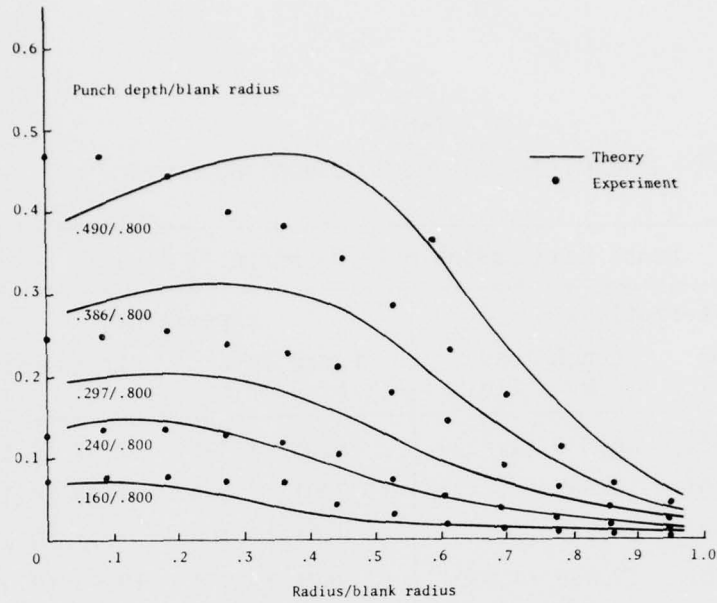
- (2) The effective stress and effective strain defined in the formulation of this report differ from the definition above by a factor such as

$$\bar{\sigma} = \sqrt{\frac{3}{2} \frac{1 + r_a}{2 + r_a}}, \quad \bar{\epsilon} = \sqrt{\frac{2}{3} \frac{2 + r_a}{1 + r_a}},$$

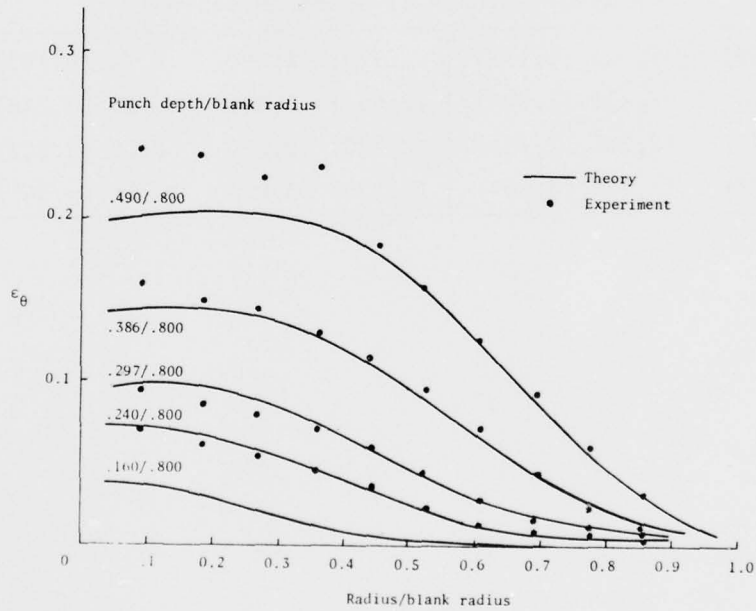
where r_a is the average r-value defined by $r_a = \frac{r_0 + 2r_{45} + r_{90}}{4}$.

Table 1
PUNCH LOAD AND DISPLACEMENT RELATIONS

Punch head radius = 19.05 mm (0.75 in.)					
Theoretical			Experimental		
Displacement mm (in.)	Punch load N (lb)		Punch load (lb)	N	Displacement
4.06 (0.160)	6,330 (1,423)		(970)	4,315	2.79 (0.110)
6.10 (0.240)	10,889 (2,448)		(1,730)	7,695	4.83 (0.190)
7.54 (0.297)	14,483 (3,256)		(2,990)	13,300	7.11 (0.280)
9.80 (0.386)	22,059 (4,959)		(4,940)	21,974	10.08 (0.397)
12.45 (0.490)	30,301 (6,812)		(6,580)	29,269	12.45 (0.490)
Punch head radius = 11.43 mm (0.45 in.)					
4.06 (0.160)	5,124 (1,152)		(920)	4,092	2.72 (0.107)
6.10 (0.240)	8,131 (1,828)		(2,000)	8,896	6.30 (0.248)
8.53 (0.336)	12,237 (2,751)		(2,770)	12,322	8.18 (0.322)
9.58 (0.377)	13,963 (3,139)		(3,130)	13,923	9.68 (0.381)

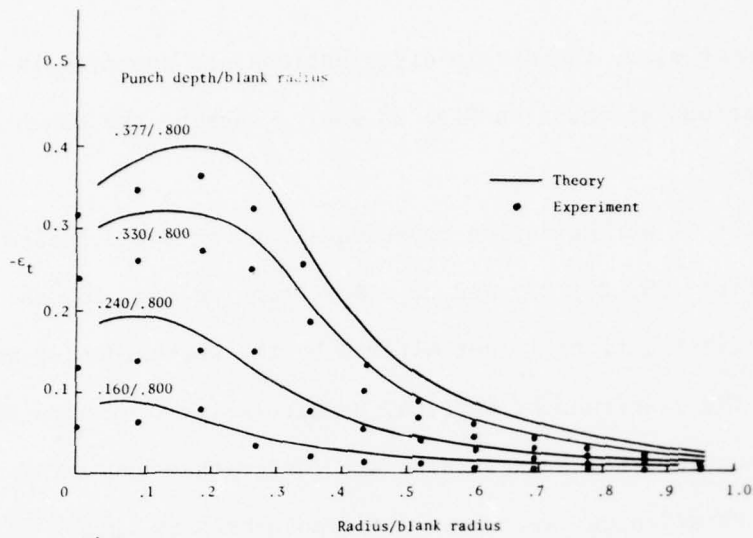


(a)

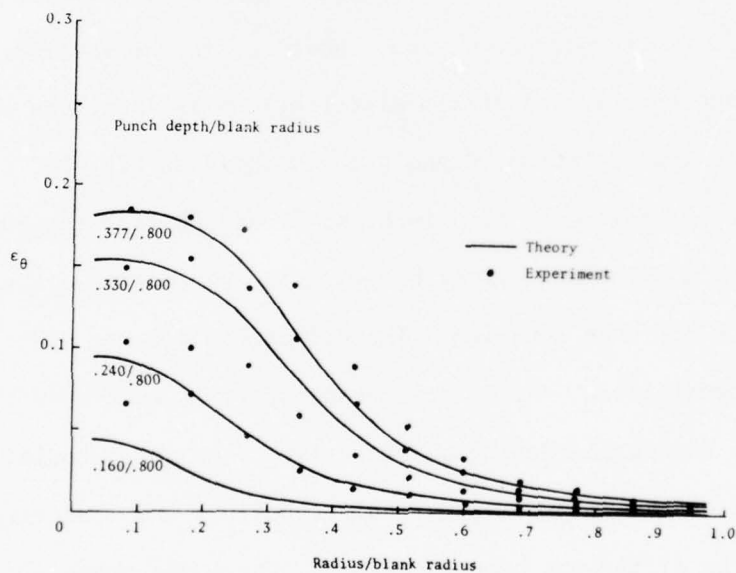


(b)

Figure 23. Experimental (Johnson's wax as lubricant) and Theoretical ($\mu = 0.2$) Strain Distributions for Punch Size ($r_p/r_0 = 0.75/0.80$).
(a) Thickness Strains; (b) Circumferential Strains



(a)



(b)

Figure 24. Experimental (Johnson's wax as lubricant) and Theoretical ($\mu = 0.2$) Strain Distributions for Punch Size ($r_p/r_0 = 0.45/0.80$).
(a) Thickness Strains; (b) Circumferential Strains

For a smaller punch size, the strain distributions are compared in Fig. 24. The same observations as those in Fig. 23 apply. Again, the punch load is in good agreement.

The influence of workhardening representations on the detailed mechanics is examined in Figs. 25, 26, 27, and 28. Referring to Fig. 25, the general trend of strain distributions is not altered by the workhardening representation. However, the magnitude of strains, particularly, peak strains, differ. With the Voce equation, the peak strains are larger than those computed by the parabolic workhardening law. This difference becomes larger as the punch penetrates.

It appears that the difference of the two is more significant for higher friction in the larger punch size. However, in the smaller punch size, the difference of the two strain distributions is about the same for the two coefficients of friction, 0 and 0.2, as shown in Fig. 26.

It is rather surprising to find in Figs. 27 and 28 that the punch load for the same punch displacement is higher with the parabolic workhardening law than that with the Voce equation. The difference becomes significant for large punch penetration. From these results, it is concluded that the representation of the workhardening characteristics of the material does have an influence on the computed strain distributions and load-displacement relationships. The difference becomes critical for large punch displacement in predicting both peak strains and the punch load. In order to determine which representation is preferable, however, more experiments with improved accuracy and control are needed.

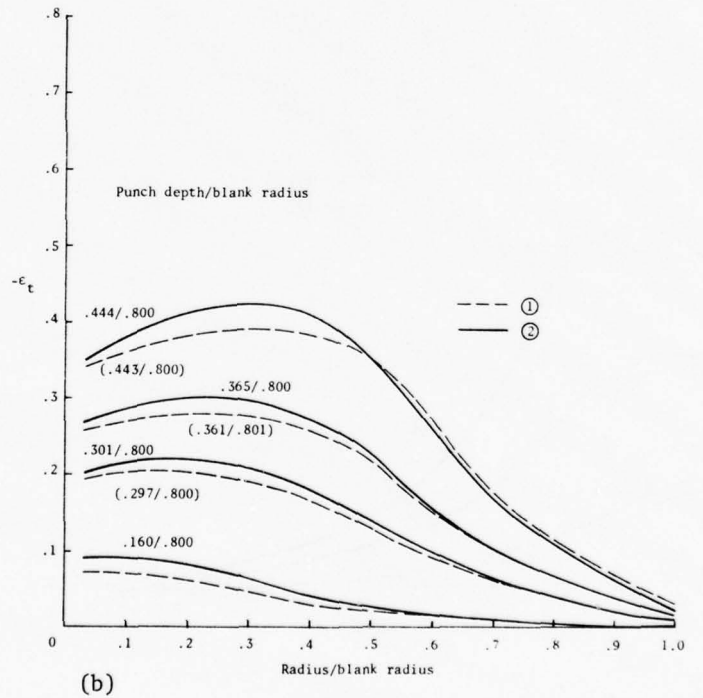
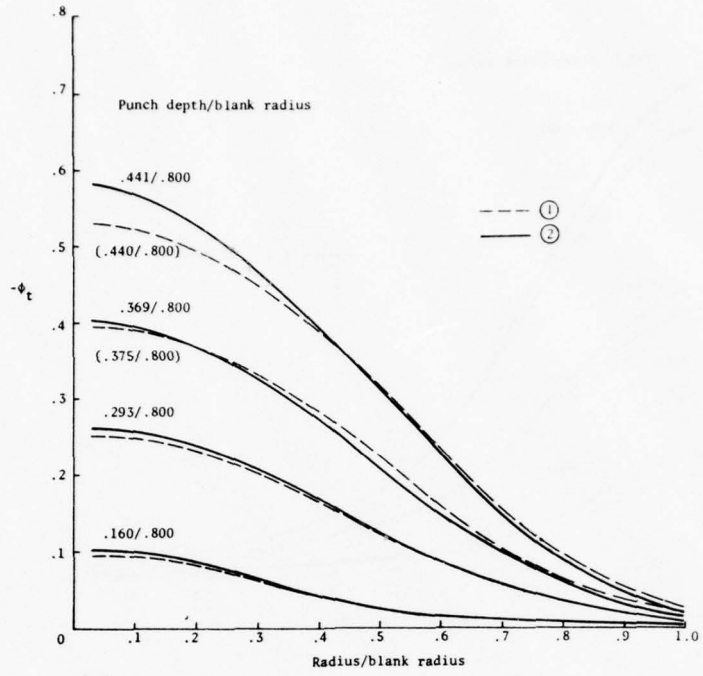
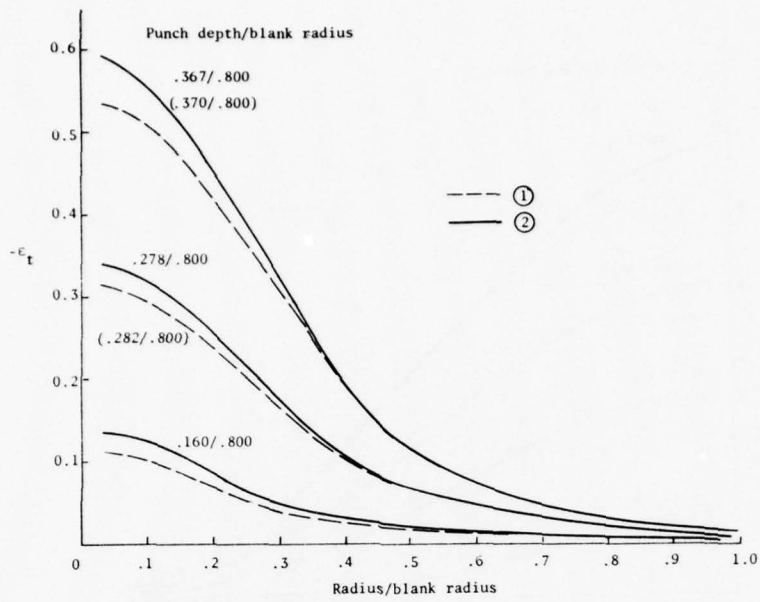
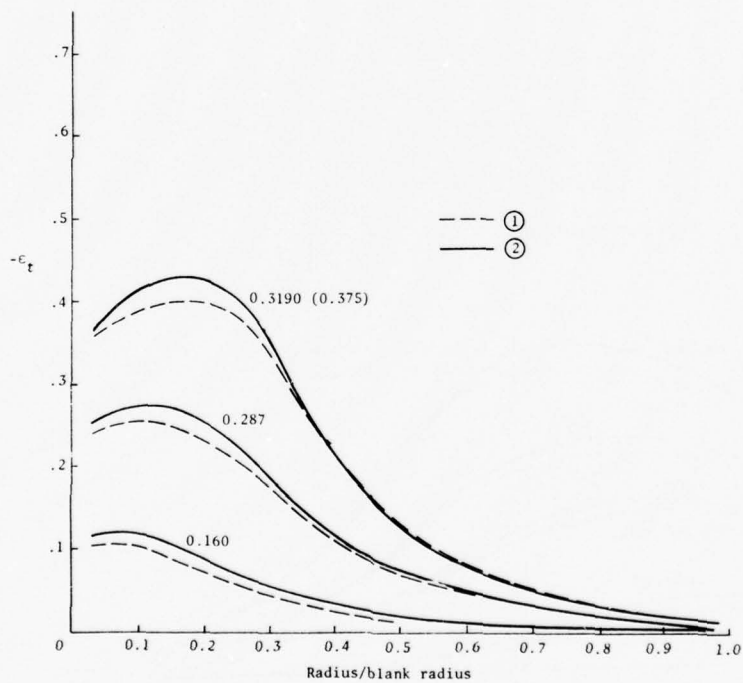


Figure 25. Comparison of Theoretical Thickness Strain Distributions Using (1) the Parabolic Work-hardening Law and (2) the Voce Equation for Punch Size ($r_p/r_0 = 0.75/0.80$) with (a) $\mu = 0$ and (b) $\mu = 0.2$



(a)



(b)

Figure 26. Comparison of Theoretical Thickness Strain Distributions Using (1) the Parabolic Workhardening Law and (2) the Voce Equation for Punch Size ($r_p/r_0 = 0.45/0.80$) with (a) $\mu = 0$ and (b) $\mu = 0.2$

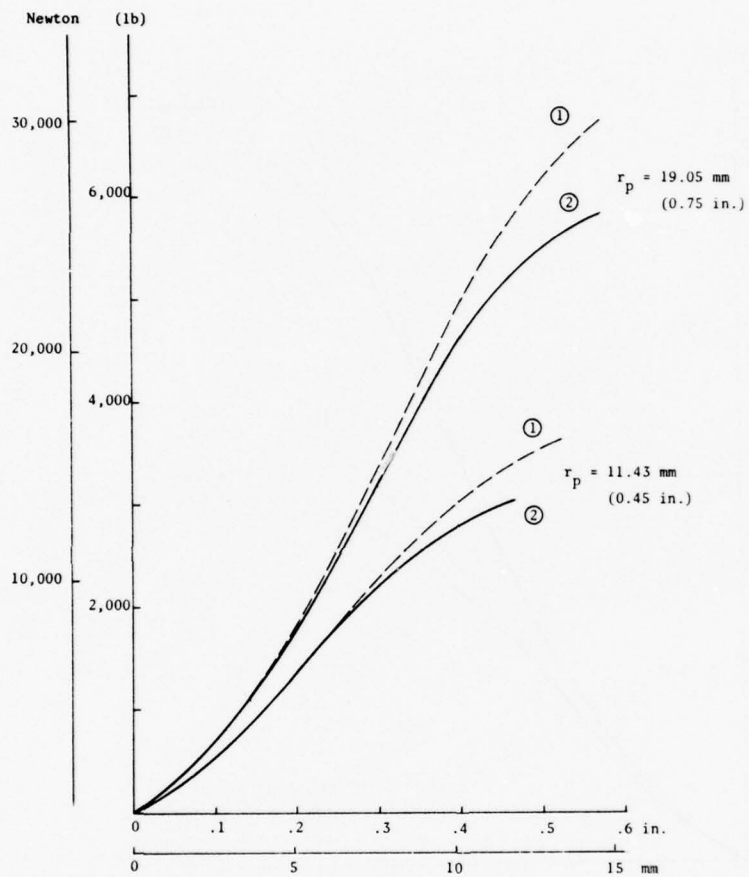


Figure 27. Comparison of Theoretical Load Displacement Curves Using (1) the Parabolic Workhardening Law and (2) the Voce Equation for $\mu = 0$

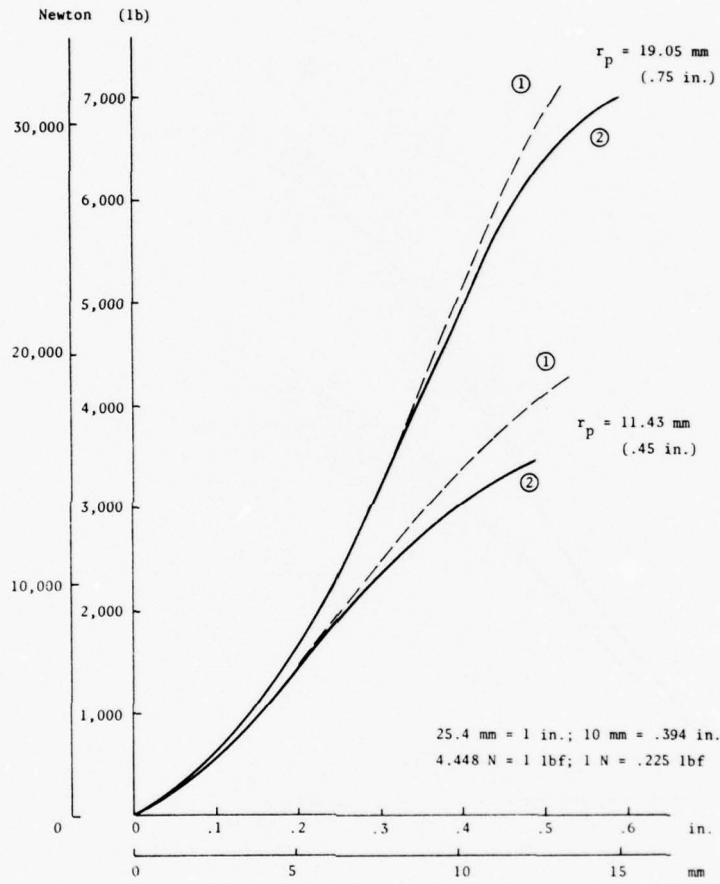


Figure 28. Comparison of Theoretical Load Displacement Curves Using (1) the Parabolic Workhardening Law and (2) the Voce Equation for $\mu = 0.2$

SECTION VI

DEEP DRAWING OF A SHEET WITH HEMISPHERICAL PUNCH

1. Introduction

In a deep drawing test a circular sheet of metal is placed between the blank holder and the die and then fully drawn into the shape of a cup. The formability is then measured by the maximum size of the blank which can be drawn without a failure, or, more often, by its ratio to the punch diameter. This ratio is called the limiting drawing ratio and this particular kind of test is called the Swift test.

Deep drawing is not only a useful method of material testing, but also one of the basic operations in sheet-metal stamping. In practice, various shapes are possible for the bottom of the punch; however, most past investigations are on deep drawing with a flat-bottomed punch [49]-[56].

Among the earlier works on deep drawing are those by Hill [13] and by Chung and Swift [52] using the incremental theory of plasticity. More refined analyses are the finite-difference solutions by Chiang and Kobayashi [57], b- Wang and Budiansky [51], and by Chakrabarty and Mellor [49]. Even though such a refinement improves the understanding of the deep drawing process, their works are not complete because they treat the deep drawing problem as an in-plane pure radial drawing and are concerned mostly with the deformation mechanics on the flange. However, it has been observed experimentally (Chung and Swift [52]) that the die profile and the punch profile significantly affect the punch load and the strain distributions and therefore a further refinement is necessary by considering these parameters in the analysis. Woo [53] performs such an analysis and then

is able to show that the solution obtained by extrapolating the strain distribution over the flange to the die throat predicts more straining than the one obtained by taking the profiles into consideration.

Contrary to these numerous investigations on deep drawing with a flat-bottomed punch, very few works are reported on the deep drawing of a sheet with a hemispherical head punch (Fig. 29). Woo [58] analyzes this problem by breaking down the deep drawing process into two component processes of the pure radial drawing over the flange and the punch stretching over the hemispherical punch head. He first obtains solutions for pure radial drawing in the flange and then uses this solution at a point initially situated near the die lip as the boundary condition for the stretching problem, and thereby essentially matched the punch stretching component with the pure radial drawing component at a particular point in the die profile region.

Instead of this tedious process of boundary matching, it is desirable to have a numerically efficient and reliable method which can treat the problem in a unified manner. The FEM is such an alternative. The finite-element model developed for the deep drawing problem is the one by Wifi [44] with a limited treatment of friction. Also, Levy et al. [59] developed the elasto-plastic finite-element program for cupdrawing based on the deformation theory of plasticity.

2. Computational procedure

The entire sheet undergoing the deep drawing process can be divided into four regions: the contact region with the punch head, the unsupported region, the contact region with the die profile, and the flange over the die. Different kinds of boundary restrictions are imposed depending upon

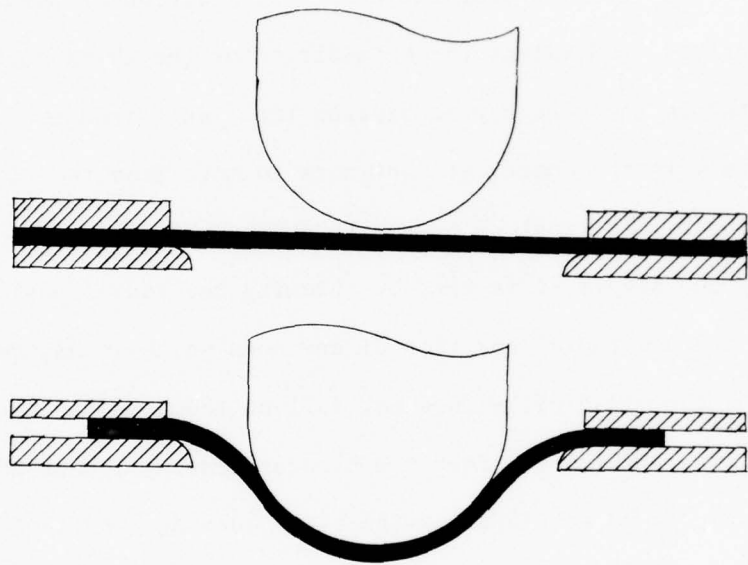


Figure 29. Schematic View of Deep Drawing of a Sheet with a Hemispherical Head Punch

the regions. For example, the flange is constrained to move only horizontally along the die face, while the contact region with the die profile or punch head should satisfy the kind of boundary conditions discussed in Section V.

The only difference in deep drawing with a hemispherical head punch from the punch stretching with a round die corner is the presence of the flange which is free to slide over the die. The addition of this moving flange is, in effect, equivalent to the addition of the third moving boundary, because, even though the boundary separating the flange from the die profile remains stationary in the space, it continues to move from the viewpoint of the deforming sheet. To treat this we make an assumption on this third moving boundary and see if it is true by checking the radial positions of the nodes. If the new radial position of any node which is assumed to lie on the flange or the die profile does not fall on the expected region after converged solution is obtained, then the boundary assumption is modified.

Another point to be mentioned is the blank holding condition of which there are two types: clearance holding and force holding. The idealization of the deformation state corresponding to the force blank holding is the plane stress state and the one corresponding to the clearance holding in the plane-strain state. The present rigid-plastic FEM is built to handle the plane stress state deformation and therefore a modification is necessary to handle the clearance blank holding. No reported work on deep drawing with a hemispherical head punch under clearance blank holding is available and therefore in the present work only the deep drawing with the force holding is analyzed. The blank holding force is implemented in the formulation as a tangential friction force acting on the last node located at the

rim of the sheet. The distribution of the blank holding force over a finite area near the rim can be handled without difficulty in the present FEM, but this distributional effect turns out to be insignificant [53]. Therefore, tangential frictional force is confined to the last node at the rim of the sheet. The increment of deformation is controlled by the punch head movement. The program is in Appendix D.

3. Results and discussion

The only available work on the complete analysis of deep drawing with the hemispherical head punch is one by Woo [58]. Along with the numerical solution by the finite-difference method, he also conducted an experiment. The parameters are:

Material: soft copper

$$\begin{aligned} \text{Stress-strain characteristics: } \sigma &= 5.4 + 27.8\epsilon^{0.504} \text{ ton/in.}^2 \\ \text{for } \epsilon < 0.36: &= (0.08208 + 0.422569\epsilon^{0.504}) \times 10^9 \text{ N/m}^2 \\ &= 5.4 + 24.4\epsilon^{0.375} \text{ ton/in.}^2 \\ \text{for } \epsilon > 0.36: &= (0.08208 + 0.37089\epsilon^{0.375}) \times 10^9 \text{ N/m}^2 \end{aligned}$$

$$\text{Blank radius: } 2.2 \text{ in.} = 5.588 \times 10^{-2} \text{ m}$$

$$\text{Radius of the die throat: } 2.123 \text{ in.} = 5.392 \times 10^{-2} \text{ m}$$

$$\text{Radius of die profile: } 0.5 \text{ in.} = 1.27 \times 10^{-2} \text{ m}$$

$$\text{Radius of punch head: } 1 \text{ in.} = 2.54 \times 10^{-2} \text{ m}$$

$$\text{Blank holding force: } 0.5 \text{ ton} = 500 \text{ kg}$$

The solution by the rigid-plastic FEM is in excellent agreement with the experiment for the flange part; however, over the punch head it predicts more straining than the experiment when the friction coefficient of 0.04 is assigned for the contact region over the punch head and over the die in the

numerical analysis. When the friction coefficient is increased to a value of 0.1 over the punch head, while the same friction coefficient of 0.04 is used for the flange, the analysis predicts less straining over the punch head than the experiment. See Figs. 30, 31, 32, and 33. The deviation of the numerical solution from the experimental data gets larger as deformation progresses, which is reflected in the punch load vs. punch depth relationship in Fig. 34.

The lubricant used in the experiment is graphite in tallow and Woo suggested the friction coefficient to be 0.04. In the analysis the practical difficulty always lies in the assignment of a reasonable value of friction coefficient because friction coefficient under a real sheet-metal forming condition is hard to measure and it may even change during deformation.

Comparison of Woo's numerical solution with the experimental data does not yield any better agreement than the present rigid-plastic FEM. In comparing his numerical solution with the experiment Woo made the correction on the circumferential strain based upon the argument that the strain value obtained from the analysis is the value at the neutral surface of the sheet, while experimental data are obtained from the outside surface and therefore a compensation for the thickness difference is necessary. There could be a question about Woo's correction because the ratio of the punch radius or die profile radius to the sheet thickness is sufficiently large in his experiment that the membrane theory is justifiable. Besides, it seems a more consistent way to consider the problem in the three-dimensional stress state instead of the plane stress condition, which is the case used in Woo's analysis, if the variation of the strain across the thickness is to be taken into account.

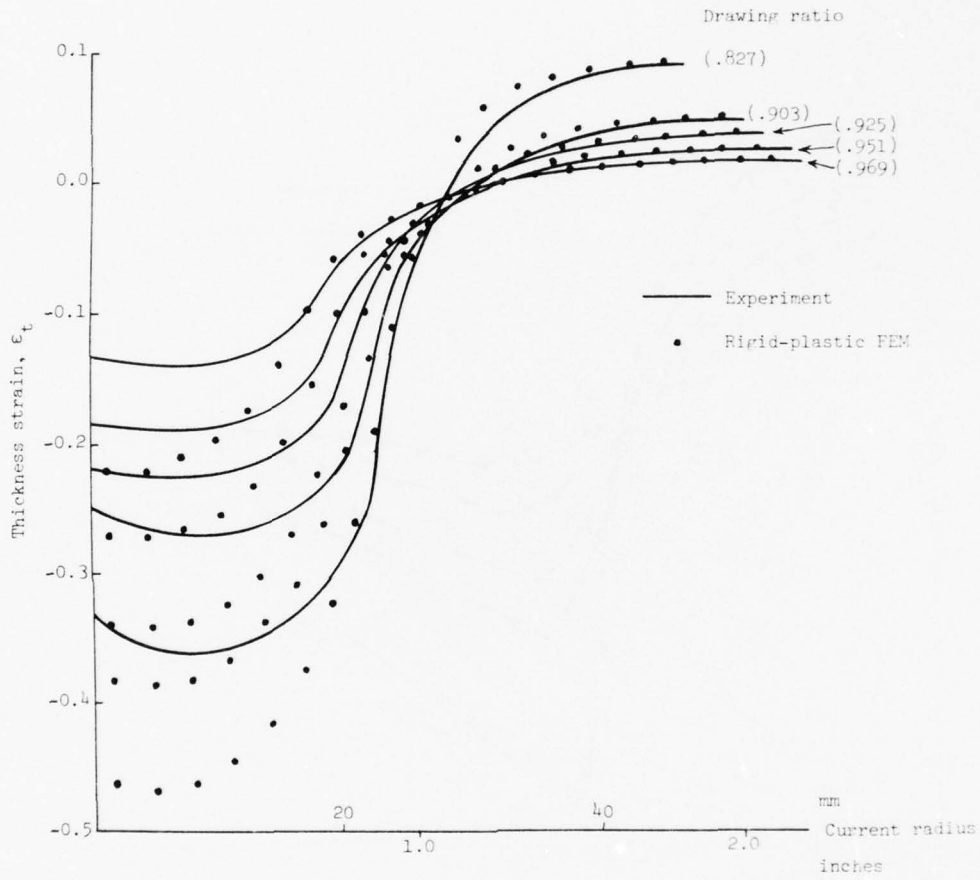


Figure 30. Distribution of Thickness Strain for $\mu_p = 0.04$,
 $\mu_d = 0.04$

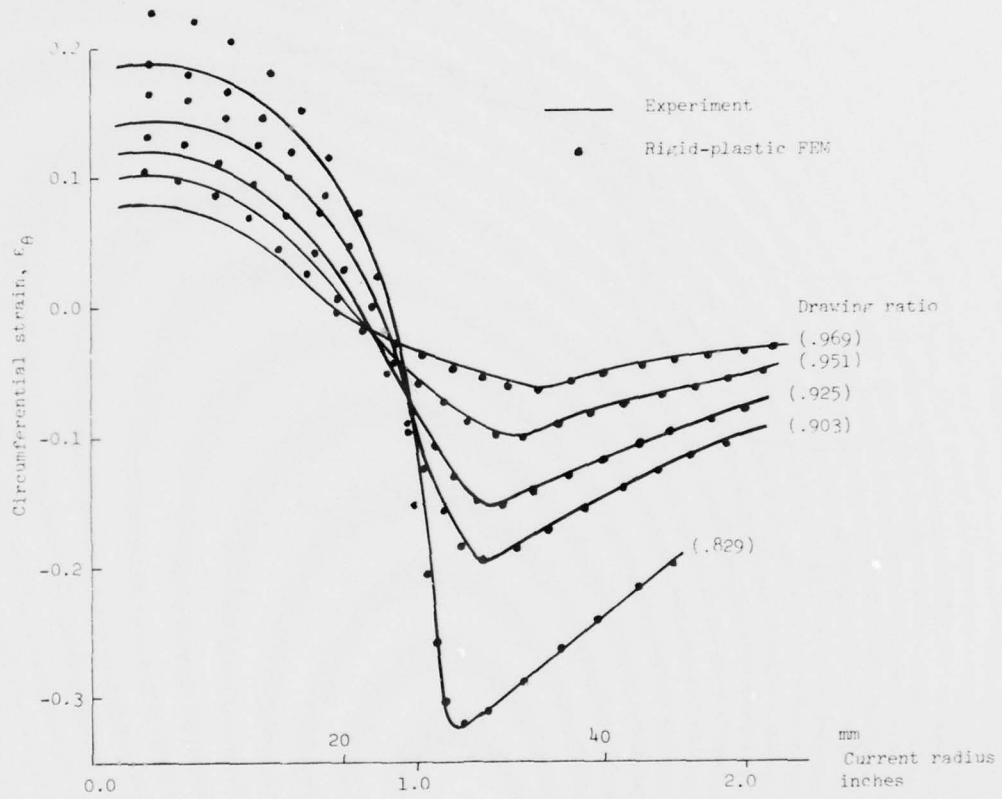


Figure 31. Distribution of Circumferential Strain for $\mu_p = 0.04$, $\mu_d = 0.04$

