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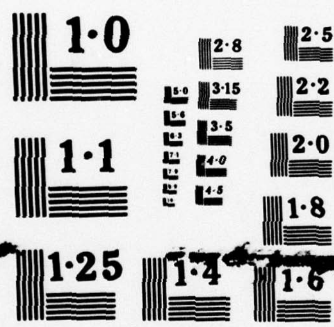
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PRESSURE RISE IN A VENTED CARGO TANK DUE TO EXTERNAL HEATING

FINAL REPORT

APRIL 1976

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16. Abstract

The normal venting capacity of marine cargo tanks appears to be adequate for relieving vapor generated when integral tanks are exposed to an external fire. However, the unwetted tank walls surrounding the ullage are subject to substantial weakening in those cases where the fire heat flux increases the wall temperature above 1000°F. Convection and radiation can adequately cool the unwetted wall only for incident heat flux less than 16,000 BTU/hr/ft^2. Thus, the failure mode of cargo tanks with an external fire may be due not to the vent system, but rather to heating of the unwetted wall.

17. Key Words		18. Distribution Statement
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ERRATA SHEET

PRESSURE RISE IN A VENTED CARGO TANK DUE TO EXTERNAL HEATING

1. Pg. 4, Par. 1, sentence 6: Replace 20 psig with 6.

2. Pg. 11, under section 4: Delete third and fourth sentences of the first paragraph. Add the following to the first paragraph:

   Using a modified theory of Clarkson (1956), estimates were made of the minimum internal pressure loadings required to initiate failure of cargo tanks of three representative vessel designs. For the offshore barge, an estimate of 6 psig was obtained; for the inland barge, 4 psig; and for a large tankship, 8 psig. The average value for these three, 6 psig, will be utilized in this report as a nominal failure pressure for tank vessel.

3. Pg. 13, fifth line: replace "20 psig" with "6 psig".
   sixth line: replace "20 psig or 3.2 psig" with "6 psig or 1 psig".

4. Pg. 27, third line from bottom: replace "20 psig" with "6 psig".

5. Pg. 36, second line from the bottom: replace "to reach 20 psig" with "to exceed 6 psig".

6. Pg. 41, last sentence: replace existing sentence with the following: "In general, our results suggest that the exposed wall area should not be more than 1000 times vent area if the pressure is not to exceed 6 psig for a typical cargo and heat flux".

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The work reported on herein was performed for the Marine Safety Technology Division of the U. S. Coast Guard Office of Research and Development. The views are those of the author(s) who is(are) responsible for the facts and accuracy of the data. This report does not constitute a specification, standard or regulation.

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1. SUMMARY AND RECOMMENDATIONS

A. BACKGROUND AND OBJECTIVES

Venting systems are installed on marine cargo tanks to relieve pressure differentials which arise between the vapor space and the ambient. The capacities of the venting systems are designed to accommodate the flow of vapor and air during cargo transfer operations under a normal back pressure of about 2 psig. However, in the event of a fire external to the tank, substantial pressure rise may occur due to vapor generation and due to heating of the trapped vapor. The hazard of pressure rise due to external fire and the design requirements of cargo venting systems to control the hazard are being reviewed by Arthur D. Little, Inc. as part of Contract No. DOT-CG-42,357-A. Three other hazardous situations which occur in marine cargo transport have been analyzed in earlier reports: Flame propagation into the vent system, blockage of venting relief valves, and overpressure due to excessive loading rates.

In the present report, the external fire problem is analyzed taking into account variations in the following parameters:

- Levals of heat flux incident on the tank;
- Exposed areas (wetted and unwetted);
- Tank wall thickness and thermal and structural properties;
- Cargo heat of vaporization;
- Molecular weight of the cargo vapor; and
- Capacity of the venting system.

The case of liquid venting (which might occur if the vessel was listing) is not included in this analysis, but has been studied for high venting rates in an earlier report.

Numerical methods were developed for predicting the pressure rise and unwetted wall temperature as a function of time for any specified combination of parameters listed above. By exercising this powerful numerical tool, it was possible to resolve such issues as:

(1) Does a tank exposed to fire fail due to inadequate venting capacity or weakening of the heated walls?

(2) What is the longest fire duration which can be tolerated before failure?

(3) What venting capacity is required for a given tank/cargo/fire situation?

B. SUMMARY OF FINDINGS

The normal venting capacity of marine cargo tanks appears to be adequate for relieving vapor generated when integral tanks are exposed to an external fire. However, the unwetted tank walls surrounding the ullage are subject to substantial weakening in those cases where the fire heat flux increases the wall temperature above 1,000F. Convection and radiation can adequately cool the unwetted wall only for incident heat flux less than 16,000 Btu/hr/ft^2. Thus, the failure mode of cargo tanks with an external fire may be due not to the vent system, but rather to heating of the unwetted wall.

Cargo boiling behind the wetted wall is not the only cause of pressure rise; heating of the confined vapor space by the unwetted wall also increases the tank pressure. These two mechanisms produce a two stage pressure history leveling off after about 20 minutes from the start of the fire. The predicted pressure rise under representative fire conditions is less than 2 psig; that is, the normal venting capacity appears to be adequate to release vapor rapidly generated by an external fire. The following unusual conditions would cause the tank pressure to exceed 10 psig during a fire:

- Ratio of vent area to the wetted wall area exposed to fire: Less than .0005.
- Ratio of heat flux (Btu/hr/ft^2) to heat of vaporization (Btu/lb): Greater than 660.

The pressure rise is also aggravated by a large vapor space and by cargo vapors of low molecular weight. Those combinations of variables leading to pressures exceeding 4 psig can be identified by expression (24) which appears on page 46.

Since the analyses employed in this work utilize well established formulas for venting and heat transfer and take into account all of the important effects that may occur during an external fire, it is expected that predictions of ullage pressure will indeed correspond closely to the pressures encountered in actual fire situations.

C. RECOMMENDATIONS

Because the weakening of cargo tank walls by an external fire can cause catastrophic failure of the tank, additional work needs to be carried out to better determine the precise conditions under which barge and tanker vessels are excessively heated and to establish means of prevention. It is recommended that studies be conducted to:

- Determine the duration and size of marine fires resulting from spills, engine room fires, ignited vent gases, and other sources;
- Establish the effect of the heat flux of each type of flame on the temperature of cargo tank walls, taking into account conduction to the supporting structure and unexposed walls; and
- Evaluate the potential for external insulation and the effects of double hulls.

Such studies should be accompanied by experiments to test the validity of the analyses and the feasibility of preventive methods.

II. INTRODUCTION

A. PROBLEM STATEMENT

Normally, the air displaced from a tank during loading is vented, along with the vapor which is generated from the liquid-cargo surface. The venting system usually includes a relief valve and represents a constriction which at normal venting rates causes the tank pressure to rise to 1 - 3 psig. Although failure points vary widely, a typical tank structure is capable of withstanding about 6 psig*(if constructed in accordance with ABS rules) and is therefore capable of handling the normal back pressure of up to 3 psig.

If there is an external heating source such as a fire, the liquid evaporation rate increases and the back pressure increases accordingly. Fire may result from collision with another vessel, resulting in the release and ignition of flammable cargo. The tank walls heat up, the liquid cargo heats up and evaporates at a faster rate, and under some conditions, boiling may occur. This rapid release of vapor with a finite venting capacity causes a continuous pressure rise. To compound the problem, the vapor is heated by the unwetted hot tank walls and this increases the tank pressure even more. Under extreme conditions the back pressure may increase to the tank failure point (which is lower than normal because of external heating). The purpose of our analysis is to determine the conditions under which tank failure could occur, and to estimate the elapsed time before failure. Key variables are the vent area, the exposed wall area, the fire characteristics, and the liquid cargo properties.

Our approach has been to perform a detailed transient heat transfer analysis of a partially-full rectangular tank exposed to lateral heating. The vapor generation rate and transient wall temperature obtained from this analysis are then used to describe the pressure rise and decreasing wall strength, respectively. As shown in Figure II-1, the cross-over of the increasing pressure and decreasing strength indicates tank wall failure.

An earlier analysis by the National Research Council (1973) contained a comprehensive survey of the heat flux to be expected from fires. The N.R.C. report also derived relief valve venting capacities by assuming that in steady state all of the heat flux goes to generating vapor. However, several

* Based on plastic deformation theory applied to actual bulkhead thickness and stiffener spacings (see Vent System and Loading Criteria for Avoiding Tank Overpressurization, part 3 of Contract DOT-CG-42,357-A).

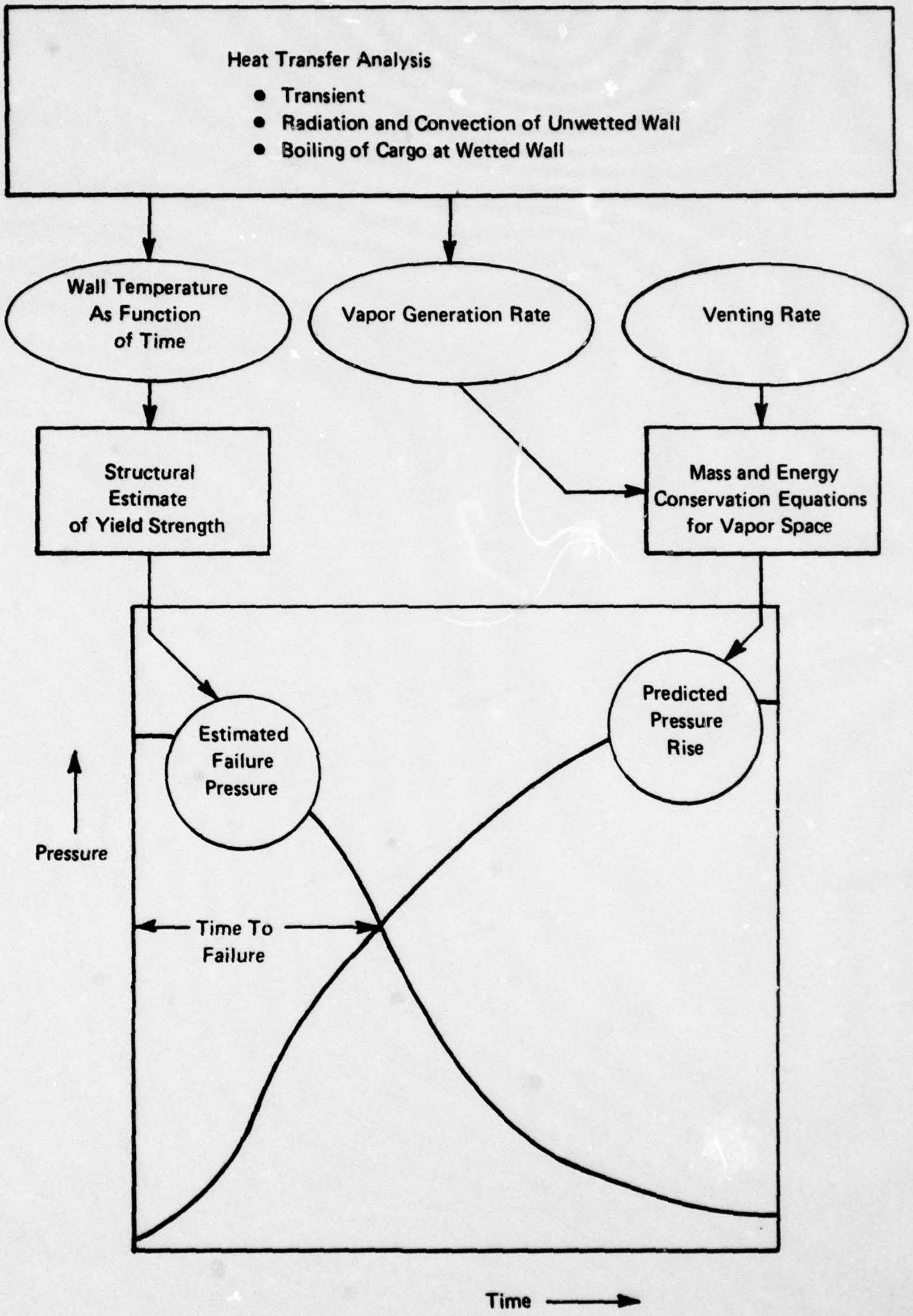


FIGURE II–1 METHOD OF ANALYSIS

limitations make the N.R.C. study inadequate to describe the transient response of an integral tank to external fire: (1) the "critical flow" venting assumption (Equation (28), p. 51) is not appropriate for low pressure venting; (2) the N.R.C. study did not include transient heatup of the tank wall; (3) secondary heating of the vapor space was not included; and (4) specific values of liquid height above sea level and unwetted wall height were not considered; instead, general factors were employed to represent the tank wall area, exposure factor, and atmospheric attenuation factors. In these respects the present analysis is more general and more readily applicable to integral tanks in barges and chemical tankers.

B. FIRE SITUATIONS

Four fire situations were considered in formulating the problem (see Figure II-2):

- Fire in the hull;
- Fire on deck;
- Adjacent fire; and
- Spot fire.

The "adjacent" fire was selected as the most useful case to analyze because (a) the fire duration is great compared to the spot or deck fire, (b) the frequency of occurrence is relatively great, (c) the fire size is larger than the spot fire and large enough to envelop the entire side wall, and (d) both wetted and unwetted portions of the tank wall are heated. A review of DOT casualty reports covering a six year period showed that the "adjacent" fire and the engine room fire were comparatively more often encountered than other types, with several casualties cited annually.

The hull fire requires adequate ventilation below the tank and occurs with independent tank construction rather than with integral tanks. This situation has been analyzed by ADL in a recent study of pressurized liquid propylene tanks (Raj [1975]). The deck fire, if prolonged, is more likely to cause simple melting of the tank wall than fracture due to inadequate venting, since the vapor generation rate (which absorbs heat and protects the wall) is relatively low.

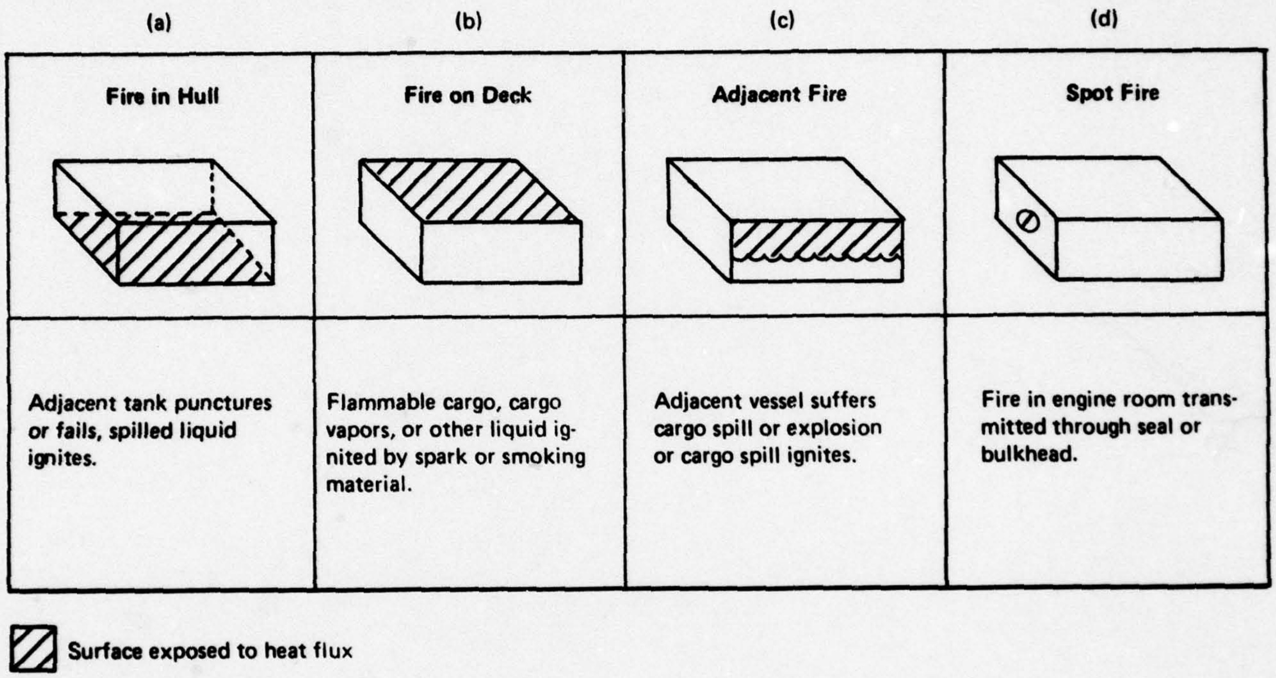


FIGURE II–2 FIRE SITUATIONS

C. RANGES OF KEY PARAMETERS

1. Heat Flux from External Fire

The incident flux from hydrocarbon/air fires has been measured and calculated by a number of investigators for widely varying conditions. Values range up to 90,000 Btu/hr/ft^2, and depend on the optimal beam length, carbon-hydrogen ratio of the fuel, and separation distance. The USCG regulation (46 CFR 54 [1971]) assumes a value of 34,500 Btu/hr/ft^2, which is reasonable since it is comparable to the heat flux to the furnace walls of an oil-fired utility boiler. The National Research Council (1973), p. 24 ff, compiled data for a number of fuels and fire sizes, supporting a representative value of 34,500 Btu/hr/ft^2 with a range from 5,400 to 47,540 Btu/hr/ft^2. Hot spots have been observed receiving up to 90,000 Btu/hr/ft^2, according to Anderson and Stresino (1963). Based on this data base, we employed the following values of heat flux:

	Heat Flux Values
Representative Value	32,000 Btu/hr/ft^2
Range of Values	16,000 to 64,000 Btu/hr/ft^2

Recent studies by Markstein (1974) and Lee (1975) on small flames confirm that the major heat flux component is radiative.

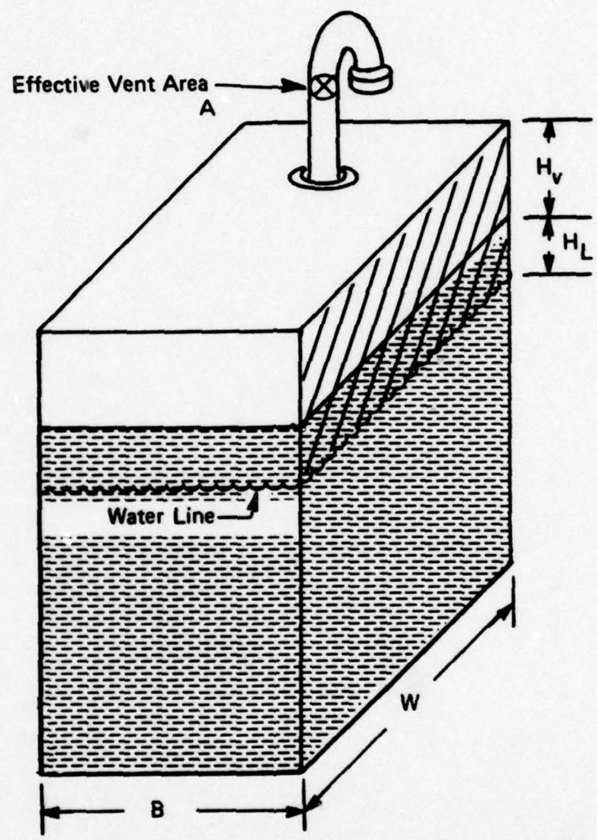
2. Tank and Vent Configuration

For the purposes of the analysis, a rectangular tank is assumed of wetted and unwetted wall heights H_L and H_v, width B and length W (as shown in Figure II-3). The effective vent area is A, with P-V valve open.