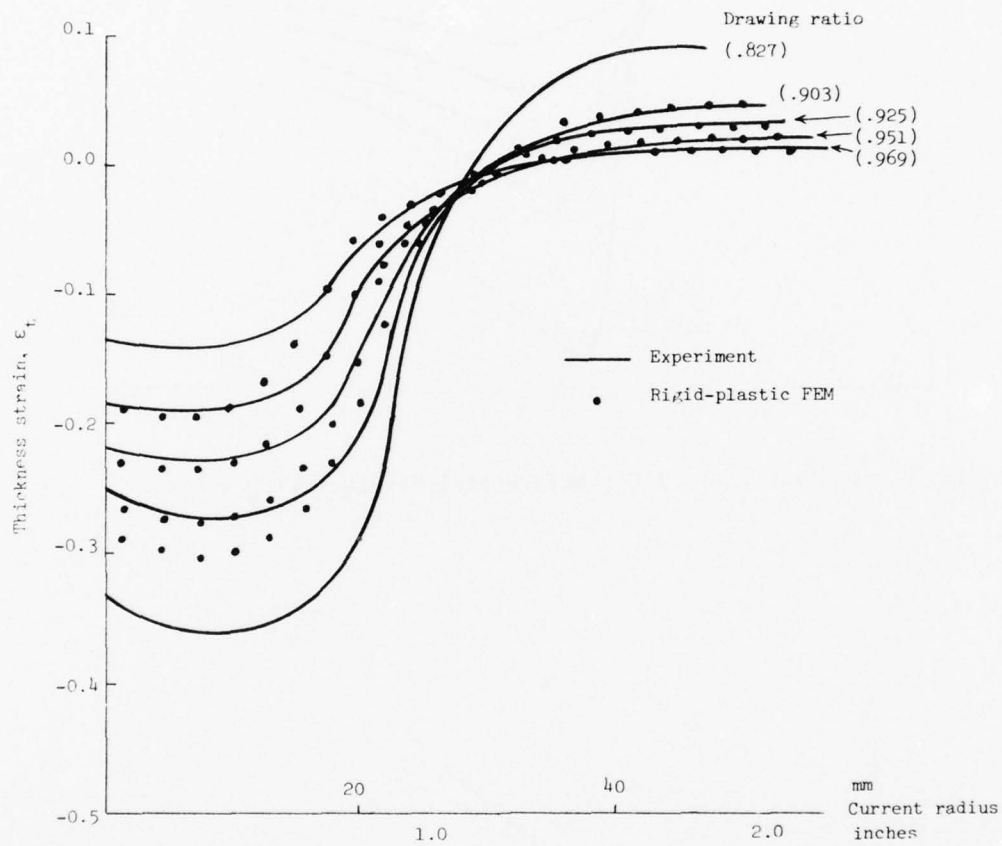


Figure 32. Distribution of Circumferential Strain for $\mu_p = 0.1$, $\mu_d = 0.04$

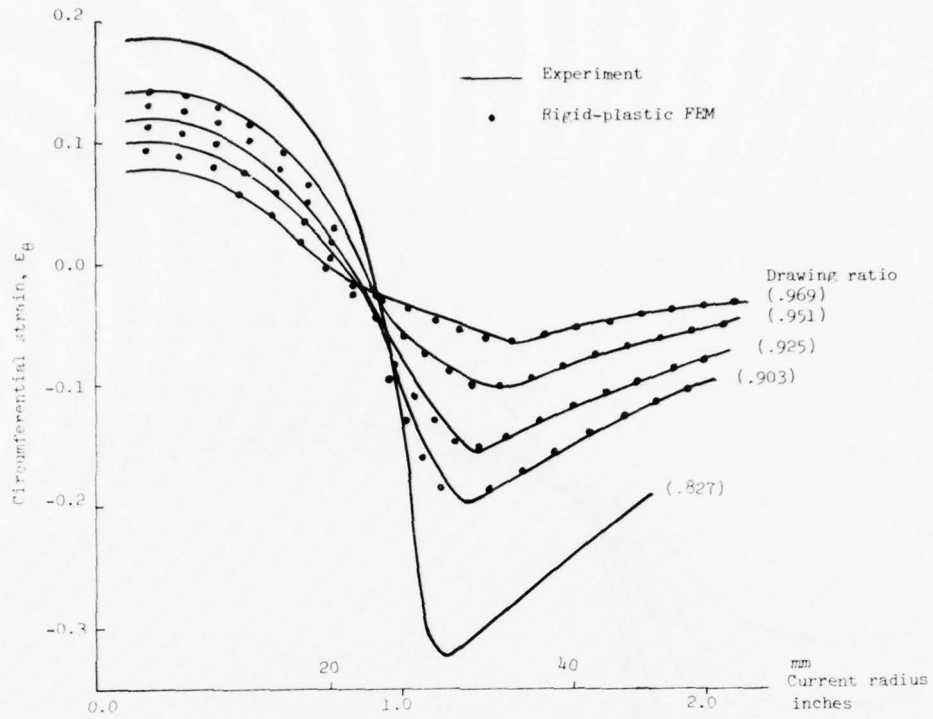


Figure 33. Distribution of Circumferential Strain for $\mu_p = 0.1$, $\mu_d = 0.04$

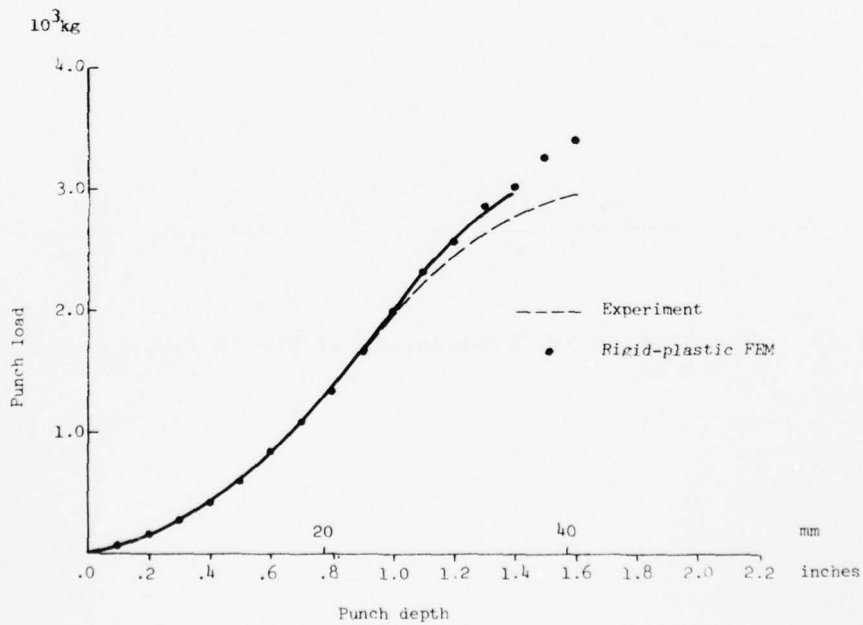


Figure 34. Punch Load vs. Punch Depth

It is necessary to have more numerical solutions and experimental data with a known friction state to assess the validity of the present rigid-plastic FEM for deep drawing problems. However, the present rigid-plastic FEM had dealt with other sheet-metal forming problems in a unified and consistent manner and therefore it seems reasonable to expect its validity for deep drawing problems when it is established for other problems.

SECTION VII
SUMMARY AND DISCUSSION

It has been made clear that classical variational formulations for the rigid-plastic solid are not appropriate for solving the sheet-metal forming problems. This is due to the nonuniqueness of the deformation mode under certain boundary conditions. This nonuniqueness, however, can be resolved by taking the workhardening rate into consideration. Such an introduction of the workhardening rate into the formulation, on the other hand, necessitates the consideration on the geometry change. The available classical formulation in which these two aspects are considered is not, however, applicable to the statically indeterminate problems, sheet-metal forming being one, because it is formulated in such a way that knowledge of stress distribution is necessary.

Within the framework of Eulerian descriptions and the hypothetical identity of the deformed configuration with the undeformed configuration, further improvement in the applicability of the variational formulations to the statically indeterminate problems is not possible. Therefore, an incremental deformation at a generic stage is considered by separating the deformed configuration from the undeformed configuration. The relevant equations are expressed with the undeformed configuration at each step as the reference frame and the variational formulation is established.

From this variational formulation a finite-element model is developed for the sheet-metal forming problems. In many sheet-metal forming processes the membrane theory is justifiable and therefore this idealization is introduced in building the model.

Three basic sheet-metal forming processes, i.e., the bulging of a sheet subject to the hydrostatic pressure, the stretching of a sheet with a hemispherical head punch, and deep drawing of a sheet with a hemispherical head punch are solved by the proposed method and its solutions are compared with the existing numerical solutions and the experimental data. The agreement is generally excellent and therefore the prime objective of the present investigation has been achieved.

In hydrostatic bulging the strain distributions and the pressure vs. polar height relationship predicted by the present rigid-plastic FEM are in excellent agreement with the available numerical solution by the elasto-plastic FEM and experimental data. The difficulty of satisfying the boundary condition along the fixed periphery experienced in the finite-difference method does not appear in the present rigid-plastic FEM.

In punch stretching, to make the problem more tractable, the presence of the die profile is neglected first so that there is only one moving boundary. This problem is successfully solved. Taking the die profile into consideration is equivalent to introducing another moving boundary, and while handling two moving boundaries simultaneously could be time consuming, the present rigid-plastic FEM again proves to be efficient and reliable. The strain distributions and the punch load vs. punch depth relationship predicted by the present rigid-plastic FEM are in excellent agreement with the numerical solutions by the finite-difference method and the experimental data.

We then investigate the influence of workhardening representation by comparing solutions, computed by both the parabolic workhardening law and the Voce equation methods. The two workhardening representations result

in the difference of peak strains and load-displacement relationships, and the difference becomes increasingly significant as punch displacement increases. It is concluded, however, that the selection of a proper work-hardening representation requires more experiments with improved accuracy and control.

The present method is further extended to the deep drawing problem. The strain distribution predicted by the present rigid-plastic FEM is in excellent agreement with the experimental data over the flange of the sheet; however, over the punch head, agreement is not as good. By assigning two different values of the friction coefficient over the punch head, two strain distributions are obtained; one predicts more straining than the experimental data, and vice versa. Therefore, an improvement in the prediction seems possible by giving the friction coefficient a proper value which is between these two bounds; however, the validity of the present rigid-plastic FEM for deep drawing analysis remains inconclusive at this stage mostly because of the lack of comparable numerical solutions and experimental data. This is apparently due to the increased sophistication and accompanying computation time when three moving boundaries are treated simultaneously and to the practical difficulty of determining proper friction coefficients.

It is concluded that the present rigid-plastic FEM can treat the sheet-metal forming problems with efficiency and reasonable accuracy.

APPENDIX A

PROGRAM FOR THE INITIAL GUESS FOR HYDROSTATIC BULGING ANALYSIS

This program is to provide the initial guess and initial geometry for Appendix B. It is based upon the analysis by Hill [23].

(I) Data preparation

1. Read NUMNP (I5)

NUMNP: Total number of nodal points to be generated

2. Read RADIUS, DIS1, DIS2 (3F 10.0)

RADIUS: Radius of the sheet to be bulged

DIS1: Polar height of the bulge in the initial geometry

DIS2: Polar height of the bulge in the new configuration

```

GRID      1          PROGRAM GRID(INPUT,OUTPUT,TAPES=INPUT,TAPES=OUTPUT,PUNCH)
GRID      2          C
GRID      3          C*****
GRID      4          C      THIS PROGRAM IS TO GENERATE THE INITIAL GEOMETRY AND VELOCITY
GRID      5          C      FIELD FOR HYDROSTATIC BULGR PROBLEM, FOLLOWING HILL
GRID      6          C*****
GRID      7          C
GRID      8          C      COMMON A(2000)
GRID      9          C
GRID     10          C
GRID     11          C*****
GRID     12          C      NUMNP=NUMBER OF NODAL POINTS TO BE GENERATED
GRID     13          C*****
GRID     14          C
GRID     15          C      READ(5,1001)NUMNP
GRID     16          C
GRID     17          C*****
GRID     18          C      N1=1
GRID     19          C      N2=N1+NUMNP
GRID     20          C      N3=N2+NUMNP
GRID     21          C      N4=N3+NUMNP
GRID     22          C      N5=N4+NUMNP
GRID     23          C      N6=N5+NUMNP
GRID     24          C      N7=N6+NUMNP
GRID     25          C      N8=N7+NUMNP
GRID     26          C      N9=N8+NUMNP
GRID     27          C      N10=N9+NUMNP
GRID     28          C      N11=N10+NUMNP
GRID     29          C
GRID     30          C
GRID     31          C      CALL GUESS(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
GRID     32          C      IA(N10),NUMNP)
GRID     33          C
GRID     34          C      1001 FORMAT(15)
GRID     35          C      STOP
GRID     36          C      END

```

```

GRID     38          SUBROUTINE GUESS(RR,ZZ,CODE,SLOF,P,Z,UR,UZ,UUR,UUZ,NUMNP)
GRID     39          DIMENSION RR(1),ZZ(1),CODE(1),SLOF(1),R(1),Z(1),UR(1),UZ(1),UUR(1)
GRID     40          ,UUZ(1)
GRID     41          C
GRID     42          C
GRID     43          C      READ(5,1001)RADIUS,DIS1,DIS2
GRID     44          C      NUMEL=NUMNP-1
GRID     45          C      K=0
GRID     46          C      DIS=DIS1
GRID     47          C      DR=RADIUS/FLGAT(NUMNP-1)
GRID     48          C
GRID     49          C
GRID     50          C      50 K=K+1
GRID     51          C      IF(K.EQ. 2)DIS=DIS2
GRID     52          C      RR=RADIUS
GRID     53          C      DO 100 I=1,NUMNP
GRID     54          C      IF(K.EQ. 1)R(I)=RR
GRID     55          C      IF(K.EQ. 1)Z(I)=0.
GRID     56          C      IF(RR.EQ. 0.1GO TO 100
GRID     57          C      R1=(RADIUS*RADIUS/RR-RR)/2.
GRID     58          C      R2=(RADIUS*RADIUS/DIS+DIS)/2.
GRID     59          C      DIS1=R2-DIS
GRID     60          C      UR(I)=(R2*R2-E1*RR-R2*DIS1)*R1/(R1*R1+R2*R2)
GRID     61          C      UZ(I)=(R2*RR+R2*R1-R1*DIS1)*R1/(R1*R1+R2*R2)
GRID     62          C      CODE(I)=0.
GRID     63          C      SLOF(I)=0.
GRID     64          C
GRID     65          C      100 RR=RR-DR
GRID     66          C      UZ(NUMNP)=DIS
GRID     67          C      UR(NUMNP)=0.
GRID     68          C      IF(K.EQ. 2)GO TO 301
GRID     69          C
GRID     70          C      DO 300 I=1,NUMNP
GRID     71          C      UUZ(I)=UZ(I)
GRID     72          C      300 UUR(I)=UR(I)
GRID     73          C

```

```

GRID 74      301 CONTINUE
GRID 75      C
GRID 76      IF(K .LT. 2)GO TO 50
GRID 77      C
GRID 78      DO 200 I=1,NUMNP
GRID 79      UR(I)=UR(I)-UUR(I)
GRID 80      UZ(I)=UZ(I)-UUZ(I)
GRID 81      200 CONTINUE
GRID 82      C
GRID 83      CODE(I)=3.0
GRID 84      CODE(NUMNP)=1.0
GRID 85      SLOP(NUMNP)=0.0
GRID 86      C
GRID 87      DO 500 I=1,NUMNP
GRID 88      RR(I)=R(I)+UUR(I)
GRID 89      ZZ(I)=Z(I)+UUZ(I)
GRID 90      WRITE(6,1000)I,RR(I),ZZ(I),UUR(I),UUZ(I),SLOP(I)
GRID 91      WRITE(6,1000)I,R(I),Z(I),UR(I),UZ(I),SLOP(I)
GRID 92      PUNCH 1011,I,CODE(I),RR(I),ZZ(I),UR(I),UZ(I),SLOP(I)
GRID 93      500 CONTINUE
GRID 94      C
GRID 95      1017 FORMAT(4F20.15)
GRID 96      1011 FORMAT(15,F5.3,EF10.7)
GRID 97      1001 FORMAT(3F10.0)
GRID 98      1000 FORMAT(15,6F10.7)
GRID 99      RETURN
GRID 100     END

```

APPENDIX B

PROGRAM FOR THE ANALYSIS OF HYDROSTATIC BULGING

This program is for the analysis of hydrostatic bulging.

(I) Data preparation

1. Read HED (A 12)
Output title
2. Read RVALUE, T, ACOEF (5F 10.0)
RVALUE: Normal anisotropy parameter
Set 1.0 for isotropic material
T: Initial thickness of blank
ACOEF: Accelerating coefficient
To start with, set 1.0
3. Read ITER, NREAD, ITCONT, NFORM, NPUNCH, NPRINT, FLIMIT (6I5, F 10.0)
The program control card
ITER: Number of iterations to be executed
NREAD: 1, if new data are to be supplied;
0, otherwise
ITCONT: 0, if computation starts at the very beginning and first/
second steps are included in the steps to be computed;
1, otherwise
NFORM: Number of steps to be computed
NPUNCH: 1, if solution is to be punched at the end of each step;
0, otherwise
FLIMIT: Value of (error norm)/(solution norm) required for
convergence. To start with, set this .000001
4. Read NUMNP (6 I 5)
NUMNP: Number of nodal points
5. Read YVALUE, PRESTN, EXPNT, PRESTS (4F 10.0)
Material characteristics are specified.
Stress = YVALUE * (Strain + PRESTN)**EXPNT + PRESTS

6. Read PRES, DPRES (4F 10.0)

PRES: Current pressure value

DPRES: Increment of pressure

7. Read N, CODE(N), R(N), Z(N), UR(N), UZ(N), SLOP(N), (15, F5.0, 5F 10.0)

Nodal information

N: Node number. Node number 1 is at the rim of the blank and the last node is at the pole

R(N): Radial position of the node

Z(N): Axial position of the node

UR(N): Increment of displacement in radial direction

UZ(N): Increment of displacement in axial direction

SLOP(N): Slope of the element
Set this 0.0

CODE(N): Type of boundary conditions:
1.0, if magnitude of UR(N) is fixed;
2.0, if magnitude of UZ(N) is fixed;
3.0, if magnitudes of UR(N) and UZ(N) are fixed;
0.0, if neither the magnitude of UR(N) nor UZ(N) are fixed

In subroutine PRELIM the interpolation of data is built in.

8. If NREAD = 1, the input data is to be placed behind nodal information cards

```

RULGE 1          PROGRAM BULGE (INPUT,OUTPUT,TAPES=INPUT,TAPE 6=OUTPUT,PUNCH)
RULGE 2          C
RULGE 3          C*****
RULGE 4          C THIS PROGRAM IS TO ANALYZE THE HYDROSTATIC BULGE)
RULGE 5          C*****
RULGE 6          C
RULGE 7          COMMON/GENCON/NUMNF,NUMEL,HEF(12),CLL,NEQ,NFORM,YIELD,TEST,ITER,
RULGE 8          INREAD,NPUNCH,NPRINT,RVALUE,T,MRAND,PNPAD,RADIUS,PRES,DPRES
RULGE 9          COMMON/MATERL/YVALUE,PRESTN,EXPNT,PRESTS
RULGE 10         COMMON/ISOTPY/RVAL1
RULGE 11         C
RULGE 12         C
RULGE 13         C*****
RULGE 14         C PROGRAM IS FOR CONTROLLING THE DIMENSION OF THE COMPLETE
RULGE 15         C PROGRAM. ITS PURPOSE IS TO PREVENT ASSIGNING A LARGER THAN
RULGE 16         C NECESSARY DIMENSION FOR ANY ARRAY THROUGH THE USE OF THE
RULGE 17         C FOLLOWING STATEMENT
RULGE 18         C*****
RULGE 19         C
RULGE 20         COMMON A(5000)
RULGE 21         C
RULGE 22         NFIELD=5000
RULGE 23         C
RULGE 24         C*****
RULGE 25         C NFIELD IS THE DIMENSION OF ARRAY A. ITS VALUE CAN BE DETERMINED
RULGE 26         C PRECISELY BY RUNNING THE PROGRAM ONCE.
RULGE 27         C*****
RULGE 28         C
RULGE 29         C
RULGE 30         TEST=1.
RULGE 31         C
RULGE 32         READ(5,1000) HEF
RULGE 33         READ(5,1004) RVALUE,T,ACOE
RULGE 34         READ(5,1003) ITER,NREAD,ITCONT,NFORM,NPUNCH,NPRINT,FLIMIT
RULGE 35         READ(5,1003) NUMNP
RULGE 36         READ(5,1004) YVALUE,PRESTN,EXPNT,PRESTS
RULGE 37         READ(5,1004) PRES,DPRES
RULGE 38         C*****
RULGE 39         C HED=OUTPUT TITLE
RULGE 40         C RVALUE=VALUE OF THE ANISOTROPY PARAMETER
RULGE 41         C ACOEF=ACCELERATING OR DECELERATING COEFFICIENT OF CONVERGENCE
RULGE 42         C NREAD=0, IF TO BYPASS THE READING STATEMENT IN SUBROUTINE PLAST
RULGE 43         C ITCONT=0, IF COMPUTATION STARTS AT THE VERY BEGINNING AND FIRST
RULGE 44         C SECOND STEPS ARE INCLUDED IN THE STEPS TO BE COMPUTED
RULGE 45         C =1, OTHERWISE
RULGE 46         C THIS INDEX IS RELATED TO THE DETERMINATION OF STEP SIZE
RULGE 47         C NFORM=NUMBER OF STEPS ASSIGNED PER RUN
RULGE 48         C NPUNCH=1, IF DATA ARE TO BE PUNCHED
RULGE 49         C =0, OTHERWISE
RULGE 50         C FLIMIT=VALUE OF (ERROR NORM)/(SOLUTION NORM) REQUIRED
RULGE 51         C FOR CONVERGENCE
RULGE 52         C NPRINT=1, IF ADDAL POINT DATA ARE TO BE PRINTED
RULGE 53         C =0, OTHERWISE
RULGE 54         C NUMNP=NUMBER OF NODAL POINTS
RULGE 55         C PRES= CURRENT PRESSURE
RULGE 56         C DPRES=INCREMENT OF THE PRESSURE
RULGE 57         C
RULGE 58         C YVALUE, PRESTN, EXPNT, PRESTS ARE TO EXPRESS THE WORKHARDENING
RULGE 59         C CHARACTERISTICS OF THE PLANK
RULGE 60         C STRESS=YVALUE*(PRESTN+STRAIN)**EXPNT+PRESTS
RULGE 61         C
RULGE 62         C NEQ=NUMBER OF EQUATIONS TO BE SOLVED
RULGE 63         C NUMEL=NUMBER OF ELEMENTS
RULGE 64         C MBAND=BAND WIDTH
RULGE 65         C
RULGE 66         C*****
RULGE 67         C
RULGE 68         C
RULGE 69         C NUMEL=NUMNP-1
RULGE 70         C RVAL1=RVALUE
RULGE 71         C MRAND=6
RULGE 72         C NEQ=NUMNF*3
RULGE 73         C NQ=NEQ
RULGE 74         C NEL=NUMEL
RULGE 75         C
RULGE 76         C N1=1
RULGE 77         C N2=N1+NUMNP

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RULGE 78      N3=N2+NUMNP
RULGE 79      N4=N3+NUMNP
RULGE 80      N5=N4+NUMNP
RULGE 81      N6=N5+NUMNP
RULGE 82      N7=N6+NUMNP
RULGE 83      N8=N7+NUMEL
RULGE 84      N9=N8+NUMEL
RULGE 85      N10=N9+NUMEL
RULGE 86      N11=N10+NUMEL
RULGE 87      N12=N11+NUMEL
RULGE 88      N13=N12+NUMEL*3
RULGE 89      N14=N13+NUMEL*4
RULGE 90      N15=N14+NUMEL*4
RULGE 91      N16=N15+NEQ
RULGE 92      N17=N16+NEQ*MPAND
RULGE 93      N18=N17+NUMEL
RULGE 94      N19=N18+NUMEL
RULGE 95      C
RULGE 96      C
RULGE 97      CALL PRELIM(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6))
RULGE 98      C
RULGE 99      C
RULGE 100     IF(N19 .LE. NFIELD) GO TO 100
RULGE 101     WRITE(6,1001) N19
RULGE 102     STOP
RULGE 103     100 CONTINUE
RULGE 104     WRITE(6,1002) N19
RULGE 105     C
RULGE 106     CALL PLAST(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
RULGE 107     1A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18),
RULGE 108     2NO,NEL,FLIMIT,ITCNT,ACCEF)
RULGE 109     C
RULGE 110     1000 FORMAT(12A6)
RULGE 111     1001 FORMAT(/** THE DIMENSION OF THE ARRAY (A) IS TOO SMALL*/
RULGE 112     ,* THE SIZE OF THE ARRAY (A) MUST BE *, I7)
RULGE 113     1002 FORMAT(/** THE NECESSARY SIZE OF THE ARRAY (A) IS*, I7)
RULGE 114     1003 FORMAT(6I5,F10.0)
RULGE 115     1004 FORMAT(4F10.0)
RULGE 116     1005 FORMAT(4I5,F10.0)
RULGE 117     C
RULGE 118     STOP
RULGE 119     END

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RULGE 121     SUBROUTINE PRELIM(P,7,UR,U7,CODE,SLOP)
RULGE 122     COMMON/CENCON/NUMNP,NUMEL,HED(12),DLL,NEQ,NFORM,YIELD,TEST,ITER,
RULGE 123     INREAD,NFUNCH,NPRINT,RVALUE,T,MPAND,PNFAC,RADIUS,PRES,PPRES
RULGE 124     C
RULGE 125     C
RULGE 126     DIMENSION R(1),Z(1),CODE(1),UR(1),U7(1),SLOP(1)
RULGE 127     C*****
RULGE 128     C READ AND PRINT OF CONTROL INFORMATION AND MATERIAL PROPERTIES      MAIN0013
RULGE 129     C*****
RULGE 130     C*****
RULGE 131     C
RULGE 132     50 CONTINUE
RULGE 133     WRITE(6,2000) HED,NUMNP,NUMEL
RULGE 134     CALL HARD(0.,YIELD)
RULGE 135     WRITE(6,2010) YIELD
RULGE 136     WRITE(6,1009) ITER
RULGE 137     C
RULGE 138     C*****
RULGE 139     C READ AND PRINT OF NODAL POINT DATA      MAIN0021
RULGE 140     C*****
RULGE 141     C*****
RULGE 142     C
RULGE 143     L=0
RULGE 144     IF(NPRINT.EQ.0) GO TO 60
RULGE 145     WRITE(6,1114)
RULGE 146     WRITE(6,2004)
RULGE 147     60 READ(5,1002) N,CODE(N),R(N),Z(N),UR(N),U7(N),SLOP(N)
RULGE 148     NL=L+1
RULGE 149     7X=N-L
RULGE 150     IF(L .EQ. 0) GO TO 70
RULGE 151     DR=(R(N)-R(L))/7X
RULGE 152     DZ=(Z(N)-Z(L))/7X
RULGE 153     DUR=(UR(N)-UR(L))/7X
RULGE 154     DUZ=(U7(N)-U7(L))/7X
RULGE 155     DS=(SLOP(N)-SLOP(L))/7X
RULGE 156     C
RULGE 157     70 L=L+1
RULGE 158     IF(N-L) 100,90,80
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RULGE 933     C
RULGE 934     C
RULGE 935     C
RULGE 936     C
RULGE 937     C
RULGE 938     C
RULGE 939     C
RULGE 940     C
RULGE 941     C
RULGE 942     C
RULGE 943     C
RULGE 944     C
RULGE 945     C
RULGE 946     C
RULGE 947     C
RULGE 948     C
RULGE 949     C
RULGE 950     C
RULGE 951     C
RULGE 952     C
RULGE 953     C
RULGE 954     C
RULGE 955     C
RULGE 956     C
RULGE 957     C
RULGE 958     C
RULGE 959     C
RULGE 960     C
RULGE 961     C
RULGE 962     C
RULGE 963     C
RULGE 964     C
RULGE 965     C
RULGE 966     C
RULGE 967     C
RULGE 968     C
RULGE 969     C
RULGE 970     C
RULGE 971     C
RULGE 972     C
RULGE 973     C
RULGE 974     C
RULGE 975     C
RULGE 976     C
RULGE 977     C
RULGE 978     C
RULGE 979     C
RULGE 980     C
RULGE 981     C
RULGE 982     C
RULGE 983     C
RULGE 984     C
RULGE 985     C
RULGE 986     C
RULGE 987     C
RULGE 988     C
RULGE 989     C
RULGE 990     C
RULGE 991     C
RULGE 992     C
RULGE 993     C
RULGE 994     C
RULGE 995     C
RULGE 996     C
RULGE 997     C
RULGE 998     C
RULGE 999     C
RULGE 1000    C

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AD-A068 250

CALIFORNIA UNIV BERKELEY DEPT OF MECHANICAL ENGINEERING F/G 13/8
ANALYSIS OF AXISYMMETRIC SHEET-METAL FORMING PROCESSES BY THE R--ETC(U)
SEP 78 F33615-77-C-5111

UNCLASSIFIED

AFML-TR-78-120

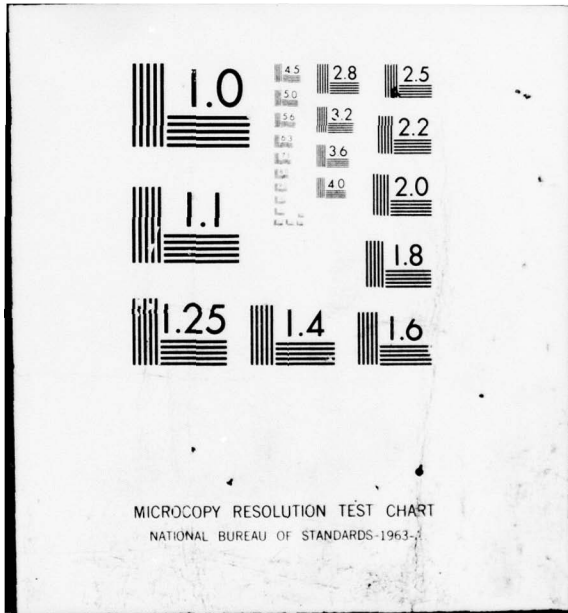
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RULGE 158      80 CODE(L)=0.0                                MAIN0043
RULGE 159      R(L)=R(L-1)+CF                               MAIN0044
RULGE 160      Z(L)=Z(L-1)+DZ
RULGE 161      UR(L)=UR(L-1)+DUR
RULGE 162      UZ(L)=UZ(L-1)+DUZ
RULGE 163      SLOP(L)=SLOP(L-1)+DS
RULGE 164      GO TO 70
RULGE 165      90 IF (NUMNP-N) 100,110,60                    MAIN0045
RULGE 166      C
RULGE 167      100 WRITE (6,2009) A
RULGE 168      CALL EXIT                                     MAIN0052
RULGE 169      C
RULGE 170      110 CONTINUE
RULGE 171      WRITE (6,2002) (K,CODE(K),R(K),Z(K),UR(K),UZ(K),SLOP(K),K=1,NUMNP)
RULGE 172      C
RULGE 173      NEQ=3*NUMNP
RULGE 174      WRITE(6,1122) NEQ,MPANC
RULGE 175      C*****MAIN0119
RULGE 176      1002 FORMAT (15,F5.0,5F10.0)                    MAIN0122
RULGE 177      1003 FORMAT(16I5)
RULGE 178      1004 FORMAT(I8,2I11,2F10.6)
RULGE 179      1005 FORMAT(2I5,4F10.0)
RULGE 180      1006 FORMAT(// * THE NODAL POINTS AT WHICH FORCE CALCULATIONS ARE DESIR
RULGE 181      1ED* // 20I5)
RULGE 182      1007 FORMAT(1H1,15X, 39H LINEARLY DISTRIBUTED BOUNDARY STRESSES/
RULGE 183      1 / 60H NODE I NODE J PRESSURE I PRESSURE J SHEAR I
RULGE 184      2 14H SHEAR J)
RULGE 185      1008 FORMAT(2I9,4E15.5)
RULGE 186      1009 FORMAT(//** MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR EACH INCREMEN
RULGE 187      1T =*, I3)
RULGE 188      1114 FORMAT(1H1, 39H NODAL POINT INFORMATION BEFORE SCALING//)
RULGE 189      1122 FORMAT(/// * NUMBER OF EQUATIONS =*, I4/
RULGE 190      1 * BANDWIDTH =*, I4/
RULGE 191      2 * DIAGONAL ELEMENTS =*, I4 )
RULGE 192      2000 FORMAT (1H 12A6/
RULGE 193      1 30H0 NUMBER OF NODAL POINTS----- I3 /
RULGE 194      2 30H0 NUMBER OF ELEMENTS----- I3 /)                    MAIN0127
RULGE 195      2002 FORMAT (I12,F12.2,2F12.3,3E24.7)
RULGE 196      2003 FORMAT (I113,4I6,1I12)
RULGE 197      2004 FORMAT (/ * NODAL POINT TYPE 9-COORDINATE Z-COORDINATE P LO
RULGE 198      1AD OR DISPLACEMENT Z LOAD OF DISPLACEMENT BETA-SLOPE #1)
RULGE 199      2005 FORMAT(//,*FORCES SPECIFIED AT NODAL POINT*,//,
RULGE 200      1 * NODAL PT. ELEMENT1 ELEMENT2 PRESSURE SHEAR #,/)
RULGE 201      2006 FORMAT (26HONODAL POINT CARD ERROR N= I5)
RULGE 202      2010 FORMAT(// * INITIAL YIELD STRESS = *, F15.7//)
RULGE 203      RETLRN
RULGE 204      END

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RULGE 205      SUBROUTINE PLAST(F,7,UF,UZ,CODE,SLOP,YY,YX,SPHI,CPHI,DL,STS,TEPS,
RULGE 206      1EPS,R,A,THICK,PFI,NO,WEL,FLIMIT,ITCON,T,ACOFF)
RULGE 207      C
RULGE 208      C*****
RULGE 209      C PLAST IS THE CONTROLLING SUBROUTINE
RULGE 210      C*****
RULGE 211      C
RULGE 212      C
RULGE 213      COMMON/GENCON/NUMNP,NUMEL,HEP(12),DLL,NEQ,NFCOM,YIELD,TEST,ITEF,
RULGE 214      1NPEAD,NPUNCH,NPRINT,PVALUE,T,MRAND,PNRAD,FRADIUS,PRES,DPRES
RULGE 215      COMMON/CONQUAD/SS(4),WT(4),D(2,2),SOFT1
RULGE 216      COMMON/FORVE/FACT,DFACT
RULGE 217      C
RULGE 218      DIMENSION R(1),Z(1),UR(1),UZ(1),CODE(1),SLOP(1),YY(1),YX(1),
RULGE 219      1TEPS(4,1),R(1),A(NO,1),THICK(1),CPHI(1),SPHI(1),DL(1),EPS(4,1),
RULGE 220      2PHI(1),STS(3,1)
RULGE 221      C
RULGE 222      C
RULGE 223      DFACT=DPRES
RULGE 224      FACT=PPRES
RULGE 225      C
RULGE 226      SS(1)=0.861136311E
RULGE 227      SS(2)=0.326981043E
RULGE 228      SS(3)=-SS(1)
RULGE 229      SS(4)=-SS(2)
RULGE 230      C
RULGE 231      DO 442 N=1, NLMEL
RULGE 232      THICK(N)=T
RULGE 233      DO 442 I=1, 4
RULGE 234      TEPS(I,N)=0.
RULGE 235      TEPS(4,N)=0.0001
RULGE 236      **2 CONTINUE
RULGE 237      C
RULGE 238      C

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BULGE 239 C*****
BULGE 240 C IF THE COMPUTATION IS INTERRUPTED AFTER A NUMBER OF STEPS
BULGE 241 C AND RESTARTED, THEN NECESSARY DATA NEED BE FEED
BULGE 242 C*****
BULGE 243 C
BULGE 244 C IF(NREAD .LE. 0) GO TO 440
BULGE 245 READ(5,1017) (UR(I), UZ(I), SLOP(I), I=1,NUMNP)
BULGE 246 IF(ITCONT .EQ. 1) GO TO 440
BULGE 247 READ(5,1017) (R(I), Z(I), I=1, NUMNP)
BULGE 248 READ(5,1017)((TEPS(I,N),I=1,4), N=1, NLMEL)
BULGE 249 READ(5,1017)(THICK(N),N=1,NUMEL)
BULGE 250 READ(5,2223)FACT
BULGE 251 440 CONTINUE
BULGE 252 C
BULGE 253 C
BULGE 254 NSTEP=0
BULGE 255 2100 NSTEP=NSTEP+1
BULGE 256 C
BULGE 257 C
BULGE 258 C*****
BULGE 259 C COMPUTE THE YIELD STRESS AND THE WORKHARDENING RATE
BULGE 260 C*****
BULGE 261 C
BULGE 262 DO 220 N=1, NLMEL
BULGE 263 CALL HARD(TEPS(4,N),YY(N))
BULGE 264 CALL HARD2(TEPS(4,N),YX(N))
BULGE 265 220 CONTINUE
BULGE 266 C
BULGE 267 C
BULGE 268 C
BULGE 269 WRITE(6,1007) NSTEP
BULGE 270 C
BULGE 271 C
BULGE 272 659 CONTINUE
BULGE 273 C
BULGE 274 C*****
BULGE 275 C DETAIL OF THE PRESENT CONFIGURATION
BULGE 276 C SPHI=SINE OF ANGLE PHI
BULGE 277 C CPHI=COSINE OF ANGLE PHI
BULGE 278 C DL=ELEMENT LENGTH
BULGE 279 C*****
BULGE 280 C
BULGE 281 WRITE(6,1011)
BULGE 282 DO 690 N=1, NLMEL
BULGE 283 NP1=NP+1
BULGE 284 DR=R(N)-R(NP1)
BULGE 285 DZ=Z(NP1)-Z(N)
BULGE 286 DL(N)=SQRT(DR*DR+CZ*CZ)
BULGE 287 SPHI(N)=DR/DL(N)
BULGE 288 CPHI(N)=DZ/DL(N)
BULGE 289 PHI(N)=ASIN(SPHI(N))*180./3.14156
BULGE 290 WRITE(6,1030)N,PHI(N),THICK(N),DL(N)
BULGE 291 690 CONTINUE
BULGE 292 C
BULGE 293 RP1=RVALUE+1.
BULGE 294 RCONST=3.*RP1/(2.*(1.+RVALUE+RVALUE))
BULGE 295 D(1,1)=RP1*RCONST
BULGE 296 D(1,2)=RVALUE*RCONST
BULGE 297 D(2,1)=D(1,2)
BULGE 298 D(2,2)=D(1,1)
BULGE 299 CLAMDA=RP1*RP1/(1.+RVALUE+RVALUE)
BULGE 300 POISON=RVALUE/RP1
BULGE 301 C
BULGE 302 DO 2000 K=1, ITER
BULGE 303 C
BULGE 304 C
BULGE 305 CALL STIFF(R,Z,UR,UZ,CODE,SLOP,YY,YX,SPHI,CPHI,DL,SPS,
BULGE 306 1THICK,A,P,NG)
BULGE 307 C
BULGE 308 C
BULGE 309 C*****
BULGE 310 C INTRODUCTION OF BOUNDARY CONDITION
BULGE 311 C*****
BULGE 312 C
BULGE 313 CALL MODIFY(CODE,A,P,NUMNP,NEC,MPANE)
BULGE 314 C
BULGE 315 C*****
BULGE 316 C BOUNDED SYMMETRIC SOLUTION
BULGE 317 C*****
BULGE 318 C
BULGE 319 CALL T3(INEO,MPANE,A)
BULGE 320 CALL BACKS(NEC,MPANE,A,P)
BULGE 321 C
BULGE 322 C
BULGE 323 C

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RULGE 324      DO 130 I=1, NUMNP
RULGE 325      IZ=3*I-1
RULGE 326      IR=I-1
RULGE 327      IS=I+1
RULGE 328      UP(I)=UR(I)+P(IR)*ACCEF
RULGE 329      UZ(I)=UZ(I)+P(IZ)*ACCEF
RULGE 330      SLOP(I)=SLOP(I)+P(IS)*ACCEF
RULGE 331      130 CONTINUE
RULGE 332      C
RULGE 333      C
RULGE 334      WRITE(6,1016) K
RULGE 335      WRITE(6,1006) K
RULGE 336      DO 776 I=1, NLMNF
RULGE 337      IZ=2*I-1
RULGE 338      IR=I-1
RULGE 339      IS=I+1
RULGE 340      WRITE(6,1002) I,B(IR),E(IZ),P(IS),UF(I),U7(I),SLOP(I),R(I),Z(I)
RULGE 341      776 CONTINUE
RULGE 342      C
RULGE 343      C
RULGE 344      C
RULGE 345      C*****
RULGE 346      C COMPUTE NORM OF ERROR AND NORM OF SOLUTION.
RULGE 347      C*****
RULGE 348      ENORM = 0.
RULGE 349      SNORM = 0.
RULGE 350      DO 134 I=1, NUMNP
RULGE 351      IZ=3*I-1
RULGE 352      IR=I-1
RULGE 353      IS=I+1
RULGE 354      ENORM = ENORM + B(IR)*B(IR) + P(IZ)*P(IZ) + P(IS)*P(IS)
RULGE 355      SNORM = SNORM + UP(I)*UR(I) + UZ(I)*U7(I) + SLOP(I)*SLOP(I)
RULGE 356      134 CONTINUE
RULGE 357      ENORM = SQRT(ENORM)
RULGE 358      SNORM = SQRT(SNORM)
RULGE 359      ESNCRM=ENORM/SNORM
RULGE 360      WRITE(6,1015) SNORM,ENORM,ESNCRM
RULGE 361      C
RULGE 362      C
RULGE 363      131 CONTINUE
RULGE 364      C
RULGE 365      C
RULGE 366      C
RULGE 367      C*****
RULGE 368      C COMPUTE STRAIN FROM THE NEW GUESS.
RULGE 369      EPS(1,N)=INCREMENT OF MERIDIAN STRAIN
RULGE 370      EPS(2,N)=INCREMENT OF TANGENTIAL STRAIN
RULGE 371      EPS(3,N)=INCREMENT OF THICKNESS STRAIN
RULGE 372      C*****
RULGE 373      C
RULGE 374      C
RULGE 375      DO 200 N=1,NUMEL
RULGE 376      NF1=N+1
RULGE 377      DLL=DL(N)
RULGE 378      SF=SPH(N)
RULGE 379      CPH=CPH(N)
RULGE 380      AU=UR(N)+UR(NF1)
RULGE 381      AR=R(N)+R(NF1)
RULGE 382      OR=O(N)+O(NF1)
RULGE 383      DZ=Z(NF1)-Z(N)
RULGE 384      DU=UR(N)-UR(NF1)
RULGE 385      DW=U7(NF1)-U7(N)
RULGE 386      EX1=1.+2.*DP*DU/DLL/DLL+2.*DZ*DW/CLL/CLL+(DU*DU+DW*DW)/DLL/DLL
RULGE 387      EPS(1,N)=SQRT(EX1)-1.
RULGE 388      EPS(1,N)=ALOG(1.+EPS(1,N))
RULGE 389      EPS(2,N)=AU/AR
RULGE 390      EPS(2,N)=ALOG(1.+AU/AR)
RULGE 391      EPS(3,N)=-EPS(1,N)-EPS(2,N)
RULGE 392      C
RULGE 393      200 CONTINUE
RULGE 394      C
RULGE 395      C
RULGE 396      TEST=0.0
RULGE 397      WRITE(6,1026) NSTEP
RULGE 398      C
RULGE 399      C
RULGE 400      C
RULGE 401      C*****
RULGE 402      C COMPUTE INCREMENT OF EFFECTIVE STRAIN
RULGE 403      C*****
RULGE 404      C
RULGE 405      DO 222 N=1, NUMEL
RULGE 406      ES=EPS(1,N)
RULGE 407      ET=EPS(2,N)
RULGE 408      PRAR=0.1*(ES*ES+ET*ET) + 2.*CVALUE*ES*ET

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RULGE 400          EPS(4,N)=SQRT(2.*ECCNST*RPAP/2.)
RULGE 410          C
RULGE 411          C
RULGE 412          C
RULGE 413          C*****
RULGE 414          C    COMPUTE STRESS DISTRIBUTION
RULGE 415          C    STS(1,N)=MERIDIAN STRESS
RULGE 416          C    STS(2,N)=CIRCUMFERENTIAL STRESS
RULGE 417          C    STS(3,N)=EFFECTIVE STRESS
RULGE 418          C*****
RULGE 419          C
RULGE 420          C    STS(1,N)=CLAMTA*(ES + POISON*ET)*YY(N)/EPS(4,N)
RULGE 421          C    STS(2,N)=CLAMTA*(ET + POISON*ES)*YY(N)/EPS(4,N)
RULGE 422          C    IF(ESNORM .LT. FLIMIT)TEST=1.0
RULGE 423          C    DO 443 I=1, 4
RULGE 424          C    447 TEPS(I,N)=TEPS(I,N)+EFS(I,N)*TEST
RULGE 425          C    ES=STS(1,N)
RULGE 426          C    ET=STS(2,N)
RULGE 427          C    EFS=ES*ES+ET*ET-2.*POISON*ES*ET
RULGE 428          C    STS(3,N)= SQRT(EFS)
RULGE 429          C
RULGE 430          C
RULGE 431          C    WRITE(6,1003)N,EPS(1,N),TEPS(1,N),EFS(2,N),TEPS(2,N),EPS(3,N),TEP
RULGE 432          C    IS(3,N),EPS(4,N),TEPS(4,N)
RULGE 433          C    IF(ESNORM .LT. FLIMIT)THICK(N)=THICK(N)*EXP(EFS(3,N))
RULGE 434          C    222 CONTINUE
RULGE 435          C    WRITE(6,1027)
RULGE 436          C
RULGE 437          C    DO 430 N=1,NUMEL
RULGE 438          C    430 WRITE(6,225)N,(STS(I,N),I=1,3)
RULGE 439          C
RULGE 440          C
RULGE 441          C    IF(ESNORM .LT. FLIMIT)FACT=FACT+DFACT
RULGE 442          C    IF(ESNORM .LT. FLIMIT)WRITE(6,1028)FACT
RULGE 443          C    IF(ESNORM .LT. FLIMIT)GC TO 428
RULGE 444          C
RULGE 445          C    2000 CONTINUE
RULGE 446          C    2200 CONTINUE
RULGE 447          C
RULGE 448          C
RULGE 449          C    438 CONTINUE
RULGE 450          C
RULGE 451          C
RULGE 452          C*****
RULGE 453          C    NEW CONFIGURATION
RULGE 454          C*****
RULGE 455          C
RULGE 456          C    DO 439 I=1, NUMNP
RULGE 457          C    IZ=7*I-1
RULGE 458          C    IP=I-1
RULGE 459          C    R(I)=R(I)+UR(I)*TEST
RULGE 460          C    Z(I)=Z(I)+UZ(I)*TEST
RULGE 461          C    439 CONTINUE
RULGE 462          C    777 CONTINUE
RULGE 463          C
RULGE 464          C
RULGE 465          C
RULGE 466          C    IF(NPUNCH .EQ. 0) GC TO 310
RULGE 467          C    PUNCH 1017, (UR(I),UZ(I), SLOF(I), I=1,NUMNP)
RULGE 468          C    PUNCH 1017, (R(I),7(I),I=1, NUMNP)
RULGE 469          C    PUNCH 1017, ( (TEPS(I,N),I=1,4), N=1, NUMEL)
RULGE 470          C    PUNCH 1017,(THICK(N),N=1,NUMEL)
RULGE 471          C    PUNCH 2223,FACT
RULGE 472          C    310 CONTINUE
RULGE 473          C
RULGE 474          C    IF(ESNORM .GT. FLIMIT)GC TO 2300
RULGE 475          C
RULGE 476          C    IF(NSTEP .LT. NFORM) GC TO 2100
RULGE 477          C
RULGE 478          C    2300 CONTINUE
RULGE 479          C    500 CONTINUE
RULGE 480          C
RULGE 481          C    1002 FORMAT(15,3F12.7,5X,3F12.7,5X,3F12.7)
RULGE 482          C    1003 FORMAT(17,11F11.6)
RULGE 483          C    1004 FORMAT(16E1)
RULGE 484          C    1005 FORMAT(1H1,* STRAIN-STRESS SOLUTION AT STEP NUMBER =*,I4//
RULGE 485          C    1 * EL. NO.,P-STRAIN..2-STRAIN..TH-STRAIN..RZ-STRAIN...EF-STRAIN
RULGE 486          C    2...R-STRES...7-STRES...TH-STRES...R7-STRES..EF-STRES..AVG-STRES...
RULGE 487          C    *)
RULGE 488          C    1006 FORMAT(/// 30X, * DISPLACEMENT SOLUTION AT ITERATION NUMBER =*,I4
RULGE 489          C    1/// 20X, * PUFTUREEC%, 26X, * TOTAL%, 20X, * DEFORMED COCRD*/
RULGE 490          C    2/ * NP DU DR DRETA U
RULGE 491          C    7 * W BETA R 7*)
RULGE 492          C    1007 FORMAT(1H1,70X,*ITERATION PROCESS FOR STEP*,I4)
RULGE 493          C    1008 FORMAT( 6CX, * TOTAL R-LOAD =*, F12.7

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BULGE 494      1      / 60X, * TOTAL Z-LOAD =*, F12.7
BULGE 495      2      / 60X, * TOTAL M-LOAD =*, F12.7)
BULGE 496      1010 FORMAT( // * NODAL POINT FORCE AT STEP =*, I4//
BULGE 497      1*.....N.F.....R-FORCE.....Z-FORCE.....Z-STRESS ON DIE SUP
BULGE 498      2FACE...*)
BULGE 499      1011 FORMAT(I5,3F10.0)
BULGE 500      1012 FORMAT(I9,6F17.5)
BULGE 501      1015 FORMAT(60X,* VELOCITY CONVERGENCE*,/
BULGE 502      1      60X, * NCRM OF SOLUTION VECTOR =*, F13.8
BULGE 503      1      / 60X, * NCRM OF ERROR VECTOR =*, F13.8
BULGE 504      2      / 60X, * FRACTIONAL NCRM
BULGE 505      1016 FORMAT( // * DISPLACEMENT SOLUTION AT ITERATION NUMBER =*, I4)
BULGE 506      1017 FORMAT(F10.7)
BULGE 507      1018 FORMAT(////* DOES NOT CONVERGE*//
BULGE 508      1* TRY AGAIN WITH DECELERATION COEFFICIENT =ACDEF= LESS THAN*,
BULGE 509      2F8.3)
BULGE 510      1020 FORMAT(20F4.1)
BULGE 511      1025 FORMAT(4X,I5,3X,F12.6,10X,I5,3X,F12.6,10X,I5,3X,F12.6)
BULGE 512      2251 FORMAT(I5,4F20.7)
BULGE 513      1026 FORMAT(///**INCREMENTAL STRAIN-TOTAL STRAIN AT STEP NUMBER=*,I4//
BULGE 514      1*EL ND....S-STRAIN.....TOTAL...THE-STRAIN.....TOTAL....THI-STF
BULGE 515      2AIN.....TOTAL....EF-STRAIN.....TOTAL...*)
BULGE 516      1027 FORMAT(///**EL. ND....S-STRESS.....THE-STRESS....EF-STRESS...*)
BULGE 517      1030 FORMAT(/I7,3F10.5)
BULGE 518      1031 FORMAT(* GEOMETRY OF PROFILE*//
BULGE 519      1 *EL NND....ANGLE.....THICKNESS...*)
BULGE 520      1028 FORMAT(* FORCE AT THIS STEP IS *,F20.7)
BULGE 521      2223 FORMAT(F20.7)
BULGE 522      C
BULGE 523      RETURN
BULGE 524      END

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BULGE 526      SUBROUTINE STIFF(R,Z,UF,UZ,CODE,SLOP,VY,YX,SPHI,CPHI,DL,EPS,
BULGE 527      1THICK,A,R,NG)
BULGE 528      C
BULGE 529      COMMON/GENCON/NUMNF,NUMEL,HED(12),DLL,NEQ,NFORM,YIELD,TEST,ITEP,
BULGE 530      INREAD,NFUNCH,NPRINT,RVALUE,T,MBAND,PNRAC,RADIUS,PPES,DPRES
BULGE 531      COMMON/STFMAT/H(6),P(6,6),TEX,TEY,TE7,THKL
BULGE 532      COMMON/CCNGUAD/SS(4),WT(4),D(2,2),SOFT1
BULGE 533      C
BULGE 534      DIMENSION R(1),Z(1),CODE(1),UP(1),UZ(1),SLOP(1),R(1),ANG(1),
BULGE 535      1 EPS(4,1),ZZ(2),UU(6),VY(1),VX(1),THICK(1),DL(1),SPH(1),CPH(1)
BULGE 536      2,RR(2)
BULGE 537      C
BULGE 538      C
BULGE 539      DO 50 N=1, NEQ
BULGE 540      R(N)=0.
BULGE 541      DO 50 M=1,MBAND
BULGE 542      50 A(N,M)=0.
BULGE 543      C
BULGE 544      WT(1)=0.3478548451
BULGE 545      WT(2)=0.6521451549
BULGE 546      WT(3)=WT(1)
BULGE 547      WT(4)=WT(2)
BULGE 548      C
BULGE 549      C
BULGE 550      DO 1000 N=1, NUMEL
BULGE 551      NP1=N+1
BULGE 552      DLL=DL(N)
BULGE 553      SPH=SPH(N)
BULGE 554      CPH=CPH(N)
BULGE 555      RR(1)=R(N)
BULGE 556      ZZ(1)=Z(N)
BULGE 557      RR(2)=R(NP1)
BULGE 558      UU(1)=UP(N)
BULGE 559      UU(2)=UP(NP1)
BULGE 560      UU(3)=SLOP(N)
BULGE 561      UU(4)=UP(NP1)
BULGE 562      UU(5)=UP(NP1)
BULGE 563      UU(6)=SLOP(NP1)
BULGE 564      THKL=THICK(N)*DLL
BULGE 565      ZZ(2)=Z(NP1)
BULGE 566      YG=YX(N)
BULGE 567      VH=VY(N)
BULGE 568      C
BULGE 569      CALL QUAD(RR,ZZ,UU,DLL,SPH,CPH,YG,VH)
BULGE 570      C
BULGE 571      C
BULGE 572      C
BULGE 573      C*****
BULGE 574      C PERFORM THE ASSEMBLY OPERATION, BECAUSE MATRIX A IS SYMMETRIC

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BULGE 575 C ONLY UPPER HALF OF THE MATRIX IS CREATED. AND THE STORAGE FOR
BULGE 576 C MATRIX A IS A SQUARE ARRAY BECAUSE OF HANDED SYMMETRIC PROPERTY
BULGE 577 C*****
BULGE 578 C
BULGE 579 C DO 200 I=1, 6
BULGE 580 C II=N*3 - 3 + I
BULGE 581 C R(II)=R(II)+M(II)
BULGE 582 C DO 200 J=1, 6
BULGE 583 C
BULGE 584 C
BULGE 585 C JJ= N*3 - 3 + J - II + 1
BULGE 586 C IF(JJ .LT. 1) GO TO 200
BULGE 587 C A(II,JJ)=A(II,JJ)+P(I,J)
BULGE 588 C 200 CONTINUE
BULGE 589 C
BULGE 590 C 1000 CONTINUE
BULGE 591 C
BULGE 592 C
BULGE 593 C 1001 FORMAT(///,* THE DIAGONAL VECTOR OF MATRIX OF STIFFNESS*/)
BULGE 594 C 1002 FOPMAT(12E11.3)
BULGE 595 C 1005 FORMAT(// 29# ELEMENT WITH NEGATIVE AREA =. I5)
BULGE 596 C
BULGE 597 C RETURN
BULGE 598 C END

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BULGE 600 SUPROUTINE GU#D(RR,ZZ,UU,DLL,SPH,CPH,S2,S1)
BULGE 601 C
BULGE 602 C COMMON/ISOTPV/RVAL1
BULGE 603 C COMMON/STEMAT/H(6),P(6,6),TEX,TEY,TEZ,THKL
BULGE 604 C COMMON/CONOUAC/SS(4),WT(4),D(2,2),SOPT1
BULGE 605 C COMMON/FORVE/FACT,DFACT
BULGE 606 C
BULGE 607 C DIMENSION RR(2),ZZ(2),UU(6),P(2,6),XX(6,6),PZERC(6),CP(2,6)
BULGE 608 C DIMENSION RA(6,6),RP(6)
BULGE 609 C
BULGE 610 C
BULGE 611 C RC=(RR(1)+RR(2))/2.
BULGE 612 C
BULGE 613 C DO 2 I=1,6
BULGE 614 C RR(I)=0.
BULGE 615 C M(I)=0.
BULGE 616 C DO 2 J=1,6
BULGE 617 C RA(I,J)=0.
BULGE 618 C 2 F(I,J)=0.
BULGE 619 C
BULGE 620 C
BULGE 621 C
BULGE 622 C RVALLE=RVAL1
BULGE 623 C DZ=ZZ(2)-Z7(1)
BULGE 624 C DR=RR(1)-RR(2)
BULGE 625 C DU=UU(1)-UU(4)
BULGE 626 C DW=UU(5)-UU(2)
BULGE 627 C AU=LU(1)+UU(4)
BULGE 628 C AR=RR(1)+RR(2)
BULGE 629 C
BULGE 630 C
BULGE 631 C C1=2.*DR/DLL/CLL
BULGE 632 C C2=2.*DU/DLL/DLL
BULGE 633 C C3=2.*DW/DLL/CLL
BULGE 634 C C4=2.*DW/DLL/CLL
BULGE 635 C C5=AU/AR/2.
BULGE 636 C C6=1.+DR*C2+DZ*C4+(RU*DU+DW*DW)/DLL/DLL
BULGE 637 C C7=2./DLL/DLL
BULGE 638 C C8=2./AR/AR
BULGE 639 C C9=1./SOPT(C6)/2.
BULGE 640 C C10=C9/C6
BULGE 641 C C11=C1+C2
BULGE 642 C C12=C3+C4
BULGE 643 C
BULGE 644 C
BULGE 645 C DES1=SOPT(C6)
BULGE 646 C DET1=2.*C5+1.
BULGE 647 C
BULGE 648 C E1=C9*C11/DES1
BULGE 649 C E2=-C9*C12/DES1
BULGE 650 C E3=-E1
BULGE 651 C E4=-E2
BULGE 652 C E5=1./AR/DET1
BULGE 653 C E6=(-C10*C11+C11/2.+C9*C7)/DES1-E1*E1
BULGE 654 C E7=-E6

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RULGE 655 FR=C10*C11*C12/2./DES1-E1*E2
RULGE 656 E9=-E4
RULGE 657 E10=E4
RULGE 658 E11=(-C10*C12*C12/2.+C5*C7)/DES1-E2*E2
RULGE 659 E12=-E5*E5
RULGE 660 C
RULGE 661 C
RULGE 662 DES=ALOG(DES1)
RULGE 663 DET=ALOG(DET1)
RULGE 664 C*****
RULGE 665 C DES=MERIDIAN STRAIN INCREMENT
RULGE 666 C DET=CIRCUMFERENTIAL STRAIN INCREMENT
RULGE 667 C COMPUTATION OF EFFECTIVE STRAIN INCREMENT
RULGE 668 C E1=DERIVATIVE OF MERIDIAN STRAIN INCREMENT WITH RESPECT TO UU(1)
RULGE 669 C E2=D(DES)/D(UU(1))
RULGE 670 C E3=D(DES)/D(UU(2))
RULGE 671 C E4=D(DES)/D(UU(4))
RULGE 672 C E5=D(DES)/D(UU(5))
RULGE 673 C E6=D(DES)/D(UU(1))
RULGE 674 C E7=D(E1)/D(UU(1))
RULGE 675 C E8=D(E1)/D(UU(2))
RULGE 676 C E9=D(E3)/D(UU(2))
RULGE 677 C E10=D(E4)/D(UU(5))
RULGE 678 C E11=D(E5)/D(UU(2))
RULGE 679 C *****
RULGE 680 C
RULGE 681 C
RULGE 682 C
RULGE 683 RVP1=RVALUE+1.
RULGE 684 RVP2=SQRT(2.*RVALUE+1.)
RULGE 685 RVP3=RVP1/RVP2
RULGE 686 RVP4=2.*RVALUE/RVP1
RULGE 687 C
RULGE 688 C *****
RULGE 689 C EFFECTIVE STRAIN
RULGE 690 C *****
RULGE 691 C
RULGE 692 EFS=DES*DES+DET*DET+RVP4*DES*DET
RULGE 693 EFS1=RVP3*SQRT(EFS)
RULGE 694 EFS2=RVP3/SQRT(EFS)/2.
RULGE 695 EFS3=RVP3/EFS/SQRT(EFS)/4.
RULGE 696 D1=(2.*DES+RVP4*DET)*E1+(2.*DET+RVP4*DES)*E2
RULGE 697 D2=(2.*DES+RVP4*DET)*E2
RULGE 698 D3=(2.*DES+RVP4*DET)*E3+(2.*DET+RVP4*DES)*E5
RULGE 699 D4=(2.*DES+RVP4*DET)*E4
RULGE 700 C
RULGE 701 F1=EFS2*D1
RULGE 702 F2=EFS2*D2
RULGE 703 F3=EFS2*D3
RULGE 704 F4=EFS2*D4
RULGE 705 F11=EFS3*D1*D1+EFS2*((2.*DES+RVP4*DET)*E1+(2.*DET+RVP4*DES)*E2
RULGE 706 1+(2.*E1+RVP4*E5)*E1+(2.*E2+RVP4*E5)*E5)
RULGE 707 F12=EFS3*D1*D2+EFS2*((2.*DES+RVP4*DET)*E1+(2.*DET+RVP4*DES)*E2
RULGE 708 F13=EFS3*D1*D3+EFS2*((2.*DES+RVP4*DET)*E1+(2.*E3+RVP4*E5)*E1
RULGE 709 1+(2.*E5+RVP4*E5)*E1+(2.*DET+RVP4*DES)*E2)
RULGE 710 F14=EFS3*D1*D4+EFS2*((2.*DES+RVP4*DET)*E1+(2.*E1+RVP4*E5)*E4
RULGE 711 F22=EFS3*D2*D2+EFS2*((2.*DES+RVP4*DET)*E1+(2.*E2*E2)
RULGE 712 F23=EFS3*D2*D3+EFS2*((2.*DES+RVP4*DET)*E1+(2.*E3+RVP4*E5)*E2)
RULGE 713 F24=EFS3*D2*D4+EFS2*((2.*DES+RVP4*DET)*E1+(2.*E4*E2)
RULGE 714 F33=EFS3*D3*D3+EFS2*((2.*DES+RVP4*DET)*E1+(2.*E1+RVP4*E5)*E3+(
RULGE 715 12.*E5+RVP4*E5)*E5+(2.*DET+RVP4*DES)*E1)
RULGE 716 F34=EFS3*D3*D4+EFS2*((2.*DES+RVP4*DET)*E1+(2.*E3+RVP4*E5)*E4
RULGE 717 F44=EFS3*D4*D4+EFS2*((2.*DES+RVP4*DET)*E1+(2.*E4*E4)
RULGE 718 C
RULGE 719 C
RULGE 720 C
RULGE 721 C *****
RULGE 722 C F1=DERIVATIVE OF EFFECTIVE STRAIN INCREMENT WITH RESPECT TO UU(1)
RULGE 723 C F2= WITH RESPECT TO UU(2)
RULGE 724 C F3= WITH RESPECT TO UU(4)
RULGE 725 C F4= WITH RESPECT TO UU(5)
RULGE 726 C F11=D(F1)/D(UU(1))
RULGE 727 C F12=D(F1)/D(UU(2))
RULGE 728 C F13=D(F1)/D(UU(4))
RULGE 729 C F14=D(F1)/D(UU(5))
RULGE 730 C F22=D(F2)/D(UU(2))
RULGE 731 C F23=D(F2)/D(UU(4))
RULGE 732 C F24=D(F2)/D(UU(5))
RULGE 733 C F33=D(F3)/D(UU(4))
RULGE 734 C F34=D(F3)/D(UU(5))
RULGE 735 C F44=D(F4)/D(UU(5))
RULGE 736 C *****
RULGE 737 C
RULGE 738 P(1,1)=(F1+S2*EFS1)*F11+S2*F1*F1)*RC*THKL
RULGE 739 P(1,2)=(F1+S2*EFS1)*F12+S2*F1*F2)*RC*THKL

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PULGE 740      P(1,4) = ((S1+S2*EFS1)*F13+S2*F1*F7)*RC*THKL
PULGE 741      P(1,5) = ((S1+S2*EFS1)*F14+S2*F1*F4)*RC*THKL
PULGE 742      P(2,2) = ((S1+S2*EFS1)*F22+S2*F2*F2)*RC*THKL
PULGE 743      P(2,4) = ((S1+S2*EFS1)*F23+S2*F2*F3)*RC*THKL
PULGE 744      P(2,5) = ((S1+S2*EFS1)*F24+S2*F2*F4)*RC*THKL
PULGE 745      P(4,4) = ((S1+S2*EFS1)*F33+S2*F3*F3)*RC*THKL
PULGE 746      P(4,5) = ((S1+S2*EFS1)*F34+S2*F3*F4)*RC*THKL
PULGE 747      P(5,5) = ((S1+S2*EFS1)*F44+S2*F4*F4)*RC*THKL
PULGE 749      P(2,1) = P(1,2)
PULGE 749      P(4,1) = P(1,4)
PULGE 750      P(4,2) = P(2,4)
PULGE 751      P(5,1) = P(1,5)
PULGE 752      P(5,2) = P(2,5)
PULGE 753      P(5,4) = P(4,5)
PULGE 754      C
PULGE 755      H(1) = -(S1+S2*EFS1)*F1*RC*THKL
PULGE 756      H(2) = -(S1+S2*EFS1)*F2*RC*THKL
PULGE 757      H(4) = -(S1+S2*EFS1)*F3*RC*THKL
PULGE 758      H(5) = -(S1+S2*EFS1)*F4*RC*THKL
PULGE 759      C
PULGE 760      C
PULGE 761      C*****
PULGE 762      C ENCLOSED VOLUME CHANGE
PULGE 763      C*****
PULGE 764      C
PULGE 765      RK1 = DLL*CPH+DN
PULGE 766      RUU1 = PR(1)+UU(1)
PULGE 767      RUU2 = PR(2)+UU(4)
PULGE 768      RK2 = 2.*RUU1+RUU2
PULGE 769      RK3 = 2.*RUU2+RUU1
PULGE 770      RA(1,1) = RK1/3.
PULGE 771      RA(1,2) = -RK2/6.
PULGE 772      RA(1,4) = RK1/6.
PULGE 773      RA(1,5) = RK2/6.
PULGE 774      RA(2,2) = 0.
PULGE 775      RA(2,4) = -RK3/6.
PULGE 776      RA(2,5) = 0.
PULGE 777      RA(4,4) = RK1/3.
PULGE 778      RA(4,5) = RK3/6.
PULGE 779      RA(5,5) = 0.
PULGE 780      RB(1) = RK2*RK1/6.
PULGE 781      RB(4) = RK3*RK1/6.
PULGE 782      RR(2) = -(DUU1*RUU1+DUU2*RUU2+PUL1*FUU2)/6.
PULGE 783      CR(5) = -RB(2)
PULGE 784      RA(2,1) = RA(1,2)
PULGE 785      RA(4,1) = RA(1,4)
PULGE 786      RA(4,2) = RA(2,4)
PULGE 787      RA(5,1) = RA(1,5)
PULGE 788      RA(5,2) = RA(2,5)
PULGE 789      RA(5,4) = RA(4,5)
PULGE 790      C
PULGE 791      C
PULGE 792      DO 108 I=1,6
PULGE 793      H(I) = P(I)+((FACT+DFACT)*RB(I))
PULGE 794      DO 108 J=1,6
PULGE 795      P(I,J) = P(I,J) - (FACT+DFACT)*RA(I,J)
PULGE 796      108 CONTINUE
PULGE 797      C
PULGE 798      C
PULGE 799      71 CONTINUE
PULGE 800      RETURN
PULGE 801      END

PULGE 803      SUBROUTINE CONDEN(A,B,NEQ,MBAND,N,U)
PULGE 804      C
PULGE 805      C*****
PULGE 806      C PERFORM THE MATRIX CONDENSATION WHEN THE VALUE OF A COMPONENT
PULGE 807      C X IN AX=B IS SPECIFIED EQUAL TO ZERO
PULGE 808      C*****
PULGE 809      C
PULGE 910      DIMENSION B(NEQ),A(NEQ,1)
PULGE 911      C
PULGE 912      DO 250 M=2,MBAND
PULGE 913      K=N-M+1
PULGE 914      IF(K) 235,235,230
PULGE 915      230 B(K)=B(K)-A(K,M)*U
PULGE 916      A(K,M)=0.0
PULGE 917      235 K=N+M-1
PULGE 918      IF(NEQ-K) 250,240,240
PULGE 919      240 B(K)=B(K)-A(K,M)*U
PULGE 920      A(N,M)=0.0
MCD 3
MCD 4
MCD 5
MCD 6
MCD 7
MCD 8
MCD 9
MCD 10
MCD 11

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RULGE 921      250 CONTINUE
RULGE 922      A(N,1)=1.0
RULGE 923      R(N)=0
RULGE 924      C
RULGE 925      RETURN
RULGE 926      END
MOD 12
MOD 13
MOD 14
MOD 16
MOD 15
MOD 17

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RULGE 829      SUBROUTINE MODIFY(CODE,A,B,NUMNP,NEQ,MPAND)
RULGE 829      C
RULGE 930      DIMENSION CODE(1),A(NEG,1),S(1)
RULGE 931      C
RULGE 932      C
RULGE 933      DO 121 I=1, NUMNP
RULGE 934      IL=3*I
RULGE 935      IZ=IL-1
RULGE 936      IR=IZ-1
RULGE 937      C=CODE(I)
RULGE 938      IF (C.EQ. 1.) GO TO 101
RULGE 939      IF (C.EQ. 2.) GO TO 102
RULGE 940      IF (C.EQ. 3.) GO TO 103
RULGE 941      CALL CONDEN(A,B,NEG,MPAND,IL,0.)
RULGE 942      GO TO 121
RULGE 943      C
RULGE 944      101 CONTINUE
RULGE 945      CALL CONDEN(A,B,NEQ,MPAND,IR,C.)
RULGE 946      CALL CONDEN(A,B,NEG,MPAND,IL,0.)
RULGE 947      GO TO 121
RULGE 948      C
RULGE 949      102 CONTINUE
RULGE 950      CALL CONDEN(A,B,NEQ,MPAND,IZ,C.)
RULGE 951      CALL CONDEN(A,B,NEG,MPAND,IL,C.)
RULGE 952      GO TO 121
RULGE 953      C
RULGE 954      103 CONTINUE
RULGE 955      CALL CONDEN(A,B,NEG,MPAND,IR,C.)
RULGE 956      CALL CONDEN(A,B,NEG,MPAND,IZ,0.)
RULGE 957      CALL CONDEN(A,B,NEG,MPAND,IL,0.)
RULGE 958      121 CONTINUE
RULGE 959      C
RULGE 960      RETURN
RULGE 961      END

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RULGE 963      SUBROUTINE TRIA(NN,MM,A)
RULGE 964      C
RULGE 965      C*****
RULGE 966      C TRIANGULIZATION OF GAUSSIAN ELIMINATION FOR THE SOLUTION
RULGE 967      C OF BANDED SYMMETRIC MATRIX
RULGE 968      C*****
RULGE 969      C
RULGE 970      DIMENSION A(NN,1)
RULGE 971      C
RULGE 972      N=0
RULGE 973      100 N=N+1
RULGE 974      IF(N.EQ.NN)RETURN
RULGE 975      IF(A(N,1).NE.C.) GO TO 150
RULGE 976      GO TO 100
RULGE 977      C
RULGE 978      150 I=N
RULGE 979      MB=MIN0(MM,NN-N+1)
RULGE 980      C
RULGE 981      DO 260 L=2,MB
RULGE 982      I=I+1
RULGE 983      C=A(N,L)/A(N,1)
RULGE 984      IF(C.EQ.0.0)GO TO 260
RULGE 985      J=0
RULGE 986      DO 250 K=L,MB
RULGE 987      J=J+1
RULGE 988      250 A(I,J)=A(I,J)-C*A(N,K)
RULGE 989      A(N,L)=C
RULGE 990      260 CONTINUE
RULGE 991      C
RULGE 992      GO TO 100
RULGE 993      C
RULGE 994      END

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RULGE 896      SUBROUTINE BACKS(NN,MM,A,B)
RULGE 897      C
RULGE 898      C
RULGE 899      C*****
RULGE 900      C  BACK SUBSTITUTION FOR SOLUTION OF BANDED SYMMETRIC MATRIX
RULGE 901      C*****
RULGE 902      C
RULGE 903      C  DIMENSION A(1),F(1)
RULGE 904      C
RULGE 905      C  MMM=MM-1
RULGE 906      C  N=0
RULGE 907      270  N=N+1
RULGE 908      C=R(N)
RULGE 909      IF(A(N).NE.0.C)B(N)=R(N)/A(N)
RULGE 910      IF(N.EC.NN)GO TO 300
RULGE 911      IL=N+1
RULGE 912      IH=MIN0(NN,N+MMM)
RULGE 913      M=N
RULGE 914      DO 285 I=IL,IH
RULGE 915      M=M+NN
RULGE 916      285  R(I)=R(I)-A(M)*C
RULGE 917      GO TO 270
RULGE 918      C
RULGE 919      300  IL=N
RULGE 920      N=N-1
RULGE 921      IF(N.EQ.0) RETURN
RULGE 922      IH=MIN0(NN,N+MMM)
RULGE 923      M=N
RULGE 924      DO 400 I=IL,IH
RULGE 925      M=M+NN
RULGE 926      400  B(N)=R(N)-A(M)*R(I)
RULGE 927      GO TO 300
RULGE 928      C
RULGE 929      END

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RULGE 931      SUBROUTINE HARD(EP, Y)
RULGE 932      C
RULGE 933      C*****
RULGE 934      C  WORKHARDENING CHARACTERISTIC CURVE
RULGE 935      C*****
RULGE 936      C
RULGE 937      C  COMMON/MATERL/YVALUE,PRESTN,EXPNT,PRESTS
RULGE 938      C
RULGE 939      C  Y=YVALUE*(PRESTN+EPS)**EXPNT+PRESTS
RULGE 940      C
RULGE 941      C  RETURN
RULGE 942      C  END

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RULGE 944      SUBROUTINE HARD2(EP, Y)
RULGE 945      C
RULGE 946      C*****
RULGE 947      C  COMPUTE WORK HARDENING RATE
RULGE 948      C*****
RULGE 949      C
RULGE 950      C  COMMON/MATERL/YVALUE,PRESTN,EXPNT,PRESTS
RULGE 951      C
RULGE 952      C  Y=EXPNT*YVALUE*(PRESTN+EPS)**(EXPNT-1.)
RULGE 953      C
RULGE 954      C  RETURN
RULGE 955      C  END