For a fully loaded tank, H_v is less than 4% of the total tank depth. Also, unless several tanks are empty and the vessel displacement is unusually low, H_L is rarely more than several feet above waterline. The range of parameters used in the present study is representative of chemical tanker operation:

Parameter:	H_v	H_L	W	A
Representative Value:	4 ft	3.3 ft	25.5 ft	.215 ft^2*
Range of Values:	2 - 31 ft	3.3 - 16.5	25.5 ft	.054 - .54 ft^2

* Approximately equal to 6" pipe diameter.



Area exposed to fire

FIGURE II–3 TANK CONFIGURATION

For the baseline vapor space height of 4 ft, the total volume of the vapor space assumed was 2,700 ft^3 with B assumed to be 26 ft.

3. Liquid Cargo Properties

The latent heat of vaporization of the cargo controls the vapor generation rate, once the wall was heated up and the heat flux to the liquid has equilibrated. The values assumed for this key parameter and for the vapor molecular weight are given below.

Parameter:	Heat of Vaporization (λ)	Molecular Weight (μv)
Representative Value:	145 Btu/lb	86 lb/lb mole
Range of Values:	86 - 259 Btu/lb	20 - 100 lb/lb mole

Representative values correspond to Hexane.

4. Failure Pressure of Unwetted Tank Wall

One can use plastic deformation theory to estimate the failure pressure at room temperature. (See Vent System and Loading Criteria for Avoiding Tank Overpressurization, part 3 of Contract DOT-CG-42,357-A, p. 14 ff.) Using a modified theory of Clarkson (1956), estimates were made of the minimum internal pressure loadings required to initiate failure of cargo tanks of three representative vessel designs. For the offshore barge, an estimate of 6 psig was obtained; for the inland barge, 4 psig; and for a large tankship, 8 psig. The average value for these three, 6 psig, will be utilized in this report as a nominal failure pressure for tank vessel.

As the wall temperature increases, the material strength decreases. The strength does not decrease substantially below about 600F, as shown in Figure II-4. The strength-temperature relationship appears to be approximately log linear between 900F and 1400F for steel, with a decrease of about 25% for each 100F above 900F. The data in Figure II-4

* The failure pressure for a particular tank may be less or greater than this value, and will depend on the structure and condition of the tank

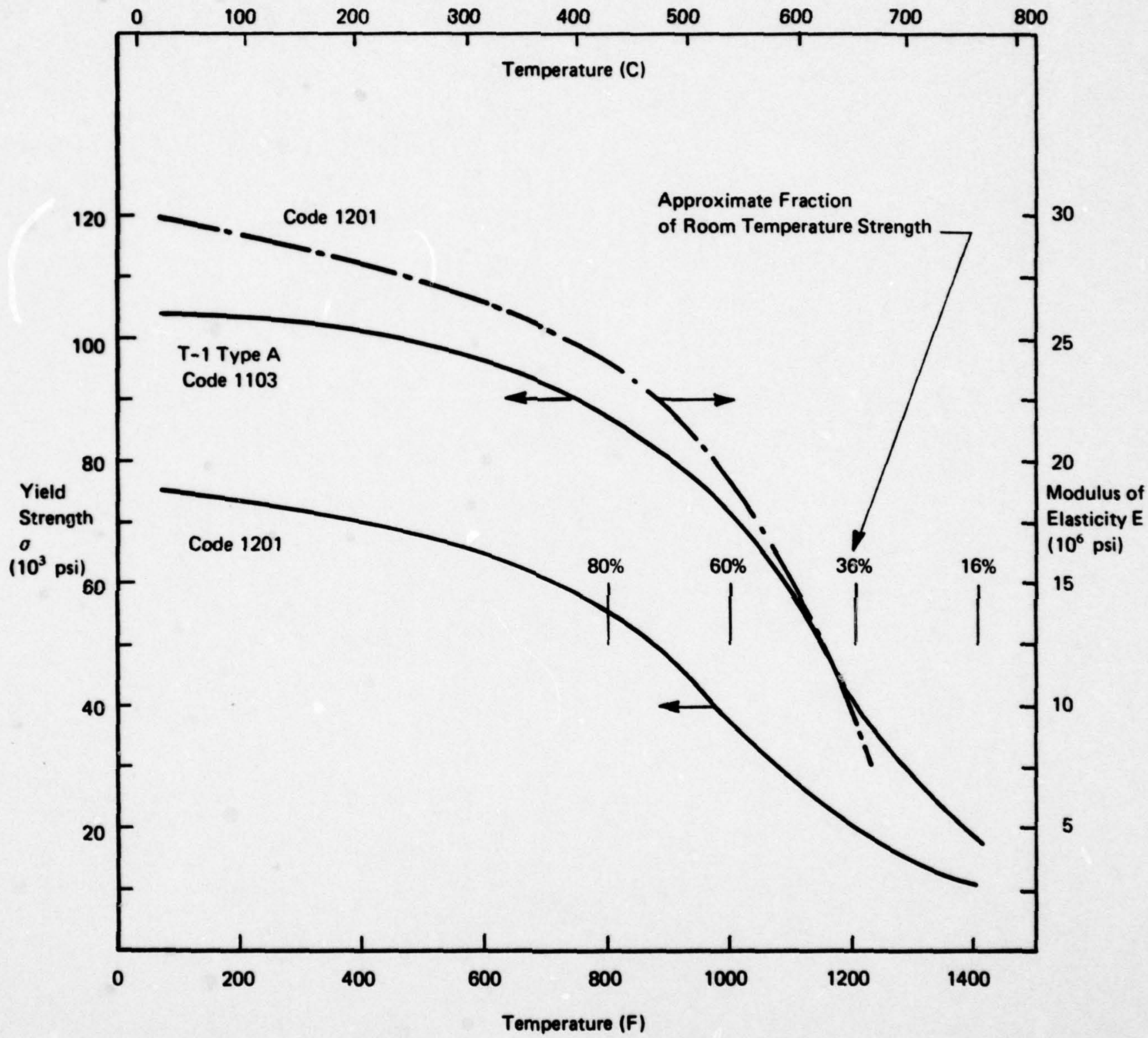


FIGURE II-4 EFFECT OF TEMPERATURE ON STEEL WALL STRENGTH

for two structural steels is taken from Weiss, et al. (1966). If we assume that Clarkson's equation holds and P_{max} is proportional to $\sigma^{4/3}/E^{1/3}$, then Figure II-4 can be used to estimate the decreasing ability of the wall to withstand pressure in the vapor space. For example, if an unwetted wall normally capable of 6 psig should reach 1400F temperature, an estimate of the failure pressure would be 16% of 6 psig or 1.0 psig. Here the 16% factor was obtained from Figure II-4 at 1400F.

An additional stress factor, which was not dealt with in the present analysis, is the steep thermal gradient along the exposed wall both at the water line and at the liquid cargo line.

III. ANALYSIS AND MODEL FORMULATION

A. Physical Mechanisms

For the purposes of discussion and calculations, we consider a rectangular tank, one wall of which is exposed to the fire. The physical system is illustrated schematically in Figure III-1, and the important physical processes are illustrated in Figure III-2.

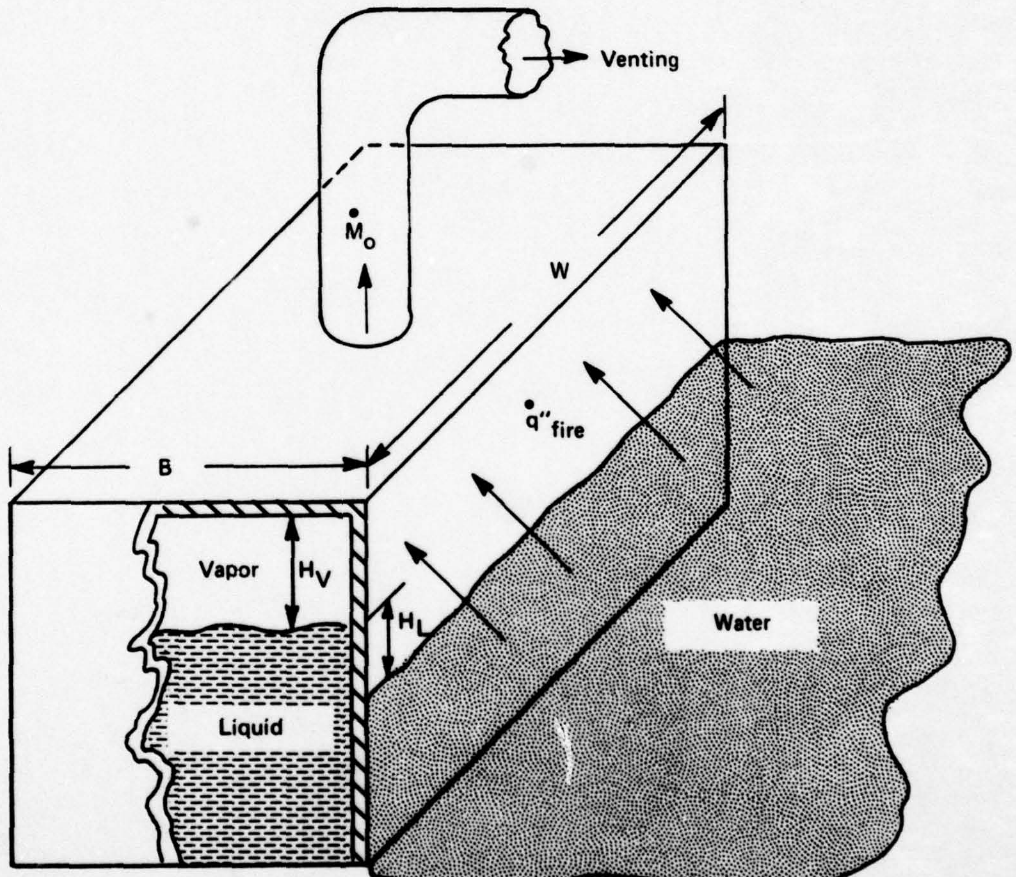


FIGURE III-1 SCHEMATIC DIAGRAM ILLUSTRATING THE SCENARIO OF ADJACENT FIRE, VAPOR AND LIQUID DEPTHS AND TANK WALL

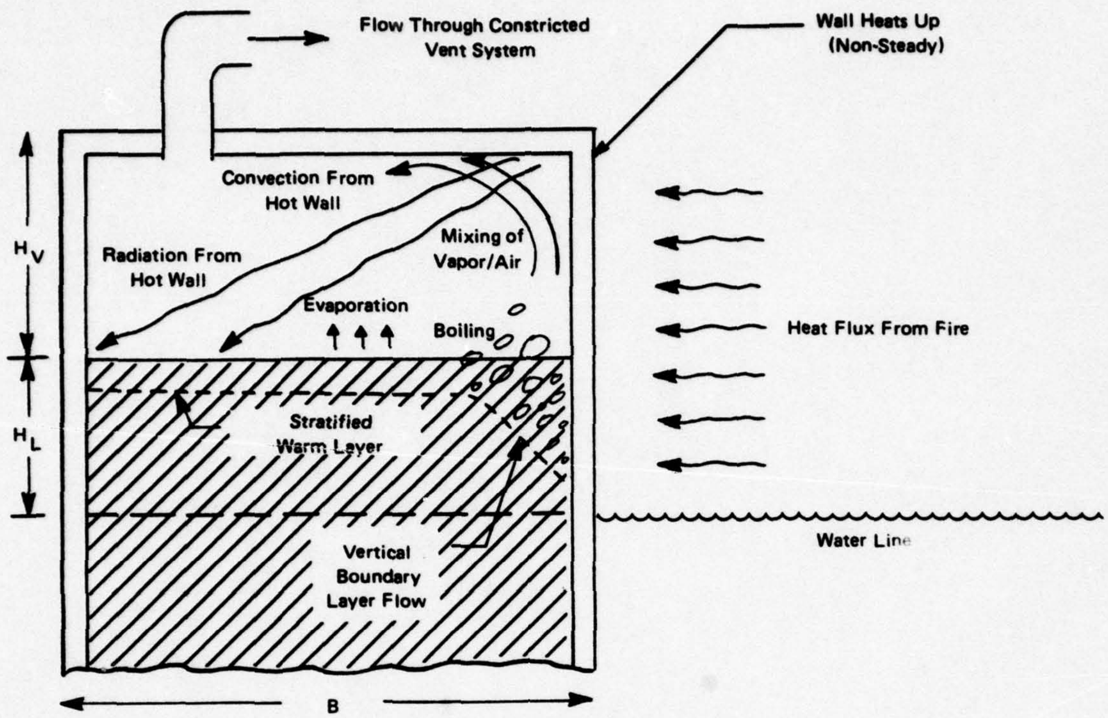


FIGURE III–2 PHYSICAL PROCESSES DETERMINING THE PRESSURE RISE DUE TO EXTERNAL HEATING

The radiation from an external fire heats up the portion of the wall which is above the water line. That part of the wall in contact with the vapor heats up more rapidly than the one in contact with the liquid. The wall in contact with the liquid transfers heat to the liquid, thereby setting a flow of the heated liquid near the wall. The difference between the heat coming from the fire into the wall and the heat expended in heating the fluids (vapor or liquid) on the inner face goes to increase the wall temperature.

When a liquid in a tank is heated from the sides, it is well known that the resulting flow of liquid produces a density stratification phenomenon which is very stable. In addition, for reasonably high heat flux input into the wall, nucleate boiling takes place.

It has been found that in spite of the liquid boiling on the hot wall surface, the stratification of the bulk of liquid persists [Anderson, et al., (1974)]. Gebhart (1971) has reported that up to 10^6 Btu/hr/ft^2 heat flux can be accepted from a submerged heater by nucleate boiling. The mass of vapor emanating from the liquid will increase substantially when boiling occurs on the wall.

Once the liquid starts boiling on the wall (and even if the bulk of liquid is not at saturation temperature), the temperature of the wall in contact with the liquid remains essentially unaltered because of the very high heat fluxes that occur in the boiling process. The average wall temperature will be between 10 and 30F higher than the saturation temperature of the liquid. Boiling results in considerably increased vapor flow into the ullage volume. Consequently, the tank pressure builds up rapidly. In the ullage volume, the confined vapor is heated directly by the hot unwetted wall by convection. Also, mixing of the vapor generated by boiling with the pre-existing ullage contents occurs, and molecular weight differences must be considered. The increased pressure, together with diminishing strength of the wall in contact with the vapor due to increased temperature, may result in situations where the tank would fail. The schematic diagram III-2 illustrates these basic physical phenomena.

In this report, we have analyzed, by suitable models, the above physical phenomena. The pressure-time history and the wall temperature-time history are calculated for a variety of tank, liquid, fire, and vent characteristics.

B. Model Formulation and Assumptions

The model developed to determine the pressure-time history involves essentially writing the appropriate mass, energy, and specie balance equations in addition to accounting for the vapor heating and liquid boiling. Figure III-3 illustrates schematically the various heat sinks in the system that absorb the heat from fire. We write the equations to describe the state of the system subject to the following assumptions:

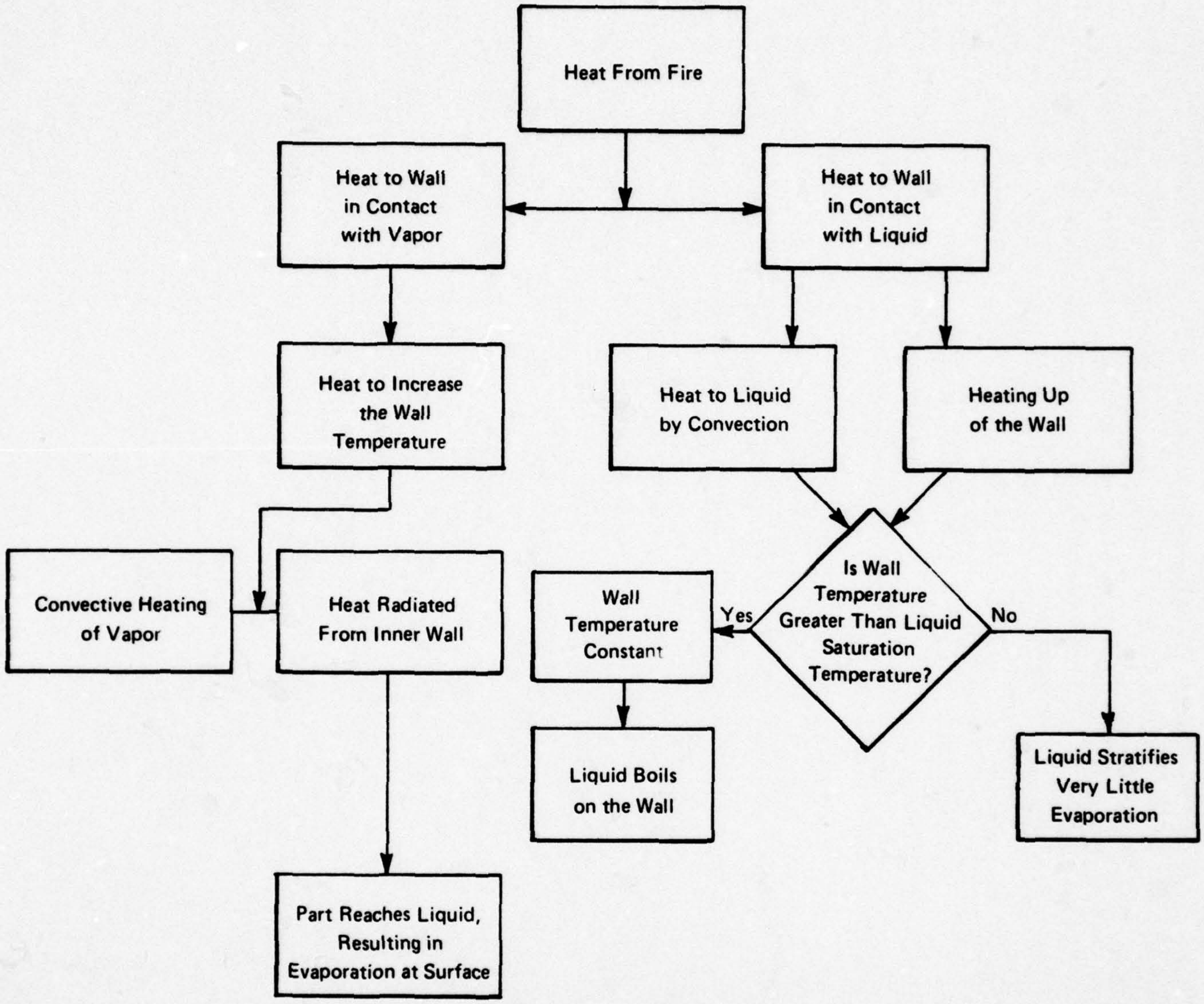


FIGURE III-3 HEAT BALANCE DIAGRAM WHICH SCHEMATICALLY INDICATES THE VARIOUS SINKS FOR THE HEAT FROM THE FIRE

Assumptions

- The liquid is characterized by a single boiling point, and its vapor has unique thermodynamic properties.
- The vapor produced and any air present in the ullage volume act like perfect gases.
- Mixing of vapor and air in the ullage occurs instantaneously so that there are no temperature or composition gradients in the mixture.
- The change in the liquid volume due to heating is small; that is, the volume of the ullage space remains a constant. No cargo loading or unloading is assumed to occur during the fire.
- Quasi-steady analysis is used to describe the free convection (turbulent) heat transfer between the wall and the cargo or vapor; that is, the boundary layer is assumed to be fully developed and assumed to correspond at every instant of time to the wall temperature and fluid temperature values at the instant under consideration.
- The radiation from the wall is not absorbed by the gas in the ullage.
- The heat flux from the fire is uniform over the entire wall face exposed to the fire and constant in time.
- There are no temperature variations across the thickness of the wall; that is, there is no spatial variation of temperature in either the wetted or unwetted wall. However, the temperature is a function of time.
- The temperature of the wall in contact with liquid remains constant when the liquid starts boiling. Nucleate boiling rather than transition or film boiling is assumed to occur.
- Heat flux reaching the cargo surface by unwetted wall reradiation is assumed to generate vapor rather than heat up the liquid.
- In spite of the large temperature gradient in the wall near the liquid meniscus, the total heat flow rate by conduction

along the wall is negligible. Alternatively, the temperatures of the walls are not coupled by heat transfer between them. Conduction to the cooler deck or supporting structure is also neglected.

- Radiative heat received by colder walls in the tank is not reradiated to the liquid.

- The cargo is carried at ambient pressure at the start of the fire.

Model Equation Formulation

We write below the various "balance" equations. The definitions of the symbols are given in the nomenclature. The various terms are also defined schematically in Figure III-4.

C. Governing Equations

1. Thermal Balance on the Steel Wall in Contact with Vapor

$$\underbrace{\rho_w c_w L_w \frac{dT_w}{dt}}_{\substack{\text{Rate of increase in the}\\ \text{internal energy of a}\\ \text{unit area of wall}}} = \underbrace{\dot{q}''_f - \dot{q}''_{rad} - \overline{\dot{q}''_c}}_{\substack{\text{Net heat coming}\\ \text{into the wall}\\ \text{per unit area}}} \qquad (1)$$

where, assuming no reradiation into the wall,

$$\dot{q}''_{rad} = \sigma T_w^4 \qquad (2)$$

and $\overline{\dot{q}''_c}$ is the mean heat flux leaving the wall due to turbulent free convection cooling by the vapor flow. The expression for $\overline{\dot{q}''_c}$ is obtained from the expression given in Appendix B (equation B-4).

$$\overline{\dot{q}''_c} = 0.0246 \frac{K(T_w - T)}{H_v} \left[\frac{g\beta(T_w - T) H_v^3}{\nu^2}\right]^{2/5} \frac{Pr^{7/15}}{[1 + 0.494\, Pr^{2/3}]^{2/5}} \qquad (3)$$

It is noted that in the above expressions, both T_w and T (the vapor temperature) are functions of time.

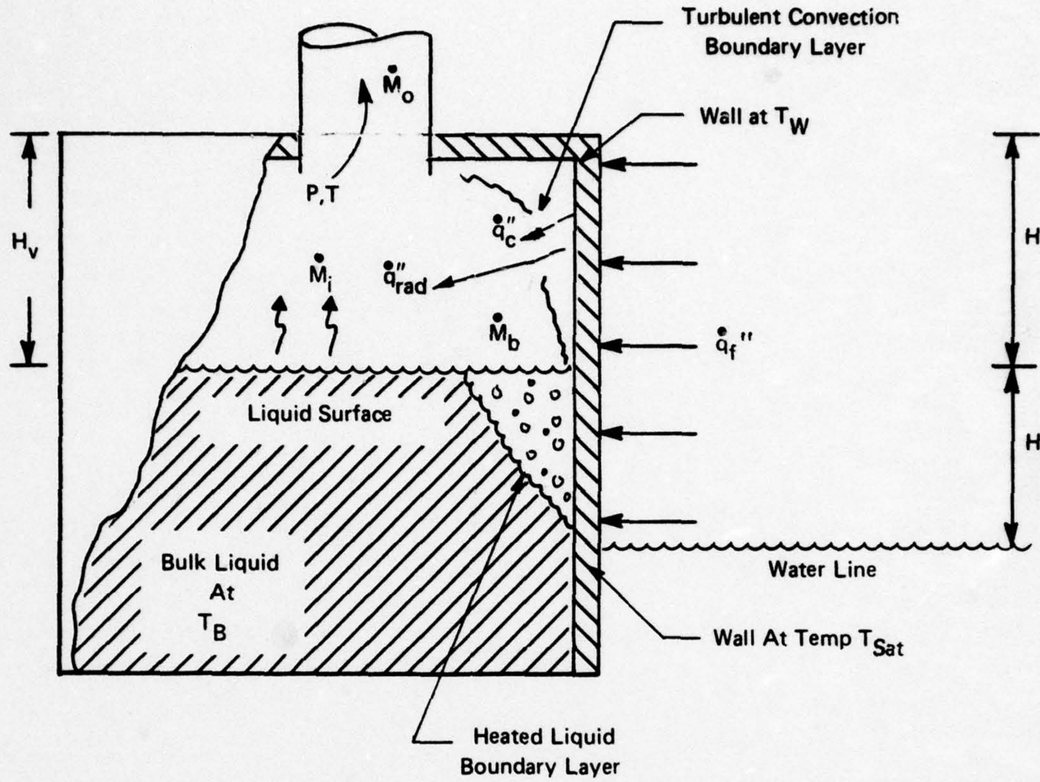


FIGURE III–4 ILLUSTRATION OF HEAT TRANSFER TERMS

2. Heating of Liquid and Its Subsequent Boiling

When the wall wetted by the liquid is exposed to radiation from fire, a part of the heat is absorbed by the wall material and the remaining part goes to heat the liquid layers adjacent to the wall. Heating of the liquid creates a free convection flow. The heated liquid accumulates in the top layers of the liquid bulk in the form of a stably stratified layer. The depth of this layer increases continuously with time. An analysis of heating of the liquid which considers this liquid stratification phenomenon is developed in Appendix A. It is found from this analysis that for a typical tank and liquid combination representing the most common types of tanks and liquids used in practice, the time of exposure to typical flame radiation (32,000 Btu/hr ft^2) necessary before the wall temperature attains boiling temperature of liquid is very small (of the order of 100 seconds).

Once the wall temperature reaches a few degrees above the liquid boiling temperature, it is reasonable to expect the liquid to boil. Since the time for the wall to attain the boiling temperature is small, we assume, for pressure calculation purposes, that boiling ensues from the instant the wall is exposed to fire radiation. The result of this assumption would be to predict, during the initial period, a quicker rate of increase of pressure than that which would occur if the transient heating of and stratification of liquid phenomena are taken into account. Therefore, using the equations given below would amount to a conservative prediction of the time of tank rupture.

Since the heat flux to the liquid during boiling is very much larger than when the liquid is being heated convectively, the wall temperature remains a constant once boiling ensues. Therefore,

$$\dot{M}''_b = \frac{\dot{q}''_f}{\lambda} \tag{4a}$$

and $$\dot{M}_b = \dot{M}''_b H_L W \tag{4b}$$

We further assume that all of this vapor generated by the liquid boiling is released into the vapor space in the tank; that is, no part of vapor dissolves during its ascent through the liquid.

Additional mass of vapor is released into the ullage volume because of the evaporation of liquid at its surface. The heat for this evaporation comes from the hot wall in contact with the vapor by direct radiation.

$$\dot{M}_{evap} = W H_v \frac{F \dot{q}''_{rad}}{\lambda} \tag{5}$$

where F is the view factor for radiative exchange between the hot wall in contact with the vapor and the liquid surface (which is assumed to be flat). The expression for calculating F has been taken from Wiebelt (1966) and is given in Appendix B. Therefore,

$$\dot{M}_i = \dot{M}_b + \dot{M}_{evap} = \text{total rate at which vapor is "injected" into the ullage volume} \tag{6}$$

Global Mass Balance Equation

We now write the mass balance equations for air and vapor species as well as the total mass of gases within the ullage volume.

$$\underbrace{\frac{dM}{dt}}_{\text{Rate of change of mass in the ullage volume}} = \underbrace{\dot{M}_i - \dot{M}_o}_{\text{Net inflow of gases}} \qquad (7)$$

Specie Mass Balance Equations

$$\underbrace{\frac{d(Y_a M)}{dt}}_{\text{Rate of change of mass of air in the ullage volume}} = \underbrace{- Y_a \dot{M}_o}_{\text{Mass outflow rate of air}} \qquad (8a)$$

Similarly, the mass balance equation on the vapor is written as

$$\frac{d(Y_v M)}{dt} = \dot{M}_i - Y_v \dot{M}_o \qquad (8b)$$

and it is noted that

$$1 = Y_a + Y_v \qquad (9)$$

In writing equations 8a and 8b, it is assumed that no air is generated by the boiling process and that in the venting gas, the proportion of air and vapor (by mass) is the same as in the ullage volume.

Using equations 7 and 8a, we can write the air mass balance equation as

$$\frac{dY_a}{dt} = - Y_a \left(\frac{\dot{M}_i}{M}\right) \qquad (10)$$

Venting Equation

According to Bernoulli's law, the mass flux through a duct of cross sectional area A can be related to the pressure drop:

$$\dot{M}_o = A\, C_d \sqrt{2(p - p_{amb})\, \rho} \qquad (11)$$

where ρ is the density of gases in the ullage space and C_d is the discharge coefficient*.

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* Note that the pressure difference is in psi, the area in ft^2, and the density lbm/ft^3 so that a conversion factor of 12 in/ft^2 must be applied to the right hand side when using equation (11).

Pressure Equation

The pressure in the gas space is due to the partial pressures of air and vapor.

$$p = p_a + p_v \tag{12a}$$

with
$$p_a = \frac{M_a}{V} \frac{R_u T}{\mu_a} \tag{12b}$$

$$p_v = \frac{M_v}{V} \frac{R_u T}{\mu_v} \tag{12c}$$

Differentiating 12a with respect to time, and using 12b and 12c, we get

$$\frac{dp}{dt} = \frac{R_u}{V} \left[\left(\frac{T}{\mu_a} \frac{dM_a}{dt} + \frac{T}{\mu_v} \frac{dM_v}{dt} \right) + \left(\frac{M_a}{\mu_a} + \frac{M_v}{\mu_v} \right) \frac{dT}{dt} \right] \tag{12d}$$

Energy Conservation Equation

In order to estimate the temperature variation of the gas with respect to time when it is subjected to heat from the walls and when there is venting, we write the energy equation. The derivation of the equation is involved and quite extensive; therefore, it is given in Appendix C. Only the final result is indicated below.

$$\frac{dT}{dt} = \frac{\dot{Q}_c + \dot{M}_i \left[\frac{R_u}{\mu_v} T + c_v^P (T_b - T) \right] - \dot{M}_o \left[\frac{Y_a}{\mu_a} + \frac{Y_v}{\mu_v} \right] R_u T}{M[Y_a c_a^V + Y_v c_v^V]} \tag{13}$$

Equations (1), (10), (12d), and (13) represent the variation with time of the four unknowns in the system; namely, T_w, Y_a, p, and T. All other equations contain parameters that can be expressed in terms of the above unknowns. The problem therefore reduces to solving for four unknowns from four simultaneous, coupled, first order, nonlinear differential equations. The solution procedure is briefly explained in the next section. The initial values of the parameters used depend on the type and dimensions of the tank, amount of air in the ullage volume, etc.

D. Solution Procedure

The solution to the above equations is obtained by first writing all of the equations in dimensionless form. This is illustrated in Appendix D. Then the nondimensionalized equations 1, 10, 12d, and 13 are solved numerically on a computer using the Hamming Predictor Corrector Technique [Hamming (1973)]. This technique is essentially a procedure which involves incrementing time in a stepwise fashion finding the rate of change of parameters at the given time step (with known values at the given time). Then the values are calculated at the next time step and errors evaluated between the calculated values and the values obtained if one half of the time step is used. If the error is greater than a specified value, the time step is halved and the procedure repeated. The program is capable of halving the time step ten times automatically to achieve the desired error bound (i.e., the lowest values of time step that the program will use will be equal to the given initial time step divided by 2^{10}).

E. STEADY STATE SOLUTION

If the fire continues indefinitely, the tank pressure vapor temperature, and unwetted wall temperature will gradually level off at steady state values. Under these conditions, the air is completely displaced by vapor so that $\mu = \mu_v$, $Y_a = 0$, and $Y_v = 1$. In addition, the unwetted wall temperature is quite large (above 800C according to results to be presented in Chapter IV), so that convection can be neglected relative to reradiation ($\dot{q}_c'' = 0$)*. Applying these assumptions and setting time derivatives equal to zero in Equations (7) and (13), one can obtain the following solution for T and p:

$$T = T_b \tag{14}$$

$$A\, C_d \sqrt{2\left(p - p_{amb}\right)\rho} = \frac{\dot{q}_f'' \, WH_L}{\lambda}\left(1 + F\frac{H_v}{H_L}\right) \tag{15}$$

Using Equation (12c) to express ρ in terms of p and T, and substituting (14) into (15), we can derive a quadratic equation for pressure:

$$A\, C_d \sqrt{2\left(p - p_{amb}\right)\frac{p\,\mu_v}{R_u\,T_b}} = \frac{\dot{q}_f'' \, WH_L}{\lambda}\left(1 + F\frac{H_v}{H_L}\right), \tag{16}$$

or

$$\left(p - p_{amb}\right) p = \frac{R_u\,T_b}{2\,\mu_v}\left(\frac{\dot{q}_f''\,WH_L}{\lambda\,A\,C_d}\left(1 + F\frac{H_v}{H_L}\right)\right)^2 \tag{17}$$

The solution to Equation (17) is as follows:

$$p - p_{amb} = p_{amb}\left[\frac{\sqrt{1 + 4\xi} - 1}{2}\right] \tag{18}$$

---

* Calculations show that at a wall temperature of 730C, $\dot{q}_c'' = 6\%$ of $\dot{q}_f''$, compared to $\dot{q}_{rad}'' = 94\%$ of $\dot{q}_f''$, if the emissivity is unity (as assumed in Equation (2)).

$$\text{where } \xi = \frac{R_u T_b}{2 \mu_v} \left(\frac{\dot{q}_f'' W H_L}{p_{amb} \lambda A C_d} \left(1 + F \frac{H_v}{H_L} \right) \right)^2$$

Under the condition $\xi >> 1/4$, Equation (18) can be simplified to read

$$p - p_{amb} = p_{amb} \left(\sqrt{\xi} - \frac{1}{2} \right). \qquad \text{(Large } \xi\text{)} \qquad (19)$$

However, for baseline values (see Chapter IV), $\xi \simeq .05$ and so the assumption $4 \xi << 1$ more applicable. Under this assumption,

$$p - p_{amb} \simeq p_{amb} \xi . \qquad \text{(Small } \xi\text{)} \qquad (20)$$

In the regime of parameter values for which Equation (20) is valid, the steady state pressure in the tank is predicted to vary as follows:

$$p - p_{amb} = \frac{R_u T_b}{2 \mu_v p_{amb}} \left(\frac{\dot{q}_f'' W H_L}{\lambda A C_d} \left(1 + F \frac{H_v}{H_L} \right) \right)^2 \qquad (21)$$

IV. RESULTS AND DISCUSSION

A. PRESSURE RISE AND UNWETTED-WALL TEMPERATURE BASELINE CASE

The computer simulation described in Part III for the cargo tank exposed to external fire was exercised for the following baseline values:

Fire heat flux	(32,000 Btu/hr/ft^2)	
Cargo latent heat	145 Btu/lb	(hexane)
Cargo molecular weight	86 lb/lb mole	(hexane)
Wetted wall area	84 ft^2	
Unwetted wall height	4. ft	
Effective vent area	.215 ft^2	(6" vent valve)

The pressure rise under these conditions levels off at about 1.5 psig in about 25 minutes, as shown in Figure IV-1. The relatively low level of pressure is significant; to the extent that the input data is realistic this means that many cargo tanks are readily capable of venting the vapor generated by fire. The pressure rise for a lighter vapor fraction (μ_v = 30, say) could be as high as 3.5 psig as will be shown below.

An interesting qualitative feature of the pressure rise is that it occurs in two stages. A rapid increase to about 0.5 psig within 20 seconds occurs due to immediate boiling from the wetted wall. A more gradual pressure rise follows, caused by the heat up of the unwetted wall (which in turn heats up the confined vapor). The indentation noticable in Figure IV-1 at about 500 sec is pressumably ascribable to the gradual shift in average molecular weight from the original tank contents (assumed μ_a = 30) to the pure vapor (μ_v = 86).