```

APPENDIX C

PROGRAM FOR THE ANALYSIS OF PUNCH STRETCHING

This program is for the analysis of the stretching of a sheet with hemispherical punch, where the die profile is neglected.

(I) Data card preparation

1. Read HED (A 12)
2. Read RVALUE, T, ACOEF (5F 10.0)
3. Read ITER, NREAD, ITCONT, NFORM, NPUNCH, NPRINT, FLIMIT (6I5, F10.0)
4. Read NUMNP (6I5)
5. Read PNRAD, RADIUS, FRITN (4F 10.0)
PNRAD: Radius of the hemispherical punch
RADIUS: Radius of the blank
FRITN: Friction coefficient between the punch head and the blank
6. Read YVALUE, PRESTN, EXPNT, PRESTS (4F 10.0)
7. Read ECONST, TDIST (4F 10.0)
ECONST: Step size in terms of the maximum magnitude of the effective strain increment. To start with, set this 0.04
TDIST: Criterion distance of the contact of the sheet with the punch head. To start with, set this 0.008
8. Read N, CODE(N), R(N), Z(N), UR(N), UZ(N), SLOP(N), (I5, F5.0, 5F 10.0)
Code (N) = 4.0 for the contact zone of the sheet with the punch head
9. If NREAD = 1, the new input data is to be placed behind the nodal information card

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STRCH 1          PROGRAM STRCH(INPUT,OUTPUT,TAPE F=INPUT,TAPE F=OUTPUT,PUNCH)
STRCH 2          C
STRCH 3          C*****
STRCH 4          C    THIS PROGRAM IS FOR THE ANALYSIS OF THE PUNCH STRETCHING. BY J.KIM
STRCH 5          C    HERE, THE RADIUS OF THE DIS PROFILE IS NEGLECTED.
STRCH 6          C*****
STRCH 7          C
STRCH 8          C    COMMON/GENCON/NUMNP,NUMEL,HED(12),DLL,NEG,NFORM,YIELD,TEST,ITER,
STRCH 9          C    INREAD,NPUNCH,NPRINT,FVALUE,T,MRAND,PNRAD,RADIUS,FRITN,
STRCH 10         C    2ECONST,PNHED,TCIST
STRCH 11         C    COMMON/MATERL/YVALUF,PRESTN,EXFNT,PRESTS
STRCH 12         C    COMMON/ISO/RVAL
STRCH 13         C
STRCH 14         C*****
STRCH 15         C    PROGRAM    IS FOR CONTROLLING THE DIMENSION OF THE COMPLETE
STRCH 16         C    PROGRAM.    ITS PURPOSE IS TO PREVENT ASSIGNING A LARGER THAN
STRCH 17         C    NECESSARY DIMENSION FOR ANY ARRAY THROUGH THE USE OF THE
STRCH 18         C    FOLLOWING STATEMENT
STRCH 19         C*****
STRCH 20         C
STRCH 21         C    COMMON A(5000)
STRCH 22         C
STRCH 23         C*****
STRCH 24         C    NFIELD IS THE DIMENSION OF ARRAY A. ITS VALUE CAN BE DETERMINED
STRCH 25         C    PRECISELY BY RUNNING THE PROGRAM ONCE.
STRCH 26         C*****
STRCH 27         C
STRCH 28         C    NFIELD=5000
STRCH 29         C
STRCH 30         C
STRCH 31         C
STRCH 32         C*****
STRCH 33         C    READ THE INPUT DATA CONTROL CARDS
STRCH 34         C*****
STRCH 35         C
STRCH 36         C    TEST=1.
STRCH 37         C    READ(5,1000) HED
STRCH 38         C    READ(5,1004) FVALUE,T,ACOFF
STRCH 39         C    READ(5,1003) ITER,NREAD,ITCONT,NFORM,NPUNCH,NPRINT,FLIMIT
STRCH 40         C    READ(5,1003) NUMNP
STRCH 41         C    READ(5,1004) PNRAD,RADIUS,FRITN
STRCH 42         C    READ(5,1004) YVALUE,PRESTN,EXFNT,PRESTS
STRCH 43         C    READ(5,1004) ECONST,TCIST
STRCH 44         C
STRCH 45         C
STRCH 46         C*****
STRCH 47         C    HED=OUTPUT TITLE
STRCH 48         C    FVALUE=VALUE OF THE ANISOTROPY PARAMETER
STRCH 49         C    ACOFF=ACCELERATING OR DECELERATING COEFFICIENT OF CONVERGENCE
STRCH 50         C    NREAD=0, IF TO BYPASS THE READING STATEMENT IN SUPPORTIVE PLAST
STRCH 51         C    ITCONT=0, IF COMPLETION STARTS AT THE VERY BEGINNING AND FIRST/
STRCH 52         C    SECOND STEPS ARE INCLUDED IN THE STEPS TO BE COMPUTED
STRCH 53         C    =1, OTHERWISE
STRCH 54         C    THIS INDEX IS RELATED TO THE DETERMINATION OF STEP SIZE
STRCH 55         C    NFORM=NUMBER OF STEPS ASSIGNED PER FUM
STRCH 56         C    NPUNCH=1, IF DATA ARE TO BE PUNCHED
STRCH 57         C    =0, OTHERWISE
STRCH 58         C    FLIMIT=VALUE OF (ERRCP NORM)/(SOLUTION NORM) REQUIRED
STRCH 59         C    FOR CONVERGENCE
STRCH 60         C    NPRINT=1, IF NODAL POINT DATA ARE TO BE PRINTED
STRCH 61         C    =0, OTHERWISE
STRCH 62         C    NUMNP=NUMBER OF NODAL POINTS
STRCH 63         C    PNRAD=RADIUS OF HEMISPHERICAL PUNCH HEAD
STRCH 64         C    RADIUS=RADIUS OF THE BLANK
STRCH 65         C    FRITN =FRICTION COEFFICIENT BETWEEN BLANK AND PUNCH
STRCH 66         C    ECONST=STEP SIZE IN MAXIMUM EFFECTIVE STRAIN INCREMENT
STRCH 67         C
STRCH 68         C
STRCH 69         C    YVALUE, PRESTN, EXFNT, PRESTS ARE TO EXPRESS THE WORKHARDENING
STRCH 70         C    CHARACTERISTICS OF THE BLANK
STRCH 71         C    STRESS=YVALUE*(PRESTN+STRAIN)**EXFNT+PRESTS
STRCH 72         C
STRCH 73         C    NEG=NUMBER OF EQUATIONS TO BE SOLVED
STRCH 74         C    NUMEL=NUMBER OF ELEMENTS
STRCH 75         C    MRAND=BAND WIDTH
STRCH 76         C*****
STRCH 77         C

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STRCH 78 C
STRCH 79 RRVAL=PVALUE
STRCH 80 NUMEL=NUMNF-1
STRCH 81 MRAND=C
STRCH 82 NEC=NUMNP*3
STRCH 83 NQ=NEQ
STRCH 84 NEL=NUMEL
STRCH 85 C
STRCH 86 C
STRCH 87 N1=1
STRCH 88 N2=N1+NUMNP
STRCH 89 N3=N2+NUMNP
STRCH 90 N4=N3+NUMNP
STRCH 91 N5=N4+NUMNP
STRCH 92 N6=N5+NUMNP
STRCH 93 N7=N6+NUMNF
STRCH 94 N8=N7+NUMEL
STRCH 95 N9=N8+NUMEL
STRCH 96 N10=N9+NUMEL
STRCH 97 N11=N10+NUMEL
STRCH 98 N12=N11+NUMEL
STRCH 99 N13=N12+NUMEL*3
STRCH 100 N14=N13+NUMEL*4
STRCH 101 N15=N14+NUMEL*4
STRCH 102 N16=N15+NEC
STRCH 103 N17=N16+NEC*MEAND
STRCH 104 N18=N17+NUMEL
STRCH 105 N19=N18+NUMNF
STRCH 106 N20=N19+NUMNP
STRCH 107 N21=N20+NUMNF
STRCH 108 N22=N21+NUMNF
STRCH 109 N23=N22+NUMNP
STRCH 110 N24=N23+NEC
STRCH 111 N25=N24+NUMNP
STRCH 112 N26=N25+NUMNF
STRCH 113 N27=N26+NUMNF
STRCH 114 C
STRCH 115 C
STRCH 116 C
STRCH 117 IF(N27 .LE. NFIELD)GO TO 100
STRCH 118 C
STRCH 119 WRITE(6,1001) N27
STRCH 120 STOP
STRCH 121 C
STRCH 122 100 CONTINUE
STRCH 123 WRITE(6,1002) N27
STRCH 124 C
STRCH 125 C
STRCH 126 CALL PPELIM(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6))
STRCH 127 C
STRCH 128 CALL PLAST(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
STRCH 129 1A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18),
STRCH 130 2A(N19),A(N20),A(N21),A(N22),A(N23),A(N24),A(N25),A(N26)
STRCH 131 7, NQ,NEL,FLIMT,ITCNT,ACCF)
STRCH 132 C
STRCH 133 1000 FORMAT(12A6)
STRCH 134 1001 FORMAT(///# THE DIMENSION OF THE ARRAY (A) IS TOO SMALL*/
STRCH 135 1# THE SIZE OF THE ARRAY (A) MUST BE *, I7)
STRCH 136 1002 FORMAT(///# THE NECESSARY SIZE OF THE ARRAY (A) IS*, I7)
STRCH 137 1003 FORMAT(6I5,F10.0)
STRCH 138 1004 FORMAT(4F10.0)
STRCH 139 1005 FORMAT(4I5,F10.0)
STRCH 140 C
STRCH 141 STOP
STRCH 142 ENC

STRCH 144 SUBROUTINE PRELIM(P,Z,UP,U7,CODE,SLOP)
STRCH 145 C
STRCH 146 C*****
STRCH 147 C READ AND PRINT OF CONTROL INFORMATION AND MATERIAL PROPERTIES MATN0013
STRCH 148 C*****
STRCH 149 C
STRCH 150 C
STRCH 151 COMMON/GENCON/NUMNF,NUMEL,HEC(12),PLL,NEQ,NFCRM,YIELD,TEST,ITEP,
STRCH 152 INREAD,NPUNCH,NPRINT,PVALUE,T,MRAND,PNPAD,RADIUS,FRITN,
STRCH 153 2ECCNST,FNHEC,TC1ST
STRCH 154 C
STRCH 155 DIMENSION R(1),Z(1),CODE(1),UP(1),U7(1),SLOP(1)
STRCH 156 C
STRCH 157 C
STRCH 158 50 CONTINUE

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STRCH 159      WRITE (6,2000) HED,NUMNP,NUMEL
STRCH 160      CALL HARC(C,,YIELD)
STRCH 161      WRITE(6,2010) YIELD
STRCH 162      WRITE(6,1009) ITER
STRCH 163      C
STRCH 164      C*****MAIN0030
STRCH 165      C  READ AND PRINT OF NODAL POINT DATA                               MAIN0031
STRCH 166      C*****MAIN0032
STRCH 167      C
STRCH 168      L=0                                                                MAIN0034
STRCH 169      IF(NPRINT.EQ.0) GO TO 60
STRCH 170      WRITE (6,1114)
STRCH 171      WRITE (6,2004)                                                    MAIN0033
STRCH 172      60 READ (5,1002) N,CCDF(N),P(N),Z(N),UP(N),UZ(N),SLOP(N)
STRCH 173      C
STRCH 174      NL=L+1                                                                MAIN0036
STRCH 175      ZX=N-L                                                                MAIN0037
STRCH 176      IF(L.EQ.0) GO TO 70
STRCH 177      DP=(P(N)-P(L))/ZX
STRCH 178      DZ=(Z(N)-Z(L))/ZX                                                    MAIN0038
STRCH 179      DUP=(UP(N)-UP(L))/ZX
STRCH 180      DUZ=(UZ(N)-UZ(L))/ZX
STRCH 181      DS=(SLOP(N)-SLOP(L))/ZX
STRCH 182      C
STRCH 183      70 L=L+1                                                                MAIN0041
STRCH 184      IF(N-L) 100,90,80
STRCH 185      C
STRCH 186      90 CODE(L)=0.0
STRCH 187      Z(L)=P(L-1)+DP
STRCH 188      SLOP(L)=SLOP(L-1)+DS
STRCH 189      Z(L)=Z(L-1)+DZ
STRCH 190      UP(L)=UP(L-1)+DUP
STRCH 191      UZ(L)=UZ(L-1)+DUZ
STRCH 192      GO TO 70
STRCH 193      C
STRCH 194      90 IF(NUMNP=N) 100,110,60
STRCH 195      C
STRCH 196      100 WRITE (6,2009) N
STRCH 197      CALL EXIT
STRCH 198      C
STRCH 199      110 CONTINUE
STRCH 200      IF(NPRINT.EQ.0) GO TO 120
STRCH 201      WRITE (6,2002) (K,CODE(K),P(K),Z(K),UP(K),UZ(K),SLOP(K),K=1,NUMNP)
STRCH 202      120 CONTINUE
STRCH 203      C
STRCH 204      NEQ=T*NUMNP
STRCH 205      WRITE(6,1122) NEQ,MPAND
STRCH 206      C
STRCH 207      1002 FORMAT (15,F5.0,5F10.0)
STRCH 208      1003 FORMAT(16I5)
STRCH 209      1004 FORMAT(18,2I11,2F10.4)
STRCH 210      1005 FORMAT(2I5,4F10.0)
STRCH 211      1006 FORMAT(// * THE NODAL POINTS AT WHICH FORCE CALCULATIONS ARE DESIR
STRCH 212      1007 FORMAT(1H1,15X, 3GH LINEARLY DISTRIBUTED BOUNDARY STRESSES/
STRCH 213      1 / 60H  NODI I  NODJ J  PRESSURE I  PRESSURE J  SHEAR I
STRCH 214      2 14H  SHEAR J)
STRCH 215      1008 FORMAT(2I9,4E15.5)
STRCH 216      1009 FORMAT(// * MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR EACH INCREMENT
STRCH 217      1T =*, I2)
STRCH 218      1114 FORMAT(1H1, 3GH NODAL POINT INFORMATION BEFORE SCALING//)
STRCH 219      1122 FORMAT(// * NUMBER OF EQUATIONS  =*, I4 /
STRCH 220      1 * BANDWIDTH =*, I4 /
STRCH 221      2 * DIAGONAL ELEMENTS =*, I4 )
STRCH 222      2000 FORMAT (1H 12A6 /
STRCH 223      1 30H0 NUMBER OF NODAL POINTS----- I3 /
STRCH 224      2 30H0 NUMBER OF ELEMENTS----- I3 /)
STRCH 225      2002 FORMAT (1I12,2F12.3,3E24.7)
STRCH 226      2003 FORMAT (1I13,4I6,1I12)
STRCH 227      2004 FORMAT (/ * NODAL POINT TYPE R-ORDINATE Z-ORDINATE F LC
STRCH 228      1AD CR DISPLACEMENT ? LCAD CR DISPLACEMENT BETA-SLOPE *)
STRCH 229      2005 FORMAT(//,*FORCES SPECIFIED AT NODAL POINT*,//.
STRCH 230      1 * NODAL PT. ELEMENT1 ELEMENT2 PRESSURE SHEAR*,/)
STRCH 231      2006 FORMAT (26HONODAL POINT CARD FROM N= IF)
STRCH 232      2010 FORMAT(// * INITIAL YIELD STRESS = *, F15.7//).
STRCH 233      RETURN
STRCH 234      END
STRCH 235

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STRCH 237      SUBROUTINE PLAST(R,7,UR,UZ,CCDF,SLOP,VY,VX,SPHI,CPhi,DL,STS,TEPS,
STRCH 238      1EPS,R,A,THICK,ALPHA,GAMMA,ETA,FRNFCE,PHI,FF,TOUCH,UUP,UU7,
STRCH 239      2NG,NEL,FLIMIT,ITCONT,ACOF,NDEX)

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STRCH 240          COMMON/GENCON/NUMNP,NUMEL,HEX(12),DLL,NEG,NFORM,YIELD,TEST,ITER,
STRCH 241          INPEAD,NPUNCH,NPRINT,PVALUE,T,WRAMP,PNFAD,PA?US,FRITN,
STRCH 242          2ECCNST,FNHED,TDIST
STRCH 243          C
STRCH 244          C*****
STRCH 245          C   PLAST IS THE CONTROLLING SUBROUTINE
STRCH 246          C*****
STRCH 247          C
STRCH 248          COMMON/CONQUAE/SS(4),WT(4),D(2,2),SQRT1
STRCH 249          DIMENSION P(1),Z(1),UR(1),UZ(1),CGDF(1),SLCP(1),YY(1),VX(1),
STRCH 250          1TEPS(4,1),R(1),A(NG,1),THICK(1),CPHI(1),SPHI(1),DL(1),FPS(4,1),
STRCH 251          2ALPHA(1),GAMMA(1),ETA(1),FRNFCE(1),PHI(1),FF(1),TOUCH(1)
STRCH 252          3,STS(3,1),UUF(1),UU7(1)
STRCH 253          COMMON/ATOUCH/NTOUCH
STRCH 254          C
STRCH 255          C*****
STRCH 256          C   THE FIRST NODE IS LOCATED AT THE RIM OF THE BLANK
STRCH 257          C   AND THE POLE IS THE LAST NODE
STRCH 258          C*****
STRCH 259          C
STRCH 260          C
STRCH 261          C*****
STRCH 262          C   A1,A2 ARE CONSTANTS RELATED TO DETERMINATION OF ACDEF
STRCH 263          C   PNHED=PRESENT POSITION OF PUNCH HEAD
STRCH 264          C   NTCUCH=FIRST NODAL POINT IN CONTACT WITH PUNCH HEAD
STRCH 265          C   NCHECK=NUMBER OF NODES FOR WHICH THE CONTACTING WILL BE CHECKED
STRCH 266          C   TCHCDF=0, IF EQUADRY IS TO ADVANCE
STRCH 267          C   =1, OTHERWISE
STRCH 268          C*****
STRCH 269          C
STRCH 270          C
STRCH 271          C   PNHED=7*(NUMNP)
STRCH 272          C   A2=2.
STRCH 273          C   A1=10.
STRCH 274          C   NCHECK=NUMEL/10
STRCH 275          C   NUM1=NUMNP-1
STRCH 276          C   NTCUCH=NUMNP
STRCH 277          C   TCHCDF=0.0
STRCH 278          C   NSTCP=0
STRCH 279          C   SS(1)=0.8611363116
STRCH 280          C   SS(2)=0.3256810436
STRCH 281          C   SS(3)=-SS(1)
STRCH 282          C   SS(4)=-SS(2)
STRCH 283          C
STRCH 284          C   DO 442 N=1, NUMEL
STRCH 285          C   FRNFCE(N)=0.0
STRCH 286          C   THICK(N)=T
STRCH 287          C   DO 442 I=1, 4
STRCH 288          C   442 TEPS(I,N)=0.
STRCH 289          C*****
STRCH 290          C   TO HANDLE THE INFINITE WORKHARDENING RATE INITIALLY VERY
STRCH 291          C   SMALL VALUE IS ASSIGNED TO THE EFFECTIVE STRAIN
STRCH 292          C*****
STRCH 293          C
STRCH 294          C   DO 450 N=1,NUMEL
STRCH 295          C   450 TEPS(4,N)=0.0001
STRCH 296          C
STRCH 297          C   FRNFCE(NUMNP)=0.00?
STRCH 298          C   DPNSTR=C.1
STRCH 299          C   DPNHED=UZ(NUMNP)
STRCH 300          C   ESTAR=1.0
STRCH 301          C
STRCH 302          C*****
STRCH 303          C   IF THE COMPUTATION IS INTERRUPTED AFTER A NUMBER OF STEPS
STRCH 304          C   AND RESTARTED, THEN NECESSARY DATA NEED BE FEED
STRCH 305          C*****
STRCH 306          C
STRCH 307          C   IF(NREAD .LE. 0) GO TO 440
STRCH 308          C   READ(5,1017) (UR(I), UZ(I), SLCP(I), I=1,NUMNP)
STRCH 309          C   READ(5,1017) ( R(I), Z(I), I=1, NUMNP)
STRCH 310          C   READ(5,1017)((TEPS(I,N),I=1,4), N=1, NUMEL)
STRCH 311          C   READ(5,1017)(THICK(N),N=1,NUMEL)
STRCH 312          C   READ(5,2223)PNHED,NTCUCH,TCHCCF,EFAC
STRCH 313          C   READ(5,2331)(FRNFCE(N),N=1,NUM1)
STRCH 314          C   READ 277,ESTAR,DPNSTE,DPNHED
STRCH 315          C   440 CONTINUE
STRCH 316          C
STRCH 317          C   NSTEP=0
STRCH 318          C   TOUCH2=TDIST
STRCH 319          C   2100 NSTEP=NSTEP+1
STRCH 320          C
STRCH 321          C*****
STRCH 322          C   DPNHED=ASSIGNED INCREMENT OF PUNCH HEAD TRAVEL
STRCH 323          C   ESTAR=ADJUSTING FACTOR
STRCH 324          C*****

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STRCH 325 C
STRCH 326 PNFED=PNFED+DPNFED
STRCH 327 DO 445 I=1,NUMNF
STRCH 328 UR(I)=UR(I)*ESTAR
STRCH 329 UZ(I)=UZ(I)*ESTAR
STRCH 330 445 CONTINUE
STRCH 331 C
STRCH 332 TOUCH3=TOUCH2
STRCH 333 DO 247 I=1,NCHECK
STRCH 334 247 TOUCH(I)=0.
STRCH 335 C
STRCH 336 2101 CONTINUE
STRCH 337 IF(TCHCCF .EG. 1.)GO TO 219
STRCH 338 NTOUCH=NTOUCH-1
STRCH 339 NTCEI=NTCEI+1
STRCH 340 FRNFCI(NTOUCH)=FRNFCI(NTCEI)/3.
STRCH 341 219 CONTINUE
STRCH 342 446 CONTINUE
STRCH 343 C
STRCH 344 C*****
STRCH 345 C UPDATING OF THE BOUNDARY CONDITION
STRCH 346 C*****
STRCH 347 C
STRCH 348 C
STRCH 349 DO 100 I=2,NUM1
STRCH 350 CODE(I)=0.0
STRCH 351 IF(I .GE. NTOUCH)CODE(I)=4.0
STRCH 352 100 CONTINUE
STRCH 353 CODE(NUMNP)=3.0
STRCH 354 NTCHM1=NTOUCH-1
STRCH 355 C
STRCH 356 C*****
STRCH 357 C COMPUTE THE YIELD STRESS AND THE WORKHARDENING RATE
STRCH 358 C*****
STRCH 359 C
STRCH 360 DO 220 N=1, NUMEL
STRCH 361 CALL HARD(TEPS(4,N),YY(N))
STRCH 362 CALL HARD2(TEPS(4,N),YX(N))
STRCH 363 220 CONTINUE
STRCH 364 C
STRCH 365 C
STRCH 366 C
STRCH 367 C
STRCH 368 WRITE(4,1007) NSTEP
STRCH 369 C
STRCH 370 C
STRCH 371 659 CONTINUE
STRCH 372 WRITE(5,1031)
STRCH 373 C
STRCH 374 C*****
STRCH 375 C DETAIL OF THE PRESENT CONFIGURATION
STRCH 376 C SPMI=SINE OF ANGLE PHI
STRCH 377 C CPHI=COSINE OF ANGLE PHI
STRCH 378 C DL=ELEMENT LENGTH
STRCH 379 C*****
STRCH 380 C
STRCH 381 DO 690 N=1, NUMEL
STRCH 382 NP1=N+1
STRCH 383 DR=R(N)-R(NP1)
STRCH 384 DZ=Z(NP1)-Z(N)
STRCH 385 DL(N)=SQRT(DR*DR+DZ*DZ)
STRCH 386 SPMI(N)=DR/DL(N)
STRCH 387 CPHI(N)=DZ/DL(N)
STRCH 388 PHI(N)=ASIN(SPMI(N))*180./3.14159
STRCH 389 WRITE(6,1020)N,PHI(N),THICK(N),DL(N)
STRCH 390 690 CONTINUE
STRCH 391 C
STRCH 392 C
STRCH 393 RPI=RVALUE+1.
STRCH 394 RCCNST=3.*RPI/( 2.*(1.+RVALUE+RVALUE) )
STRCH 395 D(1,1)=RPI*RCCNST
STRCH 396 D(1,2)=RVALUE*RCCNST
STRCH 397 D(2,1)=D(1,2)
STRCH 398 D(2,2)=D(1,1)
STRCH 399 CLAMDA=RPI*RPI/(1.+RVALUE+RVALUE)
STRCH 400 POISON=RVALUE/RPI
STRCH 401 C
STRCH 402 C
STRCH 403 K=0
STRCH 404 2001 CONTINUE
STRCH 405 K=K+1
STRCH 406 C
STRCH 407 C
STRCH 408 CALL STIFF(R,7,UR,UZ,CCCE,SLDF,YY,YX,SPMI,CPHI,DL,EPS,
STRCH 409 ITHICK,ALPHA,GAMMA,ETA,FRNFCI,FF,A,B,NC)

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STRCH 410 C
STRCH 411 C*****
STRCH 412 C INTRODUCTION OF BOUNDARY CONDITION
STRCH 413 C*****
STRCH 414 C
STRCH 415 C
STRCH 416 CALL MODIFY(CODE,A,P,ALPHA,GAMMA,ETA,NUMNF,NFQ,MBAND,FPNFCE)
STRCH 417 C
STRCH 418 C Banded symmetric solution
STRCH 419 C
STRCH 420 CALL TRIA(NEQ,MEANC,A)
STRCH 421 CALL RACKS(NEG,MBAND,A,R)
STRCH 422 C
STRCH 423 C*****
STRCH 424 C PERTURBATION OF UR IS COMPUTED FROM PERTURBATION OF UZ FOR NODES
STRCH 425 C*****
STRCH 426 C
STRCH 427 C
STRCH 428 DO 101 N=2,NUM1
STRCH 429 IR=3*N-2
STRCH 430 I7=IR+1
STRCH 431 IF(N .GE. NTOLCH)F(I7)=R(I7)/ALPHA(N)+GAMMA(N)
STRCH 432 101 CONTINUE
STRCH 433 C
STRCH 434 C*****
STRCH 435 C TO OBTAIN AN EFFICIENT CONVERGENCE ACDEF IS COMPUTED
STRCH 436 C THE ACCELERATING COEFFICIENT IS DETERMINED IN SUCH A WAY
STRCH 437 C PERTURBATIONAL TERM TIMES ACCELERATING COEFFICIENT IS NEVER
STRCH 438 C GREATER THAN THE INITIAL VALUE, BUT BE A FRACTION OF IT.
STRCH 439 C E.G. A2=2 MEANS HALF
STRCH 440 C*****
STRCH 441 C
STRCH 442 CONCOF=0.0
STRCH 443 DO 103 I=1,NUMNF
STRCH 444 UUR(I)=UR(I)
STRCH 445 UUZ(I)=UZ(I)
STRCH 446 I7=3*I-1
STRCH 447 IP=I7-1
STRCH 448 IF(E(IR) .EQ. 0.)GO TO 102
STRCH 449 COF1=UR(I)/B(IP)
STRCH 450 102 IF(E(I7) .EQ. 0.)GO TO 103
STRCH 451 COF2=UZ(I)/B(I7)
STRCH 452 COF1=ABS(COF1)
STRCH 453 COF2=ABS(COF2)
STRCH 454 A1=AMIN1(A1,COF1,COF2)
STRCH 455 103 CONTINUE
STRCH 456 C
STRCH 457 105 CONTINUE
STRCH 458 IF(CONCOF .EQ. 1.)A2=A2*5.
STRCH 459 IF(A2 .GT. 1250)GO TO 2300
STRCH 460 IF(A1 .EQ. 1.0 .AND. A2 .GT. 10.)COF1=5.
STRCH 461 IF(COF1 .EQ. 0.)COF1=2.
STRCH 462 A1=COF1/A2
STRCH 463 IF(A1 .GT. 1.0) A1=1.0
STRCH 464 ACDEF=A1
STRCH 465 WRITE(6,104)A1
STRCH 466 C
STRCH 467 C*****
STRCH 468 C OBTAIN NEW VALUE
STRCH 469 C*****
STRCH 470 C
STRCH 471 DO 130 I=1, NUMNF
STRCH 472 I7=3*I-1
STRCH 473 IP=I7-1
STRCH 474 IS=I7+1
STRCH 475 UR(I)=UUR(I)
STRCH 476 UZ(I)=UUZ(I)
STRCH 477 UR(I)=UR(I)+P(IP)*ACDEF
STRCH 478 UZ(I)=UZ(I)+F(I7)*ACDEF
STRCH 479 SLOC(I)=SLOC(I)+B(IS)*ACDEF
STRCH 480 130 CONTINUE
STRCH 481 C
STRCH 482 WRITE(6,1016) K
STRCH 483 WRITE(6,1006) K
STRCH 484 C
STRCH 485 C
STRCH 486 C*****
STRCH 487 C COMPUTE NORM OF ERROR AND NORM OF SOLUTION.
STRCH 488 C*****
STRCH 489 C
STRCH 490 ENORM = 0.
STRCH 491 SNCFM = 0.
STRCH 492 DO 134 I=1, NUMNF
STRCH 493 I7=3*I-1
STRCH 494 IP=I7-1

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STPCH 495      IS=I7+1
STPCH 496      ENCRW = ENDRW + R(IR)*E(IR) + R(IZ)*R(IZ) + R(IS)*R(IS)
STPCH 497      SNDRW = SNDRW + UR(I)*UR(I) + UZ(I)*UZ(I) + SLOP(I)*SLOP(I)
STPCH 498      174 CONTINUE
STPCH 499      ENCRW = SQRT(ENCRW)
STPCH 500      SNDRW = SQRT(SNDRW)
STPCH 501      ESNCRW=ENDRW/SNCRW
STPCH 502      WRITE(6,1015) SNDRW,ENCRW,ESNCRW
STPCH 503      C
STPCH 504      DO 776 I=1, NLMWF
STPCH 505      IZ=3*I-1
STPCH 506      IR=IZ-1
STPCH 507      IS=IZ+1
STPCH 508      WRITE(6,1002) I,R(IR),R(IZ),R(IS),UR(I),UZ(I),SLOP(I),F(I),Z(I)
STPCH 509      776 CONTINUE
STPCH 510      C
STPCH 511      131 CONTINUE
STPCH 512      C
STPCH 513      C
STPCH 514      C
STPCH 515      C
STPCH 516      C*****
STPCH 517      C COMPUTE STRAIN FROM THE NEW GUESS.
STPCH 518      C EPS(1,N)=INCREMENT OF MERIDIAN STRAIN
STPCH 519      C EPS(2,N)=INCREMENT OF TANGENTIAL STRAIN
STPCH 520      C EPS(3,N)=INCREMENT OF THICKNESS STRAIN
STPCH 521      C*****
STPCH 522      C
STPCH 523      C
STPCH 524      DO 800 N=1, NUMEL
STPCH 525      NP1=N+1
STPCH 526      DLL=DL(N)
STPCH 527      SPH=SPH1(N)
STPCH 528      CPH=CPH1(N)
STPCH 529      AU=UR(N)+UR(NP1)
STPCH 530      AR=R(N)+R(NP1)
STPCH 531      DR=R(N)-R(NP1)
STPCH 532      DZ=Z(NP1)-Z(N)
STPCH 533      DU=UR(N)-UR(NP1)
STPCH 534      DW=UZ(NP1)-UZ(N)
STPCH 535      EX1=1.+2.*DR*CL/DLL/DLL+2.*D7*DW/DLL/CLL+(DU*DU+DW*DW)/DLL/DLL
STPCH 536      EPS(1,N)=SQRT(EX1)-1.
STPCH 537      EPS(1,N)=ALOG(1.+EPS(1,N))
STPCH 538      EPS(2,N)=AU/AR
STPCH 539      EPS(3,N)=-EPS(1,N)-EPS(2,N)
STPCH 540      800 CONTINUE
STPCH 541      C
STPCH 542      C
STPCH 543      C
STPCH 544      C
STPCH 545      TEST=0.0
STPCH 546      WRITE(6,1026) NSTEP
STPCH 547      C
STPCH 548      C
STPCH 549      C
STPCH 550      C*****
STPCH 551      C COMPUTE INCREMENT OF EFFECTIVE STRAIN
STPCH 552      C*****
STPCH 553      C
STPCH 554      DO 222 N=1, NUMEL
STPCH 555      ES=EPS(1,N)
STPCH 556      ET=EPS(2,N)
STPCH 557      RPAR=EP1*(ES*ES+ET*ET) + 2.*FVALUE*ES*ET
STPCH 558      EPS(4,N)=SQRT(2.*PCCNST*RPAR/3.)
STPCH 559      C
STPCH 560      C
STPCH 561      IF(NSTEP .EQ. 0)YY(N)=YIELD
STPCH 562      STS(1,N)=CLAMDA*(ES + POISON*ET)*YY(N)/EPS(4,N)
STPCH 563      STS(2,N)=CLAMDA*(ET + POISON*ES)*YY(N)/EPS(4,N)
STPCH 564      C
STPCH 565      C*****
STPCH 566      C COMPUTE STRESS DISTRIBUTION
STPCH 567      C STS(1,N)=MERIDIAN STRESS
STPCH 568      C STS(2,N)=CIRCUMFERENTIAL STRESS
STPCH 569      C STS(3,N)=EFFECTIVE STRESS
STPCH 570      C*****
STPCH 571      C
STPCH 572      IF(ESNCRW .LT. FLIM1)TEST=1.0
STPCH 573      ES=STS(1,N)
STPCH 574      ET=STS(2,N)
STPCH 575      EFSTRS=ES*ES+ET*ET-2.*FCISON*ES*ET
STPCH 576      STS(3,N)= SQRT(EFSTRS)
STPCH 577      C
STPCH 578      WRITE(6,1003)N,(EPS(I,N),I=1,4)
STPCH 579      222 CONTINUE

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STRCH 580 C
STRCH 581 WRITE(6,1027)
STRCH 582 DO 430 N=1,NUMEL
STRCH 583 430 WRITE(6,2251)N,(STS(1,N),I=1,3)
STRCH 584 C
STRCH 585 C*****
STRCH 586 C CHECK WHETHER ACCOF IS TOO LARGE TO CAUSE A PHYSICALLY
STRCH 587 C UNACCEPTABLE SOLUTION. WHENEVER COMPUTED MERIDIAN STRESS BECOMES
STRCH 588 C NEGATIVE ADJUST ACCOF VALUE)
STRCH 589 C*****
STRCH 590 C
STRCH 591 CONCOF=0.0
STRCH 592 STS1=0.0
STRCH 593 DO 431 N=1,NUMEL
STRCH 594 IF(STS(2,N) .LT. STS1)STS1=STS(2,N)
STRCH 595 431 IF(STS(1,N) .LT. STS1)STS1=STS(1,N)
STRCH 596 IF(STS1 .LT. 0.)CONCOF=1.0
STRCH 597 IF(CONCOF .EQ. 1.0)GO TO 10*
STRCH 598 C
STRCH 599 C
STRCH 600 C
STRCH 601 C*****
STRCH 602 C CHECK WHETHER (SPECF NORM)/(SOLUTION NORM) IS LESS THAN FLIMIT
STRCH 603 C IF YES, THE SOLUTION IS FINAL
STRCH 604 C*****
STRCH 605 C
STRCH 606 C
STRCH 607 IF(ESNORM .LT. FLIMIT)GO TO 43F
STRCH 608 C
STRCH 609 IF(K .GE. ITER)GO TO 43E
STRCH 610 GO TO 2001
STRCH 611 2000 CONTINUE
STRCH 612 2200 CONTINUE
STRCH 613 C
STRCH 614 43F CONTINUE
STRCH 615 IF(ESNORM .GT. FLIMIT)GO TO 777
STRCH 616 C
STRCH 617 C
STRCH 618 WRITE(6,2800)
STRCH 619 C
STRCH 620 C EXAMINATION ON BOUNDARY ASSUMPTIONS
STRCH 621 C*****
STRCH 622 C
STRCH 623 DO 250 I=1,NCHECK
STRCH 624 N=NTOUCH-I
STRCH 625 IF(N .LE. 1)GO TO 250
STRCH 626 TOUCH(I)=(Z(N)+UZ(N)+PARAD-PNHED)**2.+(P(N)+UP(N))**2.
STRCH 627 TOUCH(I)=SQRT(TOUCH(I))-PNPAD
STRCH 628 WRITE(6,2900)N,TOUCH(I)
STRCH 629 250 CONTINUE
STRCH 630 C
STRCH 631 IF(ABS(TOUCH(I)) .LT. .0001)TOUCH(I)=0.
STRCH 632 C
STRCH 633 C*****
STRCH 634 C CHECK ON BOUNDARY OVER PUNCH HEAD
STRCH 635 C*****
STRCH 636 C
STRCH 637 IF(TOUCH(I) .GE. 0.)GO TO 300C
STRCH 638 C
STRCH 639 WRITE(6,3100)
STRCH 640 C
STRCH 641 C*****
STRCH 642 C NODE AT NCHM1 IS INSIDE PUNCH. COMPUTE AGAIN
STRCH 643 C*****
STRCH 644 C
STRCH 645 TOUCH=0.0
STRCH 646 TINSDE=0.0
STRCH 647 GO TO 2101
STRCH 648 C
STRCH 649 C
STRCH 650 300C CONTINUE
STRCH 651 TINSDE=1.0
STRCH 652 TEST=1.0
STRCH 653 C
STRCH 654 C*****
STRCH 655 C COMPUTATION OF FRICTION COEFFICIENT
STRCH 656 C*****
STRCH 657 C
STRCH 658 MUDEX=0
STRCH 659 PPNHED=PNHED
STRCH 660 WRITE(6,233)(FRNFC(N),N=NTOUCH,NUM1)
STRCH 661 IF(FRITN .GE. 0.)GO TO 234
STRCH 662 WRITE(6,231)
STRCH 663 C
STRCH 664 DO 230 I=NTOUCH,NUM1

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STRCH 665      I7=I7-1
STRCH 666      I8=I7-1
STRCH 667      DUM1=(Z(I1)+U7(I1)+PNEAD-PPNHED)/PNEAD
STRCH 668      DUM2=(F(I1)+U8(I1))/PNEAD
STRCH 669      PN=FF(I2)*DUM1+FF(I8)*DUM2
STRCH 670      PT=FF(I2)*DUM2-FF(I8)*DUM1
STRCH 671      XMU=PT/FA
STRCH 672      WRITE(5,232)I,XML
STRCH 673      XMU=XMU/FRITN
STRCH 674      IF(XMU .GT. 1.02 .OR. XMU .LT. .98)MUDEX=1
STRCH 675      FRNFC(I)=FRNFC(I)/XML
STRCH 676      230 CONTINUE
STRCH 677      234 CONTINUE
STRCH 678      C
STRCH 679      C
STRCH 680      C
STRCH 681      C
STRCH 682      C*****
STRCH 683      C      MUDEX=0, IF FRICTION CONDITION IS SATISFIED
STRCH 684      C      =1, OTHERWISE
STRCH 685      C*****
STRCH 686      C
STRCH 687      C
STRCH 688      C      IF(MUDEX .EQ. 1)TCHCOF=1.0
STRCH 689      C      IF(MUDEX .EQ. 1)GO TO 2001
STRCH 690      C      N77=3*NTOUCH-1
STRCH 691      C
STRCH 692      C
STRCH 693      C*****
STRCH 694      C      GENERALIZED NODAL FORCE NORMAL TO THE PUNCH IS COMPUTED
STRCH 695      C      TO CHECK WHETHER THE BOUNDARY IS ASSUMED TO MOVE TOO FAST.
STRCH 696      C*****
STRCH 697      C
STRCH 698      C      IF(FF(NZ7) .GT. 0.) GO TO 500
STRCH 699      C      IF(ABS(FF(NZ7)) .LT. .000001 .OR. TINSDE .EQ. 0.)GO TO 500
STRCH 700      C      T01ST=TOUCH3*.P
STRCH 701      C      WRITE(6,510)T01ST
STRCH 702      C      NTOUCH=NTOUCH+1
STRCH 703      C      TCHCOF=1.0
STRCH 704      C      GO TO 444
STRCH 705      C      500 CONTINUE
STRCH 706      C
STRCH 707      C
STRCH 708      C
STRCH 709      C
STRCH 710      C      NNTCH=NTOUCH
STRCH 711      C      DO 240 I=1,NCHECK
STRCH 712      C      IF( TOUCH(I)-T01ST)245,245,246
STRCH 713      C      245 NTOUCH=NNTCH-I
STRCH 714      C      TOUCH2=TOUCH(I)
STRCH 715      C      FRNFC(NTOUCH)=FRNFC(NNTCH)
STRCH 716      C      240 CONTINUE
STRCH 717      C      246 TCHCOF=1.0
STRCH 718      C      WRITE(6,1043)
STRCH 719      C
STRCH 720      C*****
STRCH 721      C      COMPUTE TOTAL STRAIN
STRCH 722      C*****
STRCH 723      C
STRCH 724      C      DO 444 N=1,NUMEL
STRCH 725      C      DO 443 I=1, 4
STRCH 726      C      443 TEPS(I,N)=TEPS(I,N)+EPS(I,N)*TEST
STRCH 727      C      IF(ESNORM .LT. FLIMIT)THICK(N)=THICK(N)*EXP(EPS(3,N)*TEST)
STRCH 728      C      WRITE(6,1003)N,(TEPS(I,N),I=1,4)
STRCH 729      C      444 CONTINUE
STRCH 730      C
STRCH 731      C
STRCH 732      C
STRCH 733      C
STRCH 734      C*****
STRCH 735      C      COMPUTE DPNHED FOR NEXT STEP WHICH WOULD GIVE THE INCREMENT
STRCH 736      C      OF MAXIMUM EFFECTIVE STRAIN APPROXIMATELY EQUAL TO PRESET VALUE.
STRCH 737      C*****
STRCH 738      C
STRCH 739      C      EMAX=0.0
STRCH 740      C      DO 775 N=1,NUMEL
STRCH 741      C      775 IF(EPS(4,N) .GT. EMAX) EMAX=EPS(4,N)
STRCH 742      C      EFACTA=ECONST/EMAX
STRCH 743      C      IF(NSTEP .LE. 2 .AND. ITCNT .EQ. 0)GO TO 778
STRCH 744      C      DPNHED=2./EFACTA/UZ(NUMNP)-1./EFACTA/DPNSTP
STRCH 745      C      DPNHED=1./DPNHED
STRCH 746      C      ESTAR=DPNHED/LZ(NUMNP)
STRCH 747      C      DPNSTR=UZ(NUMNP)
STRCH 748      C      778 CONTINUE
STRCH 749      C      EFACT=EFACTA

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STPCH 750 C
STPCH 751 C
STPCH 752 C
STPCH 753 777 CONTINUE
STPCH 754 IF(ESNDPM .GT. FLIMIT)TCHCOF=1.0
STPCH 755 C
STPCH 756 C*****
STPCH 757 C NEW CONFIGURATION
STPCH 758 C*****
STPCH 759 C
STPCH 760 DO 439 I=1, NUMNF
STPCH 761 IR=3*I-1
STPCH 762 IR=IR-1
STPCH 763 R(I)=R(I)+UR(I)*TEST
STPCH 764 Z(I)=Z(I)+U7(I)*TEST
STPCH 765 439 CONTINUE
STPCH 766 C
STPCH 767 C
STPCH 768 C*****
STPCH 769 C PUNCH THE SOLUTION
STPCH 770 C*****
STPCH 771 C
STPCH 772 C
STPCH 773 IF(NPUNCH .EQ. 0) GO TO 310
STPCH 774 PUNCH 1017, (UR(I),U7(I), SLOC(I), I=1,NUMNF)
STPCH 775 PUNCH 1017, (R(I),Z(I),I=1, NUMNF)
STPCH 776 PUNCH 1017, ( (TEPS(I,N),I=1,4), N=1, NUMEL)
STPCH 777 PUNCH 1017, (THICK(N),N=1,NUMEL)
STPCH 778 PUNCH 2223,PNFEC,NTCUCH,TCHCOF,EFACT
STPCH 779 PUNCH 233, (PRNFCE(N),N=1,NUM1)
STPCH 780 PUNCH 237,ESTAP,DFNSTA,DFNHED
STPCH 781 310 CONTINUE
STPCH 782 C
STPCH 783 IF(ESNDPM .GT. FLIMIT)GO TO 2300
STPCH 784 C
STPCH 785 WRITE(6,1040)
STPCH 786 DO 849 I=1,NUMNF
STPCH 787 IR=3*I-2
STPCH 788 IR=IR+1
STPCH 789 IL=IZ+1
STPCH 790 WRITE(6,1041)(I,FF(IF),FF(IZ),FF(IL))
STPCH 791 849 CONTINUE
STPCH 792 C
STPCH 793 C
STPCH 794 C*****
STPCH 795 C COMPUTE THE PUNCH LOAD FROM ENERGY BALANCE
STPCH 796 C*****
STPCH 797 C
STPCH 798 SUMF=0.
STPCH 799 DO 850 I=2,NUMNF
STPCH 800 IZ=I*3-1
STPCH 801 IR=I*3-1
STPCH 802 850 SUMF=SUMF+FF(IZ)*U7(I)+FF(IR)*UR(I)
STPCH 803 SUMF=SUMF/U2(NUMNF)/TEST/RADIUS
STPCH 804 WRITE(6,1042)SUMF
STPCH 805 IF(NSTOR .EQ. 1) GO TO 2300
STPCH 806 WRITE(6,1028)PPNHED,NNTCH
STPCH 807 WRITE(6,1029)EMAX,U2(NUMNF),EFACT
STPCH 808 IF(NSTEP .LT. NRCFM) GO TO 2100
STPCH 809 2300 CONTINUE
STPCH 810 2301 CONTINUE
STPCH 811 C
STPCH 812 C
STPCH 813 1002 FORMAT(15,3F13.7,5X,3F13.7,5X,3F13.7)
STPCH 814 1003 FORMAT(17,11F11.6)
STPCH 815 1004 FORMAT(14I5)
STPCH 816 1005 FORMAT(14I,*, STRAIN-STRESS SOLUTION AT STEP NUMBER =*,I4//
STPCH 817 1 * FL. NO.,R-STRAIN..Z-STRAIN..TH-STRAIN..F7-STRAIN..EF-STRAIN
STPCH 818 2...R-STRES...Z-STRES...TH-STRES...RZ-STRES..FF-STRES..AVG-STRES..*
STPCH 819 3)
STPCH 820 1006 FORMAT(/// 30X, * DISPLACEMENT SOLUTION AT ITERATION NUMBER =*,I4
STPCH 821 1/// 20X, * PLRTUPED*, 20X, * TOTAL*, 20X, * DEFORMED COORD*/
STPCH 822 2/ * NP DU OR DRETA U
STPCH 823 3 W BEYA F 2*)
STPCH 824 1007 FORMAT(14I,70X,*ITERATION PROCESS FOR STEP*,I4)
STPCH 825 1008 FORMAT( 60X, * TOTAL R-LOAD =*, F12.7
STPCH 826 1 / 60X, * TOTAL Z-LOAD =*, F12.7
STPCH 827 2 / 60X, * TOTAL M-LOAD =*, F12.7)
STPCH 828 1010 FORMAT( // * ADDL POINT FORCE AT STEP =*, I4//
STPCH 829 1*.....N.P.....R-FORCE.....Z-FORCE.....7-STRESS OR DIE SUP
STPCH 830 2FACE...*)
STPCH 831 1011 FORMAT(15,3F10.4)
STPCH 832 1012 FORMAT(16,F17.2)
STPCH 833 1015 FORMAT(40X,* VELOCITY CONVERGENCE*/
STPCH 834 1 60X, * NCF OF SOLUTION VECTOR =*, F13.8

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STRCH  835      1      / 50X, * NCPM OF FERRIC VECTOR =*, F13.6
STRCH  836      2      / 60X, * FRACTIONAL NCPM =*, F13.8)
STRCH  837      1016  FORMAT(      * DISPLACEMENT SOLUTION AT ITERATION NUMBER =*, I4)
STRCH  838      1017  FORMAT(8F10.7)
STRCH  839      1018  FORMAT(////* DOES NOT CONVERGE**
STRCH  840      1* TRY AGAIN WITH DECELERATION COEFFICIENT =*COEF= LESS THAN*,
STRCH  841      2F8.7)
STRCH  842      1020  FORMAT(20F4.1)
STRCH  843      1025  FORMAT(4X, I5, 3X, F12.6, 10X, I5, 3X, F12.6, 10X, I5, 3X, F12.6)
STRCH  844      2251  FORMAT(1E, 4F20.7)
STRCH  845      1026  FORMAT(////* INCREMENTAL STRAIN-TOTAL STRAIN AT STEP NUMBER=*, I4//
STRCH  846      1* EL NO....S-STRAIN.....THE-STRAIN.....THI-STRAIN.....EF-STRAIN
STRCH  847      1*1
STRCH  848      1027  FORMAT(///* EL. NO....S-STRESS.....THE-STRESS.....EF-STRESS.....*)
STRCH  849      1042  FORMAT(* PUNCH FORCE=*, F15.7)
STRCH  850      1043  FORMAT(///* EL NO....S-STRAIN.....THE-STRAIN.....THI-STRAIN....
STRCH  851      1EF-STRAIN...*)
STRCH  852      1041  FORMAT(5X, I10, 5X, 3F20.7)
STRCH  853      1040  FORMAT(///* NO. OF NODE          FORCE*)
STRCH  854      510  FORMAT(///* NTOUCH IS FORCED TO TOUCH, COMPUTE AGAIN
STRCH  855      1*/* TOIST=*, F10.7)
STRCH  856      1030  FORMAT(/17, 3F10.5)
STRCH  857      1031  FORMAT(* GEOMETRY OF PROFILE**
STRCH  858      1 * EL NNO.....ANGLE.....THICKNESS.....*)
STRCH  859      104  FORMAT(///* ACCF CALCULATED*, F10.7)
STRCH  860      2900  FORMAT(/110, F20.7)
STRCH  861      2800  FORMAT(///* CHECKING DISTANCE AWAY FROM PUNCH*, /
STRCH  862      1* ELEM NO.          TOUCH**/)
STRCH  863      3100  FORMAT(* NODE AT NCHM1 IS INSIDE PUNCH, COMP AGAIN*)
STRCH  864      231  FORMAT(///* NODAL POINT          (COEFFICIENT*/,)
STRCH  865      232  FORMAT(11G, F10.5)
STRCH  866      233  FORMAT(4F15.7)
STRCH  867      2223  FORMAT(F15.7, I5, 3F15.7)
STRCH  868      1028  FORMAT(* PUNCH HEAD DISPLACEMENT*, F10.5/* NTOUCH= *, I5)
STRCH  869      1029  FORMAT(///* MAX EFFECTIVE STRAIN INCREMENT=*, F10.7/* PUNCH
STRCH  870      1 HEAD INCREMENT=*, F10.7/* PUNCH HEAD ADJUSTING FACTOR=*, F10.7
STRCH  871      2)
STRCH  872      C
STRCH  873      RETURN
STRCH  874      END

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STRCH  876      SUBROUTINE STIFF(P, Z, UR, UZ, COCE, SLOP, VY, YX, SPHI, CPHI, DL, EPS,
STRCH  877      1 THICK, ALPHA, GAMMA, ETA, FRNCE, FF, A, P, NQ)
STRCH  878      C
STRCH  879      COMMON/GENCCN/NUMNE, NUMEL, HEN(12), DLL, NEG, NFORM, YIELD, TEST, ITRF,
STRCH  880      1 NREAD, NPUNCH, NPRINT, PVALUE, T, MBAND, CNRAD, RADIUS, FRITN,
STRCH  881      2 ECCAST, FNHEC, TOIST
STRCH  882      COMMON/STFMAT/M(6), P(F, F), TEX, TEY, TEZ, THKL
STRCH  883      COMMON/CONQUAD/SS(4), WT(4), D(2, 2), SOPT1
STRCH  884      COMMON/ATOUCH/NTCLCH
STRCH  885      C
STRCH  886      DIMENSION R(1), Z(1), CCCE(1), UR(1), UZ(1), SLOP(1), R(1), A(NQ, 1),
STRCH  887      1 EPS(4, 1), RR(2), Z7(2), UU(6), VY(1), THICK(1), DL(1), SPHI(1), CPHI(1),
STRCH  888      2 YX(1), ALPHA(1), GAMMA(1), ETA(1), FRNCE(1), FF(1)
STRCH  889      C
STRCH  890      DO 50 N=1, NEG
STRCH  891      9(A)=0.
STRCH  892      DO 50 M=1, MBAND
STRCH  893      50 A(N, M)=0.
STRCH  894      C
STRCH  895      WT(1)=0.347E54P451
STRCH  896      WT(2)=0.45214E1549
STRCH  897      WT(3)=WT(1)
STRCH  898      WT(4)=WT(2)
STRCH  899      C
STRCH  900      C
STRCH  901      DO 1000 N=1, NUMEL
STRCH  902      NP1=N+1
STRCH  903      DLL=DL(N)
STRCH  904      SPH=SPHI(N)
STRCH  905      CPH=CPHI(N)
STRCH  906      RR(1)=R(N)
STRCH  907      ZZ(1)=Z(N)
STRCH  908      RR(2)=R(NP1)
STRCH  909      UU(1)=UR(N)
STRCH  910      UU(2)=UZ(N)
STRCH  911      UU(3)=SLOP(N)
STRCH  912      UU(4)=UR(NP1)
STRCH  913      UU(5)=UZ(NP1)
STRCH  914      UU(6)=SLOP(NP1)
STRCH  915      THKL=THICK(N)*DLL

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STRCH  916          77(2)=7(NP1)
STRCH  917          YG=VX(N)
STRCH  918          YH=VY(N)
STRCH  919          D7=Z(NP1)-Z(N)
STRCH  920          C
STRCH  921          CALL QUAD(RR,ZZ,UU,ELL,SPH,CPH,YG,YH)
STRCH  922          C
STRCH  923          C
STRCH  924          C
STRCH  925          C*****
STRCH  926          C PERFORM THE ASSEMBLY OPERATION. BECAUSE MATRIX A IS SYMMETRIC
STRCH  927          C ONLY UPPER HALF OF THE MATRIX IS CREATED. AND THE STORAGE FOR
STRCH  928          C MATRIX A IS A SQUARE ARRAY BECAUSE OF BANNED SYMMETRIC PROPERTY
STRCH  929          C*****
STRCH  930          C
STRCH  931          DO 200 I=1, 6
STRCH  932          II=N*3 - 3 + I
STRCH  933          E(II)=R(II)+(I)
STRCH  934          DO 200 J=1, 6
STRCH  935          JJ=N*3-3+J-II+1
STRCH  936          IF(JJ.LT. 1) GO TO 200
STRCH  937          A(II,JJ)=A(II,JJ)+P(I,J)
STRCH  938          200 CONTINUE
STRCH  939          C
STRCH  940          1000 CONTINUE
STRCH  941          C
STRCH  942          C
STRCH  943          C
STRCH  944          C*****
STRCH  945          C TO HANDLE MIXED BOUNDARY CONDITION, FOLLOWING MATRICES ARE EVALUATED
STRCH  946          C*****
STRCH  947          C
STRCH  948          C=PNRAD-PNHE
STRCH  949          NMP1=NUMNP-1
STRCH  950          C
STRCH  951          DO 1200 N=2,NMP1
STRCH  952          DUM2=R(N)+UR(N)
STRCH  953          DUM1=C+7(N)+UZ(N)
STRCH  954          NF1=N+1
STRCH  955          NM1=N-1
STRCH  956          C DUM3=(2.*R(N)+R(NP1))/3.*DL(N)+(2.*R(N)+R(NM1))/3.*DL(NM1) /2
STRCH  957          C IF(N.EQ. NTOUCH)DUM3=(2.*R(N)+R(NP1))/3.*DL(N)+(3.*R(N)+R(NM1))/
STRCH  958          C 15.*DL(NM1)
STRCH  959          ALPHA(N)=-DUM2/DUM1
STRCH  960          GAMMA(N)=(PNRAD*PNPAC-DUM2*DUM2-DUM1*DUM1)/2./DUM2
STRCH  961          ETA(N)=FPNFC(N)*PNRAD/DUM2
STRCH  962          1200 CONTINUE
STRCH  963          C
STRCH  964          C
STRCH  965          C*****
STRCH  966          C STORE GENERALIZED NODAL FORCE
STRCH  967          C*****
STRCH  968          C
STRCH  969          NUM3=NUMNP*3
STRCH  970          DO 1300 I=1,NUM3
STRCH  971          1300 FF(I)=-P(I)
STRCH  972          C
STRCH  973          C
STRCH  974          1001 FORMAT(///,* THE DIAGONAL VECTOR OF MATRIX OF STIFFNESS*/)
STRCH  975          1002 FORMAT(12E11.3)
STRCH  976          1005 FORMAT(// 29H ELEMENT WITH NEGATIVE AREA =, 15)
STRCH  977          C
STRCH  978          RETURN
STRCH  979          END

STRCH  981          SUBROUTINE QUAD(RR,ZZ,UU,ELL,SPH,CPH,S2,S1)
STRCH  982          C
STRCH  983          COMMON/STEMAT/H(6),P(6,6),TEX,TEY,TF7,THKL
STRCH  984          COMMON/CONQUAD/ES(4),*T(4),D(2,2),SOFT1
STRCH  985          COMMON/ISO/PRVAL
STRCH  986          C
STRCH  987          DIMENSION RR(2),ZZ(2),UU(6),P(2,6),XX(6,6),PZERO(6),DB(2,6)
STRCH  988          C
STRCH  989          PRVALUE=PRVAL
STRCH  990          RC=(PR(1)+PR(2))/2.
STRCH  991          C
STRCH  992          DO 2 I=1,6
STRCH  993          H(I)=0.
STRCH  994          DO 2 K=1, 2
STRCH  995          DO 2 J=1,6
STRCH  996          2 P(I,J)=0.

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STRCH  997  C
STRCH  998  C
STRCH  999  C
STRCH 1000  C7=ZZ(2)-ZZ(1)
STRCH 1001  DR=RR(1)-RR(2)
STRCH 1002  DU=LU(1)-LU(4)
STRCH 1003  DW=UU(5)-UU(2)
STRCH 1004  AU=LU(1)+UU(4)
STRCH 1005  AR=RR(1)+RR(2)
STRCH 1006  C
STRCH 1006  C1=2.*DZ/DLL/DLL
STRCH 1007  C2=2.*DU/DLL/DLL
STRCH 1008  C3=2.*DZ/DLL/DLL
STRCH 1009  C4=2.*DW/DLL/DLL
STRCH 1010  C5=AU/AR/2.
STRCH 1011  C6=1.+DR*C2+DZ*C4+(DU*DU+DW*DW)/DLL/DLL
STRCH 1012  C7=2./DLL/DLL
STRCH 1013  CR=2./AR/AR
STRCH 1014  CG=1./SQRT(C6)/2.
STRCH 1015  C10=C9/C6
STRCH 1016  C11=C1+C2
STRCH 1017  C12=C3+C4
STRCH 1018  C
STRCH 1019  C
STRCH 1020  DES1=SQRT(C6)
STRCH 1021  DET1=2.*C5+1.
STRCH 1022  C
STRCH 1023  E1=C9*C11/DES1
STRCH 1024  E2=-C9*C12/DES1
STRCH 1025  E3=-E1
STRCH 1026  E4=-E2
STRCH 1027  E5=1./AR/DET1
STRCH 1028  F6=(-C10*C11*C11/2.+C9*C7)/DES1-E1*E1
STRCH 1029  E7=-E4
STRCH 1030  E8=C10*C11*C12/2./DES1-E1*E2
STRCH 1031  E9=-E8
STRCH 1032  E10=E8
STRCH 1033  E11=(-C10*C12*C12/2.+C5*C7)/DES1-E2*E2
STRCH 1034  E12=-E5*E5
STRCH 1035  C
STRCH 1036  C
STRCH 1037  C
STRCH 1038  C
STRCH 1039  C*****
STRCH 1040  C COMPUTATION OF EFFECTIVE STRAIN INCREMENT
STRCH 1041  C DES=MERIDIAN STRAIN INCREMENT
STRCH 1042  C DET=CIRCUMFERENTIAL STRAIN INCREMENT
STRCH 1043  C E1=DERIVATIVE OF MERIDIAN STRAIN INCREMENT WITH RESPECT TO UU(1)
STRCH 1044  C E2=D(DES)/D(UU(1))
STRCH 1045  C E3=D(DES)/D(UU(2))
STRCH 1046  C E4=D(DES)/D(UU(4))
STRCH 1047  C E5=D(DES)/D(UU(5))
STRCH 1048  C E6=D(DET)/D(UU(1))
STRCH 1049  C E7=D(E1)/D(UU(1))
STRCH 1050  C E8=D(E1)/D(UU(4))
STRCH 1051  C E9=D(E1)/D(UU(2))
STRCH 1052  C E10=D(E4)/D(UU(2))
STRCH 1053  C E11=D(E4)/D(UU(5))
STRCH 1054  C E12=D(E5)/D(UU(2))
STRCH 1055  C*****
STRCH 1056  C
STRCH 1057  C
STRCH 1058  C DES=ALOG(DES)
STRCH 1059  C DET=ALOG(DET)
STRCH 1060  C RVP1=RVALUE+1.
STRCH 1061  C RVP2=SQRT(2.*RVALUE+1.)
STRCH 1062  C RVP3=RVP1/RVP2
STRCH 1063  C RVP4=2.*RVALUE/RVP1
STRCH 1064  C
STRCH 1065  C*****
STRCH 1065  C EFFECTIVE STRAIN
STRCH 1066  C F1=DERIVATIVE OF EFFECTIVE STRAIN INCREMENT WITH RESPECT TO UU(1)
STRCH 1067  C F2=WITH RESPECT TO UU(2)
STRCH 1068  C F3=WITH RESPECT TO UU(4)
STRCH 1069  C F4=WITH RESPECT TO UU(5)
STRCH 1070  C F11=D(F1)/D(UU(1))
STRCH 1071  C F12=D(F1)/D(UU(2))
STRCH 1072  C F13=D(F1)/D(UU(4))
STRCH 1073  C F14=D(F1)/D(UU(5))
STRCH 1074  C F22=D(F2)/D(UU(2))
STRCH 1075  C F23=D(F2)/D(UU(4))
STRCH 1076  C F24=D(F2)/D(UU(5))
STRCH 1077  C F33=D(F3)/D(UU(4))
STRCH 1078  C F34=D(F3)/D(UU(5))
STRCH 1079  C F44=D(F4)/D(UU(5))
STRCH 1080  C
STRCH 1081  C*****