That the venting system can accommodate the vapor generated by a 32,000 Btu/hr/ft^2 external fire with only a 1.5 psig pressure rise does not guarantee that the tank will remain intact, of course. Figure IV-2 shows that the unwetted wall temperature exceeds 600C in about 10 minutes. In 15 minutes, the wall strength will have been reduced by heating to the point where failure at 2 psig is entirely possible for a representative tank (normally capable of 6 psig). This result suggests that the failure mode of cargo tanks is due not to the vent system, but due to simple weakening of the unwetted wall. It also suggests that the method of

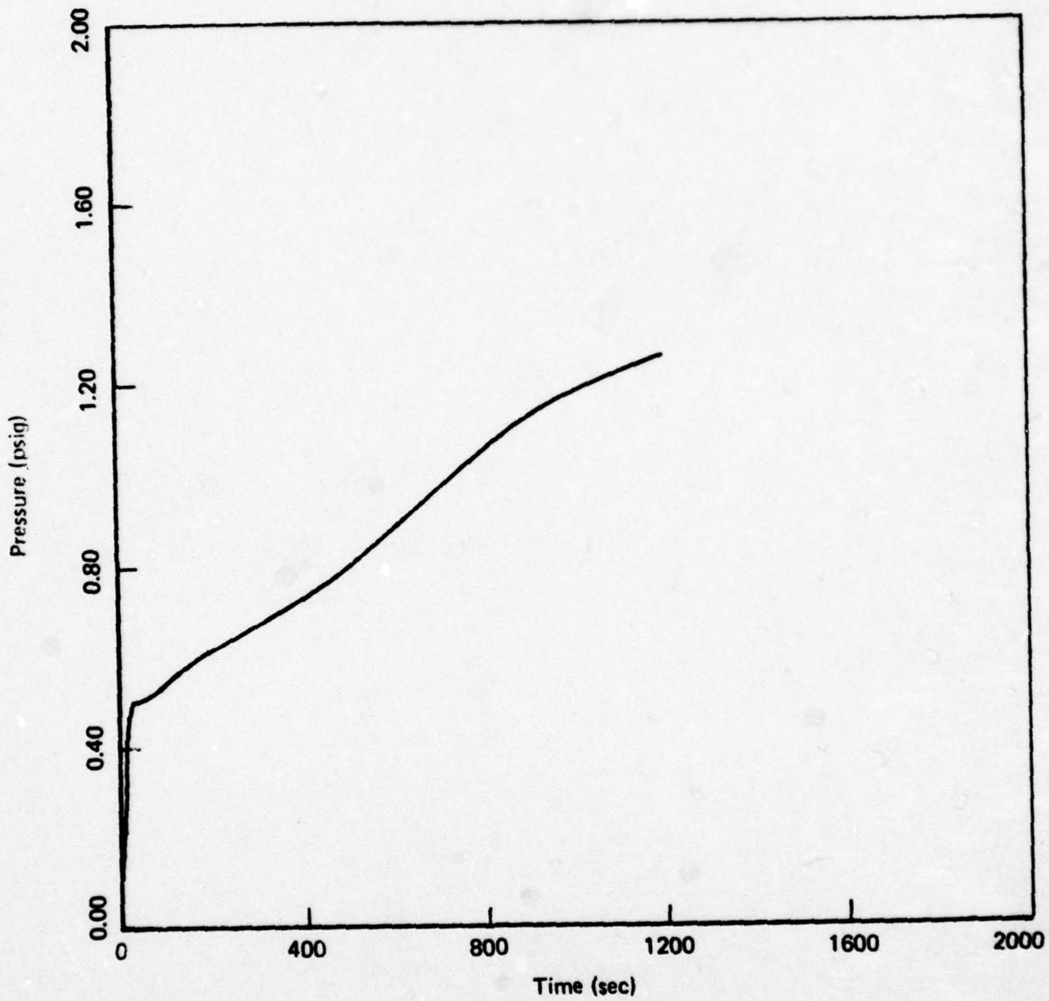

FIGURE IV-1 BASELINE CASE

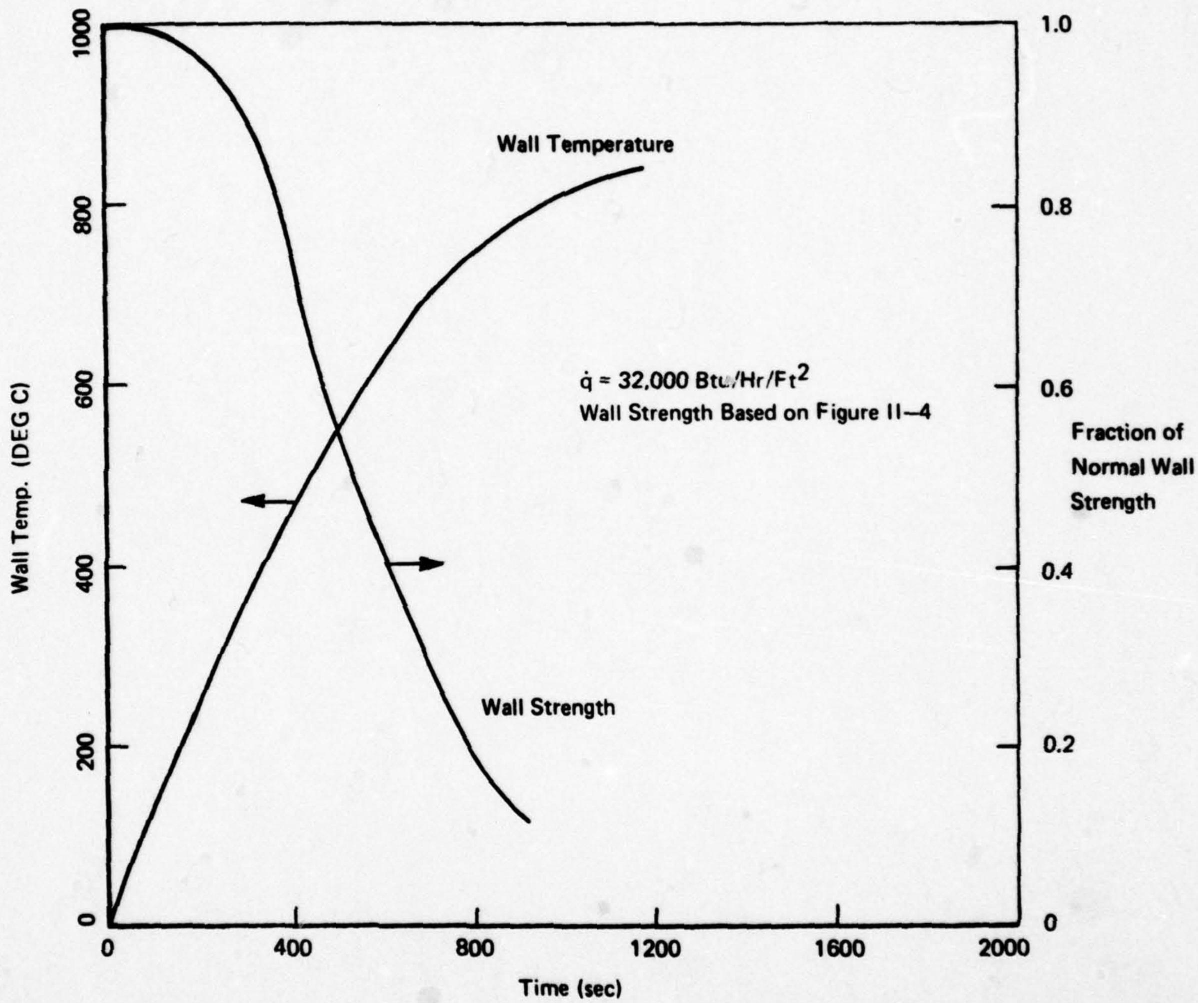

FIGURE IV-2 WALL TEMPERATURE AND STRENGTH DURING A REPRESENTATIVE FIRE analysis of unwetted wall temperature should be refined by including lateral heat conduction to supporting structure, stiffeners, and the deck plating not exposed to fire. The key input data in our prediction of 800C wall temperature can also be scrutinized: Fire flux of 32,000 $Btu/hr/ft^2$, unwetted wall height of 4 ft, and plating thickness of .062 ft (.75 in).

B. EFFECT OF HEAT FLUX LEVEL

The heat flux level reaches 32,000 $Btu/hr/ft^2$ only in the extreme case of a large fire immediately adjacent to the cargo tank. If the heat flux level is less than this, the unwetted wall temperature will be less and also the vapor generation rate will be less. In Figure IV-3 we have shown predictions of the wall temperature for heat flux ranging from 16,000 to 64,000 $Btu/hr/ft^2$. Note that at 16,000 $Btu/hr/ft^2$ the wall temperature does not exceed 550C in the first 20 minutes of the fire, and making reference to Figure II-4 one can deduce that the tank pressure can rise to 60% of the normal maximum without failure. On the other hand extreme fire fluxes of 150,000 $Btu/hr/ft^2$ are predicted to cause local wall collapse within 10 minutes (under the assumptions of the present model).

The pressure rise at elevated heat flux is displayed in Figure IV-4. If the heat flux doubles from 32,000 to 64,000 $Btu/hr/ft^2$, the pressure rise after 20 minutes apparently increases about a factor of four. The relationship $\Delta P \sim \dot{q}^2$ can be explained on the basis that ΔP is proportional to the square of the velocity in the vent, which is in turn proportional to the vapor generation rate caused by the heat flux. For a 6" vent diameter, the pressure rise is less than 6 psig even with the maximum conceivable heat flux (64,000 $Btu/hr/ft^2$). This reiterates the earlier conclusion that the external fire hazard is attributable to the unwetted wall rather than the vent system.

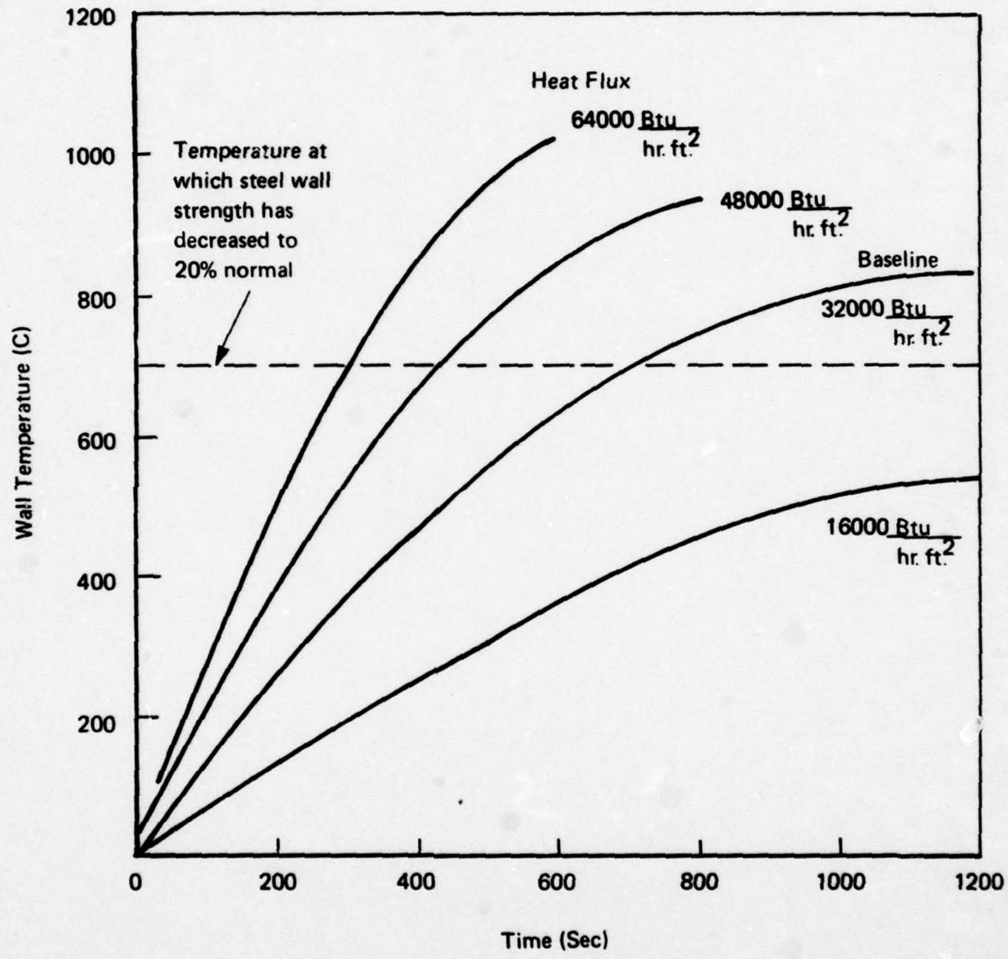

FIGURE IV-3 EFFECT OF HEAT FLUX ON UNWETTED-WALL TEMPERATURE

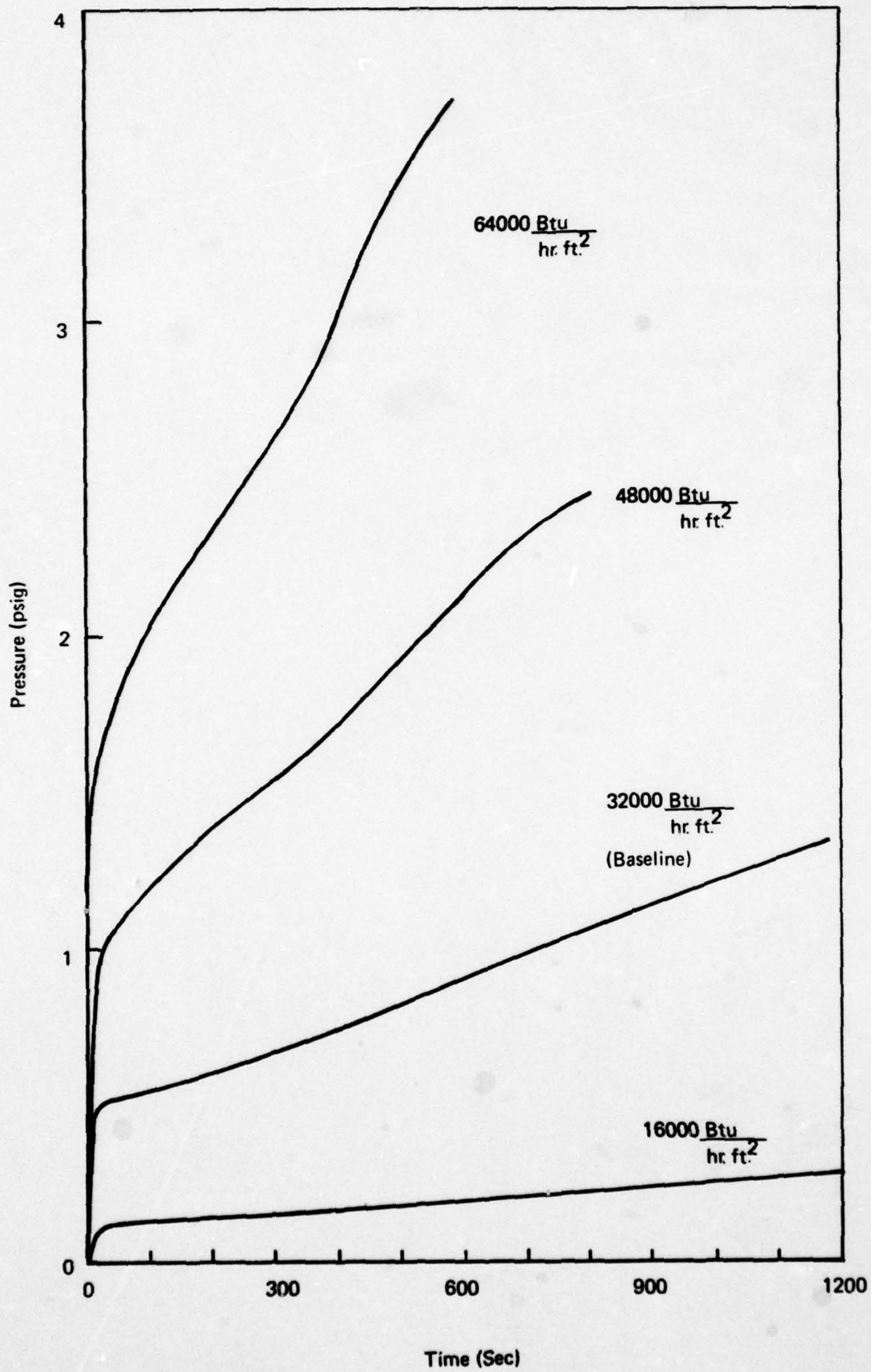

FIGURE IV-4 EFFECT OF HEAT FLUX LEVEL

C. EFFECT OF CARGO HEAT OF VAPORIZATION

In steady state after wall heat up, the rate of vapor generation is inversely proportional to the heat absorbed per unit vapor, the largest part of which is the heat of vaporization. Thus, the velocity of vent gases should be nearly inversely proportional to the heat of vaporization. Since $\Delta P \sim v^2$, one therefore expects $\Delta P \sim \lambda^{-2}$.

Since most hydrocarbons show a heat of vaporization in the range 100 to 300 Btu/lb... we have selected hexane (145 Btu/lb) for the baseline representative value. We have compared the pressure rises for several values of vaporization heat in Figure IV-5. For the lowest reasonable value (86 Btu/lb), the pressure rise is still less than 3.2 psig after 20 minutes, which is quite safe for a wall of normal strength.

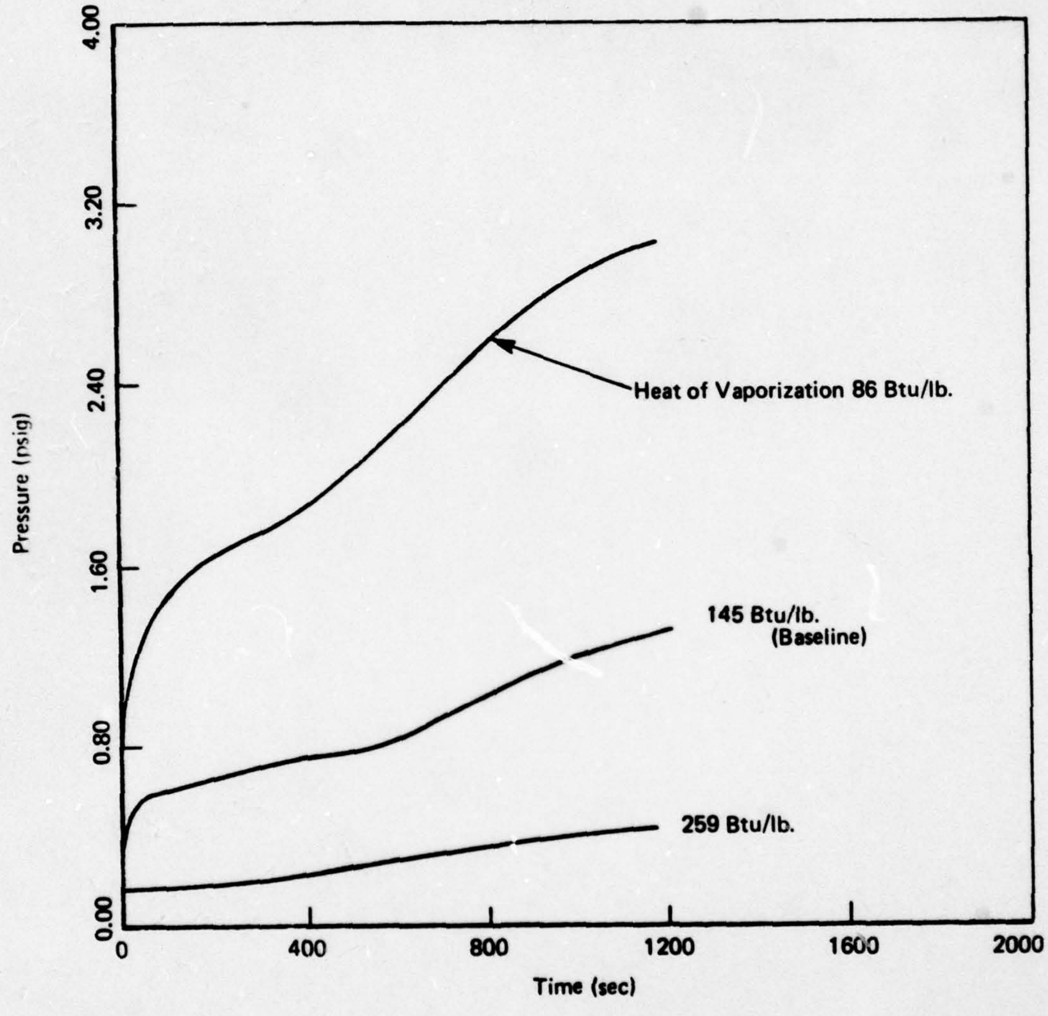

FIGURE IV-5 EFFECT OF HEAT OF VAPORIZATION

D. EFFECT OF EXPOSED CARGO-TANK-WALL AREA

In the usual case, the amount of liquid cargo stored above the waterline is not large; in fact we have assumed an exposed wetted wall area of only 84 ft^2 per tank wall. If several adjacent tanks are empty of for some other reason the tank exposed to fire protrudes well above the waterline, more vapor will be generated by the fire. Figure IV-6 shows how the exposed wetted-wall area affects the pressure rise. A factor of three increase in area leads to a factor of four increase in pressure differential after 20 minutes of fire exposure. If the exposed wetted area climbs to 420 ft^2 (16 feet above waterline for a 26 ft wide tank), the pressure is predicted to rise to values exceeding 6 psig. In view of the heat up and weakening of the unwetted wall, this would be an extremely hazardous situation.