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STRCH 1082 C
STRCH 1083 EFS=DES*DES+DET*DET+RVP4*DES*EF
STRCH 1084 EFS1=RVP3*SQRT(EFS)
STRCH 1085 EFS2=RVP3/SQRT(EFS1)/2.
STRCH 1086 EFS3=-RVP3/EFS/SQRT(EFS1)/A.
STRCH 1087 C
STRCH 1088 C
STRCH 1089 D1=(2.*DES+RVP4*DET)*E1+(2.*DET+RVP4*DES)*EF
STRCH 1090 D2=(2.*DES+RVP4*DET)*E2
STRCH 1091 D3=(2.*DES+RVP4*DET)*E3+(2.*DET+RVP4*DES)*EF
STRCH 1092 D4=(2.*DES+RVP4*DET)*E4
STRCH 1093 C
STRCH 1094 F1=EFS2*D1
STRCH 1095 F2=EFS2*D2
STRCH 1096 F3=EFS2*D3
STRCH 1097 F4=EFS2*D4
STRCH 1098 C
STRCH 1099 F11=EFS3*D1*D1+EFS2*(2.*DES+RVP4*DET)*E6+(2.*DET+RVP4*DES)*E12
STRCH 1100 1+(2.*F1+RVP4*EF)*E1+(2.*E5+RVP4*E1)*E5
STRCH 1101 F12=EFS3*D1*D2+EFS2*(2.*DES+RVP4*DET)*E9+(2.*E1+RVP4*EF)*E2
STRCH 1102 F13=EFS3*D1*D3+EFS2*(2.*DES+RVP4*DET)*E7+(2.*E3+RVP4*EF)*E1
STRCH 1103 1+(2.*E5+RVP4*E1)*E5+(2.*DET+RVP4*DES)*E12
STRCH 1104 F14=EFS3*D1*D4+EFS2*(2.*DES+RVP4*DET)*E9+(2.*E1+RVP4*EF)*E4
STRCH 1105 F22=EFS3*D2*D2+EFS2*(2.*DES+RVP4*DET)*E11+(2.*E2*E2)
STRCH 1106 F23=EFS3*D2*D3+EFS2*(2.*DES+RVP4*DET)*E9+(2.*E3+RVP4*EF)*E2
STRCH 1107 F24=EFS3*D2*D4+EFS2*(2.*DES+RVP4*DET)*E11+(2.*E2*E2)
STRCH 1108 F33=EFS3*D3*D3+EFS2*(2.*DES+RVP4*DET)*E6+(2.*E3+RVP4*EF)*E3+(
STRCH 1109 12.*E5+RVP4*E1)*E5+(2.*DET+RVP4*DES)*E12
STRCH 1110 F34=EFS3*D3*D4+EFS2*(2.*DES+RVP4*DET)*E10+(2.*E3+RVP4*EF)*E4
STRCH 1111 F44=EFS3*D4*D4+EFS2*(2.*DES+RVP4*DET)*E11+(2.*E4*E4)
STRCH 1112 C
STRCH 1113 P(1,1)=(S1+S2*EFS1)*F11+S2*F1*E1)*RC*THKL
STRCH 1114 P(1,2)=(S1+S2*EFS1)*F12+S2*F2*E1)*RC*THKL
STRCH 1115 P(1,4)=(S1+S2*EFS1)*F13+S2*F1*E3)*RC*THKL
STRCH 1116 P(1,5)=(S1+S2*EFS1)*F14+S2*F1*E4)*RC*THKL
STRCH 1117 P(2,2)=(S1+S2*EFS1)*F22+S2*F2*E2)*RC*THKL
STRCH 1118 P(2,4)=(S1+S2*EFS1)*F23+S2*F2*E3)*RC*THKL
STRCH 1119 P(2,5)=(S1+S2*EFS1)*F24+S2*F2*E4)*RC*THKL
STRCH 1120 P(4,4)=(S1+S2*EFS1)*F33+S2*F3*E3)*RC*THKL
STRCH 1121 P(4,5)=(S1+S2*EFS1)*F34+S2*F3*E4)*RC*THKL
STRCH 1122 P(5,5)=(S1+S2*EFS1)*F44+S2*F4*E4)*RC*THKL
STRCH 1123 C(2,1)=P(1,2)
STRCH 1124 P(4,1)=P(1,4)
STRCH 1125 P(4,2)=P(2,4)
STRCH 1126 P(5,1)=P(1,5)
STRCH 1127 P(5,2)=P(2,5)
STRCH 1128 P(5,4)=P(4,5)
STRCH 1129 C
STRCH 1130 H(1)=-S1+S2*EFS1)*F1)*RC*THKL
STRCH 1131 H(2)=-S1+S2*EFS1)*F2)*RC*THKL
STRCH 1132 H(4)=-S1+S2*EFS1)*F3)*RC*THKL
STRCH 1133 H(5)=-S1+S2*EFS1)*F4)*RC*THKL
STRCH 1134 C
STRCH 1135 C
STRCH 1136 71 CONTINUE
STRCH 1137 RETURN
STRCH 1138 END

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STRCH 1140 SUPROUTINE CONDEN(A,P,NEQ,MBAND,N,U)
STRCH 1141 C
STRCH 1142 C*****
STRCH 1143 C PERFORM THE MATRIX CONDENSATION WHEN THE VALUE OF A COMPONENT
STRCH 1144 C X IN AX=B IS SPECIFIED EQUAL TO ZERO
STRCH 1145 C*****
STRCH 1146 C
STRCH 1147 DIMENSION B(NEQ),A(NEQ,1)
STRCH 1148 C
STRCH 1149 DO 250 M=2,MBAND MDD 3
STRCH 1150 K=N-M+1 MDD 4
STRCH 1151 IF(K) 235,235,230 MDD 5
STRCH 1152 230 R(K)=R(K)-A(K,M)*U MDD 6
STRCH 1153 A(K,M)=0.0 MDD 7
STRCH 1154 235 K=N-M-1 MDD 8
STRCH 1155 IF(NEQ-K) 250,240,240 MDD 9
STRCH 1156 240 B(K)=E(K)-A(N,M)*U MDD 10
STRCH 1157 A(N,M)=0.0 MDD 11
STRCH 1158 250 CONTINUE MDD 12
STRCH 1159 A(N,1)=1.0 MDD 13
STRCH 1160 B(N)=U MDD 14
STRCH 1161 RETURN MDD 15
STRCH 1162 C MDD 16
STRCH 1163 END MDD 17

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STRCH 1165      SUBROUTINE MODIFY(CODE,A,P,ALPHA,GAMMA,ETA,NUMNF,NEG,MRAND,FRNFCE)
STRCH 1166      C
STRCH 1167      DIMENSION CODE(1),A(NEG,1),R(1),ALPHA(1),GAMMA(1),ETA(1),FRNFCE(1)
STRCH 1168      C
STRCH 1169      DO 121 I=1, NUMNF
STRCH 1170          IL=3*I
STRCH 1171          IZ=IL-1
STRCH 1172          IP=IZ-1
STRCH 1173          C=CODE(I)
STRCH 1174          IF (C.EQ. 1.) GO TO 101
STRCH 1175          IF (C.EQ. 2.) GO TO 102
STRCH 1176          IF (C.EQ. 3.) GO TO 103
STRCH 1177          CALL CONDEN(A,R,NEG,MRAND,IL,C)
STRCH 1178          CONSTA=ALPHA(I)
STRCH 1179          CONSTE=ETA(I)
STRCH 1180          CONSTG=GAMMA(I)
STRCH 1181          IF (C.EQ. 4) GO TO 104
STRCH 1182          GO TO 121
STRCH 1183      C
STRCH 1184      101 CONTINUE
STRCH 1185          CALL CONDEN(A,R,NEG,MRAND,IP,C)
STRCH 1186          CALL CONDEN(A,R,NEG,MRAND,IL,0.)
STRCH 1187          GO TO 121
STRCH 1188      C
STRCH 1189      102 CONTINUE
STRCH 1190          CALL CONDEN(A,R,NEG,MRAND,IZ,C)
STRCH 1191          CALL CONDEN(A,R,NEG,MRAND,IL,0.)
STRCH 1192          GO TO 121
STRCH 1193      C
STRCH 1194      103 CONTINUE
STRCH 1195          CALL CONDEN(A,R,NEG,MRAND,IP,C)
STRCH 1196          CALL CONDEN(A,R,NEG,MRAND,IZ,0.)
STRCH 1197          CALL CONDEN(A,R,NEG,MRAND,IL,0.)
STRCH 1198          GO TO 121
STRCH 1199      C
STRCH 1200      104 CONTINUE
STRCH 1201          CALL RMIX(A,R,NEG,MRAND,I,CONSTA,CONSTE,CONSTG)
STRCH 1202          121 CONTINUE
STRCH 1203      C
STRCH 1204          RETURN
STRCH 1205          END

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STRCH 1207      SUBROUTINE TRIA(NN,MM,A)
STRCH 1208      DIMENSION A(NN,1)
STRCH 1209      C
STRCH 1210      C*****
STRCH 1211      C TRIANGULIZATION OF GAUSSIAN ELIMINATION FOR THE SOLUTION
STRCH 1212      C OF Banded SYMMETRIC MATRIX
STRCH 1213      C*****
STRCH 1214      C
STRCH 1215          N=0
STRCH 1216      100  N=N+1
STRCH 1217          IF(N.EQ.NN)RETURN
STRCH 1218          IF(A(N,1).NE.C.) GO TO 150
STRCH 1219          GO TO 100
STRCH 1220      C
STRCH 1221      150  I=N
STRCH 1222          MR=MIN0(MM,NN-N+1)
STRCH 1223      C
STRCH 1224          DO 260 L=2,MR
STRCH 1225              I=I+1
STRCH 1226              C=A(N,L)/A(N,1)
STRCH 1227              IF(C.EQ.0.0)GO TO 260
STRCH 1228              J=0
STRCH 1229              DO 250 K=L,MR
STRCH 1230                  J=J+1
STRCH 1231              250  A(I,J)=A(I,J)-C*A(N,K)
STRCH 1232                  A(N,L)=C
STRCH 1233              260  CONTINUE
STRCH 1234      C
STRCH 1235          GO TO 100
STRCH 1236      C
STRCH 1237      C
STRCH 1238          ENC

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STRCH 1240      SUPROUTINE BACKS(NN,MM,A,P)
STRCH 1241      C
STRCH 1242      C
STRCH 1243      C*****
STRCH 1244      C   BACK SUBSTITUTION FOR SOLUTION OF BANDED SYMMETRIC MATRIX
STRCH 1245      C*****
STRCH 1246      C
STRCH 1247      C
STRCH 1248      C
STRCH 1249      C   DIMENSION A(1),P(1)
STRCH 1250      C
STRCH 1251      C   MM=MM-1
STRCH 1252      C   N=N
STRCH 1253      C   N=N+1
STRCH 1254      C   C=P(N)
STRCH 1255      C   IF(A(N),NE,0.0)IF(N)=P(N)/A(N)
STRCH 1256      C   IF(N.F0,NN)GO TO 300
STRCH 1257      C   IL=N+1
STRCH 1258      C   IM=MIN0(NN,N+MM)
STRCH 1259      C   M=N
STRCH 1260      C
STRCH 1261      C   DO 295 I=IL,IM
STRCH 1262      C   M=M+NN
STRCH 1263      C   295 B(I)=P(I)-A(M)*C
STRCH 1264      C
STRCH 1265      C   GO TO 270
STRCH 1266      C
STRCH 1267      C   300 IL=N
STRCH 1268      C   N=N-1
STRCH 1269      C   IF(N.E0.0) RETURN
STRCH 1270      C   IM=MIN0(NN,N+MM)
STRCH 1271      C   M=N
STRCH 1272      C
STRCH 1273      C   DO 400 I=IL,IM
STRCH 1274      C   M=M+NN
STRCH 1275      C   400 B(N)=B(N)-A(M)*P(I)
STRCH 1276      C
STRCH 1277      C   GO TO 300
STRCH 1278      C   END

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STRCH 1280      SUPROUTINE BCMIX(A,P,NEG,MPAND,N,CONSTA,CONSTB,CONSTC,CONSTG)
STRCH 1281      C
STRCH 1282      C*****
STRCH 1283      C   THIS SUBROUTINE HANDLES THE MIXED BOUNDARY CONDITION
STRCH 1284      C*****
STRCH 1285      C
STRCH 1286      C   DIMENSION A(NE0,1),P(1)
STRCH 1287      C   IR=3*N-2
STRCH 1288      C   I7=IR+1
STRCH 1289      C   A(I7,1)=A(I7,1)+2.*A(IR,2)/CONSTA+A(IR,1)/CONSTA/CONSTA
STRCH 1290      C   B(I7,1)=B(I7,1)+B(IR)/CONSTA-CONSTG*(A(IR,1)/CONSTA+A(IR,2))
STRCH 1291      C   B(I7,1)=B(I7,1)+CCNSTE
STRCH 1292      C   A(IR,1)=1.0
STRCH 1293      C   A(IR,2)=0.0
STRCH 1294      C   B(IR)=0.0
STRCH 1295      C
STRCH 1296      C   DO 100 M=3,MPAND
STRCH 1297      C   K=I7-M+1
STRCH 1298      C   IF(K.LT. 1)GO TO 100
STRCH 1299      C   MM=M-1
STRCH 1300      C   A(K,M)=A(K,M)+A(K,MM)/CONSTA
STRCH 1301      C   B(K)=B(K)-A(K,MM)*CCNSTG
STRCH 1302      C   100 A(K,MM)=0.0
STRCH 1303      C
STRCH 1304      C   MB1=MPAND-1
STRCH 1305      C
STRCH 1306      C   DO 200 M=2,MB1
STRCH 1307      C   MM=M+1
STRCH 1308      C   A(I7,M)=A(I7,M)+A(IR,MM)/CONSTA
STRCH 1309      C   I77=IR+M
STRCH 1310      C   P(I77)=P(I77)-A(IR,MM)*CONSTG
STRCH 1311      C   200 A(IR,MM)=0.0
STRCH 1312      C
STRCH 1313      C   RETURN
STRCH 1314      C   END

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STRCH 1316      SUPROUTINE HARD(EPS,Y)

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STPCH 1317 C
STPCH 1318 C*****
STPCH 1319 C   WORKHARDENING CHARACTERISTIC CURVE
STPCH 1320 C*****
STPCH 1321 C
STPCH 1322 C   COMMON/MATERL/YVALUE,PFESTN,EXFNT,PRESTS
STPCH 1323 C
STPCH 1324 C   Y=YVALUE*(PRESTN+EPS)**EXFNT+PFESTS
STPCH 1325 C
STPCH 1326 C   RETURN
STPCH 1327 C   END

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STRCH 1329 SUBROUTINE HARD2(EPS,Y)
STRCH 1330 C
STRCH 1331 C*****
STRCH 1332 C   COMPUTE WORK HARDENING RATE
STRCH 1333 C*****
STRCH 1334 C
STRCH 1335 C   COMMON/MATERL/YVALUE,PFESTN,EXFNT,PRESTS
STRCH 1336 C
STRCH 1337 C   Y=EXFNT*YVALUE*(PRESTN+EPS)**(EXFNT-1.)
STRCH 1338 C
STRCH 1339 C   RETURN
STRCH 1340 C   END

```

APPENDIX D

PROGRAM FOR THE ANALYSIS OF DEEP DRAWING AND
PUNCH STRETCHING WITH ROUND DIE CORNER

This program is for the analysis of deep drawing with a hemispherical punch head and stretching with a hemispherical punch head. In stretching, a round die profile is considered.

(I) Data preparation card

1. Read HED (A 12)
2. Read RVALUE, T, ACOEF (5F 10.0)
3. Read ITER, NREAD, ITCONT, NFORM, NPUNCH, NPRINT, FLIMIT (6I5, F10.0)
4. Read NUMNP, NDEX (6I5)
NDEX: 2, if punch stretching is to be analyzed
3, if deep drawing is to be analyzed
5. Read PNRAD, RADIUS, DIERAD, RTART (4F 10.0)
DIERAD: Radius of the die profile
RTHRT: Distance from the pole to the die throat
6. Read FRITNP, FRITND, BHFCE (4F 10.0)
FRITNP: Friction coefficient between the punch head and the blank
FRITND: Friction coefficient between the die and the blank
BHFCE: Blank holding force
Set 0.0 for punch stretching problem
7. Read YVALUE, PRESTN, EXPNT, PRESTS (4F 10.0)
8. Read TCONTC, TDIST, ECONST (4F 10.0)
TCONTC: Criterion distance of the contact with the die profile
To start with, set this 0.002
9. Read N, CODE(N), R(N), Z(N), UR(N), UZ(N), SLOP(N), (15, F5.0, 5F 10.0)
10. If NREAD = 1, the new input data is to be placed behind the nodal information card.

```

SHEET 1      PROGRAM SHEET (INPUT,OUTPUT,TAPE 5=INPUT,TAPE 6=OUTPUT,PUNCH)
SHEET 2      COMMON/GENCON/NUMNP,NUMEL,HED(12),PLL,NEQ,NFORM,YIELD,TEST,ITER,
SHEET 3      INPAD,NPUNCH,NPRINT,PVALUE,T,MRAD,PNRAD,RADIUS,FRITNP,FRITND,
SHEET 4      2ECONST,RNHED,RTHRT,DIERAD,ICONTC,TDIST,RHFCF
SHEET 5      C
SHEET 6      C*****
SHEET 7      C      PROGRAM BOTH FOR PUNCH STRETCHING WITH ROUND PROFILE AND
SHEET 8      FOR DEEP DRAWING, BY J.H.KIM
SHEET 9      C*****
SHEET 10     C
SHEET 11     COMMON/WATERL/YVALUF,PRESTN,EXPNT,PRESTS
SHEET 12     COMMON/ISOTPY/PVAL1
SHEET 13     C
SHEET 14     C
SHEET 15     C*****
SHEET 16     C      PROGRAM SHEET IS FOR CONTROLLING THE DIMENSION OF THE COMPLETE
SHEET 17     C      PROGRAM. ITS PURPOSE IS TO PREVENT ASSIGNING A LARGER THAN
SHEET 18     C      NECESSARY DIMENSION FOR ANY ARRAY THROUGH THE USE OF THE
SHEET 19     C      FOLLOWING STATEMENT
SHEET 20     C*****
SHEET 21     C
SHEET 22     COMMON A(5000)
SHEET 23     C
SHEET 24     C
SHEET 25     C*****
SHEET 26     C      NFIELD IS THE DIMENSION OF ARRAY A. ITS VALUE CAN BE DETERMINED
SHEET 27     C      PRECISELY BY RUNNING THE PROGRAM ONCE.
SHEET 28     C*****
SHEET 29     C
SHEET 30     NFIELD=5000
SHEET 31     C
SHEET 32     C
SHEET 33     C*****
SHEET 34     C      READ THE INPUT DATA CONTROL CARDS
SHEET 35     C*****
SHEET 36     C
SHEET 37     TEST=1.
SHEET 38     READ(5,1000) HED
SHEET 39     READ(5,1004) PVALUE,T,ACDEF
SHEET 40     READ(5,1003) ITER,NPAD,ITCNT,NFORM,NPUNCH,NPRINT,FLIMIT
SHEET 41     READ(5,1003) NUMNP,NDEX
SHEET 42     READ(5,1004) PNRAD,RADIUS,DIERAD,RTHRT
SHEET 43     READ(5,1004) FRITNP,FRITND,RHFCF
SHEET 44     READ(5,1004) YVALUF,PRESTN,EXPNT,PRESTS
SHEET 45     READ(5,1004) ICONTC,TDIST,ECONST
SHEET 46     C
SHEET 47     C*****
SHEET 48     C      HED=OUTPUT TITLE
SHEET 49     C      PVALUE=VALUE OF THE ANISOTROPY PARAMETER
SHEET 50     C      ACDEF=ACCELERATING OR DECELERATING COEFFICIENT OF CONVERGENCE
SHEET 51     C      NPAD=0, IF TO BYPASS THE READING STATEMENT IN SUBROUTINE PLAST
SHEET 52     C      ITCNT=0, IF COMPUTATION STARTS AT THE VERY BEGINNING AND FIRST/
SHEET 53     C      SECOND STEPS ARE INCLUDED IN THE STEPS TO BE COMPUTED
SHEET 54     C      =1, OTHERWISE
SHEET 55     C      THIS INDEX IS RELATED TO THE DETERMINATION OF STEP SIZE
SHEET 56     C      NFORM=NUMBER OF STEPS ASSIGNED PER RUN
SHEET 57     C      NPUNCH=1, IF DATA ARE TO BE PUNCHED
SHEET 58     C      =0, OTHERWISE
SHEET 59     C      FLIMIT=VALUE OF (ERROR NORM)/(SOLUTION NORM) REQUIRED
SHEET 60     C      FOR CONVERGENCE
SHEET 61     C      NPRINT=1, IF NODAL POINT DATA ARE TO BE PRINTED
SHEET 62     C      =0, OTHERWISE
SHEET 63     C      NUMNP=NUMBER OF NODAL POINTS
SHEET 64     C      PNRAD=RADIUS OF HEMISPHERICAL PUNCH HEAD
SHEET 65     C      RADIUS=RADIUS OF THE BLANK
SHEET 66     C      FRITND=FRICITION COEFFICIENT BETWEEN BLANK AND PUNCH
SHEET 67     C      FRITNP=FRICITION BETWEEN BLANK AND DIE PROFILE
SHEET 68     C      DIERAD=RADIUS OF DIE PROFILE
SHEET 69     C      RTHRT=RADIUS OF DIE TROAT
SHEET 70     C      ECONST=STEP SIZE IN MAXIMUM EFFECTIVE STRAIN INCREMENT
SHEET 71     C      NDEX=2, IF PUNCH STRETCHING WITH ROUND PROFILE
SHEET 72     C      =3, IF DEEP DRAWING
SHEET 73     C
SHEET 74     C
SHEET 75     C      YVALUF, PRESTN, EXPNT, PRESTS ARE TO EXPRESS THE WORKHARDENING
SHEET 76     C      CHARACTERISTICS OF THE BLANK
SHEET 77     C      STRESS=YVALUF*(PRESTN+STRAIN)**EXPNT+PRESTS

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SHEET 78 C
SHEET 79 C
SHEET 80 C NFG=NUMBER OF EQUATIONS TO BE SOLVED
SHEET 81 C NUMEL=NUMBER OF ELEMENTS
SHEET 82 C MRAND=RAND WIDTH
SHEET 83 C
SHEET 84 C*****
SHEET 85 C
SHEET 86 C RVAL1=RVALUE
SHEET 87 C NUMEL=NUMNP-1
SHEET 88 C MRAND=C
SHEET 89 C NEQ=NUMNP*3
SHEET 90 C NQ=NEQ
SHEET 91 C NEL=NUMEL
SHEET 92 C
SHEET 93 C*****
SHEET 94 C DETERMINE THE LOCATION OF THE STARTING POINTS OF DIFFERENT
SHEET 95 C ARRAYS ON ARRAY A
SHEET 96 C*****
SHEET 97 C
SHEET 98 C
SHEET 99 C
SHEET 100 C N1=1
SHEET 101 C N2=N1+NUMNP
SHEET 102 C N3=N2+NUMNP
SHEET 103 C N4=N3+NUMNP
SHEET 104 C N5=N4+NUMNP
SHEET 105 C N6=N5+NUMNP
SHEET 106 C N7=N6+NUMNP
SHEET 107 C NR=N7+NUMEL
SHEET 108 C N9=NR+NUMEL
SHEET 109 C N10=N9+NUMEL
SHEET 110 C N11=N10+NUMEL
SHEET 111 C N12=N11+NUMEL*3
SHEET 112 C N13=N12+NUMEL*4
SHEET 113 C N14=N13+NUMEL*4
SHEET 114 C N15=N14+NUMEL*4
SHEET 115 C N16=N15+NEQ
SHEET 116 C N17=N16+NEQ*MEAND
SHEET 117 C N18=N17+NUMEL
SHEET 118 C N19=N18+NUMNP
SHEET 119 C N20=N19+NUMNP
SHEET 120 C N21=N20+NUMNP
SHEET 121 C N22=N21+NUMNP
SHEET 122 C N23=N22+NUMNP
SHEET 123 C N24=N23+NEQ
SHEET 124 C N25=N24+NUMNP
SHEET 125 C N26=N25+NUMNP
SHEET 126 C N27=N26+NUMNP
SHEET 127 C N28=N27+NUMNP
SHEET 128 C
SHEET 129 C CALL PRELIM(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6))
SHEET 130 C IF(N28.LE.NFIELD) GO TO 100
SHEET 131 C
SHEET 132 C
SHEET 133 C WRITE(6,1002) N28
SHEET 134 C
SHEET 135 C*****
SHEET 136 C N2P IS TOTAL SPACE REQUIRED. IF NFIELD, THE INITIALLY ASSIGNED
SHEET 137 C SPACE, IS NOT ENOUGH, STOP THE PROGRAM.
SHEET 138 C*****
SHEET 139 C
SHEET 140 C STOP
SHEET 141 C 100 CONTINUE
SHEET 142 C
SHEET 143 C CALL PLAST(A(N1),A(N2),A(N3),A(N4),A(N5),A(N6),A(N7),A(N8),A(N9),
SHEET 144 C 1A(N10),A(N11),A(N12),A(N13),A(N14),A(N15),A(N16),A(N17),A(N18),
SHEET 145 C 2A(N19),A(N20),A(N21),A(N22),A(N23),A(N24),A(N25),A(N26),A(N27)
SHEET 146 C 3, NQ,NEL,FLIMIT,ITCONT,ACDEF,NDEX)
SHEET 147 C
SHEET 148 C 1000 FORMAT(12A6)
SHEET 149 C 1001 FORMAT(///# THE DIMENSION OF THE ARRAY (A) IS TOO SMALL#
SHEET 150 C 1# THE SIZE OF THE ARRAY (A) MUST BE #, I7)
SHEET 151 C 1002 FORMAT(///# THE NECESSARY SIZE OF THE ARRAY (A) IS#, I7)
SHEET 152 C 1003 FORMAT(6I5,F10.0)
SHEET 153 C 1004 FORMAT(5F10.0)
SHEET 154 C 1005 FORMAT(4I5,F10.0)
SHEET 155 C
SHEET 156 C STOP
SHEET 157 C END

SHEET 150 C SUBROUTINE PRELIM(R,Z,UR,U7,CCOE,SLOP)
SHEET 160 C COMMON/GENCON/NUMNP,NUMEL,MED(12),OLL,NEQ,NFORN,YIELD,TEST,ITER,
SHEET 161 C 1NPEAR,NPUNCH,NPRINT,PVALUE,T,MRAND,PNRAC,RADIUS,FRITNP,FRITND,

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SHEET 162          2ECONST,PNHED,PTHET,DIERAD,TCOATC,TDIST,PHFCE
SHEET 163          C
SHEET 164          C*****MAIN0
SHEET 165          C   READ AND PRINT OF CONTROL INFORMATION AND MATERIAL PROPERTIES      MAIN0013
SHEET 166          C*****MAIN0014
SHEET 167          C
SHEET 168          C
SHEET 169          C   DIMENSION R(1),Z(1),CCOE(1),UF(1),UZ(1),SLOP(1)
SHEET 170          C
SHEET 171          50 CONTINUE
SHEET 172          WRITE (6,2000) HED,NUMNF,NUMEL
SHEET 173          CALL HADD(0.,YIELD)
SHEET 174          WRITE (6,2010) YIELD
SHEET 175          WRITE (6,2011)
SHEET 176          WRITE (6,2012) RADIUS,PNRAD,DIERAD,PTHET,FRITNP,FRITND
SHEET 177          WRITE (6,2013) YVALUE,PRESTN,EXENT,PREST5
SHEET 178          WRITE (6,2014) ECONST
SHEET 179          WRITE (6,1009) ITER
SHEET 180          C
SHEET 181          C*****MAIN0030
SHEET 182          C   READ AND PRINT OF NODAL POINT DATA      MAIN0031
SHEET 183          C*****MAIN0032
SHEET 184          C
SHEET 185          C
SHEET 186          L=0                                          MAIN0034
SHEET 187          IF (NPRINT.EQ.C) GO TO 60
SHEET 188          WRITE (6,1114)
SHEET 189          WRITE (6,2004)                                          MAIN0033
SHEET 190          60 READ (5,1002) N,CODE(N),P(N),Z(N),UF(N),UZ(N),SLOP(N)
SHEET 191          C
SHEET 192          NL=L+1
SHEET 193          ZX=N-L                                          MAIN0037
SHEET 194          IF (L.EQ.0) GO TO 70
SHEET 195          DR=(R(N)-P(L))/ZX
SHEET 196          DZ=(Z(N)-Z(L))/ZX
SHEET 197          DUR=(UF(N)-UF(L))/ZX
SHEET 198          DUZ=(UZ(N)-UZ(L))/ZX
SHEET 199          DS=(SLOP(N)-SLOP(L))/ZX
SHEET 200          70 L=L+1                                          MAIN0041
SHEET 201          C
SHEET 202          IF (N-L) 100,90,80
SHEET 203          C
SHEET 204          80 CODE(L)=0.0
SHEET 205          R(L)=P(L-1)+DR
SHEET 206          SLOP(L)=SLOP(L-1)+DS
SHEET 207          Z(L)=Z(L-1)+DZ
SHEET 208          UR(L)=UF(L-1)+DUR
SHEET 209          UZ(L)=UZ(L-1)+DUZ
SHEET 210          GO TO 70                                          MAIN0045
SHEET 211          C
SHEET 212          90 IF (NUMNF-N) 100,110,60
SHEET 213          100 WRITE (6,2009) N
SHEET 214          CALL EXIT                                          MAIN0053
SHEET 215          110 CONTINUE
SHEET 216          C
SHEET 217          WRITE (6,2002) (K,CODE(K),P(K),Z(K),UF(K),UZ(K),SLOP(K),K=1,NUMNP)
SHEET 218          C
SHEET 219          C
SHEET 220          NEQ=3*NUMNP
SHEET 221          WRITE (6,1122) NEQ,MEANC
SHEET 222          C
SHEET 223          C*****MAIN0116
SHEET 224          100? FORMAT(15,F5.0,5F10.0)
SHEET 225          1003 FORMAT(16I5)
SHEET 226          1004 FORMAT(1P,2I11,2F10.6)
SHEET 227          1005 FORMAT(2I5,4F10.0)
SHEET 228          1006 FORMAT(// * THE NODAL POINTS AT WHICH FORCE CALCULATIONS ARE DESIR
SHEET 229          1E0* // 2015)
SHEET 230          1007 FORMAT(1H1,15X, 39H LINEARLY DISTRIBUTED BOUNDARY STRESSES/
SHEET 231          1 / 60H NODE I NODE J PRESSURE I PRESSURE J SHEAR I
SHEET 232          2 14H SHEAR J)
SHEET 233          1008 FORMAT(2I9,4E15.5)
SHEET 234          1009 FORMAT(/// * MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR EACH INCREMEN
SHEET 235          1T =*, 13)
SHEET 236          1114 FORMAT(1H1, 39H NODAL POINT INFORMATION BEFORE SCALING//)
SHEET 237          1122 FORMAT(/// * NUMBER OF EQUATIONS =*, 1A/
SHEET 238          1 * PANDWIDTH
SHEET 239          2 * DIAGONAL ELEMENTS =*, 1A 1)
SHEET 240          2000 FORMAT (1H 12A6/
SHEET 241          1 30H0 NUMBER OF NODAL POINTS----- 13 /
SHEET 242          2 30H0 NUMBER OF ELEMENTS----- 13 /)
SHEET 243          2002 FORMAT (112,F12.2,2F12.3,3E24.7)
SHEET 244          2003 FORMAT (1113,4I6,1112)
SHEET 245          2004 FORMAT (/ * NODAL POINT TYPE R-ORDINATE Z-ORDINATE R LO
SHEET 246          1AD OR DISPLACEMENT Z LOAD OR DISPLACEMENT BETA-SLOPE #1

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SHEET 247 2005 FORMAT(//,*FORCES SPECIFIED AT NODAL POINT*,//,
SHEET 248 1 * NODAL PT. ELEMENT1 ELEMENT2 PRESSURE SHEAR*,/)
SHEET 249 2009 FORMAT (26H NODAL POINT CARD ERROR N= 15)
SHEET 250 2010 FORMAT(// * INITIAL YIELD STRESS = *, F15.7//)
SHEET 251 2011 FORMAT(// * SPECIFICATIONS OF THE PROBLEM*//)
SHEET 252 2012 FORMAT(* RADIUS OF PUNCH IS *,F10.5/
SHEET 253 1 * RADIUS OF PUNCH HEAD *,F10.5/
SHEET 254 2 * RADIUS OF DIE PROFILE *,F10.5/
SHEET 255 3 * DISTANCE FROM POLE TO THEAT*,F10.5/
SHEET 256 4 * FRICTION COEFFICIENT OVER PUNCH HEAD*,F10.5/
SHEET 257 5 * FRICTION COEFFICIENT OVER DIE*,F10.5/
SHEET 258 6 * WORKHARDENING CHARACTERISTICS*/
SHEET 259 2013 FORMAT(*STRESS=*,F10.5,*TIMES(*,F10.5,* STRAIN)EXP(*,FE.3,
SHEET 260 1*)*,F10.3//)
SHEET 261 2014 FORMAT(// * STEP SIZE IS*,F10.5/)
SHEET 262 RETURN
SHEET 263 END

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SHEET 265 SUBROUTINE PLAST(R,7,UR,U7,CODE,SLOP,YY,YX,SPHI,CPHI,DL,STS,TEPS,
SHEET 266 1EPS,R,A,THICK,ALPHA,GAMMA,ETA,FRNFC,PHI,FF,TOUCH,CONTAC,UR,UU7,
SHEET 267 2NG,NEL,FLIMIT,ITCNT,ACCF,NDEX)
SHEET 268 C
SHEET 269 C*****
SHEET 270 C PLAST IS THE CONTROLLING SUBROUTINE
SHEET 271 C*****
SHEET 272 C
SHEET 273 COMMON/GENCON/NUMNP,NUMEL,HED(12),DLL,NEG,NFCFM,YIELC,TEST,ITEF,
SHEET 274 1NREAD,NPUNCH,NPRINT,EVALUE,T,MRAND,PNFAD,RADIUS,FRITNP,FRITND,
SHEET 275 2ECONST,PNMED,FTMPT,DIERAD,TCNTC,TDIST,RHFCF
SHEET 276 COMMON/CONQUAC/SS(4),WT(4),D(2,2),SOFT1
SHEET 277 COMMON/ATOUCH/NTOUCH,NDIE,NCONTC
SHEET 278 C
SHEET 279 DIMENSION R(1),Z(1),UR(1),U7(1),CODE(1),SLOP(1),YY(1),YX(1),
SHEET 280 1TEPS(4,1),R(1),A(NG,1),THICK(1),CPHI(1),SPHI(1),DL(1),EPS(4,1),
SHEET 281 2ALPHA(1),GAMMA(1),ETA(1),FRNFC(1),PHI(1),FF(1),CCNTAC(1),TOUCH(1),
SHEET 282 3,STS(3,1),UUR(1),UU7(1)
SHEET 283 C
SHEET 284 C*****
SHEET 285 C THE FIRST NODE IS LOCATED AT THE FIM OF THE PUNCH
SHEET 286 C AND THE POLE IS THE LAST NODE
SHEET 287 C*****
SHEET 288 C
SHEET 289 C
SHEET 290 C*****
SHEET 291 C A1,A2 ARE CONSTANTS RELATED TO DETERMINATION OF ACCF
SHEET 292 C PNMED=PRESENT POSITION OF PUNCH HEAD
SHEET 293 C NTOUCH=FIRST NODAL POINT IN CONTACT WITH PUNCH HEAD
SHEET 294 C NCHECK=NUMBER OF NODES FOR WHICH THE CONTACTING WILL BE CHECKED
SHEET 295 C TCHCOF=0, IF BOUNDARY IS TO ADVANCE
SHEET 296 C =1, OTHERWISE
SHEET 297 C*****
SHEET 298 C
SHEET 299 PNMED=7(NUMNP)
SHEET 300 NCHECK=NUMEL/10
SHEET 301 NUW1=NUMNP-1
SHEET 302 NTOUCH=NUMNP
SHEET 303 TCHCOF=0.0
SHEET 304 EFACT=1.0
SHEET 305 TEST=0.0
SHEET 306 TEST=1.0
SHEET 307 C
SHEET 308 DO 350 N=1,NUMNP
SHEET 309 IF(R(N).GE.(PTMET+DIERAD))NDIE=N
SHEET 310 350 IF(R(N).GT. FTTRY)NTHRT=N
SHEET 311 NCONTC=NDIE
SHEET 312 C
SHEET 313 SS(1)=0.861136311E
SHEET 314 SS(2)=0.334981043E
SHEET 315 SS(3)=-SS(1)
SHEET 316 SS(4)=-SS(2)
SHEET 317 C
SHEET 318 DO 442 N=1, NUMEL
SHEET 319 FRNFC(N)=0.0
SHEET 320 THICK(N)=T
SHEET 321 DO 442 I=1, 4
SHEET 322 442 TEPS(I,N)=0.
SHEET 323 C
SHEET 324 C*****
SHEET 325 C TO HANDLE THE INFINITE WORKHARDENING RATE INITIALLY VERY
SHEET 326 C SMALL VALUE IS ASSIGNED TO THE EFFECTIVE STRAIN
SHEET 327 C*****

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SHEET 328 C
SHEET 329 DO 450 N=1,NUMFL
SHEET 330 450 TEPS(4,N)=0.0001
SHEET 331 C
SHEET 332 FRNFCE(NTOUCH)=.001
SHEET 333 FRNFCE(1)=FRNFCE
SHEET 334 DO 360 N=NDIE,NCNTC
SHEET 335 360 FRNFCE(N)=.0001
SHEET 336 C
SHEET 337 DPNSTR=C.1
SHEET 338 DPNHED=UZ(NUMNP)
SHEET 339 ESTAR=1.0
SHEET 340 C
SHEET 341 C*****
SHEET 342 C IF THE COMPUTATION IS INTERRUPTED AFTER A NUMBER OF STEPS
SHEET 343 C AND RESTARTED, THEN NECESSARY DATA NEED BE FEED
SHEET 344 C*****
SHEET 345 C
SHEET 346 IF(NREAD .LE. 0) GC TC 440
SHEET 347 READ(5,1017)(UR(I),UZ(I),CODE(I),I=1,NUMNP)
SHEET 348 READ(5,1017) ( R(I), Z(I), I=1, NUMNP)
SHEET 349 READ(5,1017)((TEPS(I,N),I=1,4), N=1, NUMEL)
SHEET 350 READ(5,1017)(THICK(N),N=1,NUMEL)
SHEET 351 READ(5,2223)FRNHED,NTOUCH,TCHCCF,EFACT,NCNTC
SHEET 352 READ(5,235)(FRNFCE(N),N=1,NUM1)
SHEET 353 READ(5,233)ESTAR,DPNSTR,DPNHED,TEST
SHEET 354 440 CONTINUE
SHEET 355 C
SHEET 356 NSTEP=0
SHEET 357 TCONT2=TCONTC
SHEET 358 TOUCH2=TDIST
SHEET 359 C
SHEET 360 2100 NSTEP=NSTEP+1
SHEET 361 C
SHEET 362 C*****
SHEET 363 C BOUNDARY NODE NDIE IS UPDATED AFTER EVERY STEP
SHEET 364 C*****
SHEET 365 C
SHEET 366 DO 351 N=1,NUMNF
SHEET 367 351 IF(R(N) .GE. (PTHR+DIERAD))NDIE=N
SHEET 368 IF(FRNFCE(NDIE) .EQ. 0.)FRNFCE(NDIE)=.0001
SHEET 369 C
SHEET 370 TCONT3=TCONT2
SHEET 371 TOUCH3=TOUCH2
SHEET 372 C
SHEET 373 DO 247 I=1,NCHECK
SHEET 374 CONTAC(I)=0.0
SHEET 375 247 TOUCH(I)=0.
SHEET 376 IF(TEST .EQ. 0.)GC TC 2101
SHEET 377 C
SHEET 378 C*****
SHEET 379 C DPNHED=ASSIGNED INCRFMENT OF FUNCH HEAD TRAVEL
SHEET 380 C ESTAR=ADJUSTING FACTOR
SHEET 381 C*****
SHEET 382 C
SHEET 383 PNHED=PNHED+DPNHED
SHEET 384 C
SHEET 385 DO 445 I=1,NUMNP
SHEET 386 UR(I)=UR(I)*ESTAR
SHEET 387 UZ(I)=UZ(I)*ESTAR
SHEET 388 445 CONTINUE
SHEET 389 C
SHEET 390 2101 CONTINUE
SHEET 391 IF(TCHCCF .EQ. 1.)GO TO 219
SHEET 392 NTOUCH=NTOUCH-1
SHEET 393 NTCP1=NTOUCH+1
SHEET 394 FRNFCE(NTOUCH)=FRNFCE(NTCP1)/3.
SHEET 395 219 CONTINUE
SHEET 396 446 CONTINUE
SHEET 397 C
SHEET 398 C*****
SHEET 399 C UPDATING OF THE BOUNDARY CONDITION
SHEET 400 C*****
SHEET 401 C
SHEET 402 DO 100 I=1,NUMNP
SHEET 403 CODE(I)=0.0
SHEET 404 IF(I .GE. NTOUCH)CODE(I)=4.0
SHEET 405 IF(I .GT. NDIE .AND. I .LE. NCNTC) CODE(I)=4.0
SHEET 406 IF(I .LE. NDIE) CODE(I)=2.0
SHEET 407 100 CONTINUE
SHEET 408 CODE(NUMNP)=3.0
SHEET 409 CODE(1)=2.0
SHEET 410 IF(NDEX .EQ. 2) CODE(1)=3.0
SHEET 411 C
SHEET 412 NTCM1=NTOUCH-1

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SHEET 413 C
SHEET 414 C
SHEET 415 C*****
SHEET 416 C COMPUTE THE YIELD STRESS AND THE WORKHARDENING RATE
SHEET 417 C*****
SHEET 418 C
SHEET 419 C DC 220 N=1, NUMEL
SHEET 420 CALL HARD(TEPS(4,N),VY(N))
SHEET 421 CALL HARD2(TEPS(4,N),YX(N))
SHEET 422 220 CONTINUE
SHEET 423 C
SHEET 424 C
SHEET 425 C
SHEET 426 C
SHEET 427 C
SHEET 428 C WRITE(6,1007) NSTEP
SHEET 429 C
SHEET 430 C CONV=111111.
SHEET 431 C
SHEET 432 650 CONTINUE
SHEET 433 C WRITE(6,1031)
SHEET 434 C
SHEET 435 C*****
SHEET 436 C DETAIL OF THE PRESENT CONFIGURATION
SHEET 437 C SPHI=SINE OF ANGLE PHI
SHEET 438 C CPHI=COSSINE OF ANGLE PHI
SHEET 439 C DL=ELEMENT LENGTH
SHEET 440 C*****
SHEET 441 C
SHEET 442 C DO 690 N=1, NUMEL
SHEET 443 NP1=N+1
SHEET 444 DR=R(N)-R(NP1)
SHEET 445 DZ=Z(NP1)-Z(N)
SHEET 446 DL(N)=SQRT(DR*DR+DZ*DZ)
SHEET 447 SPHI(N)=DR/DL(N)
SHEET 448 CPHI(N)=DZ/DL(N)
SHEET 449 PHI(N)=ASIN(SPHI(N))*180./3.14159
SHEET 450 WRITE(6,1030)N,PHI(N),THICK(N),DL(N)
SHEET 451 690 CONTINUE
SHEET 452 C
SHEET 453 RP1=RVALUE+1.
SHEET 454 RCONST=3.*RP1/(2.*(1.+RVALUE+RVALUE))
SHEET 455 D(1,1)=RP1*RCONST
SHEET 456 D(1,2)=RVALUE*RCONST
SHEET 457 D(2,1)=D(1,2)
SHEET 458 D(2,2)=D(1,1)
SHEET 459 CLAMDA=RP1*RP1/(1.+RVALUE+RVALUE)
SHEET 460 PDI=DN=RVALUE/RP1
SHEET 461 C
SHEET 462 K=0
SHEET 463 2001 CONTINUE
SHEET 464 K=K+1
SHEET 465 C
SHEET 466 C
SHEET 467 CALL STIFF(R,Z,UP,U7,CCDE,SLOP,YY,YX,SPHI,CPHI,DL,EPS,
SHEET 468 ITHICK,ALPHA,GAMMA,ETA,FRNFCE,FF,A,B,NG)
SHEET 469 C
SHEET 470 C*****
SHEET 471 C INTRODUCTION OF BOUNDARY CONDITION
SHEET 472 C*****
SHEET 473 C
SHEET 474 C
SHEET 475 CALL MODIFY(CCDE,A,R,ALPHA,GAMMA,ETA,NUMNP,NEG,MRAND,FRNFCE)
SHEET 476 C
SHEET 477 C RANDED SYMMETRIC SOLUTION
SHEET 478 C
SHEET 479 CALL TRIA(NEO,MBAND,A)
SHEET 480 CALL BACKS(NEG,MRAND,A,P)
SHEET 481 C
SHEET 482 C
SHEET 483 C*****
SHEET 484 C PERTURBATION OF UP IS COMPUTED FROM PERTURBATION OF U7 FOR NODES
SHEET 485 C*****
SHEET 486 C
SHEET 487 C
SHEET 488 C
SHEET 489 C
SHEET 490 C DO 101 N=1,NUMNP
SHEET 491 IP=7*N-2
SHEET 492 IZ=IR+1
SHEET 493 IF(CODE(N).EQ.4.C) R(IR)=R(IZ)/ALPHA(N)+GAMMA(N)
SHEET 494 101 CONTINUE
SHEET 495 C*****
SHEET 496 C TO OBTAIN AN EFFICIENT CONVERGENCE ACCEF IS COMPUTED
SHEET 497 C THE ACCELERATING COEFFICIENT IS DETERMINED IN SUCH A WAY

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SHEET 498 C PERTURBATIONAL TERM TIMES ACCELERATING COEFFICIENT IS NEVER
SHEET 499 C GREATER THAN THE INITIAL VALUE, BUT BE A FRACTION OF IT.
SHEET 500 C E.G. A2=2 MEANS HALF
SHEET 501 C*****
SHEET 502 A1=10.
SHEET 503 A2=2.
SHEET 504 CONCOF=0.0
SHEET 505 DO 103 I=1, NUMNF
SHEET 506 IF (ABS(UR(I)) .LT. .000001) UR(I)=0.
SHEET 507 IF (ABS(UZ(I)) .LT. .000001) UZ(I)=0.
SHEET 508 UUR(I)=UR(I)
SHEET 509 UUZ(I)=UZ(I)
SHEET 510 IZ=3*I-1
SHEET 511 IR=I7-1
SHEET 512 IF (R(IR) .EQ. 0.) GO TO 102
SHEET 513 COF1=ABS(UR(I))/R(IR)
SHEET 514 102 IF (B(I7) .EQ. 0.) GO TO 103
SHEET 515 COF2=ABS(UZ(I))/R(I7)
SHEET 516 A1=AMIN1(A1,COF1,COF2)
SHEET 517 103 CONTINUE
SHEET 518 C
SHEET 519 C
SHEET 520 105 CONTINUE
SHEET 521 IF (CONCOF .EQ. 1.) A2=A2*5.
SHEET 522 IF (A1 .EQ. 1.0 .AND. A2 .GT. 10.) COF1=5.
SHEET 523 IF (COF1 .EQ. 0.) COF1=2.
SHEET 524 A1=COF1/A2
SHEET 525 IF (A1 .LT. .00001) GO TO 2300
SHEET 526 IF (A1 .GT. 1.) A1=1.0
SHEET 527 ACDEF=A1
SHEET 528 WRITE(5,104) A1
SHEET 529 C
SHEET 530 C
SHEET 531 C*****
SHEET 532 C OBTAIN NEW VALUE
SHEET 533 C*****
SHEET 534 C
SHEET 535 DO 133 I=1, NUMNF
SHEET 536 IZ=3*I-1
SHEET 537 IR=I7-1
SHEET 538 IS=I2+1
SHEET 539 UR(I)=UUR(I)
SHEET 540 UZ(I)=UUZ(I)
SHEET 541 UR(I)=UR(I)+R(IR)*ACDEF
SHEET 542 UZ(I)=UZ(I)+R(I7)*ACDEF
SHEET 543 SLOP(I)=SLOP(I)+R(IS)*ACDEF
SHEET 544 130 CONTINUE
SHEET 545 C
SHEET 546 WRITE(6,1016) K
SHEET 547 WRITE(6,1006) K
SHEET 548 C
SHEET 549 C
SHEET 550 C*****
SHEET 551 C COMPUTE NORM OF ERROR AND NORM OF SOLUTION.
SHEET 552 C*****
SHEET 553 C
SHEET 554 ENCRM = 0.
SHEET 555 SNCRM = 0.
SHEET 556 DO 134 I=1, NUMNF
SHEET 557 IZ=3*I-1
SHEET 558 IR=I7-1
SHEET 559 IS=I2+1
SHEET 560 ENCRM = ENCRM + R(IR)*R(IR) + R(IZ)*R(IZ) + R(IS)*R(IS)
SHEET 561 SNCRM = SNCRM + UR(I)*UR(I) + UZ(I)*UZ(I) + SLOP(I)*SLOP(I)
SHEET 562 134 CONTINUE
SHEET 563 C
SHEET 564 C
SHEET 565 ENCRM = SORT(ENCRM)
SHEET 566 SNCRM = SORT(SNCRM)
SHEET 567 ESNCRM=ENCRM/SNCRM
SHEET 568 WRITE(6,1015) SNCRM,ENCRM,ESNCRM
SHEET 569 DO 776 I=1, NUMNF
SHEET 570 IZ=3*I-1
SHEET 571 IR=I7-1
SHEET 572 IS=I2+1
SHEET 573 WRITE(6,1002) I,R(IR),B(I7),R(IS),UR(I),UZ(I),SLOP(I),R(I),Z(I)
SHEET 574 776 CONTINUE
SHEET 575 C
SHEET 576 131 CONTINUE
SHEET 577 C
SHEET 578 C
SHEET 579 C
SHEET 580 C*****
SHEET 581 C COMPUTE STRAIN FROM THE NEW GUESS.
SHEET 582 C EPS(1,N)=INCREMENT OF MERIDIAN STRAIN

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SHEET 583 C EPS(2,N)=INCREMENT OF TANGENTIAL STRAIN
SHEET 584 C EPS(3,N)=INCREMENT OF THICKNESS STRAIN
SHEET 585 C *****
SHEET 586 C
SHEET 587 C
SHEET 588 C
SHEET 589 C DO 400 N=1,NUMEL
SHEET 590 NP1=N+1
SHEET 591 DLL=DL(N)
SHEET 592 SPH=SPH(N)
SHEET 593 CPH=CPH(N)
SHEET 594 AU=UR(N)+UR(NP1)
SHEET 595 AR=R(N)+R(NP1)
SHEET 596 DR=R(N)-R(NP1)
SHEET 597 DZ=Z(NP1)-Z(N)
SHEET 598 DU=UP(N)-UR(NP1)
SHEET 599 DW=UZ(NP1)-UZ(N)
SHEET 600 EX1=1.+2.*DR*DU/DLL/DLL+2.*DZ*DW/DLL/DLL+(DU*DU+DW*DW)/DLL/DLL
SHEET 601 C
SHEET 602 EPS(1,N)=ALOG(EX1)/2.
SHEET 603 EPS(2,N)=ALOG(1.+AU/AR)
SHEET 604 EPS(3,N)=-EPS(1,N)-EPS(2,N)
SHEET 605 R00 CONTINUE
SHEET 606 TEST=0.0
SHEET 607 C
SHEET 608 C
SHEET 609 C
SHEET 610 C *****
SHEET 611 C COMPUTE INCREMENT OF EFFECTIVE STRAIN
SHEET 612 C *****
SHEET 613 C
SHEET 614 WRITE(4,1026) NSTEP
SHEET 615 DO 222 N=1, NUMEL
SHEET 616 NP1=N+1
SHEET 617 IF(CODE(N) .EQ. 3.0 .AND. CODE(NP1) .EQ. 3.0)GO TO 222
SHEET 618 ES=EPS(1,N)
SHEET 619 ET=EPS(2,N)
SHEET 620 RBAR=RP1*(ES*ES+ET*ET) + 2.*RVALUE*ES*ET
SHEET 621 C
SHEET 622 EPS(4,N)=SQRT(2.*RCCNST*RBAR/3.)
SHEET 623 C
SHEET 624 C
SHEET 625 STS(1,N)=CLAMDA*(ES + RCISON*ET)*VY(N)/EPS(4,N)
SHEET 626 STS(2,N)=CLAMDA*(ET + RCISON*ES)*VY(N)/EPS(4,N)
SHEET 627 C
SHEET 628 C *****
SHEET 629 C COMPUTE STRESS DISTRIBUTION
SHEET 630 STS(1,N)=MERIDIAN STRESS
SHEET 631 STS(2,N)=CIRCUMFERENTIAL STRESS
SHEET 632 STS(3,N)=EFFECTIVE STRESS
SHEET 633 C *****
SHEET 634 C
SHEET 635 IF(ESNCFW .LT. FLIMIT)TEST=1.0
SHEET 636 ES=STS(1,N)
SHEET 637 ET=STS(2,N)
SHEET 638 EFSTRS=ES*ES+ET*ET-2.*RCISON*ES*ET
SHEET 639 STS(3,N)= SQRT(EFSTRS)
SHEET 640 C
SHEET 641 WRITE(6,1003)N,(EPS(I,N),I=1,4)
SHEET 642 222 CONTINUE
SHEET 643 WRITE(6,1027)
SHEET 644 DO 430 N=1,NUMEL
SHEET 645 430 WRITE(6,2251)N,(STS(I,N),I=1,3)
SHEET 646 C
SHEET 647 C *****
SHEET 648 C CHECK WHETHER ACDF IS TOO LARGE TO CAUSE A PHYSICALLY
SHEET 649 UNACCEPTABLE SOLUTION. WHENEVER COMPUTED MERIDIAN STRESS BECOMES
SHEET 650 NEGATIVE ADJUST ACDF VALUE)
SHEET 651 C *****
SHEET 652 C
SHEET 653 CENCDF=0.0
SHEET 654 STS1=0.0
SHEET 655 DO 431 N=1,NUMEL
SHEET 656 NP1=N+1
SHEET 657 IF(CODE(N) .EQ. 3.0 .AND. CODE(NP1) .EQ. 3.0)STSI,N)=0.
SHEET 658 431 IF(STS(1,N) .LT. STS1)CENCDF=1.0
SHEET 659 C
SHEET 660 C *****
SHEET 661 C CHECK WHETHER (ERRDF NORM)/(SOLUTION NORM) IS LESS THAN FLIMIT
SHEET 662 IF YES, THE SOLUTION IS FINAL
SHEET 663 C *****
SHEET 664 C
SHEET 665 C
SHEET 666 C
SHEET 667 IF(ESNORM .LT. FLIMIT)GO TO 430

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SHEET 568 C
SHEET 569 IF(K .GE. ITER)GO TO 2307
SHEET 570 C
SHEET 571 C
SHEET 572 DO 1900 N=1,NUMEL
SHEET 573 IF(FPS(4,N) .GT. .000001)GO TO 1900
SHEET 574 NP1=N+1
SHEET 575 DUM=UR(N)*UR(NP1)
SHEET 576 CODE(NP1)=3.0
SHEET 577 CODE(N)=3.0
SHEET 578 1900 CONTINUE
SHEET 579 C
SHEET 580 GO TO 2001
SHEET 581 2000 CONTINUE
SHEET 582 2200 CONTINUE
SHEET 583 C
SHEET 584 438 CONTINUE
SHEET 585 IF(ESNDEM .GT. FLIMIT)GO TO 777
SHEET 586 C
SHEET 587 C
SHEET 588 WRITE(6,2800)
SHEET 589 C
SHEET 590 EXAMINATION ON BOUNDARY ASSUMPTIONS
SHEET 591 C*****
SHEET 592 C
SHEET 593 DO 250 I=1,NCHECK
SHEET 594 N=NTOUCH-I
SHEET 595 IF(K .LE. 1)GO TO 250
SHEET 596 TOUCH(I)=(Z(N)+U7(N)+PNRAD-PNHEC)*(Z(N)+U2(N)+PNRAD-PNHEC)+(R(N)
SHEET 597 +UR(N))*R(N)+UP(N))
SHEET 598 TOUCH(I)=SQRT(TOUCH(I))-PNRAD
SHEET 599 WRITE(6,2900)N,TOUCH(I)
SHEET 700 250 CONTINUE
SHEET 701 WRITE(6,265)
SHEET 702 C
SHEET 703 C
SHEET 704 C*****
SHEET 705 C COMPUTE THE DISTANCES OF THE FREE NODE FROM THE DIE PROFILE
SHEET 706 C*****
SHEET 707 C
SHEET 708 RDIE=RTERT+DIERAD
SHEET 709 DO 260 I=1,NCHECK
SHEET 710 N=NCONTC+I
SHEET 711 CONTACT(I)=(DIERAD-Z(N)-U2(N))*(DIERAD-Z(N)-U7(N))+(RDIE-R(N)-UP(N)
SHEET 712 )*(RDIE-R(N)-UP(N))
SHEET 713 CONTACT(I)=SQRT(CONTACT(I))-DIERAD
SHEET 714 260 WRITE(6,270)N,CONTACT(I)
SHEET 715 C
SHEET 716 IF(ABS(TOUCH(I)) .LT. .0001)TOUCH(I)=0.
SHEET 717 IF(ABS(CONTACT(I)) .LT. .0001)CONTACT(I)=0.
SHEET 718 C
SHEET 719 C*****
SHEET 720 C CHECK ON BOUNDARY OVER PUNCH HEAD
SHEET 721 C*****
SHEET 722 C
SHEET 723 IF(TOUCH(I) .GE. 0.)GO TO 3000
SHEET 724 C
SHEET 725 WRITE(6,3100)
SHEET 726 C
SHEET 727 C*****
SHEET 728 C NODE AT NCHM1 IS INSIDE PUNCH, COMPUTE AGAIN
SHEET 729 C*****
SHEET 730 C
SHEET 731 TCHCOF=0.0
SHEET 732 TINSDE=0.0
SHEET 733 GO TO 2101
SHEET 734 C
SHEET 735 C
SHEET 736 3000 CONTINUE
SHEET 737 TCHCOF=1.0
SHEET 738 C
SHEET 739 C*****
SHEET 740 C CHECK ON BOUNDARY OVER DIE PROFILE
SHEET 741 C*****
SHEET 742 C
SHEET 743 IF(CONTACT(I) .GE. 0.)GO TO 3500
SHEET 744 WRITE(6,3400)
SHEET 745 C
SHEET 746 C*****
SHEET 747 C NODE AT NCONTC+1 IS INSIDE DIE, COMPUTE AGAIN
SHEET 748 C*****
SHEET 749 C
SHEET 750 NCONTC=NCONTC+1
SHEET 751 NCI=NCONTC-1
SHEET 752 FRNFC(NCONTC)=FRNFC(NCI)

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SHEET 753 C
SHEET 754 GO TO 2101
SHEET 755 3500 CONTINUE
SHEET 756 IF(INDEX .EQ. 2100) GO TO 3501
SHEET 757 IF((R(NDIE)+UP(NDIE)) .GT. POF) GO TO 3501
SHEET 758 NDIE=NDIE-1
SHEET 759 WRITE(6,2450)
SHEET 760 C
SHEET 761 C*****
SHEET 762 C NDIE NODE HAS BEEN BROUGHT INTO CONTACT WITH DIE PROFILE
SHEET 763 C*****
SHEET 764 C
SHEET 765 FRNFC(NDIE)=0.0001
SHEET 766 GO TO 2101
SHEET 767 C
SHEET 768 3501 CONTINUE
SHEET 769 TINSDE=1.0
SHEET 770 TEST=1.0
SHEET 771 C
SHEET 772 C*****
SHEET 773 C COMPUTATION OF FRICTION COEFFICIENT
SHEET 774 C*****
SHEET 775 C
SHEET 776 MUDEX=0
SHEET 777 PPNED=PNED
SHEET 778 WRITE(6,233)(FRNFC(N),N=1,NUM1)
SHEET 779 WRITE(6,231)
SHEET 780 C
SHEET 781 DO 230 I=1,NUM1
SHEET 782 IZ=I-1
SHEET 783 IR=IZ-1
SHEET 784 IF(CODE(I) .EQ. 3.0) GO TO 230
SHEET 785 IF(I .GE. NTOUCH) GO TO 280
SHEET 786 IF(I .LE. NCONT) .AND. I .GT. NDIE) GO TO 281
SHEET 787 IF(I .LT. NTOUCH .AND. I .GT. NCONT) GO TO 230
SHEET 788 IF(INDEX .EQ. 2100) GO TO 230
SHEET 789 IF(I .LE. NDIE) GO TO 282
SHEET 790 C
SHEET 791 280 CONTINUE
SHEET 792 DUM1=(Z(I)+U7(I)+PNFAD-PPNED)/PNFAD
SHEET 793 DUM2=(R(I)+UP(I))/PNFAD
SHEET 794 FRITN=FRITN
SHEET 795 IF(FRITN .EQ. 0.) GO TO 230
SHEET 796 GO TO 283
SHEET 797 C
SHEET 798 281 DUM1=DIEFAD-Z(I)-U7(I)
SHEET 799 DUM2=RDIE-R(I)-UP(I)
SHEET 800 FRITN=FRITN
SHEET 801 IF(FRITN .EQ. 0.) GO TO 230
SHEET 802 GO TO 283
SHEET 803 C
SHEET 804 282 DUM1=1.
SHEET 805 DUM2=0.
SHEET 806 FRITN=FRITN
SHEET 807 IF(FRITN .EQ. 0.) GO TO 230
SHEET 808 C
SHEET 809 283 CONTINUE
SHEET 810 PN=FF(IZ)*DUM1+FF(IR)*DUM2
SHEET 811 PT=FF(IZ)*DUM2-FF(IR)*DUM1
SHEET 812 IF(PN .EQ. 0.) GO TO 230
SHEET 813 XMU=PT/PN
SHEET 814 WRITE(6,232)I, XMU
SHEET 815 XMU=XMU/FRITN
SHEET 816 IF(XMU .GT. 1.02 .OR. XMU .LT. .99) MUDEX=1
SHEET 817 FRNFC(I)=FRNFC(I)/XML
SHEET 818 IF(I .EQ. NCONT) PN=-PN
SHEET 819 230 CONTINUE
SHEET 820 234 CONTINUE
SHEET 821 C
SHEET 822 C
SHEET 823 C
SHEET 824 C*****
SHEET 825 C MUDEX=0, IF FRICTION CONDITION IS SATISFIED
SHEET 826 C =1, OTHERWISE
SHEET 827 C*****
SHEET 828 C
SHEET 829 C
SHEET 830 IF(MUDEX .EQ. 1) TCHCDF=1.0
SHEET 831 IF(MUDEX .EQ. 1) GO TO 2001
SHEET 832 C
SHEET 833 ICHECK=0
SHEET 834 C
SHEET 835 C*****
SHEET 836 C GENERALIZED NODAL FORCE NORMAL TO THE PUNCH IS COMPUTED
SHEET 837 C TO CHECK WHETHER THE BOUNDARY IS ASSUMED TO MOVE TOO FAST.

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SHEET 838 C*****
SHEET 839 C
SHEET 840 C IF(CODE(NTOUCH) .EQ. 3.)GO TO 500
SHEET 841 C
SHEET 842 NZZ=3*NTOUCH-1
SHEET 843 IF(FF(NZZ) .GT. 0.) GO TO 500
SHEET 844 IF(ABS(FF(NZZ)) .LT. .000001 .OP. TINSCE .EQ. 0.)GO TO 500
SHEET 845 TDIST=TOUCH3*.8
SHEET 846 WRITE(6,510)TDIST
SHEET 847 NTOUCH=NTOUCH+1
SHEET 848 TCHCOF=1.0
SHEET 849 ICHECK=1
SHEET 850 500 CONTINUE
SHEET 851 C
SHEET 852 C
SHEET 853 C
SHEET 854 C*****
SHEET 855 C GENERALIZED NODAL FORCE NORMAL TO DIE IS COMPUTED.
SHEET 856 C*****
SHEET 857 C
SHEET 858 IF(CODE(NCONTC) .EQ. 3.)GO TO 550
SHEET 859 IF(NCONTC .EQ. NCONC)GO TO 550
SHEET 860 IF(OPN .GT. -.000001)GO TO 550
SHEET 861 C
SHEET 862 TCONTC=TCONTC*.8
SHEET 863 WRITE(6,540)TCONTC
SHEET 864 NCONTC=NCONTC-1
SHEET 865 ICHECK=1
SHEET 866 550 CONTINUE
SHEET 867 C
SHEET 868 C*****
SHEET 869 C ICHECK=1. IF BOUNDARY ASSUMPTION NEEDS TO BE MODIFIED.
SHEET 870 C*****
SHEET 871 C
SHEET 872 IF(ICHECK .EQ. 1)GO TO 446
SHEET 873 NNCONC=NCONTC
SHEET 874 C
SHEET 875 C*****
SHEET 876 C MAKE BOUNDARY ASSUMPTION FOR NEW STEP BASED UPON
SHEET 877 C THE DISTANCE AWAY FROM PUNCH OF DIE
SHEET 878 C*****
SHEET 879 C
SHEET 880 DO 560 I=1,NCHECK
SHEET 881 IF(CONTACT(I)-TCONTC)561,562,562
SHEET 882 561 NNCONC=NNCONC+1
SHEET 883 TCONTC=CONTACT(I)
SHEET 884 550 FRNFC(NCONTC)=FRNFC(NNCONC)
SHEET 885 *62 CONTINUE
SHEET 886 C
SHEET 887 C
SHEET 888 C
SHEET 889 C
SHEET 890 C
SHEET 891 NNTCH=NTOUCH
SHEET 892 DO 240 I=1,NCHECK
SHEET 893 IF(TOUCH(I)-TDIST)245,245,246
SHEET 894 245 NTOUCH=NNTCH-1
SHEET 895 TOUCH2=TOUCH(I)
SHEET 896 FRNFC(NTOUCH)=FRNFC(NNTCH)
SHEET 897 240 CONTINUE
SHEET 898 246 TCHCOF=1.0
SHEET 899 C
SHEET 900 C*****
SHEET 901 C COMPUTE TOTAL STRAIN
SHEET 902 C*****
SHEET 903 C
SHEET 904 WRITE(6,1043)
SHEET 905 DO 444 N=1,NUMEL
SHEET 906 DO 443 I=1, 4
SHEET 907 443 TEPS(I,N)=TEPS(I,N)+EPS(I,N)*TEST
SHEET 908 IF(ESNORM .LT. FLIMIT)THICK(N)=THICK(N)*EXP(EPS(3,N)*TEST)
SHEET 909 WRITE(6,1003)N,(TEPS(I,N),I=1,4)
SHEET 910 444 CONTINUE
SHEET 911 C
SHEET 912 C
SHEET 913 C
SHEET 914 C
SHEET 915 C*****
SHEET 916 C COMPUTE OPNHED FOR NEXT STEP WHICH WOULD GIVE THE INCREMENT
SHEET 917 C OF MAXIMUM EFFECTIVE STRAIN APPROXIMATELY EQUAL TO PRESET VALUE.
SHEET 918 C*****
SHEET 919 C
SHEET 920 EMAX=0.0
SHEET 921 DO 775 N=1,NUMEL
SHEET 922 775 IF(EPS(4,N) .GT. EMAX) EMAX=EPS(4,N)

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SHEET 923      EFACTA=ECONST/EMAX
SHEET 924      IF(INSTEP .LE. 2 .AND. ITCONT .EQ. 0)GO TO 778
SHEET 925      DPNHED=2./EFACTA/UZ(NUMNP)-1./EFACT/DPNSTR
SHEET 926      DPNHED=1./DPNHED
SHEET 927      ESTAR=DPNHED/L7(NUMNP)
SHEET 928      DPNSTR=L7(NUMNP)
SHEET 929      778 CONTINUE
SHEET 930      EFACT=EFACTA
SHEET 931      C
SHEET 932      C
SHEET 933      C
SHEET 934      777 CONTINUE
SHEET 935      IF(ESNDPM .GT. FLIMIT)TCHCOF=1.0
SHEET 936      C
SHEET 937      C*****
SHEET 938      C      NEW CONFIGURATION
SHEET 939      C*****
SHEET 940      C
SHEET 941      DO 439 I=1, NUMNP
SHEET 942      IZ=3*I-1
SHEET 943      IR=IZ-1
SHEET 944      R(I)=R(I)+UR(I)*TEST
SHEET 945      Z(I)=Z(I)+UZ(I)*TEST
SHEET 946      439 CONTINUE
SHEET 947      C
SHEET 948      C
SHEET 949      C
SHEET 950      C*****
SHEET 951      C      PUNCH THE SOLUTION
SHEET 952      C*****
SHEET 953      C
SHEET 954      IF(NPUNCH .EQ. 0) GO TO 310
SHEET 955      2307 CONTINUE
SHEET 956      PUNCH 1017,(UR(I),UZ(I),CODE(I),I=1,NUMNP)
SHEET 957      PUNCH 1017,(R(I),Z(I),I=1,NUMNP)
SHEET 958      PUNCH 1017,( STEPS(I,N),I=1,4),N=1,NUMEL)
SHEET 959      PUNCH 1017,(THICK(N),N=1,NUMEL)
SHEET 960      PUNCH 2223,DPNHED,NTOUCH,TCHCOF,EFACT,NCONTC
SHEET 961      PUNCH 235,(RNFCE(N),N=1,NUM1)
SHEET 962      PUNCH 233,ESTAR,DPNSTR,DPNHED,TEST)
SHEET 963      310 CONTINUE
SHEET 964      C
SHEET 965      IF(ESNDPM .GT. FLIMIT)GO TO 2300
SHEET 966      C
SHEET 967      C
SHEET 968      WRITE(5,1040)
SHEET 969      DO 849 I=1,NUMNP
SHEET 970      IR=3*I-2
SHEET 971      IZ=IR+1
SHEET 972      IL=IZ+1
SHEET 973      WRITE(5,1041)(I,FF(IR),FF(IZ),FF(IL))
SHEET 974      849 CONTINUE
SHEET 975      C
SHEET 976      C
SHEET 977      C*****
SHEET 978      C      COMPUTE THE PUNCH LOAD FROM ENERGY BALANCE
SHEET 979      C*****
SHEET 980      C
SHEET 981      SUMF=0.
SHEET 982      DO 850 I=2,NUMNP
SHEET 983      IZ=I*3-1
SHEET 984      IR=IZ-1
SHEET 985      850 SUMF=SUMF+FF(IZ)*U7(I)+FF(IR)*UR(I)
SHEET 986      SUMF=SUMF/UZ(NUMNP)/TEST*.3.14
SHEET 987      C
SHEET 988      WRITE(6,1042)SUMF
SHEET 989      IF(INSTOP .EQ. 1) GO TO 2300
SHEET 990      WRITE(6,102F)DPNHED,NNTCH
SHEET 991      WRITE(6,1029)EMAX,U7(NUMNP),EFACT
SHEET 992      IF(INSTEP .LT. NFORM) GO TO 2100
SHEET 993      2300 CONTINUE
SHEET 994      2301 CONTINUE
SHEET 995      C
SHEET 996      C
SHEET 997      C
SHEET 998      1002 FORMAT(15,3F13.7,5X,3F13.7,5X,3F13.7)
SHEET 999      1003 FORMAT(17,11F11.6)
SHEET 1000      1004 FORMAT(16I5)
SHEET 1001      1005 FORMAT(1H1,* STRAIN-STRESS SOLUTION AT STEP NUMBER =*,I4//
SHEET 1002      1 * EL, NC...R-STRAIN...Z-STRAIN...TH-STRAIN...F7-STRAIN...EF-STRAIN
SHEET 1003      2...R-STRES...7-STRES...TH-STRES...F7-STRES...EF-STRES...AVG-STRES...*
SHEET 1004      3)
SHEET 1004      1006 FORMAT(/// 30X, * DISPLACEMENT SOLUTION AT ITERATION NUMBER =*,I4
SHEET 1005      1/// 20X, * PLYURPED*, 26X, * TOTAL*, 20X, * DEFORMED COORD*/
SHEET 1006      2/ * NP      CU      DW      DPETA      U
SHEET 1007      3      W      BETA      R      2*)