Suppose on top of the abnormally high (420 ft^2) exposed area, one assumes that the cargo liquid has a reasonably low heat of vaporization (86 Btu/lb). Then, as shown in Figure IV-7, the pressure rise is predicted to exceed 6 psig and is almost certain to cause failure even for unwetted walls of normal strength.

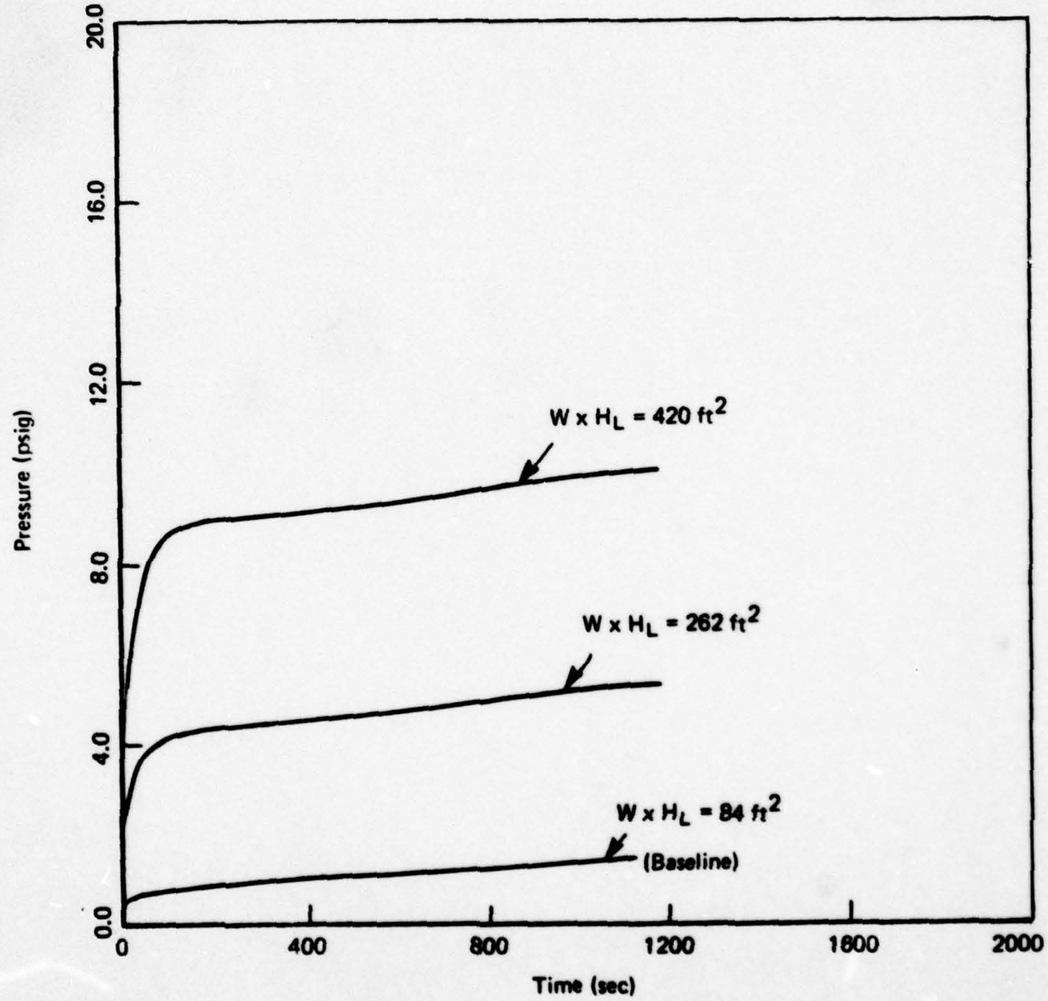

FIGURE IV-6 EFFECT OF LARGE WETTED AREA

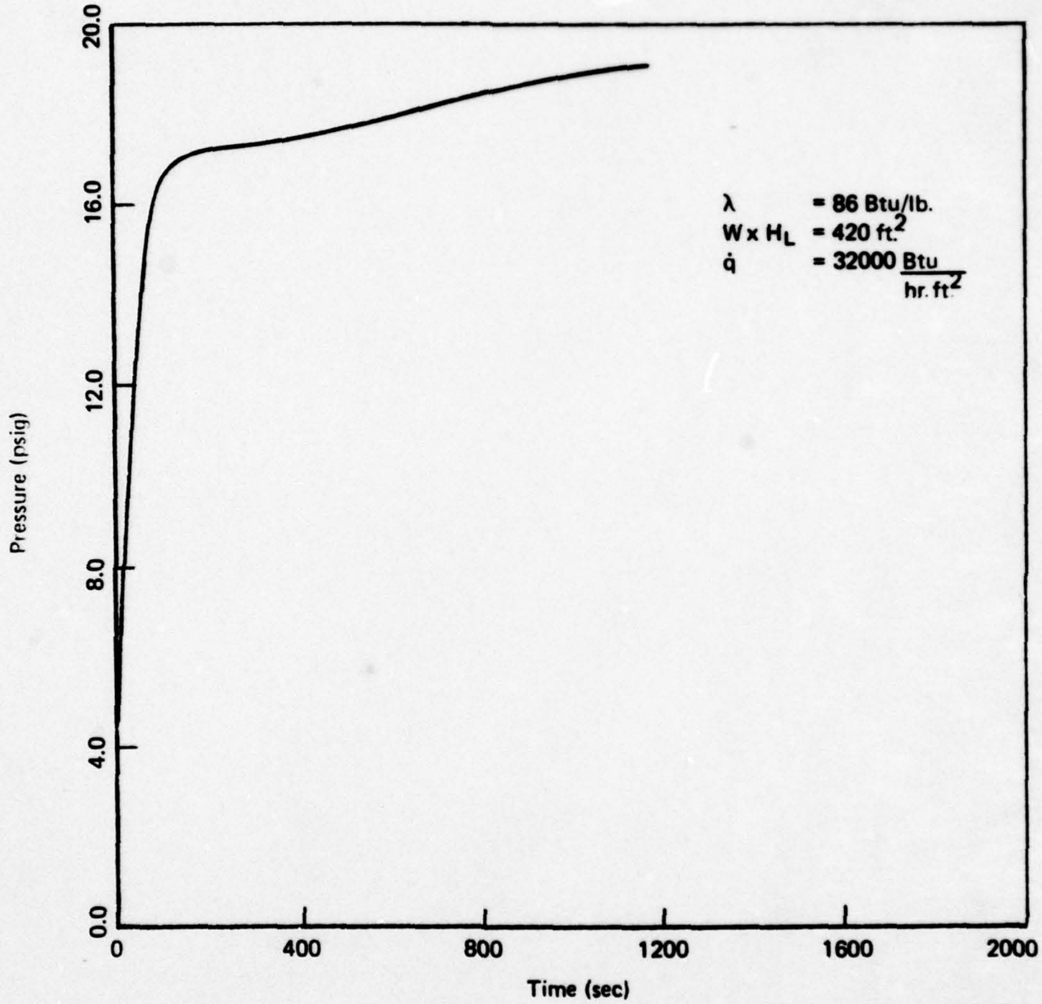

FIGURE IV-7 EFFECT OF COMBINED SMALL HEAT OF VAPORIZATION AND LARGE EXPOSED AREA

E. EFFECT OF ULLAGE VOLUME

The ullage, or portion of the tank not filled with liquid cargo, increases the pressure rise by transmitting additional heat to the confined vapor from the inside of the unwetted wall. This occurs by convection to the vapor and by radiation to the liquid surface. A series of runs was conducted at 32,000 Btu/hr/ft^2 heat flux, assuming Hexane, varying only the height of the vapor space from 2 to 32 feet (baseline 4 feet). The ullage volume only affects the later stages of pressure rise, as shown in Figure IV-8. At 7 minutes from the start of fire exposure, the pressure rise is 0.7 psig for all cases and is mainly due to vapor generation (not "afterheating" by the unwetted wall). At later stages, say after 20 minutes, the unwetted wall is transferring more heat to the confined vapor and the pressure now is quite sensitive to ullage volume. A factor of 8 increase in vapor space leads to an increase in tank pressure from 1.3 psig to 3.0 psig (both after 20 minutes).

The sensitivity of pressure to ullage volume is not so great that the venting capacity becomes limiting; the maximum pressure in the cases considered was 3.0 psig.

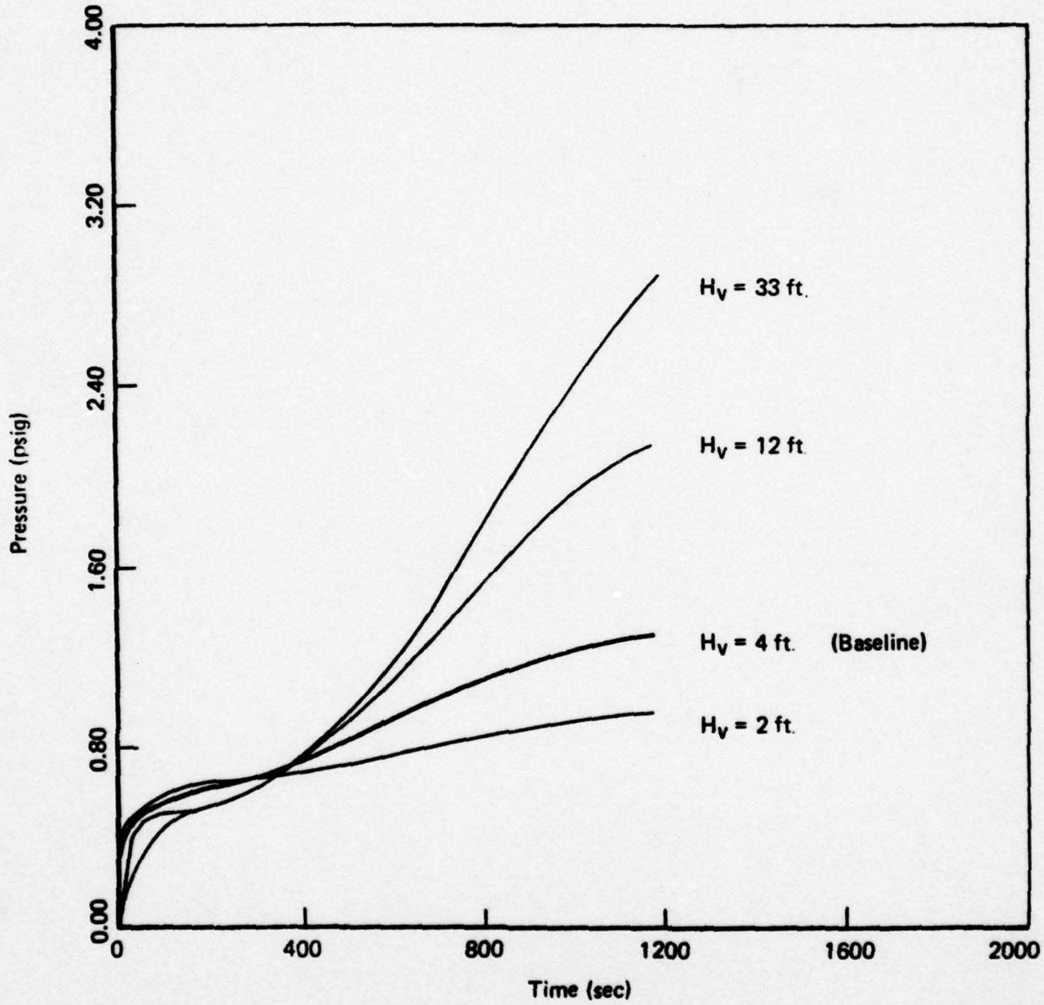

FIGURE IV-8 EFFECT OF PARTIALLY EMPTY TANK

F. EFFECT OF VENT AREA

Vent pipes are normally sized according to the cargo inlet pipes, which in turn are sized according to the tank valves so that the tank can be filled in 4 - 8 hours with about 10 ft/sec liquid velocity. The value of .215 ft^2 adopted for the baseline vent area should correspond to the inlet pipe size on a 82 x 26 x 26 ft tank since this gives 25,800 seconds loading time (7.2 hours), at 10 ft/sec loading velocity.

Smaller tanks, say on barges, would have smaller vent areas, down to .054 ft^2 (approximately 3" pipe), and still might have the exposed wetted area of 84 ft^2 we are using as baseline. The pressure rise is obviously much larger for such cases, as shown in Figure IV-9. The pressure rise may be as great as 10 psig for a .054 ft^2 vent area according to our predictions. In practice, a 3" vent would most often be found on a smaller tank with much less than 84 ft^2 exposed wetted wall, and the pressure would be correspondingly lower. In general, our results suggest that the exposed wall area should not be more than 1,000 times the vent area if the pressure is not to exceed 6 psig for a typical cargo and heat flux.

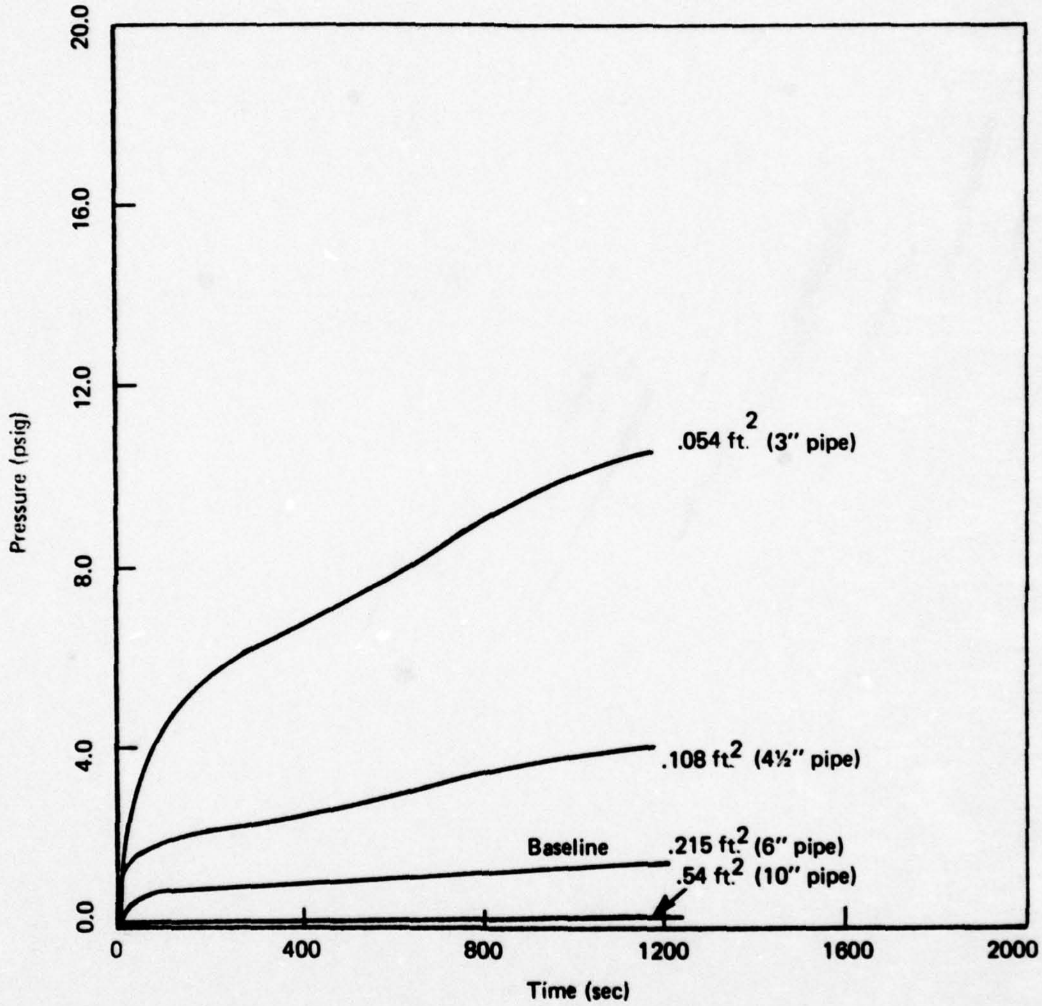

FIGURE IV-9 EFFECT OF VENT AREA

G. EFFECT OF VAPOR MOLECULAR WEIGHT

Since the volume taken up by a gas is inversely proportional to its molecular weight, the confined vapor generated by an external fire causes a greater pressure rise the lower the molecular weight. This is illustrated by Figure IV-10; the pressure rise after 20 minutes is 3.4 psig for a 30 lb/lb mole vapor (compared to a baseline of 1.3 psig for hexane at 86 lb/lb mole). The range of molecular weights has a lower limit of 16 lb/lb mole (CH_4) so that hazardous pressures are not expected due to the molecular weight variation.

No physical significance should be attached to the "spikes" which appear in Figure IV-10 at vapor molecular weights of 50 and 30. Physically, at the onset of a fire there is almost immediate boiling at the wetted wall. Numerically, the venting rate is initially zero and underpredicted for the first few time steps. This results in an overprediction of dP/dt [see Equation (12d)] initially which is magnified for low-molecular-weight vapor because the μ_v is in the demoninator.