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SHEET 1008 1007 FORMAT(1H1,70X,*ITERATION PROCESS FOR STEP*,I4)
SHEET 1009 1008 FORMAT( 60X, * TOTAL X-LOAD =*, F12.7)
SHEET 1010 1 / 60X, * TOTAL Z-LOAD =*, F12.7)
SHEET 1011 2 / 60X, * TOTAL W-LOAD =*, F12.7)
SHEET 1012 1010 FORMAT( // * NODAL POINT FORCE AT STEP =*, I4//
SHEET 1013 16.....N,F.....E-FORCE.....Z-FORCE.....Z-STRESS ON DIE SUR
SHEET 1014 2FACE...*)
SHEET 1015 1011 FORMAT(15,3F10.0)
SHEET 1016 1012 FORMAT(15,6F17.5)
SHEET 1017 1015 FORMAT(60X,* VELOCITY CONVERGENCE*,/
SHEET 1018 1 60X, * NORM OF SOLUTION VECTOR =*, F13.8)
SHEET 1019 1 / 60X, * NORM OF ERROR VECTOR =*, F13.8)
SHEET 1020 2 / 60X, * FRACTIONAL NORM =*, F13.8)
SHEET 1021 1016 FORMAT( * DISPLACEMENT SOLUTION AT ITERATION NUMBER =*, I4)
SHEET 1022 1017 FORMAT(9F10.7)
SHEET 1023 1018 FORMAT(////* DOES NOT CONVERGE**//
SHEET 1024 1 * TRY AGAIN WITH DECELERATION COEFFICIENT =ACDEF= LESS THAN*,
SHEET 1025 2FR,?)
SHEET 1026 1020 FORMAT(20F4.1)
SHEET 1027 1025 FORMAT(4X,15,3X,F12.6,10X,15,3X,F12.6,10X,15,3X,F12.6)
SHEET 1028 2251 FORMAT(15,4F20.7)
SHEET 1029 1025 FORMAT(////*INCREMENTAL STRAIN-TOTAL STRAIN AT STEP NUMBER=*,I4//
SHEET 1030 1*EL NO....S-STRAIN.....THE-STRAIN.....THI-STRAIN.....EF-STRAIN
SHEET 1031 1*)
SHEET 1032 1027 FORMAT(///*EL. NO....S-STRESS.....THE-STRESS....EF-STRESS....*)
SHEET 1033 1042 FORMAT(* PUNCH FORCE=*,F15.7)
SHEET 1034 1047 FORMAT(///*EL NO....S-STRAIN.....THE-STRAIN.....THI-STRAIN....
SHEET 1035 1EF-STRAIN...*)
SHEET 1036 1041 FORMAT(5X,I10,5X,3F20.7)
SHEET 1037 1040 FORMAT(///* NO. OF NODE FORCE=*)
SHEET 1038 510 FORMAT(///* NTOUCH IS FORCED TO TOUCH, COMPUTE AGAIN
SHEET 1039 1*,/* TDIST=*,F10.7)
SHEET 1040 1030 FORMAT(17,3F10.5)
SHEET 1041 1031 FORMAT(* GEOMETRY OF PROFILE**//
SHEET 1042 1 *EL NNO.....ANGLE.....THICKNESS.....*)
SHEET 1043 104 FORMAT(///* ACDEF CALCULATED*,F10.7)
SHEET 1044 265 FORMAT(///* CHECKING DISTANCE AWAY FROM DIE*)
SHEET 1045 270 FORMAT(15,F20.7)
SHEET 1046 2900 FORMAT(110,F20.7)
SHEET 1047 2900 FORMAT(///* CHECKING DISTANCE AWAY FROM PUNCH*,/
SHEET 1048 1 * SLEM NO. TOUCH**//)
SHEET 1049 3100 FORMAT(* NODE AT NCHM1 IS INSIDE PUNCH, COMP AGAIN*)
SHEET 1050 3400 FORMAT(* NODE AT NCONTC+1 IS INSIDE DIE, COMP AGAIN*)
SHEET 1051 3450 FORMAT(* NODE HAS BEEN BROUGHT INTO CONTACT WITH CORNER**//)
SHEET 1052 231 FORMAT(///* NODAL POINT COEFFICIENT*,/
SHEET 1053 540 FORMAT(///* NCONTC IS FORCED TO TOUCH, TCNTC=*,F10.7)
SHEET 1054 232 FORMAT(110,F10.5)
SHEET 1055 233 FORMAT(5F15.7)
SHEET 1056 235 FORMAT(4F15.7)
SHEET 1057 2223 FORMAT(F15.7,15,2F15.7,15)
SHEET 1058 1029 FORMAT(* PUNCH HEAD DISPLACEMENT*,F10.7/* NTOUCH= *,15)
SHEET 1059 1029 FORMAT(///* MAX EFFECTIVE STRAIN INCREMENT**,F10.7/* PUNCH
SHEET 1060 1 HEAD INCREMENT=*,F10.7/* PUNCH HEAD ADJUSTING FACTOR=*,F10.7
SHEET 1061 2)
SHEET 1062 C
SHEET 1063 RETURN
SHEET 1064 END

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SHEET 1066 SURFOUTLINE STIFF(F,Z,UF,UZ,CODE,SLDP,YY,YX,SPHI,CPHI,DL,EFS,
SHEET 1067 1THICK,ALPHA,GAMMA,ETA,FRNFCE,FF,A,B,NO)
SHEET 1068 C
SHEET 1069 C*****
SHEET 1070 C CALCULATION OF STIFFNESS MATRIX FOR ENTIRE SYSTEM
SHEET 1071 C*****
SHEET 1072 C
SHEET 1073 COMMON/GENCON/NUMNP,NUMEL,HEP(12),DLL,NEG,NFORP,YIELD,TEST,ITER,
SHEET 1074 1NREAD,NPUNCH,NPRINT,RVALUE,T,MRAND,PNFAD,RADIUS,FRITND,FRITND,
SHEET 1075 2ECONST,FNHED,RTHT,CIERAD,TCNTC,TDIST,PHFCE
SHEET 1076 DIMENSION R(1),Z(1),CCCE(1),UF(1),UZ(1),SLOP(1),P(1),A(N0,1),
SHEET 1077 1 EFS(4,1),RR(2),ZZ(2),UU(6),YY(1),THICK(1),DL(1),SPHI(1),CPHI(1),
SHEET 1078 2 YX(1),ALPHA(1),GAMMA(1),ETA(1),FRNFCE(1),FF(1)
SHEET 1079 COMMON/STPMAT/H(6),P(6,6),TEX,TEY,TEZ,THKL
SHEET 1080 COMMON/CONQUAD/SS(4),WT(4),D(2,2),SOFT1
SHEET 1081 COMMON/ATOUCH/NTOUCH,NDIE,NCONTC
SHEET 1082 C
SHEET 1083 C
SHEET 1084 C*****
SHEET 1085 C INITIALIZE A AND F MATRIX FOR EQUATION AX=B
SHEET 1086 C BECAUSE BANDER SYMMETRIC PROPERTY OF THE STIFFNESS MATRIX A,
SHEET 1087 C THE STORAGE OF A IS IN A SQUARE MATRIX
SHEET 1088 C*****

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SHEET 1089 C
SHEET 1090 DO 50 N=1, NEC
SHEET 1091 R(N)=0.
SHEET 1092 DO 50 M=1, MRAAD
SHEET 1093 50 A(N,M)=0.
SHEET 1094 WT(1)=0.3478548451
SHEET 1095 WT(2)=0.6521451549
SHEET 1096 WT(3)=WT(1)
SHEET 1097 WT(4)=WT(2)
SHEET 1098 C
SHEET 1099 C
SHEET 1100 C*****
SHEET 1101 C CONSTPLCT P AND M AT ELEMENT LEVEL
SHEET 1102 C*****
SHEET 1103 C
SHEET 1104 DO 1000 N=1, NUMEL
SHEET 1105 NP1=N+1
SHEET 1106 IF(CODE(N) .EQ. 3.0 .AND. CODE(NP1) .EQ. 3.0) GO TO 1000
SHEET 1107 DLL=DL(N)
SHEET 1108 SPH=SPH1(N)
SHEET 1109 CPH=CPH1(N)
SHEET 1110 RR(1)=R(N)
SHEET 1111 ZZ(1)=Z(N)
SHEET 1112 RR(2)=R(NP1)
SHEET 1113 LU(1)=UR(N)
SHEET 1114 UU(2)=U7(N)
SHEET 1115 UU(3)=SLCP(N)
SHEET 1116 UU(4)=UR(NP1)
SHEET 1117 UU(5)=U7(NP1)
SHEET 1118 UU(6)=SLCP(NP1)
SHEET 1119 THKL=THICK(N)*DLL
SHEET 1120 ZZ(2)=Z(NP1)
SHEET 1121 YG=YG(N)
SHEET 1122 YH=YY(N)
SHEET 1123 C
SHEET 1124 CALL QUAD(RR,ZZ,UL,DLL,SPH,CPH,YG,YH)
SHEET 1125 C
SHEET 1126 C
SHEET 1127 C
SHEET 1128 C*****
SHEET 1129 C PERFORM THE ASSEMBLY OPERATION, BECAUSE MATRIX A IS SYMMETRIC
SHEET 1130 C ONLY UPPER HALF OF THE MATRIX IS CREATED. AND THE STORAGE FOR
SHEET 1131 C MATRIX A IS A SQUARE ARRAY BECAUSE OF RANDOM SYMMETRIC PROPERTY
SHEET 1132 C*****
SHEET 1133 C
SHEET 1134 DO 200 I=1, 6
SHEET 1135 II=N*3 - 3 + I
SHEET 1136 A(II)=A(II)+A(II)
SHEET 1137 DO 200 J=1, 6
SHEET 1138 C
SHEET 1139 JJ=N*3-3+J-II+1
SHEET 1140 IF(JJ .LT. 1) GO TO 200
SHEET 1141 A(II,JJ)=A(II,JJ)+P(I,J)
SHEET 1142 200 CONTINUE
SHEET 1143 1000 CONTINUE
SHEET 1144 C
SHEET 1145 C*****
SHEET 1146 C TO HANDLE MIXED BOUNDARY CONDITION, FOLLOWING MATRICES ARE EVALUATED
SHEET 1147 C*****
SHEET 1148 C
SHEET 1149 C=PNRAD-DNHED
SHEET 1150 NMP1=NUMNP-1
SHEET 1151 PNRAD=PNRAD
SHEET 1152 RDIE=RTTERT+DIERAD
SHEET 1153 C
SHEET 1154 DO 1200 N=1,NMP1
SHEET 1155 IF(CODE(N) .NE. 4.0)GO TO 1200
SHEET 1156 DUM2=RDIE-P(N)-UP(N)
SHEET 1157 DUM1=DIERAD-7(N)-UZ(N)
SHEET 1158 PNRAD=DIERAD
SHEET 1159 IF(N .LE. NCONTC)GO TO 1100
SHEET 1160 ENRAD=PNRAD
SHEET 1161 DUM2=P(N)+UP(N)
SHEET 1162 DUM1=C+7(N)+UZ(N)
SHEET 1163 C
SHEET 1164 1100 CONTINUE
SHEET 1165 ALPHA(N)=-DUM2/DUM1
SHEET 1166 GAMMA(N)=(PNRAD*PNRAD-DUM2*DUM2-DUM1*DUM1)/2./DUM2
SHEET 1167 ETA(N)=FRNFCE(N)*PNRAD/DUM2
SHEET 1168 IF(N .GE. NTOUCH)GO TO 1200
SHEET 1169 GAMMA(N)=-GAMMA(N)
SHEET 1170 ETA(N)=-ETA(N)
SHEET 1171 1200 CONTINUE
SHEET 1172 C
SHEET 1173 C*****

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SHEET 1174 C STORE GENERALIZED NODAL FORCE
SHEET 1175 C*****
SHEET 1176 C
SHEET 1177 NUM3=NUMNP*3
SHEET 1178 DC 1300 I=1,NUM3
SHEET 1179 1300 FF(I)=-R(I)
SHEET 1180 C
SHEET 1181 C
SHEET 1182 C
SHEET 1183 C
SHEET 1184 1001 FORMAT(///,* THE DIAGONAL VECTOR OF MATRIX OF STIFFNESS*/
SHEET 1185 1002 FORMAT(12E11.3)
SHEET 1186 1005 FORMAT(/ / 29H ELEMENT WITH NEGATIVE AREA =. TE)
SHEET 1187 C
SHEET 1188 RETURN
SHEET 1189 END

SHEET 1191 SURROUTINE QUAD(PR,ZZ,UU,DLL,SFR,CPH,S2,S1)
SHEET 1192 COMMON/STFMAT/H(6),P(6,6),TEX,TEY,TE7,THKL
SHEET 1193 COMMON/CONJUC/SS(A),BT(A),D(2,2),SOFT)
SHEET 1194 DIMENSION RE(2),ZZ(2),UU(6),R(2,6),XX(6,6),PZERO(6),DB(2,6)
SHEET 1195 COMMON/ISOTPY/PVALUE
SHEET 1196 C
SHEET 1197 RC=(PR(1)+PR(2))/2.
SHEET 1198 C
SHEET 1199 DD 2 I=1,6
SHEET 1200 H(I)=0.
SHEET 1201 DD 2 J=1,6
SHEET 1202 2 P(I,J)=C.
SHEET 1203 TEX=0.
SHEET 1204 TEY=0.
SHEET 1205 TE7=0.
SHEET 1206 C
SHEET 1207 C
SHEET 1208 DZ=ZZ(2)-ZZ(1)
SHEET 1209 DR=PR(1)-PR(2)
SHEET 1210 DU=UU(1)-UU(4)
SHEET 1211 DW=UU(5)-UU(2)
SHEET 1212 AU=UU(1)+UU(4)
SHEET 1213 AR=PR(1)+PR(2)
SHEET 1214 C
SHEET 1215 C1=2.*DR/DLL/DLL
SHEET 1216 C2=2.*DU/DLL/DLL
SHEET 1217 C3=2.*DW/DLL/DLL
SHEET 1218 C4=2.*DW/DLL/DLL
SHEET 1219 C5=AU/AR/2.
SHEET 1220 C6=1.+DR*C2+DZ*C4+(DU*DU+DW*DW)/DLL/DLL
SHEET 1221 C7=2./DLL/DLL
SHEET 1222 C8=2./AR/AR
SHEET 1223 C9=1./SOFT(C6)/2.
SHEET 1224 C10=C9/C6
SHEET 1225 C11=C1+C2
SHEET 1226 C12=C3+C4
SHEET 1227 C
SHEET 1228 C
SHEET 1229 C
SHEET 1230 DES1=SOFT(C6)
SHEET 1231 DET1=2.*C5+1.
SHEET 1232 C
SHEET 1233 E1=C9*C11/DES1
SHEET 1234 E2=-C9*C12/DES1
SHEET 1235 E3=-E1
SHEET 1236 E4=-E2
SHEET 1237 E5=1./AR/DET1
SHEET 1238 E6=(-C10*C11*C11/2.+C9*C7)/DES1-E1*E1
SHEET 1239 E7=-E4
SHEET 1240 E8=C10*C11*C12/2./DES1-E1*E2
SHEET 1241 E9=-E8
SHEET 1242 E10=E8
SHEET 1243 E11=(-C10*C12*C12/2.+C9*C7)/DES1-E2*E2
SHEET 1244 E12=-E5*E5
SHEET 1245 C
SHEET 1246 C*****
SHEET 1247 C DES=MERIDIAN STRAIN INCREMENT
SHEET 1248 C DET=CIRCUMFERENTIAL STRAIN INCREMENT
SHEET 1249 C COMPUTATION OF EFFECTIVE STRAIN INCREMENT
SHEET 1250 C E1=DERIVATIVE OF MERIDIAN STRAIN INCREMENT WITH RESPECT TO UU(1)
SHEET 1251 C E2=D(DES)/D(UU(1))
SHEET 1252 C E3=D(DES)/D(UU(2))
SHEET 1253 C E4=D(DES)/D(UU(4))
SHEET 1254 C E5=D(DES)/D(UU(5))

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SHEET 1254 C E5=D(DET)/D(U(1))
SHEET 1255 C E6=C(E1)/D(U(1))
SHEET 1257 C E7=D(E1)/D(U(4))
SHEET 1258 C E8=C(E1)/D(U(2))
SHEET 1259 C E9=D(E3)/D(U(2))
SHEET 1260 C E10=D(E4)/D(U(5))
SHEET 1261 C E11=C(E5)/D(U(2))
SHEET 1262 C *****
SHEET 1263 C
SHEET 1264 C
SHEET 1265 C DES=ALOG(DES)
SHEET 1266 C DET=ALOG(DET)
SHEET 1267 C RVP1=0VALUE+1.
SHEET 1268 C RVP2=SQRT(2.*RVALUE+1.)
SHEET 1269 C RVP3=RVP1/RVP2
SHEET 1270 C RVP4=2.*RVALUE/RVP1
SHEET 1271 C
SHEET 1272 C *****
SHEET 1273 C EFFECTIVE STRAIN
SHEET 1274 C F1=DERIVATIVE OF EFFECTIVE STRAIN INCREMENT WITH RESPECT TO UU(1)
SHEET 1275 C F2= WITH RESPECT TO UU(2)
SHEET 1276 C F3= WITH RESPECT TO UU(4)
SHEET 1277 C F4= WITH RESPECT TO UU(5)
SHEET 1278 C F11=D(F1)/D(U(1))
SHEET 1279 C F12=D(F1)/D(U(2))
SHEET 1280 C F13=D(F1)/D(U(4))
SHEET 1281 C F14=D(F1)/D(U(5))
SHEET 1282 C F22=D(F2)/D(U(2))
SHEET 1283 C F23=D(F2)/D(U(4))
SHEET 1284 C F24=D(F2)/D(U(5))
SHEET 1285 C F33=D(F3)/D(U(4))
SHEET 1286 C F34=D(F3)/D(U(5))
SHEET 1287 C F44=D(F4)/D(U(5))
SHEET 1288 C *****
SHEET 1289 C
SHEET 1290 C EFS=DES*DES+DET*DET+RVP4*DES*DET
SHEET 1291 C EFS1=RVP3*SQRT(EFS)
SHEET 1292 C EFS2=RVP3/SQRT(EFS)/2.
SHEET 1293 C EFS3=-RVP3/EFS/SQRT(EFS)/4.
SHEET 1294 C
SHEET 1295 C
SHEET 1296 C D1=(2.*DES+RVP4*DET)*E1+(2.*DET+RVP4*DES)*E2
SHEET 1297 C D2=(2.*DES+RVP4*DET)*E2
SHEET 1298 C D3=(2.*DES+RVP4*DET)*E3+(2.*DET+RVP4*DES)*E5
SHEET 1299 C D4=(2.*DES+RVP4*DET)*E4
SHEET 1300 C F1=EFS2*D1
SHEET 1301 C F2=EFS2*D2
SHEET 1302 C F3=EFS2*D3
SHEET 1303 C F4=EFS2*D4
SHEET 1304 C
SHEET 1305 C F11=EFS3*D1*D1+EFS2*((2.*DES+RVP4*DET)*E6+(2.*DET+RVP4*DES)*E12
SHEET 1306 C )+(2.*E1+RVP4*E5)*E1+(2.*E2+RVP4*E5)*E5)
SHEET 1307 C F12=EFS3*D1*D2+EFS2*((2.*DES+RVP4*DET)*E8+(2.*E1+RVP4*E5)*E2)
SHEET 1308 C F13=EFS3*D1*D3+EFS2*((2.*DES+RVP4*DET)*E7+(2.*E3+RVP4*E5)*E1
SHEET 1309 C )+(2.*E5+RVP4*E3)*E5+(2.*DET+RVP4*DES)*E12)
SHEET 1310 C F14=EFS3*D1*D4+EFS2*((2.*DES+RVP4*DET)*E9+(2.*E1+RVP4*E5)*E4)
SHEET 1311 C F22=EFS3*D2*D2+EFS2*((2.*DES+RVP4*DET)*E11+(2.*E2*E2)
SHEET 1312 C )
SHEET 1313 C F23=EFS3*D2*D3+EFS2*((2.*DES+RVP4*DET)*E9+(2.*E3+RVP4*E5)*E2)
SHEET 1314 C F24=EFS3*D2*D4+EFS2*((2.*DES+RVP4*DET)*E11+(2.*E4*E2)
SHEET 1315 C )+(2.*E5+RVP4*E3)*E5+(2.*DET+RVP4*DES)*E12)
SHEET 1316 C F33=EFS3*D3*D3+EFS2*((2.*DES+RVP4*DET)*E10+(2.*E3+RVP4*E5)*E4)
SHEET 1317 C F34=EFS3*D3*D4+EFS2*((2.*DES+RVP4*DET)*E11+(2.*E4*E4)
SHEET 1318 C )
SHEET 1319 C
SHEET 1320 C D(1,1)=(S1+S2*EFS1)*F11+S2*F1*F1)*RC*THKL
SHEET 1321 C D(1,2)=(S1+S2*EFS1)*F12+S2*F2*F1)*FC*THKL
SHEET 1322 C D(1,4)=(S1+S2*EFS1)*F13+S2*F3*F1)*FC*THKL
SHEET 1323 C D(1,5)=(S1+S2*EFS1)*F14+S2*F4*F1)*FC*THKL
SHEET 1324 C D(2,2)=(S1+S2*EFS1)*F22+S2*F2*F2)*FC*THKL
SHEET 1325 C D(2,4)=(S1+S2*EFS1)*F23+S2*F2*F3)*FC*THKL
SHEET 1326 C D(2,5)=(S1+S2*EFS1)*F24+S2*F2*F4)*FC*THKL
SHEET 1327 C D(4,4)=(S1+S2*EFS1)*F33+S2*F3*F3)*FC*THKL
SHEET 1328 C D(4,5)=(S1+S2*EFS1)*F34+S2*F3*F4)*FC*THKL
SHEET 1329 C D(5,5)=(S1+S2*EFS1)*F44+S2*F4*F4)*FC*THKL
SHEET 1330 C D(2,1)=D(1,2)
SHEET 1331 C F(4,1)=D(1,4)
SHEET 1332 C D(4,2)=D(2,4)
SHEET 1333 C D(5,1)=D(1,5)
SHEET 1334 C D(5,2)=D(2,5)
SHEET 1335 C D(5,4)=D(4,5)
SHEET 1336 C
SHEET 1337 C
SHEET 1338 C H(1)=-S1+S2*EFS1)*F1)*RC*THKL
SHEET 1339 C H(2)=-S1+S2*EFS1)*F2)*FC*THKL

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SHEET 1340      H(A)=- (S1+S2#FFS1)#F#FC#THKL
SHEET 1341      H(S)=- (S1+S2#FFS1)#F#FC#THKL
SHEET 1342      C
SHEET 1343      C
SHEET 1344      7) CONTINUE
SHEET 1345      DETLN
SHEET 1346      END

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SHEET 1348      SUBROUTINE CONDEN(A,P,NEG,MRAND,N,U)
SHEET 1349      C
SHEET 1350      C*****
SHEET 1351      C PERFORM THE MATRIX CONDENSATION WHEN THE VALUE OF A COMPONENT
SHEET 1352      C X IN AX=B IS SPECIFIED EQUAL TO ZERO
SHEET 1353      C*****
SHEET 1354      C
SHEET 1355      DIMENSION R(NEG),A(NEG,1)
SHEET 1356      C
SHEET 1357      DO 250 M=2,MRAND                                MOD 3
SHEET 1358      K=N-M+1                                       MOD 4
SHEET 1359      IF(K) 230,235,230                               MOD 5
SHEET 1360      230 R(K)=R(K)-A(K,M)*U                               MOD 6
SHEET 1361      A(K,M)=0.0                                       MOD 7
SHEET 1362      235 K=N-M-1                                       MOD 8
SHEET 1363      IF(NEG-K) 250,240,240                               MOD 9
SHEET 1364      240 R(K)=R(K)-A(N,M)*U                               MOD 10
SHEET 1365      A(N,M)=0.0                                       MOD 11
SHEET 1366      250 CONTINUE                                       MOD 12
SHEET 1367      A(N,1)=1.0                                       MOD 13
SHEET 1368      P(N)=U                                           MOD 14
SHEET 1369      RETURN                                           MOD 15
SHEET 1370      C                                             MOD 16
SHEET 1371      END                                             MOD 17

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SHEET 1373      SUBROUTINE MODIFY(CODE,A,B,ALPHA,GAMMA,ETA,NUMNF,NEG,MRAND,FRNFCE)
SHEET 1374      DIMENSION CODE(1),A(NEG,1),R(1),ALPHA(1),GAMMA(1),ETA(1),FRNFCE(1)
SHEET 1375      DO 12) J=1, NUMNF
SHEET 1376      IL=3*I
SHEET 1377      I7=IL-1
SHEET 1378      IR=I7-1
SHEET 1379      C=CODE(I)
SHEET 1380      IF (C.EQ. 1.) GO TO 101
SHEET 1381      IF (C.EQ. 2.) GO TO 102
SHEET 1382      IF (C.EQ. 3.) GO TO 103
SHEET 1383      C
SHEET 1384      CALL CONDEN(A,P,NEG,MRAND,IL,0.)
SHEET 1385      CONSTA=ALPHA(I)
SHEET 1386      CONSTE=ETA(I)
SHEET 1387      CONSTG=GAMMA(I)
SHEET 1388      IF(C.EQ. 4)GO TO 104
SHEET 1389      GO TO 121
SHEET 1390      C
SHEET 1391      101 CONTINUE
SHEET 1392      CALL CONDEN(A,R,NEG,MRAND,IR,0.)
SHEET 1393      CALL CONDEN(A,B,NEG,MRAND,IL,C.)
SHEET 1394      GO TO 121
SHEET 1395      C
SHEET 1396      102 CONTINUE
SHEET 1397      CALL CONDEN(A,P,NEG,MRAND,I7,0.)
SHEET 1398      CALL CONDEN(A,R,NEG,MRAND,IL,C.)
SHEET 1399      R(I7)=R(I7)+FRNFCE(I)
SHEET 1400      GO TO 121
SHEET 1401      C
SHEET 1402      103 CONTINUE
SHEET 1403      CALL CONDEN(A,P,NEG,MRAND,IR,C.)
SHEET 1404      CALL CONDEN(A,B,NEG,MRAND,I7,0.)
SHEET 1405      CALL CONDEN(A,R,NEG,MRAND,IL,0.)
SHEET 1406      GO TO 121
SHEET 1407      C
SHEET 1408      104 CONTINUE
SHEET 1409      CALL RCPIX(A,E,NEG,MRAND,CONSTA,CONSTE,CONSTG,IR,I7)
SHEET 1410      C
SHEET 1411      121 CONTINUE
SHEET 1412      C
SHEET 1413      RETURN
SHEET 1414      END

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SHEET 1416      SUBROUTINE TPIA(NN,MM,A)
SHEET 1417      DIMENSION A(NN,1)
SHEET 1418      C
SHEET 1419      C*****
SHEET 1420      C TRIANGULIZATION OF GAUSSIAN ELIMINATION FOR THE SOLUTION
SHEET 1421      C OF BANDED SYMMETRIC MATRIX
SHEET 1422      C*****
SHEET 1423      C
SHEET 1424      N=0
SHEET 1425      100 N=N+1
SHEET 1426      IF(N.EQ.NN)RETURN
SHEET 1427      IF(A(N,1).NE.0.) GO TO 150
SHEET 1428      GO TO 100
SHEET 1429      C
SHEET 1430      150 I=N
SHEET 1431      MR=MINO(MM,NN-N+1)
SHEET 1432      C
SHEET 1433      DO 250 L=2,MR
SHEET 1434      I=I+1
SHEET 1435      C=A(N,L)/A(N,1)
SHEET 1436      IF(C.EQ.0.0)GO TO 260
SHEET 1437      J=0
SHEET 1438      C
SHEET 1439      DO 250 K=L,MR
SHEET 1440      J=J+1
SHEET 1441      250 A(I,J)=A(I,J)-C*A(N,K)
SHEET 1442      A(N,L)=C
SHEET 1443      260 CONTINUE
SHEET 1444      C
SHEET 1445      GO TO 100
SHEET 1446      C
SHEET 1447      C
SHEET 1448      END

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SHEET 1450      SUBROUTINE BACKS(NN,MM,A,P)
SHEET 1451      C
SHEET 1452      C*****
SHEET 1453      C PACY SUBSTITUTION FOR SOLUTION OF BANDED SYMMETRIC MATRIX
SHEET 1454      C*****
SHEET 1455      C
SHEET 1456      C
SHEET 1457      DIMENSION A(1),P(1)
SHEET 1458      C
SHEET 1459      MMM=MM-1
SHEET 1460      N=0
SHEET 1461      270 N=N+1
SHEET 1462      C=P(N)
SHEET 1463      IF(A(N).NE.0.0)B(N)=P(N)/A(N)
SHEET 1464      IF(N.EQ.NN)GO TO 300
SHEET 1465      IL=N+1
SHEET 1466      IF=MINO(NN,N+MMM)
SHEET 1467      M=N
SHEET 1468      DO 285 I=IL,IF
SHEET 1469      M=M+NN
SHEET 1470      285 B(I)=P(I)-A(M)*C
SHEET 1471      GO TO 270
SHEET 1472      C
SHEET 1473      C
SHEET 1474      300 IL=N
SHEET 1475      N=N-1
SHEET 1476      IF(N.EQ.0) RETURN
SHEET 1477      IF=MINO(NN,N+MMM)
SHEET 1478      M=N
SHEET 1479      C
SHEET 1480      DO 400 I=IL,IF
SHEET 1481      M=M+NN
SHEET 1482      400 P(N)=P(N)-A(M)*P(I)
SHEET 1483      GO TO 300
SHEET 1484      C
SHEET 1485      C
SHEET 1486      END

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SHEET 1488      SUBROUTINE HARD(ERS,V)
SHEET 1489      COMMON/WATERL/YYVAL,PPSTN,EEYENT,PEESTS
SHEET 1490      C
SHEET 1491      C*****
SHEET 1492      C WORKHARDENING CHARACTERISTIC CURVE

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SHEET 1493 C*****
SHEET 1494 C
SHEET 1495 YVALUE=YYVAL
SHEET 1496 EXPNT=EXXPNT
SHEET 1497 Y=YVALUE*(PRESTN+EPS)**EXPNT+PRESTS
SHEET 1498 RETURN
SHEET 1499 END

SHEET 1501 SUBROUTINE HARD2(EPS,Y)
SHEET 1502 COMMON/MATERL/YYVAL,PRESTN,EXXPNT,PRESTS
SHEET 1503 C
SHEET 1504 C*****
SHEET 1505 C COMPUTE WORK HARDENING RATE
SHEET 1506 C*****
SHEET 1507 C
SHEET 1508 EXPNT=EXXPNT
SHEET 1509 YVALUE=YYVAL
SHEET 1510 Y=EXPNT*YVALUE*(PRESTN+EPS)**(EXPNT-1.)
SHEET 1511 RETURN
SHEET 1512 END

SHEET 1514 SUBROUTINE RCMIX(A,R,NEQ,MBAND,CONSTA,CONSTE,CONSTG,IF,I2)
SHEET 1515 DIMENSION A(NEQ,1),B(1)
SHEET 1516 C
SHEET 1517 C*****
SHEET 1518 C THIS SUBROUTINE HANDLES THE MIXED BOUNDARY CONDITION
SHEET 1519 C*****
SHEET 1520 C
SHEET 1521 IK=I2-IR+1
SHEET 1522 A(I2,1)=A(I2,1)+2.*A(IR,IK)/CONSTA+A(IR,1)/CONSTA/CONSTG
SHEET 1523 R(I2)=R(I2)+R(IR)/CONSTA-CONSTG*(A(IR,1)/CONSTA+A(IR,IK))
SHEET 1524 B(I2)=B(I2)+CONSTE
SHEET 1525 A(IR,1)=1.0
SHEET 1526 A(IR,IK)=0.0
SHEET 1527 R(IR)=0.0
SHEET 1528 C
SHEET 1529 C
SHEET 1530 DO 300 M=2,MBAND
SHEET 1531 K=I2-M+1
SHEET 1532 IF(K.LT.1)GO TO 300
SHEET 1533 IF(K.EQ.1R)GO TO 300
SHEET 1534 IF(K.GT.1R)GO TO 350
SHEET 1535 MM=IR-K+1
SHEET 1536 A(K,M)=A(K,M)+A(K,MM)/CONSTA
SHEET 1537 R(K)=R(K)-A(K,MM)*CONSTG
SHEET 1538 A(K,MM)=0.
SHEET 1539 GO TO 300
SHEET 1540 350 CONTINUE
SHEET 1541 C
SHEET 1542 MM=K-1R+1
SHEET 1543 A(K,M)=A(K,M)+A(IR,MM)/CONSTA
SHEET 1544 R(K)=R(K)-A(IR,MM)*CONSTG
SHEET 1545 A(IR,MM)=0.
SHEET 1546 300 CONTINUE
SHEET 1547 C
SHEET 1548 MB1=MBAND-1
SHEET 1549 DO 200 M=2,ME1
SHEET 1550 MM=M+1
SHEET 1551 A(I2,M)=A(I2,M)+A(IR,MM)/CONSTA
SHEET 1552 I27=I2+M
SHEET 1553 R(I27)=R(I27)-A(IR,MM)*CONSTG
SHEET 1554 200 A(IR,MM)=0.0
SHEET 1555 C
SHEET 1556 C
SHEET 1557 RETURN
SHEET 1558 END

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