The magnitude of the steady state pressure rise can be shown to be inversely proportional to the vapor molecular weight for small pressure rises, in agreement with Figure IV-10. Equation (E-6) would take the following form for variable mixture molecular weight:

$$\frac{\bar{\mu}}{\mu_a} \frac{d\mathcal{P}}{dt} = 1 - \Delta \sqrt{\frac{\bar{\mu}\ \mathcal{P}\ (\mathcal{P}-1)}{\mu_a}}$$

Under steady state conditions the left hand side is zero, $\bar{\mu} = \mu_v$, and the dimensionless state pressure for small pressure size is given by

$$\mathcal{P}_{ss}\ (\mathcal{P}_{ss} - 1) \approx \mathcal{P}_{ss} - 1 = \frac{1}{\Delta^2}\left(\frac{\mu_a}{\mu_v}\right)$$

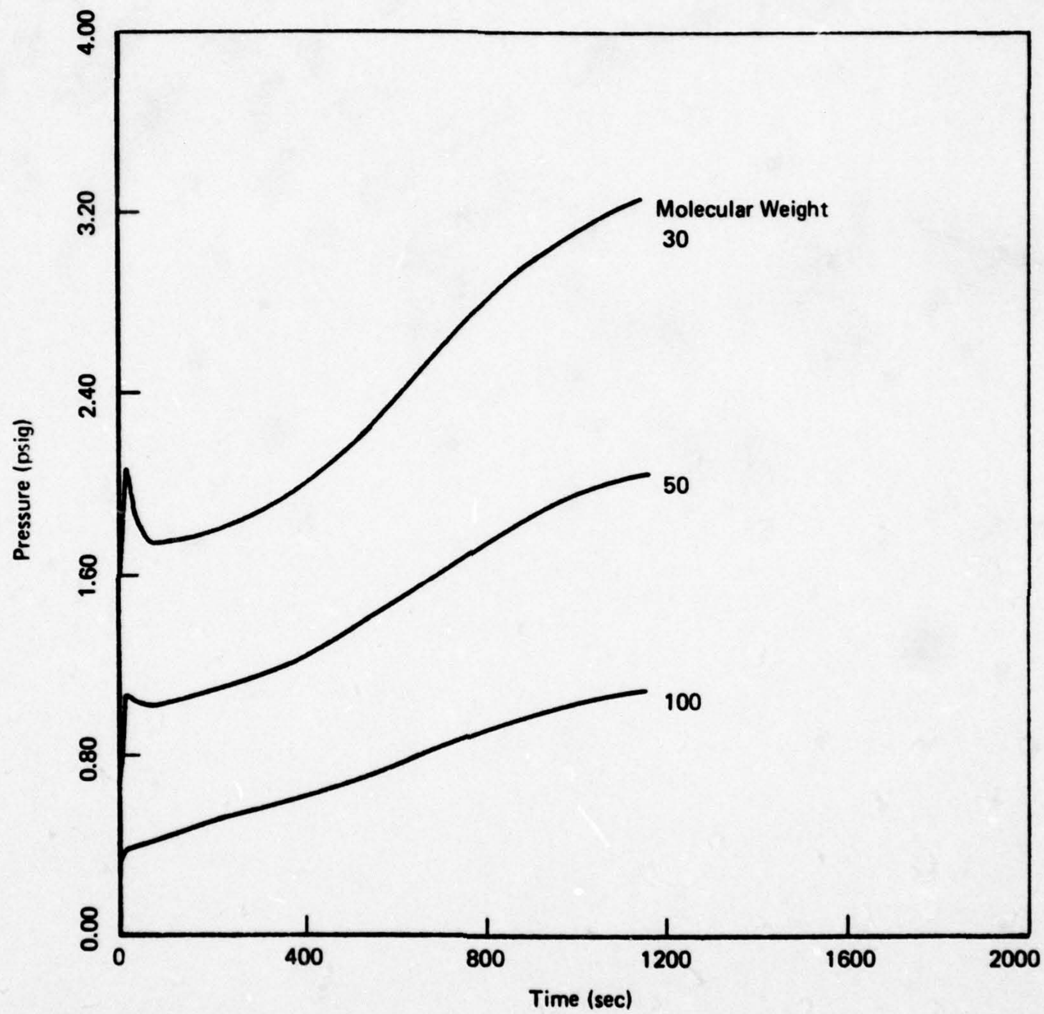

FIGURE IV-10 EFFECT OF VAPOR MOLECULAR WEIGHT

H. COMBINED EFFECT OF SEVERAL PARAMETERS AND HAZARD EVALUATION CRITERION

1. Correlation of Predicted Data

The results presented in Sections A through G describe the effect of each single variable on the pressure rise. In most cases the pressure levels off after about 20-30 minutes to a steady state value. In this section we relate this maximum pressure rise p_{ss}, to arbitrary combination of the parameters $\dot{q}_f$, λ, H_v, μ_v, H_L, W, and A. A closed form solution of Equations 1, 10, 12d, and 13 can be used to derive a general pressure rise equation applicable to any combination of variables.

A hazard criterion covering arbitrary combinations of the variables $\dot{q}_f''$, λ, H_v, μ_v, W, H_L, and A can be obtained from Equation (18), which is the asymptotic value of tank pressure reached after about 30 minutes of fire exposure:

$$p_{ss} - p_{amb} = \frac{p_{amb}}{2} \left\{ \left[1 + 4 \left(\frac{R_u T_b}{2 \mu_v} \right) \left(\frac{\dot{q}_f'' W H_L}{p_{amb} \lambda A} \left(1 + F \frac{H_v}{H_L} \right) \right)^2 \right]^{1/2} - 1 \right\}, \qquad (22)$$

where we have taken $C_d = 1$ for simplicity.

If the tank pressure $p_{ss} - p_{amb}$ must not exceed a certain value, then Equation (18) can be shown to require that

$$\left(\frac{T_b}{T_{amb}} \right) \left(\frac{\mu_a}{\mu_v} \right) \left(\frac{\dot{q}_f''}{\lambda} \right)^2 \left(\frac{W H_L}{A} \right)^2 \left(1 + F \frac{H_v}{H_L} \right)^2 < 1.3 \times 10^{10} \left\{ \left[1 + \frac{(p_{ss} - p_{amb})}{p_{amb}/2} \right]^2 - 1 \right\} \qquad (23)$$

where $\dot{q}_f''$ is in Btu/hr/ft^2, λ is in Btu/lb, and $p_{ss} - p_{amb}$ is in psi. Substituting $p_{ss} - p_{amb} = 4$ psig, we obtain

$$\left(\frac{\mu_a}{\mu_v} \right) \left(\frac{T_b}{T_{amb}} \right) \left(\frac{\dot{q}_f''}{\lambda} \right)^2 \left(\frac{W H_L}{A} \right)^2 \left(1 + F \frac{H_v}{H_L} \right)^2 < 1.8 \times 10^{10} \qquad (24)$$

Equation (24) may be termed the 4 psig criterion for venting tanks exposed to external fires. It should be noted that Equation (22) was derived on the basis that convective heat transfer was negligible, whereas the numerical results of Figures IV-1 through IV-10 include the effect of convection. Therefore, Equation (22) slightly underestimates the pressure rise. Sample calculations show $\dot{q}_c''/\dot{q}_f''$ to be from 5% to 10%, depending on unwetted wall temperature, emissivity and vapor temperature. Therefore, the correction to maximum pressure due to convection is not expected to be significant.

APPENDIX A

Thermal Stratification of Liquid Cargo

Objective

The objective of this appendix is to formulate a model for the stratification phenomenon that occurs when a liquid is heated and to obtain a time history for the temperature of the wall in contact with the liquid.

Physical Model

The physical model considered is shown in Figure A-1. The heat from the fire first heats the wall (in contact with the liquid) which in turn heats the liquid adjacent to it. Because the heated liquid becomes buoyant, it rises to the surface, setting up convection. The heated liquid rises to the surface of the liquid bulk. Because the wall temperature increases continuously with time, the temperature of the liquid in the convective flow adjacent to the wall also increases, and this liquid accumulates on the surface of the liquid bulk. The successive accumulation of hot liquid results in the formation of a stable stratified layer as shown in Figure A-1. The hottest liquid is at the top. The bulk of the liquid is not affected by this process for a considerable time. The depth of the stratified region grows with time.

Asssumptions

The above process is modeled below with the following assumptions:

- Turbulent convection in the fluid is quasi-steady; that is, at every instant the convection boundary layer is fully developed and corresponds to the wall condition existing at that time.
- Temperature of the wall in contact with the liquid is uniform in spite of the existence of the stratified layer; that is, the wall temperature predicted is a vertically averaged value.
- Heat flux from the wall into the stratified liquid layer is equal to the average heat flux (in the turbulent natural convection cooled region) over the remaining portion of the wall wetted by the fluid above the water line.

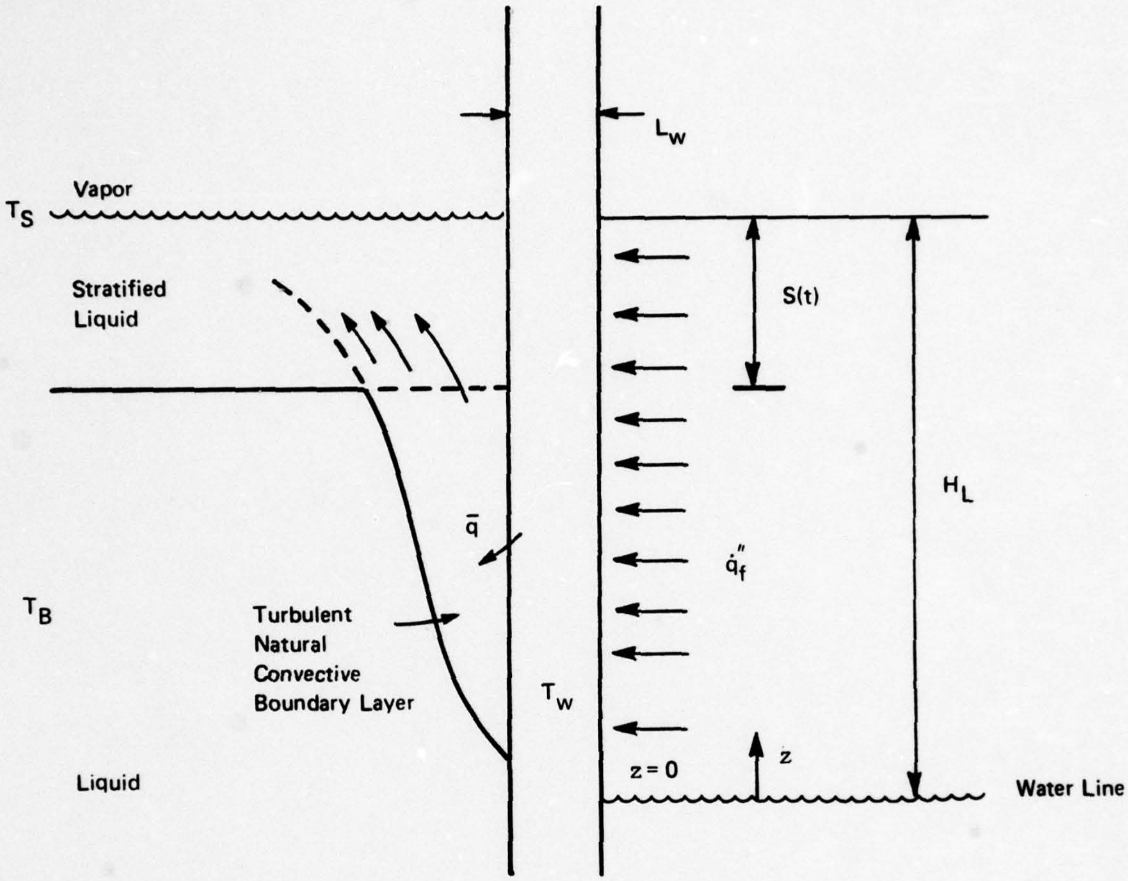

FIGURE A-1 ILLUSTRATION OF FORMATION OF STRATIFIED LAYER

- The turbulent boundary layer begins at the water line.
- The bulk temperature in the nonstratified liquid does not change.
- Evaporation (and therefore mass loss) from the liquid surface is negligible.
- Density change of liquid due to heating is small and is considered insignificant for mass balance purposes.

Governing Equations

1. Mass Balance on the Stratified Layer

$$\rho_L A_L \frac{dS}{dt} = \dot{M}_c \tag{A-1}$$

where A_L is the surface area of the liquid in the tank and $\dot{M}_c$ is the mass of heated liquid flowing into the stratified layer by convection. $\dot{M}_c$ is evaluated from the knowledge of the turbulent boundary layer characteristics. This is shown in Appendix B. Only the final answer is presented here.

$$\dot{M}_c = 0.1\, W\, \rho_L \nu_L \frac{\left[\dfrac{g\,\beta\,(H_L - S)^3}{\nu^2}\,(T_w - T_B)\right]^{0.4}}{Pr_L^{8/15}\left[1 + 0.494\, Pr_L^{2/3}\right]^{0.4}} \tag{A-2}$$

2. Energy Balance on the Stratified Layer

In writing the energy equation, we assume that the average heat flux value (over the height $H_L - S$) can be used to account for the heat into liquid over the height H_L. It is noted that this assumption is invalid if $S \simeq H_L$; that is, when the stratified layer depth has grown to an extent comparable to the depth of liquid exposed to fire. When this happens, the average heat flux is dominated by high heat flux values even though in reality such high heat fluxes cannot exist due to the influence of the heated layer in contact with the wall. However, within the above limitation (that $S \ll H_L$), we write the following energy flow equation:

$$\frac{d}{dt}\left[\rho_L c_L A_L \int_{z=0}^{S} (T - T_B)dz\right] = \overline{\dot{q}''_{c,L}}\, H_L W \qquad \text{(A-3)}$$

where T is the temperature within the stratified layer.

Inherent in equation A-3 is the assumption that the heat capacity of the small amount of liquid in the turbulent convective boundary layer is negligible. We further assume that the temperature distribution in the stratified layer is self-similar. The accuracy of this assumption has been verified by Clark (1968). Therefore, defining

$$I = \text{energy integral} = \int_{z/S=0}^{1} \left(\frac{T - T_B}{T_s - T_B}\right) d(z/S) \qquad \text{(A-4)}$$

from experimental data, it has been observed by Clark (1968). that the above energy integral is essentially constant with a value close to 0.43. Hence, substituting equation A-4 in A-3, we have

$$A_L \rho_L c_L I \frac{d}{dt} [S(T_s - T_B)] = \overline{\dot{q}''_{c,L}} H_L W \qquad \text{(A-5)}$$

3. Wall Heat Up Equation

$$L_w \rho_w c_w \frac{dT_w}{dt} = \dot{q}''_f - \overline{\dot{q}''_{c,L}} \qquad \text{(A-6)}$$

where $\overline{\dot{q}''_{c,L}}$ is the mean heat transfer rate by natural convection into the liquid. Expressions for this are derived in Appendix B in terms of the wall and liquid temperatures.

Equations A-1, A-5, and A-6 form a coupled set of nonlinear differential equations in three unknowns; namely, S, T_s, and T_w. These can be determined numerically as functions of time after expressing $\dot{M}_c$ and $\dot{q}''_{c,L}$ in terms of $T_w - T_B$ and $H_L - S$ as indicated in the next section.

Solution to Equations

Before solving the above set of equations, it is necessary to express them in dimensionless parameters. We therefore define the following:

$$\zeta = \frac{S}{H_L} = \text{dimensionless depth of stratified layer} \tag{A-7a}$$

$$t_{ch} = \frac{\rho_L c_L T_B A_L H_L}{\dot{q}''_f \, W \, H_L} = \begin{array}{l}\text{time taken to raise the temperature of}\\ \text{the liquid in the tank above the water}\\ \text{line by a temperature equal to the initial}\\ \text{bulk temperature}\end{array} \tag{A-7b}$$

$$\tau = t/t_{ch} \tag{A-7c}$$

$$\theta = (T/T_B - 1) \tag{A-7d}$$

$$q = \overline{\dot{q}''_c}/\dot{q}''_f = \begin{array}{l}\text{dimensionless mean heat flux into liquid}\\ \text{by turbulent convection}\end{array} \tag{A-7e}$$

$$\omega = \frac{W \rho_w c_w L_w H_L}{\rho_L c_L A_L H_L} = \begin{array}{l}\text{ratio of thermal capacity of steel}\\ \text{to that of the liquid}\end{array} \tag{A-7f}$$

$$\dot{M}_{ch} = 0.1 \rho_L \nu_L W \frac{\left[\dfrac{g \, \beta \, T_B \, H_L^3}{\nu^2}\right]^{0.4}}{Pr_L^{8/15}\left[1 + 0.494 \, Pr_L^{2/3}\right]^{0.4}} \tag{A-7g}$$

$$\Lambda = \frac{\dot{M}_{ch} \, t_{ch}}{\rho_L H_L \, A_L} = \frac{\text{mass of liquid that can accumulate over } t_{ch} \text{ when generated at a rate of } \dot{M}_{ch}}{\text{mass of liquid in the tank above the water line}} \tag{A-7h}$$

Using the above definitions and equation A-2, we write equation A-1 as

$$\frac{d\zeta}{d\tau} = \Lambda \, \theta_w^{0.4} \, (1 - \zeta)^{1.2} \tag{A-8}$$

Equation A-5 becomes

$$I \frac{d}{d\tau} (\zeta \, \theta_s) = q \tag{A-9}$$

and equation A-6 becomes

$$\omega \frac{d \, \theta_w}{d\tau} = 1 - q \tag{A-10}$$

with initial conditions

$$\text{at } \tau = 0; \; \theta_w = \theta_s = \zeta = 0 \tag{A-11}$$

Also, it can be shown (using the expressions for $\overline{\dot{q}''_c}$ derived in Appendix B) that

$$q = \Gamma \, \theta_w^{1.4} \, (1 - \zeta)^{0.2} \tag{A-12}$$

$$\text{where } \Gamma = 0.0246 \, \frac{K \, T_B}{H_L \, \dot{q}''_f} \left[\frac{g \, \beta \, T_B \, H_L^3}{\nu^2}\right]^{0.4} \frac{Pr^{7/15}}{[1 + 0.494 \, Pr^{2/3}]^{2/5}} \tag{A-13}$$

Substituting for q from equation A-12 in equation A-10, we have

$$\frac{d}{d\tau} (\omega \, \theta_w) = 1 - \Gamma \, \frac{(\omega \, \theta_w)^{1.4} \, (1 - \zeta)^{0.2}}{\omega^{1.4}} \tag{A-14}$$

and rewriting equation A-8, we have

$$\frac{d}{d\tau} [(1 - \zeta)^{-0.2}] = \left[\frac{0.2 \, \Lambda}{\omega^{0.4}} (\omega \, \theta_w)^{0.4}\right] \tag{A-15}$$

Also adding equations A-9 and A-10 and integrating and using initial conditions A-11, we get

$$(I \, \zeta \, \theta_s + \omega \, \theta_w) = \tau \tag{A-16}$$

Equations A-14 and A-15 form a coupled set of equations in unknowns $\omega \, \theta_w$ and $(1 - \zeta)$. These can be solved numerically, and from equation A-16, the dimensionless liquid surface temperature can be obtained.

Specific Example

A specific example is worked out with the following numerical values to illustrate the procedure of calculation as well as to obtain a feel for the value of the parameters obtained.

Tank Dimensions

Width	$W = 40$ ft
Height of liquid level above the waterline	$H_L = 16$ ft
Breadth of tank	$B = 17$ ft

Density $\rho_L = 56.2$ lbm/ft^3

Specific heat $c_L = 0.5$ Btu/lbm F

Kinematic viscosity $\nu_L = 0.03875$ ft^2/hr

Prandtl number $Pr_L = 10$

Coefficient of volume expansion $\beta = 4 \times 10^{-4}$/°F

Bulk liquid temperature $T_B = 62$°F

Steel Wall Properties

Density $\rho_w = 500$ lbm/ft^3

Specific heat $c_w = 0.13$ Btu/lbm °F

Thickness $L_w = 0.0625$ ft (3/4")

Radiation

Heat flux from fire* $\dot{q}''_f = 35{,}000$ Btu/hr ft^2

Value of energy integral $I = 0.43$
(see equation A-4)

The values of the various parameters defined in equation A-7 are:

$t_{ch} = 25640$ seconds

$\Lambda = 10.378$

$\Gamma = 2.554$

$\omega = 8.5 \times 10^{-3}$

Table A-1 shows the results obtained by solving equations A-14, A-15, and A-16. The parameters have been expressed in dimensional quantities.

It is seen that for the example considered, the wall temperature in contact with the liquid increases to more than 110°C in about 100 seconds. This illustrates the rapid temperature rise of the wall. Also by noting that the stratification depth is only a few inches, we find that our assumption $S \ll H_L$ is justified. Also noteworthy is the small surface temperature rise. This further justifies the neglect of liquid mass loss by surface evaporation.

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* Calculation was completed before the representative value of 32,000 Btu/hr/ft^2 (given on page 9) was selected. The difference has a negligible effect on the wall temperature history.

APPENDIX B

Heat and Mass Transfer Expressions

1. Turbulent Natural Convection

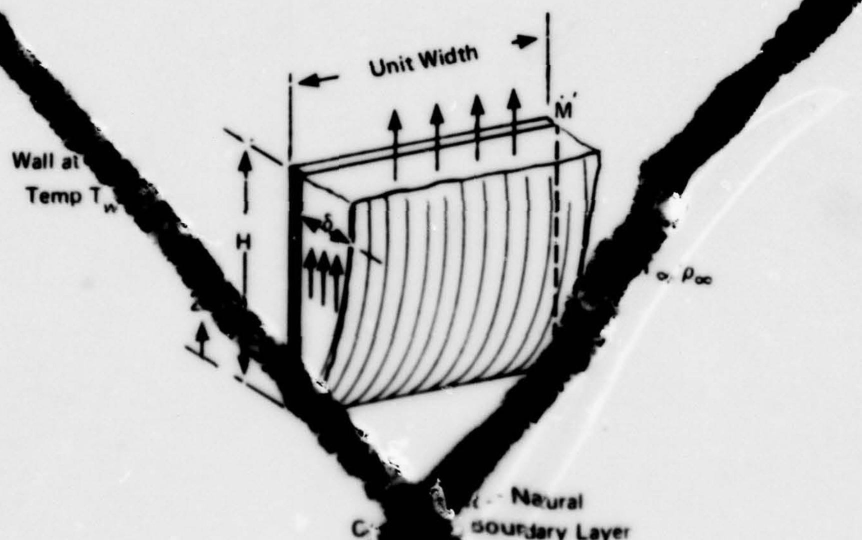

FIGURE B-1 [illegible] PRESENTATION OF THE FLOW [illegible] NATURAL [illegible] CONVECTION HEATING OF A FLUID B[illegible] A VERT[illegible] WALL

Using the [illegible] given by Rohsenow and [illegible] (1961, page 204, equations 8.60 [illegible] 8.61), it can be shown that [illegible] mass flow rate per unit width ([illegible]) in the turbulent, free convection [illegible] layer in any general fl[illegible] at any section at a height z above the [illegible] is given by

$$\dot{M}(z) = 0.1\,(\rho_\infty \nu)\,\frac{Gr_z^{\;0.4}}{Pr^{8/15}\,[1 + 0.494\,Pr^{2/3}]^{0.4}} \qquad \text{(B-1)}$$

where Gr_z = Grashoff number = $\left[\dfrac{g\beta(T_w - T_\infty)z^3}{\nu^2}\right]$ (3-2)

and Pr = Prandtl number of the fluid

where the subscript ∞ refers to the condition far from the wall.

Table A-1

Results of the Tank Heating with Stratification of Liquid Considered

t Time (Secs)	T_w Wall Temperature (C)	T_s Liquid Surface Temperature (C)	S Depth of Stratified Layer (ft)
0,000E-01	1,667E 01	1,667E 01	0,000E-01
1,282E 01	3,336E 01	2,391E 01	1,545E-02
2,563E 01	4,901E 01	2,843E 01	4,677E-02
3,845E 01	6,336E 01	3,350E 01	8,340E-02
3,885E 01	6,379E 01	3,359E 01	8,469E-02
3,925E 01	6,422E 01	3,374E 01	8,594E-02
3,965E 01	6,464E 01	3,388E 01	8,720E-02
4,005E 01	6,507E 01	3,404E 01	8,846E-02
4,045E 01	6,549E 01	3,418E 01	8,973E-02
4,085E 01	6,591E 01	3,433E 01	9,098E-02
4,165E 01	6,674E 01	3,463E 01	9,353E-02
4,245E 01	6,757E 01	3,493E 01	9,610E-02
4,325E 01	6,840E 01	3,522E 01	9,867E-02
4,406E 01	6,921E 01	3,552E 01	1,013E-01
4,566E 01	7,084E 01	3,610E 01	1,065E-01
4,726E 01	7,244E 01	3,668E 01	1,118E-01
4,886E 01	7,402E 01	3,726E 01	1,171E-01
5,046E 01	7,557E 01	3,783E 01	1,225E-01
5,367E 01	7,862E 01	3,895E 01	1,335E-01
5,687E 01	8,159E 01	4,006E 01	1,447E-01
6,008E 01	8,447E 01	4,115E 01	1,561E-01
6,328E 01	8,727E 01	4,222E 01	1,677E-01
6,969E 01	9,262E 01	4,431E 01	1,913E-01
7,610E 01	9,765E 01	4,632E 01	2,155E-01
8,250E 01	1,024E 02	4,825E 01	2,403E-01
8,891E 01	1,068E 02	5,011E 01	2,656E-01
1,017E 02	1,149E 02	5,362E 01	3,174E-01
1,145E 02	1,219E 02	5,686E 01	3,706E-01
1,274E 02	1,280E 02	5,985E 01	4,250E-01
1,402E 02	1,333E 02	6,261E 01	4,862E-01
1,658E 02	1,419E 02	6,750E 01	5,927E-01
1,914E 02	1,483E 02	7,168E 01	7,068E-01
2,171E 02	1,531E 02	7,527E 01	8,219E-01
2,427E 02	1,566E 02	7,836E 01	9,373E-01
2,683E 02	1,593E 02	8,105E 01	1,053E 00
2,940E 02	1,613E 02	8,341E 01	1,168E 00
3,196E 02	1,627E 02	8,549E 01	1,282E 00
3,452E 02	1,639E 02	8,735E 01	1,396E 00
3,709E 02	1,647E 02	8,902E 01	1,509E 00
3,965E 02	1,654E 02	9,052E 01	1,621E 00
4,221E 02	1,660E 02	9,190E 01	1,733E 00
4,478E 02	1,664E 02	9,317E 01	1,843E 00

Physically unrealistic predictions because boiling has not been included and the wall temperature exceeds the boiling point of most cargoes. (rows from t = 2,171E 02 onward)

Incident - heat flux = 35,000 Btu/hr/ft²

T_w = wall temperature in contact with the liquid

APPENDIX B

Heat and Mass Transfer Expressions

1. Turbulent Natural Convection

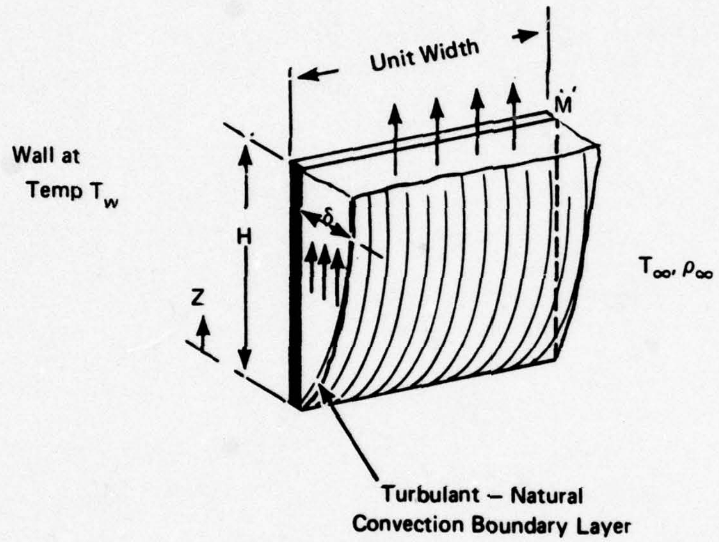

FIGURE B-1 SCHEMATIC REPRESENTATION OF THE FLOW IN A NATURAL CONVECTION HEATING OF A FLUID BY A VERTICAL WALL

Using the results given by Rohsenow and Choi (1961, page 204, equations 8.60 and 8.61), it can be shown that the mass flow rate per unit width ($\dot{M}'$) in the turbulent, free convection boundary layer in any general fluid at any section at a height z above the base is given by

$$\dot{M}'(z) = 0.1\ (\rho_\infty \nu)\ \frac{Gr_z^{\ 0.4}}{Pr^{8/15}\ [1 + 0.494\ Pr^{2/3}]^{0.4}} \qquad \text{(B-1)}$$

where Gr_z = Grashoff number = $\left[\frac{g\beta(T_w - T_\infty)z^3}{\nu^2}\right]$ (3-2)

and Pr = Prandtl number of the fluid

where the subscript ∞ refers to the condition far from the wall.

Similarly, from equation 8.62b of Rohsenow, we have, for the average Nusselt number for heat transfer over a wall height of z,

$$\overline{Nu_z} = 0.0246 \frac{Gr_z^{0.4} \; Pr^{7/15}}{[1 + 0.494\, Pr^{2/3}]^{0.4}} \qquad \text{(B-3)}$$

Therefore, the average heat flux from the wall to the fluid over a height z is

$$\overline{\dot{q}''_c} = \bar{h}(T_w - T_\infty) = \overline{Nu_z} \frac{K(T_w - T_\infty)}{z} \qquad \text{(B-4)}$$

We also define a quantity called the characteristic mean heat flux.

$$\overline{\dot{q}''_{ch}}(H) = 0.0246 \frac{K\, T_\infty}{H} \left[\frac{g\beta T_\infty H^3}{\nu^2}\right]^{0.4} \frac{Pr^{7/15}}{[1 + 0.494\, Pr^{2/3}]^{0.4}} \qquad \text{(B-5)}$$

Physically, $\overline{\dot{q}''_{ch}}(H)$ represents that average heat flux which would result over a height H when the wall temperature is twice the absolute temperature of the bulk of the fluid.

The quantity Γ defined in Appendix A, equation A-13, is therefore the ratio of the above characteristic mean heat flux $\dot{q}''_{ch}$ over the liquid height H_L to the heat flux from fire.

2. Radiation View Factor Between Two Normal, Rectangular Surfaces

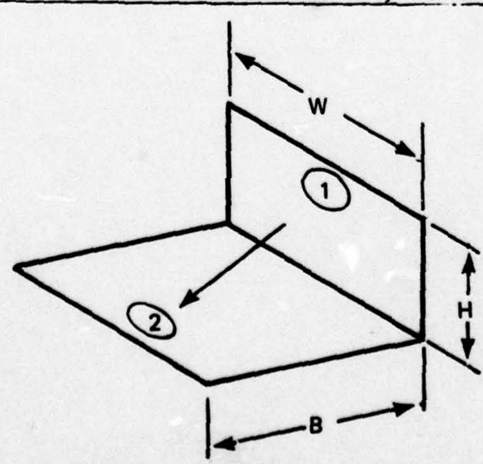

FIGURE B-2 CONFIGURATION FOR RADIATION VIEW FACTOR

With

$$h = \frac{H_v}{W} \tag{B-6a}$$

$$b = \frac{B}{W} \tag{B-6b}$$

the view factor is given by

$$F_{1\to 2} = \frac{1}{\pi h}\left[h \tan^{-1}\left(\frac{1}{h}\right) + b \tan^{-1}\left(\frac{1}{b}\right) - \sqrt{h^2 + b^2}\tan^{-1}\frac{1}{\sqrt{h^2 + b^2}}\right.$$

$$+ \frac{1}{4\pi h}\left[h^2 \ln \frac{h^2(1 + h^2 + b^2)}{(1 + h^2)(h^2 + b^2)} + b^2 \ln \frac{b^2(1 + h^2 + b^2)}{(1 + b^2)(h^2 + b^2)}\right.$$

$$\left.+ \ln \frac{(1 + h^2)(1 + b^2)}{(1 + h^2 + b^2)}\right] \tag{B-7}$$

The above formula is taken from Wiebelt (1966), page 259.

The total net heat transfer rate from surface 1 to surface 2 is

$$\dot{Q}_{rad} = F_{1\to 2}\,\sigma(T_1^{\,4} - T_2^{\,4})\,W\,H_v \tag{B-8}$$

APPENDIX C

Derivation of Energy Conservation Equation for the Gas in the Ullage Volume

Objective

The objective of the analysis presented in this appendix is to obtain expression for the change with time of the temperature of the gas in the ullage volume of the tank when exposed to fire and subjected to mass input and venting.

Analysis

In order to obtain the gas temperature variation, we write the energy conservation equation subject to the following conditions:

- The kinetic energy of flow is neglected in comparison with the thermal flow during venting.
- Gases behave as perfect gases which are mechanically mixed (no chemical reaction takes place between air and vapor).
- Direct radiative heating of gas due to thermal radiation from the wall is small compared to the gas heating by natural convection.
- The gas is in a perfectly mixed state always; that is, the gas stratification, if any, is neglected.

Figure C-1 is a schematic representation of the thermodynamic system (of constant volume) containing the ullage gas.

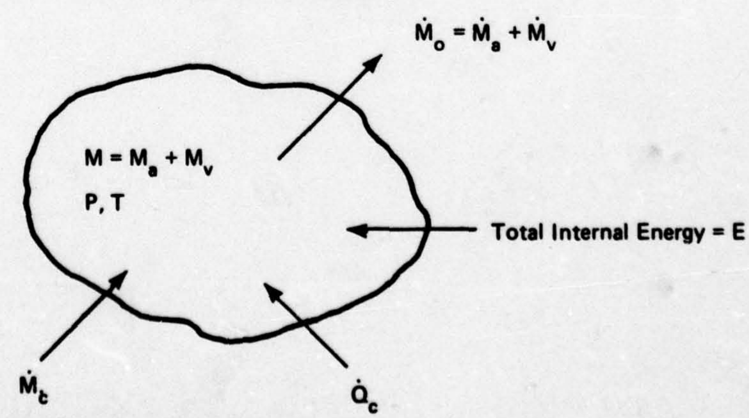

FIGURE C-1 SCHEMATIC REPRESENTATION OF THE THERMODYNAMIC SYSTEM CONTAINING THE ULLAGE GAS

with

e = specific internal energy of individual gas specie (i.e., per unit mass)

h = specific enthalpy of the individual gas specie

E = total internal energy of the gas in the ullage volume at any instant

We write the energy equation to this open system [see Keenan (1970), page 32].

$$\frac{dE}{dt} = \dot{Q}_c + \dot{M}_i\bar{h}_i - \dot{M}_o\bar{h}_o \qquad \text{(C-1)}$$

where $\bar{h}_i$ and $\bar{h}_o$ are the gas enthalpies at inlet and outlet per unit mass of the gas mixture.

$$E = M[Y_a e_a + Y_v e_v] \qquad \text{(C-2)}$$

where Y_a is the mass fraction of air in the ullage volume and Y_v is the mass fraction of vapor. Also,

$$Y_a + Y_v = 1 \qquad \text{(C-3)}$$

Differentiating Equation (C-2) and substituting the results in (C-1), we get (in view of C-3).

$$\dot{M}[Y_a e_a + Y_v e_v] + M\,\dot{Y}_a[e_a - e_v] + M[Y_a\dot{e}_a + Y_v\dot{e}_v] = \dot{Q}_c + \dot{M}_i\bar{h}_i - \dot{M}_o\bar{h}_o \qquad \text{(C-4)}$$

where the "dot" over the symbol represents its rate of change with time.

From the mass balance equation, we have

$$\dot{M} = \dot{M}_i - \dot{M}_o \qquad \text{(C-5)}$$

Substituting C-5 in C-4 and rearranging after substituting for $\dot{e}$ its equivalent $c^v \frac{dT}{dt}$, we have

$$M[Y_a c_a^V + Y_v c_v^V]\frac{dT}{dt} = \dot{Q}_c + \dot{M}_i[\bar{h}_i - Y_a e_a - Y_v e_v] - \dot{M}_o[\bar{h}_o - Y_a e_a - Y_v e_v]$$

$$- M\,\dot{Y}_a[e_a - e_v] \qquad \text{(C-6)}$$

For $\dot{Y}_a$, we substitute from equation 10 in equation C-6 and get

$$M[Y_a c_a^V + Y_v c_v^V] \frac{dT}{dt} = \dot{Q}_c + \dot{M}_i[\bar{h}_i - Y_a e_a - Y_v e_v + Y_a e_a - Y_a e_v]$$

$$- \dot{M}_o[\bar{h}_o - Y_a e_a - Y_v e_v] \qquad \text{(C-7)}$$

Noting that in the inlet gas there is only vapor and that outlet gas contains both air and vapor,

$$\bar{h}_i = h_{v,\text{ inlet}} = h_{\text{saturated}} \qquad \text{(C-8a)}$$

$$\bar{h}_o = Y_a h_a(T) + Y_v h_v(T) \qquad \text{(C-8b)}$$

we have

$$M[Y_a c_a^V + Y_v c_v^V] \frac{dT}{dt} = \dot{Q}_c + \dot{M}_i[h_{sat} - h_v(T) + \frac{p_v}{\rho_v}] - \dot{M}_o[Y_a(h_a - e_a) + Y_v(h_v - e_v)]$$

i.e.

$$M[Y_a c_a^V + Y_v c_v^V] \frac{dT}{dt} = \dot{Q}_c + \dot{M}_i\left[\frac{R_u T}{\mu_v} + c_v^P(T_b - T)\right] - \dot{M}_o\left[\frac{Y_a}{\mu_a} + \frac{Y_v}{\mu_v}\right] R_u T$$

i.e.

$$\frac{dT}{dt} = \frac{\dot{Q}_c + \dot{M}_i\left[\frac{R_u}{\mu_v} T + c_v^P(T_b - T)\right] - \dot{M}_o R_u T\left[\frac{Y_a}{\mu_a} + \frac{Y_v}{\mu_v}\right]}{M[Y_a c_a^V + Y_v c_v^V]} \qquad \text{(C-9)}$$

It is also recalled that

$$\gamma = \frac{c^P}{c^V} \qquad \text{(C-10)}$$

$$\text{and } c^P - c^V = c^V(\gamma - 1) = \frac{R_u}{\mu} \qquad \text{(C-11)}$$

Equations C-10 and C-11 apply equally to air and vapor (and have to be used with appropriate subscripts).

It is also noted that the mean values of specific heat (mass) and molecular weights in a mixture of gases (say, containing air and vapor) are expressed by

$$\overline{c} = Y_a c_a + Y_v c_v \tag{C-11}$$

and

$$\frac{1}{\overline{\mu}} = \frac{Y_a}{\mu_a} + \frac{Y_v}{\mu_v} \tag{C-12}$$

APPENDIX D

Non-Dimensionalization and Solution Procedure for the Governing Equations

In this appendix, the equations derived in the analysis of tank heating and pressurization are written in dimensionless form so as to be amenable to numerical solution on a computer. Also indicated briefly is the method of solution.

We first define the following non-dimensionalizing parameters.

$$\dot{M}_{ch} = \frac{W H_L \dot{q}_f''}{\lambda} = \text{characteristic mass generation rate} \qquad \text{(D-1a)}$$

$\dot{M}_{ch}$ represents the vapor generation rate that would occur if all of the heat incident on the liquid wetted wall face is absorbed by the liquid during the boiling process.

$$M_{in} = \frac{P_{in} V}{\frac{R_u}{\bar{\mu}_{in}} T_{in}} = \text{initial mass of the vapor-air mixture in the ullage volume} \qquad \text{(D-1b)}$$

where

$$\frac{1}{\bar{\mu}_{in}} = \frac{Y_{a,in}}{\mu_a} + \frac{Y_{v,in}}{\mu_v} \qquad \text{(D-1c)}$$

$$t_{ch} = \frac{L_w \rho_w c_w T_{in}}{\dot{q}_f''} = \text{characteristic time} \qquad \text{(D-1d)}$$

Physically, t_{ch} represents the time in which the wall temperature would increase by its initial temperature if all of the incident radiation is absorbed by the wall itself.

$$\sigma' = \frac{\sigma T_{in}^4}{\dot{q}_f''} = \text{dimensionless radiative flux from the wall initially} \qquad \text{(D-1e)}$$

$$Nu^* = 0.0246 \left[\frac{g \beta T_{in} H_v^3}{\nu_a^2}\right]^{0.4} \frac{(Pr_a)^{7/15}}{[1 + 0.494\, Pr_a^{2/3}]^{0.4}} \qquad \text{(D-1f)}$$

Nu^* is a characteristic mean Nusselt number for turbulent convective heat transfer in air over a height H_v, when the temperature difference between the wall and air is equal to the initial temperature. The product βT_{in} of Equation (D-1f) is unity for a perfect gas.

$$\mu^* = \mu_v/\mu_a \qquad \text{(D-1g)}$$

$$\Omega = \frac{H_v}{H_L} \frac{\lambda}{T_{in}(R_u/\mu_v)} \qquad \text{(D-1h)}$$

$$\Delta = \frac{C_d \ A \sqrt{2 \ p_{in} \ \rho_{in}}}{\dot{M}_{ch}} \qquad \text{(D-1i)}$$

Physically, Δ represents the ratio of the outflow rate through the vent when there is a pressure drop across the vent equal to the initial pressure in the tank to the characteristic evaporation rate defined in equation D-1a.

$$\theta_b = T_b/T_{in} = \frac{\text{liquid boiling temperature}}{\text{initial vapor temperature}} \qquad \text{(D-1j)}$$

$$\theta_B = T_B/T_{in} = 1 = \frac{\text{bulk liquid temperature}}{\text{initial temperature}} \qquad \text{(D-1k)}$$

$$\Gamma = \frac{\dot{M}_{ch} \ t_{ch}}{M_{in}} \qquad \text{(D-1}\ell\text{)}$$

$$c = \frac{c_v^V}{c_a^V} \qquad \text{(D-1m)}$$

$$p^* = p_{ambient}/p_{in} \qquad \text{(D-1n)}$$

$$\zeta = \frac{K_a T_{in}}{H_v \dot{q}''_f} = \frac{\text{heat flux that can be transferred in air by conduction for a temperature difference of } T_{in} \text{ over a length } H_v}{\text{incident fire heat flux}} \qquad \text{(D-1o)}$$

<u>Non-dimensional Variables</u>

$$\tau = t/t_{ch} = \text{dimensionless time} \qquad \text{(D-2a)}$$

$$\theta = T/T_{in} = \text{dimensionless vapor temperature} \qquad \text{(D-2b)}$$

$$\theta_w = T_w/T_{in} = \text{dimensionless wall temperature} \qquad \text{(D-2c)}$$

$$m = M/M_{in} = \frac{\text{mass of vapor-air in the ullage volume}}{\text{mass of gases initially}} \qquad \text{(D-2d)}$$

$$\beta = \frac{p}{p_{in}} = \text{dimensionless pressure} \qquad \text{(D-2e)}$$

$$\dot{m}_i = \frac{\dot{M}_i}{\dot{M}_{ch}} = \text{dimensionless vapor mass inflow rate by liquid boiling} \quad \text{(D-2f)}$$

$$\dot{m}_o = \frac{\dot{M}_o}{\dot{M}_{ch}} = \text{dimensionless mass outflow by venting} \quad \text{(D-2g)}$$

$$\bar{q}_c = \dot{q}''_c/\dot{q}''_f = \text{dimensionless natural convection heat flux} \quad \text{(D-2h)}$$

$$\bar{q}_{Rad} = \dot{q}''_{rad}/\dot{q}''_f = \text{dimensionless radiative flux leaving the wall} \quad \text{(D-2i)}$$

Using the above definitions, equations 1 through 13 are written in dimensionless form as shown below:

Equation 2 becomes

$$\bar{q}_{Rad} = \sigma' \, \theta_w^{\,4} \quad \text{(D-3)}$$

Equation 3 becomes

$$\bar{q}_c = \zeta \, Nu^* \, (\theta_w - \theta)^{7/5} \quad \text{(D-4)}$$

Equation 1 becomes

$$\frac{d\theta_w}{d\tau} = 1 - \bar{q}_{Rad} - \bar{q}_c \quad \text{(D-5)}$$

Equation 6 becomes

$$\dot{m}_i = [1 + F \frac{H_v}{H_L} \bar{q}_{Rad}] \quad \text{(D-6)}$$

Equation 11 becomes

$$\dot{m}_o = \Delta \sqrt{(\mathcal{P} - p^*)m} \quad \text{(D-7)}$$

Equation 10 becomes

$$\frac{dY_a}{d\tau} = -Y_a \Gamma \frac{\dot{m}_i}{m} \quad \text{(D-8)}$$

Equation 12d becomes

$$\frac{d\mathcal{P}}{d\tau} = \Gamma[-Y_a \dot{m}_o \, \theta + \frac{\theta}{\mu^*} (\dot{m}_i - Y_v \dot{m}_o)] + m \frac{d\theta}{d\tau} (Y_a + \frac{Y_v}{\mu^*}) \quad \text{(D-9)}$$

The energy equation, Equation (13), becomes

$$\frac{d\theta}{d\tau} = \frac{\Gamma(\gamma_a - 1)}{\mu^*} \quad \frac{[\Omega \bar{q}_c + \dot{m}_i\{\theta + \frac{\gamma_v}{(\gamma_v - 1)} (\theta_b - \theta)\} - \dot{m}_o\{Y_a\mu^* + Y_v\} \theta]}{m[Y_a + c\, Y_v]} \tag{D-10}$$

Finally, we have

$$Y_a + Y_v = 1 \tag{D-11}$$

Initial Conditions

$$\left.\begin{array}{l} Y_a = Y_{a,in} \\ \beta = \theta_w = \theta = 1 \end{array}\right\} \text{at } \tau = 0 \tag{D-12}$$

Method of Solution of the Equations

There are essentially four unknowns; namely, θ_w, Y_a, β, and θ, and four equations describing their time rate of change; namely, D-5, D-8, D-9, and D-10 respectively. However, these differential equations are coupled and nonlinear. Only a numerical solution can be obtained.

The method used is the Hamming Predictor Corrector Method enunciated by Hamming (1973). The program used is the one available in the Scientific Systems Subroutine Package developed by IBM.

APPENDIX E

Analytical Analysis of the Pressure Rise Problem
A Simple Model

Objective

The objective of the analysis developed in this appendix is to obtain a closed form solution to the variation of tank pressure as a function of time using certain simplifying assumptions.

Introduction

The calculation of pressure rise with time resulting due to the tank being exposed to a fire is a complicated problem. For most conditions encountered in practice, the solution of the equations can be obtained only by numerical techniques using a computer.

If one analyzes the effect of various physical phenomena that cause the pressure rise, one finds that the main effect is due to the tremendous vapor inflow into the ullage space due to the liquid boiling. In fact, most of the pressure rise in the initial stages is only due to this mass influx. The thermal effect (i.e., the effect of increase in temperature of the gas on the gas pressure) comes into play at a much later stage.

Therefore, in light of the above observation, the analysis can be simplified to a great extent resulting in closed form solution of pressure as a function of time. In effect, then, the analysis will be an isothermal analysis.

The following assumptions are made in this analysis:

- All gases are perfect gases.
- The temperature of the gases remains at a constant (initial) value.
- The mass inflow rate of vapor into the ullage volume by boiling of liquid is constant.
- The vapor has the same molecular characteristics as that of air.
- Volume of vapor space is a constant.
- Initial pressure is equal to the ambient pressure.

Analysis

Let $\dot{M}_i$ = constant inflow rate of vapor into the ullage space

We now write the mass balance equation on the ullage vapor:

$$\frac{dM}{dt} = \underbrace{\dot{M}_i}_{\text{inflow}} - \underbrace{C_d A \sqrt{2(p - p_{amb})\rho}}_{\text{venting}} \qquad \text{(E-1)}$$

with

$$M = \frac{pV}{RT} = \text{mass of gas in the ullage space} \qquad \text{(E-2)}$$

$$\rho = \frac{M}{V} \qquad \text{(E-3)}$$

Equations E-1, E-2, and E-3 have to be solved for p (the pressure) as a function of time with the initial condition

$$p = p_{amb} \text{ at } t = 0 \qquad \text{(E-4)}$$

In order to obtain closed form solution, we define the following parameters:

$$\rho_{in} = \frac{p_{amb}}{RT_{in}} \qquad \text{(E-5a)}$$

$$M_{in} = \rho_{in} V \qquad \text{(E-5b)}$$

$$t_{ch} = \frac{M_{in}}{\dot{M}_i} \qquad \text{(E-5c)}$$

$$\mathcal{P} = \frac{p}{p_{amb}} = \frac{M}{M_{in}} \text{ because T is constant} \qquad \text{(E-5d)}$$

$$\tau = t/t_{ch} \qquad \text{(E-5e)}$$

$$\Delta = \frac{c_d A \sqrt{2 p_{amb} \rho_{in}}}{\dot{M}_i} \qquad \text{(E-5f)}$$

Using the parameters defined in equations E-5a through E-5f, we rewrite equation E-1 as

$$\frac{d\mathcal{P}}{d\tau} = 1 - \Delta \sqrt{\mathcal{P}(\mathcal{P} - 1)} \qquad \text{(E-6)}$$

with $\mathcal{P} = 1$ at $\tau = 0$ (E-7)

From equation E-6, it is easily seen that the maximum pressure is attained when $\frac{d\mathcal{P}}{d\tau} = 0$; i.e., when the inflow is exactly equal to the outflow. Therefore,

$$1 = \Delta\sqrt{\mathcal{P}_{max}(\mathcal{P}_{max} - 1)}, \tag{E-8a}$$

or $$\mathcal{P}_{max} = \frac{1}{2}\left[1 + \sqrt{1 + \frac{4}{\Delta^2}}\right]. \tag{E-8b}$$

The exact solution to Equation (E-6) can be obtained using standard results for integrals given in Gradshteyn and Ryzhik (1973), page 107, equation 2.441-3. The solution to Equation (E-6) with

$$\phi = \cosh^{-1}\sqrt{\mathcal{P}} \tag{E-9}$$

is $$\frac{\tau\Delta}{2} = \frac{2}{\sqrt{4 + \Delta^2}} \left[\tanh^{-1}\left\{\frac{2 \tan h\ x + \Delta}{\sqrt{4 + \Delta^2}}\right\}\right]_{x=0}^{x=\phi} - \phi \tag{E-10}$$

i.e.,

$$\frac{\tau\Delta}{2} = \frac{2}{\sqrt{4 + \Delta^2}} \left[\tanh^{-1}\left\{\frac{(2 \tanh\ \phi + \Delta)}{\sqrt{4 + \Delta^2}}\right\} - \tanh^{-1} \frac{\Delta}{\sqrt{4 + \Delta^2}}\right] - \phi \tag{E-11}$$

NOMENCLATURE

a	= Coefficient in Equation (22)	$(hr\text{-}ft^2/lb)^{1.7}$ psi
A	= Cross sectional flow area of the vent	ft^2
A_L	= Liquid surface area = BW	ft^2
b	= Plating width	ft
B	= Beam width of the tank	ft
C_d	= Coefficient of discharge of the vent	
c^P	= Mass specific heat at constant pressure	Btu/lbm/°F
c^V	= Mass specific heat at constant volume	Btu/lbm/°F
c_L	= Specific heat of liquid	Btu/lbm/°F
c	= Vapor to air specific heat ratio (Equation (D-1m))	Btu/lbm/°F
D	= Diffusion coefficient	ft^2/hr
E	= Internal energy content of the gases in the ullage volume	Btu
	Also used for Modulus of Elasticity in Chapter II	psi
e	= Specific internal energy of individual gas specie	Btu/lbm
F	= Radiation view factor (Equation (B-7))	
Gr	= Grashoff number (Equation (B-2))	
g	= Acceleration due to gravity	ft/sec^2
H_L	= Depth of liquid above the water line	ft
H_v	= Depth of vapor space	ft
h	= Turbulent, natural convection transfer coefficient	Btu/hr/ft^2/°F
	Also used for specific enthalpy in Appendix C (Equation (C-1))	Btu/lbm
	Also used for plating thickness in Chapter II	ft
K	= Thermal conductivity of air	Btu/hr/ft/°F

L_w	=	Thickness of steel wall	ft
M	=	Mass of air and vapor in the ullage volume	lbm
M_a	=	Mass of air in the ullage volume	lbm
$\dot{M}_b$	=	Mass rate of liberation of vapor due to liquid boiling	lbm/hr
$\dot{M}_{evap}$	=	Mass rate of liquid evaporation at the surface due to direct radiation from the hot wall in contact with the vapor	
$\dot{M}_i$	=	Vapor mass inflow rate into ullage space due to liquid boiling and evaporation at the surface	lbm/hr
M_{in}	=	Initial mass of vapor in the ullage volume	lbm
$\dot{M}_o$	=	Mass outflow rate through the vent	lbm/hr
M_v	=	Mass of vapor in the ullage volume	lbm
Nu	=	Nusselt number = $\frac{hH_v}{K}$	
Nu*	=	Characteristic Nusselt number (see equation D-1f)	
Pr	=	Prandtl number of air or vapor	
$\mathcal{P}$	=	Dimensionless tank pressure = p/p_{in} (equation D-2e)	
p	=	Pressure in the tank	psi
p_a	=	Partial pressure of air in ullage volume	psi
p_{amb}	=	Ambient pressure	psi
p_{in}	=	Initial pressure of gases in the tank	psi
p_{ss}	=	Steady state tank pressure	psi
p_v	=	Partial pressure of vapor in the ullage volume	psi
p^{∞}	=	p_{amb}/p_{in} (Equation (D-1n))	
P_{max}	=	Gauge pressure at the tank failure limit	psig
Q_c	=	Total heat input rate into the gas by natural convection	Btu/hr
q	=	Dimensionless liquid convection heat flux (Equation (A-7e))	

$\overline{\dot{q}''_c}$	= Mean heat flux from the wall to the fluid in contact, due to turbulent free convection (equation B-4)	$Btu/hr/ft^2$
$\overline{\dot{q}''_{c,L}}$	= Mean turbulent heat flux for natural convection in liquid	$Btu/hr/ft^2$
$\dot{q}''_f$	= Heat flux at the outer surface wall due to the incoming radiation from a fire	$Btu/hr/ft^2$
$\dot{q}''_{rad}$	= Heat flux leaving the wall (in contact with the vapor) by radiation	$Btu/hr/ft^2$
R	= Individual gas constant	Btu/lbm/°F
R_u	= Universal gas constant	Btu/lbmole/°R
S	= Depth of the stratified liquid layer at any time	ft
T	= Temperature of the gases in the tank	°R
T_b	= Boiling temperature of liquid	°R
T_s	= Surface temperature of liquid	°R
T_w	= Wall temperature	°R
t	= Time	sec
t_{ch}	= Characteristic time (see equations A-7b and D-1d)	sec
V	= Volume of the ullage space	ft^3
W	= Width of tank exposed to fire radiation	ft
Y	= Mass fraction of the air or vapor specie in the ullage gas	
z	= Vertical coordinate	ft

GREEK LETTERS

β = Volumetric expansion coefficient of a fluid (for perfect gas $\beta = 1/T$) — $(°R)^{-1}$

γ = Specific heat ratio of a gas = $\frac{c^p}{c^v}$

Δ = Parameter defined in equation E-5f.

Γ = Parameter defined in equation A-13

ζ = Dimensionless liquid stratification depth. Also a parameter defined in equation D-1o

θ = Dimensionless temperature = T/T_{in} (subscripted)

λ = Heat of vaporization of liquid — Btu/lbm

Λ = Parameter defined in equation A-7h

μ = Molecular weight of a gas specie (subscripted)

$\bar{\mu}$ = Mean molecular weight of air vapor mixture (see equation C-12)

μ^* = $\frac{\mu_v}{\mu_a}$ = Ratio of vapor to air molecular weight (equation D-1g)

ξ = Parameter defined in Equation (18)

ν = Kinematic viscosity of the fluid — ft^2/hr

ρ = Density of fluid (subscripted) or wall — lbm/ft^3

σ = Stefan Boltzmann Constant — $Btu/hr/ft^2/°R^4$

Also used for yield strength in Chapter II — psi

σ' = Dimensionless radiative heat flux

τ = $\frac{t}{t_{ch}}$ = dimensionless time (see equation A-7c)

ϕ = Pressure parameter defined in equation E-9

ω = Parameter defined in equation A-7f

Ω = Parameter defined in D-1h

SUBSCRIPTS

a = Air

B = Bulk liquid

b = Boiling condition

c = Convection

ch = Characteristic value

e = Evaporation

f = Flame

i = Inlet (ullage volume) condition

in = Initial condition

L = Liquid condition

o = Outlet (vent) condition

s = Surface of liquid

ss = Steady state

sat = Saturation

v = vapor

∞ = for conditions far from the wall

SUPERSCRIPTS

. = Time rate of change

' = Per unit length

" = Per unit area

p = At constant pressure

V = At constant volume

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